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An empirical analysis of a discriminatory, closed, simultaneous, multi-object auction

BY

TOR HUGO HAUGE

To my parents

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Chapter 1

Introduction

In this dissertation, I present an empirical analysis of data from a specific auction: ocean-caught mackerel in Norway. I study the formation of market prices, test some implied restrictions from auction theory, and make some policy recommendations. In addition to providing useful information for this market, I shall document characteristics of the market and market agent's behavior that may elicit future theoretical and empirical work.

I adhere to the view expressed eloquently by Vijay Krishna [59, p. IX]: The use of the pronoun *we* in the remainder of the text is not meant to give any associations to royalty, but is used to invite the reader to see the manuscript as a dialog.

Theoretical framework. An important part of economic theory in recent decades has been devoted to the study of markets with informational asymmetries where agents behave strategically. A substantial part of this research program is concerned with auction markets. Two reasons for this interest are apparent: First, from a theoretical perspective, auctions are interesting because they constitute well-defined environments for the application of game theory. In many market models in industrial organization, the information set available to agents, and the order of moves, are not obvious; the researcher must make more or less plausible assumptions in the modelling process. At real-world auction markets, on the other hand, the "rules of the game" are written in the form of legally-binding contracts. Thus, the game is well-defined, making the modelling more realistic. Second, rich and complete datasets are available from auction markets, making them suitable for empirical analyses; both positive and normative questions can be addressed.

The theoretical study of auctions—incorporating the strategic aspects of the markets—began with the seminal paper of Vickrey [107]. The theory expanded especially rapidly after developments in game theory, in particular the work by Harsanyi [45] and his theory on noncoöperative games of incomplete information was consequential. Important early contributions, in addition to Vickrey, were Wilson [109, 110] as well as his student Ortega-Reichert [82]. With the publication of Milgrom and Weber [77], our understanding of the standard auction models reached maturity.

Positive analyses examine what happens in these markets. How are market prices determined? To what extent can prices be explained by observable product and market characteristics? Although price theory is a core subject of economics, the determination of market prices is somewhat of a black box in the traditional theory. Auction theory casts light on the price formation process by focusing on underlying, unobservable, valuations that determine market prices. Another important positive question is what the allocational consequences are in terms of goods and revenues when different auction formats are employed.

Normative issues are addressed using tools from mechanism design—a research program pioneered by Myerson [79]—in order to determine optimal ways of conducting auctions. From the seller's perspective, the issue is whether the allocation can be changed by the choice of auction format. From an economist's perspective, an even more important question is whether allocations are efficient.

Empirical methods. The fundamental idea behind auction theory is that bids—including the winning bid or the market price—are governed by bidder's underlying valuations. Valuations are modelled as random variables having probability distributions. Two approaches to empirical analysis of auction data can be distinguished. To test predictable restrictions that follows from auction theory, we may use a flexible form when modelling bid functions. If the restrictions are independent of the functional form of the probability law of valuations and there is no unobserved heterogeneity across auctions, the so-called reduced-form approach is justifiable; see Hendricks and Paarsch [47]. Reduced-form estimation to econometricians means that we examine implications of the theory without estimating the parameters of a specific economic model that represents the market behavior.

Beginning with Paarsch [83, 84], under a new structural approach it is assumed that the data can be represented by a given theoretic model. If this is the case, then unobservable valuations may be deduced. This line of research opens up a wider array of topics that can be addressed. In particular, policy recommendations concerning the auction format are possible.

Further discussion of empirical strategies for interpreting auction data is awkward without our having presented the theory of auctions and its empirical contents in detail. In the next two chapters, we develop these notions.

Market and auction format. Auction formats differ in several respects. For now, we briefly present the market and auction format we study. A detailed description is relegated to subsequent chapters. The market under study concerns the sales of pelagic fish in Norway. In the first-hand market, the owners of vessels sell the harvest to food-producing plants. By law, an association is granted monopoly to sell the raw material in the first-hand market.

The association sells the fish at auction, specifically, a discriminatory, closed, multi-object and simultaneous format with a known reserve price is used. Several objects—the catches of different vessels—are sold at the same auction. The term **multi-object** refers to objects that are not identical as opposed to offering several identical objects for sale which often is denoted a multi-unit auction. In our case, catches differ with respect to total quantity and the average fish size; they may also to some extent differ with respect to some quality variables. Given several objects for sale, there is a choice between selling them sequentially or simultaneously; in our case a **simultaneous** procedure is used. The bid process is **closed**, meaning that bids are delivered sealed as opposed to the open outcry format. The term **discrim-**

inatory relates to the price paid: each individual winning bid is a price as opposed to a uniform-price where all winning bidders pay the same price. In our case, the highest bidder wins the object (with some modifications) and pays his price, thus making it a so-called first-price or pay-as-bid auction.

In general, potential buyers have short-term capacity constraints. Therefore, bidders will want to avoid winning a quantity larger than their capacity because the fish have to be processed quickly to avoid quality deterioration. To encourage competition (as many submitted bids as possible), the auction rules permit bidders to set **quantity limits**. If a bidder wins more catches than he can take, then some catches are allocated to the next highest bidder. Other aspects of the auction format are that sellers can set a geographical **delivery sector** for their catch, and buyers can give **priorities** to their bids. All these aspects are explained in detail in subsequent chapters.

Main topics. The brief presentation above suggests that several topics and strategies for interpreting auction data exist. The range of topics we can analyse is determined by the characteristics of the market under study. Even though auction theory has made significant advances, complex markets may prove to be analytically intractable within a game-theoretic modelling approach. Any empirical analysis is constrained by the tools available. We can always analyse auction data by statistical methods. Normally, the usefulness of the analysis increases when we can relate the data to a model with general properties. At the same time, we should avoid forcing the data into a model that does not represent the data generating process reasonably well.

Given the complexity of our market, we have to be careful to avoid using too restrictive models. Several predictions of auction theory are, however, general, but for the main part we shall interpret our statistical results within an auction-theoretic model world. The **determination of prices** is a core subject. At a general level, examining the testable predictions of auction theory is worthwhile, and we shall do this. From a practical perspective, the measurement of size effects is even more important. Rather than discussing just the sign of some variable's influence on prices, we make an effort to answer the fundamental empirical question: How big is big? To wit, we ask what variables have an *economically important* effect on prices, as opposed to the exercise of revealing what variables have a *statistically significant* effect. The latter question is more a question of how reliable results are in the presence of sampling error. The market under study is large and important for a resource-based economy like Norway's, for market participants, it is useful to know the average effects that controllable quality variables have on market prices.

Auctions and bidding are about strategy. The strategic elements of our market are analysed from three perspectives:

First, we discuss one unique feature of the auction design, the **option of giving priorities to bids**. The priority option is closely linked to the option of setting quantity limits. Bidders can bid on more objects than they want. We are unaware of other auctions that give bidders this option. A more frequently used mechanism in the case of simultaneous selling is to ask bidders for demand schedules. The question we ask is whether bidders use this option strategically in order to obtain lower prices.

Next, an important empirical topic of auction markets is the vulnerability to undue coöperation among market agents. Although, we have no indication that bid cartels exist in our market, it is tempting to utilize our rich dataset in order to establish that the auction design and market characteristics are robust against **collusion**.

The new, structural approach to analysing auction data enables us, in principle, to analyse the policy question of how to set reserve prices optimally. We make an effort to answer this. Although the rewards from a successful application of a structural estimation are great, the route to the goal is through dangerous territory. More specifically, the model to be used may not capture our data well. We acknowledge that the model we use is not an exact mapping of the data generating process, but we believe it is a useful representation. Our approach is based on the fact that potential errors in using the model are one-sided, and, consequently, that our recommendation of an **optimal reserve price**, is not too high.

We adopt a practical approach and use a battery of statistical methods in order to get the data to speak. Regression analysis, correlation analysis, and nonparametric density estimation are the main tools to be used. In addition, counting, summary statistics and measures of location and scale all have their merits in empirical work.

Organization. The thesis is organized as follows: In the next two chapters, we provide background on which the empirical analysis rests. In chapter 2, we present fundamental parts of auction theory in order to establish some important concepts and the framework for subsequent analysis and discussion. The benchmark models are covered in detail, while the more complex models are discussed at a more general level. In chapter 3, the empirical contents of auction theory is discussed. The main goal is to clarify the problems facing researchers trying to understand what goes on at auction markets by use of theoretic models and market data. The remaining chapters are empirically oriented. Chapter 4 is devoted to a description of the specific market under study, a presentation of the auction format in use, and a discussion of the sales mechanism and assumptions necessary to model the market. The dataset is described in chapter 5, together with a statistical presentation of key features of the market. In the next two chapters, we present a study of prices in detail. In chapter 6, we focus on a partial analysis and analyse the responsiveness of prices to the most important product and market variables. This discussion is important background for chapter 7 where we perform a general multivariate analysis of prices, controlling for several product and market characteristics. A special property of the auction format is bidders' option of giving priorities to their bids. The price analysis of preceding chapters does not capture possible effects of this option. In chapter 8, we analyse how bidders use the option strategically. An important empirical topic of auction markets is to consider their vulnerability to undue coöperation among market agents. In chapter 9, we discuss whether our data reveal any signs of collusion. In chapter 10, we compare the actual auction format with a counter-factual format in order to analyse whether the current format could be improved with respect to reserve price setting. Finally, in chapter 11, the main results are summarized and some policy recommendations are put forward. The construction of the dataset is relegated to an appendix.

Chapter 2

Auction theory

2.1 Introduction

Many different goods are sold at auction: Stamps, fish, spectrum rights, and drilling rights for oil tracts are but a few examples. It is hard to identify a common characteristic for all of them except that auctions aid in the price-discovery process. A stable supply and demand schedule is missing, and the common price setting strategies may be impossible to use as sales mechanisms. The fundamental problem for the seller is that he does not know what values the potential buyers assign to the good. If these values were known, then the seller would maximize his revenue from the sale by a take-it-or-leave-it offer slightly below the highest valuation, given that the value exceeds the seller's own valuation. This policy ensures that a buyer finds it to his advantage to accept it, and that the seller extracts all rent from the trade. Because buyers' valuations are private information, the seller is unsure what to charge for the good, and there is a risk from the seller's perspective that the buyer ends up with the good at a price far below his willingness to pay or that no trade takes place. The popularity of auctions in markets with this characteristic derives from the fact that this trading mechanism is normally very successful in eliciting information on buyers' willingness to pay. Auctions induce competition among bidders, which to a large extent undermines the potential gains private information otherwise might provide the buyer with. There are, however, many ways to conduct an auction, and the details of the mechanism are potentially of great importance for the allocation of the total surplus involved in the trade.

Central questions addressed in the theoretical models are the following. From the bidders' perspective, given a specific auction format, what strategies (bid rules) should they follow? This question is analysed within a game-theoretic approach, and is presented in more detail in the next section. After characterizing the optimal bidding strategies, we are then able to answer the crucial question from the sellers' perspective: which auction formats will generate the highest expected price? Two approaches are possible here. For a given set of common auction formats, when bid rules are determined, it is relatively straightforward to rank them with respect to the expected revenue they generate. But a more interesting approach from a theoretic (although maybe not from a practical) perspective is to analyse what auction rules, of any conceivable, will produce the highest bids. Drawing on the theory of mechanism design, it is possible to reduce this to a relatively manageable problem. Finally, from the economist's perspective it is important to ask whether the outcome of the auction is efficient; i.e., does the chosen auction ensure that the bidder with the highest willingness to pay obtains the object? In sum, auction theory is concerned with distributional effects and efficiency considerations.

We shall concentrate on a narrow set of topics in this chapter. This will allow us to develop some fundamental results of auction theory at a rather slow pace. Two auction formats—the first-price and second-price auctions—are discussed within two different models. First, however, important elements and assumptions of auctions as games are presented. Next, the independent private-values model is described. Bid rules and expected revenue are characterized together with an analysis of the effect of introducing an optimal reserve price. The treatment of the model is concluded with analysing how relaxing one of the assumptions of this model, namely risk neutral bidders, affects seller's expected revenue. Then, we address the same questions in the more general symmetric model, which allows for interdependent valuations.

Surveys that cover a wider range of topics include McAfee and McMillan

[68], Klemperer [56], and Wilson [111]. The standard reference in auction theory is Krishna [59]. We aim at a self-contained treatment of the subject, and devote space to statistical topics like *order statistics* and *affiliated random variables*, knowledge of which is useful when studying auction theory. Moreover, the general symmetric model, in particular, is rather complex and the details become somewhat involved. Therefore, some technical derivations are included in the appendices.

2.2 Auctions as games

Auctions can be modelled as games, and only auction theory that incorporates restrictions from game theory is considered in this chapter. Depending on the detailed description of the selling scheme and bidders' preferences, information structure and payoffs, many different games will emerge. Nevertheless, a unifying theme is present in all models. The seller wants to maximize revenue while the buyers face a trade off between increasing their probabilities of winning the auction and their payoffs if they win.

2.2.1 Players and information

The players have already been introduced: a seller who wants to dispose of a good and several potential buyers who will make their bid according to the rules of the auction. Notice that the procurement situation with one buyer and many sellers is completely analogous, and can be analysed in a similar way as auctions where the seller is asking for bids; see, for example, Holt [51]. To avoid confusion, we shall only consider the traditional auction institution with one seller.

The information available to bidders is incomplete in the sense that they do not know how the other bidders value the good in question; i.e., what types they are. Neither do they know what strategies they follow, and there may be uncertainty with respect to the value of the good. Auction theory developed fast after the necessary tools for analysing these games of incomplete information was provided by John Harsanyi [45].

A critical assumption in auction-theoretic models concerns the information that bidders have with respect to the value of the good. Two polar models are named *the private-values model* and *the common-value model.* In the private-values model, bidders know their own valuations and, consequently, information concerning competing bidders' valuations does not affect their valuations, although such information may alter bids. A firm with a known cost and demand structure bidding for an input resource may be an example where this model applies. The common-value model or *natural resource model* is characterized by a common, but uncertain value of the object. An example is bidding for the right to develop an oil tract. The true value of this resource will not be known exactly. Bidders in this model are assumed to have received different signals concerning the value. A signal is construed as all relevant information a bidder would use when trying to appraise the value of the object. Information on other bidders' signals is valuable when estimating the value and deciding how much to bid. In practice, most auctions have private-values and common-value elements. Nevertheless, research has focused on these two pure models. Clear-cut conclusions from simple models are definitely more satisfactory than ambiguous conclusions from more realistic models. However, in an influential paper, Milgrom and Weber [77] formulated a more general model where the private-values and common-value models are special cases.

Another important assumption concerns the modelling of players' preferences. Most of auction theory models both sellers and bidders as risk neutral, and this is reflected in this presentation as well. The assumption of risk neutrality does not reduce the bid problem to a trivial and uninteresting one. It remains interesting because of the two factors that bidders must balance in their strategy considerations; the probability of winning and the realized gain if winning. These two elements work in opposite directions; the probability of winning is maximized when the surplus from obtaining the object is negative, and the potential gain is at the maximum when the probability of winning is virtually zero. Introducing risk aversion leads to much more complicated models, but some issues of it are, nevertheless, covered below.

2.2.2 The rules of the game

The game is played in two stages. In the first stage, the seller is assumed to have all power in deciding the sale mechanism. Once this auction mechanism has been chosen, the game in the second stage is played between the bidders.[1] Only in the literature on optimal auction design is the seller's mechanism design problem analysed in depth. In most auction theory, behavior is analysed within a restricted set of auction rules; i.e., the auction forms are given. For a treatment of optimal auction design, see the seminal paper of Myerson [79] or the rather more accessible paper by Bulow and Roberts [16].

A rich variety of auction formats, or "the rules of the game", exists. Cassady [17] has provided a description of many commonly used mechanisms. The four auction institutions most frequently encountered in the literature are the open outcry (English) and the oral descending-price (Dutch) auctions and the closed first-price and second-price auctions.

At open outcry auctions, bidders observe to some extent the actions taken by other bidders. At the English, or open ascending-price auction, oral bids are shouted out until no one is willing to increase the last bid. The good is then awarded to the bidder with the highest bid and he pays his bid. The Dutch auction is characterized by an oral descending-bid process. The auctioneer begins with an high price and successively lowers it until one of the bidders accepts the going offer. Thus, only one bid is observed. The distinguishing feature between the two open formats is that possibly valuable information about bidders' willingness to pay is revealed during the bid process at the English auction, while no such information is available at the Dutch auction. True, one finally learns about the winning bidder's strategy, but this information is received at the moment the game is over.

At the first-price, sealed-bid auction, the bidder with the highest bid wins and pays his bid. A variation of this is the second-price, sealed-bid auction where the bidder with the highest bid wins, but pays what the second-

[1] If the rules, however, specify that the seller is free to determine after the bidding process is completed whether he wants to accept the going offer, which is equivalent to a secret reservation price, then the seller is a player in the second stage as well. This situation is analyzed in Elyakime et al. [26].

highest bid amounts to. The format is referred to as a Vickrey auction in honour of William S. Vickrey who first investigated its theoretical properties in the independent private-values case. It is rarely found in practice, but is interesting for modelling reasons because it can represent the open-ascending auction, in one of its versions, and it has a simple dominant strategy as the equilibrium solution of the bid problem.

2.2.3 Solution concepts

Agents are assumed to construct bid rules that constitute a strategic equilibrium. Some auction games have weakly dominant bid strategies; in such cases, strategic equilibria are easily identified. Otherwise, we have to rely on the somewhat weaker Bayes–Nash equilibrium as our equilibrium solution. Despite the criticism of the Nash equilibrium concept and its refinements, such equilibria can be well justified when applied to auctions. The Nash equilibrium assumes that strategies actually played are based on that all players maximize their utility, given their beliefs about other players' strategies, and that these beliefs are correct. Following Milgrom [74], the maximization condition is not stronger than the usual rationality assumption in economic theory. The second condition that beliefs are correct, or that expectations are rational, is certainly more controversial and subject to criticism, but is most likely to be a sound modelling approach when analysing long-lived institutions like auctions where players are likely to have accumulated a lot of experience to base their beliefs on. In the models to be presented in this chapter, however, we assume that the game is played only once. Therefore, it might seem somewhat inconsistent to defend bidders' ability to settle on the strategic equilibrium by referring to learning through repeated play. If we were to analyse repeated auctions, then there may be more complicated strategic considerations involved than our models capture. But even if the Nash equilibrium can be criticized along this line, we shall see that the proposed solutions are quite compelling.

To be sure, there exists a different type of literature on auctions than the game-theoretic one. Beginning with Friedman [31], which generated many contributions in operations research journals, the main concern of this approach is deriving optimal bid rules. The problem formulation, however, takes a somewhat naïve view on bidders' behavior; often only one strategic bidder is assumed. For a review and critique of this literature, see Laffont [60]. The game-theoretic approach is certainly more sophisticated and satisfactory from a theoretic perspective, and it also makes possible an analysis of other and more interesting problems than just deriving bidding strategies.

2.3 The independent private-values model

The independent private-values model will serve as the benchmark model. It is the least complicated, and the most frequently analysed model in empirical works. In this section, the strategic equilibrium-bid rule at first- and second-price auctions is derived and the expected revenues which these auction formats generate are calculated. The effect of competition is discussed, and the section is concluded by an examination the effect of relaxing the assumption of risk neutrality.

First, however, we establish that the first-price, sealed-bid and the Dutch auction formats are strategically equivalent under the set of assumptions which constitute the independent private-values model. Likewise, the closed second-price auction and the English auction are also strategically equivalent; i.e., they have the same normal-form game representation.

Begin with the first-price auction. A bidder must decide what to bid in ignorance of what other bidders intend to bid. Nevertheless, it is reasonable to expect that he tender a bid under the assumption that he might be the winner. The same line of reasoning applies to the Dutch auction. In any phase of the bidding process, the fact that no one has submitted a bid does not reveal any valuable information for the bidders. They will have to make their bid conditional on being the winner. Therefore, each must decide beforehand how much to bid. The dynamic aspect of the auction format is not relevant, so the strategic considerations involved are identical to the first-price, sealed-bid auction.

If bidders at the English auction raise their bids infinitesimally above the going offer until they drop out, then the winner will have to pay the price at which the last remaining of his opponents dropped out. This drop-out level will be at his valuation. At the closed, second-price auction, the winner pays the second-highest bid. It is not difficult to prove that bidders' should submit their valuations as their bids under this auction format. This anticipates the material in section 2.3.3 and a complete discussion will be provided there. Given that strategy, however, the winning bidder ends-up paying the second-highest valuation under this auction format as well. True, the bid process at the open auction reveals information about bidders' willingness to pay. But as long as values are independently distributed, this information is irrelevant.

2.3.1 A digression on order statistics

In subsequent sections, we shall need to calculate the probability that a given bid is the highest. The idea that bids can be modelled as increasing functions of the underlying valuations, which are random variables, is fundamental. If we can assign a common probability distribution to these variables, how can we obtain the distributions of some specific values when they are ranked in decreasing order? A branch of statistics called *order statistics* is suitable for gaining insight into this problem. For an extensive treatment of the topic, see David [22].

Consider a vector of independently and identically-distributed random variables $\boldsymbol{Z} = (Z_1, Z_2, ..., Z_n)$ having a known cumulative distribution function $F_V(\cdot)$. If the elements of this vector are arranged in increasing order of magnitude, we obtain the vector of order statistics $\left(Z_{(1)}, Z_{(2)}, ..., Z_{(n)}\right)$ where $Z_{(n)}$ is the highest value. A more extensive form of notation is to write the j^{th} order statistic as $Z_{(j:n)}$ where the sample size of n is stressed. The highest order statistic from the vector that contains all variables except the i^{th} are denoted $Z_{(n:-i)}$. The probability density function (pdf) and the cumulative distribution function (cdf) of $Z_{(j)}$ are denoted respectively $f_{(j)}(\cdot)$ and $F_{(j)}(\cdot)$. The distribution of values like $Z_{(n)}$ and $Z_{(n-1)}$ is of particular interest. The cdf of the largest value, $F_{(n)}(\cdot)$, is easily calculated.

$$\begin{aligned} F_{(n)}(z) &= \Pr\left(Z_{(n)} \leq z\right) \\ &= \Pr\left(Z_1 \leq z, Z_2 \leq z, ..., Z_n \leq z\right) \\ &= \prod\nolimits_{i=1}^{n} \Pr\left(Z_i \leq z\right) = \left[F_V(z)\right]^n. \end{aligned}$$

From this expression, the pdf of $Z_{(n)}$ is

$$f_{(n)}(z) = \frac{dF_{(n)}(z)}{dz} = n\left[F_V(z)\right]^{n-1} f_V(z). \tag{2.1}$$

To calculate the distribution for a general j^{th} order statistic is slightly more complicated:

$$\begin{aligned} F_{(j)}(z) &= \Pr\left(Z_{(j)} \leq z\right) \\ &= \Pr\left(\text{at least } j \text{ of } Z_i \ \leq \ z\right) \\ &= \sum_{i=j}^{n} \Pr\left(\text{exactly } j \text{ of } Z_i \ \leq \ z\right) \\ &= \sum_{i=j}^{n} \binom{n}{i} \left[F_V(z)\right]^i \left[1 - F_V(z)\right]^{n-i} \\ &= n\binom{n-1}{j-1} \int_0^{F_V(z)} t^{j-1} (1-t)^{n-j}\, dt \end{aligned} \tag{2.2}$$

where we have made use of the fact that the binomial probability that exactly i of the values are less than or equal to z equals the term in the summand in the next to last expression. The equivalence of the two last expressions follows from repeated integration by parts of the last expression, see Dudewicz and Mishra [24]. Alternatively, they can be proved to be equal by backward induction; see Gut [40]. Differentiating equation (2.2) yields

$$f_{(j)}(z) = n\binom{n-1}{j-1}[F_V(z)]^{j-1}[1-F_V(z)]^{n-j} f_V(z).$$

From this formula the pdf of the second-highest order statistic is easily obtained

$$f_{(n-1)}(z) = n(n-1)[F_V(z)]^{n-2}[1-F_V(z)] f_V(z). \tag{2.3}$$

The order statistics are obviously neither identically-distributed nor independent, and we turn now to the derivation of the joint distribution of $Z_{(n)}$ and $Z_{(n-1)}$. In the general case of two order statistics $Z_{(s)}$ and $Z_{(t)}$, where $s < t$, the joint cdf $F_{(s,t)}$ is

$$\begin{aligned}
&F_{(s,t)}(v,w) \\
= \;& \Pr(\text{at least } s \text{ of } Z_i \le v, \text{ at least } t \text{ of } Z_i \le w) \\
= \;& \sum_{j=t}^{n}\sum_{i=s}^{j} \Pr(\text{exactly } i \text{ of } Z_i \le v, \text{ exactly } j \text{ of } Z_i \le w) \\
= \;& \sum_{j=t}^{n}\sum_{i=s}^{j} \frac{n!}{i!(j-i)!(n-j)!}[F_V(v)]^{i}[F_V(w)-F_V(v)]^{j-i}[1-F_V(w)]^{n-j}.
\end{aligned}$$

It follows that the joint pdf $f_{(s,t)}$ is

$$\begin{aligned}
f_{(s,t)}(v,w) = \;& \frac{n!}{(s-1)!(t-s-1)!(n-t)!}\times \\
& [F_V(v)]^{s-1} f_V(v)[F_V(w)-F_V(v)]^{t-s-1} f_V(w)[1-F_V(w)]^{n-t}.
\end{aligned}$$

This formula enables us to find the joint pdf of the highest and second-highest order statistic:

$$f_{(n,n-1)}(v,w) = \begin{cases} n(n-1) f_V(w) f_V(v) [F_V(w)]^{n-2} & \text{if } v \geq w; \\ \\ 0 \text{ otherwise.} \end{cases} \tag{2.4}$$

We conclude this section with the conditional density of $Z_{(n-1)}$ given a value of $Z_{(n)}$.

$$f_{(n-1|n)}(w|v) = \frac{f_{(n,n-1)}(v,w)}{f_{(n)}(v)} = \frac{(n-1)[F_V(w)]^{n-2} f_V(w)}{[F_V(v)]^{n-1}}. \tag{2.5}$$

2.3.2 First-price auctions

Assume one good for sale and $\mathcal{N}$ risk neutral potential bidders. Each bidder i has a valuation v_i of the object, $i = 1, ..., \mathcal{N}$, and only bidder i knows his own valuation. Each bidder regards all valuations except his own as independent random variables with a probability density function $f_V(\cdot)$ and a cumulative distribution function $F_V(\cdot)$ with support $[\underline{v}, \overline{v}]$. This probability distribution is common knowledge in the usual game-theoretic sense.

One consequence of the assumption of *independently* distributed valuations is that the seller, or any outside observer, cannot learn anything about the value a specific bidder has by observing the values of some other bidders. Notice that the model assumes no possibilities for profitable resale of the object. Otherwise, the valuation of any bidder would, obviously, be dependent on other bidders' valuations.

Assume that a symmetric equilibrium consists of bidding strategies $b_j = \beta(v_j), \forall j$, which are strictly increasing in valuations. Define the inverse of the bid rule to be $v_j = \beta^{-1}(b_j)$. Now take the perspective of bidder i. We want to characterize his best response or equilibrium strategy $b_i = \beta(v_i)$ given that all other bidders follow their equilibrium strategies. Therefore, bidder i seeks to maximize his expected utility, which can be written

$$U_i(v_i, b_i) = (v_i - b_i)\left\{F_V\left[\beta^{-1}(b_i)\right]\right\}^{\mathcal{N}-1}. \tag{2.6}$$

The expression $(v_i - b_i)$ is simply the utility if he wins. Since bids are increasing in valuations, the probability that all bidders tender bids below b_i is equal to the probability that all other bidders have valuations below v_i, thus $\left\{F_V\left[\beta^{-1}(b_i)\right]\right\}^{\mathcal{N}-1}$ is the probability of winning. Differentiating expression (2.6) with respect to b_i and setting the resulting expression equal to zero, we obtain the following first-order condition:

$$\begin{aligned}\frac{\partial U}{\partial b_i} = \;& (v_i - b_i)(\mathcal{N}-1)\left\{F_V\left[\beta^{-1}(b_i)\right]\right\}^{\mathcal{N}-2} f_V\left[\beta^{-1}(b_i)\right]\frac{d\,\beta^{-1}(b_i)}{d\,b_i} \\ & - \left\{F_V\left[\beta^{-1}(b_i)\right]\right\}^{\mathcal{N}-1} = 0.\end{aligned} \tag{2.7}$$

By increasing the bid incrementally, expected utility increases because of the increased probability of winning. The cost of this action is that the difference between valuation and bid is decreased. This first-order condition contains two factors which can be interpreted within the familiar framework of equating marginal gain with marginal cost. The first term measures the benefit an increased probability of winning entails while the second term is a measure of the expected loss of reducing the profit margin incrementally. In equilibrium, bidder i must choose the strategy $b_i = \beta(v_i)$. Substituting this into equation (2.7) and after simplifying and rearranging the expression, we obtain the following first-order differential equation with variable coefficient and constant term:[2]

$$\beta'(v_i) + \frac{(\mathcal{N}-1) f_V(v_i)}{F_V(v_i)}\beta(v_i) = \frac{(\mathcal{N}-1) f_V(v_i)\, v_i}{F_V(v_i)}. \tag{2.8}$$

Assume now that the seller sets a reservation price v_0; i.e., only bids at or above this value are accepted. A bidder with a valuation equal to v_0 is, therefore, excluded from achieving a positive utility from participating in the bid process. The only way he can have the slightest hope of winning the object without incurring a loss, is by submitting his valuation as the bid.

[2] Recall that $\frac{d\,\beta^{-1}(b_i)}{d\,b_i} = 1/\beta'\left[\beta^{-1}(b_i)\right] = 1/\beta'(v_i)$.

This gives a boundary condition for equation (2.8) of the form

$$U(v_0) = 0. \tag{2.9}$$

Solving equation (2.8) together with the condition (2.9) yields

$$\beta(v_i) = v_i - \frac{1}{[F_V(v_i)]^{\mathcal{N}-1}} \int_{v_0}^{v_i} [F_V(u)]^{\mathcal{N}-1}\, du. \tag{2.10}$$

Bids at a first-price auction are set to equal the valuation v_i shaved by a factor depending on this very same valuation, v_i; the number of bidders, $\mathcal{N}$; the seller's reservation value, v_0; and the distribution function, $F_V(\cdot)$. It turns out that the bid rule is equal to the expectation of the second-highest valuation, given that the highest valuation is v_i.

$$\beta(v_i) = \mathcal{E}\left[V_{(\mathcal{N}-1)} | V_{(\mathcal{N}-1)} < v_i\right].$$

This can easily be verified by using the conditional population function given by equation (2.5). In an equilibrium, no bidder will gain by single-handedly deviating from the equilibrium solution. What happens if one bidder does deviate? In the case where he underbids—his bid is less than the equilibrium bid—his expected profit and the seller's expected revenue will decrease. The players that benefit are the bidders following the equilibrium strategy, whose expected profits will increase since there is a probability that they will win the object when they in fact have the second-highest valuation. In the case where a bidder bids more aggressively than his equilibrium strategy, the effects are different. The aggressive bidder will again have lower expected profits (since he is deviating from the optimal strategy), but his action will also hurt the other bidders. A probability that they will not win the object with the highest valuation is now introduced. The seller is the one who benefits from aggressive bidding. These effects have practical importance because the equilibrium strategy is rather demanding to compute.

We now verify that the bid function $\beta(v)$ is strictly increasing in v as we assumed. Since the bid function is symmetric for all bidders, we drop the subscript i.

$$
\begin{aligned}
\beta'(v) &= 1 - \frac{[F_V(v)]^{2\mathcal{N}-2} - (\mathcal{N}-1)[F_V(v)]^{\mathcal{N}-2} f_V(v) \int_{v_0}^{v} [F_V(u)]^{\mathcal{N}-1}\ du}{[F_V(v)]^{2\mathcal{N}-2}} \\
&= (v-b)\frac{(\mathcal{N}-1) f_V(v)}{F_V(v)} > 0.
\end{aligned}
$$

At what rate will bid functions increase in valuations? Clearly, we cannot have increasing growth, since bids will always be below (or equal to) valuations. Whether bid functions are increasing concave or increasing and strictly concave in v is not distribution-free. Normally, with no reserve price, we expect bid functions to be strictly concave. If valuations, however, are drawn from the continuous uniform distribution, we get linear increasing bid functions.

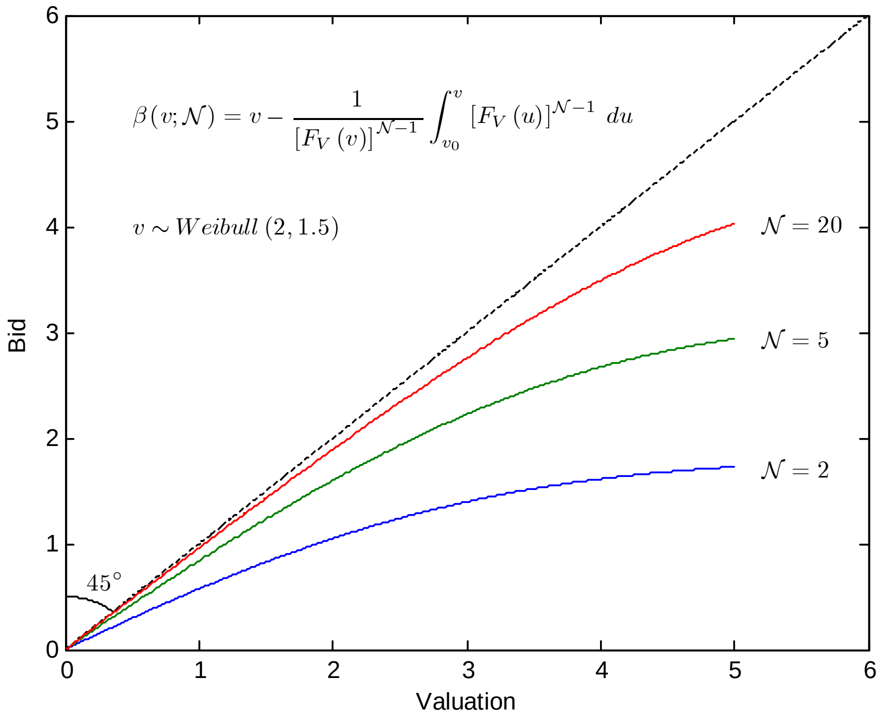
The effect of the number of bidders is easily understood by looking at the bidding rule given by equation (2.10). The last term on the right-hand side is a decreasing function of $\mathcal{N}$, and approaches zero in the limit. In other words, the bid approaches the valuation.

$$
\lim_{\mathcal{N}\to\infty} \beta(v) \to v,\ \forall\ v \in [v_0, \overline{v}]. \tag{2.11}
$$

The consequence of this is that expected revenue will approach the upper bound $\overline{v}$ when the number of bidders increases. In figure 2.1, we have sketched the bid function of equation (2.10) for different numbers of potential bidders $\mathcal{N}$ when valuations follow the Weibull distribution with location parameter 2 and scale parameter 1.5. As expected, bid shaving is severe for all valuations when $\mathcal{N} = 2$. Competition increases substantially when $\mathcal{N}$ is as low as five as witnessed by the reduced bid shaving of the drawn bid function. When $\mathcal{N} = 20$, bids are close to the 45 degree line for low valuations. The bid functions in the figure indicate clearly the asymptotic result reported in expression (2.11).

We have established from expression (2.11) that bid functions are increasing functions of $\mathcal{N}$. To see that they are, in fact, increasing *concave* functions

Figure 2.1: The bid function in first-price IPV model



of $\mathcal{N}$, we look at the second derivative of $\beta(v;\mathcal{N})$ with respect to $\mathcal{N}$. The first derivative is

$$\frac{d\beta(v;\mathcal{N})}{d\mathcal{N}} = -\int_{v_0}^{v} \log\left[\frac{F_V(u)}{F_V(v)}\right]\left[\frac{F_V(u)}{F_V(v)}\right]^{\mathcal{N}-1} du > 0. \tag{2.12}$$

Clearly, the first derivative reported in expression (2.12), is positive. The first quotient under the integral is always between 0 and 1, and, consequently, the logarithm of the quotient is negative. The second derivative is

$$\frac{d^2\beta(v;\mathcal{N})}{d\mathcal{N}^2} = -\int_{v_0}^{v} \left\{\log\left[\frac{F_V(u)}{F_V(v)}\right]\right\}^2 \left[\frac{F_V(u)}{F_V(v)}\right]^{\mathcal{N}-1} du < 0. \tag{2.13}$$

In expression (2.13), all terms under the integral is positive, resulting in a negative second derivative. Thus, by the signs of the first and second derivative, we conclude that bids are increasing and concave in $\mathcal{N}$.

Turn now to the calculation of seller's expected revenue. The expectation of sellers revenue, R^I, is equal to the expectation of the highest bid. The highest bid is a function of the first order statistic of valuations, whose density function is given by equation (2.1).

$$\begin{aligned}\mathcal{E}\left(R^I\right) &= \int_{v_0}^{\overline{v}} \beta\left(v\right) f_{V(\mathcal{N})}\left(v\right)\, dv \\ &= \mathcal{N}\int_{v_0}^{\overline{v}} \left[v f_V\left(v\right) - 1 + F_V\left(v\right)\right]\left[F_V\left(v\right)\right]^{\mathcal{N}-1}\, dv.\end{aligned} \tag{2.14}$$

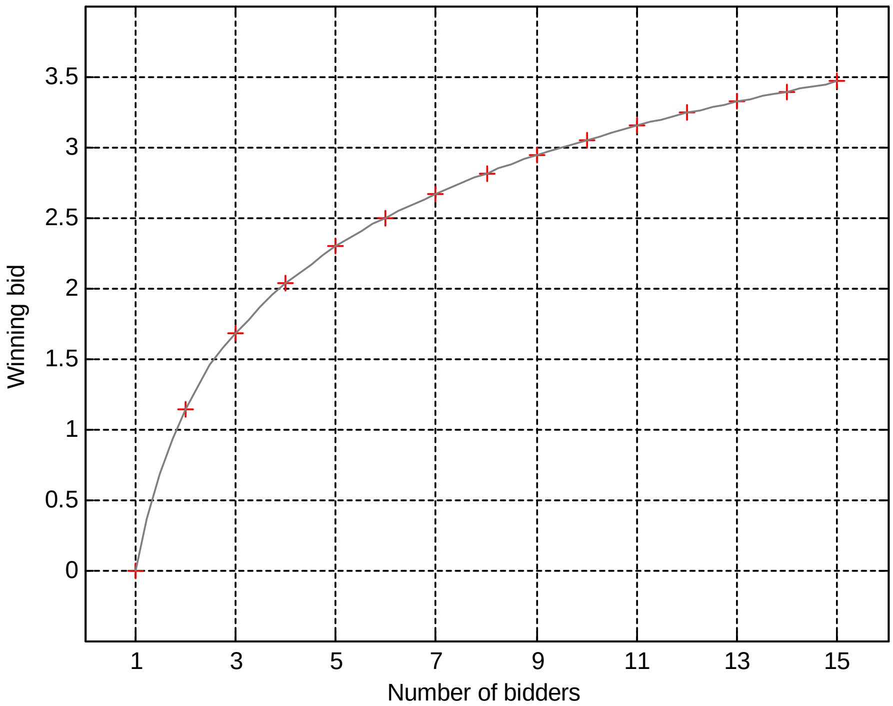
The details of the last expression are provided in the appendix to this chapter. A seller's expected revenue is a function of the same variables as the function characterizing the equilibrium-bid rule plus the upper bound on the distribution function, $\overline{v}$.

We saw above that bid functions are increasing concave functions of the number of bidders. This property will, therefore, apply to the winning bid as well. The expected revenue or the expected winning bid will be an increasing concave function of the number of bidders. For the distribution of valuations used in figure 2.1, we illustrate the increasing concave property of bids by plotting the winning bid against the number of bidders in figure 2.2. We note that the winning bid as a function of $\mathcal{N}$ is not continuous. What we find are the function values for the discrete input $\mathcal{N} \in [1, 2, 3, \ldots]$. Connecting the discrete function range by a cubic spline interpolation—as we do in figure 2.2—produces a function that is strictly concave.

2.3.3 Second-price auctions

Bidding at the second-price auctions involves a simpler strategy—in fact, a dominant strategy—as its solution: bid the valuation at the closed second-price auction and bid infinitesimally above the going offer up to one's valua-

Figure 2.2: Winning bid as a function of the number of bidders



tion at the open second-price auction. We confine attention to the sealed-bid version in this section. The best bid rule is thus

$$b_i = v_i. \tag{2.15}$$

Notice that a second-price auction implies that the winner pays the minimum amount he would have to bid and still expect to win the auction. This may appropriately be denoted a *first rejected bid payment scheme*. Thus, since expected payment is independent of a bidder's own bid, the bid rule may be interpreted as maximizing the probability of winning subject to the constraint that profits must be non-negative if winning. To establish that the bid rule (2.15) constitutes a Nash equilibrium, suppose a bidder considers bidding below his valuation, while all of his opponents follow their dominant strategies. Now, if he has the highest valuation, but the second-highest valuation (and bid) is above his bid, he will suffer a loss. Instead of making a positive profit, he earns zero. Likewise, if he were to bid above his valuation, there is a positive probability that he will end up being the winner, and that the second-highest bid is above his valuation, in which case he will make a negative profit. In all other cases the strategy of bidding above or below the valuation, will not matter. Thus, if he is to deviate from the optimal strategy, either by bidding below or by bidding above his valuation, there is a certain probability of incurring a loss while there is no possibility of gaining.

One appealing feature of the first rejected bid payment scheme is that the degree of sophistication required of bidders is moderate. A dominant, robust, and easily-understood strategy emerges. This is contrary to the first-price auction where the informational requirements are more demanding. To compute the equilibrium solution, bidders must know the distribution of valuations and the number of bidders, and the problem is obviously not trivial.

Given the optimal bid strategy at the second-price auction, what is the expected revenue to the seller under this regime? We calculate expected revenue using the joint density function of the two highest order statistics given by equation (2.4). Notice that when the second-highest valuation lies in the interval $[\underline{v}, v_0]$, while the highest valuation is above v_0, then the winner must pay v_0. Otherwise, when both valuations are above v_0, the winner pays the second-highest valuation.

$$
\begin{aligned}
\mathcal{E}\left(R^{II}\right) &= \int_{v_0}^{\overline{v}} \int_{\underline{v}}^{v_0} v_0 f_{(\mathcal{N},\mathcal{N}-1)}\left(v,u\right)\, du\, dv \\
&\quad + \int_{v_0}^{\overline{v}} \int_{v_0}^{v} u f_{(\mathcal{N},\mathcal{N}-1)}\left(v,u\right)\, du\, dv \qquad (2.16) \\
&= \mathcal{N} \int_{v_0}^{\overline{v}} \left[v f_V\left(v\right) - 1 + F_V\left(v\right)\right]\left[F_V\left(v\right)\right]^{\mathcal{N}-1}\, dv.
\end{aligned}
$$

Comparing equations (2.14) and (2.16) reveals the somewhat surprising fact that expected revenues at first- and second-price auctions are identical. Roughly explained, the second-price auction format encourages more aggressive bidding than the first-price format, and it does so to such an extent that the expected second-highest bid at the former auction equals the expected highest bid at the latter auction. In fact, within the framework of the independent private-values model, expected revenue is equal for all four common auctions; the sealed-bid, first- and second-price auctions and the open English and Dutch auctions. This result is known as the *Revenue Equivalence Theorem.* Put formally, one can state that: When bidders' values are independently distributed and private, bidder's follow strategies that constitute a noncoöperative equilibrium, the bidder with the highest valuation wins the object and bidders with lower valuations pay nothing, then *the expected revenue is equal to the expected second-highest valuation.* Notice that it is expected revenue which is equal; in actual realizations the obtained prices from first-price and second-price auctions may differ.

Another aspect of the Revenue Equivalence Theorem is that it relies critically on its assumptions and does not generalize. If values are dependent or bidders are risk averse, the various auction mechanisms will differ with respect to expected revenue. The case of dependent values are treated in section 2.4 while the case of introducing risk aversion in the model is presented in section 2.3.5.

Under the two auction formats studied thus far, the seller does not extract all rent from the trade. Because the seller is unable *ex ante* to distinguish between the bidders' types, the winner will end up with a so-called informational rent; i.e., the surplus he obtains from the trade because of his private information. How large, then, is this informational rent? This is easy to answer since we have established that the seller on average obtains a price equal to the second-highest valuation. As a matter of definition of economic rent, the winner can expect to receive a surplus equal to the difference between his own valuation and the second-highest valuation.

2.3.4 The optimal reserve price

Thus far we have considered seller's reserve price as exogenous. The bidding model allows, however, the seller to introduce an optimal reserve price. Assume the good is worth v_0 to the seller, but he sets a reserve price r. What level of r will enhance revenues most? The maximization problem at the first-price auction is

$$\max_r v_0 \left[F_V(r)\right]^{\mathcal{N}} + \mathcal{N} \int_r^{\overline{v}} \left[v f_V(v) - 1 + F_V(v)\right] \left[F_V(v)\right]^{\mathcal{N}-1} dv. \qquad (2.17)$$

The first term is the expected value to the seller if the product is unsold. The probability that all bidders submit bids below the reserve price is equal to $\left[F_V(r)\right]^{\mathcal{N}}$. The second term is the expected revenue obtained from the winner, and is taken directly from expression (2.14). The only modification is that the reserve price is now to be set optimally at r^* instead of at seller's value v_0. The global maximum of the objective function must satisfy the following first-order condition:

$$v_0 \mathcal{N} \left[F_V(r^*)\right]^{\mathcal{N}-1} f_V(r^*) - \mathcal{N}\left[r^* f_V(r^*) - 1 + F_V(r^*)\right] \left[F_V(r^*)\right]^{\mathcal{N}-1} = 0.$$

Re-arranging and simplifying this expression yields the following formula for determining the reserve price.

$$r^* = v_0 + \frac{\left[1 - F_V(r^*)\right]}{f_V(r^*)}.$$

The optimal reserve price is independent of the number of bidders, and it is strictly greater than the reservation value v_0. The same formula applies to the sealed-bid, second-price auction; see Laffont and Maskin [61] for the details.

Two possibilities must be considered when raising the reserve price above the seller's own valuation. First, there is a risk that no one has a valuation above the reserve price level, but there is at least one valuation in the interval $[v_0, v_*]$. In that case, the seller will incur a loss. Second, there is a chance that the reserve price is set in the interval between the two highest bids. Since all players submit their bids conditional on being the winner, they must all take into account the possibility that the reserve price is above the second-highest valuation. Consequently, bidders will reduce their bids less below their valuations than in the case of no reserve price since the strategy space is now narrowed down. To elaborate somewhat on this point, introducing a reserve price is equivalent to introducing a bidder with a known bid equal to the reserve price. Thus, competition will be harsher since the bidders who contemplated bidding below the reserve price before it was announced, are now forced to revise their bids upwards if they are still to have any hope of winning the auction: Some of them will find it profitable to submit a bid above the reserve price. Obviously, a reserve price is of most importance when competition is weak.

A consequence of introducing a reserve price above the seller's own valuation is that the auction format may no longer yield efficient outcomes. As we have seen, there is a certain probability that the bidder with the highest valuation will not win the object. The seller—like any monopolist—finds it to his advantage to deviate from the Pareto optimal allocation that an efficient trade mechanism constitutes. Inefficiency is introduced in order to try to capture some of the informational rent the winner otherwise obtains.

Somewhat surprising is the result that the number of bidders does not matter when setting the reserve price. The seller wants to set the reserve price in the interval between the two highest valuations. As $\mathcal{N}$ increases, these two values move towards the upper limit of the distribution. That effect should warrant the seller to increase his reserve price with $\mathcal{N}$. The interval between the two highest valuations, however, is reduced, and approaches zero in the limit when $\mathcal{N}$ increases. Thus, the importance of the reserve price is reduced with the number of bidders. Alternatively, with an increased number of bidders, it is more difficult to aim the reserve price in the relevant interval, and the risk of over-shooting; i.e. setting the reserve price above the highest valuation is imminent if the strategy of making the reserve price depend on $\mathcal{N}$ is pursued.

Let us illustrate some important facts about the effect of the reserve price by a numerical example. Assume valuations are drawn from the same distribution we used for illustrating bid functions in figure 2.1; i.e., they follow the Weibull distribution with location parameter μ equal to 2 and scale parameter σ equal to 1.5. That is, the pdf is

$$f_V(v) = \sigma\mu^{-\sigma}v^{\sigma-1}\exp\left[-\left(\frac{v}{\mu}\right)^{\sigma}\right] I_{(0,\infty)}(v),$$

and the cdf is

$$F_V(v) = 1 - \exp\left[-\left(\frac{v}{\mu}\right)^{\sigma}\right] I_{(0,\infty)}(v).$$

Set the seller's valuation, v_0, to 2. For different number of bidders, we compute the expected revenue given by expression (2.17)

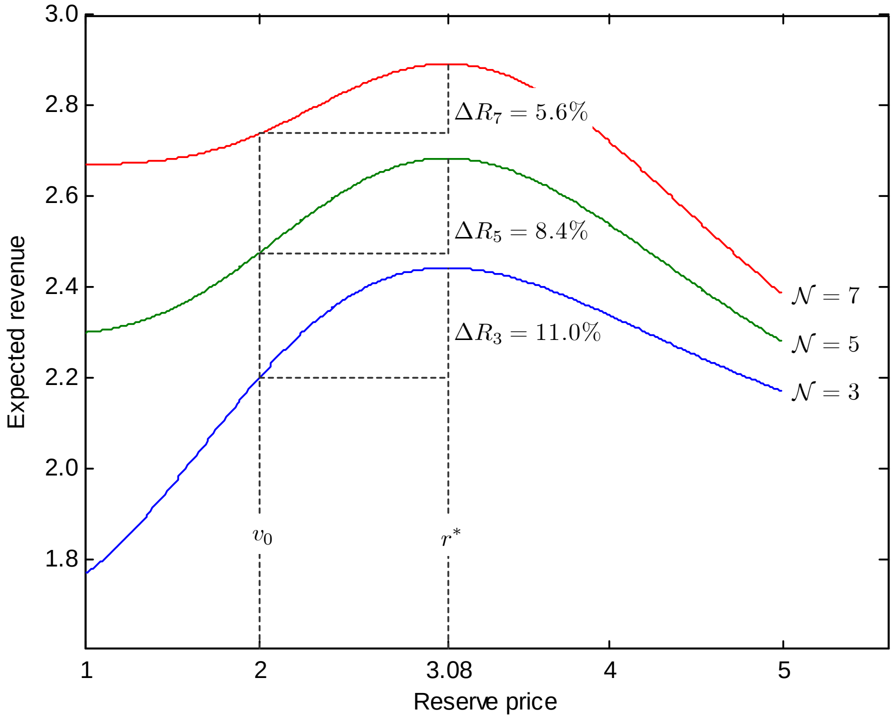
$$\mathcal{E}[R(r)] = v_0[F_V(r)]^{\mathcal{N}} + \mathcal{N}\int_r^{\overline{v}} [vf_V(v) - 1 + F_V(v)][F_V(v)]^{\mathcal{N}-1}\,dv$$

for each possible reserve price. We illustrate the expected revenue function for three different number of bidders ($\mathcal{N} \in [3, 5, 7]$) in figure 2.3.

The optimal reserve price in this example is $r^* = 3.08$. The independence of the optimal reserve price on $\mathcal{N}$ is obvious from the figure. However, loosely speaking, the importance of setting the reserve price optimally decreases with $\mathcal{N}$. The gains can be substantial when the number of bidders is low. In order to get an idea of the potential benefits within a realistic model, we have computed the percentage increase in expected revenue when going from $r = v_0$ to $r = r^*$. Denote this difference $\Delta R_{\mathcal{N}}$. When $\mathcal{N} = 7$, expected revenue increases by 5.6 percent, while for $\mathcal{N} = 3$, the corresponding increase is 11 percent.

From a practical perspective, it may be impossible to pin-down the optimal reserve price. It is, however, somewhat reassuring that there is, in general, a relatively wide range of r that will increase revenues. The expected revenue curves in figure 2.3 approach v_0 asymptotically. Looking at the behavior of the curves for high r, we see the risk of setting the reserve price too high. For sufficiently high r, the expected revenue drops below the benchmark level given by $\mathcal{E}[R(r = v_0)]$.

Figure 2.3: Expected revenue and the reserve price



2.3.5 Risk aversion

Risk aversion is a property where an individual prefers to receive a certain amount to participating in a lottery with the same expected value. How will expected revenue be affected if risk aversion exists? An important result shown by Riley and Samuelson [95] is discussed below. Assume bidders have the same utility function, $u(\cdot)$, representing risk averse preferences. Now, consider the maximization of expected utility from bidding at the first-price auction when bidders use a bidding function $\beta(x)$:

$$\max u[v_i - \beta(x)][F_V(x)]^{\mathcal{N}-1}.$$

The first-order condition can be written as

$$\beta'(v) = (\mathcal{N} - 1)\frac{f_V(v)}{F_V(v)}\frac{u[v-\beta(v)]}{u'[v-\beta(v)]}. \tag{2.18}$$

Assume now two different utility functions $u_L(\cdot)$ and $u_H(\cdot)$, where the subscripts H and L denote high and low degree of risk aversion respectively. According to the absolute risk aversion measure, this implies that

$$\frac{u_L''(\cdot)}{u_L'(\cdot)} > \frac{u_H''(\cdot)}{u_H'(\cdot)} \geq 0. \tag{2.19}$$

To analyse the effect of risk aversion from the first-order condition, the following result derived from the mean-value theorem is useful. Given two functions $\beta_H(\cdot)$ and $\beta_L(\cdot)$, if $\beta_H(v) > \beta_L(v)$ for all $v > \underline{v}$, then $\beta_H'(v) > \beta_L'(v)$. From expression (2.19) it can be deduced that $u_L(\cdot)/u_L'(\cdot) > u_H(\cdot)/u_H'(\cdot)$. Hence, it follows from equation (2.18) that $\beta_H'(v) > \beta_L'(v)$. Consequently, from the above stated result, $\beta_H(v) > \beta_L(v)$. In other words, risk aversion leads to higher bids in the first-price independent private-values model than in the case where bidders are risk neutral. Recall our discussion of the first-order condition (2.7). By reducing the bid below the valuation, the risk of losing the auction increases while the potential gain if winning also increases. Risk averse bidders tend to give more weight to the former effect, than risk neutral bidders. At the second-price auction, however, the equilibrium-bid strategy is to bid ones own valuation. Risk aversion will not affect valuations. Therefore, bids are unaffected. Thus, all things being equal, first-price auctions generate higher bids and, consequently, an higher expected revenue than second-price auctions when bidders are risk averse. This revenue ranking will not, in general, hold when we admit interdependent valuations, as discussed in the next section.

For a broader treatment of risk aversion at auctions, see Maskin and Riley [66] and Matthews [67].

2.4 The general symmetric model

In the private-values model of the preceding section, each bidder's valuation is, from the bidder's perspective, certain. In the more general model of bidding behavior developed by Milgrom and Weber [77], bidders do not know the true value of the object, but are assumed to possess some relevant private information. Denote the vector of these pieces of private information or signals, which are random variables, by $\boldsymbol{Z} = (Z_1, ..., Z_\mathcal{N})$. In addition, there are several other variables unknown to bidders which affect the object's value. We denote these value determining variables by the vector $\boldsymbol{S} = (S_1, ..., S_m)$. The true value of the object for bidder i, V_i, can thus be represented by a function

$$V_i = V\left(\boldsymbol{S}, Z_i, (Z_j)_{j \neq i}\right).$$

Notice that we now represent a bidder's valuation with V_i indicating that also bidder i conceives it as a random variable, while in the private-values model we let v_i denote the certain valuation of a bidder. We assume a symmetric equilibrium; i.e., the function $V(\cdot)$ is identical for all bidders. Furthermore, the function is assumed continuous and increasing in all its arguments and to have a finite expectation. In order to derive at optimal bid rules within this framework, there are two elements which differ from the independent private-values model. First, since a representative bidder i knows only the realization of his signal, to evaluate V_i he takes the conditional expectation of V_i. Second, bidders' signals are allowed to be dependent. In fact, a specific kind of positive dependence called *affiliation* is used.

2.4.1 Affiliated variables

To understand some of the derivations that follow, it is useful to establish a few properties of affiliated random variables. The appendix of Milgrom and Weber [77] provides a thorough treatment of this concept, and Naegelen [80] discusses several aspects and gives references to the primary literature. For our purposes, it is sufficient to understand affiliation among two variables, so the general case is not examined.

Assume two random variables X and Y having a joint density function $f_{X,Y}(x,y)$ and where $F_{Y|X}(y|x)$ and $f_{Y|X}(y|x)$ are, respectively, the conditional cumulative distribution function and the conditional density function of Y given $X = x$. A necessary condition for X and Y to be affiliated is that for all $x' \geq x$ and $y' \geq y$,

$$f_{X,Y}(x,y)\, f_{X,Y}(x',y') \geq f_{X,Y}(x',y)\, f_{X,Y}(x,y') \,. \tag{2.20}$$

We denote this expression the *affiliation inequality*. An interpretation of it is that if one of the variables is large, it is also more probable that the other variable is large than small. Milgrom [74] says that "variables are affiliated when they are positively correlated conditional on lying in any small rectangle." Thus, an high-valued signal is "good news" about the true value of the object since it is then likely that other bidders received high signals also and, consequently, that the true value is in fact "high."

A useful result we can deduce from this inequality is that for the two affiliated variables X and Y, the function $F_{Y|X}(y|x)\,/f_{Y|X}(y|x)$ is non-increasing in x.

The proof of this starts with observing that if $x' \geq x$, then we shall, according to the affiliation inequality (2.20), for any $t \leq y$, have:

$$f_{X,Y}(x,t)\, f_{X,Y}(x',y) \geq f_{X,Y}(x,y)\, f_{X,Y}(x',t) \,.$$

Re-arranging, we obtain

$$\frac{f_{X,Y}(x,t)}{f_{X,Y}(x,y)} \geq \frac{f_{X,Y}(x',t)}{f_{X,Y}(x',y)}$$

or since $f_{X,Y}(x,t) = f_{Y|X}(t|x)\, f_X(x)$,

$$\frac{f_{Y|X}(t|x)\, f_X(x)}{f_{Y|X}(y|x)\, f_X(x)} \geq \frac{f_{Y|X}(t|x')\, f_X(x')}{f_{Y|X}(y|x')\, f_X(x')}.$$

This implies

$$\frac{f_{Y|X}(t|x)}{f_{Y|X}(y|x)} \geq \frac{f_{Y|X}(t|x')}{f_{Y|X}(y|x')}. \tag{2.21}$$

The conditional density $f_{Y|X}(\cdot|\cdot)$ for which the inequality (2.21) holds is said to have a monotone likelihood ratio; see Milgrom [73]. Integrating with respect to t over the interval $(-\infty, y)$ yields

$$\int_{-\infty}^{y} \frac{f_{Y|X}(t|x)}{f_{Y|X}(y|x)}\,dt \geq \int_{-\infty}^{y} \frac{f_{Y|X}(t|x')}{f_{Y|X}(y|x')}\,dt.$$

Finally,

$$\frac{F_{Y|X}(y|x)}{f_{Y|X}(y|x)} \geq \frac{F_{Y|X}(y|x')}{f_{Y|X}(y|x')}. \tag{2.22}$$

Next, we want to establish that for the same affiliated variables X and Y, the conditional distribution function $F_{Y|X}(y|x)$ is non-increasing in x. A preliminary result we need in order to prove this is that the so called hazard rate of the distribution, $f_{Y|X}(y|x) \,/\, \left[1 - F_{Y|X}(y|x)\right]$, is non-increasing in x. As before, assume $x' > x$, but now we suppose $t > y$. The inequality sign of (2.21) is then reversed. Integrating over (y, ∞) yields

$$\int_{y}^{\infty} \frac{f_{Y|X}(t|x)}{f_{Y|X}(y|x)}\,dt \leq \int_{y}^{\infty} \frac{f_{Y|X}(t|x')}{f_{Y|X}(y|x')}\,dt$$

or

$$\frac{1 - F_{Y|X}(y|x)}{f_{Y|X}(y|x)} \leq \frac{1 - F_{Y|X}(y|x')}{f_{Y|X}(y|x')}$$

from which it follows that the hazard rate is non-increasing in x. Next, notice that

$$\frac{F_{Y|X}(y|x)}{\left[1 - F_{Y|X}(y|x)\right]} = \frac{F_{Y|X}(y|x)}{f_{Y|X}(y|x)} \times \frac{f_{Y|X}(y|x)}{\left[1 - F_{Y|X}(y|x)\right]}.$$

Since both terms on the right-hand side are positive and proved to be non-increasing, the expression on the left-hand side must be non-increasing. Thus, it is obvious that also $F_{Y|X}(y|x)$ is non-increasing in x (i.e., for all y if $x' > x$), so

$$F_{Y|X}(y|x') \leq F_{Y|X}(y|x).$$

The distribution $F_{Y|X}(y|x')$ stochastically dominates the distribution $F_{Y|X}(y|x)$; see for instance Huang and Litzenberger [52]. From this, it follows that if g is an increasing function, then

$$\mathcal{E}\left[g\left(Y\right)|X=x'\right] \geq \mathcal{E}\left[g\left(Y\right)|X=x\right]. \tag{2.23}$$

This result will prove useful because it generalizes to several variables; i.e., $g\left(\cdot\right)$ may be a function of several affiliated variables, and the conditioning may be with respect to several variables. The conditional expectation function of $g\left(\cdot\right)$ is still non-decreasing. For a formal statement and proof of this, we refer to theorem 5 in Milgrom and Weber [77].

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2.4.2 First-price auctions

The structure of the maximization problem for a representative bidder i is, as before, to maximize expected gain; i.e., to maximize the expectation of the difference between true value and bid. Since the true value is unknown to bidders, they use the expected value of their true value V_i conditional on the realized value of their signal and the highest of the opponents' signal. To save notation, we denote bidder i's signal $Z_i = X$, and the highest signal of the other bidders by $Z_{(\mathcal{N}:-i)} = Y$. The realized values of these random variables are denoted x and y respectively. We assume all bidders but i use the equilibrium strategy $\beta\left(\cdot\right)$, which is a function of their realized values of their signals. Specifically, the bidder with signal $Y = y$ tenders a bid $\beta\left(y\right)$. Bidder i seeks to maximize his expected utility $\mathcal{E}\left[u_i\left(b_i, x\right)\right]$, which is a function of his bid and his received signal and is written

$$\mathcal{E}\left[u_i\left(b_i, x\right)\right] = \mathcal{E}\left[\left(V_i - b_i\right)|X=x, \beta\left(y\right) < b_i\right]. \tag{2.24}$$

The important fact of the maximization problem is that bidders should evaluate the expectation of the true value V_i less the bid b_i not only conditional on their own signals, but also conditional on winning. This implies that they should include the information that all other bidders received lower-valued signals in their calculations. Failing to do this will result in the so-called *winner's curse.* If the expectation of V_i is taken conditional only on one's own signal, then there is an implicit, and erroneous, assumption that the highest of the other bids can be located anywhere on the support of this variable. But this cannot be true of winning. According to the result stated in equation (2.23),

$$\mathcal{E}\left[(V_i - b_i)\,|X = x, \beta(y) < b_i\right] < \mathcal{E}\left[(V_i - b_i)\,|X = x\right].$$

Thus, the bid should be reduced by a factor that takes the information $\beta(y) < b_i$ into account such that the signal x is treated as a first order statistic of the signals rather than as an unbiased estimate of V_i. The winner's curse or severe overbidding has been observed at auctions with inexperienced bidders; see Thaler [106].

To continue with the bid problem, a useful piece of notation is to define the function

$$\nu(x, y) = \mathcal{E}\left(V_i|X = x, Y = y\right). \tag{2.25}$$

We shall make use of the following identity of conditional expectations in order to proceed. For two random variables X and Y,

$$\mathcal{E}\left(Y|X\right) = \mathcal{E}\left[\mathcal{E}\left(Y|X, Y\right)|X\right]. \tag{2.26}$$

It is an extension of the well known property of conditional expectation that $\mathcal{E}(Y) = \mathcal{E}\left[\mathcal{E}(Y|X)\right]$. The proof is trivial, but tedious. Using the identity (2.26), together with the notation defined in expression (2.25), we transform the formulation of a bidder's expected payoff given in equation (2.24) to

$$\begin{aligned} & \mathcal{E}\left[(V_i - b_i)\,|X = x, \beta(y) < b_i\right] \\ = \; & \mathcal{E}\left(\mathcal{E}\left[(V_i - b_i)\,|X = x, Y = y\right]|X = x, \beta(y) < b_i\right) \qquad (2.27) \\ = \; & \mathcal{E}\left(\left[\nu(x, y) - b_i\right]|X = x, \beta(y) < b_i\right). \end{aligned}$$

After a final transformation of (2.27), the optimal bid for bidder i can be characterized as the solution to

$$\max_{b_i} \int_{\underline{x}}^{\beta^{-1}(b_i)} \left[\nu\left(x,y\right)-b_i\right] f_{Y|X}\left(y|x\right)\,dy. \tag{2.28}$$

The first-order condition of the maximization problem in (2.28) is then

$$\begin{aligned} &\nu\left[x,\beta^{-1}\left(b_i\right)\right]\, f_{Y|X}\left[\beta^{-1}\left(b_i\right)|x\right]\,\frac{d\,\beta^{-1}\left(b_i\right)}{d\,b_i} \\ &-F_{Y|X}\left[\beta^{-1}\left(b_i\right)|x\right]-b_i\, f_{Y|X}\left[\beta^{-1}\left(b_i\right)|x\right]\frac{d\,\beta^{-1}\left(b_i\right)}{d\,b_i}=0. \end{aligned} \tag{2.29}$$

Since $d\,\beta^{-1}\left(b_i\right)/\,db_i = 1/\beta'\left(\beta^{-1}\left(b_i\right)\right)$, equation (2.29) can be rearranged to yield

$$\frac{d\,\beta\left(\beta^{-1}\left(b_i\right)\right)}{d\,b_i}=\frac{\left\{\nu\left[x,\beta^{-1}\left(b_i\right)\right]-b_i\right\}\, f_{Y|X}\left[\beta^{-1}\left(b_i\right)|x\right]}{F_{Y|X}\left[\beta^{-1}\left(b_i\right)|x\right]}. \tag{2.30}$$

Now, if the strategy $\beta\left(\cdot\right)$ is a symmetric strategic equilibrium, one should expect that $b_i=\beta\left(x\right)$ or $x=\beta^{-1}\left(b_i\right)$. Substituting this into equation (2.30) yields the following first-order differential equation:

$$\beta'\left(x\right)+\frac{f_{Y|X}\left(x|x\right)}{F_{Y|X}\left(x|x\right)}\beta\left(x\right)-\nu\left(x,x\right)\frac{f_{Y|X}\left(x|x\right)}{F_{Y|X}\left(x|x\right)}=0. \tag{2.31}$$

The problem, as we have stated it, will also imply that for all x, $\nu\left(x,x\right)-\beta\left(x\right)\geq 0$. A bidder will not submit a bid which results in a negative expected payoff if he wins. Note also that $\nu\left(\underline{x},\underline{x}\right)-\beta\left(\underline{x}\right)\leq 0$. A bidder cannot expect a positive expected utility when his realized signal is the infimum of the support of X. These two conditions together result in the following boundary condition for the differential equation (2.31).

$$\beta\left(\underline{x}\right)=\nu\left(\underline{x},\underline{x}\right). \tag{2.32}$$

Consequently, the solution to the differential equation (see appendix 2.A.2) is

$$\beta(x) = \nu(x,x) - \int_{\underline{x}}^{x} L(\alpha|x)\, d\nu(\alpha,\alpha) \tag{2.33}$$

where

$$L(\alpha|x) = \exp\left[-\int_{\alpha}^{x} \frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)}\, ds\right].$$

From condition (2.22), it follows that, for all α, $L(\alpha|x)$ is a decreasing function of x. And since $\nu(x,x)$ is an increasing function, we can conclude that $\beta(x)$ is increasing in the signal x. The bid function was derived from the necessary conditions given by (2.31) and (2.32). To prove that the bid function constitutes a best-response, the following second-order considerations are sufficient. Assume a bid $\beta(z)$ is tendered when signal is x. The expected profit of such a bid takes the form

$$\Pi(z,x) = \int_{\underline{x}}^{z} [\nu(x,y) - \beta(z)]\, f_{Y|X}(y|x)\, dy.$$

Differentiating with respect to z yields

$$\begin{aligned}
\Pi_z &= [\nu(x,z) - \beta(z)]\, f_{Y|X}(z|x) - \int_{\underline{x}}^{z} \beta'(z)\, f_{Y|X}(y|x)\, dy \\
&= \left[\nu(x,z) - \beta(z)\frac{f_{Y|X}(z|x)}{F_{Y|X}(z|x)} - \beta'(z)\right] F_{Y|X}(z|x).
\end{aligned} \tag{2.34}$$

Compare this expression with the left-hand side of equation (2.31) which may be transformed to a similar expression as equation (2.34). Using equations (2.22) and (2.23), we can conclude that if $z < x$, then $\Pi_z > 0$, and if $z > x$, then $\Pi_z < 0$. This proves that the bid rule $\beta(z)$ maximizes expected profit only if $z = x$.

The bid rule (2.33) of the general symmetric model has a similar structure as the bid rule (2.10) of the independent private-values model. The conditional expectation of the true value is shaved by a factor depending on the number of bidders ($f_{Y|X}$ and $F_{Y|X}$ are functions of $\mathcal{N}$). However, while the bid rule in (2.10) was shown to equal the expectation of the second-highest valuation given that a bidder's own valuation is the highest, such an intuitive interpretation cannot be given to the bid rule in (2.33).

In the independent private-values model, we saw that bids increase with the number of bidders. The effect of increased competition is not so straightforward when there is a common-value element. More bidders make it necessary to increase bids in order to win. But there is also another factor which works in the opposite direction; the fear of the winner's curse. The shaving factor to avoid this phenomenon will increase with $\mathcal{N}$. Or, equivalently, the expected value conditional on winning will decrease with $\mathcal{N}$. Without a specific structure of the probability distributions involved in equation (2.33), it is not possible to determine the total effect of increased competition.

2.4.3 Second-price auctions

The maximization problem of a bidder at a second-price auction is a slightly modified version of the first-price problem stated in (2.24). Instead of paying his bid, the winner pays the second-highest bid. Therefore, the problem is formulated as

$$\max_{b_i} \mathcal{E}\left[\left(V_i - \beta\left(y\right)\right)|X = x, \beta\left(y\right) < b_i\right]. \tag{2.35}$$

To proceed, we state the equilibrium solution and assume all other bidders follow it. The equilibrium point of second-price auctions consists of a vector of strategies $\beta\left(z_j\right) = \mathcal{E}\left[V_j|Z_j = z_j, Y = z_j\right] = \nu\left(z_j, z_j\right)$, where the last equality follows from the definition given in equation (2.25). Bidders submit as their bid the expectation of their valuation conditional on their signal *and* that the highest of other bidders signal equals their own signal. Specifically, the bidder with signal $Y = y$, will in equilibrium bid $\beta\left(y\right) = \nu\left(y, y\right)$.

We proceed by showing that if all bidders except i follow their equilibrium-bid strategies, then it is also optimal for i to bid according to the equilibrium solution. This procedure should now be familiar. Note that the expression to be maximized can be transformed to

$$\begin{aligned}
& \mathcal{E}\left(\left[V_i - \beta\left(y\right)\right] | X = x, \beta\left(y\right) < b_i\right) \\
= \; & \mathcal{E}\left[\mathcal{E}\left(\left[V_i - \beta\left(y\right)\right] | X = x, Y = y\right) | X = x, \beta\left(y\right) < b_i\right] \\
= \; & \mathcal{E}\left[\left(\left[\nu\left(x, y\right) - \nu\left(y, y\right)\right]\right) | X = x, \beta\left(y\right) < b_i\right].
\end{aligned}$$

A final transformation of this expression gives the maximization problem

$$\max_{b_i} \int_{\underline{x}}^{\beta^{-1}(b_i)} \left[\nu\left(x, \alpha\right) - \nu\left(\alpha, \alpha\right)\right] f_Y\left(\alpha | x\right) \, dy. \tag{2.36}$$

The first-order condition of the maximization problem (2.36) is

$$\left(\nu\left[x, \beta^{-1}\left(b_i\right)\right] - \nu\left[\beta^{-1}\left(b_i\right), \beta^{-1}\left(b_i\right)\right]\right) f_{Y|X}\left[\beta^{-1}\left(b_i\right) | x\right] \frac{d\,\beta^{-1}\left(b_i\right)}{d\,b_i} = 0.$$

From this expression, it is clear that if bidder i chooses his bid to be $b_i = \beta\left(x\right)$ or $x = \beta^{-1}\left(b_i\right)$, the first-order condition is satisfied since the expression in brackets is zero; i.e., $\nu\left(x, x\right) - \nu\left(x, x\right) = 0$. Therefore, we have shown that the following strategy should be followed in equilibrium.

$$\beta\left(x\right) = \mathcal{E}\left(V_i | X = x, Y = x\right). \tag{2.37}$$

Note the similarity of this strategy and the simple weakly dominant strategy of second-price auctions under the independent private-values model where bid is equal to valuation. A similar line of reasoning can help one intuitively understand the equilibrium solution of expression (2.37). Since it is a second-price auction, we have already established that it is optimal to bid one's valuation if this is certain. In the general symmetric model, bidders have to take the expected value of the true value. This expectation should of course be calculated by incorporating the information of their own signals. But why condition on that the highest signal of opponents are equal to ones own signal? First, if assuming it was higher, then a bidder conditions on that he does not win the auction and this makes no sense. Neither can it be of advantage to condition on that Y is lower than his own signal. This will result in a lower bid submitted since the $\nu(\cdot,\cdot)$ function is increasing in both its arguments. And such a strategy results in that the probability of winning is lowered while the net gain if winning is unaltered.

