Forward curve dynamics in the Nordic electricity market

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Abstract

The purpose of this paper is to investigate the forward curve dynamics in an electricity market. Six years of price data on futures and forward contracts traded in the Nordic electricity market are analysed. For the forward price function of electricity, we specify two different multifactor term structure models in a Heath-Jarrow-Morton framework. Principal component analysis is used to reveal the volatility structure in the market. A two-factor model explains 75% of the price variation in our data, compared to approximately 95% in most other markets. Further investigations show that correlation between short- and long term forward prices is lower than in other markets. We briefly discuss possible reasons why these special properties occur, and some consequences for hedging exposures in this market.

JEL Classification Codes: C330, G130, Q490

Key words: Electricity forward market, HJM framework, principal component analysis.

1 Introduction

The purpose of this paper is to conduct an empirical investigation of electricity forward prices. With the rapid growth of derivative securities in deregulated electricity markets, the modelling and management of price risk have become important topics for researchers and practitioners. In the case of electricity contingent claims valuation and risk management were not considered important issues prior to market deregulation. Due to the special properties of this commodity volatility in deregulated electricity markets can reach extreme levels and a proper understanding of volatility dynamics is important for all participants in the market place.

There are two lines of research focusing of commodity contingent claims valuation and risk management. The traditional way has concentrated on modelling the stochastic process of the spot price and other state variables such as the convenience yield[1] (see for example Brennan and Schwartz 1985, Gibson and Schwartz 1990, Schwartz 1997 and Hilliard and Reis 1998). This approach has been adopted and modified in the recent electricity literature by, among others Deng (2000), Kamat and Ohren (2000), Philipović (1998) and Lucia and Schwartz (2000). As far as we know Lucia and Schwartz (2000) represent the first thorough empirical work on electricity spot prices.

The main problem with spot price based models is that forward prices are given endogenously from the spot price dynamics. As a result, theoretical forward prices will in general not be consistent with market observed forward prices. As a response to this, a line of research has focused on modelling the evolution of the whole forward curve using only a few stochastic factors taking the initial term structure as given. Examples of this research building on the modelling framework of Heath et al. (1992), are Clewlow and Strickland (1999a) and (1999b), Miltersen and Schwartz (1998) and Bjerksund et al. (2000).

Empirical investigations of forward curve models in commodity markets have been conducted by, among others, Cortazar and Schwartz (1994) and Clewlow and Strickland (2000). Cortazar and Schwartz (1994) studied the term structure of copper futures prices using principal component analysis and found that three factors were able to explain 99% of the term structure movements. Clewlow and Strickland (2000) investigated the term structure

[1]This direction is rooted in the theory of storage developed by Kaldor (1939), Working (1948) and (1949), Telser (1958) and Brennan (1958) and (1991). According to the theory of storage, the futures and spot price differential is equal to the cost of storage (including interest) and an implicit benefit that producers and consumers receive by holding inventories of a commodity. This benefit is termed the convenience yield. The most obvious benefit from holding inventory is the possibility to sell at an occurring price peak.

of NYMEX oil futures and found that three factors explained 98.4% of the total price variation in the 1998-2000 period. The first factor (explained 91% of total variation) shifted the whole curve in one direction. They termed this a *"shifting"* factor. The second factor, termed the *"tilting"* factor, influenced short and long term contracts in opposite directions. The third factor, coined the *"bending"* factor, moved the short and long end in opposite direction of the mid-range of the term structure.[2]

In this paper we adopt the forward curve approach and perform an empirical examination of the dynamics of the forward curve in the Nordic electricity market during the 1995-2001 period. Following the work of Cortazar and Schwartz (1994) and Clewlow and Strickland (2000) we use principal component analysis to analyze the volatility factor structure of the forward curve. This is, to our knowledge, the first study of the electricity forward curve using historical data.

This paper is organised as follows: We give a short description of the Nordic electricity market in section 2. Section 3 presents the multi-factor models and section 4 describes the data set. In section 5 we show how principal component analysis can be used in order to estimate the empirical volatility functions and section 6 reports the results. Section 7 concludes the paper.

2 The Nordic electricity market

2.1 History of the Nordic Power Exchange

From 1971 to 1993 a market called Samkjøringen coordinated the Norwegian electricity production. Every week Samkjøringen set the daily or part-of-the-day price for electricity. This price was used to decide the Norwegian electricity production and the exchange with other countries. A new Energy Law was approved by the Norwegian Parliament in 1990 and came into effect in 1991. This law introduced market-based principles for production and consumption of electricity in Norway. After England and Wales in 1989, Norway was the second country to deregulate the electricity market.

In 1993 Samkjøringen merged with Statnett SF to create a new company called Statnett Marked AS. Statnett Marked AS organised the new Norwe-

[2] The multi-factor forward approach by Heath et al. (1992) was originally developed for interest rate markets. Empirical work on factor dynamics in fixed income securities markets have been conducted by Steely (1990), Litterman and Scheinkman (1991) and Dybvig (1997). The results in these studies are quite similar to the work reported from the commodity markets. Typically, three factors explained 95%-98% of the total variation in the forward curve.

gian market place for electricity from 1993 to 1996. In 1996 the Swedish grid company, Svenske Kraftnät, bought 50% of Statnett Marked AS and became part of the power exchange area. At the same time Statnett Marked AS was renamed to Nord Pool ASA. Finland joined the power exchange area in 1998, western Denmark in 1999 and eastern Denmark in 2000. The Nordic electricity market is non-mandatory and a significant share of the physical power and financial contracts are traded bilaterally.

2.2 The physical market

Today Nord Pool organises and operates Elspot, Eltermin, Eloption, and Elclearing. Elspot is a spot market for physical delivery of electricity. Each day at noon, spot prices and volumes for each hour the following day are determined in an auction. The equilibrium price is termed the *system price*, which may be considered a one day futures contract. The following day, the national system operators organise a *regulating-* or *balance* market, where short term up- or down regulation is handled. Since 1993 the turnover in Elspot market has increased steadily from 10.2 TWh in 1993 to 96.2 TWh in 2000. In 1999, more than one fifth of the total consumption of electric power in the Nordic countries was traded via Nord Pool.

2.3 The financial market

Eloption and Eltermin are Nord Pool's financial markets for price hedging and risk management. On Eloption European options written on underlying futures and forward contracts. Asian options written on the system price do no longer trade on Eloption. This is due to low liquidity.

Financial contracts traded on Eltermin are written on the arithmetic average of the system price at a given time interval.[3] This time interval is termed the delivery period. The time period prior to delivery is called the trading period. Both futures and forward contracts are traded at Eltermin. The contract types differ as to how settlement is carried out during the trading period. For futures contracts, the value is calculated daily, reflecting changes in the market price of the contracts. These changes are settled financially at each participant's margin account. For forward contracts there is no cash settlement until the start of the delivery period.

[3] We only give a brief description of the different products traded at Nord Pool here. For a detailed description see www.nordpool.no or Lucia and Schwartz (2000). Some contracts traded in the OTC market have a different underlying reference price than the system price. Such contracts are not considered in this study.

The power contracts refer to a base load of 1 MW during every hour for a given delivery period. Futures contracts feature daily market settlement in their trading and delivery periods. Forward contracts, on the other hand, do not have settlement of market price fluctuations during the trading period. Daily settlement is made in the delivery period. None of the contracts traded at Nord Pool are traded during the delivery period.

The contracts with the shortest delivery periods are futures contracts. Daily futures contracts with delivery period of 24 hours are available for trading within the nearest week.[4] Weekly futures contracts with delivery periods of 168 hours can be traded 4-8 weeks prior to delivery. Futures contracts with 4 weeks delivery period, are termed block contracts. The forward contracts have longer delivery periods. Each year is divided into three seasons: V1 - late winter (1. January - 30. April), S0 - summer (1. May - 30. September) and V2 - early winter (1. October - 31. December). Seasonal contracts[5] are written on each of these seasonal delivery periods. In January each year, seasonal contracts on S0 and V2 the coming year, and all three seasonal contracts for the next two years are available. Furthermore, yearly forward contracts are available for the next three years. In other words, the (average based) term structure goes 3 to 4 years into the future, depending on current time of year.

In 1995 the total volume of financial contracts traded on Nord Pool and OTC was 40.9 TWh. In 2000, this number was 1611.6 TWh. The most heavily traded contracts are weekly contracts and the two nearest seasonal contracts. On average 20-30 weekly contracts and 30-80 seasonal contracts are traded each day.

3 Multifactor forward curve models

Our model setting is similar to the forward interest rate model of Heath et al. (1992). The two models we investigate in this paper are special cases of the general multifactor term structure models developed for commodity markets in Miltersen and Schwartz (1998). We consider a financial market where the uncertainty can be described by a K-dimensional Brownian motion $(W_1, ..., W_K)$ defined on an underlying probability space $(\Omega, \mathbb{F}, \mathbb{Q})$with the filtration $\mathbb{F} = \{\mathcal{F}_t : t \in [0, T^*]\}$ satisfying the usual conditions and repre-

[4]These contracts have only a short (and illiquid) history, and will not be included in our data set when analysing the volatility structure in the market.

[5]From 1995 to the end of 1999 seasonal futures contract were traded. In our empirical analysis, all contracts traded in the 1995-2001 period are used in the estimation of the models.

senting the revelation of information. The probability measure $\mathbb{Q}$ represents the equivalent martingale measure. Throughout the paper we assume constant risk free interest rate, so that futures prices and forward prices with common maturity are identical (see Cox et al. (1981)). The two terms will be used interchangeably in the following sections.

