Wage Dynamics and Career Concerns in Anarchistic Firms*

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Abstract

We consider firms where a worker's effort level is contractible, but individual output is not. We attempt to determine equilibrium degree of worker discretion in choice of task (specialization) when workers have private information about their abilities, but may not use it efficiently due to career concerns. When the market observability of task choice is low, career motives are weak, and equilibrium schemes give workers full discretion over task choice, to exploit worker private information. When the market observability is high, the firm assigns tasks to workers, as in standard principal-agent models, to avoid having workers herd to prestigious tasks (where they may be unproductive). The results may be applied to understand the recent trend towards greater worker discretion and responsibility, and to understand across-industry differences in such.

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1 Introduction

A key factor to the success of a firm is the extent to which employees work on tasks, or specialize, in accordance to their abilities. For example, law firms would like their associates to specialize in a field of law that suits their talents, and large insurance companies would like their associates with strong mathematical skills working in analysis, while associates stronger with customer relations skills working in sales.

If firms have the same, or better, information about a worker's abilities than the worker has himself, we have the classic assignment problem studied in Rosen (1982) and Waldman (1984a). However, if workers have private information about their abilities, a new set of problems appear, where firms try to design schemes that make workers voluntarily choose their efficient task. One way of ensuring an efficient choice of task is for the firm to condition a worker's wage on his marginal contribution to the firm. Barring risk concerns, the worker then voluntarily chooses his efficient task in order to maximize his expected wage. However, for firms with complex production processes, there may not exist a reliable (or verifiable) measure of an individual worker's marginal contribution to the firm. In this case, the firm must base payment on other measures.

This paper studies a setting where firms can only condition payment to a worker on the worker's choice of task and on his level of effort. Given this restriction, a profit-maximizing firm wishes to design a scheme that ensures an efficient allocation of workers without increasing costs. In a static setting, such schemes are simple to construct; simply offer the workers a wage that is independent of the worker's task choice and give them a small share of the firm. With this, all workers have an incentive to allocate themselves efficiently.

In a dynamic setting it is not so easy, because career motives can disrupt this simple solution. The worker not only cares about the immediate return from his current employer (as a function of task choice), but also on the wage he expects tomorrow as a function of his choice today. When workers are free to switch employers, such career concerns create greater problems. For example, if the most-able workers choose task A, it may be beneficial for a less-able worker, who would be more efficient in task B, to also choose task A, and thereby be associated with the most-able workers by the market. Notice that these career incentives are *endogenous* because they depend on the other workers' task choices.[1] In this paper, we build a model to see when efficient task choice can exist with career concerns, and what type of wage contracts emerge when the efficient task choice can (or cannot) be implemented.

In the model, there are two types of workers, low and high, and two types of tasks (specializations): 'easy' and 'difficult'. In the easy task, productivity is constant across workers, while in the difficult task, a high worker has a higher productivity than a low worker. An efficient allocation of workers occurs when low workers specialize in the easy task, and the high workers specialize in the difficult task. There are two periods. In the first period, workers choose which firm to work for and which task to work on. The inside firm knows with certainty which task each worker chose in period 1, while the outside firm receives less information, through an imperfect signal. Hence, if workers allocate efficiently in period 1, the inside firm has superior information to the outside firm about the true ability of the workers in period 2. In the second period, the firms make offers simultaneously to each worker. Workers then choose the firm that gives them the best offer, and choose the efficient task, since there is no incentive for misrepresentation in the final period.

Let us briefly describe the main results. There are two types of equilibria: separating and rationing. In separating equilibria, workers are given full discretion over which task to undertake, and a wage scheme is designed such that efficient tasks are chosen. When career concerns of the low workers prevent the separating equilibrium, a rationing equilibrium occurs where firms limit the fraction of workers performing the easy task. In equilibrium, firms with a low degree of outside observability are characterized by a high degree of discretion given to workers, while firms with a high degree of outside observability are characterized by a low degree of discretion given to workers. The intuition for the result is that a higher degree of market observability makes choosing the 'prestigious' difficult task more attractive for low workers. To counteract this effect, the firm must limit the entry to the easy task (and force workers to the difficult task), in order to dilute the quality

[1] Hence we accommodate interactions in the incentives of individual workers that are not due to contractual assumptions as in tournament theory (Lazear & Rosen, 1981), or due to interdependent preferences (as in Fershtman et al., 2001).

of workers in the difficult task, making these workers less attractive to outside firms. All workers prefer a situation with low outside observability to a situation with high outside observability, due to the more efficient allocation of workers in a separating equilibrium.

The novelty of the present paper is to analyze an environment where workers have private information about their abilities when hired *and* where there is an assignment problem within the firm. There is a large literature that considers each of these issues separately, and in the following we briefly describe this literature.

First, in the assignment literature, Gibbons & Waldman (1999) study an assignment model with similar technology to that in our paper. However, there the inside firm and the outside firm have symmetric information about worker abilities, hence there is no strategic assignment.[2] Bernhardt & Scoones (1993) and Bernhardt (1995) build on Waldman (1984b) by considering job assignments when employers know more about the abilities of their workers than other firms do. This creates incentives for employers to hide their able workers from outside firms, by delaying promotion (leading to inefficiency). We build on these papers by considering an environment where firms are not privy to the workers' knowledge and thus are concerned with designing schemes to induce workers to choose specialization efficiently. Despite this concern, we find that firms still may set up schemes that imply an inefficient allocation of workers (by rationing) in order to avoid the best workers being hired away. The second strand of literature considers adverse selection in the labor market. This occurs when workers know more than firms about their abilities, such as in Greenwald (1986) and Acemoglu & Pische (1998). However, this literature focuses on the hiring decision, not on how firms should try to make workers utilize their private information efficiently once hired.[3]

[2] Ignoring private information is also the case in the related literature on career concerns, as in Harris & Holmstrom (1982) and Holmstrom (1982/1999). An exception is Hvide (2000), who considers an education model where workers and firms learn asymmetrically about worker abilities. Another exception is Prendergast & Stole (1996), which operates in a setting where firms do not redesign contracts in response to the distortive career incentives.

[3] The multi-tasking literature (see Holmstrom & Milgrom, 1991), considers which tasks should be included in the description of a job, and how to give incentives such that workers undertake tasks that accord with the job description. Due to lack of worker private information, there is no notion of attempting to exploit worker's competence in designing jobs. More closely related is the work on authority by Aghion

Since this paper uses auction theory as an important solution tool, it is related to a wide range of theoretical and applied work on auctions. It is related to early theoretical work by Wilson (1967) on bidding under asymmetric information; although, to our knowledge, our analysis of the first-price sealed-bid auction is novel. It is also related to recent work that uses auction theory to determine equilibrium prices in settings that are not formally defined as auctions, but where the process that determines equilibrium price can usefully be understood through the lens of auction theory. Papers from this literature include Bulow, Huang & Klemperer (1999) on takeover battles, Bulow & Klemperer (1999) on dynamic competition between oligopolists, and Baye & Morgan (2001) on comparing prices of objects are sold over the Internet to those through dealers. Klemperer (2000) reviews this literature.

The paper is structured as follows. In Section 2, we present the model, and in Section 3 we discuss the main results. We then discuss the relation between our results and the recent trend towards greater worker discretion and responsibility in Section 4. Finally, we conclude in Section 5. Note that we relegated certain proofs to Appendices A, B, and C.

2 The Model

Let us first provide a motivating example. Take a hi-tech firm developing software, whose pool of programmers are of either (relatively) low ability or (relatively) high ability. The firm wants the most talented programmers to work with creative tasks like software development, and the less talented programmers to work with more administrative tasks, like the updating of old software, customer relations and catalogue revision. However, the engineers have private information about abilities, due to better information about the factors that created their work history, and simply assigning workers according to their claimed ability does not necessarily work. Instead, the firm attempts to design contracts that exploits the private information of workers. Individual contribution to output can be difficult to measure in software development, since development of a new product often is done in teams with extremely complex production processes. Under these conditions, the firm can only condition wage on task choice and effort level (for example, hours on the

& Tirole (1997). The relation to this paper is considered later.

job). The question is what type of contracts will be provided in equilibrium, in a dynamic setting, where workers having career motives in addition to caring about present wage, and the implications for worker discretion, efficiency, turnover, and wage dynamics.

2.1 Technology and Contracts

There is a continuum of workers and two firms. Each worker privately knows whether he has either *low* or *high* ability. The share of *high* ability workers, $\theta \in (0,1)$, is publicly known. In each firm, there are two tasks, skilled and unskilled, denoted by S and N. Task N requires the effort level e_N to be completed (for both type of workers). Given that e_N is exerted, both workers have the same productivity in the N task, π_0. Task S requires the effort level e_S to be completed. Given that e_S is exerted, the *low* type has productivity π_L in the S task, and the *high* type has productivity π_H, where $\pi_L < \pi_H$. For example, we can think of effort as the time spent on doing a certain task and π as the quality of the marginal product of a worker. We assume that the cost of effort is identical across workers. For simplicity, we normalize the cost of low effort to zero, and the cost of high effort to c, i.e., $c(e_N) = 0$, and $c(e_H) = c$.[4] Notice that if c is sufficiently high, a separating equilibrium will not be efficient. We therefore confine attention to the case $c < \pi_H - \pi_0$ (otherwise a pooling equilibrium is more efficient). Likewise, if π_0 is sufficiently low, then again a separating equilibrium will not be efficient. To avoid this, we assume that $\pi_0 > \pi_L - c$ and thus (combined with the previous assumption) $\pi_H - c > \pi_0 > \pi_L - c$. We assume that (general) human capital acquisition results in higher productivity in the second period. Label by $\bar{\pi}_1$ the productivities in the first period, (π_L, π_0, π_H). For convenience, we assume that the productivities in the second period, the vector $\bar{\pi}_2$, are given by $\bar{\pi}_2 = g(\bar{\pi}_1)$, where $g(\pi_L, \pi_0, \pi_H) = (\pi_L + h, \pi_0 + h, \pi_H + h)$, i.e., that the absolute human capital acquisition is uniform across workers and tasks.[5]

We assume that the only contractible variables are the workers choice of effort and

[4] As can easily be seen, the case $c = 0$ makes the production technology of the model into a discrete version of the production technology considered by Gibbons & Waldman (1999).