2.4.4 The linkage principle

An important result, which makes revenue comparisons over different auctions possible, is the so-called *Linkage Principle.* Milgrom and Weber [77] introduced it in a non-mathematical form (calling it "a common thread running through the results"), but in subsequent work, such as Milgrom [74], the result is given a formal presentation. The principle applies to a general class of auctions called *standard auctions.* These are auctions where the bidder with the highest valuation wins and pays a nonnegative amount, while all other bidders pay nothing. Both the first-price and the second-price auctions described above are examples of standard auctions. We shall now show how it is possible to rank the revenues generated by two different standard auctions.

Define the expected payment function of the winning bidder, c, to be a function of his signal or estimate $X = x$, and the value $\widetilde{x}$ which he bases his bid $\beta(\widetilde{x})$ on; i.e., he bids as if he received signal $\widetilde{x}$, so

$$c = c(x, \widetilde{x}).$$

The payment function is strictly increasing in both its arguments, which have support on the same interval $[\underline{x}, \overline{x}]$. At two different standard auctions, denote the auction whose payment function increases most when its first argument is increased incrementally, by A and the other by B; i.e., $c_1^A(x, \widetilde{x}) > c_1^B(x, \widetilde{x})$ where subscripts denote partial differentiation with respect to the relevant argument and superscripts represent the auction format. The partial derivative $c_1(x, \widetilde{x})$ is a measure of the increase in the payment function, or seller's expected revenue, when the bidder's estimate x increases, but his bidding strategy, which is based on $\widetilde{x}$, is held constant. Likewise, define p to be the probability of winning when bidding $\beta(\widetilde{x})$ and estimate is x, so

$$p(x, \widetilde{x}) = \Pr(Y < \widetilde{x} | X = x).$$

The expected value received conditional on the same events as above are denoted

$$\nu(x, \widetilde{x}) = \mathcal{E}(V | X = x, Y < \widetilde{x}).$$

The functions p and ν are independent of the auction mechanism. The linkage principle states that given two standard auctions, A and B, the auction A, which satisfies $c_1^A(x, x) > c_1^B(x, x)$, will yield at least as large expected selling price as auction B.

To prove this, note that a bidder, say at auction A, will maximize with respect to $\widetilde{x}$

$$\upsilon(x, \widetilde{x}) - p(x, \widetilde{x}) c^A(x, \widetilde{x}).$$

The first-order condition for this is

$$\upsilon_2(x, \widetilde{x}) - p_2(x, \widetilde{x}) c^A(x, \widetilde{x}) - p(x, \widetilde{x}) c_2^A(x, \widetilde{x}) = 0. \tag{2.38}$$

In equilibrium, a bidder will have nothing to gain by not basing his bid on the true signal. Substituting in $\widetilde{x} = x$ in (2.38) and re-arranging yields

$$c_2^A(x, x) = \frac{\upsilon_2(x, x)}{p(x, x)} - \frac{p_2(x, x)}{p(x, x)} c^A(x, x). \tag{2.39}$$

A similar expression applies to auction format B, just substitute superscripts A with B in equation (2.39). Hence, we obtain the difference

$$c_2^A(x, x) - c_2^B(x, x) = \left[c^A(x, x) - c^B(x, x)\right]\left[-\frac{p_2(x, x)}{p(x, x)}\right]. \tag{2.40}$$

Define the difference of the two payment functions to be $\triangle(x) = c^A(x, x) - c^B(x, x)$, which is the expression in the first brackets on the right-hand side. Given our assumption, we want to show that $\triangle(x) \geq 0$. The proof proceeds by showing that the converse, $\triangle(x) < 0$, cannot be true. Taking the derivative of $\triangle(x)$ yields

$$\triangle'(x) = c_1^A(x,x) - c_1^B(x,x) + c_2^A(x,x) - c_2^B(x,x). \tag{2.41}$$

Substituting from (2.40) in (2.41) yields

$$\triangle'(x) = \left[c_1^A(x,x) - c_1^B(x,x)\right] + \triangle(x)\left[-\frac{p_2(x,x)}{p(x,x)}\right]. \tag{2.42}$$

The first term in equation (2.42) is non-negative according to the fundamental hypothesis. Considering equation (2.42), we can conclude that if $\triangle(x) < 0$, then $\triangle'(x) > 0$. However, according to the mean-value theorem, if $\triangle(x) < 0$, for some x, then there will be a value of x which satisfies $\triangle'(x) < 0$. This contradicts the statement above. Hence, $\triangle(x) \geq 0$, and this concludes the proof.

The Linkage Principle is stated in a rather technical form, and its usefulness may at this point seem somewhat obscure. The implications are, however, powerful. As an example of its application, we shall use it to rank the expected revenue from the first-price and second-price auctions in the general symmetric model.

At a first-price auction the expected payment is

$$c^1(x,\widetilde{x}) = \beta^1(\widetilde{x}).$$

Hence, $c_1^1(x,\widetilde{x}) = 0$. At a second-price auction, we have

$$c^2(x,\widetilde{x}) = \mathcal{E}\left[\beta^2(Y)\,|X = x, Y < \widetilde{x}\right],$$

so according to the result on affiliated random variables stated in expression (2.23), $c^2(x,\widetilde{x})$ is an increasing function in both arguments; i.e., $c_1^2(x,\widetilde{x}) \geq 0$. From this it follows that $c_1^2(x,\widetilde{x}) \geq c_1^1(x,\widetilde{x})$. Applying the Linkage Principle, we can conclude directly that expected payment in the second-price auction is never less than for the first-price auction, but it may be larger.

The most important observation for the seller in deciding the auction format is that the winning bidder's profits are positively linked to his private information. To the extent the seller can undermine this private information, he will increase the price obtained on average. This is exactly what the Linkage Principle states. An auction mechanism that generates a link between bidders' estimates and other relevant variables, will most successfully close the gap between the highest willingness to pay and the winning bid. By submitting bids at the English auction, bidders are forced to reveal some private information they have, and this induces bidders to change their beliefs about other bidders' valuations. Bidders are more confident in their revised than in their original estimates and bid more aggressively. At the first-price, sealed-bid auction, on the other hand, there is no link between bids and other variables, thus making it possible for the winning bidder to take full advantage of his private information.

We end the discussion of the general symmetric model with a short comment on the effect of risk aversion on expected revenue. We saw that in the independent private-values model, risk aversion tends to raise bids at the first-price auction. The same effect is present in the general symmetric model; i.e., if expected profits are non-negative (as they always are in the independent private-values model), then risk aversion gives rise to higher bids. But the common-value element in this model introduces another opposing effect. In equilibrium, there is a possibility of realizing negative profits, and this effect promotes bidders to bid more cautiously if they are risk averse. Thus, the total effect is ambiguous.

2.5 Multi-unit auctions

Thus far we have surveyed auction theory developed for single-unit auctions. The two main models where valuations are either private and independent or affiliated have been covered, and bid strategies and expected revenues which emerge from first and second-price mechanisms, have been explained.

An important extension of auction theory is to consider multi-unit auctions. This is an extensive topic, but a few comments concerning the special problems they pose are offered.

At many real-world auctions, several units of the same good are for sale.

For instance, almost identical lots or units of perishable goods in agriculture and fisheries that arrive at the market one day are sold by way of auction the same day. At a single-object auction, the seller must chose, for instance, between common selling procedures like a first- or second-price auction format and between a closed or open auction format. An extra dimension is added to the seller's auction design problem in the case of multi-object auctions; a choice between selling the goods sequentially or simultaneously must be made.

At sequential auctions, a dynamic element is introduced in the strategic considerations. Consider the case where bidders only want one unit. If bidders use the equilibrium single-shot bid strategy for each good, then those with high valuations will win the first goods at relatively high prices leaving the last goods to those with lower valuations. In fact, such a declining price pattern is empirically observed in many auctions, but for other reasons than the myopic strategy rule mentioned above. Rational bidders will, of course, anticipate such a declining price pattern and be reluctant to reveal their true valuations in the first periods of the game. Thus, bid strategies become more complex at multi-unit auctions than at single-shot auctions.

Several empirical analyses on sequential multi-object auctions have observed that prices tend to decline on average. The first objects sold achieve higher prices on average than those offered later; see for instance Ashenfelter [2] and Lusht [65]. The phenomenon has been termed the *afternoon effect* (see for instance Beggs and Graddy [9]) or the *declining price anomaly* (see McAfee and Vincent [69]). Apparently, the law of one price is violated if similar objects show a decreasing price pattern, since the straightforward game-theoretic equilibrium suggests that bidders will take any effects of residual demand into consideration. Ashenfelter suggests that declining prices may be explained by risk aversion. The intuition is that if bidders are risk averse, then the prices obtained in the first round is equal to the expected price in the second round plus a risk premium for the associated price uncertainty in the second round. McAfee and Vincent show that Ashenfelter's intuition is correct for only a special case of risk averse preferences; the case of non-decreasing absolute risk aversion. McAfee and Vincent argue that this assumption is an unsatisfactory characterization of risk attitudes. Donald, Paarsch, and Robert [23], on the other hand, observe a rising price pattern, on average, at a sequential multi-unit auction. Assuming risk neutrality, but allowing multi-unit demand, their model predicts that the expected price path is increasing.

The case of simultaneous auctions have been less analysed than sequential auctions. The main problem of a *closed* simultaneous auction is that bidders might be constrained financially or in capacity, in which case they risk winning too many units if they tender bids on more units than they want. This has the unfortunate effect for the seller that it reduces competition on any lot. By allowing bidders to set a maximum number of units they want, the problem might be overcome. An interesting example of an *open* simultaneous auction format is the selling by the US. government of spectrum rights to private companies. See McMillan [72] and Milgrom [75] for a description of this format.

2.A Appendices

2.A.1 Expected revenue at first-price auctions

Expected revenue at first-price auction given by equation (2.14) are explained in some detail in this appendix. From equations (2.14), (2.10) and (2.1), we have

$$\begin{aligned}
\mathcal{E}\left(R^I\right) &= \int_{v_0}^{\overline{v}} \left\{ v - \frac{1}{\left[F_V\left(v\right)\right]^{\mathcal{N}-1}} \int_{v_0}^{v} \left[F_V\left(u\right)\right]^{\mathcal{N}-1} du \right\} \times \\
&\quad \mathcal{N}\left[F_V\left(v\right)\right]^{\mathcal{N}-1} f_V\left(v\right)\, dv \\
&= \mathcal{N} \int_{v_0}^{\overline{v}} v \left[F_V\left(v\right)\right]^{\mathcal{N}-1} f_V\left(v\right)\, dv \\
&\quad - \mathcal{N} \int_{v_0}^{\overline{v}} f_V\left(v\right) \int_{v_0}^{v} \left[F_V\left(u\right)\right]^{\mathcal{N}-1} du\, dv.
\end{aligned} \tag{2.43}$$

Consider now the outer integral in the second term of (2.43) and integrate this by parts, where $f_V(v)$ is one part and $\int_{v_0}^{v}[F_V(u)]^{\mathcal{N}-1}\,du$ is the other part. The derivative of the latter part with respect to v equals $[F_V(v)]^{\mathcal{N}-1}$, while the anti-derivative of the first part is $F_V(v)$. Recall also that $F_V(\overline{v}) = 1$. We then have

$$
\begin{aligned}
&\int_{v_0}^{\overline{v}} f_V(v) \int_{v_0}^{v} [F_V(u)]^{\mathcal{N}-1}\, du\, dv \\
= \; & F_V(v) \int_{v_0}^{v} [F_V(u)]^{\mathcal{N}-1}\, du \Big|_{v_0}^{\overline{v}} - \int_{v_0}^{\overline{v}} F_V(v) [F_V(v)]^{\mathcal{N}-1}\, dv \qquad (2.44) \\
= \; & \int_{v_0}^{\overline{v}} [F_V(u)]^{\mathcal{N}-1}\, du - \int_{v_0}^{\overline{v}} [F_V(v)]^{\mathcal{N}}\, dv.
\end{aligned}
$$

Changing the variable u with v in (2.44) and substituting into (2.43) yields

$$
\begin{aligned}
\mathcal{E}\left(R^I\right) &= \mathcal{N} \int_{v_0}^{\overline{v}} v [F_V(v)]^{\mathcal{N}-1} f_V(v)\, dv \\
&\quad -\mathcal{N} \int_{v_0}^{\overline{v}} [F_V(v)]^{\mathcal{N}-1}\, dv + \mathcal{N} \int_{v_0}^{\overline{v}} [F_V(v)]^{\mathcal{N}}\, dv \\
&= \mathcal{N} \int_{v_0}^{\overline{v}} [v f_V(v) - 1 + F_V(v)] [F_V(v)]^{\mathcal{N}-1}\, dv.
\end{aligned}
$$

2.A.2 Deriving the bid function in the affiliated model

The solution of a differential equation like (2.31) with boundary condition (2.32) is (see for instance Berck and Sydsæter [11]):

$$
\begin{aligned}
\beta(x) = \; & \nu(\underline{x}, \underline{x}) \exp\left[- \int_{\underline{x}}^{x} \frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)}\, ds \right] \\
& + \int_{\underline{x}}^{x} \nu(\alpha, \alpha) \frac{f_{Y|X}(\alpha|\alpha)}{F_{Y|X}(\alpha|\alpha)} \exp\left[- \int_{\alpha}^{x} \frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)}\, ds \right] d\alpha.
\end{aligned}
$$

The first term vanishes. Notice that from equation (2.22) it is a fact that

$$\frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)} \geq \frac{f_{Y|X}(s|\underline{x})}{F_{Y|X}(s|\underline{x})}.$$

Using this we can evaluate the exponent of the exponential function in the first term, so

$$\begin{aligned} -\int_{\underline{x}}^{x} \frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)}\, ds &\leq -\int_{\underline{x}}^{x} \frac{f_{Y|X}(s|\underline{x})}{F_{Y|X}(s|\underline{x})}\, ds \\ &= -\int_{\underline{x}}^{x} \frac{1}{F_{Y|X}(s|\underline{x})}\, dF_{Y|X}(s|\underline{x}) \\ &= -\log\left[F_{Y|X}(s|\underline{x})\Big|_{\underline{x}}^{x} \right] \qquad (2.45) \\ &= \log\left[\frac{F_{Y|X}(\underline{x}|\underline{x})}{F_{Y|X}(x|\underline{x})} \right] \\ &= -\infty, \end{aligned}$$

since $F_{Y|X}(\underline{x}|\underline{x}) = 0$. The solution of the differential equation then reduces to

$$\beta(x) = \int_{\underline{x}}^{x} \nu(\alpha, \alpha) \frac{f_{Y|X}(\alpha|\alpha)}{F_{Y|X}(\alpha|\alpha)} \exp\left[-\int_{\alpha}^{x} \frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)}\, ds \right] d\alpha.$$

Define

$$L(\alpha|x) = \exp\left[-\int_{\alpha}^{x} \frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)}\, ds \right]. \qquad (2.46)$$

Now,

$$dL(\alpha|x) = \exp\left[-\int_{\alpha}^{x} \frac{f_{Y|X}(s|s)}{F_{Y|X}(s|s)}\, ds \right] \left[\frac{f_{Y|X}(\alpha|\alpha)}{F_{Y|X}(\alpha|\alpha)} \right] d\alpha.$$

Indicating that the solution of the differential equation can be expressed as

$$\beta(x) = \int_{\underline{x}}^{x} \nu(\alpha, \alpha)\ dL(\alpha|x),$$

which can be integrated by parts to yield the following:

$$\begin{aligned}
\beta(x) &= \int_{\underline{x}}^{x} \nu(\alpha, \alpha)\ dL(\alpha|x) \\
&= [\nu(\alpha, \alpha) L(\alpha|x)]_{\underline{x}}^{x} - \int_{\underline{x}}^{x} L(\alpha|x)\ d\nu(\alpha, \alpha) \\
&= \nu(x, x) L(x|x) - \nu(\underline{x}, \underline{x}) L(\underline{x}|x) - \int_{\underline{x}}^{x} L(\alpha|x)\ d\nu(\alpha, \alpha).
\end{aligned}$$

From expression (2.46) it is obvious that $L(x|x) = 1$, and from the derivation in (2.45) it follows that $L(\underline{x}|x) = 0$. Thus, we finally obtain

$$\beta(x) = \nu(x, x) - \int_{\underline{x}}^{x} L(\alpha|x)\ d\nu(\alpha, \alpha).$$

Chapter 3

The analysis of auction data: Methodological comments

3.1 Introduction

We have seen that auctions can be modelled as games, and that they have been studied extensively for some time. What about empirical studies of auction markets? Econometric analyses in this field, as in any field, can measure effects or test hypotheses, as well as estimate structural elements in order to compare auction formats. Empirical work can also be used to detect collusion and to evaluate the impact of mergers; see Bajari [4]. Empirical analyses of auction data have attracted interest because some regard them as the best test of game theory available; see Sutton [105]. To state the purpose of empirical (econometric) analysis to be theory appraisal, however, requires clarifying what we mean by testing.

We begin by referring to another field of economic theory that uses game theory extensively as an analytical tool—oligopoly theory—because this field illustrates the reason for a growing concern among economists that game theory is unable to produce testable hypotheses. This discussion leads to a presentation of current tensions in economic methodologically about what makes a particular theory good science, and to what extent testability is a part of it. Next, we introduce a specific bit of auction theory in more detail in order to discuss its empirical content and whether it is reasonable to consider it to be a test case of game theory. In order to appreciate the difficulties and limitations of empirical analyses of auction data, we conclude the chapter by a presentation of the main empirical strategies used.

3.2 Game theory and empirical analysis

The field of industrial organization and, in particular, oligopoly theory has experienced a transformation of late with the introduction of game-theoretic models. Oligopoly theory is concerned with markets having only a few producers—how these producers decide what prices to charge and what quantities to produce. Prior to game theory's being introduced, the standard answer to what we might expect to happen in an oligopoly market was that "anything can happen." There simply were no analytical tools available that could be used to analyse markets in which strategically-dependent actors existed. Enter game theory.

Fisher [29] discussed what we now know about such markets after having experienced an explosion in theoretical work. The somewhat depressing answer is that we still can summarize the field with "anything can happen." The characteristics of the game (for example, whether oligopolists compete on price or quantity; or, to take another example from so-called entry deterrence models, whether entry is modelled as sequential or simultaneous) that the researcher is free to specify, are important for the solution. In general, applying game theory to oligopoly produces a large number of plausible solutions or equilibria. Although many stories have been told about oligopoly markets, no broad, unifying theory seems available. Such unifying (or generalizing) theories, proceeding from wide assumptions to empirical predictions, are the strengths of economics. Fisher has argued, however, that opposed to generalizing theory which tells us what *must* happen, there is what might be termed exemplifying theory which tells us what *can* happen. Typically, models in this tradition are stripped bare, concentrating on one special feature and making very simplifying and unrealistic assumptions on other important phenomena. The use of exemplifying theory is common, particularly in the field of industrial organization and oligopoly theory. The stories that are told may be interesting and illuminating, but they lack generality. In his methodological appraisal of several branches of economics, Blaug [13] was so unhappy with the current state of economics that he wrote of the "crisis in economics." He stressed that the entire field of industrial organization has very few or no definite empirical predictions about market behavior.

A similar attack came from Sutton [104]: He noticed that the game-theoretic approach in its extensive form has made possible many modelling approaches and specifications of relevant details. It may be argued that the game-theoretic modelling practice has been a success because each model apparently explains a phenomenon. The trouble is, however, that the results depend very delicately on small details, thus making robust predictions of behavior difficult. Moreover, Sutton was concerned that these games all too often have too many Nash equilibria.

Game theory is concerned with modelling rational behavior in markets with strategically-dependent participants. While standard economic theory has little to say about such markets, game theory has a lot to say. However, game-theoretic models are about concepts like *strategies* and players' *beliefs,* which are typically unobservable. Results are very sensitive to the details: slight changes in the assumptions will often produce a markedly different equilibrium solution. Finally, many possible solutions or equilibria can coexist, making it hard to predict the actual outcome. Therefore, critics have argued that it is a theory that does not generate testable hypotheses.

How do we respond to the criticisms of Fisher and Sutton? Theorists and empiricists alike with little patience for abstract methodological discussions are likely to respond: What else is new? The fact that the world is complex and that game theory confirms this obvious fact cannot be a criticism of the very same theory. In fact, the problem may be turned upside down. Is the call for simple and robust models, a call for over-simplification? Bianchi and Moulin [12] have put it this way:

> [Game theory] is perhaps better thought of as showing the poverty (degeneration?) of traditional analysis, in that game

> theory is what one gets if one admits strategic interaction into the analysis of optimizing behavior, yet game theory is unable—so far, anyway—to generate the testable propositions that most positivist defenders of the traditional analysis have implied was there.

3.3 Testing in what sense?

If it is a relevant objection to game theory that it is untestable, then we have to clarify what we mean by testing. Rather than summarizing what has been going on in the philosophy of science for the last sixty years, we limit the scope in this section to presenting an ongoing conversation in economics about these matters. In particular, we shall comment on the positions associated with Blaug [13] as well as McCloskey [71, 70].

When discussing the research practice of economists, it is necessary to distinguish between what they say they do and what they actually do. Although not many economists are interested in the finer details of methodological positions in their field, most have been exposed to a textbook chapter devoted to methodology. The view on methodology thus expressed might be summarized in what McCloskey terms the *Ten Commandments of Modernism*, of which the first two read: "1. Prediction and control is the point of science. 2. Only the observable implications (or predictions) of a theory matter to its truth" ([71, p. 143]). The method to carry out these dicta is somewhat vaguely understood by working economists, but the more methodologically inclined members of the tribe would say that the program of falsificationism associated with Karl Popper should form the basis. However, economists seldom, if ever, adhere to this program. In practice, they do what the peers of the profession (i.e., the referees of the journals) find passable. And what is passable is sometimes far from the official preaching. Now, both Blaug and McCloskey agree that what is practiced is very different from the methodological program mentioned above, but their reactions to this fact are fundamentally different. Blaug insists that economics as a science must take testing seriously by always confronting theories with data. Moreover, when actually doing this, we must do it with some risk at stake. That is, we must formulate hypotheses such that we can answer the ultimate Popperian question:

> What events if they materialised would lead us to reject that program? A program that cannot meet that question has fallen short of the highest standards that scientific knowledge can attain ([13, p. 248]).

A lot of sloppy practice proclaiming to be empirical testing goes on according to Blaug:

> But, surely, economists engage massively in empirical research? Clearly they do but, unfortunately, much of it is like playing tennis with the net down: instead of attempting to refute testable predictions, modern economists all too frequently are satisfied to demonstrate that the real world conforms to their predictions, thus replacing falsificationism, which is difficult, with verification, which is easy ([13, p. 241]).

The darker side of verification and econometric practice, is portrayed by Leamer [63] in a paper worth reading not only for its discussion of econometric practice and suggested remedies, but for the witty formulations as well.[1]

So why is falsificationism difficult? The so-called Duhem–Quine thesis offers an explanation. It states that no single hypothesis can be falsified conclusively, because we always test it together with auxiliary statements, see Cross [21]. Consequently, we do not know whether it is the hypothesis itself or some of the assumptions that cause a rejection. Now, this fact of (or problem with) testing should not paralyze us in our empirical efforts of theory appraisal. Since we can always explain an empirical refutation of theory by referring to the assumptions, and since we often can repair the same theory so that it conforms with data, we should carefully state what

[1] How about: "If you torture the data long enough, Nature will confess" (attributed to Ronald Coase) or "There are two things you are better off not watching in the making: sausages and econometric estimates."

ad hoc auxiliary assumptions (or "immunizing stratagems" which the late Popper called them) are not allowed to be introduced in this repair job.

McCloskey, on the other hand, does not regret that the official methodology of economics is not followed by economists. There is a lot more to the disciplined inquiry into a subject which constitutes a science than any rule bound methodology can capture. Moreover, believing that philosophers of science are authorities on *the* proper scientific method in all fields is somewhat naïve. To judge an academic work in economics solely by the standards of falsificationism is, therefore, highly inadequate. A fundamental part of any scientific effort is to try to persuade fellow economists working in the same field that our contribution is worth while. In so doing, we make use of many techniques including literary such as an appeal to authority or reasoning by analogy. In order to understand how economics is done, we should draw on many different sources, such as rhetoric (in the classical sense of the word, and not the common, debased meaning) and literary criticism. Scientific work is an ongoing conversation. To be sure, there are some elements, if present, which constitute a good conversation, and McCloskey describes some sound practices; be honest, don't shout, and pay attention. To encourage high standards of the cultured conversation, which is what science really is, is legitimate, but McCloskey resists proposing one, and one only Methodology (with a capital M) that dictates the proper method for all scientific investigations. In particular, if you offer the program suggested by Blaug, there is only one option left for economists on how to proceed, and that is to keep silent. Nothing can be said that will stand the test of the demanding—if not impossible—falsificationist program.

Paul Feyerabend, with his interest in the history of science, takes the argument even further. We should acknowledge that scientific practice that is in many senses irrational, nevertheless makes important contributions to the progress of scientific knowledge. So the best rule to adopt might just as well be the rule: "anything goes." Although this rule has been ascribed to Feyerabend as one of his basic rules and, accordingly, been ridiculed, his intention is rather to convince us that "all methodologies, even the most obvious ones, have their limits", [27, p. 231].

> [This rule] does not mean that there are no rational methodological principles but only that if we are to have universal methodological principles they will have to be as empty and indefinite as 'anything goes'; 'anything goes' does not express any conviction of mine, it is a jocular summary of the predicament of the rationalist, (Feyerabend [28, p 188]).

Although opinions are highly divided in this discussion, there are a lot of things on which we can agree. McCloskey encourages the practice of "confronting theories with data." In fact, when McCloskey discusses "The rhetoric of significance tests" [71, ch. 8], several useful pieces of advice on empirical practice are given. To call an empirical analysis for "testing of theory" is, however, probably a bit too pompous for McCloskey.

The practice of economic researchers, as evident in the publications in refereed journals, suggests that falsificationism is too strict a program in economics. Nevertheless, interest in testing empirically meaningful hypotheses prevails.

3.4 Auction theory and its empirical content

Having discussed the difficult task of obtaining empirically meaningful hypotheses from oligopoly theory, we want to examine the case of auctions in more detail. When describing the behavior at real-world auctions, economists have proposed models that explicitly account for the strategic considerations involved. Unlike the traditional static optimization by market participants, on which much of standard economic theory relies, this approach emphasizes the strategic interactions among the agents and, consequently, the behavior at auction markets are modelled as games; see chapter 2.

But what is it about auctions that makes auction theory better suited than oligopoly theory to produce meaningful refutable hypotheses? Sutton [105] has emphasized that the various auction formats, the "rules of the game," tighten the strategy space (the extensive form of the game) to such an extent that it permits sharp, testable predictions. In oligopoly theory, we mentioned that the model builder had to decide, for instance, in what order the various moves of the game were played, and that several alternative specifications are often reasonable. Not so in auction theory: the legal rules of the auction format will clearly specify the order of moves. So when we look at specific auction markets, we can potentially test some of the hypotheses game theory produce *conditional on* being in a given model world. Let us take a closer look at the hope expressed by Sutton.

A specific model. Several auction formats exist and several sets of assumptions regarding the players might be specified. This gives rise to many different games. For now, concentrate on the *first-price, sealed-bid* format within the symmetric independent private-values model, see section 2.3.2 of chapter 2.

We briefly repeat the model results by introducing some compact notation. Each bidder's valuation, V_i, $i = 1, ..., \mathcal{N}$, with support $[\underline{v}, \overline{v}] \subset [0, \infty)$, is distributed according to the cumulative distribution function $F_V(v_i)$ with density $f_V(v_i)$. Bids are modelled as being increasing functions of valuations, and it turns out that the number of competitors, $\mathcal{N}$, is part of the game-theoretic equilibrium strategy. In a symmetric equilibrium, we can drop the subscript i, and in chapter 2, we reported the symmetric bid function of this model, β, to have a closed form solution equal to:

$$\beta(v_i; \mathcal{N}, F_V) = v_i - \frac{1}{[F_V(v_i)]^{\mathcal{N}-1}} \int_{\underline{v}}^{v_i} [F_V(u)]^{\mathcal{N}-1} \, du. \tag{3.1}$$

The winning bid can be shown to be equal to the expectation of the second-highest valuation, hence the winning bid takes the form:

$$\max(b_i) = \mathcal{E}\left[v_{(2)}\right] = \mathcal{N} \int_{\underline{v}}^{\overline{v}} [v f_V(v) - 1 + F_V(v)] [F_V(v)]^{\mathcal{N}-1} \, dv. \tag{3.2}$$

Equations (3.1) and (3.2) tell us something about bidding behavior under a set of assumptions. The equilibrium-bid strategy is to shave ones valuation by a factor depending on the number of bidders as well as the distribution of values. What empirical predictions can we draw from this?

The empirical predictions. Following Laffont [60], who reviewed the distribution-free predictions of the main auction models, we can at least say that, conditional on being in the independent private-values model, the following predictions obtain.

First, bid functions are increasing and concave functions of the number of bidders. An obvious requirement for testing this is that we have observations on all bids. This is the case for the closed first-price auctions, but not for the open first-price format (the Dutch auction) where only the winning bid is observed. It is, however, not obvious how we can test the prediction. Binding reserve prices and dynamic effects over time complicate the test procedure.

Second, it follows immediately that winning bids are also increasing and concave functions of the number of bidders.

Third, bid functions increase when valuations increase in the strong first-order stochastic sense. This seems obvious, but is not that easy to test. We do not have observations on valuations. Developments in structural estimation techniques, pioneered by Paarsch [84, 85], have made it possible to estimate the structural elements from the winning bids. An important question is whether the unobserved values can be determined from the observed bids. It turns out that the distribution of values is identifiable under the independent private-values model and, consequently, that it is possible to uncover the unobservable strategies bidders use. There is considerable novelty in this approach, as noted by Sutton ([105]). However, the approach takes the theory as given, and is not a test of the theory. If we assume that the bid level is an increasing function of the valuation, and uncover valuations under this restriction, then we have not proved that bids increase with valuations. Laffont [60] noted that the prediction is not entirely distribution-free either.

Finally, the most important prediction is that first- and second-price auctions are revenue equivalent. While the revenue equivalence theorem is beautiful and surprising, the proposition is difficult to test. Normally, only one auction format is used at a particular market. In those rare instances where actually both mechanisms have been used in the same market, we must assume that the auctioneer considers the mechanisms to be identical with respect to revenue generated. To assume otherwise, would be to violate the axiom of rational economic agents, on which much of economic theory relies. Admittedly, this line of reasoning may be pushing the rationality assumption too far. From a Bayesian learning perspective, an auction house may want to test a different format to see whether revenues increase. If the auction house returns to the old format, then we at least have an indication that the auction house does not consider the formats to be equivalent with respect to revenues.

The prediction that bids are increasing in valuations as well as in the number of bidders (at least in the lower range of $\mathcal{N}$), have strong *a priori* support. It is a prediction that follows from basic economic axioms, and any tests that reveal the opposite, will question the application of the model to the market or the quality of the data rather than convince economists that bidding is irrational.

Above, only one simple auction model was briefly presented. A survey of different models will, however, reveal that slight changes in the assumptions will alter important results. Return again to the Revenue Equivalence Theorem—which is based on the independent private-values model. If we assume bidders to be risk averse instead of risk neutral, the Revenue Equivalence Theorem breaks down. Expected revenue will in that case be highest at the first-price auction. How to decide on which model world—what set of assumptions that apply—is, therefore, critical in any empirical analysis. Risk neutrality versus risk aversion is one critical assumption; another is whether bidders know the exact number of competitors or perceive this as a random variable. Perhaps the most fundamental assumption is whether bidders' valuations are independent or correlated. More general auction models are notoriously complex and have not yet been structurally identified for empirical analyses.

To return to the view of Sutton, are we able to test auction theory? It is true that the auction format puts structure on the modelling process; that "certain institutional features [...] allow strong constraints to be placed on the strategy space" [105, p. 317]. But important unobservable variables, such as risk aversion or the correlation of valuations, have to be assumed absent or present. Thus, auction modelling has the same inherent problems as many other fields of industrial organization economics. The results depend on the details of the assumptions. It is not the extensive form of the game that creates the problems, but rather what characterizes agents' information sets and preferences. The number of reasonable models for any given market may quickly become intractable when combining different specifications of risk aversion and correlations of valuations. At this point, we have not even introduced the problem that endogenous bid participation creates. Thus, in a sense, we are back to the complaint of Fisher: "Anything can happen."

Hansen [42], in an early paper on empirical testing of auction theory, recognized that predictions are not robust across all auction models. His careful formulation is worth noting [42, p. 156]:

> Although much of the empirical work is presented as classical hypothesis testing, it is then probably better to think about that work as informal Bayesian learning that is only guided by the structure common to all auction models.

Experimental economics. To be sure, there is one method of testing theory while at the same time controlling for the model assumptions: experimental methods. For surveys of the field, see Smith [101, 102]. The researcher in an experimental setting can assign valuations to participants that are, for example, private and independent, or private and correlated, or valuations are determined by an uncertain common value with a given probability distribution.

A representative work in this tradition is Cox et al. [19]. Subjects in their experiments were given independent, private values drawn from a uniform distribution. Monetary valuations were induced by revealing resale values of the auctioned object. Bidders in the experiment knew the number of bidders and their own monetary valuation, and the distribution of competitors' monetary valuations. The observed bid behavior was interpreted within different competing models. They found that a model of constant relative risk aversion best explained behavior. In fact, they state that the "model, tested directly by introducing lump-sum payments or charges for winning, is not falsified by the new experiments." Thus, the motivation and design of the experiments were clearly aimed at theory testing.[2]

We believe that experimental economics provides useful information and, to some extent, can be taken as tests for different competing models. But models are formulated in order to understand what goes on in the real world, and, sometimes, because we want to make practical use of them. Auction models are relevant for policy questions and it is, therefore, somewhat unsatisfactory to only be able to test them unconditionally in experimental settings. Following Harrison [44], the most obvious problem associated with many experiments is that the subjects differ from what we observe in complex real-world markets. Apart from having less experience, the compensation may prove too small for inducing optimal behavior. Most important, probably, is the fact that subjects in experiments are drawn from a nonrepresentative population, typically from a population of university students. The population of real-world markets are a selected group in the sense that they have survived in a competitive market.

3.5 Empirical strategies for analysing auction data

Empirical work concerning auction data may be classified by what the goals are or by the methods employed. Of course, the goal will determine the appropriate methods. In addition, methods, auction formats, and general market characteristics will give rise to empirical challenges. In this section, we present all the three topics (goals, methods and challenges) that current research is occupied with.

[2] As an aside, the theory was soundly rejected because none of the bids lay exactly on a bid function that theory would predict. (Thanks to Harry J. Paarsch for pointing this out.)

Methods. Let us begin by introducing the two main classes of statistical analyses that are employed using auction data. So-called reduced-form econometrics is the traditional method. Typically, bids are considered to be the dependent variable and regressed on various covariates in order to answer questions like: To what extent do bids increase with competition? Are bidders symmetric in their cost structure? Are some bidders better informed than others? In a series of papers, Hendricks and Porter examined the latter question; see for example [48, 49]. A criticism against testing reduced-form implications of bidding behavior, is that these tests often have low statistical power; see Bajari [4].

A recent approach is the structural econometric analysis of auction data. At least three statistical methods are associated with this approach: Parametric, semi-parametric, and non-parametric methods. The core of the structural approach is that there is a seamless mapping between the economic model and the statistical method used to interpret the data. The structural elements of the economic model (the unknown parameters of the model) are estimated directly. In particular, structural auction models allow us to estimate the conditional distribution of bidders' valuations, given observable covariates, nonparametrically, and to recover the individual valuations underlying submitted bids. If it is difficult or impossible to estimate the distribution of valuations (for example, because of too few observations or heterogeneity in sold objects), one may want to assume a parametric form for the distribution of valuations; see Reiss and Wolak [94]. Structural estimation is often concerned with model selection or normative issues like: Can the auction format be improved upon? What is the optimal reserve price?

In the pioneering work of Paarsch [84], the goal was to determine whether a specific auction market could best be modelled as a private-values or common-value environment. The distinction between the two environments (or paradigms as Paarsch called them) is important in order to choose the best model for addressing important and practical questions regarding auction design. In Laffont, Ossard and Vuong [62], a main finding was that their structural model failed to describe the behavior of a large bidder. Their conclusion was that their market was not composed of symmetric bidders.

Deciding on the optimal reserve price was the topic of Paarsch [85].

Challenges. In general, following Reiss and Wolak [94], a researcher in auction studies knows: (1) the auction format, (2) the winning bid, and possibly all bids, (3) item-specific information, (4) auction-specific information, and (5) bidder-specific information. Under ideal conditions, the economist has complete information; for example, there are no relevant item-specific or bidder-specific information that bidders, but not the econometrician, observe. In practice, researchers have precise information on the auction format and the submitted bids. Information on item-, auction-, and bidder-specific information, however, is most likely to be imprecise.

In addition, there are all kinds of challenges to the empiricist. Let us mention a few. First, there are technical challenges with respect to modelling. Even if we have a clear understanding of the auction format and the behavioral assumptions about bidders, the resulting model may prove to be analytically intractable.

Second, a special problem in most modelling situations is whether bid decisions are exogenous or endogenous. Normally, one assumes a known number of potential bidders since this simplifies the modelling. If a low percentage of bid opportunities actually lead to submitted bids, this questions the assumption of exogenously determined number of bidders.

Third, do we have a large enough data sample? Even a relatively large dataset consisting of 200 auctions may be too small for sensible estimation. Fourth, and related to the dataset size, is the question of possible market dynamics. The time dimension is an extensive problem in market economics. How do we define the market not only with respect to geographical space (which is normally well defined at auction markets), but also with respect to time? Structural changes in bidder behavior can be difficult to discover. Moreover, if they do occur and are handled by the researcher, they will reduce the degrees of freedom in an econometric analysis.

Finally, we mention the ideal prerequisite that auction studies should have relevance to other markets. This brings us back to the criticism of Fisher outlined in section 3.2. We think the dry remark by Hendricks and Porter

[49] is the appropriate one:

> To economic theorists ... dependence on institutional detail is less than desireable, but it appears to be the price that must be paid for relevance.

We have only mentioned a few contributions as examples of the different main strategies for analysing auction data. Given the explosion in recent empirical work on auctions, we find it advisable not to cover all important contributions. Instead, we refer the interested reader to seek out other sources. For a short survey of empirical work on auctions; see Hendricks and Paarsch [47]. Paarsch and Hong [86] is the most comprehensive presentation of empirical studies using a structural approach. In addition to giving a good background on the multiple technical aspects any researcher in this field has to be familiar with, the book presents the different contributions using a consistent and clear notation. Reiss and Wolak [94] evaluated several contributions of structural econometric modelling in industrial organization economics, including works on auctions. Athey and Haile [3] focused on nonparametric approaches to auctions.

3.6 Concluding remarks

To what extent economic theories should be subject to empirical testing is controversial. Most empirically-oriented economists will agree that it is a good thing to confront theories with data. In the case of auctions, there is considerable interest in trying to develop testable propositions, but the main problem is that of deciding among alternative assumptions for a specific auction market.

It might be argued that the formulation of theories is about developing concepts that should have explanatory power in explaining real-world behavior. But stringent, falsificationist testing of the theories may be too ambitious given the complexity of markets and the preferences of "agents." In practice, in work on auction data, theories and testing are used as model selection tools. Finding the model with associated assumptions that best fit the data, is a pragmatic and useful strategy. In turn, this can be used to understand what goes on at auction markets, and eventually—if we are lucky—we may be able to offer policy recommendations.

Chapter 4

The auction format

4.1 Introduction

At Norwegian mackerel auctions, the object for sale is the total catch captured by a vessel on a voyage. A catch may consist of several lots according to fish species or fish size. Catches from several vessels are sold *simultaneously* at a *first-price*, *sealed-bid auction* with a *known reserve price*. Catches can differ substantially in size, but the fish are basically homogenous. The most distinguishing characteristic is fish size, which is measured by average weight. However, some quality variables concerning vessel-specific harvesting, storage and preservation methods, are reported. Potential buyers may bid on specific catches, and they are free to bid on as many they like.

Bidders are linked to the auction house online, and must place their sealed bids within one hour from the commencement of the auction. During the season, up to four auctions are held each day, normally at fixed times: 06:00, 13:00, 18:00 and 22:00. As soon as the catches are allocated at the close of the auctions, the bid information is made public. Bidders learn immediately how many bids were submitted on the individual catches and what their competitors bid.

When bidding on several catches at an auction, bidders can set capacity limits and give priorities to their bids. We describe these features in detail below. The auction rules are regulated by the document *General business* *rules* [33] issued by the auction house *Norges Sildesalgslag* (NSS), which is located in the city of Bergen, Norway.

4.2 Delivery sectors

The auction is characterized as a distance auction where the captain of a vessel, which is still at sea, sends a detailed description of the catch to the auction house. In turn, the auction house provides information to all potential buyers and solicits bids. Given that the catch is sold, the seller is then told where his buyer is situated and can set sail directly to the buyer's location, which ensures that little time is lost when bringing the catch to a plant, thus maximizing the quality of the fresh fish and minimizing transportation costs to the seller.

Since the seller covers all costs involved in bringing the fish to the buyer, the location of the buyer is important. In fact, the seller sets a **preferred geographical sector of delivery** or bid area; i.e., he states a northern and southern port on the coast line. Only bidders situated inside this area can submit bids that are legally binding to the seller. Bidders situated outside the sector are entitled to submit bids, but in case one of them is declared the winner, his bid is just considered an offer to the seller, and it can be refused. Thus, an asymmetry among bidders is created. It is not an asymmetry concerning information, but rather a cost advantage for inside bidders. If an outside bidder is to win, then he must place a bid which is not only the highest, but it must be sufficiently large for the seller to consider it worthwhile to incur the extra costs involved in accepting it.[1] We use the terms *inside* and *outside* bidders to denote bidders that are inside or outside the delivery sector. Notice that it is the delivery sector for an individual catch

[1] Obviously, it may turn out to be quite a complicated problem to allocate the lots after the bidding process is over. A computer algorithm produces the result, but during the validation, some human interaction is necessary. In particular, in case an outside bidder has the highest bid, a call must be made to the vessel owner to find out if he accepts the bid. Therefore, the computer validation is halted and then continued after a response from the involved seller is received. The final allocation may take just a few minutes, but also up to an hour.

which determines whether a bidder is inside or outside. At a given auction, a bidder may then be an inside bidder for some catches and an outside bidder for other catches.

Sometimes, sellers set a wide bid area, while at other times they set a narrow sector. Attracting as many potential bidders as possible is obviously a sound strategy for sellers. To obtain that, they should set wide bid sectors. Why then do sellers sometimes find it beneficial to narrow down the number of potential bidders? The main reason is the position of the vessel. If the vessel is distant from the coast, then the differences in travelling distances to ports in a north-south direction are, generally, relatively small for several ports. On the other hand, if the vessel is close to the coast, the travelling time increases to remote ports.

4.3 Revealed product information

Revealed product information is a detailed and an important part of the distance-auction format. Because buyers cannot visually inspect a catch prior to bidding, the auction rules require that several pieces of information of the catch should be made known to potential bidders. This is in accordance with a well established recommendation from theorists; see Milgrom and Weber [77]. In particular, sellers should reveal all relevant information, both positive and negative, about the object for sale in order to maximize long term revenues. The information should be made public in a consistent way; i.e., it should be revealed in all circumstances. The importance of being earnest—apart from what follows from obvious moral reasons and contract law regulations—is that, at repeated auctions, a seller's reputation matters. The gain from a one-shot deal with somewhat imprecise description of a catch will be more than offset by the risk that buyers in the future scale down their bids to compensate for the risk involved in deals with a particular seller. In addition, the catch will be inspected by the buyer on arrival. In case of complaints that are not addressed by the seller, independent controllers can be called to settle the dispute.

The identity of the seller is reported together with a space and time variable. The space variable is the catch location, and the time variable is expected arrival time at the southern and northern port of the delivery sector. Next, some quantity variables like average fish weight and the total quantity of the lots are reported. Finally, several discrete variables concerning the quality of the catch or lots are described. We refer to the next chapter, section 5.4, for a detailed description of the variables.

4.4 Bids

Bids consist of a tuple: One or several *price quotes*, a *quantity limit*, a *vessel limit*, and a *ranking*. All elements except the price quotes are optional. In this section, we explain the price quotes while the capacity limits and rankings are described in the next two sections.

Normally, a catch consists of fish within a narrow size range. For example, all fish of a given catch are between 450 to 500 grams. In this case, only one bid is submitted, a price in NOK per kilogram (kilo). Occasionally, a catch may be divided into two or more subgroups or lots determined by significant differences in average fish weight. This happens when a catch is gathered by use of several purse-seine hauls. For example, suppose a catch is naturally divided in two lots or size classes with average individual fish weight of 400 and 600 grams, respectively. Note that the entire catch will be sold to one buyer. Pure logistics, time loss, and transportation costs make it impractical to deliver the various different lots to different buyers. Technically, bidders in this case submit **two price quotes** on the catch—a price per kilo of the large fish and a price per kilo of the small fish. The average bid is then easily calculated by weighting the two bids with the relevant quantity shares.

In principle, buyers could make these calculations themselves and only report the average bid for the whole catch, but there is a reason for the bid rule. Because no prior inspection of lots is possible, bidders must rely on the seller's information. When the fish are delivered, it is easy for the buyer to examine whether the description that has been provided is accurate. If he objects to the reported total quantity or the share of large fish in the catch, then a recalculation of the total price based on the true quantity or size grading is necessary. The two submitted bids will then dictate the new price. This happens once in a while, but is not considered a major problem; it seems that the sampling routines and the measuring technology are sufficiently sophisticated to get accurate descriptions of the catch. In general, multiple price determination is useful when the value of a commodity depends on a quality index, and no prior inspection is possible. From an econometric perspective, the required bidding procedure is also preferable since it elicits more information from buyers which is relevant with respect to buyers' valuations.

4.5 Capacity limits

The optional **quantity limit** implies that a bidder can state the maximum quantity of fish he will accept buying at a given auction.[2] Recall that the auction is a simultaneous, multi-object, first-price, sealed-bid auction. At a sealed-bid simultaneous auction, there is a risk of winning too many objects for a bidder bidding on several or all of them, causing temporary capacity problems in subsequent handling, processing, and storage. For most producers, short-term freezing capacity is the constraining factor. The possibility of stating a maximum quantity implies that a bidder, bidding on several catches, can avoid this risk. If he wins more catches than he can handle at the time, then his quantity limit comes into effect, and the auction house only allocates catches to the winner up to his limit.

To give an example, suppose a buyer has the highest bid on three catches of 40, 60, and 100 tons, respectively. If his quantity limit is 100 tons, then he will be allocated either the two smaller catches or the large catch. A catch which is not sold to the winning bidder because he is already capacity constrained is then allocated to the bidder with the second-highest bid, given that his limit is not binding, in which case the third-highest bid is considered,

[2] Actually, two quantity limits are used. Bidders may simply state the total maximum quantity independent of fish species, or they may specify quantity limits with respect to the fish species in case they bid on catches with different species. In an analysis focusing on a given fish species, the relevant quantity limit would be the latter. However, often bidders report only a total quantity limit, not bothering to specify the fish species.

and so on. From the perspective of optimal mechanism design, the option of setting a quantity limit is a sound way to encourage as many bids as possible.

In addition to a maximum ton limit, bidders may also set a minimum ton limit; i.e., if they win less quantity than their stated minimum tons, they will not be allocated any catches. This option is rarely used.

Finally, in addition to the quantity limits, bidders may also set a vessel limit; i.e., they may state the maximum number of delivering vessels they want to receive. This is equivalent to stating a maximum number of catches they want to win. A straightforward explanation for this option is that buyers may have limited quay capacity—maybe only two vessels can be handled during a day. Time delays in delivery affect the fish quality, and sellers are reluctant to have their vessels delayed in a waiting line. Another aspect of the option is that it gives the buyers the possibility to avoid winning too many small catches that will be less cost-effective than winning fewer, but larger catches. When receiving a delivering vessel, some necessary procedures are undertaken like controlling the reported average fish weight. Staff must be allocated to the job, and some dead time is likely to occur. All in all, it takes more time to empty two vessels rather than one even if the quantities are the same. Thus, one delivering vessel is more cost-effective than two.

4.6 Priority of bids

The auction house has to consider a bidder's **priority of bids** when he wins more than one catch and his capacity limit is binding. This element of the bid tuple simply means that a bidder can rank his j bids from 1 to j. The lower the ranking number, the higher is the priority. In the example above with three catches of 40, 60 and 100 tons and a quantity limit of 100 tons, giving the bid on the 100 ton catch priority 1, would ensure that the winner was allocated this catch rather than the two smaller catches.

Note that the quantity and vessel constraints described above are binding. The auction house cannot allocate catches such that these two limits are exceeded. The bid ranking option, on the other hand, can be overruled by the auction house if necessary. Most important for the auction house, as well as from the economic perspective of efficient use of scarce resources, is that the market clears. Unsold catches in this market are a concern for two reasons. First, fresh fish deteriorate quickly as time passes. An unsold catch will be put up for sale at the next few auctions, but if it is still unsold, it will have to be sold for meal production, obtaining a far lower price than in the market for fresh fish. In a worst-case scenario, the fish will be destroyed. Second, the vessel will be idle and lose harvesting time when the catch has not been sold and delivered. In addition, a vessel may, in the meantime, have to go to a specific port for various reasons, thus reducing the number of potential bidders at the upcoming auction.

An unconditional acceptance of priorities can result in unsold catches, while relaxing this constraint will find buyers for them. Suppose, for example, a buyer wins two catches, but wants only one. If his first priority catch has received another bid, while his second priority catch has no other bids, then it is clear that the latter catch goes unsold if priorities rule the allocation. Relaxing the priorities, giving the winner his second priority catch and his first priority catch to the second-highest bidder, can result in finding buyers for both catches. The complexity of the auction format is evident when the number of bidders and catches for sale is increased. Accepting a particular bidder's priorities, can produce inefficiencies several steps later in the validation process.[3] In that case, an allocation procedure that has begun will be terminated and started all over again. Thus, it is common knowledge among bidders that priorities will be considered to the extent possible, but will be overruled if they produce inefficient results.

The ranking option helps the buyer to obtain an optimal allocation—for example, one large catch instead of two small catches. Note, however, that the same result may be obtained more directly by use of the option of setting a maximum number of vessels. Why then is this an option available to buyers? One might argue that if bidders prefer some catches over others, this should be reflected in their submitted bids. One possible explanation concerns the final allocation and the leeway given to the auction house in

[3]By *validation process* we mean the stepwise allocation of catches based on the auction results.

this respect.

Normally, the auctioneer will have different final allocations to chose from because it is a simultaneous rather than a sequential auction and because of the reported capacity constraints. For example, assume three buyers, A, B, and C, and two catches, 1 and 2. Suppose bidder A wins both catches, but he only wants one, and that the second-highest bid comes from different bidders in the following way. Giving A catch number 1 results in that catch number 2 goes to B, while giving A catch 2, will give C catch 1. In the absence of bid rankings, the auctioneer can, in principle, identify all possible allocations which respect to the bids, the capacity constraints, and the market clearing condition. He could then either choose the allocation that minimizes total costs for buyers, or more likely, the allocation that maximizes total revenues for sellers. The bid ranking option removes some degrees of freedom for the auctioneer in the validation process that produces the final allocation of catches. Hence, the option gives more weight to buyers' preferences at the possible expense of reducing seller's total revenue.

4.7 Reserve price

The known **reserve price** is stated as NOK per kilo. The reserve price depends on the average weight of the fish. A given interval of average weight is said to belong to a weight class. At the beginning of the season, a range of reserve prices are set for different groups. Prices within a group last for the entire season. Initially, weight classes are allocated to these groups, but due to changing market conditions, weight classes may be moved up or down the reserve price groups. Thus, the reserve price may vary during the season, but at infrequent time intervals and at with discrete jumps at certain times. In our market, the ocean segment of the mackerel fishery, it seems that the reserve prices of the different weight classes were constant during the season under study.

An important part of the theoretical analysis of auctions is how the reserve price may be set to raise expected revenues for the seller. By setting a reserve price, the seller in fact introduces an additional bidder with a known bid.

Given information of the distributions of valuations, it has been proved in the independent private-values models (Riley and Samuelson [95]) that there is an optimal level of the reserve price that will maximize the expected selling price. Thus, from an economist's perspective, the reserve price should be allowed to vary continuously during the season in order to optimally reflect the changing market conditions. An additional option, however, offsets to some degree the possible sub-optimality of the minimum price regime in use. To wit, recently, a rule giving sellers the right to set the reserve price individually for each catch as long as it is higher than the standard reserve price has been introduced. When the gap between the market price and the reserve price is large, sellers tend to use this option. This will alleviate the problem if the constant reserve price is too low. But the problem of an inflexible reserve price during the season remains when the reserve price is too high in some periods. The result may be inefficient in the form of unsold catches. In chapter 10, we analyse the reserve price from a normative perspective; i.e., we analyse how the reserve price may be set optimally from the seller's perspective.

4.8 Simultaneous selling

One interesting feature of this auction is that catches are sold simultaneously instead of sequentially. How is this method different from the more traditional method of sequential selling? Can the various catches sold during a simultaneous auction be modelled as independent auctions or does the simultaneity lead to bid strategies that differ from the single independent case? Obviously, the option of setting capacity limits makes it possible to bid on a catch *as if* it was a single-object auction. If it turns out that bidding on individual catches can be regarded as independent, one may rely on the well-developed theory of competitive bidding at independent auctions. A less general question is whether the algorithm used for allocating the catches at this particular auction is optimal.

One reason for selling catches simultaneously may be based on considerations of fairness: The auction house wants to treat individual sellers equally.

Recall that the auctioneer represents the sellers. Several studies have reported that prices may show a declining or rising behavior during sequential selling; i.e., prices follow a path which cannot be explained by quality differences, rather it seems that bidding strategies change as the sequential selling process proceeds. Consequently, the expected selling price of a catch will differ depending on where in the sequence the catch is auctioned; see the presentation in section 2.5 and the references given there.

Understandably, the auction house wants to treat individual sellers equally and to avoid the random differences in revenues that the position at a sequential auction may result in. Otherwise, to sustain the coöperation on the seller's side may prove difficult.

The simultaneous auction raises an interesting problem when combined with a sealed-bid format. It is obvious that the bidding strategies may become very complex in this setting, at least in theory. In particular, one can easily imagine that simultaneous auctions have an inherent coördination problem. A very simple example will best describe this concept. Imagine two bidders, A and B, and two objects for sale. Suppose bidder A wants to buy both goods, while B wants only one good. This information is common knowledge. If bidders cannot set a quantity limit, then it is reasonable to argue that B bids on only one good, while A places bids on both goods. However, he would like to bid aggressively on the good which receives another bid, and he will prefer to bid the reserve price, if any, on the other good. But how is he to know which good to bid aggressively on? Hence, the expression *coördination problem*.

The option of setting constraints (maximum tons, minimum tons, maximum number of vessels) is an elegant way of avoiding the coördination problem to some extent. But even with this option, sellers have imperfect information about which catches that attract harsh competition and which that receive few bids.

Another reason for using simultaneous auctioning is a practical one. The alternative sequential selling procedure may be more time consuming, especially the use of an open English auction with increasing bids. However, it is well known that the Dutch auction—frequently used to sell more or less identical lots of flowers or agricultural produce—is quite time effective. A representative of the auction house states that from his observations he thinks the Dutch auction is too rapid given the huge amounts involved. A large catch of mackerel can obtain a total selling price above 1 million USD. A more slow-paced procedure giving buyers time to reflect on their strategies is, therefore, more appropriate according to the auctioneer. The auction house has, however, been interested in testing an increasing-bid procedure, but buyers have opposed a change of auction format.

Simultaneous selling combined with the options of setting capacity limits and priorities push the auction format in the direction of combinatorial auctions. Combinatorial auctions are auctions where bidders submit bids on specific bundles of items or packages. The analysis of package bidding is inherently complex and input from several disciplines are useful for analyses. Crampton, Shoham and Steinberg [20] have noted that the study of combinatorial auctions lies at the intersection of economics, operations research, and computer science.

4.9 Illustration of an auction

We illustrate the auction format using data from a real auction in this market. The data are described in detail in the next chapter. For now, we are interested in looking at a subset of the data in order to see how the bid vectors and "rules of the game" determine the final allocation. We chose one auction, no. 5703, in the dataset which has a small number of catches and a small number of active bidders.

Consider the description of the catches that are sold simultaneously at the auction. In table 4.1, we state some of the information presented to potential bidders. At the auction, four catches are auctioned simultaneously. Three of the four catches contain just one lot. Catch 21472 contains two lots; the two lots belong to different weight classes and, consequently, have different reserve prices. Note that the total quantity of catch 21471 is 120 tons (80 + 40). The catch with the narrowest delivery sector is catch 21471; the lower the numerical port code, the more southerly a location is.

Table 4.1: Product information given to bidders, example from auction no. 5703

Catch	Lot	Weight	Quantity	Southern port	Northern port	Reserve price
21470	1	552	120	16	25	5.25
21471	1	535	85	19	22	5.25
21472	1	570	80	13	25	5.25
21472	2	499	40	13	25	4.75
21473	1	525	230	13	25	5.25

The submitted bid vectors are reported in table 4.2. Bidders called 3, 6, 8, 9, and 12 are active at the auction. Bidders 3 and 9 bid only on the two lot catch, the other bid on all catches. Bidders 3 and 9 do not state priorities, since this makes no sense when bidding on only one catch. The value 99 is typically used for stating no priority or the lowest priority of a bid vector. Likewise, it is unnecessary to state a capacity limit when bidding on just one catch. Nevertheless, we notice that bidder 3 states as his capacity limit the quantity he bids on—i.e., 120 tons. The other bidders state complete priority vectors. Since the catch is the object for sale, the two lots of catch 21472 are given equal priorities. Bidder 12 states a capacity limit saying that he will not take more than 350 tons and not less than 85 tons. Since 85 tons is the lowest possible quantity that can be acquired at the auction, it is (strictly speaking) unnecessary to state this. In addition to a maximum and minimum ton limit, bidder 8 states that he does not want to win more than one catch (Max vessels). Thus, he is uninterested in winning two small catches, even if this is within his maximum ton limit. Bidder 6 sets no capacity limits, meaning that he can take the entire quantity auctioned. Bidder 9 is an outside bidder, since location takes the value 1. Bidder 12 is an outside bidder for catch 21471. The effect of the delivery sectors is that outside bidders may not bother to submit bids. Thus, catches with a narrow sector will, most likely, attract few bids since the number of inside bidders is

Table 4.2: Example of bid vectors, all active bidders in auction no. 5703

						Capacity limits		
Bidder	Catch	Lot	Bid	Priority	Location	Max ton	Min ton	Max vessels
3	21470	1				120		
	21471	1						
	21472	1	6.03	99	0			
	21472	2	5.55	99	0			
	21473	1						
6	21470	1	6.33	2	0			
	21471	1	6.73	1	0			
	21472	1	6.33	4	0			
	21472	2	6.03	4	0			
	21473	1	6.23	3	0			
8	21470	1	6.26	3	0	230	85	1
	21471	1	6.16	4	0			
	21472	1	6.71	1	0			
	21472	2	6.11	1	0			
	21473	1	6.31	2	0			
9	21470	1						
	21471	1						
	21472	1	6.75	99	1			
	21472	2	6.45	99	1			
	21473	1						
12	21470	1	7.17	3	0	350	85	
	21471	1	7.01	4	1			
	21472	1	7.17	2	0			
	21472	2	6.75	2	0			
	21473	1	7.07	1	0			

limited.

The prices and final allocation are reported in table 4.3. Bidder 12 has the highest bid on all catches. Since he has a capacity limit of 350 tons, he cannot take all. He is allocated two catches, number 21472 and 21473, which exactly sum up the 350 tons, in accordance with his priorities. The other two catches are allocated to the bidder with the second-highest bid, bidder 6. The seller's of these catches receive selling prices of NOK 6.33 and 6.73 per kilo rather than the highest submitted bids of 7.17 and 7.01. We see that bids, priorities and capacity limits all play a role in determining the final allocation.

Table 4.3: Auction result, auction no. 5703

			Max bid		Winning bid		
Catch	Lot	N	Bid	Bidder	Bid	Bidder	Position
21470	1	3	7.17	12	6.33	6	2
21471	1	3	7.01	12	6.73	6	2
21472	1	5	7.17	12	7.17	12	1
21472	2	5	6.75	12	6.75	12	1
21473	1	3	7.07	12	7.07	12	1

4.10 Concluding remarks

In this chapter, we have described the auction format of Norwegian pelagic fish auctions and commented in detail on some of the features. The format has several novel elements: In addition to price, bidders can also set capacity limits and priorities. These additional elements of the bid vector are introduced in order to reduce potential coördination problems that the simultaneous selling procedure would otherwise yield. Geographical space, both on the seller's side (the position of the vessel) and on the prospective buyer's side (location of plants), affects the market in the sense that delivery times and the number of potential inside bidders varies. One main goal of the auction market is to achieve efficient outcomes; i.e., that the producers with the highest willingness to pay acquire the raw material, given that this is cost-effective for the seller as well. Hence, the seller's option of delivery sectors. Another main goal is to clear the market: Unsold catches entail waste. Balancing the two main goals lead to a rather complex auction format.

In the remainder of the thesis, we shall study the auction market empirically by analysing a wealth of auction data we have gathered.

Chapter 5

The market and the data

5.1 Introduction

In this chapter, we describe the market under study—the Norwegian wholesale market for mackerel. We also present some aggregate measures of the market. Next, we describe the dataset in some detail by use of summary statistics and distributions of important variables. In addition to presenting summary statistics of the variables, we present some statistics concerning observed behavior by bidders. The dataset for the market to be analysed covers the entire 2003 season. The mackerel season begins in late summer or early fall and goes a few weeks into the next year. All tables and figures relate to data from the 2003–4 season, except figure 5.1. The construction of the dataset is documented in appendix A.

5.2 The fishery

Mackerel (*scomber scombrus*) is a pelagic fish valued for its high contents of Omega 3 fatty acids as well as vitamins B 12 and D. *Pelagic* fish live in the open sea near the surface as opposed to *demersal* fish or groundfish. Economically important pelagic species, such as mackerel and herring, have a propensity to school, perhaps in defense against predators. Schools do not decrease much in size when the total biomass of the stock is reduced, but the geographic range of schools diminishes. Although schooling contributes to survival against natural predators, it makes the fish vulnerable to modern harvesting techniques. Use of fish-finding devices, such as the sonar, means that mackerel are easy to locate; the use of modern gear, such as purse seine nets, makes the harvesting relatively efficient—see Neher [81, p. 177].

In Europe, two main stocks exist, one living to the west of the British Isles, the other living in the North Sea and the Skagerrak. Mackerel landed in Norway are mostly those who winter off the southwestern coast of Norway. During this period, nutritional intake is very low, and the average fat content gets as low as 5 percent. Spawning takes place in April and May. In summer and fall, the fish move in huge schools along the coast of Norway and into the Skagerrak and the North Sea as well as the southern parts of the Norwegian Sea. In fall, average fat content is around 30 percent. The catch season of the fishery begins in mid-August, has a peak in October, and normally ends in January.[1] The seasonality of the fishery may be explained by two factors. First, there is the issue of quality. The quality of the fish is regarded to be at its best in the peak season. Second, we cannot rule out that the economics of harvesting plays a role. When the fish are close to shore and easily accessible, it is natural that harvesting reaches a peak.

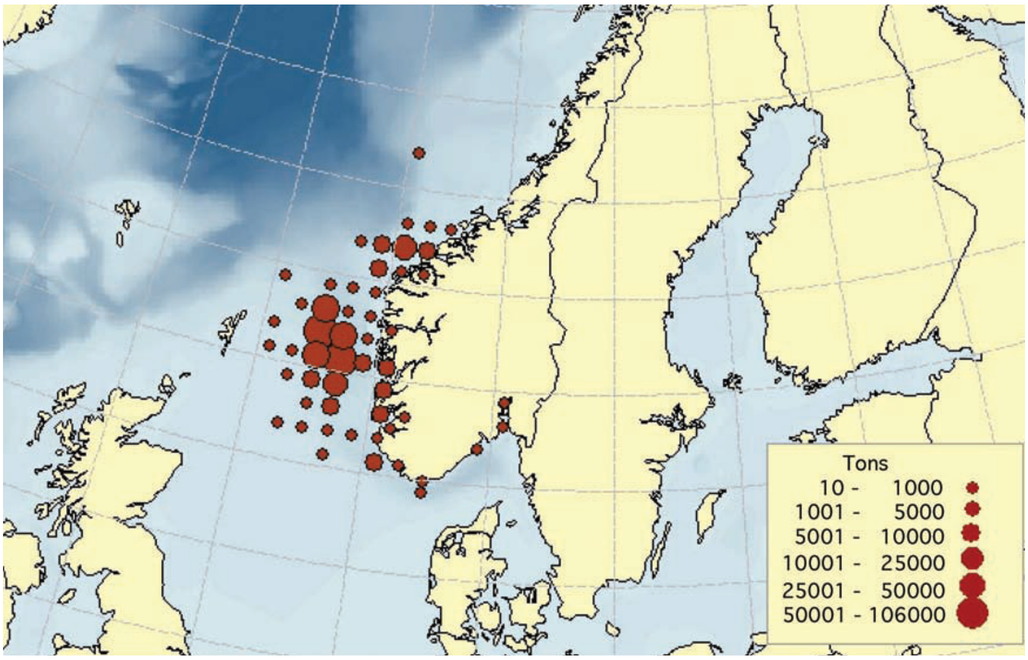
The location of the mackerel harvest in the Norwegian zone is shown in figure 5.1.[2] Most fish are harvested in a sector between the southwestern Norwegian coast and the Shetland Islands. In addition to the mackerel harvested in the Norwegian zone, our dataset contains some catches harvested in the European Union zone and around the Faroe Islands. Of the 1,405 unique catches in our dataset, 1,295 (92.17 percent) were from the Norwegian zone, 109 (7.76 percent) from the European Union zone and 1 (0.07 percent) from the Faroe Islands zone.

In order to maintain a sustainable stock, a total allowable quota (TAQ) has been negotiated among the involved nations each year. In recent years, Norway's annual quota has been in the range of 140 to 160 thousand tons.

[1] The above description relies on information from the auction house (`www.sildelaget.no`).

[2] Source: Directorate of Fisheries (`www.fiskeridir.no`)

Figure 5.1: Location of Norwegian mackerel harvest, 2004



This figure is known to all market participants, and the remaining quota is published on a daily basis during the season.

By summing the individual catch quantities offered for sale at a given time, buyers can easily deduce the total quantity supplied. Future short-term supply at any point in time is rather unpredictable because of the inherent uncertainty concerning the success of fish harvesting, but the annual total allowable catch quota and the remaining quota are common knowledge.

5.3 The market

We begin by defining the market under study precisely by classifying the market according to vessel type. Next, we describe the sellers and the buyers. Finally, we examine harvested quantities and the demand and supply over time.

5.3.1 Two different markets

The vessels used in harvesting vary from boats which use nets and trolling lines along the coast to large, ocean-going seiners. Each vessel group has a quota, which in turn is allocated to the individual vessels within the group. Once a vessel has filled its quota, it withdraws from that particular fishery. Earlier, the vessels competed for the TAQ; i.e., they were free to fish as much as they wanted so long as the total quota had not been reached. This led to a race among vessels and could involve high costs like overtime pay to crews. At the same time, competition ensured that the most effective vessels had an advantage. Now, the total quota is allocated between vessels at the start of the season.

Ocean-going seiners dominate the fishery and account for most of the harvest. The distinction between the two vessel types is important because it gives rise to two different markets: the market for "coast" mackerel and the market for "ocean" mackerel. Ocean mackerel are harvested by large, modern seiners that take the fish aboard and offer the dead fish immediately to a wide market. Catches (or lots) are, in general, large as are the individual fish weights.

Coast mackerel, on the other hand, are typically smaller than their ocean cousins. More important for our study is the fact that, in the coast mackerel market, the fish are kept alive and stored in a given location. The fish may be offered for sale some time period after harvesting. When sold, the seller ships the fish live to the buyer. In general, this shipping is expensive, mostly because quantities are small. The result is that the market for coast mackerel is far more local than for ocean mackerel, thus attracting fewer potential buyers than the mackerel delivered by ocean-going seiners or trawlers.

An interesting aspect of the two markets is the following: Ocean harvesters operate within the short-term characteristics of the market; they have to sell the fish immediately and must, thus, accept the given prices established in the market. Coast harvesters store their fish alive and have the opportunity to use a "wait and see" strategy when determining when to sell their product. In times of a large supply, they can withhold the fish from the market with the prospect of higher prices when supply is low.

In this study, we analyse the ocean market. This is a competitive auction market, by far the dominant market with respect to quantities harvested and revenues generated. The coast market is very small in comparison. Since the quantities, harvesting methods, and sellers and buyers vary considerably from the ocean fishery, it seems reasonable not to include this fishery in the analysis of the large-scale ocean fishery.

5.3.2 Sellers

The sellers are the individual vessel owners, who may come from all the North Sea nations. Norwegian vessel owners are, however, predominant. Out of the 1,405 unique catches offered in the 2003–4 season, 1,214 (86.41 percent) were from Norwegian vessels, 58 (4.13 percent) from Great Britain, 43 (3.06 percent) from the Faroe Islands, 38 (2.70 percent) from Denmark, 37 (2.63 percent) from Ireland, and 15 (1.07 percent) from Sweden. The number of sellers is large compared to the number of buyers. In our dataset, in the 2003–4 season, 303 unique vessel owners sold their catches. Sellers vary with respect to how frequently they enter the market. The "busiest" vessel owners sold 14 catches during the season, while others only sold one catch. On average, owners sold 4.65 catches during the season.

A single catch represents a considerable potential revenue for any seller. Given the large number of sellers, we can conclude that there is no monopoly power on the seller's side that would warrant government regulation. On the contrary, potential market power exists on the buyer's side. Historically, a problem for small-scale fishermen was that they had weak bargaining power when selling their catch to a buyer with local monopsony. The ocean seiners, however, have normally the opportunity to offer their catch to several buyers situated at different locations since they are still at sea when selling their catch. This notwithstanding, the wholesale market for fish is regulated by law in Norway. A coöperative, *Norges Sildesalgslag* (NSS), is granted a monopoly of selling mackerel and other pelagic fish species in the wholesale market. NSS is organized by the vessel owners, and they sell the catches exclusively at auction.

5.3.3 Buyers and export markets

The buyers are food processors or fish exporters located along the southwestern coastline of Norway. During the 2003–4 season, 25 buyers were active.[3] They may be involved in different lines of food production; mackerel, in the retail market, is sold fresh, frozen, salted, smoked or canned. But the majority of buyers in this fishery are involved in the following business: They sort the fish with respect to weight classes, then freeze the fish as it is, and ship it to resale markets abroad where the fish are processed for end market use. The main export markets for the Norwegian harvest of pelagic fish are the European Union and Eastern Europe as well as Asia. Norwegian mackerel are regarded as the highest quality available. The product is particularly popular for use in sashimi in Japan.

Buyers are involved in other fisheries: In addition to mackerel, they purchase other fish species as well. Their valuations may be influenced with respect to the alternatives of other raw material they have at hand.