Let the forward market be represented by a continuous forward price function, where $f(t,T)$ denotes the forward price at date t for delivery of the commodity at time T, where $t < T < T^*$. Given constant interest rates the futures and forward prices are by construction martingales under the measure $\mathbb{Q}$.

- **Model A: Deterministic volatility functions independent of the forward price level**

Consider a model where the dynamics of the forward price is

$$df(t,T) = \sum_{i=1}^{K} \sigma_i^A(t,T) dW_i(t) \tag{1}$$

where the $(W_1, ..., W_K)$ are independent Brownian motions, and $\sigma_i^A(t,T)$ are time dependent volatility functions.[6] The solution to (1) is

$$f(t,T) = f(0,T) + \sum_{i=1}^{K} \int_0^t \sigma_i^A(s,T) dW_i(s) \tag{2}$$

This means that the forward prices are distributed

$$f(t,T) \sim \mathcal{N}\left(f(0,T), \sum_{i=1}^{K} \int_0^t \sigma_i^A(s,T)^2 ds \right) \tag{3}$$

where $\mathcal{N}(s,v)$ denotes a normally distributed variable with mean s and variance v.

- **Model B: Deterministic volatility functions proportional to the forward price level**

Consider a model where the dynamics of the forward price is given by

$$\frac{df(t,T)}{f(t,T)} = \sum_{i=1}^{K} \sigma_i^B(t,T) dW_i(t) \tag{4}$$

[6] Volatility is a term usually associated with the (time dependent) function of the diffusion term in a lognormal model (model B above). In this paper we use the term "volatility functions" for the time dependent functions in the diffusion term in both models.

with solution

$$f(t,T) = f(0,T)\exp\left(-\frac{1}{2}\sum_{i=1}^{K}\int_0^t \sigma_i^B(s,T)^2 ds + \sum_{i=1}^{K}\int_0^t \sigma_i^B(s,T)dW_i(s)\right) \tag{5}$$

The distribution of the natural log of the forward price is given by

$$\ln f(t,T) \sim \mathcal{N}\left(\ln f(0,T) - \frac{1}{2}\sum_{i=1}^{K}\int_0^t \sigma_i^B(s,T)^2 ds, \sum_{i=1}^{K}\int_0^t \sigma_i^B(s,T)^2 ds\right) \tag{6}$$

where $\mathcal{N}(s,v)$ is defined as above.

Versions of both class A and B models have been proposed for the Nordic electricity market. Lucia and Schwartz (2000) propose a spot price model and derive analytical expressions for futures/forward prices. They consider mean reverting spot price models both in level and log form. It is easy to show that their models are consistent with forward price models with

$$\sigma_1^A(t,T) = \sigma e^{-\kappa(T-t)}$$

and

$$\sigma_1^B(t,T) = \sigma e^{-\kappa(T-t)}$$

respectively, where σ and κ are positive constants. This model produce a falling volatility curve in T, approaching zero as $T \to \infty$. Bjerksund et al. (2000) on the other hand, propose two different kinds of class B models. The one factor model is given by

$$\sigma_1^B(t,T) = \frac{a}{T-t+b} + c$$

where a, b and c are positive constants. With realistic parameter values, this specification produces a sharply falling volatility curve in T. As $T \to \infty$ the volatility converges to c. Bjerksund et. al. (2000) also propose a three factor model

$$\begin{aligned} \sigma_1^B(t,T) &= \tfrac{a}{T-t+b} \\ \sigma_2^B(t,T) &= \left(\tfrac{2ac}{T-t}\right)^{\frac{1}{2}} \\ \sigma_3^B(t,T) &= c \end{aligned}$$

with all parameters assumed positive. This three factor model allows a richer structure of the forward price dynamics. [7] They argue that the one factor

[7] We see that $\lim_{T\to t}\left(\sigma_2^B(t,T)\right) = \infty$ so that this model is in fact not well behaved in the short end.

model may be adequate for pricing contingent claims, while the three factor model is better suited for risk management purposes. Note that in all the models above, given that all the parameters are positive, forward prices of all maturities will move in the same direction. As we will see from the empirical analysis, this property of the proposed models is inconsistent with our empirical findings.

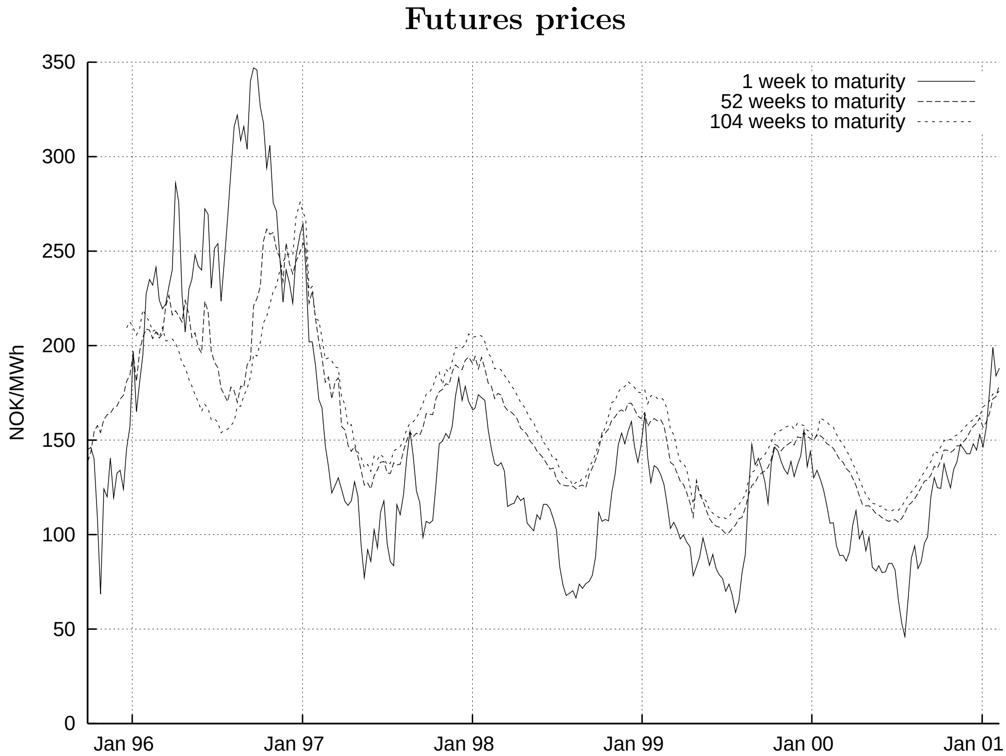

Figure 1: The graphs are time series plots of the futures prices with one week (solid line), one year - (dashed line) and two years (dotted line) to maturity.

4 Descriptive analysis and data preparation

We are interested in the volatility dynamics of the forward price function described abow. This forward price function, giving us today's price of a unit of electricity delivered at a specific instant in the future, is not directly observable in the market place. The power contracts trading on Nord Pool

Sample period: 1995 - 2001			
Maturity	W-01	W-52	W-104
Mean	145.51	159.54	163.47
Median	130.18	153.49	158.88
Min	45.25	99.91	101.24
Max	356.00	262.03	275.75
Std.dev	64.10	36.04	33.17
Skewness	1.21	0.76	0.63
Kurtosis	3.91	3.18	3.26
Nobs	1340	1340	1279

Table 1: Descriptive statistics of daily forward prices from the smoothed term structure of the total sample. The table reports statistics from three points on the term structure, the one week forward price (W-01), the one year forward price (W-52) and the two year forward price (W-104).

Sample period: 1995-2001						
Maturity	W-01	W-52	W-104	W-01	W-52	W-104
	Price differences			Price returns		
Mean	-0.32	0.00	-0.04	-0.00	0.00	-0.00
Median	-0.25	-0.02	-0.00	-0.00	-0.00	-0.00
Min	-32.75	-17.42	-29.00	-0.39	-0.07	-0.25
Max	37.25	21.36	26.80	0.22	0.09	0.23
Std.dev	6.03	2.64	2.34	0.04	0.01	0.01
Skewness	0.13	0.28	-1.06	-0.42	0.36	-1.01
Kurtosis	9.44	12.56	45.07	11.23	8.11	116.14
Nobs	1339	1339	1278	1339	1339	1278

Table 2: Descriptive statistics of daily forward price differences and forward price returns from the smoothed term structure of the total sample. The table reports statistics from three points on the term structure, the one week forward price (W-01), the one year forward price (W-52) and the two year forward price (W-104).

are all written on a future average; the delivery periods of the contracts. We need to pin down the relationship between the forward price function and the average based contracts. Let $F(t, T_1, T_2)$ be today's contract price of an average based futures contract delivering one unit of electricity at a rate of $\frac{1}{T_2 - T_1}$ in the time period $[T_1, T_2]$, where T_1 and T_2 is the beginning and the end of the delivery period of the contract, and $t \leq T_1 < T_2$. Suppose that the contract price is paid as a constant cash flow during the delivery period. Then the expression for the average contract is (see Bjerksund et al. (2000)):

$$F(t, T_1, T_2) = \int_{T_1}^{T_2} w(r, u) f(t, u) du \tag{7}$$

where

$$w(r, u) \equiv \frac{e^{-r(u-t)}}{\int_{T_1}^{T_2} e^{-r(u-t)} du} \tag{8}$$

Lucia and Schwartz (2000) note that $F(t, T_1, T_2) \approx \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t, u) du$ is a very good approximation of (7) and (8) for reasonable levels of interest rates. We use this approximation in the empirical analysis.

