How to Extend the RiskMetrics™ Market Risk Universe

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Abstract

RiskMetrics™ (RM) represents a framework for measuring market risk founded on the Value at Risk concept, and offer daily updated estimates of standard deviations and correlations of the assets within their market risk universe. Unfortunately, a company may also be exposed to other sources of market risk than the ones covered by RM. This paper shows how to extend the RM universe in a consistent way. The main challenge is to obtain the correlations between each pair of RM asset and additional asset. Simple rules apply for updating estimates of the extended universe for new daily information.

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1. Introduction

Quantitative models for managing the downside market risk exposure have gained increased attention in business as well as in legislation and regulation.[i] RiskMetrics™ (RM) represents a framework for measuring market risk founded on the Value at Risk concept[ii], and offer daily reports containing information on updated estimates of standard deviations and correlations of returns of the assets within the RM market risk universe.[iii] By assessing the company risk exposure with respect to each RM asset, the company can calculate the overall downside risk exposure due to market risk.

Unfortunately, it may be the case that a company due to its activities or its location also is exposed to other sources of market risk than the ones covered by RM. One example is the lack of data for several small counties, for instance Norway.[iv] The purpose of the paper is to show how to extend the RM market risk universe in a consistent way. The main challenge is to obtain the correlations between the returns of each pair of RM asset and additional assets. In addition, we need estimates of the standard deviation of each additional asset return as well as the correlation for each pair of additional asset returns.

For our analysis, we need information on the observed daily return for the additional assets, as well as the RM daily reports on standard deviations and correlations. In addition, it is required that we can observe the daily return time series of at least one asset within the RM universe.

To include additional assets, the idea is as follows: First, use the time series of RM daily estimates of standard deviation and correlation - in addition to the observed return time series on at least one of them - to reconstruct the daily return series for each RM asset. Second, use the observed return time series of the additional assets to estimate the current standard deviation and the correlation between each pair of them. And third, use the reconstructed and the observed daily return time series to estimate the correlation for each pair of RM asset and additional asset.

It is important to notice that once the additional assets have been included, a simple updating rule applies for updating the estimates as new information arrives. This means the following simple daily routine: Download the updated RM data. Observe the return on one asset within the RM universe, as well as the returns on the additional assets. Apply a simple rule for updating the covariances between the assets in the RM universe and the additional assets, as well as the covariances between the additional assets. Translate the covariances into standard deviations and correlations.

Section 2 translates the RM market information into a covariance matrix, which is a more convenient representation. Section 3 introduces the RM covariance estimator and the daily updating rule. Section 4 explains how to infer the underlying returns from the RM daily standard deviation and correlation reports. Section 5 explains how the covariance estimator can be approximated from a finite return time series, say 100 days, and section 6 concludes.

2. Standard deviation, correlation, and the covariance matrix

Let the RM universe consist of N assets (asset classes). RM provide daily reports where the information can be represented by a Value at Risk vector $V_N(t)$ and a correlation matrix $R_{N\times N}(t)$

$$V_N(t) \equiv \begin{pmatrix} 1.65\, s_1(t) & \cdots & 1.65\, s_N(t) \end{pmatrix} \qquad R_{N\times N}(t) \equiv \begin{pmatrix} 1 & \cdots & r_{1,N}(t) \\ \vdots & \ddots & \vdots \\ r_{N,1}(t) & \cdots & 1 \end{pmatrix}$$

where s_i is the standard deviation (per day) of daily returns of asset (class) i, and $r_{i,j}$ is the correlation between the returns of assets i and j. The number 1.65 corresponds to a 95% Value at Risk when returns are normally distributed, but is not of particular interest for our purposes.

