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**Taxing bads by taxing goods:**

**Essays on environmental protection under optimal taxation**

by

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## Preface

This inquiry is on simple principles presumed to be helpful to a benevolent planner interested in the welfare of citizens, including protecting the environment. I have thus pursued a quite old-fashioned topic in public finance, and only one of the many I find narrow but important, teasing and fun. The insights from the models of normative welfare economics need, of course, to be combined with insights from positive models of institutions, governance and of policy making, as well as with some willingness to look at the real world. I am probably unwise when spreading my personal interests and engagement over these broad areas, but it is a genuine interest.

I am privileged to take joy in what I do, and this research has not been an exception. Nevertheless, it has been going on for a long time, and my first note of warm thanks are for my wife Barbro. She has provided encouragement and support with an unimaginable patience and strength. As two miracles blessed us, Hanna Sofie and Jonas Benedict, it became more difficult to pursue this research endeavour - more tempting to deploy Gunnar in other activities. The fact that we stayed on course is testimony that I have a fabulous wife.

I also thank Agnar Sandmo, my principal advisor, who has given me strong professional support on my long and probably tedious trajectory. Agnar is firmly at the center of the topic pursued in my dissertation, giving me the privilege of having an advisor who knows what he is talking about, and the headache that he is always two steps ahead of me. My other advisors, Vidar Christiansen and Geir Asheim have also given my helpful advise - and all three have been very generous in accommodating my pace and schedule.

An ever inspiring friend is also my boss at the World Bank, Shanta Devarajan. Always willing to pursue a topic for intellectual interest only, he has been consistently supportive in terms of concrete advise, encouragement, and flexibility.

The World Bank's research department provides a great work environment for people with a research interest. Friends and colleagues at the World Bank, at Norwegian School of Economics and Business Administration, and elsewhere have been helpful and supportive, and I refrain from mentioning any for fear of forgetting some. Nevertheless, family and friends who have chipped in - say lifted a child out of my arms and chased me back to my "office" - shall know that their contribution is acknowledged.

Georgetown, Washington, D.C., August 2000.

## **Essay I**

### **Environmental protection and optimal taxation: Theory and implementation**

#### Abstract:

In practice, environmental protection is mainly about emission standards and their enforcement. In optimal tax theory, environmental protection is about a pollution tax and how it relates to a general system of taxation. We connect these worlds: helping theoreticians understand the role and strength of standards; and helping practitioners see their weaknesses - and how to compensate for the weaknesses with matching instruments. New results on how environmental protection fits in a framework of costly but optimal taxation allow us to emphasize principles that are simple and fairly intuitive, facilitating implementation and delegation.

## I. Introduction

As I engaged in discussions on how to reduce air pollution in Mexico City with technical experts and representatives of the government of Mexico City, confidence in a text-book approach was shaken by several observations. One of these observations was that the government examined in detail how classes of vehicles and fuels could be made less polluting – or “cleaner”. The textbook recommendation of an arms-length approach - an emission tax - was not only unpractical, but also not providing much guidance on principles.

It struck me then that economists did not have a plausible model for why emission standards and mandated technologies play a dominant role in practice. I would later become convinced that this blind spot in important ways have hampered our impact in practice, even in conveying well-founded insights. There were a number of guidelines I searched for, but did not find. Two important questions that I asked are explored in these essays:

- Should one stimulate emission reductions in the same way from firms and households, rich and poor?
- How should one combine instruments that make activities cleaner, with instruments that shift the economy towards less polluting activities?

Over the following years, I tried to contribute to practical advice under simplifying assumptions, while at the same time trying to develop principles under more general assumptions. The following essay reports on the lessons from this journey. As in the chronology of my own work, I start with applied analysis made with restrictive, simplifying assumptions. I then explore consequences of making less restrictive assumptions to see whether there is broader support, more general principles. In the subsequent essays, we go from quite general to more applied analysis. Essays II and III present the general theoretical analysis, while essays IV and V are applications to the problem of air pollution control in Mexico City. Thus, some readers may choose to skip essays II and III, on generalizations of the assumptions under which the analysis can be justified. The cost effectiveness analysis for Mexico City (essays IV and V) is performed under conditions of "no distortionary taxation" and "a representative consumer", while

essays II and III analyze under broader assumptions whether the same or similar analysis would apply.

One theme in this work will be that simple concepts from partial equilibrium analysis under first best have close parallels in general equilibrium with costly funding and redistribution, if one can assume that the tax structure is optimal. Another is that environmental protection is more like a problem of public goods provision than has been previously acknowledged. We see this as we introduce pollution abatement in the traditional public finance model.

The reader will see that there were some simple guideposts yet to be erected – principles to be highlighted - even though the literature in public finance and in environmental economics was quite dense with sophisticated principles.<sup>1</sup>

## II. Our platform

We shall highlight a few key building blocks in what we shall call the public finance approach to environmental protection. The first is what Arthur Cecil Pigou explained as a difference between private and social net product; Pigou used the lighthouse as one of his examples. We shall use a very specific and stylized example giving rise to such a difference; what professor Paul Samuelson (1954) called *collective consumption goods*. Later, the accepted term came to be *pure public goods* “which all enjoy in common in the sense that each individual's consumption of such a good leads to no subtraction from any other individual's consumption of that good” (Samuelson, 1954).<sup>2</sup>

We shall use air quality as our main example of a pure public good. We can all understand how the air quality is there for everyone, though some may care more than others, and some may be exposed more than others. This example serves well also to

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<sup>1</sup> I will continue in the active form, but switching here from *I* to *we*. *We* reflects in part that I hope I reason with the reader as we go along, in part recognition of what I have learned from those who have reasoned with me, worked with me.

<sup>2</sup> For the lighthouse example (later used by Ronald Coase), Pigou credits Sidgwick. Pigou also dwells on discrepancies in returns due to tenance relationships, showing awareness of: the role property rights; the challenges in structuring property rights well; and the role of government in this regard. Samuelson lets us know that he - through discussions with Richard Musgrave - learns that the principle he lays out is not new, but known. Victor Norman familiarized Norwegian students of economics with pure public goods by explaining "King services" as "superpublic goods", each person enjoying the King's services more the more others enjoy them.

demonstrate how a good is public, or "nonrivalrous", in one end - where we breathe fresh air and enjoy seeing the mountains - but "rivalrous" in the other, providing end: As one person or firm emits pollution, this subtraction from the public good must be compensated by another's reduction in emissions - if everyone's enjoyment is not to be reduced.

In the public finance approach to pollution control, environmental quality has typically been presented as a pure public good. A pollution indicator,  $e$ , may appear as an argument in a consumer  $h$ 's utility function (i.e. in her preferences),

$$(1) \quad u^h = u(x^h, e),$$

where  $x^h$  is a vector of quantities of private goods consumed by  $h$ , thus bearing her identity as a superscript, while the pollution level (or the air quality level) is the same for everyone.<sup>3</sup> This setup is used in Sandmo's seminal (1975) article "Optimal taxation in the presence of externalities", and it is used invariably in subsequent treatments, such as Bovenberg and van der Ploeg (1994), and Cremer, Gahvari and Ladoux (1998). Among others, Sandmo (1972) have analyzed collective factors of production, the analogy to pure public goods represented by a nonrivalrous input. Variations offered here are to analyze collective factors of production in a setting of costly revenue generation, and to allow environmental quality to be such a collective factor of production. This formulation should not be confounded with the typical depiction of the environment as a cost-reducing recipient of waste, which is rivalrous. We rather think of examples such as the tourism industry needing clean air, the brewer needing clean water, pharmacists needing a gene pool and farmers needing good weather.

A second key building block is that those reducing the environmental quality cannot readily be charged for their disservices, giving rise to *externalities* (equivalently: those who contribute to environmental quality cannot readily be compensated).<sup>4</sup> Since pure public goods are enjoyed by everybody to the exclusion of no-one, an exclusion mechanism does not in the outset offer itself to mobilize funds – or authority - to modify

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<sup>3</sup> See, for instance Sandmo (1975). The essential element is not that the pollution level is the same to everyone - a model characterized by Meade (1952) as atmospheric pollution. It is essential that the pollution level experienced by one person is influenced not only by herself but also (or only) by others. Often the approximation is used that the individual polluter views the pollution level as independent of her own emissions - a good approximation when the number of polluters is large.

the actions of polluters. One perspective on government is a club that can weigh costs and benefits in areas such as pollution control on behalf of members. Thus, government can take on the role of charging for the disservices - or by other means to rectify the incentive problem.<sup>5</sup>

An important representation in the public finance literature of how the environmental good is provided is to describe pollution as proportional to output in a polluting industry  $j$  or to a polluting consumption activity (Sandmo, 1975):

$$(2) \quad e = f_j \sum_h x_j^h.$$

The variation offered in the following shall be that polluters, or those who deliver to polluting activities (say, gasoline refiners and car makers) may devote resources to reduce those proportionality factors  $e = \sum_h f_j (a_j, b_j^h) x_j^h$ , where  $a_j$  and  $b_j^h$  are resources devoted to abatement of emissions for good  $j$  by producers and by consumer  $h$ , respectively. We shall see that one unexpected reward for this generalization is new insights from parallels to the traditional problem of public goods provision.

The third key building block is government, represented by a benevolent planner whose objectives can be characterized by an individualist *welfare function* in the Bergson-Samuelson tradition:

$$(3) \quad w = w(v^1, \dots, v^h).$$

In (3),  $v^h = v^h(q, I^h, e)$  is the indirect utility function corresponding to (1) for individuals  $1, \dots, h$ . The individuals are assumed to take as given a vector of consumer prices  $q$ , lump sum private incomes  $I^h$  (which may be zero), government revenue and the quality of the environment. (3) embodies two statements about the objectives of the planner: he builds on individual preferences, and can compare utility differences (utility is cardinal).

In the context of environmental protection, it may be important to highlight that (3) is an anthropocentric framework. On the one hand, there is no environmental obligation - or moral code with regard to the environment - inserted in the model from the outside. On the other hand, the framework embodies individual preferences, not only

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<sup>4</sup> The qualification that the "disservices to others" can not be charged for is due to Pigou (1932).

<sup>5</sup> Such a club would suffer under free-riding problems and would not mobilize much willingness to pay for the environment if membership were voluntary. This free-riding problem - being solved by compulsory



people's hunger to consume. It includes and aggregates what individuals find to be their obligations and interests.

The fourth key building block is a government *revenue requirement* (or a set of public expenditure opportunities) and a set of policy instruments. The policy instruments may be insufficient to fund government programs without resorting to distortionary taxation. In our case, a contribution will be to show the role of emission taxes, and what can be done by surrogate instruments such as emission standards when an emission tax is not available.

Apart from these building blocks for our models, important guideposts have been erected by prominent travelers. In the field of taxation, Frank Ramsey (1927) and Paul Samuelson (1951) laid out how linear commodity taxes should be used to raise revenue in a way minimizing welfare costs. Pigou – who instigated Ramsey's analysis - conjectured that government expenditures should be lower in the case when revenue generation is costly than they would be otherwise (and made similar observations on distributional grounds, as Sandmo, 1999, points out). Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), King (1986) and others have helped analyze and qualify Pigou's conjecture, providing insights and delineating exceptions. We shall show that this question is closely related to one raised recently in the debate on "double dividends" (Bovenberg and de Mooij, 1994). Diamond and Mirrlees (1971) and Dasgupta and Stiglitz (1972) provided the conditions under which efficiency in aggregate production is desirable even when the government must resort to distortionary revenue generation – findings of great relevance for the current study.

In the areas more closely associated with environmental protection and externalities, Ramsey (1927) noted that the task of revenue generation is distinct from the need to charge for damage for corrective purposes. Answering the challenge, Sandmo (1975) was first to deduce optimal principles under the dual objectives of correcting for externalities and mobilizing revenue. He concluded: "even in a world of distortionary taxation... there is scope for taxing externality generating commodities according to the Pigovian principle." The optimal tax structure is characterized by an "additivity

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membership - is equally serious whether citizens have the right to a clean environment - or polluters have the right to pollute. For an elaboration on this point, see Eskeland and Devarajan (1996).

property”, where the revenue motivated terms apply to all commodities in a well-known fashion, and the term motivated by the need to correct the externality applies only to the tax on the polluting good. Among his findings was that the need for distortionary revenue generation in itself does not introduce reference to complements or substitutes to the polluting goods in the corrective part of his tax formula.<sup>6</sup>

We are now ready to introduce our applied analysis into this general framework, first by making very restrictive assumptions. In section IV, we visit the more general theoretical model. This gives us a chance to check whether the applied analysis is given support under more general assumptions, and also to extend the theoretical framework.

### III. A presumptive Pigovian tax: to balance "cleaner" with "less"

*Emission standards should be matched with commodity taxes*

Drawing from Essay IV (which provides more detail), let us think about the problem in terms of a representative consumer and a government able to make lump-sum transfers, so there is no need to resort to distortionary revenue generation. Thus intervention to facilitate provision of the public good - environmental quality - is the only rationale for government intervention. Let us further think of environmental quality, or the absence of pollution, as a pure public good in the Samuelsonian sense of nonrivalry in consumption. Finally, let us assume that individual emissions,  $e$ , from a polluting activity - say driving - is determined by a technology parameter called abatement,  $a$  (say - the emission control equipment in the vehicle), and the scale of the activity,  $x$ , measured for instance by vehicle miles traveled or by gasoline consumption.

We simplify further by assuming only two private goods, so we can let  $c=c(x,a)$  represent the cost of the quantity  $x$  and abatement  $a$  in terms of the other private good. We let  $e=e(x,a)$  take the place of (2); a generalization since we do not restrict attention to costs and emissions that are proportional to consumption ( $c=c(a)x$ ;  $e=e(a)x$ ). A cost effective pollution control program now can be found by maximizing  $u(x,-c(x,a))$  subject to  $e(x,a)=\bar{e}$ . The first order conditions for optimum reduce to:

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<sup>6</sup> As Sandmo carefully points out ("our conclusion is no more than a statement about the terms in the formula"), the apparent additivity in the formula implies no independence between the environmental tax and the other taxes. This is not only because the shadow price of public revenue is a part of the corrective tax term, but also because all parameters in the formulas are functions of the tax structure.

$$(4) \quad \frac{u_x/u_y - c_x}{e_x} = -\frac{c_a}{e_a}.$$

The right hand side is the marginal cost of emission reductions through abatement.

Relative to a unit of the other good a cost effective program requires a wedge between the marginal utility and the marginal cost of  $x$  which – per unit of marginal emissions – is equal to  $-c_a/e_a$ .

As is readily known – and easily checked - an emission tax equal to the Samuelsonian sum over consumers of the marginal rates of substitution between the public good and the numeraire good can implement the optimal solution. Such a tax satisfies (4), optimally combining inducement to abatement and reduction of demand for the polluting good. Furthermore, an emission tax at any other rate will implement cost effective environmental protection (4), meaning that the pollution reductions that are attained come at minimal costs, even if the reductions are not optimal.

An instrument that is equivalent to the emission tax in models with lump sum transfers is an exogenous quota for emissions. With multiple polluters, individual quotas will also have this property, if their allocation is exogenous and they are tradable. In contrast, individual emission quotas will typically result in some flaw in the incentive framework if their allocation is not exogenous. The allocation mechanism we shall focus on here is one in which the emission quota is given in association with some output choice (or input choice). One design frequently observed in practice is an *emission standard*, as when cars are allowed a maximum of 9 grams of carbon monoxide (CO) per mile driven (Harrington, 1997). Another one, with similar implications, is an *abatement standard*, as when cars are required to come with a catalytic converter. Quite intuitively, now, pollution reductions stimulated by an abatement standard (or an emission standard) alone will not achieve cost effective pollution control, since the standard awards emission quotas conditional on expanding output. For cars, an emission standard alone may or may not increase the marginal cost of driving, but will at any rate not discourage driving in the way commanded by a cost effective program (4). Polluters could be made better off if allowed to do less abatement, compensating by reducing the scale of the polluting activity so as to leave total emissions unchanged. Similarly, an output tax alone will compress activity too much, ignoring low-cost technological abatement opportunities.

Public finance models have often made the simplifying assumption that emissions are determined by aggregate output alone, thus abstracting from the option of polluter abatement, making each unit of output less polluting.<sup>7</sup> Another consequence of such a modeling assumption is to make redundant the distinction between a corrective tax levied on emissions themselves and a corrective tax levied on the output of the polluting activity. In our context this distinction is important, and we use the term *presumptive Pigovian tax* when the corrective instrument is levied not on emissions but on an input or an output (say gasoline) in presumption of emissions.<sup>8</sup>

Observing that emission regulations typically apply to emission factors (grams emitted per mile driven, per ton of paper produced), the proposition that these empirically observed instruments should be accompanied by presumptive Pigovian taxes was made in "A presumptive Pigovian tax: Complementing regulation to mimic an emissions fee" (Essay IV). This is an alternative way of implementing the condition stated in (4), and is then given a practical illustration. The analysis of pollution control options for vehicular emissions in Mexico City allows us to focus on the dichotomy between "cleaner cars" and "fewer trips". The principle is spelled out in terms of a simple rule for cost effectiveness, to separate the message from the discussion of environmental benefits. The rule for how an optimal "matching gasoline tax" depends on the standard for abatement,  $\bar{a}$ , or the emission standard (equivalent, given our assumptions) is

$$(5) \quad \frac{t_x}{e_x(x, a)} = -\frac{c_a}{e_a}(x, \bar{a}).$$

Thus, when the two instruments "match" each other to implement pollution reductions cost effectively, the corrective tax on gasoline, per gram of emission, is equal to the marginal cost of emission reductions through abatement. If we include another polluting good, a corresponding formula applies for that good, and in addition it is required that the costs of emission reductions are the same across polluting goods.

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<sup>7</sup> Examples are Sandmo (1975), Bovenberg and van der Ploeg (1994), Bovenberg and de Mooij (1994), Cremer, Gahvari, Ladoux (1998). Diamond (1973), Sandmo (1976), Balcer (1980), and Wijkander (1985) focus on Pigovian taxes levied on goods that are imperfect as tax bases from the perspective of correcting the externality, using taxes and subsidies on associated goods as remedies for the imperfection.

<sup>8</sup> The term "presumptive" stands for a presumed relation to emissions, drawing on how the term is used in for instance presumptive income taxes. See Eskeland (1994), Eskeland and Devarajan (1996). Fullerton and Wolverton (1999) use the term in a similar way. Innes (1996), as well, proposes such an instrument to complement regulation in the context of polluting vehicles and fuels.

The tax, quoted by the left hand side of (5) per unit of emissions, is translated to a tax rate per unit of the polluting good by multiplying both sides of (5) by the emission factor,  $e_x(x, a)$ . The resulting formula - a tax proportional to the polluting good's emission factor - is equivalent to Sandmo's (1975) formula in the case when the pollution reductions attained are optimal and lump sum transfers are available.

*A positive theory of emission standards*

It is puzzling to us that the principle of a presumptive tax on outputs to complement emission standards (5) has not – to our knowledge - been highlighted in the rich literature on environmental economics. Reasonable economists probably find the proposition unsurprising, but we shall dwell a little on why the correspondence in a cost effective program between emission standards and output reductions has not been highlighted.

It is possible to reason that if monitoring costs make emission taxes unworkable, then emission standards also cannot work. It is harder, however, to argue that a regime of emission standards cannot be complemented by commodity taxes in presumptions of emission (the emissions that are presumed to remain after abatement, that is). We shall argue that there are some practically important observations that economists failed to make which allowed them - or us - to miss the point that emission standards should be accompanied by output taxation.<sup>9</sup> These omissions were related to a lack of adequate positive models for emission standards - we did not understand sufficiently why the standards were out there in the first place.<sup>10</sup>

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<sup>9</sup> For a thorough review of the literature on environmental economics, see Cropper and Oates, 1992. In public finance, see Atkinson and Stiglitz, 1980. Baumol and Oates (1988) in the chapter "Efficiency without Optimality: the charges and standards approach" develop cost effectiveness criteria. They note that standards are "somewhat arbitrary", omitting to mention, let alone model, that standards award quotas proportional to input or output. Eskeland and Jimenez (1992), in a review of instruments, also failed to make this observation. Recently, Innes (1996) recommends combinations of standards and fuel taxes, and Fullerton and Wolverton (1999) have adopted the term presumptive Pigovian taxes to highlight combinations of taxes on gasoline and instruments applied to emission factors or vehicle characteristics.

<sup>10</sup> Buchanan and Tullock (1975) provides a plausible positive model for regulation (in effect, for standards), and later authors have in the same vein seen standards as a way of distributing property rights to the environment. However, these models are rendered powerless in the context of more general policy design in which instruments include compensating transfers (In the Mexico context, such a broader design context appeared relevant: unions accepted gasoline price increases compensated by reductions in general sales taxes). Moreover, as a positive theory, these models fail to explain why the pollution quotas, once distributed, rarely are considered tradeable. Another important debate came about on the relative merits of

The first omitted observation is that the emission quotas allocated by an emission standard empirically take the shape of *conditional* property rights. Theoretically, such a conditionality – a quota if you drive a mile – can result from a particular structure for the costs of monitoring, enforcing and delivering reductions emissions. However, while economists had dealt with costly monitoring and enforcement (examples are Sandmo, 1976; Schmutzler and Goulder, 1997; Magat and Viscusi, 1990), they had not dealt with the possibility that plausible cost structures for monitoring would yield policy instruments applied to intermediate measures such as emission factors. We elaborate on this in Eskeland (1994) and in Eskeland and Devarajan (1996), but our simple contention is that quota allocation takes the shape of a regulation applied to emission factors because an emission factor is monitored at a lower cost than is an individual's cumulative emissions. An important part of this is – in the case of cars – that it is easier to associate emissions with a car than with a person. Cumulative emissions can then be addressed by the policy indirectly, as the policy maker imagines – or models - how the scale of the polluting activity is determined. For cars, the emission standard is typically a quota for emissions which expands for each mile driven (Harrington 1997), thus representing an implicit subsidy to driving. This does not imply that emission reductions become elusive, but it means that the emission reductions sought with emission standards alone could be attained at a lower cost.

The second, related observation is also both a theoretical one and a practical one: Theoretically, the proposition that allowing trade in quotas can lead only to efficiency gains no longer holds when quotas are allocated conditionally on behavior. The reason is that the allowance of trades will influence the behavior determining the allocation of quotas.<sup>11</sup> Practically speaking, if driving an old Buick in California gives you an emission quota of 9 grams of CO per mile driven (Harrington, 1997), delinking the quota

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intervening with instruments of “price” or “quantity” (See Weitzman, 1974; Roberts and Spence, 1976; Baumol and Oates, 1988), where the relevant arguments concerned aggregate uncertainty. This perspective, however, still yielded the verdict that the quantity instrument should be tradeable, thus retaining the cost effectiveness properties of the analogous price instrument.

<sup>11</sup> An illustration of this is as follows: if restaurant seats are allocated on a first come first serve basis, a norm that a given position in a line is nontradeable may appear to obstruct efficiency enhancing trades. “Removing” the norm could, however, make people line up with no intention to be seated. “Races for property rights”, “tragedies of the commons” and “overfishing” all can be seen as consequences of conditional property rights.

from the car and/or from driving would require an alternative institutional machinery; fundamentally changing the nature of the emission quota.<sup>12</sup>

Economists have - and often rightly - been harsh in their criticism of emission standards and regulatory approaches to pollution control (Baumol and Oates, 1988; Tietenberg, 1992). Perhaps because of the attention demanded to make those points, the statement on how standards should be accompanied by output taxes (5) has to this author's present knowledge not been made before the appearance of the 1994 article.

*Demand management in pollution control: Is it important?*

Pollution control agencies typically have regulated polluting activities, and typically to make them less polluting per unit of output or input.<sup>13</sup> In the public finance literature, in contrast, shifting the balance of the economy towards less polluting goods and sectors has been emphasized. Two reasons come to mind for this latter emphasis: First, shifting the balance between activities fits easily in a traditional modeling framework. Second, economists have had an important message, given the overly restricted focus on abatement - or ways to make each activity less polluting - in policy-making bodies. In terms of applied studies, several authors have analyzed the responsiveness of an economy to environmental policies - either simply considering effects on measured income or to include effects on pollution as well.<sup>14</sup> Most such studies - when they include pollution implications, concentrate on greenhouse gas emissions, for which it is fairly accurate to model emissions as strictly proportional to fuel consumption, i.e. without abatement options. As such, these studies benefited greatly from an earlier wave of applied studies in energy demand, fueled by the 1973 energy crisis (Pindyck, 1979; Fuss 1977).

Eskeland (1994) gives a detailed examination of control options and implications for emission factors, with a total of 28 measures being "admitted" to the control cost

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<sup>12</sup> Buyback programs can be seen as introducing an opportunity for a car owner to sell the remaining "emission quotas" represented by her car by scrapping it. It should be tried only for exceptional cars, of course (and typically is: see Alberini et al., 1995) since for the representative car the program would be reducing pollution only if driving up the price of cars - a job better done by tax instruments.

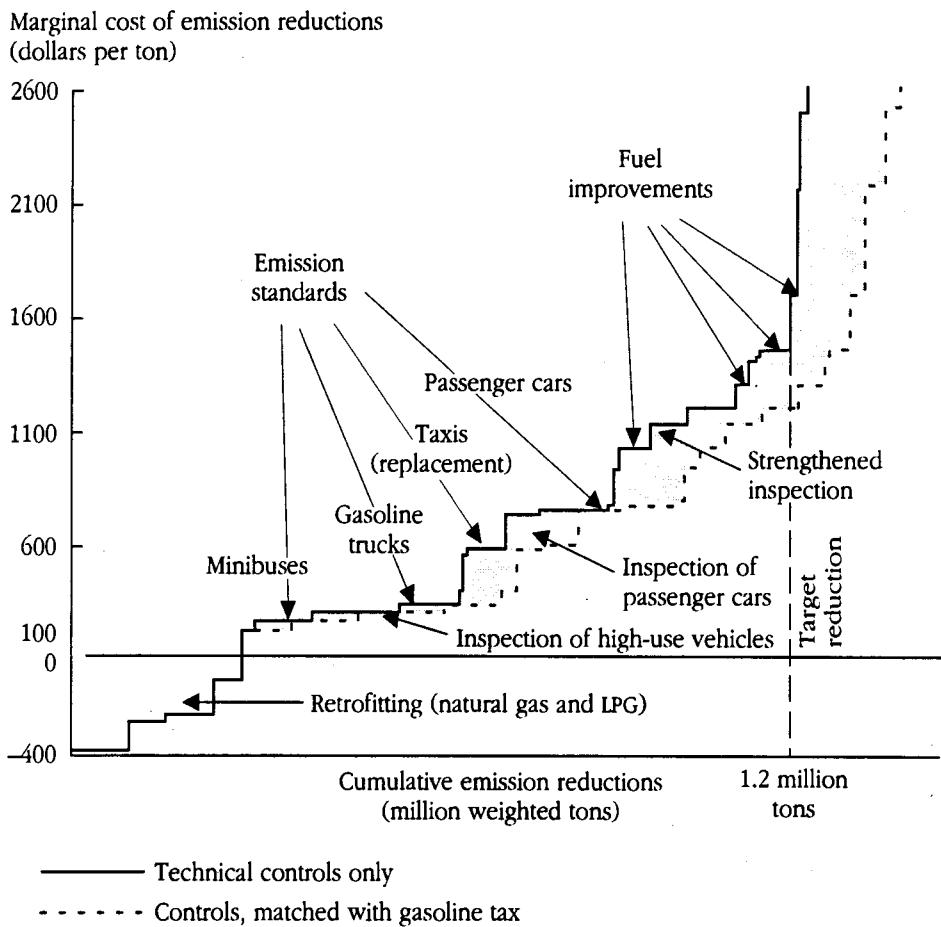
<sup>13</sup> Apart from the automobile examples (Harrington, 1997; Eskeland, 1994), see Magat and Viscusi (1990).

<sup>14</sup> Examples are Hazilla and Kopp (1990); Goulder et al. (1999); Konrad and Schroder (1991); Glomsrod, Johnsen and Vennemo (1992); Whalley and Wigle (1998); Jorgenson and Wilcoxon (1993); Eskeland, Jimenez and Liu (1998); Eskeland and Devarajan (1996); Alfsen (1992).

curve.<sup>15</sup> In other areas, the model is very simple: a representative consumer, no other taxes or tax reasons, and a general equilibrium framework with three goods: car travel, air quality, and other goods and services. Using the best available estimated demand function for gasoline (Berndt and Botero, 1985), the matching gasoline tax shifted the control cost curve down by a significant amount (figure 1).

Figure 1.

*Program to Reduce Air Pollution Emissions from Transport in Mexico City, with and without a Gasoline Tax*



<sup>15</sup> Several aspects additional to the “matching tax” result were novel: Multiple pollutants were weighed with a benefit-based metric; a cost-minimizing control cost curve was constructed. In a companion study, the proposition of a market based demand management was supplemented with a quantitative evaluation of an existing rationing scheme for driving (Eskeland and Feyzioglu, 1997b).



The conclusion was that a well-composed program of “cleaner cars” would cost 24 percent more if restricted not to include a gasoline tax in the tool chest. In terms of annual US dollars, the difference was \$111 million, or \$6 per citizen, much more per car. The proposed strategy, using (5), minimizes the welfare cost of emission reductions by viewing “cleaner cars” and “fewer trips” as competing providers of emission reductions.

In Essay V: "Is demand for polluting goods manageable? An econometric study of car ownership and use in Mexico", Eskeland and Feyzioglu (1997) make an effort to obtain more suitable estimates for the demand function. Using richer and more recent data, and techniques capable of addressing additional challenges, the estimated model resulted with a price elasticity for gasoline consumption of -1.3 to -1.1, as opposed to the original -.8 from Berndt and Botero (1985). With those results, the estimated additional cost of excluding the demand management instrument increased to 44 percent.

Our conclusion from this analysis is that demand management belongs in environmental protection not only as a matter of principle, but such as to make a significant difference quantitatively. It was interesting to have this demonstrated in a field such as automotive emissions, since it is important empirically in the world's pollution problems. Also, the field of automotive emissions combines technical controls that are quite powerful in terms of reducing emission factors with a pessimism amongst many about the manageability of demand. We made similarly encouraging findings on air pollutant emissions (Sulfur oxides, particles, others) when we estimated input demand functions from manufacturing industries in Chile and Indonesia (Eskeland, Jimenez and Liu, 1998). The estimated elasticities of different pollutant emissions with respect to the price of heavy fuels (combining differences in emission coefficients with own- and cross price elasticities) resulted in the range of -.4 to -1.3, so a forty percent price increase could reduce emissions by twenty to fifty percent.

We now turn to the more involved theoretical analysis, with the motivation to see whether the simple principles demonstrated above apply under more general assumptions, in particular regarding costly revenue generation.

#### IV. Provision of environmental quality when revenue generation is costly

An insight from the theory of the second best is that with one distortion given in the economy, it may be attractive to have others as well. Greenwald and Stiglitz (1986) forcefully demonstrated the implications when showing how *imperfections* of one policy instrument with regard to one market failure leaves it attractive to look across all instruments for compensating remedies.

In the light of that challenge, one may ask under what conditions the intuition is still correct, that the planner should use *one price* to induce emission reductions? Do the challenges of distortionary revenue generation and costly redistribution imply that the provision of the environmental good departs from simple efficiency principles? If it does, will it influence the cost effectiveness analysis and the control cost curve? Also, what does it take for us to categorize an environmental policy instrument as imperfect, in the sense that it should be combined with other instruments to protect the environment?

##### *Externalities and production efficiency*

Setting aside, for the moment, the question of how much pollution control (i.e. pollution) there should be, an important question is whether provision of pollution reductions should be efficient in the sense that the marginal rates of transformation between abatement and emission reductions are equalized.<sup>16</sup>

The question of whether the marginal cost of emission reductions shall be the same for different polluting activities (or ways to reduce emissions) is not asked in studies such as Sandmo (1975) and Cremer et al. (1998). These models include only one polluting good, and an aggregate demand reduction is the only way to reduce emissions. We introduce multiple polluting activities and resources devoted to pollution abatement (reducing emission coefficients). Furthermore, abatement can be done by producers to reduce emissions in production, or by producers and consumers and government to reduce emissions in consumption, so we can ask the question of equality in rates of transformation across many dividing lines.

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<sup>16</sup> The equalization of marginal costs of emission reductions through abatement is a concept of efficiency corresponding to what is called "cost effectiveness" in the environmental economics literature (see, for instance, Baumol and Oates, 1988; Tietenberg, 1992). The more cumbersome term (marginal rates of transformation...) is required when the comparison is across agents that may face different prices, as here.

The answer to the question of efficiency in provision of the environmental good is a quite affirmative yes (Essay II). The analysis is set under the following general assumptions: constant returns to scale; the environmental good is separable from other goods; within each activity consumers have uniform emission functions; linear taxes on inputs, outputs and emissions can be differentiated by commodity (or emission standards can be differentiated by commodity); different regimes can apply for the three sets: producers, government and the  $h$  consumers. We should notice that the assumption of constant returns to scale is more restrictive than in the previous section, since  $c(a,x)$  and  $e(a,x)$  here must be of the form  $c(a)x$  and  $e(a)x$ . The assumption of constant returns to scale plays a quite central (though not indispensable) role for traditional results on production efficiency in optimal taxation, and plays an equivalent role as we analyze their applicability in the context of externalities.

Under these assumptions, a first result is that in optimum the marginal rates of transformation between abatement and emission reductions are equal across polluting activities (i.e. goods, sectors,  $j \in N$ ), and equal for consumers, producers and government. One implication of this is that, when asking how much one should do to make a vehicle less polluting, one need not ask whether it is used by rich or poor, by households, producers or government, by the health ministry or the military.

It is implicit in Diamond and Mirrlees' (1971) result on production efficiency that the marginal rates of transformation between abatement and emission reductions shall be equal for entities within industries and government. We have simply included an additional good as relevant to consumers, and the result that efficiency in aggregate production should apply to this expanded vector of outputs is not surprising. Cremer and Gahvari (1999) also find that marginal costs of emission reductions are equal in optimum for firms with homogenous technologies, but do not place this finding in the context of the theorem on aggregate production efficiency. The part of our production efficiency result that was more unexpected was that polluting consumers, too, should abate to provide emission reductions at the same marginal rates of transformation.

These findings provide considerable relief in terms of implementation. First, if pollution reductions can be stimulated with emission taxes and abatement is untaxed, then optimal abatement can be implemented by the same tax levied on emissions

everywhere where they occur. This holds independently of whether the polluter is a producer, consumer or government, commuting to work or mowing the lawn. It also holds whether the abatement opportunities arise where emissions occur or at a prior stage, as when a car's emission factor can be reduced by the manufacturer. The intuition, here, is that auto makers and consumers who are exposed to an emission tax will work – as if together – to minimize the all-inclusive unit cost, which includes emission taxes and abatement costs at any stage.

The results are relieving also in terms of analytical simplicity. If emissions are taxed uniformly, then the formula for commodity taxes is identical to the one for optimal commodity taxes in the traditional problem without external effects. Thus, the emission tax that induces optimal abatement also induces the substitution desired towards less polluting goods, so that the formulas for optimal commodity taxes bears no evidence of the environmental problem. The formula for the emission tax, similarly, bears no evidence of the revenue generation problem, apart from through the shadow price of public funds (see below).

A second result of the listed assumptions is that if emission taxes cannot be used, then Sandmo's (1975) formula for commodity taxes that includes a term for presumptive Pigovian taxation applies. The presumptive Pigovian terms are proportional to the good's emission factor, thus uniform per unit of emissions across polluting goods. This latter result holds for any given emission factors, including when emission standards are being used to reduce emission factors. If standards can be used in combination with presumptive Pigovian taxes, then (5) is equalized across goods, and the same allocation as under emission taxation is implemented.

These results thus give support - conditional on the assumptions - to the intuition that environmental protection is much like a procurement problem - we should think of the emission tax more like a producer price than as a tax. The principle that procurement should equalize the marginal costs across potential providers is not shaken by the fact that this good is provided as a negative externality, by government, firms, and consumers; rich and poor. Thus, we find support for the cost-effectiveness analysis under more general assumptions than those originally invoked.

We shall highlight here one particular aspect that we believe may be surprising to some – hoping to assist intuition. In the Mexico City analysis, we looked across vehicle types (e.g. buses, luxury cars) and applied a representative consumer model. In Essay II, we obtain support for this equal treatment of different vehicles, even under costly taxation and redistribution. Why does the planner not differentiate emission taxes (or standards) across different vehicles for redistributive reasons, for instance to let the rich do more for the environment than the poor? The answer is simple, and shows the close links to the traditional result on aggregate production efficiency. The planner is assumed to have commodity specific commodity taxes available. These can be used to pursue the redistribution that is feasible by changing relative consumer prices, without the additional resource cost of reallocating abatement efforts in an inefficient pattern.

The production efficiency result does not apply if consumers are heterogenous in their access to pollution abatement possibilities. If consumers differ in access to (i.e. effectiveness of) abatement, consumers exposed to the same emission tax may have different emission factors for the same good.<sup>17</sup> As a consequence, the combination of an emission tax and a commodity tax confronts consumers of the same good with different unit costs, giving the planner an instrument possibly attractive for redistribution. The case of different emission factors also gives the planner a chance to price differentiate to reduce the costs of taxation. These results are presented in section IV of Essay II. Many of them are analogous to findings in the literature on imperfect corrective pricing (starting with Diamond, 1973), which prove to translate quite intuitively to a context of distortionary revenue generation.

*Pigou's conjecture about public expenditures, and the double dividend*

We have employed the assumption typically used, that there is separability in preferences between the environmental good and market goods, so that uncompensated demand for market goods  $x_j^h(q, I^h, e)$  is not influenced by changes in pollution,  $x_{je} = 0$ ,  $j = 1, \dots, n$ . This yields a rule equivalent to the established one for optimal provision of public goods

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<sup>17</sup> The same good here has a particular meaning: consumption that cannot be differentiated in the commodity tax structure (Essay II, section IV).

