Risk Taking in Selection Contests*

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Abstract

We study selection contests in which the strategic variable is degree of risk rather than amount of effort. The selection efficiency of such contests is examined. We show that the selection efficiency of a contest may be improved by limiting the competition in two ways; a) by having a small number of contestants, and b) by restricting contestant quality. The results may contribute to our understanding of such diverse phenomena as promotion processes in firms, selection of fund managers and research tournaments.

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1 Introduction

In a contest or a tournament, rewards are based on the relative performance of the contestants. Contests serve two different purposes. First, tournaments among workers can mitigate incentive problems when the effort of workers is unobservable. Second, tournaments serve as a selection mechanism. For example, since employers do not necessarily know which workers are the most able, promotions are often based on a comparison of the observed productivity of the workers; the firm promotes the top-ranked worker.

In this paper we focus on the selection aspect of contests, in the case where risk taking is the strategic variable of the contestants. Employees involved in a promotion process or tenure process, for example, may choose tasks that differ in risk profile to show off their abilities. Or even simpler, the task may be fixed but employees choose between a 'safe' working method (e.g., working thoroughly) and a 'risky' working method (e.g., working hastily).

We investigate the selection efficiency of contests in which the contestants optimize their choice of risk, given the risk taking of others. Who will come out on top, bad types or good types? In what way will the selection efficiency depend on, for example, the quality of the contestant pool? We view answering such questions as important to understand the efficiency of promotion processes in firms. Another example is the selection of fund managers in financial markets. Empirical studies show that investors tend to select fund managers with the highest rate of return previous year. Furthermore, these studies show that competition for prospective investments has impact on fund managers' risk taking (Chevalier and Ellison (1997)).

Although the case where agents choose both risk and effort seems realistic for many applications, we confine ourselves to the case where risk taking is the only strategic variable. We focus on the selection efficiency of contests along two dimensions: the number of contestants and the quality of the pool of contestants. Two natural conjectures are the following: Selection efficiency improves with the quality of the contestant pool, and selection efficiency improves with the number of contestants. Tougher competition makes tougher winners. Our two main results are negative; we show that, in our simple model, neither conjectures necessarily holds true. In a separate section, we discuss whether introduction of several prizes or several contests can solve the selection problem.

The model we work with has two types of agents, a *low* type and a *high* type, each with two possible pure strategies, $safe$ and *risky*. The *risky* strategy induces a (not necessarily mean preserving) spread in the probability distribution of individual output compared to the $safe$ strategy. For a given risk level, the *high* type's output dominates the low type's output.

We focus on what seems to be the most natural measure of selection efficiency of a contest; the probability of a *high* type agent winning it. We denote this probability by Π. We show that Π may decrease with a pool of agents of higher quality, i.e., an increase in the share of *high* ability agents in the pool. To see the underlying intuition, notice that increasing the quality of the pool has two effects. The first is the statistical effect: a higher quality of the pool increases Π, holding the strategies of the types fixed. The second effect is the equilibrium effect: increasing the quality of the pool shifts the equilibrium of the game to one with increased risk taking. The latter effect may decrease Π. Thus we show that the statistical effect's positive influence on Π may be dominated by the equilibrium effect's negative influence on Π. An implication is that a firm may discriminate against agents who are likely to be highly skilled by not allowing them to take part in the contest.[1]

A similar intuition can be applied to our discussion of the effect on Π of increasing the number of contestants, n. If n increases, the probability of a high type agent being included in the contest increases (a positive statistical effect). However, increasing the number of contestants also implies more risk taking in equilibrium (the equilibrium effect), which may harm to selection efficiency. We show that the positive statistical effect of increasing the number of contestants may be weaker than the negative equilibrium effect. Thus a firm may improve selection efficiency by limiting competition for higher-rank positions.

Although it has often been argued that contests serve both motivation and selection functions (see e.g., Lazear and Rosen (1981), Schlicht (1988)), the tournament literature has mostly focused on the case with homogenous agents, where selection problems in the sense discussed here do not arise. Papers that do consider the case with heterogeneous agents restrict the discussion to how a tournament reward structure may motivate agents to work hard. An exception is Rosen (1986) (section V), which considers both the motivation function and the selection function of contests. The present paper complements Rosen (1986) in considering selection efficiency when risk taking rather than effort is the choice variable. Also, since Rosen confines attention to the case where there is purely public information about types, our aim is in that sense broader in scope.

