The value of tax shields IS equal to the present value of tax shields[1]

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July 2005

Abstract

Fernandez (2004b) argues that the present value effect of the tax saving on debt cannot be calculated as simply the present value of the tax shields associated with interest. This contradicts standard results in the literature. It implies that, even though the capital market is complete, value-additivity is violated. As a consequence, adjusted present value formulae of a standard sort cannot be used. Also, Fernandez's argument implies that the value of the tax saving differs from conventional estimates by a considerable amount. We reconcile Fernandez's results with standard valuation formulae for the tax saving from debt. We show that, as one would expect, the value of the debt tax saving is the present value of the tax savings from interest. The apparent violation of value-additivity in the Fernandez paper comes from mixing the Miles and Ezzell and Miller and Modigliani leverage policies.

1. Introduction

Fernandez (2004b) argues that the present value effect of the tax saving on debt cannot be calculated as simply the present value of the tax shields associated with interest. Instead, he claims that the only way to obtain the correct value of the tax shields from debt is to do two present value calculations, one for the unleveraged firm and the other for the leveraged firm, and then subtract the former from the latter. He argues that the value of the tax saving from debt for a growing firm following the Miles and Ezzell (1980, ME) policy of a constant leverage ratio is not equal to the constant growth valuation formula applied to next year's tax saving from interest, as one would expect. Fernandez claims that the value is potentially considerably greater than this, by a multiple that is the ratio of the required return on the assets of the unleveraged firm to the cost of debt for the leveraged firm.

These results are potentially important, because they contradict standard results in the literature. In particular, Fernandez claims that the approaches of Modigliani and Miller (1963, MM), Myers (1974), Miles and Ezzell (1980), Harris and Pringle (1985), Brealey and Myers (2000), and Ruback (2002) are all flawed. In addition, the results imply that, even though the capital market is complete, value-additivity is violated because the value of a stream of cash flows (the tax saving from debt) is not independent of adding it to another set of cash flows (the cash flows from the unleveraged firm). As a consequence, adjusted present value formulae of a standard sort cannot be used. Finally, they imply that the value of the tax saving differs from conventional estimates by a considerable amount.

In this paper, we reconcile the Fernandez results with standard valuation formulae for the tax saving from debt. We show that, as one would expect in a complete market, the value of the debt tax saving is the present value of the tax savings from interest. The apparent violation of value-additivity in the Fernandez paper comes from mixing the Miles and Ezzell leverage policy with the Miller and Modigliani leverage adjustment. In the central case used by Fernandez, that of a growing firm following a Miles and Ezzell constant leverage policy, we show that the value of the tax saving from debt is equal to the constant growth valuation formula applied to next year's tax saving from interest, as one would expect.

One way to view what Fernandez is doing wrong is that he inserts an identity that is true in a Miller and Modigliani setting into a set of calculations that are done in a Miles and Ezzell setup. In a comment to an earlier working paper version of this paper in which this point was

made, Fernandez (2004a) claims that the setup in Fernandez (2004b) is subtly different from Miles and Ezzell. It is possible that the leverage policy he studies is a growing version of the Miller and Modigliani policy, in which debt grows independently of the firm's realized cash flows. This appears to be an implausible scenario, because it assumes that a firm with stochastic cash flows can have a growing non stochastic amount of debt. However, it is a theoretical possibility. We also investigate this case and show that the error in Fernandez (2004b) is now the importation of a result that is true in a Miles and Ezzell setting into a Miller and Modigliani setup.

The remainder of the paper proceeds as follows. Section 2 derives the value of a the tax saving from interest with constant growth under the ME debt policy and compares this with Fernandez's result. Section 3 gives a detailed explanation of the errors in Fernandez's derivation under both the ME and MM assumptions. The practical implications of the analysis are presented in Section 4, and Section 5 discusses the relationship between our results and other results in the literature. Section 6 concludes.

