Pervasive Liquidity Risk

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Abstract

While there is no equilibrium framework for defining liquidity risk per se, several plausible arguments suggest that liquidity risk is pervasive and thus may be priced. For example, market frictions increase the cost of hedging strategies requiring frequent portfolio rebalancing. Also, liquidity risk is likely to play a role whenever the market declines and investors are prevented from hedging via short positions. Using monthly return data from 1963–2000, and a broad set of test assets, we examine six candidate factor representations of aggregate liquidity risk, and test whether any one of these are priced. The results are interesting. First, with the surprising exception of the recent measure proposed by Pastor and Stambaugh (2001), liquidity factor shocks induce co-movements in individual stocks' liquidity measure (commonality in liquidity). The commonality is similar to that found in the extant literature (Chordia, Roll, and Subrahmanyam (2000)), which so far has been restricted to a single year of data. Second, again with the exception of the Pastor-Stambaugh measure, the liquidity factors receive statistically significant betas when added to the Fama-French model. Third, maximum-likelihood estimates of the risk premium are significant for the measure based on bid-ask spreads, contemporaneous turnover, as well as the Pastor-Stambaugh measure, which exploits price reversals following volume shocks. Overall, the simple-to-compute, "low-minus-high" turnover factor first proposed by Eckbo and Norli (2000) appears to do as least as well as the other factor measures.

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1 Introduction

Liquid assets trade with small direct transaction costs such as commissions and bid-ask spreads, with a minimal time delay in execution, and with little or no price impact of the trade.[1] Consistent with the notion that market liquidity affects individual stock prices, there is evidence that publicly listed firms trade at a premium over private companies, and that individual bid-ask spreads affect expected stock returns.[2] Recently, Eckbo and Norli (2000) and Pastor and Stambaugh (2001) also show that over the past three decades, *aggregate* stock liquidity factors, measured as functions of trading volume, are significant determinants of monthly expected returns to large portfolios of NYSE/AMEX listed stocks. This evidence raises important questions concerning the existence and pricing of a pervasive liquidity risk factor. What is the best way to represent pervasive liquidity risk? If such a risk factor exist, what is the best estimate of its risk premium? In this paper, we examine these and related issues using monthly stock returns for the universe of NYSE/AMEX/Nasdaq-traded companies over the period 1963-2000,

In classical equilibrium asset pricing models, markets are assumed to be perfectly liquid.[3] Thus, liquidity risk plays no role. However, as explored by Lo and Wang (2000), with trading frictions, investors' hedging demands give rise to concerns about market liquidity.[4] This powerful intuition notwithstanding, the literature has yet to provide a general model that precisely defines a pervasive liquidity risk factor. Given the paucity of both theoretical and empirical research in this area, our approach is agnostic, and we present what amounts to an empirical 'horse race' among a priori interesting factor representations of liquidity risk.

Eckbo and Norli (2000) and Pastor and Stambaugh (2001) estimate liquidity betas for large stock portfolios using monthly returns from the past four decades.[5] Analogous to the construction

[1]Various aspects of firm-specific liquidity are addressed in the inventory control models (with risk-averse market makers) of, e.g., Garman (1976), Stoll (1978), Amihud and Mendelson (1980), and Grossman and Miller (1988), and in asymmetric information (strategic trading) settings of, e.g., Copeland and Galai (1983), Glosten and Milgrom (1985), Kyle (1985), Easley and O'Hara (1987), and Admati and Pfleiderer (1988).

[2]See, e.g., Wruck (1989) and Hertzel and Smith (1993) for comparisons of public and private firms, and Stoll and Whaley (1983), Amihud and Mendelson (1986), and Chen and Kan (1996) for the impact of bid-ask spreads.

[3]This is true for the single-periods model asset pricing model (CAPM) of Sharpe (1964)-Lintner (1965)-Mossin (1966), the arbitrage pricing theory (APT) of Ross (1976), and the intertemporal model (ICAPM) of Merton (1973). In the latter, investors continuously and costlessly rebalance their portfolios to maintain optimal hedges against unexpected changes in consumption and investment opportunities.

[4]Theoretical relationships between liquidity and asset prices are also discussed in, e.g., Amihud and Mendelson (1986), Constantinides (1986), Heaton and Lucas (1996), Vayanos (1998), Huang (2001), and Lo, Mamayski, and Wang (2001). Baker and Stein (2001) extend the analysis to a setting with irrational investors.

[5]Breen, Hodrick, and Korajczyk (2000) study a liquidity factor using short-horizon, high-frequency (intra-day)

of the book-to-market (B/M) factor of Fama and French (1993), the liquidity factor in Eckbo and Norli (2000) sorts the universe of NYSE/AMEX stocks first on equity-size and then on turnover (percentage of shares traded). The factor is a portfolio long in the lowest-turnover stocks and short in the highest-turnover stocks. The average return to this "low-minus-high" liquidity factor is positive and significant over the 1973-2000 period. Of the twenty-five size- and B/M-sorted portfolios of Fama and French (1993), a majority receives significant liquidity betas in a model that also contains the three Fama-French and the Carhart (1997) momentum factors.[6]

Following Campbell, Grossman, and Wang (1993), who find that returns accompanied by high volume tend to be reversed more strongly, Pastor and Stambaugh (2001) postulate that lower liquidity corresponds to stronger volume-related return reversals. Using NYSE/AMEX stocks from the 1962-1999 period, they construct a liquidity factor by first estimating, using daily returns within a month, the sensitivity of individual stock returns to lagged, signed dollar trading volume. The liquidity factor is the monthly innovations in the cross-sectional average of the individual sensitivity estimates.[7] Each stock is then assigned a "predicted liquidity beta". The predicted liquidity beta consists of the historical time-series estimate of the stock's slope coefficient against the liquidity factor when this factor is added to the Fama-French model, as well as other firm-specific characteristics. Sorting stocks into decile portfolios based on the predicted liquidity beta, they document a large return spread between the two portfolios with lowest and highest liquidity betas.

The empirical analysis in Jones (2001) and Avramov, Chao, and Chordia (2002) also suggests that accounting for aggregate liquidity risk improves asset pricing. Jones (2001) provides information on quoted spreads and turnover for the 30 Dow Jones Industrial Average stocks for the period 1898-1998. He concludes that spreads and turnover predict monthly stock returns up to one year ahead, suggesting that liquidity is an important determinant of conditional expected returns. Avramov, Chao, and Chordia (2002) show that including either the Eckbo and Norli (2000) measure or the Pastor and Stambaugh (2001) measure of liquidity takes the market portfolio closer to

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data.

[6] In that paper, we also show that the liquidity factor contributes to *reduce* the expected return to IPO stocks relative to matched, non-IPO stocks. This is consistent with the paper's finding that IPO stocks are in fact significantly more liquid than the stocks of the matched firms.

[7] As shown below, they scale this factor to eliminate serial correlation and the upward trend in dollar volume over the period.

multifactor efficiency in the sense of Merton (1973) and Fama (1996). In a scenario where short positions are not allowed, they find that including a liquidity proxy in an intertemporal asset pricing model significantly improves the multifactor efficiency of the market portfolio. They explain this result by noting that liquidity risk is typically a major concern whenever the market declines and investors are prevented from hedging via short positions.

In light of the extant evidence, what is the best way to represent pervasive liquidity risk? The factor representations of Eckbo and Norli (2000) and Pastor and Stambaugh (2001) have very different economic interpretations and statistical properties, and neither measure use bid-ask spreads and market depth—key liquidity characteristics in the market microstructure literature.[8] In this paper, we add a third candidate liquidity factor based on bid-ask spreads, with monthly observations going back to 1963. Moreover, we investigate to what extent this and earlier factor representations exhibit what Chordia, Roll, and Subrahmanyam (2000) term "commonality in liquidity". They use intra-day data to examine whether shocks to aggregate liquidity help explain the cross-sectional variation in individual firms' liquidity changes.[9] In our view, a finding of significant commonality strengthens a specific factor's interpretation as truly representing pervasive liquidity risk.

Furthermore, we examine risk premium estimates associated with each of the proposed liquidity factors. Since the Eckbo and Norli (2000) factor is a traded portfolio, a consistent estimate of the associated risk premium is given by the average realized return on their "low-minus-high" turnover portfolio. This average is 1.7% (annualized) over the entire 1973-2000 sample period. We update this estimate by extending the sample period back to 1963, and by performing a new, joint estimation of the liquidity betas and the risk premium. Pastor and Stambaugh (2001) achieve a greater spread than Eckbo and Norli (2000) between portfolios of low- and high-liquidity stocks. However, their liquidity factor price estimate is less than 1% (annualized). As they recognize, since their factor is not a traded assets, this estimate is subject to a scaling problem and therefore difficult to interpret. To remedy this problem, we re-examine a liquidity measure similar to that of Pastor-Stambaugh using a factor mimicking portfolio.

[8] See. e.g., the survey by O'Hara (1995).

[9] Accounting for asset specific determinants of liquidity (volatility, price, and volume), Chordia, Roll, and Subrahmanyam (2000) find that market wide liquidity, measured by averaging individual spreads and depths, is a significant determinant of the liquidity of individual assets. Huberman and Halka (2001) studies the innovations in aggregate measures of liquidity similar to the aggregate measures used by Chordia, Roll, and Subrahmanyam (2000), and find positive correlations between innovations from independent samples. Using principal component analysis, Hasbrouck and Seppi (2001) also provide evidence consistent with commonality in liquidity.

One of the more intriguing findings of Pastor and Stambaugh (2001) is that the liquidity factor contribution to expected return (liquidity beta times the risk premium) is as much as 7.5% per year for stocks with high sensitivities to liquidity risk. However, this factor contribution estimate is driven primarily by their lowest-liquidity decile portfolio. This portfolio, which coincides with the very smallest stocks, also happens to be the only decile portfolio with a significant predicted liquidity beta, given that the prediction is based on historical liquidity betas alone. This suggests that the 7.5% spread is in fact driven by firm-specific characteristics other than liquidity betas per se (such as size). In this paper, we perform new beta-pricing tests on each of the candidate liquidity risk factors using the twenty-five Fama and French (1993) portfolios as test assets, and we rank the factors on their respective contribution to expected portfolio return.

The rest of the paper is organized as follows. Section 2 describes six alternative definitions of firm-specific liquidity when using monthly data, and how these are aggregated into six candidate risk factors. Section 3 tests for commonality suing each of the six factors, while Section 4presents the results of our asset pricing tests. Section 6 concludes the paper.

2 Factor representations of liquidity risk

The construction of a liquidity risk factor starts with a definition of firm-specific liquidity risk. The market microstructure literature suggests several firm-specific measures. Below, we focus on six such measures, first defined at the firm-specific level and then aggregated to represent pervasive liquidity risk. Each measure require monthly stock returns, and our sample period starts in 1963 and ends in December 2000.

2.1 Bid-ask spreads

Market-wide forces may influence inventory risk and the degree of asymmetric information between the market maker and investors. A liquidity shock can increase the risk of holding inventory, or the likelihood that a market maker faces a trader with superior information. The result is a wider bid-ask spread, and a reduction in the quoted depth. If the liquidity shock is market wide, bid-ask spreads and quoted depths for different assets should move together. Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001) all provide evidence showing that spread and depth exhibit such commonality. This renders spread and depth as reasonable components in the construction of a liquidity factor.

While it is common to study bid-ask spreads and quoted depth using intraday data within a single year, our goal is to create liquidity factors that span long time periods. We know of no publicly available source for quoted depth over long periods. However, bid-ask spreads are available for an interesting subset of stocks for the whole sample period. CRSP records daily bid and ask prices for those stocks that are not traded on a particular day. Thus, in any given month there exists a cross-section of stocks that has not been traded for one or more days during the month. To avoid problems with stale prices, we impose a minimum trading activity of 10 days a month, and that the stock price exceeds \$1.[10] In sum, the spreads that we use are for non-trading days of stocks that trade relatively frequently each month.[11]

We compute a monthly firm-specific bid-ask spread by averaging daily bid-ask spreads for the given month. As it turns out, these monthly, firm-specific averages typically contain spreads from one to three days. Since there is ample evidence that trading activity in a stock is related to liquidity, this sample contains stocks with below average liquidity. However, if liquidity is a priced factor, it is reasonable to expect liquidity shocks to *a fortiori* affect the bid-ask spreads of the stocks on these illiquid days.

