Modeling the dynamics of regulated resource systems
A fishery example

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Abstract

In this paper we develop a discrete-time model of jointly managed fisheries. Most modern real world fisheries are shared or jointly exploited and are under some kind of regulation. The regulatory part of the fishery in this paper is partitioned into two stages. In the first stage, which is our main focus, total allowable catch quotas (TACs) based on biological and/or economic considerations are determined in a way that guarantees the safety of the stock from a conservation viewpoint. In addition, we assume that a target biomass level is set by the management authorities to be achieved over a given time horizon to satisfy an economic objective. We propose a gradual approach to the target biomass level via a simple recursive rule.

Keywords: discrete-time, jointly managed, target biomass level, TACs.

JEL classification: Q22

1 Introduction

A survey of the literature in fisheries economics leaves one with the notion that fisheries fall under either an open access or optimized rent maximizing paradigm (Homans and Wilen, 1997). Existing models of fisheries have predominantly been formulated as if all fisheries were open access or optimally managed resources (Gordon 1954; Clark 1990). Most real world fisheries do not operate under the conditions of pure open access or rent maximizing. However, fisheries economists have in the past ignored this fact and have persisted in casting most problems in the framework of maximizing net revenue streams subject to constraints. Such a framework is compatible with use of the sophisticated mathematical tools of calculus of variations and optimal control theory (see, Kamien and Schwartz 1984).

Modern fisheries have since the UN Law of the Seas Convevtion of the late 1970s, operated under the so-called regulated open access regime. The majority of the important fisheries around the world are now jointly managed by two or more coastal states, e.g. Norway and Russia in the case of the Norwegian cod fishery (NCF). This law has transformed most fisheries problems from the two extreme regimes, i.e. from pure open access and sole ownership to shared and joint management problem.

A game-theoretic framework has been used to analyze many modern fisheries since the mid-1970s (see Clark 1990). Cooperative and noncooperative solutions have often been investigated (Armstrong 1994). We assume, as in Missios and Plourde (1997), that the countries involved consider a cooperative agreement, since it is generally agreed that cooperation is more diserable with respect to conservation and welfare of the states. If the exploiting parties agree to manage the resource jointly and have similar views about the resource, then the resource is basically under one management body which usually consists of members from the participating states. This body may have responsibility for quota determination as well as assignment of quotas to individual states. For example, the International Council for the Exploration of the Sea (ICES) has since the late 1970s given annual advice on total allowable catch quotas (TACs) and/or fishing mortalities for the Norwegian fisheries (Nakken 1998).

The focus of this present paper is on the determination of TACs and how the target biomass is achieved, but not on how harvests are divided or how fishers combine inputs in the production process. We develop a discrete-time model for calculation of annual quotas. There are several quota rules which regulators use. The commonly used and analytically simple rule is the one in which the allowable quota is assumed to be linearly related to the stock biomass (Homans and Wilen 1997; Ussif et al. 2000(a,b)). A proportional fishing rule is proposed by Ussif et al. (2000(a,b)) for the analysis of the NCF stock.

In this paper, we construct a dynamic recursive rule for quota calculations. It can be used in the analysis of both an overexploited and an unexploited stock. For a depleted species, the fastest stock rebuilding strategy is the imposition of a complete moratorium on fishing, which in most situations is considered a rather drastic measure. This can lead to large unemployment in the fishery sector and sometimes political and social problems ensue. However, a gradual approach may sometimes be more desirable due to nonmalleable capital, loss of skills and maintenance of markets (Hannesson 1993; Haakon 1998). The model we employ in this paper is a deterministic model. However, extension to the stochastic version should be straightforward (Pindyck 1984). It is also quite easy to extend this model to multispecies analysis (Clark 1990). Multispecies models are more realistic because ecosystem effects such as predator-prey and symbiotic association can be included in the models.

This paper is motivated by the leading works of Smith (1969) and more recently by Homans and Wilen (1997). Smith developed an open access model that according, to Clark, described a real world fishery phenomenon (Clark 1979). The regulated open access model developed by Homans and Wilen assumes that regulators are goal oriented, choosing target harvest levels according to a safe stock concept. They emphasized operational and regulatory characteristics of the industry.