The RM implicitly assumes that USD is the relevant home currency. In case the company uses another home currency, we take the standard deviations and correlations above to be the ones using the home currency as base.[v]

For convenience and with no loss of generality, we translate the above information into a covariance matrix $C_{N\times N}(t)$ where each element $s_{i,j}(t)$ is related to the standard deviations $s_i(t)$ and $s_j(t)$ and the correlation $r_{i,j}(t)$ above as follows

(1) $\quad s_{i,j}(t) \equiv s_i(t) s_j(t)\, r_{i,j}(t) \quad$ for all i, j.

By definition, $s_{i,i}$ corresponds to the return variance of asset i. Furthermore, observe from Equation (1) that $s_{i,j} = s_{j,i}$, i.e., the covariance matrix $C_{N\times N}(t)$ is symmetric around the diagonal.

Hence, we may disregard the elements northeast of the diagonal, and write the RM covariance matrix as

$$C_{N\times N}(t)\equiv\begin{pmatrix} s_{1,1}(t) & & \\ \vdots & \ddots & \\ s_{N,1}(t) & \cdots & s_{N,N}(t)\end{pmatrix}.$$

Now, suppose that we want to extend the RM universe by n additional asset. This leads to an extended covariance matrix, $C_{(N+n)\times(N+n)}(t)$, defined by

$$C_{(N+n)\times(N+n)}(t)\equiv\begin{pmatrix} C_{N\times N}(t) & \\ \tilde{C}_{N\times n}(t) & \tilde{C}_{n\times n}(t)\end{pmatrix},$$

where $\tilde{C}_{N\times n}(t)$ is the covariance matrix between the RM universe and the additional assets, i.e.,

$$\tilde{C}_{N\times n}(t)\equiv\begin{pmatrix} \tilde{s}_{N+1,1}(t) & \cdots & \tilde{s}_{N+1,N}(t) \\ \vdots & \ddots & \vdots \\ \tilde{s}_{N+n,1}(t) & \cdots & \tilde{s}_{N+n,N}(t)\end{pmatrix},$$

and $\tilde{C}_{N\times n}(t)$ is the covariance matrix of the additional assets, i.e.,

$$\tilde{C}_{n\times n}(t)\equiv\begin{pmatrix} \tilde{s}_{N+1,N+1}(t) & & \\ \vdots & \ddots & \\ \tilde{s}_{N+n,N+1}(t) & \cdots & \tilde{s}_{N+n,N+n}(t)\end{pmatrix}.$$

3. EWMA zero-mean covariance estimator

The RM covariance estimation based on the exponentially weighted moving average (EMWA), where geometric weights are assigned to each observation. In addition, RM considers the return deviations from zero, i.e., as if returns have zero mean.

To formalize, consider the following historical return time series of assets i and j

Table 1: Infinite return series

Day	Return asset *i*	Return asset *j*
$\vdots$	$\vdots$	$\vdots$
$t-k$	$X_i(t-k)$	$X_j(t-k)$
$\vdots$	$\vdots$	$\vdots$
$t-2$	$X_i(t-2)$	$X_j(t-2)$
$t-1$	$X_i(t-1)$	$X_j(t-1)$
t	$X_i(t)$	$X_j(t)$

The zero-mean covariance EWMA estimator for the returns on assets i and j is defined by[vi]

(2) $$s_{i,j}(t) \equiv (1-l)\sum_{k=0}^{\infty} l^k X_i(t-k)X_j(t-k),$$

where $0 < l < 1$.[vii] In Equation (2) above, $(1-l)l^k$ represents the positive weight assigned to the observation at day $(t-k)$, where the weights add up to unity.[viii] We may interpret $X_i(t)$ and $X_j(t)$ as the return deviation from zero for assets i and j, i.e., the deviation *as if* both assets have zero expected return (zero mean).

It can be shown that the EWMA estimate of Equation (2) leads to[ix]

(3) $$s_{i,j}(t) = (1-l)X_i(t)X_j(t) + l\,s_{i,j}(t-1),$$

which may be interpreted as a daily updating rule for the estimated covariance. On the other hand, note that if the two covariance estimates are known, Equation (3) can be used to infer (partial) information on the returns $X_i(t)$ and $X_j(t)$.