(e.g. Atkinson and Stern, 1974; Auerbach, 1985; King, 1986), or with  $h$  identical consumers

$$(6) \quad -\frac{1}{f_{jb}} = h\alpha \frac{\beta}{\mu}, \text{ all } j=1, \dots, n.$$

On the left hand side we have the marginal rate of transformation between emission reductions and abatement in sector  $j$ , whether abatement is by producer or by consumer (see the generalization indicated for equation 2). On the right hand side,  $h\alpha$  is aggregate marginal willingness to pay for the public good (i.e. for pollution reductions), and  $\beta/\mu$  is the ratio between the marginal utility of income to the consumer,  $\beta$ , and the shadow price of the government's budget constraint,  $\mu$ . In the context of an environmental program, (6) can be implemented either by an emission tax or by an emission standard. An emission tax will be combined with commodity tax rates satisfying the formulas for optimal taxes in the traditional problem without externalities, while emission standards will be combined with commodity taxes satisfying a generalized Sandmo (1975) formula, including presumptive Pigovian terms (see Essay II).

The generalization under non-separability again results in a rule equivalent to the one for optimal provision of public goods (King, 1986). The formulas for the optimal commodity taxes are unchanged, but the optimality formula characterizing optimal provision of environmental quality (6) in the case of two taxed goods changes to:<sup>18</sup>

$$(7) \quad -\frac{1}{f_{jb}} = h\alpha \frac{\beta}{\mu} - \sum_{i=1,2} t_i x_{ie}, j=1, \dots, n.$$

In other words, provision of the public good, minus  $e$ , is adjusted as if it were credited with contributions that the public good makes to the proceeds from commodity taxes. This equivalence between the traditional problem of public goods provision and environmental protection (equations 6 and 7) has not formerly been highlighted, since models of environmental externalities without abatement render no expressions corresponding the left hand sides, the marginal rate of transformation.

Pigou conjectured that costly revenue generation reduces public programs (see Essay III). We shall see that one of the questions raised in the double dividend debate is a

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<sup>18</sup> See annex to Essay III.

question addressed by Pigou's conjecture, well sorted out by Atkinson and Stern (1974) for the case of pure public goods.

The first factor that may cause Pigou's conjecture to be turned around is if nonseparability results in a positive marginal contribution from provision to the tax base (in our case that  $-\sum_i t_i x_{ie} > 0$ , so that emissions reduce demand for taxed goods). More than a curiosity, this term is worth noticing in the present context for several reasons. First in environmental economics, benefit estimation methods such as hedonic price models, wage-amenity studies and the travel cost method are based on the assumption that willingness to pay for environmental quality is reflected in market prices and behavior (see, for instance, Cropper and Oates, 1992, for a review). Second, in particular in the context of tax jurisdictions competing for highly mobile factors (residents), the tax interaction terms make possible the case that some environmental protection (or other public goods provision) can be justified on the narrow grounds that it contributes to revenues.<sup>19</sup> Third, in a developing country setting, the scope for providing public goods that stimulate participation in the taxable economy may be significant.

The second factor that may invalidate Pigou's conjecture is  $\beta/\mu$  (the inverse of the marginal cost of funds).<sup>20</sup>  $\beta/\mu$  can be expected to be less than one – in support of Pigou's conjecture – but may be larger than one if the taxed goods on average are inferior goods in the sense that  $\sum_j t_j x_{ji} < 0$ . Then, the income effects from taxation cause consumers to shift demand so as to reduce the costs of funds.

We shall proceed with a few additional assumptions, to comment upon results in the so-called double dividend literature. The question examined is whether – in the context of costly revenue generation – the emission tax rate be set at a level higher than the first best rule (i.e.  $\tau_1 = f_1 h \alpha$ , where  $\tau_1$  the emission-presumptive rate on good 1,  $f_1$  the emission factor). Bovenberg and de Mooij (1994) assume that wage income is used to purchase a clean and a dirty good and that a wage tax is the revenue instrument.

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<sup>19</sup> Tiebout's (1956) equilibrium in local public goods postulates *efficient provision* without a benevolent planner based solely on the non-separability terms: In Tiebout's model, landowners cum government have a revenue base capable of capturing *all* benefits.

<sup>20</sup> With heterogenous consumers, as Sandmo 1999 points out, the covariance between the vector  $\beta$  and the household consumption of taxed goods likely implies a tendency that redistributive considerations raise provision.

Moreover, there is separability in preferences between other market goods and leisure, and separability again between this aggregate and environmental quality (so  $\sum_j t_j x_{je} = 0$ ). They then define as "Pigovian" the first best tax for the dirty good  $\tau_1 = f_1 h \alpha$ , to ask whether welfare would be improved by moving in a revenue neutral fashion to increase this tax or to decrease it. For a marginal reduction in the tax on the dirty good they find that the answer hinges upon whether labor supply will increase or decrease as a (broader) labor tax is reduced to substitute for a narrower tax on the dirty good. They show that the tax on the dirty good will be lower than the "first best" level under the assumption that labor supply is not in a backward-bending region. They then argue that an upward-sloping labor supply is to be assumed based on empirical studies.

We may use our own framework and an additional restrictive assumption to analyze this problem in optimum. Our result that the optimum Pigovian tax can be characterized in the presence of commodity taxes that satisfy the optimality conditions for the traditional, non-environmental tax problem is useful for this. For a tax on labor only to be optimal from a non-environmental perspective, we need to restrict preferences for the subaggregate of market goods, 1 and 2 to be homothetic (Sandmo, 1974). We can then bring with us from (6) above that marginal costs of abatement can be greater than the benefits if and only if  $\beta/\mu$  is greater than one, and proceed to check under what conditions this can be the case.

The first order condition for the optimal labor tax for the traditional non-environmental problem is

$$(8) \quad \frac{\beta}{\mu} = -\frac{t}{L} \frac{\partial L}{\partial w} + 1,$$

where  $L$  is labor (endowment minus leisure) and  $t$  is the tax on labor income.<sup>21</sup> Labor productivity is a constant, so  $\partial L/\partial w = -dL/dt$ . For the optimal tax problem which includes pollution, we know that (8) should be satisfied together with

$$(9) \quad \tau_1 = f_1 h \alpha \frac{\beta}{\mu},$$

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<sup>21</sup> Rearranging,  $\beta/\mu = 1 - (t/w)\varepsilon_{Lw}$ , so the cost of funds,  $\mu/\beta$ , increases in the labor supply elasticity,  $\varepsilon_{Lw}$ , and in the tax level.



for the tax on the polluting good. From (9), we know that  $\tau_1 > f_1 h \alpha \Leftrightarrow \beta / \mu > 1$ . From (8), assuming a revenue requirement beyond  $\tau_1 x_1$ , so that  $t > 0$ , we confirm the result that the marginal costs in optimum cannot be greater than benefits  $h \alpha$  unless if the uncompensated labor supply curve is backward bending,  $\partial L / \partial w < 0$  (i.e. unless if the income effect of a wage increase dominates over the substitution effect). However, the labor supply curve can be backwardbending only in a region for which proceeds from the labor tax are declining in  $t$ . The optimal labor tax rate is not found in such a region under our preference assumptions (since substitution from labor towards leisure means away from “dirty”). Thus, we may rule out *on theoretical grounds* the possibility that in optimum the environmental tax be set at a level higher than what the first-best parameters indicate,  $f_1 h \alpha$ . We may say that under these assumptions Pigou’s conjecture applies and rejects the proposition that the environmental tax be set at higher levels than the benefits of pollution reduction.

*When a public good such as the environment benefits production*

In essay III we make the variation that environmental quality also benefits production by reducing production costs, in addition to the pure public good benefiting consumers directly. Examples may be the brewer whose costs are lower when his water source is unpolluted, or the tourism industry that needs good air quality. We retain the assumption of constant returns to scale, so the benefits in terms of cost savings are not accruing to firms or their owners, but are passed on to consumers if they are not captured by government in the form of taxation. The rule for optimal provision corresponding to (6), written to accommodate heterogenous consumers and costly redistribution, is:

$$(10) \quad -\frac{1}{f_{jb}} = \frac{\sum_h \alpha^h \beta^h}{\mu} + \sum_j (h \bar{x}_j + x_j^P) c_{je}.$$

In (10),  $c_{je}$  is the marginal increase in unit costs of good  $j$  as pollution increases and  $h \bar{x}_j + x_j^P$  is the total consumption of  $j$  by the  $h$  consumers and government. For a program which only has benefits in production, of course, the first sum in (9) is zero. Comparing (9) with (6), we can see that the generalization is in the spirit of a generalized Samuelsonian summation of marginal benefits, the benefits of reduced production costs

now added to the more familiar summation of benefits based directly on preferences (1). Interestingly, though, the benefits that originate in production costs are not “adjusted” with  $\beta/\mu$ , the ratio of the marginal utility of income to the shadow price of public revenue. Thus, we conclude, public provision with benefits in production rather than as public goods (equation 1) defies Pigou's conjecture – and the double dividend debate - all together. Put in a different way, while Samuelson's (1954) rule for optimal provision of public goods applies only with adjustments in the context of costly revenue generation, the analogous rule for provision of collective factors of production (see Sandmo, 1972) applies directly even when revenue generation is costly.

This result, too, serves well to illustrate the links to Diamond and Mirrlees' result on efficiency in aggregate production. (10) is written in a form valid also with heterogenous consumers, and one might ask again why one should not value cost reductions differently according to who consumes the goods that benefit from a better environment. Again, the answer is that the social planner is assumed to have policy instruments with which consumer prices can be changed with specificity for each good. Thus, it is not attractive to make inefficient (for revenue or redistributive reasons) a program that saves costs for producers.

There is also a parallel to our case of nonseparability. In (7), we show how an adjustment to the rule for optimal provision occurs if provision interacts with the tax base. In (9), for the cost reductions to industries, interaction with the tax base is the whole story. Cost reductions to industries can be captured in their entirety without distortionary costs, by matching them with tax increases for the benefiting goods, so as to leave the all-inclusive consumer price unchanged. It is indeed another result implicit in Diamond and Mirrlees' (1971) analysis that public provision which benefits production should be subject to aggregate production efficiency, in this case meaning that the benefits be accounted for fully. For consumer provision in contrast, as with consumer abatement, the result is not hinted at by earlier findings.

Qualified by the assumptions, the findings have direct policy implications. To illustrate, if a road maintenance project were to save vehicle operators a dollar per passage, one might suggest to count the benefits as 50 cents with reference to the high distortionary costs of funding public budgets. The present analysis indicates that no such

adjustment should apply for the share of vehicles that are commercial or of government.<sup>22</sup> Finally, we should emphasize that it would be wrong to construe this principle as reflecting a judgment that production is more important than consumption; only consumption and consumer preferences matter in this model. Rather, it reflects assumptions implying that benefits accruing in production are easily taxable.

## V. Conclusion

We have tried to summarize the lessons from a journey that included policy recommendations in a practical setting as well as development of policy principles. Working in an applied setting provides good discipline, helping not only to communicate principles better, but also to go back to the theory with reformulated questions.

It sounds odd to many theoreticians that economists had omitted to make the recommendation that a pollution control program emphasizing emission standards and "cleaner technologies" should be complemented by presumptive Pigovian taxes - to shift the economy towards "fewer polluting trips" as well. As we emphasized that this is a good principle in theory and also implementable in practice, we also quantified important aspects of demand management. First, we used results from technical studies to compute a marginal cost curve for emission reductions in the form of "cleaner cars and fuels". Then, we estimated a demand model for cars and driving and used our rule for a matching tax to combine these two instruments in a cost effective way. The result indicates that the cost of pollution reductions in Mexico City increases by 44 percent if a program of emission standards and a presumptive Pigovian tax on gasoline is restricted to not employ the gasoline tax.

These results come about under the assumption that revenue and redistributive transfers bear no premia. Our subsequent theoretical analysis indicates that this approach to policy analysis is supported under a plausible set of more general assumptions. A positive shadow price of revenue influences the optimal environmental quality, and would typically reduce it (as in Pigou's conjecture for public expenditures). However, neither the shadow price of revenue nor redistributive considerations would change the shape of the program, since commodity taxes are assumed to be available, and they are

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<sup>22</sup> Christiansen (1981) shows that benefits to a household, when they are represented by savings in terms of

better suited for redistribution and revenue generation than is a modification of a cost effective emission reduction program.

The qualifications to these generalizations are - roughly - the assumptions that support the recommendation of efficiency in aggregate production (Diamond and Mirrlees, 1971). Under these conditions, we find that firms, consumers and governments should be pushed in the same way, and equally hard, to reduce emissions, i.e. so that marginal costs of emission reductions are the same. Marginal costs of emission reductions shall also be the same across different polluting activities, or goods.

When abatement is untaxed, such pressure can be implemented by a uniform tax on emissions where they occur - combined with commodity taxes satisfying the optimality conditions for a traditional optimal tax problem without an environmental good. The emission tax will then not only induce optimal abatement - reducing emission coefficients in each activity - but also shift the economy optimally towards less polluting activities, complementing a commodity tax structure satisfying a formula that bear no evidence of the externality. If monitoring costs are such as not to allow emission taxes, but allow emission standards or abatement standards, then these standards combined with commodity taxes that include presumptive Pigovian taxes (the structure given by Sandmo, 1975) can implement the same allocation under favorable assumptions. This scenario is the more general scenario in which we support our Mexico City analysis, with the matching tax playing the role of the presumptive Pigovian part of the commodity tax structure.

The examination allowed us to shed light on other questions on the tour. Allowing for differences across consumers in access to pollution control technology, results are of two types. First, emission taxes, even though they are still first-best from an environmental perspective, they take on additional roles in lieu of the planner's objectives of revenue generation and redistribution. Second, without emission taxes, standards and presumptive Pigovian taxes will display qualities of "imperfect corrective pricing". In this case too, results under costly revenue generation prove to be fairly intuitive extensions of results developed under lump sum taxation.

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a market good (say, gasoline), shall be valued at the producer price.

A theme in our theoretical analysis is that environmental protection is different from government provided public goods only in the means of intervention, not in the more basic optimality principles, such as the wedge between marginal costs and benefits. This insight emerges now in the optimal tax model because we introduce abatement. This means that, in a setting with two public goods, one which is a negative environmental externality and another which is provided by government expenditure, the optimality conditions are the same. Intuition for this is given by noting that the difference between the two public goods in terms of revenue requirements may be substantial, but at the margin the relationships between additional provision and government revenue are identical. An area in which we benefit from this parallel is when we show that a question in the double dividend debate boils down to an old question about Pigou's conjecture for public expenditures. Pigou's conjecture applies directly, to reject the proposition of an emission tax exceeding the marginal benefits.

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## **Essay II**

### **Externalities and production efficiency**

Abstract:

How should the environment be protected when revenue generation and redistribution is costly? Building on Diamond and Mirrlees [1971], Sandmo [1975], and Cremer and Gavhari [1999], we find that the marginal rates of transformation between abatement and emission reductions should be equal for firms, consumers and government, within and across polluting activities. Furthermore, a combination of emission standards and presumptive Pigovian taxes can mimic the emission tax. These results ease implementation: With optimal commodity taxes, one emission tax - or standards and presumptive taxes - can be levied only on polluters, supporting optimal substitution and abatement efforts from everyone. Generalizations yield results known as imperfect corrective pricing.

## I. Introduction and Summary

In this paper, we combine two challenges of government: taxation for revenue generation and environmental protection. First-best intuition would say that emissions be taxed neutrally according to their marginal damages, without reference neither to the type of activity that pollutes nor to whether the polluter is rich or poor, consumer or producer, private or public. However, there has yet been no theoretical basis for assuming such simplicity if government has a revenue need that requires distortionary taxation.

Our study builds in particular on two important contributions. First, Diamond and Mirrlees (1971) demonstrated conditions under which optimal taxation involves production efficiency. Their findings imply that the input-output vector is at the aggregate production frontier, a solution that can be implemented by confronting all producers, private as well as public, with the same producer prices. Thus, a social planner may want to insert distorting tax wedges between consumers, and between the set of consumers and the set of all producers, but not between producers, whether private or public. With an external effect from producers and government, a production efficiency result follows directly from Diamond and Mirrlees' treatment. The environmental good is an additional output (or input) of productive sectors, and shall consequently be provided at the same marginal rates of transformation for the set of producers and government seen as a whole. However, the literature including externalities in models of optimal taxation has focused on cases with only one type of polluting activity or source, thus putting aside the question of production efficiency in environmental protection.

The second contribution on which the present study builds is Sandmo's seminal study "Optimal taxation in the presence of externalities" (Sandmo, 1975). Apart from providing the analytical framework used subsequently – and in this study – Sandmo made two findings we shall highlight here. First, he concluded that "even in a world of distortionary taxation...there is scope for taxing externality-generating commodities according to the Pigovian principle." Second, he noted that the optimal commodity tax structure is "characterized by what might be called an additivity property; the marginal social damage of commodity  $m$  enters the formula for that commodity only..." ( $m$  is the externality-creating good) and "the optimal tax rate on the externality creating commodity is a weighted average of two terms, of which the second is the marginal

social damage of commodity  $m$ . The first term...is composed of the efficiency terms familiar from the theory of optimal taxation”.

An important aim of the present study is to understand conditions under which optimal taxation – and Sandmo’s framework - requires production efficiency, including in environmental protection, when such protection can take many avenues. Diamond and Mirrlees considered briefly whether production efficiency would hold if there is an external effect between consumers, but then without including environmental protection in the concept of production efficiency. They concluded “it seems quite likely that efficiency will be desired in realistic settings”. The concept of production efficiency tested here is a broader one, since we include the environmental good in the vector proposed to be at the aggregate frontier.<sup>1</sup> For Sandmo, the proposition of production efficiency in environmental protection was not at the table, since there was only one polluting activity.

To examine the question of whether production efficiency can include efficiency in the protection of the environment, a key assumption is to include *pollution abatement* as an additional avenue for pollution reductions: a polluting consumer (or producer) may spend resources – say on a filter – to reduce emissions per unit consumed (or produced). This allows us to test a proposition of production efficiency more broadly defined: Under what conditions will marginal costs of abatement – per unit of emission reductions achieved - be equalized across activities and agents?

In our model, the set of polluters is not only producers (as in Cremer et al., 1998; Cremer and Gahvari, 1999), or only consumers (as in Sandmo, 1975; Diamond and Mirrlees, 1971), but comprise consumers, producers and government. Briefly put, we ask whether the social planner would tax emissions from different activities (or from producers, consumers, government) differently.

Our model (Section II of the paper) is simple - a structure with fixed coefficients of transformation between private goods is expanded with an external effect, a public good. The public good is in the outset provided by nature, but is reduced as a negative external effect (we call it pollution) results from consumption and production activities. The model involves five minor modifications to Sandmo’s (1975) model. First,

government, firms and consumers are all polluters. Second, in addition to substitution towards non-polluting goods and services, the model allows the polluter to expend resources on *pollution abatement* to reduce emissions. This term includes efforts such as the consumer's maintenance of her car, the producer's installation of a catalytic converter in her product or a filter in her smokestack, modifications of practices or of compounds such as fuels and detergents, and finally cleanup efforts. Third, we allow multiple polluting goods, or activities (we use the word activity to comprise consumption *and* production). Fourth, we allow nonuniformity across polluters in how much they pollute per unit of activity (more precisely, they differ in their costs of pollution abatement). Finally, these modifications themselves invite expansions of the set of policy instruments relative to those allowed by Sandmo and subsequent authors. On the one hand, emission taxes no longer are mere extensions of commodity taxes when pollution abatement is possible (this distinction is also used by Cremer and Gahvari, 1999). Also, we show, standards for abatement (or for emission per unit) can play a role under plausible restrictions on the observability of emissions.

Several studies have provided approaches preparing the ground for this treatment. Bovenberg and van der Ploeg (1994) introduce abatement, but as public production rather than related to own emissions (a good example might be a municipal wastewater treatment plant). Their discussion is centered on how increased environmental concern influences provision of public goods and consumption of private goods. Goulder et al. (1999) allow abatement amongst producers, and focus on the interaction between environmental instruments and pre-existing taxes. Both these studies employ assumptions giving the labor/leisure choice, not only the environmental good, a particular role in preferences. Cremer et al. (1998) analyze optimal taxation and focus on the interaction between the environmental tax and other instruments, much in the same way as did Atkinson and Stiglitz (1976) for the interaction between direct and indirect instruments in the traditional problem without externalities. Cremer and Gahvari (1999), closest to the questions asked here, allow several polluting industries with uniform technology. Amongst their findings is a uniform emissions tax, implicitly showing how Diamond and

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<sup>1</sup>Since they were proposing an external effect from consumers to consumers, it was quite natural in their context not to expand with the environmental good the vector of inputs and outputs proposed to be at the aggregate frontier.

Mirrlees' production efficiency result must apply if the input or output of producers is expanded with one element - the environmental good.

In section III, we characterize optimal policy assuming equal access to technology for all consumers. We investigate when instruments such as emission taxes will be applied "neutrally" as expected according to Pigovian principles, including under plausible restrictions on the monitoring of emissions.<sup>2</sup> These results can be viewed as generally extending those of Sandmo (1975). Also, they extend the results of Eskeland (1994) on the combination of emission standards and presumptive Pigovian taxes (levied on inputs and outputs), to a case in which taxation is costly.

For polluting producers, production efficiency applies as expected even when firms differ in their access to abatement technology (section III). In section IV of the paper, we introduce nonuniform emission functions for consumers. If the planner can differentiate emission taxes across polluting activities, when are the emission taxes equal across activities, and equal to the one applied to producers? We find that the "one tax" breaks down if the pattern of nonuniformity across consumers in emission functions lends itself to nonenvironmental goals of the planner, such as to redistribute, or to minimize the distortionary effects of taxation. Certain covariance formulas identify these cases. In the case when emission taxes are not available, presumptive Pigovian taxes on goods and emission standards are no longer 'first-best' when emission functions are nonuniform, so results are modified for that reason. While these results are new in a setting of distortionary taxation, they naturally extend results from a literature examining indirect Pigovian instruments when the externality generating good is itself unavailable or imperfect as a base for a corrective instrument.<sup>3</sup>

## II. The Model

We introduce some variations to existing treatments of optimal taxation in the presence of externalities (Sandmo, 1975, in particular, but also Cremer et al., 1998). The importance of these variations lie in their practical relevance, and we will thus intersperse the text with some examples for illustration.

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<sup>2</sup> As is the tradition in the literature, we use the words tax and taxation whether the rate is positive or negative (in everyday use, the negative rates would be called subsidies).



### *Preferences*

As is the tradition in the public finance literature, we analyze a setting in which consumers have preferences over private goods as well as a public good (with several public goods, results extend straightforwardly). As a matter of terminology, a term such as “a public bad” could be used for pollution, but we may also think of the social planner as procuring a public good when using taxes or regulation to stimulate pollution reductions. Polluters, who may be consumers or producers, then become potential providers of the polluting good. It is sometimes convenient to speak of goods in general, and then let a vector of quantities include pollution as a public good, even though consumers prefer less pollution to more.

Let  $H$  denote the set of consumers, and let  $h$  be a consumer,  $h \in H$ .  $h$ 's utility depends on her consumption  $x_j^h$  of a set  $N$  of market goods,  $j = 0, 1, \dots, n$ , as well as on a pollution indicator,  $e$ :

$$(1) \quad u^h = u^h(x_0^h, x_1^h, \dots, x_n^h, e).$$

We assume that the utility function is continuous, twice differentiable, quasiconcave, and that  $u_e^h \leq 0$ .<sup>4</sup> In addition, we shall assume that preferences are separable between the basket of market goods,  $j \in N$ , and pollution, so that the marginal rates of substitution between market goods are independent of pollution levels. A sufficient condition for this to hold is that individual preferences can be described by a separable utility function:  $u^h = u^h(\psi^h(x_0, x_1, \dots, x_n), e)$ . In the literature on taxation in the presence of external effects, separability is typically assumed.<sup>5</sup>

In assuming that pollution is experienced at the same level by all consumers, we combine two properties. The important one is that pollution (or its absence) is a pure public good in the sense of Samuelson (1954), that there is no rivalry in its consumption. If one person enjoys the low level of pollution, this does not reduce another person's enjoyment. The less significant implied property is homogenous dispersion (which James

<sup>3</sup> Starting with Diamond (1973).

<sup>4</sup> Whenever possible without risking confusion, we shall use subscripts to denote partial derivatives.

$u_e^h \leq 0$  is necessary for us to use words such as ‘negative external effects’ and ‘pollution’, but the results are equally applicable also to positive external effects. An equilibrium in which Pigovian taxation makes distortionary taxation unnecessary is less plausible with positive externalities.

Meade, 1952, termed atmospheric pollution). It simplifies notation, by ensuring that the marginal damages from emissions (or the benefits from emission reductions) are independent of who or where the polluter is. If damages per unit of emissions vary, say by location or by stack height, accommodation of this fact must be made, and results extend (the question should be asked, however, whether instruments can be differentiated accordingly).

### *Emissions and pollution abatement*

In the world we try to capture with our model, emissions of pollution are caused by several activities, in production stages as well as consumption stages. Examples that we all know of are that emissions are caused in the production of gasoline, cars, and detergents, and also as households and firms use car services and do their laundry. Also, *pollution abatement*, or efforts to make each activity less polluting, may be undertaken by producers or consumers. Reduced emissions from cars, for instance, can result as the manufacturer changes his product by adding a catalytic converter, as the refinery changes the gasoline characteristics, and as the driver drives more carefully, buys a “cleaner” gasoline, and improves her maintenance. As these examples illustrate, efforts to abate emissions may well be exerted at a production stage even if emissions occur later, for instance in consumption.

The traditional treatment of externalities in the theoretical literature has been to view substitution in consumption (towards non-polluting goods and services) as the only way to reduce pollution (Cremer and Gahvari, 1999, made advances beyond this). In such a case, it is of no importance whether emissions result from production or consumption – since the assumption of equilibrium ensures that production and consumption move in parallel. Sandmo’s important (1975) contribution described emissions as caused by consumption, but with results directly applicable for emissions caused by producers.

In a context with *pollution abatement* in contrast, it could be material whether emissions occur in consumption or in production. To illustrate, if in optimum producers and households face different price vectors and abatement options (they do), are marginal abatement costs in optimum different for car manufacturers and users? Similarly, if a car

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<sup>5</sup> An assumption of separability between leisure and other private goods is also often included (Cremer et al., 1998; Cremer and Gahvari, 1999; Bovenberg and Goulder, 1996; Bovenberg and van der Ploeg, 1994).

owner can reduce emissions, should policy stimuli depend on whether she is a consumer, an enterprise or government?

Commodities are potentially polluting in both production and consumption, so we may think of a set of 2  $(n+1)$  *polluting activities*. We choose consumer abatement and emissions as our main presentational vehicle, in part because consumer abatement is novel and poses more interesting questions in a welfare economic perspective. To save on notation, we do not introduce emissions from producers before later in this section.

Individual emissions are caused in association with the consumption of polluting goods and services, as represented by *emission factors*  $f_j^h = f_j(b_j^h)$ :

$$(2) \quad e_j^h = f_j(b_j^h) \cdot x_j^h \text{ for all goods } j=1,\dots,n, h \in H,$$

and equivalently for government. (2) reflects that the consumer may expend resources on pollution abatement,  $b_j^h$ , in order to reduce  $f_j^h$ , the emissions per unit consumed of good  $j$ . We assume, until we generalize in section IV, that consumers have access to the same abatement technology, so that emission functions  $f_j(b_j^h)$  are uniform across consumers. The assumption that emissions display proportionality with the quantity consumed (conditional on the good in question, and abatement) is restrictive, but allows us to place our results in a literature based on constant returns to scale. To simplify, we assume that the numeraire good is not polluting:  $f_o = 0$ , and we describe all other goods as polluting: for  $j \neq 0: f_j > 0$ . Abatement  $b_j$  is nonnegative and continuous and  $f_j$  is assumed to be continuous and differentiable. We assume that abatement reduces emissions at a decreasing rate, so that the marginal cost of emission reductions  $-1/f_{jb}$  is positive and increasing.<sup>6</sup>

The pollution level, the argument in each consumer's utility function, is simply emissions aggregated across polluters and polluting goods:<sup>7</sup>

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<sup>6</sup> The assumption that goods  $j = 1, \dots, n$  are polluting simplifies notation. A nonpolluting good is thus approximated as one with trivially low emissions (and emission taxes) at trivially low abatement levels.

<sup>7</sup> We skip important detail here, and some deserve mention: i) the pollution level, here a scalar, may be a vector (concentrations of dust and of ground-level ozone). Extension of results to several 'public goods' (or bads: dust, ozone), with one set of Pigovian taxes for each, is straightforward; ii) whether or not the pollution level is a scalar (say, parts per million of ozone), emissions contributing to the pollution level may be a vector (as with the precursors of ozone: tons emitted of nitrogen oxides and of volatile organic compounds); iii) we abstract from how the pollution indicator translates into damages (health effects,

$$(3) \quad e = \sum_{j \in N} \left( \sum_{h \in H} e_j^h + e_j^P \right),$$

where  $e_j^P$  denotes emissions resulting if the government uses good  $j$  (the generalization with producer emissions is straightforward, and will follow).

### *The consumer's problem*

Let  $t_j$  and  $\tau_{e_j}$  be linear taxes levied respectively on consumption and emissions of good  $j$ ,  $j = 0, \dots, n$ . The consumer faces a price  $p_j + t_j$  for each good.  $p_0 = 1$ ,  $t_0 = 0$ , so the numeraire good is untaxed.<sup>8</sup> Our model is general in its treatment of private goods, so it is not important whether one thinks of the numeraire good as leisure. In this respect, our model differs from a number of recent contributions on Pigovian taxation in which results are based in part on preferences that are separable in leisure versus other private goods (additional results following from making that assumption are rather obvious)<sup>9</sup>.

We model consumer  $h$  as maximizing her utility  $u^h$  with respect to consumption and abatement, subject to her budget constraint:<sup>10</sup>

$$(4) \quad \text{Max}_{x_j^h, b_j} : u^h(x_0^h, x_1^h, \dots, x_n^h, e) \text{ s.t. } \sum_{j \in N} [p_j + t_j + b_j^h + \tau_{e_j} f_j(b_j^h)] x_j^h = 0.$$

In  $h$ 's maximization, we shall assume that she considers the level of pollution,  $e$  (the sum of what is generated by all polluters), and also public sector revenue to be independent of her own actions. This will be either accurate descriptions or close approximations if the number of individuals,  $H$ , is large. These assumptions are rather natural extensions of the assumptions of competitive equilibrium, under which producers

species extinction, corrosion, etc.), when we describe willingness to pay simply as a function of the pollution level.

<sup>8</sup> We use the terms 'consumer' and 'household' synonymously, and are thus unable to handle distributional issues within the household.

<sup>9</sup> In a model with constant transformation coefficients, like ours (see production technology, below), the choice of untaxed commodity is immaterial: it is easily checked that the relative prices obtained here (including those inducing abatement) can be replicated with another choice of untaxed good.

<sup>10</sup> Our budget constraint is consistent with the traditional:  $\sum_{j=1}^n (p_j + t_j) x_j = I - l$  where  $I$  is

endowment and  $l$  is leisure. With  $x_0 \equiv I - l$  ( $x_0$  is a negative figure), we have  $\sum_{j=0}^n (p_j + t_j) x_j = 0$ .

When we introduce consumer abatement and emission taxes, we obtain

$$\sum_{j=0}^n (p_j + t_j + b_j^h + \tau_{e_j} f_j) x_j = 0.$$

and consumers take prices as given. In the present model, they consider two additional variables as independent of their actions: total pollution and public revenue.<sup>11</sup>

The first-order conditions for  $h$ 's individual optimum are her budget constraint and, for all goods  $j=1,\dots,n$ :

$$(5) \quad \frac{u_j^h}{u_0^h} = p_j + t_j + b_j^h + \tau_{ej} f_j(b_j^h) \equiv q_j^h \quad \text{and}$$

$$(6) \quad -\frac{1}{f_{jb}(b_j^h)} = \tau_{ej}.$$

The first equality in (5) shows how the consumer will set marginal rates of substitution between private goods equal to the relative marginal costs of these goods. In the second equation in (5), we have taken advantage of the fact that these marginal costs are independent of consumption levels, so that the marginal cost is also a unit cost, and introduced the symbol  $q_j^h$  to represent this 'all-inclusive consumer price'. In (6), the consumer sets her marginal cost of emission reductions equal to the emission tax rate. These marginal costs would be equal across consumers even if emission functions differed across consumers. However, with homogenous emission functions, abatement  $b_j^h$  and emission factors  $f_j^h$  will also be the same across consumers, ensuring that the all-inclusive consumer prices are uniform across consumers. We shall use this property to suppress individual superscripts for  $b_j$  and  $q_j$  until section IV, in which we adopt heterogeneous emission functions.

Finally we may sketch a generalization. If emissions occur at production stages as well, and if producers in sector  $j$  face emission taxes and abatement opportunities, then the producer price in (5) will itself be a sum components, to include the producer's abatement and taxes on emissions in production. Then, self interested producers will join the consumer in an effort to minimize the all-inclusive consumer price.

### *Production technology*

To describe the economy's technological constraint - its ability to transform one bundle of consumption goods into another - let capitalized variables without superscripts

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<sup>11</sup> See Sandmo (1975) or Eskeland (1994) for some further treatment. If there are  $H$  individuals who take into account the effect of their own actions, then our approximation error is to set  $H/(H-1)$  equal to one.

denote aggregate quantities:  $X_j = \sum_{h \in H} x_j^h + x_j^p$ , with  $x_j^p$  denoting government

consumption. A rather general description of technology would be:

$$(7) \quad F(X_1, \dots, X_n) = Y_0$$

where  $Y_0$  is work, or endowment less leisure and abatement:

$$(8) \quad Y_0 = I - l - \sum_{j=1}^n b_j X_j = -X_0 - \sum_{j=1}^n b_j X_j.$$

One assumption embodied in (7) and (8) is that the damages from pollution do not affect production possibilities. Thus, the motivation for pollution abatement is found solely in the way pollution affects household utility (1). In Eskeland (2000), we generalize to include productive sectors amongst the beneficiaries of pollution reductions.

Our model of the production side of the economy shall involve additional restrictions: fixed factors of transformation between market goods (see, for instance, Sandmo, 1975, or Cremer et al., 1998):<sup>12</sup>

$$(9) \quad \sum_{j=1}^n c_j X_j = Y_0,$$

where the vector  $c$  consists of the constant transformation coefficients. Though we have used aggregate quantities, we may think of (9) as describing a generally available conversion technology, possessed and controlled by many independent producers. When these producers compete in input and output markets, each handling their share of the aggregate quantities, profits will be zero and producer prices will be equal to marginal costs:

$$(10) \quad p_j = c_j, \text{ all } j=1, \dots, n, \text{ and } p_0 = 1.$$

### III. Optimal taxation

#### *A benevolent planner*

Let a benevolent planner's objectives be represented by a welfare function defined over individual utility levels,  $w = w(v^1, v^2, \dots, v^h)$ , where  $v^h = v^h(q^h, I^h, e)$  is

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<sup>12</sup> The assumption of fixed coefficients of transformation – or of constant producer prices – is motivated by our desire to compare with standard results in the optimal taxation literature. Diamond and Mirrlees (1971) showed that the results based on constant producer prices apply also to the more general case of constant returns to scale. Dasgupta and Stiglitz (1972) and Mirrlees (1972) analyze the extensions and qualifications to the case with non-constant returns to scale.

the indirect utility function corresponding to (1), and  $q^h$  is given by  $p, t, \tau$ , as described by (5) and (6). He maximizes welfare subject to a constraint that revenues are equal to a predetermined minimum - enough to finance exogenous public sector expenditures<sup>13</sup> (we initially assume that government abatement is exogenously given). Apart from the instruments used here, we assume that the planner does not have available other instruments for redistribution or revenue mobilization. However, our analysis applies also to the case when there are other instruments with redistributive and revenue implications. Such other instruments could be uniform poll taxes as well as non-linear income taxes.<sup>14</sup>

Commodity taxes are in the literature typically restricted to be linear, with a brief justification being that the planner's information includes aggregate quantities  $\sum_h x_j^h$ , but not the  $H$  vector  $x_j^h$ .<sup>15</sup> Thus, the tax man may observe liters of gasoline exiting the refinery gate or the gas station, but not individual purchases to an extent attributable to individual consumers. We may add that nonlinear commodity taxes (or commodity taxes differentiated by personal characteristics) would involve not only costly information and administration, but also distortions, as consumers would engage in costly exchange of goods and services. Thus, the restriction that commodity taxes be linear can rest on a broader set of considerations than only information availability.

This latter, broader justification is more appropriate when we assume that emission taxes may be differentiated by polluting commodity (gasoline versus heating oil, or driving versus lawn-mowing), but are confined to be linear and uniform across consumers. We may think of emissions as in principle observable by the planner at the emitting source (a meter on each car, for instance, displaying at year end the car's cumulative emissions), but that the attribution of emissions to households would be

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<sup>13</sup> Individual budget constraints add up to the technology constraint (9) if we include that of the planner:

$$\sum_h \sum_j (t_j + \tau_{ej} f_j) x_j^h = \sum_j (p_j + b_j^P) x_j^P, \text{ where the right hand side is public expenditures.}$$

<sup>14</sup> The analysis applies by viewing the presented sufficient conditions as a subset of conditions for optimal policy. In the tradition of Mirrlees (1971), nonlinear income taxes are introduced by assuming that individuals differ in endowment of time in productivity units, but that only income (work times wage) is observed and taxable by the planner. One approach is to assume a discrete number of types and nonlinear income taxes subject to self-selection constraints. Cremer et al. (1998) demonstrates analysis of instruments such as emission taxes in a broader context which includes non-linear taxes.

<sup>15</sup> "nonlinear taxation can be restricted to commodities in which retrading is impossible or perfectly observed" (Mirrlees, 1976).

costly and lead to distortions under nonlinear taxation.<sup>16</sup> Apart from these considerations, our assumptions are motivated by our practical aim of checking whether optimal emission taxes would apply neutrally when they can be differentiated by commodity - a question which is less well defined for non-linear instruments.