Recent experience from, and observations of, most fisheries indicate that a significant number of fisheries are either overexploited or even depleted. The FAO reported that about 60% of the world's marine fish stock are either fully to heavily exploited, over-exploited, depleted and are in need of urgent conservation and management measures (United Nations FAO 1995). For depleted stocks, the most appropriate and beneficial management measure is rehabilitation of the stock to a substainable level. This paper develops a simple deterministic discrete-time model that will gradually direct the stock to the short- or long-run target, i.e. the safe biomass. The discrete-time representation of the system gives rise to a set of difference equations. It appears that difference equations are reasonably good descriptions of observed time evolution of real phenomena because most measurements of time evolving variables are discrete (see Conrad and Clark 1987).

The plan of the paper is as follows. Section 1 contains a discussion of the bioeconomic model of fisheries. The dynamics of the exploitable resource are developed and discussed in this section. In section 2, we give a simple example of the behavioral model developed in section 1 and illustrate the potential usefulness of such a model. Section 3 gives a brief history of the NCF during the period of regulation. We consider a method of identifying the model parameters in section 4, and discuss the results of the parameter estimation. Section 5 concludes the paper.

2 Bioeconomic Modeling

All bioeconomic models have two fundamental components, i.e. a biological and an economic part. The first component has often been described by the generalized population dynamics model (Clark 1990). It determines the ecological and other natural constraints imposed on the biological species. The second component describes the industry and regulatory behavior of the fishery. Several mathematical models have been discussed in the literature. Gordon's model of rent dissipation assumes that fishers enter the fishery in response to rents and entry continues until effort is earning its opportunity cost (Gordon 1954). Clark and many others have developed models of optimally managed fisheries. Feedback control laws have been derived both in analytical and numerical forms (see, for example Sandal and Steinshamn (1997b); Grafton et al. 1999; Conrad and Clark (1987)). Dynamic commercial models of fisheries have also been discussed by Smith (1969) and Ussif et al. (2000(a, b)). These two relevant parts of bioeconomic models will be discussed in due course.

2.1 The biological dynamics

A discrete-time model of the population dynamics is proposed for this analysis. The population dynamics are given by the difference equation (for the theory of difference equations, see, Lakshmikantham and Trigiante 1987)

$$x_{t+1} - x_t = F(x_t) - h_t \tag{1}$$

where x_t is the biomass at time t, h_t is the rate of harvesting from the stock and F is the surplus growth function. For this analysis, we will assume that this function follows the logitisc growth law. The Schaefer logistic model has the usual capsized u-shape and is symmetric about one-half the environmental carrying capacity. This model assumes that the change in the stock depends only on the current biomass size. Notice that when $h_t = 0$, the stock will grow at its natural growth rate which is the fastest rate.

2.2 Regulations in fisheries science

The concept of regulation in fisheries is becoming even more important over time. Since the introduction of the Extended Fisheries Jurisdiction (EFJ) in the late 1970s many important fisheries are under the jurisdictions of two or more coastal states. This then quickly transforms the problem into a game-theoretic setting (see, Armstrong 1994; Missios and Plourde 1997). Several Studies have since been done to investigate different solutions (cooperative and noncooperative).

Note that here we are assuming that all political, socioeconomic and other differences have been resolved and the parties have in a way come to a common understanding of what is in their best interests. In this paper we shall not concern ourselves with the details of the game type and how the negotiations happen. It will be assumed that interested countries have come to an agreement about the objective(s) of the management. For example the resource managers have the same views on the appropriate social rate of discounting and so on.

The theory of regulations have been discussed in some detail in Clark (1990). Economists have proposed different regulatory instruments such as taxes, TACs and individual transferable quotas (ITQs) for the efficient management of fisheries. TACs have been used in practice as regulatory instruments in the management of many important fisheries.

Another instrument which has been proposed by some economists is the ITQs. Economists see this regulatory instrument as a means of creating property rights. With ITQs each fishery is given a certain proportion of the TACs which may be transferable in whole or in part. This scheme has often been used to address the problem of overcapitalization. It has been implemented in several countries, e.g. Australia, Canada, Iceland, New Zealand and the USA (Bjorndal and Munro 1998). Although ITQs have been used as instruments for promoting economic efficiency, they may also be used for conservation purposes.