4.1 Smoothed data

Instead of working directly with the different financial contracts with various delivery periods, we compute a continuous forward price function from each day's futures and forward prices. The forward price function is given by the smoothest function that prices all traded assets within the bid/ask spread using (7). The smoothed forward price functions were computed using a software with the name ELVIZ developed by Viz Risk Management Services AS.[8] The forward market representation in ELVIZ is founded on maximum smoothness with a sinusoidal prior continuous forward price function. The result of this smoothing procedure on March 27. 2000 is illustrated in figure 2. The horisontal dotted lines are closing prices on weekly, block and seasonal contracts. We have computed the smoothed forward price function on each of the 1340 trading days in our sample using all the contracts available each day. In figure 3 we have plotted weekly forward curves during the 1995-2001 sample period. Note the clear annual seasonal variation with high winter and low summer prices. The contract with the longest time to maturity increases from 80 weeks in 1995 to 208 weeks in 2001. Table 1 shows descriptive

[8] For a comprehensive description of the maximum smoothness approach see Adams and van Deventer (1994), Bjerksund and Stensland (1996) and Forsgren (1998). For more information of the ELVIZ software, see www.viz.no.

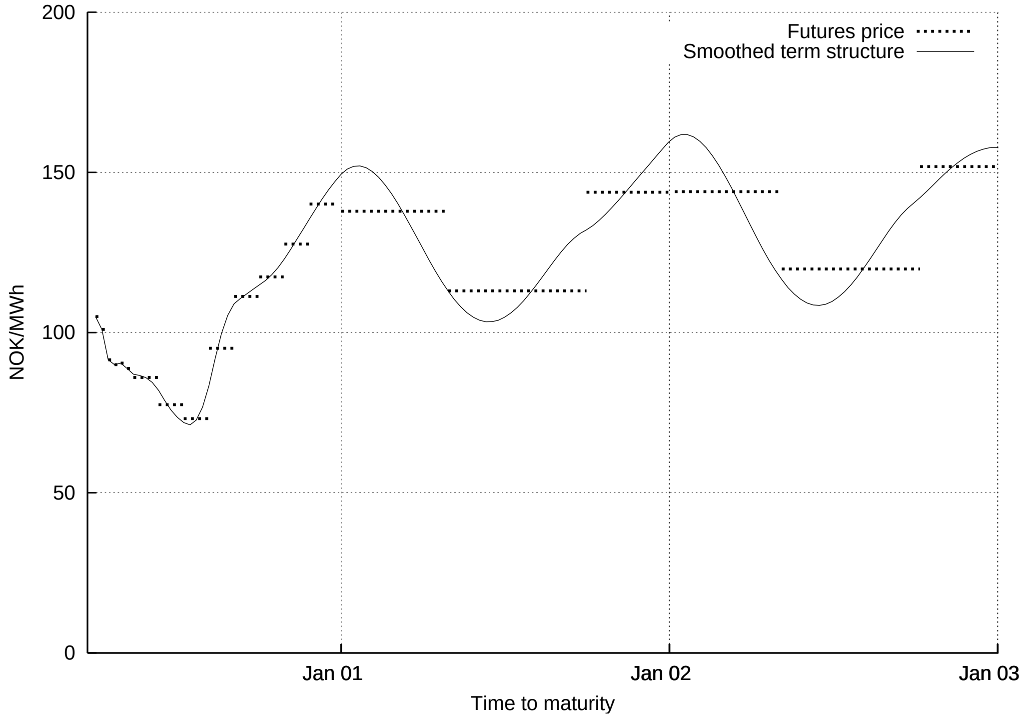

Figure 2: The futures and forward contracts on March 27. 2000 are represented by the dotted lines, and the length of the dotted lines corresponds to the delivery period on which the contracts are written. The weekly contracts (one dot) and block contracts (four dots) are futures contracts, and the seasonal contracts are forward contracts. The solid line is the smoothed term structure.

statistics on three different points on the term structure; W-01 (one week to maturity), W-52 (one year to maturity) and W-104 (two years to maturity). We note that the mean forward price is increasing with maturity. This means that the market on average can be described by normal backwardation[9] (a positive risk premium). We note that the one week forward price has fluctuated substantially during the sample period. The fluctuations decrease with time to maturity. To further examine the time series properties of the data, we have plotted the forward price with the same three maturities. It is obvious that the one week contract is much more erratic than the one- and two year contract. Note that the short-term price varies around the long-term price indicating some sort of mean reversion. Roughly speaking the market

[9]Normal backwardation is used to describe the relationship $f(t,T_2) \geq f(t,T_1)$ when $T_2 > T_1$. We must be careful when using this relationship in markets with seasonal price variation. By choosing maturities exactly one year apart, forward prices on the same time of the year are compared and seasonal variation is no longer a problem.

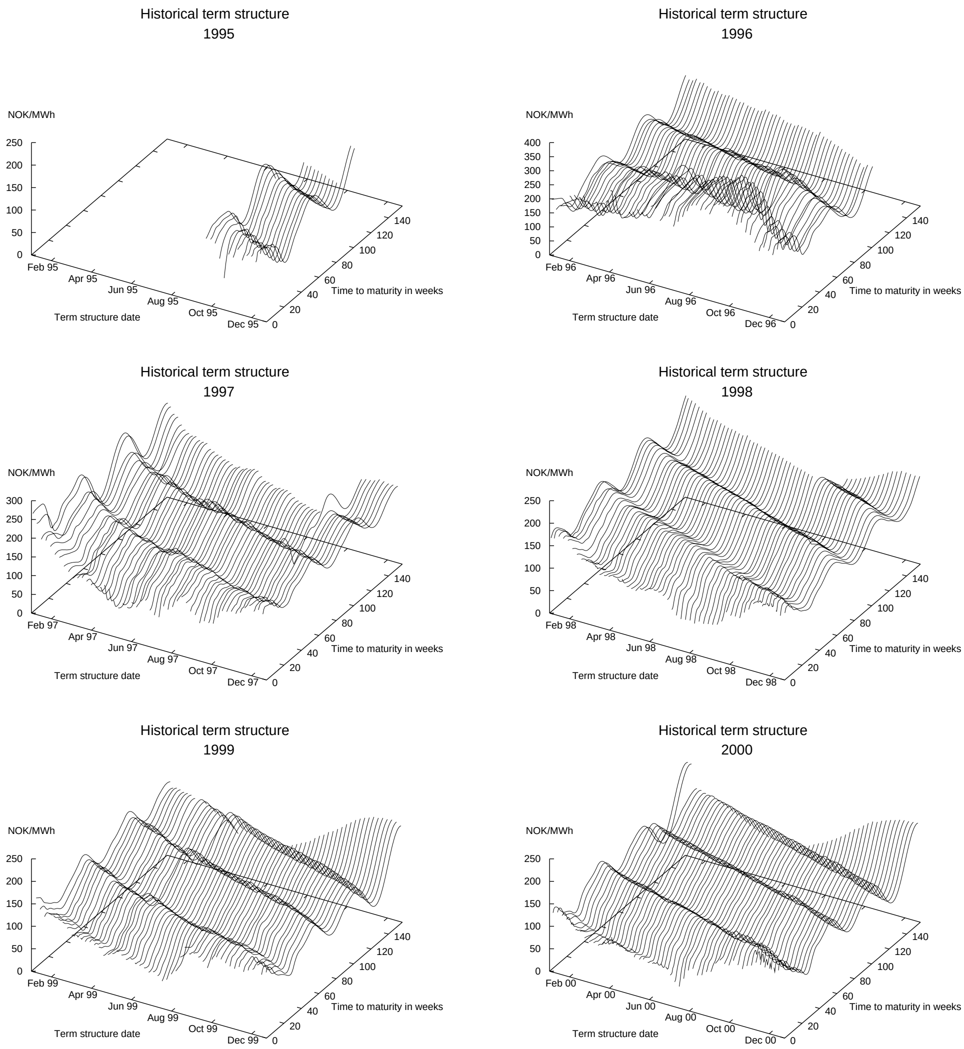

Figure 3: Surface plots of smoothed forward curves (each wednesday) for each of the years in our sample.

was in contango in 1996 and in normal backwardation in the 1997-2001 period.

4.2 Constructing two data sets

The forward price models in (2) and (5) describe the stochastic evolution under an equivalent martingale measure, and not under the real world measure where observations are made. Although there may be risk premia in the market that cause futures prices to exhibit non-zero drift terms, the diffusion terms are equal under both measures. So the volatility functions in (2) and (5) can be estimated from real world data. As noted by Cortazar and Schwartz (1994), this is only strictly correct when observations are sampled continuously. In our analysis we use daily observations as a proxy to a continuously sampled data set. Let $f(t_n, T_m)$ denote the forward price at date t_n with maturity at date T_m, where $t_n < T_m$. Our discrete approximations of model A and B are

$$df(t_n, T_m) \approx f(t_n, T_m) - f(t_{n-1}, T_m) = x^A_{n,m} \tag{9}$$

and

$$\frac{df(t_n, T_m)}{f(t_n, T_m)} \approx \frac{f(t_n, T_m) - f(t_{n-1}, T_m)}{f(t_{n-1}, T_m)} = x^B_{n,m} \tag{10}$$

where $n = 1, ..., N$ and $m = 1, ..., M$. We construct 2 different data sets from the smoothened data, $\mathbf{X}^A_{(N\times M)}$ with *forward price differences*

$$\mathbf{X}^A_{(N\times M)} = \begin{bmatrix} x^A_{1,1} & x^A_{1,2} & \cdots & x^A_{1,M} \\ x^A_{2,1} & x^A_{2,2} & \cdots & x^A_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ x^A_{N,1} & x^A_{N,2} & \cdots & x^A_{N,M} \end{bmatrix} \tag{11}$$

and $\mathbf{X}^B_{(N\times M)}$ with *forward price returns*

$$\mathbf{X}^B_{(N\times M)} = \begin{bmatrix} x^B_{1,1} & x^B_{1,2} & \cdots & x^B_{1,M} \\ x^B_{2,1} & x^B_{2,2} & \cdots & x^B_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ x^B_{N,1} & x^B_{N,2} & \cdots & x^B_{N,M} \end{bmatrix} \tag{12}$$

