Assimilation of real time series data into a dynamic bioeconomic fisheries model: An application to the Norwegian cod fishery stock

Al-Amin M. Ussif[1], Leif K. Sandal[2] and Stein I. Steinshamn[3]

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Abstract

This paper combines the new and elegant technique of inverse methods and a Monte Carlo procedure to analyze real data for the Norwegian cod fishery (NCF) stock. A simple nonlinear dynamic resource model is calibrated to real time series of observations using the adjoint parameter estimation method of data assimilation and the Monte Carlo technique. By exploring the efficient features of the adjoint technique coupled with the Monte Carlo method, optimal or best parameter estimates together with their error statistics are obtained. Thereafter, the weak constraint formulation resulting in a stochastic ordinary differential equation (SODE) is used to find an improved estimate of the dynamical variable(s). Empirical results show that the average fishing mortality imposed on the NCF stock is 16 % more than the intrinsic growth rate of the biological species.

[1]Corresponding author: Department of Finance and Management Science, Norwegian School of Economics and Business Administration, Bergen Norway. Email:ussif@hamilton.nhh.no.

[2]Department of Finance and Management Science, Norwegian School of Economics and Business Administration, Bergen Norway.

[3]Foundation for Research in Economics and Business Administration, Bergen Norway.

1 Introduction

Two important sources of information to bioeconomists and other researchers are the model on one hand and the data on the other. The model is an embodiment of the scientific beliefs of the researcher. Mathematical or numerical models have been used extensively by economists to gain useful insights in the analysis of natural resource problems (Clark, 1990; Hannesson, 1993; Sandal and Steinshamn, 1997). The other source is the observations obtained from field measurements. Unfortunately, this vital source has not been fully exploited thus far. Advanced and efficient techniques of combining these two sources of information need to be developed. This paper employs the technique of data assimilation and inverse methods (Bennett, 1992) in which all the available information is used in the analysis of the Norwegian cod fishery (NCF) stock.

Inverse methods are a set of methods employed to extract useful inferences about the real world from measurements. In other words inverse methods can be defined as a set of mathematical techniques for reducing data to acquire useful information about the physical world on the basis of inferences drawn from observations (Menke, 1984). In data assimilation, observations are merged together with a dynamical model in order to determine, as accurately as possible, a description of the state of the system. It can be used to estimate the variables of the dynamical model and/or the parameters of the model. It also leads to the resolution of mathematically ill-posed modeling problems (Bennett, 1992).

In general, there are two forms of assimilation, sequential assimilation and variational assimilation. In sequential assimilation, the model is integrated forward in time and the model solution updated whenever measurements are available. A typical example is the Kalman filter (Kalman, 1960; Gelb, 1974) which is an optimal algorithm for linear dynamics. Variational assimilation, on the other hand aims at globally adjusting a model solution to observations available over the assimilation time interval. Two different formalisms exist in variational methods: the method of strong constraint popularly known as the adjoint method and the method of weak constraint which is related to the penalty methods (Smedstad and O'Brien, 1991). The strong constraint formulation is shown to be the limiting case of the latter where the model is assumed to be perfect.

In this paper, a variational inverse formulation will be employed to estimate the generalized inverse of the stock and the poorly known input parameter(s) of a bioeconomic fisheries model. This technique has been applied with success in parameter estimation in an Ekman flow model (See Eknes and Evensen, 1997; Yu and O'Brien, 1991).

During the last quarter of the century, many important developments have taken place which have affected the structural setup of fisheries management. An important example is the U.N. law of the sea in the late 1970s. The law resulted in the Extended Fisheries Jurisdiction (EFJ) from maximum of 12 to 200 nautical miles, for coastal states. It empowered, for example, Norway to manage the Barent sea cod together with Russia, Iceland and The Faroe Islands. This calls for annual quotas being determined a priori at the inception of each fishing season. This paper will aim at addressing the question of quota determination. The recent influx of data from both fisheries biologists and economists due to improved observational and measurement methods necessitate the development of techniques in which as much information as possible can be extracted.

Inverse methods and data assimilation methods have broad application and a wide range of advantages which is demonstrated by its extensive usage in operational meteorology, oceanography and other fields. These advantages can be explored in bioeconomics to a great extent. First, it can be used to analyze the incoming data to extract useful information which will lead to important policy implications about the operation of a fishery. This in effect will help answer some of the unanswered questions in this area.

Traditionally, data assimilation and inverse methods are used to estimate variables of dynamical models, using all the available information from the model and the information about the true state from the data. However, these techniques have also been proposed as a tool for parameter estimation in dynamical models (Evensen et al., 1998). The basic idea is that it should be possible to use mathematical tools to formulate inverse problems for parameter estimation given additional information in a form of measurement data. Thus, one may attempt to search for model parameters resulting in a model solution that is closest to the observations. Notice here that this technique is new and has obvious advantages compared with the traditional methods. The technique can be applied with equal force to both an open access fishery and the sole owner fishery. It is highly suitable for complex and high dimensional problems. Multidimensional fisheries models are more realistic as ecosystem effects may be incorporated.

