

An empirically based implementation and evaluation of a network model for commuting flows

Jens Petter Gitlesen, Gisle Kleppe and Inge Thorsen,
University of Stavanger, Stord/Haugesund College,
Kristine Bonnevis vei 30, Bjørnsonsgt. 45,
N-4021 Stavanger, Norway. N-5528 Haugesund, Norway.

Jan Ubøe,
Norwegian School of Economics and Business Administration,
Helleveien 30,
N-5045 Bergen, Norway.

Abstract

In this paper we present empirical results based on a network model for commuting flows. The model is a modified version of a construction introduced in Thorsen et al. (1999). Journeys-to-work are determined by distance deterrence effects, the effects of intervening opportunities, and the location of potential destinations relative to alternatives at subsequent steps in the transportation network. Calibration is based on commuting data from a region in Western Norway. Estimated parameter values are reasonable, and the explanatory power is found to be very satisfying compared to results from a competing destinations approach. We also provide theoretical arguments in favor of a network approach to represent spatial structure characteristics.

1 Introduction

The main purpose of this paper is to offer an empirical implementation of a network model for journeys to work. The basic construction of the network model was presented in Thorsen et al. (1999). The model was constructed for pure trip distribution problems, and it focuses on the impact of spatial structure and road network characteristics rather than on socioeconomic

variables and various structural aspects of the labour market. In short, the network description of the geography was combined with a set of hypotheses on spatial labour market behaviour and some balancing procedures.

In this paper we introduce some extensions and modifications of the modeling framework that was presented in Thorsen et al. (1999). Based on central issues in the literature of spatial interaction and destination choice we briefly evaluate the individual search strategy and the delineation of choice sets involved in the network model formulation. We argue that the network approach offers a more transparent interpretation of the decision procedure than standard models within the gravity tradition, and that the delineation of the true choice set of individual workers comes out as a more natural result of the model structure.

We focus on pure trip distribution problems, and hence ignore other aspects that might influence commuting flows. For a review of modeling attempts to integrate location, land-use and transportation flows, see for instance Wilson (1998).

In the empirical parts of this paper we first discuss the quantitative effects of network and spatial structure characteristics on destination choice. The estimation of parameters is based on information of job and residential location for workers in a region in southern parts of Western Norway. Our results are further compared with estimates based on a gravity based model specification, and the models are evaluated with respect to their ability to explain commuting flows. The estimates resulting from the gravity based model specification are presented in Thorsen and Gitlesen (1998), and for the sake of comparison we now use the same data in our estimation of the network model. In particular it is interesting to note that the 5-parameter network model performs better than the 7-parameter gravity based model used in Thorsen and Gitlesen (1998).

As will be clear in subsequent sections the gravity family of spatial interaction models represents a benchmark for theoretical and empirical evaluations of our network approach. For this reason some aspects of such interactions models are reviewed in Section 2. Section 3 deals with the network approach to model commuting flows, emphasizing in particular some revisions of the modeling framework formulated in Thorsen (1999). Section 4 refers to some theoretical aspects concerning the choice between a gravity type of model and a network approach. The region and the data is presented in Section 5. Parameter estimates are presented in Section 6,

and the results are compared with the corresponding estimation results from a gravity model specification. Finally, we offer some concluding remarks in Section 7.

2 The gravity modeling tradition of spatial interaction models

The models most commonly used in applied analysis of trip distribution problems are those belonging to the tradition of gravity modelling. In a gravity model it is assumed that spatial interaction is explained by the distance between an origin and a destination, and by two aggregate measures: one to account for generativity of origins and another that addresses the attraction of destinations. In studies of journeys to work the generativity of origins is in general defined in terms of the number of workers, while the attraction of destinations is usually measured by total employment. One way of formulating a traditional gravity model is:

$$T_{ij} = A_i O_i B_j D_j \exp(-\beta d_{ij}) \quad (1)$$

$$A_i = \left[\sum_j B_j D_j \exp(-\beta d_{ij}) \right]^{-1} \quad (2)$$

$$B_j = \left[\sum_i A_i O_i \exp(-\beta d_{ij}) \right]^{-1} \quad (3)$$

Here:

T_{ij} is the estimated number of travellers from origin i to destination j , $i, j = 1, \dots, n$

O_i is the observed number of trips originating from zone $i = 1, \dots, n$

D_j is the observed number of trips destinating in zone $j = 1, \dots, n$

d_{ij} is travelling time by car, from origin i to destination j ; $i, j = 1, \dots, n$

β is a distance deterrence parameter. A_i and B_j are the balancing factors that ensure the fulfillment of the marginal total constraints; $\sum_j T_{ij} = O_i$ and $\sum_i T_{ij} = D_j$. Consequently, this doubly constrained model specification is constructed for a pure trip distribution problem. For a discussion of the theoretical foundation of this model, see for example Erlander and Stewart (1990) or Sen and Smith (1995). It is a well known fact that the doubly constrained gravity

model is equivalent to the multinomial logit model. Hence, the gravity model can be derived from stochastic utility theory, in addition to approaches dominated by the macro-oriented tradition that is inspired by social physics and based on the entropy concept.

Several authors within the spatial interaction literature point out that gravity models might be misspecified due to an inability to account for relevant aspects of spatial structure. This especially applies to unconstrained or production constrained model formulations, see for example Fotheringham (1981). In several publications by Fotheringham (for example Fotheringham (1983, 1986, and 1988) it has been argued that account should be taken to the possibility that destination choice is affected by competition or agglomeration effects. For this purpose, Fotheringham (1983) introduced the competing destinations model. In this approach, the clustering system of destinations enters into the model through a term defining the accessibility of a destination, perceived from the specific origin. Within a doubly constrained framework the structural equation of the competing destinations model is formulated as follows

$$T_{ij} = A_i O_i B_j D_j S_{ij}^\rho e^{-\beta d_{ij}} \quad (4)$$

The balancing factors A_i and B_j are defined similarly to the expressions (2) and (3). Accessibility is measured by S_{ij} , which is defined as the accessibility of destination j relative to all other destinations, as perceived from i

$$S_{ij} = \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^w D_k e^{-\beta d_{jk}} \quad (5)$$

Here, w is the number of potential destinations. The standard reference to this kind of accessibility measures is Hansen (1959). The sign of the parameter ρ in the interaction Equation (4) is a matter of empirical research, rather than theoretical considerations. When agglomeration kind of forces are dominant, the sign will be positive, while the parameter takes on a negative value if competition like forces are dominant.