Panel A of Table 1 reports descriptive statistics for proportional quoted spread, measured as

$$\text{pqspr} \equiv 100(P_A - P_B)(.5PA + .5P_B), \tag{1}$$

where P_A is the ask price and P_B is the bid price. The average proportional quoted spread is 4.5, while the median is 3.1. The high average quoted spread relative to the median is caused by extreme spreads. The largest proportional quoted spread is 171.4. Notice that both the average and median spreads in this sample are large relative to spreads reported by others. Amihud and Mendelson (1986) find an average proportional quoted spread for the period 1961–1980 of 1.4, while Chordia, Roll, and Subrahmanyam (2000), report that the average dollar spread in 1992 is \$0.32, representing a proportional spread of 1.6. Thus, as expected, our selection procedure captures

[10] We also require the stock price to be below \$1,000. Note that Nasdaq listed stocks are not included.

[11] We are currently in the process of collecting information on bid-ask spreads for the entire CRSP universe back to 1985.

relatively illiquid stocks.

While the bid-ask spread is widely used as a measure of liquidity, it has certain shortcomings. As pointed out by Hasbrouck (1991a), a tick size of 1/8 (which was the rule for most of our sample period) limits the number of values the spread can take. Price discreetness tend to obscure the effect of liquidity shocks in the cross section of firms. Perhaps more importantly, the bid and ask quotes are only good for limited quantities. Brennan and Subrahmanyam (1996) argue that bid-ask spread is a noisy measure of liquidity because large trades tend to occur outside the spread while small trades tend to occur inside. We therefore turn to an alternative measure of liquidity based on the price impact of a trade.

2.2 Stock turnover

In an intertemporal setting with zero transaction costs, investors continuously rebalance their hedge portfolios in response to changes in the consumption-investment opportunities. Transaction costs slow trade and thus reduce liquidity. Following this intuition, firm-specific trading volume may serve as a proxy for liquidity. Empirical evidence (see for example Stoll, 1978; Brennan and Subrahmanyam, 1995) show that trading volume is indeed an important determinant of other liquidity measures. At the firm level, we follow Eckbo and Norli (2000) and Lo and Wang (2000) and use the sum of daily share turnover (measured as number of shares traded divided by number of shares outstanding) as our monthly turnover measure. The second column of Panel A of Table 1 reports descriptive statistics for the cross-section of monthly turnover. Over the sample period 1963–2000, a typical firm had an average monthly turnover of 5.78% with a median of 4.05%.

2.3 Price impact of trade

At the transaction level, the price impact of a trade is driven by inventory control effects and the degree of information asymmetry between trader and market maker. It follows that the larger the price impact for a given trades size, the less liquid is the stock. Motivated by the model of Campbell, Grossman, and Wang (1993), Pastor and Stambaugh (2001) (PS) construct a liquidity measure based on the idea that a given order flow should be followed by greater return reversal for stocks that are illiquid than for stocks that are liquid. To capture this idea, they measure liquidity as the ordinary least squares (OLS) estimate of $\mathrm{ps}_{\$i}$ in the following regression (henceforth referred to as PS-liquidity):

$$r_{it+1}^{\text{AR}} = \gamma_0 + \gamma_1 r_{it} + \text{ps}_{\$i}\left[\text{sign}(r_{it}^{\text{AR}}) \times \text{vol}_{it}\right] + \epsilon_{it}. \tag{2}$$

where r_{it+1}^{AR} is abnormal return for firm i on day $t+1$, r_{it} is return for firm i on day $t-1$, vol_{it} is volume measured in millions of dollars. The abnormal return is estimated as $r_{it} - r_{mt}$, where r_{mt} is the return on the CRSP value weighted portfolio of all NYSE, American exchange (AMEX), and Nasdaq firms. Equation (2) is estimated for all firms in all months using daily data. Thus, the maximum number of observations for any given regression is the number of trading dates in a month (between 20 and 22 days). Firm-months with less than 15 daily return observations are not used.

The specification in (2) is consistent with the view that inventory control effects on stock prices are inherently transient (Hasbrouck, 1991a,b). However, it is inconsistent with the view that asymmetric information is permanently impounded into the stock price. Barring any effects of public news, it is impossible to say which of these effects will dominate in daily data, so, the specification in (2) may or may not pick up a liquidity effect. For the purpose of this study, we leave this as an empirical question to be answered and will return to this in section 3.

Moreover, as recognized by Pastor and Stambaugh (2001), the specification in (2) is somewhat arbitrary. Instead of measuring the liquidity using order flow induced return reversal, one could presumably capture the same liquidity effect by looking at the effect of order flow on the concurrent return. Breen, Hodrick, and Korajczyk (2000) (BHK) relies on this idea to construct a measure of liquidity using high frequency data and a linear regression of percent change in price (return) on net turnover (defined as buyer initiated volume less seller initiated volume). The coefficient on net turnover is then taken as the liquidity measure. We follow a similar approach, measuring liquidity as the ordinary least squares (OLS) estimate of $\text{bhk}_{\$i}$ in the following regression (henceforth referred to as BHK-liquidity):

$$r_{it}^{\text{AR}} = \psi_0 + \psi_1 r_{it-1} + \text{bhk}_{\$i}\left[\text{sign}(r_{it}^{\text{AR}}) \times \text{vol}_{it}\right] + \epsilon_{it}. \tag{3}$$

This model is estimated using the same procedure and restrictions as when estimating equation 2.[12] In equation (3), $\text{bhk}_{\$i}$ is a liquidity parameter, reflecting the idea that daily abnormal returns

[12]We have also estimated equations 2 and 3 computing abnormal as $(r_{it} - \hat{\beta}_i r_{mt})$, where r_{mt} is the return on the

should be less sensitive to trading volume the more liquid the stock.

We also estimate models (2) and (3) using turnover, measured as daily volume divided by number of shares outstanding (scaled by 10,000), as an alternative order flow measure. The liquidity measures from these alternative models are referred to as $\text{ps}_{\text{to}i}$ and $\text{bhk}_{\text{to}i}$, respectively.

The last four columns of Panel A of Table 1 report descriptive statistics for the cross-section of liquidity estimates from model (2) and model (3) when order flow is measured using either dollar volume ($) or turn over (to). As pointed out by Pastor and Stambaugh (2001), the model of Campbell, Grossman, and Wang (1993) imply that liquidity induced return reversal should be increasing in trading volume. Thus, we should expect to see mainly negative estimates of $\text{ps}_{\$i}$ and $\text{ps}_{\text{to}i}$. Consistent with the findings of Pastor and Stambaugh (2001), we find that both the cross-sectional mean and median estimates are negative.

In Table 1, the columns with heading $\text{bhk}_{\$i}$ and $\text{bhk}_{\text{to}i}$ report descriptive statistics for the cross-section of estimates from model (3), when order flow is measured using dollar volume and turnover respectively. The estimates of both liquidity measures are typically positive: the median estimate of $\text{bhk}_{\$i}$ is 0.18 and the median estimate of $\text{bhk}_{\text{to}i}$ is 0.1. This is consistent with the notion that a large order move the stock price by more than a small order.[13]

Panel C of Table 1 reports average firm-specific correlations between liquidity measures. That is, correlations are computed using the monthly time-series of liquidity measures for each firm, these correlations are then averaged to get the reported numbers. As should be expected, the correlations between the two measures of PS-liquidity and the correlations between the two measures of BHK-liquidity is high—0.9 and 0.85 respectively. However, the correlations between PS-liquidity and BHK-liquidity is very low. This is consistent with findings in Pastor and Stambaugh (2001), who report that the characteristics of their liquidity measure is very sensitive to the specification of model (2). One possible reason for why we find low correlation between PS-liquidity and BHK-liquidity, is that such a correlation would require a strong serial correlation in returns at the individual firm

CRSP value weighted portfolio of all NYSE/AMEX/Nasdaq firms, and $\hat{\beta}_i$ is the beta of firm i estimated using a time-series of up to 252 trading days (minimum 120 days) ending the month prior to the estimation month. Our results are not affected in any material way using this approach.

[13]When the model in equation (3) is estimated using high frequency transaction data, the arrival of public information can be ignored since it will be a relatively uncommon event. With daily data, however, information events are common, and the effect of such events are reflected in r_{it}^{AR}. To gage the potential impact of this issue, we apply a filter to returns, removing all daily returns exceeding 10% and all daily returns below −10%, before estimating the BHK-liquidity. Our result turn out not to be sensitive to this issue.

level. Although returns exhibit a slight serial correlation, it is not nearly strong enough to make PS-liquidity and BHK-liquidity correlated.

2.4 Factor aggregation

In each month, we aggregate each firm-specific liquidity measure across firms, creating a monthly time series for the aggregate risk factor. As pointed out by Pastor and Stambaugh (2001), the estimate of $\mathrm{ps}_{\$i}$ from equation (2) will tend to decrease over time for all firms due to a general increase in turnover and dollar volume over the sample period. The same will, of course, apply to the coefficients $\mathrm{ps}_{\mathrm{to}i}$, $\mathrm{bhk}_{\$i}$, and $\mathrm{bhk}_{\mathrm{to}i}$. To get a more homogeneous time-series, Pastor and Stambaugh scale the aggregate liquidity measures by multiplying the average liquidity for month t by the factor (m_t/m_1), where m_t is the total dollar value of the firms in the sample at the end of month $t-1$ and m_1 is the dollar value in July 1962.

We follow the same approach for aggregate liquidity derived from the individual measures $\mathrm{ps}_{\$i}$ and $\mathrm{bhk}_{\$i}$. For aggregate liquidity derived from individual stock turnover, $\mathrm{ps}_{\mathrm{to}i}$and $\mathrm{bhk}_{\mathrm{to}i}$, we scale the time-series of average liquidity using (o_t/o_1), where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover. Aggregate monthly turnover is scaled by (o_1/o_t) while aggregate liquidity derived from proportional quoted spread is unscaled.

Aggregate liquidity for month t is computed using the cross-section of firm specific liquidity measures:

$$L_t = \frac{1}{N_{lt}} \sum_{i=1}^{N_{lt}} l_{it} S_t \tag{4}$$

where l_{it} is one of the six firm-specific liquidity measures discussed above for firm i in month t, N_{lt} is the number of firms with a non-missing observation on liquidity measure l in month t, and S_t is the scaling factor as described above.

For ease of exposition, we use lower case notation to refer to firm specific measures and upper case notation to refer to the corresponding aggregate measure. Thus, PQSPR refers to the aggregate version of pqspr. To avoid undue influence by extreme observation we exclude the two most extreme observations at both ends of the cross-section before we average to get the aggregate liquidity measure.[14]

Panel A of Table 2 provides descriptive statistics for the time-series of the six aggregate liquidity measures. The average quoted spread over the sample period is 35 cents, which on average represents a percent quoted spread of 4.47%. The average monthly turnover is 4.61%. That is, over the sample period 1963–2000 it took the average firm approximately 22 months to have all its common shares turned over once. Comparing averages with medians for all the liquidity measures in Panel A, it appears to be no dramatic skewness in the aggregate liquidity series.

Turning the attention to Figure 1, we see that there is a positive trend in proportional quoted spreads. The same applies to turnover, however since turnover is scaled by (o_t/o_1), it appears as if turnover has been stable throughout the sample period. The increase in turnover is as expected given the steady increase in market liquidity over the sample period. This general increase in market liquidity, as well as the evidence in Jones (2001) of a decrease in bid ask spreads for stocks in the Dow Jones Industrial Average over our sample period, lead us to expect a reduction in quoted spreads as well. In contrast, Figure 1 shows an increase in the quoted spread factor. One interpretation of this finding is that, as market liquidity increases (as measured by turnover), the probability of experiencing a non-trading day is reduced, making a non-trading event a more extreme outcome. Thus, the conditional value of the quoted spread on a non-trading day widens over the sample period.