[5] Specific human capital acquisition has a similar effect to introducing switching costs, in that any positive level of turnover would be inefficient. Proportional human capital acquisition, of the form $\bar{\pi}_2 = h\bar{\pi}_2$, where $h > 1$, would yield the same type of results as the specification chosen.

their choice of task. Conditional on the correct effort level being exerted, firms offer one wage for the S task and one wage for the N task.[6] If an incorrect level of effort is exerted, it is assumed that the wage to a worker is zero. The case when individual output is contractible is considered in Appendix C, where we show that our basic results (under certain conditions) are robust to such a modification.

All workers and firms are risk neutral. For simplicity, we assume that if the incentive scheme is such that a worker is indifferent between doing the N task or the S task, he will choose the efficient task. This may be due to an (unmodeled) option plan, or alternatively due to increased job satisfaction in the efficient task.

2.2 Timing

In the first period, workers are born knowing their ability (high or low) and the two firms compete in attracting them. Firms only know the probability of a worker being high (θ), and furthermore are only able to commit to contracts lasting one period. Assuming that workers exert the correct level of effort, a firm offers workers w_S^1 for the S task and w_N^1 for the N task. Given the offers, workers choose for which firm to work. Importantly, before workers choose their task, a firm has the option to *raise* any of the wages $\{w_S^1, w_N^1\}$ offered. In other words, firms can commit to not lowering wages, but may choose to raise one of them.[7] Although such raises will not occur in equilibrium, it will turn out to have an impact on equilibrium. Finally, workers choose task, and production takes place.

After the first period, the two firms bid for the workers. The *inside firm* (the worker's first employer) is assumed to be fully informed about the task choice of the worker. The *outside firm* (the competitor of the worker's first employer), however, receives some public, imprecise, information about the task choice of the worker (and thereby on wages). Formally, the public information about task choice is an independent realization of a random variable X. For simplicity, it is assumed that X can take just two values, N and S. If the worker is in N, then $X = N$ occurs with probability p, and $X = S$ occurs

[6] It may seem awkward that an offer by a firm is a vector of wages, rather than just a wage. However, we can interpret the vector as reflecting differences in overtime payment or fringe benefits between the possible tasks.

[7] In technical terms, this is the criterion of renegotation-proofness.

with probability $1-p$. But if the worker is in S, then $X = S$ with probability p, and $X = N$ occurs with probability $1-p$. As usual, $\frac{1}{2} \leq p \leq 1$, where the larger p the more informative the signal is. Notice that $p = 1$ is the case of symmetric information between the inside firm and the outside firm about which task the worker performed in the first period. The case when first period wages can be observed, in addition to task, is qualitatively similar, and will be discussed later on.

Given the informational structure, the inside firm and the outside firm compete for the workers before the second period. We assume that the bidding follows a first-price sealed-bid auction. In other words, each firm gives a single offer to a worker, in ignorance of the other firm's offer, and the worker accepts the highest offer. The simultaneous structure of the bidding process is realistic for situations where firms may bid in turn, but where workers have no way of verifying the offer made by one firm to the other firm. Hence firms make secret or unverifiable offers to workers, so that a worker cannot start a 'bidding war' by presenting one firm with the offer from the other firm.[8]

Other papers model the competition for workers as a sequential auction.[9] For example, Greenwald (1986) and Acemoglu & Pische (1998) assume that the inside firm can always match the offer made from the outside firm. Such a structure creates a winner's curse for the outside firm so extreme that it offers a wage assuming the worker has the lowest ability. This leads all the workers to stay with the inside firm, unless the cost for switching to a different firm is negative, and hence this approach is unable to generate turnover without adding assumptions about 'utility shocks'. In contrast, our approach endogenously creates turnover without assuming utility shocks (or firm heterogeneity).[10]

[8] It seems plausible to assume that firms can *choose* whether to give verifiable offers to workers or not. Hvide & Kaplan (2001) models such a situation and finds that neither the inside firm nor the outside firm would wish to give verifiable offers in equilibrium, in fear of starting a bidding war. That result provides a justification for the use of simultanous auction rather than a sequential auction as wage determining mechanism.

[9] Scoones & Bernhardt (1998) apply a (simultanous) ascending second-price auction as the wage setting mechanism. But since there is no auctioneer present in the labor market, this auction form is essentially just a technique to reproduce the full information, competitive wages.

[10] The important properties of the auction considered are those described in Proposition 1. Since it is not transparent what the rules of the bidding game are empirically, it is comforting that these properties are also satisfied in more general auction models, for example in certain hybrid versions of the auction

3 Results

Recall that workers have no incentive to misrepresent themselves in the second period, and hence choose their efficient task in that period. We first present results that focus on the *separating equilibria*, where both type of workers also choose their appropriate task in period 1. We then examine cases where the equilibria are non-separating. For some results, the proofs in the text confine attention to the cases $p = \frac{1}{2}$ and $p = 1$, while the case $p \in (\frac{1}{2}, 1)$ has been confirmed numerically.

In order to solve for the strategies in the first period, we use backward induction and start out by analyzing the equilibrium bidding for workers in the second stage, given that a separating equilibrium is played in the first stage. Recall that when the sorting is efficient at time 1, the inside firm knows the ability of a worker before the second period, while the outside firm receives a noisy signal (whose reliability is p) about the task choice of a worker.

Let w_N^2 and w_S^2 denote the expected second-period wage of a worker that chose the respective N and S task in the first period (which equals the expected *maximum* second-period offer). For convenience, we derive the following result assuming $c = h = 0$.

Lemma 1 *Given that a separating equilibrium is played in the first period,*

(i) $\pi_0 \leq w_N^2 < w_S^2 \leq \pi_H$*, with strict inequalities for* $p < 1$*.*

(ii) $\dfrac{\partial(w_S^2 - w_N^2)}{\partial p} > 0$.

Proof. For (i), see Appendix A, and for (ii), see Appendix B. ■

The intuition for (i) is that both the inside firm and the outside firm will bid more aggressively for the high workers than for the low workers; the inside firm because it knows the ability of a worker, and the outside firm because it receives an informative signal about ability. Hence, the equilibrium wage in period 2 is higher for a high worker than for a low worker, given that a separating equilibrium is played.[11] The intuition for (ii) is that the

studied and the auction considered by Greenwald (1986). Notice, however, that the standard ascending auction does not satify these properties (except with weak inequalities, see the previous footnote).

[11] When $p = 1$, the firms bid equally aggressively for both types of workers, and wage must equal productivity for both types. When the signal is completely uninformative ($p = \frac{1}{2}$), the high workers also receive a higher wage than the low worker in the second period. In this case, the outside firm must bid

outside firm will bid more aggressively for the workers with signal S the more informative the signal, and in response the inside firm will also bid more aggressively for those workers (and conversely for the workers with signal N). Hence an increased informativeness of the signal X increases wage differences in period 2, given that a separating equilibrium is played.[12]

Since the auction equilibrium is in mixed strategies, identical workers sometimes receive different wages in the second period. Empirical work have found substantial heterogeneity in wage profiles for workers, controlling for match, education, years of experience, and job level (see Gibbons & Waldman, 2000, for an overview). While this finding is usually attributed to unobservable worker (or firm) heterogeneity, Lemma 1 shows that such differences in wages may in fact result from equilibrium bidding behavior alone.[13]

The following proposition describes the contracts, wage dynamics and turnover of separating equilibria.

Proposition 1 *A separating equilibrium has the following properties:*

- *Workers are given full discretion over task choice.*
- *Low (high) workers get a wage that is higher (lower) than their marginal product in both periods.*
- *Both type of workers have positive turnover, however, high type workers have a lower turnover than low type workers.*

  For sufficiently high c and h, a separating equilibrium satisfies:

- *High workers earn more than low workers in both periods.*

equally aggressive for both type of workers. The inside firm, however, bids more aggressively for the high workers than for the low workers, since the former has a higher value to the firm.

[12] Despite the intuitive nature of Lemma 1, we were able to prove the second part analytically only for $\theta = \frac{1}{2}$, see Appendix B for details.

[13] If the auction were almost common value (see Klemperer, 1998), rather than common value, and had a small private component, then the pure strategies in the resulting Bayes-Nash equilibrium will follow these mixed strategies, arbitrarily closely. Thus, wage dispersion can also be attributed to privately observed attributes that have an insignificant effect on the value. For more on this issue, see Hvide & Kaplan (2001).

- *Wages increase over time for both types of workers.*

  For not too large c or h, a separating equilibrium also satisfies,

- *High workers have a steeper wage dynamics than low workers.*

Proof. For the first part of the proof, we assume for brevity that $c = 0$ and $h = 0$. Furthermore, we normalize by setting $\pi_0 = 0$ and $\pi_H = 1$. In order for a low worker to choose the right task in the first period, the wage over a low worker's career for choosing the N task must be at least as large as the wage over the career for choosing the S task,

$$w_N^1 + w_N^2 \geq w_S^1 + w_S^2 \tag{1}$$

Applying the same argument for a high worker, such a worker chooses the right task if and only if,

$$w_S^1 + w_S^2 \geq w_N^1 + w_S^2 \tag{2}$$

Combining (2) and (3), we get that a separating equilibrium implies that,

$$w_N^1 + w_N^2 = w_S^1 + w_S^2 \tag{3}$$

If this condition does not hold, either a low worker or a high worker has incentive to allocate himself *inefficiently*. The only way to ensure an efficient allocation of workers is to set wages such that (4) holds, and allow workers to choose their task. Hence workers are given full discretion over task choice in a separating equilibrium.