In Homans and Wilen (1997), regulation is split into two stages. In the first stage, a target harvest quota is chosen to ensure stock safety. The quota is then distributed among all the exploiting states. In the second stage, regulatory instruments are selected to achieve the target determined from the quota rule. The quota rule they used is quite simple and is linearly related to the stock biomass. We propose a more general rule which incorporates a realistic hypothesis about the long- or short-run objective of management in regard to the safe biomass level. The model developed here is more general and equally easy to analyze. Simulations using this model require a few lines of computer code and can be done using any available software.

2.3 The harvesting sector

Modeling the exploitation of a fish stock is one of the critical and most difficult aspects of bioeconomic analysis. It often requires good judgement and a profound understanding of the system by the analyst(s). Describing the dynamics of the system can be rather demanding since serious consideration must be given to the technological, operational, economic, political and even social factors when modeling the system. The harvesting or TAC submodel of the fishery is defined as

$$h_{t+1} - h_t = \eta\psi(x_t, h_t; x^\tau) \tag{2}$$

where ψ is a function which measures the performance of the fishery and can be seen as a form of reaction function for the management, x^τ is a parameter and η is a constant to be discussed shortly. This function must incorporate the characteristics of the fishery and assumptions about the resource exploitation, e.g., the optimal biomass level, cost and demand conditions. Various forms of the above function exist. The interesting question is what form it should take?

We shall demonstrate this using some examples which have often been employed in the literature (Clark 1990; Hannesson 1993). First, we consider a fishery where the management has the objective of simply exploiting the stock at a fixed rate, then $\psi = 0$ and $h_{t+1} = h_t = h$, where h is constant. A commonly used analytical TAC rule assumes that the harvest rate is linearly related to the stock biomass and takes a more general form of $max(0, a + bx_t)$. In this example, $h_t = a + bx_t$ whenever the stock is above a certain biologically safe minimum, else, $h_t = 0$ is the rule. a/b is some minimum safe biomass level which can be set to guard against stock collapse. For such models the drastic measure of closure of the fishery is taken if the stock falls below the minimum safe biomass level. Such measures are not uncommon in real world fisheries. Two important examples are the Norwegian Spring Spawning Herring (Bjorndal et al. 1999) and the Northern cod on the Grand Banks of Newfoundland (see, Lauck et al. 1998).

3 An Example

In this section we present a more realistic example of the behavioral model developed in the previous section. A simple parameterization of the ψ function will be used to demonstrate the usefulness of this modeling approach. The discrete-time nonlinear system of equations is written as

$$
\begin{aligned}
x_{t+1} - x_t &= F(x_t) - h_t \\
h_{t+1} - h_t &= \eta\psi(x_t, h_t; x^\tau) \\
x_0, h_0 &> 0
\end{aligned}
$$

where x_t is the biomass defined previously, η is a proportionality constant and $\psi(x_t, h_t; x^\tau)$ is a reaction function for the fishery managers. The proportionality coefficient is not very well known for specific fisheries but can be estimated from historical data. It may be seen as a factor that determines the velocity with which we wish to reach the target level. This parameter is a decision coefficient which may reflect the political or socioeconomic orientations of management. In a way this coefficient can be compared with the managers social rate of discounting. In the traditional rent or utility maximizing context, optimality and/or efficiency is often assumed regardless of whether that is an observed behavior or not. When the goals of the management are not explicitly the optimization of discounted rents or utility of consumption, then a different modeling approach should be used. The major impetus of this paper is that we attempt to model the fishery system based on the postulated practices of management.

A simple but good example of ψ is $(1 - x_t/x^\tau)h_t$. We assume that the per annual rate of change of the harvest depends explicitly both on the current stock and the harvest rate and has a similar form as the proportional growth of the logistic growth function. Notice that ψ can assume several different forms and ought to be modeled according to the understanding of the specific fishery. The form proposed here is quite simple but very attractive in many respects. It is intuitively appealing because, it incorporates the target biomass level and uses the distance from this point as a timing device. This is a sort of adaptive approach to setting TACs. To further clarify this modeling approach, we note that the yield or catch equation is derived in a rather different fashion from the traditional models in bioeconomic analysis. The easiest way to interpret the harvest constraint equation is to think of the management as varying the percent annual catches, i.e. $(h_{t+1} - h_t)/h_t$ in proportion to a particular indicator. This indicator must be simple and easy to measure if it has to be useful. We are therefore proposing a very simple example that has several interesting features.