Parameter estimation has been used extensively in economics and other fields. However, very few studies have so far been reported in which the techniques in this paper have been used. For all we know, no such study has been made in fisheries bioeconomics. The reason may be attributed to lack of data and computer power in the past.

The purpose of this study is threefold. First to introduce the powerful tool of data assimilation in the management of renewable resources such as fisheries. Second, to exploit the elegant and efficient properties of the inverse methods in order to extract the best information from the available measurements. Third, to estimate the parameters of the growth and production relations and thereby estimate the stock and harvest quotas under a dynamic constraint.

The remainder of the paper is structured as follows. Section two presents the general formulation of the inverse methods and discusses the null hypotheses. In section three, the estimator is defined and a comprehensive discussion of the solution method presented. The least squares method is used to define a scalar objective functional emphasizing the link between this technique and the theory of statistical estimation. In the fourth section, a simple bioeconomic model is defined. The biological base is the Schaefer growth function. It is tied to the economics by a catch per unit effort type of production function. Section five is a historical discussion of the cod fishery and a sensitivity analysis of the parameters of the model. Finally, the results are presented in section six with an equilibrium analysis using the estimated parameters.

2 The Variational Inverse Formulation

A variational inverse problem can be formulated as either a strong constraint problem where the model is assumed to be perfect, i.e., the model holds exactly or the weak constraint formalism (Sasaki, 1970) where the model is allowed to contain errors. In modeling a system, several assumptions are often made both for mathematical convenience and tractability. Several uncertain inputs are also used in the model resulting in a model that approximately represents the real system. Modeling errors are unavoidable in many situations. Thus, adding a term to the model that quantifies the errors makes the model more realistic. It is sometimes a common practice among some researchers to assume a model that is perfect then vary some of the free parameters such as the initial conditions of the model in order to find the solution which best fit the data (Yu and O'Brien, 1992). Such a formulation is known as the strong constraint problem. It is shown that the strong constraint problem is a limiting case of the weak constraint problem (see Bennett, 1992).

In this paper, the adjoint technique will be employed to fit the dynamic model to the observations. We then use the estimated parameters in an inverse calculation using the weak constraint formulation. In the first problem the control variables are the input parameters. Using the adjoint method the gradients of the cost functional with respect to the control variables are efficiently calculated through the use of the Lagrange multipliers. The gradients are then used to find the parameters of the model dynamics which best fit the data. In the second case however, the model variables are the control parameters. The gradients of the variables at each grid point are calculated and the values used to search for the minimum of the cost functional (see Bennett, 1992).

2.1 The data and the model

To formulate the problem, a general nonlinear scalar dynamic model together with the initial condition is defined as

$$\frac{dx}{dt} = g(\beta; x) + q(t) \qquad (1)$$

$$x(0) = u + a \qquad (2)$$

$$\beta = \beta_0 + \hat{\beta} \tag{3}$$

where g is a nonlinear operator, β is a parameter(s) to be estimated and is assumed poorly known. The terms a, $\hat{\beta}$ and $q(t)$ are random white noise terms and are defined as the errors in the first or best guess of the initial condition (u), the parameter(s) (β_0) and the model formulation respectively. Such a formulation is referred to as the weak constraint general inverse problem (Evensen et al., 1998). The task involves solving for the optimal dynamical variables while updating the model parameters. The result is a solution of the model that is closest to the observations and simultaneously satisfies the model constraints approximately. If the errors in the initial condition and/or the model formulation are assumed to vanish identically, i.e., $a \equiv 0$ and $q \equiv 0$, then we retrieve the strong constraint parameter estimation problem.

The model is one source of information which in general is the physical laws governing the system, e.g., the population dynamic model of the Schaefer (1964) type. Additional available information is the set of observations given by

$$\mathbf{d} = \mathcal{H}[x] + \epsilon \tag{4}$$

where $\mathbf{d}$ is the measurement vector, $\mathcal{H}$ is a linear operator that relates the observations to its model counterpart and ϵ is the vector of measurement errors. The errors may be due to instrumental imprecision and from other sources.