The competing destinations model has been object to empirical testing based on data for different kinds of spatial interaction problems. We will not review the empirically based evaluation of the model specification here. Relevant references can be found in Pellegrini and Fotheringham (1999). In Thorsen and Gitlesen (1998) the competing destinations model is evaluated from the

same data set that is described in Section 5. Based on commuting flow data and a doubly constrained model specification it was found that the competing destinations formulation performs significantly better than the traditional gravity model. It was also found that competition effects were dominating for this kind of spatial interaction.

A theoretically based criticism that can be directed towards the traditional gravity model formulation is related to the property of independence of irrelevant alternatives (IIA property). This property is in general undesirable for spatial choice situations, see for instance Liaw et al. (1986) for a discussion of the problem. The IIA property follows as a result of the unilevel simultaneous destination choice evaluation of the multinomial logit model and, equivalently, the standard gravity model. One way to deal with this problem is the approach in Liaw and Ledent (1987), who introduce a hierarchical processing strategy by distinguishing between a departure model and a destination choice model in a nested logit formulation. The competing destinations model represents an alternative way to deal with the IIA property. By introducing the accessibility measure the odds ratio corresponding to two destination alternatives will in general no longer be independent of changes in other, surrounding, destination alternatives.

Spatial choices are in general based on many alternatives of which only a portion is represented in the true choice set of individual decision makers. A conventional model formulation that does not distinguish between the universal and the true choice set involves specification errors. Analyses that demonstrate how a misspecification of choice sets might produce misleading parameter estimates and predictions can be found in for example Pellegrini et al. (1997), Thill (1992), and Thill and Horowitz (1997). Many of the spatial interaction studies dealing explicitly with the problem of restricted choice sets are along lines suggested by Manski (1977), with a probabilistic choice set generation. The probability assigned to a specific alternative is defined to be the product of the choice set generation probability and the (conditional) probability that this alternative is chosen from destination opportunities in the relevant choice set. In Fotheringham (1988) the probability of an alternative being in the true choice set is assumed to be determined by a measure of potential accessibility relative to alternative destination opportunities. This set-up produces the competing destinations model. This means that the competing destinations model can be interpreted to result from a two-stage decision process. First, the decision makers select the set of alternatives which are relevant destination choices. Second, a specific destination

is selected from this set of alternatives. Hence, the competing destinations model corresponds to a hierarchical decision process. Gitlesen and Thorsen (2000) offer a more detailed theoretical interpretation of the competing destinations modeling framework in modeling commuting flows.

Both the nested logit and the competing destinations model can be explained from a hypothesis of a hierarchical processing search strategy rather than a simultaneous evaluation of all alternatives. This means that such models involve a delineation of choice sets of individual workers. As Fotheringham (1988) points out, the nested logit model is based on an apriori specified choice structure, where alternatives are assigned to clusters prior to model calibration. In other words the modeling framework assumes that the modeller has knowledge to individual choice sets. Pellegrini and Fotheringham (1999) argue that this is a drawback of the nested logit model as a spatial choice model. Contrary to the nested logit model the competing destinations model is based on a procedure which considers the likelihood of an alternative being in the true choice set. This likelihood is according to the similarity, or position, of that alternative relative to the other alternatives. Pellegrini and Fotheringham (1999) argue that this flexible multistage approach is one important argument in favour of the competing destinations model.

The specification of appropriate choice sets is related to aggregation aspects. One such aspect is the spatial aggregation level. According to Horner and Murray (2002) zonal commuting flow data should be as disaggregate as possible. We will not enter into this discussion in this paper, our data do not allow for real experiments with the spatial aggregation level. There is another kind of potential aggregation problem, however, that is not accounted for in the recommendations in Horner and Murray (2002). Workers are not homogeneous, and the response to variations in distance might differ, for example with respect to gender, age, income, and/or profession. Deterrence parameters that are estimated from aggregate data reflect the effect of varying preferences across worker categories, as well as of a spatial mismatch between categories of workers and relevant job opportunities, see Ubøe (2004), Jörnsten et al. (2004), and O’Kelly and Lee (2005). If a system is divided into smaller units spatial aggregation problems might get less severe, but the possibility increases of a serious spatial mismatch between relevant job offers and worker categories. To some degree such aggregation aspects have influenced the specification of a distance deterrence function in this paper. For more details on those matters, see Ubøe (2004).

3 A network approach to commuting

As mentioned in the introduction the empirical results to be presented in this paper follow from a modified version of a model formulation proposed by Thorsen et al. (1999). In this model commuting flows are determined through a procedure based on a network description of the geography. The individuals adapt to the environment through a sequence of choices, and destinations are selected according to the level in the network. The specification of the network is origin-specific, it starts out to consider the elements of one row in the journey-to-work matrix at a time. As in gravity models commuting flows are determined both by the individual preferences and by a set of basic restrictions on the system. The models might differ, however, with respect to how large part of the predicted commuting flows that is left for the balancing procedure. We will return to this problem in the section where the results are presented.

In this section we start by introducing a general distance deterrence function that appears at several stages in the network model construction. As a next step we present the module accounting for within-zone journeys-to-work, before we explain the mechanism determining commuting flows to specific levels of remaining destinations in the transportation network. Finally, the balancing procedure is explained. Basically, the construction is the same that was presented in Thorsen et al. (1999). The empirical implementation of the modeling framework demonstrated, however, that some modifications were required. In the presentation to follow most weight is put on the parts of the model construction representing changes from the framework proposed by Thorsen et al. (1999).