The two bottom panels of Figures 1 pictures the scaled time-series of aggregate PS-liquidity and BHK-liquidity. As should be expected, the time-series of aggregate PS-liquidity and BHK-liquidity are near mirror images. Low liquidity measured using $\text{ps}_{\$i}$ implies a large negative coefficient while low liquidity measured using $\text{bhk}_{\$i}$ implies a large positive coefficient. It is a lot more surprising that proportional quoted spread and BHK-liquidity display very similar time-series patterns. Both measures identify four or five periods that stand out as having noticeable low liquidity. There seems to be adverse liquidity shocks during the 1973 oil crises and the following 1973–1975 recession, the dollar panics of 1978 and 1979,[15] the stock market crash of October 1987, the 1990 recession, and the Russian debt crisis of 1998.

[14]For the individual measures $\widehat{\psi}_{to}$, $\widehat{\psi}_{\$}$, $\widehat{\gamma}_{to}$, $\widehat{\gamma}_{\$}$, and turnover, the number of cross-sectional observations range between 1,500 and 2,300. Thus, removing 4 observations constitute between 0.17 and 0.27 percent of the cross-section. For pqspr, the number of firms in the cross-sections range from about 140 to 1,100.

[15]The Carter administration had to raise $30 billion of hard foreign currency in support of the sinking dollar.

Returning to Table 2, Panel B shows the correlation between the aggregate liquidity measures. To the extent that the liquidity proxies developed in this paper is related to the same underlying liquidity factor, we should expect the empirical proxies to be correlated. For turnover and PS-liquidity, lower values represent less liquidity while for quoted spreads and BHK-liquidity lower numbers represent higher liquidity. For example, a general decrease in quoted spreads or a decrease in the sensitivity of prices to order flow (BHK-liquidity) represent an increase in liquidity. Thus, we would expect quoted spreads and BHK-liquidity to be positively correlated, PS-liquidity and turnover to be positively correlated, and PS-liquidity and turnover should be negatively correlated with quoted spreads and BHK-liquidity. Panel B of Table 2 confirms all the expected correlations. The correlation coefficient of 0.579 between $BHK_{\$}$and PQSPR reported in Panel B, confirms the observation from Figure 1 that $BHK_{\$}$and PQSPR seems to exhibit a similar time-series pattern.

The autocorrelations in Panel C of Table 2 show that all the aggregate liquidity series are persistent.[16] For quoted spread and turnover, this is not very surprising. However, it is less obvious that PS-liquidity and BHK-liquidity should be persistent over time. In section 5.1 we deal with this persistency by creating time-series of innovations in these series.

We now turn to tests for commonality in each of our six candidate liquidity risk factors.

3 Commonality in liquidity

To the extent that there exists a common underlying liquidity factor affecting stock prices, an empirical proxy correlated with this factor should induce co-movements in liquidity. Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001) pursue this argument using data for one year. All three studies find that daily changes in firm specific measures of liquidity tend to move together. For example, using bid-ask spreads and quoted depth, Chordia, Roll, and Subrahmanyam (2000) find that market-wide liquidity is a significant determinant of the liquidity of individual assets, even after accounting for firm specific determinants of liquidity, such as volatility, price, and volume.

We extend the extant tests of commonality in liquidity to several factors and to monthly data over a much longer sample period than previously. Specifically, for each of the seven liquidity

[16]Based in the Ljung-Box Q statistic, not reported, the autocorrelation coefficients in Panel C of Table 2 are all statistically significant at the 1% level.

measures, we estimate the covariation between firm specific liquidity and market wide liquidity, using the following regression equation:

$$\frac{l_{it} - l_{it-1}}{l_{it-1}} = a + b_i \frac{S_t L_t - S_{t-1} L_{t-1}}{S_{t-1} L_{t-1}} + \epsilon_{it} \tag{5}$$

where l_{it} is a measure of individual stock liquidity for month t, L_t is the corresponding market wide liquidity measure, and S_t is a scalar constructed to remove time-trends in L_t.[17] In other words, the change in firm specific liquidity is regressed on a constant term and the change in the corresponding scaled aggregate liquidity measure.

Table 3 reports descriptive statistics for the cross-section of estimates of b_i. The first and second rows show that individual stock liquidity tend to move together, inducing a positive relationship between aggregate liquidity and firms specific liquidity. Except for PS-liquidity, the majority of slope coefficients, b_i, are positive—ranging from 79% positive slopes for concurrent price impact of order flow ($\text{bhk}_{\text{to}i}$) to 96% positive slope coefficients for turnover (to).

For quoted spreads, BHK-liquidity, and turnover the number of t-statistics that exceed the critical value for a one-tailed test ranges from 31.5% to 73.7%. Interestingly, these percentages are very similar to the ones reported by Chordia, Roll, and Subrahmanyam (2000) in daily data over a sample period of one year. For all liquidity measures except PS-liquidity, we reject the null hypothesis that the average slope coefficient is zero.

Panel B of Table 3 reports the results from a commonality regression where other firm specific variables, known to be related to liquidity, is used as control variables. We follow Chordia, Roll, and Subrahmanyam (2000) and use the standard deviation of the daily stock returns for the current month (STD), the average daily price for the current month (PRC), and the aggregate dollar volume for the current month (DVOL). All variables are measured in natural logs. The results are qualitatively the same as without the firm specific control variables. In sum, over the sample period 1963–2000, Table 3 indicates significant commonality in liquidity for all but the PS liquidity

[17] When the aggregate liquidity measure, L_t, is the Pastor-Stambaugh (PS) or the Breen-Hodrick-Korajczyk (BHK) measure and order flow is measured using dollar volume, the scalar $S_t = (m_t/m_1)$, where m_t is the total dollar value at the end of month $t-1$ of all firms included in the computation of the average PS liquidity measure and m_1 is the dollar value in July 1962. When L_t is the proportional quoted spread, $S_t = 1$. When L_t is the average monthly turnover, $S_t = (o_1/o_t)$, where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover, and o_1 is the monthly turnover in July 1962. When L_t is PS-liquidity or BHK-liquidity and order flow is measured using turnover, $S_t = (o_t/o_1)$.

factor.

4 Asset pricing test methodology

4.1 Generic factor model framework

Let r_{pt} denote the return on portfolio p in excess of the risk-free rate, and assume that expected excess returns are generated by a K-factor model,

$$E(r_{pt}) = \beta_p' \lambda, \tag{6}$$

where β_p is a K-vector of risk factor sensitivities (systematic risks) and λ is a K-vector of expected risk premiums. This model is consistent with the APT model of Ross (1976) and Chamberlain (1988) as well as with the intertemporal (multifactor) asset pricing model of Merton (1973).[18] The excess-return generating process can be written as

$$r_{pt} = E(r_{pt}) + \beta_p' F_t^{\epsilon} + e_{pt}, \tag{7}$$

where $F_t^{\epsilon} \equiv F_t - E(F_t)$ is a K-vector of risk factor shocks, F_t and $E(F_t)$ are K-vectors of factor realizations and expected values, and e_{pt} is the portfolio's idiosyncratic risk with expectation zero.

Combining equations (6) and (7) yields:

$$r_{pt} = \beta_p' \lambda + \beta_p' F_t^{\epsilon} + e_{pt}. \tag{8}$$

Consider now the excess return F_{kt}^M on a "factor-mimicking" portfolio that has unit factor sensitivity to the kth factor and zero sensitivity to the remaining $K-1$ factors. Since this portfolio must also satisfy equation (6), it follows that $E(F_{kt}^M) = \lambda_k$ for all the K elements in the vector F_t^M of factor mimicking portfolios. Substituting λ for $E(F_t)$ in (8) gives the following regression equation:

$$r_{pt} = \beta_p' F_t^M + e_{pt}. \tag{9}$$

[18] Connor and Korajczyk (1995) provide a review of APT models.

Equations (9) and (8) are nested in the following regression:

$$r_{pt} = \alpha_p + \beta_p' F_t + e_{pt}. \tag{10}$$

Thus, if F_t in (10) contains factor mimicking portfolios, the intercept α_p should be zero. If F_t contain innovations in the factors, the model framework implies that $\alpha_p = \beta_p' \lambda$.

It follows from the above that, when factors are represented by factor mimicking portfolios, the risk premium is simply the unconditional expected portfolio return. However, risk premiums estimated this way ignore the fact that empirical factors will be correlated. When factors are correlated, a positive risk premium on factor k can be driven by correlation between factor k and another priced factor. A partial solution to this problem is to follow the alternative route and test whether the factor sensitivity (beta) to factor k in equation (10) is significant when the beta is estimated in the presence of other factors.[19] The procedure that we use for this β-pricing approach is outlined below. An additional advantage of this approach is that the model takes the same form under the null hypothesis regardless of whether the factors are mimicking portfolios or innovations.

4.2 Test statistics for β-pricing

Suppose returns on N assets are generated by the model in (10). Stacking the N excess returns from period t into the vector r_t, the model in (10) can be written:

$$r_t = \alpha + \beta F_t + \epsilon_t, \tag{11}$$

where α is an N-vector of intercepts, β is an N by K matrix of factor sensitivities, and F_t is a K-vector of factors. With T observations on r_r and F_t, equation (11) can be written as:

$$R = XB + E \tag{12}$$

where R is T by N with typical row r_t', X is T by $(K+1)$ with typical row $[1 \;\; F_t']$, B is the $(K+1)$ by N matrix $[\alpha \;\; \beta]'$, and E is T by N with typical row ϵ_t'. If ϵ_t, conditional on X, is independent

[19] The reason why this is only a partial solution is that the estimate of beta on k will reflect a correlation between k and some unobserved factor that is not included in the regression.

and identically distributed (i.i.d) jointly normal with zero mean and variance-covariance matrix Σ, then the hypothesis that the factor sensitivities of the N assets with respect to factor k, β_k, is equal to zero can be tested using the exact test:[20]

$$F_1 = \left(\frac{T-K-N}{NT}\right)(F_k' M F_k)\hat{\beta}_k'\hat{\Sigma}^{-1}\hat{\beta}_k \sim F_{N,T-K-N} \tag{13}$$

where F_k is column k from X, M is an idempotent matrix constructed using the first k columns of X, and $\hat{\Sigma}$ and $\hat{\beta}_k$ are both ordinary least squares (OLS) estimates. When the error term ϵ_t is i.i.d. but not normal the test statistic F_1 still performs well in small samples. However, when the error term no longer is i.i.d. but conditionally heteroskedastic, F_1 show low power to reject a false null hypothesis in small samples. An asymptotically valid test that takes conditional heteroskedasticity into account is:

$$J_2 = \hat{\beta}_k'\hat{G}_a^{-1}\hat{\beta}_k \overset{A}{\sim} \chi^2_N, \tag{14}$$

where

$$\hat{G}_a^{-1} = \sum_{t=1}^{T} e'(X'X)^{-1}x_t x_t'(X'X)^{-1}e \ \otimes \hat{\epsilon}_t\hat{\epsilon}_t',$$

and e is a column vector where all elements are zero except for the last which is one. That is, $e'Ae$ picks out the lower right element of the square matrix A. To improve on the small sample properties of J_2, we also use:[21]

$$F_2 = \left(\frac{T-K-N}{NT}\right)\hat{\beta}_k'\hat{G}_a^{-1}\hat{\beta}_k \sim F_{N,T-K-N} \tag{15}$$

In sum, the three test statistics F_1, F_2, and J_2 are all designed to test the null hypothesis that assets has zero sensitivity to a given factor k. The test statistic F_1 gives an exact test of the null when the error terms from (11) are i.i.d. normal. The test is asymptotically valid when the error terms are i.i.d. but not normal. When the error terms exhibit conditional heteroskedasticity, F_1 is not asymptotically valid and does not perform well in finite samples. For this case, the tests statistics F_2 and J_2 are more appropriate.

[20] See Anderson (1984) and p. 410 in Seber (1984).

[21] The test statistic F_2 is only approximately true, but have been shown to have better small sample properties. We thank Raymond Kan for pointing out the tests statistics F_1, F_2, and J_2.

5 Asset pricing test results

5.1 Tests using factor innovations

5.1.1 Factor construction and properties

We compute factor innovations using a time series model for the expected liquidity factor value. Panel C of Table 2 shows that all the aggregate liquidity measures are very persistent—all with positive autocorrelation coefficients on lags one through five.[22] To remove this persistence in the liquidity time-series and hence get a time-series of innovations, we follow Pastor and Stambaugh (2001) and estimate the following model:

$$\Delta L_t = a + b\Delta L_{t-1} + cS_t L_{t-1} + \epsilon_t, \tag{16}$$

where L_t is an aggregate liquidity measure and $\Delta L_t = S_t(L_t - L_{t-1})$.