The matrices above deserve a thorough description. We first compute daily forward price functions from the observed market prices. From these forward functions we compute 104 weekly midpoint prices, one price for each week along a two years term structure. Within each week these maturities are held constant. Next we compute $N = 1339$ time series observations on price returns and price differences. The contracts are rolled over each Friday. Let us illustrate our approach using the contract with maturity in one week: The daily returns and differences from Monday to Friday are computed from the contracts with maturity the following week (T_1). On Friday we observe the price of the contact with maturity two weeks ahead (T_2). The return and difference on this contract is calculated from Friday to Monday. Reaching Monday, this contract has now become the new one week contract. We use this approach of fixing the time to maturity to avoid problems of seasonality in prices over the year. Finally we pick $M = 21$ price returns and differences with different maturities among the 104 weekly prices. If we scale T_m in "weeks-to-maturity" the specific maturities chosen are $T_1, \ldots, T_M = [1, 2, 3, 4, 5, 6, 7, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 70, 88, 104]$. The maturities are chosen in such a way that they reflect the actual traded contracts. In the shortest end we pick 7 maturities with weekly intervals, mimicking the weekly contracts. The next 11 maturities are 4 weeks apart. There are only three maturities in the last year of the term structure, representing seasonal contracts. In table 2 we report descriptive statistics on the one week-, one year- and two year forward price differences and forward price returns for the whole sample period. The standard deviation of both price returns and price differences is sharply falling with time to maturity. We also note that kurtosis is high, and that skewness is different from zero. The sign of the skewness changes along the term structure. In tables 5 and 6 we report descriptive statistics on semi annual and seasonal sub-interval of forward price differences and forward price returns respectively. We note that the standard deviation of price differences is markedly higher in the 1995-1996 sub-period than in 1997-1998 and 1999-2001.

5 Principal component analysis and volatility functions

Principal component analysis (PCA) is concerned with the identification of structure within a set of interrelated variables. It establishes dimensions within the data, and serves as a data reduction technique. The aim is to determine factors (i.e. principal components) in order to explain as much of the total variation in the data as possible. In order to use principal component analysis to estimate the volatility functions in (2) or (5) we assume that these functions only depend on time to maturity $\tau = (T - t)$. Assume that we have a total of N observations of M different variables contained in vectors $\mathbf{x}_1$, $\mathbf{x}_2, ..., \mathbf{x}_M$ all of which dimension is $(N \times 1)$.[10] Let the data matrix, $\mathbf{X}$, be given by

$$\mathbf{X}_{(N\times M)} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_M \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1M} \\ x_{21} & x_{22} & \cdots & x_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{NM} \end{bmatrix} \tag{13}$$

The corresponding sample covariance matrix, of order M, is denoted $\mathbf{\Psi}$**.** The orthogonal decomposition of the covariance matrix is

$$\mathbf{\Psi} = \mathbf{P\Lambda P}' \tag{14}$$

where

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_M \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix}$$

and

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & 0 & \cdots & 0 \\ 0 & \lambda_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{MM} \end{bmatrix}$$

$\mathbf{\Lambda}$ is a diagonal matrix whose diagonal elements are the eigenvalues λ_{11}, $\lambda_{22}, ...,$ λ_{MM}, and where $\mathbf{P}$ is an orthogonal matrix of order M whose ith column, $\mathbf{p}_i$, is the eigenvector corresponding to λ_{ii}. $\mathbf{P'}$ is the transpose of $\mathbf{P}$. The matrix $\mathbf{Z} = \mathbf{XP}$ is called the matrix of principal components. Its columns, $\mathbf{z}_i$, are linear combinations of the columns of $\mathbf{X}$ with the weights given by the elements of $\mathbf{p}_i$. That is, the ith principal component is

$$\mathbf{z}_i = \mathbf{X}\mathbf{p}_i = \mathbf{x}_1 p_{1i} + \mathbf{x}_2 p_{2i} + ... + \mathbf{x}_M p_{Mi} \tag{15}$$

where p_{ij} is the element in the jth row and ith column of $\mathbf{P}$.

[10]Througout this section we write matrices in bold upper case letters, vectors in bold lower case letters and elements in plain text. The principal component analysis is conducted on both forward price differences $\left(\mathbf{X}^A\right)$ and forward price returns $\left(\mathbf{X}^B\right)$. We supress superscripts for notational convenience througout this section.

The sample covariance matrix of $\mathbf{Z}$ is given by

$$var(\mathbf{Z}) = \mathbf{P}'\mathbf{\Psi}\mathbf{P} = \mathbf{P}'\mathbf{P}\mathbf{\Lambda}\mathbf{P}'\mathbf{P} = \mathbf{\Lambda} \tag{16}$$

since $\mathbf{PP}' = \mathbf{P}'\mathbf{P} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, hence the $\mathbf{Z}$ variates are uncorrelated, and the variance of $\mathbf{z}_i = \lambda_{ii}$. The eigenvectors on the diagonal of $\mathbf{\Lambda}$ are of convention ordered so that $\lambda_{11} \geq \lambda_{22} \geq ... \geq \lambda_{MM}$. To explain all the variation in $\mathbf{X}$, we need M principal components. Since the objective of our analysis is to explain the covariance structure with just a few factors, we approximate the theoretical covariance matrix using the first $K < M$ eigenvalues in (14). Unfortunately we lack any solid statistical criterion to determine the number of factors that constitute the theoretical covariance matrix. Hair et al. (1995) discuss several criteria:

1. Eigenvalue criterion; only factors eigenvalues greater than 1 are considered significant.

2. Scree test criterion; the test is derived by plotting the eigenvalues against the number of factors in their order of extraction, and the shape of the curve is used to evaluate the cutoff point.

3. Percentage of variance criterion; additional factors are added until the cumulative percentage of the variance explained reach a prespecified level.

We consider all of these criteria, but the latter criterion is the one frequently employed in the finance literature. The K factors should explain a "big" part of the total covariance of the underlying variables (typically around 95%). The proportion of total variance accounted for by the first K factors is

$$\text{Cumulative contribution of first } K \text{ factors} = \frac{\sum_{i=1}^{K} \lambda_i}{\sum_{i=1}^{M} \lambda_i}$$

Component loadings are often computed to facilitate interpretation of the results from a principal component analysis. Here, we instead plot the empirical volatility function, $\widehat{\sigma}_i(.)$, directly from the eigenvalue decomposition as

$$\widehat{\sigma}_i(t, T_m) = \sqrt{\lambda_i} p_{mi}$$

where $i = 1, \ldots, K$. In this way easy-to-interpret volatility functions can be graphed.

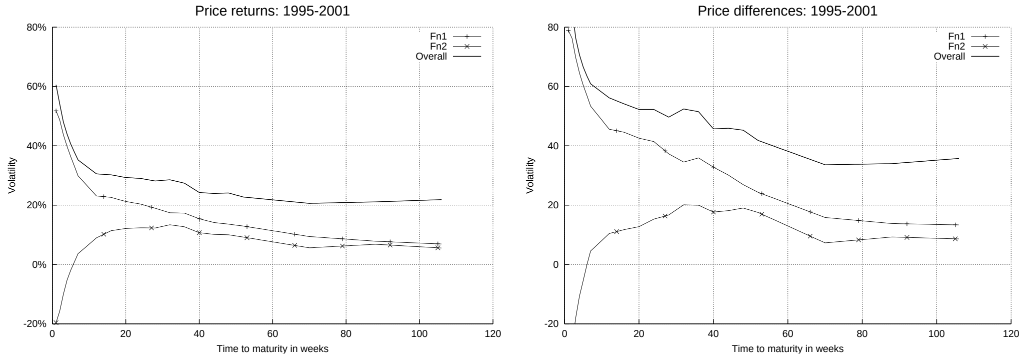

Figure 4: The first two volatility functions and overall volatility in the full sample period 1995-2001. The volatility functions on the left hand side are computed from price returns and the volatility functions on the right hand side are computed from price differences. The functions are annualized using a factor of square root of 250 (number of trading days).

6 Empirical results

In table 3 we report the results from the PCA analysis conducted on the full sample. We note that a one factor model is able to explain 68% and 70% of the variation of price returns and price differences respectively. The eigenvalue and scree test criteria both agree upon a two factor model for both returns and differences with a total of 75% and 78% variation explained respectively. This is considerably lower than in other markets. Furthermore, we see that we need more than 10 factors to explain 95% of the variation in the data for both models. Combined our results suggests that the variance in the term structure movements common to all maturities represents 75% of the total variance. The rest of the variance is specific to each maturity. The fact that as much as 25% of the variance is maturity specific is, as far as we know, a feature unique to the electricity market.