The Lagrangian of the planner's maximization problem is:

$$(11) \quad L_{t_i, \tau_{ej}} = w(v^1, \dots, v^h) + \mu \sum_{j \in N} \left\{ \sum_{h \in H} (t_j + \tau_{ej} f_j(b_j)) x_j^h - (p_j + b_j^p) x_j^p \right\}, \quad i=1, \dots, n.$$

### Optimal taxation

To simplify exposition, we introduce the following definitions:

$$\beta^h = \frac{\partial w}{\partial v^h} \frac{\partial v^h}{\partial \mathcal{I}^h} \quad \text{and} \quad \alpha^h \equiv - \frac{\partial v^h}{\partial e} / \frac{\partial v^h}{\partial \mathcal{I}^h}.$$

$\beta^h$  is the marginal value of additional income to individual  $h$  as valued in optimum by planner's welfare function.  $\alpha^h$  is  $h$ 's willingness to pay - in terms of the numeraire good - for pollution reductions, a non-negative number by assumption.<sup>17</sup>

We shall initially consider government abatement as given. Partially differentiating (11) with respect to the  $n$  commodity tax rates and the  $n$  emission tax rates, first order conditions for optimal taxation are, for all  $i = 1, \dots, n$ :

$$\frac{\partial \mathcal{L}}{\partial t_i} = 0 \Leftrightarrow$$

$$(12) \quad - \sum_{h \in H} \beta^h \left[ x_i^h + \alpha^h \frac{de}{dt_i} \right] + \mu \sum_{g \in H} \left[ x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) \frac{dx_j^g}{dt_i} \right] = 0, \quad \text{and}$$

<sup>16</sup> Examples to illustrate such difficulties: Multiple car households, (temporary) exchange of cars and car services, multihousehold heating and municipal waste-water discharge. The impossibility of using potential information on individual emissions for nonlinear taxation is particularly clear in the case of polluting producers in a model with constant returns to scale, as ours. Nonlinear taxation of emissions would be ruled out by the costless replication (or merger) of firms. For households, on the other hand, non-linear taxation of emissions could with plausibility be feasible and attractive, save for reasons of administrative difficulty and distortions. We learn something relevant for non-linear taxation of household emissions when we treat differences in emission factors in section IV.

<sup>17</sup> Use the definition of the indirect utility function and the envelope theorem:

$$u(x(q, I, e), e) = v(q, e, I) \Leftrightarrow - \frac{\partial u}{\partial e} = - \frac{\partial v}{\partial e}. \quad \text{Divide by minus the marginal utility of income to express}$$

$$\text{willingness to pay in terms of the numeraire: } - \frac{\partial u}{\partial e} / \sum_j \frac{\partial u}{\partial x_j} \frac{\partial x_j}{\partial \mathcal{I}} = - \frac{\partial v}{\partial e} / \frac{\partial v}{\partial \mathcal{I}}.$$



$$\frac{\partial \mathcal{L}}{\partial \tau_{ei}} = 0 \Leftrightarrow$$

$$(13) \quad - \sum_{h \in H} \beta^h \left( f_i x_i^h + \alpha^h \frac{de}{d\tau_{ei}} \right) + \mu \sum_{g \in H} \left[ f_i x_i^g + \tau_{ei} f_{ib} b_{ir} x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) \frac{dx_j^g}{d\tau_{ei}} \right] = 0.$$

In (12), we have used the fact that producer prices are independent of commodity taxes:

$$\frac{dp_j}{dt_i} = 0, \Rightarrow \frac{\partial v^h}{\partial \alpha_i} = \sum_j \frac{\partial v^h}{\partial q_j} \frac{dq_j}{dt_i} = \frac{\partial v^h}{\partial q_i}, \text{ and Roy's identity: } \frac{\partial v^h}{\partial q_i} = -x_i^h \frac{\partial v^h}{\partial I^h}.$$
 In (13), we

have used  $\frac{dq_i}{d\tau_{ei}} = f_i$  and  $\frac{dq_j}{d\tau_{ei}} = 0$  for  $j \neq i$ . This follows from differentiation of  $q_i$  (see equation 5) and the envelope theorem. To develop these expression further, we may use:

$$(14) \quad \frac{de}{dt_i} = \sum_j \sum_g f_j (b_j) \frac{dx_j^g}{dt_i} \text{ and } \frac{de}{d\tau_{ei}} = \sum_g \left( f_{ib} b_{ir} x_i^g + \sum_j f_j \frac{dx_j^g}{d\tau_{ei}} \right).$$

The  $2n$  equations (12) and (13), with (14) describe an optimal tax structure for the  $n$  commodity tax rates and the  $n$  emission tax rates.

The uncompensated demand functions are in general defined over prices, income, and the quantity of the public good,  $x_j^g = x_j^g(q, I^g, e)$ . Let us now employ the assumption of separability between pollution and other goods:<sup>18</sup>  $x_{je}^g = 0 \Rightarrow$

$$(15) \quad dx_j^g / dt_i = x_{ji}^g \text{ and } dx_j^g / d\tau_{ei} = x_{ji}^g f_i, \text{ all } i, j, g.$$

When using (15) and (14), (12) and (13) simplify to, for all  $i = 1, \dots, n$ :

$$(16) \quad - \sum_{h \in H} \beta^h \left[ x_i^h + \alpha^h \sum_{g \in H} \sum_{j \in N} f_j x_{ji}^g \right] + \mu \sum_{g \in H} \left[ x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) x_{ji}^g \right] = 0, \text{ and}$$

$$(17) \quad - \sum_{h \in H} \beta^h \left[ f_i x_i^h + \alpha^h \sum_{g \in H} \left( f_{ib} b_{ir} x_i^g + \sum_{j \in N} f_j x_{ji}^g f_i \right) \right] \\ + \mu \sum_{g \in H} \left[ f_i x_i^g + \tau_{ei} f_{ib} b_{ir} x_i^g + \sum_{j \in N} (t_j + \tau_{ej} f_j) x_{ji}^g f_i \right] = 0.$$

In order to gain further insights into the tax structure implied by (16) and (17), we shall go via simplifying assumptions.

<sup>18</sup> In Eskeland (2000), we show that results extend for a simple case to non-separability.

Assumption 1: No abatement available: emission factors are exogenously given

The case with an exogenously given emission factor was analyzed by Sandmo (1975). When polluting goods cannot be made less polluting per unit (in our model's terminology, when  $f_{ib} = 0$ , all  $i \in N$  for any  $b_i$ ), pollution reductions will rely solely on changes in consumption patterns towards goods that are less polluting (say: from motorcycles to bicycles, from cigars to cigarettes). The model with exogenously given emission factors is – fortunately – unrealistic in most practically interesting cases, but provides important insights even for the more general case.<sup>19</sup> We show it here as a basis for comparison with existing literature, and also to generalize to several polluting activities (or goods).

With no abatement available, the equations in (6) do not apply, and every equation in (17) is simply  $f_i$  times the corresponding equation in (16). Thus, the  $2n$  by  $2n$  coefficient matrix is at most of rank  $n$  and at most  $n$  instruments are required to implement the optimal allocation. Assuming that the  $n$  equations in (16) are linearly independent, we use this redundancy to set emission taxes all equal to zero and implement the optimal solution using commodity taxes only (as in Sandmo's treatment). Substituting  $\tau_{ej} = 0$ , all  $j \in N$  into (16), we obtain

$$(18) \quad \sum_{j \in N} t_j \sum_{g \in H} x_{ji}^g = - \sum_{h \in H} \left[ x_i^h \left( \frac{\beta^h}{\mu} - 1 \right) + \frac{\alpha^h \beta^h}{\mu} \sum_{j \in N} f_j \sum_g x_{ji}^g \right], \text{ all } i \in N.$$

Insights are gained by rearranging to have tax rates on the left hand side. We display the solution for the case with four goods (0,1,2,e), two tax rates:<sup>20</sup>

$$(19) \quad t_1 = \frac{\sum_h \left( \frac{\beta^h}{\mu} - 1 \right) (x_1^h \bar{x}_{22} - x_2^h \bar{x}_{21})}{H(x_{11} \bar{x}_{22} - x_{21} \bar{x}_{12})} + f_1 \frac{\sum \beta^h \alpha^h}{\mu} \text{ and}$$

<sup>19</sup> It is not unrealistic in *all* interesting cases, and realism depends on the level of generality in the model. In the example of CO<sub>2</sub>, there are virtually no abatement technologies available to users if we examine fuel-efficient combustion technologies for each fuel (say: coal fired power plant). Thus, for a model disaggregating to the individual fuel, the assumption of fixed emission factors would be quite appropriate. In contrast, for a model with an energy aggregate only, one could represent the flexibility within this aggregate (towards fuels that are less CO<sub>2</sub> intensive) as abatement options.

<sup>20</sup>In the more general case, we have  $t_k = \sum_h \left( (\beta^h / \mu) - 1 \right) \sum_i x_i^h F_{ik} / |E| + f_k \sum_h \beta^h \alpha^h / \mu$ , where  $E$  is the coefficient matrix in (18), and  $F_{ik}$  is the cofactor of row  $i$ , column  $k$ .

$$t_2 = \frac{\sum_h \left( \frac{\beta^h}{\mu} - 1 \right) (x_2^h \bar{x}_{11} - x_1^h \bar{x}_{12})}{H(x_{11} x_{22} - x_{21} x_{12})} + f_2 \frac{\sum \beta^h \alpha^h}{\mu},$$

which is the solution given by Sandmo (1975). In (19), we have used consumer averages ( $\bar{x}_{ij} = \sum_h x_{ij}^h / H$ ) to highlight which terms in these formulas are weighted by the vector  $\beta$ . The optimal tax formula “cares” about individual consumption  $x^h$  and willingness to pay  $\alpha^h$ , but about demand responsiveness only in aggregate.

Sandmo describes the optimal tax structure (19) as giving commodity taxes in the presence of externalities an ‘additivity property’ (page 92). Of the two terms, the first is equal to the formula for optimal commodity taxes in the traditional problem with no external effects (see below), and the second is motivated by the need to correct for external effects. The term for the corrective tax is, as Sandmo noted, zero for commodities that are not polluting, and it is zero for all commodities if there is no willingness to pay for pollution reductions ( $\sum_h \beta^h \alpha^h = 0$ ). The addition to Sandmo’s result given here is only that with several polluting goods, the corrective tax element in each tax formula is uniform per unit of public good (the emission factor in each formula ensures this). This result, it can be argued, follows so directly from Sandmo’s analysis, it is implicit.

We shall now use the redundancy in tax instruments to explore a specific alternative way to implement this allocation. Let us examine a solution including the following tax rate levied on emissions uniformly across polluting goods:

$$(20) \quad \tau_{ek} = \tau_{el} \equiv \tau_e = \frac{\sum \beta^h \alpha^h}{\mu}, \text{ all } k, l = 1, \dots, n.$$

Substituting (20) into (16), we have:

$$(21) \quad -\sum_h \beta^h x_i^h + \mu \sum_g [x_i^g + \sum_j t_j x_{ji}^g] = 0, \text{ all } i = 1, \dots, n.$$

(20) and (21) also solves (17), so this system of commodity taxes and a uniform emission tax implements the optimal allocation. Solving for the commodity tax rates, and again assuming two taxed goods, (21) is satisfied for:<sup>21</sup>

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<sup>21</sup> The more general case corresponds to the formula in footnote 20, eliminating the Pigovian element.

$$(22) \quad t_1 = \frac{\sum_h \left( \frac{\beta^h}{\mu} - 1 \right) (x_1^h \bar{x}_{22} - x_2^h \bar{x}_{21})}{H(x_{11} x_{22} - x_{21} x_{12})} \quad \text{and} \quad t_2 = \frac{\sum_h \left( \frac{\beta^h}{\mu} - 1 \right) (x_2^h \bar{x}_{11} - x_1^h \bar{x}_{12})}{H(x_{11} x_{22} - x_{21} x_{12})}.$$

The formulas for the commodity taxes in (21) and (22) (and also those for the non-Pigovian terms in equations 18, 19) are equivalent to those of the solution to the traditional problem of optimal commodity taxation without pollution (i.e. Samuelson, 1951), though the actual tax rates in models with and without pollution will in general not be the same. To highlight the implied structure for the non-Pigovian part, let us follow Samuelson and use the Slutsky equation and the symmetry of the compensated demand derivatives  $s_{ij}^h(q, u^h) = s_{ji}^h(q, u^h)$  to see that (21)  $\Rightarrow$

$$(23) \quad \sum_j t_j \sum_h s_{ij}^h = \sum_h x_i^h \left[ \sum_j t_j \frac{\partial x_j^h}{\partial I^h} + \left( \frac{\beta^h}{\mu} - 1 \right) \right], \quad \text{all } i = 1, \dots, n.$$

As Samuelson pointed out, if one assumes an arbitrarily small revenue requirement and identical consumers, (23) gives the same proportionate reduction in compensated demand for all commodities. Sandmo (1976) highlighted that this feature of (23) extends to hold for substantive revenue requirements if all taxed goods have equal income elasticities. Simplifications often used to illustrate the implications of (21) (or 23) are to assume that the displayed cross price responses are zero, implying that taxes are inversely proportional to own-price elasticities.<sup>22</sup> The structure is also equivalent to the one analyzed by Corlett and Hague (1953), who showed that with two taxed goods the good be taxed at a higher rate which has a higher degree of complementarity with the untaxed good.

Thus, it can be seen, the standard and recognized results for optimal commodity taxes extend to the case with an environmental externality, as long as the externality is taken care of by an appropriate tax levied on emissions. This is in itself not an interesting

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<sup>22</sup> Samuelson's (1951) rule should in part be attributed to F. P. Ramsey (1927) who finds: "the production of each commodity should be diminished in the same proportion". As noted in Munk (1978), Ramsey's changes in production are equal to changes in uncompensated demand, equal to those of compensated demands if income elasticities are zero. Samuelson writes: "Aspects of the right answer have been hinted at by Ramsey (1927)". Diamond (1975) proposes to use the concept social marginal utility of income,  $\gamma^h$ , rather than our  $\beta^h$ , with  $\gamma^h = \beta^h + \mu \sum_j t_j \partial x_j^h / \partial I^h$ . This is intuitively an equally attractive concept and simplifies expression of certain results: "for each good the change in aggregate compensated demand is proportional to the covariance between individual quantities demanded and social marginal utility of income" (page 338). A good orientation in this literature is provided by Auerbach (1985).

observation, first because there is redundancy in instruments,<sup>23</sup> and second because the formal equivalence of commodity tax formulas with and without presence of external effects in no way would imply equivalence in tax rates. However, there are two aspects of this solution giving the emission tax an intuitive interpretation as a price. First, uniformity across polluting activities hint that emission reductions are elicited at the same marginal cost wherever they can be found - alluding to a simple procurement rule. Second, the expression itself consists of a weighted sum of the willingness to pay for emission reductions, reminiscent of the Samuelson (1954) rule for optimal provision of public goods.

Definition: When the purpose is to internalize external effects such as emissions, we shall use term *Pigovian tax* in the traditional way – to mean a corrective tax - if the tax/subsidy is levied directly on emissions (or more generally on a measured contribution to the public good). We shall use the term *presumptive Pigovian tax* if the corrective tax is levied on a commodity (such as an input or an output in the externality generating activity) with the rate per unit of the commodity motivated by a presumed emission factor.<sup>24</sup>

*Proposition 1: Fixed emission factors and presumptive Pigovian taxation*

*With fixed emission factors, two alternative tax structures implementing the optimal allocation are*

- a) *as in Sandmo (1975), a commodity tax structure in which the formula is the sum of presumptive Pigovian taxes and the formula for optimal commodity taxes in the traditional problem without external effects (equation 19, or more generally from 18).*
- b) *a combination of a Pigovian tax (20) uniformly applied to emissions from all polluting goods and services, and a commodity tax structure satisfying the formula*

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<sup>23</sup> See Cremer et al. (1998), who operate with nonlinear instruments more general than ours.

<sup>24</sup> Terms and definitions: We thus associate the term *Pigovian tax* with the objective of internalizing externalities, but not a rule or a level (contrasting, for instance, Cremer et al., 1998). An *indirect Pigovian tax* typically means a corrective tax levied not on the externality causing good itself, but on substitutes and complements (Sandmo, 1976b). Outside the realm of Pigovian taxation, *indirect taxes* have a different meaning (see, for instance Atkinson and Stiglitz, 1976). The term *presumptive* is for income taxes established, with a meaning parallel to ours for corrective taxes (See, for instance Musgrave and Musgrave, 1984, or articles in Newbery and Stern, 1987, and Gillis, 1989).

*for optimal commodity taxes in the traditional problem without external effects (equation 22 or more generally from 21).*

The proof is given above.

A historical note is worthwhile. Ramsey wrote in the introduction to his (1927) treatment of the traditional problem without externalities: "I shall suppose that, in Professor Pigou's terminology, private and social net products are always equal, or has been made so by State interference not included in the following." Sandmo (1975) followed up to solve the twin tasks thus referred to by Ramsey. In his concluding paragraphs on how to assess real-world taxes, Ramsey wrote: "In the case of motor taxes we must separate off so much of the taxation as is offset by damage to the road. This part should be so far as possible equal to the damage done. The remainder is a genuine tax and should be distributed according to our theory;". Thus, we may say Ramsey had in mind something like Sandmo's 'additivity property'. Another aspect in Sandmo's formula was that the damage component (reflecting benefits of public good provision) is adjusted by the shadow price of public revenue. This adjustment points back to an important idea of Pigou's, that when revenue generation is costly "expenditure ought not to be carried so far as to make the real yield of the last unit of resources expended by the government equal to the real yield of the last unit left in the hands of the representative citizen."<sup>25</sup> Pigou's conjecture later was found to require qualification, but the indicated adjustment is assured if there is separability between the public good and taxed goods (our model) and taxed goods are not predominantly inferior goods (See Atkinson and Stern, 1974).

*Assumption 2: Endogenous emission factors*

When abatement technologies are available for a set  $M$  of polluting goods ( $f_{ib} < 0, i \in M$ ), the system (16) and (17) is at most of rank  $n+m$ . For simplicity, let us assume that the rank is  $2n$  so all  $n$  polluting goods have abatement technologies.

We may immediately substitute the Pigovian tax (20) into (16) and (17) to see that this reduces to a set of  $n$  equations; a formula equal to the one defining the optimal

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<sup>25</sup> Pigou (1947, 1949 reprint, page 34).

commodity tax structure in the traditional problem without external effects.<sup>26</sup> Thus, we may conclude with:

*Proposition 2: Endogenous emission factors and Pigovian taxation*

*A combination of a Pigovian tax (20) and a commodity tax structure satisfying the formula for optimal commodity taxes in the traditional problem without external effects (equation 22, or more generally 21) is optimal also in the context of endogenous emission factors.*

One way of looking at this result is that one introduces a good-specified emission tax to induce abatement. First, this tax is to be the same across polluting goods. Second, in a context of commodity taxes satisfying the formula for optimal taxes in the traditional problem without externalities, this emission tax also induces optimal substitution towards cleaner goods and services. Another way to communicate the result is to suggest that the presumptive Pigovian part of Sandmo's formula be replaced - when possible - by a tax levied on emissions. Viewing Sandmo's formula as a sum of two taxes, the presumptive Pigovian tax is transformed to a Pigovian tax by moving the emission factor from the tax rate to the base of a new tax. This scheme is strictly preferred in a context in which abatement can be induced, and thus optimal in a wider range of circumstances.

*Assumption 3: Emissions are not observed (or not taxable) at the individual level*

We here examine briefly the implications of two crudely defined constraints on the monitoring of emissions. Let us first assume that the planner is not able to tax emissions, but he can regulate abatement and levy commodity taxes. Standards for emission factors (or for abatement) are often seen in the real world, and one interpretation of this is that it is less costly to monitor emission *factors* or technology than it is continuously to monitor emissions (or to obtain a measure of cumulative emissions, say at year-end).<sup>27</sup> Examples

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<sup>26</sup> In an earlier version of this paper, direct derivation of this tax structure was provided. It can be made available upon request.

<sup>27</sup> The existence of emission standards has been given several interpretations. The interpretation compatible with our treatment here is that monitoring emissions at the source is prohibitively costly, but that abatement (or technology, or emission *factors*) can be monitored cheaply *ex ante* (as a car model is approved by the authorities) or periodically (at annual vehicle inspections) or even randomly. Such a structure is analyzed in Eskeland (1994), where it was shown that standards then should be combined with presumptive Pigovian taxes, levied for instance on a car's odometer, or on a variable input, such as gasoline. A more comprehensive practical discussion of these principles in the light of monitoring and enforcement problems

are that vehicles and industries face emission standards which either mandate a particular technology or define a maximal rate for emissions per unit of output (say: grams per mile or per gallon of fuel, for vehicles).<sup>28</sup> Simultaneously, the vehicles may be subject to odometer charges (by mile, or kilometer) or fuel taxes, and industries may be subject to taxes on inputs and outputs.

With these assumptions, the planner has two instruments for each good ( $t_j, b_j$ ), again a total of  $2n$  instruments at his disposal. Modifying the Lagrangian (11) with the applicable budget constraint  $\sum_h \sum_j t_j x_j^h = \sum_j (p_j + b_j^p) x_j^p$  and instruments, we consider government abatement given (see assumption 4, below). The first order conditions for optimum are the budget constraint and, for all  $i=1, \dots, n$ ,

$$(24) \quad - \sum_{h \in H} \beta^h \left[ x_i^h + \alpha^h \sum_{g \in H} \sum_j f_j(b_j) x_{ji}^g \right] + \mu \sum_{g \in H} \left[ x_i^g + \sum_j t_j x_{ji}^g \right] = 0, \text{ and}$$

$$(25) \quad - \sum_{h \in H} \beta^h \left[ x_i^h + \alpha^h \sum_{g \in H} \left( \sum_j f_j(b_j) x_{ji}^g + f_{ib}(b_j) x_i^g \right) \right] + \mu \sum_{g \in H} \left[ \sum_j t_j x_{ji}^g \right] = 0.$$

where we have used  $\partial q_j / \partial b_j = 1, j=1, \dots, n$  (see equation 5).

It is easily checked that if the planner sets abatement or emission standards such that marginal costs equal social benefits adjusted for the shadow price of public revenue:

$$(26) \quad \frac{\sum_{h \in H} \alpha^h \beta^h}{\mu} = \frac{-1}{f_{ib}},$$

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is given in Eskeland and Devarajan (1996). Another important interpretation of standards and regulation in general, as opposed to the tax treatment, is that they give the planner a way to distribute emission permits (See Buchanan and Tullock, 1975, and Baumol and Oates, 1988).

<sup>28</sup> Standards for emission factors and for abatement (or technology) have equivalent implications in our model, but more generally instruments should be as open and flexible as possible. Thus, standards will be more effective, *ceteris paribus*, if they define maximum emission factors than if they specify a technology which meets that goal.

For reviews including discussion of monitoring costs and their consequences, see, for example, Eskeland and Jimenez (1992). Using a model with monitoring costs, Schmutzler and Goulder (1997) conclude "Pure output taxes are optimal under sufficiently high monitoring costs, sufficiently limited options for emission reductions by means other than output reduction, and sufficiently high substitutability of the output".

For cars, a potentially important advantage of odometer charges (relative to fuel taxes) that is not exploited in the literature nor in the real world is that it could implement a system where the corrective tax is raised conditional on vehicle characteristics or emission factors. With fuel taxes, such differentiation will be constrained.



then (25) reduces to (24) times the vector  $f$ , to be satisfied if (24) is satisfied. Then, the formula for optimal commodity taxes including presumptive Pigovian taxes (19, or more generally 18) satisfies (24) and (25).

*Proposition 3: Emission standards and presumptive Pigovian taxation*

- a) If the planner cannot tax emissions, but he can set standards for emission factors (or abatement) and levy commodity taxes, then the optimal allocation is the same as when emission taxes are available. The marginal cost of abatement per unit of emissions is the same across activities, as if driven by the optimal emission tax (20). Commodity taxes will satisfy the formula for optimal commodity taxes including presumptive Pigovian taxes (equation 19, or more generally 18), as in Sandmo (1975).*
- b) If the planner cannot address abatement in any way, then the optimal allocation is one of commodity taxes including presumptive Pigovian taxes (19 or more generally 18) as in Sandmo (1975).*

Part *a*) of Lemma 3 (the allocation is the same as with emission taxes) is seen by noticing that abatement is identical, and that such abatement and the level of presumptive Pigovian taxes result in the same all-inclusive consumer prices and the same public revenue. Part *b*) of Lemma 3 is seen by noticing that when the planner has only  $n$  commodity tax rates as instruments, optimality is characterized only by the budget constraint and the  $n$  equations in (24), equivalent to (18). Under the assumptions of *b*), emission factors are higher and the sum of abatement and Pigovian taxes weigh more heavily in the 'all-inclusive consumer price' than in *a*).

The contribution of Lemma 3 is a modest one, since it is well known that the efficiency properties of a quota for emissions can be the same as those for an emission tax (See, for instance, Baumol and Oates, 1988; or Tietenberg, 1992). What we do here is to introduce a generalizing and a restrictive feature. We generalize by looking at the use of quotas (or standards) in a context with distortionary revenue generation. One of the lessons thus learned is that such a system of optimal standards and presumptive taxes in our model has the same allocative and distributive impacts as a system with emission taxes. On the restrictive side, as we generalize to introduce constraints on monitoring and

enforcement, we assume that these allow a separate policy instrument to make activities less polluting per unit of activity. “Emission quotas” often come in the form of a standard for emissions per unit of output, a fact that has formerly been afforded scant notice and interpretation in the public finance literature.<sup>29</sup> A contribution of Eskeland (1994) was to show that emission quotas of this kind make activities cleaner, but fail to give appropriate incentives to reduced consumption, and thus (under lump sum transfers) should be accompanied by a presumptive Pigovian tax.<sup>30</sup>

Finally, we should emphasize that Sandmo’s (1975) result can be read as holding for any given level of abatement. Building on this, Lemma 3 contributes with a rule for optimal abatement.

*Assumption 4: Producers and government abate and pollute*

*Proposition 4: Production efficiency*

*Optimal abatement is efficient in the sense that the marginal cost of abatement per unit of emissions is the same not only across activities but also across agents: government, households and firms.*

Consider first the case of nonpolluting consumption in which production of good  $j$  involves firms with the same costs, abatement opportunities  $a_j$  and emission consequences  $f_j(a_j)$ . The above results and these combine if we let  $e_j^h$  be the sum of emissions from producers and consumer  $h$ , determined by abatement from producers and consumers,  $e_j^h = f_j(a_j, b_j^h)x_j^h$ . Let us assume that abatement in production influences production-stage emissions (at the car-maker’s smoke-stack, rather than at his customer’s tail-pipe). Then the producer price for good  $j$  (see equation 10) will include not only the producer’s costs of abatement but also his emission taxes,  $\tau_{aj}f_j$ :

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<sup>29</sup> Important textbooks such as Baumol and Oates, 1988, and Tietenberg, 1992, do not mention monitoring costs as possibly favoring (or explaining) emission standards. *Uncertainty* in estimates of benefits and costs is an accepted consideration in quantity instruments versus prices (Weitzman, 1974), but that argument does not rely on costly monitoring of emissions.

<sup>30</sup> One can argue that such emission standards implicitly award emission quotas to operators of polluting processes: You may emit more, but the same amount per unit, if you drive more, or if you produce more steel. That perspective is even more important when existing facilities are ‘grandfathered’ (given more lenient treatment than new ones) in regulations. For analysis of such differential treatment, see Crandall et al. (1986) and Harrington (1997) on automobiles, and Nelson et al. (1993) on EPA’s new source emission standards. Grandfathering has positive and negative connotations: ‘Grandfather clauses allow rents to be shifted to those grandfathered without distorting supply responses’ (Wittman, 1989).

$$(27) \quad p_j = c_j + a_j + \tau_{aj} f_j(a_j).$$

It is easily checked that an emission tax  $\tau_{aj} = \tau_e$  as in (20) is optimal (substitute (27) into (5), simplify by setting consumer emissions and abatement to zero, and modify (11) accordingly).

By the same argument, if two producers of  $j$  are active but have different technologies and emission factors, their emissions are taxed at the same rate in optimum. Note that two (or more) firms with different emission functions and different emission factors can be active at only one level of the emission tax, since if the emission tax is raised slightly, the producer with the higher emission factor shuts down (by the envelope theorem). However, if there are latent technologies, then at any emission tax level technologies with different emission factors can be active, and we have shown they shall be taxed at the same rate per unit of emissions. Thus, the marginal cost of abatement per unit of emissions reduced will be the same in productive sectors as amongst consumers:

$$(28) \quad \frac{-1}{f_{ja}} = \frac{-1}{f_{ia}} = \frac{-1}{f_{jb}} = \frac{\sum_h \beta^h \alpha^h}{\mu}.$$

For government, the Lagrangian (11) assumed that the government consumption vector  $x^P$  as well as the government abatement vector  $b^P$  was given. Modify (11) to reflect a choice of government abatement, and partially differentiate with respect to  $b_j^P$  in addition to the previously applied instruments  $t_e, \tau_e$ . No changes in expressions are implied for the previously established set of first order conditions. For the additional first order conditions reflecting optimal abatement for government, we have for all  $j=1, \dots, n$ :

$$(29) \quad \sum_h \beta^h \alpha^h \frac{de}{db_j^P} - \mu x_j^P = 0.$$

Using  $\frac{de}{db_j^P} = f_{jb}^P x_j^P$ , from (2) and (3) we can see that

$$(30) \quad \frac{\sum_h \beta^h \alpha^h}{\mu} = -\frac{1}{f_{jb}^P}.$$

Thus, in optimum, the marginal costs of abatement per unit of emissions reduced will be equalized across firms, government and households.

As a matter of implementation, if government agencies are geared to pursue their respective goals while maximizing some appropriate “profit” function, then these agencies should be exposed to the same emission tax (or abatement requirements) as the one levied on consumers and firms.

We have now shown that abatement should be stimulated by the same emission tax (or emission standard) when abatement reduces own emissions. The generalization remaining is to allow abatement at any stage to influence emissions or abatement opportunities at other stages as well, as when the car’s emissions can be reduced by abatement efforts in the car-factory, at the service station, in the oil refinery and by the driver. It is intuitive, now, that the emission tax (20) provides optimum stimulus in this more general setting. As producers and consumers join forces to minimize private costs – including emission taxes, emission reductions are provided effectively. Showing this involves additional notation, and is left to the reader.<sup>31</sup>

We are now ready to summarize our findings:

Summary of central findings: Pigovian principles and production efficiency

*Assume constant returns to scale, that the environmental good is separable from other goods, that within each activity consumers have uniform emission functions, that linear taxes on inputs, outputs and emissions can be differentiated by commodity (or that emission standards can be differentiated by commodity), and that different regimes can apply for consumers, producers and government.*

*i) Welfare optimum is characterized by the marginal rates of transformation between abatement and emission reductions a) equal across polluting activities (i.e. goods, sectors,  $j \in N$ ), b) equal for consumers, producers and government, and c) equal to*

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<sup>31</sup> It may be worthwhile to revisit with a practical perspective the issue of the untaxed good. Our formulation states that consumer abatement is through application of the untaxed good. It is this feature which allows equal rates of transformation between abatement and emission reductions to be implemented by one emission tax faced by producers and consumers (since producers face pretax prices). Assume now that leisure has to be the untaxed good and that consumers may abate emissions with leisure (using time to drive more carefully, or to perform more laborious laundry with less polluting detergents) and by changing filters, and that producers may abate by installing filters and through many other actions. If leisure is the untaxed good, and filters can be taxed at zero rates when used in abatement, then the efficient solution can be implemented by confronting consumers and producers with the same emission tax. If filters cannot be taxed at zero rates (when used in abatement by consumers), then the optimal allocation – still equalizing the marginal costs of emission reductions - is implemented by a separate emission tax for consumers.

the welfare weighted sum of willingness to pay across consumers adjusted by the shadow

price of public revenue: 
$$-\frac{1}{f_{jb}} = -\frac{1}{f_{ja}} = -\frac{1}{f_{jb}^P} = \frac{\sum_h \beta^h \alpha^h}{\mu}.$$

ii) When abatement is untaxed and emissions are observable, such abatement can be implemented by a tax levied uniformly on all emissions (20):

$$\tau_e = \frac{\sum_h \beta^h \alpha^h}{\mu}.$$

iii) An emission tax satisfying this formula combined with commodity taxes satisfying the formula for optimal commodity taxes in the traditional problem without externalities (21) implements the optimal allocation.

iv) When emissions are not taxable, but emission standards or abatement standards can be used, the same allocation can be implemented by a combination of emission standards (as in i), above) and commodity taxes that include presumptive Pigovian taxes, as in Sandmo (1975).

v) When abatement cannot be induced by the planner, the optimal allocation is implemented by commodity taxes which include presumptive Pigovian taxes, as in Sandmo (1975).

With pollution just from producers and government, the equality of marginal rates of transformation between abatement and emission reductions is a predictable consequence of Diamond and Mirrlees' (1971) result. They showed that the set of producers and government should be treated as one, to all have equal marginal rates of transformation between goods. This clearly should apply even when an additional input valued by consumers (the environment, equation 1) is included in the model. Amongst the findings of Cremer and Gahvari (1999) is that an emission tax should apply uniformly across industries with homogenous technologies. We add that this holds also for pollution from government, from firms with heterogeneous emission functions, and from consumers - only the latter one of which does not follow almost directly from Diamond and Mirrlees' treatment. Also, we show how standards can take the place of emission taxes under some plausible restrictions on monitoring.

The result least to be expected, that production efficiency shall include pollution abatement amongst consumers, must be understood in a context of assumptions about

available instruments. Also, that result depends on the assumption that consumers face the same abatement opportunities, implying that they have the same emission factors when exposed to the same emission taxes. We relax this assumption in the section to follow.

#### IV. Non-Uniform Emission Functions

In the previous section, we established that producers shall be taxed uniformly on emission irrespective of whether they have uniform emission functions. For consumers, the analysis till now has assumed uniform emission functions. We now investigate the consequences of heterogeneity across consumers in emission functions, reintroducing individual superscripts for emission functions, and thus abatement:  $f_j^g(b_j^g)$ .

We should note that the additivity in the relationship between emissions and the environmental good (equation 3) is retained. What we now allow implies only that consumers may differ in terms of emissions per unit consumed of the polluting good. An alternative formulation - also important in practice - could be that polluters differ in the relationship between emissions and the environmental good (so the damages could differ per unit emitted, rather than per unit consumed, which is our formulation). Results would be very similar in nature, though with qualifications regarding instrument availability. The reason is that an emission tax is still “first best” with respect to environmental protection when emission functions differ. If damages per unit emitted are different, in contrast, the emission tax is first best only if each polluter can be taxed at the same rate per unit of damages. With presumptive Pigovian taxes levied on each unit of the polluting good, the parallel is more direct, since that instrument loses its first best properties in both formulations.

Uniformity of emission functions for a given commodity is more plausible the more narrowly one can define each polluting commodity. Examining the model, this is a question of whether consumption with different emission functions can be differentiated in the commodity tax structure. If consumption *can* be differentiated in the commodity tax structure, so that within each “commodity” uniform emission functions result, then the results of the previous section apply.

To give a practical example, assume first that emissions are taxable, and that car travel is more polluting when using leaded gasoline than when using unleaded, but with

emission functions that are uniform among users of leaded gasoline, and among users of unleaded gasoline. If the two fuels *can* be taxed separately in the commodity tax structure, then the results of the previous section apply. Assume in contrast, that emission functions differ by car or user characteristics (old versus new/young, male versus female). To the extent that commodity taxes cannot be conditioned on these (perhaps they could, if odometer charges were used), one has set the scene for the topic of this section.<sup>32</sup>

We assume polluters are exposed to non-individualized linear instruments: commodity taxes and emission taxes or uniform abatement requirements,  $b_j^h = \bar{b}_j$ . Under regulation, then, abatement is uniform by assumption, and emission factors may differ if emission functions are heterogeneous. Under an emission tax, consumers equalize marginal abatement costs  $-1/f_{jb}^h(b_j^h(\tau_{ej})) = -1/f_{jb}^g(b_j^g(\tau_{ej})) = \tau_{ej}$  (re equation (6)), and abatement as well as emission factors may differ if emission functions are heterogeneous.

As an important background, in a setting with costless redistribution and revenue generation, an emission tax is a first-best corrective instrument even when emission functions are heterogeneous. In contrast, presumptive taxation of goods would be an imperfect corrective instrument under heterogeneous emission functions. This difference should be on our mind as we set out to analyze the cases with and without emission taxes separately.

#### *Emission taxes available*

Corresponding to (16) and (17), our first order conditions for optimum are, for all  $i=1, \dots, n$ :

$$(31) - \sum_{h \in H} \beta^h \left[ x_i^h + \alpha^h \sum_{g \in H} \sum_j f_j^g x_{ji}^g \right] + \mu \sum_{g \in H} \left[ x_i^g + \sum_j (t_j + \tau_{ej} f_j^g) x_{ji}^g \right] = 0, \text{ and}$$

<sup>32</sup> On empirical aspects of vehicle characteristics and emission factors, see Eskeland (1994), Innes (1996) and Harrington (1997). In Eskeland and Kong (1998), the distributional consideration is examined in detail. One stylized fact found is that the expansion path in household energy use is toward energy carriers with lower emission factors (say, from coal and wood to electricity and natural gas). Another is that emission factors are lower for newer equipment, both because designs improve with vintage and because of age- and use- deteriorating functions (emission control, combustion).

$$(32) \quad - \sum_{h \in H} \beta^h \left[ f_i^h x_i^h + \alpha^h \sum_{g \in H} \left( f_{ib}^g b_{ir}^g x_i^g + \sum_{j \in N} f_j^g x_{ji}^g f_i^g \right) \right] \\ + \mu \sum_{g \in H} \left[ f_i^g x_i^g + \tau_{ei} (f_{ib}^g b_{ir}^g x_i^g) + \sum_j (t_j + \tau_{ej} f_j^g) x_{ji}^g f_i^g \right] = 0.$$

The reader may verify that straightforward application of Pigovian principles

( $\tau_{ei} = \sum \beta^h \alpha^h / \mu$ , all  $i$ ) leaves (31) solved by commodity taxes satisfying the formula for optimal commodity taxes in the traditional problem without pollution, but that this emission tax is inconsistent with solving the set as a whole with only  $n$  remaining instruments. Thus, emission taxes cannot in general comply with this simple Pigovian principle when emission functions are heterogeneous across consumers. We proceed to qualify and interpret these deviations from Pigovian principles.