This function has the following properties: (1) $\psi(x^\tau, h^\tau, x^\tau) = 0$, i.e. it is zero at the target stock level, (2) it is positive whenever we are below the target and negative otherwise. Note that x^τ can be any reasonable level of stock biomass that the management might set as desirable. The initial conditions require that the resource is available and that it is subject to human predation. It is important to note that x_0 can be above or below x^τ which is the target biomass level. If the initial stock is less than the target biomass, then the fishery may be considered overfished and the goal is to invest in the stock otherwise disinvest as fast as possible.

Appropriate choice of the target biomass level will depend on the objective(s) of the management. The much criticized biologically oriented maximum sustainable yield (MSY) objective will imply that the TAC should be set so that, the stock is gradually directed to $x^\tau = K/2$. Different management objectives yield various target biomass levels. In this formulation, x^τ can be an endogenous or exogenous parameter in the problem. In practice however, the target biomass may be assumed to be known a priori. That is to say resource managers will, in consultation with scientists, decide on a predetermined target biomass level.

Let us introduce the difference operator Δ such that, $\Delta x_t = x_{t+1} - x_t$. Then, at equilibrium, $\Delta x = 0$ and $\Delta h = 0$, which implies $h = rx(1 - x/K)$ and $x = x^\tau$ and $h = h^\tau$. This yields a unique steady state which corresponds to the target biomass level.

Notice that these difference relations contain four parameters, i.e. (r, K) for the biological and (η, x^τ) for the ahrvesting submodel. Methods for estimating these parameters are available, although input parameters of the biological and ahrvesting submodels such as growth and mortality rates and the velocity of adjustments towards the target biomass, are very difficult to measure in general.

Figure 1 is a plot of the growth function and the corresponding harvest rate against the stock biomass. This exemplifies a fishery that is originally depleted and a moratorium was put in place. Fishing mortality is set to zero for biomass level of up to 1000 kilo-tons which is assumed to be the minimum safe biomass level in this case. The recursive rule is then applied to determine the harvest rate which incrementally adjusts the stock towards the target level. The following hypothetical values of the parameters are used: $K = 6000$ kilo-tons, $r = 0.35$ per year, $x^{\tau} = 2K/3$ kilo-tons and $\eta = 0.05$. That is, the target biomass is set equal to two-thirds of the carrying capacity.

In the next graph, we show the approach of the biomass towards the target level. The figure shows how the fishery can gradually be revitalized. For a stock of less than 1000 kilo-tons, no fishing mortality was applied and the stock grew at its natural rate. This is shown by the kink in the graph.

The approach used in this paper permits some level of fishing activities at the same time as the fishery is being rebuilt. This may be a better and more acceptable alternative solution than the often proposed bang-bang approach in the literature. That is, when the stock is below the sustainable level, the rule is not to harvest until the stock builds up to the sustained biomass level. It is also easy to see that the dynamics of the exploitation rate (2), is serving as an exogenous constraint imposed on the industries in the fishery.

4 The Norwegian Cod Fishery (NCF)

In this section we present a brief history of the NCF since the inception of regulations in 1975 when a TAC was introduced. The first TAC set by the ICES was considered far too high and hence this measure was concluded ineffective (Nakken 1998). Effective TAC came into operation after the establishment of the national Exclusive Economic Zones (EEZs) in 1977, even though the ICES had given management advice since the early 1960s. To give a more elaborate picture, we present a table of the stock biomass, the advised, agreed, and the actual catches since 1978 (see table 1). These figures are taken from Nakken (1998).

Nakken investigated the extent to which the advice on TACs is heeded. The observation is that there is a general tendency that the agreed TAC is higher than the advised. Also, the actual catch often surpassed the agreed TAC.

5 Model Calibration

Parameter identification has been one of the most difficult aspects of the application of biomass dynamics models in management schemes. Realistic models of renewable resources contain many important input parameters such as the natural mortality and the intrinsic growth rate which are very difficult or even impossible to measure. It is however possible to estimate these parameters if historical data are available.