That $w_N^2 > \pi_0$ and $w_S^2 < \pi_H$ are shown in Lemma 1. We now show that $w_S^1 < \pi_H$ and that $w_N^1 > \pi_0$. As can be seen from the auction equilibrium described in Appendix A, the maximum average profit per worker made by a firm in the second period (which occurs for $p = \frac{1}{2}$) is equal to $\theta(1-\theta)$. It follows that the maximum average wage in the first period equals $\theta + \theta(1-\theta)$, due to the zero profit condition.[14] As can easily be

[14]Zero profits across periods imply that,

$$2\theta = (1-\theta)(w_N^1 + w_N^2) + \theta(w_S^1 + w_S^2)$$

where 2θ is just the total productivity across periods, and the expression on the right hand side is the total wage bill.

seen from this expression, the maximum average wage in the first period cannot exceed 1. Furthermore, from Lemma 1 it follows that $w_N^1 > w_S^1$ in a separating equilibrium, and hence $w_S^1 < 1$. Second, the maximum average profit per worker *in the first period* is 0 (which occurs for $p = 1$), and hence average wages must exceed θ in the first period. Since $w_N^1 > w_S^1$, it follows that $w_N^1 > 0$. Hence low (high) workers are paid more (less) than their marginal productivity in both periods. The same type of argument applies for $c, h > 0$. The turnover result is shown in Appendix A.

Now consider the second part of the proposition, where we introduce $c, h > 0$. Briefly, $c > 0$ plays the role of ensuring that high workers are paid more than low workers in the first period in a separating equilibrium, and $h > 0$ plays the role of ensuring that wages are increasing through time for both type of workers.

Assume that there exists a separating equilibrium for the exogenous parameters $\{c = 0, h = 0, \pi_H, \pi_0, \pi_L, \theta\} = \Pi_1$, given by the equilibrium wage vector $\{w_N^1, w_S^1, w_N^2, w_S^2\} = \Omega_1$. Further suppose that there exists a separating equilibrium for the exogenous parameters $\{\hat{c} > 0, \hat{h} > 0, \pi_H + \hat{c}, \pi_0, \pi_L, \theta\} = \Pi_2$, with equilibrium wages given by Ω_2. Notice that with $\hat{c} > 0$, firms must condition period 2 wages on task choice in period 2 (in addition to the information about task choice in period 1) to obtain efficient allocation, in contrast to the case when $c = 0$. Specifically, to obtain an efficient allocation of workers at time 2, firms will offer the workers that choose the S task an 'overtime payment', or bonus, of $\hat{c}$. The wage vector Ω_2 is characterized by four elements, $\{\hat{w}_N^1, \hat{w}_S^1, \hat{w}_N^2, \hat{w}_S^2\}$, where $\hat{w}_N^1$ ($\hat{w}_S^1$) is the period 1 equilibrium wage for a worker that chooses the N (S) task in period 1, and where $\hat{w}_N^2$ ($\hat{w}_S^2$) is the expected wage in period 2 when choosing the N (S) task in period 1, *conditional on choosing the N (S) task in period 2.* We then have that $\Omega_2 = \{w_N^1, w_S^1 + \hat{c}, w_N^2 + \hat{h}, w_S^2 + \hat{c} + \hat{h}\}$. The reason for this is twofold. First consider the effect of the human capital acquisition factor h. As can easily be confirmed from the auction equilibrium of Proposition 1, the effect of introducing h to second period wages is simply to increase wages by h, independently of ability and independently of the task choice. Moreover, wages in the first period are not affected by h, because the wage difference in the second period is not affected by h. Now consider the effect of the positive cost of effort in the S task, $\hat{c}$. Taking into account the effect of h, the productivities in Π_2 *net of effort* is the same as the productivities in Π_1. Therefore, taking into account

h, the equilibrium wages net of effort must be the same. It can easily be shown, and is hence omitted, that given that a separating equilibrium exists for Π_1, there must exist a separating equilibrium for Π_2.

To show that $\hat{w}_S^1$ can be higher than $\hat{w}_N^1$, provided c large enough, notice that for a separating equilibrium it must be the case that

$$\hat{w}_N^1 + \hat{w}_N^2 = \hat{w}_S^1 + \hat{w}_S^2 - 2c \tag{4}$$

which holds if $c > \frac{\hat{w}_S^2 - \hat{w}_N^2}{2}$. However, since $\hat{w}_S^2 - \hat{w}_N^2 < \pi_H - \pi_0$, there must exist a range of c such that a separating equilibrium exists (see Proposition 2), and moreover where $\hat{w}_S^1 > \hat{w}_N^1$. To show that $\hat{w}_N^2$ ($\hat{w}_S^2$) can be larger than $\hat{w}_N^1$ ($\hat{w}_S^1$) for high enough h is trivial and hence omitted.

Now we prove the third part of the result, where we show that a high workers have a steeper wage dynamics than a low worker in a separating equilibrium, provided c and h are not too high. Define the slope of the wage dynamics of a low worker as,

$$\Psi_N = \frac{w_N^2 - w_N^1}{w_N^1} \tag{5}$$

and for a high worker as,

$$\Psi_S = \frac{w_S^2 - w_S^1}{w_S^1} \tag{6}$$

We show that $\Psi_N < \Psi_S$ for c or h not too high. Clearly, for $c = h = 0$, the denominator of Ψ_N is higher than the denominator of Ψ_S, since $w_N^1 > w_S^1$ in that case. Also, from $w_N^1 > w_S^1$ and the fact that $w_N^2 < w_S^2$ it follows that the numerator of Ψ_N is smaller than the numerator of Ψ_S. Hence it follows that $\Psi_N < \Psi_S$ for $c = h = 0$. We now consider the effect of introducing c, $h > 0$ on Ψ_i. Assuming that a separating equilibrium exists for $c, h > 0$, we have that

$$\Psi_N' = \frac{w_N^2 + \frac{h}{1-c} - w_N^1}{w_N^1} \tag{7}$$

and,

$$\Psi_S' = \frac{w_S^2 + \frac{h}{1-c} - w_S^1}{w_S^1 + \frac{c}{1-c}} \tag{8}$$

As can easily be seen from these expressions, $\Psi'_S > \Psi'_N$ for any c given that h is zero, and $\Psi'_S > \Psi'_N$ for any h given that c is zero. $\Psi'_S < \Psi'_N$ requires that *both* c and h are larger than zero. ■

Under separation, the low (high) workers have bad (good) career prospects, due to the partial revelation of their abilities. To be willing to separate, low workers must be compensated by a relatively high wage in the first period. Hence the wage profile of high workers is steeper than the wage profile of the low workers. The reason why turnover rates are lower for the high workers is that the inside firm will be more keen to keep such workers, and will, due to asymmetric information, retain a larger share of high ability workers than low ability workers.[15] Hence there will be a 'lemons problem' in equilibrium, but not to the extent that trade breaks down, as in Akerlof (1970).

Moreover, it is reassuring that Proposition 1 is consistent with (nominal) wage decreases being rare, and with increasing wage dispersion over time through time, both strongly corroborated empirical findings from the careers in organizations literature (Baker, Gibbs & Holmstrom, 1994a,b, and Gibbons & Waldman, 2000).

A central property of separating, efficient equilibria is that they are 'anarchistic' in the sense that workers themselves choose which task to work in, instead of being assigned to one.[16] As we will return to later, this seems to be a good approximation to what occurs in knowledge-intensive firms, with individual workers having a great deal of discretion with which projects to pursue. It is also consistent with the extensive use of matrix organization through project groups, where the choice of project groups is to some extent voluntary.

We now turn to characterize worker discretion when there does not exist a separating equilibrium. First we explain the conditions for existence of a separating equilibrium in the following remark.

[15] Since some workers receive the same offer from the two firms before the second stage, the turnover rate is indeterminate. The result described on turnover holds for any indifference rule chosen by workers.

[16] If the principal and the agents can communicate without costs, we could also construct separating equilibria through the revelation principle, by letting the workers report their type to the principal. The principal would then let payment be conditional on the report ($w_{\mathsf{N}}^{\mathbf{1}}$ for a worker reporting that he is the low type, and $w_{\mathsf{S}}^{\mathbf{1}}$ for a worker reporting that he is the high type) and furthermore assign workers according to their reported type.

Proposition 2 *A separating equilibrium is more likely to exist for lower p.*

Proof. We start out by comparing the case $p = 1/2$ with the case $p = 1$, and show that the conditions for existence of a separating equilibrium is more restrictive in the latter case. For simplicity of exposition, we assume that $c = h = 0$ and $\theta = 1/2$.