3.1 The distance deterrence function

The network model construction introduced by Thorsen et al. (1999) is based on three basic principles:

- Journeys-to-work are determined by random choice when distances are short
- Journeys-to-work are determined by minimal cost when distances are long
- Commuting flows matrices can be expressed as convex combinations of extreme states

We refer to Thorsen et al. (1999) for complete details. The extreme states correspond to the cases where all the distances within the system are either zero or infinitely large. Roughly

speaking, distance deterrence is defined as the total level of attraction towards the minimum costs state, that is a state where the total traveling cost in the system is as small as possible. Equipped with such ideas it is obviously important to find an appropriate specification of a distance deterrence function. It is reasonable that any distance deterrence function should satisfy the following properties:

- $D(0) \approx 0$
- $d \mapsto D(d)$ is increasing
- $\lim_{d \rightarrow \infty} D(d) \approx 1$

Thorsen et al. (1999) proposed a logistic function to represent the distance deterrence effect. Based both on empirical experiments and theoretical considerations, however, we introduce a convex combinations of exponentials. This means that the distance deterrence function to be exploited in this paper is given by the specification:

$$D(x) = 1 - \left(\alpha e^{-\beta_1 x} + (1 - \alpha) e^{-\beta_2 x} \right) \quad (6)$$

Here, α , β_1 , and β_2 are parameters to be estimated from observations of commuting flows. Ubøe (2004) offers a theoretical rationale for the use of this specification of a distance deterrence function. The basic idea is that it adjusts for possible specification errors resulting from the possibility that workers are not homogeneous, neither with respect to the qualifications in the labor market, nor with respect to their response to distance. As made clear by Ubøe (2004) workers can for example be grouped together according to age, income, and/or profession. Deterrence parameters that are estimated from aggregated data reflect the effect of varying preferences across worker categories, as well as of a spatially varying mismatch between categories of workers and relevant job categories. Ubøe (2004) argues, and demonstrates through numerical experiments, that the convex combination of exponentials is appropriate to deal with the presence of different population groups that are non-interacting in the labor market. The exponential terms allow the distance deterrence function to shift its shape as the distance changes, reflecting the possibility that distinct population groups respond differently to changes in distance. Ubøe (2004) also argues that such effects are satisfactorily captured by a function with two exponential terms even if the real population consists of many non-interacting groups of workers. It is of

course in particular important to adjust for the relevant kind of aggregation problems in cases where the geography is subdivided into zones with strongly asymmetric distributions of different categories of workers that respond substantially different with respect to changes in distance.

For simplicity of notation we also include the following definition

$$\bar{D}_\kappa(x) = (1 - D(x))^\kappa \quad (7)$$

where κ is another parameter to be estimated from our data, reflecting to what degree the distance deterrence effect excludes zones as destination alternatives. In what follows, we will try to model the journey-to-work matrix as the result of an hierarchical sequence of choices in which each alternative is weighed according to the level of the distance deterrence. If there are alternatives within short or medium distance from the residential location of an agent, these will be the preferred choices, and no weight should be put on other alternatives. If all alternatives are at long range, however, such alternatives must be put into practice. All efforts in this paper are then directed towards using the functions D and \bar{D} to account for such principles at all stages of the decision process.

3.2 Within-zone journeys-to-work

The “zero-th” level, or step, in the decision process concerns the propensity to work and live in an area that is represented by the same node in the network. Consider node A . A^c then denotes the collection of all other nodes. Thorsen et al. (1999) view this as a two node system, and they define two extreme states of the corresponding commuting matrix. T_A^0 represents the case of random commuting (corresponding to so short distances that the whole system is effectively acting as a single node), while T_A^∞ represents the minimal cost solution that corresponds to long distances between the nodes in question. The extreme states can be found from the explicit expressions

$$T_A^0 = \begin{bmatrix} \frac{L_A E_A}{E_A + E_{A^c}} & \frac{L_A E_{A^c}}{E_A + E_{A^c}} \\ \frac{L_{A^c} E_A}{E_A + E_{A^c}} & \frac{L_{A^c} E_{A^c}}{E_A + E_{A^c}} \end{bmatrix} \quad T_A^\infty = \begin{bmatrix} \min\{L_A, E_A\} & L_A - \min\{L_A, E_A\} \\ L_{A^c} - \min\{L_{A^c}, E_{A^c}\} & \min\{L_{A^c}, E_{A^c}\} \end{bmatrix} \quad (8)$$

In Thorsen et al. (1999) L_{A^c} and E_{A^c} were defined as the total labor/employment outside A . If the system is very large, the within-zone commuting flows would then be nearly zero in the

random case. The term $\frac{E_A}{E_A+E_{A^c}}$ can be interpreted as the probability of choosing employment in zone A at random. As a modification we now define E_{A^c} to mean relevant job opportunities outside A. This means that we introduce a procedure where zones are weighted according to their *total relevance* as job destinations for workers living in zone A, and vice versa for workers with residence in other zones. To achieve this, we define

$$E_{A^c} = \sum_{i \in A^c} \bar{D}_\kappa(d_{Ai}) E_i \quad (9)$$

The parameter κ introduced in Equation (7) is controlling for the size of this effect. Commuting from zone A is now defined as a convex combination of the two extreme states:

$$T^A(d_{AA^c}) = T_A^0(1 - D(d_{AA^c})) + T_A^\infty D(d_{AA^c}) \quad (10)$$

where d_{AA^c} corresponds to the generalized distance to the other nodes of the system. We define the generalized distance from a node A to a set B through the expression

$$d_{AB} = \sum_{j \in B} \frac{W_{Aj}}{\sum_{k \in B} W_{Ak}} d_{Aj} \quad i = 1, 2, \dots, N \quad (11)$$

Here, $\frac{W_{Aj}}{\sum_{k \in B} W_{Ak}}$ is the *relative relevance* (weight) of potential destination j . One possibility is that the variable W_{Aj} is defined by the number of job opportunities in zone j ; $W_{Aj} = E_j$. Then d_{AB} is equal to the average euclidean distance to the job opportunities in set B . As a more refined measure, however, our weights in addition reflect both the position of the alternative destinations relative to zone A ($\bar{D}_\kappa(d_{Aj})$), and the labor market situation within the alternative zones ($\frac{E_j}{L_j}$), i.e.:

$$W_{Aj} = \bar{D}_\kappa(d_{Aj}) E_j \frac{E_j}{L_j} \quad (12)$$

Notice that the total and the relative relevance of destination alternatives are conceptually different. The first is used to exclude irrelevant alternatives from the calculation of the extreme states. The second is used as a measure of how alternative zones outside A contribute to the value of the generalized distance from zone A. Assume as an example that the labor market situation is particularly favorable in zones located close to zone A, with relatively few job opportunities in more distant locations from A. The value of the generalized distance will then be low, and

total commuting flows from zone A will not differ substantially from the situation with random commuting, represented by T_A^0 .