2.2 Some statistical assumptions

To describe the errors in the model, the data and the parameters, we require some statistical hypotheses. For our purpose in this paper the following hypotheses will suffice

$$\bar{q}(t) = 0, \qquad \overline{q^T q} = w_q^{-1}$$

$$\bar{a} = 0, \qquad \overline{a^2} = w_a^{-1}$$

$$\bar{\epsilon} = 0, \qquad \overline{\epsilon^T \epsilon} = w^{-1}$$

$$\bar{\hat{\beta}} = 0, \qquad \overline{\hat{\beta}^T \hat{\beta}} = w_\beta^{-1}$$

where the scalars $w's$ are the weights and the T denotes matrix transpose operator. That is, we are assuming that the errors are normally distributed with zero means and constant variances (homoscedastic) which are ideally the inverses of the optimal weights. The assumption of unbiasedness is very common in the literature (see Bennett, 1992). The overbar denotes the mathematical expectation operator. It will be, however, more realistic to make the variances more general by allowing cross-variances, but this will not be used in this paper. The linear measurement operator may be defined as

$$\mathcal{H}_i[x] = \int_0^{T_f} x(t)\delta(t - T_i)dt = x(T_i) \tag{5}$$

where T_i is the measurement location in time, T_f is the time horizon, δ is the Dirac delta function and i denotes a component of the measurement functional which is a vector with dimension equal to the number of observations. In the subsequent sections, we shall present a simple but detailed discussion of the strong and the weak constraint formalisms.

3 The Least Squares Estimator

In data assimilation, the goal is to find a solution of the model which is as close as possible to the available observations. Several estimators exist for fitting models to data. In this paper, we seek residuals that result in model prediction that is in close agreement with the data. Hence the fitting criterion is the least squares loss function which is the sum of the model, data, initial residuals and parameter misfits. This is given by

$$\mathcal{J} = w_\beta \hat{\beta}^T \hat{\beta} + w\epsilon^T \epsilon + w_q \int_0^{T_f} q(t)^2 dt + w_a (x(0) - u)^2 \tag{6}$$

where w_β, w_q, w_a and w are scalar constants. We have thus formulated a nonlinear unconstrained optimization problem. The last two terms in (6) are penalty terms on the dynamics and the initial condition respectively.

To derive the strong constraint problem as a special case, define $\lambda = w_q q$ and $\lambda_a = w_a a$ where $q \equiv 0$ and $a \equiv 0$, i.e., both the dynamics and the initial condition are perfect. This is equivalent to assigning infinitely large weights to the dynamics and the initial condition. The cost functional reduces to

$$\mathcal{J}_s = w_\beta \hat{\beta}^T \hat{\beta} + w\epsilon^T \epsilon \tag{7}$$

where $\mathcal{J}_s$ is the cost function for the strong constraint problem. Inserting λ and λ_a in (6) we obtain the Langrage functional for the adjoint method. The necessary condition for an optimum (local) is that the first variations of the cost function with respect to (wrt) the controls vanish $\partial \mathcal{J} = 0$.

There are many efficient algorithms for solving unconstrained optimization problems

(Luenberger, 1984). The once used most are the classical iterative methods such as the gradient descent, the quasi-Newton and the Newton methods. These methods require the derivatives of the cost functional and the Hessian for the case of the Newton methods. However, nonconventional methods could be used. For example, methods of optimization without derivatives and statistical methods such as simulated annealing could be used to find the minimum of the cost functional at a greater computational cost. Their advantage is that a more general cost functional including discontinuous functions could be used. The inherent problem of local solutions in the line search methods is said to be absent in simulated annealing (see Goffe et al., 1992; Matear, 1995).

In order to make the paper accessible to more readers we avoid the mathematical and computational details but give a comprehensive verbal explanation of the methods.

One approach of solving the inverse problem is to derive the Euler-Lagrange (E-L) systems of equations and solve them. The E-L systems derived from calculus of variations or optimal control theory (see Kamien and Schwartz, 1980) are generally coupled and nonlinear and require simultaneous integration of the forward and the adjoint equations. The task easily becomes arduous and very often impractical. Such a procedure is called the integrating algorithm. In the adjoint formulation, the assumption of a perfect model leads to the decoupling of the E-L equations. The forward model is then integrated followed by the backward integration of the backward equations. For the weak constraint inverse problem, the approach here avoids solving the forward and backward models but uses the gradient information to efficiently search for the control variables that minimize the loss function subject to the constraints. Given the cost functional, which is assumed to be continuous with respect to the controls, find the derivatives wrt the controls and then use the gradients to find the minimum of the cost function. The second procedure is referred to as the substituting algorithm and is generally efficient in finding the local minimum. In the case of the adjoint method, the algorithm is as follows:

- Choose the first guess for the control parameters.
- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the cost function.
- Integrate the adjoint equation backward in time forced by the data misfits.
- Calculate the gradient of $\mathcal{J}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control parameters which make the cost function move towards a minimum.
- Check if the solution is found based on a certain criterion.
- If the criterion is not met, repeat the procedure until a satisfactory solution is found.

The solution algorithm for the weak constraint inverse problem is similar except that the gradients are not calculated from the backward integration of the adjoint equations but are obtained directly by substitution. The procedure is outlined below.