In equation (16), S_t is a scalar that depends on the specific form of the liquidity measure L_t. When L_t is either the Pastor-Stambaugh (PS) or the Breen-Hodrick-Korajczyk (BHK) measure of aggregate liquidity, and when order flow is measured using dollar volume, $S_t = (m_t/m_1)$. Here m_t is the total dollar value at the end of month $t-1$ of all firms included in the computation of the average PS liquidity measure, and m_1 is the dollar value in July 1962. When L_t is the proportional quoted spread, $S_t = 1$. Finally, when L_t is the average monthly turnover, $S_t = (o_1/o_t)$, where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover, and o_1 is the monthly turnover in July 1962.

Table 4 and Figure 2 show that five out of the six innovations time-series generated by the model in (16) do not exhibit drift or persistence. None of the first order autocorrelations are statistically significant. The quoted spread time-series shows statistically significant autocorrelation coefficients for lags 2 through 5, and BHK-liquidity computed using order flow measured as dollar volume show a statistically significant second order autocorrelation and a marginally significant third order autocorrelation.

Figure 2 shows that the innovations exhibit peeks or troughs in periods where there likely were shocks to liquidity. Innovations in percent quoted spread, greater contemporaneous response of

[22] Based on non-reported Ljung-Box test statistics, all lags are statistically significant at conventional levels.

return to order flow (BHK-liquidity), and greater return reversals (PS-liquidity) all show larger than normal shocks to liquidity during the oil-crisis of 1973 and the following 1973–1975 deep recession. Figure 2 also conveys what appear to be liquidity shocks related to the stock market crash of October 1987 and the debt crisis of 1998. These patterns found in the innovations to aggregate liquidity is consistent with similar patterns found by Jones (2001) and Pastor and Stambaugh (2001).

5.1.2 Test results using innovations

We use as test assets the 25 size and book-to-market ratio sorted portfolios as test assets.[23] These portfolios are reformed in June each year using NYSE, AMEX, and Nasdaq listed stocks and held for one year starting in July. Table 5 documents raw returns and CAPM betas on the test assets.

The null hypothesis is first tested in the framework of the CAPM. That is, we regress the 25 test assets on an intercept, the market factor, and one liquidity factor. This procedure is repeated for all liquidity factors. Table 6 reports factor sensitivities (betas), t-statistics associated with the beta estimates, and the test statistics F_1, J_2, and F_2 (see equations 13, 14, and 15). Panel A of the table shows that most betas for the two quotes spread and turnover factors are statistically significant. Viewed individually, the betas on the other factors are typically not statistically significant at conventional levels. Panel B in the table shows that we reject the null hypothesis that the betas are jointly zero for all factors when using the F_1 test. However, if we allow for conditional heteroskedasticity, we cannot reject the null of jointly zero liquidity betas for the PS-liquidity factors.

Second, we test the null hypothesis using the Fama and French (1993) three-factor model. This is an interesting exercise since we know that the Fama-French factors explain a large proportion of the variation in the returns on the test assets—which implies that the liquidity factors face a much "tougher job" in terms of providing additional explanatory power.

Panel B of Table 7 reports the betas on the liquidity factors when they are added to the Fama-French model. As expected, the number of betas that are statistically significant is much lower. In Panel B, the joint tests show mixed evidence. The exact test F_1 reject the null hypothesis for all

[23]The 25 size and book-to-market ratio sorted portfolios downloaded from Ken French's web-site at Dartmouth College. Details on how the portfolios are formed are available from the same web-site.

but the PS factor that measures order flow using turnover (PS_{to}). However, the tests that account for conditional heteroskedasticity in the model errors typically do not reject the null hypothesis.

Given the evidence of commonality in several of our liquidity factors, one interpretation of the results in Panel B is that the Fama-French factors themselves capture liquidity. That is, factors constructed using innovations in aggregate liquidity may be"forced out" by the Fama-French factors since because the latter are constructed using stock returns. This possibility leads us to use liquidity-sorted portfolios to represent the liquidity factors, as described next.

5.2 Tests using liquidity-sorted portfolios as mimicking factors

Fama and French (1993) construct their SMB and HML factors with the idea to "mimic the underlying risk factors in returns related to size and book-to-market equity." Their procedure accomplishes this goal by making sure that the average size for the firms in the three book-to-market portfolios is the same, while also maintaining the same average book-to-market ratio for the two size portfolios. We construct liquidity factors using a similar motivation and procedure.

Since liquidity by most measures will be increasing in firm size, we control for size when creating a factor mimicking portfolio for liquidity (with the exception of the proportional spread, see below). Specifically, for all liquidity measures except the proportional quoted spread, we construct a mimicking portfolio for aggregate liquidity measure L as follows: Starting in December 1962 we form two portfolios based on a ranking of the end-of-month market value of equity for all NYSE stocks, and three portfolios formed using NYSE/AMEX stocks ranked on firm specific liquidity measure l. Next, six portfolios are constructed from the intersection of the two market value and the three liquidity portfolios. Monthly equally-weighted returns on these six portfolios are calculated starting in January 1963. Portfolios are reformed in June held for one year starting in July. The factor mimicking liquidity portfolio is the difference between the equal-weighted average return on the two portfolios with low liquidity and the equal-weighted average return on the two portfolios with high liquidity.

The factor mimicking portfolio for proportional quoted spreads is constructed without sorting the cross-sections on size. The reason for this is that the firms for which we observe the quoted spread are all very small firms. Hence, there are almost no observations in the "Big" firm sort. The return on this mimicking portfolio is computed as the difference between the equally weighted return on a portfolio of the one-third least liquid firms in the cross-section less the equally weighted return on a portfolio of the one-third most liquid firms.

Table 8 reports descriptive statistics for the liquidity factor mimicking portfolios together with the factors from the Fama-French three-factor model.

Table 9 reports the results of regressing each of the 25 test assets on an intercept, the market factor, SMB, HML and one the liquidity factors. Panel A of the table shows that the majority of betas for all factors except the PS factors are statistically significant. This is dramatically different from the results reported in Panel A of Table 7, where very few of the individual betas were statistically different from zero. Moreover, the test statistics reported in Panel B of 9 strongly reject the null hypothesis that betas are zero for the factors constructed using quoted spreads, turnover, and concurrent price impact of order flow (BHK-liquidity).

Overall, the results for the PS measure of liquidity are largely consistent with the results of our commonality regressions in Table 3 above. That is, whether we use a commonality in liquidity approach, or a beta-pricing approach, the hypothesis that the PS measure pervasive liquidity risk is rejected. At the same time, based on the other factor representations, and in particular TO, there is significant evidence of the existence of a pervasive risk factors, both from the commonality regressions and the beta-pricing tests above.

5.3 Risk premiums and factor contributions to expected returns

In this section we report test for the pricing of a liquidity factor using estimate risk-premiums. Risk premiums can be estimated as the coefficients in a multivariate regression of returns on estimates of betas. This approach is commonly known as the Fama and MacBeth (1973) two-step procedure. However, rather than first estimating betas using time-series data and then estimating the risk-premiums, betas and risk premiums are estimated simultaneously using maximum likelihood. This approach has the advantage that, at least asymptotically, we avoid the errors-in-variables problem created by having to estimate betas in the first step to identify the risk-premiums in the second step.

Table 10 reports risk-premiums estimated using this approach. The risk premiums are estimated using factors that are innovations in aggregate liquidity and 25 size and book-to-market ratio sorted portfolios as test assets. Panel A reports the risk premium estimates when the liquidity factor is added to the CAPM, while Panel B reports the risk premium estimates when the liquidity factor is added to the Fama-French three factor model. For the maximum likelihood (ML) estimates, we find statistically significant risk-premiums for all but one factor ($BHK_{\$}$) when factors are added to the CAPM, and for all but two factors ($PS_{to}$and BHK_{to}) when factors are added to the Fama-French model. This is significant evidence in favor of the existence of a pervasive liquidity risk factor.

To gauge the economic significance of the risk-premiums, Panel C of Table 10 reports the product of beta and risk-premium (i.e., the factor's contribution to expected return) for five size portfolios and five book-to-market portfolios. The betas are computed as average betas. For example, for the portfolio of the smallest firms (row named Small in Table 10) the beta is computed as the average beta on the five small-firm portfolios (P11, P12, ..., P15) as reported in Table 6. Average betas are computed in a similar way for the other four size portfolios and the five book-to-market ratio portfolios.

For the size sorted portfolios, the liquidity betas are monotonically increasing or decreasing in size. Taking the sign of the estimated risk-premium into account, we see that the contribution to expected return is decreasing in size. Thus, it appears that investors require a higher return on small stocks because of their exposure to liquidity risk. As expected, given the small liquidity betas from Table 7, Panel D of Table 10 shows that the contribution to expected return is much smaller in the presence of the Fama-French factors.

6 Conclusion

While the literature contains substantial evidence that stock liquidity affect individual stock prices and expected returns, evidence on liquidity as a pervasive risk factor has only recently started to emerge (Eckbo and Norli (2000), Pastor and Stambaugh (2001),Jones (2001), Avramov, Chao, and Chordia (2002)). While this evidence suggests that liquidity is priced, important questions remain. Absent theoretical guidance, what is the "best" empirical factor representation for liquidity risk? Do shocks to the aggregate factor value on average affect induce co-movements in liquidity at the firm level (as in Chordia, Roll, and Subrahmanyam (2000))? Is there evidence of liquidity risk being priced when using a more diverse set of test assets than in previous studies?

We examine these issues using monthly stock returns for the universe of NYSE/AMEX/Nasdaq-traded companies over the period 1963-2000, Our analysis incorporate several a prior plausible factor representations of liquidity risk, including information on bid-ask spreads, stock turnover, and price reversals. A factor based on price reversals, which is used by Pastor and Stambaugh (2001) and Breen, Hodrick, and Korajczyk (2000), is in part motivated by the finding of Campbell, Grossman, and Wang (1993) that returns accompanied by high volume tend to be reversed more strongly. A factor based on (contemporaneous) turnover, as in Eckbo and Norli (2000), is consistent with dynamic hedging strategies in a multifactor world. For example, liquidity risk is likely to play a role whenever the market declines and investors are prevented from hedging via short positions.

For a given liquidity factor definition, we represent that factor two ways: either using the innovations from a time series model for the factor (e.g., as in Chen, Roll, and Ross (1986), Ferson and Harvey (1991)), or, alternatively, the returns to a factor mimicking portfolio. The factor mimicking portfolios are constructed as zero-investment portfolios that are long illiquid stocks and short liquid stocks (analogous to the procedure used by Fama and French (1993)). These portfolios have the advantage that the unconditional mean portfolio return represents a good estimate of the factor risk premium. For each liquidity factor, and using both the CAPM and the Fama and French (1993) three-factor model as points of reference, we perform maximum-likelihood tests of the joint hypothesis that the factor betas and the associated risk premiums are significantly different from zero.

Our results are interesting. First, with the surprising exception of the Pastor and Stambaugh (2001) liquidity factor, there is significant evidence that liquidity factors based on either bid-ask spreads or contemporaneous turnover exhibit commonality (i.e., induces co-movements in individual stocks). Second, again with the exception of the Pastor-Stambaugh measure, the liquidity factors receive statistically significant betas when added to the Fama-French model. Third, maximum-likelihood estimates of the risk premium are significant for the measure based on bid-ask spreads, contemporaneous turnover, as well as the Pastor-Stambaugh measure, which exploits price reversals following volume shocks. Overall, the simple-to-compute, "low-minus-high" turnover factor first proposed by Eckbo and Norli (2000) appears to at least as well as the other candidate factor measures.