The reservation wage related to job offers is a declining function of the distance to job opportunities elsewhere in the spatial system. The job opportunities in zone j are not, however, reflected by E_j alone. In addition it can be argued that the labour supply L_j originating from zone j should be incorporated, to account for the tendency that many workers prefer jobs in the neighborhood. As argued by Thorsen and Gitlesen (1998) the alternative of being offered a job in the neighborhood might for some individuals be to leave the labor force, for instance due to problems of running a two-worker household. Local firms might to some degree exploit a monopsonistic labor market position, resulting in a tendency to employ workers who are spatially and professionally immobile, with a relatively low reservation wage for local job offers. Such a labor market situation might explain a tendency that jobs are occupied with workers residing nearby, and this is the reason why the factor $\frac{E_j}{L_j}$ is included in the definition of relative relevance.

3.3 The commuting flow to the first order level in the transportation network

In Equation (10) T^A is a 2×2 matrix, and the element $T_{AA} := T_{11}^A$ defines the internal commuting in zone A . Its complement $T_{AA^c} := T_{12}^A$ defines the traffic flow directed outwards in the system. The next part of the construction is to distribute the commuting flow T_{AA^c} among the zones outside of A . The first step in this direction is to distribute mass to first order neighbors (adjacent nodes). The basic idea is to measure how much better the first order neighbors are positioned compared to the rest of the system. To this end Thorsen et al. (1999) introduced a neighboring distance deterrence function, $n(d_{AA_1}, d_{AA_1^c})$. Here A_1 denotes the first order neighbors, and A_1^c denotes all the neighbors further out in the transportation network. The idea is to compare the relevance of these two groups of alternatives for workers living in zone A . Technically the neighboring distance deterrence function is defined by:

$$n(d_{AA_1}, d_{AA_1^c}) := \sigma \frac{\overline{D}_\gamma(d_{AA_1})}{\overline{D}_\gamma(d_{AA_1^c})} \quad (13)$$

Here σ and γ are parameters to be estimated from data on commuting flows. If some members of A_1 and some members of A_1^c are positioned close to A , then the generalized distances d_{AA_1} and $d_{AA_1^c}$ will be roughly equal, and the factor $\frac{\overline{D}_\gamma(d_{AA_1})}{\overline{D}_\gamma(d_{AA_1^c})} \approx 1$. A tendency might still exist that

first order neighbors are preferred to higher order alternatives. This corresponds to the idea in the so called intervening opportunities model, stating that spatial interaction between two zones is proportional to the total number of trips from the origin, and inversely proportional to the number of intervening opportunities, see for instance Wilson (1970). The parameter σ represents the absorption, reflecting the effect of intervening opportunities and search costs. This measures the tendency that $n(d_{AA_1}, d_{AA_1^c}) > 1$ even when $d_{AA_1} \approx d_{AA_1^c}$. Another kind of absorption effect arises if the generalized distance to relevant first order neighbors is significantly lower than the generalized distance to relevant higher order alternatives, $\frac{\bar{D}_\gamma(d_{AA_1})}{\bar{D}_\gamma(d_{AA_1^c})} > 1$. The parameter γ measures the size of this effect; the impact of differences between generalized distances to specific levels in the transportation network will be moderated by a low estimate for this parameter.

Assume that the distances to subsequent levels in the transportation network are quite similar and that commuting is determined by physical distances alone. In such a case the extreme state of random commuting offers a reasonable estimate of commuting flows to first order neighbors. Thorsen et al. (1999) introduced the following fraction for this purpose:

$$\text{RF}_1 = \frac{\sum_{i \in A_1} E_i}{\sum_{i \in A^c} E_i} \quad (14)$$

This definition fails its purpose in large networks, however, since $\text{RF}_1 \approx 0$ in that case. Once again we therefore modify the modeling framework according to the relevant job opportunities. The modified version of the random fraction is defined as follows:

$$\text{RF}_1 = \frac{\sum_{i \in A_1} \bar{D}_\kappa(d_{Ai}) E_i}{\sum_{i \in A^c} \bar{D}_\kappa(d_{Ai}) E_i} \quad (15)$$

As a next step this random fraction is combined with the neighboring distance deterrence function to determine the fraction of commuting flows from zone A with a destination at the first order neighboring level. Hence, the estimated commuting flows reflect the two kinds of absorption effects underlying the neighboring distance deterrence function. To be specific the commuting flow T_{AA_1} from A to the first order neighbors is determined by the following equation:

$$T_{AA_1} = T_{AA^c} \cdot f_\delta(n(d_{AA_1}, d_{AA_1^c})) \cdot \text{RF}_1 \quad (16)$$

where f_δ is defined as follows

$$f_\delta(x) = \frac{x}{(1+x^\delta)^{\frac{1}{\delta}}} \quad (17)$$

The function f_δ is introduced as a consistency requirement, to ensure that $T_{AA_1} \leq T_{AA^c}$. The function has the following properties:

i) $0 \leq f_\delta(x) \leq 1$ when $x \geq 0$, ii) $f_\delta(x) \approx x$ when $x \approx 0$ and iii) $\lim_{x \rightarrow \infty} f_\delta(x) = 1$

The parameter δ controls the speed of which $f_\delta(x)$ approaches 1 when $x > 1$. If δ is large, almost all traffic will be directed to the first order neighbors whenever $n(d_{AA_1}, d_{AA_1^c}) \cdot RF_1 > 1$. A small δ , however, will slow down this effect.

3.4 The distribution of first order journeys-to-work to alternative zones at this level

The next step in the model construction is to distribute the commuting flow T_{AA_1} to the individual first order neighbors. First, however, it is necessary to test and account for two consistency requirements:

- i) If T_{AA_1} exceeds the total number of jobs in A_1 , we put $T_{AA_1} = \sum_{i \in A_1} E_i$, and distribute a traffic flow $T_{Ai} = E_i$ to each zone in A_1 .
- ii) If $T_{AA_1^c}$ exceeds the total number of jobs in A_1^c , we put $T_{AA_1} = T_{AA^c} - \sum_{i \in A_1^c} E_i$, and distribute a traffic flow $T_{Ai} = E_i$ each zone in A_1^c .