In this section, we use a nonlinear parameter estimation method (Greene 1997) to estimate some of the input parameters of the dynamic model developed in the previous section. The estimation is carried out in a systems context using version 8.0 of Shazam. Many alternative frameworks exist for the estimation of simultaneous system of non-linear equations. Instrumental Variables (IVs) and Nonlinear three stage least squares (N3SL) have been employed in the literature (Greene 1997).

The NCF is used as a test case on the model. It is important to note that we are not sure that this fishery has been managed strictly according to the hypotheses of the model. What is certain is that history tells us that from the late 1970s, some form of regulation has been in place and TACs have since been determined in accordance with a decision rule (Nakken 1998).

We use data from 1978-1999 on the actual catch and the estimated biomass for the NCF. We do not claim that this model exactly represents the NCF. We shall however assume that the assumptions are consistent with the management practices of the authorities and then try to answer questions of the form: what are the parameters of the model which give predictions that are as close as possible to the observed data? What is the target biomass level that is implied by the existing policy?

The nonlinear econometric model is given by

$$x_{t+1} = x_t + r x_t (1 - x_t/K) - h_t + \epsilon_{xt} \quad (3)$$

$$h_{t+1} = h_t + \eta(1 - x_t/x^\tau) h_t + \epsilon_{yt} \quad (4)$$

where ϵ_{xt} and ϵ_{yt} are the random noise terms. The parameter set is $[r, K, \eta, x^\tau]$.

Three experiments will be performed in order to analyze the available data from the NCF. First, all the parameters in the model will be estimated. Our goal is to find out if there is some reasonable set of parameters that makes the model comformable with the data. Then we shall claim that we have been able to unveil the policy objective of the management body. Second, we will assume that the intrinsic growth rate is known and set to 0.35 per year and then estimate the remaining subset of the parameters. In the third experiment, we guess the target biomass level a priori. That is to assume that the management authorities have explicitly informed us about the target biomass level and hence we know actual value. For the the sake of simplicity, we shall set this value to one-half the carrying capacity, i.e. $x^\tau = K/2$. We hope that this will not be a big source of concern for those economists who have been quite skeptical of the MSY objective. This implies that $(1 - 2x_t/K)h_t$ and we therefore have three disposable parameters due to the linear restriction $x^\tau - K/2$ we imposed on the target biomass. The results of these experiments will be discussed below.

5.1 Results

The results of the above empirical tests of our model is presented in this subsection. The estimated parameters and their standard errors (in parentheses) are given in Table 2. Data from the NCF show that immediately after World War II, the stock biomass was around 4.2 million tons. This figure is considered to be below the carrying capacity of the stock but beyond the MYS level ($K/2$) and is therefore probably not overfished. During the World War II period, there was apparently a cessation of almost all commercial fishing activities hence the stock can be assumed to be in a healthy condition.

A guess of the carrying capacity of the NCF may reasonably lie within 4.5-6.0 million tons and the intrinsic growth rate can be somewhere between 0.2-0.5 per year. These values are considered reasonable by some researchers (Ussif et al, 2000(a,b)). No strong scientific evidence is available to us to support this claim.

Estimating all the four parameters simultaneously did not seem to give reliable values. The carrying capacity was a bit low while the intrinsic growth rate was very close to the upper limit of the chosen interval in the first experiment. The cause of this underestimation of the carrying capacity and overestimation of the intrinsic growth rate is still unclear. It appears that, the adjustment parameter η is about the same (with respect to size and sign) in all the three experiments. The target biomass levels are also quite close in all the cases. The values of the K in the other two cases are reasonable, i.e. they are within the expected limits.

To make the usefulness of this model more apparent, we compare the optimal stock biomass and the harvet rate/TACs pictorially. Note that optimal in this case means estimates based on the appropriate choice of the initial harvest quantity and the adjustment coefficient. Here, we assumed that the biological parameters from experiment two are the reasonable ones for the NCF. Since we do not know very well the target biomass, we shall assume that the target set by the management is the MSY biomass level. The decision coefficient and the optimal harvest rate/TAC are then chosen using simulations. The parameters used are $r = 0.35$, $K = 5684.0$, $x^{\tau} = K/2$ and $\eta = 0.22$. For this simulation, the actual estimates of the stock in 1978 is used and since the actual harvest/TAC for this year is considered too high, we chose the initial estimate of the TAC to be 300 kilo-tons which is about 50% of the actual harvest for that year. Notice that, initial harvest rate is must lie between zero inclusive and the growth of the biomass i.e $0 \geq h_0 < f(x_0)$ Figures 3 and 4 show the results of the simulations. The results indicate that a more gradual approach could be used to rebuild the stock to a set target level. The TACs have generally been too high leading to decline in the stock biomass over the years. The outputs have fluctuated very violently. These random fluctuations are not conducive conducive to the fishermen.