Without loss of generality, let us split the taxes levied on emissions in (31) and (32) in two parts: one "environmental tax"  $\tau_e$  which we set according to  $\tau_e = \sum \beta^h \alpha^h / \mu$ , and a supplementary emission tax (or subsidy)  $\tau_i$  which we leave for further investigation:

$$(33) \quad \tau_{ei} \equiv \tau_e + \tau_i,$$

$$(34) \quad \tau_e = \sum_h \beta^h \alpha^h / \mu.$$

Also, to simplify exposition, let us introduce the following expression (it is the derivative of revenue from consumer  $g$  with respect to  $t_i$ , except the part  $\tau_e \sum_g \sum_j f_j^g x_{ji}^g$ ):

$$(35) \quad \frac{\partial R^g}{\partial t_i} \equiv \left[ x_i^g + \sum_j (t_j + \tau_j f_j^g) x_{ji}^g \right].$$

Substituting (33), (34) and (35) into (31) and (32), we have, for all  $i=1, \dots, n$ ,

$$(36) \quad \sum_h \left( \beta^h x_i^h - \mu \frac{\partial R^h}{\partial t_i} \right) = 0, \text{ and}$$

$$(37) \quad \sum_{h \in H} \left( \beta^h f_i^h x_i^h - \mu \left[ \frac{\partial R^h}{\partial t_i} f_i^h + \tau_i f_{ib}^h b_{ir}^h x_i^h \right] \right) = 0.$$



(37) can be rewritten using averages and covariances across consumers as follows:

$$(38) \quad \bar{f}_i \sum_{h \in H} \left( \beta^h x_i^h - \mu \frac{\partial R^h}{\partial t_i} \right) - H \left[ \overline{\mu \tau_i f_{ib} b_{ir} x_i} - \text{Cov}(\beta x_i, f_i) + \mu \text{Cov} \left( \frac{\partial R}{\partial t_i}, f_i \right) \right] = 0.$$

We can see that if the covariances in (38) are zero, then we can set  $\tau_i = 0$ , all  $i$ , and each equation in (38) is simply  $\bar{f}_i$  times (36). Thus, when those covariances are zero, emissions are taxed according to the Pigovian principle (34) only, and a commodity tax structure which solves (36) is optimal. When  $\tau_i$  is zero for all polluting goods, (36) is also the solution to the traditional optimal commodity tax problem (i.e. without pollution).

More generally, let us observe that (38) is a sum of two terms, where the first is simply  $f$  times (36), so we may think of the optimal tax structure as follows. (36) is at most of rank  $n$ , so the  $n$  commodity tax rates can be reserved to solve (36), conditional on a set of supplementary emission tax rates. Thus, the system (36) and (38) has a solution for which the supplementary emission tax rates render zero the bracket term in (38). Assuming that  $\overline{\mu f_{ib} b_{ir} x_i} \neq 0$ , and using

$\text{Cov}(f_i, \partial R / \partial t_i) = \text{Cov}(f_i, x_i) + \sum_j t_j \text{Cov}(f_i, x_{ji}) + \sum_j \tau_j \text{Cov}(f_i, f_j x_{ji})$ , the term in brackets of (38) is zero for

$$(39) \quad \tau_i = \frac{\text{Cov}(f_i, \beta x_i) - \mu \left[ \text{Cov}(f_i, x_i) + \sum_j t_j \text{Cov}(f_i, x_{ji}) + \sum_j \tau_j \text{Cov}(f_i, f_j x_{ji}) \right]}{\overline{\mu f_{ib} b_{ir} x_i}},$$

all  $i=1, \dots, n$ . This is no explicit solution: not only are there tax rates on the right hand side, but all the expressions may be functions of the tax rates. Nevertheless, from (39) we learn that it is a specific set of covariances that gives a potential role to “non-Pigovian” supplementary taxation of emissions. To gain some additional insight, let us make the assumption that only one activity is polluting:  $\bar{f}_j = 0$ ,  $j \neq i$ , and assume that

$$1 + \frac{\mu \text{Cov}(f_i, f_i x_{ii})}{\overline{f_{ib} b_{ir} x_i}} \neq 0:$$

$$(40) \quad \tau_i = \frac{\text{Cov}(f_i, \beta x_i) - \mu \left[ \text{Cov}(f_i, x_i) + \sum_j t_j \text{Cov}(f_i, x_{ji}) \right]}{\mu \left( \overline{f_{ib} b_{ir} x_i} + \text{Cov}(f_i, f_i x_{ii}) \right)}.$$

(40) is still a complicated combination of effects, but all intuitively play a role given that the social planner compares the effects of supplementary emission taxes to the effects of commodity taxes. The two terms in the denominator represent the responsiveness of abatement and emissions to the emission tax. These responses are, from a first-best perspective, wasteful when emission taxes differ from Pigovian principles, so the absolute value of their sum *ceteris paribus* reduces the absolute value of the supplementary emission tax. In the numerator, the first covariance represents the planner's evaluation of the distributive pattern of the emission tax (as compared to the commodity tax). As an illustration, assume that the denominator is negative ( $\overline{f_{ib} b_{ir} x_i} < 0$ , by assumption) and that the bracket term is zero. If the emission factor falls with income (as when wealthier have newer cars and these are less polluting) and  $\beta x_i$  falls (rises) with income, the supplementary non-Pigovian emission tax will be negative (positive).<sup>33</sup>

The combined term in brackets distinguishes between the emission tax and the commodity tax in terms of the marginal effect on revenue. If the bracket term is negative, then it means that increasing the emission tax on good  $i$  raises revenue less than  $f_i$  times a change in  $t_i$ , an effect which *ceteris paribus* reduces the emission tax (assuming the denominator  $< 0$ ).

To focus on revenue and redistributive considerations, assume that the denominator is negative and the tax weighted term with demand responsiveness in (40),  $\sum_j t_j \text{Cov}(f_i, x_{ji})$  is zero: For  $\tau_i$  to have a determined sign *a priori*,  $\text{Cov}(f_i, \beta x_i)$  and  $\text{Cov}(f_i, x_i)$  must be of opposite sign. For a normal good, a "steep"  $\beta$  is sufficient to ensure a sign, and the sign is given by whether  $f_i$  is increasing or declining with income. Giving a practical illustration, emission factors will often be declining in income. Assuming  $\beta$  steep enough that  $\text{Cov}(f_i, \beta x_i)$  is positive even though  $\text{Cov}(f_i, x_i)$  is negative, these effects lead to a downward adjustment in emission taxes from Pigovian levels.

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<sup>33</sup> Intuition: In case  $\beta x_i$  and  $f_i$  fall with income, the poor are hurt more by the emission tax than by the commodity tax. A slight change in taxation from emissions to the commodity redistributes from rich to poor.

Let us finally focus on the possible covariance between the emission factor and demand responsiveness. To illustrate simply, assume that in (40) all cross price elasticities are zero, and that  $Cov(x_{ii}, f_i) > 0$ , so that the more polluting consumers are less responsive in their demand. Assume further that the denominator is negative and that the other covariances are zero. If  $t_i$  is positive (negative), then non-Pigovian taxation of emissions is positive (negative). In this case, the planner takes the opportunity for “price discrimination” simply to reduce distortions (this effect does not depend on the vector  $\beta$ ): If more polluting consumers are less price responsive (for good  $i$ ), they should face a higher effective price for reasons known in the literature on Ramsey-pricing. A distortionary effect of this is that emissions are taxed “too heavily”, so too much abatement is executed.

We should highlight again that these perspectives often will point us back to ask for a more differentiated commodity tax structure, rather than to actually modifying emission taxes with non-Pigovian objectives. When emission taxes are brought to differ from Pigovian principles, here, it is because they take on roles in redistribution and revenue generation that are left unsolved by other instruments. Using emission taxes for these purposes have separate, identifiable costs, and can be attractive only if other available instruments entail costs as well.

#### *Presumptive Pigovian taxes*

In the case of non-uniform emission functions, taxation of commodities in presumption of emissions has a weaknesses in addition to the potential weakness of not inducing abatement: consumption by dirtier consumers and cleaner consumers is discouraged with ‘equal pressure’. This may present a problem of fairness and distribution, but also of efficiency, since the emission factor determines the emission reductions ‘bought’ when consumption is reduced.

Let us initiate this analysis by assuming that emission factors are given. This problem is similar to the problem of imperfect corrective pricing analyzed by Diamond (1973).<sup>34</sup> Our results compare with studies on indirect Pigovian instruments (substitutes

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<sup>34</sup> In the literature, ‘corrective taxation’ is used synonymously with ‘Pigovian taxation’, and corrective pricing refers to prices that include corrective elements, equivalent to prices that include our ‘presumptive Pigovian taxes’. Diamond’s problem is more general than ours in the sense that he makes no separability

and complements to the polluting good) when the ideal corrective instrument is not available.<sup>35</sup> Analogously to (16), first order conditions for optimal commodity taxes are:

$$(41) \quad - \sum \beta^h \left[ x_i^h + \alpha^h \sum_g \sum_j f_j^g x_{ji}^g \right] + \mu \left[ \sum_g x_i^g + \sum_j (t_j + t_{ej}) x_{ji}^g \right] = 0, \quad i=1, \dots, n.$$

In (41), we have followed steps in previous sections to 'artificially' split the commodity tax rates in two parts. The system is then indeterminate, and we can choose one part arbitrarily. Let us choose  $t_j$ ,  $j = 1, \dots, n$  such as to solve the traditional problem of optimal commodity taxes when there is no pollution:

$$(42) \quad - \sum_h \beta^h x_i^h + \mu \sum_g \left[ x_i^g + \sum_j t_j x_{ji}^g \right] = 0, \quad \text{all } i = 1, \dots, n.$$

Then, for (41) to be solved, we must have

$$(43) \quad \sum_h \beta^h \alpha^h \sum_g \sum_j f_j^g x_{ji}^g = \mu \sum_j t_{ej} \sum_g x_{ji}^g, \quad i = 1, \dots, n.$$

Assuming that the coefficient matrix in (43) is nonsingular, we may use Cramer's rule to develop more explicit expressions. For the two-good case the presumptive Pigovian tax on good one is

$$(44) \quad t_{e1} = \left( \frac{\sum_g f_1^g (x_{11}^g \overline{x_{22}} - x_{12}^g \overline{x_{21}})}{H(x_{11} x_{22} - x_{12} x_{21})} + \frac{\sum_g f_2^g (x_{21}^g \overline{x_{22}} - x_{22}^g \overline{x_{21}})}{H(x_{11} x_{22} - x_{12} x_{21})} \right) \frac{\sum_h \beta^h \alpha^h}{\mu},$$

where the denominator is positive.

Corresponding formulas, for  $t_{e2}$  or for systems with more goods are straightforward to derive. We may consider a system consisting of traditional commodity taxes (42) and presumptive Pigovian taxes (44) as a generalized version of commodity taxes which include presumptive Pigovian taxes (19). The large parenthesis in (44) then plays the role of the emission factor  $f_1$ , and the first fraction in this parenthesis is indeed a

assumption, so his 'public good' (absence of congestion) may influence demand. On the other hand, our problem is more general in including distortionary revenue generation, and in including effects across markets in the external effects as well. These differences are illustrated in one of his concluding passages: "...the optimal surcharge will be small relative to the average externality when individuals who contribute greatly to congestion per unit demanded .. tend to have demands which are congestion sensitive .. and price insensitive ..". In our model, the analogue to congestion sensitivity is zero due to separability (the case for such sensitivity is more compelling for congestion), and our results with respect to price sensitivity will be less clear cut, due to cross price effects both in revenue generation and in external effects.

<sup>35</sup> Notable contributions are Green and Sheshinski (1976), Sandmo (1976b), Balcer (1980), Wijkander (1985), Greenwald and Stiglitz (1986).

weighted average for  $f_1$ , which equals  $\bar{f}_1$  if the covariances  $Cov(f_1, x_{11})$  and  $Cov(f_1, x_{12})$  are zero (as when  $f_1$  is uniform). The second fraction in the parenthesis represents emission spillovers via cross-price elasticities to good 2, and is zero if the covariances  $Cov(f_2, x_{21})$  and  $Cov(f_2, x_{22})$  are zero. Interestingly, good 1 may be taxed with reference to the Pigovian objective even if only good 2 is polluting, a result observed in the literature on indirect instruments (see below). To focus on covariances, let us rearrange. (44) =>

(45)

$$t_{e1} = \left( \bar{f}_1 + \frac{\bar{x}_{22}[Cov(f_1, x_{11}) + Cov(f_2, x_{21})] - \bar{x}_{21}[Cov(f_1, x_{12}) + Cov(f_2, x_{22})]}{x_{11}x_{22} - x_{12}x_{21}} \right) \frac{\sum \beta^h \alpha^h}{\mu}$$

The four covariances all are between an emission factor and the price responsiveness of the good to which it applies. If we assume that own price elasticities are negative and cross price elasticities are positive, then positive covariances result in a tax rate lower than under unweighted average emission factors. The reason for this is that positive covariances raise the marginal cost of emission reductions relative to that indicated by the average coefficient (illustration:  $Cov(f_1, x_{11}) > 0 \Rightarrow Cov(f_1, |x_{11}|) < 0$ , so individuals with high emission factors adjust consumption less than average).

To enhance intuition further and to compare with the literature on indirect Pigovian instruments, let us consider again the case with two taxed goods and assume that good 1 is not polluting. The Pigovian parts of the commodity tax rates in this context are

$$(46) \quad t_{e1} = \frac{\bar{x}_{22}Cov(f_2, x_{21}) - \bar{x}_{21}Cov(f_2, x_{22})}{x_{11}x_{22} - x_{12}x_{21}} \frac{\sum \beta^h \alpha^h}{\mu}, \text{ for the nonpolluting good, and}$$

$$(47) \quad t_{e2} = \left[ \bar{f}_2 + \frac{x_{11}Cov(f_2, x_{22}) - x_{12}Cov(f_2, x_{21})}{x_{11}x_{22} - x_{12}x_{21}} \right] \frac{\sum \beta^h \alpha^h}{\mu}, \text{ for the polluting good.}$$

Our model has greatest similarity with that of Balcer (1980). The literature we refer to does not include distortionary revenue generation, but compares with our formula for the presumptive Pigovian tax which bares evidence of distortionary taxation only through  $\sum \beta \alpha / \mu$ . Balcer focuses on the dimensions of “large offenders” (consumers  $g$ , for whom  $f_2^g > \bar{f}_2$ , in our terminology) and “large offender complementarity” ( $Cov(f_2, x_{21}) < 0$ ).

If we make the assumption of 'aggregate independence' in demand ( $\overline{x_{12}} = \overline{x_{21}} = 0$ ), we can tabulate results analogous to some of those in Balcer's table 1:<sup>36</sup>

Table 1: Presumptive Pigovian Taxation  
under Aggregate Independence ( $\overline{x_{12}} = \overline{x_{21}} = 0$ )

Assumption for "large offenders":	Own -Price Responsive $Cov(f_2, x_{22}) < 0$	Own-price Neutral $Cov(f_2, x_{22}) = 0$	Own-Price Non-responsive $Cov(f_2, x_{22}) > 0$
Result for Direct Instrument	$t_{e2} > \frac{\overline{f_2} \sum \beta \alpha}{\mu}$	$t_{e2} = \frac{\overline{f_2} \sum \beta \alpha}{\mu}$	$t_{e2} < \frac{\overline{f_2} \sum \beta \alpha}{\mu}$
Assumption for "large offenders"	Complementarity $Cov(f_2, x_{21}) < 0$	Neutrality $Cov(f_2, x_{21}) = 0$	Substitutability $Cov(f_2, x_{21}) > 0$
Result for Indirect Instrument	$t_{e1} > 0$	$t_{e1} = 0$	$t_{e1} < 0$

The results under aggregate independence are quite intuitive. As examples, for the direct instrument  $t_{e2}$  the tax is raised by own - price responsiveness for large offenders, since this reduces the costs of emission reductions when the price is raised equally for all. For the indirect instrument,  $t_{e1}$ , the level will be negative if there is large offender substitutability, since a subsidy then reduces consumption of good 2 amongst large offenders but not for average offenders, thus providing emission reductions at low costs. As an illustration, assume one group (say, the young) would substitute metro- for car travel if metro fares were lower, but that for the old the two are complements, so that aggregate demand for car travel is independent of metro-fares. If young people pollute

<sup>36</sup> Note: Additional results in the absence of aggregate independence are found by examining (46) and (47). Examples: Average complementarity ( $\overline{x_{12}} < 0$ ,  $\overline{x_{21}} < 0$ ): the direct instrument  $t_{e2}$  will be raised (reduced), if there is large offender substitutability (complementarity). The indirect instrument  $t_{e1}$  will be raised (reduced) if large offenders are own-price nonresponsive.

more than old when traveling by car, the commodity tax rate for metro travel would be adjusted downwards for Pigovian reasons.<sup>37</sup>

Our results for the indirect instrument,  $t_{e_1}$ , are similar to those of Balcer, though his results are sharper due to more restrictive assumptions.<sup>38</sup> For the direct instrument,  $t_{e_2}$ , we report how the tax level compares to  $\bar{f}_2 \sum \beta^h \alpha^h / \mu$ , whereas Balcer compares to the tax level without any taxation of the associated good (i.e.  $t_{e_1} = 0$ ). For this reason, his results are not qualified by own-price responsiveness ( $x_{22}$  and  $Cov(f_2, x_{22})$ ).

*Emission standards and commodity taxes including presumptive Pigovian taxes*

Equation (41) defines optimal commodity taxes including presumptive Pigovian taxes for any given abatement levels. If we assume that the planner can regulate abatement but must do this uniformly for all consumers (though emission functions differ), then first order condition for optimal abatement are, for all  $i=1, \dots, n$ :

$$(48) \quad - \sum_n \beta^h \left[ x_i^h + \alpha^h \sum_g \left( f_{ib}^g x_i^g + \sum_j f_j^g x_{ji}^g \right) \right] + \mu \sum_g \left[ \sum_j (t_j + t_{ej}) x_{ji}^g \right] = 0.$$

Commodity taxes are optimal, so we may subtract (43), to obtain

$$(49) \quad - \sum_n \beta^h \alpha^h \sum_g f_{ib}^g x_i^g - \mu \sum_g x_i^g = 0.$$

This results in ( $f_{ib}^g x_i^g < 0$  by assumption)

$$(50) \quad \frac{\sum_h \beta^h \alpha^h}{\mu} = - \frac{\sum_g x_i^g}{\sum_g f_{ib}^g x_i^g} = \frac{-\bar{x}_i}{x_i f_{ib} + Cov(f_{ib}, x_i)},$$

which simplifies to (26)  $\sum_n \beta^h \alpha^h / \mu = -1/\bar{f}_{ib}$  if the covariance between consumption of the polluting good and the marginal cost of abatement is zero. (50) reflects a rather

<sup>37</sup> Little systematic knowledge exists about demand responsiveness for polluting goods, let alone for disaggregate groups. In practical discussions of the air pollution control program for Mexico City (Eskeland, 1994), the responsiveness of demand for travel, including how it might vary by groups of vehicles, was one of the issues on the agenda. Eskeland and Feyzioglu (1997b) estimated responsiveness in demand for gasoline and vehicles in Mexico. Eskeland and Feyzioglu (1997) found very unfortunate consequences of a scheme to ration trips in Mexico City: Households bought additional, used cars at unexpected rates to circumvent the regulation. Initially, the program rationing car use was seen as politically attractive because of its assumed distributional implications.

<sup>38</sup> Balcer has zero income effects, so  $x_{12} = x_{21}$ . His results on the indirect instrument are not qualified by aggregate independence.

intuitive relationship between the instrument at hand and the costs of emission reductions. The planner has to ask high consumption individuals to abate proportionally more than average consumption individuals (they must abate equally per unit). If the covariance  $Cov(f_{ib}, x_i)$  is negative (positive), so that high – consumption individuals have low (high) marginal costs, then optimum is found in a point with – *ceteris paribus* – higher (lower) abatement standards and lower (higher) emissions than if the covariance were zero.

Concluding, when presumptive taxes and standards are used, covariances with demand patterns influence instruments, but only for reasons related to efficiency in environmental protection. The demand patterns are important for efficiency because - for both instruments - they determine the marginal costs of reducing emissions when these instruments are unable to equalize costs across consumers. The reason why distributional considerations do not directly affect these instruments under nonuniform emission functions (though they do for emission taxes) is simply that in this model neither instrument can do anything towards redistribution that commodity taxes cannot.

## V. Discussion

Our aim with this study was to examine whether intuition about ‘pricing’ the environment applies in more general contexts than explored earlier. Does Sandmo’s “additivity property” (1975) apply in such a way that different polluting activities be treated in the same fashion? If negative externalities can be reduced not only by changes in consumption patterns, but also by making each activity cleaner (abatement efforts), how shall optimal policy combine inducements to these various approaches? Finally, if negative externalities are caused by agents as different as consumers, producers and government, how does optimal policy combine efforts from these to reduce pollution?

Three assumptions are critical when we show that the marginal rates of transformation between abatement and emission reductions shall be the same across activities (goods, sectors) and across polluters. The assumption of constant returns to scale is widely applied in the literature, and is required in the present context since we want to see how established results on production efficiency extend. Second, we assume that consumers have equal access to pollution abatement opportunities (but also examine results of relaxing this assumption). Third, we assume that the planner can differentiate



his policy instruments (emission taxes or abatement standards) by polluting good, and by whether the polluter is a consumer, a producer or government, but he cannot differentiate such instruments - or the commodity taxes - by personal characteristics, or make them non-linear in individual emissions.

Comparing our results with Sandmo's results, they represent generalizations that are very simple at a formal level: One may replace the presumptive Pigovian part of his commodity tax rates with an emission tax applied uniformly across agents and goods: The emission factors that are part of the expression for Sandmo's tax rates will now form the base of an emission tax. Such a tax, combined with commodity taxes that satisfy the formula for optimal taxation in the traditional problem without external effects, induces optimum substitution towards less polluting activities as well as optimal abatement everywhere.

The paper adds that the applicability of these principles is not limited to contexts in which emissions are monitored at the source. Emission standards (or abatement standards) may be implemented with more limited monitoring capabilities (car model certifications, for instance), and a combination of emission standards and commodity taxes that include presumptive Pigovian taxes can under the applied assumptions implement the same allocation as the one implementable by commodity taxes and emission taxes.

The results also extend the production efficiency result of Diamond and Mirrlees, to include efficiency in environmental protection. For polluters that are producers and in government, production efficiency (in a sense that includes equal marginal rates of transformation between abatement and emission reductions) is to be expected. As an additional public good – the environment – is included in the relevant input-output vector, the result that the optimal vector is at the aggregate production frontier prevails.

When production efficiency applies also for polluting consumers – in the sense that they too shall abate pollution at the same marginal rate of transformation as firms and government – it is more surprising. One might expect that the planner – in his desire to redistribute or collect revenue at minimal distortionary costs - would choose to apply different pressures to abate pollution in different activities in order to pursue these goals. When consumers have equal access to abatement technology, however, emission taxes differentiated according to polluting activity (i.e. goods) are redundant instruments for

redistribution and revenue generation. They have the same dimensionality as commodity taxes, and commodity taxes dominate since they do not induce additional resource costs by making abatement deviate from efficient patterns.

When emission functions differ across producers, no deviation from Pigovian principles result. The consequences when emissions functions are heterogeneous across consumers are of two kinds, both related to covariances between emission factors and consumption patterns and demand responses. First, when emission taxes are available, the planner has a Pigovian instrument that is first best, and deviations from Pigovian principles come if the differentiated pattern of emission taxes and abatement costs lend themselves to his goals of redistribution and revenue generation. Second, when emission taxes are not available but emission standards are used, non-uniformity of emission functions influence policy because the instruments are no longer first best from a Pigovian perspective. These resulting adjustments are related merely to the objective of correcting externalities at least cost – not to revenue generation or redistribution. As an example, if for a good  $j$  marginal costs of emission reductions covary negatively with consumed quantities, then this enhances the cost effectiveness of the emission standard, relative to the case with no covariance.

Finally, simplicity in principles in this case also seems to simplify implementation. Think about how to stimulate pollution reductions from those making cars, roads, tires and fuels, and from those using cars. First, it simplifies implementation that the stimulus given to abatement at one stage (say at the factory) is independent of whether the abatement yields i) emission reductions at that stage (the factory), ii) emission reductions at some other stage (in the refinery, in the commute), or iii) enhanced abatement opportunities at some other stage (the refinery, the commute). This allows abatement efforts at all stages optimally to be stimulated by a uniform emission tax levied where emissions occur. Second, it simplifies things that optimal abatement is independent of whether the car is used by government, firms or households, for weddings or for work.

The outlined principles could be helpful also in simplifying the organization of intervention for revenue and environmental protection, and perhaps in reducing the scope for wasteful political battles in environmental policy making. As an illustration, notice that the emission tax that induces optimal abatement in its formula refers only to benefits of environmental protection, not to price elasticities for polluting goods. Nevertheless, it

also induces optimal substitution towards less polluting goods, in the sense that this emission tax should be combined with commodity tax rates satisfying the formula for optimal taxation in the traditional problem without external effects. Thus, at a very intuitive level, the environmental minister is concerned about pricing the environment - and the finance minister may think about him as such. The revenues will contribute to the general treasury and thereby influence the shadow price of public revenue. *Thereby*, the environmental minister's agenda influences the optimal commodity tax problem of the finance minister, but the finance minister need not think about the environmental costs or opportunities in each activity. Similarly, the environmental minister need not think about whether he - when taxing polluting sectors - tax sectors that are important for other reasons, such as revenue or redistribution.

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## **Essay III**

### **The irrelevance of the cost of funds when public provision – or the environment - benefits production**

Abstract:

Pigou's conjecture -- that optimal public provision is lower when taxation is costly -- does not apply when the program provides cost reductions for industries, rather than pure public goods. The reason is that benefits accruing to productive sectors can be taxed, while provision of public goods is reduced - as if they were.



### Summary

Assume that a public program - whether in the form of government provision or as environmental protection induced by the government's emission taxes - provides not only a public good to consumers but also a collective input (say, for brewers: a less polluted water source, or better roads for their trucks). In a context of optimal taxation and constant returns to scale, we show that only the direct benefits to consumers in terms of a public good are adjusted by the shadow price of public revenue (typically by downward, as Pigou conjectured) before benefits are aggregated to establish optimal provision. This holds also for optimal environmental protection, which is our example. Put differently, while the Samuelson (1954) condition for optimal provision of public goods applies only when taxation is costless, the analogous rule for collective inputs applies even under costly revenue generation.

One example is greenhouse gas limitations, believed in large part to benefit agriculture. Another is infrastructure investments such as roads, to the extent that they provide services to firms, rather than public goods for households. Optimal provision in these types of programs requires that marginal costs are equal to the cost savings for productive sectors - in a way independent of the shadow price of public revenue. The intuition is that output markets enable the government to capture these cost savings through taxation, so it is not worth it to have inefficient programs for revenue reasons.

This result is independent of whether the good in question is provided by government (widening a road, say) or it is damaged/generated by firms or households, rich or poor (say, emitting pollution/engaging in pollution control). Also, it is independent of whether the final users of the outputs from the benefiting industries are rich or poor, government or firms.

Apart from being of policy relevance in itself, our results assist in the interpretation of established principles. The well-known adjustment of benefits in the form of public goods plays the role of taxing a good that otherwise would not be taxed (King, 1986). In contrast, benefits in the form of cost savings in production are derived from demand in markets that can be taxed directly, and thus need no such adjustment. Christiansen (1981) makes a finding much like ours - but with a very different twist on standard assumptions.

Interestingly, the intuition that commodity taxes make the benefits taxable supports both findings.

Results along lines similar to the ones shown here are given in Bovenberg and van der Ploeg (1994) and in Williams (2000). Amongst the contributions of the present paper is to highlight how these results are closely related to Diamond and Mirrlees' (1971) result on production efficiency. This link to central questions in the optimal tax literature is seen more easily in our treatment since our analysis is more general, and in particular because we treat in a parallel fashion programs pursued through government expenditures and programs pursued by soliciting efforts from the private sector (as with pollution abatement). For applied analysis as well as for principles, an advantage of our approach is to highlight when the tax system is better suited for redistribution than is the modification of public programs.

## I. Introduction

In the public finance literature, environmental quality is typically introduced as a pure public good in the sense of Samuelson (1954), directly represented in consumer preferences.<sup>1</sup> This public good is provided at a certain level by nature and in addition influenced by human activities, most typically depleted as a negative external effect of consumption and/or production activities. Important contributions that illustrate this approach are Sandmo (1975) and - more recently - Cremer et al. (1998).

In our treatment, there are a number of parallels between provision via public expenditures – e.g. to procure public goods – and provision via other government powers, as when regulation and taxation is used by the government to protect the environment.<sup>2</sup> We shall use the term *public program* for the general case when provision is either by government directly – as with a public road – or induced by government policies – as when government acts with emission standards or taxation to stimulate emission reductions from the private sector.

Our more important generalization, however, is in terms of how the provided items are useful. We let the public program generate cost reductions to producers in addition to the benefits enjoyed directly by consumers in the form of a pure public good. We conduct this analysis while describing the public program as providing an environmental good - i.e. addressing a negative externality by inducing emission reductions. However, our central result applies equivalently when a good that is collectively available to households and/or producers is provided directly by government.

It is not new to suggest that publicness - in the sense of nonrivalry amongst beneficiaries - can apply to a factor of production as well as to goods that play a role in

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<sup>1</sup> Samuelson's term was *collective consumption goods* "which all enjoy in common in the sense that each individual's consumption of such a good leads to no subtraction from any other individual's consumption of that good". Later, the terms "pure public good" has come to be accepted for goods that are non-rivalrous in consumption, and terms like "non-rivalry" or "non-exhaustability" are often used as criteria for pure public goods (see, for instance, Laffont, 1988). Interestingly, in Samuelson's article, the word 'pure' was used to qualify his contribution to pure theory, not to describe the type of goods in question.

<sup>2</sup> One of these parallels are shown in Eskeland (2000), in which the environment is a pure public good. Pollution abatement by the firms (firms, households) and by government is shown in optimum to yield the same marginal emission reductions. For firms and government, this is also an implication of Diamond and Mirrlees' (1971) result on aggregate production efficiency. Government abatement is equivalent to provision of a public good via public expenditures. Thus, externalities and government provided public goods differ in the means of intervention but not in the more basic optimality conditions, such as the wedge – if any - between marginal rates of transformation and marginal benefits.

consumer preferences. First, as A. C. Pigou introduced what later came to be known as “external effects”, or externalities, his words were: “a person A, in the course of rendering some service, for which payment is made, to a second person, B, incidentally also renders services” .. “to other persons (not producers of like services), of such sort that payment cannot be exacted from the benefited parties...”. Thus, with his parenthesis, Pigou included producers in his notion of beneficiaries (Pigou, 1932).<sup>3</sup> Further, in his concrete examples, the affected third parties were producers whose costs are affected as well as households whose wellbeing are affected: The lighthouse generates benefits to ships, and smoke from a chimney results in nuisance and higher costs for neighboring households as well as firms (Pigou 1932, pages 183-184; 1947, pages 94-95). Second, when Meade (1952) gave name to the influential concept of “atmospheric externalities”, he used effects between producers as examples. Third, among others, Sandmo (1972) and Oakland (1987) have treated “collective factors” of production as a case parallel to pure public goods. In a context without costly revenue generation, the result is an intuitively appealing rule for optimal provision analogous to the Samuelson condition (1954) for optimal provision of a pure public good, with vertical summation over beneficiaries of marginal benefits.

In a context of costly revenue generation, in contrast, treatments of external effects and of optimal provision have emphasized pure public goods directly represented in consumer preferences, as opposed to collective factors of production. Thus our contribution lies in setting collective factors of production in a context of distortionary revenue generation.

We should conclude this introduction by visiting the literature that feeds our priors. First, we should highlight what may be called *Pigou's conjecture*: That public expenditures should be lower in the context of costly revenue generation than they would be under lump sum taxation (Pigou, 1947, page 34). Atkinson and Stern (1974), following Stiglitz and Dasgupta (1971), qualified Pigou's conjecture by classifying the exceptions that apply for the case with a pure public good. Second, a more recent

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<sup>3</sup> We follow the tradition in the literature not to press sign generalizations into the text: the word tax is used even if it may be negative (a subsidy). Similarly, we may talk about the choices of polluters as benefiting the polluted (polluters are potential providers of pollution reductions, i.e. of the public good). Thereby, we

literature focuses on the interaction between an environmental pure public good and costly revenue generation (See, for instance, Bovenberg and de Mooij, 1994). What differentiates that problem is additional restrictive assumptions about preferences and technology; the fact that provision is induced by taxation turns out not to be important in terms of basic principles, as long as the government still needs to resort to costly revenue generation.

To this author's knowledge, no direct analysis has been made of Pigou's conjecture in the case when the public expenditures benefit production with a "collective factor of production" (Sandmo, 1972). Diamond and Mirrlees' result on production efficiency (obtained under general assumptions equivalent to ours), and also the treatment of Dasgupta and Stiglitz (1972) would seem to indicate that "effects" between producers, and between producers and government entities should reflect fully the benefits to the recipients - with no distorting wedges - whether the goods/inputs "delivered" are private or non-rivalrous to the benefiting entities. But what if a public program benefits producing sectors as well as consumers, as does for instance the expansion of a road? If in addition the good in question can be generated by consumers *and* producers, as with emission reductions, the answer does not seem to be indicated by the result on aggregate production efficiency.

In benefit cost analysis, two factors may have contributed to the fact that benefits to firms have not been highlighted. First, there is a tradition in that literature to model net effects, the analyst taking program benefits through producing sectors to evaluate them as they give households expanded consumption opportunities. Our treatment supplements this approach by highlighting what it implies in terms of valuation of the more direct cost savings accruing in production, and also provides a critique in terms of policy principles. Second, cost benefit analysis is often not conducted under the assumptions that lead to the production efficiency result; it may not even be assumed that the tax structure is optimal.

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facilitate the analogy with public goods provision. Here, we skipped Pigou's sign generalization: "services or disservices".

## II. The Model

We shall modify the traditional model of an environmental good by allowing not only consumers but also productive sectors to benefit from pollution reductions (equation 3, below). Since other aspects of the model are the same as in Eskeland, 2000, we shall be brief in our description of the model.

Preferences. A consumer  $g$ 's utility  $u^g$  depends on her consumption  $x_j^g$  of  $n+1$  private goods,  $j = 0, 1, \dots, n$ , as well as on the aggregate level of emissions,  $e$ , in her environment.  $e$  is a public good (we might have called it a "public bad", since it has a negative effect on utility, see below), experienced at the same level by all consumers. For convenience of notation, we assume that the  $h$  consumers are identical, and we may thus suppress personal superscripts (we briefly treat the case with heterogeneous consumers in section IV). Thus,

$$(1) \quad u = u(x_0, x_1, \dots, x_n, e) \equiv u^g(x_0^g, x_1^g, \dots, x_n^g, e).$$

We assume that the utility function is continuous, twice differentiable, quasiconcave, and that  $u_e \leq 0$ .<sup>4</sup> Government consumption is assumed constant and is therefore suppressed in (1).

Emissions, aggregated across agents, is linked linearly by emission factors  $f_j$ ,  $j=1, \dots, n$  to total output in each sector (i.e. to consumption or production - or any combination - of each good). We shall introduce endogenous emission factors later (in equation 32), and initially simplify by assuming exogenously given emission factors.

$$(2) \quad e = \sum_j f_j \cdot (hx_j + x_j^P),$$

where  $x_j^P$  is the exogenously given government consumption (or use) of good  $j$ .<sup>5</sup> To simplify, we assume that the numeraire good is not polluting:  $f_0 = 0$ , and we describe all other goods as possibly polluting,  $f_j \geq 0$ ,  $j = 1, \dots, n$ .

Producers are harmed if there is pollution in the environment, but apart from this, production possibilities are characterized by constant marginal rates of transformation

<sup>4</sup> Whenever possible without risking confusion, we shall use subscripts to denote partial derivatives.

<sup>5</sup> If not otherwise indicated, summation indicated for instance by  $\Sigma_j$  is over goods  $j=1, \dots, n$ .

$$(3) \quad hx_0 + x_0^p + \sum_{j=1}^n c_j(e) \cdot (hx_j + x_j^p) = K,$$

where  $K$  is a constant. (3) can be thought of as a general conversion technology possessed by many producers, and we shall assume perfectly competitive behavior.

It is in (3) that the somewhat novel element of this optimal tax analysis is introduced.  $c_j(e)$ , continuous, twice differentiable and convex, describes how the unit cost (and marginal cost – they are equal) in each sector (i.e. for each good) depends on the aggregate level of pollution. Sectors may vary in their sensitivity to pollution, but we assume for simplicity that  $c_{je} \equiv \partial c_j / \partial e \geq 0$ , all  $j = 1, \dots, n$ .<sup>6</sup> We describe pollution as not affecting sector zero, but this is no restrictive assumption, since the role of  $c$  simply is to describe the  $n$  marginal rates of transformation between the  $n+1$  goods, and how these change as the level of pollution changes.

The consumer has a nontaxable lump sum income  $I$  and faces consumer prices  $q = q_0, q_1, \dots, q_n$  equal to producer prices plus linear commodity taxes

$$(4) \quad q_0 = 1, \\ q_i = c_i(e) + t_i, \quad i = 1, \dots, n.$$

We assume that consumers and producers take as given the levels of pollution and government revenue. The first-order conditions for the consumer's individual optimum are her budget constraint  $\sum_{j=0}^n q_j x_j = I$  and, for  $j=1, \dots, n$ ,

$$(5) \quad \frac{u_j}{u_0} = q_j.$$

Let

$$(6) \quad x_j = x_j(q, I, e)$$

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<sup>6</sup> To clarify, we retain the classical description of what *causes* emissions: each polluter derives benefits from her own *individual* emissions, exploiting the environment's services as a waste recipient. The dis-services thus provided by polluters are analogous to the services from those who provide a traditional public good in the sense that both are rivalrous. For polluters, when one emits more, another must emit less, if environmental quality is not to fall. For providers, when one provides less, another must provide more, if provision of the public good is not to fall. The novel aspect introduced here is that producers are "harmed" by the *aggregate* level of pollution in the environment, a producer analogy to the consumer side of a traditional public good, which is nonrivalrous.

denote the Marshallian demand function consistent with the consumer's first order conditions for optimum.