For $p = 1$, it follows that in a separating equilibrium, we must have that

$$\begin{aligned} w_N^2 &= \pi_0 \\ w_S^2 &= \pi_H, \end{aligned} \tag{9}$$

By the zero-profit condition of firms and the incentive condition of workers to reveal their type, we have

$$\begin{aligned} w_N^1 &= \pi_H \\ w_S^1 &= \pi_0 \end{aligned} \tag{10}$$

We now check under which circumstances these wage offers are consistent with equilibrium in the game between firms. Suppose that firm 1 sticks to the wage schedule (w_N^1, w_S^1) and firm 2 deviates by offering the wage schedule (w'^1_N, w'^1_S), where $w'^1_S = w_S^1$ and $w'^1_N < w_N$. In that case, firm 2 would attract a share of the high workers while all the low workers choose firm 1. Since w'^1_S is less than the marginal productivity of the high worker, firm 2 would run a profit, and hence the deviation (w'^1_N, w_S^1) would be profitable. However, suppose a low worker also chooses to work for firm 2. Taking this possibility into account, firm 2 may wish to revise w'^1_N. Denote this revised offer for w''^1_N. The point with offering w''^1_N instead of w'^1_N would be to give incentives for low workers to self allocate themselves efficiently. The productivity gain from making a low worker choose the N task instead of the S task would be $\pi_0 - \pi_L$. The wage increase required to make this low worker prefer the N task to the S task would be $w_S^2 - w_N^2 = \pi_H - \pi_0$. Hence, a firm would prefer to set $w''^1_N = w_S^1 + (\pi_H - \pi_0) = \pi_H$ if

$$\pi_H - \pi_0 < \pi_0 - \pi_L \tag{11}$$

But in that case, $(w''^1_N, w'^1_S) = (w_N^1, w_S^1)$, and the deviation by firm 2 is not credible. Hence, firm 2 cannot *only* attract high ability workers and does not have additional profits, and there exists a separating equilibrium when equation (7) holds. On the other hand, when $2\pi_0 < \pi_L + \pi_H$, the firm can commit to setting $w"^1_N < w_N$ and hence only attract high workers.

We now use the same type of argument as above to show that the conditions for existence of a separating equilibrium is less restrictive when $p = \frac{1}{2}$ than when $p = 1$. Suppose that a separating equilibrium exists, and label the corresponding wages for ($\hat{w}^1_N$, $\hat{w}^1_S$, $\hat{w}^2_N$, $\hat{w}^2_S$). Then, since there is asymmetric information in the bidding before the second stage,

$$\begin{aligned} \hat{w}^2_N &> \pi_0 \\ \hat{w}^2_S &< \pi_H \end{aligned} \tag{12}$$

For a separating equilibrium to be played, zero profits (across the two periods) imply,

$$\begin{aligned} \hat{w}^1_N &< \pi_H \\ \hat{w}^1_S &> \pi_0 \end{aligned} \tag{13}$$

Suppose that firm 2 deviates by offering the wage schedule ($\hat{w}'^1_N$,$\hat{w}'^1_S$), where $\hat{w}'^1_S = \hat{w}^1_S$ and $\hat{w}'^1_N < \hat{w}_N$. The productivity gain from making a low worker choose the N task instead of the S task would, as before, be $\pi_0 - \pi_L$. The wage increase required to make a low worker prefer the N task to the S task would, however, be $\hat{w}^2_S - \hat{w}^2_N < \pi_H - \pi_0$. Hence, a firm would prefer to set $\hat{w}"^1_N = \hat{w}^1_S + (\hat{w}^2_S - \hat{w}^2_N) = \hat{w}^1_N < \pi_H$ if

$$\hat{w}^2_S - \hat{w}^2_N < \pi_0 - \pi_L, \tag{14}$$

in which case a separating equilibrium exists. Since $\hat{w}^2_S - \hat{w}^2_N < \pi_H - \pi_0$, the condition for existence of a separating equilibrium is less restrictive for $p = \frac{1}{2}$ than for $p = 1$.

For general p, to prove the result it is necessary that $w^2_S - w^2_N$ increases with p in a separating equilibrium, which is shown in Lemma 1.[17] ■

[17] One may notice that an inside firm generates profits from a worker switching to the N task by both the increase of efficiency and the usefulness of the knowledge gained. Why in deviation condition do we only take into account the former and not the latter? The answer rests in that the gain from the knowledge is solely from the outside firm's beliefs about f. The outside firm's strategy is a mixed strategy with support starting from π_0. An informed inside firm can extract all the surplus of his knowledge for

With an efficient allocation of workers, the low workers get paid *more* than their marginal productivity while the high workers get paid *less* than their marginal productivity, as shown in Proposition 1. This creates a potential incentive for firms to deviate in order to attract only high workers, by lowering the wage for the N task. However, when it is sufficiently inexpensive for firms to make low workers choose the N task instead of the S task, once workers have entered the firm, then a deviating firm cannot credibly offer a wage schedule that only attracts the high workers.

We now consider equilibrium when there does not exist separating equilibria. Such equilibria entails that workers are given less discretion over task choice than in separating equilibria.

Proposition 3 *(i) If there does not exist a separating equilibrium, there exists a rationing equilibrium, where the number of slots in the N task is restricted in each firm. (ii) There does not exist a rationing equilibrium where the number of slots in the S task is restricted. (iii) The degree of rationing is increasing in p.*

Proof. We start out by proving the existence of a rationing equilibrium where the number of slots in the N task is restricted, and then prove the impossibility of a rationing equilibrium where the slots in the S task is rationed. Finally, we prove that the degree of rationing is increasing in p. We start out by assuming $p = 1$ and then consider the case $p = \frac{1}{2}$. The case $p \in (\frac{1}{2}, 1)$ is considered in Appendix B.

For $p = 1$, when $\pi_0 < (\pi_L + \pi_H)/2$ then a deviating firm will have incentive to higher the wage of the N task once the workers have chosen that firm, and hence there does not exist a separating equilibrium. Suppose that a firm chooses a schedule so that the high workers prefer to work in the S task, and the low workers prefer to work in the N task. However, the firm allows only a fraction f of the workers that prefer the N task to enter the N task. The complementary fraction of workers, (1-f), is forced to work in the S task (the admission to the N task is being allocated in a way such that the firm does not learn

an S task worker by placing a bid at $\pi_0 + \epsilon$. Likewise, an uninformed inside firm can also make this bid and lose ϵ for all the N task workers he would have avoided. As one can see, this extra cost is negligable for small ϵ. Thus, the inside firm's value of information is actually worthless. All that matters is that the outside firm thinks he has such information.

the type of those workers that are not admitted to the N task). In that case, we still have that

$$w_N^1 + w_N^2 = w_S^1 + w_S^2$$

Moreover, second period wages must satisfy,

$$\begin{aligned} w_N^2 &= \pi_0 \\ w_S^2 &= \frac{\theta\pi_H + (1-\theta)(1-f)\pi_0}{1-f(1-\theta)} \end{aligned} \tag{15}$$

Any value of f makes the equations consistent, and we now put restrictions on f. If f is high, then a deviating firm can make a profit by the procedure described in the previous result. On the other hand, if f is low, the firm will lose money on mis-allocation. So, equilibrium is a situation where f is the maximal value that is consistent with there not existing a profitable deviation. A deviating firm can only make a profitable deviation if,

$$w_S^2 - w_N^2 \geq \pi_0 - \pi_L \tag{16}$$

Hence f^* is the value of f such that this condition holds with equality. Simplifying, we get that,

$$f^* = \frac{(1+\theta)\pi_0 - \theta\pi_H - \pi_L}{(\pi_0 - \pi_L)(1-\theta)} \tag{17}$$

Notice that when $\pi_0 > \frac{\pi_L + \pi_H}{2}$, then $f^* > 1$, and we get a separating equilibrium. The case $f^* \leq 0$ is considered in a remark below.

We now prove (ii), that there cannot be rationing equilibrium where the number of slots in the S task is restricted. If the number of slots in the S task is restricted, there are two possibilities. First, it can be the case that both types wish to work in the S task. In that case, the proportion of workers should be the same in both jobs. If this happens, there are no career concerns since no information inferred by task choice. Because of this, the firm can induce a high worker switch from the N task to the S task, by paying the same wage in the S task as in the N task. Such a scheme would increase productivity without increasing costs. So in equilibrium, it cannot be the case that both types of workers wish to work in the S task. The second possibility is that the low type wishes to work in the N task, while the high type workers wish to work in the S task. In that case, total wages must be equalized across tasks. But then, the firm can increase profits by allowing a higher fraction of workers in the S task, by allowing workers to move from the N task to the S task (since only high workers would wish to move). This occurs since both the wage in the S task is lower than in the N task (since the fraction of high workers in the S task is higher than in the N task) and productivity of high workers is higher in the S task. Hence a situation where the slots in the S task is rationed cannot be an equilibrium.

That the degree of rationing is higher for $p = 1$ than for $p = \frac{1}{2}$ follows from a very similar argument to why $w_S^2 - w_N^2$ is higher for $p = 1$ than for $p = \frac{1}{2}$ (Proposition 2).[18] The case with general p numerically yields the same type of results, and is considered in Appendix B. ■

When there is incentive for a firm to deviate from a separating equilibrium, equilibrium must have the feature that firms assign workers to tasks, in order to make the market know less about ability through the worker allocation. An alternative interpretation of rationing equilibria is that of job rotation; all interested workers are allowed to do the easy job, but only a certain amount of time.[19] The intuition for why there cannot be a rationing equilibrium where the number of slots in the S task is restricted is that if S slots are rationed then the firm could increase productivity without increasing costs, by letting more (high) workers do the S task.

While in separating equilibria workers have full discretion over which task to choose, there is also a certain discretion in rationing equilibria. There also exist equilibria where the firms are unable to construct a scheme that makes workers exploit their private information, and must force the workers to choose one of the tasks. This case is considered in the following remark. We then consider welfare properties of the different equilibria.

[18] The outline of the proof goes as follows. Given a certain degree of rationing, f, the wage difference $w_{\mathsf{S}}^2 - w_{\mathsf{N}}^2$ is greater at $p = 1$ than at $p = \frac{1}{2}$. The wage difference $w_{\mathsf{S}}^2 - w_{\mathsf{N}}^2$ at $p = 1$ is also increasing in f. Since the equilibrium f is the f such that $w_{\mathsf{S}}^2 - w_{\mathsf{N}}^2 = \pi_{\mathsf{0}} - \pi_{\mathsf{L}}$, the equilibrium f has to be decreasing from $p = \frac{1}{2}$ to $p = 1$.