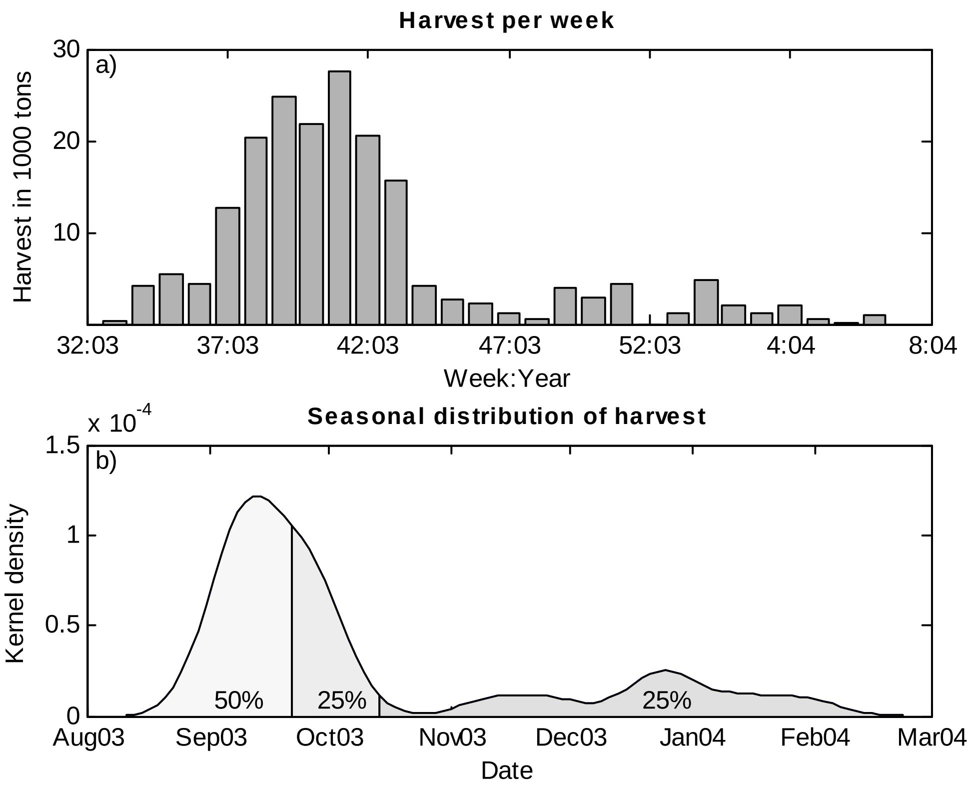
5.3.4 The harvest of mackerel

The season spans close to six months—from mid-August to January and February. The 2003 season stretched over a longer period than normal, starting 11 August 2003 and ending 17 February 2004. Harvesting effort is far from evenly distributed during this period. Harvesting is most intense during the first two and a half month, where roughly three-quarters of the total quota is filled. The peak in harvesting occurs a few weeks into the season. In figure 5.2a, the harvested quantity in tons per week is shown. In figure 5.2b, we have smoothed the data from (a) and transformed them into an empirical distribution by calculating a kernel density over time. In addition, we have

[3] In fact, we observed a single bid from three additional bidders in 2003. We included their bids in the analyses, but found it misleading to say that active bidders are 28 rather than 25. The three extra bids were likely submitted by mistake since they are located far from the relevant delivery zone.

indicated the first half of the mass under the density function, and the next two quartiles of the mass. The peak is in week 41. 50 percent of the quota is harvested by mid September or only six weeks into the season. Within the next four weeks, an additional 25 percent of the total harvest is captured. The last 25 percent of the quota is captured at lower harvest effort over a prolonged period. Most of the mackerel from the Norwegian zone are harvested by November. The last part of the season is dominated by fish from the European Union zone, and these fish are often harvested west of Ireland.

Figure 5.2: Mackerel harvest in 2003–04 season



Given that vessel owners have a quota, why do they not distribute their harvesting effort more evenly over the period? Probably the most important reason is the quality of the fish. The average fat content of the fish is high in the peak season. High fat contents require high prices, and will obviously affect harvesting efforts. Another reason for the intense harvesting efforts in September and October is to minimize harvesting costs. Because the fish move in large schools, harvesting is most cost-effective when the fish are close to land and the schools are large. When the fish move further away, schools get smaller and catch-costs increase. Thus, Nature and the rapidly shrinking quantities of available fish, explain the seasonal harvesting pattern. In addition, we may add that different fisheries have similar peaks in fish availability. Fishermen have to gear their efforts to specific species in given periods. After the peak of the mackerel season, the season of another important pelagic species, herring, begins. Thus, there is an alternative cost aspect of harvesting decisions as well. Effort is allocated to the most profitable fishery at the time.

5.3.5 Supply and demand during the season

Harvest in figure 5.2 equals market supply per week with one minor modification. In a few cases, an offered catch may go unsold at an auction and to be sold at the next auction, perhaps. When counting the harvests per week, we have excluded the subsequent offers of the same catch in order to avoid double counting of catches or lots. On the other hand, when calculating supply, we find it reasonable to include all offered catches since they are part of supply at a given time. Below, where we present figures of market demand and supply, catches offered several times are included in the supply figures. The difference between the harvest and supply figures are, however, small, since about 95 percent of catches are sold the first time they are offered.

Demand is not as easily measured as supply. To begin, there will be a latent unobservable demand for mackerel when the supply is zero. At times of positive supply, we have a point observation of demand. To be precise, what we can identify is the revealed aggregate demand at the submitted bids when mackerel is offered. Demand from bidders with valuations below the reserve price is not observed at all.

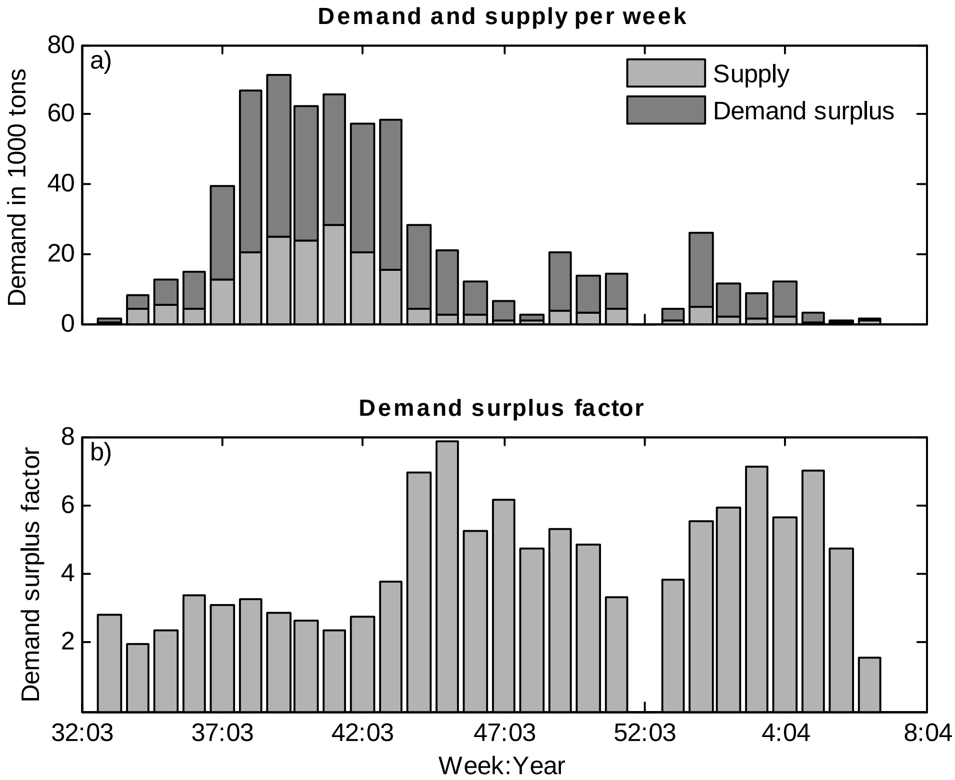
Bids are formed under strategic considerations. The marginal quantity demanded is represented by the bidder with the lowest bid. Since bidders do not submit demand schedules stating their price for different quantities, we cannot infer a demand function for different price levels. As standard economic theory tells us, it is likely that some bidders will reveal larger demand at a lower bid level than the chosen bid. Consequently, the revealed demand figure tells us that real market demand is at least as large as the observed quantity demanded at the price level that corresponds to the minimum observed bid.

Relevant demand figures are demand per auction, per day or for any other time period, such as demand per week. Demand per catch, however, is not that easy to describe. Aggregate demand is not evenly distributed over catches. Recall that bidders may bid on as many catches as they like, but due to limitations in short-term production capacity, may set a maximum ton number as part of their bid vector. The stated capacity constraint is useful in determining demand for an auction or time periods like day or week. Demand for an individual catch is not directly linked to the capacity constraint. For example, when a bidder bids on two catches, but states through his maximum ton limit that he can only take one, then we cannot determine a demand for each individual catch.

We illustrate demand and supply as we did for harvest; i.e., we report it per week in order to have a measure that is easy to visualize. For bidders that set no capacity limits at a given auction, we take their demand to equal the sum of quantities they bid on. For bidders that do set a capacity limit, we measure their demand to be equal to their capacity limit. In a few cases, bidders bid on less than their capacity limit, making the capacity limit unnecessary to state. In these cases, we resort to the method used for bidders with no stated quantity limits, or in other words, we set demand equal to the minimum of stated capacity and demanded quantity. In figure 5.3a, we depict the demand and supply per week in the 2003 season. The exact numbers the plots are based on, are those in table 5.6 on page 111.

The lower part of each bar in figure 5.3a shows the supply per week, while the upper part is how much demand exceeds supply; i.e., the demand-surplus. Thus, the total height of each bar is equal to demand since *demand-surplus = Demand – Supply.* In figure 5.3b, we have calculated the demand-surplus

Figure 5.3: Demand and supply in 2003–04 season



factor defined by: *demand-surplus factor = Demand/Supply.* The mean of the demand-surplus factor is 4.33, telling us that demand per week is, on average, 4.33 times higher than supply per week. The minimum and maximum demand-surplus factors observed in 2003 is 1.57 and 7.83, respectively.

When interpreting these figures, bear in mind that they are weekly aggregates. Such aggregate measures, although widely used in all realms of empirical economics, should be interpreted with some caution. In the extreme, each catch with its time, geographical location and quality space dimension constitutes a single market. Demand for individual catches varies, not only because of the system of delivery zones, but also because of other catch-specific characteristics. But the reported demand figures *is* a measure, if imperfect, of competition in the market. Based on traditional demand-supply analysis, looking in particular at the high average demand-surplus factor, we may conclude that, in general, competition is high in this market. We shall, however, analyse this question from another perspective as well. Moving away from aggregate demand and supply, we shall see that at auction markets, the number of competitors is a good measure of competition at the individual lot level.

5.4 Product-specific variables

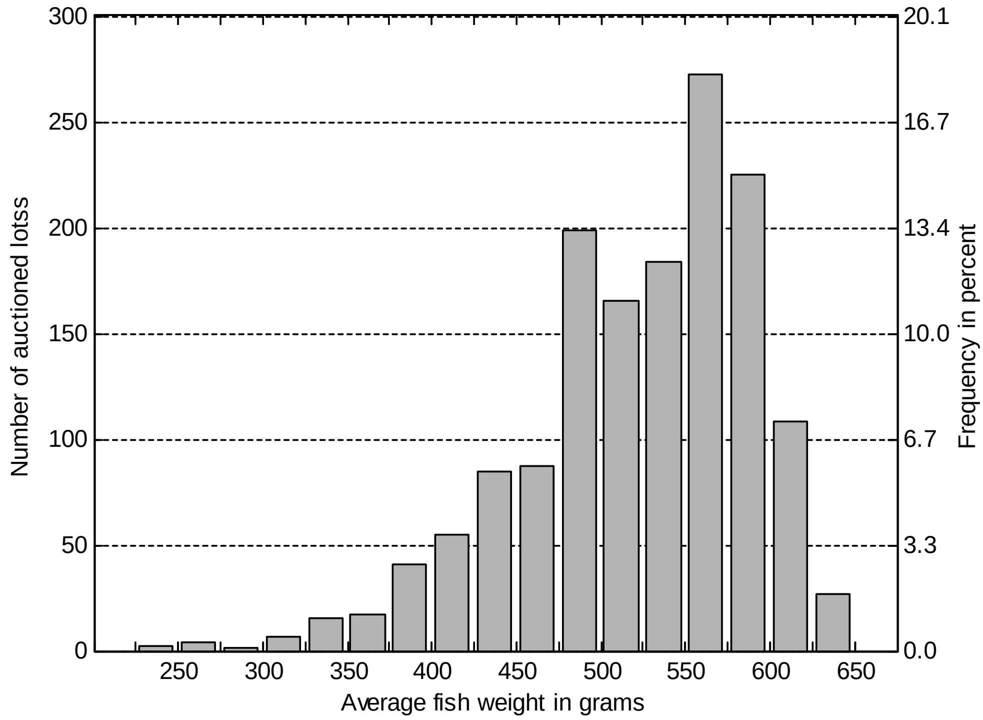
The seller describes his registered catch and lots by several variables. The identity of the seller is known to bidders and may influence the price. Some sellers may have a better reputation for handling fish than others, or it may even be the case that some buyers prefer some sellers due to individual personal relations. The number of sellers is relatively large, and it seems inconvenient to incorporate the identity of the seller in the analysis. More important are probably some quality characteristics that sellers report and other generally relevant variables, in particular total quantity supplied.

Before we turn to a description of the characteristics of the specific lots or catches that comprise the dataset, recall the distinction between catches and lots. A catch may consist of several lots for two reasons. Lots may consist of different species. This is rarely observed in the mackerel fishery; for example, only three lots in the dataset are from multi-species catches. The most obvious reason for several lots in a catch is that the size of the fish differs in such a way that it is natural to divide it into two or more lots. In the dataset, 118 catches consisted of two lots, no catches had three lots and one catch had four lots. The remaining 1337 catches (91.83 per cent) had only one lot. We analyse the data for the most part at the lot level since this is the most detailed level. Some reported variables, like average fish weight, are transformed into an average of averages when going from the lot level to the catch level. When reporting statistical measures of catches, we shall clearly state that we are looking at catches rather than lots; otherwise, statistical measures are based on lots.

5.4.1 Continuous variables

As a practical matter, we may regard reported fish weight and lot quantity as continuous variables. Fish weight is measured in grams and lot quantity in tons.

Figure 5.4: Distribution of average fish weight



Average fish weight. The most important variable is the average fish size. Reserve prices are linked to this variable. Large fish entail less waste in the production process. Consequently, buyers are willing to pay more for large fish than for small fish. In case a catch consists of several lots, the average fish weight is reported for each lot. The relationship between prices and average fish weight is analysed in section 6.2, page 127.

In figure 5.4, we depict the distribution of average fish weight over lots.

Average weight ranges from 230 to 650 grams. Very few lots belong to weight classes below 300 grams. Roughly 66 percent of lots have an average fish weight of 500 grams or above. This segment, requiring the highest reserve price, is obviously the economically most important part of the fishery.

Total vessel quantity. Catch weights range from below 20 to up to 800 tons. The distribution of catch or vessel quantities is shown in figure 5.5. Forty-six percent of catches are below 100 tons, 61 percent are below 150 tons, and close to 93 percent of the catches are below 300 tons. Thus, catches above 300 tons are relatively rare.

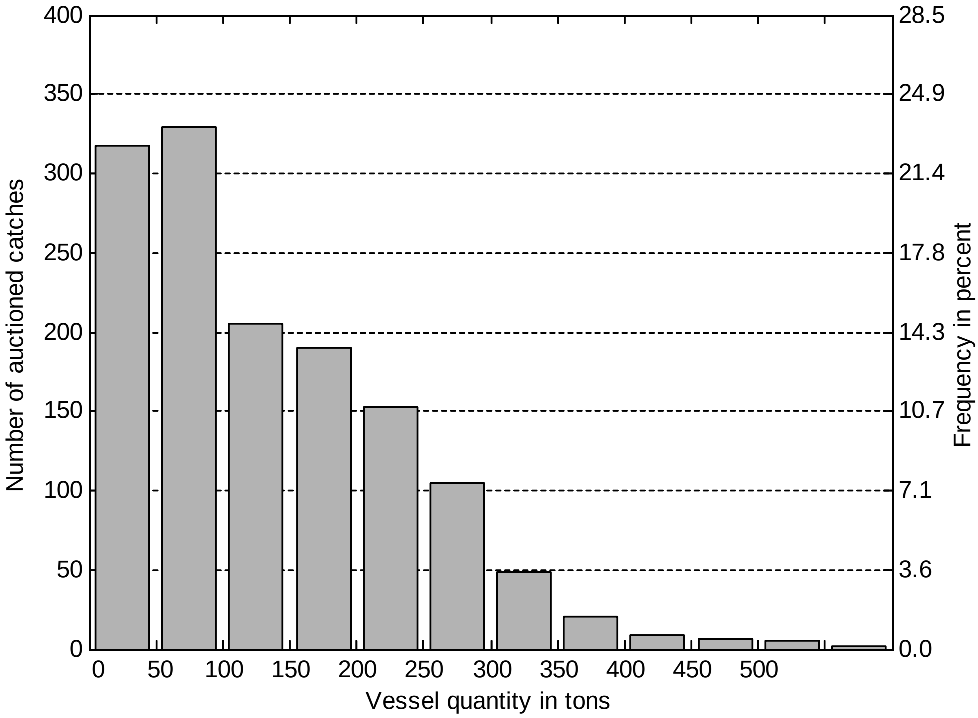
Cost considerations mean that a buyer will prefer to obtain his desired quantity by as few catches as possible. For instance, receiving one large catch of 100 tons rather than two smaller catches of 50 tons each, will be more cost-effective for the buyer. Fixed costs and scale economies are associated with each catch received.

The total quantity of a catch or lot is important because quantity limits are frequently used by bidders. Quantity will affect the final allocation of catches when capacity constraints are binding. In the case of very large catches, it will also affect the number of potential bidders, since some buyers can find the catch too large to handle.

5.4.2 Discrete variables

In addition to the obviously interesting quantitative measures of fish weight and catch quantity, a catch is described in several other respects. The objective of most of the required information is to give buyers a good understanding of the quality of the fish and when it can be delivered. Sellers are obliged to report the following variables to prospective buyers: (1) fishing gear used; (2) the number of hauls used to secure a catch; (3) whether the fish contain feed; (4) whether the fish have been constrained to reduce feed level; (5) preservation method; (6) the number of storage tanks for the catch; (7) the catch field, and (8) estimated arrival time. In table 5.1, we have summarized the dataset with respect to the reported discrete variables. We present the

Figure 5.5: Distribution of vessel quantity



number of records for each category together with the percentage share. The column entitled *Code* in the table, is simply the internal code used by the auction house for the different categories.

Gear. The dominant fishing method used is the purse seine: 86.7 percent of lots were caught using this method. In the data base, a distinction is made between *purse seine* which relates to the ocean-going seiners, and *purse seine coast* which relates to smaller vessels using the very same gear. The other method used is the trawl. This method is grouped into three sub categories; bottom-trawl, floating trawl, and floating-pair trawl. In general, the purse seine is preferred because this method is the most gentle; trawling causes more damage to the fish.

Feed. One characteristic of the harvested fish is whether they contain feed. No feed in the fish is preferred. Feed can ruin the fish quality if it takes a long time before the fish are delivered to the producer and frozen. If the fish contain a lot of feed, the harvester may choose to constrain the fish. This means that after the fish are properly secured inside the net, it is left swimming for a few hours in the sea in order to reduce feed contents. The procedure is rarely observed; constrain times ranging from one to five hours are reported for only eight lots. The feed contents descriptions are classified in four groups ranging from *no feed* to *full of feed*. Our dataset does not contain any records classified as *full of feed*. The intermediary classifications *very little feed* and *some feed* are predominant.

Preservation. Seafood is highly perishable and needs to be cooled rapidly in order to slow spoilage. Bacterial, enzymatic, and chemical processes quickly reduce the quality of the fish if they are not handled properly. Immediate chilling of the fish to a temperature just above freezing point is the best preservation method. Historically, crushed ice was used in seafood preservation, and it is still used to some extent. But these days, two more common methods for seafood chilling and storage are to use either refrigerated seawater (RSW) or slurry ice (CSW), which is a mixture of finely crushed ice (micro particles of ice with size 0.25–0.50 mm) and water. Using RSW, it takes relatively long time to bring the temperature of the fish down to the desired level. In addition, a concern is that the fish take up salt from the seawater. Slurry ice is considered a faster chilling method. Compared to the alternative of just using crushed ice, slurry ice covers the fish without bruising and leaves no air spots. Consequently, slurry ice is the preferred chilling method.[4] Slurry ice is, however, costly which explains its rare use in the dataset. By far the most common method is to use refrigerated seawater.

Tanks. The number of storage tanks used for a given catch is reported. In the dataset, the number ranges from one to six. The number of tanks used depends on how large the catch is. The more tanks used for a fixed quantity,

[4]Source: Eurofish (`www.eurofish.dk`)

Table 5.1: Distribution of quality variables

Variable	Category	Code	Records	Percent
Gear	Purse seine coast	10	399	26.71
	Purse seine	11	896	59.97
	Bottom trawl	51	4	0.27
	Floating trawl	53	178	11.91
	Floating trawl pair	54	17	1.14
Feed	No feed	1	70	4.69
	Very little feed	2	1184	79.25
	Some feed	3	240	16.06
Preservation	Ice	9	153	10.24
	RSW[a]	11	1325	88.69
	RSW + ice	18	10	0.67
	RFW + acid + ozone	24	1	0.07
	CSW[b]	25	5	0.33
Hauls	One haul	1	1318	88.22
	Two hauls	2	157	10.51
	Three hauls	3	19	1.27

[a] RSW: Refrigerated seawater
[b] CSW: Slurry ice

the better, since the fish are then less packed, and even more important, they are chilled faster. Preliminary analysis revealed no effect of this variable on prices. Consequently, we did not incorporate it in the analysis.

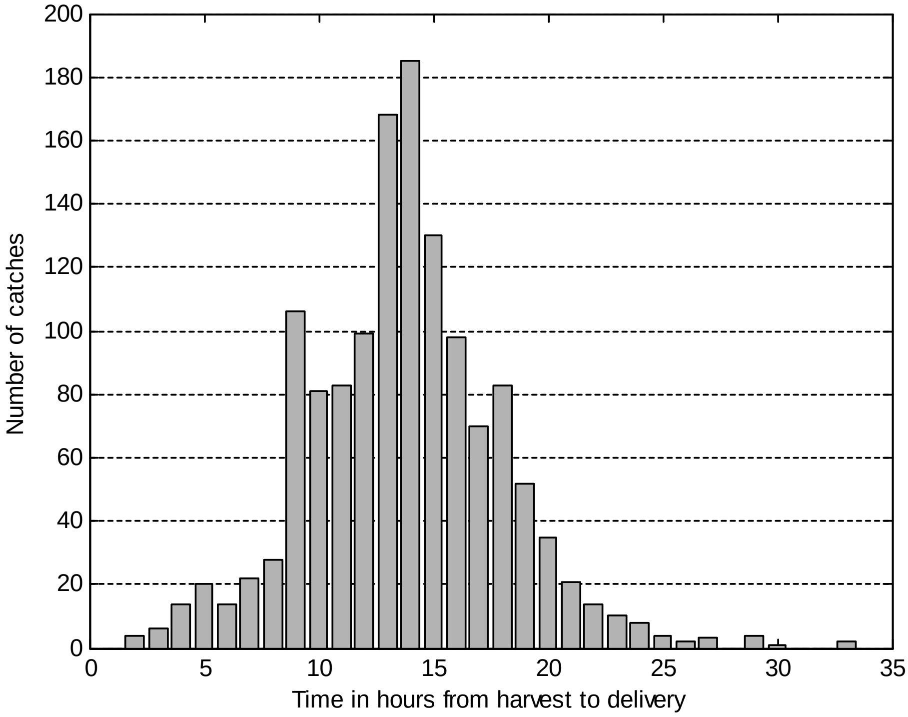
Hauls. Maneuvering the purse seine into the sea is referred to as making a *haul*. Typically, the number of hauls is one or two; occasionally, three are made. One haul is preferred because the fish are then as fresh as possible. If two or more hauls are necessary, then part of the catch will have been caught several hours before the fish are offered on the market. Like the number of tanks, the number of hauls does not seem to have any impact on prices.

Delivery time. The catch field or sector is reported together with the delivery sector. The field in itself is probably of no particular importance, but the distance between the vessel and a potential buyer may be of interest. The vessel reports expected arrival time at the southern and northern ports in the preferred delivery sector. If a long time passes before arrival at port, then the quality of the fish will suffer. Moreover, a specific arrival time can be considered more or less convenient for a plant's work flow schedule.

Data on other reported variables are also available, but they are mostly relevant when comparing different fisheries. Since they tend to have the same value at the mackerel auctions, we do not include them in the analysis.

In figure 5.6, we depict the distribution of the delivery time of the catches. For each catch, we have two delivery times, one for the southern and one for the northern port of the delivery sector. We take the mean of these two delivery times. (The picture does not change much if we alternatively plot the southern or northern delivery time.) Two outlier observations with mean delivery times of 36 and 64 hours are not depicted in the histogram. We see that most catches have a delivery time between 10 and 20 hours. The median delivery time is 14 hours.

Figure 5.6: Distribution of delivery time



5.4.3 The reserve price

Reserve prices are linked to average fish weight. Fish belonging to a certain weight class demand a corresponding reserve price. Five weight classes and reserve prices were used in 2003. In table 5.2, we present some statistics of the reserve price in our dataset. The sample size (i.e., the number of lots for each reserve price) is shown in column 2. In columns 3–5 of the table, we have calculated some percentage shares of each reserve price class. First, we present the percentage of the number of lots in each class. In the next column, we look at the percentage of total supplied quantity in each class. Finally, we find the percentage of total value for each reserve price class calculated by auction prices. We see that the largest fish, having a reserve price equal to 5.25 NOK, are the economically most important class, accounting for around 68 percent of the total quantity and close to 71 percent of total value. This explains our emphasis on this reserve price class in the analysis to follow in the next two chapters. In the last two columns of table 5.2, we show the weight intervals that correspond to the reserve prices. The minimum observed weight in the dataset is 230 grams, and the maximum observed weight is 650 grams.

Table 5.2: Reserve price statistics

		Percentage shares			Weight (g)	
Reserve price	Records	Lots	Quantity	Value	From	To
1.50	29	1.94	1.71	1.13	230	349
2.50	58	3.88	3.87	3.07	350	399
3.50	140	9.37	9.69	8.77	400	449
4.75	286	19.14	16.53	16.22	450	499
5.25	981	65.66	68.20	70.81	500	650

5.5 Market-specific variables

The most important market-specific variable is the number of bidders. We have observations on all bids, and hence the number of actual bidders N. We examine the distribution of actual bidders. Since there is a reserve price, some bidders with valuations below the reserve price will not bid. The number of potential bidders is not observable. The distribution between inside and outside bidders is also described.

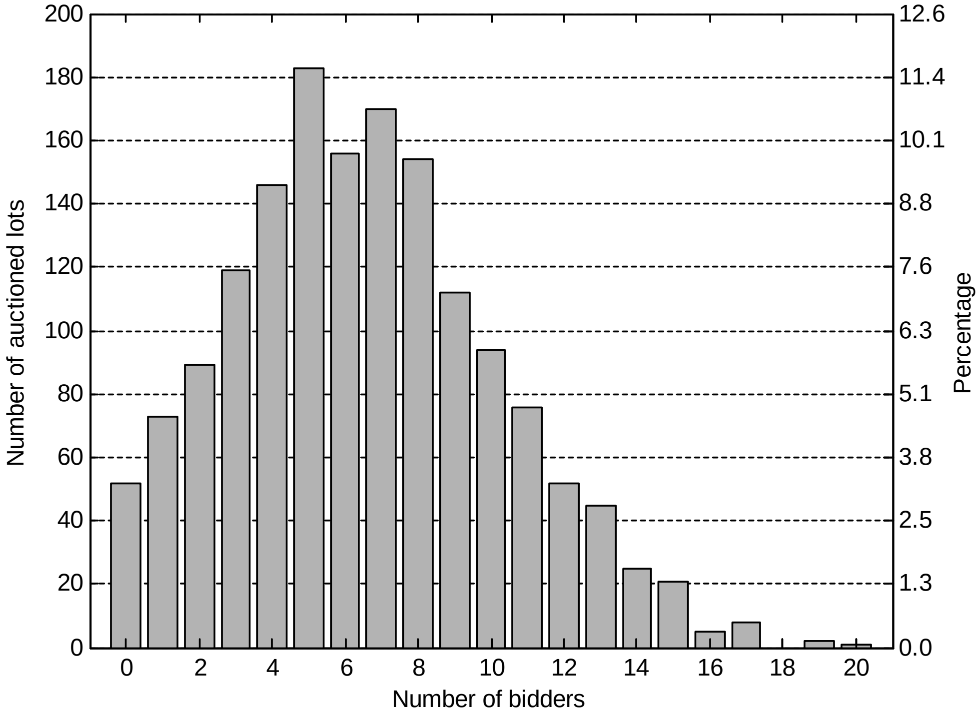
5.5.1 Distribution of N

The number of submitted bids varies considerably in the dataset, from 0 to 20. Most of the mass is located below 8 bidders. Two reasons may explain this: First, the auctions are quite frequent. Also, during the peak season, large quantities are offered on the market. Buyers who have just recently acquired raw material by winning a lot (mackerel or other species), are perhaps less likely to participate at the following auction due to short-term capacity problems. Typically, a winner which is occupied with freezing a catch, will be out of a "bid position" for 24 hours. Second, the fact that sellers set a preferred geographical zone of delivery will to a large extent determine the number of potential bidders. Although outside bidders are eligible to bid, they do this relative rarely and win even less often. We return to this below where we discuss the delivery sectors, see in particular table 5.7.

Using all bids, the distribution of submitted bids is depicted in figure 5.7. The exact numbers the figure is based on is reported in table 5.11 on page 116. The median number of submitted bids is 6. Since bidders can set capacity constraints, the number of submitted bids will contain some bids that are not binding. If we remove those bids that are not binding and concentrate on the ones that actually competed on a lot (*ex post*), the distribution of N moves left as seen in figure 5.8 on page 117. The median number of submitted bids then drops to 5. We discuss the distinction between all bids and binding bids in more detail in chapter 6, section 6.5.

In interpreting the economic impact of N, we may, somewhat roughly,

Figure 5.7: Distribution of N, all bids



make a distinction between large N and small N. As we shall see in chapter 6, when N is around 9 or higher, we reach a competitive level, where prices obtained do not depend on the number of submitted bids. Thus, for large N, some worries the seller typically has at an auction market, are of less concern, for instance, determining the reserve price and guarding against possible collusion among bidders. For small N, however, all the characteristics and interesting aspects of an auction market are present.

5.5.2 Delivery sectors and inside bidders

The stated delivery sector of a catch offered for sale defines whether a bidder is an inside or outside bidder. Inside bids are binding for the seller, while outside bids are an offer, which the seller can refuse. In our dataset, 8.22 percent of all bids are from outside bidders. If an outside bid is equal to an inside bid or just marginally higher, the seller will prefer the inside bid since the transport and time related costs are lower in this case. Thus, in order to win the auction, an outside bid should be sufficiently higher than any inside bid to compensate the seller for his extra costs. Looking at all lots that attracted bids, 8.97 percent of the maximum bids were from outside bidders. Compared to the share of total outside bids of 8.22 percent, this indicates that outside bidders tend to bid a bit, but not much, higher than inside bidders. Most outside bids are, however, rejected by the seller. In only 2.54 percent of the cases (38 lots), was the outside bid accepted by the seller. In 9 out of these 38 lots, the outside bid was the only binding bid. We have summarized the percentages in table 5.3.

Table 5.3: Percentage of inside versus outside bids

	All bids	Max. bid	Winning bid
Inside bidders	91.78	91.03	97.46
Outside bidders	8.22	8.97	2.54

Narrow sectors will reduce the number of potential inside bidders. In principle, since outside bidders are eligible, we may say that the number of potential bidders is constant and equal to the number of all bidders. In practice, however, some catches are unavailable for outside bidders. A catch only offered to, say, port 25, means that the vessel is probably close to or has already arrived at this port. In such a case, the vessel will not set course to a remote destination, such as the southernmost port 12. Costs rule-out this option as well as the concern that the time loss will reduce the quality of the fish.

We shall take a quick look at the distribution of delivery sectors. Recall that the delivery sector is the sector between a southern and northern port. In table 5.7 on page 112, we show how many lots fall into a given combination of a southern and northern port. The numeric code for ports are explained in appendix A, see page 313. Some sectors appear more frequently than others. The most common southern ports are Agnefest (12), Egersund (13), Haugesund (16), Bergen (19), Florø (20), Måløy (22) and Harøysund (25). The most common northern ports are Bergen, Måløy and Harøysund. We note that as many as 208 lots are offered to port 25; i.e., Harøysund is both the southern and northern port of the sector.

5.6 Bidders and bids

In this section, we present bid frequencies and the success rate for the different bidders. This will tell us something about asymmetries among bidders. Information analysis and competence in interpreting the information is crucial for any bidder at an auction market. The buyers are all regarded as expert bidders. They are professionals, well-experienced in bidding. In addition, they are, in general, well-informed. In the case of asymmetric information about a common-value good, a bidder with superior information to that of its rivals, may obtain informational rents. We shall argue later that bidders' valuations are predominantly private. It appears that informational rents are not very relevant in this market.

5.6.1 Bid frequencies of buyers

Not all potential buyers bid on any catch. The number of actual bidders varies a lot from catch to catch. An obvious reason for this is the aspect of delivery sectors. Most bidders do not bother to bid when they are defined as outside bidders. In addition, short-term capacity constraints are important. A producer will not be at the auction market if he is capacity constrained. A third reason for not submitting a bid on a catch might be that his valuation is below the reserve price. Finally, a bidder may refrain from bidding because he has preferences over sellers.

In table 5.9 on page 114, we present how frequently the 25 buyers bid in the 2003–4 season. Under the heading *offered catches*, we report the number of bid opportunities and make the distinction between inside and outside catches. Next, we looked at the number of submitted bids for all buyers. We report the number of total bids, and the number of inside and outside bids. Finally, we calculated the bid frequencies in percent.

We notice that bidding activity varies among bidders. Some are active and bid on many catches, while others are infrequent participants at the auctions. Bidders with a low number of submitted bids may have a location that often makes them outside bidders. Bidders located at core delivery sectors, will mostly be inside bidders. Some bidders, in particular bidders 9 and 11, but also bidders 3, 21 and 23, have locations that are frequently outside the delivery sector. Differences in bid frequencies are somewhat puzzling. At least we see that the delivery sectors cannot explain the pattern alone. Low frequent bidders may be small-scale producers or have a production line geared to other species than mackerel. Bidders may have valuations below the reserve price, but this explanation is closely linked to being capacity constrained. The economics of the trade suggests that when a bidder does not have free-freezing capacity, his valuation of a catch drops to zero. Otherwise, when a producer has free capacity, it may be the case that his valuation is still below the reserve price because better opportunities exist (the season for herring will overlap the mackerel season).

Apart from differences in bid frequencies and bid opportunities, are bidders asymmetric in their bid behavior? Let us examine the success rate of bidders. In table 5.10 on page 115, we repeat the number of submitted bids from table 5.9. In column 5, we report the number of bids for each bidder that turned out to be the highest bid. The percentage of highest bids in terms of submitted bids is reported in column 6 (Score). Since the highest bid does not necessarily win the auction—the bid is not binding due to capacity constraints or an outside location of the bidder—we examined the number of winning bids as well. In column 7, the number of winning bids for each bidder is reported, and in the last column the score of winning bids is reported.

From the scores reported, we may state that some bidders are more aggressive than others. Bidders 6, 7 and 17 have a winning score above 25 percent, while bidder 1 has a score as low as 5 percent. From this alone, we cannot conclude that bidders are very asymmetric. Asymmetric bidders, in the auction-theoretic sense, will have different valuations. To what extent some bidders systematically have higher valuations than others will have to be analysed by looking at the market price when controlling for observable differences between catches. We analyse this in chapter 7. Alternatively, instead of looking just at winning bids, we may analyse differences in bid levels using information from all bids. An index of bidder differences is reported in section 9.5.

In figure 5.9 on page 122, we visualize the bid frequencies of all bidders. Each subplot in the figure shows a bar when a bidder bids on a specific catch, and the height of the bar indicates how large the catch is in tons. Bidder numbers are reported in the upper left of the subplots. In figure 5.10, we show the actual quantities the different bidders won, and how these are distributed over time.

5.6.2 Capacity differences

Closely related to differences in bid activity are differences in production capacities. Short-term capacities take the form of stating a maximum ton limit when bidding on several catches at an auction. Infrequent bidders may often have a location outside the delivery sector, but another possible cause is that they generally have low production capacities. A bidder with alot of production capacity may have an advantage because he can take the residual demand after low capacity producers have been allocated their desired quantities. In principle, he can use his position to acquire catches at a lower price.

From figure 5.3 on page 90 and table 5.6 on page 111, we saw, however, that demand generally exceeds supply. No evidence exists to suggest that some bidders can act as residual demanders. Nevertheless, we cannot rule out that large buyers have an advantage. In table 5.13 on page 119, we report summary statistics of capacity limits; the median together with the minimum and maximum. To see the differences between "small" and "large" producers, focus on bidder 14 and 7. For bidder 14, a small-scale producer, the median submitted capacity limit is 110 tons and the largest submitted capacity limit is 250 tons. Bidder 7 on the other hand, has a median capacity limit of 700 tons and his maximum reported capacity is 2,085 tons.

5.6.3 Position of winning bids

The highest bidder does not always win the auctioned lot for two reasons. First, he might be an outside bidder, and the seller may refuse his bid. Second, and most frequently, an high bid may not be binding because of capacity constraints. In this case, the first binding bid may be the second highest or have a lower position. We might call these two rules *exclusion rules*. What effect does this aspect of the auction format have on the realized winning bids? Let us summarize the position of the winning bid in our dataset, see table 5.4.

For each auctioned lot, we identified the position of the winning bid; i.e., whether the highest, second-highest, or any lower ranking bid was allocated the object. First, we used all bids, including outside bids, in the calculation. This gave us the total effect of the two exclusion rules. The relevant numbers are in columns 2–4 of table 5.4. Roughly 68 percent of lots were allocated to the highest bid, and 19 percent to the second-highest bid. Thus, 87 percent

Table 5.4: Position of winning bid at lot level

	All bids			Inside bids		
Position	Lots	%	Cum. %	Lots	%	Cum. %
1st	1022	68.41	68.41	1064	73.08	73.08
2nd	281	18.81	87.22	245	16.83	89.90
3rd	110	7.36	94.58	84	5.77	95.67
4th	50	3.35	97.93	33	2.27	97.94
5th	16	1.07	99.00	18	1.24	99.18
6th	8	0.54	99.53	6	0.41	99.59
7th	2	0.13	99.67	2	0.14	99.73
8th	4	0.27	99.93	3	0.21	99.93
9th	0	0.00	99.93	1	0.07	100.00
10th	1	0.07	100.00	0	0.00	100.00

of the lots were allocated to the highest or second-highest bid.

Second, we examined the position of the winning bid after excluding outside bids. Recall from table 5.3 that although 8 percent of all bids are outside bids, only 2.5 percent of outside bids (38 lots) do in fact win. Outside bids, are therefore considered to have a marginal effect on market prices. From columns 5–7 in table 5.4, we see that the highest bid now wins in 73 percent of the cases. The total number of lots included when outside bids are excluded are 38 less than the case using all bids, since there are 38 lots were an outside bidder won.

A possible cause for noise in the distribution of winning bids at the lot level is that the average catch bid, in fact, determines the winner. A bidder may have the second-highest bid on lot one and the highest bid on lot two. If lot two has the largest quantity, then he will have the highest catch bid. We examine the distribution of winning bids at the catch level in table 5.8 on page 113. Thirty-six catches had outside winners. Without going into the details of the table, we can safely conclude that the distribution of winning bids do not change much if we summarize it at the catch level rather than at the lot level. This is to be expected, since close to 92 percent of the catches consist of only one lot. The most notable difference between tables 5.4 and 5.8 is that the percentage of maximum bids that win an object goes from 68.41 percent at the lot level to 70.10 percent at the catches level when including all bids. The corresponding percentages when excluding outside bids, are 73.08 and 75.45 percent.

5.6.4 Simultaneous selling

An important aspect of the auction format is that lots are sold simultaneously. The number of catches varies from auction to auction. At some auctions, only one catch is offered. The largest number of catches offered at an auction was 31. The aspect of simultaneous selling is summarized in table 5.12 on page 118. Column 1 represents the number of catches offered at a given auction while column 2 is the count of auctions where the event in column 1 happens. For example, we see from column 2 that the event *four offered catches* occurs at 23 auctions or, from column 3, in close to 8 per cent of all auctions. At around 31 percent of all auctions, only one catch is offered, while two catches are offered at around 16 percent of the auctions.

One hypothesis with respect to bidding under such simultaneous format is that bids are spread thin on catches when the number of auctioned catches is large. The result would be that each catch attracted a low number of bids. We investigate this in column four to six in table 5.12. In general, there is no pattern that supports the hypothesis of few bids on catches when the number of offered catches is large. Looking in particular at column five, the largest median number of bids, in fact, occurs when relatively many catches are offered simultaneously. If there is a pattern, one might distinguish between the first half of the table where the number of auctioned catches is below 12, and the lower part of the table. For the first half, the median number of bids varies from 4 to 8. When the number of catches is greater or equal to 12, the variability of the median number of bids increases. In this case the median number of bids ranges from 1.5 to 10.

Two points should be mentioned. First, in construction of table 5.12, we did not consider the effect of differences in bid sectors for the number of bids reported. Given the large number of catches offered, we found it likely that the number of potential inside bidders does not vary much for the different entries of catches. Second, the fact that the number of submitted bids on each individual catch was not lower when the number of catches is high, is obviously caused by the capacity constraint option. This option ensures that bidders are free to bid on as many catches they want without the risk of winning too many. It seems that bidders in fact use the option.

More interesting than the number of bids submitted is the price effect of simultaneous selling. Do prices tend to be lower when many catches are offered at the same time? We investigate this in the chapter 7.

5.6.5 Use of capacity constraints and priorities

Two options offered bidders under this simultaneous auction format are to set capacity constraint as well as priorities to the catches they bid on; see section 4.5 and 4.6 in chapter 4 for a presentation of the options. Setting a capacity constraint is only relevant for a bidder when bidding on more catches than he can take. Likewise, setting priorities is, obviously, only relevant when bidding on two or more catches. How frequently are the options used? To answer, we identified for every bidder at every auction the cases where a bidder submits bids on two or more catches. Then we looked at whether he sets an effective capacity constraint, and whether he sets priorities to the catches he competes for. A capacity constraint is said to be effective when a bidder bids on catches with a total quantity superseding his capacity constraint. In table 5.14 on page 120, we report the results. The percentages reported in column 4 and 6 are simply the percentage of the count of auctions in columns 3 and 5 of the total number of relevant auctions in column 2.

The use of the maximum ton limit ranges from 23 percent to 100 percent among bidders. Counting all relevant bid vectors for all bidders, the maximum ton limit is used in 76.2 percent of the cases. The priority option is used slightly more frequently, in 82.3 percent of the cases. All bidders use it at 50 percent or more of the relevant auctions. For differences among bidders, we refer to table 5.14. We conclude that both the capacity limit and the priority option are used frequently.

5.A Appendix: Tables and figures

Table 5.5: Distribution of vessel quantity in tons

From weight	To weight	Count	Percent	Cum. percent	Mean	Min.	Max.
0	50	317	22.59	22.59	27.71	5	48
50	100	329	23.45	46.04	68.81	50	98
100	150	205	14.61	60.66	119.55	100	145
150	200	190	13.54	74.20	166.95	150	195
200	250	153	10.91	85.10	216.60	200	245
250	300	105	7.48	92.59	266.25	250	295
300	350	49	3.49	96.08	314.08	300	340
350	400	21	1.50	97.58	365.95	350	390
400	450	9	0.64	98.22	416.67	400	440
450	500	7	0.50	98.72	472.86	460	490
500	550	5	0.36	99.07	513.00	500	530
550	600	2	0.14	99.22	562.50	550	575
600	650	5	0.36	99.57	604.00	600	620
650	700	1	0.07	99.64	690.00	690	690
700	750	2	0.14	99.79	715.00	700	730
750	800	3	0.21	100.00	773.33	760	790

Table 5.6: Demand and supply in tons per week

Year	Week	Supply	Demand	Demand surplus	Demand surplus factor
2003	33	415	1171	756	2.82
2003	34	4207	8170	3963	1.94
2003	35	5450	12862	7412	2.36
2003	36	4432	14975	10543	3.38
2003	37	12792	39330	26538	3.07
2003	38	20363	66653	46290	3.27
2003	39	24714	71178	46464	2.88
2003	40	23570	62119	38549	2.64
2003	41	28154	65938	37784	2.34
2003	42	20608	57163	36555	2.77
2003	43	15606	58486	42880	3.75
2003	44	4100	28530	24430	6.96
2003	45	2680	20985	18305	7.83
2003	46	2295	11985	9690	5.22
2003	47	1075	6635	5560	6.17
2003	48	590	2790	2200	4.73
2003	49	3865	20538	16673	5.31
2003	50	2880	13900	11020	4.83
2003	51	4290	14155	9865	3.30
2003	52	0	0	0	
2004	1	1100	4220	3120	3.84
2004	2	4725	26050	21325	5.51
2004	3	1930	11450	9520	5.93
2004	4	1240	8840	7600	7.13
2004	5	2105	11930	9825	5.67
2004	6	440	3080	2640	7.00
2004	7	180	850	670	4.72
2004	8	940	1480	540	1.57

Table 5.7: Delivery zone combinations, all catches

		Southern ports								
		12	13	16	19	20	22	23	24	25
Northern ports	13	3	0	0	0	0	0	0	0	0
	16	1	10	1	0	0	0	0	0	0
	19	19	61	27	8	0	0	0	0	0
	20	1	2	2	1	0	0	0	0	0
	22	19	73	98	50	25	0	0	0	0
	23	0	0	0	0	1	0	0	0	0
	24	0	0	0	0	0	1	0	0	0
	25	142	227	126	152	117	62	1	8	207
	29	0	1	1	0	4	0	0	0	2
	30	0	0	0	0	0	1	0	0	0
	31	0	0	1	1	0	0	0	0	0

Table 5.8: Position of winning bid at catch level

	All bids			Inside bids		
Position	Catches	%	Cum. %	Catches	%	Cum. %
1st	957	69.91	69.91	998	74.87	74.87
2nd	251	18.33	88.24	214	16.05	90.92
3rd	97	7.09	95.33	73	5.48	96.40
4th	41	2.99	98.32	26	1.95	98.35
5th	11	0.80	99.12	12	0.90	99.25
6th	5	0.37	99.49	4	0.30	99.55
7th	2	0.15	99.63	2	0.15	99.70
8th	4	0.29	99.93	3	0.23	99.92
9th	0	0.00	99.93	1	0.08	100.00
10th	1	0.07	100.00	0	0.00	100.00

Table 5.9: Bid frequencies of buyers

	Offered cathces			Submitted bids			Bid frequency		
Bidder	In[a]	Out[b]	All	In	Out	All	In	Out	All
1	812	644	1456	407	91	498	50.1	14.1	34.2
2	1102	354	1456	141	3	144	12.8	0.8	9.9
3	559	897	1456	239	20	259	42.8	2.2	17.8
4	1102	354	1456	411	0	411	37.3	0.0	28.2
5	1053	403	1456	511	31	542	48.5	7.7	37.2
6	1012	444	1456	376	0	376	37.2	0.0	25.8
7	1053	403	1456	334	0	334	31.7	0.0	22.9
8	1012	444	1456	376	5	381	37.2	1.1	26.2
9	185	1271	1456	57	79	136	30.8	6.2	9.3
10	1053	403	1456	617	62	679	58.6	15.4	46.6
11	2	1454	1456	2	16	18	100.0	1.1	1.2
12	1053	403	1456	832	103	935	79.0	25.6	64.2
13	1053	403	1456	768	78	846	72.9	19.4	58.1
14	1102	354	1456	55	0	55	5.0	0.0	3.8
15	1053	403	1456	574	27	601	54.5	6.7	41.3
16	1053	403	1456	741	46	787	70.4	11.4	54.1
17	1102	354	1456	450	2	452	40.8	0.6	31.0
18	1053	403	1456	92	5	97	8.7	1.2	6.7
19	812	644	1456	324	76	400	39.9	11.8	27.5
20	1012	444	1456	209	8	217	20.7	1.8	14.9
21	559	897	1456	228	24	252	40.8	2.7	17.3
22	1102	354	1456	113	1	114	10.3	0.3	7.8
23	559	897	1456	170	112	282	30.4	12.5	19.4
24	1053	403	1456	459	6	465	43.6	1.5	31.9
25	1053	403	1456	250	4	254	23.7	1.0	17.4

[a] In: Bidder is located inside delivery sector.

[b] Out: Bidder is located outside delivery sector.

Table 5.10: Bidders' scores

	Submitted bids			Max. bids		Winning bids	
Buyer	All	Inside	Outside	Count	Score[a]	Count	Score[b]
1	498	407	91	20	4.02	25	5.02
2	144	141	3	12	8.33	12	8.33
3	259	239	20	53	20.46	50	19.31
4	411	411	0	24	5.84	31	7.54
5	542	511	31	76	14.02	64	11.81
6	376	376	0	79	21.01	106	28.19
7	334	334	0	72	21.56	99	29.64
8	381	376	5	43	11.29	51	13.39
9	136	57	79	47	34.56	24	17.65
10	679	617	62	102	15.02	91	13.40
11	18	2	16	1	5.56	0	0.00
12	935	832	103	156	16.68	163	17.43
13	846	768	78	133	15.72	106	12.53
14	55	55	0	20	36.36	10	18.18
15	601	574	27	91	15.14	63	10.48
16	787	741	46	80	10.17	65	8.26
17	452	450	2	86	19.03	120	26.55
18	97	92	5	9	9.28	17	17.53
19	400	324	76	30	7.50	37	9.25
20	217	209	8	58	26.73	53	24.42
21	252	228	24	26	10.32	34	13.49
22	114	113	1	14	12.28	14	12.28
23	282	170	112	85	30.14	58	20.57
24	465	459	6	32	6.88	34	7.31
25	254	250	4	57	22.44	42	16.54

[a] A bidder's percentage of maximum bids to all his submitted bids.

[b] A bidder's percentage of winning bids to all his submitted bids.

Table 5.11: Distribution of N^a

	All bids[b]			Binding bids[c]		
N	Lots	Percentage	Cum. perc.	Lots	Percentage	Cum. perc.
0	52	3.28	3.28	88	5.56	5.56
1	73	4.61	7.90	153	9.67	15.22
2	89	5.62	13.52	170	10.74	25.96
3	119	7.52	21.04	173	10.93	36.89
4	146	9.22	30.26	186	11.75	48.64
5	183	11.56	41.82	207	13.08	61.72
6	156	9.85	51.67	173	10.93	72.65
7	170	10.74	62.41	128	8.09	80.73
8	154	9.73	72.14	93	5.87	86.61
9	112	7.08	79.22	68	4.30	90.90
10	94	5.94	85.15	66	4.17	95.07
11	76	4.80	89.96	37	2.34	97.41
12	52	3.28	93.24	21	1.33	98.74
13	45	2.84	96.08	13	0.82	99.56
14	25	1.58	97.66	5	0.32	99.87
15	21	1.33	98.99	2	0.13	100.00
16	5	0.32	99.31	0	0.00	100.00
17	8	0.51	99.81	0	0.00	100.00
18	0	0.00	99.81	0	0.00	100.00
19	2	0.13	99.94	0	0.00	100.00
20	1	0.06	100.00	0	0.00	100.00

[a] N: Number of active bidders.
[b] Total number of all bids = 10397.
[c] Total number of binding bids = 7669.

Figure 5.8: Distribution of N, binding bids

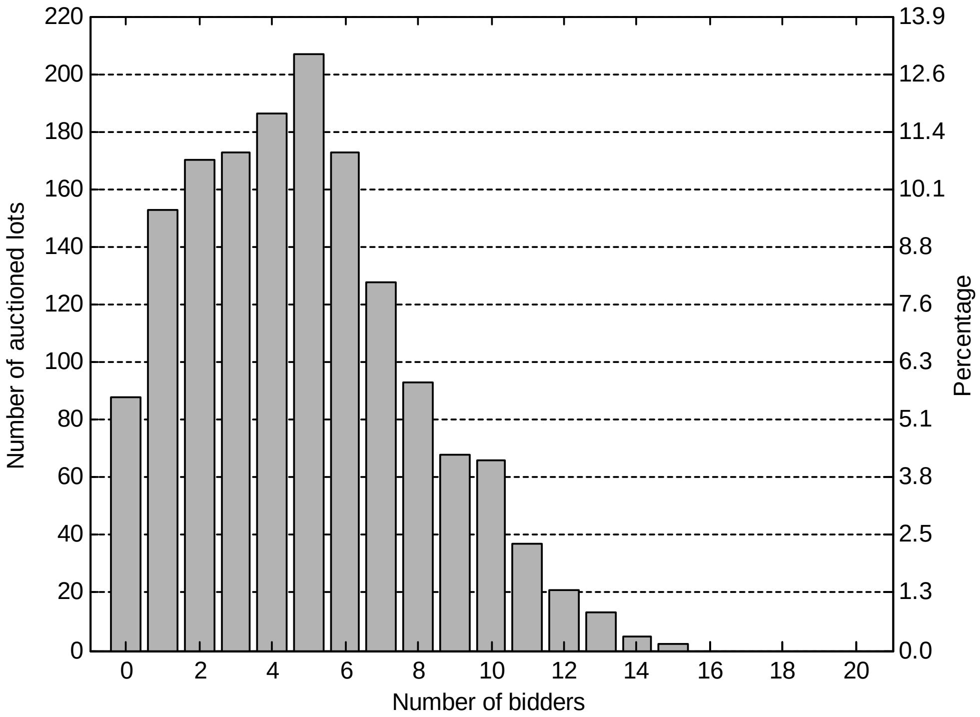


Table 5.12: Number of catches auctioned simultaneously

	Auctions		Number of bids		
Catches	Count	Percent	Min	Median	Max
1	89	30.90	0	5.0	13
2	44	15.28	1	6.0	11
3	27	9.38	0	6.0	14
4	23	7.99	0	6.0	17
5	15	5.21	0	4.0	11
6	10	3.47	0	8.0	15
7	17	5.90	0	6.0	14
8	15	5.21	0	6.0	17
9	7	2.43	1	7.0	17
10	5	1.74	1	7.0	17
11	4	1.39	0	6.5	12
12	3	1.04	0	1.5	12
13	3	1.04	1	6.0	20
14	10	3.47	0	10.0	17
16	1	0.35	1	7.0	12
19	3	1.04	0	5.0	17
20	3	1.04	2	7.0	13
21	1	0.35	0	2.0	7
23	1	0.35	1	5.0	8
24	1	0.35	3	8.0	13
25	3	1.04	0	6.0	14
26	1	0.35	4	8.0	12
27	1	0.35	1	10.0	17
31	1	0.35	1	9.0	13

Table 5.13: Summary statistics of maximum ton limits

Sorted by bidder (B)					Sorted by median (Med.)				
B	Count	Med.	Min.	Max.	B	Count	Med.	Min.	Max.
1	115	250	60	770	14	11	110	43	250
2	41	140	35	470	22	41	130	35	310
3	104	220	30	790	2	41	140	35	470
4	108	200	45	440	5	135	150	10	1000
5	135	150	10	1000	11	15	150	50	330
6	39	350	85	1200	18	15	150	55	1000
7	43	700	80	2085	23	87	150	15	600
8	96	220	25	800	25	60	198	75	400
9	30	200	90	220	4	108	200	45	440
10	131	200	18	800	9	30	200	90	220
11	15	150	50	330	10	131	200	18	800
12	185	250	20	800	24	91	200	20	550
13	180	240	30	530	20	102	210	50	800
14	11	110	43	250	3	104	220	30	790
15	152	225	25	950	8	96	220	25	800
16	155	270	70	650	15	152	225	25	950
17	103	426	35	2000	13	180	240	30	530
18	15	150	55	1000	19	118	245	20	770
19	118	245	20	770	1	115	250	60	770
20	102	210	50	800	12	185	250	20	800
21	17	300	100	380	16	155	270	70	650
22	41	130	35	310	21	17	300	100	380
23	87	150	15	600	6	39	350	85	1200
24	91	200	20	550	17	103	426	35	2000
25	60	198	75	400	7	43	700	80	2085

Table 5.14: Use of capacity limits and priorities

		Max. ton option		Priority option	
Bidder	Auctions[a]	Count[b]	Percent	Count[c]	Percent
1	85	66	77.65	49	57.65
2	25	24	96.00	25	100.00
3	57	50	87.72	44	77.19
4	73	72	98.63	48	65.75
5	84	73	86.90	49	58.33
6	68	26	38.24	51	75.00
7	49	24	48.98	45	91.84
8	67	51	76.12	56	83.58
9	32	28	87.50	27	84.38
10	101	71	70.30	93	92.08
11	2	2	100.00	1	50.00
12	138	109	78.99	121	87.68
13	126	105	83.33	121	96.03
14	8	8	100.00	8	100.00
15	104	91	87.50	97	93.27
16	131	112	85.50	96	73.28
17	81	34	41.98	77	95.06
18	15	12	80.00	13	86.67
19	74	53	71.62	70	94.59
20	42	40	95.24	40	95.24
21	64	15	23.44	44	68.75
22	24	19	79.17	17	70.83
23	64	51	79.69	61	95.31
24	72	65	90.28	50	69.44
25	49	45	91.84	43	87.76

[a] Number of auctions where bidder bids on two or more catches.
[b] Number of relevant auctions where bidder sets a capacity limit.
[c] Number of relevant auctions where bidder sets priorities.

Table 5.15: Demanded and acquired quantities in tons

	Total quantity			Individual catches[a]			
Buyer	Demanded	Acquired	Percent[b]	Count	Mean	Min.	Max.
1	31400	2900	9.24	25	116.0	40	250
2	6367	775	12.17	12	64.6	30	245
3	25420	11058	43.50	50	221.2	20	760
4	21365	3805	17.81	31	122.7	30	265
5	22012	3336	15.16	64	52.1	9	200
6	41303	17171	41.57	106	162.0	20	770
7	36595	16227	44.34	99	163.9	14	700
8	23267	6157	26.46	51	120.7	15	800
9	7948	3178	39.98	24	132.4	40	200
10	33601	9280	27.62	91	102.0	5	690
11	2500	0	0.00	0	0.0	0	0
12	49026	19396	39.56	163	119.0	10	550
13	42516	13963	32.84	106	131.7	5	470
14	1868	855	45.77	10	85.5	45	240
15	37727	10073	26.70	63	159.9	25	360
16	46323	11070	23.90	65	170.3	20	360
17	53991	19434	35.99	120	161.9	18	800
18	5612	1932	34.43	17	113.6	25	250
19	34982	5682	16.24	37	153.6	10	790
20	24446	10034	41.05	53	189.3	22	600
21	42186	6466	15.33	34	190.2	45	460
22	5175	1180	22.80	14	84.3	30	170
23	17107	7993	46.72	58	137.8	20	600
24	21043	2822	13.41	34	83.0	8	240
25	11323	4171	36.84	42	99.3	15	255

[a] Summary statistics of acquired catches.

[b] The percentage of acquired quantity to demanded quantity.

Figure 5.9: Catch quantities bid on during the season for each bidder

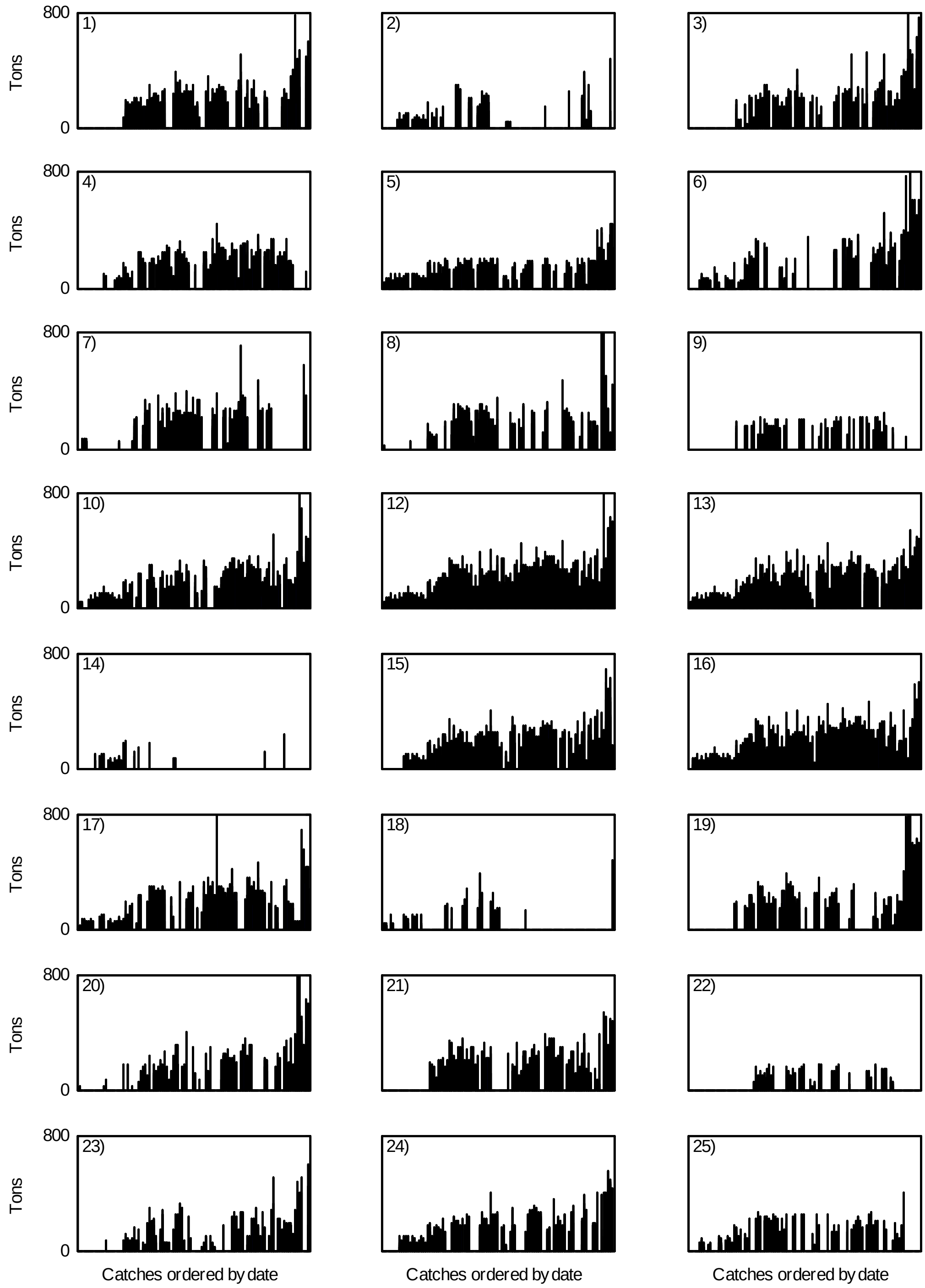
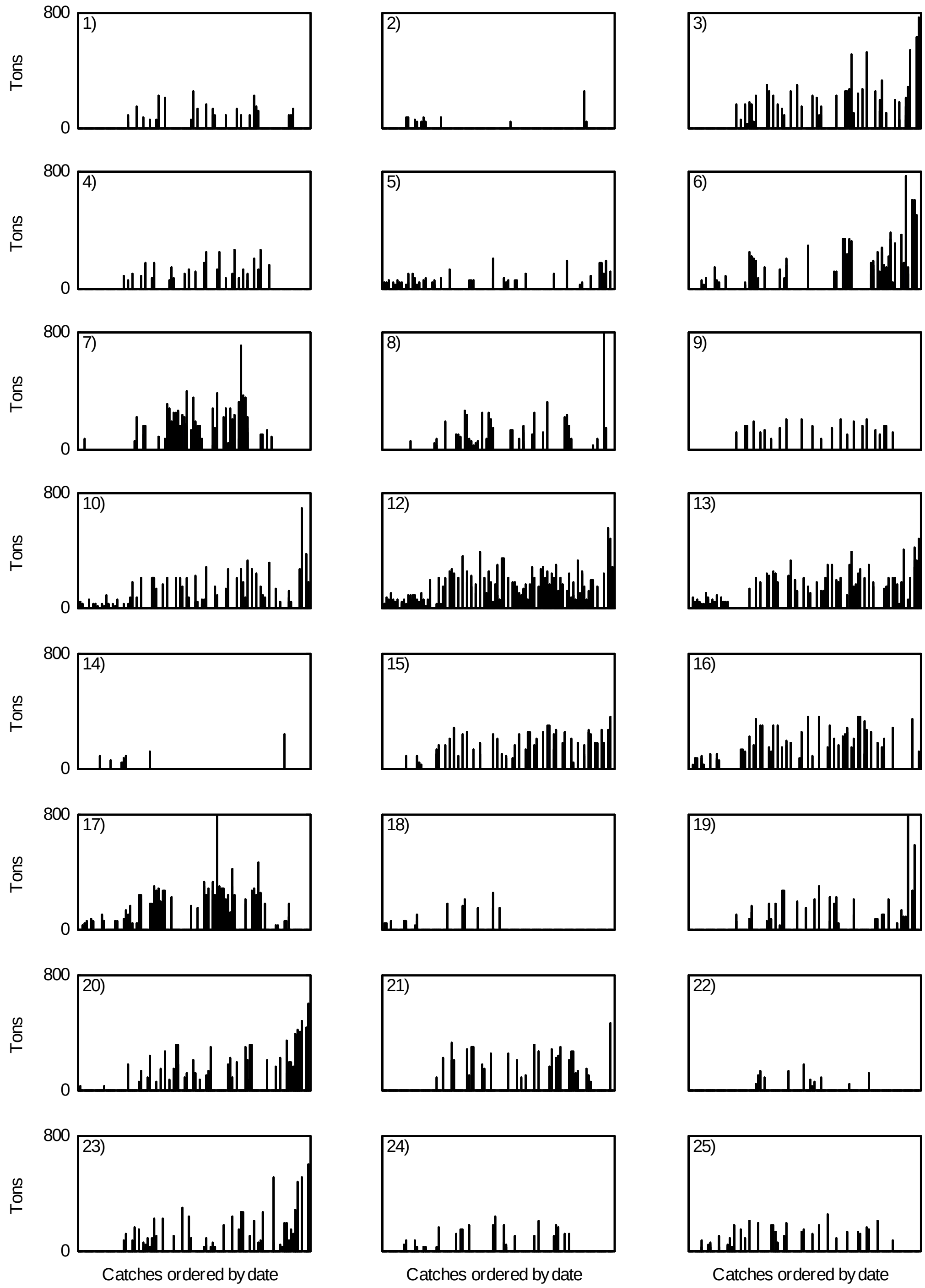


Figure 5.10: Catch quantities acquired during the season for each bidder



Chapter 6

Price formation: Partial analysis

6.1 Introduction

In this chapter, as an introduction to the relevant variables of a multivariate analysis, we examine how prices vary partially with some potentially important variables. In addition to being a prelude to a multivariate analysis, the partial approach has the merit of being simple. It may also uncover important and stable relationships that do not depend critically on controlling for all details of a lot.

Two product-specific covariates are analysed. We focus first on the most important product-specific variable, which is average fish weight of a lot. Next, we examine how the delivery times of catches affect winning bids. We expect a negative relationship between delivery times and prices. Then, we turn our attention to the effect of market measures. We examine general market forces at work from two perspectives: We aim first at an analysis of the effect of market supply and demand in quantities. Second, we examine an alternative measure of competition which is specific to auction data, to wit, how the number of active bidders influences obtained prices. Finally, we present the so-called "money left on the table" measure. This is a measure of how much the winning bidder overshoots. A traditional analysis of how prices as the explained variable is determined by several explanatory variables will be given in the next chapter.

The boundaries of the market. What constitutes the "aggregate market"? Supply is clearly defined to be all catches sold through the auction house NSS, but can we conclude that this is supplied to a single market? The characteristics of the auction mechanism introduce several complexities. In particular, different delivery zones create several geographical submarkets. For a couple of reasons, however, it is appropriate to treat the entire market as a whole. First, although the delivery zones sets a preferred geographical sector for buyers' locations, recall that outside bidders are permitted, thus making the entire buyer population potential market participants. Second, delivery zones are partly overlapping. The price level in any delivery zone will be influenced by common market forces. Moreover, it is unrealistic that the major submarkets are associated with significant different price levels. If this was the case, then sellers would be encouraged to set wider delivery zones. More importantly, competition from outside bidders would level differences in prices. The rule stating that outside bidders are eligible to bid effectively puts a bound on the price differences between a wide and narrow delivery sector that otherwise could pertain. In a way, our market can be compared to several retailers selling a product to their local customer bases, which may be distinct or partly overlapping. Customers will only tolerate small differences in prices. If this is not the case, then they will make the effort to travel to a retailer with lower prices.

Another feature of the auction is that capacity constraints can result in bidders having submitted the second highest, or lower ranking bids, winning a catch, which adds complexity from an auction-theoretic perspective. From a traditional market perspective, however, it is the familiar condition that the marginal buyer determines price.

Competitive regime. To begin any price analysis one must look at the competitive regime. Is it a competitive or regulated market? Do any agents on either side of the market have market power? Sellers have organized a common sales mechanism—a complex auction—with the goal of obtaining competitive prices by utilizing competition on the buyer's side of the market. The coöperative on the supply side may, in particular through the minimum price option, exercise market power. Any downright monopolistic adjustment—reducing the quantity supplied in order to increase total profits—is ruled out, however, because the major concern is to clear the market; all offered catches should find buyers.

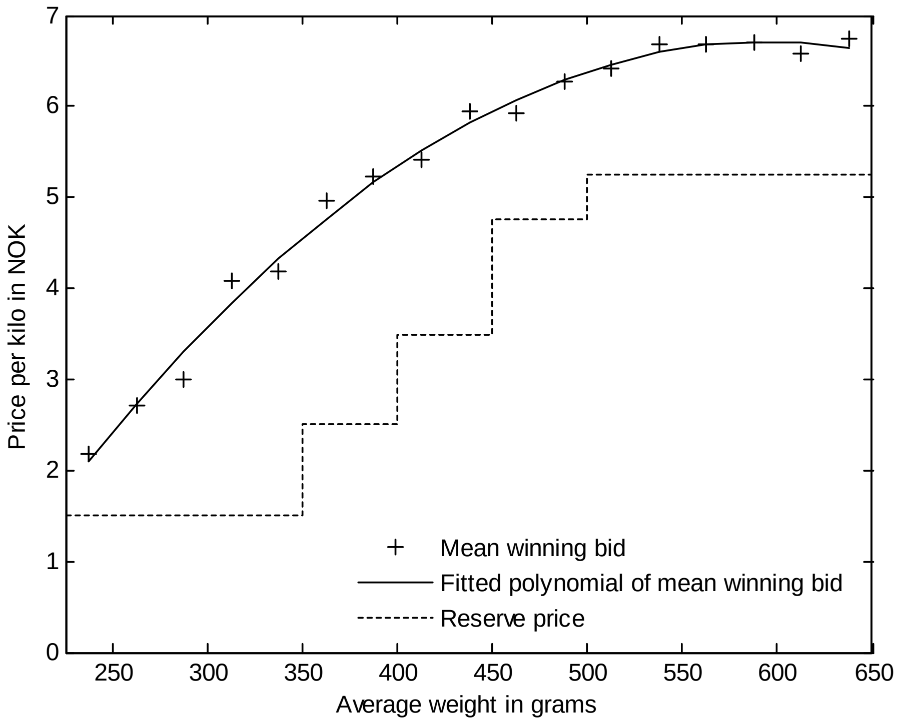
A limited number of buyers exist in this market. This is explained by the fact that the product is a natural resource in short supply. An optimal structure of fish processing plants, both with respect to total capacity and with respect to the number and location of plants, is likely to emerge due to the effects of competition. Sufficient competition for the fish does not, however, require a very large number of demanders. One important goal of the analysis is to establish the number of competitors necessary for competitive prices to emerge.

Since the number of bidders varies a lot from catch to catch, we may conclude that some catches attracting a large number of bidders are characterized by a competitive price regime. The selling of other catches, attracting very few bidders, are more subject to the exercise of market power on the buyers' side. Bids will in this case be formulated to ensure a larger profit for the buyer than a more competitive regime would make possible.