- Choose the first guess for the control variables.
- Calculate the misfits and hence the cost function.
- Calculate the gradient of $\mathcal{J}$ with respect to the control variables.

- Use the gradient in a descent algorithm to find an improved estimate of the control variables which make the cost function move towards a minimum.

- Check if the solution is found based on a certain criterion.

- If the criterion is not met repeat the procedure until a satisfactory solution is found.

4 The Bioeconomics

Fisheries management and bioeconomic analysis have been given considerable attention in the last two decades. Fisheries economists have for the past years combined biological and economic theory to understand and address management issues concerning the most important renewable resource stock, i.e., the fish. Questions about efficient exploitation and conservation measures are being raised both in the academic literature and in the media.

The mainstay of bioeconomic analysis is the mathematical models. In this paper we advance a little further by combining information both from the theoretical model of a fishery and the actual field observations. In formulating the bioeconomic model, we require a reasonable biological submodel as a basis. Following the tradition in the literature, we propose an aggregated growth model of the Schaefer (1964) type. Let x denote the total stock biomass and h denote the rate of harvesting from the stock. We represent the dynamics of the stock as

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) - h \tag{8}$$

where r, K are the intrinsic growth rate per unit time and the environmental carrying capacity in 10^3 tons respectively. The growth law for this fishery is assumed to follow the logistic law (Schaefer, 1964). The dynamics of the stock depends on the interplay between terms on the right hand side of the equation. The stock will increase if h is less than the growth term and decreases if h is greater. If human predation ceases, i.e., $h = 0.0$ then the stock will increase at a rate equal to the natural growth of the stock. The stock biomass will increase towards the maximum population size K. This simple model describes a year-class model of the Gordon-Schaefer type. It basically describes the dynamics of an exploited fishery by linking the biological dynamics and the economics through the general production function $h(t)$.

4.1 The production function h

In this paper the general Cobb-Douglas production function $h(e, x)$ is defined as

$$h = qe^b x^c \tag{9}$$

where e is the fishing effort, q, b and c are constants. The production function quantifies the rate of production of the industry and describes how the inputs are combined in the production process. It depends on two important inputs, the stock biomass and the level of effort expended in fishing. In the fisheries economics literature it is often assumed that harvest is linear in effort and stock level, i.e., $b = c = 1$. The harvest function then reduces to $h = qex$ where q is the catchability coefficient. This results in the catch per unit effort which is proportional to the biomass. Several implicit assumptions underly the hypothesis including uniform distribution of fish, etc. The natural way to link the biology and economics of fishing is through the mortality parameter or fishing intensity rate f. Where $f = qe$ is generally a function of time. In this paper we will specialize a bit by assuming a nonvarying f over time. That is a constant fishing mortality rate or proportional removal rate of the standing stock policy is applied. This yields a simple harvest law which can be used by the management authorities to set total allowable catch quotas (TAC).

To understand the nature and kind of policy used in the management of the NCF, we apply a simple feedback relation to analyze the data. The assumption may be unrealistic, but we still hope much practical insight will be gained and will lead to better understanding of the fishery. Thus, the harvest function for the linear case is

$$h = fx \tag{10}$$

where f is the unknown, or poorly known economic parameter, to be estimated. This formulation appears quite simple but may be of immense contribution to our understanding of the practical management of the NCF. It can be considered as a first order linear approximation of the true harvest function. The function proposed is by no means supposed to be the complete and absolute characterization of the feedback specifications but is considered as a useful and practical approximation of the true one. To reiterate, our purpose is to be simple and to construct a model that is tractable which will lead to some important policy implications.

4.1.1 Some remarks about the model

Linking the biology of the exploited species and the simple approximate harvesting or TAC rule above yields

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) - fx \tag{11}$$

put in another form gives

$$\frac{dx}{dt} = \gamma x - \frac{rx^2}{K} \tag{12}$$

where $\gamma = (r - f)$ is the difference between the intrinsic growth rate and the fishing mortality rate. Let us call this the residual growth rate of the species. The residual growth rate can be positive, zero or negative at least theoretically. If no fishing mortality is imposed on the stock ($\gamma = r, f = 0.0$) then it grows to its maximum population level K at a rate equal to the natural growth. If f is positive but less than r the population will settle at a level less than K. For the critical scenario where fishing mortality balances the intrinsic growth rate ($\gamma = 0.0$) the population is driven to extinction. This case can be seen mathematically as

$$\frac{dx}{dt} = -\frac{rx^2}{K} \tag{13}$$

It is also the case where f exceeds r and γ becomes negative. The population will be driven to zero even faster. The dynamics are shown as

$$\frac{dx}{dt} = -\gamma x - \frac{rx^2}{K} \tag{14}$$

The predictions of this simple model are evident in the case of most commercial fisheries. Many important fisheries have collapsed in recent times. An example is the Norwegian spring spawning herring (Bjorndal and Munro, 1998).