A benevolent planner uses linear commodity tax rates  $t = t_1, \dots, t_n$  to maximize the indirect utility function  $v(q, I, e)$  corresponding to (1), (4) and (6) subject to the government's budget constraint  $h \sum_j t_j x_j = x_0^p + \sum_j c_j(e) x_j^p$ .<sup>7</sup> The Lagrangian of the planner's maximization problem is

$$(7) \quad L = h v(q, I, e) + \mu \sum_j \left[ h t_j x_j - (x_0^p + c_j(e) x_j^p) \right].$$

### III. Optimal provision

We may partially differentiate (7) with respect to the  $n$  tax rates to find first order conditions for welfare optimum. For all  $i = 1, \dots, n$ , we have:

$$(8) \quad \frac{\partial L}{\partial t_i} = -h\beta \left[ \sum_j x_j \frac{dq_j}{dt_i} + \alpha \frac{de}{dt_i} \right] + \mu \left[ h \left( x_i + \sum_l t_l \frac{dx_l}{dt_i} \right) - \sum_j x_j^p c_{je} \frac{de}{dt_i} \right] = 0,$$

where  $\beta$  is the marginal value of income (in terms of the numeraire) to the consumer and

$\alpha$  is the consumer's marginal willingness to pay for pollution reductions,  $\alpha = -\frac{\partial v}{\partial e} / \frac{\partial v}{\partial I}$ .

Using (4), we have

$$(9) \quad \frac{dq_i}{dt_i} = 1 + c_{ie} \frac{de}{dt_i}, \text{ and } \frac{dq_j}{dt_i} = c_{je} \frac{de}{dt_i}, \text{ for } j \neq i.$$

From (2) and the assumption that government consumption is exogenously determined,

$$(10) \quad \frac{de}{dt_i} = h \sum_j f_j \frac{dx_j}{dt_i}.$$

Substituting (9) and (10) into (8), we have, for all  $i=1, \dots, n$

$$(11) \quad \frac{\partial L}{\partial t_i} = -\beta \left[ x_i + \left( \alpha + \sum_j x_j c_{je} \right) h \sum_l f_l \frac{dx_l}{dt_i} \right] + \mu \left[ x_i + \sum_l t_l \frac{dx_l}{dt_i} - \sum_j x_j^p c_{je} \sum_l f_l \frac{dx_l}{dt_i} \right] = 0.$$

Reordering to have the tax rates and their coefficients on the left hand side, we have

<sup>7</sup> In the case of fixed emission coefficients, assumed here (equation 2), the role of emission taxes can be assumed by commodity taxes (see Sandmo, 1975, and Eskeland, 2000). Emission taxes play a role, however, in the case with endogenous emission coefficients, later in section III.



$$(12) \quad \sum_i t_i \frac{dx_i}{dt_i} = \left( \frac{\beta}{\mu} - 1 \right) x_i + \sum_i f_i \frac{dx_i}{dt_i} \left[ h \frac{\beta}{\mu} \left( \alpha + \sum_j x_j c_{je} \right) + \sum_j x_j^p c_{je} \right], \text{ all } i=1, \dots, n.$$

It will prove useful to have introduced matrix notation. Let  $G$  be the following matrix of total derivatives:

$$G = \begin{bmatrix} \frac{dx_1}{dt_1} & \dots & \frac{dx_n}{dt_1} \\ \vdots & & \vdots \\ \frac{dx_1}{dt_n} & \dots & \frac{dx_n}{dt_n} \end{bmatrix}$$

We can now write (12) as

$$(13) \quad G \cdot t = \left( \frac{\beta}{\mu} - 1 \right) x + G \cdot f \left[ h \frac{\beta}{\mu} \left( \alpha + \sum_j x_j c_{je} \right) + \sum_j x_j^p c_{je} \right],$$

where  $t$  and  $f$  are  $n$ -vectors of tax rates and emission factors, respectively. Assuming that  $G$  is non-singular (to be explored below, footnote 9), we may pre-multiply both sides by the inverse of  $G$  to have the tax rates in a more explicit form:

$$(14) \quad G^{-1} G t = t = G^{-1} \left( \frac{\beta}{\mu} - 1 \right) x + f \left[ h \frac{\beta}{\mu} \left( \alpha + \sum_j x_j c_{je} \right) + \sum_j x_j^p c_{je} \right].$$

(14) characterizes the optimal tax structure.

To obtain further insight, we need to analyze the matrix  $G$ , and to do this, we shall assume that the environmental good is separable in preferences (1) from the  $n+1$  market goods, so that  $x_{je} = 0$ , all  $j = 1, \dots, n$  (see equation 6).<sup>8</sup> Let us examine the differentials  $dx_j / dt_i$ . Using separability, (6) and (9),

$$(15) \quad \frac{dx_k}{dt_i} = \sum_j \frac{dx_k}{dq_j} \frac{dq_j}{dt_i} = x_{ki} + \sum_j x_{kj} c_{je} \frac{de}{dt_i}.$$

<sup>8</sup> This separability assumption, typically invoked in models with public goods, is discussed further in the annex to this essay, and Eskeland (2000b). Under nonseparability, optimal provision of the public good, minus  $e$ , is expanded (the emission tax is raised) to the extent that provision contributes to demand for taxed goods ( $\sum_j t_j x_{je} < 0$ ) and vice versa. When the public good is provided directly by government (or when an emission tax is used to induce environmental protection), the expressions for optimal commodity taxes are not changed. The plausibility of the separability assumption clearly depends on the externality. Consider two negative externalities from driving; pollution and congestion. Separability appears more plausible for the former.

For illustration, let us briefly visit the example with three private goods, or two relative prices;  $x_j = x_j(q_1, q_2)$  and two tax rates. Only the goods that are taxed and pollute appear in  $G$ , and we have two equations in two unknowns,  $dx_1/dt_1$  and  $dx_2/dt_1$  and two additional equations for  $dx_1/dt_2, dx_2/dt_2$ . The first of these four equations is, as an

example:  $\frac{dx_1}{dt_1} = x_{11} + x_{11}c_{1e} \frac{de}{dt_1} + x_{12}c_{2e} \frac{de}{dt_1}$ , or on a more standardized form:

$$\frac{dx_1}{dt_1} \left( 1 - hf_1 \sum_{j=1}^2 x_{1j} c_{je} \right) + \frac{dx_2}{dt_1} \left( -hf_2 \sum_{j=1}^2 x_{1j} c_{je} \right) = x_{11}. \text{ The system has one solution if and}$$

only if the coefficient matrix ( $F$ , below) is nonsingular. Making this assumption, let us display the solution for one example of these four total differentials:

$$\frac{dx_1}{dt_1} = \frac{-hf_2 c_{2e} (x_{11} x_{22} - x_{12} x_{21})}{1 - h \sum_i f_i \sum_j x_{ij} c_{je}},$$

where the denominator, the determinant of the coefficient matrix, is written in a form applicable for a general number of goods.

For the case of a general number of goods, let  $A$  be the transpose of the Jacobian matrix of the Marshallian demand functions for goods  $1, \dots, n$ :

$$(16) \quad A = \begin{bmatrix} x_{11} & \cdots & x_{n1} \\ \vdots & & \vdots \\ x_{1n} & \cdots & x_{nn} \end{bmatrix}.$$

Working with the individual elements  $dx_j/dt_i$  in the general case, we find that

$$(17) \quad GF = A, \text{ where}$$

$$F = \begin{bmatrix} 1 - hf_1 \sum_j x_{1j} c_{je} & -hf_1 \sum_j x_{2j} c_{je} & \cdots & -hf_1 \sum_j x_{nj} c_{je} \\ -hf_2 \sum_j x_{1j} c_{je} & 1 - hf_2 \sum_j x_{2j} c_{je} & \cdots & -hf_2 \sum_j x_{nj} c_{je} \\ \vdots & \vdots & \ddots & \vdots \\ -hf_n \sum_j x_{1j} c_{je} & -hf_n \sum_j x_{2j} c_{je} & \cdots & 1 - hf_n \sum_j x_{nj} c_{je} \end{bmatrix}.$$

Assuming that  $A$  and  $F$  are nonsingular, we have

$$GFF^{-1} = AF^{-1} \Rightarrow G = AF^{-1} \Rightarrow G^{-1} = (AF^{-1})^{-1} \Rightarrow$$

$$(18) \quad G^{-1} = FA^{-1}.$$

The inverse of  $A$  involves its determinant and the transpose of the matrix of its cofactors:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix},$$

where  $A_{ij}$  is the cofactor of  $x_{ij}$  in  $A$ .<sup>9</sup> Thus

$$(19) \quad G^{-1} = \frac{1}{|A|} \begin{bmatrix} 1 - hf_1 \sum_j x_{1j} c_{je} & \cdots & -hf_1 \sum_j x_{nj} c_{je} \\ \vdots & & \vdots \\ -hf_n \sum_j x_{1j} c_{je} & \cdots & 1 - hf_n \sum_j x_{nj} c_{je} \end{bmatrix} \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix}.$$

We may write  $F$  as the sum of the identity matrix and one with elements  $-hf_i \sum_j x_{kj} c_{je}$  in row  $i$ , column  $k$ . We then obtain:

$$(20) \quad G^{-1} = \frac{1}{|A|} \left\{ \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix} + h \begin{bmatrix} -f_1 \sum_j c_{je} \sum_l A_{1l} x_{lj} & \cdots & -f_1 \sum_j c_{je} \sum_l A_{ln} x_{lj} \\ \vdots & & \vdots \\ -f_n \sum_j c_{je} \sum_l A_{1l} x_{lj} & \cdots & -f_n \sum_j c_{je} \sum_l A_{ln} x_{lj} \end{bmatrix} \right\}.$$

The elements of the last matrix are of the form  $f_m \sum_j c_{je} \sum_l A_{lk} x_{lj}$ . By central results for expansion of determinants by cofactors,  $\sum_l A_{lk} x_{lj} = |A|$  for  $j = k$ , and  $\sum_l A_{lk} x_{lj} = 0$  for  $j \neq k$ , so

<sup>9</sup> From  $G=AF^{-1}$  we can elaborate on the condition that  $G$  is nonsingular. First,  $G$  here plays a role analogous to the one played by  $A$  in the more traditional problem of optimal commodity taxes with a public good separable from private goods in preferences.  $A$  is negative semi-definite by the second order conditions for consumer optimum, and the additional assumption that  $A$  is of full rank ( $|A| \neq 0$ ) is standard and ensures that there are  $n$  independent instruments in the  $n+1$  commodity problem. The interesting question is thus about the transformation performed by  $F^{-1}$  of  $A$  into  $G$ . The determinant of  $F$  is  $1 - h \sum_l \sum_j x_{lj} c_{je}$ , and if externalities are "big",  $|F|$  can thus be zero. If  $|F|$  is zero,  $G$  is of less than full rank despite  $A$  being of full rank, and in that case the optimal tax problem has at most  $n-1$  independent instruments. An illustration of this problem is as follows: Say only good one is polluting and only good one is affected (as with congestion). Then, since  $(dx_1/dt_1) \cdot (1 - hf_1 x_{11} c_{1e}) = x_{11}$ ,  $|F| = 1 - hf_1 x_{11} c_{1e} = 0$  would mean that congestion influences demand for good one so strongly that it cancels the direct effect of a tax on good one, and changes in the tax on good one does not influence demand for good one. While we do not pursue this question further, to avoid anomalies, stronger conditions than  $|F| \neq 0$  are required. In this case,

$1 > hf_1 x_{11} c_{1e}$  would ensure that  $dx_1/dt_1$  and  $x_{11}$  take the same sign (We here benefit from insights in an inquiry started by Buchanan and Kafoglis, 1963, continued by Diamond and Mirrlees, 1973, Diamond, 1973, and Sandmo, 1980).

$$\begin{aligned}
 (21) \quad G^{-1} &= \frac{1}{|A|} \left\{ \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix} - h \begin{bmatrix} f_1 c_{1e} |A| & \cdots & f_1 c_{ne} |A| \\ \vdots & & \vdots \\ f_n c_{1e} |A| & \cdots & f_n c_{ne} |A| \end{bmatrix} \right\} \\
 &= A^{-1} - h \begin{bmatrix} f_1 c_{1e} & \cdots & f_1 c_{ne} \\ \vdots & & \vdots \\ f_n c_{1e} & \cdots & f_n c_{ne} \end{bmatrix} = A^{-1} - h f c_e',
 \end{aligned}$$

where  $c_e'$  is the row vector  $c_{1e}, \dots, c_{ne}$ . Using (21) in (14), we have

$$(22) \quad t = A^{-1} \left( \frac{\beta}{\mu} - 1 \right) x - h f c_e' \left( \frac{\beta}{\mu} - 1 \right) x + f \left[ \frac{h\beta}{\mu} \left( \alpha + \sum_j x_j c_{je} \right) + \sum_j x_j^p c_{je} \right],$$

or for all  $k = 1, \dots, n$ :

$$(23) \quad t_k = \left( \frac{\beta}{\mu} - 1 \right) \frac{\sum_j A_{jk} x_j}{|A|} - \left( \frac{\beta}{\mu} - 1 \right) h f_k \sum_j x_j c_{je} + f_k \left[ \frac{h\beta}{\mu} \left( \alpha + \sum_j x_j c_{je} \right) + \sum_j x_j^p c_{je} \right].$$

Elements in the second term cancel against elements in the third term, and the expression simplifies to

$$(24) \quad t_k = \left( \frac{\beta}{\mu} - 1 \right) \frac{\sum_j A_{jk} x_j}{|A|} + f_k \left( \frac{\beta}{\mu} h \alpha + \sum_j (h x_j + x_j^p) c_{je} \right) \quad \text{all } k = 1, \dots, n.$$

This corresponds to Sandmo's (1975) formula (and the generalization in Eskeland, 2000) in the case when  $c_{je} = 0$ , all  $j = 1, \dots, n$ . The optimal tax structure combines the formulas from the traditional problem of optimal taxation without external effects with an externality motivated term  $f_k \left( \frac{\beta}{\mu} h \alpha + \sum_j (h x_j + x_j^p) c_{je} \right)$ , where the latter is included for each polluting good according to the emission factor  $f_k$ . Thus, the emission motivated term gives the same inducement – in terms of dollars per unit of emissions – across all polluting goods.

#### *An alternative implementation*

As with the optimal allocation in Eskeland (2000), the one supported by  $n$  commodity taxes in (24) can be implemented in a more intuitive manner by an alternative

set of  $n+1$  instruments: an emission tax  $\tau$  levied uniformly on emissions from all activities, in combination with  $n$  commodity taxes:

$$(25) \quad \tau = \frac{h\beta\alpha}{\mu} + \sum_j (hx_j + x_j^P)c_{je}, \text{ and}$$

$$(26) \quad t_k = \left( \frac{\beta}{\mu} - 1 \right) \frac{\sum_j A_{jk} x_j}{|A|}, \text{ all } k = 1, \dots, n.$$

(26) is equivalent to the solution given by Samuelson (1951) for the problem of optimal linear commodity taxes in the problem without an environmental externality. Special cases of (26) are known as “inverse elasticity” rules and Corlett and Hague’s (1953) rule, which emphasizes complementarity with the untaxed good.

To see that the system (25), (26) is consistent with optimum, proceed as follows: Restate the Lagrangian (7) to have proceeds from the emission tax  $\tau$  included in the government budget constraint,

$$(27) \quad L = h\lambda(q, I, e) + \mu \sum_j [h(t_j + \tau f_j)x_j - (x_0^P + c_j(e))x_j^P].$$

Nothing has changed with the underlying economic problem, of course: We still assume it is of rank  $n$ , so there is redundancy in instruments and one can be chosen arbitrarily. We shall proceed by taking a choice for the emission tax as given by (25), to check whether this in combination with (26) is consistent with the first order conditions for maximum of (27). Partially differentiating (27) with respect to the  $n$  commodity tax rates, the first order conditions for maximum of (27), corresponding to (12) are:

$$(28) \quad \sum_i t_i \frac{dx_i}{dt_i} + \tau \sum_i f_i \frac{dx_i}{dt_i} = \left( \frac{\beta}{\mu} - 1 \right) x_i + \sum_i f_i \frac{dx_i}{dt_i} \left[ h \frac{\beta}{\mu} (\alpha + \sum_j x_j c_{je}) + \sum_j x_j^P c_{je} \right],$$

all  $i=1, \dots, n$ .

As we substitute (25) into (28), several terms cancel, and we obtain

$$(29) \quad \sum_i t_i \frac{dx_i}{dt_i} = \left( \frac{\beta}{\mu} - 1 \right) x_i + \sum_i f_i \frac{dx_i}{dt_i} \left( \frac{\beta}{\mu} - 1 \right) h \sum_j x_j c_{je}, \text{ all } i=1, \dots, n, \text{ or}$$

$$(30) \quad G \cdot t = \left( \frac{\beta}{\mu} - 1 \right) x + G \cdot f \left( \frac{\beta}{\mu} - 1 \right) h \sum_j x_j c_{je} \Rightarrow$$

$$(31) \quad t = A^{-1} \left( \frac{\beta}{\mu} - 1 \right) x - h f c_e \left( \frac{\beta}{\mu} - 1 \right) x + f \left( \frac{\beta}{\mu} - 1 \right) h \sum_j x_j c_{je} = A^{-1} \left( \frac{\beta}{\mu} - 1 \right) x,$$

which is the same as (26), so (25),(26) implements the optimal allocation.

In the more general model with endogenous abatement (see below) the implementation with an emission tax (25), (26) represents an essential implementation mechanism, rather than an alternative one. In the present model with exogenous emission factors, it contributes by separating the instrument implementing environmental protection from the commodity tax rates, thereby emphasizing parallels to the more basic problems in the literature. For these reasons, we view the implementation with an emission tax (25), (26) as the reference point from now on.

#### *Abatement technologies available*

We now consider the more realistic and interesting case when it is possible in each activity  $i=1,\dots,n$  to reduce emissions per unit by expending resources on abatement,  $b_i$ , so  $f_i = f_i(b_i)$ . This model is described in greater detail in Eskeland, 2000.<sup>10</sup> An important result provided there is that abatement shall be efficient across abatement opportunities for producers, consumers and government, and across polluting goods and activities. This implies that emission reductions from consumers and producers can be stimulated by a uniform tax levied on emissions where they occur. We use this result here by describing abatement as performed by the user of a polluting good, focusing first on the consumer who faces an emission tax. This covers the general case if we interpret  $b_i$  as the sum of abatement by the producer and the consumer of a unit of good  $i$ , and  $f_i$  as the sum of emissions from the maker and the user. We allow the government as a user to choose its abatement level independently (equation 32 and 46, below).

For emissions, recalling that subscripts index goods,  $i,j=1,\dots,n$ , we now have

$$(32) \quad e = \sum_j (hf_j(b_j)x_j + f_j^P(b_j^P)x_j^P)$$

replacing (2). We assume the marginal cost of emission reductions,  $-1/f_{ib}$ , is positive and increasing in abatement. As we let the "all-inclusive consumer prices" include abatement and the emission tax  $\tau$ , (4) is replaced by

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<sup>10</sup> A familiar example is emissions from motor vehicles: Makers of cars and fuels can reduce the car's emission factors by modifying the vehicle and its fuels; drivers/owners can reduce emission factors for driving through maintenance, tuning and style of driving.

$$(33) \quad q_i = c_i(e) + t_i + \tau f_i(b_i) + b_i, \quad i=1, \dots, n.$$

The polluter (now represented by the consumer) is assumed to abate to minimize the "all-inclusive consumer price" (33). This implies that abatement is a function of the emission tax only, and marginal costs of emission reductions equal the emission tax:

$$(34) \quad \tau = -1/f_{ib}(b_i), \quad i=1, \dots, n.$$

The Lagrangian (27) still applies (modified with government abatement, assumed given), and partial differentiation with respect to the  $n$  commodity taxes yields  $n$  expressions identical to (28). Moreover, upon examination, (9) and (10) still apply, and the proof from the previous section that the  $n$  first-order conditions for the commodity taxes are satisfied with tax rates (25), (26) still applies.

With abatement, however, the underlying problem has changed. There is no redundancy in a set of  $n+1$  instruments, since  $\tau$  influences abatement, and no other instrument does. We shall now need:

$$(35) \quad \frac{dq_m}{d\tau} = f_m + c_{me} \frac{de}{d\tau}, \quad \text{all } m=1, \dots, n, \text{ from (33), and the envelope theorem, and}$$

$$(36) \quad \frac{dx_l}{d\tau} = \sum_m x_{lm} \frac{dq_m}{d\tau}, \quad \text{all } l=1, \dots, n, \text{ from (6), and separability.}$$

Partial differentiation of (27) under the assumption that abatement is available yields the following first order condition for instrument number  $n+1$ :

$$(37) \quad -h\beta \left[ \sum_j x_j f_j + \left( \alpha + \sum_j x_j c_{je} \right) \frac{de}{d\tau} \right] + \mu \left[ h \left( \sum_j x_j f_j + \sum_l t_l \frac{dx_l}{d\tau} \right) + \left( \tau + \sum_j x_j^p c_{je} \right) \frac{de}{d\tau} \right] = 0.$$

It remains to be checked whether (25), (26) satisfies (37). (37) $\Rightarrow$

$$(38) \quad h \sum_l t_l \frac{dx_l}{d\tau} = h \left( \frac{\beta}{\mu} - 1 \right) \sum_j x_j f_j + \frac{de}{d\tau} \left[ h \frac{\beta}{\mu} \left( \alpha + \sum_j x_j c_{je} \right) - \tau + \sum_j x_j^p c_{je} \right].$$

We substitute in (25) and simplify to obtain

$$(39) \quad \sum_l t_l \frac{dx_l}{d\tau} = \left( \frac{\beta}{\mu} - 1 \right) \sum_j x_j \left[ f_j + c_{je} \frac{de}{d\tau} \right].$$

Using (35) and (36), we have

$$(40) \quad \sum_l t_l \frac{dx_l}{d\tau} = \sum_l t_l \sum_m x_{lm} \frac{dq_m}{d\tau} = \sum_l t_l \sum_m x_{lm} \left( f_m + c_{me} \frac{de}{d\tau} \right), \text{ and thus (39) } \Rightarrow$$

$$(41) \quad \sum_i t_i \sum_m x_{im} \left[ f_m + c_{me} \frac{de}{d\tau} \right] = \left( \frac{\beta}{\mu} - 1 \right) \sum_m x_m \left[ f_m + c_{me} \frac{de}{d\tau} \right].$$

For both sides, let us take one element in the sum over  $m=1, \dots, n$ , looking only at the first term in the brackets:

$$(42) \quad \sum_i t_i x_{im} f_m = \left( \frac{\beta}{\mu} - 1 \right) x_m f_m.$$

If we divide by  $f_m$  on both sides, we have one of the first order conditions in the traditional problem of optimal commodity taxation (without external effects) - the simple problem that is solved by (26). Thus, (42) is satisfied as (26) is satisfied, and we may subtract (42) from (41) for  $m = 1, \dots, n$ . What remains is

$$(43) \quad \sum_i t_i \sum_m x_{im} c_{me} \frac{de}{d\tau} = \left( \frac{\beta}{\mu} - 1 \right) \sum_m x_m c_{me} \frac{de}{d\tau}.$$

Here, we conduct similar steps again: For each element in the sum over  $m=1, \dots, n$ , we have

$$(44) \quad \sum_i t_i x_{im} c_{me} \frac{de}{d\tau} = \left( \frac{\beta}{\mu} - 1 \right) x_m c_{me} \frac{de}{d\tau},$$

which is just  $c_{me} \frac{de}{d\tau}$  times (42) divided by  $f_m$ .

Thus, we have shown that a tax structure satisfying the set (25), (26) satisfies the first order conditions for optimum also in the case with endogenous abatement.

We can now state the condition for optimal environmental protection given by (25) and (34):

$$(45) \quad \frac{-1}{f_{ib}} = h\alpha \frac{\beta}{\mu} + \sum_j (hx_j + x_j^p) c_{je}, \quad i=1, \dots, n.$$

On the left hand side is the marginal rate of transformation between emission reductions and the numeraire good. On the right hand side is an aggregation of marginal benefits which – for the benefits accruing as a public good to consumers - includes an adjustment factor  $\beta/\mu$ , the inverse of the marginal cost of funds (see below).

We can now use a finding from Eskeland (2000) to show that an alternative implementation of this allocation is one with emission standards or abatement standards in combination with commodity taxes like those of Sandmo (1975), which include presumptive emission taxes. For a particular structure of monitoring costs, plausible for



cars, for instance, emission standards or abatement standards are feasible when emission taxes are not, so this alternative implementation can be an attractive one.

*Direct provision of a public good*

We only state briefly that the same condition applies for government provision of a public good, formulated here as government abatement of emissions ( $b_i^P$  in equation 32). This formulation is unusual in the way public provision is linked to public consumption of  $j$ ,  $x_j^P$  (assumed given), but the result is easily seen to be general:

$$(46) \quad \frac{-1}{f_{ib}^P(b_i^P)} = h\alpha \frac{\beta}{\mu} + \sum_j (hx_j + x_j^P)c_{je}, \text{ all goods } i=1, \dots, n.$$

Thus, marginal costs of provision are equal to those for consumers and producers in (45), and we may note that this equality does not depend on the government emission function  $f_i^P$  being equal to those of producers or consumers (see equation 32).

In the current context, it is obvious that the marginal costs of emission reductions will also be the same across consumers (consumers are identical and are exposed to the same emission tax). Eskeland (2000) shows that the planner would want emission taxes to be uniform across polluting goods even with heterogenous consumers as long as consumers have the same access to abatement technologies (i.e. that  $f_i^g(b_i^g) = f_i^h(b_i^h)$  for  $b_i^g = b_i^h$ , so that if Peter and Paul install the same catalytic converter, then their emissions are reduced by the same amount).

We are now ready to state our main result:

*Proposition:*

*When optimal public provision or environmental protection is benefiting*

- A) consumers with a pure public good, a wedge (the ratio of the marginal utility of income to the shadow price of public revenue to) separates the Samuelsonian sum of marginal benefits and the marginal rate of transformation;*
- B) production with a nonrivalrous input, so that provision reduces marginal costs in production, there is production efficiency in the sense that no wedge applies between the sum of marginal benefits and the marginal rate of transformation.*

Equations (45) and (46) present the optimality condition for a public program that combines these two types of benefits. The observation that benefits in terms of productivity will not be adjusted by the shadow price of public funds has also been made - in more restrictive models - by Bovenberg and van der Ploeg (1994) and Williams (2000). However, these do not place the result in the context of production efficiency or of Pigou's conjecture (see below).<sup>11</sup>

### *Discussion*

Sandmo (1972) and others have shown that a condition analogous to the Samuelson condition for optimal provision of public goods applies to “collective factors of production” under lump sum taxation. We know (from Stiglitz and Dasgupta, 1971, for instance) that the Samuelson condition for optimal provision of public goods applies only after an adjustment by the ratio  $\beta/\mu$  in the case with distortionary taxation. Thus, a way of restating our result is that the condition for optimal provision of a collective factor of production – in contrast to the condition for public goods - extends without any adjustment to the case with distortionary taxation.

The factor  $\beta/\mu$  in (25) which adjusts the benefits associated with consumer preferences  $h\alpha$  but not the benefits associated with production  $\sum_j (h_j x_j + x_j^P) c_{je}$  is the subject of a literature on a topic we may call *Pigou's conjecture*. Pigou's much cited statement was:

“Where there is indirect damage” (from mobilizing revenue, Eskeland's remark) “it ought to be added to the direct loss of satisfaction involved in the withdrawal of the marginal unit of resources by taxation, before this is balanced against the satisfaction yielded by the marginal expenditure. It follows that, in general, expenditure ought not be carried so far as to make the real yield of the last unit of resources expended by the

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<sup>11</sup> This may be the right place to apologize that we use an adjustment factor  $\beta/\mu$ , rather than its better known inverse, often called the marginal cost of funds, MCF. MCF typically adjusts the cost side of the Samuelson condition for optimal provision of public goods (see, for instance, Auerbach, 1985). In our case, the domain of benefits is expanded beyond public goods, and then only a strict subset of benefits is to be adjusted (45), so the adjustment cannot equivalently be done on the cost side of the optimality condition. Besides, as seen in the following, adjusting benefits “as if they were taxed” (King, 1986) contributes to intuition. Sandmo (1998) highlights that some aspects of what is sometimes referred to with the MCF concept relates to whom the beneficiaries are, another reason to view adjustment of the cost side as too restrictive (see also section IV, below).

government equal to the real yield of the last unit left in the hands of the representative citizen”.

Starting with Stiglitz and Dasgupta (1971), Pigou’s conjecture that optimal provision should be lower when revenue generation is costly has been analyzed assuming the program provides pure public goods (see Atkinson and Stern, 1974, or Auerbach, 1985). For programs providing pure public goods, the exceptions to the rule that provision be adjusted downward when taxation is costly are now well known. We have assumed separability ( $x_{je} = 0$ , all  $j$ , see the annex), which rules out one class of exceptions. The remaining class is when taxed goods are predominantly inferior goods, in such a way as to make  $\sum_j t_j \partial x_j / \partial I < 0$ . In that case, the excess burden may be falling in the level of taxation due to an income effect shifting consumption towards taxed goods. In other, perhaps more frequently observed cases, the shadow price of public revenue plays the role of cutting optimal provision short of the point where the sum of marginal benefits equal marginal costs.

The discovery in the present context is that such an adjustment (whether upward or downward) does not apply to benefits derived as cost savings in production sectors. This result is in line with Diamond and Mirrlees’ classical result on production efficiency, but there are a number of reasons not to see this as a widely recognized consequence. First, our result applies independently of whether consumers or producers are the providers in the public program (e.g. whether it is consumers or producers that can reduce emission factors). Thus, if we thought of the production efficiency result as saying there should not be wedges between producers, we might have expected a wedge to apply in the case when consumers are providers – say of pollution abatement - and producers are benefiting. Diamond and Mirrlees themselves (1971) venture to discuss externalities only to the extent that they occur between consumers. Second, such an implication has to this author’s knowledge not been highlighted, and studies such as Bovenberg and van der Ploeg (1994) and Williams (2000) do not make this connection. Third, applied analysis, say benefit cost analysis of roads, to this author’s knowledge does not typically distinguish between benefits to consumers and benefits to producers, even when it applies a shadow price of public funds in the analysis. It follows from the current analysis that if the benefits of a road project accrue directly to households as a pure public good and to

producing sectors in terms of reduced transportation costs, then Pigou's conjectured adjustment applies to the direct household benefits but not to transportation cost savings of firms and government.<sup>12</sup>

We venture now to provide intuition to this result. When the public program provides a pure public good for consumers, the role of the correction factor  $\beta/\mu$  in the principle for optimal provision is typically to reduce provision of a good that would otherwise not have been taxed.<sup>13</sup> When the public program provides an input into production, in contrast, the benefits from provision are derived from a taxable market. Cost savings can be captured as government revenue at no distortionary costs (matching the savings by tax increases) so that distorting a program providing such benefits is redundant and costly. Put differently, for the program providing cost savings, the benefits are taxed, and for the program providing a pure public good, provision is adjusted according to the shadow price of public funds, to the same effect.

#### IV Heterogenous individuals and distribution

We briefly address the implications of heterogenous consumers. In this case, the welfare function takes the form  $w(v^1, \dots, v^h)$ , with consumer prices and the environment experienced at the same level by all, so  $v^h = v^h(q, I^h, e)$ . Now with  $\beta^h = \partial w^h / \partial v^h \cdot \partial v^h / \partial I^h$ , and  $\alpha^h = \partial v^h / \partial e / \partial v^h / \partial I^h$ , the expressions equivalent to (25) and (26) take the form:

$$(47) \quad \tau_e = \frac{\sum_g \beta^g \alpha^g}{\mu} + \sum_j (h \bar{x}_j + x_j^p) c_{je},$$

$$(48) \quad t_k = \frac{\sum_g \sum_j \left( \frac{\beta^g}{\mu} - 1 \right) A_{jk} x_j^g}{h|A|}, \quad \text{all } k=1, \dots, n,$$

<sup>12</sup> If households benefit not through a pure public good, but with savings of a taxed good (say gasoline, as the road improves), then those savings are to be valued by the producer price (Christiansen, 1981).

<sup>13</sup> We say typically (the correction factor may be greater than one, see below), just as we think of the vector of commodity taxes as "typically" having positive elements (though goods may be taxed at negative rates). In a context with an income tax, this becomes simpler: King, 1986, notes "Treasury should instruct those responsible for project appraisal to calculate benefits as if they were taxed at the same rate as private incomes".

where  $\bar{x}_j$  denotes average consumption of good  $j$ ,  $A$  now consists of average responses for consumers  $g=1,\dots,h$ ,  $\Sigma_g x_{ij}^g / h$ , and  $A_{jk}$  is the cofactor of column  $k$ , row  $j$ , of matrix  $A$ . Thus, the weighting scheme that applies in the condition for optimal provision (47) applies to the individual marginal benefits for the public good, but not to the distribution of consumption for the goods that benefit through cost reductions. The way distributional considerations apply in the commodity tax structure (48) is well known (See, for instance, Feldstein, 1972, or Sandmo, 1975). It is useful to notice, however, that it matters who consumes how much of a good, but demand responsiveness matters only in aggregate across consumers.

So optimal provision - or more precisely the divergence between aggregate marginal benefits and the marginal costs of provision - is independent of who consumes the goods that benefit in terms of cost reductions. The intuition behind this is exactly the same as in the preceding case with identical consumers, and illustrates how the result on aggregate production efficiency applies: To the redistributive planner, of course, it matters who consumes which goods, since the distributional characteristics of goods give her means with which to redistribute, in this case with linear commodity taxes, as given by the equations (26). Nevertheless, precisely because the planner has available a tax instrument for each commodity, programs that influence costs in industries do not need to be distorted by distributional considerations.<sup>14</sup>

This particular implication will easily get lost in applied cost benefit analysis if the analysis is conducted by taking the cost reductions directly through the productive sectors and to households (with distributional weights) according to their consumption of goods from benefiting industries. That approach may have a role to play if tax rates are considered exogenously given, or if for other reasons the assumptions behind our analysis

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<sup>14</sup> We may here insert a qualification that applies more generally: The program is influenced indirectly by distributional considerations (and in the previous section, by the shadow price of public revenue), in the sense that all "parameters" of our solution are functions, and thus endogenous. However, the program is not influenced directly - i.e. in terms of the intervention in the market itself. For instance, we point out that marginal costs equal aggregate marginal cost savings in industries - so there is no wedge applying in that market. But there are many such points - and the point associated with the optimal allocation depends, *inter alia*, on the shadow price of public revenue, and on distributional considerations. For a public good, provision may be expected to fall (rise) with redistribution to the poor if the elasticities with respect to income of willingness to pay for the public good is higher (lower) than one. Cost savings to industry  $j$  are worth more the higher is output of  $j$ , and provision is thus influenced by redistribution according to the income elasticities for good  $j$ .

do not apply. However, under the assumptions of the present analysis, benefits accruing through cost reductions in productive sectors shall be accounted for equally whether the goods in question are eventually consumed by households that are rich or poor, or by government.

## V. Summary and conclusions

We have analyzed optimal provision of pure public goods and nonrivalrous inputs in a context of constant returns to scale and optimal taxation. When public provision benefits productive sectors with cost savings, we show that provision shall be such that the marginal costs of provision is equal to the marginal reduction in costs in benefiting sectors. That result is as can be expected in a context of Diamond and Mirrlees' result on production efficiency.

Thus, programs which benefit production shall not be scaled down by the "penalty" from the shadow price of public revenue. This is in contrast to programs providing pure public goods (i.e. valued directly by consumers), for which an adjustment of benefits "as if they were taxed" is appropriate, typically leading to reduced provision, according to Pigou's conjecture.

The intuition behind the absence of such an adjustment when program benefits are in the form of cost savings is that these benefits are derived from markets that are otherwise taxable. The cost savings can be captured by government at no distortionary cost by increasing the tax rates for each good to match the provided cost savings.

As we have shown in Eskeland (2000), optimal abatement requires that marginal abatement costs are the same for polluting consumers, polluting producers, and government. Thus, our rules for optimal abatement in the private sector hold irrespective of whether it is consumers who pollute and abate or producers who pollute and abate - or a combination. The equivalence in marginal abatement costs between polluting producers and government (and also the equivalence with more traditional government provision, such as for roads) is a natural implication of Diamond and Mirrlees' result from 1971 on aggregate production efficiency. The equivalence between the marginal costs of emission reductions and benefits in terms of cost reductions (observed also by Bovenberg and van der Ploeg, 1994, Williams, 2000) also is such a natural implication (though not

highlighted before, we believe), since production efficiency implies that no wedges shall apply within the aggregate sector of government and production. The results for optimal provision by consumers - in the form of pollution abatement, for instance - are to our knowledge new.

We may conclude with some reflection over whether benefits of public programs are mostly for consumers or mostly benefiting producers. For environmental protection, most problems that we can think about have aspects of both: Viable ecosystems may play a role directly in our preferences, with non-use values as well as use-values, but also as inputs in the production of commodities such as timber and fish, and pharmaceuticals; present or future. Clean air and water, similarly, have value both directly to consumers and as inputs for producers. Studies of global climate change (the greenhouse problem) seem to reflect a general assumption that a significant proportion of the implications of global climate change will be reflected in production systems, and in particular in damage to agriculture (See, for example, Mendelsohn, Nordhaus and Shaw, 1994). Our approach indicates, thus, that it is important to expand the normal treatment of "the environmental good" to separate public goods in preferences from benefits in production systems.

For non-environmental programs too, an approach limited to public goods provision often will be too narrow, though stylized facts likely differ according to context. For roads, for instance, the share of commercial vehicles (trucks, buses) is predominant in poor countries. Road provision should - in an optimal tax context - not be reduced by the shadow price of public revenue to the extent that traffic is from commercial vehicles. To the extent that this is provocative (poor countries often are assumed to have high marginal costs of public revenue), the explanation comes along with the qualification: The assumption is that taxation is optimal, and that the markets served by the commercial vehicles - or the vehicles themselves - can be taxed. In some countries (in Africa, in particular) with low funding to roads as well as to other sectors, potholes are the size of small cars and truckers have accepted - or even proposed - that price increases for fuels etc. be used to fund road improvements via earmarked taxes. Thus, as a model assuming a benevolent planner and optimal taxation should be seen

merely as shedding light from one particular angle, forces may emerge in her absence to cut into gross inefficiencies, as much as they may emerge to create inefficiencies.