6.2 Price versus weight

The average fish weight of a lot is probably the most important variable determining prices. This is emphasized by the construction of the reserve price scheme. Recall that the reserve price depends on the average fish weight as a step function; see table 5.2 on page 99. In order to examine the functional relation between prices obtained and weight, we divided our sample into 17 weight classes ranging from 225 grams (the minimum observed weight is 230 grams) to the maximum observed weight of 650 with an increment of 25 grams. (The average fish weight reported to buyers are in grams; it is not rounded to the nearest 25 or 50 grams.) We then calculated the mean winning bid within each weight class. The scatter plot of mean winning bids together with the reserve price is depicted in figure 6.1.

Figure 6.1: Price as a function of average fish weight



The discrete points of mean winning prices for different weight classes depicted in figure 6.1 exhibit a clear functional pattern—a close to monotonically increasing and concave function—which we describe by a polynomial curve fit. A polynomial approximation of degree 2 that describes the data best in a least squares sense, turns out to be sufficient. The resulting function of mean prices $\bar{P}$ as a function of average fish weight, w, is

$$\bar{P}(w) = -6.04 + 0.043w - 0.000036w^2.$$

The figure reveals quite a good fit; the relative errors are small. Notice that the plotted domain of $\bar{P}(w)$ in the figure contains the relevant domain. Average weight below 225 or above 650 is unobserved in our complete dataset. Thus, we do not have to worry about the properties of the function outside the plotted domain. Somewhat surprising is the good fit between points and function for low values of w since the data sample in this domain is small. The details of the data for figure 6.1, together with additional statistics, are given in table 6.1 in the appendix to this chapter.

Note, for use in the next chapter, that for weights above 500 grams, the relation between price and weight flattens. The flat section above 500 grams is the source of the non-linear relation of price and weight. Under 500 grams, the relation is quite linear. We conclude that prices definitely vary with weight, but the relationship is consistent with an increasing concave function. Consequently, weight should enter our multivariate linear analysis by both w and w^2.

6.3 Price versus delivery time

Fish deteriorate in quality with time. In particular, bidders express the view that if the fish contain feed, then it is important that the vessel arrives at port quickly. We have information on the date and time of the harvest and on the estimated arrival time at the northern and southern port in the delivery sector. We have computed the time interval in hours from harvest to delivery, denoted delivery time. We investigated the relationship between prices and delivery times. A negative correlation is *expected*; on average, the longer the delivery time, the lower the price.

Surprisingly, there is no strong correlation between prices and delivery times. In fact, contrary to expectations, it turns out to be slightly positive. For lots with a reserve price of NOK 5.25, the correlation is 0.13 between prices and delivery time to the northern port. In figure 6.8 on page 151, we plot the mean price for lots with reserve price 5.25 against delivery time. In addition, we illustrate the linear relationship between them. In table 6.5 on page 150, we report the underlying numbers the figure is based on.

At first sight, the relationship between mean prices and delivery time seems rather erratic. Dividing the delivery time into three subsamples, however, revealed some patterns in the data. Up to a delivery time of 15 hours, mean prices rise with delivery time, contrary to what is expected. One interpretation of this is that bidders do not distinguish between short delivery times, and that other factors determine prices on lots with short delivery times. One such alternative factor is average fish weight. In table 6.5, we report the average fish weight of the different samples with a given delivery time. We see that this obvious candidate for explaining the rising pattern of prices, cannot shed any light on the puzzle. Average fish weight does not vary much between the subsamples and it does not increase with delivery time. Most price records, about 95 percent of observations, have delivery times between 6 and 24 hours. Thus, small-sample variability cannot explain the rising prices for short delivery times.

From delivery times roughly between 16 and 28 hours, the expected negative relationship between prices and delivery time is, however, confirmed. For long delivery times—more than 29 hours—we have few observations, and we note that the high outlier observations for prices of 31 and 37 hours is based on only two observations and one observation respectively, see table 6.5. Thus, small-sample variability can explain this phenomenon.

Another possible candidate for the high prices of very long delivery times is the combined effect of catch location and supply. Late in the season, when supply is low, most catches are harvested to the west of Ireland, a long distance from buyers located on the Norwegian southwestern coast. The positive effect that short supply and the end of season period have on prices may dominate the negative effect of a long delivery time.

We conclude that, at an aggregated level, there is no evidence that high delivery times lead to significantly lower prices. That is, although there might be a negative relation between price and delivery time, the effect is so weak that it is not discovered in a partial analysis. We cannot, however, rule-out that an high delivery time is highly negative for a few catches. In particular, some catches with very high delivery time, go unsold.

6.4 Price versus demand and supply

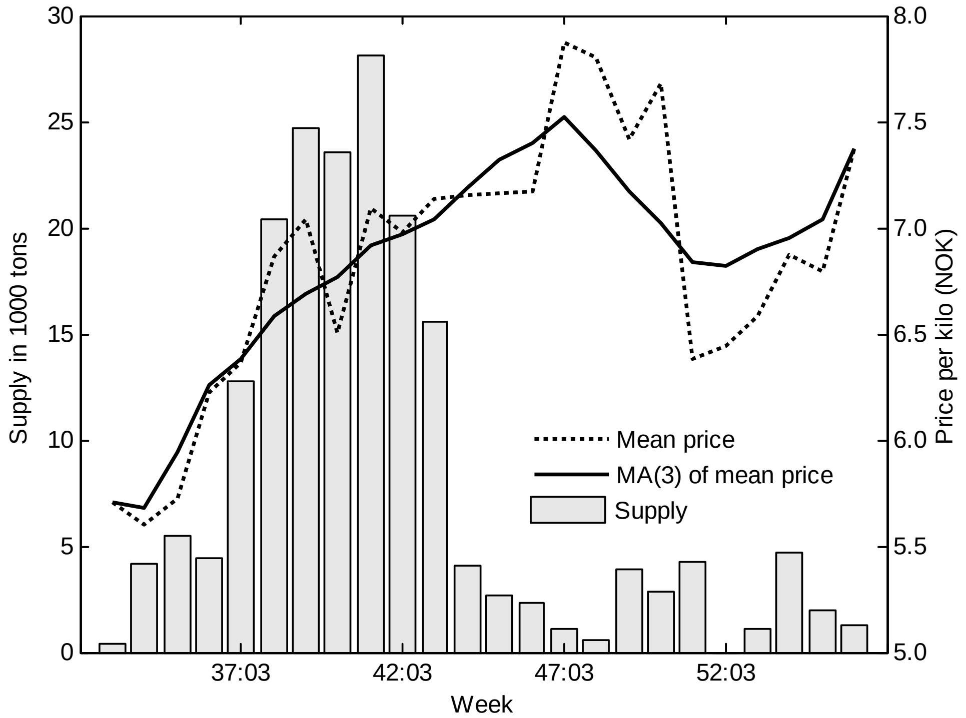
Auction prices vary during the season. Two reasons for price variability over time are: First, general market conditions may affect prices. The highly seasonal pattern of supply is an obvious candidate for price variability. In periods of massive supply, do the prices tend to fall? Demand, likewise, is also subject to shifts, probably caused by competition from other seasonal fisheries and the level of exogenously given output prices. Second, prices may vary because the fish quality changes during the season. For mackerel, average fat content fall late in the season, and this may cause a drop in prices.

Supply and price over time. In order to measure price over time, we need to determine an appropriate time interval in which to compute mean prices. We may examine mean prices from each auction, which will give us a frequency of up to four periods per day. An alternative is to measure mean prices for each day. Using auction or day intervals, however, results in samples within intervals that frequently are too small for reliable statistical measures. We decided on using weekly prices in order to have large enough samples. Since average fish weight is such an important explanatory variable, we controlled for this by concentrating on weekly prices for the most important weight class; i.e., lots with average weight above 500 grams or, equivalently, lots with a reserve price equal to 5.25.

In figure 6.2, we plot the total weekly supply as vertical bars. The mean price of lots with a reserve price equal to 5.25 for each week is shown as a dotted line. For two weeks, we cannot calculate the mean price. In week 52, there was no supply at all, and in week 44, there were no lots offered with relevant reserve prices. We interpolated the mean price for these two weeks by a piecewise Hermite cubic interpolation; see Judd [53, ch. 6.8]. The data on supply in figure 6.2 are from table 5.6, and the price data with additional statistics on frequency and variability measures are reported in table 6.2 on page 147.

In order to smooth the underlying price pattern, we have plotted a three-period moving-average, denoted MA(3), of the mean price as well. Prices dropped significantly in week 40, which is not easy to explain. Apart from this, prices rose during the first part of the season, well past the period where supply peaks. Thus, we observed no downward pressure on prices when supply increased. The steadily-rising mean prices from the beginning of the

Figure 6.2: Price and supply, weekly data



season to around week 47, may be explained by the quality (fat contents) of the fish. The remaining period, with little weekly supply, is characterized by an erratic price pattern. A slight fall in mean prices at the end of the year, and a moderate increase in prices at the end of the season is observed. Weekly samples of lots are largest when the supply is high. Consequently, a larger variability is associated with mean prices at the beginning and the end of the season; see also figure 7.1 on page 162 where we present the price pattern over time for alternative aggregate measures to weekly average prices.

The demand-surplus factor. Assuming the demand-surplus factor (DSF) is a better measure of general market forces than supply alone, let us look at how prices vary with this measure. Although our revealed demand-surplus (see section 5.3.5) is an imperfect measure of demand, we rely on it in order to have an alternative measure of relevant market forces at work. We disregard the time effect now and sort the weekly data by the demand-surplus factor in increasing order. For a plot of the demand-surplus factor over time, see figure 5.3b on page 90.

Will prices tend to increase when the demand-surplus factor increases? Weekly DSF and mean prices are plotted in figure 6.3. Mean prices show an highly-volatile pattern with respect to the DSF level. This happens because we do not plot the data by time. Thus, the time effect from figure 6.2 is now mixed in between the observations in figure 6.3. The tendency, however, is clear enough. Looking at the best linear-predictor (BLP) of the mean prices, we see that prices do tend to rise with the demand-surplus.

It is somewhat reassuring that predictions from general economic demand and supply analysis, is not without power in this market. But we note the high price volatility, and must conclude that traditional market analysis—using the data we have available for such an analysis—have a limited explanatory power of the price formation process at this auction market.

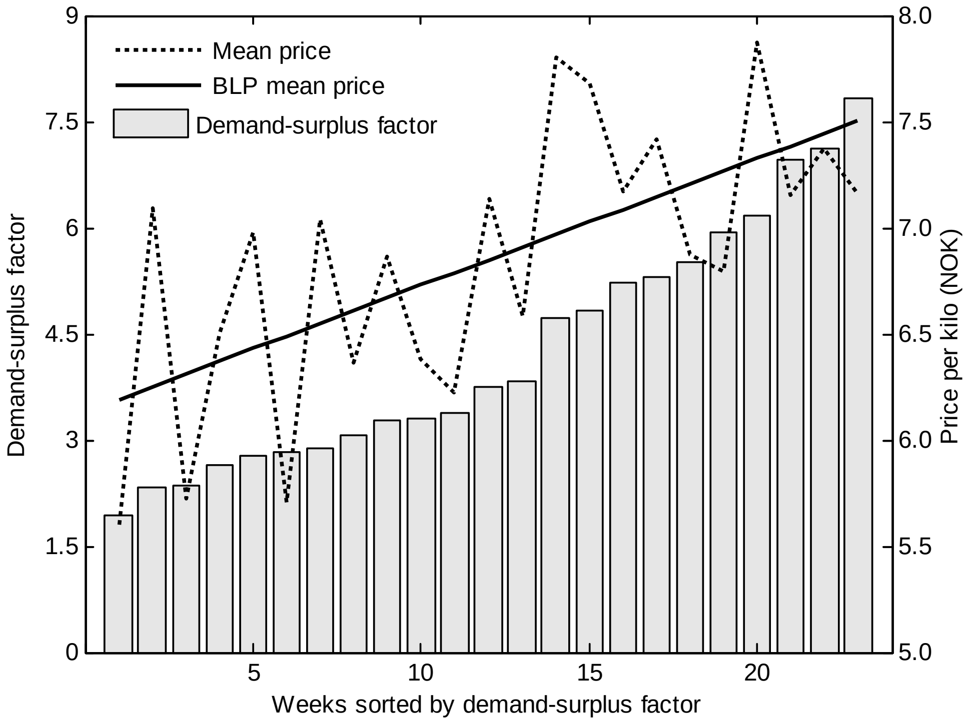
6.5 Price versus the number of bids

The number of submitted bids N is a measure of competition. The more buyers that are bidding for a catch of fish, the more likely, *ceteris paribus*, the price will be high. This follows from both the general economic theory of demand and supply and from game theory if valuations are predominantly private.

From the general demand-supply analysis, the effect of increased N is straightforward and transparent: At a given time, the supply is fixed. Adding buyers will shift the demand curve outward, and result in a predicted higher price. In the traditional model, we do not assume any strategic interactions between buyers.

In game-theoretic auction models, on the other hand, the players' strategies are considered. Bids are formed by balancing the probability of winning and the profit conditional on winning such that expected profit is maximized. The effect of potential buyers is in these models incorporated in the "prob-

Figure 6.3: Price and the demand-surplus factor, weekly data



ability of winning" part of the model in the following way: With increased number of bidders, the likelihood that some competitors have high private valuations increases. Hence, a bidder will have to reduce the span between his bid and his private valuation in order to reduce the possibility that a competitor sets a bid above his bid but below his valuation.

The case of private values. The exact relation between the number of bids and winning bid is well understood in the benchmark models of auction theory. Under the first-price, sealed-bid auction format, with independent and identically-distributed valuations, the number of bidders enters the bid function as a variable that monotonically increases bids, see equation (2.10) on page 19. The effect was illustrated in figure 2.1 on page 21. The theoretical relationship is valid for a homogenous product. Notice, however, that all standard auction formats within the independent private-values paradigm—open and closed, and first-price, second-price, third-price, and so forth—will have in common that expected revenue to the seller increases by increased competition.

When a reserve price is present, a distinction between the potential number of bidders $\mathcal{N}$ and the actual number of submitted bids N must be made. We do not observe $\mathcal{N}$. Although bid strategies such as equation (2.10) states the effect of $\mathcal{N}$ on bids, below we rely on that N will have a similar effect in expected terms.

Introducing affiliated valuations complicates matters. Pinkse and Tan [89] have shown that the bid function may not be increasing in $\mathcal{N}$ at first-price, sealed-bid auctions when valuations are private, but affiliated. Although the competitive effect of increased $\mathcal{N}$ monotonically increases bids, they identify an opposing affiliation effect that affects bidders' expectations about the competitive intensity downwards.

The case of a common value. If bids are predominantly determined by a common, uncertain value, then the relationship between prices and competition is more involved. In addition to the two factors—the probability of winning which increases with bid level, and the utility of winning which decreases with bid level—a third factor enter bid considerations, the so-called *winner's curse*. We explained this phenomenon in chapter 2, section 2.4.2. To summarize briefly, there is an adverse selection problem when bidding for an object with an uncertain common value since the winning bidder will be the one with the most optimistic estimate of the value. Therefore, rational bidding behavior will involve adjusting the bid downward in the presence of many competitors. As noted by Paarsch [84], the empirical implication is that bids will not be a monotone function of the number of bids. Instead, we may have the following scenario: First, bids increase with competition since bid shaving decreases in line with the private-values model. Ultimately, however, bids will stop increasing with competition, since failing to adjust bids downward will expose the winner to the adverse selection effect. A stronger prediction is that optimal bids eventually decrease with the number of bidders. Laffont [60] noted that there seems to be no general proof available for this prediction.

Discussion. Is the assumption of a common value realistic in our market? Production costs are likely to be stable. Firms run production regularly, although they will have to reduce efforts during periods with low supply of raw material. The one factor that could introduce an uncertain common value is the end price of mackerel in export markets. From what we know, the time span between bid decision and realized revenues is relatively short. For producers involved in the main business of selling whole and frozen mackerel, it typically takes 3–4 weeks before the acquired fish are sold in the second-hand market. Although we do not have data on export prices, it is reasonable to assume that export prices do not fluctuate a lot in the short interval. Thus, the uncertainty in future export prices is small. Given this, we expect private values to best describe the auction market, but we notice that the empirical relationship between mean winning prices and N will indicate what model world we are in. Results will, however, have to be interpreted with caution. Pinkse and Tan have noted that the affiliation effect can occur in both private-values and common-value models. Thus, if bids start to decrease for large N, we cannot conclude that we are in a common-value world. We may as well be in a world with private, but affiliated values.

Thus, in both model worlds—the traditional economic model of demand and supply and the auction model with private values—capture the common sense belief that increased demand or harsher competition on the demand side will have an effect on realized market prices. If valuations are private, then we expect prices to be monotonically increasing in N, while prices will drop for large N if valuations are determined by a common value. But the central question of how large the effect of competition is must be determined by an empirical analysis. In this section, we focus narrowly on the relationship by giving a rough, but revealing, indication of the relationship. A more detailed numerical relationship is obtained in the next chapter where we control for several covariates when examining price formation.

Choice of sample. In our case, we have already established that fish weight makes auctioned objects heterogenous, and possible other covariates may as well introduce diversity. In order to analyse how prices vary with respect to the number of active bidders or equivalently the number of submitted bids, we partially adjust for the most important covariate; average weight. To bring in other covariates and to keep them constant, thus satisfying the *ceteris paribus* assumptions, is difficult.

We have to keep the number of different weight classes restricted in order to have a sufficient number of observations. Rather than looking at mean prices obtained for each N and weight class, we look at the price-N relation for each group of reserve prices. Recall that the reserve prices are increasing in average fish weight. We concentrate on the economically most important weight class—average weight above 500 grams—which corresponds to a reserve price equal to 5.25 NOK.

Measure of N. Two questions emerge at this point. What price measure should we use? What measure of N is appropriate? As far as the price is concerned, we have two alternatives. Either we use the maximum bid observed, or we use the bid that was allocated the lot (the winning bid). Although the maximum bid reveals interesting information on underlying valuations and there is some randomness in how lots are allocated to bidders with capacity constraints making the resulting validated prices random in a sense also, we dismiss this measure at this point. Since this is an empirical market analysis, we regard the winning bid as the relevant price variable. This is the price obtained by the seller and is interpreted as the willingness to pay for the marginal buyer when buyers with higher bids have their demand satisfied by other lots.

Given that we choose to use the winning bid rather than the maximum bid as a price measure, this has consequences for how we measure N. The auction format encourages, through the capacity constraint option, buyers to bid on as many lots they like. When bidders bid on more lots than they can take, however, the number of submitted bids becomes dubious for our purpose.

Fundamental to the analysis is the idea that N is a measure of competition. The more bids a lot attracts, the more likely it is that an "high" bid is submitted since competition forces bids upwards. The correct measure of competition, thus, should measure the real competition a lot is subject to. In the validation process, it turns out that some bids are refuted because the bidders have fulfilled their demand elsewhere. Thus, we find it reasonable to adjust the number of submitted bids by subtracting the bids higher than the winning bid for each lot. We do this by identifying for each lot the position of the winning bid; i.e., whether it was the highest, second-highest, and so forth. Then we subtract the number of bids above the winning bid. By this procedure, we obtain a better measure of N since bids refuted in the validation process are not counted.

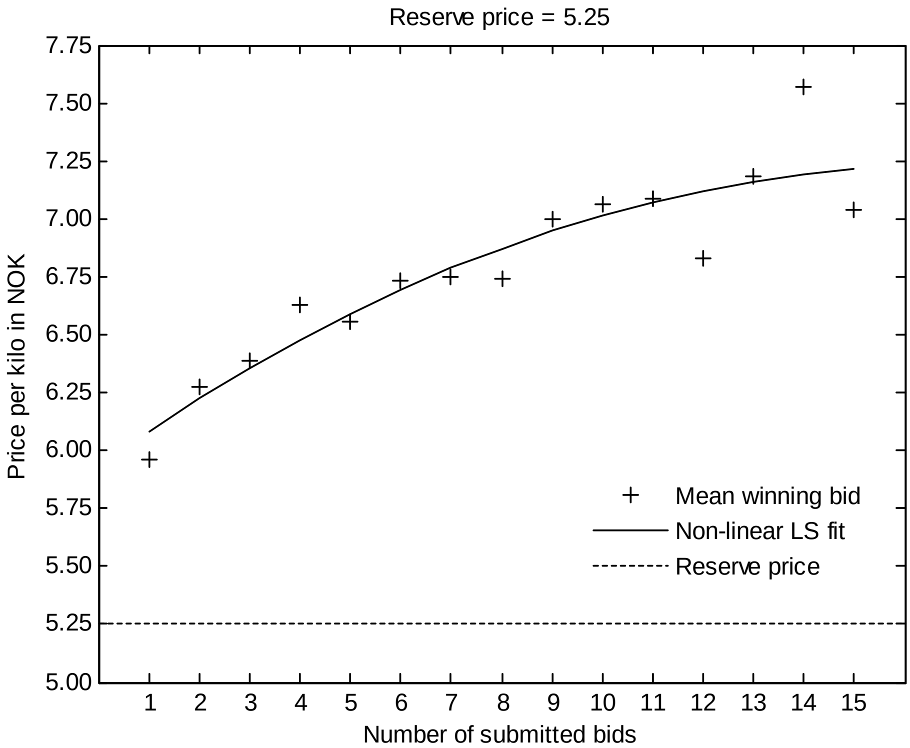
A bid can be refused because the relevant bidder has (1) reached his maximum ton limit; (2) not obtained his minimum ton limit; or (3) reached his maximum vessel limit. In addition, (4) the bidder may be a refused outside bidder. By far, the most common cause for an high bid is being refused is that the bidder's maximum ton constraint binds.

Although we can take out all high irrelevant bids, some bids lower than the winning bids may also turn out to be irrelevant when the agents who tendered those bids have binding constraints. We can partially adjust for this. Since we have information on both the maximum ton constraint and how much quantity a bidder was allocated after validation, we can take out the bids from bidders with binding maximum ton limits. For each lot and each bidder that did not win the lot, we compared the allocated quantity with the maximum quantity limit. If the allocated quantity plus the quantity of the lot in question exceeds the set limit, then we do not count the bid since the bidder could not take the lot anyway if winning.

The total effect of this procedure is that 24.35 percent of bids are removed. An unwanted side effect is that the number of observations for some groups is small.

Results. We depict in figure 6.4 the resulting relation of obtained price per kilo and the number of bidders for lots with a reserve price equal to

Figure 6.4: Price as a function of N, binding bids



NOK 5.25. We plot the mean of winning bids for each N, and draw the least squares line of a polynomial fit of degree 2. A reasonable fit with small errors is observed for N up to and including 11. The volatility of the mean prices seems to increase for $N > 11$. This is probably explained by the fact that the number of observations is small at this level for N, making the resulting statistics uncertain. Only 2.74 percent of all lots have a number of submitted bids above 11. In particular, the observed "outlier" price values for the maximum values of submitted bids—N equal to 14 and 15—have only 5 and 2 observations respectively.

The illustrated functional relation of the second degree polynomial approximation of mean winning bids is:

$$\bar{P}(N) = 5.94 + 0.153N - 0.0045N^2. \tag{6.1}$$

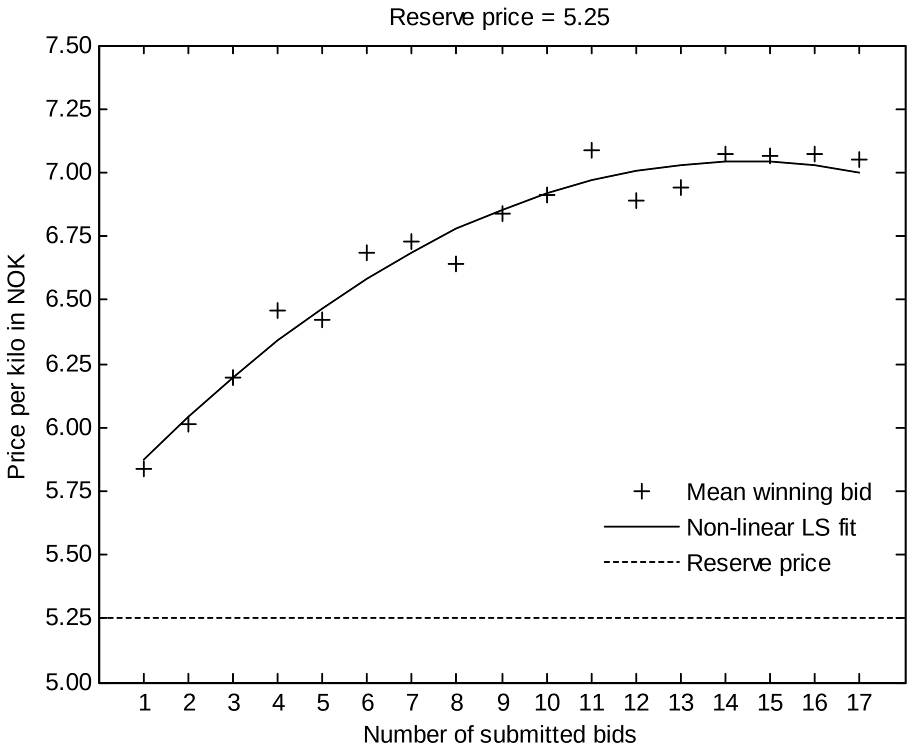
The curvature of $\bar{P}(N)$ is low. The relation is close to linear, the corresponding linear approximation is $\bar{P}(N) = 6.144 + 0.08N$. The appealing feature of a second degree approximation is that it captures the expected effect of competition. Increasing N from 1 to 2 is a significant increase in competition, while an extra competitor when N is, say, 12 has a marginal effect on competition. Competition sets in at a certain level of N in the sense that additional increases in N do not increase prices.

Admittedly, the problem of adjusting N is an involved one. If we do think that some information is lost when taking away bidders with binding constraints *ex post*, then it is somewhat assuring that the relation between mean price and N is rather stable anyway. In figure 6.5, we plot the same relation as in figure 6.4, but now using all bids for the classifications of N. This results in less volative mean prices for high values of N since the number of observations is increased. Note that the competitive effect is now more pronounced; the mean price tapers off at the upper end. The functional form of the second degree polynomial least squares fit of figure 6.5 is: $\bar{P}(N) = 5.73 + 0.156N - 0.0045N^2$. The corresponding function when forcing a linear fit on the data points is: $\bar{P}(N) = 6.00 + 0.074N$. The slope for N above 8 is close to horizontal. A reasonable interpretation of this is that (sufficiently) competitive prices are obtained at $N \geq 9$.

At the risk of being pedantic, we note that the second-order polynomial approximation function has the unfortunate property of reaching a peak and then actually take on quickly decreasing values for N larger than we have in our sample. Although decreasing prices for large N are predicted if we have a common-value auction, we argued against this above. And even if assuming a bid generating process of common-value, prices will not fall as much as predicted by equation (6.1). The predictive properties of equation 6.1 is therefore weak for large N. The easy solution to this is simply to say that mean prices for large N are expected to be close to the competitive price level that seems to characterize mean prices when the number of bidders is between 14 and 17.

We conclude from figures 6.4 and 6.5 that there is a fairly stable relation between mean auction prices and the number of submitted bids in our

Figure 6.5: Price as a function of N, all bids



dataset. The pattern does not contradict the hypothesis that valuations are predominantly private. We do not observe a clear drop in prices for large N. The relatively low price for the last observation ($N = 15$) in figure 6.4 is probably the effect of a small sample size rather than an indication of an adverse selection effect which is predicted by the common-value model.

So far we have worked with mean prices. We computed mean prices for different weight classes and for different samples of N, and for both cases, we found a reasonable good fit to an increasing concave price function. However, predicting *single-lot prices* based on N and average weight is entirely another matter. A lot of volatility is introduced when moving the analysis from mean lot prices to single lot prices. The range of prices and measures of volatility with respect to average fish weight are reported in table 6.1. A couple of related tables reporting statistics on the mean prices grouped by the number of submitted bids, are tables 6.3 (all bids) and 6.4 (binding bids) on pages 148 and 149.

In order to depict the volatility measures in tables 6.3 and 6.4, we present some box plots in figure 6.7. A box plot summarizes effectively some important statistics and outliers of a data material. In our case, the box plot figures show a box and whisker plot for the variable "Winning bid" grouped by the variable "Number of submitted bids"; i.e., we group all lots with a reserve price equal to NOK 5.25 by the number of bids they received and then present the statistics of the winning bid for each sub group. The box has lines at the lower quartile (lower horizontal line), median, and upper quartile (upper horizontal line) values. The median is preferred over the mean since it is invariant to extreme values. The whiskers are lines extending from each end of the box to show the extent of the rest of the data that are within 1.5 times the interquartile range from the ends of the box. Outliers are data with values beyond the ends of the whiskers and are represented by a plus. If there is no data outside the whisker, a dot is placed at the bottom whisker. The notches of each box around the medians represent a robust estimate of the uncertainty about the medians for box-to-box comparison. Boxes whose notches do not overlap, indicate that the medians of the two groups differ at the 5 percent significance level.[1]

Although figures 6.4 and 6.5 show a nice relationship between mean winning bids and the number of submitted bids, we see from figure 6.7 that the relationship between all winning bids and N is characterized by severe volatility. We can conclude from the notches around the medians in the box plots, that the medians differ between the lowest and the highest values of N. Apart from that, the main message of figure 6.7 is that the volatility of mean or median prices warrants a multivariate analysis in order to gain an understanding of what determines prices in this market.

[1] The description of the box plot is based on the source: <http://www.matworks.com/access/helpdesk/help/toolbox/stats/> Web accessed: April 30, 2008.

6.6 Money left on the table

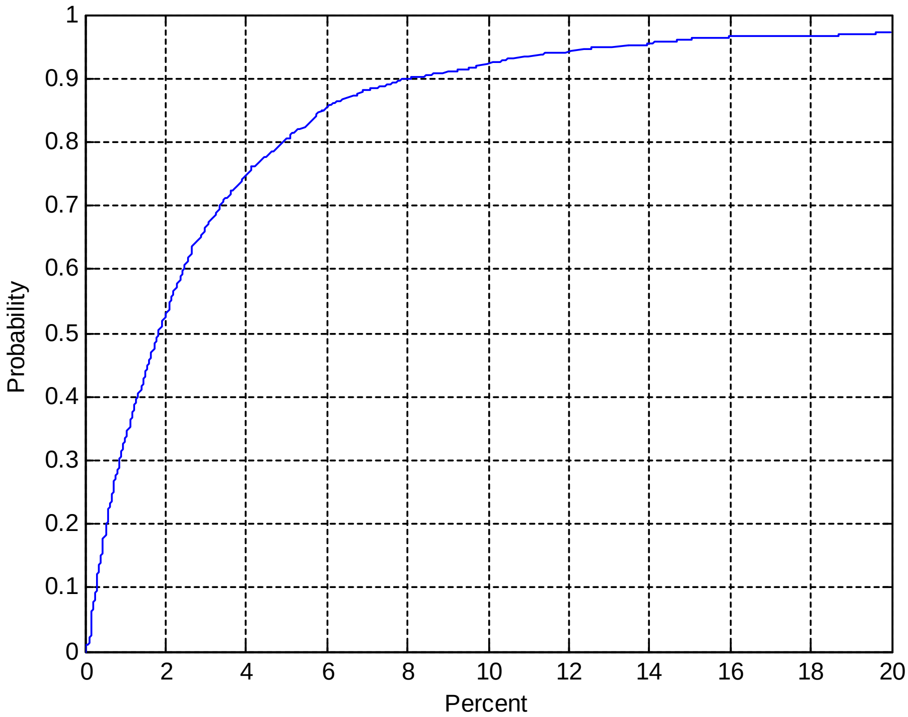
We end this chapter by examining an alternative concept of competition, the so-called *money left on the table* (MLOT) measure. This is defined to be the difference between the winning bid and the closest non-winning bid. In our case, where bids are per kilo and quantities of lots or catches differ, we may express the money left on the table for a single catch in NOK or in percent. Due the variability in prices between different weight classes, we prefer to measure it in terms of percent. If there is only one bid, we measure MLOT as the difference between the winning bid and the reserve price.

In a common-value model, the measure may indicate something about the uncertainty associated with the value of the object. In a private-values model, the interpretation is different; the "money left on the table" is in some sense a measure of competition.

In figure 6.6, we have plotted the empirical distribution function of MLOT for all catches. 60.7 percent of the catches leave less than 2.5 percent on the table. The percentage of catches leaving less than 10 percent is 92.2 percent. The 2.5 percent of catches with MLOT larger than 20 percent are not plotted in figure 6.6 (for purely visual reasons; it would extend the x-axis too much). Most of these outliers are characterized by having a low reserve price that does not reflect the market price *and* only one submitted bid. A mere 11 out of 1144 catches have a MLOT measure above 40 percent. Four catches leave more than 100 percent of the table. The maximum percentage left is 188 percent.

The MLOT figures indicate that this is a fairly competitive market. Bidders are experienced, and leave little money on the table. Alternatively, the figures tell us that there is not much evidence of the winner's curse or "trembling hand" mistakes.

Figure 6.6: Empirical cumulative distribution function of “Money left on the table” in percent



6.A Appendix: Tables and figures

Table 6.1: Price as a function of weight

From weight	To weight	Lot count	Mean price	Std. price	Min. price	Max. price	Reserve price
225	249	2	2.19	0.85	1.59	2.79	1.50
250	274	4	2.70	1.06	1.50	4.07	1.50
275	299	1	2.99	0.00	2.99	2.99	1.50
300	324	7	4.08	1.28	1.52	5.80	1.50
325	349	15	4.20	0.79	2.55	5.09	1.50
350	374	17	4.97	0.73	3.55	6.21	2.50
375	399	41	5.23	0.84	3.50	6.51	2.50
400	424	55	5.41	1.01	3.50	7.06	3.50
425	449	85	5.95	0.93	3.50	7.14	3.50
450	474	87	5.94	0.85	4.75	7.61	4.75
475	499	199	6.26	0.79	4.75	7.59	4.75
500	524	165	6.41	0.74	5.25	7.81	5.25
525	549	184	6.68	0.68	5.25	7.69	5.25
550	574	272	6.67	0.69	5.28	7.86	5.25
575	599	225	6.70	0.59	5.29	7.70	5.25
600	624	108	6.59	0.55	5.44	7.77	5.25
625	650	27	6.75	0.76	5.53	7.99	5.25

Figure 6.7: Box plot of winning bids versus the number of submitted bids

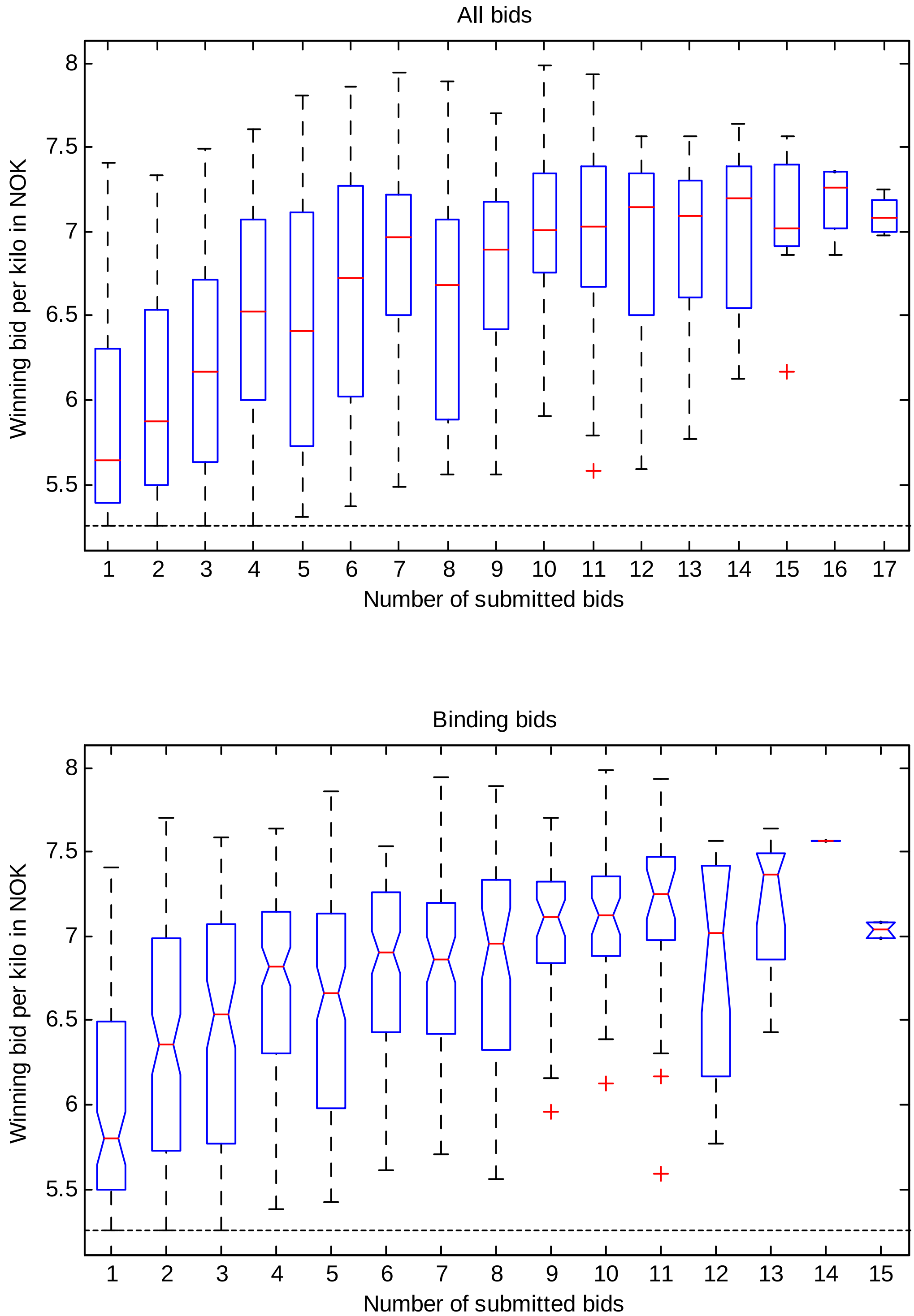


Table 6.2: Price per week

Year	Week	Count lots	Mean price	Max. price	Min. price	Std. price	Std. mean price	MA(3) mean price[a]
2003	33	11	5.71	6.02	5.28	0.26	0.08	5.71
2003	34	66	5.60	6.02	5.25	0.18	0.02	5.68
2003	35	90	5.73	6.16	5.29	0.22	0.02	5.94
2003	36	45	6.22	6.91	5.44	0.33	0.05	6.26
2003	37	100	6.36	6.93	5.25	0.40	0.04	6.38
2003	38	145	6.86	7.27	5.61	0.30	0.03	6.58
2003	39	146	7.04	7.70	5.30	0.47	0.04	6.69
2003	40	101	6.50	7.63	5.25	0.72	0.07	6.77
2003	41	105	7.09	7.64	5.31	0.46	0.04	6.91
2003	42	101	6.98	7.66	5.88	0.41	0.04	6.97
2003	43	17	7.14	7.69	5.81	0.55	0.13	7.04
2003	44	0[b]	7.15	na	na	na	na	7.18
2003	45	5	7.16	7.41	6.89	0.20	0.09	7.32
2003	46	6	7.17	7.45	6.82	0.21	0.09	7.40
2003	47	4	7.88	7.94	7.77	0.08	0.04	7.53
2003	48	3	7.80	7.99	7.61	0.19	0.11	7.36
2003	49	12	7.42	7.55	7.23	0.11	0.03	7.17
2003	50	5	7.68	7.89	7.41	0.22	0.10	7.02
2003	51	11	6.38	7.15	6.06	0.38	0.11	6.83
2003	52	0[b]	6.44	na	na	na	na	6.82
2004	1	1	6.58	6.58	6.58	0.00	0.00	6.90
2004	2	2	6.88	7.12	6.63	0.35	0.25	6.95
2004	3	1	6.79	6.79	6.79	0.00	0.00	7.04
2004	4	4	7.37	7.55	7.16	0.17	0.09	7.37

[a] MA(3): Three-period moving average.

[b] Mean price is interpolated.

Table 6.3: Summary statistics of winning and maximum bids grouped by the number of all submitted bids

	Winning bids					Maximum bids				
N	Count	Mean	Std.	Min.	Max.	Count	Mean	Std.	Min.	Max.
1	31	5.89	0.66	5.25	7.41	40	5.75	0.64	5.25	7.41
2	44	6.04	0.59	5.25	7.33	48	6.05	0.57	5.28	7.33
3	70	6.24	0.66	5.25	7.49	76	6.28	0.66	5.25	7.49
4	89	6.51	0.64	5.25	7.61	91	6.55	0.64	5.26	7.61
5	121	6.40	0.72	5.31	7.81	121	6.47	0.69	5.42	7.81
6	99	6.64	0.69	5.37	7.86	99	6.72	0.67	5.52	7.86
7	100	6.83	0.55	5.49	7.94	100	6.92	0.55	5.56	7.94
8	100	6.57	0.63	5.56	7.89	100	6.66	0.62	5.69	7.89
9	77	6.79	0.55	5.56	7.70	77	6.87	0.54	5.59	7.70
10	65	7.00	0.43	5.91	7.99	65	7.07	0.40	6.07	7.99
11	59	6.96	0.52	5.58	7.93	59	7.08	0.44	6.07	7.93
12	38	6.91	0.56	5.59	7.57	39	7.02	0.51	5.59	7.59
13	36	6.92	0.49	5.77	7.57	36	7.02	0.48	5.77	7.57
14	20	7.01	0.51	6.13	7.64	20	7.11	0.41	6.33	7.64
15	18	7.05	0.41	6.17	7.57	18	7.09	0.42	6.17	7.57
16	4	7.19	0.24	6.86	7.36	4	7.21	0.25	6.87	7.41
17	7	7.10	0.11	6.98	7.25	7	7.22	0.17	6.99	7.42
19	2	6.78	0.16	6.67	6.89	2	6.91	0.04	6.88	6.94
20	1	6.86	0.00	6.86	6.86	1	6.88	0.00	6.88	6.88

Table 6.4: Summary statistics of winning and maximum bids grouped by the number of binding (*ex post*) submitted bids

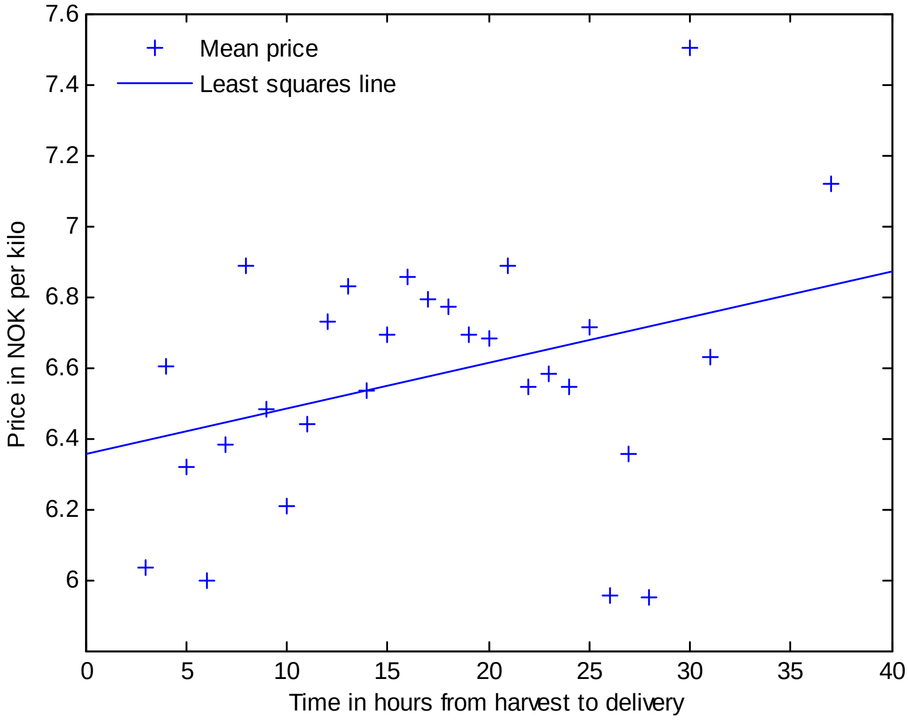
	Winning bids					Maximum bids				
N	Count	Mean	Std.	Min.	Max.	Count	Mean	Std.	Min.	Max.
1	98	6.02	0.60	5.25	7.41	98	6.15	0.65	5.25	7.41
2	123	6.35	0.70	5.25	7.70	123	6.49	0.72	5.26	7.70
3	106	6.46	0.70	5.25	7.59	106	6.53	0.69	5.25	7.59
4	126	6.69	0.59	5.38	7.64	126	6.78	0.58	5.42	7.64
5	136	6.62	0.65	5.42	7.86	136	6.69	0.65	5.42	7.86
6	108	6.78	0.56	5.61	7.53	108	6.83	0.56	5.61	7.54
7	77	6.80	0.56	5.71	7.94	77	6.85	0.58	5.71	7.94
8	56	6.80	0.62	5.56	7.89	56	6.85	0.59	5.59	7.89
9	46	7.02	0.43	5.96	7.70	46	7.07	0.42	6.16	7.70
10	48	7.08	0.37	6.13	7.99	48	7.14	0.32	6.48	7.99
11	26	7.13	0.53	5.59	7.93	26	7.15	0.53	5.59	7.93
12	17	6.89	0.63	5.77	7.57	17	6.92	0.63	5.77	7.57
13	10	7.21	0.41	6.43	7.64	10	7.21	0.41	6.43	7.64
14	2	7.57	0.00	7.57	7.57	2	7.57	0.00	7.57	7.57
15	2	7.04	0.06	6.99	7.08	2	7.04	0.06	6.99	7.08

Table 6.5: Summary statistics of prices grouped by delivery time. Lots with reserve price 5.25

		Price per kilo				
Time[a]	Count	Mean	Std.	Min.	Max.	Weight[b]
3	5	6.03	0.64	5.44	7.11	540
4	5	6.60	0.75	5.67	7.34	555
5	7	6.32	0.69	5.57	7.34	581
6	14	6.00	0.68	5.36	7.36	559
7	12	6.38	0.75	5.49	7.31	567
8	7	6.89	0.54	5.79	7.49	546
9	25	6.48	0.69	5.31	7.44	545
10	70	6.21	0.69	5.36	7.51	556
11	59	6.44	0.75	5.25	7.57	555
12	58	6.73	0.77	5.25	7.86	554
13	65	6.83	0.62	5.36	7.70	558
14	126	6.53	0.73	5.28	7.89	562
15	95	6.69	0.64	5.28	7.64	566
16	98	6.86	0.57	5.32	7.99	562
17	79	6.79	0.47	5.53	7.81	564
18	47	6.77	0.56	5.25	7.94	572
19	79	6.69	0.60	5.40	7.64	561
20	40	6.68	0.62	5.25	7.52	563
21	26	6.89	0.42	5.78	7.51	553
22	16	6.55	0.70	5.55	7.49	570
23	11	6.58	0.64	5.43	7.40	545
24	12	6.54	0.61	5.50	7.31	567
25	5	6.71	0.48	6.35	7.33	549
26	7	5.96	0.57	5.25	6.81	552
27	5	6.35	0.67	5.50	7.16	542
28	4	5.95	0.43	5.31	6.23	570
30	2	7.50	0.06	7.46	7.55	555
31	1	6.63	0.00	6.63	6.63	550
37	1	7.12	0.00	7.12	7.12	560

[a] Delivery time in hours.
[b] Mean fish weight in grams.

Figure 6.8: Mean price versus delivery time. Lots with reserve price 5.25



Chapter 7

Price formation: Multivariate analysis

7.1 Introduction

In this market, with relatively few agents on the buyer's side, prices are determined by three main factors: product-specific characteristics, market-specific characteristics in a broad sense, and firm-specific characteristics. Product-specific characteristics are common to all buyers, market-specific characteristics are also relatively common given that the majority of buyers are engaged in the same export business of selling unprocessed frozen fish. Firm-specific factors, in particular cost structure and capacity, are by nature more diverse.

Quality of data. The dataset presented in chapter 5 and in appendix A is rich and detailed. In fact, it represents an entire season and all information concerning auctioned objects given to potential bidders is available. Data on final prices and quantities are available for each market transaction as well. This puts us in a position where we can analyse price formation in a market with very detailed micro data. Contrast this to the far more usual situation in economics: When analysing the market of a specific product, a researcher often does not even have a good measure of price, not to mention the underlying variables determining prices. For example, in many areas of industrial organization, prices that enter the analyses are traditionally based on broad aggregates, both with respect to time (monthly data at best, often quarterly or yearly data is used) and with respect to quality of products. A common approach to analysing fish price elasticities to export markets is to obtain prices by simply calculating so-called unit prices by dividing the total export value of a product within a time period by the relevant total export quantity. Obviously, such aggregated measures of price introduce a lot of noise into any empirical price analysis.

Empirical approach. One fundamental market factor is the derived-demand and supply conditions as well as prices obtained in second-hand (export) markets. Obviously, the prices of all kinds of end-products put a ceiling on the prices in the wholesale market. Retail prices are formed under a complex set of constraints where demographics, the more or less erratic change of consumer tastes and substitute products are but a few of the explanatory variables. Clearly, trying to incorporate these general market variables, quickly becomes very complex, if not impossible. The effort is not worthwhile since data noise and lack of relevant information make the results suspicious. The beauty of auction data is that, in principle, bids reveal all relevant information available to the price formation process. Buyers form their bids based on underlying valuations which are formed under rational expectations about all relevant market variables. To uncover the valuations, we have to resort to a structural approach. A structural approach requires us to commit to a specific auction-theoretic model that explains winning bids. Given the complexities of the aggregate market, we do not set up a structural auction-theoretic model for explaining bids in this chapter. Instead, we utilize the dataset to determine the effects that several variables with potential explanatory power have on realized market prices. However, since we perform reduced-form estimation, we have to avoid drawing strict causal inferences. Potential omitted-variables bias can lead to estimates that are larger than the causal effect.

The next factor in price formation, firm-specific characteristics, may be revealed to a certain degree by gathering information from individual firms. Public accounting information and information on capacities, and so forth can be of use in this context. Although we have some firm-specific information at hand, we use a direct approach that summarizes any asymmetries on the buyers' side effectively: We test whether the identity of the winning bidder has any effect on prices.

The task then is to regress prices on several covariates. We are mainly interested in analysing the effect of directly observable covariates like weight and other quality variables. To pin-down the effect of these with best precision, we need to control for several other covariates. In particular, we should control for short-term market conditions. This implies that we include several regression variables that empirically turn out to have explanatory power.

7.2 Regression model and interpretation of coefficients

The explained variable in our model is the winning bid for each lot, which we denote by P_t for lot t. For now, denote the j binary variables (dummy variables) by D_j and the i other discrete and continuous variables by X_i. We propose a regression model of the general form:

$$P_t = \exp\left[\beta_0 + \sum_i \left(\beta_i X_{it} + \gamma_i X_{it}^2\right) + U_t\right] \times \prod_j \left(1 + d_j\right)^{D_{jt}} \tag{7.1}$$

where U_t is a residual. We suppress the observation-specific subscript t in the remainder. Equation (7.1) may be transformed to a left-side semi-logarithmic (log-linear) functional form:

$$\begin{aligned} \log P &= \beta_0 + \textstyle\sum_i \left(\beta_i X_i + \gamma_i X_i^2\right) + \sum_j \log\left(1 + d_j\right) D_j + U \\ &= \beta_0 + \textstyle\sum_i \left(\beta_i X_i + \gamma_i X_i^2\right) + \sum_j \delta_j D_j + U. \end{aligned} \tag{7.2}$$

Note that we have set $\delta_j \equiv \log\left(1 + d_j\right)$ above. Only a couple of squared terms of covariates will be considered incorporated in the regression equation. We have argued that N (submitted bids) and average fish weight are reasonable candidates since they exhibit a concave increasing relationship to mean prices. The motive for using a semi-logarithmic functional form is that interpretations of estimated coefficients are intuitive with respect to elasticities. The semi-logarithmic form is common to use when some covariates are binary since the alternative log-log form (which also gives easily interpretable coefficients) is inappropriate in that case. The price elasticity of a continuous variable, E_{P,X_i}, i.e., the percentage effect on the expected price of a given percentage increase in X_i, will then take the form:

$$E_{P,X_i} \equiv \frac{\partial P}{\partial X_i}\frac{X_i}{P} = P \times (\beta_i + 2\gamma_i X_i)\,\frac{X_i}{P} = \beta_i X_i + 2\gamma_i X_i^2.$$

In order to find the percentage effect on the expected price due to the presence of a dummy variable, a bit more care must be exercised since the derivative of a binary variable does not exist. Let P_{j0} and P_{j1} be the values of the dependent price variable when binary variable D_j is equal to 0 and 1 respectively. The percentage effect on expected P when the dummy effect is present is then, using equation (7.2), equal to

$$100 \times \frac{P_{j_1} - P_{j_0}}{P_{j_0}} = 100 d_j.$$

The naïve—but not uncommon—interpretation of $100\delta_j$ as the percentage effect on the dependent variable is not correct, as noted by Halvorsen and Palmquist [41]. Estimating equation (7.2) gives us δ_j while we are interested in d_j for purposes of interpretation. Since $\delta_j = \log(1 + d_j)$, we have that $d_j = \exp(\delta_j) - 1$, or equivalently that the percentage effect on P when D_j is present, is:

$$100 \times \frac{P_{j_1} - P_{j_0}}{P_{j_0}} = 100\,[\exp(\delta_j) - 1]\,. \tag{7.3}$$

Kennedy [54] has noted that estimating d_j by $[\exp(\hat{\delta}_j)-1]$ produces a biased estimator. Assuming errors to be lognormally distributed, and relying on a result from Goldberger [36], Kennedy has proposed the following estimator:

$$\hat{d}_j = \exp\left[\hat{\delta}_j - 0.5\hat{\mathcal{V}}\left(\hat{\delta}_j\right)\right] - 1 \tag{7.4}$$

where $\hat{\mathcal{V}}\left(\hat{\delta}_j\right)$ is the usual unbiased estimator of the variance of $\hat{\delta}_j$. Although this estimator is still biased, it is less biased than $[\exp(\hat{\delta}_j)-1]$. The estimated elasticity is then equal to $100\hat{d}_j$. Giles [34] showed that Kennedy's estimator yields interpretations of dummy variables that differ negligibly from those that would result from using the minimum variance unbiased estimator.

Finally, it is desirable to have an estimate of the corresponding uncertainty of the estimated dummy variable elasticity. Notice that $\hat{d}_j$ is a non-linear function of the parameter estimate $\hat{\delta}_j$. Garderen and Shah [32] have claimed that the traditional approach of obtaining the variance of a non-linear function of parameter estimates—the Delta method—will lead to over-estimation of the variance in the present context. Instead, they propose a simple approximation to the exact minimum variance unbiased estimator of the variance which works well in practice. Expressed in terms of the standard deviation of $\hat{d}_j$, their estimator—which we shall rely on—is:

$$\widehat{se}\left(\hat{d}_j\right) = \left[100^2 \exp\left(2\hat{\delta}_j\right)\left\{\exp\left[-\hat{\mathcal{V}}\left(\hat{\delta}_j\right)\right] - \exp\left[-2\hat{\mathcal{V}}\left(\hat{\delta}_j\right)\right]\right\}\right]^{0.5}. \quad (7.5)$$

7.3 Estimation method

We have formulated a linear regression model in equation (7.2). Let n be the number of observations and p the number of explanatory variables or covariates (including a constant term). The linear regression model is expressed as $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{U}$ where $\boldsymbol{Y}$ and $\boldsymbol{U}$ are $(n \times 1)$ vectors of explained variables and residuals respectively, $\boldsymbol{X}$ is an $(n \times p)$ matrix of explanatory variables (the design matrix), and $\boldsymbol{\beta}$ is the $(p \times 1)$ vector of coefficients to be estimated. A single observation i of the explained variable is denoted Y_i while the corresponding $(1 \times p)$ vector of covariates for observation i is $\boldsymbol{x}_i$.

Typically, characteristics of the residual vector are important for the choice of estimation method. If the errors have the same distribution whatever the realizations of $\boldsymbol{X}$, then the work horse of linear regression models, ordinary least squares (LS), will give us the conditional mean function $\mathcal{E}\left[Y|\boldsymbol{X}\right] = \boldsymbol{X\beta}$; i.e., how the mean of $\boldsymbol{Y}$ changes with the covariates. However, the prevalence of some outliers—in particular, some extra high prices from time to time, and some prices at the minimum price—suggests that we estimate our relationship by a robust regression technique as well. By robust we mean that the estimator is unaffected by outliers. Relying on the same assumptions for the errors as above, the least absolute deviations (LAD) estimator is an estimate of the conditional *median* function; i.e., how the expected median of $\boldsymbol{Y}$ changes with the covariates. In the same way as the median (rather than the mean) is a robust location parameter of an empirical distribution, is LAD robust compared to LS. For large samples, the need for a robust estimator is less critical.

The standard assumption of ordinary least squares that errors are spherical (the twin assumptions of homoskedasticity and nonautocorrelation) implies that $\boldsymbol{X}$ only affects the location of the conditional distribution of $\boldsymbol{Y}$. Covariates may, however, affect other aspects of the distributional shape of the conditional distribution of $\boldsymbol{Y}$. Going from the assumption of homoskedastic to heteroskedastic errors, is just one of the many ways that affects the stochastic relationship between variables.

Quantile regression, first introduced by Koenker and Bassett [58], is an estimation method that potentially may provide a more informative empirical analysis than least squares alone. The quantile regression estimates the conditional quantile of the dependent variable. In general, the θ^{th} quantile regression ($0 < \theta < 1$) solves (see Buchinsky [15]):

$$\min_{\boldsymbol{\beta}} \left\{ \sum_{i:Y_i \geq \boldsymbol{x}_i\boldsymbol{\beta}} \theta \left| Y_i - \boldsymbol{x}_i\boldsymbol{\beta} \right| + \sum_{i:Y_i < \boldsymbol{x}_i\boldsymbol{\beta}} (1-\theta) \left| Y_i - \boldsymbol{x}_i\boldsymbol{\beta} \right| \right\}. \tag{7.6}$$

The LAD estimator is a special case of the optimization problem (7.6) as it is the solution of (7.6) when $\theta = 0.5$. Following Buchinsky [15], some attractive features of quantile regression are: (1) the regression model can be used to characterize the entire conditional distribution of $\boldsymbol{Y}$ given $\boldsymbol{X}$; (2) because the quantile regression objective function is a weighted sum of absolute deviations, the estimator is robust to outlier observations on $\boldsymbol{Y}$; (3) quantile regression estimators may be more efficient than least squares estimators if errors are non-normal.

The problem in (7.6) can be solved by linear programming. Portnoy and Koenker [92] (see also Koenker [57, section 6.4.2]) have described an interior point algorithm, denoted the Frisch–Newton method, that is effective for large-scale computations. We relied on an implementation of this algorithm in the computations.

Standard errors of the quantile estimates may either be estimated from the asymptotic covariance matrix or computed by Efron's [25] bootstrap. We prefer to make no particular assumptions on the distribution of errors. We assume only that errors are independent. The so-called design matrix bootstrapping estimator, analysed and recommended by Buchinsky [15], is common. Samples of size n of pairs $(Y_i, \boldsymbol{x}_i)$ are drawn at random from the original dataset with replacement. For each sample, we compute the parameter estimate. After resampling R times, we use the empirical distributions of the samples of bootstrapped parameters to compute standard errors or confidence intervals.

For the LAD estimator, we chose to compute the asymptotic standard errors as well. Greene [38] reports a sample estimator for the asymptotic covariance matrix of the LAD estimator equal to:

$$\left[\frac{0.5}{f\left(0\right)}\right]^2 \left(\mathbf{X}^\top \mathbf{X}\right)^{-1}$$

where $f\left(0\right)$ is the density of the disturbances evaluated at 0. A convenient estimator of $f\left(0\right)$, again according to Greene, is

$$\hat{f}\left(0\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h}\kappa\left(\frac{\varepsilon_i}{h}\right)$$

where ε_i is the residual for observation i, $\kappa\left(\cdot\right)$ is a kernel function, and h is the bandwidth.

7.4 Explanatory variables and expected sign

Our purpose is to identify the determinants of winning bids. Three different categories of variables will be considered. First, we have the product-specific variables presented to buyers before the auction, see section 5.4 for a description. The product-specific variables relate to commonly agreed upon quality characteristics of the product and are public information. Next, we have what we term market-specific variables, which come in two forms. We have general market-specific variables that capture unobservable market changes over time or information specific to an auction. In addition, we have market characteristics of lots that are revealed *ex post*. An example, is the number of submitted bids a lot attracted. Finally, we do not want to restrict our model—i.e., equation (7.2)—by imposing symmetry on the potential buyers.

7.4.1 Product-specific covariates

The partial analysis in the preceding chapter suggests that weight is an important variable in explaining prices. Weight in linear form will obviously be expected to have a positive sign. This follows from the economics of the food processing trade; large fish are associated with less waste. In addition, although related to the "less waste" argument, large fish are normally more valuable in end-markets. The structure of the minimum price scheme clearly indicates that the market acknowledges the positive relation. However, since the partial analysis also indicates that weight is an increasing concave function of price, we introduce weight squared (Weight^2) as well, and expect the estimated coefficient sign to be negative.

Vessel quantity (VesQ) is also expected to increase with price, at least up to a certain level. Handling one large lot, rather than two smaller lots, is more cost-effective due to administrative costs. In addition, it simply takes more time to empty two vessels rather than one. It might be argued that very large catches can induce less competition since not all potential buyers can handle a very large catch. In that case, the linearity of the covariate VesQ is questionable. We expect the effect of VesQ to be small, but positive.

The variable gear was presented in section 5.4.2, see in particular table

5.1. For parsimony, we divide gears into three groups. The first gear group is *purse seine*. This is the gear used by the ocean going seiners, and the one most frequently used. The next gear group is *coastal purse seine*. In the final group, we bundle all *trawl* gear. We made three dummy variables for the three gear groups, and since we have a constant in our model, we incorporate two of the dummies; i.e., we do not incorporate the purse seine dummy. The expected sign of the coastal purse seine (GearPSC) and trawl (GearT) dummy variables is not obvious. The predominant use of purse seine may be based on cost considerations. However, one hypothesis is that purse seine is the most careful harvesting method which damages the fish the least. Consequently, it will give the best price on average. In that case, the estimated sign on GearT will be negative.

Feed is constructed as a dummy variable. The benchmark is *no feed* or *very little feed*. The included dummy variable is then *some feed*, and fish with this characteristic are expected to achieve a lower price than fish with no feed or very little feed. The estimated feed coefficient is expected to be negative.

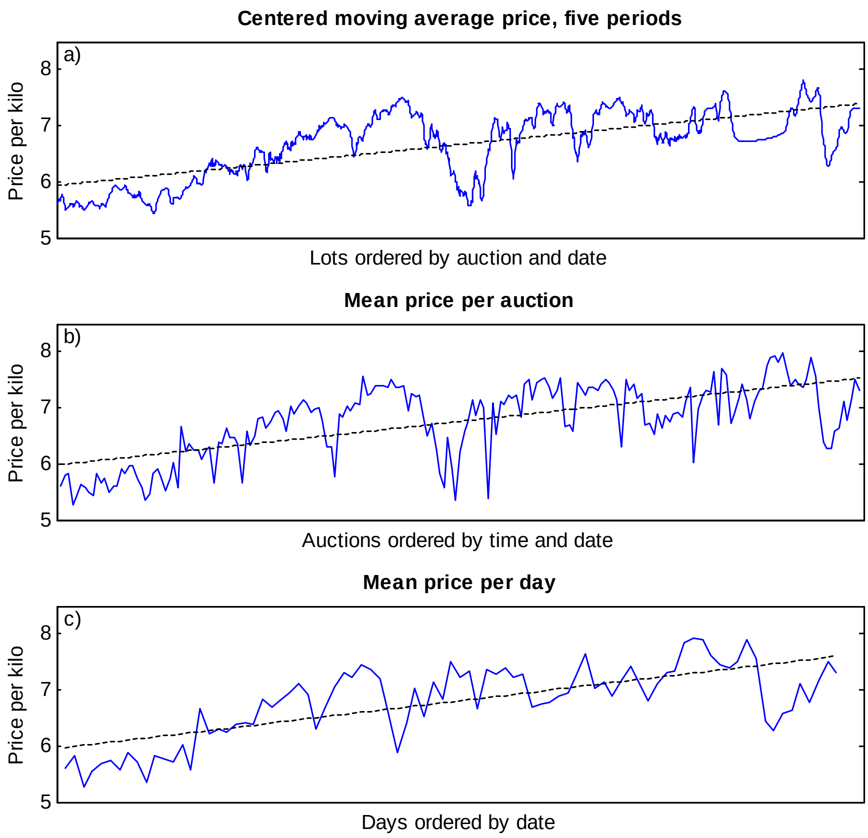
The last product-specific binary variable we include is preservation or cooling method. We group the preferred cooling methods refrigerated seawater (RSW) and slurry ice (CSW) together since we have just a few records with CSW and use the resulting variable as benchmark. Our included dummy variable is Ice, which we expect to have a negative sign.

Although we have information on the number of hauls, preliminary analysis showed no effect of the number of hauls on prices. We did not include this variable.

7.4.2 Market-specific variables

Market forces and constraints influencing the bid process can take several forms. We included several covariates that control for different characteristics of the market at a given time or time interval; three of them are not linked to the specific lot, but rather measure market forces in different ways. The simultaneous selling of catches at auctions together with the associated option of setting capacity constraints, is a central aspect of the market. In the preliminary analysis, we examined the effect of simultaneity by introducing the number of simultaneously auctioned lots as a covariate.

Figure 7.1: Price over time, lots with reserve price 5.25 NOK



General market variables. At the most general level, we include variables measuring structural changes in prices during the season. Next, we want a measure of demand and supply conditions for a given time interval which should neither be too frequent nor too wide. And finally with respect to general variables, we include a variable that measures supply at the auction level.

Are there any structural changes in price formation during the season to consider? Figure 6.2 depicts average weekly prices and suggests that prices seems to rise some time into the season. Let us look at price development over time in more detail. Prices at lot level are very volatile; some smoothing is useful at this point when presenting prices over time. In figure 7.1, we plot price measures of different frequency for lots with a reserve equal to 5.25 NOK as the solid lines while the linear dotted lines represent the mean trend. In figure 7.1a, we plot a centered five-period moving-average of prices for all lots in the sample. Any missing values—a missing value occurs at auctions where no lots with reserve price 5.25 were sold—are interpolated. Note that the price line cannot be interpreted as a pure time series since some lots are sold simultaneously at the same auction. In figure 7.1b and c, we depict the mean prices obtained per auction and per day for the same lots with reserve price 5.25. Thus, we have three pictures of how prices develop over time with different degrees of frequency. The price lines all tell the same story. The beginning of the season is characterized by lower prices than later in the season. A period of increasing prices follows, and then a clear fall in prices appear. The last part of the season, where supply is low, is quite volatile in the price pattern. The low prices in the beginning of the season are explained by fish quality. It is common knowledge that fish harvested in the beginning of the season have a lower quality (lower average fat content) than fish in the peak season.

The pattern indicates that there is a potential for explaining prices by suitable parameters for structural changes, some piecewise regression parameters for both intercept and slope should be considered. Preliminary analysis, where we tried to estimate the increasing prices and sharp fall in prices that appear into the season by use of a reasonable number of piecewise regressions, was, however, unsuccessful. We conclude that the volatility at the lot level in prices is too high for estimating statistically reliable piecewise structural parameters.

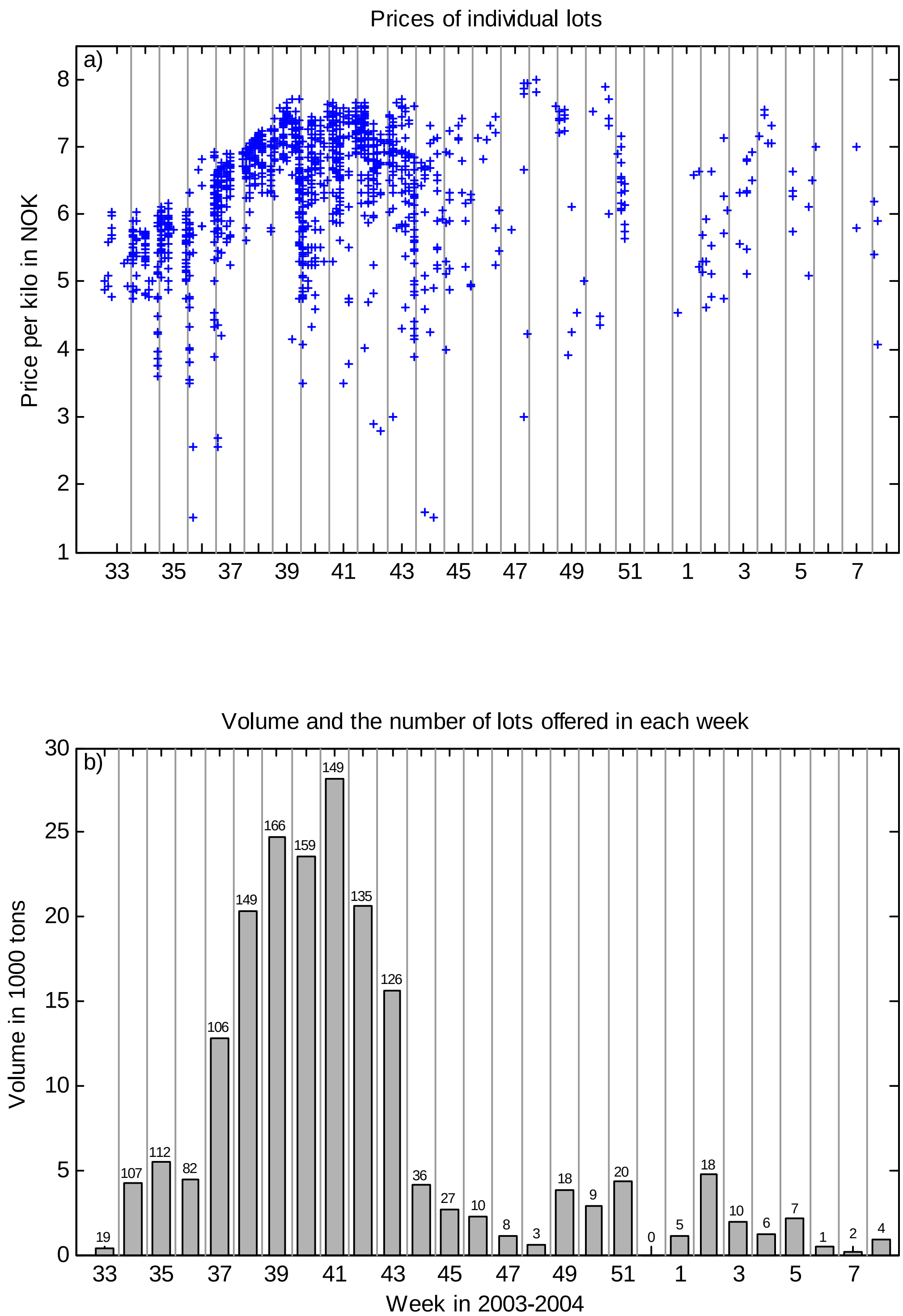
Mean prices are just part of the picture. Let us look at all prices. To see all data points of winning bids, we refer to figure 7.2. The prices and their distribution over time, and in particular within weeks, are plotted in figure 7.2a. Although all data points are plotted, notice that since we use date on the x-axis, identical prices on the same day will overlap exactly. Thus, some data points are "hidden" in figure 7.2a. The number of sold lots each week together with the weekly volume; i.e., the total quantities sold are reported in figure 7.2b. Since the lot quantities differ, the week with the highest volume does not necessarily entail that the number of sold lots is the highest in this week as well. For example, the week of peak volume is week 41 where 149 lots are sold, while week 39 has the highest number of sold lots. Obviously, the average lot quantity is higher in week 41 than in week 39.

Figure 7.2a depicts the distribution of prices within weeks for all lots. Breaking this overall picture down by reserve price is useful. In figures 7.3 to 7.7 on pages 193–197, we depict all prices of the relevant reserve price category in part a of the figure, and in part b we depict the mean prices and price range. We learn that price distribution over time is similar for several classes. To focus on the two most important classes—lots with a reserve price of 5.25 and 4.75 NOK—we see in figures 7.3 to 7.4 that the patterns have a lot in common. Notice, in particular, the wide price range that occurs in the transition from week 39 to week 40. One conclusion to be drawn from this observation is that large price ranges within weeks are not explained by the reserve price classes.

One hypothesis is that in weeks with large price ranges, lots with very different weight were offered. This hypothesis is refuted. Both lots with reserve price 5.25 and lots with reserve price 4.75 exhibit large price ranges in the weeks 39 and 40. Similarly, low prices at the beginning of the season are not explained by a domination of lots with small average fish weight being offered in this period. On the contrary, most lots offered in the first two weeks belong to the two largest weight classes, and all reserve price classes are characterized by low prices in the first weeks of the season. This is easily seen from figures 7.3 to 7.7.