2. Leverage policy and the value of the tax saving

Like Fernandez, we start with an unleveraged firm that generates stochastic cash flows, which, in expectation, are an after-tax growing perpetuity starting at FCF and growing at the rate g. There are no investor taxes, and the required return on the unleveraged equity is K_U. The value of the unleveraged firm is

$$V_U = FCF/(K_U - g). \tag{1}$$

The leveraged firm has equity value E and debt value D. We ignore costs of financial distress, so the value of the tax saving from debt, VTS, is defined as the difference between the enterprise value of the leveraged firm and the value of the unleveraged firm:

$$E + D = FCF/(K_U - g) + VTS. \tag{2}$$

The two main approaches to leverage policy are the Modigliani and Miller (1963) and the Miles and Ezzell (1980). The difference is that ME assume that the amount of debt is adjusted to maintain a fixed market value leverage ratio, whereas MM assume that the amount of debt in each future period is set initially and not revised in light of subsequent developments. The standard versions of the MM formulas apply to the case in which there is no growth, so $g = 0$.

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The ME case that is closest to the MM analysis is when $g = 0$. Even then, the ME leverage policy still differs from that assumed by MM. In particular, the ME policy generates future tax savings from interest that are proportional to the future value of the firm. The firm value varies over time because the cash flows are stochastic. For example, when $g = 0$ they could follow a random walk with no drift. In contrast, the MM policy, in which the amount of leverage is set to a particular level and then not revised in light of later developments, generates a tax saving from interest that does not vary as the value of the firm varies.

Because the level of risk in the tax savings is different, relations between key parameters are different for the two assumed leverage policies. For the MM policy, the relation between the cost of equity K_E and K_U is given by

$$K_E = K_U + (D/E)[K_U - K_D](1 - T), \tag{3}$$

where T is the tax rate and K_D is the cost of debt. For the ME leverage policy with continuous rebalancing, it is given by (see, for instance, Taggart, 1991)

$$K_E = K_U + (D/E)[K_U - K_D].^1 \tag{4}$$

Similarly, with the MM assumptions, the discount rate adjusted for the tax effect is given by

$$K_L = K_U(1 - TD/(E + D)), \tag{5}$$

whereas, with the ME assumptions, it is

$$K_L = K_U - TK_D D/(E + D). \tag{6}$$

The value of the tax savings when the MM policy is followed is

$$VTS^{MM}(g = 0) = DK_D T/K_D = DT. \tag{7}$$

All the above are standard. The novel part of the Fernandez analysis is the derivation of the value of the tax saving for a growing firm pursing the ME leverage policy of a constant debt to value ratio. We now derive this value.

The leveraged firm generates the same operating free cash flow as the unleveraged firm. One way to value the leveraged firm is by using the tax-adjusted discount rate given by Eq. (5) or Eq.

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[1]These expressions are derived assuming that debt is riskless. The general case, assuming risky debt, is derived in Cooper and Nyborg (2004b).

(6) to discount the unleveraged cash flows. An alternative is to use Eq. (2), value the tax saving from debt directly, and add it to the value of the unleveraged firm. Whichever approach is taken, whether the MM or the ME leverage policy is being used must be specified from the outset.

The value of the leveraged firm, using the ME formula Eq. (6) for K_L, is

$$
\begin{aligned}
E + D &= FCF/(K_L - g) \\
&= FCF/(K_U - TK_DD/(E+D) - g).
\end{aligned}
\tag{8}
$$

If the company had no leverage, then its value would be

$$
V_U = FCF/(K_U - g).
\tag{9}
$$

The value of the tax saving is the difference between these values:

$$
\begin{aligned}
VTS^{ME} &= FCF/(K_U - TK_DD/(E+D) - g) - FCF/(K_U - g)) \\
&= DK_DT/(K_U - g).
\end{aligned}
\tag{10}
$$

Thus, the value of the tax saving is the value of a growing perpetuity starting at DK_DT, growing at g, with risk the same as the unlevered assets. This is what we get if we value the tax saving directly. The first period tax saving is equal to the interest charge, DK_D, multiplied by the tax rate. With the ME constant debt to value leverage policy, the tax saving changes at the same rate as the unleveraged cash flows, and the risk of the tax saving is the same as the risk of the firm. Thus, if unleveraged cash flows are represented by a growing perpetuity, the tax saving is valued as a perpetuity starting at DK_DT, growing at g, and discounted at K_U.

We can contrast this with the value of the tax saving under the MM assumptions by setting g equal to zero. Then the value is

$$
VTS^{ME}(g = 0) = DK_DT/K_U.
\tag{11}
$$

In contrast, the value of the tax savings when the MM policy is followed is

$$
VTS^{MM}(g = 0) = DK_DT/K_D = DT.
\tag{12}
$$

The ratio of the two values is K_U/K_D.