We now want to take a closer look at the volatility dynamics for the first two factors that affect the whole term structure. In figure 4 we have plotted the empirical volatility functions corresponding to the two largest eigenvalues along with the overall volatility. The scaling on the vertical axes are annualised volatilities. We have used data for the whole sample period in these calculations. Volatility falls rapidly with time to maturity, and after approximately one year it stabilises. The first factor is positive for all maturities, shifting all forward prices in the same direction. It causes much bigger movements in the short end than in the long end. The second factor causes short and long term forward prices to move in opposite directions, a

Sample period: 1995 - 2001				
Data	Price returns		Price differences	
Sample	1995-2001			
Factor	Ind.	Cum.	Ind.	Cum.
Fn1	0.68	0.68	0.70	0.70
Fn2	0.07	0.75	0.08	0.78
Fn3	0.05	0.80	0.05	0.83
Fn4	0.03	0.83	0.02	0.86
Fn5	0.03	0.86	0.02	0.88
Fn6	0.02	0.88	0.02	0.89
Fn7	0.02	0.89	0.02	0.91
Fn8	0.02	0.91	0.01	0.92
Fn9	0.01	0.92	0.01	0.93
Fn10	0.01	0.94	0.01	0.94

Table 3: Principal component analysis of forward price differences and price returns. The analysis is performed on the whole data set, 1339 observations from September 1995 to March 2001. The table reports the individual contribution (Ind.) of each factor (Fn.) of the total variance, and the cumulative effect (Cum.) of adding an additional factor.

so called *"tilting"* or *"steepening"* factor.

Some authors have claimed that correlations between forward prices with different maturities seem to be lower than in other markets. In order to take a closer look at this, we conducted the PCA analysis once again, but this time with a twist. First we compute 10 principal components. Then, for each of the 21 maturities, we sort the components according to size for each maturity. The results are given in tables 7 and 8. Since the results are qualitatively similar, we only comment on price return data in table 8. The number of each individual principal component is given in superscript. We note that factor number 1 is the factor explaining most of the variation for each maturity within the first year. Factors number 1 and 2 are among the 4 most important factors for every maturity. However, in the long end of the term structure, factors number 9 and 6 are the most important ones. On average very little is gained in terms of percentage variation explained, by increasing the number of factors beyond 5. Combined, this evidence supports the conjecture made by Philipović (1998) that electricity prices exhibit "split personalities". The most important factors driving the long end of the curve have very little impact on price changes in the short end. Why do we see this kind of forward curve behaviour in the electricity market? The answer possibly lies in the non-storable nature of electricity. For example, assume that the Swedish government makes a final decision to phase out their nuclear electricity production and decides to start cutting production two years from now. This would lower future supply, resulting in rising futures prices with more than 2 years to maturity. In a market where storage is possible, speculator would buy for storage (or producer would hold back production), as a reaction to the anticipated rise in electricity prices in the future. This would in turn result in a positive shift in spot and short-term futures prices as well as long term futures prices. Since buying for storage is impossible[11] in electricity markets, the price on electricity will stay low until the date of reduced production. Consequently, only futures contracts with maturity after the production cut will react to this information.

Using the whole sample period in our calculations, we implicitly assume that volatility dynamics have been constant in the 1995-2001 period. Investigating the validity of this assumption, we plotted the volatility series from the shortest maturity for each of our models in figure 5. To compute the series, we calculated the annualised volatility of price differences and price returns of the one week forward price using a 30 day moving window. Volatility of price differences is measured on the left vertical axis, and volatility of price returns is measured on the right vertical axis. The volatility of price differences was high in the period 1995-1997 and relatively much lower in the 1998-2001 period. We also note that the volatility is all but constant. The volatility of price returns was not especially high the first years. In this model we see a relatively regular pattern; volatility peaks during summer. We want to investigate yearly and seasonal differences further. In table 9 we report the results from PCA analysis on two years sub-intervals and seasonal sub-intervals for model A and B. The two first volatility functions and overall volatility for each sub-sample are plotted in figures 6 and 7. From table 9 we see that the V1 and S0 sub-periods, fewer factors are needed to explain 95% of the variation in the data. Dividing into semi-yearly samples resulted in increased explanatory power of the 10 factors. This indicates that volatility dynamics changes both seasonally and from one year to the other[12]. Still, from the volatility function in figures 6 and 7 we recognise the shifting and

[11] A large part of the electricity consumed the Nordic market is produced in hydropower based production units. Many of these units have reservoir facilities that, to some extent, enables them to move energy between periods. Such reservoir facilities provide a relatively high level of operating flexibility. Still, the capacity reservoir are not big enough for producers to shut down production for long periods of time without spilling water.

[12] We also computed the non-parametric Kolmogorov-Smirnoff test on equality of distributions across seasons and years. The test results, not reported here, showed rejections of equal distributions on 1% level in all cases.

tilting factor as the most important factors driving the forward curve.

Finally, we are interested in which of the two models, A or B, best resembles the data generating process. We know that model A assumes normally distributed price differences and model B assumes normally distributed price returns. In table 10 we report statistics on skewness, kurtosis and the combined effect of the two (the Jarque-Bera test) under the null hypothesis of normality of price differences and price returns respectively. The tests are conducted on 3 points on the term structure, with one week, one year and two years to maturity. We note that both price differences and price returns are positively skewed in the short end, and negatively skewed in the long end. Excess kurtosis is substantially different from zero for both models and increases with maturity for both specifications. The high degree of kurtosis may indicate that jumps are present in the data. Not surprisingly, the Jarque-Bera tests reject the null hypothesis of normality for both models, and so further modifications and testing of the models are necessary to decide upon the winning candidate.

7 Conclusions and suggestions for further research

In this paper we have conducted an exploratory investigation of the volatility dynamics in the Nordic futures and forward market in the period 1995-2001. We have used smoothed data and performed a principal component analysis to reveal the factor structure of the forward price curve. We specified two different models in the framework of Heath et al. (1992); one model where the volatility was independent of the forward price level and one model where the volatility was proportional to the price level.

The main results are: Two factors are common across all maturities. A two factor model explains around 75% of total variation in the data. The first two factors governing the forward curve dynamics are comparable to other markets. The first factor is positive for all maturities, hence it shifts all forward prices in the same direction. The second factor causes short and long term forward prices to move in opposite directions. In contrast to other markets, more than 10 factors are needed to explain 95% of the term structure variation. Furthermore, the main sources of uncertainty affecting the movements in the long end of the forward curve, have virtually no influence on variation in the short end of the curve. We argue that this behaviour may occur because electricity is a non-storable commodity. Note that the endpoint of the forward curve we examined is 2 years. One might suspect that

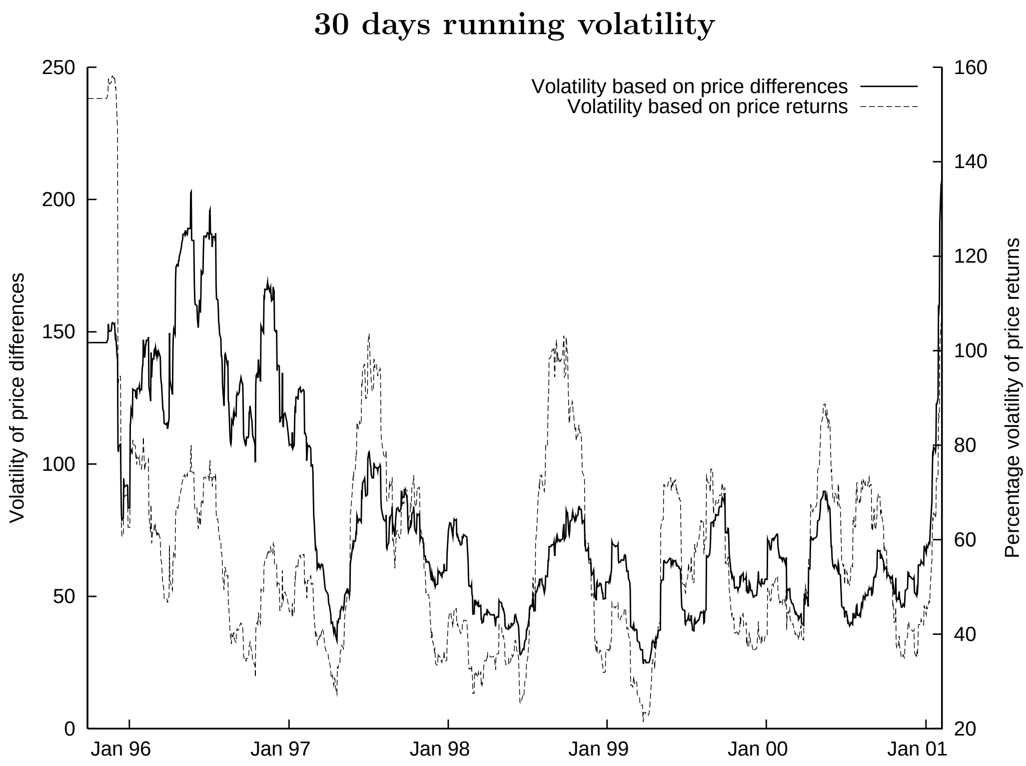

Figure 5: The volatility series are computed from daily data of the one week ahead forward price returns (dashed line) and forward price differences (solid line). Both volatility series are simple arithmetic average of the last 30 trading days. We have annualized the series by the square root of 250 (number of trading days).

contracts sold in the OTC market with maturities further into the future are even less correlated with short term contracts. These results indicate that modelling the whole forward curve has less merit in this market than others. For example, hedging long term commitments using short term contracts may prove disastrous.

The results reported above apply to both models. Both models fail the normality test, and so neither of them is completely satisfactory. Results from semi-yearly and seasonal sub-intervals suggest that volatility is not constant through time. Hence extending the basic model to include stochastic volatility, possibly with a seasonally time-dependent component, may be fruitful.