4. How to infer return from the RM reports

One of our basic premises is that we can observe the return of at least one RM asset, say asset $i = 1$. Examples of such asset classes are the S&P-500 stock index, the euro/dollar rate, or the 1-year U.S. zero-coupon yield.

From the RM daily reports, we know the current variance estimate $s_{i,i}(t)$ as well as yesterdays estimate $s_{i,i}(t-1)$ of the RM assets. It follows from Equation (3) that the absolute return deviation (from zero) for asset i is

(4) $$|X_i(t)| = \sqrt{\frac{1}{1-l} s_{i,i}(t) - \frac{l}{1-l} s_{i,i}(t-1)} \quad \text{for} \quad i = 2, \cdots, N.$$

Consequently, we can use Equation (4) to determine the absolute value of returns, $|X_i|$, of the RM assets from the diagonals of the two RM covariance matrices $C_{N\times N}(t-1)$ and $C_{N\times N}(t)$.

The remaining problem is to determine the sign of the returns of these RM assets. Insert $j = 1$ in Equation (3), and rearrange to obtain

(5) $$(1-l) X_i(t) X_1(t) = s_{i,1}(t) - l\, s_{i,1}(t-1) \quad \text{for} \quad i = 2, \cdots, N.$$

With $0 < l < 1$, it follows immediately from Equation (5) that if the right hand side is positive, $X_i(t)$ and $X_1(t)$ must have the same sign, and opposite signs otherwise.[x]

Consequently, we have demonstrated that the return $X_i(t)$ at day t on RM asset $i = 2, \cdots, N$ can be inferred from the covariance matrices $C_{N\times N}(t)$ and $C_{N\times N}(t-1)$ and the observed return $X_1(t)$. It follows immediately that for a given period, we may reconstruct the entire RM underlying return time series from the daily covariance reports and the observed return series on at least one of the RM asset classes. [xi]

5. How to approximate the covariance from a finite time return series

Armed with the time series of inferred RM returns and the observed additional asset returns, the problem boils down to estimating the covariance between returns of each RM and additional assets, as well as the covariance between returns of each pair of additional assets, i.e., the matrices $\tilde{C}_{N\times n}(t)$ and $\tilde{C}_{n\times n}(t)$. In section 3 above, we implicitly assumed an infinite time series. Assume the following finite daily return time series from day 1 to day t

Table 2: Finite return series

Day	Return asset *i*	Return asset *j*
1	$X_i(1)$	$X_j(1)$
2	$X_i(2)$	$X_j(2)$
$\vdots$	$\vdots$	$\vdots$
$t-2$	$X_i(t-2)$	$X_j(t-2)$
$t-1$	$X_i(t-1)$	$X_j(t-1)$
t	$X_i(t)$	$X_j(t)$

From equation (2), it follows that[xii]

(6) $$s_{i,j}(t) \equiv (1-l)\sum_{k=0}^{t-1} l^k X_i(t-k) X_j(t-k) + l^t s_{i,j}(0),$$

where the weights assigned to the observations from day 1 to day t add up to the sum

(7) $$(1-l)\sum_{k=0}^{t-1} l^k = 1 - l^t.$$

However, the initial covariance $s_{i,j}(0)$ in Equation (6) is unknown. We suggest that the zero-mean covariance is approximated by

(8) $$\tilde{s}_{i,j}(t) \equiv (1-l)\sum_{k=0}^{t-1} l^k X_i(t-k) X_j(t-k) .$$

The approximation in Equation (8) may be interpreted as using the daily updating rule of Equation (6) above, where the "history" at the initial day is taken to be $s_{i,j}(0) = 0$, i.e., as if returns before that day were riskless (no deviation from zero). Consequently, the covariance estimator $\tilde{s}_{i,j}(t)$ suggested in Equation (8) represents a downward biased approximation to the "true" (but unknown) covariance estimator $s_{i,j}(t)$.