How do we account for the varying prices over time? We suggest to incorporate weekly time dummies. Using week as time period has the benefit of being a sufficiently narrow time span in order to capture otherwise unex-

Figure 7.2: All individual lot prices, volume and number of sold lots in each week



plained market characteristics. At the same time, it adds a limited number of explanatory variables to our regression matrix. Using narrower time dummies might expand the regression matrix by too much, resulting in singularity problems when estimating.

Recall that we ran two regressions; in the first, we used all observations, while in the second we sampled only lots with a reserve price of 5.25 NOK. In the first regression, we used the first week of the season—week 33 in 2003—as the benchmark dummy. Each week in the main part of the season acts as a dummy; i.e., weeks 34–45. The subsequent weeks have far less observations, and in order to avoid singularity problems in estimation, we bundled some weeks.[1] The weeks 46–49 are bundled into one dummy as are weeks 50–51, 1–2, and 3–8. Thus, we included in total 16 time-specific dummy variables.

In the second regression, we used week 33 in 2003 as the benchmark dummy as well. The weeks 34–43 represent individual dummy variables, while we bundled weeks 44–50 and 51–04; in total this resulted in 12 time-specific dummy variables.

Next, we turn our attention to a traditional measure of supply and demand within a given time period. To measure the effect of quantities supplied and demanded, we used the demand-surplus factor per day (DayDSF). Recall that in chapter 5 we defined the demand-surplus factor as the revealed demanded quantity divided by the supplied quantity within a given time period. Demand-surplus factor per auction is quite volatile, and will introduce too much noise as an explanatory variable. Since we already have introduced weekly time dummies, we did not use demand-surplus per week as a variable. Presumably, a weekly demand-surplus factor would add little information which is not already provided by the week dummies. We believe a per day measure better captures a general underlying tendency of the supply and demand conditions within a short, but not too short, time interval. The higher the demand-surplus factor is, the higher will competition be. Thus, we expect a positive sign for this variable.

[1] Singularity problems are, in particular, likely to occur under bootstrapping. If a dummy variable has few positive observations, drawing samples with replacement may result in some dummy variable vectors where all records are zero.

Given that supply at the auction level is too volatile, we have to seek an alternative measure for controlling for what goes on at the auction level. In addition, a central aspect at the auction level is the fact that catches are sold simultaneously. A simple measure of the degree of simultaneity is how many lots are sold simultaneously at the same auction. We included this as a covariate, denoting it Lots. Will the number of catches offered simultaneously, have any impact on prices? One hypothesis is that when many catches are put up for sale, the strategy space for bidders increases. They can by setting capacity constraints and rankings try to acquire catches at a lower price than when supply is low. On the other hand, it might turn out that many lots are offered when prices are high. We have earlier argued that supply is to a large extent driven by the characteristics of the fishery. Although market prices to some extent will determine activity and supply, the availability of the fish and their quality at a given time are also important—perhaps dominant—inputs to harvesting decisions. If prices are high when the quality of the fish and the supply is high, then we shall not necessarily observe a negative relation between prices and the number of offered lots. Note that the variable Lots will also, in some sense, be a measure of supply at auction level.

To conclude this section, we may view the three variables introduced to represent market forces of three different frequencies: per week, per day and per auction. At the most general level, we used weekly time dummies to capture all unexplained market forces within a relatively long time interval. Market forces at work in the somewhat shorter run are captured by the demand-surplus per day. Finally, at the shortest run possible, we used the number of offered lots per auction. Preliminary analysis revealed that the variable Lots did not have any impact on prices. Consequently, we omitted the variable in the final regression model to be reported below.

Lot-specific market variables. At the lot level, an obvious candidate for explaining the price is the number of active bidders, N. We described the effect in some detail in section 6.5. We could model the number of active bidders as dummy variables. Given the low number of observations for high

N, we chose to model it as a discrete variable. The number of active bidders on a lot has a similar effect on price as weight. Prices are increasing in N but at a decreasing rate. We therefore introduce in the model both a linear N, which is expected to be positive, and N squared which is expected to be negative. Note that if we do not introduce N squared, we shall impose the same effect on prices when N increase from one to two and when N increase from, say, ten to eleven. In the case where the relevant variable is an ordered qualitative variable, this will normally be a mistake (see Kennedy [55, p. 403]). The variable N^2 is an effective way of measuring the decreasing competitive effect of increased N. A disadvantage, however, may be that estimated coefficients give counter-intuitive elasticity measures for large N; i.e., elasticities may not go to zero as we expect when N is large, but can actually take on negative values.

We have stated several times that the number of active bidders, or equivalently, the number of submitted bids, is a measure of competition. We know that observed N varies quite a lot. The underlying reasons for this—and whose effects to some extent will be captured by N as an explanatory variable—are the changing delivery sectors and changes in available capacities. When N is large, then the delivery sector is probably wide, and in addition, many buyers have available capacities.

We return at this point to the question of what effect the simultaneous selling has on prices. We have already introduced the number of lots offered simultaneously as an explanatory variable. For the individual lot or catch, the details of bidders' capacity constraints (see section 4.5) and bid rankings (see section 4.6) will determine the winner and, thus, the price. One way of measuring the potential effect of simultaneous selling and capacity constraints is to bring in the winning bid's position among all submitted bids for a lot. Recall that winning bids—the market price—is our explained variable, not the highest bid. We showed in table 5.8 on page 113 that 31.59 percent of winning bids are not equal to the highest bid. Therefore, we introduced the position of the winning bid as a covariate. In principle, we can introduce the position of the winning bid, which ranges from 1 to 10, as a slope parameter, or we can use dummy variables for all positions in order to measure different intercepts associated with the position of the winning bid. Because the number of records decreases sharply for each lower position of the winning bid, we found it reasonable to represent the effect by simply using two dummies. We used the case where the winning bid is equal to the highest bid as the benchmark dummy which is excluded from the regression matrix. One included dummy variable, denoted WinBid2nd, represents the case where the winning bid is the second-highest bid. The other, denoted WinBid3rd, takes care of the case where the winning bid is the third-highest or lower. The coefficients are both expected to be negative; WinBid3rd even more than WinBid2nd.

Table 7.1: Description of covariates

Name	Short name	Description	Data type[a]	Econ. type[b]
Weight	W	Average fish weight in grams	c	P
Weight^2	W^2	Weight squared	c	P
VesQ	VesQ	Vessel quantity in tons	c	P
GearPSC	GPSC	Gear is purse seine coast	b	P
GearT	GT	Gear is trawl	b	P
Feed	Feed	Fish contain some feed	b	P
Ice	Ice	Fish are kept in ice	b	P
N	N	Number of submitted bids	d	M
N^2	N^2	N squared	d	M
WinBid2nd	P2	Winning bid is 2nd-highest	b	M
WinBid3rd	P3+	Winn. bid lower than 2nd-high.	b	M
DayDSF	DDSF	Demand surplus factor per day	c	M

[a] b = binary, c = continuous, d = discrete.
[b] P = product-specific variable, M = market-specific variable.

7.4.3 Buyer-specific variables, asymmetries

Does the buyer's identity matter with respect to realized prices? Do some buyers bid aggressively and end up paying more than other buyers? A simple (and naïve) competitive model of the demand side, will lead us to conclude that competition ensures that buyers do not differ much with respect to cost structure and capacities. Plants with a cost disadvantage are driven out of competition in such a model world. Looking at all the buyers, they do, however, differ with respect to total capacity, and bid activity in the mackerel market. We saw in tables 5.9 and 5.10 on pages 114 and 115 that the activity of buyers differs as does the winning score of buyers. One obvious reason is that location matters. Moreover, from table 5.13 on page 119, we learn that the median reported capacity limit differs somewhat between buyers. Processing capacity is largest for producers located within the most frequently observed delivery sectors.

From this alone, we cannot conclude that prices are affected significantly due to asymmetric buyer strategies. Because we know the identity of bidders and winners, we can effectively control for any buyer-specific variables that affect prices. Given that we have a large dataset (1,494 lot records) and 25 bidders, we can incorporate buyer identities as dummy covariates in order to measure the possible effect of asymmetric buyers. Buyer number 11 never wins, and was removed, one additional buyer should be removed to avoid the dummy-variable trap because we have a constant coefficient in our model. This leaves us with 23 dummy variables representing the winning buyer. This way of structuring buyers as covariates will shift the intercept associated with each buyer. We chose the buyer with the median average winning price, number 14, as our benchmark that is omitted from the regression.

7.4.4 Summary of covariates

We summarize the covariates included in the regression in table 7.1. We state the data type—whether the variable is continuous, discrete or binary—since this is of importance in interpreting coefficients. Continuous variables (weight, vessel quantity, and weekly demand-surplus factor) in this setting are variables with a large number of different values, while the discrete variables have a limited number of observed integer values. The traditional elasticity of a continuous variable is straightforward: it measures the percentage increase in price when the continuous covariate increases by one percent. For discrete variables, the effect of a small change in the covariate is not that meaningful. More interesting is the question of how much price increases when, for instance, the number of bidders increases from two to four. An empirical elasticity measure for the different values is, consequently, more appropriate. The interpretation of dummy variables in our semi-logarithmic model follows from equation (7.3).

We summarize the discussion of variables and expected sign in table 7.2.

Table 7.2: Expected sign on coefficients

Covariate	Expected sign	Explanation
Weight	+	$P'(\text{Weight}) > 0$
Weight^2	−	$P''(\text{Weight}) < 0$
VesQ	+	$P'(\text{VesQ}) > 0$
GearPSC	−	P(purse seine coast) < P(purse seine)
GearT	−	P(trawl) < P(purse seine)
Feed	−	P(some feed) < P(little feed)
Ice	−	P(ice) < P(RSW/CSW)
N	+	$P'(N) > 0$
N^2	−	$P''(N) < 0$
WinBid2nd	−	P(2nd) < P(1st)
WinBid3rd	−	P(3rd) < P(1st)
DayDSF	+	$P'(\text{DayDSF}) > 0$

See table 7.1 for explanation of covariate names.

In the appendix to this chapter, several tables are gathered. Descriptive statistics of discrete and continuous variables are in table 7.15. Frequencies of dummy variables are in table 7.5. See table 7.7 for the matrix of correlation coefficients of the regression variables and the corresponding p-values of correlation coefficients.

7.5 Results: All lots

We choose to run two main regressions. In the first, we included all lots. In the second, we sampled only lots with a reserve price equal to 5.25 NOK. The motivation for analysing this weight class—apart from being the economically most important weight class—is that we then neutralize to a large extent the most important covariate: weight. It is interesting to see whether coefficients remain stable between the two regressions.

For each main regression, we run two regressions, one without considering buyer asymmetries, and one where we specifically incorporate the possibility that the level of the winning bid depends on the buyer. Results were stable between the two regressions (with and without bidder dummy variables). We do not report the results from the regression without bidder dummy variables.

In addition to the least squares estimate, we ran 19 quantile regressions; $\theta \in [0.05, 0.10, \ldots, 0.95]$. The LAD estimate is the quantile estimate where $\theta = 0.5$. For inference purposes, we performed a bootstrap with 500 replications for each quantile regression. We also estimated asymptotic standard errors for the LAD estimator by using the logistic density function as the kernel and bandwidth $h = 0.9\hat{\sigma}n^{-1/5}$ where $\hat{\sigma}$ is the standard error of the sample of size n.

In this section, we concentrate on the regression of all lots where fish weight is expected to dominate while the next section covers the results of the regression of lots with a reserve price equal to 5.25 NOK. Our regression variables are suitably divided into three categories; (1) the covariates summarized in table 7.1, (2) the bidder dummies, and (3) the weekly time dummies. Results concerning the covariates and the bidder dummies are reported and discussed below. The weekly time dummies are important for controlling the whole regression, but estimated coefficients are of less interest. The week dummies hardly generalize, and represent to a large extent the unexplained part of the price level. We do not report them.

7.5.1 Product and market-specific variables

The result of the regression of logarithmic prices on covariates are reported in table 7.3. We report both the LS and the LAD estimates. All coefficients have the same sign in the two estimates. Differences in levels are discussed below when we investigate the quantile regression results. For the LAD estimate, we report t- and p-values based on two different estimates; asymptotic and bootstrapped standard errors. Differences between the two measures are, in general, small.

The LS estimate is a useful reference and produces some interesting statistics like adjusted R-square. Adjusted R-square is 0.8431 for the LS estimate. The unadjusted R-square of 0.8485 is just slightly higher, indicating that we have not populated our regression matrix with unnecessary variables—i.e., variables with little explanatory power. Thus, roughly 85 percent of the total variation in logarithmic prices can be explained by the variability in the explanatory variables. The standard error of regression s is 0.0044.

All in all, was the robust estimation necessary? Robust estimation makes sense when it turns out that a few outlier values change the estimate significantly. In general, as noted by Anscombe [1], "we are happier if the regression relation seems to permeate all the observations" rather than being influenced much by a few. In order to determine the influence of outliers, we may look at the leverage values h_{ii}; i.e., the diagonal elements of the hat matrix $\boldsymbol{H} = \boldsymbol{X}\left(\boldsymbol{X}^{\top}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^{\top}$. The further away from the mean of the regression matrix a particular observation is, the larger is the leverage of that observation. A rule of thumb according to Goodall [35] is to consider leverage values to be high if $h_{ii} > 2\left(p+1\right)/n$ where p is the number of covariates and n is the number of observations. It turns out that 88 out of 1,494 observations—or 5.9 percent of the observations—have leverage values exceeding this (somewhat arbitrary) critical value. We think this is sufficiently many "outliers" to warrant a robust estimation in order to understand the effect of the bulk of observations. The most noteworthy difference between the LS and the LAD estimate is perhaps the coefficient Feed; the LAD estimate is considerably smaller than the LS estimate.

In figure 7.8, on page 198, we report the LS and the quantile regression estimates. In the 12 panels, we depict the same variables as in table 7.3 except for the intercept. The quantiles θ ranging from 0.05 to 0.95 are on the horizontal axis while the estimated coefficient values are on the vertical axis. The 19 quantile regressions coefficients are depicted as the solid curves (rather than depicting the 19 distinct points, we draw the line between them.) The shaded gray area represents a 90 percent confidence interval for the quantile regression estimates. The confidence intervals were computed by taking the empirical 5 and 95 percent quantiles of the bootstrapped sample estimates. The dashed (horizontal) line in each panel is the LS estimate while the two dotted lines represent conventional 90 percent confidence intervals for the least squares estimate.

When interpreting the quantile regressions, it is useful to imagine how they would look like under standard least squares assumptions. If we have a correct linear regression model (no specification error) and spherical disturbances, all the quantile regression coefficients would, in principle, coincide with the respective LS coefficients; the only exception would be the intercept term which typically rises with the quantile.

The quantile estimates of the variables Weight and N and their squared cousins differ clearly from the LS estimate. Notice how most of the quantile estimates for these four variables—including the LAD estimates—are outside the least squares 90 percent confidence band.

The quantile estimates for *Feed* and *Ice* are consistently above the LS estimates. Since the coefficients are negative, the quantile estimates mean that the impact on prices is less than that predicted by the LS estimate. The impact of vessel quantity (VesQ) is reduced for the highest quantiles compared to the least squares prediction; otherwise the quantile estimates for VesQ are close to the LS estimate.

For most quantiles, trawlers (GearT) obtain lower prices than the LS estimate indicates. The difference to the LS estimate is, in particular, marked at the left tail of the distribution. For GearPSC and DayDSF, the results are mixed, but for most quantiles the estimates are within the 90 percent LS confidence band.

The effect on price when the second-highest or lower bids win is represented by P2 and P3+ respectively. Apart from some low estimates for the lower quantiles, the variables have a quite uniform effect over the entire range of the distribution. The estimates are close to the LS estimates. The two variables stand out as variables that have a pure location shift effect on the conditional distribution.

Table 7.3: Determinants of winning bids, all lots

	LS			LAD				
					Asymptotic		Bootstrap	
Coefficient	Est.	t-val.	p-val.	Est.	t-val.	p-val.	t-val.	p-val.
Constant	-0.9110	-12.34	0.000	-0.3842	-5.27	0.000	-2.94	0.003
Weight	0.0092	32.01	0.000	0.0071	25.18	0.000	14.84	0.000
Weight^2	-0.0000	-26.18	0.000	-0.0000	-19.91	0.000	-12.67	0.000
VesQ	0.0002	8.74	0.000	0.0002	8.35	0.000	8.10	0.000
GearPSC	-0.0456	-6.86	0.000	-0.0530	-8.07	0.000	-6.32	0.000
GearTrawl	-0.1021	-11.55	0.000	-0.1221	-13.98	0.000	-8.51	0.000
Feed	-0.0199	-3.32	0.001	-0.0072	-1.22	0.222	-1.63	0.103
Ice	-0.0405	-5.61	0.000	-0.0349	-4.90	0.000	-4.92	0.000
N	0.0294	12.77	0.000	0.0207	9.10	0.000	9.55	0.000
N^2	-0.0016	-9.34	0.000	-0.0012	-6.67	0.000	-7.89	0.000
WinBid2nd	-0.0161	-3.48	0.001	-0.0155	-3.40	0.001	-4.35	0.000
WinBid3rd	-0.0237	-4.29	0.000	-0.0238	-4.35	0.000	-5.04	0.000
DayDSF	0.0109	5.74	0.000	0.0136	7.29	0.000	5.06	0.000

Dependent variable: Logarithm of winning bid.
The regression model includes a set of 23 buyer and 16 week-specific dummy variables.
Number of observations: 1494. Degrees of freedom: 1442.
LS estimation: R-squared = 0.8485. Adjusted R-squared = 0.8431.

The remainder of this section is concerned with interpretation of the size of coefficients or how influential covariates are for price determination. The discussion refers to the robust LAD estimates.

Economic significance. In table 7.3, we have estimated the empirical price equation on all lots. Signs on coefficients are as expected. Given the mix of covariate types (continuous, discrete, binary), it is not obvious what covariates are most economically influential by looking at the size of coefficients. A more meaningful analysis is to make the effects independent of scale and interpret coefficients in terms of elasticities. We do so below for the variable weight and all binary variables. For now, a couple of ways to evaluate better how economically significant covariates are, is to look at the standardized coefficients and coefficients multiplied by the mean of the relevant covariate. We compute these statistics in table 7.9.

For covariate k, let $\hat{\beta}_k$ be the estimated coefficient, $\bar{X}_k$ the mean and s_{X_k} the standard deviation of the covariate. The standard deviation of the explained variable $\log(P)$ is denoted s_Y. The standardized coefficients reported in column 2 and 5 in table 7.9, can be obtained by regression on standardized variables or, in the least squares case, simply by multiplying the unstandardized coefficients by s_{X_k}/s_Y. While the unstandardized coefficients show the net effect in logarithmic price which follows from a unit change in the covariate, the standardized coefficients show how many standard deviations the explained variable changes when the covariate changes by one standard deviation. Thus, we have a measure that is independent of the units that covariates are measured by.

The net effect of weight is around 0.7 $(3.67 - 3.01 = 0.66)$, i.e., when weight change by one standard deviation, the logarithmic price changes by 0.7 standard deviation. The other continuous or integer variables give more moderate responses in prices.

For interpretation of binary variables, we prefer to look at elasticities. Although interpretations are eased by looking at standardized coefficients, probably a better way to examine how influential the covariates are, is to multiply the coefficient by the mean of the relevant covariate. This will show the individual effects of the covariates in explaining the mean of the explained variable. This is a numerical property of the least squares estimate, the relationship is not exact for LAD estimation. Looking at column 3 and 6 in table 7.9, we see that the by far most important variable is weight. Compared to weight, all other covariates have a moderate economic significance in explaining prices.

In the lower part of table 7.9, in the row named *Sum covariates*, we sum columns 3 and 6. Then we add to this in the row named *Sum bidders* the combined effect of bidders—i.e., $\sum_b \delta_b \bar{D}_b$ from the regression equation (7.2) where b denotes the subset of dummy variables that represents bidder dummies. Likewise, in the row named *Sum weeks*, we add the combined effect of weekly time dummies—i.e., $\sum_w \delta_w \bar{D}_w$ from the regression equation where w denotes the subset of dummy variables that represents week dummies. Finally, in *Sum total*, we have the total sum of all estimated coefficients multiplied by the mean of the variables. We see that the total sum in the LS case equals the mean of the explained variable. Again, weight is the predominant variable that explains prices.

Effect of binary variables on average price. Elasticities of dummy variables are reported in table 7.13 on page 192. Elasticities reported in the second column are calculated by the transformation (7.4) multiplied by 100, and the standard deviations of elasticities shown in the third column are calculated by use of equation (7.5). Conditional on that we have appropriately controlled for relevant covariates that explain prices, we may interpret the estimation results for included dummy variables. Coastal vessels using purse seine obtain prices that are on average around 5 percent lower than when purse seine is used. Correspondingly, trawlers obtain on average 11.5 percent lower prices than purse seiners. Feed in the fish seems to have a small impact on prices. Fish containing some feed obtain only 0.72 percent lower prices than the case where the fish contain no or very little feed. Preservation method, on the other hand, affects prices. The inferior cooling method by use of ice gives lot prices that are 3.4 percent lower than for lots where refrigerated sea water (RSW) or slurry ice (CSW) are used. We note that lots that were won by the second-highest bidder, on average had 1.5 percent lower prices than lots won by the highest bidder. Finally, lots won by the third or lower highest bidder obtained on average prices 2.4 percent lower than the benchmark high bid lots.

As expected, elasticities of some dummy variables are relatively large, since dummy variables represent a significant shift in "treatment." We notice from the low standard deviations of elasticities that estimated elasticities are rather precise.

The price elasticity of weight. Weight is so important as an explanatory variable that it warrants a table of the elasticities. In figure 6.1, we depicted the relation between prices and weight based on the empirical mean prices of the weight classes. In the multivariate analysis, we have controlled for several covariates and can present the estimated price elasticities of weight, see table 7.4. In column 1, we show the weight range with appropriate intervals, and in column 2 the relevant estimated mean prices of the given weights are calculated. The point elasticities in column 3 show the percentage effect on price when weight increases by 1 percent. More interesting, from a practical perspective, is probably the percentage effect on price when weight increases by some discrete jump like 25 grams. In column 4, the interval elasticities are calculated using the midpoint rule. We see that mean price is estimated to increase by 11 percent when average weight increase from 225 to 250 grams (or more precisely from 237.5 to 262.5 grams since we used the mid point rule). The elasticities for the higher weight classes decline, but are still of significant size up to 500 grams. From 525 to 575 grams, price elasticities are moderate. For lots with average fish weights above 575 grams, prices do not change much.

Elasticities exhibit a declining pattern that is in accordance with the price path we illustrated in figure 6.1 on page 128; the price responds more when the weight increases from a low level than from an high level. The fact that price elasticities are small but negative for the highest weight classes is probably caused by our modelling approach where the squared term dominates the linear term for large weights, see figure 6.1.

Table 7.4: Price elasticity of weight, all lots

Weight	Estimated price[a]	Point elasticity[b]	Discrete elasticity[c]
225	2.78	1.02	
250	3.11	1.07	11.00
275	3.44	1.09	10.28
300	3.79	1.11	9.57
325	4.14	1.10	8.85
350	4.49	1.09	8.14
375	4.84	1.06	7.42
400	5.18	1.02	6.70
425	5.49	0.96	5.99
450	5.79	0.88	5.27
475	6.06	0.80	4.55
500	6.30	0.70	3.83
525	6.50	0.58	3.12
550	6.66	0.45	2.40
575	6.77	0.30	1.68
600	6.83	0.15	0.96
625	6.85	−0.03	0.25
650	6.82	−0.22	−0.47

[a] Price in NOK per kilo. Other variables than weight are evaluated at their means in the price equation.

[b] Percentage effect on price when weight (measured in grams) increases by one percent.

[c] Percentage effect on price when going from one weight class to the next.

7.5.2 Firm-specific variables: Bidder asymmetries

An important question remains: Are bidders asymmetric in the sense that some, due to firm-specific characteristics, tend to bid higher on average than others? We have established that bidders are different with respect to bid activity, see table 5.9 on page 114, and with respect to average reported capacities, see table 5.13 on page 119. Both tables reveal that bidders may be categorized into the groups: small, medium and large bidders. The delivery sectors will partly explain why some bidders more frequently submit bids than others. On the other hand, when examining whether winning bids differ amongst bidders, we find no significant differences, neither in terms of economic significance (which is the important measure) nor in terms of statistical significance.

We have summarized our results in table 7.11 on page 190. Columns two to four give statistics on winning bids; i.e., the number, the mean and standard deviation of winning bids for each bidder. In column five we compute the percentage difference in mean price from bidder 14. We use bidder 14 as the benchmark bidder since he has the median mean winning bid. Differences in mean bids range from −8.58 percent (bidder 5) to 8.67 percent (bidder 7). The mean winning bid differs from bidder 14 by more than 2 percent for 16 bidders. A naïve interpretation of these differences in mean prices would be that bidders are asymmetric.

However, when examining whether the identity of the winning bidder has any effect on prices, we cannot simply compare mean winning prices for each bidder. Rather, we have to control for the product- and market-specific variables. When we control for theses variables by running equation (7.2) with the variables reported in table 7.3 and dummy variables for all winning bidders (but one), the picture changes dramatically.

Estimated differences in mean prices are calculated by transforming the estimated coefficient by use of equation (7.4). First, we note that the sign of percentage difference in price changes for several bidders when going from the uncontrolled to the controlled case. Consider, for instance, bidder 23 whose mean price is 3.59 percent lower than bidder 14. Controlling for other variables, we estimate that his mean price is in fact 1.17 percent higher than bidder 14's. Second, and more important, the controlled differences in mean prices in column six are small. Only three bidders now have mean winning bids that differ from bidder 14 by more than 2 percent. Bidder 22 has a mean winning bid which is 5.35 percent lower than bidder 14's. Bidder 1's mean winning bid is 3.02 percent lower while bidder 6's mean winning bid is 2.49 percent lower than bidder 14's. Column seven reports the standard deviation associated with estimated differences in mean prices from column 6. In computing standard deviations, we used equation (7.5).

A traditional approach would suggest that comparisons by pairs of the winning bids do not reveal asymmetries. A standard formulation would be that standard deviations indicate that we cannot reject the hypothesis that bidders' mean winning bids are equal, and consequently that bidders are symmetric in valuations. However, assuming that we are dealing with the population of winning bids, rather than a sample, we conclude that winning bids show some—but small—differences amongst bidders.

An alternative procedure for analysing asymmetries between buyers would be to use a multiple comparison procedure, see Hochberg and Tamhane [50] and Searle, Speed, and Milliken [96], on the mean bids. The procedure is, however, less useful in our case because the problem with controlling for co-variates remains. We dismiss this popular method, and rely on the regression approach that effectively controls for other variables. In chapter 9, we shall analyse asymmetries between bidders based on all submitted bids rather than only winning bids, which is the topic of this chapter.

7.6 Results: Lots with reserve price 5.25

All tables for the case where we estimate prices for lots with reserve price 5.25 are relegated to the appendix of this chapter. Summary statistics of continuous variables are in table 7.16. The corresponding table for binary variables is table 7.6. Correlation coefficients for regression variables are in table 7.8. The discussion below will concentrate on the differences to the previous regression on all lots.

Turning to the regression results, we report the estimated coefficients in table 7.17. For interpretation of the effect of the variables, we refer to table 7.10 as well. The signs on coefficients are the same as in the previous regression on all lots. The variability in the explained variable—the logarithm of winning bids—is reduced in this regression where we sample more homogenous lots. The effect on estimated coefficients compared to the estimate on all lots can roughly be summarized as follows: Some variables have stable coefficients. These include VesQ, GearTrawl, WinBid2nd, WinBid3rd, and DayDSF. The other variables have less effect on prices—either positive or negative—than the estimation of all lots identified. As expected, the weight coefficients are now considerably smaller. Weight, being the dominant explanatory variable in the case where we used all lots, now turns out to have a more moderate impact on prices. To see this, compare table 7.10 with 7.9. The effect of GearPSC, Feed, Ice, and N is reduced. Since most variable coefficients are stable or have smaller absolute values, the constant term picks up the main effect. Elasticities of dummy variables are in table 7.14.

The quantile regressions are depicted in figure 7.9 on page 199. The weight coefficients are now within the least squares 90 percent confidence band. Coefficients and differences among quantiles are small. Notice that the increasing pattern from figure 7.8 panel (a) and the decreasing pattern from panel (b) are now actually reversed. The Feed coefficients in panel (f) of figure 7.9 are close to zero and actually above zero for low quantiles. Other quantile estimates, and in particular the coefficients for N and N^2, are quite similar to estimated coefficients reported in figure 7.8. Thus, most coefficients are stable between the two main regressions.

Considering again the question of any asymmetries between bidders, we report in table 7.12 the estimated price differences among bidders. Only four bidders have an estimated price difference from the benchmark bidder 14 of more than two percent. The largest controlled percentage difference from bidder 14 is -3.9. We cannot conclude that bidders are asymmetric with respect to the level of winning bids.

7.7 Concluding remarks

We have investigated the empirical relationship between prices and covariates. We included bidder dummies and time dummies that capture otherwise unobserved, but relevant, variables. Our discussion has focused on the robust least absolute deviations estimator. In the absence of a structural estimation, we cannot conclude on the causal effects. Some empirical regularities are, however, evident. Weight is the dominant variable that explains prices. Next, the number of bidders is important. The quantile regressions make a case for not relying solely on models for the conditional mean or median in empirical analyses.

7.A Appendix: Tables and figures

Table 7.5: Summary of dummy variables, all lots

Variable	Dummy	Status*	Records	Percent
Gear	Purse seine	B	896	59.97
	Purse seine coast	I	399	26.71
	Trawl	I	199	13.32
			1494	100.00
Feed	No/little feed	B	1254	83.94
	Some feed	I	240	16.06
			1494	100.00
Preservation	RSW/CSW	B	1341	89.76
	Ice	I	153	10.24
			1494	100.00
Winning bid	Highest bid won	B	1022	68.41
	2nd-highest bid won	I	281	18.81
	3rd-high. or lower won	I	191	12.78
			1494	100.00

* B: Benchmark dummy variable, not included in the regression.
I : Dummy variable is included in the regression.

Table 7.6: Summary of dummy variables, lots with reserve price 5.25

Variable	Dummy	Status*	Records	Percent
Gear	Purse seine	B	654	66.67
	Purse seine coast	I	260	26.50
	Trawl	I	67	6.83
			981	100.00
Feed	No/little feed	B	827	84.30
	Some feed	I	154	15.70
			981	100.00
Preservation	RSW/CSW	B	900	91.74
	Ice	I	81	8.26
			981	100.00
Winning bid	Highest bid won	B	640	65.24
	2nd-highest bid won	I	196	19.98
	3rd-high. or lower won	I	145	14.78
			981	100.00

* B: Benchmark dummy variable, not included in the regression.
I : Dummy variable is included in the regression.

Table 7.7: Correlation coefficients[a] and corresponding p-values[b] of regression variables, all lots

	$\log(P)$	W	W^2	VesQ	GPSC	GT	Feed	Ice	N	N^2	P2	P3+	DDSF
$\log(P)$		0.57	0.53	0.34	−0.49	−0.15	−0.32	−0.42	0.32	0.29	0.01	0.01	0.09
W	0.00		1.00	−0.03	0.01	−0.36	0.02	−0.10	0.04	0.05	0.03	0.10	−0.32
W^2	0.00	0.00		−0.03	0.01	−0.35	0.02	−0.10	0.04	0.05	0.03	0.10	−0.31
VesQ	0.00	0.25	0.22		−0.54	0.34	−0.29	−0.35	0.02	0.01	−0.01	0.00	0.15
GPSC	0.00	0.70	0.72	0.00		−0.24	0.42	0.52	−0.23	−0.21	−0.00	−0.02	−0.27
GT	0.00	0.00	0.00	0.00	0.00		−0.17	−0.08	0.02	0.01	−0.09	−0.10	0.45
Feed	0.00	0.56	0.49	0.00	0.00	0.00		0.28	−0.03	−0.04	−0.02	0.02	−0.10
Ice	0.00	0.00	0.00	0.00	0.00	0.00	0.00		−0.19	−0.18	−0.00	−0.05	−0.13
N	0.00	0.17	0.14	0.55	0.00	0.37	0.29	0.00		0.96	−0.04	−0.10	0.45
N^2	0.00	0.08	0.07	0.80	0.00	0.67	0.16	0.00	0.00		−0.05	−0.09	0.41
P2	0.63	0.20	0.25	0.61	0.88	0.00	0.46	0.87	0.12	0.05		−0.18	−0.08
P3+	0.66	0.00	0.00	0.86	0.48	0.00	0.48	0.05	0.00	0.00	0.00		−0.14
DDSF	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

[a] Correlation coefficients are in the upper triangular part of the table.
[b] p-values are in the lower triangular part of the table.

Table 7.8: Correlation coefficients[a] and corresponding p-values[b] of regression variables, lots with reserve price 5.25

	$\log(P)$	W	W^2	VesQ	GPSC	GT	Feed	Ice	N	N^2	P2	P3+	DDSF
$\log(P)$		0.10	0.10	0.54	−0.73	0.13	−0.34	−0.45	0.41	0.36	−0.01	−0.09	0.42
W	0.00		1.00	−0.03	0.01	−0.12	0.07	−0.05	0.09	0.08	−0.02	0.03	0.01
W^2	0.00	0.00		−0.03	0.02	−0.12	0.07	−0.04	0.09	0.08	−0.02	0.03	0.02
VesQ	0.00	0.33	0.30		−0.58	0.27	−0.25	−0.34	0.03	0.01	0.02	0.01	0.18
GPSC	0.00	0.67	0.59	0.00		−0.16	0.33	0.49	−0.25	−0.22	−0.04	−0.04	−0.27
GT	0.00	0.00	0.00	0.00	0.00		−0.11	−0.07	0.06	0.03	−0.02	−0.06	0.46
Feed	0.00	0.04	0.03	0.00	0.00	0.00		0.19	−0.02	−0.03	−0.03	0.04	−0.02
Ice	0.00	0.16	0.18	0.00	0.00	0.04	0.00		−0.18	−0.17	−0.04	−0.04	−0.16
N	0.00	0.00	0.00	0.39	0.00	0.08	0.52	0.00		0.96	−0.05	−0.10	0.45
N^2	0.00	0.01	0.01	0.74	0.00	0.29	0.42	0.00	0.00		−0.06	−0.08	0.41
P2	0.82	0.51	0.48	0.59	0.21	0.45	0.41	0.23	0.13	0.06		−0.21	−0.04
P3+	0.01	0.33	0.36	0.77	0.27	0.08	0.20	0.19	0.00	0.01	0.00		−0.14
DDSF	0.00	0.70	0.63	0.00	0.00	0.00	0.47	0.00	0.00	0.00	0.21	0.00	

[a] Correlation coefficients are in the upper triangular part of the table.

[b] p-values are in the lower triangular part of the table.

Table 7.9: Importance of covariates, all lots

	LS			LAD		
	$\hat{\beta}_k$ [a]	$\hat{\beta}_k s_{X_k}/s_Y$ [b]	$\hat{\beta}_k \bar{X}_k$ [c]	$\hat{\beta}_k$	$\hat{\beta}_k s_{X_k}/s_Y$	$\hat{\beta}_k \bar{X}_k$
Constant	−0.9110	0.0000	−0.9110	−0.3842	0.0000	−0.3842
Weight	0.0092	3.6596	4.7830	0.0071	2.8438	3.7168
Weight^2	−0.0000	−3.0056	−2.1107	−0.0000	−2.2577	−1.5854
VesQ	0.0002	0.1260	0.0263	0.0002	0.1189	0.0248
GearPSC	−0.0456	−0.1200	−0.0122	−0.0530	−0.1394	−0.0141
GearTrawl	−0.1021	−0.2064	−0.0136	−0.1221	−0.2468	−0.0163
Feed	−0.0199	−0.0434	−0.0032	−0.0072	−0.0158	−0.0012
Ice	−0.0405	−0.0731	−0.0041	−0.0349	−0.0630	−0.0036
N	0.0294	0.5136	0.1508	0.0207	0.3618	0.1063
N^2	−0.0016	−0.3594	−0.0573	−0.0012	−0.2535	−0.0404
WinBid2nd	−0.0161	−0.0374	−0.0030	−0.0155	−0.0361	−0.0029
WinBid3rd	−0.0237	−0.0471	−0.0030	−0.0238	−0.0472	−0.0030
DayDSF	0.0109	0.1024	0.0349	0.0136	0.1284	0.0438
Sum covariates			1.8769			1.8405
Sum bidders			−0.0048			−0.0085
Sum weeks			−0.0371			0.0095
Sum total			1.8349			1.8414
Mean of log (P)			1.8349			1.8349

[a] $\hat{\beta}_k$ is the estimated coefficient for variable k.

[b] $\hat{\beta}_k s_{X_k}/s_Y$ is the standardized coefficient. s_{X_k} is the standard deviation of variable X_k, and s_Y is the standard deviation of the dependent variable.

[c] $\hat{\beta}_k \bar{X}_k$ is the coefficient multiplied by the mean of variable k, $\bar{X}_k$.

Table 7.10: Importance of covariates, lots with reserve price 5.25 NOK

	LS			LAD		
	$\hat{\beta}_k$ [a]	$\hat{\beta}_k s_{X_k}/s_Y$ [b]	$\hat{\beta}_k \bar{X}_k$ [c]	$\hat{\beta}_k$	$\hat{\beta}_k s_{X_k}/s_Y$	$\hat{\beta}_k \bar{X}_k$
Constant	1.0455	0.0000	1.0455	1.0260	0.0000	1.0260
Weight	0.0019	0.5974	1.0733	0.0020	0.6238	1.1207
Weight^2	−0.0000	−0.4367	−0.3915	−0.0000	−0.4715	−0.4227
VesQ	0.0001	0.1328	0.0191	0.0001	0.1418	0.0205
GearPSC	−0.0440	−0.1869	−0.0117	−0.0376	−0.1600	−0.0100
GearTrawl	−0.1227	−0.2981	−0.0084	−0.1338	−0.3251	−0.0091
Feed	−0.0043	−0.0149	−0.0007	−0.0031	−0.0107	−0.0005
Ice	−0.0125	−0.0331	−0.0010	−0.0115	−0.0306	−0.0010
N	0.0185	0.5335	0.0951	0.0158	0.4560	0.0813
N^2	−0.0010	−0.3574	−0.0346	−0.0008	−0.3011	−0.0292
WinBid2nd	−0.0163	−0.0628	−0.0033	−0.0138	−0.0530	−0.0027
WinBid3rd	−0.0286	−0.0978	−0.0042	−0.0253	−0.0863	−0.0037
DayDSF	0.0121	0.1495	0.0348	0.0153	0.1895	0.0441
Sum covariates			1.8124			1.8137
Sum bidders			0.0089			0.0038
Sum weeks			0.0649			0.0737
Sum total			1.8862			1.8912
Mean of log (P)			1.8862			1.8862

[a] $\hat{\beta}_k$ is the estimated coefficient for variable k.
[b] $\hat{\beta}_k s_{X_k}/s_Y$ is the standardized coefficient. s_{X_k} is the standard deviation of variable X_k, and s_Y is the standard deviation of the dependent variable.
[c] $\hat{\beta}_k \bar{X}_k$ is the coefficient multiplied by the mean of variable k, $\bar{X}_k$.

Table 7.11: Price differences among bidders, all lots

	Winning bids			Price differences[a]		
Bidder	Count	Mean	Std.	Uncon-trolled	Cont-trolled	Std.[b]
1	26	6.08	0.76	−3.24	−3.02	3.56
2	12	5.77	0.44	−8.20	−1.93	5.15
3	61	6.52	0.82	3.67	−0.43	3.18
4	32	6.81	0.48	8.42	−0.02	3.35
5	66	5.75	0.83	−8.58	−1.09	3.52
6	126	6.36	0.73	1.25	−2.49	3.11
7	108	6.83	0.62	8.67	−0.89	3.10
8	55	6.42	0.80	2.18	−0.32	3.62
9	24	6.64	0.67	5.71	−0.15	5.18
10	98	5.79	1.17	−7.96	−1.60	3.05
12	175	6.16	0.97	−1.98	−0.53	2.88
13	119	6.27	0.95	−0.18	−0.94	2.78
14	11	6.29	0.17	0.00	0.00	0.00
15	70	6.50	0.92	3.37	−0.66	2.97
16	69	6.65	0.82	5.72	−1.04	3.19
17	127	6.52	0.71	3.76	−1.23	2.95
18	18	6.35	0.84	1.00	−1.29	3.07
19	40	6.36	1.23	1.13	−0.25	3.83
20	60	6.82	0.64	8.48	0.97	4.01
21	43	6.57	0.62	4.51	0.29	3.92
22	14	6.33	0.70	0.72	−5.35	3.90
23	62	6.06	1.19	−3.59	1.17	3.59
24	35	5.82	1.26	−7.47	−1.06	3.71
25	43	6.34	0.71	0.93	−0.38	3.07

[a] Percentage difference in mean price compared to bidder 14.
[b] Std. = standard deviation of controlled difference in mean price.

Table 7.12: Price differences among bidders, lots with reserve price 5.25 NOK

	Winning bids			Price differences[a]		
Bidder	Count	Mean	Std.	Uncon-trolled	Cont-trolled	Std.[b]
1	19	6.39	0.58	2.22	−2.55	2.43
2	8	5.77	0.29	−7.73	−3.39	14.05
3	33	6.86	0.61	9.71	1.25	1.92
4	28	6.85	0.51	9.54	1.13	1.97
5	37	6.05	0.58	−3.20	0.33	2.32
6	72	6.56	0.66	4.91	−0.93	1.96
7	90	6.88	0.57	10.10	0.27	1.78
8	34	6.69	0.50	7.04	−0.85	2.37
9	15	6.94	0.41	10.96	1.10	3.91
10	53	6.25	0.78	0.02	−0.11	1.82
12	99	6.53	0.71	4.42	0.33	1.69
13	78	6.46	0.83	3.33	0.08	1.64
14	9	6.25	0.17	0.00	0.00	0.00
15	42	6.90	0.58	10.28	0.79	2.03
16	57	6.73	0.72	7.67	0.80	1.87
17	102	6.68	0.60	6.86	0.95	1.56
18	15	6.56	0.76	4.86	0.70	1.79
19	27	6.92	0.47	10.58	1.33	2.28
20	39	6.96	0.52	11.32	0.89	2.07
21	23	6.93	0.43	10.75	3.05	3.41
22	14	6.33	0.70	1.24	−3.90	3.77
23	34	6.69	0.63	6.99	1.68	2.83
24	24	6.42	0.54	2.67	0.98	2.03
25	29	6.56	0.56	4.96	1.04	1.95

[a] Percentage difference in mean price compared to bidder 14.

[b] Std. = standard deviation of controlled difference in mean price.

Table 7.13: Price effect of some dummy variables, all lots

Dummy variable	Elasticity	Std. of elasticity	Benchmark dummy
Gear: Purse seine coast	−5.16	0.63	Purse seine
Gear: Trawl	−11.50	1.61	Purse seine
Feed: Some feed	−0.72	0.19	Little or no feed
Preservation: Ice	−3.44	0.47	RSW or CSW
2nd-highest bid won	−1.54	0.12	Highest bid won
3rd-high. or lower won	−2.35	0.21	Highest bid won

Interpretation: When the given factor is present, the elasticity is the percentage change in estimated price compared to the case where the benchmark dummy variable is present.

Table 7.14: Price effect of some dummy variables, lots with reserve price 5.25 NOK

Dummy variable	Elasticity	Std. of elasticity	Benchmark dummy
Gear: Purse seine coast	−3.70	0.74	Purse seine
Gear: Trawl	−12.55	4.34	Purse seine
Feed: Some feed	−0.31	0.19	Little or no feed
Preservation: Ice	−1.15	0.37	RSW or CSW
2nd-highest bid won	−1.37	0.14	Highest bid won
3rd-high. or lower won	−2.50	0.18	Highest bid won

Figure 7.3: Individual prices and mean prices and ranges for lots with reserve price 5.25

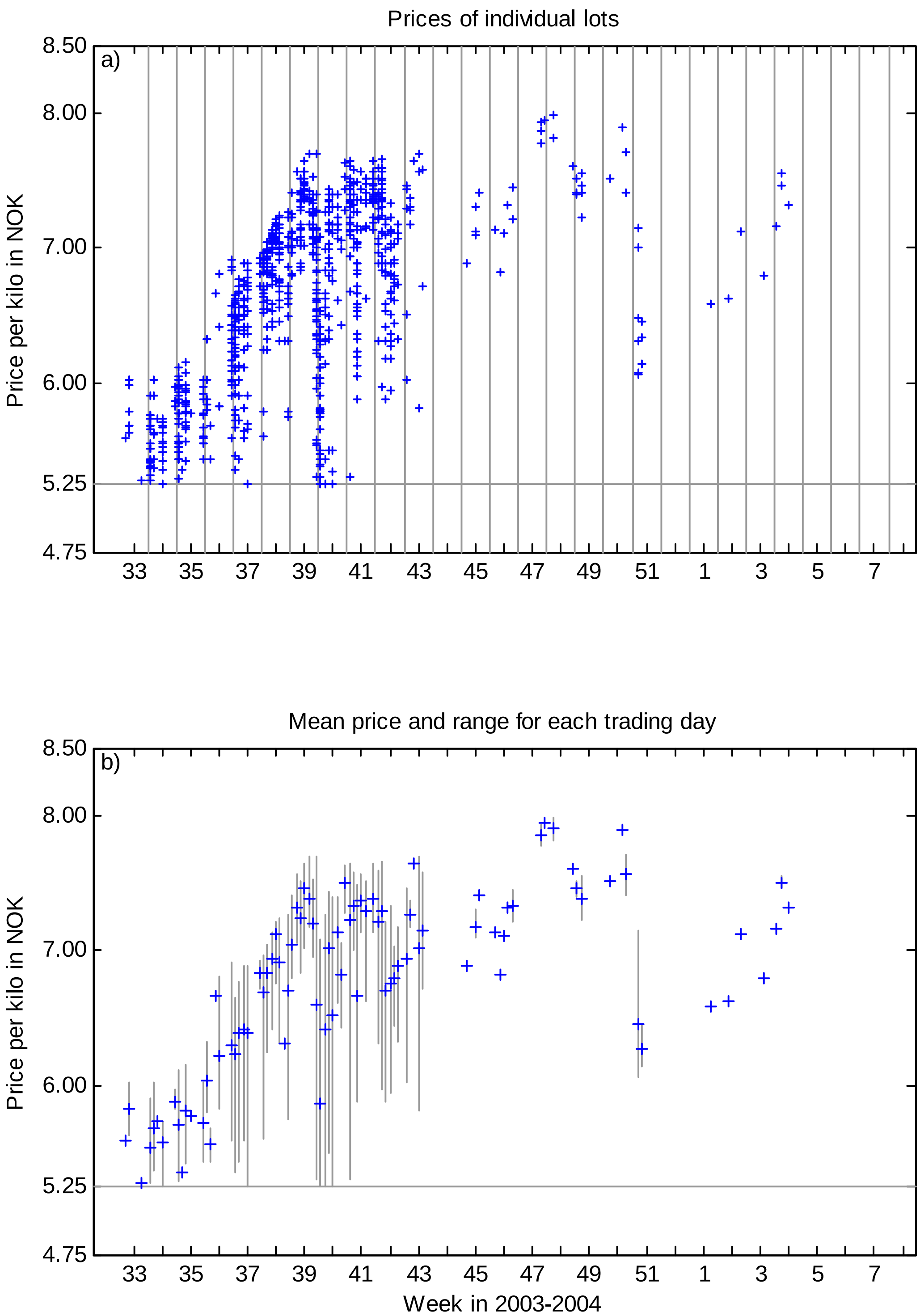


Figure 7.4: Individual prices and mean prices and ranges for lots with reserve price 4.75

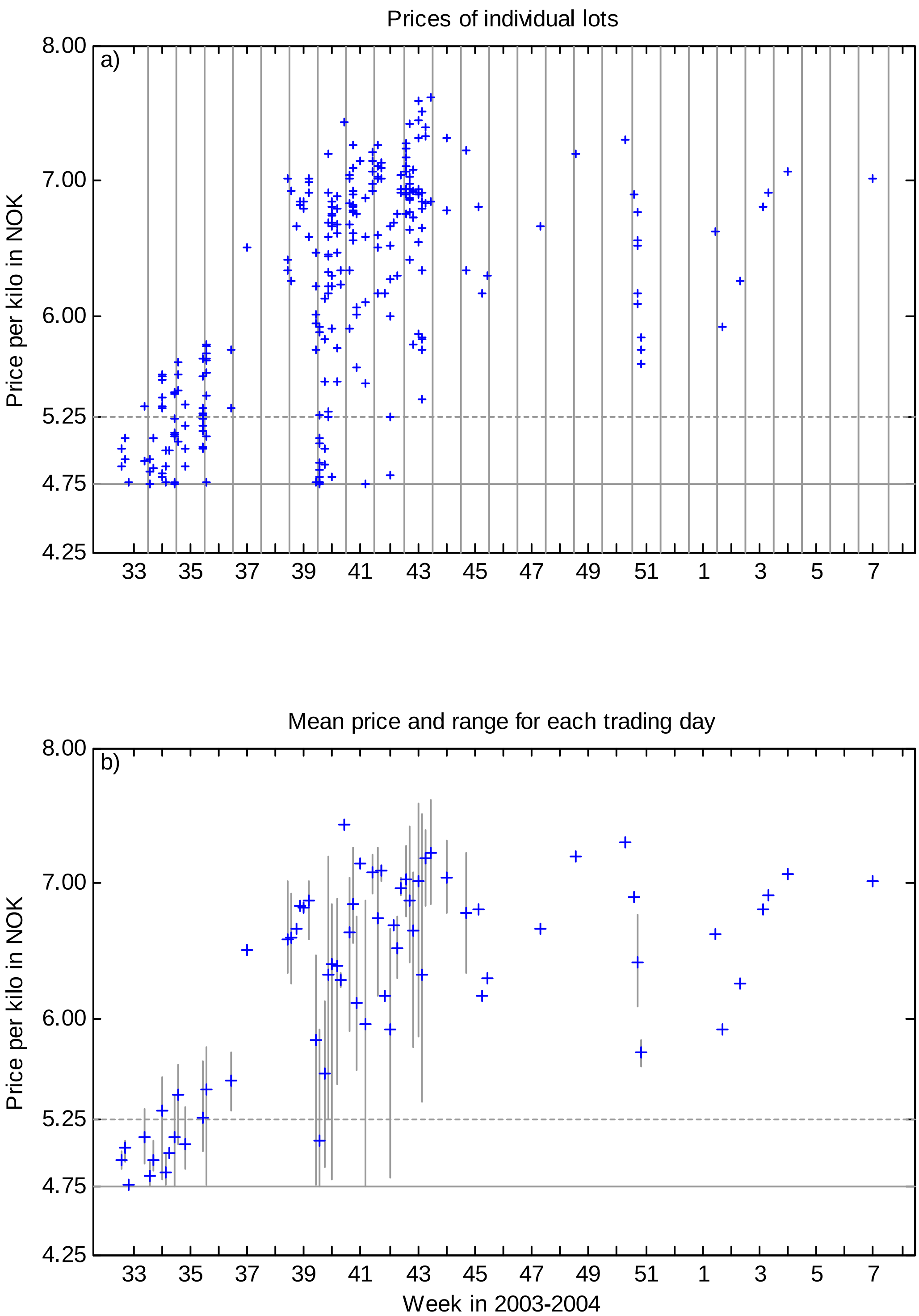


Figure 7.5: Individual prices and mean prices and ranges for lots with reserve price 3.50

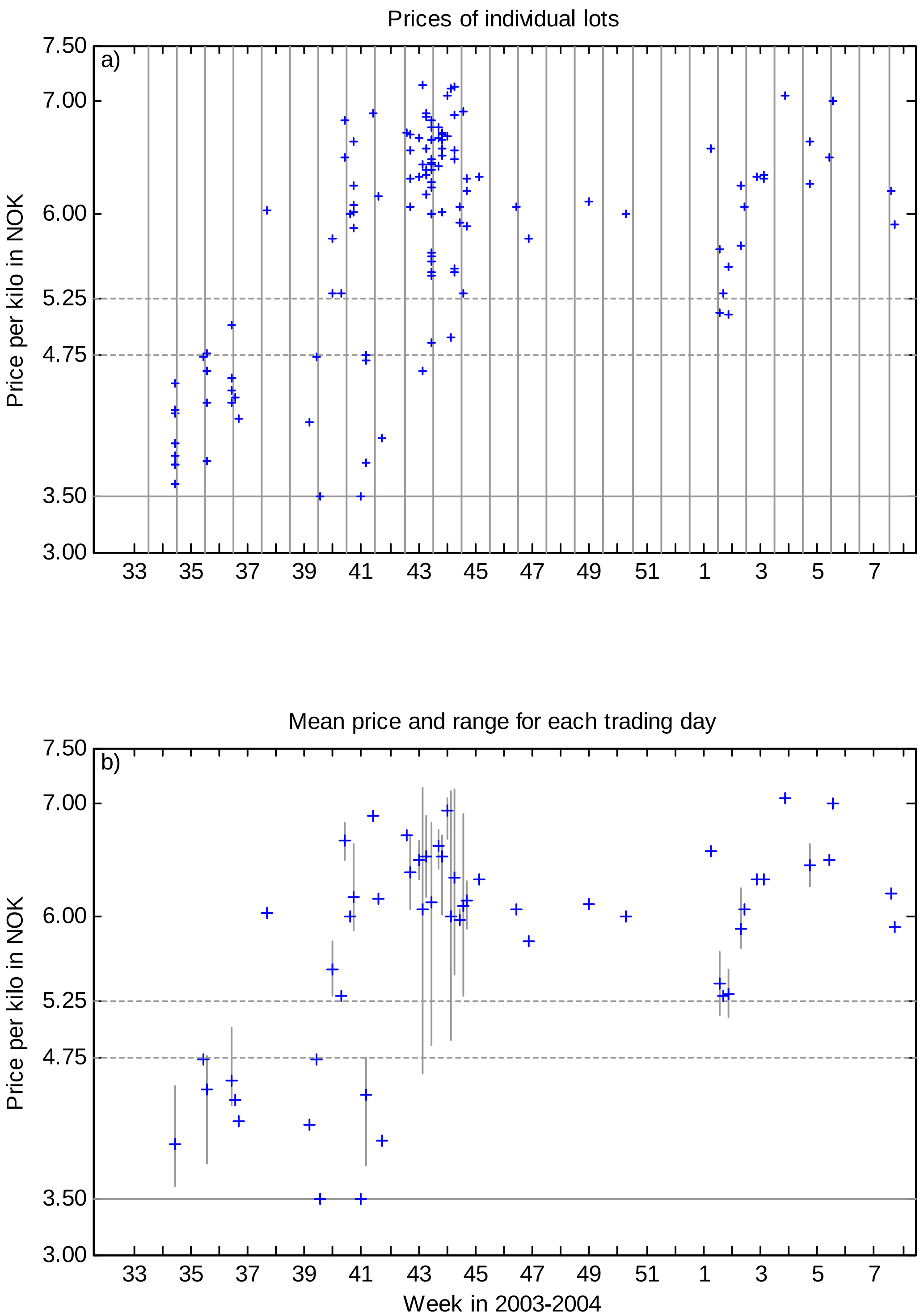


Figure 7.6: Individual prices and mean prices and ranges for lots with reserve price 2.50

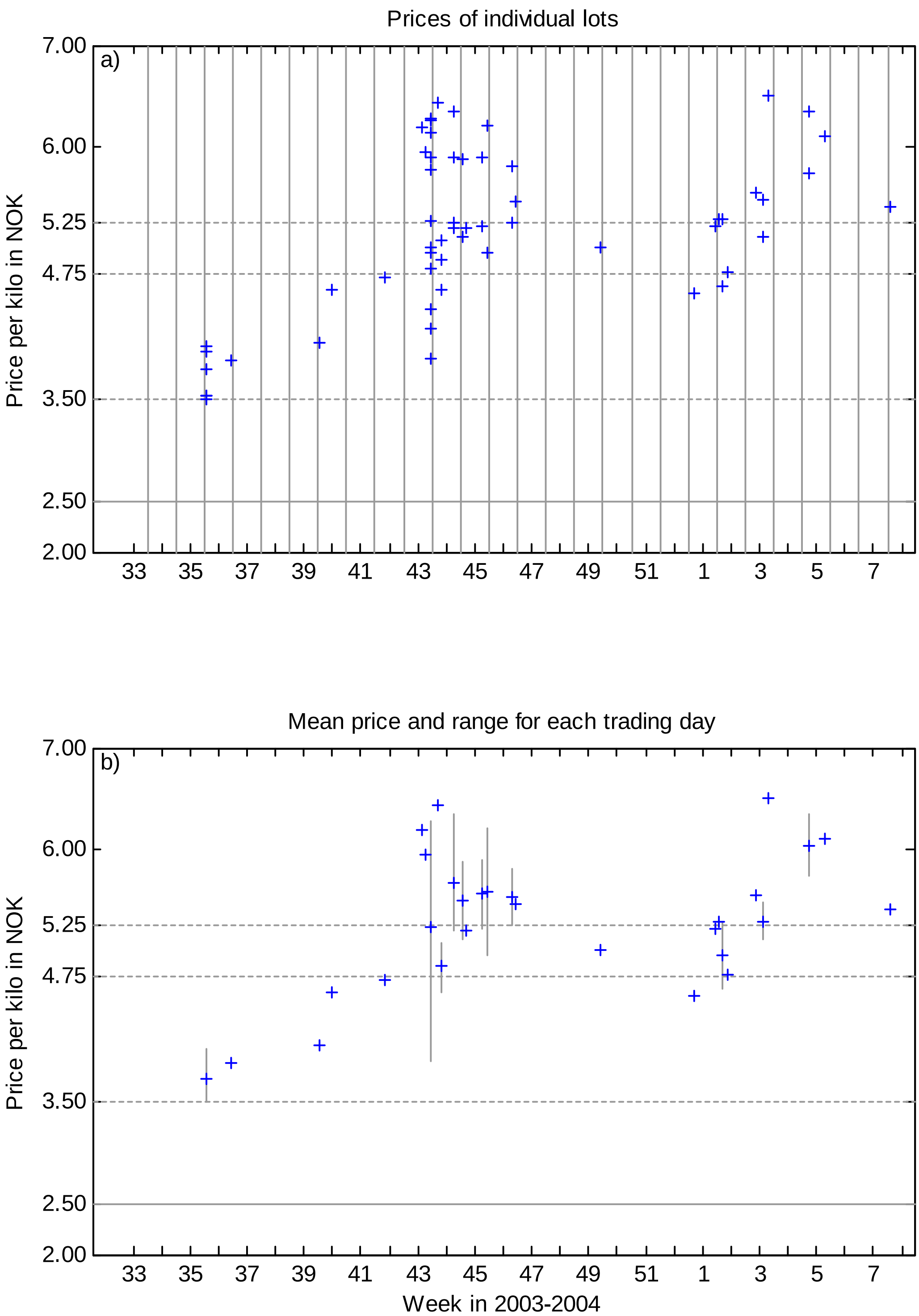


Figure 7.7: Individual prices and mean prices and ranges for lots with reserve price 1.50

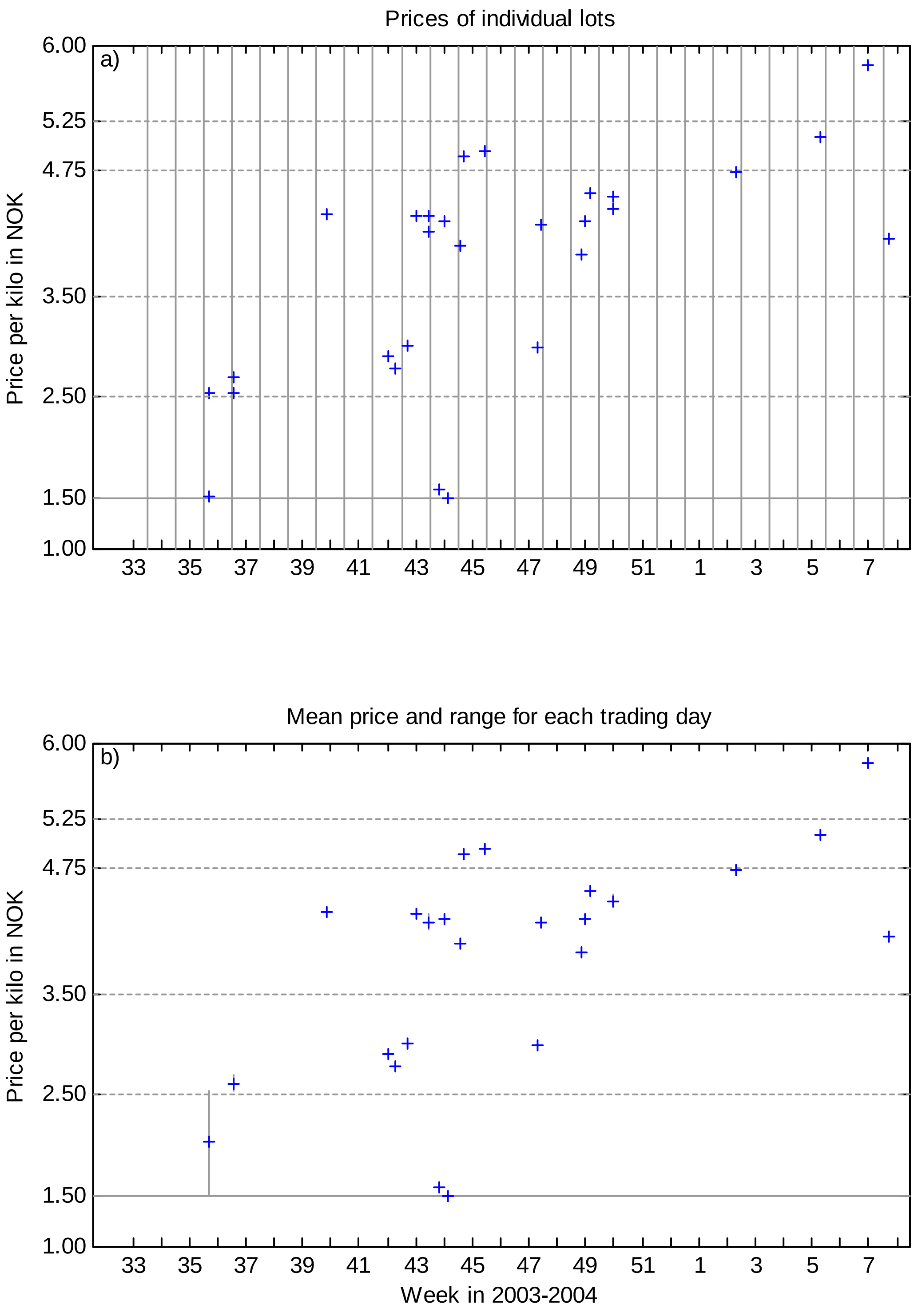


Figure 7.8: Least squares and quantile regression estimates, all lots

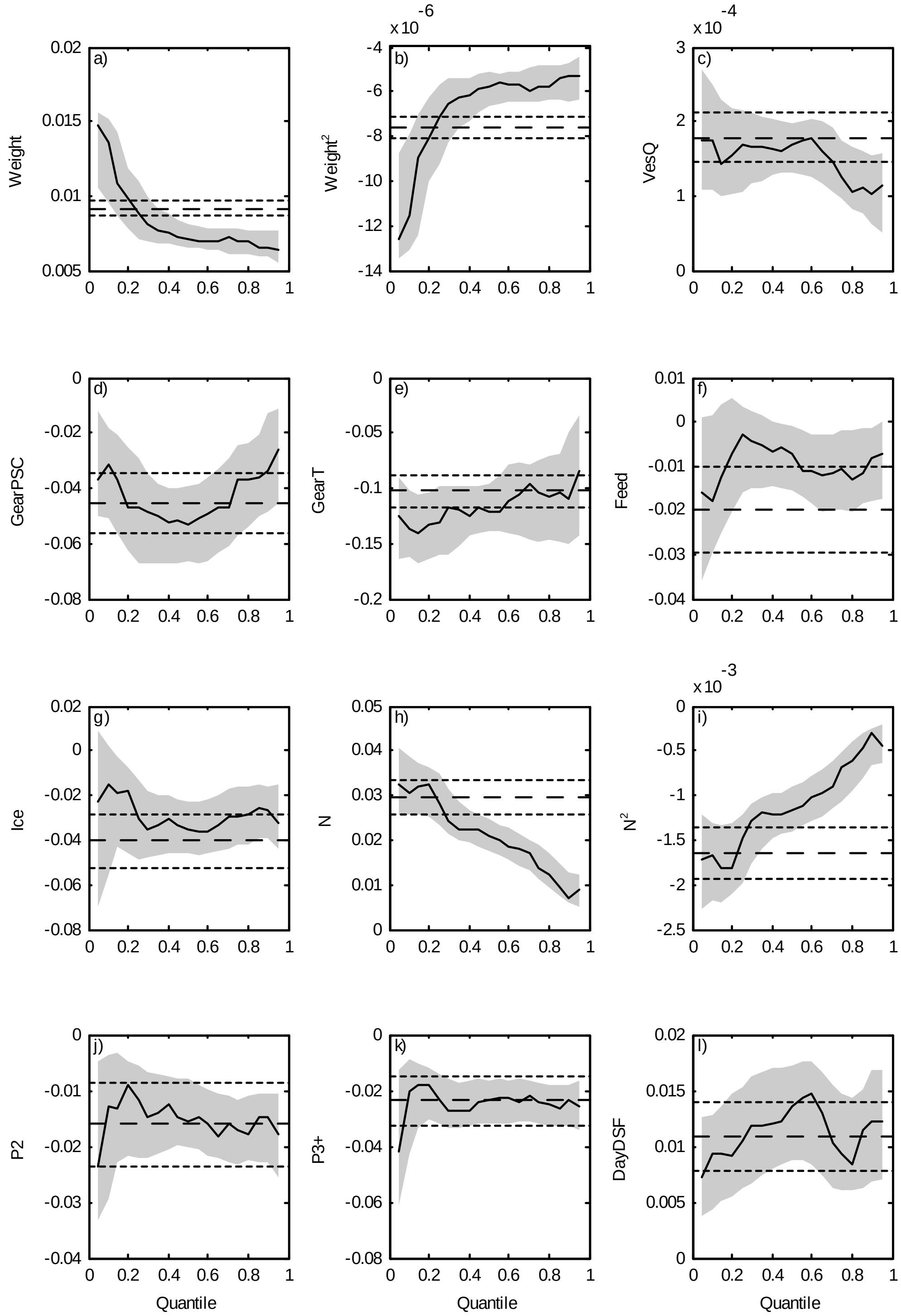


Figure 7.9: Least squares and quantile regression estimates, lots with reserve price 5.25 NOK

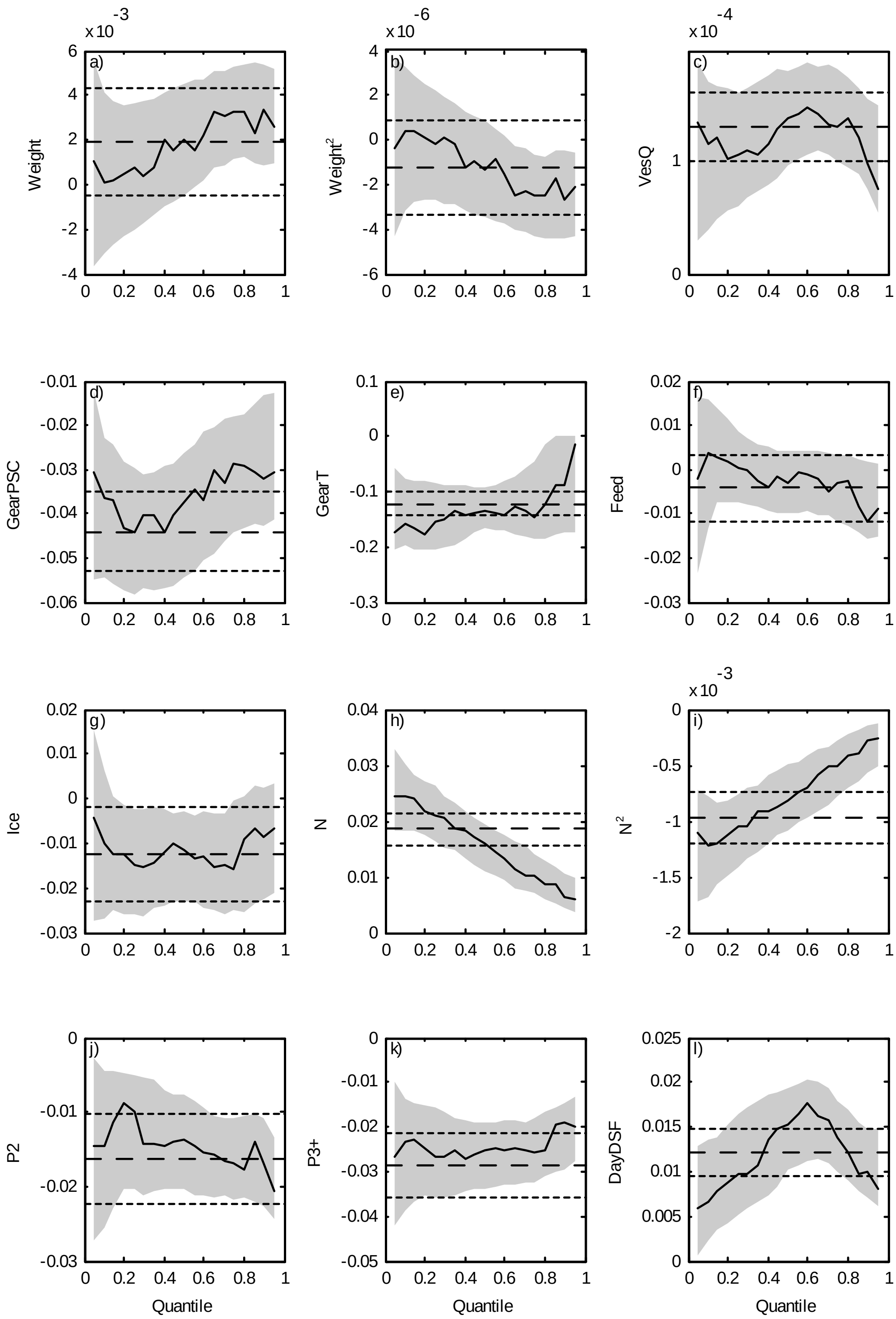


Table 7.15: Summary statistics of continuous and discrete variables, all lots (sample size = 1494)

	P	$\log(P)$	Weight	VesQ	N	DDSF
Minimum	1.50	0.41	230	5	1	1.00
25th percentile	5.76	1.75	483	55	3	2.34
50th percentile	6.54	1.88	535	125	5	3.03
75th percentile	7.05	1.95	570	200	7	3.58
Maximum	7.99	2.08	650	800	15	13.00
Mean	6.34	1.83	521.2	146.9	5.1	3.22
Standard deviation	0.92	0.17	67.05	118.41	2.94	1.58
Skewness	−1.10	−2.36	−0.91	1.74	0.60	1.74
Kurtosis	4.89	14.82	3.91	8.03	2.84	7.17

Table 7.16: Summary statistics of continuous and discrete variables, lots with reserve price 5.25 NOK (sample size = 981)

	P	$\log(P)$	Weight	VesQ	N	DDSF
Minimum	5.25	1.66	500	5	1	1.00
25th percentile	6.04	1.80	535	60	3	2.19
50th percentile	6.76	1.91	560	130	5	2.92
75th percentile	7.17	1.97	585	206	7	3.28
Maximum	7.99	2.08	650	800	15	13.00
Mean	6.63	1.89	560.4	146.0	5.2	2.88
Standard deviation	0.67	0.10	32.41	105.23	3.00	1.29
Skewness	−0.38	−0.51	0.17	1.60	0.63	2.22
Kurtosis	1.97	2.08	2.29	8.82	2.79	12.77

Table 7.17: Determinants of winning bids, lots with reserve price 5.25 NOK

	LS			LAD				
					Asymptotic		Bootstrap	
Coefficient	Est.	t-val.	p-val.	Est.	t-val.	p-val.	t-val.	p-val.
Constant	1.0455	2.60	0.010	1.0260	2.27	0.024	2.45	0.015
Weight	0.0019	1.33	0.182	0.0020	1.24	0.215	1.34	0.179
Weight^2	-0.0000	-0.98	0.329	-0.0000	-0.94	0.348	-1.03	0.304
VesQ	0.0001	7.08	0.000	0.0001	6.73	0.000	5.28	0.000
GearPSC	-0.0440	-8.15	0.000	-0.0376	-6.21	0.000	-4.23	0.000
GearTrawl	-0.1227	-9.25	0.000	-0.1338	-8.98	0.000	-5.62	0.000
Feed	-0.0043	-0.93	0.353	-0.0031	-0.60	0.552	-0.69	0.488
Ice	-0.0125	-1.94	0.052	-0.0115	-1.60	0.110	-1.88	0.060
N	0.0185	10.04	0.000	0.0158	7.64	0.000	6.42	0.000
N^2	-0.0010	-6.99	0.000	-0.0008	-5.24	0.000	-4.78	0.000
WinBid2nd	-0.0163	-4.40	0.000	-0.0138	-3.30	0.001	-3.62	0.000
WinBid3rd	-0.0286	-6.72	0.000	-0.0253	-5.28	0.000	-5.75	0.000
DayDSF	0.0121	7.60	0.000	0.0153	8.57	0.000	5.29	0.000

Dependent variable: Logarithm of winning bid.
The regression model includes a set of 23 buyer dummies and 12 week-specific dummies.
Number of observations: 981. Degrees of freedom: 933.
LS estimation: R-squared = 0.8327. Adjusted R-squared = 0.8241.