Once the consistency requirements are fulfilled, the following multinomial logit model expression is used to distribute mass to the individual nodes in the network:

$$T_{Ai} = T_{AA_1} \cdot \frac{W_{Ai} e^{-\beta d_{Ai}}}{\sum_{j \in A_1} W_{Aj} e^{-\beta d_{Aj}}} \quad (18)$$

where β is a parameter to be estimated from our data. These expressions, too, must be tested for consistency. The idea is to distribute as much mass as we can to each node, and then redistribute excess mass proportionally to consistent alternatives. This is done through the following algorithm:

Put $p_i^{\text{old}} = \frac{W_{Ai} e^{-\beta d_{Ai}}}{\sum_{j \in A_1} W_{Aj} e^{-\beta d_{Aj}}}$, $i \in A_1$. Find the set S of all $i \in A_1$ where $E_i \geq T_{AA_1} \cdot p_i$. We define $p_i^{\text{new}} = \frac{E_i}{T_{AA_1}}$ if $i \in A_1 \setminus S$, and $p_i^{\text{new}} = K \cdot p_i^{\text{old}}$ if $i \in S$, and compute a value for K such

that $\sum_{i \in A_1} p_i^{\text{new}} = 1$. This process is then repeated until $S = A_1$. We can then define

$$T_{Ai} = T_{AA_1} \cdot p_i^{\text{final}} \quad (19)$$

and all these flows are consistent. This completes the distribution of mass to the first order neighbors.

3.5 The commuting flow to higher order neighbors and the balancing procedure

After the derivation of commuting to first order neighbors we are left with a flow $T_{AA_2} = T_{AA_c} - T_{AA_1}$ to be distributed among neighbors at higher order levels. This is done in exactly the same way as for the first order level. The various steps are repeated until we reach neighbours of maximal order in the transportation network. At this stage there is no mass to distribute to alternatives further out in the system, so at this last level we only use the multinomial logit model expression to distribute the mass. Because of all the previous consistency checks, the system will always be consistent on this last level, and that completes the distribution of mass from node A .

The function $f_\delta(x)$ in Equation (17) was not incorporated in the model formulation presented in Thorsen et al. (1999). The commuting flow from a zone might then in general be totally absorbed by the destination alternatives at a specific level in the transportation network, with zero probabilities of selecting alternatives further out in the network. The functional form $f_\delta(x)$ is introduced to avoid that the model assigns zero probabilities of choosing specific destination alternatives. The choice probabilities will be insignificant for distant alternatives far out in the transportation network, but no alternatives will be totally ruled out by definition. Even at very long distances, a positive probability will in general exist that commuting finds place, for example due to rigidities concerning the hiring and firing procedure, and slow adjustment to residential relocations.

If we assume that the network consists of N nodes, we carry out the above construction using $A = 1, 2, \dots, N$ to define a journey-to-work matrix $T = \{T_{ij}\}_{i,j=1}^N$. This matrix satisfies

the condition

$$\sum_{j=1}^n T_{ij} = L_i \quad (20)$$

It does not, however, necessarily satisfy the condition

$$\sum_{i=1}^n T_{ij} = E_j \quad (21)$$

Thorsen et al. (1999) solved this problem using a projection procedure. By this projection procedure the final journey-to-work matrix is defined by minimizing the squared deviations between the entries of the temporary and the final journey-to-work matrix, subject to the constraints of given row and column sums, and the constraint that all the entries of the resulting matrix are non-negative. In this paper we propose to replace the projection with a penalty method to balance the system. The idea is to penalize the weights of the nodes that is allocated too much commuting, and to enhance the weights of the nodes where the structural model mechanism has allocated too small commuting. Technically we compute the factors

$$F_j = \frac{E_j}{\sum_{i=1}^n T_{ij}}, \quad j = 1, 2, \dots, N \quad (22)$$

and redefine the weights using

$$W_{ij}^{\text{new}} = F_j \cdot W_{ij}^{\text{old}} \quad (23)$$

The process is then repeated until $\sum_{j=1}^N (F_j - 1)^2$ is sufficiently small. At this stage all the factors F_j will be sufficiently close to 1, i.e., the balancing condition

$$\sum_{i=1}^n T_{ij} = E_j \quad (24)$$

is satisfied.

One problem of the model that is proposed by Thorsen et al. (1999) appears for spatial systems where one zone (or a few zones) represents substantially more attractive employment opportunities than the remaining zones. The argument is that the relevant sequence of steps allocates too much commuting to the central node(s) in the spatial system. The number of workers that was allocated to the central node(s) tends to exceed the limited number of employment opportunities here. Hence, the relevant mechanisms fail to take adequately into account

the competition between workers with respect to attractive employment opportunities. The projection procedure in Thorsen et al. (1999) is motivated to fulfill the basic restrictions that $\sum_i T_{ij} = E_j$ rather than by theoretical considerations based on plausible mechanisms. In this paper we have chosen an approach that is based on a more plausible balancing mechanism. The weights (W_A) in the model are adjusted by a factor that is based on a comparison between the column sums that follow from our row-wise model construction and the column sums that are known from the data set. This correction factor takes the competition between attractive employment opportunities into account, and represents a “penalty method” that balances the system.

4 Theoretical aspects related to the choice between a gravity model formulation and a model based on a network approach

As for the gravity model and the intervening opportunities model the basic ideas in our network approach to commuting are straightforward and easy to explain. Still, the specification of the geography and the balancing procedures result in a more complex model structure. In our opinion this drawback of increased complexity can be defended by a more appropriate modeling of relevant spatial structure characteristics, individual choice sets, and the spatial labour market behaviour.

In the network approach spatial structure characteristics appear explicitly in the basic construction of the model. Gravity based models are not constructed to capture other spatial structure characteristics than one dimensional distances. Spatial structure effects to some degree enter into the model through the balancing factors. The nature of those factors are diffuse and aggregate, however, and it is difficult to keep track of how specific changes in spatial structure influence spatial interaction through different mechanisms. From a behavioural point of view spatial structure effects should appear through reasonable hypotheses on the destination choice procedure rather than implicitly through possibly relevant effects related to the balancing of the system.