[19] Under this interpretation, the inside firm should only know the identity of a certain fraction of those workers participating in job rotation.

Remark 1 *For $p = 1$, the rationing equilibrium can be a pooling equilibrium where all workers are forced to work in the S task.*

Proof. If $p = 1$, then offering a wage of $\theta\pi_H + (1-\theta)\pi_L$ for the S task is an equilibrium. If a firm tries to get workers to self-select at least partially, the second period wage for workers who chose the S task must be greater than the wage for workers who chose the N task by at least $\theta\pi_H + (1-\theta)\pi_0 - \pi_0 = \theta(\pi_H - \pi_0)$. Productivity gain for each low ability worker that switches tasks is $\pi_0 - \pi_L$ which is less than $\theta(\pi_H - \pi_0)$. Thus, there is no incentive to try to get the worker to self-select. Also notice that $(1+\theta)\pi_0 < \pi_L + \theta\pi_H$ implies $\theta\pi_H + (1-\theta)\pi_L > \pi_0$ (since $\pi_L < \pi_0$). Thus, one would not have incentive to offer a slightly higher wage to workers to take the N task. ■

Clearly rationing implies an efficiency loss, since some low workers are allocated to the S task. Since a separating equilibrium is more likely to exist the lower outside observability, we have the surprising result that a higher degree of competitiveness (higher outside observability) for workers leads to reduced efficiency, due to the misallocation that occurs from career concerns.

A related question is whether welfare of workers is improved or deteriorated when p increases. Intuitively, one would think that at least the high workers prefer a high p to a low p. We have the following result.

Proposition 4 *Both type of workers prefer a low p to a high p.*

Proof. We confine attention to comparing the case $p = \frac{1}{2}$ to the case $p = 1$. First notice that in all equilibria, the two types of workers enjoy the same level of lifetime utility. Since firms make zero profits, and since allocation is more efficient the higher level of f, it is sufficient that f must be higher for $p = \frac{1}{2}$ than for $p = 1$, which follows from Proposition 1 and Proposition 3. The case with general p is considered in Appendix B. ■

The intuition for the result goes in two steps. First, notice that both types of workers prefer a separating equilibrium to a rationing equilibrium. The reason is the following. Total wages are equal across workers in both separating and rationing equilibria. And since the total production of the firm is higher in a separating equilibrium than in a rationing equilibrium, both type of workers must be better off in a separating equilibrium. And since a separating equilibrium is more likely to exist for low p, both type of workers prefer a low p to a high p. We conclude that when ability becomes more observable, career motives becomes a more serious *obstacle* to an efficient sorting of workers. This result should be contrasted to the results of Fama (1980) and Holmstrom (1982/1999), who demonstrate how career concerns can *promote efficiency*.

In the present two-period model, direct wage information would not make a qualitative difference since the inside firm is not informed about ability of a worker before bidding at the first stage, and hence cannot reveal information about ability through the wage offer to a worker. Thus in a two-period setting, adding independent information about wages for the outside firm to act upon would be equivalent to increasing p, and would not make a qualitative difference to the results. In a three-period setting, however, the wage offered to a worker before the second stage would reveal information about the ability of a worker, since the inside firm has private information at that point, and would open up for strategic wage-setting. If (second-period) wages are observable, the inside firm knows that bidding aggressively for a high worker before the second period has two effects. The first effect, as before, is to increase the likelihood of retaining the worker. The second effect is to give the outside firm information that the worker is high, which is potentially useful for the outside firm before bidding at the third stage. Specifically, if the outside firm receives information that the second period wage of a worker was high, the outside firm will bid more aggressively for that worker (before the third period). Assuming that a separating equilibrium is played, the inside firm responds by bidding less aggressively for high workers before the second period. Hence when the outside firm receives independent information about wages, the wage difference between high and low workers in the second period will be less (and also less in the first period). Apart from that, there would be no qualitative difference to the results.

In the next section we consider numerical examples to illustrate the results.

3.1 Numerical example

We now present a typical numerical example, to illustrate the results of the previous section.

Example 1 *Suppose* $\theta = 1/2$, $\pi_0 = 1$, $\pi_h = 6$, $\pi_l = 1, c = 1, h = 1.$

Notice that even though gross productivity for a low worker is the same in the two tasks, his net productivity is higher in the N task, due to the higher cost of effort in the S task. Let us now illustrate the equilibria of the example by the following figure.

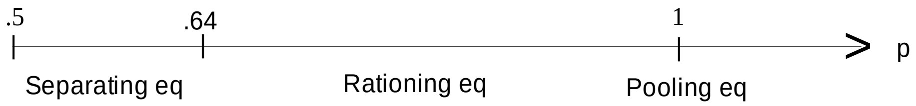

Figure 1

The figure depicts the structure of equilibrium for varying p. For a low p, there exists a separating equilibrium where the worker allocation is efficient and where welfare for *both* type of workers is maximized, which confirms Proposition 1 and Proposition 4. When p increases to .64, there only exists a rationing equilibrium, where slots in the N task is restricted, due to the possibility of cream-skimming with separation: for a high p, it becomes credible to pay a low wage for the N task, because it is expensive to make low workers switch tasks in the interim. Therefore, on the interval $(.64, 1)$ there exists a rationing equilibrium where only a fraction f of those workers that prefer to work in the N task are actually allowed to work in the N task (which confirms Proposition 3 (i)). The fraction f is decreasing in p, due to the increased threat of cream-skimming (which confirms Proposition 3 (iii)), and also welfare. When p goes to 1, the rationing equilibrium becomes a pooling equilibrium, where no workers are allowed to enter the N task, which confirms Remark 1.

High workers earn more than low workers in both periods, and both type of workers experience a wage increase between the two periods (for sufficiently high values of p). Moreover, high workers have a steeper wage schedule than low workers, for any value of p, and the turnover rate is higher for low workers than for high workers. These findings confirm Proposition 1.

We summarize the findings of the example in the following remark.

Remark 2 *For example 1, there exists a separating equilibrium for $p < .64$. For $p > .64$, there exists a rationing equilibrium where only a fraction f of the low workers are allowed into the N task. The fraction f is decreasing in p, and for $p = 1$ there only exists a pooling equilibrium, where no workers are allowed into the N task. High workers earn more than low workers in both periods, and for sufficiently high values of p, both type of workers* *experience a wage increase between the two periods. High workers have a steeper wage dynamics than low workers.*[20]

4 Discussion

Here we first discuss the plausibility of separating equilibria in light of documented management practices, and then discuss the main empirical prediction of the paper, that the degree of discretion given to workers should be decreasing in the degree of outside observability.

Baron & Kreps (1999) reports on the management practices of Sun Hydraulics Corp., a company founded in 1970 to manufacture fluid power products. The founder of Sun, Robert Koski, deemed standard management tools such as organization charts to be destructive, by restricting worker initiative and information. To deal with such problems, Koski designed the organization to eschew with almost all forms of hierarchy (to accord with State of Florida law, there is a President and a Controller). As Baron & Kreps (1999), p. 87, put it : 'Work [at Sun] is self-organized. Natural teams have formed (and reformed as necessary) spontaneously to organize work, but individual workers retain

[20] In the following table, we report the equilibrium wages and rationing fraction for varying p.

p	f	w^1_{N}	w^1_{S}	w^2_{N}	w^2_{S}	T_{L}	T_{H}
.5	1	4.39	4.61	2.61	4.39	.75	.25
.8	.22	2.81	2.81	2.93	4.93	.54	.45
1	0	x	3.5	x	5	.5	.5

For $p = .5$, there is a separating equilibrium where w^1_{N} is the wage offered to workers entering the N task, and w^2_{N} is the expected wage in the second period conditional on choosing the N task in the first period. As can easily be verified, expected payment subtracted the cost of effort c, is identical for the two tasks. T_{L} is the turnover rate for the low type workers and T_{H} is the turnover rate for the high type workes. Since .75>.25, the turnover rate is higher for the low type workers.

For $p = .8$, there is a rationing equilibrium where f is the fraction of low workers permitted into the N task and where w^1_{N} is the wage in the first period conditional on being admitted into the N task, and w^2_{N} is the expected wage in the second period conditional on being admitted to the N task in the second period and on choosing the N task in the second period. That the wage dynamics is steeper for high workers than for low workers for the given values of p can be easily verified.

The program used for generating the numbers is available on request from the authors.

the right and responsibility to choose how they spend their own time.'[21] In 1997, its products apparently enjoyed a higher margin than competitors, and had a reputation for outstanding quality.[22]

The model fits to several features of Sun. The model predicts that full discretion, separating equilibria are more likely to occur the lower outside observability. And Sun seems to be characterized by both a high degree of worker discretion and a low outside observability. For example, since job titles are non-existent at Sun, and the pay to individual workers is covert (Baron & Kreps, 1999, p. 295) it is hard for outside firms to assess the productivity of a single employee. As a consequence, Sun probably has to worry less about being outbid for inside workers than standard firms, due to the large winner's curse problem associated with bidding for Sun's workers. Furthermore, the winner's curse problem favors a low turnover at Sun, compared to firms with a higher outside observability, which accords with findings by Kaftan (1984).[23]

The model seems to capture some important aspects of modern personnel management, as exemplified by the (arguably extreme) practices of Sun Hydraulics. Importantly, these aspects are not covered by standard principal-agent models of organization, which emphasizes assignments to tasks rather than worker discretion.

There are other features of the human resource practices at Sun that our model is not rich enough to capture. For example, since production in the model is separable in the contribution of each worker, there is no notion of duplication of work between workers. In reality, such coordination costs seem important: in expansion periods both Sun and Gore seem to prefer 'cloning' existing plants rather than expanding them (Kaftan, 1984,

[21] The degree of discretion given to workers at Sun can be illustrated by a case where an engineer had been hired with a product development function in mind but had 'become intrigued with the computer in his first days on the job, and since had concentrated entirely on creating new programming applications.' (Kaftan, 1985).