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Figure 1
Aggregate liquidity measures computed using NYSE/AMEX stocks 1963–2000

The sample used to compute firm specific liquidity measures is NYSE/AMEX stocks 1963–2000. Time-series with a monthly frequency are constructed using monthly averages of firm specific liquidity measures. That is,

$$L_t = \frac{1}{N_{lt}} \sum_{i=1}^{N_{lt}} l_{it}$$

where l_{it} is the Pastor-Stambaugh (PS) liquidity measure or the Breen-Hodrick-Korajczyk liquidity measure (BHK) and N_{lt} is the number of firms with a non-missing observation on liquidity measure l. Before computing average liquidity for a given month, the cross-sections are trimmed by removing the two most extreme observations at both ends. The plotted time-series are scaled with S_t.When the aggregate liquidity measure, L_t, is the Pastor-Stambaugh (PS) or the Breen-Hodrick-Korajczyk (BHK) measure and order flow is measured using dollar volume, the scalar $S_t = (m_t/m_1)$, where m_t is the total dollar value at the end of month $t-1$ of all firms included in the computation of the average PS liquidity measure and m_1 is the dollar value in July 1962. When L_t is the proportional quoted spread, $S_t = 1$. When L_t is the average monthly turnover, $S_t = (o_1/o_t)$, where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover, and o_1 is the monthly turnover in July 1962.

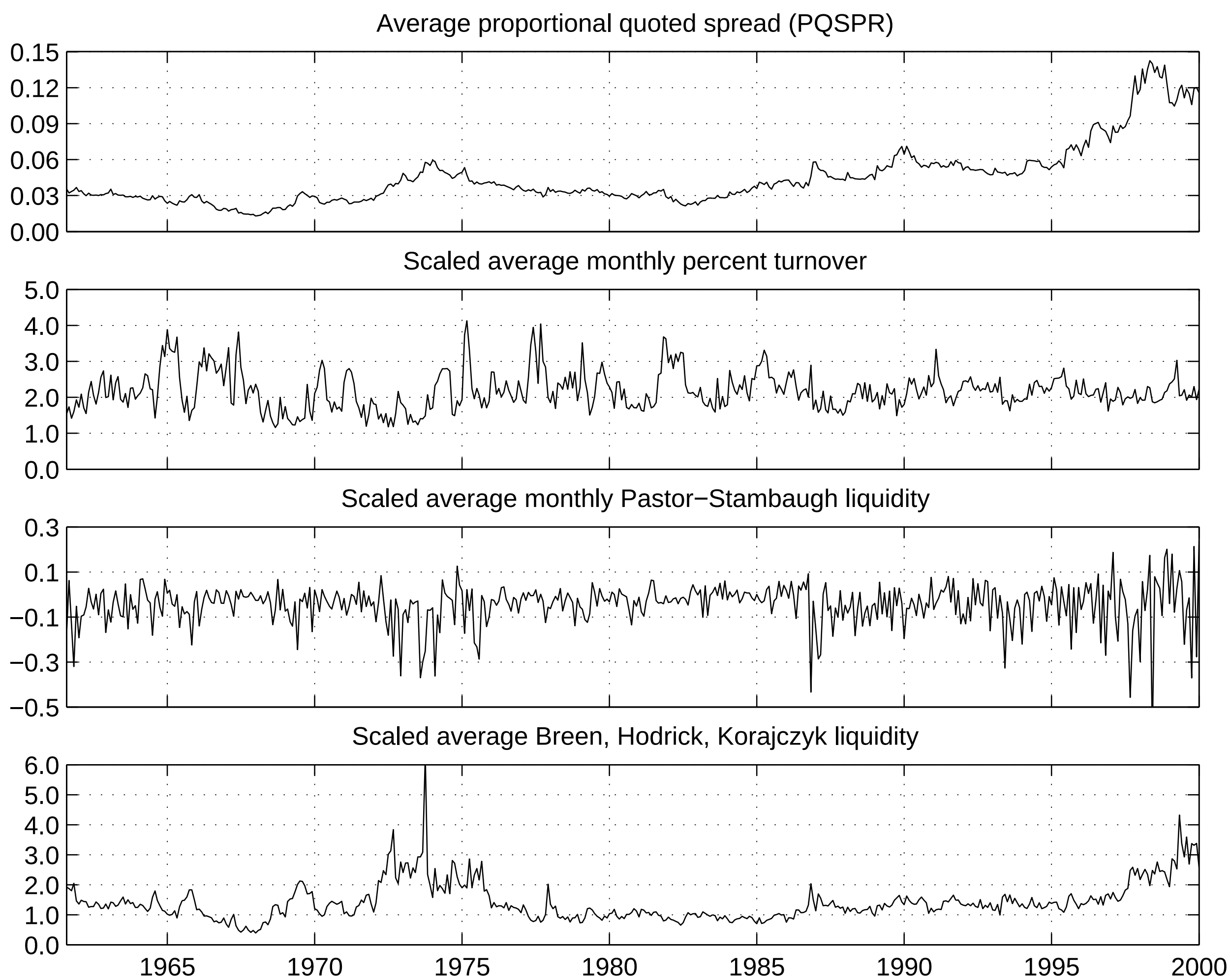

Figure 2
Innovations in measures of aggregate monthly liquidity, 1963–2000

Aggregate liquidity measures are computed using NYSE/AMEX stocks. Innovations in liquidity measure L_t is the residual from the estimation of the model:

$$\Delta L_t = a + b\Delta L_{t-1} + cS_t L_{t-1} + \epsilon_t,$$

where L_t is an aggregate liquidity measure, $\Delta L_t = S_t(L_t - L_{t-1})$. When the aggregate liquidity measure, L_t, is the Pastor-Stambaugh (PS) or the Breen-Hodrick-Korajczyk (BHK) measure and order flow is measured using dollar volume, the scalar $S_t = (m_t/m_1)$, where m_t is the total dollar value at the end of month $t-1$ of all firms included in the computation of the average PS liquidity measure and m_1 is the dollar value in July 1962. When L_t is the proportional quoted spread, $S_t = 1$. When L_t is the average monthly turnover, $S_t = (o_1/o_t)$, where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover, and o_1 is the monthly turnover in July 1962.

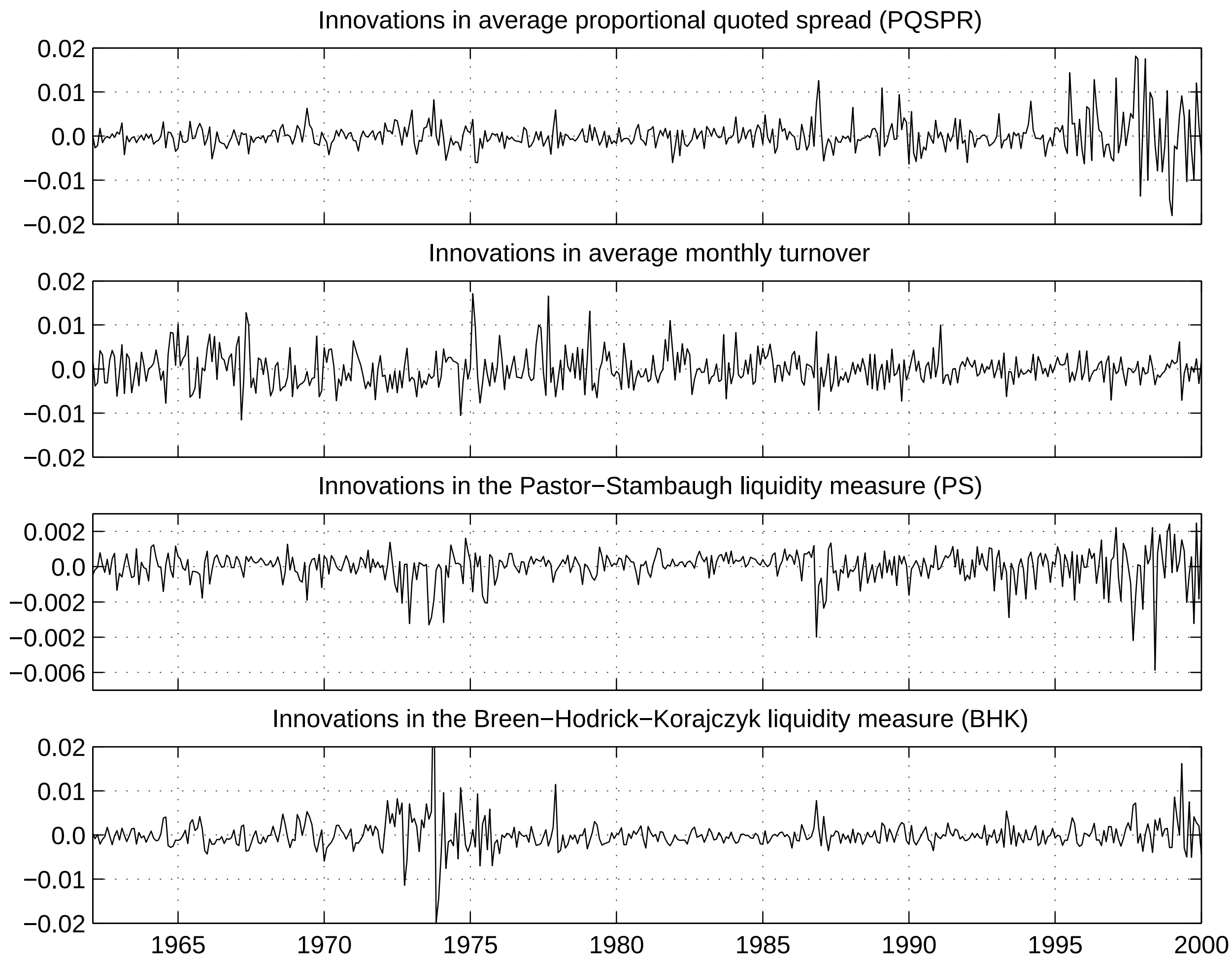

Table 1
The cross-section of firm specific liquidity measures 1963–2000

This table reports descriptive statistics for the cross-section of liquidity measures for NYSE/AMEX firms. The liquidity measures are proportional quoted spreads (pqspr) computed as $(P_A - P_B)/(0.5P_A + 0.5P_B)$ where P_A is the ask price and P_B is the bid price, monthly turnover (to) measured as the sum of the daily percent turnovers during the month, Pastor-Stambaugh liquidity ($\text{ps}_{\$i}$) which measures the delayed price impact of order flow, where order flow is measured using daily dollar volume, Pastor-Stambaugh liquidity ($\text{ps}_{\text{to}i}$) when order flow is measures as daily volume, and Breen-Hodrick-Korajczyk which measures the contemporaneous price impact of order flow ($\text{bhk}_{\$i}$and $\text{bhk}_{\text{to}i}$). Panel A shows descriptive statistics for the cross-section of firm specific time-series averages. In panel B, correlations are computed between pairs of firm level liquidity time-series. The reported correlations are average correlations for all firms in the sample. We require that there are at least 60 non-missing observations in the time-series before the correlation is included in the average. The numbers in parentheses are numbers of observations used to compute the average.

	pqspr	to	$\text{ps}_{\$i}$	$\text{bhk}_{\$i}$	$\text{ps}_{\text{to}i}$	$\text{bhk}_{\text{to}i}$
A. Descriptive statistics						
Mean	4.50	5.78	0.001	0.659	−0.008	0.158
Median	3.10	4.05	−0.001	0.180	−0.002	0.100
Standard dev.	5.20	22.56	2.018	2.325	0.271	0.805
Minimum	0.00	0.00	−51.237	−0.538	−19.278	0.000
Maximum	171.40	1843.63	135.47	133.75	4.988	61.19
Mean num. obs.	43	134	137	137	137	137
B. Correlations						
to	−0.129 (1042)					
$\text{ps}_{\$i}$	−0.023 (977)	0.019 (4326)				
$\text{bhk}_{\$i}$	0.353 (975)	−0.388 (4323)	−0.051 (4454)			
$\text{ps}_{\text{to}i}$	−0.016 (977)	0.022 (4326)	0.900 (4459)	−0.040 (4454)		
$\text{bhk}_{\text{to}i}$	0.263 (975)	−0.475 (4323)	−0.047 (4454)	0.845 (4454)	−0.048 (4454)	

Table 2
Descriptive statistics for aggregate liquidity measures, 1963–2000

The sample used to compute firm specific liquidity measures is NYSE/AMEX stocks 1963–2000. Time-series with a monthly frequency are constructed using monthly averages of firm specific liquidity measures. That is,

$$L_t = \frac{1}{N_{lt}} \sum_{i=1}^{N_{lt}} l_{it}$$

where l_{it} is one of the six firm specific liquidity measures described in Table 1, N_{lt} is the number of firms with a non-missing observation on liquidity measure l. Upper case notation is used to refer to the aggregate version of firm specific liquidity measures in lower case notation. Thus, PQSPR refers to the aggregate version of pqspr. Before computing average liquidity for a given month, the cross-sections are trimmed by removing the two most extreme observations at both ends.When the aggregate liquidity measure, L_t, is the Pastor-Stambaugh (PS) or the Breen-Hodrick-Korajczyk (BHK) measure and order flow is measured using dollar volume, the scalar $S_t = (m_t/m_1)$, where m_t is the total dollar value at the end of month $t-1$ of all firms included in the computation of the average PS liquidity measure and m_1 is the dollar value in July 1962. When L_t is the proportional quoted spread, $S_t = 1$. When L_t is the average monthly turnover, $S_t = (o_1/o_t)$, where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover, and o_1 is the monthly turnover in July 1962. When L_t is PS-liquidity or BHK-liquidity and order flow is measured using turnover, $S_t = (o_t/o_1)$.