The difference here arises because the tax saving in ME is discounted at the required return on assets, whereas, in MM, it is discounted at the required return on debt. MM does not represent simply the ME assumption with zero growth. It is a completely different financing strategy. Even with cash flows that are expected to be perpetuities, the MM and ME assumptions differ. MM assume that the amount of debt will not change, regardless of whether the outcome of the risky perpetuity is higher or lower than its expected value, whereas ME assume that it will rise and fall in line with the expected cash flow.

3. Comparison with the Fernandez result

Fernandez (2004b) values the tax shield for a growing firm, just as we did in Section 2. In contrast to Eq. (10), he derives a value for the tax saving under a constant debt to value ratio (the ME financing strategy) of

$$VTS^{ME} = DK_U T/(K_U - g). \hspace{2cm} \textit{Fernandez (28')}$$

This equation differs from Eq. (10) in that the growing perpetuity being valued does not start at a level of $DK_D T$, the tax saving in the first period, but at the higher value of $DK_U T$. Fernandez's value of the tax saving under a constant debt to value ratio with constant growth is a factor of K_U/K_D times ours. The difference is substantial, because this ratio could be as much as two or more, depending on the levels of interest rates, equity risk premia, and asset betas.

The flaw in Fernandez's argument depends on how one interprets his setup. The most obvious interpretation is that he is working in a standard ME framework. But Fernandez (2004a) suggests that there is an alternative interpretation.

3.1. Standard Miles and Ezzell setup

The reconciliation of the standard result Eq. (10) and *Fernandez (28')* lies in the assumption that Fernandez makes in setting the limit of relation derived under the ME assumptions when the expected growth rate is zero. Although he assumes that the leverage ratio is constant, he assumes that the value of the tax saving is DT, as given by Eq. (12). With a constant leverage ratio, however, the tax shield is risky and its value is less than DT, as explained above and also in Miles and Ezzell (1985). The problem is that Eq. (12) assumes an MM financing strategy. Under the ME constant leverage ratio policy, VTS is given by Eq. (11) when g is zero. The ratio between this

and the value assumed by Fernandez is K_U/K_D, the ratio between his value of the tax saving for the growing firm and ours. This ratio arises from assuming that the tax saving has risk equivalent to K_D, when its risk is equivalent to K_U.

The critical passage in Fernandez's paper, in which the ME and MM policies are mixed, lies between his Eqs. (37) and (38), where he asserts: "We know from Eq. (16) that for $g = 0$, $VTS = DT$" (Fernandez, 2004b, p. 152). This is right only under the MM financing policy of a constant amount of debt. But at this point in his paper, Fernandez is working under a constant debt to value ratio.[2] If he had used the correct expression, Eq. (11), Fernandez would have found that the tax shield for his growing perpetuity firm would be given by our Eq. (10) and not *Fernandez (28')*.

The source of confusion in Fernandez's derivation can be traced back to his Eq. (12), in which he claims that, when $g = 0$, the taxes paid by the leveraged firm, $Taxes_L$, are proportional to cash flows to equity, ECF: $Taxes_L = T\,ECF/(1-T)$. This allows him to conclude that the value of taxes for the leveraged firm is $G_L = T\,E/(1-T)$. Using the result that the value of taxes for the unleveraged firm is $G_U = T\,V_U/(1-T)$, he then arrives at the result that $VTS = G_U - G_L = DT$.

But Fernandez's Eq. (12) assumes that the debt is constant, i.e., it assumes the MM debt policy. The equation would not hold under the ME debt policy because equity cash flow at date t would then depend on the change in the amount of debt at time t, whereas taxes at time t would not. To see this, note that the following two relations hold:

$$Taxes_{L,t} = (FCF_{0,t} - K_D D_{t-1})T \quad \text{and} \tag{13}$$

$$ECF_t = (FCF_{0,t} - K_D D_{t-1})(1 - T) + D_t - D_{t-1}, \tag{14}$$

where $FCF_{0,t}$ is what the enterprise cash flow would be without taxes at date t (i.e., EBITD$_t$).[3] $Taxes_{L,t}$ is proportional to ECF_t if and only if $D_t = D_{t-1} = D$ (the MM policy). When $D_t \neq D_{t-1}$, it is much harder to put a value on G_L, because taxes are no longer proportional to equity flows. One case that can be solved is when D_t/V_t is constant (the ME financing policy). In this case, when $g = 0$ and the time between refinancing goes to zero, $G_L = \frac{T}{1-T}V_U - \frac{DK_DT}{K_U}$. Thus,

$$VTS = G_U - G_L = \frac{DK_DT}{K_U}. \tag{15}$$

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[2]Working in a pricing kernel framework, Arzac and Glosten (2005) also identify this error in Fernandez (2004b).