[22] The following statement from W. L. Gore, founder of Gore & Associates (which produces the Gore-Tex© products) is an echo from Sun: 'In Gore & Ass., one of our basic principles is to encourage maximum freedom for each employee. There is no need for bosses, assignment of tasks, establishing lines of command, defining channels of permitted communication, and the like' (Gore, 1990).

[23] The pay policy at Sun seems to accord quite well with the absence of performance contracts in the model: 'Contrary to industry wide practices there would be no standard production times or procedures and no piece rate pay incentives at Sun Hydraulics' (Kaftan, 1984).

and Gore, 1990).[24]

At a more general level, it is interesting to link our work to developments in the organization of work. Here, an important recent trend includes the increased use of team work and job rotation, blurring of occupational barriers, a reduction in the number of management levels, decentralization of responsibility, and increased participation of employees in decision making such as through self-directed work teams and quality groups (Osterman, 1994, and Lindbeck & Snower, 1996, 2000, 2001). In short, we can refer to this development as increased worker discretion (or authority).

From the present paper, we expect that degree of worker discretion for a given job level within a firm to be decreasing in the degree of outside visibility, since greater outside visibility means that the most able workers need to be hidden through some degree of assignment. We are not aware of any empirical studies on the relation between the degree of visibility and the degree of worker discretion. Casual observation gives some support to our hypothesis. First, in hi-tech firms (such as Sun Hydraulics), where the line of operations commonly are sufficiently diffuse to make outside bidding for workers difficult, a high degree of discretion to workers is often implemented. On the other hand, in more traditional firms, the line of operations (and the tasks undertaken by individual workers) is more visible, and some degree of assignment is undertaken, as in standard principal-agent models. Second, based on survey data, Osterman (1994) reports that (p. 381) 'It is also apparent that higher-level employees have much more autonomy than do blue-collar workers.' Since the tasks of higher-level employees typically are harder to observe from the outside than that of blue-collar workers, this finding also seems confirmatory of our hypothesis.

A more thorough empirical investigation should take into account that firms with a high degree of discretion are typically rather small, which suggests that coordination costs are important in determining degree of worker discretion.[25] It should also take

[24] Coordination costs are also emphasized by the human resource literature (e.g., Milgrom & Roberts, 1990, Baron & Kreps, 1999) as a negative aspect of decentralization.

[25] In a large firm, it is more difficult for workers to have knowledge of who is doing what.

One can see this in a different context– as researchers, we enjoy almost full discretion on the projects that we attempt. Naturally, we try to avoid duplicating work of others. In a smaller field, it is easier to avoid this possible duplication since it is easier for us to be aware of all of our colleagues' working papers.

into account that different types workers are less mobile than others. For example, older workers can be expected to have higher moving costs than younger workers. This should make firms less anxious about older workers being bid away, and hence we can expect a more efficient allocation within the firm for older workers than for younger workers, controling for the fact that more is known about the ability of older workers.

5 Conclusion

One of the basic questions in labor economics is whether the employee defines the job or whether the job defines the employee (Lazear, 1995). Traditional human capital theory, such as expressed by Mincer (1974), tend to think of workers as carrying their skills to the workplace, and the job as being of minimal importance in determining productivity; a description of the job is simply not included in the theory. In contrast, more institutional views of the labor market, as in matching theory (Jovanovic, 1979), or in principal-agent theory expressed by Holmstrom (1979) and Holmstrom & Milgrom (1991), a well-defined description of a job comes first, and workers are simply hired to fill the vacant slots. This paper, instead of taking a stance on which comes first, lets the relation between a worker and a job be endogenous.[26]

One type of equilibrium in the model (separating equilibrium) is characterized by the firm hiring workers and then giving them full discretion in defining their job, while in other types of equilibria, the firm constructs a scheme where the worker discretion is either limited (rationing equilibrium) or absent (pooling equilibrium). There are two underlying forces that determine the equilibrium degree of worker discretion. On one hand, workers having private information favors the worker coming first, because the worker is then

[26] Aghion & Tirole (1997) models a different type of setting where degree of worker discretion, or authority, is endogenous. Under P-authority, the principal chooses which project the firm should implement, while under A-authority, the agent chooses the project. The basic trade-off determining whether a firm will be characterized by P-authority or A-authority is that giving the agent more discretion increases his effort but also increases the probability that a bad project (from the principal's viewpoint) is implemented. In a similar setting to Aghion & Tirole (1997), Zabojnik (2001) argues that worker discretion could be high even in a situation where the principal is better informed, due to problems with enforcing the first best contract under limited liability.

better able to judge his appropriate task. On the other hand, career concerns create problems, because a worker may have incentives to create his job in a fashion that makes him look good to the market rather than a job that coincides with the best interest of the firm.

When the outside observability is low, the career motives are weak, and the firm can construct a scheme that gives workers full discretion over defining their job, and an efficient allocation of workers follows. When the outside observability is high, however, the career motives are strong, and the firm must define the job for the worker, and a misallocation of workers follows. Hence, we find firms with high outside observability to have a low degree of discretion to workers, and firms with low outside observability to have a high degree of discretion to workers. This conclusion seems to have some empirical support, but more research is called for.

There are several other paths for future research. One would be to view the degree of outside observability as a choice variable for the firms. This could create the trade-off that less observability implies less compensation costs to able workers, but also increased coordination costs inside the firm due to e.g., duplication of work. Another possible extension would be to consider whether increased worker discretion can lead to a higher innovation rate, both with respect to product and work method improvements. This would include what is possibly another motivation for free management practices – such practices lead to an improved innovation rate compared to more standard organizations.

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7 Appendix A: Auction equilibrium

Proof. of Proposition 1.

Clearly there cannot exist a pure strategy auction equilibrium. We here derive the mixed-strategy equilibrium. Consider first the equilibrium offers for a worker who receives a good signal. Let us say that the outside firm uses a mixed strategy with cumulative distribution $G^g(x)$ and support $S^g_{outside} = [\pi_0, \bar{\pi}^g]$, where $\bar{\pi}^g < \pi_H$. The inside firm will also use a mixed strategy with cumulative distribution of F_l for the low worker and F_h

for high worker. For a low worker, F_L will simply be the distribution degenerate at π_0. As can easily be shown, $S^g_{inside} = S^g_{outside} = S^g$. Given that the inside firm offers x to a high worker with a good signal, the inside firm will get,

$$G^g(x)(\pi_H - x),\ x \in S^g \tag{A1}$$

where $G^g(x)$ is the probability that the inside firm will win the auction, and $(\pi_H - x)$ is the surplus he gets in the case he wins. Since the inside firm must be indifferent at all points in his support, we have that,

$$G^g(x)(\pi_H - x) = k^g_{insideh},\ x \in S^g \tag{A2}$$

where $k^g_{insideh}$ is a constant. By integration, this constant equals the profits the inside firm makes on high workers that get a good signal. Now define the probability of a worker being a high type conditional on a good signal as θ^g and the bad signal as θ^b. As can easily be verified,

$$\begin{aligned} \theta^g &= \frac{p\theta}{p\theta + (1-p)(1-\theta)} \\ \theta^b &= \frac{(1-p)\theta}{p(1-\theta) + (1-p)\theta} \end{aligned} \tag{A3}$$

Given that the outside firm offers y to a worker with a signal i, the outside firm will get,

$$\theta^g F^g_h(y)(\pi_H - y) + (1-\theta^g)F^g_l(y)(\pi_0 - y),\ y \in S^g \tag{A4}$$

By the same argument as for the inside firm, the outside firm must be indifferent at all points in his support. From Milgrom (1981), we know that the profits of the outside firm must be zero. Hence the above expression must be zero. By inserting for $y = \bar{\pi}^g$, we can determine the upper end of the support, $\bar{\pi}^g$, as,

$$\bar{\pi}^g = \theta^g \pi_H + (1-\theta^g)\pi_0 \tag{A5}$$

We now determine the cdf's. From (A2) and inserting for $x = \bar{\pi}^g$ in (A1) to get $k^g_{insideh} = \pi_H - \bar{\pi}^g$, we get that,

$$G^g(x) = \frac{\pi_H - \bar{\pi}^g}{\pi_H - x},\ \text{where } x \in S^g \tag{A6}$$

Notice that this cdf places an atom at $x = \pi_0$, where the magnitude of the atom equals $\frac{\pi_H - \bar{\pi}^g}{\pi_H - \pi_0}$. To determine F_h^g, insert for $F_l^g = 1$ in (A4) to get,

$$F_h^g(y) = \frac{1 - \theta^g}{\theta^g} \frac{y - \pi_0}{\pi_H - y}, \; y \in S^g \tag{A7}$$

Notice that this distribution does not place an atom at the lower end of the support. For a worker with a bad signal, we use exactly the same procedure to get,

$$G^b(x) = \frac{\pi_H - \bar{\pi}^b}{\pi_H - x}, \text{ where } x \in S^b \tag{A8}$$

where the magnitude of the atom at $x = \pi_0$ equals $\frac{k^g_{insideh}}{\pi_H - \pi_0}$. Notice that we have that,

$$k^b_{insideh} > k^g_{insideh} > 0, \tag{A9}$$

since $\bar{\pi}^g > \bar{\pi}^b = \theta^b \pi_H + (1 - \theta^b)\pi_0$. Hence, as expected, the informed firm makes a higher profit on a (high) worker that receives a bad signal than a (high) worker that receives a good signal. Finally, we get,

$$F_h^b(y) = \frac{1 - \theta^b}{\theta^b} \frac{y - \pi_0}{\pi_H - y}, \; y \in S^b \tag{A10}$$

which is an atomless distribution. Now the equilibrium (expected) wage for an agent of type i. Clearly his expected wage just equals the expectation of the maximum offer conditional on the signal. First a low ability person. His expected wage equals,

$$w_N^2 = \pi_0[p(1 - \theta^b) + (1 - p)(1 - \theta^g)] + p \int_{\pi_0}^{\bar{\pi}b} z g^b(z) dz + (1 - p) \int_{\pi_0}^{\bar{\pi}^g} z g^g(z) dz > \pi_0 \tag{A11}$$

since the offer from the outside firm fully determines his wage. On the other hand, the expected wage for a high ability person equals,

$$w_S^2 = p \int_{\pi_0}^{\bar{\pi}g} z dG^g(z) F^g(z) + (1 - p) \int_{\pi_0}^{\bar{\pi}b} z dG^b(z) F^b(z) < \pi_H \tag{A12}$$