To some degree the gravity family of model can be modified to take spatial structure effects into account. It was clear from Section 2 that the competing destinations model represents one such modification, where a Hansen accessibility measure is introduced as an additional variable

in the model formulation. In this paper the measure of generalized distance is preferred as a measure of spatial structure. From a theoretical point of view we think that the idea of marginal and deterring distances capture important aspects in the choice of job locations, and the relevant parameters represent a flexibility that does not apply to the Hansen type of accessibility measure.

According to the discussion and the literature review in Section 2 a misspecification of choice sets might produce misleading parameter estimates and predictions. Both the competing destinations model and the nested logit model formulation represent methods where the unilevel decision process is substituted by a hierarchical specification of destination alternatives, introducing the probability of an alternative being in the true choice set. Still, those model formulations are not based on a sequential interpretation of the search procedure, through the specification of a search path. For some kinds of spatial interaction the sharp distinction between choice set delineation and the choice of a particular destination might seem somewhat artificial. With a network specification of the geography a decision process comes out natural where individuals consider different levels in the network at a time. Due to the presence of search costs we think that this specification of a link-based spatial sequential decision procedure is appealing from a behavioural point of view. It is also appealing that the delineation of the true choice set follows from the specification of the search procedure, including a stopping rule.

Summarized, the network approach benefits from the fact that both relevant aspects of the spatial structure, the delineation of individual choice sets, and a reasonable sequential decision procedure, follow explicitly from the basic model construction. Hence, we think that the fundamental mechanisms underlying the spatial labour market behaviour are more transparent and realistic in a network approach than for the gravity family of models.

5 The region and the data

As stated in the introduction one important ambition in this paper is to compare results based on the network modeling framework to results based on more traditional gravity model specifications. For this purpose we use the same data that was used by Thorsen and Gitlesen (1998), in an empirical evaluation of alternative specifications of gravity and competing destinations models. The data represent information of job and residential location in 1989, for workers in a region in southern parts of western Norway. The region does in some respect come close to

what Paelinck and Nijkamp (1975, pp. 173) refer to as a polarized region (core region, or nodal region), “a connex area, in which the internal economic relationships are more intensive than the relationships with respect to regions outside the area”. This high degree of intra-dependency within the region is very much due to physical, topographical, transportation barriers, that lengthen travel distances, and thereby deter economic relationships with other regions. The region also comes close to what is defined as “an economic area” in Barkley et.al. (1995), with a relatively self-contained labor market, and a relatively large central place (Haugesund) which influences on economic activity in a peripheral region. There are seven municipalities in the region, and each municipality is divided into postal delivery zones. All in all the region is divided into 43 (postal delivery) zones. The corresponding transportation network is illustrated in Figure 1. The total number of inhabitants in the region was 88000 in 1989. Far the biggest zone is the regional center Haugesund, with 27250 inhabitants in 1989.

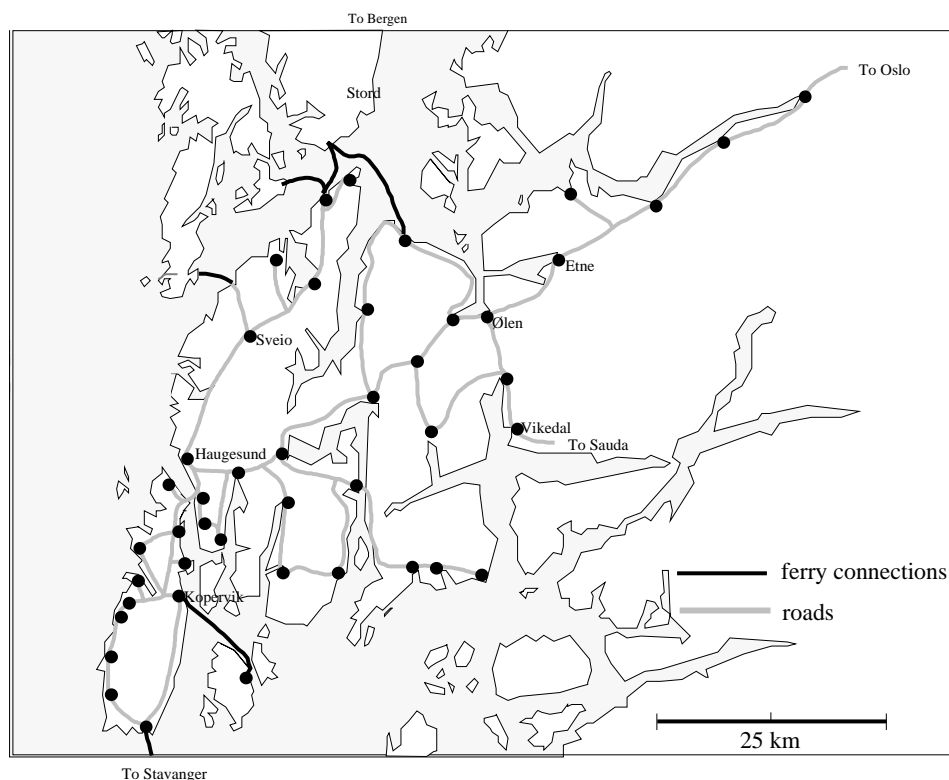


Figure 1: The main transportation network in the region in 1989.

This division of the region into zones corresponds to the most detailed level of information which is available on residential and work location of each individual worker within the region.

The information is based on the so called AA-register, which belongs to the National Insurance Administration of Norway, and includes a code for the postal delivery zone of both residence and work location. A trip distribution matrix was constructed after a correction for some faults in the data that was provided for us.

The spatial pattern of population and employment in this region is very appropriate for our problem. Population and employment is concentrated to the zonal centers rather than more evenly dispersed, and most centers are not too isolated and distant from each other to prevent a considerable interzonal commuting. The division of zones corresponds to a natural kind of clustering, where the interzonal distances are in general significantly longer than intrazonal distances. Distances are measured as traveling time by car, and they refer to the shortest route from an origin to the specific destinations.

6 Results

The parameters are estimated by the method of maximum likelihood. Maximum likelihood was found through an irregular simplex iteration sequence (see Nelder and Mead (1965)). Standard errors were estimated by numerical derivation. In accordance with Thorsen and Gitlesen (1998) we have used the assumption of quasi independence for cells with no observations.