	PQSPR	TO	$PS_{\$}$	$BHK_{\$}$	PS_{to}	BHK_{to}
A. Descriptive statistics for unscaled times-series of average liquidity						
Mean	4.470	4.613	-0.016	0.442	-0.004	0.140
Median	3.684	4.502	-0.006	0.249	-0.002	0.107
Std	2.522	2.013	0.037	0.435	0.007	0.090
Min	1.303	1.342	-0.331	0.051	-0.052	0.055
Max	14.254	11.420	0.063	3.911	0.014	1.097
B. Correlations for scaled times-series of average liquidity						
PQSPR	1.000					
TO	−0.069	1.000				
$PS_{\$}$	−0.043	0.159	1.000			
$BHK_{\$}$	0.579	−0.371	−0.277	1.000		
PS_{to}	−0.132	0.204	0.777	−0.239	1.000	
BHK_{to}	0.261	−0.507	−0.298	0.760	−0.303	1.000
C. Autocorrelations for scaled times-series of average liquidity						
Lag						
1	0.989	0.915	0.283	0.896	0.148	0.845
2	0.981	0.886	0.268	0.853	0.074	0.779
3	0.976	0.866	0.166	0.825	0.087	0.741
4	0.965	0.836	0.240	0.815	0.029	0.721
5	0.957	0.836	0.230	0.785	0.123	0.680

Table 3
Commonality in liquidity in monthly data, 1963–2000

The sample used to compute firm specific liquidity measures is NYSE/AMEX stocks 1963–2000. Time-series with a monthly frequency are constructed using monthly averages of firm specific liquidity measures. The model estimated in Panel A is:

$$\frac{l_{it} - l_{it-1}}{l_{it-1}} = a + b_i \frac{S_t L_t - S_{t-1} L_{t-1}}{S_{t-1} L_{t-1}} + \epsilon_{it}$$

where l_{it} is one of the seven firm specific liquidity measures described in Table 1 and L_t is the corresponding aggregate liquidity measure (see Table 2), and S_t is a variable that scales the aggregate liquidity. That is, the change in firm specific liquidity is regressed on a constant term and the change in the corresponding scaled aggregate liquidity measure. When the aggregate liquidity measure, L_t, is the Pastor-Stambaugh (PS) or the Breen-Hodrick-Korajczyk (BHK) measure and order flow is measured using dollar volume, the scalar $S_t = (m_t/m_1)$, where m_t is the total dollar value at the end of month $t-1$ of all firms included in the computation of the average PS liquidity measure and m_1 is the dollar value in July 1962. When L_t is the proportional quoted spread, $S_t = 1$. When L_t is the average monthly turnover, $S_t = (o_1/o_t)$, where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover, and o_1 is the monthly turnover in July 1962. When L_t is PS-liquidity or BHK-liquidity and order flow is measured using turnover, $S_t = (o_t/o_1)$.Panel A reports descriptive statistics for the cross-section of estimates of b_i. In Panel B, firm specific variables are added to the regression: STD is the standard deviation of the daily stock returns in month t, PRC is the average daily price in month t, and DVOL is the aggregate dollar volume for month t. The three variables STD, PRC, and DVOL are measured in natural logs.

	Dependent variable					
	pqspr	to	$ps_{\$}$	$bhk_{\$}$	ps_{to}	bhk_{to}
A. Regression with no firm specific control variables						
Average $\hat{b}_i$	0.680	1.412	0.028	3.135	0.016	3.056
Percent $\hat{b}_i > 0$	0.844	0.957	0.496	0.819	0.496	0.793
Percent t-values > 1.65	0.315	0.737	0.024	0.364	0.031	0.320
H_0: Average $\hat{b}_i = 0$	26.351	22.971	1.026	10.530	0.288	9.319
Average number TS observations	112	195	200	200	200	200
Average Adjusted R^2	0.015	0.057	-0.000	0.021	0.001	0.019
Number of CS observations	1043	4445	4428	4427	4432	4425
B. Regression with firm specific control variables						
STD	0.063	0.425	0.000	0.847	-0.407	0.777
t	14.161	15.091	0.001	6.423	-1.256	9.994
PRC	0.035	-0.410	-0.690	0.198	0.015	0.382
t	7.676	-9.807	-1.417	1.109	0.031	3.528
DVOL	-0.019	0.509	0.117	-0.728	-0.143	-0.709
t	-8.559	24.888	0.793	-16.575	-0.922	-17.669
$(S_t L_t - S_{t-1} L_{t-1})/S_{t-1} L_{t-1}$	0.655	1.107	0.023	2.891	-0.002	2.801
t	24.897	17.918	1.262	9.982	-0.128	9.852
Average number of TS observations	112	195	201	200	201	200
Average Adjusted R^2	0.013	0.185	0.002	0.066	0.003	0.062
Number of CS observations	1040	4435	4352	4424	4354	4423

Table 4
Autocorrelation in innovations in aggregate liquidity, 1963–2000

The sample used to compute firm specific liquidity measures is NYSE/AMEX stocks 1963–2000. Time-series with a monthly frequency, L_t, are constructed using monthly averages of firm specific liquidity measures. Innovations in liquidity measure L_t is the residual from the estimation of the model:

$$\Delta L_t = a + b\Delta L_{t-1} + cS_tL_{t-1} + \epsilon_t,$$

where L_t is an aggregate liquidity measure, $\Delta L_t = S_t(L_t - L_{t-1})$. When the aggregate liquidity measure, L_t, is the Pastor-Stambaugh (PS) or the Breen-Hodrick-Korajczyk (BHK) measure and order flow is measured using dollar volume, the scalar $S_t = (m_t/m_1)$, where m_t is the total dollar value at the end of month $t-1$ of all firms included in the computation of the average PS liquidity measure and m_1 is the dollar value in July 1962. When L_t is the proportional quoted spread, $S_t = 1$. When L_t is the average monthly turnover, $S_t = (o_1/o_t)$, where o_t is the 24-month moving average, computed over months $t-24$ through $t-1$, of the average monthly turnover, and o_1 is the monthly turnover in July 1962.

	PQSPR	TO	$PS_{\$}$	$BHK_{\$}$	PS_{to}	BHK_{to}
A. Autocorrelations at lags 1 through 5						
Lag						
1	−0.019	−0.005	−0.006	−0.030	−0.010	−0.016
2	−0.130	−0.034	0.003	−0.117	−0.006	−0.099
3	0.251	0.042	0.053	0.011	0.068	0.002
4	−0.105	−0.045	0.052	−0.002	−0.040	0.065
5	−0.140	0.060	0.072	0.006	0.074	0.028
B. p-values for Ljung-Box Q statistic						
Lag						
1	0.677	0.930	0.863	0.525	0.827	0.738
2	0.023	0.678	0.982	0.039	0.969	0.103
3	0.000	0.655	0.773	0.088	0.558	0.209
4	0.000	0.651	0.620	0.162	0.634	0.163
5	0.000	0.543	0.453	0.256	0.434	0.231
C. Correlations						
PQSPR	1.000					
TO	0.013	1.000				
$PS_{\$}$	−0.034	0.063	1.000			
$BHK_{\$}$	0.251	−0.339	−0.279	1.000		
PS_{to}	−0.030	0.127	0.770	−0.201	1.000	
BHK_{to}	0.235	−0.345	−0.266	0.939	−0.199	1.000

Table 5
Descriptive statistics for the cross-section of 25 size and book-to-market ratio sorted portfolios, 1963–2000

The 25 size and book-to-market ratio sorted portfolios are downloaded from Ken French's website at Dartmouth College. The reported betas are estimated using the CAPM.

	Raw returns				Betas			
Period	Mean	Std.	Min	Max	Mean	Std.	Min	Max
1963:07–2000:12	1.222	0.224	0.722	1.571	1.058	0.173	0.782	1.426
1963:07–1969:12	1.197	0.392	0.286	1.832	1.249	0.237	0.765	1.720
1970:01–1979:12	0.934	0.320	0.277	1.517	1.147	0.190	0.802	1.502
1980:01–1989:12	1.491	0.312	0.441	1.899	0.999	0.157	0.754	1.344
1990:01–2000:12	1.253	0.223	0.451	1.502	0.952	0.230	0.615	1.435

Table 6

Testing for β-pricing using factors that are innovations in aggregate liquidity and using 25 size and book-to-market ratio sorted portfolios as test assets, 1963–2000

The model is:

$$R = XB + E$$

where R is T by N matrix of excess returns on N portfolios, X is T by $(K+1)$ matrix of factors with typical row $[1 \;\; F_t']$, B is the $(K+1)$ by N matrix $[\alpha \;\; \beta]'$ where α is a N-vector of intercepts and β is a N by K matrix of factor sensitivities, and E is T by N with typical row $[\epsilon_{1t} \; \epsilon_{2t} \; \ldots \; \epsilon_{Nt}]$. The liquidity factors are innovations in monthly cross-sectional averages of firm specific liquidity measures (See Table 4 for a description). The table reports test statistics F_1, J_2, and F_2 for the null hypothesis that the factor sensitivities of the N assets with respect to liquidity factor L_t is equal to zero. The test statistics are as reported in equations (13), (14), and (15). The parentheses in Panel A contains t-values. The parentheses in Panel B contains p-values.