[3] We use Fernandez's notation but have added time subscripts to the variables. As is common in the literature, Fernandez's analysis and these relations ignore the fact that, in practice, tax savings could be (partially) deferred if earnings fall below interest due. The correct way to value tax savings is arguably using contingent claims techniques. But this would lead to a degree of complexity that adds little to our main points here.

This is our Eq. (11).

3.2. Alternative setup: Modigliani and Miller with growth

The discussion in Section 3.1 assumes that the debt policy is the ME policy. However, in a later clarification of Fernandez (2004b), Fernandez (2004a, p. 2) claims:

> There is also a subtle difference between the Miles-Ezzell (1980) assumption about the capital structure . . . and the assumption that I use in my paper. . . . Miles-Ezzell (1980) assumption requires continuous debt rebalancing, while my assumption does not.

It is not clear what this means. Given that Fernandez is analyzing a firm whose expected value grows, debt has to be rebalanced over time because debt levels have to grow, too. Assume that what Fernandez means is that, in contrast to an ME policy, debt levels of his firm are not a function of future cash flow realizations. Instead, the amount of future debt at every time t is fixed initially, as it would be under an MM policy.[4] This might make sense because, as just discussed, for Fernandez's argument to go through it is necessary that the leverage policy is consistent with MM for $g = 0$.

Given this understanding of the Fernandez (2004b) setup, we show that the five assumptions he makes are not internally consistent.

1. Expected unlevered cash flows grow at $g > 0$.

2. Future debt levels are fixed at time 0 (our interpretation of the quotation above)

3. $L = D/V$ is a constant, independent of growth, g, and time.

4. K_U, K_E, and K_D are all constants, independent of growth, g, and time.

5. $K_U > K_F$, where K_F is the risk-free rate.[5]

The independence of these parameters of g is critical. Without this, Fernandez could not substitute $VTS = DT$ into Eq. (37), as he notes himself in Fernandez (2004b), and his argument would break down. Fernandez (2004b) asserts this independence of g without proof between his

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[4] More generally, we could have D_t being stochastic provided that $E[D_t] = D_t + \eta_t$, where η_t is a non priced risk. Our analysis below handles this. See footnote 8.

[5] If there is no priced risk, there is no argument. Fernandez (28') collapses to $VTS = \frac{TDK_F}{K_F - g}$. So the value of the tax shield is its present value (as it should be).

Eq. (32) and (33). While such independence would be true in the standard ME setting, it is not under the alternative interpretation of Fernandez's setup.

Let $\tilde{c}_t$, $t = 0, 1, \ldots$, denote the (stochastic) cash flows to the unlevered firm at time t, with realization c_t. Let $E[\tilde{c}_t]$ be the unconditional expected value, and let $E_{t-1}[\tilde{c}_t]$ be the expected value conditional on information available at time $t-1$. Specifically, this information includes the realized value of the cash flow at time $t-1$. Let V_t be the unlevered value of the firm at time t, ex cash flow. The cash flow is distributed to investors in full. Let V_t^L be the levered value. For the debt level at time t to be known at time 0 and satisfy $D_t = LV_t^L$, it is necessary that V_t^L is known at time 0. This also means that V_t and VTS_t must be known at time 0. Therefore, given Fernandez's constant growth assumption, it is necessary that

$$E[\tilde{c}_t] = E_{t-1}[\tilde{c}_t] = c_0(1+g)^t. \tag{16}$$

This does not mean that $\tilde{c}_t$ is deterministic. We have

$$\tilde{c}_t = E[\tilde{c}_t] + \tilde{\varepsilon}_t, \tag{17}$$

where $\tilde{\varepsilon}_t$ is a random variable with conditional and unconditional expected value equal to 0.[6]

3.2.1. Is K_U independent of g?

We will focus initially at the unlevered firm and conditions under which K_U is independent of g. From Eq. (16), it follows that

$$E[V_t] = E_{t-1}[V_t] = V_t = \frac{c_0(1+g)^{t+1}}{K_U - g}. \tag{18}$$

As a result, the capital gain every period is known with certainty. Specifically, it is

$$\text{capital gain}_t = V_t - V_{t-1} = \frac{c_0(1+g)^t g}{K_U - g} = V_{t-1}g. \tag{19}$$