Uniqueness follows directly from the argument. ■

Proof. of Proposition 2 (Turnover part)

The turnover rate for the low workers equals the fraction of low workers that receives a (strictly) higher bid from the outside firm than from the inside firm at time 2, and half of the workers that receive the same offer from the two firms. Recall that the inside firm always bids π_0 for the low workers. Hence T_L equals,

$$\begin{aligned} T_L &= p[1 - \tfrac{1}{2}G^b(\pi_0)] + (1-p)[1 - \tfrac{1}{2}G^g(\pi_0)] \\ &= \frac{1}{2} + \frac{\theta^b p + (1-p)\theta^g}{2} \end{aligned} \tag{A13}$$

where θ^b and θ^g are defined as in the previous proof. It follows immediately from (A13) that this expression is positive. Now to the turnover rate of the high workers, which equals,

$$\begin{aligned} T_H &= p\int_{\pi_0}^{\bar{\pi}g} F^g(z)dG^g(z) + (1-p)\int_{\pi_0}^{\bar{\pi}b} F^b(z)dG^b(z) \\ &= \frac{\theta^g p + (1-p)\theta^b}{2} \end{aligned} \tag{A14}$$

As with the low workers, it follows immediately from the expression that the turnover rate for the high agents is positive. Notice that both T_L and T_H, and hence total turnover, are increasing in θ, since both θ^b and θ^g are increasing in θ. Expressing the difference, we find that,

$$\begin{aligned} T_L - T_H &= \frac{1}{2} + \frac{\theta^b p + (1-p)\theta^g}{2} - \frac{\theta^g p + (1-p)\theta^b}{2} \\ &= \frac{1}{2} + \theta^b p + \frac{1}{2}\theta^g - \theta^g p - \frac{1}{2}\theta^b \\ &= \frac{1}{2} + (p - \frac{1}{2})(\theta^b - \theta^g) \end{aligned} \tag{A15}$$

Since the second term on the right hand side always exceeds (-$\frac{1}{2}$), the turnover is always higher for the low type workers than for the high type workers. Intuitively, if the realization of the signal is N, the inside firm bids π_0 if the worker is low, and $F_h^g(y)$ if the worker is high. So, conditional on the signal being N, the turnover is higher for low workers than for high workers. The same type of argument applies if the value of the signal is S. ■

8 Appendix B

In this appendix, we numerically analysis of the claims made in Lemma 1 and Proposition 3, part (iii).[27] First, for the claim of Lemma 1, we show that $\Delta w^2 := w_S^2 - w_N^2$ is increasing in p (from which Proposition 2 follows). Second, for the claim of Proposition 3, we show that the degree of rationing is increasing in p.

8.1 Existence of separating equilibrium

To show that a separating equilibrium is more likely to exist the lower p (Proposition 2), we need to check that $\Delta w^2 := w_S^2 - w_N^2$ is increasing in p (the higher this amount the lower the incentive for firms to induce low-skilled workers to switch from the difficult task to the easy task).[28]

We start out by simplifying Δw^2, then prove analytically that $\left.\frac{\partial \Delta w^2}{\partial p}\right|_{\theta=\frac{1}{2}}$ is increasing in p, and finally consider numerical analysis for general θ. From Appendix A, we know that,

$$w_N^2 = \pi_0[p(1-\theta^b) + (1-p)(1-\theta^g)] + p\int_{\pi_0}^{\bar{\pi}^b} zg^b(z)dz + (1-p)\int_{\pi_0}^{\bar{\pi}^g} zg^g(z)dz > \pi_0 \tag{B1}$$

and that,

$$w_S^2 = p\int_{\pi_0}^{\bar{\pi}^g} zdG^g(z)F^g(z) + (1-p)\int_{\pi_0}^{\bar{\pi}^b} zdG^b(z)F^b(z) < \pi_H \tag{B2}$$

Observe that,

$$\begin{aligned}
\int_{\pi_0}^{\bar{\pi}^i} zdG^i(z) &= (\pi_H - \pi_0)(1-\theta^i)\int_{\pi_0}^{\bar{\pi}^i} zd\left(\frac{1}{\pi_H - z}\right) \\
&= (\pi_H - \pi_0)(1-\theta^i)\left[\frac{z}{\pi_H - z} + \ln(\pi_H - z)\right]_{\pi_0}^{\bar{\pi}^i} \\
&= \pi_H\theta^i + (\pi_H - \pi_0)(1-\theta^i)\ln(1-\theta^i)
\end{aligned} \tag{B3}$$

[27] All calculations and graphs are generated in Maple V. The worksheets are available from the authors.

[28] The reason why it is difficult to prove analytically that Δw^2 is increasing in p is that while w_S^2 is always increasing in p, surprisingly w_N^2 is not always monotonic in p.

where $i = b, g$, and where θ^b, θ^g are as in equation (A3). We can use (B3) to simplify w_N^2 into,

$$\begin{aligned} w_N^2 &= \pi_0 + (\pi_H - \pi_0)\left[p(\theta^b + (1-\theta^b)\ln(1-\theta^b)) + (1-p)(\theta^g + (1-\theta^g)\ln(1-\theta^g))\right] \\ &= \pi_0 + (\pi_H - \pi_0)\left[p\theta^b + (1-p)\theta^g\right] + \\ &\quad (\pi_H - \pi_0)[p(1-\theta^b)\ln(1-\theta^b) + (1-p)(1-\theta^g)\ln(1-\theta^g)] \end{aligned} \tag{B4}$$

Moreover notice that,

$$\int_{\pi_0}^{\overline{\pi}^i} z dG^i(z) F^i(z) = \pi_H(2\theta^i - 1) + 2\pi_0(1-\theta^i) - \frac{(1-\theta^i)^2}{\theta^i}(\pi_H - \pi_0)\ln(1-\theta^i) \tag{B5}$$

Hence,

$$\begin{aligned} w_S^2 &= 2\pi_0 - \pi_H + 2(\pi_H - \pi_0)(p\theta^g + (1-p)\theta^b) - \\ &\quad (\pi_H - \pi_0)[p\frac{(1-\theta^g)^2}{\theta^g}\ln(1-\theta^g) + (1-p)\frac{(1-\theta^b)^2}{\theta^b}\ln(1-\theta^b)] \end{aligned} \tag{B6}$$

We then have that,

$$\begin{aligned} \Delta w^2 &= w_S^2 - w_N^2 = \pi_0 - \pi_H + (\pi_H - \pi_0)[(3p-1)\theta^g + (2-3p)\theta^b] - \\ &\quad (\pi_H - \pi_0)[(1-\theta^g)(\frac{p}{\theta^g} + 1 - 2p)\ln(1-\theta^g) + (1-\theta^b)(\frac{1-p}{\theta^b} + 2p - 1)\ln(1-\theta^b)] \\ &= \pi_0 - \pi_H + (\pi_H - \pi_0)[(3p-1)\theta^g + (2-3p)\theta^b] - \\ &\quad \frac{\pi_H - \pi_0}{\theta}[(1-\theta^g)(1-p)\ln(1-\theta^g) + (1-\theta^b)p\ln(1-\theta^b)] \end{aligned} \tag{B7}$$

Notice that for $\theta = \frac{1}{2}$, we have that $\theta^g = p$ and $\theta^b = 1 - p$, and hence (B7) reduces to (we normalize by setting $\pi_0 = 0$ and $\pi_H = 1$),

$$\Delta w^2_{\theta=\frac{1}{2}} = -1 + (3p-1)p + (2-3p)(1-p) - 2[(1-p)^2\ln(1-p) + p^2\ln p] \tag{B8}$$

Differentiating this expression with respect to p we obtain,

$$\left.\frac{\partial \Delta w^2}{\partial p}\right|_{\theta=\frac{1}{2}} = 4[(1-p)\ln(1-p) - (1-p) - (p\ln(p) - p)] > 0 \tag{B9}$$

This expression is greater than zero because $xln(x) - x$ is decreasing in x for $x \in (0, 1)$. Hence we have shown that Δw^2 is increasing in p for $\theta = \frac{1}{2}$. We expect a proof of the case with general θ to be along the same lines, but significantly more cumbersome. In absence of an analytical proof, we now plot Δw^2 for other values of θ (still using the normalization $\pi_0 = 0$ and $\pi_H = 1$),

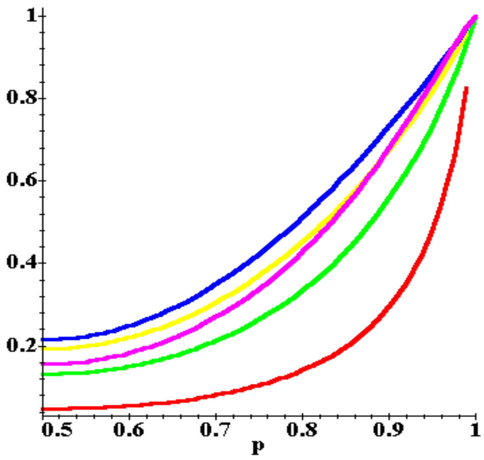

Figure 2

The figures depicts Δw^2 as a function of p for $\theta = .1$ (bottom line), $\theta = .3$, $\theta = .5$, $\theta = .7$, $\theta = .9$ (top line). As can be seen from the figure, Δw^2 is increasing in p for all the values of θ. This finding has been confirmed by extensive numerical analysis.