It may be worthwhile in the future to pursue formulations that further examine “intermediate cases” between goods that benefit consumers directly and those that reduce production costs. Christiansen (1981) provides one important step in this direction. Interesting examples would be health and education, both major public programs in all countries. Health effects of pollution, for instance, may be described partly as raising the costs of keeping us healthy, partly as simply reducing the quality of life (interestingly, the applied literature on benefit estimation uses both perspectives). With a different angle, education probably can be described partly as directly improving the well-being of the educated individual, partly as instilling social values and finally as raising the individual's labor productivity. The latter will in part raise the individual's earnings, but to the extent the price of human capital falls (it likely will, if rising education is wide-spread) - the gains are passed on via firms as cost reductions for their outputs. The latter case, if important, could indicate that broad public investments in human capital not be directly burdened by the shadow price of public revenue. We leave these as ideas for pursuit in the future.

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### Annex to Essay III: Non separability, or when pollution influences demands

We relax the assumption of separability in preferences between the set of private goods and the environmental good in a model with four goods:  $u = u(x_0, x_1, x_2, e)$ . For simplicity of exposition, we describe the problem in terms of a representative consumer, with  $h$  identical consumers.  $\beta/\mu$  is the value of income to the consumer relative to shadow price of public funds, and  $\alpha$  is the marginal rate of substitution (in preferences) between environmental quality (or emission reductions) and the numeraire good. Good one pollutes and the emission coefficient is endogenous:  $e = f_1(b_1)hx_1$ . Policy instruments are commodity taxes on goods 1 and 2 and an emission tax<sup>15</sup>. In the case with endogenous emission coefficients, instrument number  $n+1$ , the emission tax, is not redundant, since abatement,  $b_1$ , responds to the emission tax,  $\tau_e$ , but not to the  $n$  commodity tax instruments (see equations 33 and 34). The first order conditions for the commodity taxes are identical to the case with exogenous emission coefficients, and thus correspond to (12):

$$(A.1) \quad \frac{\partial L}{\partial t_i} = -\beta \left( x_i + h\alpha \frac{de}{dt_i} \right) + \mu \left[ x_i + t_1 \frac{dx_1}{dt_i} + t_2 \frac{dx_2}{dt_i} + \tau_e \frac{de}{dt_i} \right] = 0, \quad i=1,2.$$

The first order condition for the emission tax, corresponding to (37) is:

$$(A.2) \quad \frac{L}{\partial \tau_e} = -\beta \left( f_1 x_1 + h\alpha \frac{de}{d\tau_e} \right) + \mu \left[ f_1 x_1 + t_1 \frac{dx_1}{d\tau_e} + t_2 \frac{dx_2}{d\tau_e} + \tau_e \frac{de}{d\tau_e} \right].$$

Rewriting these three equations to have tax rates on the left hand side, we have

$$(A.3) \quad \begin{aligned} t_1 \frac{dx_1}{dt_1} + t_2 \frac{dx_2}{dt_1} + \tau_e \frac{de}{dt_1} &= \left( \frac{\beta}{\mu} - 1 \right) x_1 + ha \frac{\beta}{\mu} \frac{de}{dt_1}, \\ t_1 \frac{dx_1}{dt_2} + t_2 \frac{dx_2}{dt_2} + \tau_e \frac{de}{dt_2} &= \left( \frac{\beta}{\mu} - 1 \right) x_2 + ha \frac{\beta}{\mu} \frac{de}{dt_2} \quad \text{and} \\ t_1 \frac{dx_1}{d\tau_e} + t_2 \frac{dx_2}{d\tau_e} + \tau_e \frac{de}{d\tau_e} &= \left( \frac{\beta}{\mu} - 1 \right) f_1 x_1 + h \frac{\beta}{\mu} \frac{de}{d\tau_e}. \end{aligned}$$

Let us name the coefficient matrix  $A$ :

<sup>15</sup> Generalizations (more goods, more polluting goods) should be confirmed but appear to apply (examine matrix  $A$ ).

$$A = \begin{bmatrix} \frac{dx_1}{dt_1} & \frac{dx_2}{dt_1} & \frac{de}{dt_1} \\ \frac{dx_1}{dt_2} & \frac{dx_2}{dt_2} & \frac{de}{dt_2} \\ \frac{dx_1}{d\tau_e} & \frac{dx_2}{d\tau_e} & \frac{de}{d\tau_e} \end{bmatrix}.$$

These total derivatives describe how the quantities  $x_1, x_2$  and  $e$  respond to tax changes, with three underlying functional relationships: First, the uncompensated demand functions now are defined over the all-inclusive consumer prices, emissions and private income:  $x_i = x_i(1, q_1, q_2, e, I)$ . Second, there is the emission function, above, and finally there is equation (33) describing how the all-inclusive consumer prices for good one include tax rates and abatement,  $t_1, \tau_e f_1, b_1$ , and for good two includes  $t_2$ . Solving (first for the differentials of  $q$ , then for the quantities as functions of  $q$  and  $\tau_e$ ), the elements in  $A$  are as follows:

$$(A.4) \quad A = \begin{bmatrix} \frac{x_{11}}{1 - f_1 x_{1e}} & x_{21} + x_{2e} \frac{f_1 x_{11}}{1 - f_1 x_{1e}} & \frac{f_1 x_{11}}{1 - f_1 x_{1e}} \\ \frac{x_{12}}{1 - f_1 x_{1e}} & x_{22} + x_{2e} \frac{f_1 x_{12}}{1 - f_1 x_{1e}} & \frac{f_1 x_{12}}{1 - f_1 x_{1e}} \\ \frac{x_{11} f_1 + x_{1e} f_{1b} b_{1\tau} x_1}{1 - f_1 x_{1e}} & x_{21} f_1 + x_{2e} \frac{f_1 x_{11} f_1 + f_{1b} b_{1\tau} x_1}{1 - f_1 x_{1e}} & \frac{f_1 x_{11} f_1 + f_{1b} b_{1\tau} x_1}{1 - f_1 x_{1e}} \end{bmatrix}$$

It can be shown that the determinant of  $A$  is

$$(A.5) \quad |A| = \frac{f_{1b} b_{1\tau} x_1}{1 - f_1 x_{1e}} (x_{11} x_{22} - x_{12} x_{21}).$$

For the commodity tax rate  $t_1$ , we have

(A.6)

$$t_1 = \frac{\begin{pmatrix} \frac{\beta}{\mu} - 1 \end{pmatrix} \begin{vmatrix} x_1 & \frac{dx_2}{dt_1} & \frac{de}{dt_1} \\ x_2 & \frac{dx_2}{dt_2} & \frac{de}{dt_2} \\ f_1 x_1 & \frac{dx_2}{d\tau_e} & \frac{de}{d\tau_e} \end{vmatrix} + ha \frac{\beta}{\mu} \begin{vmatrix} \frac{de}{dt_1} & \frac{dx_2}{dt_1} & \frac{de}{dt_1} \\ \frac{de}{dt_2} & \frac{dx_2}{dt_2} & \frac{de}{dt_2} \\ \frac{de}{d\tau_e} & \frac{dx_2}{d\tau_e} & \frac{de}{d\tau_e} \end{vmatrix}}{|A|} = \frac{\begin{pmatrix} \frac{\beta}{\mu} - 1 \end{pmatrix} \begin{vmatrix} x_1 & \frac{dx_2}{dt_1} & \frac{de}{dt_1} \\ x_2 & \frac{dx_2}{dt_2} & \frac{de}{dt_2} \\ f_1 x_1 & \frac{dx_2}{d\tau_e} & \frac{de}{d\tau_e} \end{vmatrix}}{|A|}$$

The determinant in the numerator can be shown to be  $f_{1b}b_{1r}x_1(x_1x_{22} - x_2x_{21})/(1 - f_{1x_{1e}})$ , so

$$(A.7) \quad t_1 = \frac{\left(\frac{\beta}{\mu} - 1\right)(x_1x_{22} - x_2x_{21})}{x_{11}x_{22} - x_{12}x_{21}}, \text{ and correspondingly}$$

$$t_2 = \frac{\left(\frac{\beta}{\mu} - 1\right)(x_2x_{11} - x_1x_{12})}{x_{11}x_{22} - x_{12}x_{21}}.$$

Thus, in terms of the formulas for optimal commodity taxation, the proposition that emissions and their taxation do not interfere with commodity tax principles generalizes to the case of an environmental good nonseparable from private goods.

We go through similar steps to arrive at an expression for the emission tax:

$$(A.8) \quad \tau_e = h\alpha \frac{\beta}{\mu} + \frac{\left[-x_{1e}\left(\frac{\beta}{\mu} - 1\right)(x_1x_{22} - x_2x_{21}) - x_{2e}\left(\frac{\beta}{\mu} - 1\right)(x_1x_{12} - x_2x_{11})\right]}{x_{11}x_{22} - x_{12}x_{21}}.$$

Additional insight can be gained by substituting the expressions for the commodity tax rates (A.7) into (A.8):

$$(A.9) \quad \tau_e = h\alpha \frac{\beta}{\mu} + (t_1 \cdot (-x_{1e}) + t_2 \cdot (-x_{2e})).$$

In (A.9), we have placed  $-x_{1e}$  inside an inner parenthesis, to see the tax rates multiplied by the response to a public good, as opposed to a bad. (A.9) shows quite clearly how adjustments of the emission tax from Pigovian levels arise because of the effects that public goods provision has on proceeds from commodity taxes. If we think of the finance ministry and the environment ministry as two departments in a corporation, then it appears as if the environment ministry is credited with the support that its product (environmental quality, or minus  $e$ ) gives to the revenues from commodity taxes. Using the corporation analogy further, one could envisage the corporation using such principles in its internal processes to make sure that appropriate resources are allocated to a department marketing the brand name of the corporation as a whole. The analogy between the benevolent planner's public sector and the corporation departs at one point:

For the planner, these effects via revenues represent adjustments to a Pigovian principle, while for the corporation, the effects via product revenues is the whole story. For the corporation, the separability assumption would demolish the budget for a corporate brand name: if the value of the program is not captured through profits from sold products, then it is of no value to the corporation.

Using the Slutsky equation and the symmetry of the compensated demand derivatives  $s_{12} = s_{21}$ , the commodity tax structure (A.7) implies<sup>16</sup>:

$$(A.10) \quad \frac{\sum_j t_j s_{1j}}{x_1} = \frac{\sum_j t_j s_{2j}}{x_2} = \frac{\beta}{\mu} - 1 + \sum_j t_j x_{j1}, \quad j = 1, 2,$$

where  $x_{j1}$  is the marginal propensity to spend on good  $j$  out of personal income.

(A.10) states that for an allocation near the first best (i.e. near the one implemented when there are no distortions  $\beta/\mu = 1$ ), the compensated demand reductions caused by the commodity taxes shall be in equal proportions for the taxed goods (as shown by Samuelson, 1951).

Also for the emission tax, let us examine implications further by replacing the demand derivatives with the Slutsky equation:

$$(A.11) \quad x_{ie} = s_{ie} - h\alpha x_{i1}.$$

Here,  $s_{ie}$  is the compensated change in demand for good  $i$  as the public bad  $e$  increases.

Using the above equation and that the marginal cost of emission reductions is equal to the emission tax,  $\tau_e = -1/f_{1b}$ , we now have:

$$(A.12) \quad \tau_e = \frac{-1}{f_{1b}} = h\alpha \frac{\beta}{\mu} - [t_1(s_{1e} - h\alpha x_{11}) + t_2(s_{2e} - h\alpha x_{21})]$$

Using symmetry  $s_{ie} = s_{ei}$  yields

$$(A.13) \quad \frac{t_1 s_{e1} + t_2 s_{e2}}{h\alpha} = \frac{\beta}{\mu} - \frac{-1/f_{1b}}{h\alpha} + t_1 x_{11} + t_2 x_{21}.$$

The left hand side of this expression is analogous to the left hand side of (A.10) above. It expresses the proportionate reduction in compensated willingness to pay for the public good (defining the good as a reduction in pollution) that is caused by the commodity

<sup>16</sup> This is traditional, from Samuelson, 1951, so we suppress deduction.

taxes. The right hand side is similar – but not equal – to the right hand side of (A.10), but the two are close (as noted by King, 1986, in the case of a government provided public good), when we examine allocations close to the first best equilibrium. In first best ( $\tau_e = -1/f_{1b} = h\alpha$ ), the two right hand sides are equal, and the commodity taxes cause the same proportionate reduction in willingness to pay for the public good as in demand for the taxed goods. In the case of a public good provided directly by government, the allocation near the first best would have to be supported by exogenous government revenue. In the case of pollution, positive revenue can be generated by the instrument(s) implementing the allocation (as pointed out by Sandmo, 1975), and we could imagine the proceeds redistributed as lump sum transfers or being used to provide another public good.

## Essay IV

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# A Presumptive Pigovian Tax: Complementing Regulation to Mimic an Emissions Fee

Gunnar S. Eskeland

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*If regulations are used to make cars and fuels cleaner, should gasoline taxes be used to manage demand for trips that pollute? Analysis of a well-composed program for Mexico City indicates that the emission reductions would cost 24 percent more if a tax on gasoline was not introduced.*

*A simple analytical framework is developed to analyze the use of abatement requirements to make cars cleaner, and a gasoline tax to economize on the use of cars. The two instruments should be combined to mimic the incentives that would have been provided by an emissions fee. Thus, cleaner cars and fewer trips are analogous to competing suppliers of emission reductions; the planner should buy from both so that marginal costs are equal. Applying that rule, the marginal cost of emission reductions is, simply, the gasoline tax rate divided by emissions per liter.*

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This article is prompted by the practical challenge of reducing air pollution from transport in a metropolitan area such as Mexico City while keeping an eye on the welfare costs of doing so. A least-cost solution to such a problem could involve behavioral change, such as modified travel patterns, as well as a number of technical modifications, whether in the form of tune-ups and retrofitting of existing capital equipment or in the form of new configurations of machinery (for example catalytic converters) or improved fuels.

These details have not been of great interest to economists in the public finance tradition (with some notable exceptions) because a fee levied on individual emissions would provide perfect incentives. Firms and households exposed to such a fee would self-select, taking (only) those measures that are most effective from society's point of view, irrespective of whether they are technical modifications, changes in input mix, or changes in the consumption basket. Using such a fee, or tradable pollution permits, the detailed actions that can be

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taken to reduce pollution need be known only to the economy's microagents, because the market can help the planner find cost-effective abatement (Baumol and Oates 1988 provides good coverage of this topic).

If a social planner were to possess data on how much pollution each individual caused through the year, then a year-end tax bill based on emissions or related damages would provide appropriate incentives for pollution reduction. When continuous monitoring of individual emissions is not applied, however (and it is not yet feasible for motor vehicles), the planner needs to investigate which sectors are polluting, what options exist within those sectors, and how to best stimulate each option. This is the context in which the analysis of a program to control air pollution from motor vehicles in Mexico City takes place.

Real-world programs to control pollution rely almost entirely on abatement, or technical controls, aimed at reducing emissions per unit of production or consumption. Abatement measures, such as the use of (costlier but) cleaner fuels and catalytic converters, will then generally be induced by regulatory and price-based policies, the design of which may have a great impact on the efficiency of the program. One example is emission standards with periodic emissions testing. The effect of the policy will obviously depend on whether the test result is a reasonable proxy for emissions in use, which again will depend on technical, institutional, and behavioral conditions.<sup>1</sup> Technical standards, such as mandating the use of catalytic converters and unleaded gasoline, may be easier to monitor but are less flexible and less directly related to emissions and may thus be costlier. Both emission standards and technical standards may be enforced by policies imposing penalties or revoking privileges. Other inducement mechanisms may be lower taxes on cleaner fuels and on cars equipped for natural gas. The costs and benefits of these measures will depend on how well the planner knows the field and, in particular, on how much the planner knows about the individuals whose behavior is to change.

Even when well designed, a program that emphasizes technical controls may be improved in a variety of ways. The most obvious way would be to use the car's emissions factor (grams emitted per liter or per mile, as determined from biannual emission tests), multiply it by the odometer reading (as a proxy for the utilization of the "pollution plant" since the last test), and apply an emissions fee to the result. The fee could be paid upon testing, or it could be uniformly paid as a presumptive tax, at the gas station, to be refunded in part to the owners of vehicles that tested to be cleaner than presumed. The efficiency gains from such a reform would come through several channels. First, all owners would have a continuous incentive to drive less, and owners of the more polluting cars would have a greater incentive. Second, all owners would have continuous incentives to make their cars cleaner, but owners that rarely use their cars would be subject to

1. Lawson and others (1990) used a test technology different from those used in mandatory test programs, and surprise roadside tests, and found that the length of time since the last periodic emissions test had little influence on whether a car's emissions were within the compliance range.

less of this pressure. As a consequence, society would waste few resources cleaning cars that are rarely used. As an added benefit, the car market would facilitate the exchange of vehicles to make sure that households that use their vehicles intensively end up with the cleaner ones.

This article investigates the gains to be made from a less ambitious reform of a traditional program. A traditional program does little or nothing to discourage the use of goods that pollute. The article assesses the advantages of including such discouragement, without trying to differentiate this discouragement according to how clean the car is (or how easy it is to clean it). The proposed reform is a gasoline tax, presumptive of emissions. It is shown that a gasoline tax, even when uniformly applied, makes sense.

The practical motivation for suggesting such a modest reform is the general suspicion that administrative and technical systems for monitoring and enforcement are still weak and vulnerable, so that it is doubtful whether emission tests can be used as major tax-collecting devices. Of course, when monitoring technology and technical capacity so allow, the program can (and should) be improved. The most immediate direction would be to use emissions test results and utilization rates to collect an emissions fee, so as to increase pressure on high polluters and to reduce wasteful pressure on low polluters. The proposed uniform increase in the variable costs of polluters could also be a reform that would allow such refinements to gain momentum over time.

Section I briefly reviews the theoretical literature. Section II develops the theoretical background for analyzing cost-effectiveness from the perspective of very simple, general equilibrium, welfare economics. Section III applies the analytical framework to data from a program to contain pollution in Mexico City and shows how inclusion of a gasoline tax in the program would reduce the costs of attaining the targeted emission reductions. Section IV offers conclusions.

## I. THEORETICAL BACKGROUND

The theory of optimal taxation has mainly been concerned with minimizing the distortionary costs of revenue-raising taxes (see, for instance, Mirrlees 1976). The broader normative public-finance literature has provided a case for an authoritative government and intervention through public expenditures, taxation, and regulation, with the two main rationales being market failure and concerns about income distribution (see Atkinson and Stiglitz 1980; Starrett 1988, for broad coverage). The result of greatest relevance for this study was provided by Pigou (1932), whose recommendation that pollution problems could best be taken care of by taxes gave rise to the term "Pigovian taxes" (the term "corrective taxes" is also used). The theory prescribes that taxes be applied so that individuals are confronted with the full marginal social costs of their activities. If taxes are applied this way, and if the definition of social costs

includes such effects as the problems caused by pollution, then pollution control would be efficient in the sense that there would be no net benefits to society from different or further prevention of pollution or from more pollution. The position that authoritative intervention, for instance through Pigovian taxation, is necessary for efficiency when there are external effects was later challenged by Coase (1960). Coase argued that voluntary negotiations between those causing and those affected by an external effect could provide for efficiency. Later literature has emphasized that negotiations, as well as an intervening, poorly informed bureaucrat, may be costly and inefficient (see Farrell 1987 for a simple exposition and discussion).

Sandmo (1975) combines the motive of revenue generation with the need to discourage pollution when he analyzes how a revenue-motivated optimal tax structure would be modified when a negative external effect, such as pollution, is associated with one of the taxed commodities. He shows that traditional, distortion-minimizing revenue formulas will prevail but that a Pigovian element will be contained in the formula for the polluting good. As a special case, if the revenue requirement is sufficiently low, taxation of the polluting good may be sufficient so that revenues can be raised without causing distortions.

Other theoretical contributions concerned with Pigovian taxes have generally abstracted from the need to generate revenues through distortionary taxes. These theoretical contributions could be interpreted as effectively assuming that it is not costly to fund the public sector or, simply, that the topics can be analyzed separately. Sandmo (1975) may provide some support for such a separation, although the pollution-control agency would need to coordinate with the revenue-generating agency.

Many analysts have, however, been concerned with the distortionary effects of Pigovian taxes when the taxes do not perfectly correct the external effects. Notable among these are Sandmo (1976), Balcer (1980), and Wijkander (1985), all of whom ask whether taxes and subsidies levied on complements and substitutes can be helpful when taxation of the polluting good is either not feasible or not perfect. They find that such supportive instruments can be helpful when (a) the polluting good is used both in a polluting and in a nonpolluting activity (Sandmo 1976), (b) some users of the polluting good cause more harm per unit consumed than others (Balcer 1980), and (c) taxing the polluting good directly is not feasible (Wijkander 1985). These results can all be read as special cases of the point made by Greenwald and Stiglitz (1986) that market equilibria in economies with market failures are not constrained Pareto optimal and that a demand system, with all its own- and cross-price elasticities, can provide opportunities to seek Pareto improvements.

Designing pollution-control policies may involve more complex mechanisms than those discussed here, in particular when the costs of pollution reductions are better known to the individual than to the planner. The literature on incentives under asymmetric information and revelation mechanisms discusses whether optimal pollution control can still be induced (or whether the losses

arising from the information asymmetry will be great).<sup>2</sup> Generally, the planner wants less pollution control from firms with high pollution-control costs. However, sending out such a signal would give firms incentives to exaggerate their control costs. The planner thus wants a mechanism that induces the firm to truthfully reveal its costs, or that induces self-selection based on true characteristics. Much of this literature centers on problems caused by small numbers of polluters, in which case the position of their individual control cost curves can be of great relevance for the desired total level of pollution.

For several reasons, however, it may be less important to construct mechanisms more complex than a straight fee when emissions are caused by many polluters—as with millions of vehicles causing urban smog. When there are many polluters, communication costs for sophisticated mechanisms may be higher. Also, the uncertainty with respect to each polluter's control cost will be of less relevance to the planner, unless the hidden parts of individual control costs are highly correlated (in which case more information through sampling of the population might be valuable). Dasgupta, Hammond, and Maskin (1980) show that the planner can do almost as well with knowledge about the population of polluters as with (additional) information about individual polluters when the number of polluters is large and the disturbance terms are uncorrelated. Hammond (1979: 263) points to an important feature of economies with many agents: "In a large economy, no agent has sufficient influence to be able to distort the terms of trade in his favor by distorting his true characteristics." When efficiency is not achieved in models of asymmetric information, constraints such as the participation constraint (that agents prefer to sign the contract with the principal) and the balanced budget constraint (that the contract neither generates nor requires funds) often play a role. In the model to be presented, in contrast, it is assumed not only that the planner has authoritative powers—and thus can impose new costs on polluters (the polluter-pays principle)—but also that a mechanism that generates or uses revenue is acceptable. Furthermore, risk aversion plays no role in the model.

In a traditional control program (which emphasizes making fuels and vehicles cleaner) the planner undertakes costly efforts to estimate the costs of pollution control for various groups of vehicles and users. These efforts are mostly based on surveys, sample tests, and engineering estimates and serve to narrow the planner's prior distribution of cost estimates for each of the groups. This information is used to estimate what the total of emission reductions should be and to design mechanisms for inducing change. Sometimes, although not always, a mechanism can be chosen that is sensitive to the particular circumstances of a vehicle or a vehicle owner in a subgroup (as when the price of conversion kits and the price of natural gas are used to make high-use vehicles self-select for conversion to natural gas).

2. See, for instance, Baron and Myerson (1982) or Besanko and Sappington (1987). For a review of results with emphasis on pollution control, see Laffont (1993).

A program consisting mostly of mandated abatement requirements has many potentially important weaknesses. The improvement proposed here—the uniform taxation of a major input or output of the polluting activity—merely removes one of these weaknesses, namely, that abatement requirements do not efficiently discourage demand for polluting goods. The gasoline tax is an indirect instrument that, through one-way communication, reveals privately held information about which trips can be sacrificed at a low social cost and encourages firms and individuals to sacrifice those. (The term “sacrifice of trips” is used figuratively for options that reduce pollution through reduced demand for the polluting good. Among other such options are more efficient cars.)

The analysis here makes the assumption that the pollution-control agency has all the existing knowledge about the status of vehicles and the efficiency of various abatement options. Removing this assumption would, obviously, open the door to further improvements through instruments that more closely mimic a true emissions fee. Consequently, the proposed program is poorer than a theoretically conceivable program in which, for instance, a pollution tax would reveal and exploit all relevant privately held information. How much poorer the program is depends on how important these remaining information gaps are, assuming that the agency exploits rationally the information that it holds. It is good to know, however, that the additional information upon which the proposed improvement relies—gasoline consumption—is readily available at the pump.

Lastly, in the theoretical literature the distinction between the optimal scale of polluting activities and optimal abatement has been treated only tangentially. The point has been made that pollution taxes are superior to abatement subsidies because the latter may lead to too much of the polluting activity (see, for instance, Baumol and Oates 1988). However, making polluters pay for abatement (as advocated by the Organization of Economic Cooperation and Development; see OECD 1975 and Opschoor and Vos 1989) does not imply optimal discouragement if they do not also pay for damages. Making polluters pay for damages would imply optimal discouragement; polluters would then choose to pay for optimal abatement. In the present study, two instruments are assumed available to the planner: an abatement requirement and a tax on a variable input (the one most strongly associated with pollution generation) in the polluting activity. Unless the emissions or the polluting good is taxed, the polluting activity is too large, even when polluters pay for abatement.<sup>3</sup> The use of more than one instrument to deal with only one negative external effect is driven by a monitoring problem. When monitoring of individual contributions to pollution is costly, indirect instruments should be used to influence the different choices that can affect pollution (see Eskeland and Jimenez 1992).

3. Some insight into the role that can be played by changes in the level of activity in polluting sectors is provided by Jorgenson and Wilcoxon (1990) and Hazilla and Kopp (1990). However, they explore changes in sectoral activity levels as result of abatement costs, rather than as a result of pollution taxes, input taxes, or output taxes.

## II. A SIMPLE MODEL WITH DEMAND MANAGEMENT AND ABATEMENT

The model must not only allow for behavioral responses to policies that can influence demand, but must also provide a measure of the social costs of such demand manipulation. The models proposed in the literature on welfare economics are tailored to these purposes. Ideally, the model would have many consumers or groups of consumers. This would allow for analysis of the distribution of costs and benefits across economic agents, apart from efficiency aspects.

To focus on efficiency, the model used here is one with a representative consumer. Such a framework has two principal shortcomings. First, it cannot be used to analyze the effects on income distribution. The use of a representative consumer can be justified only by assuming that the effects of the air pollution control strategy on income distribution is not of major interest because, for instance, the planner can use other instruments that can cheaply transfer income between groups. Second, in practice consumers differ along other dimensions, for instance, by owning unevenly polluting vehicles. The model can best be interpreted as one in which a representative consumer owns a composite of the vehicle fleet in Mexico City.

The model employed here is separable along two lines in the direct utility function, as in Balcer (1980) and Wijkander (1985), and has a representative consumer, as in Sandmo (1976) and Wijkander (1985). Finally, it is assumed that generating public revenue is not costly in itself. This assumption is reasonable only if the requirement for public sector revenue does not exhaust the potential of instruments available for costless transfers to the public sector.

### *The Consumer's Problem*

Let consumers be numbered 1 through  $n$  and let individual  $j$ 's emissions depend on the individual's consumption of the polluting good and the abatement applied. The individual's preferences are represented by a utility function, with utility depending on the quantities of polluting goods and nonpolluting goods consumed, as well as on the total amount of emissions from all  $n$  individuals. It is assumed that the utility function satisfies the traditional regularity conditions: it is quasi-concave, continuous, and twice differentiable. Furthermore, it is assumed that the quantities consumed of polluting goods and nonpolluting goods,  $x$  and  $y$ , respectively, are constrained to non-negative values, as is abatement,  $a$ , and that the individually optimal solution does not involve either of the corners  $y = 0$  or  $x = 0$ . Furthermore, in this section, it is assumed that initial expenditures on abatement are very productive (abatement is produced at constant returns to scale, but its effect on emissions is declining), so that the corner  $a = 0$  does not occur in the planner's optimum unless in combination with  $t_x = 0$ , where  $t_x$  is the rate of tax on the polluting good. The latter assumption is relaxed in section III.

It is assumed that individual  $j$  takes consumer prices as given and chooses a consumption vector that maximizes utility,  $u$ , under a budget constraint that requires that the total value of the individual's consumption not exceed the individual's income. Letting  $\beta^j$  be the shadow price of  $j$ 's budget constraint, the Lagrangian of  $j$ 's maximization problem can be written

$$(1) \quad \mathcal{L}^j = u^j \left[ y^j, x^j, \sum_{i=1}^n e^i(x^i, a^i) \right] \\ - \beta^j \left[ y^j + (p_x + t_x)x^j + p_a a^j - \left( I^j + \frac{1}{n} t_x \sum_{i=1}^n x^i \right) \right],$$

where superscripts denote individuals and  $\sum e^i(x^i, a^i)$  is the sum of emissions,  $e$ , generated by all individuals. In the budget constraint,  $(p_x + t_x)$  and  $p_a$  are the consumer prices of the polluting good and of abatement, respectively, whereas  $p_x$  and  $p_a$  are the producer prices. The nonpolluting good is untaxed, and its price is normalized to one. Furthermore, the budget constraint reflects the assumption that tax revenues are redistributed to consumers as transfers, to be added to the consumer's lump-sum income,  $I^j$ . The consumer, if expanding the consumption of the taxed good, will share the generated tax revenues with all the other individuals. Thus, public and private income at the margin have the same social value, so that there is no need for costly revenue generation. For simplicity of exposition, it is furthermore assumed that an individual's abatement has little or no value to that individual compared with the price of the abatement. Thus, the consumer applies as little abatement as possible: zero or the level mandated by the planner. Then, as abatement is chosen by the planner, the first-order condition for consumer optimum is found by setting the partial derivatives of equation 1 with respect to  $x^j$  and  $y^j$  equal to zero:

$$(2) \quad u_{x^j}^j / u_{y^j}^j + u_{e^j}^j e_{x^j}^j / u_{y^j}^j = p_x + t_x - t_x / n,$$

where subscripts to the function symbols denote partial derivatives, and the equation has been solved for the shadow price of income for consumer  $j$ . Notice that there are superscripts for only one individual in the first-order conditions. We assume that individuals are equal, in order to be able to work with a representative consumer, and may thus eliminate individual superscripts.

Additional assumptions are that individuals do not take into account the effect of their own pollution on themselves and that they do not take into account that a share of their own tax payments will be returned to them. Both are either theoretically correct descriptions or minor approximations if  $n$ , the number of individuals who pollute each other and share public revenues (here assumed to be the same), is large (Sandmo 1975). Then, from the perspective of individual optimization, the second term and the term  $t_x/n$  in equation 2 are both zero, so the first-order condition for individual optimum is

$$(3) \quad u_x / u_y = p_x + t_x.$$

Generally, the Marshallian demand functions  $x(\cdot)$  and  $y(\cdot)$  consistent with equation 3 will depend on the consumer's income, consumer prices, the mandated abatement, and the level of pollution. However, to simplify exposition and focus on the policy instruments, prices and income are suppressed. Also, the simplifying assumption that demand does not depend on the level of pollution gives the demand functions  $x = x(a, t_x)$  and  $y = y(a, t_x)$ .<sup>4</sup>

### *The Planner's Problem*

The planner affects abatement through regulation, whereas consumption decisions are influenced by the regulation and by the tax rate levied on the polluting good. It is assumed that the technology is such that production costs (and thus producer prices) are constant, that is, not influenced by the manipulation of consumer prices. As is demonstrated in the literature, the analysis extends to the case with responsive producer prices as long as there are constant returns to scale (see, for instance, Diamond and Mirrlees 1971 or Atkinson and Stiglitz 1980: 373).

In advising a benevolent planner whose objective is to maximize consumer utility, the relevant resource constraint is that of the economy as a whole because it is assumed there is no need for distortionary taxation. The problem is formulated as one of maximizing the utility of the representative consumer, and the budget constraint can be written net of taxes and transfers. The Lagrangian of this problem, with mandated abatement and a tax on the polluting good as instruments, can be written

$$(4) \quad \mathcal{L} = u\{y(a, t_x), x(a, t_x), ne[x(a, t_x), a]\} \\ - \gamma [y(a, t_x) + p_x x(a, t_x) + p_a a - I],$$

where  $u(y, x, ne)$  is substituted for  $u(y, x, \Sigma_i e^i)$ . Comparing equations 1 and 4, the difference between the individual's objective function and the planner's is that the individual does not take into account his effect on emissions, whereas the planner takes into account the effect of emissions on all individuals. A similar difference is present in the constraints of the two problems: whereas the individual looks at tax payments as costs, the planner takes into account that they are all redistributed. Thus, to the planner, taxes paid are not lost and involve costs only to the extent that they distort resource use.

An optimal program is characterized by the partial derivatives of equation 4 with respect to the abatement requirement and the tax rate both being equal to zero. Using also the partial derivatives of the resource constraint (which ties the demand responsiveness for the two consumption goods to each other), and

4. To see how the results extend, notice first that prices will be determined by the use of these policy instruments and that if producer prices are constant,  $x_i = dx/d(p_x + t_x)$ , and so on. Let  $x = x(a, t_x, e)$ ,  $y = y(a, t_x, e)$ , and  $e = e(x, a)$ . Totally differentiating and solving,  $dx/da$ ,  $dy/da$ ,  $dx/dt_x$ , and  $dy/dt_x$  can substitute for  $x_a$ ,  $y_a$ ,  $x_t$ , and  $y_t$ , and the subsequent analysis and results apply.



assuming that demand for the polluting good is not completely insensitive to its price ( $x_t \neq 0$ ), we find that the optimal allocation is characterized by<sup>5</sup>

$$(5) \quad u_x/u_y + nu_e e_x/u_y = p_x$$

$$(6) \quad nu_e/u_y = \frac{p_a}{e_a}$$

Equation 6 requires that the sum across individuals of the marginal rates of substitution be equal to the marginal rates of transformation, consistent with Samuelson's (1954) result for optimal provision of public goods. Air quality, or absence of pollution, is an ideal example of a public good according to Samuelson's definition that consumption of a public good is nonexclusive.

Using the fact that marginal rates of substitution in consumption will equal consumer prices (equation 3), the optimal allocation is induced by an appropriate abatement requirement and a tax to be levied on the polluting good equal to

$$(7) \quad t_x/(p_x + t_x) = -nu_e e_x/u_x$$

Thus, the consumer price of the polluting good shall be such as to incorporate the social costs that its consumption imposes on others (notice that no such tax on the polluting good is desirable if emissions themselves are taxed).

Solving for  $nu_e$ , optimality requires that

$$(8) \quad t_x/e_x = -p_a/e_a$$

Equation 8 states that the optimal tax rate on the polluting good, per unit of emissions from the polluting good, is equal to the direct marginal cost of abatement per unit of achieved emission reductions. This will prove a useful comparison in the next subsection, in the characterization of a cost-effective program.

The optimal program, as completely characterized by equations 5 and 6, could be implemented by one instrument: an emissions fee, if it were available. This fact is easily checked by replacing the instruments in equation 4 with a tax levied on emissions and modifying the individual budget constraint accordingly.

### *Cost-effective Pollution Control*

In the optimal program, abatement and demand management are pursued to the point where marginal benefits equal marginal costs. If benefit estimates are unavailable, or in dispute, it is helpful to ask how a specified target for emissions

5. In general, if consumption of the polluting good is completely insensitive to its price (meaning that the adjustments to price changes will be in the consumption of nonpolluting goods only), then  $c_a/e_a = nu_e$  characterizes the optimal program, whereas  $t_x$  is not determined by pollution-control objectives, because it has no effect on pollution.

(or emission reductions) can be achieved at lowest possible costs.<sup>6</sup> The following shows how the concept of cost-effectiveness, emphasizing the costs of manipulating demand, fits into a traditional framework of welfare analysis.

Starting from an arbitrary set of policies—an abatement requirement,  $a$ , and tax rate,  $t_x$ —welfare and emissions will be given as functions of  $a$  and  $t_x$ :  $w(a, t_x) = u\{y(\cdot), x(\cdot), e[a, x(\cdot)]\}$  and  $e(a, t_x) = e[a, x(\cdot)]$ . The estimated marginal effect on welfare from a small change in the tax rate, per unit of associated reductions in emissions, is found by partial differentiation and division:

$$(9) \quad \frac{\partial w}{\partial t_x} \bigg/ \frac{\partial e}{\partial t_x} = \frac{u_y t_x}{e_x} + nu_e.$$

In conventional terminology, the first element in equation 9 is the marginal cost of a change in the tax rate, and the second is the marginal benefit. Following the same procedure, but this time differentiating with respect to the abatement requirement, the marginal impact on welfare of an adjustment in the abatement requirement, per unit of associated reductions in emissions, is

$$(10) \quad \frac{\partial w}{\partial a} \bigg/ \frac{\partial e}{\partial a} = \frac{(t_x x_a - p_a) u_y}{e_x x_a + e_a} + nu_e$$

where similar comments apply for the two elements.

Equations 9 and 10 are valid expressions for the net marginal impact on welfare of a change in the tax rate and the abatement requirement, respectively, even when the instruments are not applied cost-effectively or optimally. Furthermore, should the use of one of the instruments be constrained to some value, then the optimal policy (as opposed to cost-effective pollution control), conditional on the actual application of one instrument, is characterized by the available (unconstrained) instrument's net marginal impact on welfare being equal to zero.

Composing a cost-effective program requires the comparison of marginal costs of emission reductions across instruments. It is now easily seen that a comparison of the two instruments—abatement and taxation—is robust to imprecision in the benefit estimate, because the benefit estimate is added in the same way to the expressions for the marginal impact on welfare.