Now, one can decompose a holding of a share of the firm into a holding of a share in the cash flow and a share in the capital gains. Because capital gains are known with certainty, the appropriate discount rate for them is the risk-free rate, K_F. Cash flows, though, are uncertain. For any time t, suppose the discount rate for the cash flow is K_c. Thus, for any $t-1$, the expected rate of return of the unlevered firm is

$$K_U = K_c \frac{E[\tilde{c}_t]/(1+K_c)}{V_{t-1}} + K_F \frac{V_{t-1} - E[\tilde{c}_t]/(1+K_c)}{V_{t-1}}. \tag{20}$$

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[6]Eq. (17) is completely general because ε_t could be a function of past information, including $E[\tilde{c}_t]$. For example, this includes a multiplicative error specification, where $\tilde{c}_t = E[\tilde{c}_t]\eta_t$ and η_t has an expected value of 1.

Using Eq. (16) and Eq. (18), this becomes[7]

$$K_U = \frac{K_F(1 + K_c) - (K_c - K_F)g}{1 + K_F}.$$ (21)

Unlike the Fernandez assumption, K_U is not independent of growth unless $K_c = K_F$. In particular, K_U is decreasing in g. This is intuitive; the higher the growth rate, the larger is the fraction of returns that come as risk-free capital gains. In the case that $K_c = K_F$, $K_U = K_F$, too, which violates Fernandez's assumption that there is priced risk (number 5).

We conclude that, if the discount rate for the cash flows is a constant that is independent of growth, Fernandez's assumptions are internally inconsistent.

Next, we therefore consider the case in which the discount rate for the cash flows is a function of growth and write it $K_c(g)$. For K_U to be independent of growth, Eq.(21) implies that $K_c(g)$ must be given by

$$K_c(g) = \frac{K_U(1 + K_F) - K_F(1 + g)}{K_F - g}.$$ (22)

This shows that as g increases, $K_c(g)$ must increase for K_U to be independent of g. This is intuitive. Because the proportion of returns that come as risk free capital gains increases as g increases, the discount rate for the cash flow must increase to keep the weighted average of the two the same.

A critical feature of the setup here is that we must have $g < K_F$. This shows that the stochastic process that Fernandez needs for his result does not exist for all growth rates $g < K_U$. Thus, at best, his result lacks generality.

3.2.2. Are K_D and K_E independent of g?

Regarding the riskiness of the debt, if there were no tax shield (e.g., as a result investor taxes), the debt would be risk free if and only if $D_{t-1}(1 + K_F) \leq V_t$. Otherwise, the repayment of the debt at t would depend on the risky cash flows. Because $D_{t-1} = LV_{t-1}$, using Eq. (18), the debt is risk-free if and only if $L \leq (1 + g)/(1 + K_F)$. If this is not satisfied for $g = 0$, the riskiness of the debt depends on g and so K_D would not be independent of g, in contradiction with Fernandez's assumption (number 3). Therefore, suppose that the debt is risk-free. Then $K_D = K_F$ regardless of g. The debt is risk-free if $L < 1/(1 + K_F)$ regardless of the value of the tax shield.

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[7]This could also be found by solving for K_U in

$$V_{t-1} = \frac{E[\tilde{c}_t]}{1 + K_c} + \frac{V_t}{1 + K_F}.$$

Given that the debt is risk-free and growing deterministically at a rate of g, standard arguments would say that

$$VTS^{MM} = \frac{TK_DD}{K_D - g},$$ (23)

where $K_D = K_F$.[8] *Fernandez (28')* gives a value to the tax shield that is less than this.

Now, we have narrowed Fernandez's setup down to a case in which all of his assumptions are satisfied, except possibly one, namely K_E being independent of growth. We will show that this assumption is violated.

With K_U, K_D, and L being constants (independent of growth), if $VTS = 0$, then K_E would be a constant, too. But this is not necessarily true when $VTS > 0$. If the tax shield is risk-free, its discount rate should be K_F. In this case, the expected rate of return to the levered firm, K_L, is less than K_U (being a blend of the unlevered assets and the tax shield). What is required for K_E to be independent of g is that K_L is independent of g. We will show that this is not the case.