8.2 Rationing increases with p

We now show that the degree of rationing is increasing in p, or in other words that f^* is decreasing in p (Proposition 3 (iii)).

Denote the wages in the second period of a rationing equilibrium as $\hat{w}_S^2$ and $\hat{w}_N^2$. To determine $\hat{w}_S^2$ and $\hat{w}_N^2$, we work with the same equations as before, except that θ is replaced by $\hat{\theta}$, and π_H is replaced by $\hat{\pi}_H$, where

$$\begin{aligned} \hat{\theta} &= \theta + (1-f)(1-\theta) \\ \hat{\pi}_H &= \frac{\theta \pi_H + (1-f)(1-\theta)\pi_0}{\theta + (1-f)(1-\theta)} \end{aligned} \tag{B10}$$

where $\hat{\theta}$ is the expected share of workers that choose the S task in the first period, and where $\hat{\pi}_H$ is the expected productivity of those workers *in the second period* (since workers choose their efficient task in the second period, π_L does not enter the expression). As explained in the proof of Proposition 3, the equilibrium f^* is the value of f such that the wage difference $\hat{w}_S^2 - \hat{w}_N^2$ equals $\pi_0 - \pi_L$. Hence,

$$f^* = \{f : \hat{w}_S^2 - \hat{w}_N^2 = \pi_0 - \pi_L\} \tag{B11}$$

For the expressions we have checked, f^* is unique, and we expect this to hold generally. We now plot f^* against p, using (B8), (B10), and (B11), and insert the parameter values used in Example 1, except that we let θ be a free parameter (in addition to p).

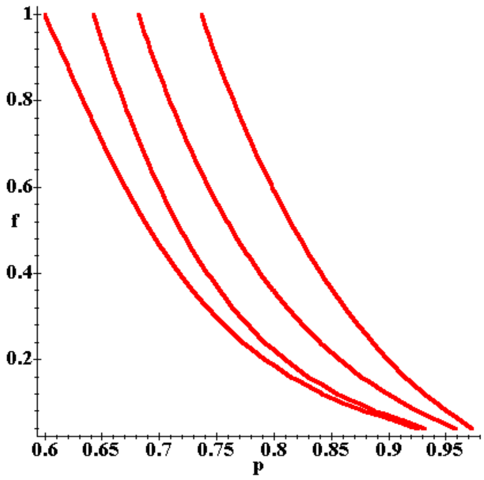

Figure 3

The figure plots f^* against p for varying values of θ [$\theta = .3$ (top line), $\theta = .5$, $\theta = .7$, and $\theta = .9$ (bottom line)]. The figure shows that f^* is decreasing in p for all values of θ. Extensive numerical analysis confirms that point. Hence we have substantiated that the degree of rationing is increasing in p.

9 Appendix C: Performance Contracts

Our justification for not having (a measure of) individual performance as a contractible variable is that for many production processes, measuring individual contribution to profits that go beyond the measurement of effort can be very costly and noisy task. Moreover, the assumption is consistent with a large empirical literature that shows that real-life payment schemes to a little extent depends on such measures (see Prendergast, 2000). The purpose of the appendix is to show that even when individual output is contractible, the equilibrium contracts can be similar or identical to the (fixed-wage) contracts analyzed in the main text. We illustration, we consider the case when effort is supplied inelastically, and where the wage to a worker can *only* be made conditional on a measure of his individual output.

For simplicity, we assume that there is only one period, and moreover that $c = 0$. To make our task harder, we assume a very weak form of risk aversion: workers maximize expected (total) wages, but for a given level of (expected) wages, workers prefer a lower-risk scheme to a higher-risk scheme.[29] If workers are indifferent given this criterion, they choose their efficient task.

From the assumptions made on risk preferences, we can confine attention to contracts of the following (linear) form,

$$w = \beta_0 + \beta_Y Y \tag{C1}$$

where Y is the (observed) output of an agent, β_0 is the salary, and β_Y is the bonus. We assume that in the N task, the output π_0 is certain, while the output in the S task is not perfectly observable. Y_i has two possible levels, π_g and π_b,where $\pi_b < \pi_H < \pi_g$. An H worker in the difficult task has a p_H probability of π_g, while an L worker in the difficult task has a p_L probability of obtaining π_g, where $p_L < p_H$. We assume that,

$$\begin{aligned} p_H \pi_g + (1 - p_H)\pi_b &= \pi_H \\ p_L \pi_g + (1 - p_L)\pi_b &= \pi_L \end{aligned} \tag{C2}$$

As can readily be seen, the deterministic technology studied in the previous sections is obtained for the special case $p_H = 1$, $\pi_g = \pi_H$, $p_L = 0$, and $\pi_b = \pi_L$. We have the following result.

Proposition 5 *For sufficiently high π_0 equilibrium contracts will be arbitrarily close fixed-wage (as in the main part of the paper).*

Proof. We assume that there exists an equilibrium where the L workers choose the N job and get the wage π_0, while the H workers choose the S job and get the scheme $w = \beta_0 + \beta_Y Y$. For (β_0, β_Y) to be consistent with equilibrium, it must maximize the utility of the high type, given zero profits and given that the low type prefers to work in

[29] More general risk preferences give qualitatively the same type of result, but would add technical problems with existence of equilibrium (similar to in Rotschild & Stiglitz, 1975)

the N job. Self-selection of low workers implies that,

$$\beta_0 + \beta_Y(p_L\pi_g + (1 - p_L)\pi_b) \leq \pi_0 \tag{C3}$$

while zero profit in the S job implies that,

$$\beta_0 + \beta_Y(p_H\pi_g + (1 - p_H)\pi_b) = \pi_H \tag{C4}$$

The second condition determines the salary as,

$$\beta_0 = \pi_H(1 - \beta_Y) \tag{C5}$$

Since high workers prefer a lower risk to a higher risk, the self-selection constraint is binding, and we get that,

$$\beta_Y = \frac{\pi_0 - \beta_0}{p_L\pi_g + (1 - p_L)\pi_b} \tag{C6}$$

Solving the system, we get that equilibrium contracts (β_0^*, β_Y^*) must satisfy,

$$\begin{aligned} \beta_0^* &= \frac{\pi_H(\pi_b + p_L\pi_g - \pi_b p_L - \pi_0)}{p_L\pi_g + \pi_b - \pi_H - \pi_b p_L} \\ \beta_Y^* &= \frac{\pi_0 - \pi_H}{p_L\pi_g + \pi_b - \pi_H - \pi_b p_L} \end{aligned} \tag{C7}$$

From those expressions, it can easily be seen that (β_0^*, β_Y^*) converges to $(\pi_0, 0)$, as π_0 approaches π_H. ■

Intuitively, when π_0 is high, the self-selection constraint becomes easier to satisfy, and hence β_Y can be lowered and still self-selection occurs. When π_0 approaches π_H, we get that β_Y can be close to zero without self-selection being violated, and since workers are risk averse, the equilibrium β_Y will in fact be close to zero as π_0 increases.

Since there is no intrinsic reason why π_0 should be close to π_H, the result does not seem too strong. However, by adding a cost of monitoring, m, where $0 < m < \pi_H - \pi_0$, for obtaining a performance measure, one can add realism to the result. As can easily be verified, the result is that when m is sufficiently high (so that $\pi_H - \pi_0$ is close to zero), the equilibrium performance contracts are close to the (fixed-wage) contracts considered in the previous sections.

Even if the sufficient condition outlined in Proposition 5 does not hold, there are circumstances under which performance contracts will not be used in equilibrium even if individual output is contractible. One set of circumstances is when workers can commit ex-ante to contractual form (e.g., through labor unions), as the following result shows.

Proposition 6 *If workers can commit to contractual form ex-ante to discovering their ability, equilibrium contracts may consist of fixed wages even if performance contracts were available.*

Proof. Assume that $(0 < p_l < (\pi_n - \pi_b)/(\pi_g - \pi_b) < p_h < 1)$, that is, for efficiency H workers should be in the S job, while L workers should be in the N job. In addition, assume that the proportion of H workers θ is such that $\theta(p_h\pi_g+(1-p_h)\pi_b)+(1-\theta)(p_l\pi_g+(1-p_l)\pi_b) \leq \pi_n$; that is, it is better that *all* the workers go in job N than *all* the workers going into job S. (This helps guarantee the existence of a separating equilibrium.) If firms competed over contingent contracts for the workers wages over workers in job N would be paid π_n and workers in job S would be paid w_g^*, w_b^* that solves

$$\begin{aligned} &\max_{w_g, w_b} Eu_h(w_g, w_b) \\ \text{s.t. } & p_h w_g + (1-p_h)w_b \leq p_h\pi_g + (1-p_h)\pi_b) \\ & Eu_l(w_g, w_b) \leq u(\pi_n) \end{aligned} \tag{C8}$$

This type of equilibrium is similar to Rothschild & Stiglitz (1975) except we include the moral-hazard of job selection as well. If one cannot write contracts contingent upon outcome, then the equilibrium contract would be w^* such that $w^* = \theta(p_h\pi_g+(1-p_h)\pi_b)+(1-\theta)\pi_n$. Notice that when workers are risk-averse $u(w^*) > \theta Eu(w_g, w_b) + (1-\theta)u(\pi_n)$. Such a contract cannot occur when contingent contracts are available, since one would be able to skim the good workers. This implies that it is best for the ex-ante for the workers to prevent contingent contracts. ■