6 Conclusion

This paper has in the first place developed a dynamic discrete-time fishery model that describes a more realistic fishery system. The model incorporates some practical assumptions about the decisions, goals and actions of the management authorities. This modeling approach is quite different from existing approaches in many ways. An additional advantage of this model is that it allows for short term planning in the sense that managers can make management decsisons in a time consistent manner.

Existing models have mostly advocated rent maximization as the main objective of the management body, which renders them inappropriate for most practical management situations. This is because management are often not concerned about getting biomass stocks to dynamically optimal long run levels. Hence, instead of expending too much effort on normative analysis as it was posited in Scott's pioneering work (Scott 1955), economists ought to do more in the development of predictive models if issues of vital interests to managers/fishers are to be resolved.

Dynamic TACs are determined through a simple adaptive recursive rule. Management authorities are considered to be target oriented, selecting annual harvest limits in a gradual fashion in order to achieve the desired/optimal biomass level. By adjusting TACs until a steady state is reached, the resource rent is maximized as the system induces the fishermen to include the user cost into their objective function, given that the fishery is perfectly monitored and enforced both for holders and others (Haakon 1998; Hannesson 1993).

The model is used in an application to the NCF. This fishery is an example of a real world fishery for which TACs have been determined since around 1978. Unknown parameters in the model are estimated using nonlinear parameter estimation technique. These estimates are quite reasonable except in the first experiment where the parameters were somehow biased. The results should however be interpreted with caution. First, available data are sparse and noisy. Second, the model is quite simple.

To summarize, we stress the point that this model has numerous advantages. It is simple and adaptive and also very easy to analyze on a personal computer. Its major pitfalls are that we are still modeling the system as if nature was deterministic even though it is not. Also, fisheries systems are inherently multidimensional but this model is cast in a single species setting. Nonetheless, there is a lot to be learned from simple models that have the basic features of the system to analyzed.

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TAC	Stock	Advised	Agreed	Actual
1978	1800	850	850	699
1979	1488	600	700	441
1980	1205	390	390	382
1981	1201	—	300	399
1982	1012	<432	300	365
1983	750	<380	300	290
1984	866	150	220	290
1985	1011	170	220	308
1986	1247	<446	400	430
1987	1064	<645	560	518
1988	825	530[a]	590	435
1989	918	335	451	332
1990	961	172	160	212
1991	1508	215	215	319
1992	1937	250	356	513
1993	2510	256	500	582
1994	2342	649	700	771
1995	2103	681	700	740
1996	2041	746	700	740
1997	1630	850	762	762
1998	1300	654	593	593
1999	1490	480	480	4832

Table 1: Stock biomass and the TACs for the NCF since 1978. Source Nakken, 1998.

Source: ICES, 1980-1996- Reports of the Advisory Committee for the Fisheries Mangement

Table 2. Model parameter estimates of the model

Parameter Estimates			
	Experiment 1	Experiment 2	Experiment 3
r	0.4883(0.2207)	0.35^a	.4237(0.1493)
K	3700(1776.0)	5684.3(3307.7)	4489.0(938.0)
η	-0.262(0.2542)	-0.3892(0.1581)	-0.3222(0.1848)
x^τ	(2539.6(1356.5)	2297(657.0)	2244.5^b