Now, let K_{TS} be such that

$$VTS = \frac{TK_{TS}D}{K_{TS} - g}.$$ (24)

This formulation captures both the standard valuation formula Eq.(23) and *Fernandez (28')*. In the standard formula, $K_{TS} = K_D = K_F$, and in Fernandez' formula, $K_{TS} = K_U$. Using this general formulation, K_L is

$$K_L = \frac{V}{V + VTS} K_U + \frac{VTS}{V + VTS} K_{TS} = \frac{\frac{c_0(1+g)}{K_U - g} K_U + \frac{TK_{TS}D}{K_{TS} - g} K_{TS}}{\frac{c_0(1+g)}{K_U - g} + \frac{TK_{TS}D}{K_{TS} - g}}.$$ (25)

K_L is independent of g if and only if $K_{TS} = K_U$. Thus by assuming K_E and therefore K_L being independent of g, Fernandez has forced the result that $K_{TS} = K_U$. This is erroneous because a risk free tax shield must be discounted at the risk free rate. We conclude that all of Fernandez's assumptions cannot be simultaneously satisfied.

We could have misunderstood what debt policy Fernandez meant to assume. However, our general point here is this: Unlike what Fernandez (2004b) assumes, it does not follow from K_U, K_D, and L being independent of growth that K_E is so, too. This will hold if $K_{TS} = K_U$, but not in general. One cannot simply assume that K_E is independent of growth; it has to be shown. One case in which we know this holds is under the ME policy with continuous rebalancing, which is the

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[8] If $D_t = LV_t + \eta_t$, where η_t is a non priced risk (see footnote 4), we also get Eq. (23). As in Fernandez (2004b), Eq. (23) ignores the option-like nature of tax savings. See footnote 3. If we factored in deferred tax savings, which would be uncertain, VTS and therefore V_t^L could not be known at time 0. Thus L would not be known, and we would be done.

scenario discussed in Section *3.1*. However, as just demonstrated, it does not hold in the MM with growth scenario. Here, K_E depends on the growth rate, through the value of the tax saving. VTS as a fraction of total value varies with the growth rate. Because the tax shield accrues to equity and because the riskiness of the tax shield differs from that of the operating assets, this implies that the riskiness of flows to equity varies with the growth rate. If the dependence of K_E on the growth rate is allowed for, the Fernandez analysis would give a value for the tax saving equal to Eq. (23), as one would expect.

4. Practical implications

Two simple sets of assumptions are used in practice to incorporate into valuations the effects of the tax saving from interest. One is the MM assumption of a constant amount of debt combined with an expected operating cash flow that is a level perpetuity. This gives a value for the tax saving of

$$VTS^{MM} = DT. \tag{26}$$

To calculate an adjusted present value, this amount should be added to the unlevered value of the firm, which can be calculated using a discount rate set with the unlevered beta. The unlevered beta cannot be observed directly. Assuming riskless debt, it can be estimated from the observable equity beta by

$$\beta_U^{MM} = \beta_E/(1 + (1 - T)(D/E)). \tag{27}$$

Alternatively, one can use the ME assumption of a constant leverage ratio. This could be used with any profile of future operating cash flows, so there is no general formula for the present value of the tax saving. The merit of the ME assumption is that it is consistent with the standard formula for the weighted average cost of capital whatever the future cash flow profile. Therefore, valuation including the present value of the tax saving from interest can be achieved by simply discounting the operating cash flow at the weighted average cost of capital. In the case in which an adjusted present value calculation is necessary, the formula for the unlevered equity beta is

$$\beta_U^{ME} = \beta_E(E/(D + E)). \tag{28}$$

In the commonly assumed case in which the expected operating cash flow is a growing perpetuity, the result we have shown above is that the present value of the tax saving from interest is

$$VTS^{ME} = DK_DT/(K_U - g). \tag{29}$$

For both cases, there are equivalent formulas when debt is risky and corresponding formulas for unlevering discount rates. These are given in Cooper and Nyborg (2004a).

No consensus exists as to which set of assumptions to use. For example, Grinblatt and Titman (2002, p. 466) use the MM formula for the unlevered beta, whereas Brealey and Myers (2003, p. 229) use the ME formula. The difference in the value of the tax saving resulting from the two different assumptions can be substantial but can go either way. The ME tax saving grows at the rate g and is higher than the MM tax saving because of that. However, it is riskier than the MM tax saving, and its value is reduced as a consequence.