	PQSPR	TO	$PS_{\$}$	BHKDV	PS_{to}	BHK_{to}
A. Liquidity betas when a liquidity factor is added to the CAPM						
P11	−3.95 (−6.3)	3.71 (5.7)	0.87 (0.3)	−1.85 (−2.5)	0.17 (0.8)	−0.13 (−2.7)
P12	−3.77 (−6.9)	3.27 (5.7)	−0.69 (−0.3)	−0.96 (−1.5)	0.14 (0.7)	−0.08 (−1.9)
P13	−2.43 (−5.3)	2.98 (6.5)	1.36 (0.7)	−0.46 (−0.9)	0.32 (2.1)	−0.04 (−1.3)
P14	−2.41 (−5.6)	3.04 (7.0)	1.50 (0.8)	−0.63 (−1.2)	0.38 (2.7)	−0.06 (−1.7)
P15	−2.45 (−5.2)	3.25 (6.9)	1.52 (0.8)	−0.62 (−1.1)	0.45 (2.9)	−0.06 (−1.8)
P21	−2.38 (−4.9)	1.74 (3.5)	−0.76 (−0.4)	−0.79 (−1.4)	0.03 (0.2)	−0.07 (−1.9)
P22	−1.67 (−4.4)	1.86 (4.8)	0.36 (0.2)	−0.48 (−1.1)	0.24 (2.0)	−0.05 (−1.6)
P23	−1.73 (−5.1)	1.80 (5.2)	1.26 (0.9)	−0.46 (−1.2)	0.31 (2.8)	−0.04 (−1.5)
P24	−1.16 (−3.4)	1.49 (4.3)	0.70 (0.5)	0.01 (0.0)	0.29 (2.6)	−0.02 (−0.6)
P25	−1.39 (−3.4)	1.99 (4.8)	−1.65 (−1.0)	0.25 (0.5)	0.21 (1.6)	0.00 (0.1)
P31	−1.94 (−4.9)	1.09 (2.7)	−1.04 (−0.6)	−0.78 (−1.7)	−0.09 (−0.7)	−0.06 (−2.0)
P32	−0.63 (−2.1)	0.94 (3.1)	1.78 (1.5)	−0.37 (−1.1)	0.27 (2.8)	−0.02 (−1.1)
P33	−0.29 (−1.0)	1.19 (3.9)	0.57 (0.5)	−0.01 (−0.0)	0.30 (3.2)	−0.01 (−0.3)
P34	−0.44 (−1.4)	1.10 (3.5)	−0.32 (−0.3)	0.28 (0.8)	0.25 (2.5)	0.01 (0.5)
P35	−0.59 (−1.5)	1.86 (4.9)	−0.78 (−0.5)	0.43 (1.0)	0.22 (1.8)	0.01 (0.3)
P41	−1.31 (−4.3)	0.31 (1.0)	−0.16 (−0.1)	−0.66 (−1.9)	−0.14 (−1.4)	−0.05 (−2.0)
P42	−0.07 (−0.3)	0.61 (2.4)	0.73 (0.7)	−0.14 (−0.5)	0.20 (2.6)	−0.01 (−0.4)
P43	0.47 (1.7)	0.44 (1.6)	−0.05 (−0.0)	0.67 (2.2)	0.22 (2.6)	0.04 (2.2)
P44	−0.15 (−0.5)	0.87 (2.9)	−1.54 (−1.3)	0.82 (2.5)	0.10 (1.0)	0.04 (2.0)
P45	0.06 (0.2)	1.15 (2.9)	−1.40 (−0.9)	0.99 (2.2)	0.16 (1.3)	0.05 (1.7)
P51	0.35 (1.6)	−0.44 (−1.9)	0.21 (0.2)	−0.11 (−0.4)	−0.09 (−1.3)	−0.01 (−0.7)
P52	0.92 (4.3)	−0.55 (−2.4)	0.73 (0.8)	0.53 (2.1)	0.06 (0.9)	0.03 (2.2)
P53	0.91 (3.4)	−0.56 (−2.0)	0.26 (0.2)	0.85 (2.8)	0.09 (1.1)	0.07 (3.4)
P54	1.40 (4.6)	−0.12 (−0.4)	−2.39 (−1.9)	1.55 (4.5)	−0.02 (−0.2)	0.09 (4.3)
P55	1.12 (2.9)	0.64 (1.6)	−3.75 (−2.4)	1.16 (2.6)	−0.05 (−0.4)	0.07 (2.6)
B. Test statistics under the null of jointly zero betas						
$F1_{(25,426)}$	3.4 (0.000)	2.8 (0.000)	1.6 (0.038)	2.4 (0.000)	1.6 (0.032)	2.2 (0.001)
$J2_{(25)}$	49.4 (0.003)	52.9 (0.001)	30.3 (0.212)	59.9 (0.000)	31.8 (0.165)	58.8 (0.000)
$F2_{(25,426)}$	1.9 (0.008)	2.0 (0.003)	1.1 (0.292)	2.3 (0.001)	1.2 (0.238)	2.2 (0.001)

Table 7

Testing for β-pricing using factors that are innovations in aggregate liquidity and using 25 size and book-to-market ratio sorted portfolios as test assets, 1963–2000

The model is:

$$R = XB + E$$

where R is T by N matrix of excess returns on N portfolios, X is T by $(K+1)$ matrix of factors with typical row $[1 \;\; F_t']$, B is the $(K+1)$ by N matrix $[\alpha \;\; \beta]'$ where α is a N-vector of intercepts and β is a N by K matrix of factor sensitivities, and E is T by N with typical row $[\epsilon_{1t} \; \epsilon_{2t} \; \ldots \; \epsilon_{Nt}]$. The liquidity factors are innovations in monthly cross-sectional averages of firm specific liquidity measures (See Table 4 for a description). The table reports test statistics F_1, J_2, and F_2 for the null hypothesis that the factor sensitivities of the N assets with respect to liquidity factor L_t is equal to zero. The test statistics are as reported in equations (13), (14), and (15). The parentheses in Panel A contains t-values. The parentheses in Panel B contains p-values.

	PQSPR	TO	$PS_{\$}$	BHKDV	PS_{to}	BHK_{to}
A. Liquidity betas when a liquidity factor is added to the Fama-French three-factor model						
P11	−0.15 (−0.5)	1.14 (3.9)	−1.11 (−1.0)	0.04 (0.1)	−0.09 (−1.0)	0.00 (0.1)
P12	−0.41 (−1.8)	0.57 (2.5)	−2.12 (−2.4)	0.46 (1.9)	−0.16 (−2.4)	0.02 (1.5)
P13	0.24 (1.4)	0.61 (3.5)	0.50 (0.8)	0.45 (2.4)	0.04 (0.7)	0.03 (2.3)
P14	−0.05 (−0.3)	0.73 (4.2)	0.92 (1.4)	0.02 (0.1)	0.10 (1.8)	−0.00 (−0.0)
P15	−0.16 (−0.8)	0.72 (3.9)	1.20 (1.7)	−0.18 (−0.9)	0.13 (2.3)	−0.02 (−1.6)
P21	0.51 (2.3)	−0.04 (−0.2)	−2.41 (−2.8)	0.77 (3.2)	−0.13 (−1.9)	0.04 (2.5)
P22	0.46 (2.4)	0.03 (0.1)	−0.39 (−0.5)	0.29 (1.4)	0.04 (0.6)	0.01 (1.0)
P23	−0.13 (−0.7)	0.08 (0.4)	0.96 (1.4)	−0.10 (−0.5)	0.09 (1.7)	−0.00 (−0.3)
P24	0.20 (1.1)	−0.23 (−1.3)	0.70 (1.0)	0.12 (0.6)	0.07 (1.3)	0.00 (0.2)
P25	0.16 (0.9)	−0.08 (−0.4)	−1.56 (−2.3)	0.30 (1.5)	−0.07 (−1.3)	0.02 (1.7)
P31	0.24 (1.1)	−0.16 (−0.7)	−2.40 (−2.9)	0.48 (2.0)	−0.20 (−3.0)	0.03 (1.7)
P32	0.58 (2.5)	−0.21 (−0.9)	1.48 (1.7)	−0.04 (−0.2)	0.13 (1.9)	0.00 (0.2)
P33	0.52 (2.4)	0.04 (0.2)	0.73 (0.9)	−0.09 (−0.4)	0.14 (2.2)	−0.00 (−0.2)
P34	0.04 (0.2)	−0.01 (−0.1)	0.14 (0.2)	−0.05 (−0.2)	0.08 (1.3)	0.00 (0.0)
P35	0.17 (0.8)	0.37 (1.7)	−0.28 (−0.3)	0.08 (0.3)	0.00 (0.0)	−0.00 (−0.2)
P41	−0.05 (−0.2)	−0.19 (−0.9)	−1.17 (−1.4)	0.25 (1.1)	−0.16 (−2.5)	0.01 (0.9)
P42	0.28 (1.2)	0.11 (0.5)	0.81 (0.9)	−0.18 (−0.7)	0.13 (1.8)	−0.01 (−0.4)
P43	0.55 (2.4)	−0.14 (−0.6)	0.43 (0.5)	0.30 (1.2)	0.13 (1.9)	0.03 (1.6)
P44	−0.06 (−0.3)	0.13 (0.6)	−0.98 (−1.2)	0.39 (1.7)	−0.03 (−0.4)	0.02 (1.4)
P45	0.12 (0.4)	0.17 (0.6)	−0.57 (−0.5)	0.34 (1.1)	−0.01 (−0.1)	0.02 (0.8)
P51	−0.05 (−0.3)	0.32 (1.8)	0.00 (0.0)	0.03 (0.2)	0.01 (0.3)	−0.01 (−0.7)
P52	0.20 (1.0)	−0.08 (−0.4)	1.16 (1.5)	0.13 (0.6)	0.11 (1.8)	0.01 (0.5)
P53	−0.01 (−0.0)	−0.13 (−0.6)	0.92 (1.0)	0.26 (1.0)	0.12 (1.8)	0.03 (1.7)
P54	0.42 (2.2)	−0.01 (−0.0)	−1.33 (−1.8)	0.65 (3.0)	−0.05 (−0.9)	0.04 (2.8)
P55	0.45 (1.6)	0.31 (1.1)	−2.63 (−2.4)	0.21 (0.7)	−0.15 (−1.8)	0.02 (0.9)
B. Test statistics under the null of jointly zero betas						
$F1_{(25,424)}$	1.6 (0.045)	1.8 (0.011)	1.6 (0.045)	1.9 (0.006)	1.4 (0.088)	1.7 (0.022)
$J2_{(25)}$	32.3 (0.148)	36.7 (0.061)	30.8 (0.195)	33.2 (0.125)	23.2 (0.565)	27.1 (0.353)
$F2_{(25,424)}$	1.2 (0.223)	1.4 (0.109)	1.2 (0.278)	1.2 (0.195)	0.9 (0.650)	1.0 (0.448)

Table 8
Descriptive statistics for factor mimicking portfolios 1963–2000

For all liquidity measures except the proportional quoted spread, factor mimicking portfolios are constructed using a procedure similar to the one used by Fama and French (1993) to construct HML. To construct a factor mimicking portfolio for aggregate liquidity measure L, we start in December 1962 and form two portfolios based on a ranking of the end-of-month market value of equity for all NYSE stocks and three portfolios formed using NYSE/AMEX stocks ranked on firm specific liquidity measure l. Next, six portfolios are constructed from the intersection of the two market value and the three liquidity portfolios. Monthly equally-weighted returns on these six portfolios are calculated starting in January 1963. Portfolios are reformed in June and held for one year starting in July. The factor mimicking liquidity portfolio is the difference between the equal-weighted average return on the two portfolios with low liquidity and the equal-weighted average return on the two portfolios with high liquidity. The factor mimicking portfolio for proportional quoted spreads is constructed without sorting the cross-sections on size. The return on this mimicking portfolio is computed as the difference between the equally weighted return on a portfolio of the one-third least liquid firms in the cross-section less the equally weighted return on a portfolio of the one-third most liquid firms.

	PQSPR	TO	$PS_{\$}$	$BHK_{\$}$	PS_{to}	BHK_{to}	$R_m - R_f$	SMB	HML
A. Descriptive statistics									
Mean	0.29	0.13	0.06	0.00	0.07	0.02	0.51	0.16	0.40
Median	−0.56	0.12	0.06	0.01	0.04	0.01	0.74	0.01	0.42
Standard dev.	4.66	2.85	0.83	2.78	0.71	1.68	4.41	3.26	2.90
Minimum	−12.25	−9.37	−3.70	−8.61	−3.79	−6.15	−23.09	−16.69	−12.03
Maximum	25.41	10.48	2.73	9.73	2.92	5.56	16.05	21.49	13.11
B. Average monthly returns									
Mean 1963:07–1969:12	1.21	-0.19	-0.01	0.11	-0.01	-0.13	0.45	0.74	0.32
Mean 1970:01–1979:12	0.44	0.23	0.08	0.37	0.09	0.25	0.10	0.20	0.67
Mean 1980:01–1989:12	-0.35	0.38	0.11	-0.17	0.14	0.05	0.68	0.09	0.45
Mean 1990:01–2000:12	0.21	-0.01	0.03	-0.25	0.03	-0.13	0.89	-0.11	0.01
C. Correlations									
PQSPR	1.00								
TO	−0.28	1.00							
$PS_{\$}$	0.09	−0.00	1.00						
$BHK_{\$}$	0.32	0.39	0.12	1.00					
PS_{to}	0.08	−0.05	0.85	0.09	1.00				
BHK_{to}	−0.05	0.89	0.02	0.56	−0.03	1.00			
$R_m - R_f$	0.22	−0.69	0.05	−0.29	0.11	−0.54	1.00		
SMB	0.59	−0.50	0.07	0.24	0.06	−0.31	0.30	1.00	
HML	0.00	0.47	−0.05	0.14	−0.09	0.40	−0.41	−0.29	1.00

Table 9

Testing for β-pricing using firm characteristics based factors and using 25 size and book-to-market ratio sorted portfolios as test assets, 1963–2000

The model is:

$$R = XB + E$$

where R is T by N matrix of excess returns on N portfolios, X is T by $(K+1)$ matrix of factors with typical row $[1 \;\; F_t']$, B is the $(K+1)$ by N matrix $[\alpha \;\; \beta]'$ where α is a N-vector of intercepts and β is a N by K matrix of factor sensitivities, and E is T by N with typical row $[\epsilon_{1t} \; \epsilon_{2t} \; \ldots \; \epsilon_{Nt}]$. For all liquidity measures except the dollar and percent quoted spreads, factor mimicking portfolios are constructed using a procedure similar to the one used by Fama and French (1993) to construct HML (See Table 8 for a more elaborate description of this procedure). The factor mimicking portfolio for proportional quoted spreads is constructed without sorting the cross-sections on size. The return on this mimicking portfolio is computed as the difference between the equally weighted return on a portfolio of the one-third least liquid firms in the cross-section less the equally weighted return on a portfolio of the one-third most liquid firms. The table reports test statistics F_1, J_2, and F_2 for the null hypothesis that the factor sensitivities of the N assets with respect to liquidity factor L_t is equal to zero. The test statistics are as reported in equations (13), (14), and (15). The parentheses in Panel A contains t-values. The parentheses in Panel B contains p-values.