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Panel A: Yearly sub-intervals									
Sample	1995-1996			1997-1998			1999-2001		
Maturity	W-01	W-52	W-104	W-01	W-52	W-104	W-01	W-52	W-104
Mean	230.28	201.75	198.15	126.03	159.84	168.93	113.20	133.93	141.35
Median	236.50	203.83	201.61	119.43	157.12	169.43	116.00	135.22	144.13
Min	66.50	138.19	151.18	54.86	122.71	101.24	45.25	99.91	107.95
Max	356.00	262.03	275.75	264.50	255.42	271.56	249.00	182.01	178.54
Std.dev	66.49	31.27	29.64	34.96	25.77	28.01	30.97	19.30	20.16
Skewness	-0.35	0.17	0.38	0.62	0.81	0.58	0.35	-0.01	-0.08
Kurtosis	2.50	2.13	2.51	3.78	3.82	3.48	3.32	1.96	1.76
Nobs	315	315	254	500	500	500	525	525	525
Panel B: Seasonal sub-intervals									
Sample	early winter (V2)			late winter (V1)			summer (S0)		
Maturity	W-01	W-52	W-104	W-01	W-52	W-104	W-01	W-52	W-104
Mean	159.95	175.32	182.06	149.40	169.59	177.56	131.76	139.72	140.32
Median	144.25	163.09	175.44	133.25	163.69	175.06	101.00	131.63	138.96
Min	66.50	137.10	145.82	78.26	109.27	115.51	45.25	99.91	101.24
Max	337.50	262.03	275.75	290.50	255.42	271.56	356.00	231.04	200.26
Std.dev	52.48	35.43	31.80	49.15	31.68	29.33	78.60	30.42	21.07
Skewness	1.60	1.33	1.04	0.78	0.32	0.17	1.47	1.08	0.41
Kurtosis	4.93	3.46	3.19	2.65	2.38	2.99	3.87	3.40	2.43
Nobs	382	382	321	432	432	432	523	523	523

Table 4: Descriptive statistics of daily forward prices from the smoothed term structure. In panel A the analysis is performed on each two year sub-interval of the total sample. In panel B the data set is re-shuffled, and the analysis is performed on 3 seasonal sub-intervals, V2 (early winter), V1 (late winter) and S0 (summer) (see the text for exact period specifications). The table reports statistics from three points on the term structure, the one week forward price (W-01), the one year forward price (W-52) and the two year forward price (W-104).

Panel A: Yearly sub-intervals									
Sample	1995-1996			1997-1998			1999-2001		
Maturity	W-01	W-52	W-104	W-01	W-52	W-104	W-01	W-52	W-104
Mean	-0.10	0.42	0.16	-0.64	-0.26	-0.22	-0.16	0.01	0.02
Median	0.00	0.27	0.09	-0.50	-0.19	-0.03	-0.25	-0.02	-0.00
Min	-31.50	-17.42	-11.31	-17.11	-13.95	-29.00	-32.75	-6.79	-6.57
Max	34.50	21.36	10.77	16.20	9.32	26.80	37.25	7.84	13.48
Std.dev	9.16	4.05	2.70	4.49	2.48	2.92	4.80	1.37	1.29
Skewness	-0.12	0.33	0.00	-0.22	-0.65	-1.58	1.06	0.57	2.64
Kurtosis	4.97	7.50	5.69	4.43	7.09	44.09	16.91	10.29	30.16
Nobs	315	315	254	500	500	500	524	524	524
Panel B: Seasonal sub-intervals									
Sample	Late winter (V2)			Early winter (V1)			Summer (S0)		
Maturity	W-01	W-52	W-104	W-01	W-52	W-104	W-01	W-52	W-104
Mean	-0.86	-0.14	-0.05	-0.18	-0.00	-0.14	-0.00	0.13	0.05
Median	-0.50	-0.15	-0.05	-0.25	0.01	-0.00	-0.00	0.02	0.01
Min	-31.50	-17.42	-11.31	-32.75	-13.95	-23.01	-20.97	-9.81	-29.00
Max	27.50	21.36	10.77	37.25	12.11	13.48	29.79	15.03	26.80
Std.dev	6.04	2.75	2.14	6.47	3.02	2.60	5.57	2.17	2.22
Skewness	-0.65	0.17	0.21	0.68	0.01	-1.70	0.19	1.12	-0.86
Kurtosis	9.53	19.64	9.18	10.88	6.65	21.54	6.61	12.55	97.07
Nobs	382	382	321	431	431	431	523	523	523

Table 5: Descriptive statistics of daily forward price differences from the smoothed term structure. In panel A the analysis is performed on each two year sub-interval of the total sample. In panel B the data set is re-shuffled, and the analysis is performed on 3 seasonal sub-intervals, V2 (early winter), V1 (late winter) and S0 (summer) (see the text for exact period specifications). The table reports statistics from three points on the term structure, the one week forward price (W-01), the one year forward price (W-52) and the two year forward price (W-104).

Panel A: Yearly sub-intervals									
Sample	1995-1996			1997-1998			1999-2001		
Maturity	W-01	W-52	W-104	W-01	W-52	W-104	W-01	W-52	W-104
Mean	-0.00	0.00	0.00	-0.01	-0.00	-0.00	-0.00	0.00	0.00
Median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00
Min	-0.39	-0.07	-0.04	-0.17	-0.07	-0.25	-0.16	-0.04	-0.04
Max	0.22	0.09	0.04	0.15	0.06	0.23	0.18	0.06	0.09
Std.dev	0.05	0.02	0.01	0.04	0.01	0.02	0.04	0.01	0.01
Skewness	-1.38	0.41	0.06	-0.25	-0.14	-1.27	0.54	0.94	2.72
Kurtosis	17.83	6.14	4.23	5.55	6.03	91.45	6.19	10.89	26.86
Nobs	315	315	254	500	500	500	524	524	524
Panel B: Seasonal sub-intervals									
Sample	Late winter (V2)			Early winter (V1)			Summer (S0)		
Maturity	W-01	W-52	W-104	W-01	W-52	W-104	W-01	W-52	W-104
Mean	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00
Median	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00
Min	-0.39	-0.07	-0.04	-0.16	-0.06	-0.09	-0.17	-0.07	-0.25
Max	0.22	0.09	0.04	0.18	0.06	0.09	0.16	0.07	0.23
Std.dev	0.04	0.01	0.01	0.04	0.02	0.01	0.04	0.01	0.02
Skewness	-1.65	0.22	0.25	0.63	0.33	-0.10	-0.15	0.48	-1.48
Kurtosis	24.96	12.24	6.03	6.82	5.86	14.08	4.67	8.44	134.31
Nobs	382	382	321	431	431	431	523	523	523

Table 6: Descriptive statistics of daily forward price returns from the smoothed term structure. In panel A the analysis is performed on each two year sub-interval of the total sample. In panel B the data set is re-shuffled, and the analysis is performed on 3 seasonal sub-intervals, V2 (early winter), V1 (late winter) and S0 (summer) (see the text for exact period specifications). The table reports statistics from three points on the term structure, the one week forward price (W-01), the one year forward price (W-52) and the two year forward price (W-104).

Relative importance of factors across maturities for price differences										
Maturity	Cumulative variance explained (%)									
	1^{th}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
W-01	0.85^{1}	0.92^{2}	0.94^{4}	0.95^{3}	0.96^{8}	0.96^{9}	0.96^{7}	0.97^{5}	0.97^{6}	0.97^{10}
W-02	0.89^{1}	0.94^{2}	0.95^{4}	0.96^{3}	0.96^{9}	0.96^{8}	0.96^{6}	0.96^{7}	0.96^{10}	0.96^{5}
W-03	0.91^{1}	0.94^{2}	0.95^{8}	0.96^{7}	0.96^{9}	0.96^{5}	0.96^{6}	0.96^{10}	0.96^{4}	0.96^{3}
W-04	0.91^{1}	0.93^{2}	0.94^{4}	0.94^{5}	0.95^{8}	0.95^{3}	0.96^{7}	0.96^{9}	0.96^{10}	0.96^{6}
W-05	0.91^{1}	0.93^{4}	0.95^{3}	0.96^{5}	0.96^{2}	0.96^{6}	0.96^{10}	0.96^{8}	0.96^{9}	0.96^{7}
W-06	0.90^{1}	0.93^{4}	0.95^{3}	0.95^{8}	0.96^{5}	0.97^{7}	0.97^{6}	0.97^{9}	0.97^{10}	0.97^{2}
W-07	0.88^{1}	0.91^{4}	0.92^{8}	0.94^{3}	0.94^{7}	0.95^{5}	0.95^{2}	0.95^{9}	0.96^{6}	0.96^{10}
W-12	0.81^{1}	0.88^{5}	0.90^{9}	0.92^{2}	0.92^{3}	0.93^{4}	0.94^{8}	0.94^{7}	0.94^{10}	0.94^{6}
W-16	0.82^{1}	0.89^{5}	0.92^{2}	0.93^{4}	0.93^{8}	0.94^{7}	0.94^{9}	0.94^{3}	0.94^{10}	0.94^{6}
W-20	0.81^{1}	0.87^{5}	0.90^{2}	0.92^{9}	0.92^{4}	0.92^{10}	0.93^{6}	0.93^{8}	0.93^{3}	0.93^{7}
W-24	0.79^{1}	0.85^{2}	0.89^{9}	0.91^{5}	0.92^{10}	0.92^{7}	0.93^{6}	0.94^{3}	0.94^{8}	0.94^{4}
W-28	0.75^{1}	0.82^{2}	0.85^{7}	0.87^{3}	0.88^{6}	0.89^{9}	0.89^{10}	0.89^{4}	0.89^{5}	0.89^{8}
W-32	0.66^{1}	0.82^{3}	0.91^{2}	0.93^{10}	0.93^{9}	0.94^{7}	0.94^{5}	0.94^{4}	0.95^{8}	0.95^{6}
W-36	0.70^{1}	0.82^{3}	0.91^{2}	0.93^{5}	0.94^{7}	0.95^{9}	0.95^{10}	0.95^{8}	0.95^{4}	0.95^{6}
W-40	0.72^{1}	0.82^{2}	0.88^{7}	0.90^{8}	0.91^{5}	0.92^{4}	0.93^{3}	0.93^{10}	0.94^{6}	0.94^{9}
W-44	0.66^{1}	0.77^{2}	0.84^{4}	0.89^{3}	0.91^{7}	0.93^{8}	0.93^{6}	0.93^{5}	0.93^{9}	0.93^{10}
W-48	0.59^{1}	0.74^{4}	0.85^{2}	0.94^{3}	0.95^{5}	0.95^{7}	0.95^{6}	0.96^{10}	0.96^{8}	0.96^{9}
W-52	0.58^{1}	0.72^{2}	0.81^{4}	0.86^{3}	0.90^{7}	0.92^{8}	0.93^{5}	0.93^{10}	0.94^{9}	0.94^{6}
W-70	0.59^{10}	0.76^{1}	0.86^{8}	0.88^{2}	0.91^{9}	0.92^{5}	0.94^{7}	0.94^{3}	0.94^{4}	0.94^{6}
W-88	0.49^{6}	0.64^{1}	0.71^{9}	0.76^{2}	0.78^{4}	0.80^{10}	0.81^{3}	0.82^{8}	0.83^{5}	0.83^{7}
W-104	0.72^{6}	0.81^{1}	0.85^{2}	0.87^{4}	0.89^{10}	0.90^{7}	0.90^{5}	0.90^{3}	0.90^{9}	0.90^{8}
Avg.	0.76	0.84	0.89	0.91	0.92	0.93	0.93	0.94	0.94	0.94