Chapter 8

Strategic bidding: Bids and priorities

8.1 Introduction

Bidding is about strategy. In the two previous chapters, we have examined how prices are determined by observable market and product characteristics. In addition, because our dataset is large, we were able to examine asymmetries between bidders. We concluded that, although bid frequencies vary between bidders, there is no evidence that bidders tend to be asymmetric in the sense that their winning bids differ much. In this and the next chapter, we tie-up a couple of loose ends by analysing whether bidders use the priority option strategically and whether the data reveal any signs of collusion among bidders.

In this chapter, we study an interesting strategic option that bidders have, the option of setting priorities to their bids. The option of priorities is closely linked to the option of setting quantity limits. To illustrate, suppose a bidder has the highest bid on two catches, say, 40 and 60 tons, and he sets a capacity limit equal to 70 tons. He will then only be allocated one of them, and the remaining catch will go to the bidder with the second-highest bid. If the bidder in this example, gives the 40-ton lot higher priority than the 60-ton lot, then he will be allocated the smaller lot should he win both auctions.

We first discuss why the option is part of the auction mechanism. Then, we discuss whether the priority option might be used strategically in order to increase profits. We do this by first proposing a strategy, and then setting-up and solving a model of equilibrium bidding relevant to our market. Finally, we examine empirically how the option is used by bidders and interpret our findings.

8.2 Rationale of the priority option

One reason for the priority rule is to give buyers an opportunity to optimize their bundles. Given the nature of supply, most notably the different delivery sectors of individual sellers, bundling catches before offering them on the market is impractical.[1] But buyers do have preferences over bundles. Since catches differ in size, not all bundle combinations are equally attractive. For a given bidder at a given time, winning one large and one small lot may be better than winning two small or two large lots. In general, bidders will probably prefer to obtain total quantities as close to their capacities as possible.

Another rationale for the priority option is that it gives structure to the potentially complex program of allocating the objects after the end of the bidding process. With no priorities, some discretion is left to the auctioneer with respect to the order of solving for capacity constraints. In principle, this could be used for the purpose of maximizing revenues at the cost of reduced utility for buyers.

To give an example, suppose a bidder A has the highest bid of 7 on two lots, but his stated capacity is to take only one. Now, if the second-highest bids are 6 and 5 on lots 1 and 2, respectively, then the auctioneer, in order to maximize revenues, would give lot 2 to A and sell lot 1 for 6. In so doing, he would realize 13 in revenue rather than 12. In turn, we might suspect that such discretion, if left to the auction house, would influence

[1] Chakraborty [18] analyses whether auctioneers should bundle different objects before selling them. Under the standard auction formats, he finds that when the number of bidders is above a critical level, the seller prefers unbundled sales.

bidding behavior. Bidders would take into account the risk of ending-up with suboptimal bundles. In most cases, with priorities, however, the allocation will be obvious. Thus, the option of setting priorities, as argued in the case of the capacity constraints, frees bidders from basing their bids on strategic considerations with respect to obtaining optimal bundles. Note, however, that buyers bid on each lot independently. The priority option is an imperfect instrument in order to get the preferred bundle. This, in turn, provides incentives to bid more aggressively on the preferred lot.

8.3 Strategic aspects of the priority option

We have seen that the priorities make the allocation mechanism transparent, and remove a strategic instrument from the seller. Given preferences over bundles, the priority option seems to be an instrument that helps to solve a complex allocation process, and helps bidders to acquire optimal quantities or bundles of catches.

Assume now that preferences over bundles of catches are unimportant. A natural question under this assumption is whether bidders can use priorities strategically; i.e., can they use priorities to obtain catches at a lower price than without this option? At a sealed-bid auction, imperfect information concerning competitors and their bids are important. In fact, the equilibrium first-price, sealed-bid function with independent valuations requires a rather difficult calculation and a precise information set in order to arrive at an equilibrium-bid strategy. Dropping the assumption of independent valuations, bid strategies do not get simpler. Although the theoretical bid function is the solution to an economic model, and not a blue-print of reality, it is safe to say that the informational requirements are demanding under our auction mechanism. It seems that the priority option can be used to reduce the uncertainty.

Recall that at an open, second-price auction because bidding one's valuation is a dominant strategy, the winning bid is equal to the second-highest valuation. Similarly, at a first-price auction the highest bid is equal to the *expected* second-highest valuation. Thus, the equilibrium solution is that winning bids will be above the second-highest *bid*, but equal to the second-highest *valuation*. This makes intuitive sense, but the argument to prove it is somewhat elaborate; see Krishna [59, section 2.3]. In the case of first-price auctions, bidders are not assumed to predict perfectly the second-highest valuation; after all, we model the situation as draws from a probability distribution. A bidder is assumed to know the distribution of competitors' valuations; he does not know the individual valuations. For winning bids to equal the expected value, they will necessarily from one auction to another fail on either side of the target. Sometimes, the winning bid will be below the second-highest valuation and sometimes above.[2]

Now, to the point, the bid ranking option enables bidders to hit the target of bidding slightly above the second-highest bid. Typically, bidders will only want some catches, not all offered for sale. During peak season, no one has the capacity to handle the entire quantity for sale. With this in mind, and recalling that bidders can bid on as many catches they like and set capacity limits, the bid-ranking option provides bidders the opportunity to test different bid levels. More precisely, bidders may diversify their bids over seemingly-equal lots, and then rank their bids, giving highest priority to their low bids. In that case, the result is a positive correlation between bids and priorities because an high priority is a low number. This strategy, combined with setting a quantity limit, will have the effect that of all winning bids a bidder submits, he is certain of being allocated the catch corresponding to his lowest winning bid because this will have higher priority than his even higher bids.[3]

Let us illustrate the argument by an example. Consider a bidder who wants two of four objects. One way of using the priorities strategically is the following: First, set a capacity limit for two catches. Next, submit four different bids, say, in descending order, so that object one has the highest bid and object four the lowest bid. Finally, set priorities in reverse order of

[2] It can be shown that the variance of the winning bid is larger at open, second-price auctions than at closed, first-price auctions. With increased number of bidders the difference between the two variances will be reduced.

[3] The expression "certain" should be somewhat modified since the auction house can over-rule priorities if necessary in order to clear the market.

bid levels; i.e., give priority one to the lowest bid and priority four to the highest bid. If all four objects are won, then a bidder will obtain the two lots assigned the lowest bids. The high bid may be interpreted as the best estimate of the equilibrium bid. Given the uncertainty about competitors valuations and bids, our bidder tests whether lower bids are winning as well.

Table 8.1: Use of priorities, numerical example

			Strategy S_1		Strategy S_2	
Catch	Quantity[a]	Bid[b]	Priority	TC[c]	Priority	TC
1	150	6.1	1	915	4	
2	150	6.3	2	945	3	
3	150	6.5	3		2	975
4	150	6.7	4		1	1005
Sum:				1860		1980

[a] Quantity in tons
[b] Bid in NOK per kilo
[c] TC: Total cost in 1000 NOK

To make the idea clear, we provide a simple numerical example of the situation by setting actual quantities and bids for a given bidder and examine the effect of two different priority rankings, denote them S_1 and S_2. The figures of our example is reported in table 8.1. To focus on the effect of priorities, we set the quantity of the catches to be equal. Recall that our bidder has the highest bid on all lots.

Under priority strategy S_1, the bidder is allocated catch 1 and 2, while he wins catches 3 and 4 under strategy S_2. Using strategy S_2 will increase the average price by 6.45 percent or involve a total cost 120,000 NOK higher than if strategy S_1 was used. Consequently, under ideal conditions, the priority option can be used strategically by a bidder to reduce the price paid. The way he obtains this is by testing different bid levels in order to reduce the difference between his winning bid and the second-highest bid. One condition is that the bidder is indifferent between the catches with respect to all factors but price. Given this prerequisite, he will have to hope that the bid process gives the result that he is the winner of the catches with low bids.

To a certain degree, however, some factors will reduce the possible impact of the bid ranking option. First, if catches are heterogenous, then using the bid ranking option in the way described above becomes less attractive, because other aspects of the catches will dominate the bid priorities. Second, systematic bid ranking strategies are most valuable when there is a sufficient number of catches offered on the market. The opportunity to test different bid levels is weakened when there are only a few comparable catches available. In this case, a bidder may want all catches. Note that it only takes two catches offered in order to implement the strategy of correlating priorities with the bid levels.

8.4 An auction model with priorities

We have pointed out a potential bidding strategy for increasing expected profits: Bidders may vary bids and give the lowest bid the highest priority. One might object that a dominant bid strategy in the single-object case should be repeated at the multi-object auction since bids in a sense are independent due to the option of setting a capacity limit. In terms of optimization, the problem of maximizing total profits is obviously equivalent to the problem of maximizing the independent parts that constitute the whole. An optimal bid in the single-object case is based on balancing the probability of winning with the profit margin if winning. If a bidder does not follow this optimal rule for all bids, but instead strategically varies bids over a given range, then he deviates from the strategy of maximizing the individual parts.

However, the optimality of the dominant bid strategy rests on the information set available to bidders. With the bid ranking option, the bidding procedure is, to some extent, equivalent to the following two stage process. First, bidders are invited to bid on some or all catches, and then they are *ex post* given the opportunity to decide what winning bids are binding. It is clear that the information set in this case differs from the case with no bid ranking option.

Can a strategy where bid levels are correlated with priorities be an equilibrium solution of the game? We analysed this by formulating a model where the allocation depends on the priorities. Although we consider the simplest possible model that captures the characteristics of our auction market, the model is too complex to analyse analytically, so we chose to solve for the equilibrium numerically.

Benchmark model. As a benchmark, consider first a traditional discriminatory, simultaneous auction model: Three bidders—denoted A, B and C—with single-unit demand compete for two identical units. Each bidder draws a value independently from the same distribution F_V. Each bidder submits only one bid, and the two highest bidders obtain a unit each.

The equilibrium bid is to bid the expected second-highest value of the competitors conditional on that the bidder itself has drawn the highest value. In general, according to Krishna [59, p. 195], with $\mathcal{N}$ bidders and $K < \mathcal{N}$ units, the equilibrium strategy, $\beta(v)$, is

$$\beta(v) = \mathcal{E}\left[Y_{(K:\mathcal{N}-1)} | Y_{(K:\mathcal{N}-1)} < v\right] \tag{8.1}$$

where $Y_{(K:\mathcal{N}-1)}$ is the K^{th}-highest order statistic of $(\mathcal{N}-1)$ draws from the distribution function F_V. In the case of three bidders and two units, we see that equation (8.1) tells us to bid the second-highest, equivalently, the lowest order statistic from two draws from F_V conditional on that one's own value v is higher:

$$\beta(v) = \mathcal{E}\left[Y_{(2:2)} | Y_{(2:2)} < v\right]. \tag{8.2}$$

The population density function of the second-highest order statistic, $f_{(2)}(v)$, is from equation (2.3):

$$f_{(2)}(v) = 2\left[1 - F_V(v)\right] f_V(v).$$

The cumulative distribution function of the second-highest order statistic,

$F_{(2)}(v)$, is from equation (2.2):

$$F_{(2)}(v) = F_V(v)\left[2 - F_V(v)\right].$$

Since the conditional expectation of a random variable V with pdf f_V and cdf F_V given that $V < v$ is

$$\mathcal{E}\left[V|V < v\right] = \frac{1}{F_V(v)} \int_0^v u f_V(u)\, du,$$

we find that the equilibrium-bid function in equation (8.2), can be expressed in closed-form as

$$\beta(v) = \frac{1}{F_V(v)\left[2 - F_V(v)\right]} \int_0^v 2u\left[1 - F_V(u)\right] f_V(u)\, du. \tag{8.3}$$

Below, we refer to equations (8.2) or (8.3) as the benchmark bid.

Model with two submitted bids and priorities. We now extend the benchmark model by changing the following characteristics: Bidders compete separately on each object. They still have single-unit demand. Thus, they are allowed to set a capacity limit equal to one unit, and to give priorities to their bids. We see that the auction format resembles our empirical auction market. In principle, it is possible to have this situation in our real-world market. For most auctions, however, the number of potential bidders will vary and so will their capacity limits.

The allocation mechanism is then as it is at our real-world auction market: If the same bidder has the highest bid on both objects, then he is allocated his preferred object, and the other object goes to the second-highest bidder on that good. On the other hand, if different bidders have the highest bids, then they are allocated their respective objects. Notice that priorities do not play any role in the latter case. Priorities only come into consideration when the same bidder wins both objects. Let a bid vector for representative bidder

i be defined to be a tuple of four:

$$\boldsymbol{b}_i = \{B_1, B_2, P_1, P_2\}$$

where B_j is the bid and P_j is the priority for object j. In this model, there is no reason to bid on only one unit. The probability of winning cannot decrease if bidding on both. Moreover, as long as one submits bids equal to the equilibrium bid from the benchmark model, this is equivalent to bidding in the benchmark model. Thus, bidders will submit tenders for both units.

What will equilibrium bidding look like under this format? If bidders use the benchmark strategy of equation (8.2) on both objects, then the outcome will be the same as in the benchmark model. Expected profits and the seller's revenue will not change. Bidding lower than the benchmark bid on both objects cannot constitute an equilibrium. A bidder will then find it profitable to deviate by scaling his bid upwards towards the equilibrium bid.

We propose that an equilibrium might be to bid the benchmark strategy on the low priority object and to scale down the benchmark bid by a factor on the high-priority object. The two high value bidders will then ensure that they obtain a unit for at most the benchmark bids, and the auction remains efficient. However, if submitted priorities are favorable, then the high-value bidder may obtain a unit for a lesser price than the benchmark bid.

We proceed by looking for an equilibrium when a constant scaling factor is used; i.e., the scaling factor is not a function of valuations. Let α represent a number between 0 and 1. Thus, we propose to examine the following completely mixed bidding strategy:[4]

$$\boldsymbol{b}_i = \begin{cases} \{\beta(v), \alpha\beta(v), 0, 1\}, & \text{with probability } \frac{1}{2}; \\ \\ \{\alpha\beta(v), \beta(v), 1, 0\}, & \text{with probability } \frac{1}{2}. \end{cases} \tag{8.4}$$

Bidders do not know the priorities of their competitors. Thus, we analyse a game where coördination is a strategic problem. We assume each bidder

[4] A *mixed* strategy maps each of a player's possible information sets to a probability distribution over actions. A *completely* mixed strategy puts positive probability on every action; see Rasmusen [93, pp. 66–67].

chooses his high-priority unit—the one with a scaled down bid—at random (by the flip of a coin). A bidder's priority of a unit is equal to 1 if he prefers the unit, and 0 if he prefers the other unit. Denote the submitted priorities on unit 1 by a vector $\left(P_1^A, P_1^B, P_1^C\right)$; i.e., P_1^A is bidder A's priority for unit 1, and so forth. Denote the set of possible priority vectors by Pri_j for object j. For each auction t, we then have eight possible priority vectors for the first unit:

$$\mathrm{Pri}_1 = \{(111), (110), (101), (011), (100), (010), (001), (000)\}.$$

The priority vector (111) signifies that all bidders prefer unit 1, while the vector (010) represents the case where only bidder B prefers unit 1. Pri_2, the possible priorities for the second unit, equals the complement of Pri_1.

What potential gains might there be to using the strategy stated in equation (8.4)? Denote the benchmark bid from the bidder with the K^{th}-highest valuation by $\beta\left[v_{(K)}\right]$. For the bidder with the highest valuation, there is no risk in using the strategy. He will always win one unit. In case all bidders have identical priorities, which happen with probability (1/4), he will certainly obtain the unit at the scaled down bid. In the remaining cases, whether the high-value bidder obtains a unit at the scaled down bid $\alpha\beta\left[v_{(1)}\right]$ depends on the scaling factor. For moderate scaling factors, we can safely assume that the high-value bidder will have the highest bid on both objects, regardless of how competitors choose their priorities and corresponding high bids. The bidder with the second-highest valuation has, however, some worries. If he has the same priority as the high-value bidder, but opposite priority from the third-highest-value bidder, then he may miss winning a unit if the scaling factor is sufficiently high. To be specific, for priority vectors {(110),(001)} which happen with probability (1/4), there is a risk of losing the unit for the second-highest valuation bidder. This happens only if the scaled bid of the second-highest valuation bidder is less than the equilibrium bid of the third-highest valuation bidder; i.e., $\alpha\beta\left[v_{(2)}\right] < \beta\left[v_{(3)}\right]$.

To summarize the strategic considerations of the model, we note: Bidders are assumed to compute the benchmark bid $\beta(v)$ of equation (8.3). Next, what object to prioritize is determined randomly. Finally, the percentage α which determines the high-priority bid is the important strategic element that we solve the model for.

If we want to solve the model analytically, then we shall have to set-up the expected profit function. We quickly realize that the different probabilities of winning make the model messy and probably impossible to solve for a closed form equilibrium bid. Thus, we opt for a numerical analysis of the model.

Solving the model numerically. We examine the large-sample properties of the model by simulating T auctions. We first draw T priority vectors from the eight possible combinations in Pri_1 with equal probability. We ensure that the drawn priority vectors are represented exactly the same number of times; each vector is represented in (1/8) of the T draws.

In a second, independent simulation, we draw $(T \times 3)$ values from the uniform distribution on [0,1]. For each bidder and valuation, we compute the two bids $\beta(v)$ and $\alpha\beta(v)$ for a reasonable set of combinations of α and allocate the two bids to the correct objects according to the priorities. In principle, we should test all possible strategy combinations where $\alpha \in [0, 1]$. Given the continuous strategy space, we have to settle for a discrete approximation of the strategy space in a numerical analysis. Obviously, using $\alpha = 0$ cannot be part of an equilibrium since an α infinitesimal above 0 is a profitable deviation. We examine 20 different strategies for $\alpha \in [0.05, 0.10, 0.15, \ldots, 1]$ for each bidder in the numerical analysis and rely on that the response functions are smooth and continuous for the strategies not tested. The result is a game in normal form with 8,000 strategy combinations since each of the three bidders has 20 strategies to choose between.

Next, we allocate the objects to the winning bidders according to the allocation mechanism described above. After setting up the allocation table, we map the strategies into payoffs (expected profits).

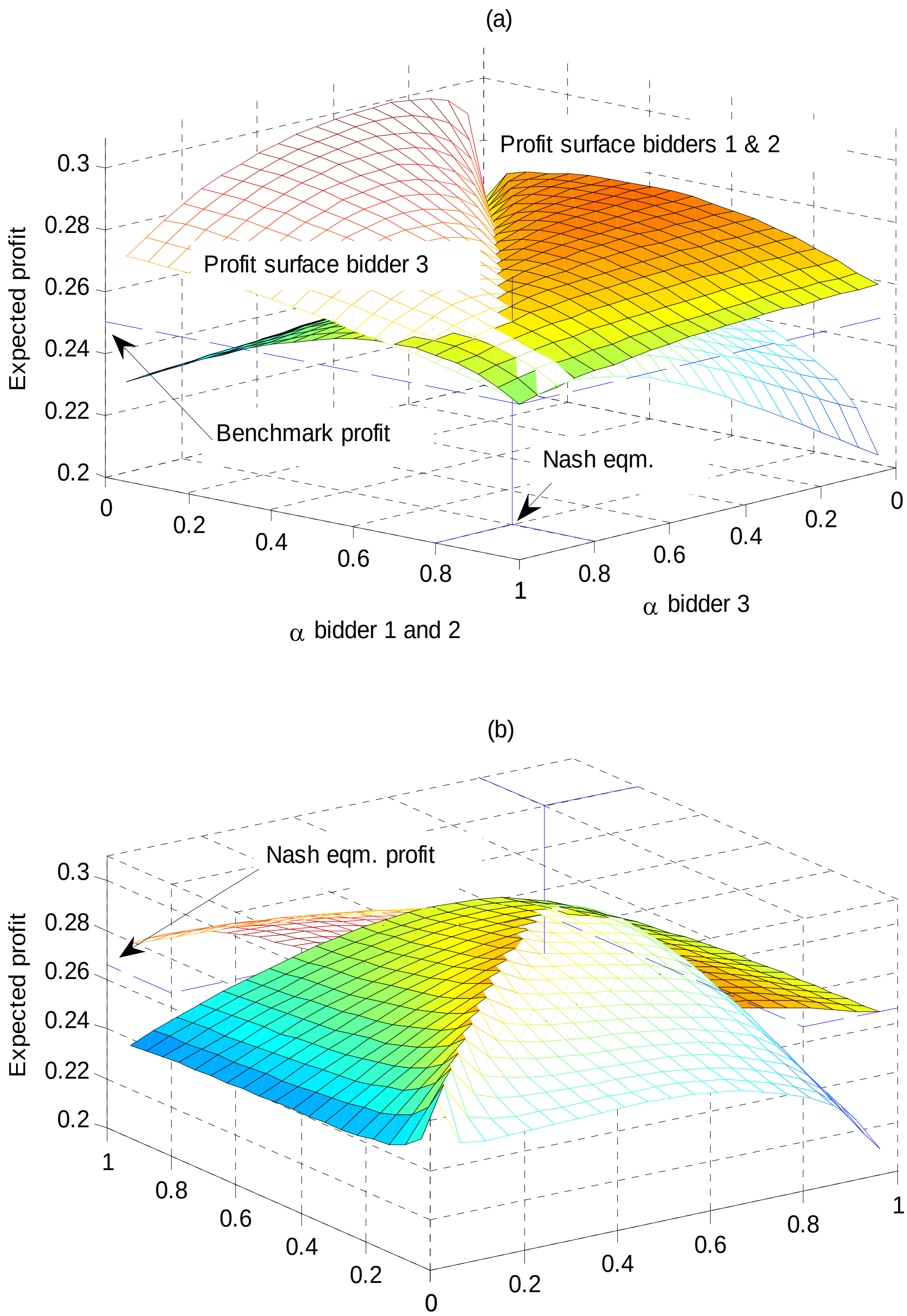
In the simulation, we used $T = 100,000$. The $(T \times 3)$ valuations and priority vectors are drawn once, and the $(T \times 3)$ low priority bids $\beta(v)$ are computed once. The $(T \times 3)$ high-priority bids $\alpha\beta(v)$, however, are calculated 8,000 times; i.e., they are calculated over again for each strategy combination of scaling factors. The numerical simulation and the identification of the Nash equilibrium is documented by the script written in MATLAB which is in appendix B.[5]

Searching the strategic form of expected profits for Nash equilibria results in a unique symmetric equilibrium where bidders use $\alpha = 0.80$. In figure 8.1, we plot the payoff surfaces for bidders.

The profit surface of bidder 3 is the surface with light squares in the grid, while the surface with filled squares represents bidder 1 and 2. The latter case is constructed by taking the mean expected profit of bidder 1 and 2 when they use the same (symmetric strategy) α. In figure 8.1a, we notice that when all bidders use $\alpha = 1$ (no scaling of prioritized bid), the expected profit is 0.25 for all. This is the same as expected profit in the benchmark model. If bidder 3 maintains the strategy $\alpha = 1$, then the two other bidders will profit from reducing their α, but only up to a point. For sufficiently low α, however, we see that their payoff falls well below the benchmark profit of 0.25. To be precise, when $\alpha = 1$ for bidder 3 and $\alpha = 0.05$ for bidders 1 and 2, expected profits are roughly 0.27 and 0.23 respectively. Obviously, when a bidder uses a very low α, he will almost never win his prioritized object. The unique symmetric Nash equilibrium is illustrated in both figure 8.1a and 8.1b. In 8.1b, we show the equilibrium profit which equals 0.266. The profit surfaces at the Nash equilibrium is sloping downward in the relevant directions. To see this, fix α at 0.8 for bidder 3 and look at the profit for bidders 1 and 2 for different values of α. The maximum profit along the surface where bidder 3's α is 0.8 is at $\alpha = 0.8$. A similar argument shows that bidder 3 reaches an optimum at the Nash solution when bidder 1 and 2's α is fixed at 0.8. No bidder has an incentive to deviate from the proposed solution if the other players do not deviate.

[5] MATLAB is a registered trademark of The Mathworks, Inc.

Figure 8.1: Payoff surfaces and Nash equilibrium



A clearer picture of the Nash equilibrium is obtained by the two dimensional figure 8.2 on page 223. Here, we fix bidder 1's and 2's α at 0.8, and show bidder 2's profit for different strategies of bidder 3. Bidder 1 will have a similar profit function like that depicted for bidder 2. Likewise, we plot the profit function for bidder 3 for different choices of scaling factor. The maximum profit for bidder 3 is clearly at $\alpha = 0.8$. The Nash equilibrium is robust in the sense that whatever strategy bidder 3 chooses, bidder 1 and 2 will obtain an higher profit than the benchmark profit of 0.25.

The seller's revenue suffers in this model compared to the benchmark model, where it can be shown that the expected revenue is equal to 0.5. In the equilibrium, the sellers expected revenue is 0.45; i.e., a 10 percent decrease from the benchmark model.

Concluding remarks on the model. We stated in section 8.3 that bidders might use a strategy where bids are correlated to priorities, but it was far from obvious that down-scaling high-priority bids would constitute an equilibrium. In this section, we have shown that a unique symmetric equilibrium exists for the case of three bidders competing for two identical objects when valuations are drawn independently from the standard uniform distribution.[6] The equilibrium is characterized by bidding lower on the high-priority bid. An α equal to 0.8 as the equilibrium strategy is obviously a result of the chosen distribution of values. The important result is that bidders differentiate the two submitted bids and scale down the bid on the high-priority object. We have not shown that this result is distribution-free. We conjecture, however, that it generalizes to other distributions. The driving force of the model is that the options of setting capacity constraints and priorities open up a possibility of testing different bid levels.

[6] To be precise, what we proved was that single-handedly deviating from the symmetric strategy $\alpha = 0.8$ by discrete jumps of 0.05 in either direction is suboptimal for all players.

8.5 Correlations of bids and priorities

Anecdotal evidence from representatives of the auction house suggests that they sometimes, but not frequently, observe bidders using the strategy of systematically linking low bids to high priorities, or "trying to be smart" as they call it. We now turn to an empirical examination of the use of priorities using some statistical measures. If bidders use priorities strategically as described in the example above, then high priorities will correspond to low bids. In order to examine this hypothesis, we propose to examine the correlation between bids and priorities. Note that an high priority in our case corresponds to a low number; the preferred catch is given priority 1 and less preferred catches will have priorities 2, 3, and so forth. Correlation coefficients will, consequently, be positive when high priorities (low priority numbers) correspond to low bids. Correspondingly, high bids accompanied by high priorities result in a negative correlation coefficient.

Correlation measures. Different measures of correlation exist; see, for example, Myers and Well [78]. In our case, a natural candidate for a correlation coefficient measure is Spearman's rank correlation. Spearman's rank correlation is a non-parametric measure of correlation; no assumptions concerning the frequency distribution of the variables are required. The correlation measure is between ranks of the variables. In our case, the priorities are already ranked. Bids will have to be transformed to a ranking. Given a bid vector [5.25 6.10 5.35] for three catches, the ranking vector of bids is [1 3 2], indicating that the first bid is the lowest, the second bid is the highest and the third bid is the medium bid.

The formula of Spearman's rank correlation is

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

where d_i is the difference between each rank of corresponding values of X and Y, and n is the number of pairs of values. In the data, however, we sometimes have ties both between bids and between priorities. With respect to priorities, a bidder may give priorities [1 2 2] indicating that he prefers the first catch, but is indifferent between catch 2 and 3. Likewise, the bid vector [6.10 5.35 6.10] produces the ranking [2 1 2]. The simple formula of Spearman's rank correlation coefficient is inappropriate to use when ties between values occur. We can, however, always obtain the Spearman's rank correlation by using the formula of the classic Pearson product-moment correlation coefficient to the ranked data:

$$\rho = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^2 - \left(\sum y_i\right)^2}}. \tag{8.5}$$

Spearman's correlation coefficients are equivalent to computing the Pearson correlation coefficient using rank vectors as input; again see Myers and Well [78, p. 508].

For every bidder and every relevant auction in the dataset, we computed the correlation coefficient between ranked bids and priorities. By relevant auction, we mean that correlation coefficients are only appropriate to compute when the bid vector is sufficiently large. Obviously, a bidder has to bid on at least two catches for the procedure to be meaningful.

Given the high number of statistics thus generated, we need to summarize our results appropriately. We chose to report for each bidder the number of correlation coefficients that falls into the following seven categories: (1) perfect negative, $\rho = -1$, (2) large negative, $\rho \in (-1, -0.5)$, (3) moderate negative, $\rho \in [-0.5, 0)$, (4) no correlation, $\rho = 0$, (5) moderate positive, $\rho \in (0, 0.5]$, (6) large positive, $\rho \in (0.5, 1)$, and (7) perfect positive, $\rho = 1$.

Since most bid vectors are short, computed correlations will typically have high p-values; correlation coefficients are statistically insignificant for standard levels of significance. We see no point in reporting the p-values. Dismissing all "insignificant" correlations will simply take away a lot of valuable data points. If a bidder consistently has a perfect positive correlation between bids and priorities when his bid vector has length 3, this pattern is interesting, but we shall be unable to conclude that this pattern actually reveals a strategy if we dismiss all insignificant correlations. The suggested summary of the distribution of correlations for every bidder will, however, reveal any underlying tendency of correlations.

Discussion of correlation tables. All tables are relegated to an appendix of this chapter. In table 8.2 on page 224, we report the results when we take all relevant catches and auctions where a bidder submits two or more bids.[7] The case *no correlation* ($\rho = 0$) between bids and priorities appears frequently. Four reasons exist for a zero correlation to appear for a given vector of bids and priorities: (1) all priorities and all bids are equal; (2) all priorities are equal, but some or all bids are unequal; (3) priorities are unequal but all bids are equal; and (4) priorities and bids are both unequal. Note that equal priorities will normally mean that no priorities are set. Only case (4) is a true zero correlation. This case can only appear when the bid vector has an odd length above or equal to 5. The remaining cases are set to zero, since if we apply the formula of equation (8.5), we end up with a zero standard deviation of one of the variables.

Of the 440 zero correlations reported in table 8.2, case (1) accounts for 12.7 percent, case (2) 53.0 percent, case (3) 30.0 percent, and case (1) 4.3 percent. A true zero correlation is rare. The most frequent cause of a zero correlation is simply that bidders bid the same for two catches, and the second important explanation for a zero correlation is that no priorities were set. Looking at the distribution of negative versus positive correlations, we see that the distribution is left skewed; far more correlations are negative than positive meaning that bidders prefer catches they bid high on. Let us analyse this table in depth.

Before proceeding, we shall discuss one possible explanation for negative correlations. If bidders prefer catches with high average fish weight, then they will tend to prefer the high bids since realized prices increase with fish weight. Therefore, in the following, we shall compute correlations for bid vectors that exclusively relate to catches with reserve price 5.25 as well. This will partly control for the effect of average fish weight on priorities. Note that the procedure will, in some instances, change the length of bid vectors.

[7] Length of a bid vector is shortened ℓ in the list of tables.

In many cases, however, bidders bid on only large weight catches, and the correlations will be the same as when we sample all catches. In the cases where a bidder bids on two catches with reserve price 5.25 and one catch with reserve price 4.75, his bid vector used for computing the correlation changes from length three to two. In table 8.3 on page 225, we report the correlations when we sample all bid vectors larger than two but with reserve price 5.25.

A striking fact is the high number of perfect negative correlations; 16.5 percent in table 8.2 and 15.5 percent in table 8.3. This is the result of calculating correlations for bid vectors with length equal to two. For bid vectors with few bids—two or three bids—only a few values of Spearman's rank correlations are possible. When a bidder bids on only two catches at an auction, and gives different priorities to them, then the correlation coefficient will take only two values. The only possible outcomes are a perfect negative correlation or a perfect positive correlation; correlations will either take the value -1 or 1. In table 8.4 on page 226, we show the distribution of correlations when we examine bid vectors of length two. The most common choice is not to give priorities at all, resulting in $\rho = 0$, but when bidders do, they tend to end up with a negative correlation. This is the case where we sample all relevant catches, but also when we sample only catches with reserve price 5.25. Comparing table 8.2 with table 8.4, we conclude that perfect correlations are predominantly explained by bid vectors with length two. In total, we have 270 perfect negative correlations (table 8.2). Bid vectors of length 2 account for 201 (table 8.4), and bid vectors of length 3 account for 53 of them (table 8.5).

When the bid vector has length three, we have four possible outcomes if there are no ties between bids or between priorities: $(-1, -0.5, 0.5, 1)$. In the presence of ties, correlations can take the values $(-0.87, 0, 0.87)$. In tables 8.5 and 8.6 on pages 227–228, we have summarized the number of correlations for the two samples—all catches and catches with reserve price 5.25—when bid vectors have length three. Roughly a third of the correlations are zero, the remaining correlations are predominantly negative.

Given the few possible value outcomes of correlation coefficients when only two or three bids are submitted, we examine correlations for bid vectors of length four or higher. The results are reported in tables 8.7 and 8.8 on pages 229 and 230. Again, we find that correlations are left skewed. A rare case of a perfectly positive correlation is observed for bidder 16 in table 8.8, while quite a few of the bidders have some correlations that are perfectly negative.

To summarize our results, note that many correlations are zero. This is explained for the most part by bids or priorities (or both) being equal. Dismissing zero correlations and looking at the percentage of negative correlations versus positive correlations, we report the percentages for each bidder in table 8.9. All bid vectors discussed above are reported. Some differences between bidders exist. For bidders 3 and 16, we observe the percentage of negative correlations dropping below 50 percent when three bids are submitted at an auction. All in all, the evidence of predominantly negative correlations are firm.

So why are so few positive correlations observed? Why do bidders tend to give high priorities to their high bids? This is somewhat counter-intuitive; at least in the light of the reduced cost the strategy explained in the example of table 8.1 can lead to. One possible explanation is that priorities are determined by average fish weight of catches rather than the bids. Given the significant correlation between bids and weight, we expect the correlations between average fish weight and priorities to show a similar pattern as the tables reporting the correlations between bids and priorities. For the case of bid vector larger than 1 and all catches, we report correlations between weight and priorities in table 8.10 on page 232. Our hypothesis is confirmed. Bidders do tend to give high priorities to catches with high average fish weight. Priorities act as an energizer to the bids; in addition to submitting an high bid on a catch, the bidder gives the catch an high priority as well.

The reason is, however, somewhat puzzling. In chapter 6, we found that the effect on prices when weight increases, is weak for weights above 500 grams, see in particular the regression results of table 7.17. Given that prices do not rise much when weight increases from an high level, why does the percentage of negative correlations not decrease when we sample only catches with reserve price equal to 5.25? The best explanation is probably that priorities are used strategically to get preferred bundles of lots. When bidders prefer their high bid catches, it indicates that preferences are reflected in the bids. This again indicates that heterogeneity of catches determines differences in bids. If we observed positive correlations between priorities and bids—the case where bidders preferred their low bid catches—then it could indicate that uncertainty about the value of the catch was a dominant factor, and, consequently, that there was room for the randomizing strategy discussed above.

8.6 Concluding remarks

We have found little evidence supporting the hypothesis that bidders use the priority option strategically in order to reduce the winning bid. Our findings are consistent with the hypothesis that bidders use priorities to obtain optimal bundles of catches in order to reduce total costs. Thus, it seems that priorities are used the way they are meant to be used. If priorities were used strategically, as in the model of section 8.4, then inefficiencies would have been introduced. In some cases, the bidder with the highest market-clearing willingness-to-pay would not win the object. From an economic perspective, it is reassuring that inefficiencies do not seem to be introduced by the auction format.

8.A Appendix: Tables and figures

Figure 8.2: Nash equilibrium illustrated

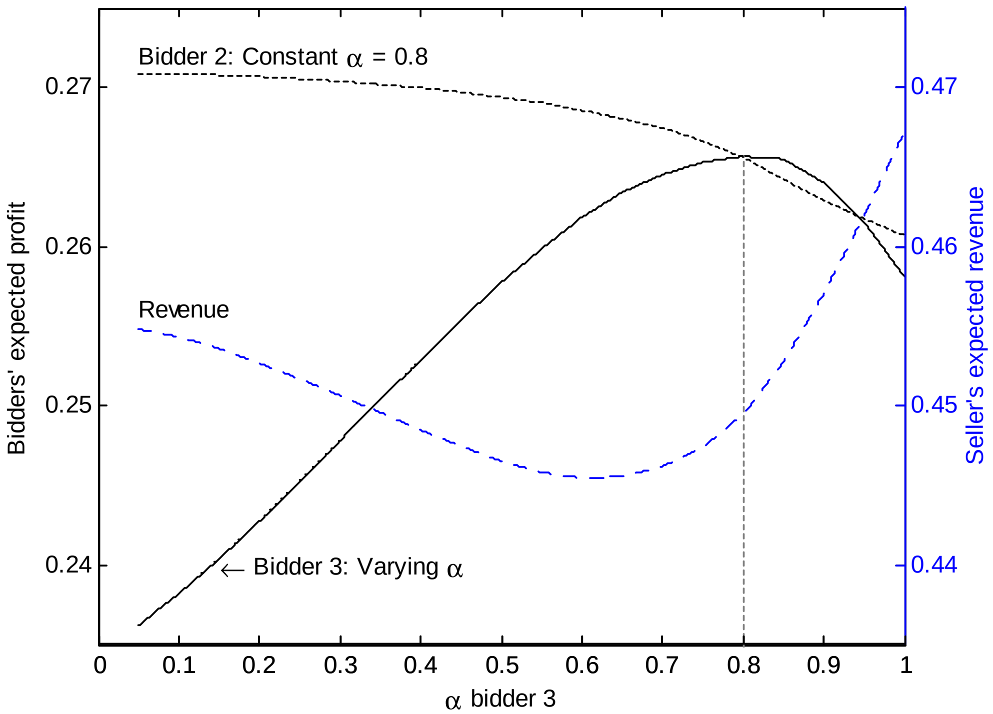


Table 8.2: Correlation of bids and priorities. All catches, bid vectors larger than 1

	Number of $\rho \in$							
Bidder	$[-1]$	$(-1,-.5)$	$[-.5,0)$	$[0]$	$(0,.5]$	$(.5,1)$	$[1]$	Count
1	11	31	3	38	1	0	1	85
2	6	15	2	2	0	0	0	25
3	8	8	8	18	5	5	5	57
4	15	25	4	26	1	0	2	73
5	7	20	11	39	3	1	3	84
6	13	31	2	19	1	0	2	68
7	4	27	6	7	0	4	1	49
8	14	19	12	12	7	0	3	67
9	7	5	6	7	3	2	2	32
10	16	21	19	26	7	8	4	101
11	1	0	0	1	0	0	0	2
12	22	60	18	29	5	3	1	138
13	27	55	18	15	6	0	5	126
14	0	7	0	1	0	0	0	8
15	17	23	16	24	11	5	8	104
16	13	33	14	46	11	11	3	131
17	15	46	6	12	0	0	2	81
18	1	8	1	4	0	1	0	15
19	23	26	4	15	1	1	4	74
20	14	12	6	5	2	1	2	42
21	7	12	8	24	4	3	6	64
22	2	8	1	11	0	2	0	24
23	10	11	12	15	5	5	6	64
24	7	12	7	30	6	5	5	72
25	10	17	2	14	2	3	1	49
Sum:	270	532	186	440	81	60	66	1635
Percent:	16.5	32.5	11.4	26.9	5.0	3.7	4.0	100

Table 8.3: Correlation of bids and priorities. Lots with reserve price 5.25, bid vectors larger than 1

	Number of $\rho \in$							
Bidder	$[-1]$	$(-1,-.5)$	$[-.5,0)$	$[0]$	$(0,.5]$	$(.5,1)$	$[1]$	Count
1	9	24	0	23	1	0	2	59
2	3	9	2	3	0	0	0	17
3	6	9	5	7	4	4	4	39
4	12	19	3	16	1	0	3	54
5	2	18	9	19	2	1	3	54
6	6	16	1	14	2	0	0	39
7	6	24	5	7	0	1	1	44
8	9	16	9	6	2	0	3	45
9	3	4	3	6	2	2	2	22
10	5	14	9	31	3	2	1	65
11	0	0	0	0	0	0	0	0
12	18	41	14	25	3	3	1	105
13	18	41	13	11	3	2	7	95
14	0	6	0	1	0	0	0	7
15	12	17	9	18	9	4	4	73
16	13	29	7	29	4	9	10	101
17	11	32	5	13	0	0	2	63
18	1	7	1	4	0	1	0	14
19	13	21	0	8	1	1	1	45
20	10	8	3	5	1	0	3	30
21	5	11	4	13	0	4	1	38
22	1	8	0	6	0	2	0	17
23	6	9	6	11	3	3	2	40
24	4	6	7	25	3	6	4	55
25	7	10	1	14	2	3	1	38
Sum:	180	399	116	315	46	48	55	1159
Percent:	15.5	34.4	10.0	27.2	4.0	4.1	4.7	100

Table 8.4: Correlation of bids and priorities. Bid vectors of length 2

	All catches				Catches with $r = 5.25$			
Bidder	$\rho = -1$	$\rho = 0$	$\rho = 1$	Count	$\rho = -1$	$\rho = 0$	$\rho = 1$	Count
1	8	17	1	26	7	10	2	19
2	5	2	0	7	2	2	0	4
3	7	14	2	23	5	5	1	11
4	11	6	2	19	7	2	3	12
5	7	18	3	28	2	8	3	13
6	9	9	2	20	5	9	0	14
7	2	4	1	7	4	3	1	8
8	6	10	3	19	7	5	3	15
9	4	5	2	11	2	2	2	6
10	13	11	4	28	5	11	1	17
11	0	1	0	1	0	0	0	0
12	12	16	1	29	14	14	1	29
13	20	6	5	31	14	6	7	27
14	0	0	0	0	0	0	0	0
15	15	12	7	34	11	9	4	24
16	9	17	2	28	10	13	6	29
17	12	6	1	19	8	4	1	13
18	1	2	0	3	1	2	0	3
19	17	8	4	29	11	4	1	16
20	11	3	2	16	9	2	3	14
21	6	14	5	25	4	4	1	9
22	2	7	0	9	1	3	0	4
23	9	10	5	24	4	5	2	11
24	7	10	5	22	3	11	4	18
25	8	5	1	14	7	4	1	12
Sum:	201	213	58	472	143	138	47	328
Percent:	43	45	12	100	44	42	14	100

Table 8.5: Correlation of bids and priorities. All catches, bid vectors of length 3

	Number of ρ equal to							
Bidder	-1	-0.87	-0.5	0	0.5	.87	1	Count
1	2	3	3	9	0	0	0	17
2	0	3	0	0	0	0	0	3
3	1	1	1	4	0	2	3	12
4	2	1	1	4	1	0	0	9
5	0	4	0	8	1	0	0	13
6	3	5	0	4	0	0	0	12
7	1	1	0	1	0	0	0	3
8	7	2	3	1	1	0	0	14
9	2	1	2	0	2	0	0	7
10	3	3	3	7	0	4	0	20
11	1	0	0	0	0	0	0	1
12	8	3	0	6	1	2	0	20
13	3	6	4	4	0	0	0	17
14	0	4	0	0	0	0	0	4
15	2	3	0	6	2	2	1	16
16	3	2	1	14	2	1	1	24
17	3	3	1	4	0	0	1	12
18	0	2	0	1	0	0	0	3
19	5	4	1	5	0	1	0	16
20	3	1	2	2	0	0	0	8
21	1	1	4	6	2	0	1	15
22	0	3	0	0	0	1	0	4
23	1	2	5	2	0	1	1	12
24	0	0	1	8	0	0	0	9
25	2	3	2	1	0	0	0	8
Sum:	53	61	34	97	12	14	8	279
Percent:	19.0	21.9	12.2	34.8	4.3	5.0	2.9	100

Table 8.6: Correlation of bids and priorities. Lots with reserve price 5.25, bid vectors of length 3

	Number of ρ equal to							
Bidder	-1	-0.87	-0.5	0	0.5	.87	1	Count
1	2	1	0	3	0	0	0	6
2	0	3	0	0	0	0	0	3
3	1	2	0	2	2	3	3	13
4	4	1	1	3	1	0	0	10
5	0	3	1	4	1	0	0	9
6	1	3	0	3	0	0	0	7
7	1	2	0	2	0	0	0	5
8	2	1	2	0	0	0	0	5
9	0	1	1	4	2	0	0	8
10	0	2	0	5	0	2	0	9
11	0	0	0	0	0	0	0	0
12	3	2	1	3	0	2	0	11
13	2	4	4	2	0	1	0	13
14	0	3	0	0	0	0	0	3
15	1	2	1	3	1	1	0	9
16	2	2	0	7	1	1	3	16
17	3	4	0	4	0	0	1	12
18	0	1	0	1	0	0	0	2
19	1	1	0	3	0	1	0	6
20	1	1	0	3	0	0	0	5
21	1	1	1	5	0	0	0	8
22	0	2	0	0	0	1	0	3
23	2	2	0	4	1	1	0	10
24	1	0	0	4	0	0	0	5
25	0	1	1	2	0	0	0	4
Sum:	28	45	13	67	9	13	7	182
Percent:	15.4	24.7	7.1	36.8	4.9	7.1	3.8	100

Table 8.7: Correlation of bids and priorities. All catches, bid vectors larger than 3

	Number of $\rho \in$							
Bidder	$[-1]$	$(-1,-.5)$	$[-.5,0)$	$[0]$	$(0,.5]$	$(.5,1)$	$[1]$	Count
1	1	28	0	12	1	0	0	42
2	1	12	2	0	0	0	0	15
3	0	7	7	0	5	3	0	22
4	2	24	3	16	0	0	0	45
5	0	16	11	13	2	1	0	43
6	1	26	2	6	1	0	0	36
7	1	26	6	2	0	4	0	39
8	1	17	9	1	6	0	0	34
9	1	4	4	2	1	2	0	14
10	0	18	16	8	7	4	0	53
11	0	0	0	0	0	0	0	0
12	2	57	18	7	4	1	0	89
13	4	49	14	5	6	0	0	78
14	0	3	0	1	0	0	0	4
15	0	20	16	6	9	3	0	54
16	1	31	13	15	9	10	0	79
17	0	43	5	2	0	0	0	50
18	0	6	1	1	0	1	0	9
19	1	22	3	2	1	0	0	29
20	0	11	4	0	2	1	0	18
21	0	11	4	4	2	3	0	24
22	0	5	1	4	0	1	0	11
23	0	9	7	3	5	4	0	28
24	0	12	6	12	6	5	0	41
25	0	14	0	8	2	3	0	27
Sum:	16	471	152	130	69	46	0	884
Percent:	1.8	53.3	17.2	14.7	7.8	5.2	0.0	100

Table 8.8: Correlation of bids and priorities. Lots with reserve price 5.25, bid vectors larger than 3

	Number of $\rho \in$							
Bidder	$[-1]$	$(-1, -.5)$	$[-.5, 0)$	$[0]$	$(0, .5]$	$(.5, 1)$	$[1]$	Count
1	0	23	0	10	1	0	0	34
2	1	6	2	1	0	0	0	10
3	0	7	5	0	2	1	0	15
4	1	18	2	11	0	0	0	32
5	0	15	8	7	1	1	0	32
6	0	13	1	2	2	0	0	18
7	1	22	5	2	0	1	0	31
8	0	15	7	1	2	0	0	25
9	1	3	2	0	0	2	0	8
10	0	12	9	15	3	0	0	39
11	0	0	0	0	0	0	0	0
12	1	39	13	8	3	1	0	65
13	2	37	9	3	3	1	0	55
14	0	3	0	1	0	0	0	4
15	0	15	8	6	8	3	0	40
16	1	27	7	9	3	8	1	56
17	0	28	5	5	0	0	0	38
18	0	6	1	1	0	1	0	9
19	1	20	0	1	1	0	0	23
20	0	7	3	0	1	0	0	11
21	0	10	3	4	0	4	0	21
22	0	6	0	3	0	1	0	10
23	0	7	6	2	2	2	0	19
24	0	6	7	10	3	6	0	32
25	0	9	0	8	2	3	0	22
Sum:	9	354	103	110	37	35	1	649
Percent:	1.4	54.5	15.9	16.9	5.7	5.4	0.2	100

Table 8.9: Percentage of negative correlations. Zero correlations not counted

	All catches				Catches with $r = 5.25$			
Bidder	> 1	$= 2$	$= 3$	> 3	> 1	$= 2$	$= 3$	> 3
1	96	89	100	97	92	78	100	96
2	100	100	100	100	100	100	100	100
3	62	78	38	64	63	83	27	80
4	94	85	80	100	89	70	86	100
5	84	70	80	90	83	40	80	92
6	94	82	100	97	92	100	100	88
7	88	67	100	89	95	80	100	97
8	82	67	92	82	87	70	100	92
9	72	67	71	75	63	50	50	75
10	75	76	69	76	82	83	50	88
11	100	na	100	na	na	na	na	na
12	92	92	79	94	91	93	75	93
13	90	80	100	92	86	67	91	92
14	100	na	100	100	100		100	100
15	70	68	50	75	69	73	67	68
16	71	82	60	70	68	63	44	74
17	97	92	88	100	96	89	88	100
18	91	100	100	88	90	100	100	88
19	90	81	91	96	92	92	67	95
20	86	85	100	83	84	75	100	91
21	68	55	67	75	80	80	100	76
22	85	100	75	86	82	100	67	86
23	67	64	80	64	72	67	67	76
24	62	58	100	62	57	43	100	59
25	83	89	100	74	75	88	100	64

Table 8.10: Correlation of average weight and priorities. All catches and bid vectors larger than 1

	Number of $\rho \in$							
Bidder	$[-1]$	$(-1,-.5)$	$[-.5,0)$	$[0]$	$(0,.5]$	$(.5,1)$	$[1]$	Count
1	12	21	14	36	1	0	1	85
2	9	9	3	2	1	0	1	25
3	8	2	14	14	10	2	7	57
4	13	8	19	26	3	2	2	73
5	9	14	13	36	5	1	6	84
6	13	18	10	18	4	0	5	68
7	6	14	9	5	8	5	2	49
8	10	7	16	14	13	4	3	67
9	6	5	5	5	5	2	4	32
10	16	16	31	12	15	2	9	101
11	1	0	0	1	0	0	0	2
12	20	27	44	19	16	5	7	138
13	30	40	30	9	8	1	8	126
14	3	4	1	0	0	0	0	8
15	19	17	27	11	14	5	11	104
16	15	36	17	39	11	6	7	131
17	15	27	22	6	4	1	6	81
18	2	3	4	3	1	2	0	15
19	32	20	8	6	3	0	5	74
20	12	5	7	4	9	1	4	42
21	9	6	9	24	7	1	8	64
22	8	3	1	8	4	0	0	24
23	10	6	17	6	5	4	16	64
24	14	11	8	22	10	3	4	72
25	16	11	9	7	1	3	2	49

Chapter 9

An examination of possible collusion

9.1 Introduction

Collusion at auctions, or bid rigging, is like price fixing; it involves the auction participants' forming a ring whose members agree not to bid against one another, either by avoiding the auction or by placing phony bids; see Shor [98]. Possible collusion among bidders is a major concern at many auctions and, in general, at most oligopoly markets; see Pesendorfer [87]. The topic is difficult to analyse, because of the wide range of ways in which to collude. Collusion may take the form of downright illegal contracts between bidders. In our market, we can imagine bidders following bid rotating schemes to allocate catches. Another common way of colluding is the use of side payments, but this seems less likely in our case. A third method of collusion involves dividing the market into geographical territories, but this is hardly an option in our case; this form of collusion is more relevant at procurement auctions.

Markets with repeated auctions are also vulnerable to tacit agreements among bidders. Although it may be seen as the outcome of a game-theoretic equilibrium, the effect can be identical to explicit collusion. Is tacit collusion illegal? The answer is yes, at least for most European countries. According to Articles 85 and 86 of the Treaty of Rome, which many national legislations have incorporated, tacit collusion (or what the treaty text calls "concerted practices") is illegal because it is a form of abuse of a dominant position; see Phlips [88] for a further discussion of the legal framework.

We have no indications that collusion has taken place in the mackerel market. Given our rich dataset, however, we find it reasonable to examine the possibility. If collusion exists, then it is important to detect it because it has associated costs and victims. The relevant costs are the market inefficiencies and social loss that result from deviations from a competitive equilibrium. In the present market, with a fixed supply that is shipped to buyers, we have no distortions of sold quantities. Moreover, inefficiencies on the producers' side may not be that serious, since a successful ring at least must win against bidders outside the ring. Nevertheless, collusion may slow down necessary restructuring of the industry. The most obvious effect of collusion is that the revenues of sellers are reduced.

In the remainder of the chapter, we first review some of the factors that facilitate or inhibit collusive schemes. Then, we discuss suggested ways to detect it. Finally, we examine our dataset and perform some tests for collusion. An important by-product of the analysis is that we estimate a bid function using all bids and create a buyer index showing each individual buyer's bid level compared to the average bid level.

9.2 Likelihood of collusion

Although collusion or bid rigging is a potential problem, the central question is whether it is likely that bid rigging is prevalent at our auction market. We start the discussion by reviewing three problems experienced by colluders. In particular, our discussion draws on Porter [90].

First, collusion can be detected by antitrust authorities or the victims. To our knowledge, Norwegian antitrust authorities (*Konkurransetilsynet*) have not paid any attention to this market. This is understandable since there have been no complaints from sellers, or any dissident ring member. Representatives of the auction house state outright that they do not believe collusion is a problem at the pelagic fish auctions. In general, competition is considered so strong that collusion would be hard to sustain.

A measure of competition is the number of active bidders. We reported in section 5.5.1 that the median number of active bidders is 6 when counting all bids, and 5 when counting only binding bids. Several catches will have a number of submitted bids close to what we regard as a competitive level, around 8 or 9 bids. The more bidders, the more difficult for a bidding ring to achieve any advantages from conspiracy. From the median number of bidders and from figure 5.7 on page 101, we see that for around 50 percent of the catches, there are six or fewer active bidders. The potential for successful collusion increases under such circumstances. The distribution of submitted bids shows that competition is not necessarily very strong all the time and, consequently, the competition argument alone cannot rule out the collusion risk in the market.

Second, all collusive schemes face the problem of unilateral defection. The motivation for cheating on the agreement can be so strong that the bid ring breaks down. Typically, defection will be punished by other ring members. At open, second-price auctions, it is easy to sustain collusion because defection can be immediately detected and punished. In our case, we have a closed, first-price auction, making a one-shot defection easy and tempting. But the frequency of auctions—giving rise to a repeated-game format—makes it easy to respond to defection at subsequent auctions. Similarly, the gains from a one-shot defection are probably small compared to the gains that a prolonged period as a ring member bring. We notice again, as in the case of the competition argument, that our market may be vulnerable to collusion because ring members can effectively respond to defection.

Third, Porter has noted that a successful ring may be characterized by higher profits than normal. High profits will attract new producers and effectively undermine the bidding ring. Fish processing plants and buyers have, loosely speaking, not experienced high profits in recent years; see Bendiksen [10]. Although extraordinary profits might indicate that something fishy is going on, the opposite—low profits—cannot be taken as evidence that there is no collusion among bidders. We do regard the observed low profits on the buyer's side as weak evidence that collusion is not a large problem, but we also note that in a market characterized by low profits, and where restructuring may be wanted, the motivation for collusion is high. In sum, the fish processing industry, like most industries, may have motives for collusion or a "conspiracy against the public."

9.3 Tests for collusion

No test for collusion is infallible. In general, markets differ so much that we have to consider the characteristics of the market in detail in order to examine potential collusion. The main idea, familiar from studies of market power, is to identify the observable implications of competitive and collusive behavior and test for differences; see Porter and Zona [91].

In the Porter and Zona [91] study of highway construction contracts, collusion took the form of placing phoney bids. Ring members determined who should win the object, and the higher phoney bids were submitted in order to give an air of competition. Their procedure for detecting the practice was based on an analysis of observed bid levels and on the ranking of bids. Estimated bid levels, controlling for auction-specific and other relevant public information, were used for detecting abnormalities in behavior. In particular, tests of differences among cartel and non-cartel members in coefficients of the bid functions, could suggest collusion. However, Porter and Zona noted that detecting collusive schemes directly from bid data is not easy and, in any case, inconclusive evidence. Differences in bidding behavior across firms may be explained by insufficient control for auction-specific or firm-specific characteristics.

They noted that analysing bid rankings among cartel and non-cartel members may be a more direct and less qualified test. Phoney bids are placed in order to hide the collusive agreement. The key point is that the ordering of cartel member bids will not necessarily reflect observable cost differences while this is to be expected from competitive bids. The analysis of Porter and Zona, like that of Pesendorfer [87], was made easy by the fact that it was legally proven that collusion took place, and the identities of cartel members were known.

A more general approach was taken by Bajari and Ye [5, 6, 7] in their analysis of procurement auctions. Their approach is general, in the sense that it can be applied to all auction markets within a certain model world. The limitation is that the auction market under study actually has to fit the model; outside the model, results will not necessarily be robust. In their benchmark competitive model, the following is assumed: (1) the auction has a first-price, sealed-bid format; (2) bidders have private information about their costs, but costs are asymmetric and this is common knowledge; (3) bidding strategies follow a Bayes–Nash equilibrium. Given these assumptions, Bajari and Ye showed that two conditions, termed **conditional independence** and **exchangeability**, are both necessary and sufficient for bids to be considered competitive.

Conditional independence implies that bids should be uncorrelated after adjusting for auction- and firm-specific public information. If, however, firms coördinate their bids, then their bids will normally be correlated in a way that can be detected. Bid levels will typically be correlated, unless we are in a model of independent private values. Hence, the need to adjust bids for auction- and firm-specific characteristics before looking at correlations.

Exchangeability between bids means that bidders behave identically when facing the same cost structure. The requirement rests on—what is typically observed in many procurement projects—that characteristics of the project, in particular location affect the costs of firms in an asymmetric way. A firm located close to the project will have lower costs than more distant firms and tend to submit a low bid. Bidders should, however, respond to project characteristics in a consistent way under competition; if firms exchange costs, they should exchange bids. On the other hand, if bidding is collusive, then the exchangeability between costs and bids breaks down. A low-cost firm for the relevant project will place an high-cost bid if the bid is phoney, with the purpose to let another cartel member win. Both conditional independence and bidder exchangeability can be examined by reduced-form estimation of bid functions.

In principle, a clever cartel can submit bids that pass both the test for conditional independence and that for bidder exchangeability. Bajari and

Summers [8, p. 145] state, however, that

> ... in all case studies of collusion of which we are aware, failures of conditional independence or exchangeability accompanied collusion.

From a practical perspective, a more serious objection of using the two tests for collusion is that the tests may incorrectly reject the hypothesis that bids are conditionally-independent and exchangeable. Two reasons for this to occur are either the competitive model is not sufficiently general to nail down the determinants of bidder behavior or the necessary process of adjusting bids for relevant public information is imprecise.

Since the two tests may be inconclusive, Bajari and Ye [6] supplemented them with a third test, a simulation that identifies whether the model of competition explains the data better than alternative collusive models. A necessary first step for the simulation is to set-up more detailed structural models that explain bids. A key feature of the modelling approach is to elicit distributions of unobservable private markups and costs from industry experts. The traditional approach in much of economics is to let the data speak and we hope the data will reveal information about important unobservable variables. The advantages of relying on information from "combat proven" experts—their survival in unforgiving competitive markets is an indication of sound judgement—is that it may give superior small-sample properties as opposed to basing the analysis on asymptotic approximations. The simulation test is based on that the two former tests have identified a collusion cartel. For a further discussion of the approach, see Bajari and Ye [7] as well as Bajari and Summers [8].

9.4 Potential collusion in the market

From the definition of collusion given in the introduction of this chapter, we see that collusion might take two forms. Either some colluding bidders, as part of a bid rotating scheme, are inactive, or they are active but place phoney bids. Inactive colluding bidders cannot be detected by the tests described in the previous section. We propose to look at bid patterns between bidders to investigate the potential practice of agreeing not to bid against each other.

When examining the possibility of phony bids, however, we draw on the work by Bajari and Ye. This raises the question of whether their model and, consequently, the two conditions of conditional independence and bidder exchangeability, is appropriate to use for our market. Recall that their auction model has a first-price, sealed-bid format with asymmetric private-values distributions that are common knowledge, and that bid strategies constitute a Bayes–Nash equilibrium. How do these assumptions fit our auction market? First, we have to assume a Bayes–Nash equilibrium; we cannot test for this directly. Responses to important parameters of the bid function, like the number of submitted bids, at least do not contradict the assumption. Second, we have a first-price, sealed-bid format, with the modifications that the capacity and priority options introduce. Costs are private, but whether they are symmetric or asymmetric is unclear. A bidder index based on all submitted bids—to be reported at the end of the chapter—indicates that these are fairly symmetric, so, we have some evidence of symmetric costs.

As we see it, the test for conditional independence does not rest critically on asymmetric costs. The usefulness of testing for bidder exchangeability, on the other hand, is based on the assumptions that bids can in part be explained by observable cost differences, and that this distribution is common knowledge. We have no obvious variable in the dataset that can reveal asymmetric costs. Although distance from the vessel to prospective buyers differ, catches are brought to the buyer's destination at the seller's cost. Other variables prove to have too little variability or do not make any economic sense to incorporate. We elaborate on this point when discussing the estimated bid function below. Thus, we cannot test for bidder exchangeability. The suggested third test, running a simulation, will not be incorporated. We regard the information necessary for this test to be too demanding to gather in our market.

9.4.1 Testing for phoney bids: Conditional independence

Recall the definition of conditional independence: Bids should be uncorrelated after adjusting for public information. We examine conditional independence by looking at the fitted residuals of estimated bids. Let us, however, begin by looking at correlations of unadjusted (the actual) bids in order to be able to measure the effect on correlations when going from unadjusted to adjusted bids.

Correlations of bids. If underlying valuations are drawn independently from the same distribution, then correlations of bids will tend to be distributed symmetrically around zero. This is the case of pure private values. However, we identified in chapter 7, that some covariates have an influence on prices, in particular weight. When all bids are positively dependent on weight, a common factor is introduced which will tend to make bid correlations positive.

In the upper triangular part of table 9.7, on pages 258 and 259, we report the correlation coefficient, $r(i,j)$, of bids for every pair of bidders i and j. Only complete bid vectors are used—i.e., catches where both bidders submit bids are sampled. In table 9.8, on pages 260 and 261, the numbers of pair of bids that correlations are based on, are reported.

We computed p-values, $p(i,j)$, of correlation coefficients for testing the hypothesis of no correlation. Each p-value is the probability of getting a correlation at least as large as the observed value by random chance, when the true correlation is zero. When $p(i,j)$ is small, say less than a standard significance level of 0.05, then the correlation coefficient $r(i,j)$ is significant. The p-values are computed by transforming the correlation r to create a t-statistic, $t = r\sqrt{(k-2)/(k-r^2)}$, with $k-2$ degrees of freedom where k is the number of observations. Two-tailed p-values in the lower triangular part of table 9.7 on pages 258 and 259 are then calculated by the standard formula using Student's t cumulative distribution function. The two-tailed p-values are computed by doubling the most significant of the two one-tailed p-values. Thus, in the case where the right-tailed p-value is more significant than the left-tailed p-value, the two-tailed p-value is:

$$p = 2 \times \left[\int_t^\infty \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)} \frac{1}{\sqrt{\upsilon\pi}} \frac{1}{\left(1 + x^2/\upsilon\right)^{\frac{\upsilon+1}{2}}} \, dx \right]$$

where Γ is the gamma function

$$\Gamma(\upsilon) = \int_0^\infty t^{\upsilon-1} \exp(-t) \, dt$$

and υ are the degrees of freedom. We note that p-values are exact if bids are normal, a questionable assumption, but the test procedure is probably relatively robust for non-normal variables.

Typically, high absolute correlations are insignificant if the number of observations is low. Correlations coefficients close to zero, say between -0.2 and 0.2 will turn out to be insignificant even if the number of observations is relatively high.[1]

Bid vectors are complete in the sense that we used all observed bids. This raises again the question of how to interpret statistical inference measures like p-values. Some way or another, we have to take account of the possibility that realized correlations are not governed by an underlying law, but rather are random outcomes. The length of bid vectors is obviously important in this respect. With only two or three bids to compute correlations from, there will be a strong correlation one way or the other, whether bids are governed by the same considerations or are purely random. We could exclude bid vectors with a length, say below four. However, we chose to report all correlations, but report the length of bid vectors in a separate table. In the tables summarizing the distribution of correlation coefficients, we report all correlations and only significant correlations. Thus, p-values will, in the cases with short bid vectors, reveal whether we can conclude that bids are, indeed, correlated and not just random realizations.

[1] For instance, for an observed correlation of 0.25 with 20 observations, we get a p-value of 0.29.

Tables 9.7–9.8 contain a lot of information. We need some summary of the data in order to understand to what extent bids are correlated. With 25 bidders, we have potentially 300 correlations of bids. It turns out that in nine cases, there is no match of bids by pairs. For example, bidder 11 is an infrequent bidder and never bids against bidder 2, 7, 9, 14, 18, 22 and 25. This leaves us with 291 correlation coefficients to examine. We report the distribution of correlations in table 9.5 on page 256.

Looking at all pairwise bid correlations, we see that all but three correlations are positive. The small samples that some correlation coefficients are based on make the distribution of all correlations dubious. For instance, for two bidder pairs we have a perfect positive correlation—between bidders 11 and 3, and between bidders 11 and 17—see table 9.7. From table 9.8, we observe that the length of those bid vectors is only two. Thus, the distribution of significant correlations gives a more reliable picture. Ninety percent of all correlations are above 0.50. If we count only significant correlations, then all of them are well into the positive realm. There are no negative correlations and 93 percent of them exceed 0.50. Thus, correlations of bids are positive, suggesting that covariates have an influence on prices, which we already know from the analysis of chapter 7.

Correlations of residuals of bid functions. The test of conditional independence requires us to control bids for observable public information, to clean the bid data for the effect of covariates. The procedure is to set up a common bid function and to estimate it for all bids from all bidders. If bids are conditionally independent after controlling for covariates, then the residuals, which can be interpreted as unobservable private cost shocks, will be uncorrelated. In chapter 7, we set up a bid function for winning bids using covariates with explanatory power and *ex post* observable variables like the position of the winning bid as explanatory variables. What we now need is a bid function for all bids.

Using all bids, we suggest that bids may be explained by the product of a constant term, a bidder-specific term and a catch-specific term. Thus, given the fact that we have observations on all bids, we can disregard the observable covariates, and need not worry about the effect of unobservable public information about catches that affect bids. We have $\mathcal{N}$ bidders and T catches in the dataset. For bidder i, let B_i be a binary variable that takes the value 1 if the bid belongs to i. Likewise, for each catch t, let the binary variable C_t be equal to one if the catch is t. Thus, we propose the bid for bidder i and catch t, Y_{it}, to be estimated by:

$$Y_{it} = a \times \prod_i (1 + b_i)^{B_i} \times \prod_t (1 + c_t)^{C_t} \times \exp(U_{it}) \tag{9.1}$$

where a, b_i, and c_t are the $(1 + \mathcal{N} + T)$ parameters to be estimated, and U_{it} is an error term. Introducing a full set of bidder and catch dummy variables, exploits efficiently the structure of the bid data and will capture both observable and unobservable bidder- and catch-specific characteristics. Including particular catch-specific covariates like weight and the number of bidders is, hence, not necessary from an economic perspective, since such variables do not add to the information captured by the catch-specific dummy variables. More seriously, from an econometric perspective, with no variability between covariates and the catch dummy variables, including both set of variables would lead to singularity.

Ideally, in order to increase the explanatory power, we would want to include buyer-specific information that varies over time. We have few variables in the dataset of such a nature. In principle, we could include whether the buyer is inside or outside. Because some buyers have no variability in this variable, we cannot use it. Capacity limits or quantity bid at an auction is another variable to entertain in this respect. Since the purpose of capacity limits is to free bids from the constraints in the production process, we do not find it useful to introduce it.

In order to have a linear relation between bids and explanatory variables, we take the logarithm of the model in (9.1), and obtain:

$$\log Y_{it} = \log a + \sum_i \log(1 + b_i) B_i + \sum_t \log(1 + c_t) C_t + U_{it}$$

or

$$\log Y_{it} = \alpha + \sum_i \beta_i B_i + \sum_t \gamma_t C_t + U_{it} \tag{9.2}$$

where $\alpha = \log a$, $\beta_i = \log(1 + b_i)$ and $\gamma_t = \log(1 + c_t)$.

Given the large number of catches and, hence, the large number of catch dummy variables, estimating relation (9.2) by any estimator but least squares is not advisable. Thus, equation (9.2) is a least squares dummy variable model; see Greene [38, p. 287]. Instead of showing the structure of the general regressor matrix, which would involve some cumbersome notation, it is probably more informative to use a small example. Suppose we have three active bidders and four catches offered. Recall that Y_{it} is bidder i's bid on catch t. Bidders do not bid on all catches. In the example below, we have a total of seven bids on the four catches. We then have an unbalanced panel data where the setup of the regressor matrix looks like:

$$\log \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{13} \\ Y_{21} \\ Y_{24} \\ Y_{32} \\ Y_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \alpha + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} + \begin{bmatrix} U_{11} \\ U_{12} \\ U_{13} \\ U_{21} \\ U_{24} \\ U_{32} \\ U_{33} \end{bmatrix}.$$

When estimating the coefficients of equation (9.2), we omit one dummy variable in B and one dummy variable in C to avoid a singular regressor matrix. In our actual regressor matrix, we have 24 bidder dummy variables, 1,368 catch dummy variables, and 9,443 bids.

Results. After regressing the pooled bid function for all 25 bidders, we saved the estimated residuals which we denote ε_{it}. Next, we calculated the correlation coefficients of residuals between bidders. Correlation coefficients and the corresponding p-values are reported in table 9.9 on pages 262 and 263. As we did in the case of bid correlations, we summarize the distribution of the correlation coefficients in a table, see table 9.6.

In principle, both negative and positive correlations; i.e., deviations from the predicted zero correlations of our model, can be interpreted as signs of collusion. High positive correlations indicate that bidders coördinate their bids to be equal. It is, however, not obvious why bidders are interested in such a strategy. The more obvious reason for observing positive correlations is that we have insufficiently corrected the bids for important public information.

A negative correlation has the most intuitive explanation. If some bidders tend to have negative correlations, it can indicate that bids are arranged in the sense that when one bidder bids "high", the other agrees to bid "low"; a typical example of placing phoney bids. Thus, given a competitive regime, large and significant negative correlations are puzzling.

Looking at table 9.6, we note: Correlation coefficients of residuals are distributed over a larger range than the correlation coefficients of bids. The difference from the numbers reported in table 9.5 is striking. Apparently, the adjustment for public information seems to have had a considerable effect. If the hypothesis of conditional independence is correct, we would expect zero correlations, or at least a distribution of correlations around zero. The distribution of all correlations seems to be distributed reasonable symmetric on either side of zero, with most of the mass close to zero. A slight left skewed pattern is observed, i.e., correlations are symmetric around -0.1 rather than around zero. Focusing on distribution of significant correlations in table 9.6, we see that just a few of correlations are positive. Most correlations are moderately negative; 83.1 percent of the correlations are larger than -0.40. We observe no correlations that are less than -0.70. Although the tendency of moderately negative correlations is a bit surprising—we would expect a more symmetric distribution around zero—the distribution does not lend support to the hypothesis of arranged bids between any bidders.

We conclude that, although the modelling of bid functions could be improved, the testing for conditional independence does not suggest that any pair of bidders place phoney bids.

9.4.2 Testing for absent bids: Bid patterns

If collusion takes the form of agreeing not to bid against one another, then bid correlations is of no use in detecting it. We suggest examining bid patterns between bidders. In particular, we shall look at how many times bidders bid against one another compared to when only one bid. If the number of cases where both bidders submit bids is small when compared to the case when there is only one bid, then this could be an indication of collusion.