5. Relationship to other results

In general, the value of the leveraged firm including the tax effect of debt is the unlevered value plus the present value of the tax savings from debt:

$$E + D = V_U + \sum_{t=1}^{\infty} E(T_t I_t)/(1 + K_{TS}(t))^t, \tag{30}$$

where $E(.)$ is the expectations operator, I_t is the interest payment at date t, T_t is the tax that is saved at date t per dollar of interest charges, and $K_{TS}(t)$ is the discount rate appropriate to the tax saving at date t.[9] To use this equation in practice, we must estimate the unlevered value, the discount rate for the tax shield, and the expected net tax saving from interest deductions in each future period.

The approaches of Modigliani and Miller (1963), Myers (1974), Miles and Ezzell (1980), Harris and Pringle (1985), Miles and Ezzell (1985), Brealey and Myers (2000), and Ruback (2002) make different assumptions about some or all of the assumptions that underlie this expression, including the level of risk of the cash flows of the unlevered firm, the rate of growth of these cash flows, and the financing strategy.[10] However, all satisfy the basic relationship Eq. (30), as they should in a complete capital market.

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[9]This assumes that capital markets are complete, so that any cash flow stream has a well-defined value.

[10]For a discussion of these and other approaches, see Cooper and Nyborg (2004a), See also Grinblatt and Liu (2004) who develop a general contingent claims formulation for valuing tax shields.

6. Summary

Fernandez (2004b) claims to demonstrate that the present value effect of the tax saving on debt cannot be calculated as simply the present value of the tax shields associated with interest. This contradicts standard results in the literature. It implies that, even though the capital market is complete, value-additivity is violated. As a consequence, adjusted present value formulae of a standard sort cannot be used. Also, he implies that the value of the tax saving differs from conventional estimates by a considerable amount.

In this paper, we reconcile the Fernandez results with standard valuation formulae for the tax saving from debt. We show that, as we would expect in a complete market, the value of the debt tax saving is the present value of the tax savings from interest. We show that regardless of how one interprets Fernandez's setup with respect to whether the firm is following Miles and Ezzell- or Miller and Modigliani-style debt policies, the mistake in his derivations arises from confusing these policies. It is either inserting a result that is true under MM into an ME setting, or vice versa.

References

Arzac, E. R., Glosten, L. R., 2005. A reconsideration of tax shield valuation. European Financial Management 11, 453-461.

Brealey, R. A., Myers, S. C., 2003. Principles of Corporate Finance McGraw-Hill, New York.

Cooper, I. A., Nyborg, K. G., 2004a. Discount rates and tax. Unpublished working paper London Business School, London, UK.

Cooper, I. A., Nyborg, K. G., 2004b. Tax-adjusted discount rates with investor taxes and risky debt. Unpublished working paper London Business School, London, UK.

Fernandez, P., 2004a. Reply to 'the value of tax shields is equal to the present value of tax shields.' Unpublished working paper IESE Business School, University of Navarra, Barcelona, Spain.

Fernandez, P., 2004b. The value of tax shields is not equal to the present value of tax shields. Journal of Financial Economics 73, 145-165.

Grinblatt, M., Liu, J., 2004. Debt policy, corporate taxes, and discount rates. Unpublished working paper Anderson School of Management, University of California at Los Angeles.

Grinblatt, M., Titman, S., 2002. Financial Markets and Corporate Strategy McGraw-Hill, New York.

Harris, R. S., Pringle, J., 1985. Risk-adjusted discount rates: extensions from the average risk case. Journal of Financial Research 8, 237-244.

Miles, J., Ezzell, J. R., 1980. The weighted average cost of capital, perfect capital markets, and project life: a clarification. Journal of Financial and Quantitative Analysis 15, 719-730.

Miles, J., Ezzell, J. R., 1985. Reformulating the tax shield: a note. Journal of Finance 40, 1485-1492.

Modigliani, F., Miller, M. H., 1963. Corporate income taxes and the cost of capital: a correction. American Economic Review 53, 433-443.

Myers, S. C., 1974. Interactions of corporate financing and investment decisions: implications for capital budgeting. Journal of Finance 29, 1-25.

Ruback, R. S., 2002. Capital cash flows: a simple approach to valuing risky cash flows. Financial Management 31, 85-103.

Taggart, R. A, 1991. Consistent valuation and cost of capital expressions with corporate and personal taxes. Financial Management 20, 8-20.