6.1 Goodness-of-fit and parameter estimates

In this section we will present estimation results based on the following model formulations:

M0= the basic model formulation

M1= a model formulation where the weight attached to the distance deterrence function is the same on different steps in the decision process ($\gamma = \kappa$)

M2= a model formulation with only one exponential term in the distance deterrence function ($\alpha = 1$), estimated by finding the maximum likelihood solution

We report both parameter values and values of some goodness-of-fit indices. According to Knudsen and Fotheringham (1986) the Standardized Root Mean Square Error (SRMSE) is the most accurate measure to analyze the performance of two or more models in replicating the same

data set, or for comparing a model in different spatial systems. SRMSE is defined by $SRMSE = \sqrt{\frac{\sum_{i,j} (T_{ij} - \hat{T}_{ij})^2}{I \cdot J}} \cdot \frac{\sum_{i,j} T_{ij}}{\sum_{i,j} T_{ij}}$, where I denotes the number of rows (origins) in the trip distribution matrix, while J is the number of columns (destinations). A measure with a more obvious interpretation, but inappropriate for statistical testing, is the Relative Number of Wrong Predictions, $RNWP = \frac{\sum_{i,j} (|\hat{T}_{ij} - T_{ij}|)}{\sum_{i,j} T_{ij}}$.

Table 1: Estimations results based on alternative model formulations. M0 and M1 are based on a distance deterrence function represented by a convex combination of exponentials, while M2 is based on the distance deterrence function $D(x) = 1 - e^{-\beta_1 x}$

	M0	M1	M2
$\hat{\alpha}$	0,3248 (0,1032)	0,7212 (0,0832)	- (-)
$\hat{\beta}_1$	0,0208 (0,0021)	0,0146 (0,0014)	0,0107 (0,0003)
$\hat{\beta}_2$	0,0069 (0,0010)	0,0023 (0,0014)	- (-)
$\hat{\beta}$	0,0819 (0,0039)	0,0833 (0,0038)	0,0829 (0,0038)
$\hat{\kappa}$	8,4396 (0,3458)	8,6738 (0,3708)	8,6714 (0,3569)
$\hat{\gamma}$	9,1867 (0,2369)	8,6738 (0,3708)	8,6714 (0,3569)
$\hat{\delta}$	0,4424 (0,0097)	0,4475 (0,0081)	0,4487 (0,0092)
$\hat{\sigma}$	6,1026 (0,1696)	6,7733 (0,6270)	6,3658 (0,6981)
RNWP	0,1924	0,1925	0,1918
SRMSE	0,6992	0,7024	0,6793
L	-121768,44	-121770,03	-121779,14

Note: Standard errors in parentheses. $\kappa = \gamma$ for the model specifications M1 and M2.

Notice first from Table 1 that the estimates of κ and γ resulting from model M0 are of similar size. Those parameters reflect to what degree the distance deterrence function excludes zones as relevant destination alternatives at the specific steps in the decision process. κ influences the tendency that workers choose job opportunities within in the zone where they reside, while γ affects the tendency to be absorbed in an occupation at a specific neighboring level rather than searching for jobs further out in the transportation network. According to the results presented in Table 1 we find no strong empirical support for distinguishing between the effect of distance deterrence at separate steps in the decision process. In comparing models M0 to model M1 it follows that the explanatory power is not significantly reduced in the case where $\kappa = \gamma$ a priori. The value of the likelihood ratio test statistic is 2,92, which is lower than the critical value of a

chi square distribution with one degree of freedom at the 5% level of significance.

The parameter estimates resulting from specification M0 have reasonable interpretations in terms of commuting behavior. As was clear in Section 3 the parameters β_1 , β_2 , and α operate through the distance deterrence function, and affect the probability that a worker is absorbed in an occupation at a specific level in the transportation network. Estimated values of $\hat{\alpha} \approx 0,38$ and $\beta_1 \neq \beta_2$ mean that the distance deterrence function effectively is estimated to be a convex combination of exponentials, and that the population consists of non-interacting groups with different spatial labor market response to variations in distance. The idea is that the parameter estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ can be interpreted as the distance responsiveness of two categories of workers. According to the estimates resulting from specification M0 the responsiveness with respect to variations in distance is almost four times higher for category 2 than for category 1.

The estimates of β_1 , β_2 , and α are found to be relatively sensitive with respect to model specification. By comparing the model specifications M0 and M1 to the specification M2 it follows from the relevant likelihood ratio test that the introduction of the convex combination adds significantly to the explanatory power. Still, the values of the remaining two goodness-of-fit measures (SRMSE and RNWP) indicate that explanatory power even is deteriorated through the introduction of the more complex distance deterrence function. Hence, empirical results based on our model specification and our data set give no unambiguous support for the hypothesis that distance deterrence effects are best represented by a convex combination of exponentials. This hypothesis calls for more empirical testing.

Our network approach distinguishes between two kinds of distance deterrence effects. The parameters involved through the distance deterrence function $D(x)$ refer to decisions where relevant job opportunities in the entire transportation network are considered. The parameter β refers to how variations in distance affect choices between alternatives at the same neighboring level. The estimates in Table 1 indicate that distance has a considerably stronger impact on the choice of destination at a specific level in the transportation network than on the general choice between system-wide alternatives.

Also the parameters κ , δ , and σ reflect the response to network characteristics of the geography. The estimated value of κ (and γ in M0) is very high, and corresponds to a case with myopic spatial behavior, where only nearby destination alternatives affect the tendency that commuting

flows are absorbed at a specific level in the network. Given this delimitation of the true choice set the relatively low value of $\hat{\delta}$ means that the absorption tendency is only moderately sensitive to the value of the neighboring distance deterrence function. In other words, the decision to choose a job opportunity at a specific level in the transportation network is not very sensitive to the location of relevant alternatives at subsequent levels. The significantly positive estimate of σ means that commuting flows between two zones depend not only on distances, but also on the number of intervening opportunities. The order in the transportation network matters for how commuting flows are distributed to relevant zones.