5 The Norwegian Cod Fishery

The NCF is the most important demersal species along the coast of Norway and Northern Russia. This fishery has played an important economic role within the coastal communities for the past thousand years. The NCF has for the past half century experienced large variations which result in a corresponding variation in the annual harvest quantities. The stock size fell from its highest level in 1946 of 4.1 million tons to the lowest in 1981 of .75 million tons. However, the stock seems to be recovering from the depleted state in the 1990s due to improved management strategies. In this study, a time series of observations from 1946 to 1996 is used. The variables are the annual stock and harvest measured in 10^3 tons. In what follows, we present a brief qualitative description of the data.

Figure 1., is a plot of the stock divided by a factor of three and the harvest. The stock and the harvest have generally a downward trend with periodic oscillations. Apart from the first few years the directions of fluctuation in both the stock and the harvest are the same. It may be observed from the graph that there exists some proportional relationship between the harvest rate and the level of stock.

5.1 Sensitivity analysis

Input parameters of bioeconomic models are crucial in the analysis of the system. To provide good simulations, precise and reasonable parameters are required. Unfortunately, the values of these parameters are highly uncertain which translate into the output of the models. Sensitivity is a measure of the effect of changes in the given input parameter on a model solution. It quantifies the extent that uncertainties in parameters contribute to uncertainties in the model results (Navon, 1997). Several analytical techniques of sensitivity analysis exist. To quantify the uncertainties of the k^{th} parameter, we define the following sensitivity index IS_k

$$IS_k = \frac{\sum_0^T (z_t - z_t^k)^2}{\sum_0^T z_t^2} \tag{15}$$

where z_t is the original model prediction and the z_t^k is the perturbed prediction. The results of the sensitivity of the biological and economic parameters are shown below. The parameters are each perturbed to 90 percent of their original values. These parameters are ranked in an increasing order of importance.

The fishing intensity parameter is the most important and the growth rate is the least. The maximum population of the species is the more sensitive biological parameter which confirms the results of an earlier paper (Ussif et al., 1999a). The results indicate the fishing mortality rate is in fact very critical in the model. This outcome is used in the subsequent experiments to guide us in regard to which parameters to vary and which to give more attention.

6 Results

The empirical results of the research are discussed and shown in this section. All the results are based on actual observations of the NCF for the period from 1946 to 1996. The results of the adjoint parameter estimation are presented. They are followed by the weak constraint inverse results and then a steady state equilibrium analysis is performed.

6.1 Estimation of the growth and yield functions

The combined adjoint-Monte Carlo technique was used to fit the bioeconomic model to the observations assuming that the fishery is exactly governed by the simple model. The model contains three input parameters: the intrinsic growth, the carrying capacity and the human predation coefficient. These are all important to a fisheries manager. Estimating all the parameters at the same time for this simplified model may pose a problem of identification. To obviate the bottleneck, the least sensitive parameter in the model is exogenously but randomly selected and then the other two, namely the carrying capacity and the fishing intensity rate, are optimally determined using adjoint methods. Relying on some physical information from experts, a range of r values between .25 and .45 is chosen. A subsample of 3005 was randomly drawned from the population. Using this sample, the adjoint method is used to find the optimal estimates of the parameters. The statistic of choice in this paper is the mean even though there are other estimators that are efficient. In table 2., we show the parameter estimates and their standard deviations.

These estimates are all reasonable and intuitively appealing. What is astounding is that the model has been able to capture the salient features of the NCF. The estimated rate of capture of the stock exceeds the intrinsic growth rate of the species even when the population was highly vulnerable.

6.2 State estimation of the stock biomass

Inverse methods and data assimilation can be used to estimate the variables of a dynamical system or the parameters of a dynamical model, using all the available information from the model formulation and the set of observations. The former embodies all the beliefs of the modeler about the system she or he is interested in studying. They may use economic and biological theory as well as intuitive reasoning in order to construct a model that approximately represents the system. In the weak constraint formulation, the model dynamics are assumed to approximately hold. The fisheries model employed in this paper is quite oversimplified. Many important variables such as the environmental effects and predation from other species are disregarded. The harvest function is also a simple first order approximation. All these factors make the model quite unrealistic. To remedy this, we accept a certain unknown level of error in the model by adding a term that quantifies the errors and their uncertainties.