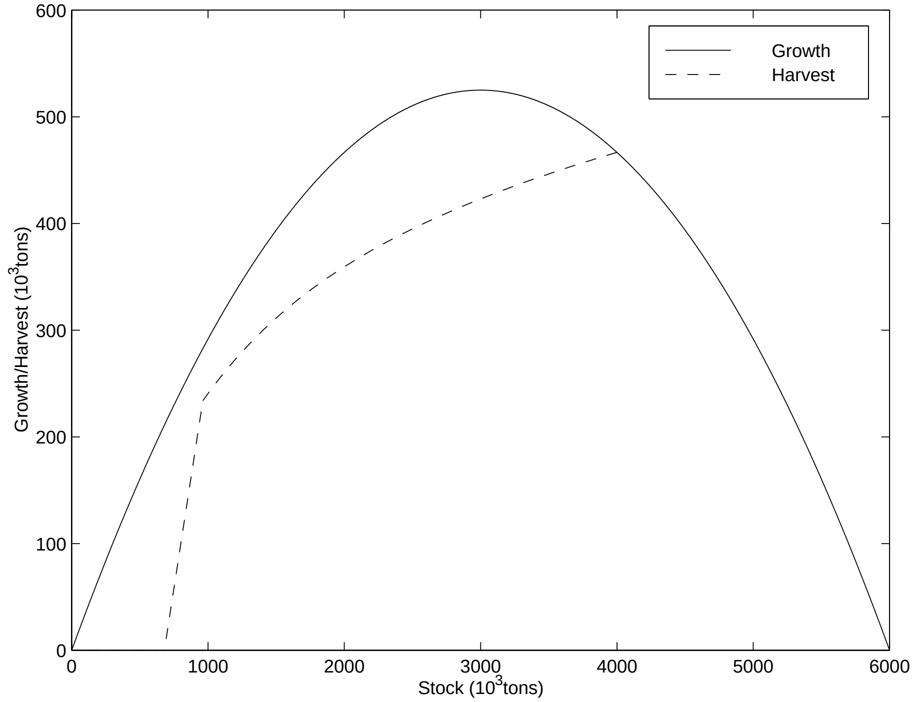

Figure 1: Graph of actual harvest and the stock biomass.

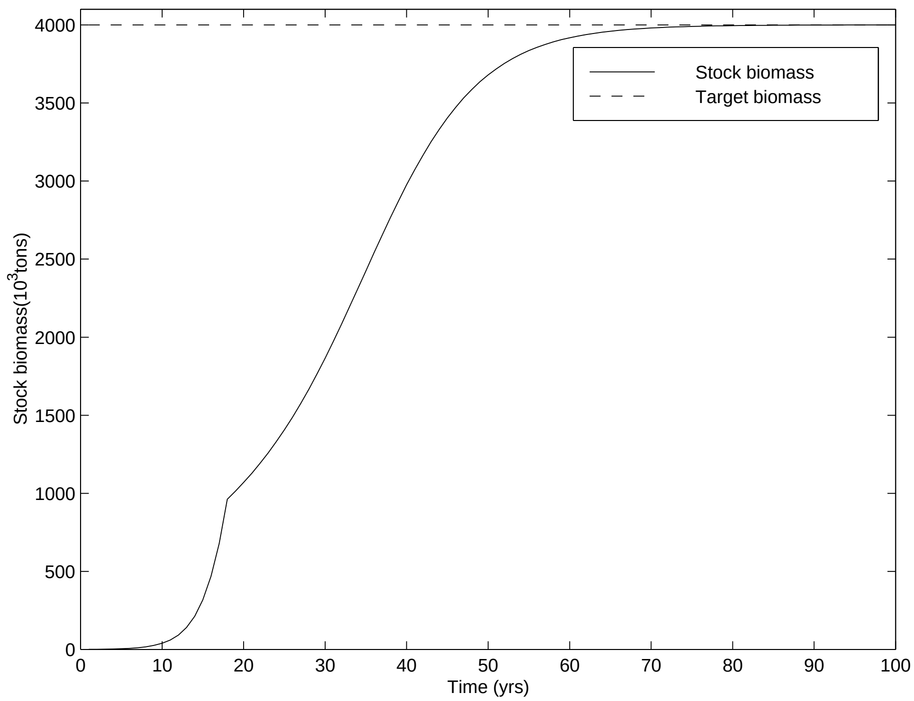

Figure 2: Graph of actual harvest and the stock biomass.