Note, however, that at day t, the weight l^t is assigned to the initial covariance $s_{i,j}(0)$. Hence, the downward bias tends to wash out with the number of observations. As an illustration, note that with a weight of $l = 0.94$ (used by RM) and 100 daily observations, the history preceding this 100-days period is assigned a weight of $l^{100} \approx 0.0021$. Now, if the return in the two periods were equally risky, the approximated covariance estimator would be downward biased by a factor[xiii] $(1- l^{100}) \approx 0.9979$.

It can be shown[xiv] that with a finite time series, the approximated covariance estimate above translates into the following daily updating rule

(9) $$\tilde{s}_{i,j}(t) = (1-l) X_i(t) X_j(t) + l\,\tilde{s}_{i,j}(t-1) ,$$

which is similar to the updating rule of Equation (3) above. Hence, yesterdays approximated covariance and the current (observed or reconstructed) returns represent sufficient information for updating the approximated covariance.

6. The extended RM market risk universe

The final step is to apply the identities

(10) $\tilde{s}_i(t) = \sqrt{\tilde{s}_{i,i}(t)}$ for $i = N+1, \cdots, N+n$,

(11) $\tilde{r}_{i,j}(t) = \dfrac{\tilde{s}_{i,j}(t)}{\tilde{s}_i(t)\tilde{s}_j(t)}$ for $j = 1, \cdots, i-1$ and $i = N+1, \cdots, N+n$,

and translate the estimated covariances of the extended covariance matrix $C_{(N+n)\times(N+n)}(t)$ into the following extended Value at Risk vector $V_{N+n}(t)$

$$V_{N+n}(t) \equiv \left(1.65\, s_1(t) \quad \cdots \quad 1.65\, s_N(t) \quad 1.65\tilde{s}_{N+1}(t) \quad \cdots \quad 1.65\tilde{s}_{N+n}(t)\right),$$

and the extended correlation matrix $R_{(N+n)\times(N+n)}$

$$R_{(N+n)\times(N+n)}(t) = \begin{pmatrix} 1 & & & & & \\ \vdots & \ddots & & & & \\ r_{N,1}(t) & \cdots & 1 & & & \\ \tilde{r}_{N+1,1}(t) & \cdots & \tilde{r}_{N+1,N}(t) & 1 & & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \\ \tilde{r}_{N+n,1}(t) & \cdots & \tilde{r}_{N+n,N}(t) & \tilde{r}_{N+n,N+1}(t) & \cdots & 1 \end{pmatrix}.$$

This information can now be used, in combination with the risk exposure of the company with respect to each asset class, to calculate the total downside risk of the company.

6. Conclusion

In this paper, we discuss a method for including additional assets in the RM concept, i.e., extending the RM asset universe. The idea is to use the daily RM reports and the observed return time series of at least one of these assets, to reconstruct the entire set of underlying return time series. The reconstructed return time series and the observed return time series of the additional assets are thereafter used to estimate the desired volatilities and correlations. Once an additional asset has been included, a simple rule applies for updating the estimates as new information arrives in terms of RM reports and returns on the additional assets and at least one RM asset.

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[i] For instance the recommendations of the Basel Committee (Bank for International Settlements), see http://www.bis.org

[ii] For a discussion of the Value at Risk concept, see for instance Chapter 14 in Hull (2000) or Jorion (1997).

[iii] Documentation is available at http://www.riskmetrics.com

[iv] In fact, this research was initiated by discussions with the largest Norwegian bank, DnB.

[v] For a consistent translation of standard deviations and correlations, see Singer, Terhaar and Zerolis (1998).

[vi] See the RiskMetrics™ Technical Document or Chapter 15 in Hull (2000).

[vii] RM uses the weight $l = 0.94$.

[viii] Formally, $\sum_{k=0}^{\infty}(1-l)\,l^{k}=1$ when $0<l<1$.