A cost functional measuring the disagreements between the data and the model predictions was defined, and a penalty term appended which penalizes the model misfits. A model prediction that is as close as possible to the data, is sought in a least squares sense. The optimization procedure used in this paper is the classical quasi-Newton method (Gilbert and Lemarechal, 1991). The results are shown for two cases. The first case uses the solution of the adjoint method as the first guess solution. That is the parameter estimates from the first method were used to solve the model and the solution is taken as the best guess to start the optimization. To show that the algorithm is robust to the initial guess of the solution, a constant equal to the average of the first case is used as the first guess. The results are shown in the figures below. The circles denote actual data, the broken line is the first guess which is the solution of the strong constraint problem in case 1. The fit of the model is good in general. The algorithm is also robust and did not depend very much on the initial guesses. However, the convergent rate is slightly affected by the choice of the initial guesses.

6.3 Equilibrium analysis using the deterministic model

The use of the population dynamic equation assumes the existence of equilibrium in the model. This section briefly discusses this concept in this application. At the steady state time is no longer important and the stock biomass becomes constant at a level x^*. This implies the time rate of change of the population is identically zero, i.e., the net growth of the stock balances the rate of harvesting

$$\frac{dx}{dt} = 0, \qquad h^* = rx^*(1 - \frac{x^*}{K})$$

It follows then that for the linear harvest function we have

$$fx^* = rx^*(1 - \frac{x^*}{K}) \tag{16}$$

where f is as defined previously. Hence the steady state biomass is

$$x^* = K(1 - \frac{f}{r}) \tag{17}$$

Ideally, the fishing mortality rate should not exceed the intrinsic growth rate of the biological species, i.e, $f < r$ for a fishery that is overexploited and is under rehabilitation. If the fishery is unexploited and initial stock is to the right of the maximum sustained biomass level then higher mortality rates may be applied in order to quickly adjust it to the desired optimal state. The equilibrium stock is a function of the biological and economic parameters. It is clear that if the carrying capacity (e.g., the aquatic environment) increases, x^* will increase and vice versa. The effect of small change in r is similar. However, increasing fishing mortality will result in a decline in the equilibrium biomass.

The concept of maximum sustainable yield has been the practical management objective for many fisheries (Clark, 1990). The NCF is not an exception to the rule. For the compensation model used in this paper, the $x_{msy} = K/2$, i.e., 2634.2 10^3 tons. The figure below shows the historical state of the NCF from 1946 to 1996 and the x_{msy}. A careful study of the time series reveals some interesting observations. The fishery was in 1946 at a level of about 80 percent (4231.9 10^3 tons) of the carrying capacity. It was fished down to about 50 percent (x_{msy}) of K by 1958. It then remained at about that level forming a window until the late 1970s when the situation got completely out of control. However, due to the inherent stochastic nature of the biological species coupled with inadequate knowledge of the biology and economics of fishers/managers, the goal failed to yield results. This occurrence might also be attributed to the shortsightedness of the politicians and also the conflict of interest between the two major participants (Norway and Russia) in the exploitation of the stock.

The state of the stock continued to dilapidate and by 1983 it was at its worst level of less than 20 percent of the carrying capacity. The trend has, however, changed and the 1996 estimate of stock indicates a sign of recovery. Recent observations, however, indicate that the stock is again in deep trouble.

6.4 Conclusion

This paper uses a novel approach of data assimilation into dynamical models to analyze real data for the NCF. Model parameters were estimated by the adjoint technique in combination with a Monte Carlo procedure. The adjoint technique provides an efficient way of calculating the gradients of the loss functional with respect to the control parameters. The estimates are as expected, the fit to the data is also good with the model amazingly capturing the trend in the data but failing to capture the oscillations. This is not surprising because the model is deterministic and does not have the ability to absorb the random events in the system. The estimated parameters are then used in an inverse calculation to find an improved estimate of the stock using the full information available in the form of observations and the model dynamics. The weak constraint model however does very well in capturing the stochasticity in the data.

The key results of the paper are that for the NCF the average intrinsic growth rate is about 0.35 per year and the maximum population that the environment can support is about 5.3 million tons. The fishing intensity rate is about 0.41 per year which is greater than the intrinsic growth rate. This implies the annual harvest or production from the fishery is consistently above the net growth curve. This is intuitively supported by the persistent decline of the stock since 1946. It is important to be reserved in generalizing the findings in this paper. The reason is that the model used in this paper is very simple and does not absolutely represent the fishery. Finally, the inverse and data assimilation methods have proven very efficient and can be very useful in analyzing, testing and improving resource models.

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Parameters	Original values	New values	IS_k
r	0.450	0.405	1.50
K	6000.0	5400.0	1.68
f	0.400	0.360	5.09

Table 1: Sensitivity index of model parameters.

Parameters	r	K	f
Estimates	0.3499	5268.4	.4076
se	(0.0578)	(868.3)	(0.0579)

Table 2: Estimated parameters and their standard deviations.