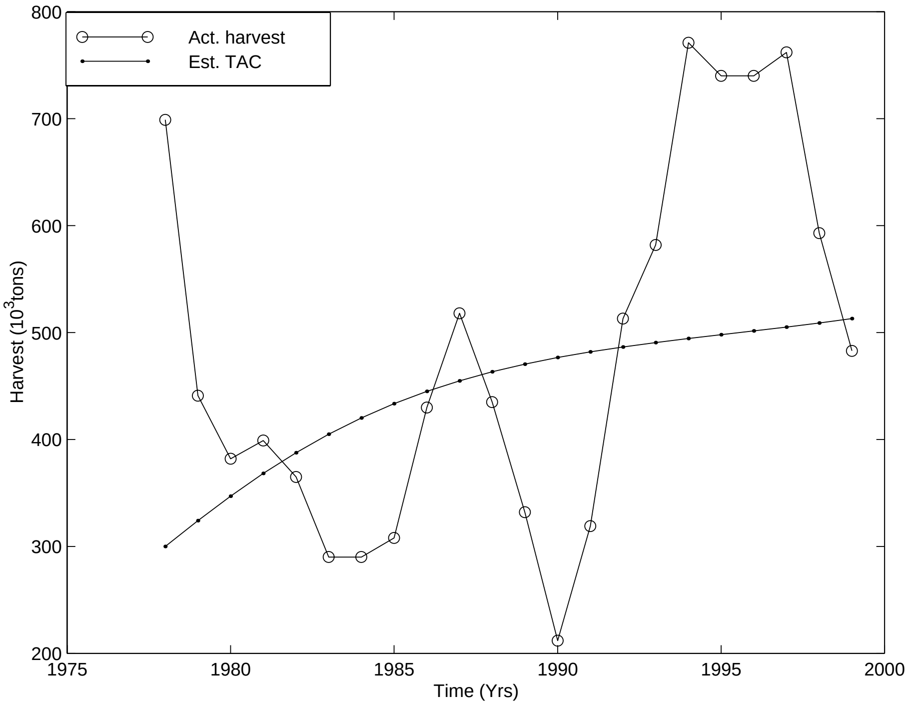

Figure 3: Graph of actual harvest and the stock biomass.

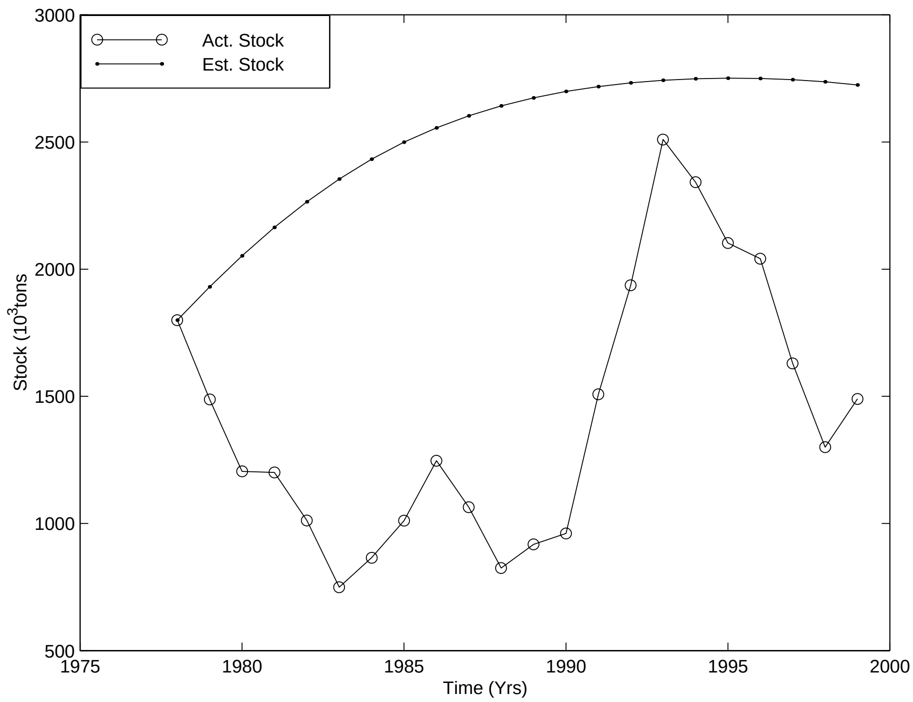

Figure 4: Graph of actual harvest and the stock biomass.