Several reasons exist for not bidding on a particular catch. The most obvious reasons are: The bidder may be an outside bidder, in which case most bidders, most of the time, do not consider it worthwhile bidding. In addition, the bidder may not be in the market at all, his capacity is fully used. Even if a bidder has free capacity, a particular catch may be too large for his capacity. Finally, the reserve price of the catch may exceed a bidder's willingness to pay. We cannot rule out other reasons. Labor costs may play a role; allocating manpower to follow the auction market and submitting bids is a cost-benefit question—for example, a single small catch is offered at an inconvenient time of the day. It may not be worthwhile for a small factory to pay employee-overtime to be ready for submitting bids on that catch. Bidders may also be uncertain about a specific seller and choose not to bid.

The most important reason for no bid is the delivery sector. To avoid the problem of missing bids from outside bidders, we concentrate on bidders located at the major port—i.e., port 25. There is less reason to examine collusion between bidders located far from each other, since often they will not both be eligible bidders. We sample only catches that are offered to port 25. This gives us ten bidders and their bid vectors to work with. All ten bidders will be inside bidders for all sampled catches. To adjust the sample for bidders having full capacity or a capacity smaller than the vessel quantity of a given catch is not possible. In principle, we could sample only catches from auctions where a bidder is active. This would indicate that he has free capacity. However, such a sampling procedure is arbitrary. We do not know why a bidder is not active at an auction. In general, the number of auctions and catches a bidder does not bid on is simply too large to be explained by capacity limits alone. Next, taking away catches that exceed a stated quantity limit is possible, but again we only partially correct for the reason for not bidding. At the auctions where a bidder is not active, he will not state a capacity limit; we have no data on capacities in these cases. Consequently, we prefer to count all catches offered to port 25, but observe that a non-bid can be explained by several factors.

For each pair of ten bidders we sample catches that meet the requirement that both bidders are inside bidders for the relevant catch. With a sample of 10 bidders, this gives us 45 samples to examine ($9 \times 10/2$). For each sample we count the number of catches that falls into one of the following categories: (1) Both bidders submit bids, (2) just one bidder submits a bid, and (3) neither bidder submits a bid.

In table 9.10, on page 264, we report the results. In the upper part of the table, part (a), we show the case where both bidders bid. Note that the diagonal elements give the total number of bids for the given bidder, given the sampling scheme we have used. For example, bidder 5 submits 510 bids on catches (not lots) when he is an inside bidder. The corresponding number for bidder 7 is 257. Comparing bidder 5 to bidder 7, we see that they compete on 102 catches. Look now at the middle part, part (b), of table 9.10 where we count the number of catches where only one bidder submits a bid, although both are eligible to bid. Read row-wise, the table shows the number of catches where the "row" bidder, but not the "column" bidder, bids. Likewise, the columns show the number of catches where the column bidder does not bid, but the row bidder bids. Thus, bidder 5 submits 408 bids where bidder 7 does not bid, and bidder 7 submits bids on 155 catches where bidder 5 is not bidding. Finally, in the lower part of table 9.10, part (c), we show the number of catches where no bidder submits a bid.

The cases where both bid and neither bids result in symmetrical tables, while the case where just one bid is asymmetrical, since we have to distinguish the case where bidder i but not j bids and the case where bidder j but not i bids. All catches where two bidders can submit bids are found by summing one of the numbers in part (a) and (c) and the two numbers in the part (b) of the table. The table reveals that we have 1053 catches offered to port 25.

Are bid frequencies independent? Before discussing and interpreting the bid patterns reported in table 9.10, we discuss the bid frequencies from another perspective. The question we ask is whether bid participation is independent events from one bidder to another. If capacity limits or other individual characteristics governed the decision whether to bid or not on a given catch, we would expect bid frequencies to be independent. However, if characteristics of the object for sale partly govern the decision—and bidders value these characteristics in a more or less consistent way—then a bidder's decision to bid on a given catch is not independent from another bidders' bid decision.

The question of independent bid frequencies can be analysed by a traditional test for independent events. For each pair of bid vectors we create (2×2) contingency tables and count the actual number of catches that fall into the four events: both bidders bid (a_{11}), just one bid (a_{12} and a_{21}), and neither bid (a_{22}). The tables will then have the structure shown in table 9.1.

Table 9.1: Count of bids: Actual outcomes

	Bid from j	No bid from j
Bid from i:	a_{11}	a_{12}
No bid from i:	a_{21}	a_{22}

Given the observed individual bid frequencies, we then create (2×2) contingency tables of the expected number of bids (e_{ij}) that fall into the four categories if events are independent. The formula for computing the expected outcomes are shown in table 9.2.

The null hypothesis is that the relative proportions of bids and non-bids for one bidder are independent of the second bidder. In order to test for independence, we compute the familiar test statistic where we look at the sum of squared differences between actual and expected outcomes. The test

Table 9.2: Count of bids: Expected outcomes if independence

	Bid j	No bid j
Bid i:	$e_{11} = \sum_j a_{1j} \sum_i a_{i1} / \sum_i \sum_j a_{ij}$	$e_{12} = \sum_j a_{1j} \sum_i a_{i2} / \sum_i \sum_j a_{ij}$
No bid i:	$e_{21} = \sum_j a_{2j} \sum_i a_{i1} / \sum_i \sum_j a_{ij}$	$e_{22} = \sum_j a_{2j} \sum_i a_{i2} / \sum_i \sum_j a_{ij}$

is equal to the χ^2 goodness-of-fit test:

$$\chi^2 = \sum_i \sum_j \frac{(a_{ij} - e_{ij})^2}{e_{ij}}.$$

The test statistic is distributed as a χ^2 variable with one degree of freedom because we have two outcomes for each variable. The test for independence is a right-tail test. We compute p-values for the test statistic:

$$p = \int_{\chi^2}^{\infty} \frac{t^{(\upsilon-2)/2} e^{-t/2}}{2^{\upsilon/2} \Gamma\left(\frac{\upsilon}{2}\right)} \, dt$$

where Γ is the Gamma function and υ are the degrees of freedom. If the p-value is low, say below 0.05, then we conclude that the probability of observing the test statistic; i.e., the difference between actual and expected frequencies, (or a more extreme value) under the null hypothesis is low. Consequently, we cannot accept the null hypothesis of independence.

The test for independence of bid frequencies is reported in table 9.3. Bid frequencies are, in general, not independent for the chosen bidders, although they are all inside bidders for the sampled catches. Out of 45 sampled pairs of bidders, only six pairs of bidders have independent bid frequencies if we set the significance level to 0.05. This is the following pairs: $\{(5, 12)$, $(5, 15)$, $(5, 16)$, $(7, 18)$, $(7, 24)$, $(10, 16)\}$.

Why are bid frequencies not independent? If we examine the relationship between actual and expected number of bids for the four events, we find that the events where (1) both bid and (2) neither bids are consistently too low for expected outcomes. The exact figures are reported in table 9.4. Of the 45 samples, 42 of the samples have a positive difference between the number of actual and expected catches where both or neither bid. The mean difference is +32 catches. The other two events—just one bidder bids—are underestimated by 32; i.e., the mean difference is −32. The symmetry between the the two mean differences follows from the structure of table 9.2.

Some catches are popular and attract a large number of bids, while other catches are unpopular attracting few bids. Looking at the 45 samples and the three events and then calculating the mean number of submitted bids, we find: The mean number of submitted bids over the 45 samples in the event "Both bid" is 10.1, in the event "Just one bid" is 7.4, and in the event "Neither bid" is as low as 4.4. The mean number of submitted bids are calculated by using all bids from all bidders. Although we are concentrating on the ten bidders and the catches where they are all inside bidders, making the sample size of eligible catches equal for all ten, other bidders will for some of the catches be inside as well. When we calculated the mean number of submitted bids $\bar{N}$ counting only the bids from the ten bidders at port 25, it was 6.8 for "Both bid", 4.9 for "Just one bid", and 2.8 for "Neither bids."

We already know from chapter 5 that the variability of the number of submitted bids is large, see figure 5.7 on page 101. The variability could possibly be explained by the delivery sectors; setting a narrow delivery sector implies few potential buyers. Capacity limits of individual buyers, on the other hand, are not expected to systematically affect the number of submitted bids. We have now established that even for catches offered to the same location, the variability of the number of submitted bids is relatively large. The reason seems to be that some catches attracts more competition than others. Whether this was caused by quality characteristics of the catches or by other factors (catches that attract few bids are offered at an inconvenient time, and so forth) remains an open question.

Discussion of bid patterns. Returning now to table 9.10, in order to interpret the numbers better, we tabulate the same information as percentages in table 9.11 on page 265. From the diagonal of part (a) of the table, we see that bidder 18 has the lowest bid frequency, he bids on only 8.7 percent of the catches offered to port 25. Bidder 12, on the other hand, is active and bids on 79 percent of the catches.

In case of agreements between bidders to abstain from bidding against one another, we may expect a balanced agreement where bidders divide their catches between them in a roughly equal number. We find no pair of combinations in part (b) where both the numbers of just one bidding are high and the numbers are roughly equal. On the contrary, an high percentage in part (b) for two bidders, is accompanied by a low percentage for the complement event. For example, bidder 12 bids on 70.9 percent of the catches where bidder 18 does not bid. Bidder 18, however, bids on a mere 0.7 percent of the catches bidder 12 does not bid on. The reason is obvious, bidder 18 has a low bid frequency in general, and the pattern cannot be taken as evidence to refrain from bidding against one another.

Based on the statistics introduced, we cannot conclude that agreements on not bidding against one another are prevalent. The high number of potential bidders makes it difficult to secure any advantages of mutual agreements among ring members.

9.5 The bidder index

An interesting by-product of the regression equation (9.2)—we were primarily interested in the residuals of the regression—is that we are able to estimate a bidder index. The interpretation of the estimated $\hat{\beta}_i$ of equation (9.2) is that it—when multiplied by 100—roughly equals the percentage difference in bidder i's bid compared to the omitted bidder. A slightly more precise measure of the percentage difference is 100 times Kennedy's estimator in equation (7.4) on page 156. Expressing bidders' responses as a ratio to the omitted bidder is obtained by computing the $(\mathcal{N} - 1)$ values of $\exp(\hat{\beta}_i)$. Again, taking into consideration that $\exp\left(\hat{\beta}_i\right)$ is a biased estimator of $\exp\left(\beta_i\right)$, a better estimator of the ratio is $\exp[(\hat{\beta}_i - 0.5\hat{\mathcal{V}}(\hat{\beta}_i)]$; see Kennedy [54].

A more interesting measure (than the percentage difference from or the ratio to the omitted bidder) is the percentage difference of bidders' bids compared to the average. For a given catch, the constant term adjusted for the effect of the catch-specific dummy variable, will represent the average bid for that catch. Individual bid levels are distributed around this average. If bidders are more or less symmetric in their costs and risk attitude, their bids will be distributed narrowly around the average. In order to deduce how individual bidders deviate from the average, we create an index that measures each bidder's deviation from the average bid level. The average of such an index should be 1. Notice that we now introduce the effect the omitted bidder has in the data generating process as well. In the estimation, the coefficient of the omitted bidder is contained in the constant term α. We now want to clean the constant term α of the effect of the omitted bidder and catch dummy, and instead report an index of all bidder effects.

The procedure for constructing a bidder index, suggested by Suits [103], is to add a constant k_1 to each $\hat{\beta}_i$ and force the average to be 1. We extend the procedure of Suits; instead of using $\hat{\beta}_i$, we use the less biased estimator of Kennedy defined on page 156. Thus, we want a set of indexes such that

$$\sum_i \exp\left[\hat{\beta}_i - 0.5\hat{\mathcal{V}}\left(\hat{\beta}_i\right) + k_1\right] = \mathcal{N}. \tag{9.3}$$

Solving for the constant k_1, we have:

$$k_1 = \log\left(\mathcal{N} \;/ \sum_i \exp\left[\hat{\beta}_i - 0.5\hat{\mathcal{V}}\left(\hat{\beta}_i\right)\right]\right). \tag{9.4}$$

Index values $(1 + b_i)$ are then estimated by:

$$\left(1 + \hat{b}_i\right) = \exp\left[\hat{\beta}_i - 0.5\hat{\mathcal{V}}\left(\hat{\beta}_i\right) + k_1\right]. \tag{9.5}$$

A similar procedure may be used for the set of catch dummy variables. Thus, we compute:

$$k_2 = \log\left(T \;/ \sum_t \exp\left[\hat{\gamma}_t - 0.5\hat{\mathcal{V}}\left(\hat{\gamma}_t\right)\right]\right).$$

The index for catches is not that interesting in itself. For adjusting the constant term, however, we need k_2 as well, since we recover an estimate of the constant term a in equation (9.1) by:

$$\hat{a} = \exp\left(\hat{\alpha} - k_1 - k_2\right).$$

The estimated bidder dummy variables and the bidder index are reported in table 9.12. Bidder index values range from 0.947 (bidder 16) to 1.049 (bidder 9). This corresponds to average percentage deviations in bid levels from the average bid (of individual catches) of -5.3 percent and $+4.9$ percent respectively. 21 of 25 bidders have index values that is less than 2.7 percent away from the average bid level. We conclude that asymmetries in bid levels, are moderate.

9.6 Concluding remarks

We have examined the dataset for possible evidence of collusion. Two approaches were used. In order to test for phoney bids, we examined the so-called requirement of conditional independence. To test for agreements on not bidding at all against one another, we looked at the bid patterns between bidders.

Our procedure is not suited for detecting "small-scale" collusion. If some bidders collude sometime, then a suspicious pattern will not be revealed by our procedure since the majority of non-colluding bids will dominate the reported statistics. For the individual seller, with just a few catches delivered during the season, the effect of collusion may be annoying, but at an aggregated level, we may safely conclude that collusion is not a large problem. Since the topic of this chapter is somewhat delicate—merely suggesting to investigate for collusion may offend honorable professionals—we state our clear conclusion: We have found no evidence of collusion.

An interesting result of the analysis of bid patterns was that bid frequencies are not independent. This may indicate that the number of competitors should be modelled as an endogenous variable rather than considered exogenously given. Finally, the creation of the bidder index revealed that any asymmetries between bidders are moderate.

9.A Appendix: Tables

Table 9.3: Test of independence of bid frequencies

		Bidder									
		5	7	10	12	13	15	16	18	24	25
Bidder	5		1.4	63.9	.4	34.0	2.8	3.5	24.0	23.2	28.6
	7	.001		12.1	49.5	14.5	15.3	59.6	.8	2.1	17.7
	10	.000	.001		15.5	14.9	31.9	1.6	1.6	33.6	21.3
	12	.541	.000	.000		108.6	45.5	148.5	1.9	5.0	39.8
	13	.000	.000	.000	.000		87.0	9.3	19.3	55.4	57.9
	15	.097	.000	.000	.000	.000		74.2	6.1	117.6	86.3
	16	.061	.000	.202	.000	.000	.000		4.9	29.6	4.4
	18	.000	.368	.001	.001	.000	.013	.027		8.1	13.2
	24	.000	.149	.000	.000	.000	.000	.000	.005		21.9
	25	.000	.000	.000	.000	.000	.000	.000	.000	.000	

Chi-square statistics are in the upper triangular part of the table.
p-values are in the lower triangular part of the table.

Table 9.4: Independence of bid frequencies: Differences between actual and expected outcomes

		Bidder									
		5	7	10	12	13	15	16	18	24	25
Bidder	5		−22	64	4	42	13	−14	22	39	37
	7	22		−24	40	24	27	49	4	10	25
	10	−64	24		26	27	45	9	15	46	31
	12	−4	−40	−26		61	44	74	12	46	35
	13	−42	−24	−27	−61		67	63	18	53	47
	15	−13	−27	−45	−44	−67		64	11	87	64
	16	14	−49	−9	−74	−63	−64		9	40	40
	18	−22	−4	−15	−12	−18	−11	−9		13	14
	24	−39	−10	−46	−46	−53	−87	−40	−13		32
	25	−37	−25	−31	−35	−47	−64	−40	−14	−32	

Upper triangular part of the table: Differences in the cases *Both bid* and *Neither bid*. Lower triangular part of the table: Differences in the case *Just one bid*.

Table 9.5: Distribution of correlation coefficients of pairwise bids

		All correlations			Significant* correlations		
From	To	Count	%	Cum. %	Count	%	Cum. %
0.90	1.00	43	14.8	14.8	22	9.1	9.1
0.80	0.90	87	29.9	44.7	81	33.6	42.7
0.70	0.80	65	22.3	67.0	59	24.5	67.2
0.60	0.70	45	15.5	82.5	42	17.4	84.6
0.50	0.60	24	8.2	90.7	20	8.3	92.9
0.40	0.50	15	5.2	95.9	14	5.8	98.8
0.30	0.40	4	1.4	97.3	3	1.2	100.0
0.20	0.30	2	0.7	97.9	0	0.0	100.0
0.10	0.20	3	1.0	99.0	0	0.0	100.0
0.00	0.10	0	0.0	99.0	0	0.0	100.0
−0.10	0.00	1	0.3	99.3	0	0.0	100.0
−0.20	−0.10	2	0.7	100.0	0	0.0	100.0
−0.30	−0.20	0	0.0	100.0	0	0.0	100.0
−0.40	−0.30	0	0.0	100.0	0	0.0	100.0
−0.50	−0.40	0	0.0	100.0	0	0.0	100.0
−0.60	−0.50	0	0.0	100.0	0	0.0	100.0
−0.70	−0.60	0	0.0	100.0	0	0.0	100.0
−0.80	−0.70	0	0.0	100.0	0	0.0	100.0
−0.90	−0.80	0	0.0	100.0	0	0.0	100.0
−1.00	−0.90	0	0.0	100.0	0	0.0	100.0

* Significance level 0.05. Bid vectors less than 30 not counted.

Table 9.6: Distribution of correlation coefficients of estimated residuals

		All correlations			Significant* correlations		
From	To	Count	%	Cum. %	Count	%	Cum. %
0.90	1.00	6	2.1	2.1	0	0.0	0.0
0.80	0.90	1	0.3	2.4	0	0.0	0.0
0.70	0.80	1	0.3	2.7	0	0.0	0.0
0.60	0.70	3	1.0	3.8	0	0.0	0.0
0.50	0.60	1	0.3	4.1	1	0.8	0.8
0.40	0.50	3	1.0	5.2	2	1.7	2.5
0.30	0.40	7	2.4	7.6	5	4.2	6.8
0.20	0.30	15	5.2	12.7	7	5.9	12.7
0.10	0.20	19	6.5	19.2	2	1.7	14.4
0.00	0.10	38	13.1	32.3	0	0.0	14.4
−0.10	0.00	46	15.8	48.1	0	0.0	14.4
−0.20	−0.10	35	12.0	60.1	13	11.0	25.4
−0.30	−0.20	39	13.4	73.5	29	24.6	50.0
−0.40	−0.30	43	14.8	88.3	39	33.1	83.1
−0.50	−0.40	14	4.8	93.1	11	9.3	92.4
−0.60	−0.50	9	3.1	96.2	6	5.1	97.5
−0.70	−0.60	8	2.7	99.0	3	2.5	100.0
−0.80	−0.70	2	0.7	99.7	0	0.0	100.0
−0.90	−0.80	0	0.0	99.7	0	0.0	100.0
−1.00	−0.90	1	0.3	100.0	0	0.0	100.0

* Significance level 0.05. Bid vectors less than 30 not counted.

Table 9.7: Correlation coefficients[a] and p-values[b] of bids on catches

	1	2	3	4	5	6	7	8	9	10	11	12
1		.82	.78	.81	.68	.77	.69	.82	.79	.77	.90	.68
2	.00		.90	.88	.73	.79	.99	.96	.76	.81	na	.76
3	.00	.00		.83	.75	.75	.68	.85	.66	.68	1.00	.58
4	.00	.00	.00		.81	.77	.62	.87	.42	.62	na	.61
5	.00	.00	.00	.00		.88	.85	.89	.72	.91	.86	.91
6	.00	.00	.00	.00	.00		.79	.81	.82	.81	.94	.84
7	.00	.00	.00	.00	.00	.00		.88	.45	.86	na	.82
8	.00	.00	.00	.00	.00	.00	.00		.50	.90	.98	.75
9	.00	.00	.00	.00	.00	.00	.00	.00		.36	na	.37
10	.00	.00	.00	.00	.00	.00	.00	.00	.01		.98	.86
11	.00	na	na	na	.00	.00	na	.00	na	.00		1.00
12	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	
13	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
14	.02	.00	.70	.00	.00	.00	.00	.00	.07	.00	na	.00
15	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
16	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.00
17	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	na	.00
18	.02	.00	.04	.00	.00	.02	.00	.32	.09	.00	na	.00
19	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
20	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
21	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.05	.00
22	.00	.00	.99	.00	.00	.00	.00	.00	.25	.00	na	.00
23	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.34	.00
24	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.00
25	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	na	.00

[a] Correlation coefficients are in the upper triangular part of the table.
[b] p-values are in the lower triangular part of the table.

Table 9.7: Correlation coefficients and p-values of bids on catches (cont.)

13	14	15	16	17	18	19	20	21	22	23	24	25
.82	.55	.57	.68	.59	.45	.81	.70	.85	.83	.81	.63	.80
.85	.92	.77	.74	.79	.82	.85	.65	.83	.83	.83	.80	.78
.70	-.15	.62	.64	.68	.66	.86	.82	.81	-.00	.91	.71	.60
.70	.75	.68	.74	.59	.73	.56	.42	.75	.89	.86	.86	.85
.91	.89	.90	.83	.77	.93	.82	.67	.79	.86	.87	.93	.90
.87	.94	.85	.75	.84	.93	.73	.67	.67	.89	.81	.83	.81
.86	.99	.76	.73	.67	.73	.60	.56	.81	.47	.59	.75	.74
.88	.90	.81	.76	.41	.20	.91	.76	.83	.45	.83	.91	.79
.73	.63	.44	.48	.53	.51	.66	.57	.67	.24	.73	.42	.48
.91	.87	.85	.76	.55	.94	.75	.67	.66	.59	.68	.85	.89
.99	na	.90	.90	1.00	na	.93	.98	1.00	na	.86	.99	na
.90	.85	.83	.76	.55	.96	.63	.34	.64	.52	.57	.87	.85
	.90	.87	.81	.57	.93	.71	.71	.81	.78	.72	.84	.87
.00		.72	.79	.73	.31	-.11	1.00	.99	na	.15	.92	.79
.00	.00		.73	.58	.92	.55	.50	.60	.53	.45	.81	.79
.00	.00	.00		.64	.91	.66	.63	.69	.69	.74	.84	.81
.00	.00	.00	.00		.97	.62	.51	.43	.40	.70	.62	.85
.00	.54	.00	.00	.00		.75	.95	.85	.72	.22	.91	.96
.00	.68	.00	.00	.00	.00		.71	.80	.64	.86	.79	.76
.00	na	.00	.00	.00	.00	.00		.71	.20	.70	.63	.78
.00	.00	.00	.00	.00	.00	.00	.00		.71	.69	.83	.62
.00	na	.00	.00	.01	.11	.00	.40	.00		.87	.78	.74
.00	.68	.00	.00	.00	.47	.00	.00	.00	.00		.79	.82
.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00		.85
.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	

Table 9.8: Number of pairwise bids between bidders

	1	2	3	4	5	6	7	8	9	10	11	12
1	494	71	155	195	161	108	156	233	84	195	8	305
2	71	144	45	76	79	47	37	63	16	96	0	116
3	155	45	258	104	77	69	81	112	79	104	2	158
4	195	76	104	411	142	114	153	151	65	227	1	320
5	161	79	77	142	527	160	101	126	60	369	9	405
6	108	47	69	114	160	375	50	87	33	212	12	273
7	156	37	81	153	101	50	331	127	42	132	0	263
8	233	63	112	151	126	87	127	377	77	174	9	244
9	84	16	79	65	60	33	42	77	136	52	0	95
10	195	96	104	227	369	212	132	174	52	670	10	545
11	8	0	2	1	9	12	0	9	0	10	18	4
12	305	116	158	320	405	273	263	244	95	545	4	921
13	306	115	134	286	417	265	228	232	90	490	11	693
14	18	31	9	30	45	18	7	13	9	50	0	54
15	254	100	126	255	287	194	173	192	83	383	12	506
16	274	101	138	301	348	249	237	199	85	455	7	680
17	210	69	80	196	164	48	66	149	60	229	2	358
18	27	13	10	29	67	5	28	27	12	70	0	86
19	245	52	138	173	152	122	105	182	64	190	15	225
20	106	22	45	95	62	62	66	107	29	90	5	123
21	125	37	105	94	56	68	81	100	62	94	3	158
22	72	17	24	54	43	31	43	52	25	23	0	77
23	159	37	98	108	66	58	72	133	50	107	3	132
24	182	77	83	167	261	143	120	127	51	319	4	406
25	111	49	60	112	156	96	86	96	40	180	0	236

Table 9.8: Number of pairwise bids between bidders (cont.)

13	14	15	16	17	18	19	20	21	22	23	24	25
306	18	254	274	210	27	245	106	125	72	159	182	111
115	31	100	101	69	13	52	22	37	17	37	77	49
134	9	126	138	80	10	138	45	105	24	98	83	60
286	30	255	301	196	29	173	95	94	54	108	167	112
417	45	287	348	164	67	152	62	56	43	66	261	156
265	18	194	249	48	5	122	62	68	31	58	143	96
228	7	173	237	66	28	105	66	81	43	72	120	86
232	13	192	199	149	27	182	107	100	52	133	127	96
90	9	83	85	60	12	64	29	62	25	50	51	40
490	50	383	455	229	70	190	90	94	23	107	319	180
11	0	12	7	2	0	15	5	3	0	3	4	0
693	54	506	680	358	86	225	123	158	77	132	406	236
835	46	494	619	295	89	226	135	154	77	126	387	228
46	55	47	44	42	6	17	2	8	2	10	48	31
494	47	595	473	239	63	204	109	120	56	82	333	198
619	44	473	781	317	75	215	112	147	56	116	364	215
295	42	239	317	450	16	144	63	89	42	97	190	88
89	6	63	75	16	97	27	7	10	6	13	53	36
226	17	204	215	144	27	399	100	103	50	136	143	77
135	2	109	112	63	7	100	217	59	20	59	63	43
154	8	120	147	89	10	103	59	252	20	84	74	51
77	2	56	56	42	6	50	20	20	111	37	49	30
126	10	82	116	97	13	136	59	84	37	277	65	36
387	48	333	364	190	53	143	63	74	49	65	461	140
228	31	198	215	88	36	77	43	51	30	36	140	251

Table 9.9: Correlation coefficients[a] and p-values[b] of residuals

	1	2	3	4	5	6	7	8	9	10	11	12
1		-.66	-.39	.19	-.35	-.33	-.22	-.32	-.01	-.08	.45	-.54
2	.00		.32	-.35	-.41	-.02	.18	.18	.69	-.45	na	-.07
3	.00	.03		.15	-.28	-.15	-.00	.10	-.06	-.13	1.00	-.05
4	.01	.00	.14		-.22	-.19	-.25	.12	-.19	-.49	na	-.33
5	.00	.00	.01	.01		-.19	-.31	.16	.20	-.03	-.37	-.17
6	.00	.87	.23	.05	.02		.57	-.23	.26	-.39	.21	.01
7	.01	.28	.99	.00	.00	.00		.06	.02	.25	na	-.17
8	.00	.15	.29	.14	.07	.03	.50		-.01	.09	-.36	-.18
9	.91	.00	.57	.13	.12	.14	.91	.91		-.36	na	-.09
10	.29	.00	.21	.00	.55	.00	.00	.23	.01		-.10	-.01
11	.26	na	na	na	.32	.51	na	.34	na	.78		-.99
12	.00	.47	.57	.00	.00	.88	.00	.00	.39	.80	.01	
13	.00	.45	.00	.00	.53	.34	.66	.07	.03	.00	.02	.00
14	.01	.67	.02	.00	.19	.13	.00	.00	.07	.26	na	.19
15	.00	.83	.84	.02	.62	.90	.13	.89	.05	.56	.03	.32
16	.00	.19	.00	.12	.00	.01	.00	.00	.00	.00	.68	.00
17	.00	.77	.07	.01	.00	.03	.09	.07	.66	.00	na	.00
18	1.00	.01	.44	.92	.06	.27	.00	.00	.89	.84	na	.08
19	.06	.00	.17	.00	.23	.00	.05	.00	.58	.33	.46	.01
20	.39	.82	.00	.00	.12	.69	.06	.59	.30	.98	.02	.00
21	.28	.02	.95	.15	.44	.26	.39	.85	.56	.78	.42	.46
22	.02	.01	.02	.02	.71	.21	.00	.01	.60	.06	na	.00
23	.22	.89	.00	.00	.99	.00	.00	.48	.50	.00	1.00	.00
24	.00	.00	.84	.00	.74	.02	.01	.74	.89	.00	.26	.02
25	.19	.59	.00	.55	.11	.00	.01	.00	.90	.00	na	.00

[a] Correlation coefficients are in the upper triangular part of the table.

[b] p-values are in the lower triangular part of the table.

Table 9.9: Correlation coefficients and p-values of residuals (cont.)

13	14	15	16	17	18	19	20	21	22	23	24	25
-.26	-.58	-.33	-.20	-.33	-.00	.12	.08	-.10	.28	-.10	-.35	.12
.07	-.08	.02	-.13	-.04	-.70	.39	-.05	-.38	-.60	-.02	-.42	-.08
-.28	-.75	-.02	-.38	-.21	-.28	.12	-.49	-.01	-.49	.42	-.02	-.59
-.51	-.62	-.14	.09	-.18	.02	-.27	-.31	-.15	.31	.27	.33	.06
-.03	.20	-.03	-.21	-.26	-.23	-.10	-.20	-.11	-.06	.00	.02	.13
-.06	.37	.01	-.18	-.31	-.61	-.25	-.05	-.14	.23	-.42	-.20	-.39
.03	.93	.11	-.22	-.21	-.54	-.19	-.23	-.10	-.54	-.39	-.23	-.30
.12	.84	.01	-.23	-.15	-.57	-.26	-.05	-.02	-.36	.06	.03	-.40
.22	-.64	-.21	-.38	-.06	-.05	.07	.20	.08	-.11	.10	.02	-.02
.29	-.16	.03	-.35	-.27	-.02	-.07	-.00	-.03	-.40	-.32	-.26	.22
.68	na	-.63	-.19	1.00	na	-.21	.94	.79	na	.00	-.74	na
.27	-.18	.04	-.34	-.17	.19	-.17	-.43	-.06	-.52	-.38	-.12	-.32
	.10	.14	-.28	-.31	-.29	-.28	-.08	.10	-.40	-.56	-.33	-.06
.52		-.13	-.40	-.28	.19	-.18	1.00	.99	-1.00	-.44	.22	-.08
.00	.39		-.29	.00	-.25	-.30	-.34	-.18	-.60	-.33	-.35	-.23
.00	.01	.00		-.08	-.12	-.36	-.32	-.26	.23	-.37	.03	-.29
.00	.07	.95	.17		.34	-.40	-.09	-.18	-.13	.01	-.11	.08
.01	.73	.05	.30	.19		.08	.60	-.44	-.03	-.34	-.47	.03
.00	.49	.00	.00	.00	.69		.08	-.36	.06	-.13	.00	.02
.37	na	.00	.00	.46	.15	.43		-.33	-.26	.08	-.30	.34
.21	.00	.04	.00	.09	.20	.00	.01		.06	-.29	.20	-.37
.00	na	.00	.09	.41	.95	.67	.27	.80		.21	-.01	.46
.00	.21	.00	.00	.90	.26	.14	.53	.01	.22		.08	-.25
.00	.14	.00	.51	.12	.00	.97	.02	.08	.95	.50		-.29
.40	.67	.00	.00	.49	.87	.88	.03	.01	.01	.14	.00	

Table 9.10: Pairwise comparison of bid patterns: Count

		Bidders									
		5	7	10	12	13	15	16	18	24	25
Part (a): Both bid	5	510	102	365	407	414	289	345	67	261	158
	7	102	257	128	243	211	166	230	26	122	86
	10	365	128	622	517	481	381	447	69	317	179
	12	407	243	517	832	668	494	659	85	409	233
	13	414	211	481	668	768	482	603	85	388	229
	15	289	166	381	494	482	569	464	61	335	199
	16	345	230	447	659	603	464	741	74	363	216
	18	67	26	69	85	85	61	74	92	53	36
	24	261	122	317	409	388	335	363	53	459	141
	25	158	86	179	233	229	199	216	36	141	250
Part (b): Only one bid	5	0	408	145	103	96	221	165	443	249	352
	7	155	0	129	14	46	91	27	231	135	171
	10	257	494	0	105	141	241	175	553	305	443
	12	425	589	315	0	164	338	173	747	423	599
	13	354	557	287	100	0	286	165	683	380	539
	15	280	403	188	75	87	0	105	508	234	370
	16	396	511	294	82	138	277	0	667	378	525
	18	25	66	23	7	7	31	18	0	39	56
	24	198	337	142	50	71	124	96	406	0	318
	25	92	164	71	17	21	51	34	214	109	0
Part (c): Neither bid	5	543	388	286	118	189	263	147	518	345	451
	7	388	796	302	207	239	393	285	730	459	632
	10	286	302	431	116	144	243	137	408	289	360
	12	118	207	116	221	121	146	139	214	171	204
	13	189	239	144	121	285	198	147	278	214	264
	15	263	393	243	146	198	484	207	453	360	433
	16	147	285	137	139	147	207	312	294	216	278
	18	518	730	408	214	278	453	294	961	555	747
	24	345	459	289	171	214	360	216	555	594	485
	25	451	632	360	204	264	433	278	747	485	803

Table 9.11: Pairwise comparison of bid patterns: Percentages

		Bidders									
		5	7	10	12	13	15	16	18	24	25
Part (a): Both bid	5	48.4	9.7	34.7	38.7	39.3	27.4	32.8	6.4	24.8	15.0
	7	9.7	24.4	12.2	23.1	20.0	15.8	21.8	2.5	11.6	8.2
	10	34.7	12.2	59.1	49.1	45.7	36.2	42.5	6.6	30.1	17.0
	12	38.7	23.1	49.1	79.0	63.4	46.9	62.6	8.1	38.8	22.1
	13	39.3	20.0	45.7	63.4	72.9	45.8	57.3	8.1	36.8	21.7
	15	27.4	15.8	36.2	46.9	45.8	54.0	44.1	5.8	31.8	18.9
	16	32.8	21.8	42.5	62.6	57.3	44.1	70.4	7.0	34.5	20.5
	18	6.4	2.5	6.6	8.1	8.1	5.8	7.0	8.7	5.0	3.4
	24	24.8	11.6	30.1	38.8	36.8	31.8	34.5	5.0	43.6	13.4
	25	15.0	8.2	17.0	22.1	21.7	18.9	20.5	3.4	13.4	23.7
Part (b): Only one bid	5	0.0	38.7	13.8	9.8	9.1	21.0	15.7	42.1	23.6	33.4
	7	14.7	0.0	12.3	1.3	4.4	8.6	2.6	21.9	12.8	16.2
	10	24.4	46.9	0.0	10.0	13.4	22.9	16.6	52.5	29.0	42.1
	12	40.4	55.9	29.9	0.0	15.6	32.1	16.4	70.9	40.2	56.9
	13	33.6	52.9	27.3	9.5	0.0	27.2	15.7	64.9	36.1	51.2
	15	26.6	38.3	17.9	7.1	8.3	0.0	10.0	48.2	22.2	35.1
	16	37.6	48.5	27.9	7.8	13.1	26.3	0.0	63.3	35.9	49.9
	18	2.4	6.3	2.2	0.7	0.7	2.9	1.7	0.0	3.7	5.3
	24	18.8	32.0	13.5	4.7	6.7	11.8	9.1	38.6	0.0	30.2
	25	8.7	15.6	6.7	1.6	2.0	4.8	3.2	20.3	10.4	0.0
Part (c): Neither bid	5	51.6	36.8	27.2	11.2	17.9	25.0	14.0	49.2	32.8	42.8
	7	36.8	75.6	28.7	19.7	22.7	37.3	27.1	69.3	43.6	60.0
	10	27.2	28.7	40.9	11.0	13.7	23.1	13.0	38.7	27.4	34.2
	12	11.2	19.7	11.0	21.0	11.5	13.9	13.2	20.3	16.2	19.4
	13	17.9	22.7	13.7	11.5	27.1	18.8	14.0	26.4	20.3	25.1
	15	25.0	37.3	23.1	13.9	18.8	46.0	19.7	43.0	34.2	41.1
	16	14.0	27.1	13.0	13.2	14.0	19.7	29.6	27.9	20.5	26.4
	18	49.2	69.3	38.7	20.3	26.4	43.0	27.9	91.3	52.7	70.9
	24	32.8	43.6	27.4	16.2	20.3	34.2	20.5	52.7	56.4	46.1
	25	42.8	60.0	34.2	19.4	25.1	41.1	26.4	70.9	46.1	76.3

Table 9.12: Estimate of bids on bidder dummies

	Estimate[a]		Elasticity[b]			
Bidder	Coef.	Std.	Coef.	Std.	Bids	Index[c]
1	−0.0803	0.0061	−7.72	0.3215	494	0.951
2	−0.0189	0.0082	−1.88	0.6529	144	1.012
3	−0.0165	0.0071	−1.64	0.4854	258	1.014
4	−0.0554	0.0063	−5.39	0.3556	411	0.976
5	−0.0372	0.0060	−3.65	0.3375	527	0.993
6	−0.0217	0.0065	−2.15	0.3996	375	1.009
7	−0.0255	0.0066	−2.52	0.4201	331	1.005
8	−0.0171	0.0065	−1.69	0.4032	377	1.014
9	0.0168	0.0085	1.69	0.7432	136	1.049
10	−0.0352	0.0058	−3.46	0.3145	670	0.995
11	−0.0473	0.0198	−4.64	3.5705	18	0.983
12	−0.0273	0.0056	−2.69	0.2932	921	1.003
13	−0.0120	0.0056	−1.20	0.3080	835	1.019
14	−0.0055	0.0117	−0.55	1.3488	55	1.025
15	−0.0296	0.0059	−2.91	0.3234	595	1.001
16	−0.0848	0.0057	−8.13	0.2704	781	0.947
17	−0.0318	0.0062	−3.13	0.3659	450	0.999
18	−0.0341	0.0095	−3.36	0.8471	97	0.996
19	−0.0522	0.0064	−5.09	0.3706	399	0.979
20	−0.0097	0.0075	−0.97	0.5443	217	1.021
21	−0.0357	0.0071	−3.51	0.4731	252	0.995
22	−0.0568	0.0091	−5.52	0.7409	111	0.974
23	−0.0038	0.0071	−0.38	0.4980	277	1.027
24	−0.0507	0.0061	−4.95	0.3396	461	0.980
25	na	na	na	na	251	1.031

[a] Dependent variable: Logarithm of catch bid. The regression includes a constant and 1369 catch dummies. Dfe = 8050. Adj. R-square = 0.8162.

[b] Coef. represents the percentage difference in mean bid compared to bidder 25, see equation (7.4). Std. is the standard error of the elasticity, calculated by use of equation (7.5).

[c] Index shows the level of a bidder's mean bid compared to the average bidder whose index value is 1, see equation (9.5).

Chapter 10

The optimal reserve price

Models are to be used, not believed.

Henri Theil

10.1 Introduction

Empirical analyses of auctions draw on the theory of games of incomplete information. In the case of first-price, sealed-bid auctions, Riley and Samuelson [95] showed that there is a closed-form Bayes–Nash equilibrium solution of bidder's strategies under the assumptions that buyers have independent and identically-distributed private values (symmetric IPV) and are risk neutral. The symmetric equilibrium strategy characterized the bid function as a monotonically increasing function of bidders' private valuations of the object for sale. In this chapter, we assume that the Riley and Samuelson model of independent private values is, as an approximation, a reasonable theoretical framework for an analysis of a carefully chosen sample of the Norwegian mackerel market. This makes it possible to analyse the auction market from the perspective of optimal mechanism design.

A central question is how the seller should design his auction format in order to garner the most revenues. We restrict our analysis to the auction format that is being used currently. Only improvements in the closed discriminatory auction are considered. At closed, discriminatory auctions, the only mechanism the seller has control over is the reserve price. Myerson [79] and Riley and Samuelson [95] derived an implicit relation from which the optimal reserve price can be obtained in the symmetric IPV model. Our purpose is to estimate the necessary elements of the formula in order to apply it on our real-world market.

In order to implement the optimal reserve price, we need information concerning the distribution of private values on which bidders base their bids. Typically, at auction markets, bids are observed, while private values and their distribution are unobserved. Advances in the structural econometrics of auction data have focused on recovering unobservable elements of bid strategies; Paarsch and Hong [86] provided a comprehensive presentation of the literature. The benefit of this novel approach is that it enables researchers to address policy questions regarding a given format and to compare the actual format with counter factual formats.

We rely on the nonparametric two-step estimator of Guerre, Perrigne, and Vuong [39] for estimating the underlying distribution of private values implied by observed bids and, thus, to obtain an estimate for a lower bound on the optimal reserve price. This estimator is derived under quite restrictive assumptions. From an empirical perspective, we need a sample of homogenous goods and a fixed set of potential bidders. From a theoretical perspective, the estimator is developed for the sale of one object within the independent private-values model. Our main contribution in this chapter is to show that the estimator can also be used under more complicated auction formats and assumptions, given appropriate interpretations of results.

We acknowledge that our real-world market does not fit into the theoretical model without friction. Some elements of the auction format and the market—notably the simultaneous sales format, and, possibly, an endogenously determined number of participants—may give rise to an equilibrium solution that differs from the model. However, we shall argue that the potential error from our empirical model is one-sided. In particular, we show that the estimation procedure produces results that consistently underestimates true valuations; the procedure will not overestimate valuations. Relying on a result from the theory of stochastic orders, we then show that our estimate of the optimal reserve price will also consistently underestimate the true optimal reserve price. Therefore, our estimate will represent a lower bound on the optimal reserve price. Thus, one contribution of the paper is to show that relevant policy recommendations can be obtained by applying a seemingly restrictive model to a more complex real-world market.

An important paper analysing the optimal reserve price by applying a structural econometric approach is Paarsch [85]. He studied timber sales in British Columbia conducted by an open, ascending-price auction. His estimates of the optimal reserve price suggested that the current reserve prices used at the time should be substantially raised. The work seems to have had an impact as the government increased reserve prices at subsequent auctions.

The remainder of the chapter is organized in the following way: First, we present the theoretical framework of our auction market. The bid function and optimal reserve price are presented briefly, since they are discussed in chapter 2. The important issue of identifying private values from observed bids is addressed. Next, the assumptions of the theoretical model are discussed in detail in order to determine whether the model is appropriate for the real-world market under study. In section 10.4, we present the empirical strategy used for obtaining estimates of the relevant elements in the model. In section 10.5, we present and discuss our results. In the last section, we provide some concluding remarks.

10.2 The model

The theoretical model we "force" our data into is discussed in chapter 2, see section 2.3. In the following, we use the same notation as in chapter 2.

10.2.1 The bid function and the optimal reserve price

The allocation rule is that the bidder with the highest bid above the reservation price r wins the object and pays his bid. The equilibrium-bid function of a first-price, sealed-bid auction for which risk neutral bidders receive valuations independently from a symmetric distribution was derived from the first-order condition (2.7) on page 18. A slight reformulation of equation (2.7) is the key to the identification problem in section 10.2.2 below. Since $b = \beta(v)$, $d\beta^{-1}(b)/db = 1/\beta'(v)$ and $\beta^{-1}(b) = v$, we can rearrange terms in equation (2.7) to obtain

$$[v - \beta(v)](\mathcal{N} - 1)\frac{f_V(v)}{\beta'(v)} = F_V(v). \tag{10.1}$$

As shown by Riley and Samuelson [95], this ordinary differential equation has a closed-form solution. The equilibrium-bid function of the first-price, sealed-bid auction with a known reserve price is:

$$b = \beta(v; r, \mathcal{N}, F_V) = v - \frac{1}{[F_V(v)]^{\mathcal{N}-1}} \int_r^v [F_V(u)]^{\mathcal{N}-1} du. \tag{10.2}$$

The bidding function imposes structure on our auction data. The optimal reserve price was presented in section 2.3.4. The optimal reserve price—for both first-price and second-price auctions—is the solution to the following equation:

$$r^* = v_0 + \frac{[1 - F_V(r^*)]}{f_V(r^*)}. \tag{10.3}$$

Interestingly, the optimal reserve price is independent of the number of bidders and is strictly greater than the seller's reservation valuation v_0. A key assumption for this result is that valuations are private, and not correlated. Levin and Smith [64] analysed the case with correlated valuations. Under specific rules governing how valuations are correlated, assuming exogenous entry and that the seller commits to not re-offer unsold objects, they find that the optimal reserve price converges to the seller's private value v_0 as the number of bidders increases.

We note for later use that the last term on the right-hand side of equation (10.3) is the inverse hazard rate of V. In terms of actually computing the optimal reserve price, we express the optimization problem as

$$r^{*} = \arg\max \left\{ (r - v_0) \left[1 - F_V(r) \right] \right\}.$$

10.2.2 Identification of valuations

In the sample of auctions we study, all bids and the reserve price are observed, while valuations and their underlying distribution are unobserved. The issue is whether the unobserved private values can be expressed in terms of variables that are either observed directly or can be estimated. Guerre, Perrigne and Vuong [39] showed that this is indeed the case.

Since bids are observed, we can estimate their distribution directly. Let $G_B(b)$ be the distribution of observed bids with support $[r, \beta(\overline{v})]$. It can be shown that $G_B(b)$ and the corresponding density function, $g_B(b)$, can be expressed in terms of unobservable elements of the first-order condition given by (10.1). We have that

$$G_B(b) = \frac{\left[F_V(v) - F_V(r) \right]}{\left[1 - F_V(r) \right]} \tag{10.4}$$

and

$$g_B(b) = \frac{f_V(v)}{\beta'(v)} \frac{1}{\left[1 - F_V(r) \right]}. \tag{10.5}$$

The details of arriving at the above probability functions are in appendix 10.A.1. Notice that G_B and g_B are conditional probability functions while F_V and f_V are unconditional probability functions. Using equation (10.4), substitute $\left[G_B(b) \left[1 - F_V(r) \right] + F_V(r) \right]$ into equation (10.1) for $F_V(v)$. Using equation (10.5), substitute $\left\{ g_B(b) \left[1 - F_V(r) \right] \right\}$ for $f_V(v) / \beta'(v)$. After some algebraic manipulations, we get

$$v = b + \frac{1}{\mathcal{N} - 1} \left\{ \frac{G_B(b)}{g_B(b)} + \frac{F_V(r)}{g_B(b) \left[1 - F_V(r) \right]} \right\}. \tag{10.6}$$

We now have an expression that links the unobservable private values v to variables that can be estimated from the observable bids. Equation (10.6) forms the basis for the estimator used in section 10.4.

10.3 The market

Having established that there is a theoretical framework for interpreting our auction data, we discuss how well the market under analysis fits the maintained assumptions of the theoretical model. The auction format was described in chapter 4. For a sound empirical analysis, we need to demonstrate that our theoretical model reasonably approximates the real-world auctions we study. We employ a structural empirical approach for our analysis of these auctions. A structural approach enables us to recover the unobservable objects in which we are interested. A motivating factor for the development of structural analysis of auction data was to enable researchers to address policy questions like: What is the best auction design in a given market? This is in contrast to the reduced-form approach in which standard hypothesis testing allows applied researchers to consider theory. The structural approach is a powerful one, but it hinges critically on the assumption that our model correctly characterizes the specific attributes of the world. Forcing our data into an incorrect model will produce dubious results. Thus, we discuss in this section how our market relates to the assumptions of the theoretical model.

Our strategy for ensuring consistency between data and the structural estimation of the theoretical model is based on two observations. First, we sampled auctions that most closely represent the single-object symmetric IPV model. Second, any remaining elements of our sample of auctions that deviate from the theoretical model actually reinforce our conclusions on the optimal reserve price.

The symmetric IPV model is based on one single object for sale. Bidders are symmetric and have independent and private valuations. Models with more complex information structures quickly become inherently difficult to analyse within the structural empirical approach. Consequently, the bulk of empirical work has been devoted to the symmetric IPV model. We now discuss each of the fundamental assumptions and clarify our arguments for why we consider the maintained assumptions reasonable in our market. For the details of the auction format to be discussed below, see chapter 4.

10.3.1 The effect of multi-object sales

Since bid strategies at single-object and multi-object auctions normally differ, we have to take a closer look at possible effects that multi-object sales in our mechanism might have on equilibrium bidding compared to the solution of the single-object sale.

Under a simultaneous-*independent* format, a classification of Weber [108], the sale of one object does not depend on the outcome of other sales. In our case, the highest bidder on any object normally wins the object. The only modification to the simultaneous-independent format is that, if capacity constraints are binding, the allocation of objects will, to some extent, be interdependent. Recall that the expected winning bid at the standard first-price, sealed-bid auction is the conditional expected second-highest private value. If a bidder knows that, on a given lot, the high bidder will not take the lot due to a capacity constraint, then he would condition his bid on this information and bid an amount equal to the conditional expected third-highest private value. But bidders have no information that makes it possible to predict reliably such occurrences. It is likely, however, that bidders expect capacity constraints to play a role at some auctions, and, consequently, bid less aggressively.

Below, we establish that some important elements of standard multi-unit auctions are likely to be absent from our mechanism.

Constant marginal values. Standard models of multi-unit auctions are formulated under the assumption that the marginal value of each acquired unit is decreasing; see Krishna [59, section 12.1]. For example, suppose a bidder that demands three units submits a bid vector $(8, 7, 5)$, meaning that he will pay 8 for one unit, $8+7$ for two units, and $8+7+5$ for three units. This gives rise to the so-called demand reduction effect: When bidders have multi-unit demand, and the marginal value of units decreases, then bid shaving increases for each additional demanded unit; see, for instance, Milgrom [76, p. 258].

We consider the assumption of declining marginal values inappropriate in our market since bidders can explicitly state capacity constraints. Plants have a fixed capacity per day, notably cooling or freezing capacities, that cannot be adjusted in the short run. As long as they are operating within their capacity, marginal costs can reasonably be modelled as being constant. Thus, at a given auction, the marginal value of obtaining raw materials to be used for output in competitive markets is also constant. In other words, buyers have a flat demand within their capacity limit.

The driving factor that explains differences in marginal values for a given bidder over time, is the fluctuations in available capacity. When a buyer is replete with raw material, then his marginal value of obtaining more units drops significantly, perhaps close to zero.

Similarly, differences between different buyers' marginal values at a given time may also be explained by the short-term variation in their available capacities. Thus, the theoretical construct—that bidders draw valuations from the same distribution—seems especially appropriate in this market. The randomness of valuations—or the types—is explained by the fluctuations in capacities that give rise to variance in marginal values. To be precise on this essential point; we assume marginal values vary over time for a given bidder, and between types at a given time, but that a single distribution function of valuations captures this randomness.

The acceptance of the present auction format among buyers supports our maintained hypothesis that marginal values are constant when operating below the stated capacity constraints. Under the auction format, there is no direct way of formulating bids that reveal declining marginal values. Bids are independent; one "risks" winning *any* object one bids on, but due to the capacity constraint option, one does not risk winning *all* objects one bids on.

Increased bid shaving. Will bid shaving increase under a multi-object, simultaneous format? We start the discussion by introducing the notion of residual demand. The residual demand curve facing a bidder, is equal to the total supply less the sum of the quantities demanded by other bidders given that this difference is positive, otherwise the residual demand is zero. In our market, residual demand is almost always zero for all bidders. On average, demand exceeded supply by a factor of 4.33 in this market in the 2003–4 season.

In general, however, it is not the case that every buyer individually can take away all supply. At several auctions in the dataset, it takes the aggregate demand from two or three buyers to "consume" the entire supply. Since each individual multi-unit demand in these cases is less than the total supply, bid shaving increases compared to the case of single-unit auctions.

The equilibrium price at an auction market will—as in the case of traditional demand-supply analysis—be determined by the marginal buyer; i.e., the buyer with a marginal value equal to the market clearing price. Consider first the case of single-unit auctions. At a first-price auction, the expected second-highest private value will determine the market price. In the case of multi-unit sales with unit demand, this reasoning carries directly over. Suppose there are four objects for sale, and each bidder only wants one unit. Then the equilibrium strategy is to bid equal to some conditional expectation of the fourth highest valuation. If $\mathcal{N} > 4$, then the equilibrium bid for bidder i is: $\beta_i(v_i) = \mathcal{E}\left[V_{(\mathcal{N}-3:\mathcal{N})} | v_i \geq v_{(\mathcal{N}-3:\mathcal{N})}\right]$. Under the assumption of single-unit demand, bids are, consequently, lower and bid shaving is larger at multi-unit than at single-unit auctions. The driving mechanism for this result is that buyers with fulfilled demand drop out and the competition for remaining objects is reduced. Under rational expectations, bidders will anticipate this effect and all bids at the auction will be adjusted downwards.

In our case, however, we have multi-unit demand; most bidders want more than one catch. This increases demand and competition compared with single-unit demand. The consequence is that bid shaving is somewhere between single-object and multi-object auctions with unit demand. To see this, consider the case of four objects for sale where each bidder wants two units. Assume the same number of participants as above in order to compare the prices obtained. In this case, the market price equals $\mathcal{E}\left[V_{(\mathcal{N}-1:\mathcal{N})}\right]$, which is strictly higher than $\mathcal{E}\left[V_{(\mathcal{N}-3:\mathcal{N})}\right]$. The conclusion is that since we have multi-unit demand, the bids will deviate from the single object case to a lesser degree than under multi-object, unit-demand auctions.

Recall the two essential elements of the auction format; it is a discriminatory auction with capacity limits. Capacity limits imply that sometimes the second-highest or third-highest bid will win. But the discriminatory part of the mechanism ensures that a winner always pays his bid. We do not have to consider bid strategies based on second-price or third-price formats where bids will typically be closer to valuations than under the first-price format. In a third-price format, equilibrium bids can actually be higher than valuations.

Modelling the bid strategies with random multi-unit demand is messy. In addition to requiring a distribution over valuations, one needs a distribution over demand in order to model the appropriate number of bidders. Making the strategies excessively complex also leads us to question how appropriate the models are in reflecting real-world behavior. We proceed under the assumption that the bid function at single-object auctions captures the essential strategic considerations, but sometimes will underestimate bid shaving.[1] Bids will still be below valuations since it is a first-price auction, bids will still be increasing in $\mathcal{N}$ and r, and bids will still depend on the prior beliefs of other bidders valuations represented by F_V. The main difference is that expected revenue may depend on a lower order statistic from F_V than in the single-object case where expected revenue is given by $\mathcal{E}\left[V_{(\mathcal{N}-1:\mathcal{N})}\right]$.

Effect on optimal reserve price. Finally, what effect does increased bid shaving have on the formula for the optimal reserve price given by equation (10.3)? Valuations will not change under multi-object sales since we assume constant marginal values, only strategies may change. The optimal reserve price depends on the distribution of valuations, not the distribution of bids. Consequently, the optimal reserve price will not change when the equilibrium solution predicts increased bid shaving. In fact, the importance of setting a reserve price is likely to increase when bid shaving increases.

The optimal reserve price depends on the shape of F_V and f_V. One might wonder if the finer details of the distributions have an unpredictable impact on calculating the optimal reserve price r^* based on an estimate $\hat{F}_V$ that un-

[1] We say *sometimes*, because in several auctions there is actually just one lot for sale and, at other times, some or several buyers demand the total supply.

derestimates the true F_V. Basic economic and statistical reasoning suggests the answer is no. If buyers have constant marginal values for objects at a simultaneous-independent discriminatory auction, then the optimal reserve price r^* based on the true F_V will not be less than the estimated optimal reserve price $\hat{r}^*$ based on $\hat{F}_V$. The reasoning supporting this proposition is as follows: First, demand is fixed. Consider one unit offered at a single-unit, first-price auction. The expected revenue is $\mathcal{E}\left[V_{(\mathcal{N}-1:\mathcal{N})}\right]$. Next, increase demand by introducing several units. It follows from fundamental economic theory and the principle of purposeful behavior that with demand fixed and supply increased, the new equilibrium price cannot be higher than under the single-unit format.

What are the consequences if we model bids by (10.2) and derive the optimal reserve price by (10.3), and the actual bid shaving taking place is more severe than our model predicts? In that case, identifying values by incorrectly assuming the single object format underestimates values, since actual bid shaving is more severe than what follows from the theoretical bid shaving factor we use. In consequence, our estimate of $\hat{F}_V$ is first-order stochastically dominated by the true F_V—i.e., for all $v \in [\underline{v}, \overline{v}]$, $\hat{F}_V(v) \geq F_V(v)$. Thus, the estimated inverse hazard rate $(1 - \hat{F}_V)/\hat{f}_V$ first-order stochastically dominates the true $(1 - F_V)/f_V$. Since the optimal reserve price at the single-object auction is defined by

$$r - v_0 - \frac{[1 - F_V(r)]}{f_V(r)} = 0,$$

it follows that $r^* \geq \hat{r}^*$. This is formally shown in appendix 10.A.2. Thus, the estimated reserve price will represent a lower bound for the optimal reserve price as far as model misspecification is concerned. A statistical estimation error will, however, always be present.

10.3.2 Symmetry and independence of valuations

A key element of auction models is to what extent bidders' valuations of the auctioned good are correlated. Two polar cases are private-values and common-value. Valuations are private if they are not dependent; i.e., the value one bidder places on a good does not depend on how other bidders value it. Information on competitors' valuations do not influence one's own valuation of the good. On the other hand, valuations are common if all value the good at the same price. A distinguishing characteristic between the two concepts concerns the degree of certainty of valuations. Typically, a common value is uncertain. The prime example of this situation is oil companies bidding for the right to exploit oil tracts with uncertain contents. Most real-world auctions have a private-values and a common-value element. In the following, however, we shall argue that the auction under study is predominantly a private-values auction. A closer look at what characterizes buyers' input and output markets and a study of what kind of uncertainty that is present, is necessary to proceed.

To begin, consider the output market. The buyers are food producers who use the fish as an input to various end-products. If bidders face an uncertain future end-product price at the time of bidding for raw material, then a common-value element is present. The price uncertainty in the end-product markets is, however, probably relatively small. For most of these products, established competitive prices are only to a small degree sensitive to fluctuations in the supply side of inputs in one market. The end-products face competition both from other complementary food products, and from similar products from food producers who get their inputs in other markets. In case the end-product is a conserved good, like canned products, the price variability is known to be small. In case the fish are shipped unprocessed as fresh or frozen to export markets, the time span between raw costs and revenues is, in this industry, small. Thus, presumably, the true value of the end-product is quite certain, and the common-value element is negligible.

There *is* a common-value element in the cost structure in the sense that producers all face some common costs and constraints in production. Wage levels and capital costs may not differ that much, and public taxes and charges are similar for all. However, there is no particular uncertainty with respect to these costs. The interesting part of the common-value aspect, the uncertainty involved, is, hence, not an issue. The common-value part of costs defines a base level for all producers.

If we are willing to invoke a certain degree of rationality on the sellers, an additional argument for private valuations follows from the analysis of Levin and Smith [64]. As mentioned, they showed that if valuations are correlated, the optimal reserve price converges, often rapidly, to the seller's private value when the number of bidders increases. The fact that sellers commit to a reserve price that is considerably higher than their private value, suggests that buyers' valuations are predominantly private.

Having argued that valuations are private, we next turn to the question of whether bidders are symmetric in the sense that bidders' valuations are identically distributed and drawn from the same distribution. This does not mean that bidders are identical, just that their valuations are drawn from the same distribution. The distribution is said to encompass all relevant information bidders have about their competitors. Bidders entertaining a specific bid make assumptions about the probability that competitors' valuations exceed a certain level. The assumption of identically-distributed valuations entails that a bidder has no reason to believe that a particular competitor has a certain valuation with higher probability than others. The assumption has important consequences for the empirical analysis. The theoretical model we have employed is a one-shot game. In order to analyse auction games empirically we have to aggregate observations from a sequence of auctions. But how can we defend the assumption that valuations are identically distributed over time?

To begin, we notice that such an assumption is, at most real-world markets with repeated auctions, a simplification. Over time, given the information previous auctions reveal with respect to individual bidder behavior, it is likely that some learning takes place. During this learning process, bidders' information sets might be transformed as follows: At first, they start with a crude assumption on the distribution of valuations, associated with a common distribution with relatively large or wide scale and location parameters. Later, as experience is growing, the information set changes to a more sophisticated representation where individual bidders or groups of bidders are associated with more concentrated individual distributions of valuations.

If bidders' cost structures differ in a systematic way, then there will be asymmetries. We invoke the paradigm of competitive markets, and assume that buyers, operating within their capacity, will more or less have the same expected long-run profit margins. But if variation in buyers' profit margins is relatively small, then why do they submit different bids? To answer, we must pay attention to the one important specific characteristic of production: the short-term capacity constraints of buyers. Since the object for sale is dead fish, processing and freezing must take place within a short time span in order to avoid spoilage. A buyer who, at the time when a specific auction is held, has lots of raw material, will not be as keen to bid high as a buyer who suffers shortage of raw material. Private values will differ in this respect both among different buyers and for a given buyer over time. The dynamics of the market will shift a buyer's private value within the distribution from auction to auction. Since variations in short-term capacities are random, using a common distribution of valuations, seems especially appropriate in this market.

10.3.3 Risk attitude

Next, we turn to the question of whether bidders are risk averse or risk neutral. Although most applied studies simply assume risk neutrality, we devote some space to discuss the assumption. Risk aversion is the normal case; several studies, especially in the finance literature, have shown that human decisions are best modelled by use of risk aversion. Various notions of risk aversion have been developed in the literature. Constant risk aversion implies that bidders have the same attitude or aversion for risk irrespective of the amount at stake, while increasing risk aversion means that the negative preference for risk increases with the amount.[2] Recall from auction theory that a bid may be seen as composed of two factors. A bidder will balance (1) the probability of winning or equivalently the risk of losing with, (2) the realized profit if winning. These factors work in opposite ways; an high bid

[2]A distinction is made between *absolute* and *relative* risk aversion. The above explanation refers to the concept of absolute risk aversion.

increases the probability of winning while at the same time reducing the profit margin. Riley and Samuelson [95] showed that a bidder with increasing risk aversion will place more weight on the probability of losing-factor when the amount at stake increases compared to a risk neutral bidder. Consequently, since the probability of losing increases with decreasing bids, equilibrium bids are higher if we model bidders as risk averse rather than risk neutral.

To begin, we assume that bidders at this auction are risk averse. In the benchmark model, on the other hand, bidders are modelled as risk neutral. This is a simplifying assumption making the model tractable, since introducing risk aversion complicates the model substantially. A possible defense for the simplifying assumption of risk neutrality is that, if at a given auction, the risk involved is marginal, then risk neutrality may be an acceptable modelling strategy.

The central question then becomes: How risk averse are bidders at the mackerel auction? We shall argue that there is relatively little risk involved at the mackerel auction. Bidding on a single lot does not entail a large financial burden for the winner, relative to the total turnover.[3] The argument rests on the fact that increasing risk aversion seems to be the norm. People are generally not risk averse with respect to small amounts, but less inclined to gambles when large amounts, relative to wealth, are involved. This is supported by the close to universal fact that people do not insure small value items, while most people purchase insurance for valuable assets.

Moreover, during the season, many catches are offered on the market within frequent time intervals; in the peak season, up to four auctions are held each day. In addition, plants have the opportunity to buy other species. The risk of losing at a given auction is therefore offset by other opportunities. We conclude that risk neutrality seems to be a reasonable approximation to risk aversion of small order.

[3] To be more precise, we should compare the risk involved with the plant owners' wealth since this is the relevant factor in the theory of risk; see, for example, Gollier [37].

10.3.4 Fixed and known number of participants

An important assumption in most auction models is that the number of potential bidders, $\mathcal{N}$, is known. Moreover, when analysing auction data empirically using the structural approach, we need to aggregate comparable auctions. This raises two questions to be addressed by the researcher. First, is the assumption of a known $\mathcal{N}$ plausible at a single auction? Second, is $\mathcal{N}$ stable over sequential auctions?

At single-shot auctions, assuming a certain number of participants may in some instances be a bit off the wall. One case in question is where a large and complicated sale involves substantial pre-contract costs. Firms considering bidding will have to weigh the potential benefits and the costs from participating. Under this scenario, bidder participation can be explained by stating that, in equilibrium, the expected profit from participating is equal to the sunk costs that pre-contract efforts involve; see French and McCormick [30]. In our case, we have frequently repeated auctions, where the competition for raw materials is routinely undertaken. Participation costs are likely to be negligible.

Another case in question, is when there is a general uncertainty about the number of competitors. This can be modelled by introducing a distribution over $\mathcal{N}$. Harstad, Kagel, and Levin [46] showed that at first-price auctions, with risk neutral agents, the unique symmetric equilibrium-bid function is a weighted average over the bid functions with a known $\mathcal{N}$. Formally, if $b_{\mathcal{N}}$ is the bid function with a certain number of bidders $\mathcal{N}$, then the bid function with uncertainty in the number of participants at the auction is:

$$b\left(v\right)=\sum_{\mathcal{N}}w_{\mathcal{N}}\left(v\right)b_{\mathcal{N}}\left(v\right), \tag{10.7}$$

where the weight $w_{\mathcal{N}}$ is the probability of $\mathcal{N}$ bidders conditional upon winning with bid $b_{\mathcal{N}}$. Each bid $b_{\mathcal{N}}$ is the standard Bayes–Nash equilibrium bid presented in equation (10.2). Obviously, the weighted bid over $\mathcal{N}$ will be lower than a bid based on the maximum $\mathcal{N}$.

In the present market, we chose to analyse one delivery sector, one of the most frequently observed. Lots offered in this sector will have a stable set of potential bidders since it is a industry with high entry costs; the competitors know about each other. It is a fact, however, that the number of actual bidders N varies from one auction to another. Formally, this could be defended by saying that bidders' valuations vary over time, within the same distribution $F_V(v)$. A specific draw from the distribution of valuations results in N bidders with valuations above the reserve price. It seems, however, somewhat unrealistic to assume that bidders' strategies are not affected by the observed variability in N.

Another approach is to assume that the number of potential bidders varies from auction to auction because some plants are not in a buyer position due to capacity constraints; i.e., their valuations drop to zero in this case. Although bidders do not have exact information on competitors' capacities at a given time, the total supply will be an indicator of whether many capacity constraints are binding. This situation seems to invite bidders to form their bids on a distribution over $\mathcal{N}$ like the model considered by Harstad, Kagel, and Levin [46]. We analyse the market under the condition that the number of participants is the full set of observed potential bidders; i.e., we use the maximum N observed in the empirical specification of the model. If bidders use a weighted average bid to account for numbers uncertainty, then their bids will be lower than what is predicted by our model. The error is one-sided, and the consequence is that valuations will be consistently underestimated. We see that the error is of the same nature as discussed in section 10.3.1. Thus, we rely on the same stochastic order result (see page 276) that says that the estimated optimal reserve price will represent a lower bound on the true optimal reserve price.

10.4 Empirical specification

We proceed by explaining how we can utilize the identification result of equation (10.6) to estimate bidders' private values, and the distribution of valuations, empirically. The approach is based on the idea that these estimates represent a lower bound on true values.

The strategy for finding $F_V(v)$ and $f_V(v)$, necessary to compute the optimal reserve price r^*, is to follow the two-step estimator of Guerre et al. [39]: First, in order to uncover unobservable private values using relation (10.6), we need estimates of $g_B(b)$, $G_B(b)$, $\mathcal{N}$ and $F_V(r)$. Straightforward estimators of $\mathcal{N}$ and $F_V(r)$ can be obtained, while the estimation of $G_B(b)$ and $g_B(b)$ are complicated by the presence of a binding reserve price, since observed bids represent a truncated sample of the full set of potential bids without a reserve price. A transformation of bids is necessary in order to proceed. Once we have obtained the estimates, we calculate pseudo-valuations by using a modified version of relation (10.6) on page 271, see relation (10.11) below. The next step is then to use the calculated pseudo-valuations to estimate the conditional functions $F_{V|V\geq r}(v|v \geq r)$ and $f_{V|V\geq r}(v|v \geq r)$. Finally, we transform the conditional probability functions to estimated unconditional functions $F_V(v)$ and $f_V(v)$.

10.4.1 Estimation of the bid distribution

A technical problem concerning the estimation of $g_B(b)$ and $G_B(b)$ is that the density $g_B(b)$ is unbounded at $b = r$ since $\beta(v < r) = 0$. When the data at hand is not unbounded—in our case observed bids are bounded—then straightforward kernel density estimation will fail, since the kernel estimate near the boundary is not consistent; see Simonoff [100, section 3.2.1]. In our setting, this means that $g_B(b) \to \infty$ as $b \searrow r$. Guerre, Perrigne and Vuong [39] note that $g_B(b)$ is proportional to $1/\sqrt{b-r}$ when $b \searrow r$. The solution they suggest is to transform observed bids B to a variable

$$B(r) = \sqrt{B - r}. \tag{10.8}$$

Under this transformation, the cdf and pdf of the transformed bids, $G_{B(r)}$ and $g_{B(r)}$, can be expressed in terms of the corresponding functions of observable bids, G_B and g_B, as:[4]

$$G_{B(r)}\left[b\left(r\right)\right] = G_B\left[r + b\left(r\right)^2\right] \tag{10.9}$$

and

$$g_{B(r)}\left[b\left(r\right)\right] = \frac{dG_B}{db\left(r\right)} = 2b\left(r\right) g_B\left[r + b\left(r\right)^2\right]. \tag{10.10}$$

From (10.8) we have that $b = b\left(r\right) + r^2$. Substituting this, together with the expressions in (10.9)–(10.10), into (10.6) yields the result that valuations are identified by

$$v = b\left(r\right)^2 + r + \frac{2b\left(r\right)}{\left(\mathcal{N} - 1\right)} \left\{ \frac{G_{B(r)}\left[b\left(r\right)\right]}{g_{B(r)}\left[b\left(r\right)\right]} + \frac{F_V\left(r\right)}{g_{B(r)}\left[b\left(r\right)\right]\left[1 - F_V\left(r\right)\right]} \right\}. \tag{10.11}$$

Let T be the number of sampled auctions, and N_t the number of observed bids at auction t. A straightforward estimator of $G_{B(r)}$ is the empirical cdf of the transformed observed bids

$$\hat{G}_{B(r)}\left[b\left(r\right)\right] = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\mathbf{1}\left[B_{it}\left(r\right) \le b\left(r\right)\right].$$

We estimate the truncated density function of bids by a kernel-smoothed density estimator

$$\hat{g}_{B(r)}\left[b\left(r\right)\right] = \frac{1}{Th_g}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\kappa\left[\frac{B_{it}\left(r\right) - b\left(r\right)}{h_g}\right] \tag{10.12}$$

where $\kappa\left(\cdot\right)$ is a kernel function satisfying some standard assumptions, and h_g is a smoothing parameter, also called the bandwidth or window width. We discuss the appropriate choice of kernel function and bandwidth in section 10.5.2.

[4] $G_{B(r)}\left[b\left(r\right)\right] = \Pr\left[\sqrt{B - r} \le b\left(r\right)\right] = \Pr\left[B \le b\left(r\right)^2 + r\right] = G_B\left[r + b\left(r\right)^2\right]$.

10.4.2 Estimation of $\mathcal{N}$ and $F_V(r)$

With no reserve price, the number of potential bidders $\mathcal{N}$ equals the number of observed bidders. When introducing a reserve price, $\mathcal{N}$ is unobserved since bidders with valuations below r, do not bid. We observe the number of actual bidders N, a subset of the number of potential bidders. Following Paarsch and Hong [86], we describe the relationship between $\mathcal{N}$ and N and present suitable estimators for $\mathcal{N}$ and $F_V(r)$. They noted that the number of active bidders is the sum of a Bernoulli sequence:

$$N = \sum_{i=1}^{\mathcal{N}} I_i \quad \text{where} \quad I_i = \begin{cases} 1 & \text{if } v_i \geq r, \text{ with probability } [1 - F_V(r)]; \\ 0 & \text{if } v_i < r, \text{ with probability } F_V(r). \end{cases}$$

Thus, N has a binomial distribution with parameters $\mathcal{N}$ and $[1 - F_V(r)]$, and the probability mass function is then:

$$f_N(n) = \binom{\mathcal{N}}{n} F_V(r)^{\mathcal{N}-n} [1 - F_V(r)]^n, \qquad n = 0, 1, \ldots, \mathcal{N}.$$

A natural estimator for $\mathcal{N}$ is

$$\hat{\mathcal{N}} = \max_{t=1,\ldots,T} N_t. \tag{10.13}$$

It can be shown that $\hat{\mathcal{N}}$ converges almost surely to $\mathcal{N}$.