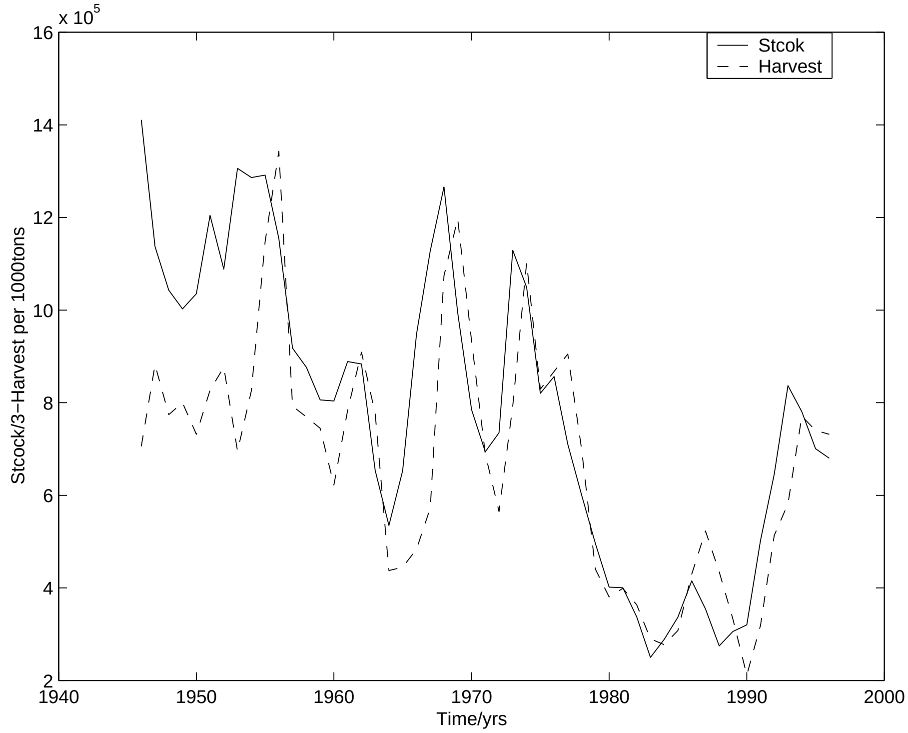

x 10^5
16
14
12
10
8
6
4
2
1940
1950
1960
1970
1980
1990
2000
Stcok
Harvest
Stcock/3−Harvest per 1000tons
Time/yrs

Figure 1:

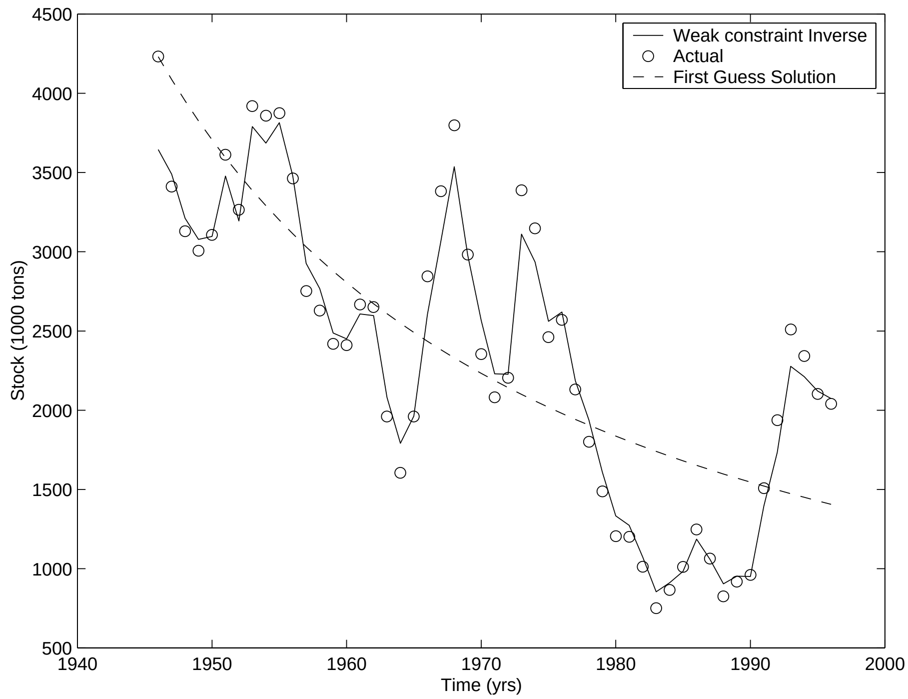
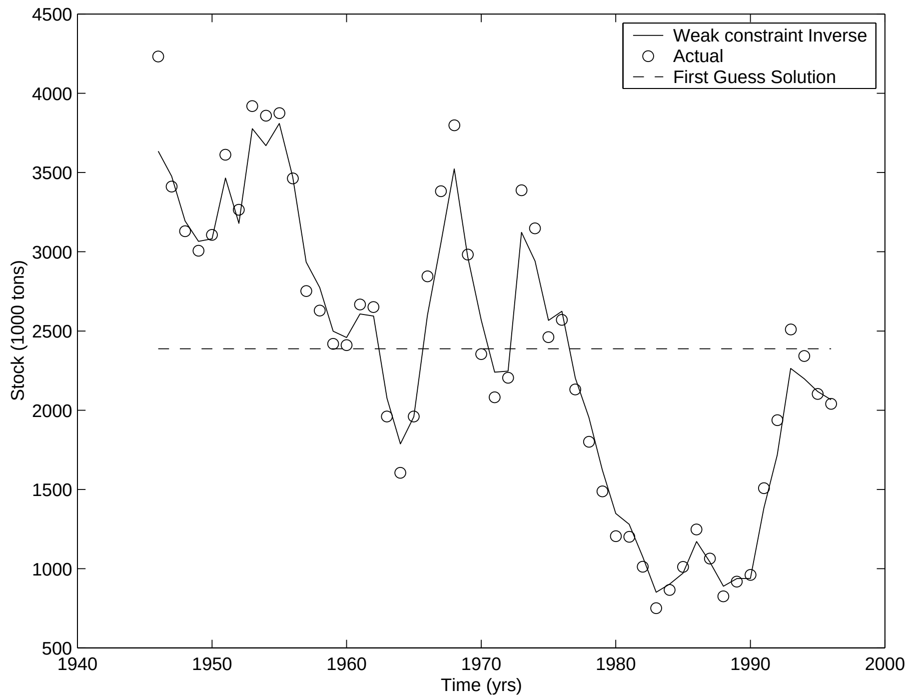
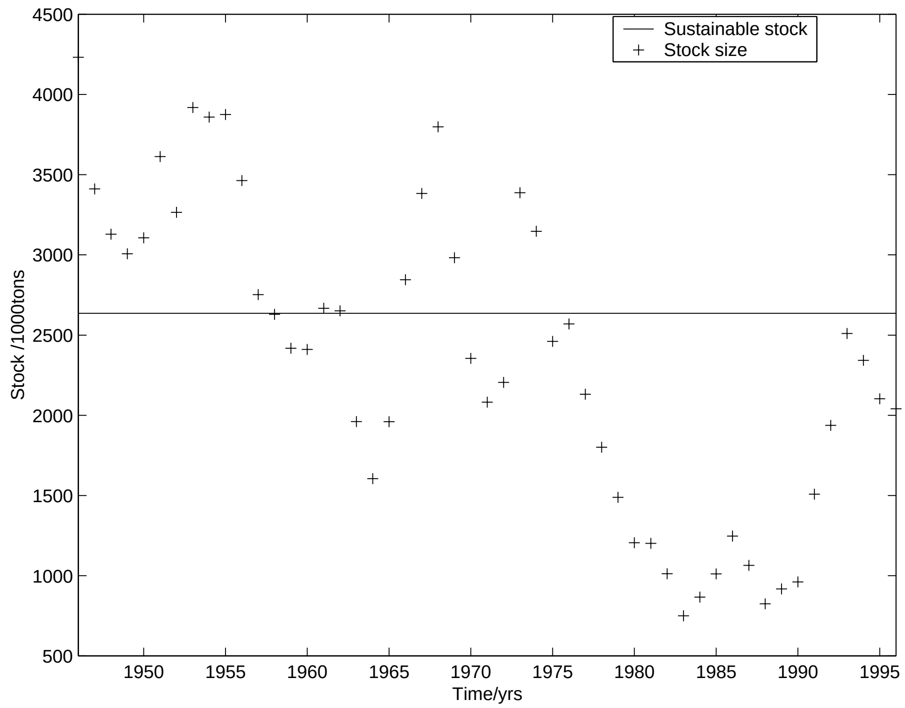

Weak constraint Inverse
Actual
First Guess Solution
Stock (1000 tons)
Time (yrs)
4500
4000
3500
3000
2500
2000
1500
1000
500
1940
1950
1960
1970
1980
1990
2000

Figure 2:



Weak constraint Inverse
Actual
First Guess Solution
Stock (1000 tons)
Time (yrs)
4500
4000
3500
3000
2500
2000
1500
1000
500
1940
1950
1960
1970
1980
1990
2000

Figure 3:



Sustainable stock
Stock size
Stock /1000tons
Time/yrs
4500
4000
3500
3000
2500
2000
1500
1000
500
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995

Figure 4:

Lists of graphs

1. Time series plot of the stock and harvest rates of the NCF.

2. Graph of the stock biomass: case 1.

3. Graph of the stock biomass: case 2.

4. Plot of the stock and harvest rates (tons) vs. time (yrs) .