6.2 A comparison with results based on a gravity model formulation

Based on the same data set that was presented in Section 5, Thorsen and Gitlesen (1998) tested several versions of gravity based models for journeys-to-work. The model specification that was found to be most satisfying in explaining the observations is represented by the structural equation Equation (4) in Section 2, extended by some terms representing the tendency that workers reside and work in the same zone:

$$T_{ij} = A_i O_i B_j D_j S_{ij}^{\rho} \left(O_i^{\alpha_1} D_j^{\alpha_2} \right)^{\delta_{ij}} e^{(-\beta d_{ij} + \mu \delta_{ij})} \quad (25)$$

Here, δ_{ij} is the Kronecker delta and μ is a parameter representing a start up cost to be incurred if work and residence are not in the same zone. The term $O_i^{\alpha_1} D_j^{\alpha_2}$ represents the possibility that the frequency of within-zone journeys-to-work depends on local labor market characteristics in addition to the distance deterrence effect and the start up cost. The introduction of this term is motivated by a hypothesis that local sectors tend to employ workers who are spatially and professionally immobile, for instance due to practical problems of running two-worker households. A high value of O_i and a low value of D_i is expected to result in a relatively high value of T_{ii} . This hypothesis is supported by the empirical results in Thorsen and Gitlesen (1998); α_1 is estimated to be significantly positive, while α_2 is found to be significantly negative. The hypothesis of a positive start up cost is supported by a significantly positive estimate of μ . Commuting flows are found to be negatively related to the accessibility of a destination relative to other destinations ($\hat{\rho} < 0$); competitive forces are found to dominate. Hence, it affects commuting flows to a destination negatively if this destination is located within a cluster of potential destinations

rather than in a more isolated location, *ceteris paribus*.

Thorsen and Gitlesen (1998) introduced two additional parameters through the specification of the accessibility measure. First, a parameter γ accounts for the possibility that total inflow D_k is not necessarily the correct value to represent the contribution of destination k . Second, the weight attached to distance in the accessibility measure is not necessarily the same that the distance deterrence parameter β appearing in the specification of commuting flows in Equation (25). The accessibility measure is then defined by $S_{ij} = \sum_{\substack{k=1 \\ k \neq i, k \neq j}}^w D_k^\gamma e^{-\sigma d_{jk}}$. Both parameters are found to contribute significantly to the explanatory power of the model, with $\gamma > 0$ and $\hat{\sigma} \approx 0,031 < \hat{\beta} \approx 0,067$.

This version of the competing destinations model has 7 structural parameters. As another alternative the accessibility measure S_{ij} and the three corresponding parameters (ρ, γ and σ) can be ignored. In Table 2 those two gravity based models are denoted by:

CDM= the competing destinations model represented by the structural Equation (25)

GM= a gravity model, with no account taken to the accessibility measure S_{ij}

The corresponding values of the goodness-of-fit indices in Table 2 are taken from Thorsen and Gitlesen (1998). It follows from the table that our network approach results in more satisfying values of all the indices. Notice in particular that a network model with only 5 structural parameters performs better than the 7-parameter competing destinations model, CDM.

Table 2: The value of goodness-of-fit indices for alternative spatial interaction models.

	M0	M1	M2	CDM	GM
RNWP	0,1924	0,1925	0,1918	0,1993	0,2312
SRMSE	0,6992	0,7024	0,6793	0,7409	0,9202
L	-121768,44	-121770,03	-121779,14	-121880,8	-122193,0
# parameters	8	7	5	7	4

Such findings have to be interpreted with care. In both modeling frameworks it is probably possible to account for additional effects and alternative parametric specifications that would increase the explanatory power. The results nevertheless indicate that a network approach is at least an interesting alternative to standard gravity based approaches. In the choice of approach such results have to be supplemented by theoretical considerations on how alternative modeling frameworks capture spatial structure characteristics and relevant aspects of the decision process.

It is obvious from the results presented in Thorsen and Gitlesen (1998) that spatial structure characteristics are not adequately represented by the distance deterrence effect and the balancing factors, nor by the fraction between labor supply and labor demand within each zone. If the accessibility measure S_{ij} and the three corresponding parameters (ρ, γ and σ) are ignored in the gravity based model formulation, it follows from Table 2 that goodness-of-fit is considerably reduced. Still, the question is whether the accessibility measure captures the relevant spatial structure characteristics adequately, or if a network approach is more appropriate.

The empirically based explanatory power is of course one important factor in the evaluation of a model. In addition the evaluation has to consider the reliability of the basic mechanisms. Summarized, the competing destinations model in Thorsen and Gitlesen (1998) is based on distance deterrence effects, the fraction between labor supply and labor demand within each zone, and spatial structure characteristics represented by the accessibility measure.

The network approach presented in this paper is based on a hypothesis that commuting flows result from sequential decisions, where alternative destinations are considered successively, relative to their location in the transportation network. The parameter estimates determine the relative importance of the steps in this decision process. Our sequential network approach means that journeys-to-work are determined by

- distance deterrence effects
- the effects of intervening opportunities
- the location of potential destinations relative to alternatives at subsequent steps in the transportation network

We also find it to be very important that irrelevant alternatives are excluded from the choice set (a high value of $\hat{\kappa}$). The relative location of relevant destinations is accounted for through the sequential model specification and the use of generalized measures of distance. Though our approach is based on a few, well-established, behavioral principles, it captures effects of complex spatial structure characteristics. We think that this is at least an appealing alternative to introducing explicitly measures of spatial structure into structural equations in a gravity modeling framework. In some cases it is far from obvious how such measures relate to basic behavioral principles.

7 Concluding remarks

The empirical results presented in this paper do of course not provide evidence for categorical conclusions on the choice of modeling framework in studies of journeys-to-work. As mentioned in the previous section both the gravity based framework and the network approach probably could be extended to account for additional effects and alternative parametric specifications that would increase the explanatory power. Still, both theoretical considerations and our empirical results at least introduce the network approach as an appealing alternative to the more established gravity based approaches. Relevant characteristics of spatial structure can be argued to be better represented in a network approach than through the introduction of separate measures in a gravity based approach.

In addition to more empirical testing a natural next step in the evaluation of modeling alternatives is to consider the predictability. How do, for instance, alternative model formulations predict implications of specific changes in the road transportation network? Are parameter estimates autonomous to specific characteristics of spatial structure? Can parameter estimates based on data from one region be used for making reliable predictions of commuting flows in other regions? Are parameter estimates reasonably constant across both space and time? The predictability of a network model specification might be acceptable even if individual parameter estimates varies considerable. To some degree the relevant mechanisms are represented by the simultaneous effect of several parameters, covariation problems might cause variation in individual estimates. We leave empirical testing of such aspects for future research.

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