[ix] Obtain Equation (3) from Equation (2) as follows:

$$
\begin{aligned}
s_{i,j}(t) &= (1-l)\sum_{k=0}^{\infty} l^{k} X_i(t-k)X_j(t-k) \\
&= (1-l)\left[X_i(t)X_j(t) + lX_i(t-1)X_j(t-1) + l^{2}X_i(t-2)X_j(t-2)+\cdots\right] \\
&= (1-l)X_i(t)X_j(t) \\
&+ l\left\{(1-l)\left[X_i(t-1)X_j(t-1) + lX_i(t-2)X_j(t-2)+\cdots\right]\right\} \\
&= (1-l)X_i(t)X_j(t) + l\left\{(1-l)\sum_{k=0}^{\infty} l^{k} X_i(t-1+k)X_j(t-1+k)\right\} \\
&= (1-l)X_i(t)X_j(t) + l\,s_{i,j}(t-1)
\end{aligned}
$$

[x] To formalize, the inferred return is

$$X_i(t)=\operatorname{sgn}\left(X_1(t)\right)\cdot\operatorname{sgn}\left(s_{i,1}(t)-l\,s_{i,1}(t-1)\right)\cdot\left|X_i(t)\right| \quad \text{for} \quad i=2,\cdots,N$$

where $\operatorname{sgn}(\cdot)$ denotes the sign function.

[xi] As demonstrated above, one observed return series is sufficient to determine the signs. In practice, problems due to zero return and missing observations may arise, hence several RiskMetrics™ return series should be observed (if possible).

[xii] Obtain Equation (6) from Equation (2) as follows:

$$
\begin{aligned}
s_{i,j}(t) &\equiv (1-l)\sum_{k=0}^{\infty} l^{k} X_i(t-k)X_j(t-k) \\
&= (1-l)\sum_{k=0}^{t-1} l^{k} X_i(t-k)X_j(t-k) + (1-l)\sum_{k=t}^{\infty} l^{k} X_i(t-k)X_j(t-k) \\
&= (1-l)\sum_{k=0}^{t-1} l^{k} X_i(t-k)X_j(t-k) + l^{t}(1-l)\sum_{k=0}^{\infty} l^{k} X_i(0-k)X_j(0-k) \\
&= (1-l)\sum_{k=0}^{t-1} l^{k} X_i(t-k)X_j(t-k) + l^{t}s_{i,j}(0)
\end{aligned}
$$

Next, obtain the identity stated in Equation (7) by considering the two equations

$$\sum_{k=0}^{t-1} l^k = 1 + l + l^2 + \ldots + l^{t-2} + l^{t-1},$$

$$l \sum_{k=0}^{t-1} l^k = l + l^2 + \ldots + l^{t-2} + l^{t-1} + l^t.$$

Subtract the two equations, and rearrange, to obtain the desired result.

xiii Combine Equations (6) and (8) to obtain $s_{i,i}(t) = \tilde{s}_{i,i}(t) + l^t s_{i,i}(0)$, insert $s_{i,i}(0) = s_{i,i}(t)$, and rearrange to obtain the factor $\tilde{s}_{i,i}(t) / s_{i,i}(t) = (1 - l^t)$.

xiv We obtain Equation (9) from Equation (8) as follows:

$$\begin{aligned}
\tilde{s}_{i,j}(t) &\equiv (1-l)\sum_{k=0}^{t-1} l^k X_i(t-k) X_j(t-k) \\
&= (1-l) X_i(t) X_j(t) + (1-l)\sum_{k=1}^{t-1} l^k X_i(t-k) X_j(t-k) \\
&= (1-l) X_i(t) X_j(t) + l(1-l)\sum_{k=0}^{t-2} l^k X_i((t-1)-k) X_j((t-1)-k) \\
&= (1-l) X_i(t) X_j(t) + l\tilde{s}_{i,j}(t-1)
\end{aligned}$$