The cost expression in equation 9 is very simple: marginal costs depend only on the tax rate on the polluting good (assuming that other goods are priced at marginal costs) and on the marginal impact on emissions of consuming the polluting good (grams of pollutants emitted per liter of gasoline con-

6. Quantifiable estimates of environmental benefits can be hard to come by, both in physical terms (for example, improved visibility or reduced mortality) and in value terms (for example, willingness to pay for improved visibility or reduced mortality). For a recent, general discussion, see Cropper and Oates (1992). Briefly, on what is applicable to Mexico, see Margulis (1991). For a methodology based on health effects, see Ostro (1994).

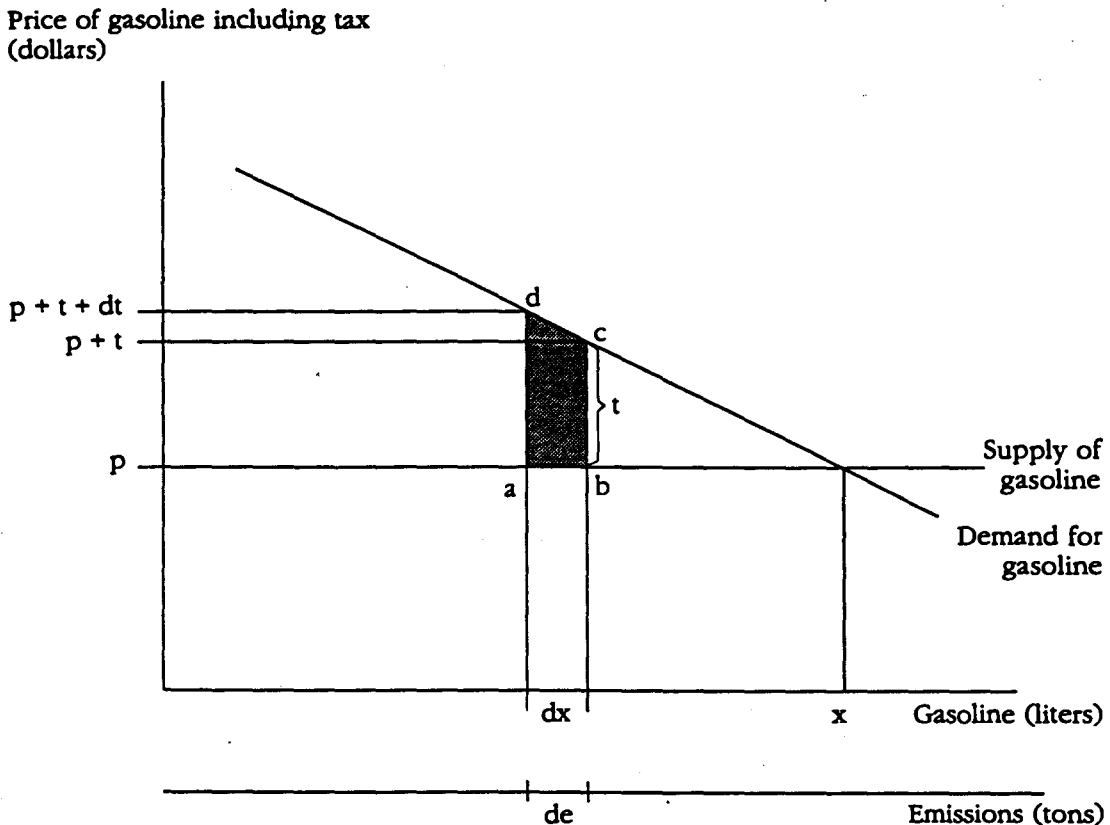
sumed).<sup>7</sup> Thus, the marginal cost of using tax rate changes to reduce emissions does not depend on the elasticity of demand for polluting goods. This result is illustrated in figure 1, which is drawn for a given level of abatement and consequently a given  $e_x$ . The welfare cost (emission benefits excluded) of a tax change,  $dt$ , is the trapezoid  $abcd$ , approximated by the rectangle  $tdx = tx, dt$  for small tax changes. Emission reductions,  $de$ , will equal  $e_x dx = e_x x, dt$ , and  $x$ , cancels out in the ratio between the two, that is,  $(dw/dt)/(de/dt) = t/e_x$ , which is the expression for marginal costs. Thus, the part of the gasoline demand curve that lies above the supply curve can be seen as a supply curve for emission reductions (emissions per liter of gasoline,  $e_x$ , is shown as an alternative unit of measurement along the  $x$  axis). This result does not say that the amount of emission benefits offered by a given tax change is independent of the demand elasticity. It says that the marginal welfare costs, per unit of obtained emission reductions, are independent of the demand elasticity. As an example, if the elasticity were small, the emission reductions would be small, but so would be the costs from sacrificed consumption, because changes in consumption would be small. The result should be of no surprise. A basic result of welfare economics says that efficiency is ensured when agents face the marginal social costs and benefits of their actions. In the absence of other distortions, that result does not depend on demand elasticities.

In comparison with equation 9, the expression for marginal welfare costs of abatement requirements, equation 10, is considerably more complicated. In particular, the responsiveness of demand to stricter abatement requirements,  $x_a$ , remains a determinant both of the welfare costs (in the numerator) and of the emission reductions (in the denominator). Somewhat paradoxically, the cost of emission reductions through abatement depends on the demand responsiveness, whereas the cost through changing the tax rate, the demand management instrument, does not.<sup>8</sup>

Figure 1 also shows, however, that the cost of achieving a given emissions reduction is higher, the lower the demand elasticity. This result carries over to the case in which abatement is available. With abatement available, the implication is that the cost of not applying a gasoline tax is higher, the higher the

7. The assumption that the responsiveness of emissions to small changes in gasoline prices will be proportional to the responsiveness of gasoline consumption, that is, that  $e_x(a, x)x = kx$ , is probably fair, although conservative (Krupnick 1992 provides some analysis). Proportionality is assumed in the main emissions projection models, such as the U.S. Environmental Protection Agency's Mobile 4 and AP-42 models. In this analysis, the use of different fuels and the relative prices of fuels are suppressed so that the results apply to a general price level for automotive fuels. In practice, relative prices between fuels may not be available for manipulating demand between fuels. Technical considerations may give the planner preferences for a specific match between car type and fuel type. (The concern in Mexico City was to reserve limited supplies of unleaded gasoline for cars with catalytic converters.)

8. Several authors have addressed the issue that abatement requirements also affect emissions through demand responsiveness, but I have not seen noted that this responsiveness affects welfare costs as well. An effect explored in the literature is that the higher costs of new cars decelerate replacement of older, dirtier cars (Crandall and others 1986; Berkovec 1985). Equation 10 does not include such effects on fleet demographics, which will, to some extent, wash out in the long run.

Figure 1. *The Welfare Cost of an Increase in the Tax on Gasoline*

demand elasticity. As an example, if abatement is cheap and demand inelastic, a cost-effective program would take only small emission reductions from demand reductions, so losses in a program that failed to stimulate demand reductions would not be large. One may notice, here, an important distinction between Pigovian and revenue-motivated taxes. For Pigovian purposes, it is particularly important to tax goods if they are elastic in demand, because one seeks reductions in demand. For revenue generation, one seeks to tax goods inelastic in demand, to minimize demand distortions.

Minimizing the welfare costs of targeted emission reductions, one would utilize the two instruments (the gasoline tax and mandated abatement) so that their marginal costs are equalized (just as one would procure goods from two suppliers). Setting the marginal-cost expressions, equations 9 and 10, equal to each other, some elements cancel out, and a cost-effective program is characterized by

$$(11) \quad t_x/e_x = -p_a/e_a.$$

Equation 11 is the solution to the maximization of welfare subject to an emissions constraint. Constrained maximization would, in addition, yield a shadow price equal to the two expressions in equation 11 on the emissions

constraint. The indirect method used here also derives the marginal cost for the two instruments when they are not exploited cost-effectively (equations 9 and 10). Equations 9 through 11 illustrate that the attractiveness of a tax on the polluting good does not depend on the availability of benefit estimates. The mere application of mandated abatement reveals that welfare costs can be saved by taxing polluting goods.

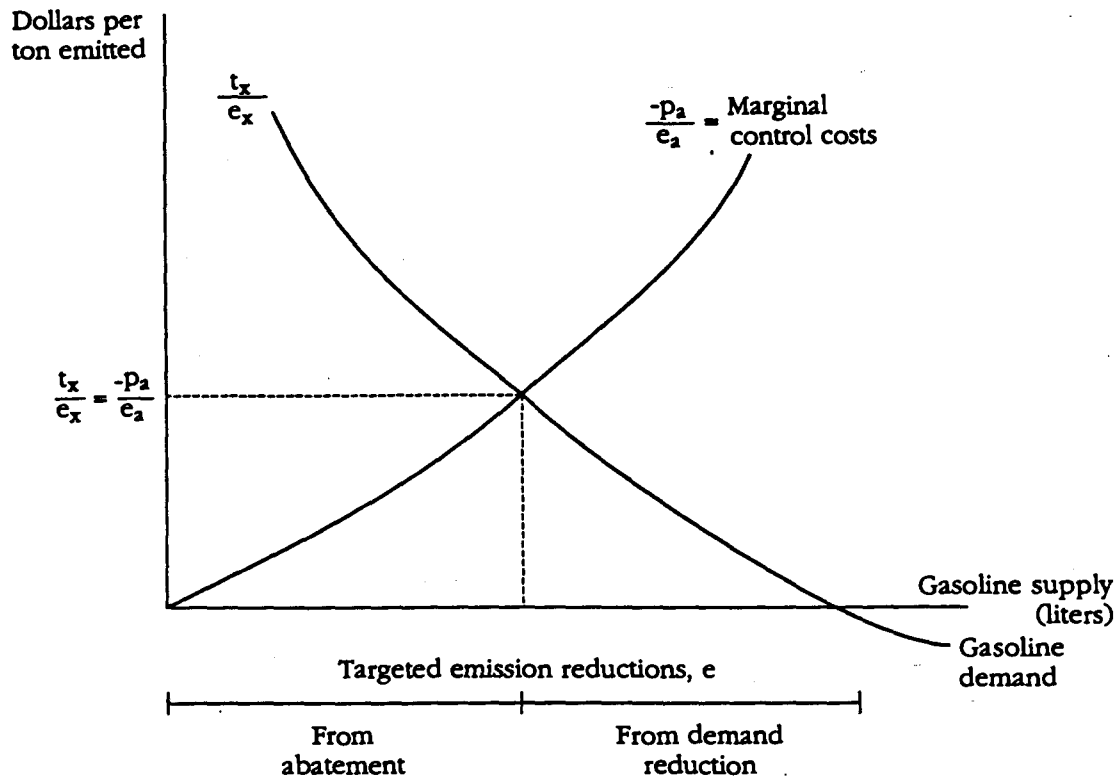
To interpret this result in light of a first-best program, notice that a program with direct taxation of individual emissions (or with tradable emission permits) optimally combines discouragement of the polluting activity with incentives to make the activity cleaner. Mandated abatement, instead, needs to be accompanied by instruments discouraging activity levels to minimize welfare costs of emission reductions. Also, the left side of equation 11 is the marginal cost measure for the tax on polluting goods, and the right side is the simple, or direct, marginal cost for abatement expenditures. Thus, this simplistic measure of cost-effectiveness, often used in applied studies, is valid, but only if the polluting good is taxed accordingly (otherwise, equation 10 gives a different measure, which is the correct one). Equation 11, which is a complete characterization of a continuum of cost-effective programs, is equal to equation 8, which, together with equation 6, gives a complete characterization of the optimal program. Thus, the optimal program is a special case among cost-effective programs.

Figure 2 illustrates a cost-effective program. The horizontal line is the amount of emission reductions targeted. The marginal cost curve for emission reductions through abatement expenditures, equation 10, is drawn from left to right (for simplicity, it is assumed that  $x_a = 0$ ). The part of the gasoline demand curve that lies above the marginal cost of supply, recalculated to be quoted per gram of implied emissions, is a supply curve for emission reductions provided by the other instrument, the gasoline tax (equation 9). A cost-effective program is found where the two curves intersect. For any other combination of abatement and tax rate that satisfies the target, the difference between the two marginal cost curves can be saved by substituting, at the margin, the cheaper for the more expensive instrument, holding emissions constant.

There is another way of exploiting the results of this section, however. Equation 11 states that knowledge of the marginal costs per unit of emissions reduced through technical controls implies knowledge of the gasoline tax rate with which it should be combined for the program to be cost-effective. This perspective is applied in the following application to data on pollution-control options in Mexico City.

### III. APPLICATION TO AN AIR POLLUTION CONTROL PROGRAM

In an analysis of emissions control options for motor vehicles in Mexico City, technical control options were ranked according to incremental costs per unit of

Figure 2. *Abatement and Demand Reduction in a Cost-Effective Program*

weighted emission reductions (table 1).<sup>9</sup> The list is thus sorted in the sequence in which measures would be implemented if the ambition level of the control program (or, equivalently, the willingness to pay for emission reductions) were gradually increased. However, demand responsiveness is not incorporated in the figures, which simply show the direct incremental costs of abatement divided by the increment in emission reductions,  $-p_a/e_a$ . The figures in table 1 are, however, valid estimates of marginal costs if the abatement initiatives are accompanied by a gasoline tax that is optimal, conditional on the extent of abatement (equation 11). Such a matching gasoline tax is shown in the fourth column.<sup>10</sup>

9. The term "weighted emission reductions" refers to the prioritization of air pollution control programs that address several kinds of emitted pollutants simultaneously. In the World Bank's analysis of the Mexico City program, weights attempted to reflect both the desirability of achieving ambient standards and the contribution of each emitted gram of a particular pollutant to total ambient concentrations of pollutants. The following weights were applied: lead, 85/g; nitrogen oxides, 4.7/g; respirable dust, 2.3/g; dust, 0.9/g; sulphur oxides, 1.4/g; carbon monoxide, 0.04/g; and nonmethane hydrocarbons, 1.8/g (see Weaver 1991).

10. For simplicity, these calculations assume that the abatement requirement does not affect demand, that is, that  $x_a = 0$ , and that the cost of abatement,  $-p_a/e_a$ , is unaffected by the gasoline tax. The latter assumption may be valid even when sizable changes in instrument use are considered, but the assumption

An example may illustrate the calculations in table 1. If the measure called "Mandate '1993 standards' for passenger cars" was the costliest applied in a program, then the cost of abatement to be matched by the gasoline tax would be \$669 per weighted ton of emissions. With this and all the cheaper measures in effect, emissions per liter for the fleet as a whole would average 60 weighted grams, and the gasoline tax should be 4 cents a liter, as calculated by equation 11.<sup>11</sup> These tax rates represent optimal discouragement of gasoline use, given the burden placed on gasoline users to make their use cleaner. Any combination of technical controls with a lower gasoline tax than suggested implies that, keeping total emissions unchanged, consumers could be better off by spending less on abatement and sacrificing more trips in return.

The tax rate per liter of gasoline in table 1 increases less than proportionally with the costs of applied technical measures. The explanation for this is that the technical measures reduce emissions per liter, so the tax base for a presumptive Pigovian gasoline tax declines with increasing control costs. Therefore, there are several reasons why the gasoline tax becomes an increasingly expensive instrument the more aggressive the program is. One is that each liter carries fewer grams of emissions as successive control measures are undertaken, so the sacrifice of a liter in consumption offers less in terms of emission reductions the cleaner the average vehicle is. Another is that, the higher the rate of the gasoline tax, the more valuable are the trips that households and firms have already sacrificed.

An estimate of the elasticity of gasoline demand is needed to estimate the emission reductions resulting from the gasoline tax. Berndt and Botero (1985) estimated demand equations based on pooled regional (1973–78), as well as national (1968–79), time-series data for gasoline sales in Mexico. On the basis of several models, they concluded with price elasticity estimates in the range of  $-0.2$  to  $-0.7$ .<sup>12</sup> Eskeland and Feyziglu (1994), using an improved

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that abatement requirements affect demand in the same way that output taxes do would be more appropriate, particularly in the long run (abatement requirements affect fixed costs of vehicle ownership more than they affect short-term variable costs). This alternative assumption would increase the emission reductions offered at any of the suggested policy combinations and thus not change the way the curve is shifted to the right when a matching gasoline tax is included in the program.

11. For the 1993 standard, annualized toxicity-weighted emissions are calculated to be 0.036 tons a year, whereas the baseline alternative would give 0.191 tons a year, so the emissions reduction is calculated to be 0.155 tons a year. Annualized costs, including fuel savings but also a higher maintenance bill, are calculated to be \$104. The 1993 standard thus offers emission reductions at  $\$104/0.155 = \$669/\text{ton}$ . To calculate the matching gasoline tax, observe that when emission controls cheaper than and including \$669 a ton are applied, the emissions coefficient is calculated to be 60 grams a liter, that is,  $(t_x \text{ dollars a liter}/60 \text{ grams a liter}) \times 10^6 \text{ grams a ton} = \$669/\text{ton}$ , which implies that  $t_x = (669 \times 60)/10^6 = 0.04$ .

12. Some other empirical studies indicate the same range. Pindyck (1979) uses pooled data and finds that for OECD countries, the price elasticity exceeds  $-0.4$  when the time for adjustment is four years or more; for Brazil and Mexico, estimates are  $-0.12$  for the short run and  $-0.55$  for the long run. Sterner, Dahl, and Franzen (1992) report estimation of various models for 21 OECD countries (time series and pooled), with an average of  $-0.25$  for short-run elasticities and  $-0.8$  for long-run elasticities.

Table 1. Mexico City: Abatement Measures and Matching Gasoline Tax Rates

<i>Abatement measure</i>	<i>Cost of weighted emission reductions (U.S. dollars per ton)</i>	<i>Cumulative weighted emission reductions (thousands of tons)</i>	<i>Cumulative costs of abatement (millions of U.S. dollars)</i>	<i>Matching gasoline tax (cents per liter)</i>
Retrofit trucks for liquid petroleum gas	-379	90	0	-4.4
Retrofit minibuses for compressed natural gas	-248	148	0	-2.8
Retrofit trucks for compressed natural gas	-225	231	0	-2.4
Recover gasoline vapor	-80	275	0	-0.8
Provide light buses with new engines	140	299	3	1.4
Bring minibuses to "1992 standards"	181	391	20	1.7
Mandate inspection and maintenance of high-use vehicles	209	545	52	1.8
Mandate "1993 standards" for gasoline trucks	264	632	75	2.1
Mandate "tier-1 standards" for taxis	322	641	78	2.5
Provide R-100 buses with new engines	482	651	83	3.7
Replace taxis to conform to "1993 standards"	510	714	115	3.7
Test emissions for passenger cars	651	771	152	4.4
Mandate "1993 standards" for passenger cars	669	883	227	4.0
Provide special diesel	699	893	234	4.2
Lower vapor pressure to 7.5	836	904	243	4.9
Provide regular unleaded gasoline	923	954	289	5.1
Decentralize inspection and maintenance of passenger cars	1,034	1,018	356	5.3
Replace gasoline trucks	1,114	1,096	442	5.0
Require 5 percent MTBE <sup>a</sup> in regular gasoline	1,201	1,116	467	5.3
Lower vapor pressure in premium unleaded to 7.5	1,313	1,128	482	5.6
Pave roads (1000 km)	1,335	1,136	498	5.7
Require "1991 standards" for passenger cars	1,367	1,180	508	5.4
Reduce sulphur to 0.1 percent in diesel	1,371	1,187	569	5.3
Require "tier-1 standards" for passenger cars	1,629	1,201	578	6.2
Conform to U.S. specifications for diesel fuel	2,097	1,207	601	7.9
Require 11 percent MTBE <sup>a</sup> in regular gasoline	2,447	1,219	613	9.0
Require 5 percent MTBE <sup>a</sup> in premium gasoline	13,487	1,222	643	49.0
Require 11 percent MTBE <sup>a</sup> in premium gasoline	14,728	1,226	686	53.2

a. MTBE is a fuel oxygenator, as an alternative to lead for raising octane levels.

Source: World Bank 1992.

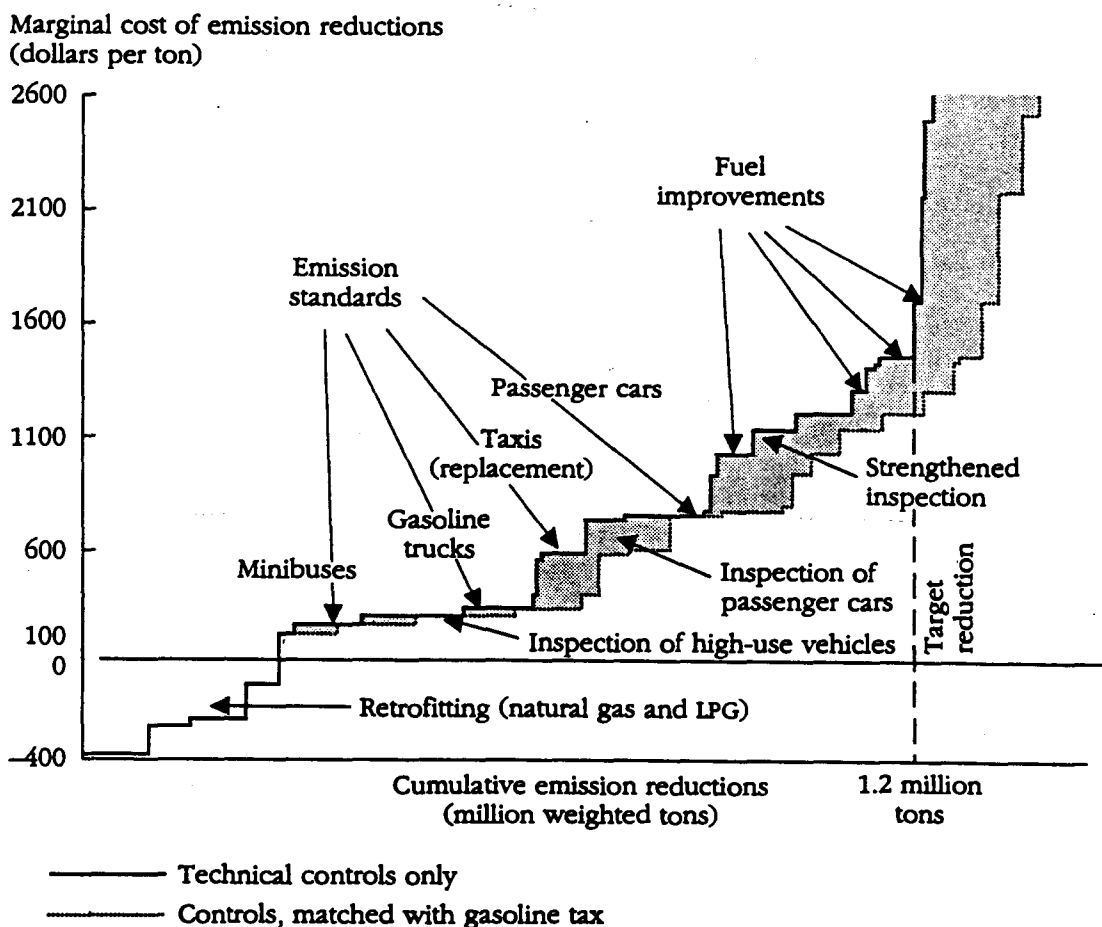


methodology and more recent data, estimate short- and long-term elasticities for total gasoline consumption of  $-0.79$  and  $-0.8$ , respectively. Thus, the most important difference in terms of estimated parameters is a higher short-term elasticity; the longer-term effects are quite similar. To estimate the effects on the 1995 emissions inventory, a price elasticity of  $-0.8$  is employed.

Because the gasoline tax will induce demand to contract, more emission reductions will be provided at every cost level, and the result will be a more moderately sloped control cost curve. The two control cost curves are shown in figure 3, with the area between the curves representing the difference in total costs between a strategy based solely on technical controls and a strategy including demand management with the help of a gasoline tax.

Under these assumptions, a gasoline tax of 6.2 cents a liter (26 percent, ad valorem) reduces demand by about 20.8 percent for a program targeted to reduce weighted emissions by 1.2 million annual tons by 1995. Applying such a

Figure 3. Program to Reduce Air Pollution Emissions from Transport in Mexico City, with and without a Gasoline Tax



Note: Calculations are based on  $-0.8$  elasticity of demand for gasoline.

tax thus allows for 20.8 percent additional emission reductions at a willingness to pay of \$1,629 a ton. Not one of the abatement measures offers emission reductions of that magnitude. Alternatively, settling for a target of 1.2 million tons in emission reductions would make unnecessary the use of measures escalating in costs from \$1,114 to \$1,629 a ton. The cost savings would be an estimated \$111 million annually, or 19.2 percent of the estimated total control costs.

The following can highlight the interdependency between the two sets of instruments. When control costs reach \$1,629 a ton, average emission coefficients are reduced by 70 percent, reducing the base for the presumptive emissions tax on gasoline to 30 percent of its precontrol level. Thus, at a willingness to pay of \$1,629 a ton, the optimal gasoline tax rate would be 20, rather than 6.2, cents a liter if the gasoline tax was the only available instrument.

A higher gasoline tax could be justified by a number of alternative assumptions, but not (as shown in section II) by a higher (or lower) demand elasticity. First, because the cost curve for technical controls is assumed to be steep for reductions exceeding 1.2 million tons, a further rise in the gasoline tax is one of the very few instruments that are effective if further reductions are needed. Second, reduction in usage also has benefits in terms of reduced congestion, noise, and accidents, none of which are accounted for in this analysis. It might be tempting to add that attaching a separate value to the transfer of funds from the private sector to the public sector would also justify a higher rate and that such transfers are to be valued in an economy that has suffered severely under strained public finances. However, such a change in modeling assumptions would motivate broadly based taxes on all goods without necessarily raising the part of the rate levied on gasoline that is motivated by the emissions control objective. (But the use of Pigovian taxes would reduce the distortionary costs of revenue generation; see Sandmo 1975). Although the present model has been developed under the assumption that generating public revenues is not costly and thus cannot be used to gauge the importance of revenue generation, it might be of interest that the tax rate indicated by the narrowly focused model would generate an estimated \$350 million in annual revenue in Mexico City alone.

#### IV. CONCLUDING REMARKS

Can demand management instruments such as a gasoline tax play a role in a cost-effective pollution control program? An analytical framework was presented that allows the comparison of demand management instruments with mandated abatement requirements. The framework provided the following results:

- Adding mandated abatement requirements to a program consisting of indirect taxes—or vice versa—will improve the program.

- The set of programs in which abatement and demand management are combined in a cost-effective fashion is characterized without knowledge of the demand elasticity for gasoline (equation 11).
- The cost associated with not including gasoline taxes in the tool kit for the control program is larger, the higher the demand elasticity.

To investigate the practical significance of these findings, the framework was applied to a recently analyzed program of technical interventions to reduce air pollution from urban transport in Mexico City. It was found that a tax of 6.2 cents a liter (26 percent, ad valorem) would be suitable to complement abatement in a program aimed at reducing emissions from the 1995 vehicle fleet by about 70 percent. Using a demand elasticity of  $-0.8$ , the inclusion of a gasoline tax in the program would make the targeted emission reductions attainable at 19.2 percent lower social costs, including the welfare costs of demand manipulation. The low level of the tax is partly explained by the fact that abatement will, by then, have reduced average emission coefficients by 60 to 70 percent, so marginal emissions per liter, the base of a presumptive Pigovian tax on gasoline, are also diminished.

The recommended tax could have been higher if higher emission reductions were targeted or if reduced congestion, accidents, and road damage were valued as well. For a city with a persistent problem of air pollution, the tax rate could decrease over time if reductions in emission coefficients so warrant. Alternatively, the tax rate could increase over time if the increase in demand for the polluting good is such that increasingly expensive measures must be undertaken.

After recent policy-induced increases in gasoline prices of 40 to 50 percent, implicit tax rates in Mexico are higher than those suggested above. The higher tax rate may well be justified by the reasons mentioned, as well as by the fact that average emission coefficients are still much higher than those assumed above for 1995. More important, the actual setting of tax and price policy in Mexico is one of a multitude of objectives and interests, including the important one of funding public budgets. The model presented here is far too modest in scope to judge a complex tax structure in a more general context.

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## Essay V



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# Is demand for polluting goods manageable? An econometric study of car ownership and use in Mexico

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### Abstract

Our motivation for estimating a demand system for gasoline and cars is its strategic relevance to policy objectives such as pollution control: if demand is responsive to pricing, demand reductions for polluting goods will provide an important share of the pollution reductions; otherwise, cleaner technologies will have to do most of the job. We estimate a model of gasoline demand and car ownership in Mexico, using a panel of annual observations by state. Key features that we introduce include instrumental variables on differenced data and the treatment of possible dynamics, measurement errors in the data, and unobserved individual state characteristics. We use tests of serial correlation in the residuals to model the dynamics properly. The resulting demand system is quite responsive to pricing even in the short term ( $-0.6$  for the own-price elasticity of gasoline), but we emphasize a medium- to long-term perspective of 5–10 years as most relevant for policy. Five- to ten-year elasticity estimates are in the range of  $-1.25$  to  $-1.13$ . Applying these elasticity estimates to data on pollution control options for the vehicle fleet in Mexico City, the costs of reaching a target for pollution reductions would be 45% more expensive if one

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424 G.S. Eskeland, T.N. Feyzioğlu / *Journal of Development Economics* 53 (1997) 423–445

were not willing to use a demand management instrument such a gasoline tax in the control program. © 1997 Elsevier Science B.V.

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*Keywords:* Demand estimation; Dynamic panel data; Pollution; Gasoline taxes

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## 1. Introduction and background

Empirical studies of energy demand systems received a wave of attention in the 1970s. Oil price shocks provided the experiments and a perception of national management priorities. This spurred an interest in empirical magnitudes and thus, development in techniques. More recently the topic has had its renaissance due to awareness of environmental externalities and its close association to energy consumption<sup>1</sup>. Our motivation is the relevance of demand relations for pollution control policies: a management challenge high on the agenda in Mexico. We engage in an empirical investigation because existing studies are outdated in terms of data, and could be improved in terms of methodology. The study should be of interest also for those interested in empirical methods, or demand for energy and transport energy and transport for other reasons.

This introductory section first explains the relevance of demand parameters when policy makers have a management objective such as pollution control, and first-best policy instruments are not available. We then review briefly the capital of relevant empirical studies, and explain how the present investigation contributes. Section 2 introduces the economic model and presents the treatment of dynamics. Section 3 discusses data and econometric issues, and Section 4 presents the empirical findings. Summary and conclusions are found in a brief Section 5.

### 1.1. Demand management in pollution control

A control strategy can deliver pollution reductions either by making each activity 'cleaner' per unit of input or output (illustratively, we may call this 'cleaner cars and fuels', or technical controls), or by scaling down the level of polluting activities (we may call this 'fewer polluting trips'). A least cost program could, theoretically at least, be induced by 'first-best' instruments such as tradeable emission permits or emission taxes, based on monitoring of individual emissions. However, obstacles such as monitoring and enforcement costs often will make it costly to use first-best instruments. Then, the policy maker may need

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<sup>1</sup> See, for instance, Pindyck (1979) and Sterner (1990) who review the developments and report results. For Mexico, see Berndt and Samaniego (1984) and Berndt and Botero (1985). For the more recent interest, see Jorgenson and Wilcoxon (1990) and Viscusi et al. (1994).

to evaluate the various ways by which emission reductions can be provided, to stimulate them separately <sup>2</sup>. For instance, fees or sanctions associated with initial certification and/or periodic tests of emission rates can stimulate cars and fuels to be cleaner (these instruments also reduce demand, but in a rather costly way if used alone). In contrast, gasoline and road taxes, mass transport policies and parking fees can manage the demand for polluting trips <sup>3</sup>.

Most of these instruments change the effective price of cars and their services. Empirical estimates of demand elasticities can inform the policy maker of the role of demand management in a cost-effective control strategy. For example, a low gasoline price elasticity would signal that a gasoline tax would not deliver much of a reduction in gasoline consumption; thus, in pollution. As in the tradition in the recent literature, we introduce a structure that decomposes changes in total demand into changes in demand for vehicles and demand for fuel per car <sup>4</sup>.

### 1.2. *The empirical literature*

There is a rich body of econometric studies of demand for vehicles and fuels. General studies of demand for energy, and specific fuels among them, bloomed in the years following the first oil price shock in 1973. Among studies focusing on demand for energy, the study of Fuss (1977) on energy use in Canadian manufacturing and the book of Pindyck (1979), 'The structure of world energy demand' probably are the most important: Fuss (1977), for demonstrating methodological breakthroughs concerning interfuel substitution; and Pindyck (1979), for a broad inquiry based on data from many countries, including developing countries.

Pindyck (1979) compares results he obtained from a developing country subsample (Mexico and Brazil) with those from developed countries. For gasoline, he finds the results to be consistent with his expectations of lower price elasticities and higher income elasticities in developing countries: "The estimated price elasticity of demand is  $-0.55$  as compared to the estimate of about  $-1.3$  obtained

<sup>2</sup> In Eskeland and Jimenez (1992), this point is elaborated in their distinction between direct instruments (based on monitoring of individual emissions) and indirect instruments (based on indicators of emissions, such as emission test results or the characteristics of cars and other machinery as a proxies for 'dirtiness', and fuel use or other measures as proxies for the output). Eskeland and Devarajan (1996), in 'Taxing Bads by Taxing Goods: Pollution Control with Presumptive Charges' synthesizes findings on taxation of goods and inputs used in polluting activities.

<sup>3</sup> The optimal stimulus to cleaner cars and fewer trips is examined in detail in Eskeland (1994) using a simple model with no other distortions and no revenue premium. (Hau, 1992a,b, and Newbery et al. (1988) discuss charging road users, to discourage road wear and congestion. McConnell and Harrington (1992), Hahn (1995), Anderson (1990) and Faiz et al. (1990) are examples of detailed studies of technical control options and costs.

<sup>4</sup> Related issues that could be analyzed with more disaggregate data are effects on the composition of the car stock, and effects by household income groups. While such issues are of great interest to us, we do not pursue them in this study, which is based on aggregate data. Disaggregate data are used in an analysis of car usage restrictions applied in Mexico City: Eskeland and Feyzioğlu (1995).



for the developed countries... the income elasticity is 1.22 as compared to 0.8 for the developed countries". Comparing his results with those of many others, Pindyck (1979) notes to have found higher elasticities in general. Arguing for data sets pooling cross-section and time series, he notes "use of data for a single country is more likely to elicit short-run or intermediate-run elasticities" (p. 233).

Another, not entirely independent development of the 1970s was regulatory changes to enhance environmental quality and fuel efficiency. Of relevance to our topic, these developments gave emphasis to the distinction between fuel efficiency, measured for instance by liters consumed per vehicle kilometer and kilometers traveled by the average household. Manski (1983) proposed an elegant model of vehicle scrappage, and Berkovec (1985) estimated a model of vehicle demand by type including such scrappage model. Using this model, he could estimate the likely effect of vehicle regulations and the associated price increases for new vehicles on the turnover and properties of the vehicle stock. Broader studies of the behavior of auto ownership and use are found in, *inter alia*, Winston et al. (1987), Crandall et al. (1986), and Grad et al. (1975). General equilibrium treatments of the effects of energy price increases and environmental regulations, with less emphasis on transportation and a particular fuel, are found in Jorgenson and Wilcoxon (1990), and in Hazilla and Kopp (1990).

There is also a literature of empirical studies based on discrete choice models and microdata, emphasizing the sensitivity of mode choice for individual trips to, *inter alia*, pricing and travel times (see, for instance, Ben-Akiva and Lerman (1985)). Results from this literature are not generally comparable to those from aggregate data — one of the most obvious reasons for this is that the mode-choice models usually assume many variables as given in the outset (residential location, workplace location, car ownership). Due to these and other important differences between the two empirical bodies of literature, one should not be surprised that estimates of such parameters as the elasticity of car use to car operating costs will usually be much lower in these models than in aggregate models.

Two recent reviews that highlight findings in empirical models are Oum et al. (1990) and Krupnick (1992). Another recent study with both a review of results and empirical estimates is Sterner (1990). Sterner (1990) surveyed close to a hundred different papers with 360 different estimated demand equations, and reestimated the models using a larger data base than those used in the studies he summarized. He points to differences in results, but concludes that there is consistency in the results and that demand does "adapt to changes in both income and prices". For OECD countries, the short-run elasticities from the dynamic models "appear to be around  $-0.2$  to  $-0.3$  and  $0.35$  to  $0.55$  for price and income respectively". The long-run elasticities were around  $-1.0$  to  $-1.4$ , and  $0.6$  to  $1.6$  for price and income, respectively. For OECD countries, the results on price elasticities are consistent with those obtained by Pindyck (1979), but the wide range for income elasticities cast a doubt on the claim that they should be systematically higher for developing countries.

Of special interest is, of course, Berndt and Botero (1985), who obtain elasticities for Mexico close to those reported for developed countries by Sterner. They present a model of vehicle stock and gasoline demand from Mexico, very similar to the objective of our study. They utilize a pooled cross-section time series data set and use the dynamic gasoline demand model discussed in Drollas (1984). For the short-run, their estimates are  $-0.23$  for the price elasticity and  $0.31$  for the income elasticity. Long-run price and income elasticities are  $-0.96$  and  $1.25$ , respectively.

There are several key issues that Berndt and Botero, 1985 do not address. First, they use pooled cross-section time series data, aggregated to 14 regions; however, they do not make allowance for possible differences among these regions (say geographical, institutional, infrastructural). If these differences across states are correlated with income and gasoline consumption the estimates of the elasticities will be biased and inconsistent<sup>5</sup>. Second, they do not test whether the dynamics are adequately modeled. As a consequence, there could still be important dynamics left as residuals in their model. Such omitted dynamics would result in biased estimates for the short-term and long-term elasticities. In fact, long-term elasticities in this study turned out to be lower than what we would have predicted in the absence of proper tests of dynamics of consumption. Third, they do not consider the effect of gasoline prices on new car sales. This results in the omission of the indirect effect of gasoline prices on total gasoline consumption, thus ignores an empirical effect of policy relevance.

We address these and other issues that arise due to the nature of the data. We utilize a pooled cross-section time series data set with annual observations from the 31 states and the federal district in the Mexican Federation. We address the problem of unobservability in the state specific characteristics by differentiating the data. We explicitly take into account the possible dynamics in behavior by incorporating it into the model and testing the residuals. We also deal with measurement error problems, specifically in income by state, by using instrumental variable techniques.