In order to find an estimator for $F_V(r)$, we note that $[1 - F_V(r)]$ is one of the parameters in the probability mass function of N. Using the fact that the expectation of a binomially distributed variable is equal to the product of its parameters, we get an expression for $F_V(r)$: $\mathcal{E}(N) = \mathcal{N}[1 - F_V(r)]$, or $F_V(r) = 1 - [\mathcal{E}(N)/\mathcal{N}]$. We estimate $\mathcal{E}(N)$ by the sample mean $\bar{N}$ and $\hat{\mathcal{N}}$ by expression (10.13). Hence, an estimator for $F_V(r)$ is

$$\hat{F}_V(r) = 1 - \frac{\bar{N}}{\hat{\mathcal{N}}} = 1 - \frac{T^{-1}\sum_{t=1}^{T} N_t}{\left(\max_t N_t\right)}. \tag{10.14}$$

10.4.3 Uncovering valuations and estimating f_V and F_V

Having presented estimators for $G_{B(r)}$, $G_{B(r)}$, N, and $F_V(r)$, we are now in a position to calculate an estimate of the valuations that bidders base their bids on, the so-called pseudo-values. For each observation it, we use (10.11) and recover valuations from bids by

$$\begin{aligned}\hat{V}_{it} &= B_{it}(r)^2 + r \\ &+ \frac{2B_{it}(r)}{\left(\hat{\mathcal{N}}-1\right)} \left\{ \frac{\hat{G}_{B(r)}\left[B_{it}(r)\right]\left[1-\hat{F}_V(r)\right]+\hat{F}_V(r)}{\hat{g}_{B(r)}\left[B_{it}(r)\right]\left[1-\hat{F}_V(r)\right]} \right\}. \end{aligned} \tag{10.15}$$

We now want to find the density and distribution of estimated valuations in order to reach the stated goal of using equation 10.6 to estimate a lower bound on the optimal reserve price. The truncated pdf of valuations is again estimated by a kernel estimate

$$\hat{f}_{V|V\geq r}(v|V\geq r) = \frac{1}{Th_g}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\kappa\left(\frac{\hat{V}_{it}-v}{h_g}\right), \tag{10.16}$$

and an estimate of the truncated cdf is given by the empirical cumulative distribution function of estimated valuations

$$\hat{F}_{V|V\geq r}(v|V\geq r) = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\mathbf{1}\left(\hat{V}_{it}\leq v\right).$$

Finally, to arrive at the unconditional probability functions, we use the following transformation:

$$\hat{f}_V(v) = \hat{f}_{V|V\geq r}(v|V\geq r)\left[1-\hat{F}_V(r)\right]$$

and

$$\hat{F}_V(v) = \hat{F}_{V|V\geq r}(v|V\geq r)\left[1-\hat{F}_V(r)\right]. \tag{10.17}$$

Notice that this estimation procedure only gives us information about the shape of f_V and F_V above the reservation price. Below r, we have no observed bids, and, thus, no information to infer f_V and F_V from. The practical consequence is that, if the reserve price in use is initially set above the optimal level, then we cannot estimate the correct level. The only conclusion we can reach is that the reserve price should be somewhere between v_0 and r.

10.5 Estimation and results

10.5.1 The data sample

The construction of the dataset is documented in the appendix of the thesis. For the 2003–4 season, beginning in mid-August and ending in February, we have observations of all submitted bids. In order to have a well-defined market with a stable set of potential buyers, we chose to analyse one of the delivery sectors most frequently observed. The sector has the city Bergen with numerical code 19 as the southern border and a specific location in the county of Møre represented by numerical code 25 as the northern border. We then chose rather homogenous catches with a stated reserve price equal to NOK 5.25. A reserve price of 5.25 refers to fish with average weight equal to or above 500 grams. This is the weight class of the largest fish, and the most important product, both in terms of the number of offered catches and in terms of total quantity. Consequently, since the reserve price and the realized price are increasing with average weight, it is also the most important weight class with respect to revenues.

In principle, buyers outside the delivery sector are entitled to submit bids. If an outside bidder has the highest bid, the seller is free to refuse it. A few bidders seem to bid routinely on outside catches. Normally, the extra cost incurred in serving an outside bidder with the highest bid outweighs the increased revenue. In fact, of all the catches offered in 2003–4 (all sectors and all weight classes), only 2.4 percent of the catches were allocated to an outside bidder, and in our sample of catches an outside bidder is never allocated a catch. We remove outside bids from our sample since they are not considered realistic bids, and since including them, would break down the assumption of a stable set of potential bidders. This gives us a sample of 57 auctions, 97 objects for sale and a total of 708 submitted bids. The distribution of the number of catches per auction, which has a reserve price equal to 5.25, in the relevant sector is:

Table 10.1: Distribution of lots per auction

Number of lots per auction:	1	2	3	4	5	7
Frequency:	35	13	2	2	3	1

We see that 61 percent of the auctions have only a single catch offered in this delivery sector at the reserve price. However, there will, in some cases, be catches with the same reserve price offered to other sectors. Some of these will partly overlap the sector under study here. In consequence, some of the buyers in sector 19–25 will have opportunities to bid on other catches. In addition, catches with fish of lighter weight and a lower reserve price will also be offered both to the same sector and to other partly overlapping sectors. One particular auction which contained 8 relevant objects and that otherwise met the two requirements—catches offered to (1) sector 19–25 with (2) a reserve price equal to 5.25—were not included in the data sample since, at this auction, several similar catches were offered to a partly overlapping sector. If we concentrate on the current sector, at the sampled auctions, 25 objects are not included because they do not meet the weight requirement and have a lower reserve price. Our sampled objects comprise 80.0 percent of the total objects and 79.6 percent of the quantity offered to this sector at the same auctions. In terms of realized revenues, the equivalent measure is 81.3 percent.

Laffont, Ossard and Vuong [62], in their analysis of a French eggplant auction market, made a point of sampling auctions where only one lot was offered each day to avoid the influence of the "dynamics of the market" on bidder strategies. It is not obvious that such a sampling strategy is sufficient to avoid the bulk of the universal noise that every empirical analysis is ridden with when confronting real-world data with theoretical models. Every market is influenced by substitutes and complements and several general variables. Some way or another, we have to define our market clearly. We chose to emphasize that the sample consists of homogenous products with identical reserve prices and a stable set of potential buyers; these are the requirements of the estimation procedure used. The influence of other, dissimilar catches offered in the market is considered of less importance. The competition in the market, characterized by the excess demand and the number of competitors, narrows down the strategy space of bidders, both in the given market and in the substitute goods markets. Thus, the errors invoked in estimated valuations are relatively small compared to environments where competition is weaker. Moreover, the error is one-sided, and we can interpret our estimated valuations as lower bounds on true valuations.

Table 10.2: Summary statistics of bids

	Max bids	Winning bids	All bids
Sample size	97.00	94.00	708.00
Minimum	5.25	5.25	5.25
25th percentile	6.65	6.58	6.23
50th percentile	7.15	7.10	6.88
75th percentile	7.42	7.39	7.16
Maximum	7.94	7.94	7.94
Mean	6.97	6.93	6.68
Standard deviation	0.60	0.57	0.63
Skewness	−1.25	−0.94	−0.73
Kurtosis	3.98	3.18	2.52

Some summary statistics of bids are reported in table 10.2. The first column in table 10.2 shows the statistics for the maximum bid of an object, and the third column shows the same statistics for all bids. The second column shows the statistics of the bid that was allocated the object; recall that some objects will go to lower order bids if the high bidder has reached his stated limit in terms of quantity. The number of observations for this variable is 94, since there are three objects in the dataset that went unsold although they received bids. While the mean of the maximum bid is 4.19 percent higher than the mean of all bids, it is only 0.58 percent higher than the mean of winning bids.

10.5.2 Choice of kernel and bandwidth

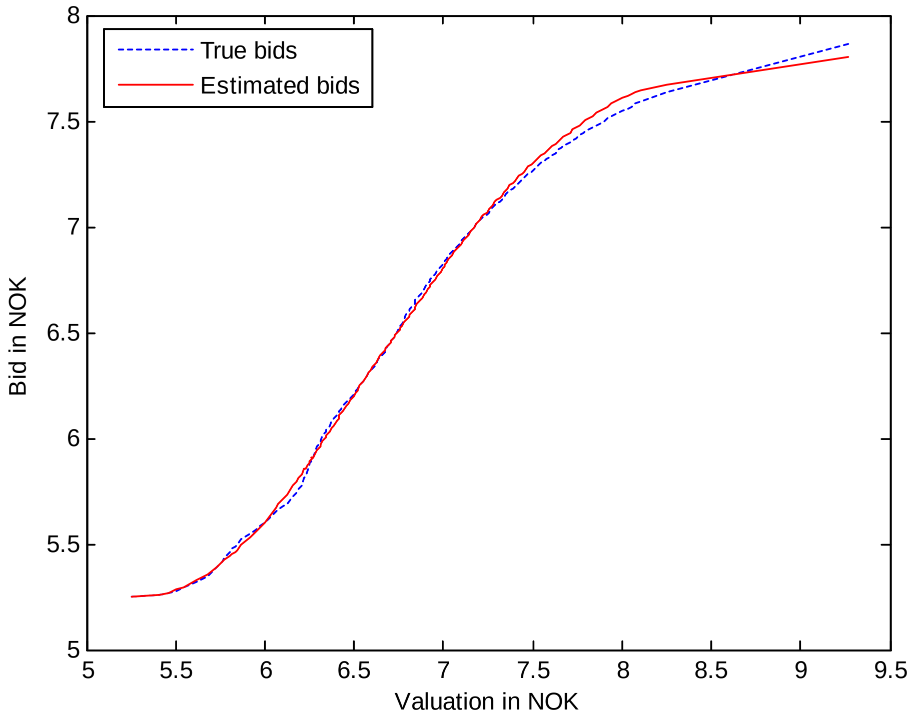
In equations (10.12) and (10.16), we use a kernel function $\kappa(\cdot)$ to obtain the conditional probability functions $\hat{g}_{B|B\geq r}$ and $\hat{f}_{V|V\geq r}$. A kernel function is defined to be symmetric around 0 and must integrate to one. Since the kernel is a density function, the kernel estimate will also be a density. For further characterizations of some standard requirements for kernel functions, we refer to Härdle [43]. Several kernel functions are known to produce reliable results, the exact choice is not critical since the differences in efficiencies are very small for the commonly used kernel functions. We used the Epanechnikov kernel defined by $\kappa(u) = \frac{3}{4}(1-u^2)\,\mathbf{1}(|u| \leq 1)$.

The choice of bandwidth, on the other hand, is critical for the kernel estimator. A bandwidth that is too narrow, will oversmooth the density function, while a broad bandwidth will undersmooth it. No universal agreement on the optimal choice of bandwidth seems to exist. Silverman [99] reports a rule-of thumb of $h_g = 1.06\hat{\sigma}S^{-1/5}$, where $\hat{\sigma}$ is the standard error of the sample of size S. This is frequently used; for example, Guerre et al. used it in their simulation analysis demonstrating their two-step estimator. We used this bandwidth as well, although a data driven bandwidth selector, such as the one proposed by Sheather and Jones [97] is preferable.

10.5.3 Results

Fit between true bids and estimated bids. A key output of the estimation procedure are the estimated underlying valuations from equation (10.15) and the associated cumulative distribution function from equation (10.17). How well do our estimates of valuations explain bids given our model? Using

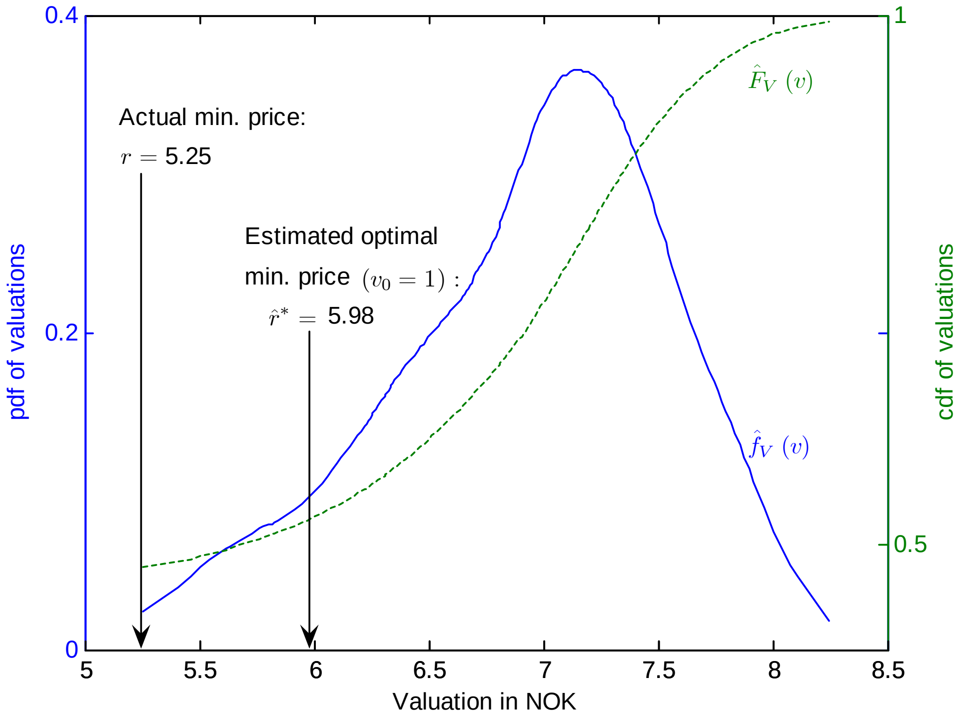
Figure 10.1: True and estimated bids



our estimated valuations, we can estimate bids by using the theoretical bid function of equation (10.2) and compare them to the observed bids. In figure 10.1, we plot the estimated valuations against both the true observed bids and the estimated bids. The fit between true and estimated bids is quite good. The estimation error is below 1 percent for all observations; the range is from -0.52 percent to 0.98 percent.

The optimal reserve price. The number of average bidders per catch is around 7 (arithmetic mean $= 7.30$). The maximum number of bidders observed at an auction, is 14. According to (10.14) this gives us $\hat{F}_V(r) = 0.48$. When we estimate the cumulative distribution function and population density function of valuations, we obtain the functions $\hat{f}_V(v)$ and $\hat{F}_V(v)$ shown in figure 10.2.

Figure 10.2: Estimated probability functions

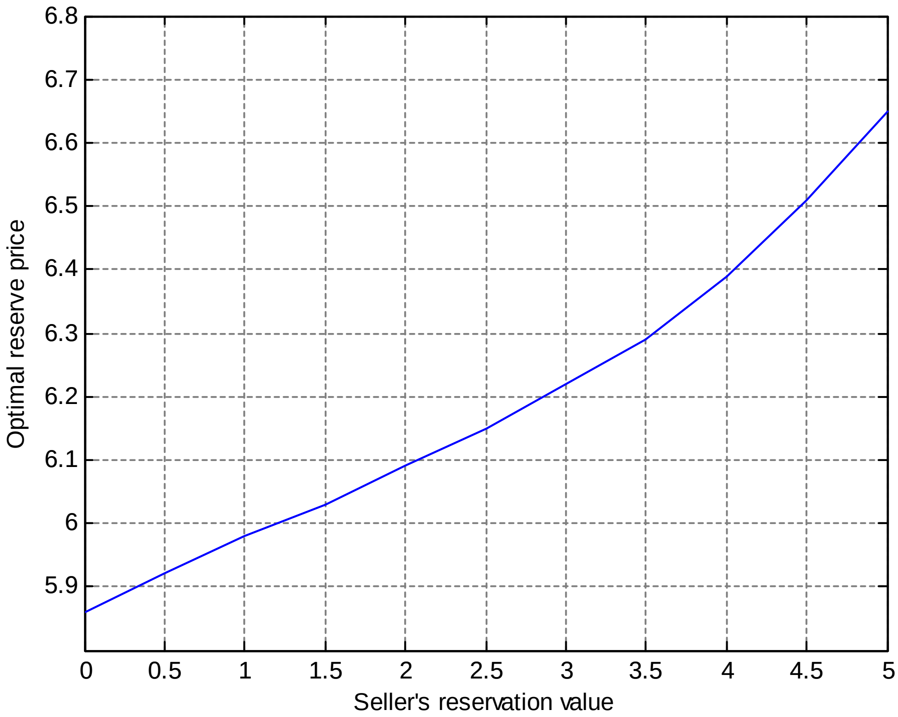


With estimates of F_V and f_V, we can numerically solve for the optimal reserve price. Since r^* depends on the seller's reservation value v_0, we must account for this in the calculation. If a catch goes unsold, it might be sold for meal production instead of for human consumption. In that case, it obtains a price considerably lower than the current reserve price. In figure 10.2, we have indicated a lower bound for $r^* = 5.98$ when $v_0 = 1$. This suggests that the reserve price should be raised by at least 13.9 percent.

We choose to calculate $\hat{r}^*$ for different assumptions of v_0. Obviously, $\hat{r}^*$ is increasing in v_0. The relationship between $\hat{r}^*$ and v_0 is shown in figure 10.3.

Effects on shaving factor and revenues. Next, we turn to the question of how an increase in the reserve price will affect revenues, if implemented. Let us first look at the degree to which shaving factors are reduced when an

Figure 10.3: The optimal reserve price as a function of seller's own valuation, v_0



higher reserve price is introduced. With as many as 14 potential bidders, we expect this market to be quite competitive, so the shaving factors will likely be moderate. Using the estimated valuations and true bids for all 708 observations, we find that the average shaving factor of bids is as low as 3.85 percent (minimum 0 percent and maximum 27.16 percent).

In order to compare actual and counter-factual shaving factors, we focus on the lots that are sold in the latter case, and the case where $v_0 = 1$ and $\hat{r}^* = 5.98$. In table 10.3, we report the estimated shaving factors for all bids and for only the winning bids. Actual shaving factors with a reserve price equal to 5.25 are, for all relevant statistics, larger than the shaving factors observed when the reserve price is set to 5.98.[5] This is to be expected; the point of raising the reserve price is to decrease shaving factors. The difference in the mean shaving factor is 0.64 percent for all bids and 0.56 percent for winning bids. This indicates that the possible gains from raising the reserve price are moderate.

Table 10.3: Shaving factors in percent, $v_0 = 1$

	All bids[a]		Winning bids[b]	
	Actual $r = 5.25$	Estimated $r = 5.98$	Actual $r = 5.25$	Estimated $r = 5.98$
Minimum	2.42	0.46	2.43	1.59
25th percentile	2.56	2.50	2.79	2.59
50th percentile	3.14	2.91	3.63	3.16
75th percentile	4.52	3.49	4.65	3.60
Maximum	27.16	28.89	27.16	28.89
Mean	3.72	3.08	4.14	3.58
Standard deviation	1.71	1.38	2.75	2.84

[a] Number of observations: 641
[b] Number of observations: 91

[5] The only exception is for the maximum statistic case. The estimate of the maximum valuation bid seems to underestimate the true bid.

Before we conclude that the seller should raise the reserve price, we examine possible efficiency effects of raising the reserve price by calculating counter-factual bids and revenues. We have seen that an increased reserve price decreases the shaving factor. When calculating counter-factual revenue, we respect the realized allocation. For example, if a catch was sold to the second-highest bid, then the counter-factual bid from that same bidder is used. The estimation procedure of underlying valuations and corresponding counter-factual bids ensures that the ranking between counter-factual bids is not altered relative to the true bids. In cases where valuations drop below the new reserve price, however, all the associated bids will equal zero. The optimal reserve price is derived by balancing the risk of having unsold objects with the gains from reduced shaving on the objects that do attract bids. At one-shot auctions, the outcome can indeed be suboptimal in the sense that an object may go unsold. However, at repeated auctions with a sufficiently large sample, it is likely that the realized outcome is close to the expected revenue predicted.

Counter-factual revenue depends on what optimal reserve price $\hat{r}^*$ we use, which, in turn depends on the seller's true reservation value v_0. Previously, we argued that a reservation value of one is likely. In table 10.4, the estimated percentage change in revenues is reported for a range of different reservation values.

The important result in table 10.4 is column 3 where the percentage change in revenue from going from a reserve price equal to 5.25 to the reserve price in column 2, is reported. The result is that total revenues slightly decrease for most v_0 below 4. This is due to inefficiencies that a raised reserve price entails. In the sample, we have three unsold lots. In column 5 of table 10.4, the additional unsold lots, due to an increase in a reserve price, are reported. We see that some additional 3–4 lots go unsold in the counter-factual case when v_0 is below 3.50. For reservation values above 3.50, the effect of inefficiencies is more severe, both in terms of unsold lots, and in terms of reduced revenue. In this sample, and given our assumptions, the total effect of raising the reserve price is that the costs associated with unsold lots that will have to end up as meal production at value v_0, outweigh the

Table 10.4: Estimated revenue effects

		Change in revenue (%)		
v_0 [a]	$\hat{r}^*$ [b]	All lots	Sold lots	Unsold [c]
0.00	5.86	−0.33	0.44	3
0.50	5.92	−0.23	0.47	3
1.00	5.98	−0.12	0.50	3
1.50	6.03	−0.03	0.52	3
2.00	6.09	−0.18	0.55	4
2.50	6.15	−0.05	0.58	4
3.00	6.22	0.14	0.66	4
3.50	6.29	−0.11	0.69	6
4.00	6.39	−1.56	0.63	10
4.50	6.51	−1.54	0.66	14
5.00	6.65	−2.04	0.58	19

[a] v_0: Seller's own valuation.

[b] $\hat{r}^*$: Estimated optimal reserve price.

[c] Number of unsold lots due to increase in the reserve price.

increase in revenues from higher bids on the sold lots. The *ex post* optimality of the calculated reserve price rests on large sample properties. Thus, care must be taken when considering changes in the reserve price in a given market with a limited number of objects for sale.

The differences between actual and counter-factual revenues are, however, very small. It is fair to say that our estimates of valuations are conservative. As discussed previously, both the general demand reduction effect due to the multi-sales format, and the uncertainty with respect to the number of bidders—see equation (10.7)—make the estimate a lower bound of true valuations. If true valuations were sufficiently underestimated, then possible inefficiency effects would not bind. In column 4 of table 10.4, the revenue effects on those lots that will be sold under both regimes, are reported. The increase in total revenue is somewhere between 0.44 percent and 0.69 percent. Admittedly, this is not a substantial increase, but even a small increase in revenues is worth considering since the absolute amounts involved in a market with a turnover of several billion NOK, are large.

10.6 Concluding remarks

Theory suggests that there may be a potential for raising revenues by adjusting the reserve price upwards. In our sample of auctions, however, inefficiencies created by the auctioneer when aiming for an optimal extraction of buyers' surpluses outweigh the gains. Only if valuations are sufficiently underestimated, will sellers increase revenues by raising the reserve price in our sample. In the long run, we estimate an increase in revenues of about 0.50 percent. Even though the observed reserve price is quite far from the estimated optimal, the revenue effects are moderate, and in that sense, the reserve price in use is, probably, not far from the target.

The analysis rests on the assumption that the theoretical model captures the driving elements of bid shaving behavior at these auctions reasonably well. Some assumptions of the analysis may oversimplify certain elements. First, the two-step estimator of Guerre et al., on which our analysis is based, considers bidders' prior beliefs to be symmetric. An interesting extension of the model is provided by Brendstrup and Paarsch [14], which allows for asymmetric bidders. The data requirements for computing individual probability functions for each bidder, may, however, prove to be too demanding in our case. Second, the analysis may benefit from a more sophisticated treatment of the number of potential bidders. The number of active bidders varies considerably across auctions. Given the high number of potential bidders we use, the market may appear more competitive than it actually is for some lots. Finally, although we have tried to sample homogenous lots, heterogeneity may be present. Investigating heterogeneity requires a parametric rather than a nonparametric, structural approach.

10.A Proofs

10.A.1 Probability distributions of bids

In this section, we show how the formula for the probability distributions of bids reported in equations (10.4) and (10.5) are derived. The cumulative distribution function (cdf) of observed bids, when a binding reserve price is present, is conditional on private values being above r:

$$G_B(b) = \Pr[\beta(v) \leq b | v \geq r] = \Pr\left[v \leq \beta^{-1}(b) | v \geq r\right] = F_V\left[\beta^{-1}(b) | v \geq r\right].$$

The conditional density above will, by definition of a cdf, depend on a constant c such that:

$$c \int_r^{\beta(\overline{v})} f_V(u)\, du = 1.$$

Since the antiderivative of $f_V(u)$ is $F_V(u)$ and $F_V[\beta(\overline{v})] = 1$, we have that

$$c = \frac{1}{[1 - F_V(r)]}.$$

Thus, the cdf reported in equation (10.4), is:

$$G_B(b) = \frac{1}{[1 - F_V(r)]} \int_r^v f_V(u)\, du = \frac{[F_V(v) - F_V(r)]}{[1 - F_V(r)]}.$$

Recalling that $d\beta^{-1}(b)/db = 1/\beta'(v)$, the pdf of observed bids of equation (10.5) is:

$$\begin{aligned} g_B(b) &= \frac{dG_B(b)}{db} = d\left\{\frac{\left[F_V\left[\beta^{-1}(b)\right] - F_V(r)\right]}{[1 - F_V(r)]}\right\}/db \\ &= [1 - F_V(r)]^{-1} f_V\left[\beta^{-1}(b)\right] \frac{d\beta^{-1}(b)}{db} = \frac{f_V(v)}{[1 - F_V(r)]\,\beta'(v)}. \end{aligned}$$

10.A.2 Stochastic dominance result

Let $\hat{r}^*$ denote the optimal reserve price calculated using estimated functions $\hat{F}_V(r)$ and $\hat{f}_V(r)$, and let r^* denote the true optimal reserve price calculated using true functions $F_V(r)$ and $f_V(r)$.

Proposition *If for all r, $\hat{F}_V(r) \geq F_V(r)$, then $\hat{r}^* \leq r^*$.*

Proof. First, we note that the optimal reserve price, r^*, is the solution to

$$r - v_0 - \frac{[1 - F_V(r)]}{f_V(r)} = 0, \tag{10.18}$$

while we estimate $\hat{r}^*$ by finding the value of r that satisfies

$$r - v_0 - \frac{\left[1 - \hat{F}_V(r)\right]}{\hat{f}_V(r)} = 0. \tag{10.19}$$

From (10.18) and (10.19), if $\frac{\left[1-\hat{F}_V(r)\right]}{\hat{f}_V(r)} \leq \frac{[1-F_V(r)]}{f_V(r)}$, then $\hat{r}^* \leq r^*$. Thus, we need to show that $\frac{\left[1-\hat{F}_V(r)\right]}{\hat{f}_V(r)} \leq \frac{[1-F_V(r)]}{f_V(r)}$ implies $\hat{F}_V(r) \geq F_V(r)$. We proceed by noting that

$$\frac{\left[1 - \hat{F}_V(r)\right]}{\hat{f}_V(r)} \leq \frac{[1 - F_V(r)]}{f_V(r)} \Rightarrow -\frac{\hat{f}_V(r)}{\left[1 - \hat{F}_V(r)\right]} \leq -\frac{f_V(r)}{[1 - F_V(r)]}.$$

Since the exponential function is monotonically increasing, this implies that

$$\exp\left(\int_{\underline{v}}^{r} -\frac{\hat{f}_V(t)}{\left[1 - \hat{F}_V(t)\right]}\,dt\right) \leq \exp\left(\int_{\underline{v}}^{r} -\frac{f_V(t)}{[1 - F_V(t)]}\,dt\right). \tag{10.20}$$

Further, since

$$-\frac{f_V(r)}{[1 - F_V(r)]} = \frac{d}{dr}\log[1 - F_V(r)],$$

we have that the right-hand side of inequality (10.20) can be written:

$$\exp\left(\int_{\underline{v}}^{r} \frac{d}{dt}\log\left[1 - F_V(t)\right]\, dt\right) = \left[1 - F_V(r)\right]. \qquad (10.21)$$

We have a similar expression for the left-hand side of (10.20). Consequently, from inequality (10.20) and equation (10.21), it follows that

$$\left[1 - \hat{F}_V(r)\right] \le \left[1 - F_V(r)\right]$$

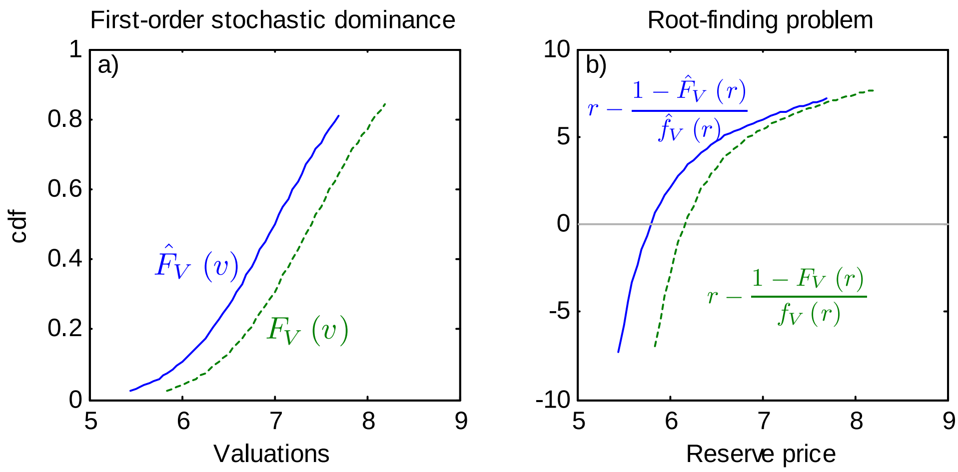
or

$$\hat{F}_V(r) \ge F_V(r).$$

■

Appendix B of Krishna [59] proved useful for the above proof. In figure 10.4, we illustrate the proof by sketching the relationship between the distribution functions in part a, and the corresponding root finding problem for an optimal reserve price in part b. As we see, and what was the point of the above proof, if $\hat{F}_V(v) \ge F_V(v)$ for all v, then $\hat{r}^* \le r^*$.

Figure 10.4: Illustration of stochastic dominance result



Chapter 11

Conclusion

Simultaneous selling mechanisms require bidders to set capacity constraints in order to avoid coördination problems that can cause efficiencies. Bidders also have the option of setting priorities to their bids. This option is unnecessary for allocation of catches, but may be important for optimal bundling of catches. The auction format is complex, but interesting from a theoretical perspective. The dimensionality of the bid vector (bids, priorities and capacity limits) combined with the effect that delivery sectors have on the potential number of bidders make it difficult to model the market analytically. Consequently, the scope for a structural empirical analysis of the market is limited.

The auction format can, however, be analysed by other available tools. Numerical simulation is one promising line of research. We analysed the effect of the priority option in chapter 8 in a simplified, but relevant setting. We identified a clear effect on the game-theoretic equilibrium strategy. Bidders will find it profitable to set a reverse relation between bid levels and priorities. Empirically, however, we found few traces of the effect. The effect is of less importance when the number of bidders increases. Heterogeneity of catches can also explain the observed predominant positive relation between bid levels and priorities.

We analysed the determinants of market price in chapter 6 and 7. The determinants may be classified as object-specific, auction-specific, and bidder-specific. Marketing of seafood products through the entire value chain have been a research topic of considerable interest. We have examined the wholesale market for pelagic fish. Quality variables that the sellers control are of particular interest. Although the primary objective of preservation methods is to secure that the seafood is as fresh and healthy as possible, cost considerations enter the picture as well. If we can identify that the best preservation method gives a rise in prices that more than offset the extra costs incurred, then the effort is worthwhile. Results show that the preservation method, in fact, has an influence on prices. The computed price elasticity of preservation method is an input to the decision on preservation method.

The overall picture is that variables outside the scope of individual harvesters—in particular, average fish weight of a given catch—are most important for winning bids. This is hardly a surprise, but the contribution of this work is a better understanding of how important the different quality variables are in determining prices.

The nature of asymmetries between bidders was also investigated. We showed in chapter 5 that bidders differ with respect to capacities, bid frequencies and to some extent with respect to bid success. However, bid levels examined in chapter 7 (winning bids) and chapter 9 (all bids) showed small average differences between bidders, and we concluded that bidders' average valuations seem to be roughly equal.

At the individual bid level, however, our understanding of bid determinants is far from perfect. We were able to explain about 85 percent of the variation in winning bids by our explanatory variables. In the regression on all bids, about 81 percent of the variation were accounted for. The remaining variation is a puzzle. The high volatility of the number of submitted bids across catches and auctions is partly explained by the system of setting delivery sectors. But even if we control for this, as we did in chapter 9, utilization of bid opportunities varies considerably between bidders. We suspect that both variation in bid levels and in auction participation may better be explained if individual capacity limits and opportunities in other fisheries are incorporated. In addition, we may speculate that liquidity squeezes or short-term financial constraints have an impact on buyer's bid decisions. Since we observe high variability in bids, a natural interpretation is that buyers' valuations are predominantly private and independent.

Given the simultaneous selling mechanism, it is unclear whether the number of competitors is an object-specific or auction-specific variable. From an economic perspective, the main interest is the question of efficiency. Auction formats may prove to be inefficient. Increased competition will reduce the effects of any inherent inefficiencies the format may give rise to. A quite robust result is that when the number of bidders for a given catch is around eight, we reach a competitive level. The important message is that as long as a catch attracts reasonably many bidders, the finer details of the auction format, is of less concern.

The dataset was examined for any signs of bid coördination. We found no evidence of such a practice among bidders.

A careful interpretation of our analysis on the optimal reserve price suggests that minimum prices are not set too high from the sellers' perspective. The allocation of revenues between private businesses may not be that vital. We note that profits have been low on the buyer's side while sellers have experienced far better operating margins in recent years. The better results on the sellers' side may be attributed to the economic rent sellers obtain through harvesting a natural resource owned by the people.

Some topics for future research are suggested. Our understanding of bid behavior may be improved in order to reduce the unexplained variability in bids. The complexity of the market and the information sets, however, may prove to be so involved that we are close to the limit with respect to predicting bid behavior. Likewise, our understanding of decisions on bid participation is less than desirable. More information, in particular on bidder-specific private information is necessary for making progress.

The geographical aspects of the market may be investigated. At the core is the question of how does the market clear optimally. The importance of the location of buyers, and the distance between vessels and delivery port with respect to total costs may give rise to a solvable programming problem. The solution may affect the auction format, and the details of the validation process.

Appendix A

Construction of dataset

A.1 Introduction

In this appendix, we describe how the dataset was constructed. We have a complete set of observations from the auction market for an entire season. The auction house, *Norges Sildesalgslag* (NSS), Bergen, Norway, provided us with all of their data concerning pelagic fish auctions from the beginning of the 2003 season (August 11, 2003) to the end of the 2004 season (January 20, 2005).

We used a relational database management system to organize the data, and then imported the final dataset into MATLAB for subsequent analysis. Data are at the auction house stored in the xml format, but were exported to a MS Access 2002 database file for us to use. Extensive reorganizing of the data was necessary because the database was constructed for the daily running of auctions and efficient validation of the bid process, not for *ex post* dataset construction for researchers. The necessary tables with appropriate formats were created by use of programming in MS VBA. Then several queries in the SQL language were used in order to fill the tables. In addition, some organizing and reformatting, in particular of date and time variables, were done in MATLAB after importing the final database tables. Extensive checking of the tables was undertaken to detect typing errors.

The database consists of three main tables and in addition several support tables that explain numeric code values used in the main tables. The first table called the *lots table*, contains records of each individual lot offered for sale. The other main table, the *bids table*, contains all bids elements. The final table, the *constraints table*, contains the relevant quantity or vessel limits set by a bidder for an auction. Thus, the number of records in the constraints table equals the number of auctions.

A.2 Quality of data

In general, the quality of any empirical analysis depends on the quality of data. Several dimensions of the data may cause trouble for the researcher. Data can be aggregated to such an extent that important short-time fluctuations are hidden. Variables may be measured at different time intervals, making it difficult to estimate consistently relationships for a given time period. Moreover, measurement errors are a major concern in empirical economics. To a large extent, our dataset is immune to these quality deteriorating features. We have micro data, there is no aggregation or inconsistent time intervals that bring in noise. Data presented to bidders were typed in electronically. Although it is conceivable that a wrong number was provided bidders, they all receive identical information and base their bids on this information. Likewise with bids: bids are submitted electronically. Normally, typing errors are understood to mean that the researcher receives information different from what took place. This is not the case for us. If a bidder meant to submit a bid of 5.95 and entered 5.59, this is a binding bid. Thus, a typing error on the bidder's part is more of a trembling-hand mistake. Admittedly, such trembling-hand mistakes may bring in noise when modelling bids to be governed by a Bayes–Nash equilibrium, but we regard the problem to be very small. A few errors, however, were detected and corrected.

A.3 The sample

Several pelagic fish species are auctioned by the auction house NSS which has been granted monopoly to sell all pelagic fish in the wholesale market of Norway. In addition, fish harvested in other national economic zones are sometimes offered at the NSS auction because this is the best-organized market in the North Sea region. In this thesis, we chose to analyse the product mackerel. The mackerel market is comprised of two segments, the sea and the coastal segments. The latter is a small-scale fishery, where harvesting occurs close to the coast or in the Norwegian fjords. The fish in this segment are small and have a local-buyer market; in addition, the fish are commonly penned alive. We concentrate on the large-scale sea segment with a much broader buyer market. We chose to analyse the 2003–4 season.

The distinction between lots and catches is important. A catch may be comprised of several lots. If the total catch is comprised of significantly different average weight classes or of different species, then the catch is divided into lots. Each lot is described to potential buyers who will be asked to submit separate bids for each lot. The total catch of a vessel will have to be sold to a single buyer. The winner will be the one who has the highest weighted average bid on the catch.

After querying our database for mackerel harvested at sea in the 2003–4 season, our dataset contained 1,582 lots offered for sale at 289 auctions. Most lots were offered only once, but some were offered several times if they first went unsold. The number of unique lots is 1,531 and the number of unique sold lots is 1,490. The number of sold lots is 1,494. We noticed that there were four more sold lots than unique sold lots. One lot was reported sold, but then sold again. Two other lots were reported sold, but then sold again later. Either the auction house made a mistake and had to cancel the first sales, or the first buyer was allowed by the auction house to cancel and the lot was put up for a resale before he had received the lot. In that case, the first buyer would probably be responsible for any negative price difference between the first and second sale. We did not correct the price data and reported buyers for these three lots.

The corresponding numbers of catches in the dataset are: We have 1,456 records of catches. A total of 1,369 catches were reported sold, while the number of unique catches is 1,405 and the number of unique sold catches is 1,365. For the discrepancy between unique sold catches and sold catches, see the comment in the paragraph above.

A.4 Transformations of data

Numeric quality variables. A catch is described by different quality variables like harvesting method and storage conditions. These qualities are represented by a code number. For example, if the number is 11 for harvesting gear, then the gear used is purse seine. For the purpose of regression analysis, in order to measure the impact of such variables on price, it is convenient to transform the variables into classical dummy variables.

Quantity, date/time and price variables. No transformations of the quantity or price variables were necessary. Quantities of catches and lots are expressed in tons, average fish weight is reported in grams, and prices and bids are expressed in NOK per kilo. Date and time variables were in full string formats like "yyyy-mm-dd HH:MM:SS.s". We reformatted them to a consistent format for our purpose and software choice. Date variables were set to the string format "yyyy-mm-dd" and time variables were set to the string format "HH:MM". Next, date and time variables were set to numeric time vectors of the form [yyyy mm dd HH SS]. This enabled us to compute time differences.

Computed variables. Below, we report the most important variables given to us from the auction house. Variables are not necessarily stored in the same order as we present them. Given these variables, we computed several variables to be used in the statistical analysis.

A.5 Tables

LotsN

The table LotsN ("Lots Numeric") contains numeric values that describes the individual lots. The most important variables are:

1. **KeyNo:** Individual lot ID number where the AucNo is the integral part and FormNo is the decimal part; see below for explanations of variables AucNo and FormNo. This field was created by us and used as the unique identification number in the SQL queries.

2. **AucNo:** Auction number ID. All lots sold simultaneously have the same auction number.

3. **FormNo:** Individual lot ID number irrespective of auction number. In case a lot goes unsold, it may turn up at the next auction with a new AucNo, but the same FormNo.

4. **FormNoPart1:** The first part of the FormNo identifying the catch. Equals FormNo with the last integer missing.

5. **FormNoPart2:** The second part of the FormNo identifying the different lots of a catch. A catch consisting of two lots will have FormNoPart2 equal to 1 and 2 respectively for the two lots.

6. **FormDate:** Date of auction. Originally recorded date string is converted to serial date number.

7. **HasBid:** Binary variable. Equals 0 if a lot received no bids, equals 1 if the lot received bids.

8. **MaxBid:** The highest bid in NOK per kilo.

9. **PriceSold:** The realized selling price in NOK per kilo; not necessarily equal to the highest bid if the quantity limit is binding for the "high" bidder.

10. **MinPrice:** The reserve price in NOK per kilo. Reserve price is determined by the average fish weight, see the variable AvgWeight below.

11. **Value:** The total bid of the lot in NOK calculated by multiplying the bid in NOK per kilo by the quantity of the lot.

12. **CatchLocation:** The field of harvesting identified by a map sector number.

13. **EcZoneId:** Number representing the economic zone of the catch location; 0: Norwegian zone, 1: European Union zone, 6: Faroe Islands zone.

14. **Quant:** Quantity of lot in tons (1000 kilos).

15. **VesQuant:** Quantity of catch in tons.

16. **AvgWeight:** Average fish weight in the lot in grams.

17. **GearId:** Harvesting gear used represented by a numeric code value: 10 = Purse seine coastal vessel, 11 = Purse seine, 51 = Bottom trawl, 53 = Floating trawl, 54 = Floating trawl, pair trawling.

18. **Preservation:** Variable describing the preservation method on board the vessel. Numeric code values in use are: 9: iced, 11: refrigerated sea water (RSW), 13: salted, 18: RSW + ice, 21: RFW + ice, 24: RFW + acid + ozone, 25: CSW.

19. **Hauls:** Number of purse seine hauls used to obtain the catch.

20. **Tanks:** Number of storage tanks used.

21. **Swim:** Describes whether the fish were held alive in sea in order to reduce feed contents before taken on board. The number reported represents the number of hours the fish were kept "swimming".

22. **Feed:** Integer variable describing the feed contents of the fish. Numeric code values in use are: 1: no feed, 2: insignificant feed, 3: significant feed, and 4: full of feed.

23. **AreaIdS and AreaIdN:** A number identifying the southernmost port (AreaIdS) and the northernmost port (AreaIdN) the seller prefers delivery to. Port codes reveal the relative position of the port along a north-south axis. If a seller sets his delivery sector to AreaIdS = 16 (south) and areaIdN = 25 (north), then a bidder with location 20 will be an inside bidder, while buyers located at 13 and 30 will be outside bidders. Location 13 is south of the delivery sector and 30 is north of it. Ports in the dataset are: 12 Agnefest; 13 Egersund and Sirevåg; 16 Haugesund, Karmøy, and Utsira; 19 Bergen and Austevoll; 20 Florø and Kalvåg; 22 Måløy and Iglandsvik; 23 Moltu; 24 Vedde; 25 Moltu and Harøysund; 29 Averøy; 30 Kristiansund; 31 Smøla.

LotsC

The table LotsC ("Lots Character") contains character strings that describe the individual lots. The most important variables are:

1. **KeyNo:** Unique identification number; same as KeyNo in LotsN.
2. **VesId:** Registration code of vessel.
3. **VesName:** Name of vessel.
4. **VesFlag:** Nationality of vessel. NO: Norway, DK: Denmark, FO: Faroe Islands, GB: Great Britain, IE: Ireland, SE: Sweden.
5. **VesOwnerName:** Name of vessel owner.
6. **BuyerName:** Name of buyer.
7. **PlantUse:** Describes whether the fish are meant for consumption (K) or meal production (M). All are K in the sample.
8. **HarvestDate:** Date of harvest.
9. **HarvestTime:** Time of harvest.

10. **AucDate:** Date of auction.

11. **AucTime:** Time of auction.

12. **AreaDateS:** Expected arrival date at southernmost port of delivery area, see variable AreaIdS in table LotsN.

13. **AreaDateN:** Expected arrival date at northernmost port of delivery area, see variable AreaIdN in table LotsN.

14. **AreaTimeS:** Expected arrival time at southernmost port, see variable AreaIdS in table LotsN.

15. **AreaTimeN:** Expected arrival time at northernmost port, see variable AreaIdN in table LotsN.

A few typing errors were detected for the last four variables in the list above; i.e., the expected date and time for arrival at port. A typical error is that arrival is set before the auction is held, and the most common explanation is that arrival mistakenly was recorded to be the same date as the auction while in fact it should be set to the next day. Data were corrected by comparing the date with auction date and auction time and with the location of the catch field. Some discretion was necessary to correct the data.

Bids

Each bid at the auction house is stored in a record (or row). This way of organizing the bid data follows from sound design rules of normalized databases following the rule that "rows are cheap, columns are expensive." Our purpose (i.e., analysing the data in matrix oriented software package) required that we organize all bids of a given catch in one record. This was achieved by creating a table with as many columns as the number of active bidders in the dataset, and then filling the records with the bids. Records of inactive bidders for a given lot were filled with NaNs (NaN represents "not a number").

A bid is a multiple of potentially six elements: the actual bid, a priority, and a location parameter in addition to three reported capacity constraints. Bids are given for each lot and priorities are given for each catch. The location parameter—i.e., whether the bidder has a location inside or outside of the delivery sector is recorded by the auction house. The capacity constraints are given for each auction and are stored in the table Constraints, see below. Of the six elements in the bid vector, only the actual bid in NOK per kilo is mandatory; the other elements are optional.

We have organized these elements as different tables in a three dimensional matrix. All tables have an identification field that uniquely identifies the corresponding records of the tables and links the table Bids to the key number in the table LotsN. Lots are recorded in rows and each bidder in the dataset is aligned a column in the tables. The different tables then hold the following information:

1. **Bids:** Bid in NOK per kilo.

2. **Pri:** Priority of bid, optional. The highest priority is set to 1, the second-highest priority is set to 2, and so forth. Lots belonging to the same catch will have the same priority number. Two catches can have the same priority number meaning the bidder is indifferent between them. A bidder will not necessarily give priorities to all his bids. He may set priorities on just one or two catches and leave the rest blank. An alternative to a blank priority is to assign the priority number 99 meaning that these catches will have the lowest priority.

3. **Loc:** Location parameter is 0 if the bidder has a location inside the delivery sector and 1 if the bidder is outside of the delivery sector. The auction house uses the value 1 for inside bidders and the value 2 for outside bidders; we changed this to 0 and 1 respectively.

4. **AvgB:** The bid on the catch, i.e., the weighted average of the bids on the lots comprising the catch. Weighting is by the quantity of the lots, see variable Quant in the table LotsN.

Constraints

For each auction and each bidder, we store five variables in the table Constraints. The table is organized as a 3D matrix where each sheet contains the relevant variable. Auctions are in rows and bidders in columns. The first column of each sheet stores the auction number which can be linked to AucNo in LotsN. The variables in the table are:

1. **MaxT:** Maximum quantity in tons a bidder wants to receive from a given auction, optional, set by the bidder.

2. **MinT:** Minimum quantity in tons a bidder wants to receive, optional, set by the bidder.

3. **MaxV:** Maximum number of vessels (or catches) a bidder wants to receive, optional, set by the bidder.

4. **CurT:** The number of tons allocated to the bidder after validation, recorded by the auction house.

5. **CurV:** The number of vessels (or catches) allocated to the bidder after validation, recorded by the auction house.

CatchesN

The table CatchesN ("Catches Numeric") contains the same variables as the table LotsN. When constructing the table, we put the lots of the same catch together. Depending on the nature of the variable, we computed the sum or the weighted average of the lot variables. Some variables have the same value, so we kept the first record; taking the mean would be an equivalent procedure. In addition, some variables did not make sense to incorporate. A few examples of the calculations are reported: Quantity of catch is the sum of the quantities of lots. As a check, it should be the same as the vessel quantity. Average fish weight and bids are weighted average of the corresponding lot variables using Quantity of lots as the weights. The number of bids is the same for all lots of a specific catch since bidders bid on the entire catch. A quality variable such as Gear is consistently the same for all lots of a catch and we could just record the average or first record. On the other hand, a quality variable such as Feed may be different for the lots of a catch, and because it takes the integer values 1–4, we did not record any weighted average of this variable.

At one auction, there were multi-species catches; i.e., the different lots of a catch consisted of different species. Three lots in the table LotsN are from multi-species catches. Since we only sample mackerel lots, we removed the multi-species catches of that auction when creating the table CatchesN.

Appendix B

Simulation script

In this appendix, we document the computer code used to perform the auction simulation of chapter 8.

Computer code

```
clc; clear all;

% Matlab: Version 7.0.1.15 (R14) Service Pack 1
% Necessary package: Symbolic Math

%% Simulation of auction model:
% 2 objects for sale, 3 bidders with single unit demand.
% Each bidder submits two bids and give priorities to their bids.
% If the same bidder has the highest bids on both objects, then
% the allocation is according to his priority. The other object
% is allocated to the second-highest bidder on that object. Other-
% wise, the highest bidders are allocated their respective objects.

% The purpose of the simulation is to examine whether a Nash
% equilibrium exists when bidders scale down their preferred lot.

```

```
% We record expected profits to bidders and expected revenue to
% the seller when all bidders scale down their preferred lot.
% We examine all possible combinations of alpha strategies for
% all 3 bidders.

%% Set some parameters
n = 2;      % Number of competitors and objects
N = n + 1;  % Number of bidders
T = 100000; % Number of auctions

ScalingFactor = (.05:.05:1);
L = length(ScalingFactor);

% Preassign tables for storing results:
Bidder1 = NaN(L,L,L);
Bidder2 = Bidder1;
Bidder3 = Bidder1;

%% Draw valuations (V):
% Set the seed (state) of the random generator;
% this ensures that results can be reproduced:
State = 591; rand('state', State);

% Draw N valuations in T auctions from U(0,1):
% Bidder 1 in col 1, bidder 2 in col 2, etc.
V = rand(T,N);

%% Calculate bids:
% Benchmark bid model: Discriminatory simultaneous.
% Ref: Vijay Krishna. 2002. Auction theory, page 195.
% Bid = E[Y(k)|Y(k)<x] where Y(k) is the k-th order
% statistic of n draws.
```

```
% Define pdf and cdf of the second-highest order statistics:
syms t z
g2 = n*(n-1)*(t^(n-2))*(1-t); % pdf: 2(1-t)
G2 = n*(z^(n-1))-(n-1)*(z^n); % cdf: 2z - z^2 = z(2-z)

BidFn = (1/G2)*int(t*g2,0,z); % Theoretical bid function
Bidz = subs(BidFn,V);         % Put valuations in bid function

% The table Bidz contains the benchmark bids. Make bid tables
% that will be updated depending on priorities and scaling
% factor:
Bids1 = Bidz; % (TxN) bids for object 1
Bids2 = Bidz; % (TxN) bids for object 2

%% Generate priorities:
% 8 possible combinations of priorities for object 1:
% Example: pri = 110 => Bidder 1 and 2 prefer object 1
% while bidder 3 prefers object 2.
State = 195; rand('state', State);
pri = [1 1 1
       1 1 0
       1 0 1
       0 1 1
       1 0 0
       0 1 0
       0 0 1
       0 0 0];

% Draw a random vector from the uniform distribution, U(0,1).
x = rand(T,1);
% Get percentiles of random vector x:
y = prctile(x,[12.5 25 37.5 50 62.5 75 87.5]);
y = [[0 y]' [y 1]'];
```

```
Pri = NaN(T,3); % Preassign a table of priorities

% Randomly assign a priority vector to all auctions.
% The 8 priority vectors in pri are assigned to the auctions
% where x belong to the percentile defined by y. Example:
% Priority vector (1 1 1) will be assigned to the auctions
% where x has a value that belongs to the lowest 12.5% in x.
% Thus, the procedure below ensures that exactly T/8 of the
% T auctions have a specific priority vector from pri:
for i = 1:8
    r = (y(i,1)<x & x<=y(i,2));
    Pri(r,:) = repmat(pri(i,:),sum(r),1);
end

Pri = logical(Pri);

%% Run auction simulations
% Strategy: A bidder bids the benchmark bid on the object
% with priority 0 while he scales down his benchmark bid
% with the scaling factor for the object with priority 1.

for i = 1:length(ScalingFactor)
    tic
    for j = 1:length(ScalingFactor)
        for k = 1:length(ScalingFactor)

Scale1 = ScalingFactor(i); % Scaling of bidder 1
Scale2 = ScalingFactor(j); % Scaling of bidder 2
Scale3 = ScalingFactor(k); % Scaling of bidder 3

% Bid lower on preferred lot:
```

```
% Bidder 1 scales down by Scale1:
Bids1(Pri(:,1),1)  = Scale1*Bidz(Pri(:,1),1);
Bids2(~Pri(:,1),1) = Scale1*Bidz(~Pri(:,1),1);

% Bidder 2 scales down by Scale2:
Bids1(Pri(:,2),2)  = Scale2*Bidz(Pri(:,2),2);
Bids2(~Pri(:,2),2) = Scale2*Bidz(~Pri(:,2),2);

% Bidder 3 scales down by Scale3:
Bids1(Pri(:,3),3)  = Scale3*Bidz(Pri(:,3),3);
Bids2(~Pri(:,3),3) = Scale3*Bidz(~Pri(:,3),3);

% Sort bids on object 1:
[B1,IX1] = sort(Bids1,2,'descend');

% Sort bids on object 2:
[B2,IX2] = sort(Bids2,2,'descend');

% Winning bidder is now in column 1 of IX1 and IX2
% Second-highest bidder is in column 2 of IX1 and IX2

%% Allocation
% Preassign allocation tables. Allocation tables
% will be filled with ones in the winner's position.
A1 = zeros(size(Bids1));
A2 = A1;

%% Case 1: Different winners
% Allocation: Give them their respective objects.

% Identify rows with different winners:
DiffWin = IX1(:,1) ~= IX2(:,1);

```

```
% Identify winners of each object:
Win1 = Bids1 == repmat(B1(:,1),1,3);
Win2 = Bids2 == repmat(B2(:,1),1,3);

% Fill allocation for rows with different winners:
A1(DiffWin,:) = Win1(DiffWin,:);
A2(DiffWin,:) = Win2(DiffWin,:);

%% Case 2: Same winner on both
% Allocation: If same bidder wins both, then give
% him the object he prefers. Give the other object
% to the second highest bidder.

% Allocation of high bidder:
% Put a 1 in winner's column if he prefers object 1
High1 = Win1.*Pri;
% Put a 1 in winner's column if he prefers object 2
High2 = Win2.*(~Pri);

% Allocation of second-highest bidder:
% Get the auctions where the highest bidder prefers object 1:
% Mnenomic: HC1 = High Chooses 1
HC1 = sum(High1,2) == 1;

% Get the auctions where the highest bidder prefers object 2:
HC2 = sum(High2,2) == 1;

Sec1 = zeros(size(High1));
Sec2 = Sec1;

% When high bidder prefers object 2, then
% second-highest is allocated object 1:
Sec1(HC2,:) = Bids1(HC2,:) == repmat(B1(HC2,2),1,3);
```

```

% When high bidder prefers object 1, then
% second-highest is allocated object 2:
Sec2(HC1,:) = Bids2(HC1,:) == repmat(B2(HC1,2),1,3);

% Update allocation for rows with same winner
A1(~DiffWin,:) = High1(~DiffWin,:) + Sec1(~DiffWin,:);
A2(~DiffWin,:) = High2(~DiffWin,:) + Sec2(~DiffWin,:);

%% Calculate expected profit for each bidder:
Surplus1 = V - Bids1; % Surplus = value - bid
Surplus2 = V - Bids2;
Surplus1(~A1) = 0;  % Set surplus to 0 if not winning
Surplus2(~A2) = 0;

% Expected profits:
BidderProfits = sum(Surplus1+Surplus2)/T;

% Store the results for scaling factor i, j and k:

Bidder1(i,j,k) = BidderProfits(1);
Bidder2(i,j,k) = BidderProfits(2);
Bidder3(i,j,k) = BidderProfits(3);

        end
    end
 fprintf('Round %3.0f finished \n',i)
 toc
end

%% Find Nash equilibrium
Z1 = Bidder1;
Z2 = Bidder2;
```

```
Z3 = Bidder3; disp(' ')
SF = ScalingFactor;

Nash = zeros(size(Z1));
for i = 1:length(Z1)
    for j = 1:length(Z2)
         for k = 1:length(Z3)
             if Z1(i,j,k) == max(Z1(:,j,k)) & ...
                Z2(i,j,k) == max(Z2(i,:,k)) & ...
                Z3(i,j,k) == max(Z3(i,j,:))
                % Fill Nash with 1 if Nash equilibrium
                % is identified:
                Nash(i,j,k) = 1;
                disp('Nash equilibrium:')
                fprintf('Bidder 1 scales: %4.2f \n',SF(i))
                fprintf('Bidder 2 scales: %4.2f \n',SF(j))
                fprintf('Bidder 3 scales: %4.2f \n',SF(k))
             end
        end
    end
end
```

References

[1] Anscombe, F.J. 1973. Graphs in statistical analysis. *The American Statistician* 27: 17–21.

[2] Ashenfelter, O. 1989. How auctions work for wine and art. *Journal of Economic Perspectives* 3: 23–36.

[3] Athey, S. and P.A. Haile. 2005. Nonparametric approaches to auctions. In *Handbook of Econometrics*, volume 6A, edited by J.J. Heckman and E. Leamer. Amsterdam: North–Holland.

[4] Bajari, P. 1998. Econometrics of sealed-bid auctions. *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*: 41–49.

[5] Bajari, P. and L. Ye. 2001. Competition versus collusion in procurement auctions: Identification and testing. Working Paper, Stanford University.

[6] Bajari, P. and L. Ye. 2001. Deciding between competition and collusion. Working Paper, Stanford University.

[7] Bajari, P. and L. Ye. 2003. Deciding between competition and collusion. *The Review of Economics and Statistics* 85: 971–989.

[8] Bajari, P. and G. Summers. 2002. Detecting collusion in procurement auctions. *Antitrust Law Journal* 70: 143–170.

[9] Beggs, A. and K. Graddy. 1997. Declining values and the afternoon effect: Evidence from art auctions. *RAND Journal of Economics* 28: 544–565.

[10] Bendiksen, B.I. 2007. Driftsundersøkelsen i fiskeindustrien – resultater for driftsåret 2006. Arbeidsnotat. Norsk institutt for fiskeri- og havbruksforskning.

[11] Berck, P. and K. Sydsæter. 1991. *Economists' Mathematical Manual.* New York: Springer-Verlag.

[12] Bianchi, M. and H. Moulin. 1991. Strategic interactions in economics: the game theoretic alternative. In *Appraising Economic Theories,* edited by N. de Marchi and M. Blaug. Aldershot, Hants: Edward Elgar.

[13] Blaug, M. 1992. *The Methodology of Economics. Or How Economists Explain*, second edition. Cambridge: Cambridge University Press.

[14] Brendstrup, B. and H.J. Paarsch. 2003. Nonparametric estimation of Dutch and first-price, sealed-bid auctions with asymmetric bidders. Typescript. Iowa City: Department of Economics, University of Iowa.

[15] Buchinsky, M. 1998. Recent advances in quantile regression models: A practical guideline for empirical research. *Journal of Human Resources* 33: 88–126.

[16] Bulow, J. and D.J. Roberts. 1989. The simple economics of optimal auctions. *Journal of Political Economy* 97: 1060–1090.

[17] Cassady, R. 1967. *Auctions and Auctioneering.* Berkely: University of California Press.

[18] Chakraborty, I. 1999. Bundling decisions for selling multiple objects. *Economic Theory* 13: 723–733.

[19] Cox, J.C, V.L. Smith, and J.M. Walker. 1988. Theory and individual behavior of first-price auctions. *Journal of Risk and Uncertainty* 1: 61–99.

[20] Crampton, P., T. Shoham, and R. Steinberg. 2006. Introduction to combinatorial auctions. In *Combinatorial Auctions*, edited by P. Crampton, T. Shoham, and R. Steinberg. Cambridge, MA: The MIT Press.

[21] Cross, R. 1982. The Duhem–Quine thesis, Lakatos and the appraisal of theories in macroeconomics. *The Economic Journal* 92: 320–340.

[22] David, H.A. 1981. *Order Statistics*. New York: John Wiley & Sons.

[23] Donald, S.G., H.J. Paarsch, and J. Robert. 2006. An empirical model of the multi-unit, sequential, clock auction. *Journal of Applied Econometrics* 21: 1221–1247.

[24] Dudewicz, E.J. and S.N. Mishra. 1988. *Modern Mathematical Statistics*. New York: John Wiley & Sons.

[25] Efron, B. 1982. *The Jackknife, the Bootstrap, and Other Resampling Plans*. Philadelphia: SIAM.

[26] Elyakime, B., J-J. Laffont, P. Loisel, and Q.H. Vuong. 1994. First price sealed bid auction with secret reservation price. *Annales d'Economie et Statistique* 34: 115–141.

[27] Feyerabend, P. 1993. *Against Method,* third edition. London: Verso.

[28] Feyerabend, P. 1978. *Science in a Free Society*. London: NLB.

[29] Fisher, F.M. 1989. Games economists play: a noncooperative view. *RAND Journal of Economics* 20: 113–124.

[30] French, K.R. and R.E. McCormick. 1984. Sealed bids, sunk costs, and the process of competition. *Journal of Business* 57: 417–442.

[31] Friedman, L. 1956. A competitive bidding strategy. *Operations Research* 4: 104–112.

[32] Garderen, K.J. van and C. Shah. 2002. The exact interpretation of dummy variables in semilogarithmic equations. *Econometrics Journal* 5: 149–159.

[33] General business rules. 1994. Bergen: Norges Sildesalgslag.

[34] Giles, D.E.A. 1982. The interpretation of dummy variables in semilogarithmic equations. *Economics Letters* 10: 77–79.

[35] Goodall, C.R. 1993. Computation using the QR decomposition. In *Handbook of Statistics 9: Computational Statistics*, edited by C.R. Rao. Amsterdam: North–Holland.

[36] Goldberger, A.S. 1968. The interpretation and estimation of Cobb–Douglas functions. *Econometrica* 36: 464–472.

[37] Gollier, C. 2001. *The Economics of Risk and Time*. Cambridge, MA: The MIT Press.

[38] Greene, W.H. 2003. *Econometric Analysis*, fifth edition. Upper Saddle River, NJ: Prentice Hall.

[39] Guerre, E., I. Perrigne, and Q.H. Vuong. 2000. Optimal nonparametric estimation of first-price auctions. *Econometrica* 68: 525–574.

[40] Gut, A. 1995. *An Intermediate Course in Probability*. New York: Springer-Verlag.

[41] Halvorsen, R. and R. Palmquist. 1980. The interpretation of dummy variables in semilogarithmic equations. *American Economic Review* 70: 474–475.

[42] Hansen, R.G. 1985. Empirical testing of auction theory. *American Economic Review, Papers and Proceedings* 75: 156–159.

[43] Härdle, W. 1991. *Smoothing Techniques with Implementations in S*. New York: Springer-Verlag.

[44] Harrison, G. 1989. Theory and misbehavior of first-price auctions. *American Economic Review* 79: 749–762.

[45] Harsanyi, J.C. 1967/1968. Games of incomplete information played by 'Bayesian' players, Parts I–III. *Management Science* 14: 159–182, 320–334, 486–502.

[46] Harstad, R.M., J.H. Kagel, and D. Levin. 1990. Equilibrium bid functions for auctions with an uncertain number of bidders. *Economic Letters* 33: 35–40.

[47] Hendricks, K. and H.J. Paarsch. 1995. A survey of recent empirical work concerning auctions. *Canadian Journal of Economics* 28: 403–426.

[48] Hendricks, K. and R.H. Porter. 1988. An empirical study of an auction with asymmetric information. *American Economic Review* 78: 865–883.

[49] Hendricks, K. and R.H. Porter. 1993. Bidding behaviour in OCS drainage auctions. Theory and evidence. *European Economic Review* 37: 320–328.

[50] Hochberg, Y. and A.C. Tamhane. 1987. *Multiple Comparison Procedures.* New York: John Wiley & Sons.

[51] Holt, C.A. 1980. Competitive bidding for contracts under alternative auction procedures. *Journal of Political Economy* 88: 433–445.

[52] Huang, C. and R.H. Litzenberger. 1988. *Foundations for Financial Economics.* New York: North–Holland.

[53] Judd, K.L. 1998. *Numerical Methods in Economics.* Cambridge, MA: The MIT Press.

[54] Kennedy, P.E. 1981. Estimation with correctly interpreted dummy variables in semilogarithmic equations. *American Economic Review* 71: 801.

[55] Kennedy, P. 2003. *A Guide to Econometrics*, fifth edition. Oxford: Blackwell Publishing.

[56] Klemperer, P. 2004. *Auctions: Theory and Practice*. Princeton, NJ: Princeton University Press.

[57] Koenker, R. 2005. *Quantile Regression*. Cambridge, UK: Cambridge University Press.

[58] Koenker, R. and G. Bassett. 1978. Regression quantiles. *Econometrica* 46: 33–50.

[59] Krishna, V. 2002. *Auction Theory*. San Diego: Academic Press.

[60] Laffont, J.-J. 1997. Game theory and empirical economics: The case of auction data. *European Economic Review* 41: 1–35.

[61] Laffont, J.-J. and E.S. Maskin. 1980. Optimal reservation price in the Vickrey auction. *Economic Letters* 6: 309–313.

[62] Laffont, J.-J., H. Ossard, and Q.H. Vuong. 1995. Econometrics of first price auctions. *Econometrica* 63: 953–980.

[63] Leamer, E.E. 1983. Let's take the con out of econometrics. *American Economic Review* 73: 31–43.

[64] Levin, D. and J.L. Smith. 1996. Optimal reservation prices in auctions. *The Economic Journal* 106: 1271–1283.

[65] Lusht, K. 1994. Order and price in a sequential auction. *Journal of Real Estate Finance and Economics* 8: 259–266.

[66] Maskin, E.S. and J.G. Riley. 1984. Optimal auctions with risk averse buyers. *Econometrica* 52: 1473–1518.

[67] Matthews, S.A. 1983. Selling to risk averse buyers with unobservable tastes. *Journal of Economic Theory* 30: 370–400.

[68] McAfee, R.P. and J. McMillan. 1987. Auctions and bidding. *Journal of Economic Literature* 25: 699–738.

[69] McAfee, R.P. and D. Vincent. 1993. The declining price anomaly. *Journal of Economic Theory* 60: 191–212.

[70] McCloskey, D.N. 1994. *Knowledge and Persuasion in Economics*. Cambridge: Cambridge University Press.

[71] McCloskey, D.N. 1998. *The Rhetoric of Economics,* second edition. Madison, WI: University of Wisconsin Press.

[72] McMillan, J. 1994. Selling spectrum rights. *Journal of Economic Perspectives* 8: 145–162.

[73] Milgrom, P.R. 1981. Good news and bad news: Representation theorems and applications. *Bell Journal of Economics* 12: 380–391.

[74] Milgrom, P.R. 1987. Auction theory. In *Advances in Economic Theory,* edited by T.F. Bewley. Cambridge: Cambridge University Press.

[75] Milgrom, P.R. 2000. Putting auctions theory to work: The simultaneous ascending auction. *Journal of Political Economy* 108: 245–272.

[76] Milgrom, P.R. 2004. *Putting Auction Theory to Work*. New York: Cambridge University Press.

[77] Milgrom, P.R. and R.J. Weber. 1982. A theory of auctions and competitive bidding. *Econometrica* 50: 1089–1122.

[78] Myers, J.L and A.D. Well. 2003. *Research Design and Statistical Analysis*, second edition. London: Lawrence Erlbaum.

[79] Myerson, R.B. 1981. Optimal auction design. *Mathematics of Operations Research* 6: 58–73.

[80] Naegelen, F. 1988. *Les Mechanismes d'Encheres*. Paris: Economica.

[81] Neher, P.A. 1990. *Natural Resource Economics. Conservation and Exploitation.* New York: Cambridge University Press.

[82] Ortega-Reichert, A. 1968. *Models for Competitive Bidding under Uncertainty.* Ph.D. thesis, Department of Operations Research, Technical report no. 8, Stanford University.

[83] Paarsch, H.J. 1989. Empirical models of auctions within the independent private values paradigm and an application to British Columbian timber sales. Discussion paper 89–14. Department of Economics, University of British Columbia.

[84] Paarsch, H.J. 1992. Deciding between the common and private values paradigms in empirical models of auctions. *Journal of Econometrics* 51: 191–215.

[85] Paarsch, H.J. 1997. Deriving an estimate of the optimal reserve price: An application to British Columbia timber sales. *Journal of Econometrics* 78: 333–357.

[86] Paarsch, H.J. and H. Hong. 2006. *An Introduction to the Structural Econometrics of Auction Data.* Cambridge, MA: The MIT Press.

[87] Pesendorfer, M. 2000. A study of collusion in first-price auctions. *Review of Economic Studies* 67: 381–411.

[88] Phlips, L. 1996. On the detection of collusion and predation. *European Economic Review* 40: 495–510.

[89] Pinkse, J. and G. Tan. 2005. The affiliation effect in first-price auctions. *Econometrica* 73: 263–277.

[90] Porter, R.H. 2005. Detecting collusion. *Review of Industrial Organization* 26: 147–167.

[91] Porter, R.H. and J.D. Zona. 1993. Detection of bid rigging in procurement auctions. *Journal of Political Economy* 101: 518–538.

[92] Portnoy, S. and R. Koenker. 1997. The Gaussian Hare and the Laplacian Tortoise: Computability of squared-error versus absolute error estimators, with discussion. *Statistical Science* 12: 279–300.

[93] Rasmusen, E. 2001. *Games and Information*, third edition. Oxford: Blackwell Publishing.

[94] Reiss, P.C. and F.A. Wolak. 2007. Structural econometric modeling: Rationales and examples from industrial organization. In *Handbook of Econometrics*, volume 6A, edited by J.J. Heckman and E. Leamer. Amsterdam: North–Holland.

[95] Riley, J.G. and W.F. Samuelson. 1981. Optimal auctions. *American Economic Review* 72: 381–392.

[96] Searle, S.R., F.M. Speed, and G.A. Milliken. 1980. Population marginal means in the linear model: an alternative to least squares means. *American Statistician* 34: 216–221.

[97] Sheather, S.J. and M.C. Jones. 1991. A reliable data-based bandwidth selection method for kernel density estimation. *Journal of Royal Statistical Society* 53: 683–690.

[98] Shor, M. 2008. Collusion in auctions. *Dictionary of Game Theory Terms, Game Theory .net*, URL: <`http:// www.gametheory.net / dictionary / Auctions / AuctionCollusion.html`> [Web accessed: April 4, 2008].

[99] Silverman, B.W. 1986. *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.

[100] Simonoff, J.S. 1996. *Smoothing Methods in Statistics*. New York: Springer-Verlag.

[101] Smith, V.H. 1982. Microeconomic systems as an experimental science. *American Economic Review* 72: 923–955.

[102] Smith, V.H. 1994. Economics in the laboratory. *Journal of Economic Perspectives* 8: 113–131.

[103] Suits, D.B. 1984. Dummy variables: Mechanics v. interpretation. *Review of Economics and Statistics* 66: 177–180.

[104] Sutton, J. 1990. Explaining everything, explaining nothing? Game theoretic models in industrial organisation. *European Economic Review* 34: 505–512.

[105] Sutton, J. 1993. Much ado about auctions. Some introductory remarks. *European Economic Review* 37: 317–319.

[106] Thaler, R.H. 1994. *The Winner's Curse: Paradoxes and Anomalies of Economic Life*. Princeton, NJ: Princeton University Press.

[107] Vickrey, W. 1961. Counterspeculation, auctions and competitive sealed tenders. *Journal of Finance* 16: 8–37.

[108] Weber, R.J. 1983. Multiple-object auctions. In *Auctions, Bidding, and Contracting: Uses and Theory*, edited by R. Engelbrecht-Wiggans, M. Schubik, and R.M. Stark. New York: New York University Press.

[109] Wilson, R.B. 1969. Competitive bidding with disparate information. *Management Science* 15: 446–448.

[110] Wilson, R.B. 1977. A bidding model of perfect competition. *Review of Economic Studies* 4: 511–518.

[111] Wilson, R.B. 1992. Strategic analysis of auctions. In *Handbook of Game Theory with Economic Applications*, volume 1, edited by R.J. Aumann and S. Hart. Amsterdam: North–Holland.

Changes in this edition

This edition, to be published online, was produced in August 2010. Compared to the printed Ph.D. thesis as of February 2010, a few grammatical mistakes and typographical errors were corrected. Table 5.8 in the printed edition, was produced by using a sample that was slightly incorrect since it included some observations from the 2004 season. The table has been updated by using the correct sample.