Table 7: Relative importance of factors across maturities for price differences. We have first conducted a principal component analysis using 10 factors. Then the importance of each factor is sorted for each maturity. The table reports the cumulative variance explained when adding one additional factor. The factor number is in superscript. The bottom row reports the the average cumulative variance explained.

Relative importance of factors across maturities for price returns (%)										
Maturity	Cumulative variance explained									
	1^{th}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
W-01	0.86^{1}	0.91^{2}	0.95^{3}	0.96^{6}	0.96^{7}	0.97^{5}	0.97^{8}	0.97^{9}	0.97^{4}	0.97^{10}
W-02	0.90^{1}	0.95^{2}	0.96^{3}	0.96^{6}	0.96^{7}	0.96^{5}	0.96^{8}	0.96^{9}	0.96^{4}	0.96^{10}
W-03	0.91^{1}	0.93^{2}	0.95^{6}	0.96^{7}	0.96^{5}	0.96^{3}	0.96^{8}	0.96^{9}	0.96^{10}	0.96^{4}
W-04	0.91^{1}	0.93^{3}	0.94^{2}	0.95^{6}	0.95^{7}	0.96^{5}	0.96^{8}	0.96^{9}	0.96^{4}	0.96^{10}
W-05	0.89^{1}	0.95^{3}	0.96^{2}	0.96^{6}	0.96^{5}	0.96^{9}	0.96^{7}	0.96^{8}	0.96^{10}	0.96^{4}
W-06	0.88^{1}	0.94^{3}	0.96^{6}	0.97^{7}	0.97^{2}	0.97^{8}	0.97^{5}	0.97^{10}	0.97^{9}	0.97^{4}
W-07	0.85^{1}	0.90^{3}	0.93^{6}	0.94^{7}	0.95^{2}	0.95^{5}	0.95^{8}	0.95^{9}	0.95^{10}	0.95^{4}
W-12	0.76^{1}	0.81^{2}	0.86^{5}	0.88^{10}	0.89^{7}	0.91^{9}	0.92^{8}	0.92^{6}	0.92^{4}	0.92^{3}
W-16	0.75^{1}	0.84^{2}	0.89^{5}	0.91^{7}	0.92^{8}	0.92^{4}	0.92^{9}	0.93^{6}	0.93^{10}	0.93^{3}
W-20	0.72^{1}	0.83^{2}	0.87^{5}	0.88^{4}	0.89^{9}	0.90^{10}	0.90^{7}	0.91^{3}	0.91^{6}	0.91^{8}
W-24	0.70^{1}	0.82^{2}	0.86^{9}	0.89^{8}	0.90^{10}	0.91^{5}	0.92^{4}	0.93^{3}	0.93^{6}	0.93^{7}
W-28	0.67^{1}	0.80^{2}	0.85^{8}	0.87^{4}	0.88^{7}	0.89^{3}	0.89^{9}	0.89^{10}	0.89^{6}	0.89^{5}
W-32	0.61^{1}	0.77^{2}	0.85^{4}	0.88^{10}	0.90^{5}	0.92^{3}	0.93^{9}	0.94^{7}	0.94^{8}	0.94^{6}
W-36	0.63^{1}	0.78^{2}	0.85^{5}	0.89^{4}	0.92^{8}	0.93^{3}	0.94^{7}	0.94^{9}	0.94^{6}	0.94^{10}
W-40	0.63^{1}	0.77^{2}	0.85^{8}	0.88^{5}	0.90^{9}	0.91^{4}	0.92^{10}	0.93^{3}	0.93^{6}	0.94^{7}
W-44	0.59^{1}	0.77^{4}	0.88^{2}	0.90^{8}	0.91^{9}	0.92^{3}	0.92^{5}	0.92^{6}	0.92^{7}	0.93^{10}
W-48	0.61^{4}	0.83^{1}	0.93^{2}	0.94^{8}	0.94^{3}	0.95^{7}	0.95^{10}	0.96^{6}	0.96^{9}	0.96^{5}
W-52	0.57^{1}	0.75^{4}	0.86^{2}	0.89^{8}	0.91^{9}	0.92^{10}	0.92^{7}	0.93^{3}	0.93^{6}	0.93^{5}
W-70	0.55^{9}	0.76^{10}	0.89^{1}	0.93^{2}	0.94^{8}	0.95^{6}	0.95^{7}	0.95^{4}	0.95^{5}	0.95^{3}
W-88	0.38^{6}	0.53^{1}	0.64^{7}	0.72^{2}	0.76^{5}	0.79^{3}	0.80^{8}	0.81^{10}	0.81^{9}	0.81^{4}
W-104	0.53^{6}	0.73^{7}	0.79^{1}	0.83^{2}	0.85^{3}	0.86^{5}	0.87^{10}	0.89^{4}	0.89^{9}	0.89^{8}
Avg.	0.71	0.83	0.88	0.90	0.92	0.92	0.93	0.93	0.93	0.93

Table 8: Relative importance of factors across maturities for price returns. We have first conducted a principal component analysis using 10 factors. Then the importance of each factor is sorted for each maturity. The table reports the cumulative variance explained when adding one additional factor. The factor number is in superscript. The bottom row reports the the average cumulative variance explained.

Panel A: Analysis of forward price differences

Sample	V2		V1		S0		1995-1996		1997-1998		1999-2001	
Factor	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.
Fn1	0.60	0.60	0.72	0.72	0.76	0.76	0.73	0.73	0.53	0.53	0.73	0.73
Fn2	0.08	0.67	0.07	0.79	0.07	0.84	0.08	0.80	0.11	0.63	0.07	0.81
Fn3	0.05	0.73	0.06	0.85	0.04	0.88	0.05	0.85	0.07	0.70	0.05	0.85
Fn4	0.04	0.77	0.04	0.89	0.03	0.91	0.03	0.89	0.06	0.76	0.02	0.88
Fn5	0.03	0.80	0.03	0.92	0.01	0.93	0.03	0.91	0.05	0.81	0.02	0.90
Fn6	0.03	0.83	0.02	0.94	0.01	0.94	0.01	0.93	0.04	0.85	0.02	0.91
Fn7	0.02	0.86	0.02	0.95	0.01	0.95	0.01	0.94	0.03	0.87	0.01	0.93
Fn8	0.02	0.88	0.01	0.96	0.01	0.96	0.01	0.95	0.02	0.90	0.01	0.94
Fn9	0.02	0.90	0.01	0.97	0.01	0.97	0.01	0.96	0.02	0.92	0.01	0.95
Fn10	0.02	0.92	0.01	0.97	0.01	0.97	0.01	0.97	0.02	0.93	0.01	0.96

Panel B: Analysis of forward price returns

Sample	V2		V1		S0		1995-1996		1997-1998		1999-2001	
Factor	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.	Ind.	Cum.
Fn1	0.59	0.59	0.70	0.70	0.80	0.80	0.70	0.70	0.58	0.58	0.73	0.73
Fn2	0.09	0.68	0.08	0.78	0.06	0.87	0.08	0.78	0.09	0.67	0.08	0.81
Fn3	0.05	0.73	0.05	0.83	0.04	0.91	0.06	0.83	0.07	0.74	0.05	0.86
Fn4	0.04	0.77	0.03	0.86	0.02	0.93	0.03	0.86	0.06	0.80	0.02	0.88
Fn5	0.04	0.81	0.03	0.89	0.01	0.95	0.03	0.89	0.04	0.83	0.02	0.90
Fn6	0.03	0.84	0.02	0.91	0.01	0.95	0.02	0.91	0.04	0.87	0.02	0.92
Fn7	0.02	0.86	0.02	0.93	0.01	0.96	0.02	0.92	0.02	0.89	0.01	0.93
Fn8	0.02	0.89	0.01	0.94	0.01	0.97	0.01	0.94	0.02	0.91	0.01	0.94
Fn9	0.02	0.90	0.01	0.95	0.01	0.98	0.01	0.95	0.02	0.93	0.01	0.95
Fn10	0.02	0.92	0.01	0.96	0.00	0.98	0.01	0.96	0.01	0.94	0.01	0.96

Table 9: Principal component analysis of forward price differences and price returns. In panel A the analysis is performed on each two year sub-interval of the total sample. In panel B the data set is re-shuffled, and the analysis is performed on 3 seasonal subintervals, V2 (early winter), V1 (late winter) and S0 (summer) (see the text for exact period specifications). The table reports the individual contribution (Ind.) of each factor (Fn.) of the total variance, and the cumulative effect (Cum.) of adding an additional factor.