## 2. Economic model and elasticities

### 2.1. *The model*

To understand how the total demand for gasoline responds to income and price changes, we decompose it into gasoline consumption per car and number of cars.

<sup>5</sup> For example, if the presence of mountains are negatively correlated with income and positively correlated with gasoline consumption, lower income levels will be correlated with higher gasoline consumption. So, even if in each state consumption may increase as income increases, if we lump all states together, we will include the negative correlation between consumption and income, and the income elasticities estimated from the full sample will be biased downwards.

428 G.S. Eskeland, T.N. Feyzioğlu / *Journal of Development Economics* 53 (1997) 423–445

This decomposition lets us analyze the role of the car stock and the average utilization rate separately<sup>6</sup>. The model is a per capita model, so that all quantities are divided by population (except for consumption per car, for which population cancels out). We start with the identity that the average gasoline consumption per car is equal to the total consumption divided by the number of vehicles registered, for each state and time period<sup>7</sup>. Elementary calculations show that for total gasoline consumption, the elasticities will be the sum of the respective elasticities of gasoline consumption per car and elasticities of the car stock. As an example, for the income elasticity,

$$\eta_y = \eta_{c,y} + \eta_{s,y} \quad (1)$$

where,  $\eta_{i,y}$  is the income elasticity of total gasoline consumption,  $\eta_{c,y}$  is the income elasticity of gasoline consumption per car, and  $\eta_{s,y}$  is the income elasticity of the car stock.

First, we model gasoline consumption per vehicle, which we can view as a short-term utilization decision. We assume a representative consumer with a utility function separable in services rendered by a car and other goods and services. We assume that the services from the car are proportional to gasoline usage. We also assume that consumers estimate their relevant income via their current and past incomes and use this measure to determine their consumption level.

In addition to prices and income, there may be differences between states, due to geography and infrastructure that affect gasoline consumption. More specifically, more roads per car may encourage more travel and more per car gasoline usage, or may decrease per car gasoline consumption due to less congestion. We capture such effects by including miles of highway per car. There may be additional differences among states, like mountains, that affect usage per car. These additional effects are not observable to us, but we can summarize them in a state-specific variable,  $\alpha_i$ , that is constant throughout years, but differs across states. We also incorporate such effects as habit persistence by considering the lagged values of the dependent variable, and write the consumption function in the following form:

$$C_{it} = f(\text{Lag}C_{it}, \text{GASPR}_t, \text{CARPR}_t, \text{PY}_{it}, \text{HW}_{it}, \alpha_i) \quad (2)$$

where,  $C_{it}$  is the average gasoline consumption,  $\text{GASPR}_t$  is the gasoline price,  $\text{CARPR}_t$  is a price index for new cars,  $\text{PY}_{it}$  is the relevant income vector,  $\text{Lag}C_{it}$  is the vector of past consumption rates,  $\text{HW}_{it}$  is miles of highway per car,  $\alpha_i$  is a scalar that allows for the state-specific characteristics.

<sup>6</sup> Throughout we shall work with three market goods and their prices; gasoline, cars and other goods and services. We normalize each price by the price of other goods and services, thus reducing the analysis to two prices only.

<sup>7</sup> Average consumption per car is for each year, and for each state, but we do not have data on the vintage (or other) characteristics of cars in each state.

Second, we model the car stock. Current car stock is equal to depreciated car stock that remained from the previous year plus the new car purchases:

$$S_{it} = (1 - \delta)S_{it-1} + I_{it} \quad (3)$$

where,  $S_{it}$  is the stock of cars,  $I_{it}$  is the new car purchases, and  $\delta$  is the depreciation factor. New car purchases depend on the current optimal stock<sup>8</sup>. By optimal stock, we mean the optimal level of cars that consumers in each state would prefer to hold, given prices, incomes and infrastructure. The reason why we include gasoline prices is that consumers may take operating costs into account in their purchasing decisions. They may calculate the total discounted cost of gasoline consumption into the car price. This implies that as gasoline prices increase, we should expect a decrease in the car stock. However, since new cars are more fuel-efficient, an increase in gasoline prices may induce more new car sales, and perhaps an increase in the depreciation of the stock (scrappage). Considering these potential effects, we do not know a priori which direction the gas prices may affect new purchases and the stock. We capture the differences across the states by miles of highway per car and an unobserved state specific constant.

We can summarize these in the following optimal stock equation:

$$S_{it}^* = s(\text{CARPR}_t, \text{GASPR}_t, \text{PY}_{it}, \text{HW}_{it}, \alpha_i) \quad (4)$$

where  $S_{it}^*$  is the optimal car stock level.

Investment in car stock should be a function of the optimal stock and the depreciated stock from the previous year. With stock adjustment costs, when the optimal stock changes due to a lasting shift in income, investment changes permanently to a level that builds up the car stock continually, until depreciation level catches up<sup>9</sup>. This can be captured in the following investment equation (where, instead of the optimal stock, we use its determinants):

$$I_{it} = I(\text{CARPR}_t, \text{GASPR}_t, \text{PY}_{it}, \text{HW}_{it}, S_{it-1}, \alpha_i). \quad (5)$$

For depreciation, we consider two alternatives. One is a constant depreciation rate, which is independent of explanatory variables. While this is a commonly used assumption, we believe it should be tested. It can be argued that the higher the new car prices, the higher the value of the used cars would be, and the lower the number of cars to be scrapped. An elegant model is given by Manski (1983),

<sup>8</sup> Berndt and Botero (1985), among others, use a partial adjustment model. A linear partial adjustment model implies a negative, close to unity relationship between the new car purchases and the previous year's stock, simply because replacement needs are equal to a given fraction the stock. We do not use such a model, as we, Pindyck (1979) and Berndt and Botero (1985) find evidence against these models.

<sup>9</sup> This type of investment behavior is based on investment models with adjustment costs. See Auerbach and Hassett (1992) for fixed investment with adjustment costs in the United States.

and applied successfully to the US market by Berkovec (1985). In addition, as gas prices go up, if older cars have lower fuel efficiency, scrappage should increase. Similar reasoning goes for an income increase: the higher incomes are, the less one would be willing to use and repair old cars (the result could go the other way if households in income ranges to buy used cars experienced much of the income growth). We test a model allowing the depreciation rate to depend on these factors:

$$\delta_{it} = d(\text{CARPR}_t, \text{GASPR}_t, \text{PY}_{it}). \quad (6)$$

This completes our model.

Several caveats are in order. First, we should emphasize that we assume that gasoline prices and car prices are exogenously determined. Apart from believing that these assumptions are plausible, testing them would require more supply side information, and is outside the scope of this study.

Second, since we are using aggregate data at the state level in this study, we are not able to study heterogeneity in the consumption and investment behavior at a lower (individual or household) level. We characterize each state by a state-specific unobserved variable that does not change through time, its highway system and its income. But we are unable to analyze phenomena such as the importance of variation of within-state income distribution, household size, age distribution, etc.

Third, when assuming separability between car services and other consumption, we cannot consider particular changes in prices among other goods and services, such as changes in the availability or price of alternative transportation modes. A substantial change in public transportation capacity and price would likely change the pattern of new car purchases and average gasoline consumption in a particular way, but is only captured through its effect on the overall price of other goods and services in our model. Consider, for example, a decrease in the price of public transportation coupled with a capacity increase. Some consumers would then be induced to use public transportation, and the demand curve for car services would shift inwards. In consequence, the average income of the people who have cars would appear to have risen. If such an incidence occurred simultaneously with income growth, our analysis would only capture the fact that the incomes at which cars are bought was shifted upward, and that the cross price effect to other goods and services was substantial. In consequence, our estimates would be too low for the income elasticity and too high for the price elasticity, should the separability assumption be inappropriate<sup>10</sup>.

## 2.2. Elasticities

We assume constant income and price elasticities of gasoline consumption and investment in cars by estimating functions linear in logarithms. We also assume

<sup>10</sup> The opposite would be true, of course, if price changes for other goods and services occurred predominantly among goods and services of little relevance to car and gasoline demand.

that consumers consider their current and previous incomes, and therefore include as many lagged incomes as are statistically significant. We differentiate between the short-run and the long-run elasticities by formulating the equations in a dynamic form. This is done by including lagged dependent and independent variables as explanatory variables.

From the consumption behavior defined in Eq. (2) we obtain the following utilization equation (or gasoline consumption per car):

$$\ln C_{it} = \beta_0 + \alpha_i + \sum_{j=1}^m \beta_j \ln C_{it-j} + \theta_1 \ln \text{GASPR}_t + \theta_2 \ln \text{CARPR}_t + \theta_3 \ln \text{HW}_{it} \\ + \sum_{j=0}^l \lambda_j \ln Y_{it-j} + \epsilon_{it} \quad (7)$$

where  $C_{it}$  is gasoline consumption,  $m$  is the lag length for the dependent variable,  $\text{GASPR}_t$  is the gasoline price,  $\text{CARPR}_t$  is the car price index,  $Y_{it}$  is income,  $l$  is the lag length for adjustment in income,  $\alpha_i$  are the individual state effects, and  $\epsilon_{it}$  is the idiosyncratic error term that is assumed to be uncorrelated through time and across states. The parameters  $\theta_1$ ,  $\theta_2$  and  $\lambda_0$  can be interpreted as short-run price and income elasticities, respectively. The lag length of the dependent variable,  $m$ , is determined by the minimum number of lags that are necessary to obtain an error term that does not have any serial correlation (see Arellano and Bond, 1991). The number of lagged income values,  $l$ , is also determined by statistical tests.

Similarly, we assume that the investment equation for new car purchases is linear in logarithms:

$$\ln I_{it} = \gamma_0 + \omega_i + \sum_{j=1}^k \gamma_j \ln I_{it-j} + \phi_1 \ln \text{GASPR}_t + \phi_2 \ln \text{CARPR}_t + \phi_3 \ln S_{it-1} \\ + \phi_4 \ln \text{HW}_{it} + \sum_{j=0}^n \tau_j \ln Y_{it} + v_{it} \quad (8)$$

where  $I_{it}$  is the investment variable,  $\text{CARPR}_t$  is the car price index,  $k$  and  $n$  are the appropriate lag lengths,  $\omega_i$  are the individual state effects, and  $v_{it}$  is the idiosyncratic error term that is assumed to be uncorrelated through time and across states. Arguments similar to those for the utilization Eq. (7) also follow. The parameters  $\phi_1$ ,  $\phi_2$  and  $\tau_0$  are interpreted as short-run price and income elasticities for new car sales.

Depreciation may also be responsive to changes in prices and income, and therefore can affect the elasticity calculations for the total stock. We let

$$S_{it} = (1 - \delta_{it}) S_{it-1} + I_{it} + \eta_{it} \quad (9)$$

where

$$\delta_{it} = \delta_0 + \delta_1 \ln \text{GASPR}_t + \delta_2 \ln \text{CARPR}_t + \delta_3 \ln Y_{it}$$

Elasticities for depreciation would be the coefficients  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  divided by depreciation rate.

432 G.S. Eskeland, T.N. Feyzioğlu / *Journal of Development Economics* 53 (1997) 423–445

While short-run elasticities for gasoline consumption per car are readily seen as the coefficients of the price and income variables, the long-run elasticities have to be calculated from the dynamics of the utilization equation. We calculate the implied long-run elasticities for gasoline consumption per car by adding the coefficients of the income variable and solving the difference equations defined by setting the errors to zero.

It is trickier to calculate the elasticities for the car stock. For the short run, the elasticities are the price and current income elasticities of investment times the ratio of investment to car stock, plus elasticities for depreciation multiplied with depreciation rate and ratio of previous to current stock. In steady state, stocks should be equal to the desired stock level  $S^*$ , and depreciation level should be equal to the investment level. If we assume that depreciation rate is not responsive to changes in prices and income, the magnitude of the rate itself does not affect the elasticities. Since lasting shifts in prices or income change the investment levels permanently, Eq. (3) implies that a percentage change in investment, accumulated, leads to the same percentage change in the stock level<sup>11</sup>. For the steady state, the relevant investment elasticity of income, of course, is the sum of all the coefficients of the current and lagged income variables. If the depreciation rate is responsive to changes in prices and income, we should subtract its elasticities from the investment elasticities to obtain the long-run elasticities for the car stock.

Elasticities for total gasoline consumption, which are given by Eq. (1), are the sums of the elasticities of gasoline consumption per car and number of cars. While sums are trivial for the short run, we should be cautious in adding the long-run elasticities. The reason is that stocks tend to converge at a much slower rate than consumption; it would be informative to explicitly spell out how long it would take for these components to converge.

### 3. Data and econometric issues

#### 3.1. Data

The data is collected across 31 states and the Federal District in Mexico from 1982 through 1988. National income data is available annually but disaggregated income for each state is published only every fifth year. The disaggregated data on

<sup>11</sup> When we solve Eq. (3) for long-run stock, stock is equal to the long-run investment divided by the depreciation rate. Another way of looking at the same result is that, in the long run, depreciation rate is equal to the ratio of investment to stocks, since investment level is equal to the depreciation level:  $S = I/\delta$ . This, in turn, implies that (assuming depreciation is constant),  $\partial S/\partial Y = (1/\delta)\partial I/\partial Y$ . But since  $\delta = I/S$ , adding income to both sides show that income elasticity of stock is equal to income elasticity of investment.

income is obtained from a publication of the Bureau of Statistics (Escudero and Rivas, 1989, INEGI), which uses the Chow and Lin (1971) method to model income levels by state for the years not published. The Chow and Lin (1971) method lets us form unbiased estimates of the income levels for each state for each year by using other variables such as bank deposits that change over time and across the states, as well as aggregate income for the nation and the disaggregated figures for each fifth year. By utilizing data generated by this process, we increase our sample, and obtain data on consecutive years. We treat this data as data with measurement errors, and use instruments to obtain consistent coefficient estimates<sup>12</sup>.

Gasoline consumption per car is calculated by dividing the total gasoline consumption for each state by the corresponding number of vehicles in stock. Vehicle stock data is based on registration data from INEGI, the national bureau of statistics. Throughout this study, only cars are considered<sup>13</sup>. The gasoline price GASPR is the price of 'nova', and does not include 'extra' or diesel<sup>14</sup>. New car sales is from the association of automobile manufacturers, which publishes sales by state. The index for new car prices is published by Banco de Mexico, and kilometers of roads by Secretaria de Comunicación y Transporte. Imports of cars were zero.

Whenever an 'ln' precedes a variable name, it means that variable is used in logarithmic form. We use population figures by state to estimate a 'per capita' model.

### 3.2. *Econometric issues*

Simple application of the ordinary least squares (OLS) method would result in parameters that are biased and inconsistent. This is due to the combination of three econometric issues: (i) the unobservability of the state specific individual effects; (ii) the dynamic specification that allows for habit persistence; and (iii) measurement errors in the data set. A method that is capable of remedying these three problems is the instrumental variable (IV) estimation method. In the rest of this section, we discuss these issues and remedies in detail.

The first issue is the possibility of individual, unobservable characteristics that influence a state's demand, given prices and income. Ideally, variables represent-

<sup>12</sup> Without the state-specific annual data, we would have only two years of data, five years apart, and could not estimate long-run income elasticities without imposing a priori restrictions on the lag structure.

<sup>13</sup> The registry and sales data exclude only heavy duty passenger and cargo trucks (camiones), and motorcycles. The latter represents 2.5% of the stock registered in 1988. Only heavy-duty vehicles use diesel in Mexico.

<sup>14</sup> As of 1988, nova amounted to 99.5% of the gasoline consumption for cars.



434 G.S. Eskeland, T.N. Feyzioğlu / *Journal of Development Economics* 53 (1997) 423–445

ing these characteristics should be included, to avoid the omitted variable problem. A state-specific constant is introduced to summarize the effect of such differences between states, to the extent that the characteristics do not change over time. To the extent that demand is influenced by state characteristics that change over time (in a way that is not fully reflected in changes in income or highways, for example), our model is unable to capture this.

The second econometric problem is due to the dynamic nature of the model<sup>15</sup>. If there is a lagged endogenous variable among the explanatory variables, then the variance components estimator under the random effects model and the least squares dummy variable estimator under the fixed effects model are biased, and for fixed time series are also inconsistent.

The third issue is the errors we have in the income variable. Even if we assume that the disaggregate income figures are correct for the years that are published, intermediate years are only estimates of the actual figures, and therefore have errors in them. Due to the interpolation methodology, the errors are uncorrelated across time and across states, but nevertheless, any error is sufficient to cause the OLS estimators to be inconsistent.

We can solve the first problem by using the differenced data, i.e., by redefining the variables to be changes across years, or by using the least squares dummy variable estimation (LSDV) or covariance estimation (CV) methods. As pointed out by Hsiao, 1986, if we difference the data, the individual effects, whether they are fixed or random, will cancel out because these effects do not change over time<sup>16</sup>. If this were the only problem, after differencing, OLS estimation method would have given unbiased and consistent estimates. However, having a lagged dependent variable as an explanatory variable or having measurement errors, render OLS, LSDV and CV estimation methods invalid.

The second and the third problems can be solved through the method of IV estimators. If we select instruments that are highly correlated with the explanatory variables, but not correlated with the errors, we can obtain consistent estimates of the coefficients, even for panel data with short time series.

The instruments we have chosen are lagged values of the gasoline consumption and lagged values of income. Since we do not have measurement error problem for the prices, we use the current prices. The data is differenced for estimation; therefore, the second lag of gas consumption will be correlated with the lagged differenced gas consumption that shows up as an explanatory variable, but it will not be correlated with the error term. Similarly, the second lag of income will be correlated with the differenced income variable, but because the measurement

<sup>15</sup> For a good exposition, see Hsiao (1986).

<sup>16</sup> Hsiao (1986, pages 75 and 89).

Table 1  
Elasticities for gasoline consumption per car

	Short run	Implied long run (5 yr)
Gasoline price elasticity	-1.04	-1.39
Car price elasticity	-0.04	-0.05
Income elasticity	0.63	0.84

errors are uncorrelated, this instrument will also be uncorrelated with the error term.

#### 4. Results

The model developed in section two links changes in prices and income to total gasoline consumption through their effect on gasoline utilization per car and the stock of cars. The stock of cars is in turn a function of new car sales and depreciation. For each of these components, we estimate the dynamic effects of prices and income by utilizing the equations defined in Section 2.1 and techniques that are discussed in Section 3. The coefficient estimates of the regressions are given in Appendix A; in this section, we report the relevant elasticities and calculations (Table 1).

For gasoline consumption per car, we observe a rapid, but not instantaneous adjustment to changes in prices and income. The resulting elasticities for gasoline consumption per car are in Table 1<sup>17</sup>. Most of the impact is within the first year and the rest is spread to no more than five years. The income elasticities are below unity, and the gasoline price elasticity is minus one for the first year, and is equal to minus 1.39 for the long-run. Lagged income variables are not significant, and we take this as an indicator that consumers do not consider their income beyond their current income (consistent with the view that the consumption per car decision is a model of utilization, a short-term decision). Car prices, as one might expect, do not have more than a slight effect on gasoline consumption per car. An interesting result is that gasoline consumption per car is positively correlated with miles of highway per car (see Appendix A). Thus, new highway construction increases car utilization more than it improves fuel efficiency via better roads and less congestion.

Our short-term elasticities are quite different from those estimated by Berndt and Botero (1985). They do not include car prices in their model, but report elasticities of -0.23 and 0.23 with respect to gasoline price and income. Their reported long-run elasticities are less distinct from ours, at -0.96 and 0.94. In their study, they use only one lag of the dependent variable. It is estimated with a

<sup>17</sup> Short-run elasticities are the regression coefficients; long-run elasticities are obtained by solving the difference equations implied.

Table 2  
Elasticities for investment in new cars

	Short run	Long run (3 yr)
Gasoline price elasticity	0.77	0.77
Car price elasticity	-0.58	-0.58
Income elasticity	4.71	2.50

high coefficient, which implies that the total effect of a change in income (on utilization) spreads over more than 15 years. We test for optimal lags, conclude with two, resulting in more rapid adjustment and larger elasticities<sup>18</sup>.

The model for investment in new cars displays large income elasticities (Table 2), confirming our expectations that developing countries tend to have a higher income elasticity for car purchases than developed countries<sup>19</sup>. The lag structure and its interpretation is as follows. Buyers adjust investment to new levels, using income changes over the last three years as basis for optimal investment. They tend to overestimate the persistence of a change in income, and correct it in the following two years<sup>20</sup>. After all the adjustments, from the third year onwards, investment has stabilized at a new level that reflects a long-term income elasticity of investment of 2.50.

The elasticity of new car purchases with respect to own price is -0.58. The elasticity with respect to gasoline prices is positive. This result cannot be ruled out a priori if we allow for the possibility that part of the attraction with new cars is

<sup>18</sup> With only one lag, our estimated elasticities would be smaller, with no statistically significant difference between the long and the short run. Berndt and Botero, 1985 do not report testing for alternative lag structures.

<sup>19</sup> One would believe income elasticities for vehicles to be highest when a high density of households enter into vehicle owning income ranges. In the US, however, modelers overestimated a downturn in vehicle demand partly because (a) the tendency towards multivehicle households was underestimated and (b) the average household size declined through the 1970s and 1980s. Pindyck (1979) conjectures, and finds, higher aggregate income elasticities for commercial energy in developing than in industrialized countries. He believes 'recruitment' is the driving force (modernization recruits more households to the classes holding equipment: vehicles, appliances, electricity connections, etc.). For gasoline, he does not test this conjecture by including vehicle registration in his LDC models. For electricity demand in Mexico, Berndt and Samaniego (1984) finds the income elasticity of new connections to partly explain the high income elasticities in aggregate demand.

<sup>20</sup> The structure of the lags means that investment does not reflect partial adjustment to an optimal stock, often proposed for 'sluggish' purchases of integer-type durable items (investment should, then, initially underestimate the permanency of an income change). Pindyck (1979) rejects evidence of stock adjustment in his aggregate data (p. 230), since the lagged stock coefficient is small enough to reflect depreciation only. Berndt and Botero, 1985 find some evidence that can be interpreted as partial adjustment, but with a lagged stock coefficient so small (2.4%) that it may reflect the depreciation of last year's stock only. Our data yields a lagged stock coefficient of 2%, but subsequent lags are inconsistent with partial adjustment.

Table 3  
Elasticities for stock of cars

	Short run	5 yr	10 yr	Very long run
Gasoline price elasticity	0.03	0.14	0.26	0.77
Car price elasticity	-0.02	-0.11	-0.19	-0.58
Income elasticity	0.19	0.58	0.93	2.50

that they are more fuel-efficient, so that there is some substitutability between new cars and gasoline <sup>21</sup>

When we estimate a depreciation model, neither income nor prices have a significant effect on depreciation. A model of depreciation as a constant share of the stock is thus not rejected, and we use an estimated constant depreciation rate of 3% (estimation results are given in Appendix A).

The model for investment in new cars, with a constant depreciation ratio, leads to a model of the stock of vehicles (Table 3). We calculate the short-run stock elasticities by multiplying the investment elasticities by the ratio of investment to stock. The short-run income elasticity of car stock is calculated to be 0.19, which is the average investment to stock ratio, 0.04, times the income elasticity of investment, 4.71. Similarly, the short-run elasticities of the stock with respect to own and gasoline prices are -0.02 and 0.03, respectively.

For long-term developments in the stock, the model has the following property. If an exogenous variable such as income (or a price) changes from one to a new permanent level, investment changes to settle on a new permanent level within three years. The stock will continue to change, converging to a new level as the depreciation level (always 3% of last year's stock) approaches the new investment level. As convergence is slow (95% within 70 years), the more relevant parameters will be intermediate figures, and we report five-year and ten-year elasticities <sup>22</sup>. In the very long run, the car stock changes 2.5% for a one percent change in income (equivalent to the income elasticity of investment), whereas in five and ten years stocks change 0.58% and 0.93% respectively. We calculate these elasticities by accumulating the difference between investment and depreciation for the indicated intervals. The results for prices and income are given in Table 3.

We can use the results presented above to calculate elasticities for total gasoline consumption. So far, only the gasoline consumption elasticities per car have been

<sup>21</sup> Kahn (1986) finds support for an 'asset pricing model' on US data for used cars (evidence that the price premium for fuel efficient cars is increasing in fuel prices). The model of Manski (1983) yields a similar result if fuel efficiency is negatively correlated with the probability of repair requirements.

<sup>22</sup> Relevance could be seen as given by the immediacy of the policy objective, for instance reflected in a discount rate. Long-term elasticities reflecting this would converge faster (Discounting at 5%, 95% is reached in 35 years) and at lower values, corresponding roughly to the 10-year (undiscounted) elasticities reported here.

Table 4  
Elasticities for the total gasoline consumption

	Short run	5 yr	10 yr	Very long run
Gasoline price elasticity	-1.01	-1.25	-1.13	-0.62
Car price elasticity	-0.06	-0.16	-0.24	-0.63
Income elasticity	0.82	1.42	1.77	3.34

presented. Total gasoline consumption varies not only due to changes in consumption per car, but also due to changes in the car stock. Eq. (1) shows how the elasticities for total consumption are obtained by simple addition of the corresponding elasticities for consumption per car and for the stock (Table 4).

From a policy management perspective, such as the management of pollution or congestion associated with car use, Table 4 provides the most relevant inputs. Total gasoline consumption is a luxury good apart from in the very short time perspective: a five-year income elasticity of 1.42 results as the sum of an income elasticity of gasoline consumption per car, 0.84, and of car stocks, 0.58. Elasticities with respect to car prices and income display the familiar feature that longer term elasticities are larger in absolute value than shorter term elasticities. For the own price elasticity of gasoline, this pattern is broken, due to the positive cross price elasticity between gasoline prices and the car stock (Table 3). An example may illustrate the effect. Higher gasoline prices, apart from suppressing consumption per car, also have a stimulating effect on stocks. The reason is that they lead to higher investments in new cars, and the depreciation rate is unaffected. The effect on total gasoline consumption becomes more important over the years, as increased investments increasingly have an effect on the stock.

Again from a policy perspective, the 5-year and 10-year elasticities should probably be seen as the most important ones. Three arguments for this view are as follows: Discounting certainly makes a very long time perspective irrelevant. Also, the deviation of the 'very long-run' elasticities rests heavily on the constancy of the depreciation rate. One may suspect that depreciation rates will be endogenous in the long run, even though our tests did not reject the hypothesis of constancy. The effect of endogenous depreciation, suggested by models such as that of Manski, would be a model with elasticities for the very long run less extreme than those reported, perhaps more like those reported for 5–10 years in Tables 3 and 4. Finally, referring to practically implementable strategies in pollution control and congestion, such as emission standards, mandatory inspection and maintenance programs, vehicle stock conversion and replacement programs, toll road and infrastructure investments — all of these require an intermediate time perspective (haste makes them very costly, while if one is not interested in the short- to medium-term effects, one should wait). Thus, if one wants to combine demand management with other strategies, the elasticities for 5–10 years would be appropriate.

For tests of sensitivity, we identified three states that had incomes substantially higher than the rest. We used dummy variables to differentiate these states and let the dummy variables interact with the income and price variables. The results were that none of the coefficients of the dummy variables were significantly different from zero. We thus conclude that there is no significant difference in elasticities between the rich states and the rest. We present these results without regression outputs not to crowd the exposition.

Finally, we should reiterate that the estimated model is in per capita terms. Thus, quantities will be scaled up by population growth with elasticities of one as long as GNP per capita is held constant, while the model parameters show how total consumption per person changes with changes in income per person.

## 5. Summary and conclusions

Assuming that demand for cars and their use is determined, predominantly, by income, road availability, prices of vehicles, fuels and other goods and services, we have used data from 31 states and the federal district over 7 years to estimate a demand model. The model incorporates adjustments in vehicle stock as well as in consumption per car. Moreover, it is estimated as a per capita model, and thus assumes that the vehicle stock and total gasoline consumption will grow in parallel with the economy if the economy grows but maintains per capita income constant.

The model estimates short-term elasticities of total gasoline consumption (similar to short-term elasticities for gasoline consumption per car) of  $-1$  and  $0.82$  with respect to own price and income, respectively. For the longer term, developments in car stocks are important. Investments in new cars are found to adjust rather quickly from one level to another as a result of a permanent shift in income or prices. Stock levels, in contrast, converge only in the very long term — implying that elasticities for the ‘very long term’ are quite different from those for the intermediate to long term (say, five to ten years). We argue that one should focus on five- to ten-year elasticities: First, it reflects a suitable time perspective for policy purposes — this is evident if one discounts future quantity changes. Secondly, the ‘very long-term effects’ rest on our result that the depreciation rate is not influenced by prices and income. We suspect that in reality it would be — although the hypothesis of a constant depreciation rate is not rejected in our seven-year data panel. Within a time horizon of five to ten years, the own-price and income elasticities for total gasoline consumption vary with 15 to 20 percent, and come out quite large: the five-year elasticities are  $-1.25$  and  $1.42$ , respectively.

A long-run income elasticity of 1.4 and above is in the upper range of a review and estimates given by Sterner (1990). However, Pindyck, and Berndt and Botero also find higher long-run income elasticities, the latter in a study for Mexico. Our estimates of five- to 10-year price elasticities for total gasoline demand are higher

than those found for the long run by Pindyck, and close to those reported by Sterner. Our five- to 10-year elasticity is also somewhat larger than that of Berndt and Botero, and our short-term price elasticity is larger.

When medium to long-term price elasticities are large, as in our case, pricing matters a great deal for resource allocation. In the case of polluting goods and services, it shows that demand management will be important in delivering emission reductions in a low-cost control program. Another way of stating this fact is that the social costs of adopting pricing policies that do not reflect social costs (costs of production and pollution, for instance) will be high, because the consequent behavioral adjustments will be large. When income elastic stocks of durable goods plays a role, as in the gasoline market, appropriate pricing becomes particularly important.

In a recent study, Eskeland (1994) explored the value of including a gasoline tax in the tool-kit of a pollution control agency. The tax would be adjusted so that the marginal costs of emission reductions would be equalized for mandated abatement (cleaner cars) and the gas tax (demand reduction). Such a matching of instruments, commanded by cost-effectiveness, is necessary to mimic the incentives that would have been given by an emission tax. The gasoline demand elasticity feeds into the analysis of a pollution control program in the following way: if a control cost curve describes options for pollution control via technical controls, then this curve is shifted downwards when a matching gasoline tax is included in the program. The area between the two curves is the welfare cost of not including a gasoline tax in the control strategy, and this area is greater, the greater the elasticity. Using our estimated price elasticity of  $-1.25$ , in an otherwise well designed program for Mexico City, failure to use the gasoline tax would make the program 44.9% more expensive, since more expensive technical controls would have to be applied.

Eskeland (1994), in contrast, assumed a price elasticity of  $-0.8$ , from Berndt and Botero (1985). With this lower assumed elasticity, failure to use the gasoline tax would make the control program only 24% more expensive (with lower elasticities, the program as a whole would be costlier under either regime, of course). The comparison between 44.9% and 24% thus illustrates how demand management instruments are more useful — more costly to ignore — the more manageable is demand.

The tax rate would be 26% ad valorem, producing 19.5% of the emission reductions. None of the individual abatement initiatives in the program produced emission reductions of that magnitude.

The fact that demand is responsive may also be used as input in discussion of other demand management instruments, such as parking fees, subway fares, tolls, cordon pricing, etc. As pointed out elsewhere, the slope of the demand curve can be viewed as an expression of the costs to consumers of sacrificing a marginally attractive trip. In that context, one need be careful with certain aggregation issues. The most important one is, perhaps, the fact that the slope of estimated the

demand curve is an aggregate demand curve, and that there are income distribution effects associated with demand management instruments. Thus, the curve reflects how trips would be sacrificed according to willingness to pay at different price levels, with a self-selection of trips between households, as well as for each household. The incidence among households requires analysis of data at the household level. Also, if revenue generating instruments such as gasoline taxes are used, incidence analysis would require assumptions about how the revenues are to be used.

Our motivation was to find out whether demand for these goods and services is at all responsive to pricing, and the results yield little support for 'elasticity pessimism': demand is responsive, and pricing matters.

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### Appendix A

The estimation results are presented in this appendix. For all results, the following apply:

The standard errors are robust to heteroscedasticity.

The Wald tests are used for the significance of the overall regression.

The Sargan test is used, with the null hypothesis that there is no specification error, including the choice of the instruments. The test statistic is distributed  $\chi^2$  under the null hypothesis. A Sargan Test Statistic that is too high with respect to the degrees of freedom indicates misspecification.

Robust test for serial correlation tests for serial correlation in the error terms. In differenced data, we expect first order serial correlation, but not second order serial correlation. This test statistics is distributed standard normal under the null hypothesis of no serial correlation. A statistic that is greater than 2 in absolute value indicates serial correlation.

We use a Generalized Method of Moments procedure to estimate the model (see Hansen (1982), MacKinnon and Davis (1993) and Arellano and Bond (1991)). Regressions are run with Dynamic Panel Data programs written in GAUSS by Arellano and Bond. Differences are used in the utilization and investment equations and levels are used in the stock equations. Results given here are from the per capita model.



442 G.S. Eskeland, T.N. Feyzioğlu / Journal of Development Economics 53 (1997) 423–445

Lag length of the dependent variable is chosen to eliminate any second order serial correlation in the error terms.

Utilization equation (gasoline consumption per car)

Dependent variable:  $\ln C_{it}$

Exogenous variable	Coefficient estimate	Standard error	<i>p</i> -value
$\ln C_{it-1}$	0.192	0.035	0.000
$\ln C_{it-2}$	0.059	0.028	0.038
$\ln \text{GASPR}_t$	-1.039	0.164	0.000
$\ln \text{CARPR}_t$	-0.039	0.008	0.000
$\ln Y_{it}$	0.625	0.098	0.000
$\ln \text{HW}_{it}$	0.775	0.067	0.000

Wald test of joint significance: 5563.591 (df = 6)

Sargan test: 26.503 (df = 22)

Robust test for first order serial correlation: -1.845

Robust test for second order serial correlation: 0.154

The heteroscedasticity consistent standard errors and *p*-values indicate that all the coefficients are significantly different from zero at 95% confidence level. Wald test rejects the hypothesis that all the coefficients are jointly equal to zero. Sargan test accepts the set of instruments used in the estimation, and the robust test for second-order serial correlation indicate that there is no detectable correlation in the error term.

Lagged income variables did not have significant coefficients.

Investment equation: (new car purchases)

Dependent Variable:  $\ln I_{it}$

Exogenous variable	Coefficient estimate	Standard error	<i>p</i> -value
$\ln \text{GASPR}_t$	0.771	0.131	0.007
$\ln \text{CARPR}_t$	-0.584	0.033	0.000
$\ln Y_{it}$	4.714	0.374	0.000
$\ln Y_{it-1}$	-1.127	0.147	0.000
$\ln Y_{it-2}$	-1.091	0.144	0.000
$\ln \text{STOCK}_{it-1}$	0.111	0.041	0.000

Wald test of joint significance: 1975.947 (df = 6)

Sargan test: 20.534 (df = 18)

Robust test for first order serial correlation: -2.688

Robust test for second order serial correlation: -0.188

The heteroscedasticity consistent standard errors and *p*-values indicate that all the coefficients are significantly different from zero at 95% confidence level. Wald test rejects the hypothesis that all the coefficients are jointly equal to zero. Similar to the utilization equation, Sargan test accepts the set of instruments used in the estimation, and the robust test for second order serial correlation indicate that there is no detectable correlation in the error term.

For this model, although dynamics were allowed in this regression, there are no lagged dependent variables because the robust statistics indicated that there were no dynamics detectable in the error terms, and lagged dependent variable coefficients were insignificant. Similarly, the coefficient of the highway variable was insignificant; therefore, we eliminated it in the final estimation. We included income up to two lags because additional lagged income made the current income insignificant.

Depreciation equation: (with constant depreciation)

Dependent variable: $(S_{it} - I_{it})$			
Exogenous variable	Coefficient estimate	Standard error	<i>p</i> -value
Constant	4936.129	582.505	0.001
$S_{it-1}$	0.970	0.008	0.000
Wald test of joint significance: 12.762 (df = 1)			
Robust test for first order serial correlation: -0.176			
Robust test for second order serial correlation: -0.969			

The heteroscedasticity consistent standard errors and *p*-values indicate that all the coefficients are significantly different from zero at 95% confidence level. Wald test rejects the hypothesis that all the coefficients are jointly equal to zero. Robust test for second-order serial correlation indicate that there is no detectable correlation in the error term.

The coefficient of lagged stock variable indicates that the depreciation rate is equal to 3% ( $= 1 - 0.97$ ).

Depreciation equation: (with variable depreciation)

Dependent variable: $(S_{it-1} - (S_{it} - I_{it}))/S_{it-1}$			
Exogenous variable	Coefficient estimate	Standard error	<i>p</i> -value
$\ln \text{GASPR}_t$	-0.380	0.564	0.501
$\ln \text{CARPR}_t$	-0.027	0.036	0.450
$\ln Y_{it}$	3.641	2.016	0.071
Wald test of joint significance: 3.455 (df = 4)			
Robust test for first order serial correlation: -3.024			
Robust test for second order serial correlation: -0.429			

The heteroscedasticity consistent standard errors and *p*-values indicate that none of the coefficients are significantly different from zero at 95% confidence level. Wald test does not reject the hypothesis that all the coefficients are jointly equal to zero. The robust test for second-order serial correlation indicate that there is no detectable correlation in the error term.

This regression is derived from Eq. (9) in Section 2.1:

$$S_{it} = (1 - \delta_{it})S_{it-1} + I_{it}$$

where

$$\delta_{it} = \delta_0 + \delta_1 \ln \text{GASPR}_t + \delta_2 \ln \text{CARPR}_t + \delta_3 \ln Y_{it}.$$

When we solve for depreciation, we get

$$\begin{aligned} (S_{it-1} - (S_{it} - I_{it})) / S_{it-1} = \delta_{it} = \delta_0 + \delta_1 \ln \text{GASPR}_t \\ + \delta_2 \ln \text{CARPR}_t + \delta_3 \ln Y_{it}. \end{aligned}$$

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*G.S. Eskeland, T.N. Feyzioglu / Journal of Development Economics 53 (1997) 423-445 445*

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