	PQSPR	TO	$PS_{\$}$	BHKDV	PS_{to}	BHK_{to}
A. Liquidity betas when a liquidity factor is added to the Fama-French three-factor model						
P11	0.24 (9.8)	−0.17 (−3.2)	0.15 (1.2)	0.13 (3.3)	0.17 (1.2)	−0.03 (−0.4)
P12	0.12 (6.0)	0.05 (1.2)	0.11 (1.2)	0.12 (3.8)	0.04 (0.4)	0.11 (2.0)
P13	0.10 (6.0)	0.03 (1.0)	0.13 (1.8)	0.06 (2.3)	0.17 (2.0)	0.08 (1.9)
P14	0.12 (7.9)	0.05 (1.4)	−0.08 (−1.1)	0.05 (2.1)	−0.04 (−0.5)	0.05 (1.2)
P15	0.22 (15.1)	0.08 (2.2)	0.06 (0.7)	0.13 (5.4)	0.14 (1.5)	0.18 (3.9)
P21	−0.03 (−1.3)	−0.35 (−8.7)	−0.11 (−1.2)	−0.13 (−4.3)	−0.20 (−1.8)	−0.32 (−5.8)
P22	−0.06 (−3.3)	−0.12 (−3.2)	−0.01 (−0.1)	−0.08 (−3.0)	0.00 (0.0)	−0.10 (−2.0)
P23	−0.09 (−5.2)	−0.10 (−2.9)	−0.19 (−2.5)	−0.12 (−4.6)	−0.18 (−2.0)	−0.17 (−3.6)
P24	−0.09 (−5.7)	−0.06 (−1.7)	−0.07 (−1.0)	−0.11 (−4.5)	−0.08 (−0.9)	−0.11 (−2.4)
P25	−0.01 (−0.5)	−0.08 (−2.2)	0.12 (1.6)	−0.06 (−2.6)	0.15 (1.7)	−0.09 (−2.0)
P31	−0.10 (−5.0)	−0.25 (−6.2)	−0.01 (−0.2)	−0.17 (−5.7)	−0.08 (−0.8)	−0.33 (−6.1)
P32	−0.11 (−5.3)	−0.22 (−5.0)	0.02 (0.2)	−0.15 (−4.8)	0.06 (0.5)	−0.29 (−4.9)
P33	−0.10 (−5.0)	−0.14 (−3.5)	−0.19 (−2.0)	−0.13 (−4.3)	−0.09 (−0.8)	−0.21 (−3.8)
P34	−0.10 (−5.1)	−0.11 (−2.7)	−0.09 (−1.0)	−0.12 (−4.1)	−0.05 (−0.5)	−0.15 (−3.0)
P35	−0.01 (−0.6)	−0.21 (−5.0)	−0.03 (−0.3)	−0.14 (−4.5)	0.05 (0.5)	−0.23 (−4.1)
P41	−0.06 (−2.8)	−0.17 (−4.1)	−0.00 (−0.0)	−0.12 (−4.0)	−0.08 (−0.8)	−0.24 (−4.5)
P42	−0.12 (−5.3)	−0.18 (−4.0)	−0.15 (−1.5)	−0.18 (−5.5)	−0.04 (−0.3)	−0.19 (−3.1)
P43	−0.09 (−4.0)	−0.20 (−4.7)	−0.15 (−1.5)	−0.16 (−5.2)	−0.11 (−1.0)	−0.23 (−3.9)
P44	−0.06 (−2.8)	−0.11 (−2.8)	0.11 (1.2)	−0.11 (−3.7)	0.13 (1.2)	−0.19 (−3.5)
P45	−0.06 (−2.1)	−0.32 (−6.2)	−0.07 (−0.6)	−0.10 (−2.7)	−0.11 (−0.8)	−0.40 (−5.7)
P51	0.06 (3.4)	0.02 (0.7)	−0.05 (−0.7)	−0.02 (−0.8)	−0.07 (−0.8)	−0.04 (−0.9)
P52	−0.03 (−1.8)	−0.02 (−0.6)	−0.02 (−0.2)	−0.03 (−1.1)	0.04 (0.4)	0.05 (0.9)
P53	−0.02 (−0.9)	−0.04 (−0.9)	−0.03 (−0.3)	−0.04 (−1.2)	0.03 (0.3)	0.03 (0.5)
P54	0.02 (1.1)	−0.15 (−4.1)	−0.10 (−1.2)	−0.05 (−1.8)	−0.12 (−1.3)	−0.12 (−2.4)
P55	0.02 (0.6)	−0.23 (−4.2)	−0.12 (−1.0)	−0.03 (−0.8)	−0.18 (−1.3)	−0.20 (−2.7)
B. Test statistics under the null of jointly zero betas						
$F1_{(25,424)}$	15.2 (0.000)	7.2 (0.000)	1.7 (0.017)	4.7 (0.000)	1.8 (0.012)	4.8 (0.000)
$J2_{(25)}$	85.9 (0.000)	70.3 (0.000)	34.5 (0.098)	72.6 (0.000)	29.7 (0.234)	62.8 (0.000)
$F2_{(25,424)}$	3.2 (0.000)	2.6 (0.000)	1.3 (0.160)	2.7 (0.000)	1.1 (0.323)	2.4 (0.000)

Table 10
Risk-premiums estimated using factors that are innovations in aggregate liquidity and 25 size and book-to-market ratio sorted portfolios as test assets, 1963–2000

The model is:

$$R = XB + E$$

where R is T by N matrix of excess returns on N portfolios, X is T by $(K+1)$ matrix of factors with typical row $[1\ (f_{1t}+\lambda_1)\ \ldots\ (f_{Kt}+\lambda_K)]$, B is the $(K+1)$ by N matrix $[\alpha\ \ \beta]'$ where α is a N-vector of intercepts and β is a N by K matrix of factor sensitivities, and E is T by N with typical row $[\epsilon_{1t}\ \epsilon_{2t}\ \ldots\ \epsilon_{Nt}]$. Panels A and B report OLS and ML estimates for the risk-premiums λ_k. Parentheses in Panels A and B contain t-values for the risk-premium estimates. Panel C and D reports liquidity factor contributions to expected return computed as beta times the ML risk premiums reported in Panel A and B. The beta for the portfolio of the smallest firms (row named Small) is computed as the average beta on the five small-firm portfolios (P11, P12, ..., P15) as reported in Table 6. Average betas are computed in a similar way for the other four size portfolios and the five book-to-market ratio portfolios. Parentheses in Panel C and D contain the average betas.

	PQSPR	TO	PS$_{\$}$	BHK$_{\$}$	PS$_{to}$	BHK$_{to}$
A. Adding liquidity factor to the CAPM						
OLS	−0.12 (−2.2)	0.09 (1.7)	0.02 (1.2)	−0.10 (−1.2)	0.77 (2.5)	−2.42 (−1.7)
ML	−0.22 (−4.4)	0.15 (2.7)	0.27 (2.3)	4.23 (0.6)	3.12 (2.4)	−9.38 (−2.8)
B. Adding liquidity factor to the Fama-French three factor model						
OLS	−0.14 (−2.0)	−0.15 (−1.8)	0.02 (1.0)	−0.03 (−0.5)	0.07 (0.3)	−1.68 (−1.5)
ML	−0.48 (−3.6)	−0.79 (−2.7)	0.20 (2.6)	1.00 (2.1)	−6.23 (−1.3)	−30.35 (−1.1)
C. Contribution to expected return: Liquidity beta times ML risk-premium from the CAPM						
Small to Big market value of equity						
Small	0.66 (−3.00)	0.47 (3.25)	0.25 (0.91)	−3.82 (−0.90)	0.90 (0.29)	0.70 (−0.07)
2	0.36 (−1.66)	0.26 (1.77)	−0.01 (−0.02)	−1.24 (−0.29)	0.67 (0.22)	0.31 (−0.03)
3	0.17 (−0.78)	0.18 (1.24)	0.01 (0.04)	−0.39 (−0.09)	0.59 (0.19)	0.13 (−0.01)
4	0.04 (−0.20)	0.10 (0.67)	−0.13 (−0.48)	1.42 (0.34)	0.33 (0.11)	−0.16 (0.02)
Big	−0.21 (0.94)	−0.03 (−0.21)	−0.27 (−0.99)	3.37 (0.80)	−0.01 (−0.00)	−0.48 (0.05)
Low to High book-to-market ratio						
Low	0.40 (−1.84)	0.19 (1.28)	−0.05 (−0.18)	−3.54 (−0.84)	−0.08 (−0.02)	0.58 (−0.06)
2	0.23 (−1.04)	0.18 (1.22)	0.16 (0.58)	−1.20 (−0.28)	0.57 (0.18)	0.23 (−0.02)
3	0.13 (−0.61)	0.17 (1.17)	0.18 (0.68)	0.50 (0.12)	0.77 (0.25)	−0.04 (0.00)
4	0.12 (−0.55)	0.19 (1.28)	−0.11 (−0.41)	1.72 (0.41)	0.62 (0.20)	−0.14 (0.02)
High	0.14 (−0.65)	0.26 (1.78)	−0.33 (−1.21)	1.87 (0.44)	0.61 (0.20)	−0.13 (0.01)
D. Contribution to expected return: Liquidity beta times ML risk-premium from the Fama-French model						
Small to Big market value of equity						
Small	0.05 (−0.11)	−0.59 (0.75)	−0.02 (−0.12)	0.16 (0.16)	−0.01 (0.00)	−0.21 (0.01)
2	−0.12 (0.24)	0.04 (−0.05)	−0.11 (−0.54)	0.28 (0.28)	0.00 (−0.00)	−0.44 (0.01)
3	−0.15 (0.31)	−0.00 (0.01)	−0.01 (−0.06)	0.08 (0.08)	−0.20 (0.03)	−0.15 (0.01)
4	−0.08 (0.17)	−0.01 (0.02)	−0.06 (−0.30)	0.22 (0.22)	−0.08 (0.01)	−0.43 (0.01)
Big	−0.10 (0.20)	−0.06 (0.08)	−0.08 (−0.38)	0.26 (0.26)	−0.05 (0.01)	−0.51 (0.02)
Low to High book-to-market ratio						
Low	−0.05 (0.10)	−0.17 (0.21)	−0.29 (−1.42)	0.31 (0.31)	0.71 (−0.11)	−0.45 (0.01)
2	−0.11 (0.22)	−0.06 (0.08)	0.04 (0.19)	0.13 (0.13)	−0.31 (0.05)	−0.26 (0.01)
3	−0.11 (0.23)	−0.07 (0.09)	0.14 (0.71)	0.17 (0.17)	−0.66 (0.11)	−0.46 (0.02)
4	−0.05 (0.11)	−0.09 (0.12)	−0.02 (−0.11)	0.23 (0.23)	−0.21 (0.03)	−0.38 (0.01)
High	−0.07 (0.15)	−0.23 (0.30)	−0.16 (−0.77)	0.15 (0.15)	0.13 (−0.02)	−0.20 (0.01)