Sample period 1995 - 2001						
Data	Price differences			Price returns		
Maturity	W-01	W-52	W-104	W-01	W-52	W-104
Skewness	0.15	0.25	-1.06	0.08	0.33	-1.01
Sign.	(0.04)	(0.00)	(0.00)	(0.26)	(0.00)	(0.00)
Std. Kurt.	6.53	9.49	42.07	2.65	5.07	113.14
Sign.	(0.00)	(0.00)	(0.00)	(0.20)	(0.00)	(0.00)
Jarque-Bera	2277.17	4809.96	94483.59	374.69	1394.68	681867
Sign.	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Nobs	1339	1339	1278	1339	1339	1278

Table 10: Results from normality tests. $Std.skewn. = \frac{(skewness)}{(std.deviation)^3}$, $Std.kurt. = \frac{(kurtosis)}{(std.deviation)^4} - 3$, and $JarqueBera(JB) = \frac{T}{6}(std.skewn)^2 + \frac{T}{24}(std.kurt)^2$. Sign. calculates significance level at which the null hypothesis of normality can be rejected using a 2-sided test.

Volatility functions: Model A

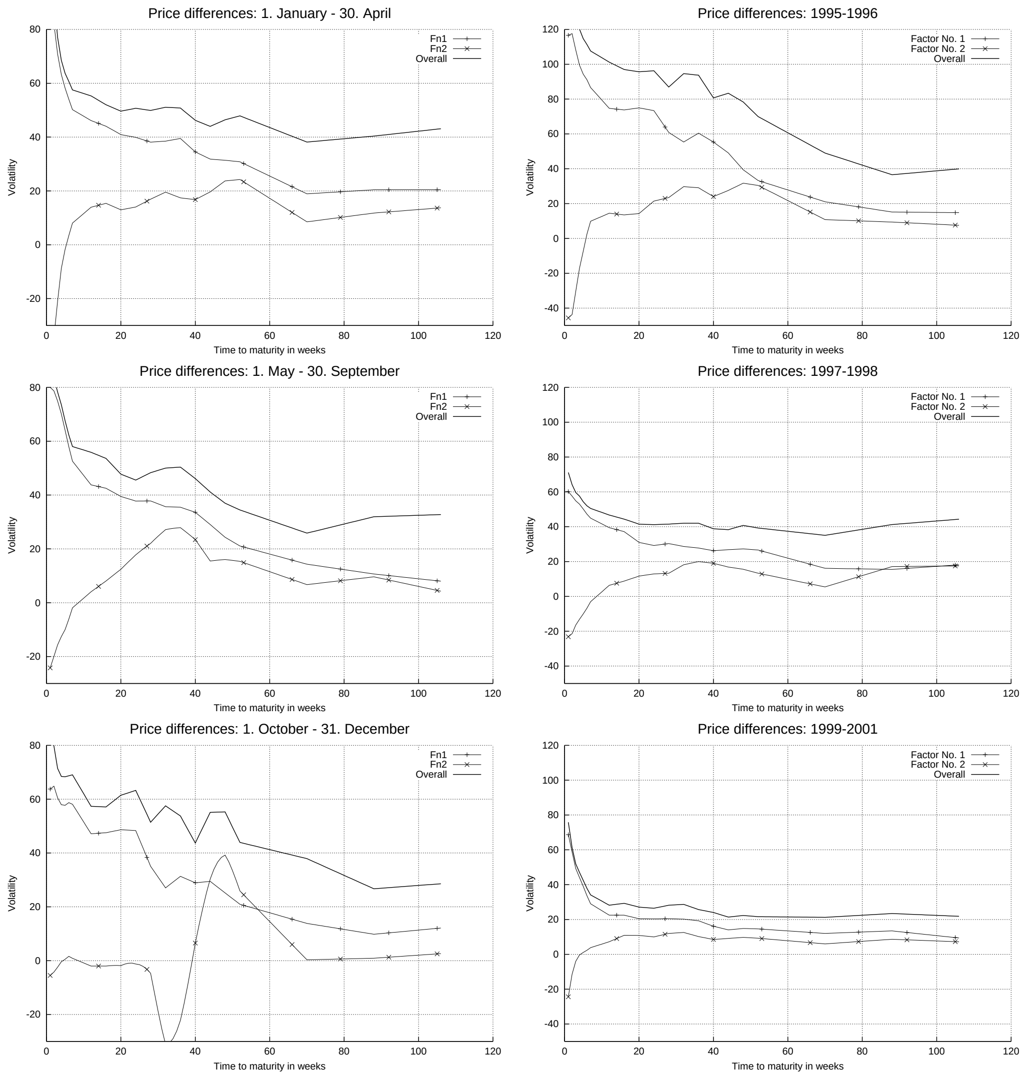

Figure 6: The two first volatility functions and overall volatility. The volatility functions on the left hand side are computed from different seasons corresponding to seasonal contracts traded at Nord Pool and the functions on the right hand side are computed from the time periods 1995-1996, 1997-1998 and 1999-2001. The functions are annualized using a factor of square root of 250 (number of trading days).

Volatility functions: Model B

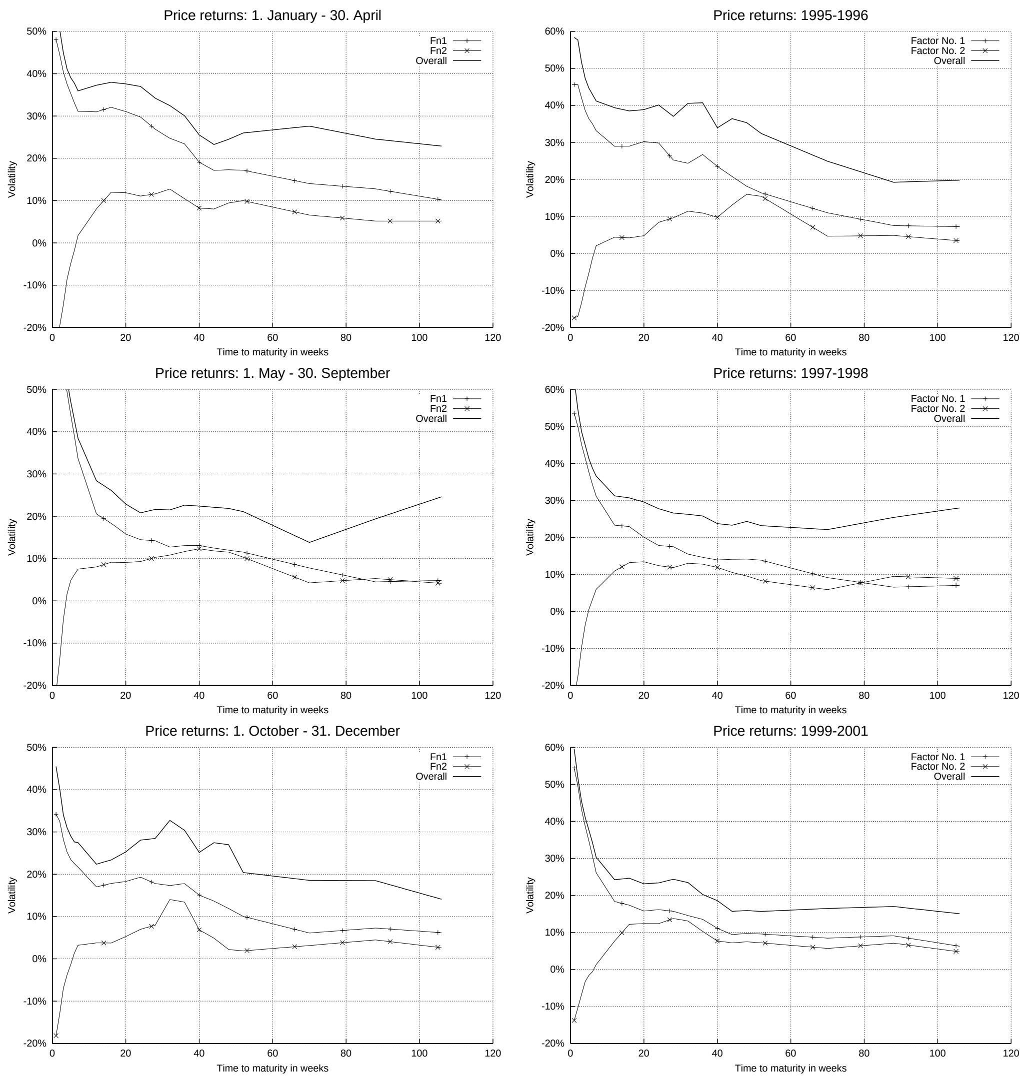

Figure 7: The two first volatility functions and overall volatility. The volatility functions on the left hand side are computed from different seasons corresponding to seasonal contracts traded at Nord Pool and the functions on the right hand side are computed from the time periods 1995-1996, 1997-1998 and 1999-2001. The functions are annualized using a factor of square root of 250 (number of trading days).