Harrington (1998, 1999) consider a promotion game where agents with the highest output are promoted to a higher level in an organization. Harrington (1998) shows that if agents are endowed with simple behavior rules, agents with rules that are unresponsive to changes in the environment reach the top of the organization. Harrington (1999), on

[1] Baye et al. (1993) reports a related exclusion result in a complete information setup for all-pay auctions. Auction revenue may increase if agents with high valuations are excluded.

the other hand, allows agents to act strategically and shows that the "rigidity" result of Harrington (1998) can be reversed. While Harrington (1998) does not consider strategic actions and Harrington (1999) assumes that agents are homogenous, the present paper considers heterogenous agents that act strategically.[2]

The efficiency of various selection procedures is a main topic in the statistical decision theory (see e.g., Gibbons et al. (1977)). By focusing on selection efficiency as the measure of the success of a contest, instead of e.g., aggregate output, our work is in that sense closer to statistical decision theory than to the tournament literature. However, the strategic element makes the noise in the selection process we study endogenous, while the noise in the selection processes studied by statistical decision theory is exogenous. Thus, the statistical decision theory literature only considers statistical effects, while we consider the interaction between statistical and equilibrium effects.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 performs the analysis. Section 4 considers design issues, and Section 5 concludes.

2 The Model

A principal arranges a contest in order to identify a talented agent. We assume that the principal can only observe the rank of the agents, and awards a prize to the agent with the highest rank, or output.[3] There are n risk-neutral agents competing for the prize,

[2]Using tools from evolutionary game theory Dekel and Scotchmer (1999) find an evolutionary pressure towards risk loving preferences provided that those who breed in a population is determined by a contest (and where a child inherits the risk preferences of its parents). The focus of Dekel and Scotchmer (1999) is very different from our focus (there is e.g., no discussion of selection efficiency in Dekel and Scotchmer (1999)), but the models applied are similar.

A patent race is a kind of contest in which there is only one prize – the patent. Selection issues in patent races have to our knowledge not been analyzed.

[3]As pointed by e.g., McLaughlin (1988), cases where the principal mainly has ordinal information on individual output, or where only ordinal information is verifiable (Malcomson (1984)), are common in practice. If cardinal information on individual output is available and verifiable, an interesting question,

whose value is normalized to 1. The individual output space Z consists of four elements; $Z := \{z_1, z_2, z_3, z_4\}$, where $z_1 < z_2 < z_3 < z_4$ (tied winners have an equal chance of obtaining the prize). There are two types of agents, *low* (l) and *high* (h), with θ denoting the share of the h type in the pool from which the n agents are drawn. Both types have an opportunity cost of participation equal to zero, and hence the group of contestants is a true random sample from the pool. Agents of each type have two pure strategies, $safe$ (s) and $risky$ (r). If an l type agent chooses s then her output is z_2 with certainty. If an h agent chooses s then her output is z_3 with certainty. If a l type agent chooses r then her output is z_1 with probability $1 - x$, and z_4 with probability x. If an h type agent plays r then her output is z_1 with probability $1 - y$, and z_4 with probability y, where $y > x$. We do not exclude mixed strategies, and thus the (mixed) strategy space has the usual continuity properties. Outputs are assumed to be statistically independent. We assume that there are no costs associated with risk taking, and hence expected utility for an agent equals her win probability.[4]

Notice that the discrete output space restricts the possible risk taking, in that risk can only be increased by putting more probability weight on the endpoints z_1 and z_4, something that would not be the case with a continuous output space. In Appendix B, we consider the case where output is normally distributed, and where agents can choose the level of variance of their output. The results here show that our main results hold also when the output space is continuous.

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that goes beyond the aims of the present paper, is whether such cardinal information can make schemes where the prize goes to an agent with an output in the 'middle' optimal. (Notice that such non-monotonic schemes have the weakness that they give agents incentives to dispose with parts of their output in equilibrium. For example, fund managers have an incentive to inflate trading costs.)

[4]The case when the distribution of output under r is a mean preserving spread of the distribution of output under s is a special case of the model.

3 Equilibrium Analysis

We consider the incomplete information game $\Gamma(n, \theta)$, where an agent does not know the type of the other contestants, but she knows n and θ and her own type. A strategy is a mapping from the type space T, where $T := \{l, h\}$, to the action space C, where $C := \{s, r\}$. We denote the set of symmetric pure strategies S, where $S := \{(s, s)$, (s, r), (r, s), $(r, r)\}$, with the l type's action written first. We confine our attention to symmetric Bayes-Nash equilibria (BNE), i.e., strategy tuples where all agents maximize their probability of winning given the strategy of the other agents, and where all agents of the same type play the same strategy. The key endogenous variable is the probability of a h type agent winning the prize in a BNE, denoted by $\Pi(\Gamma)$.

3.1 Quality of Contestant Pool

To see the effect of increasing the quality of the contestant pool,[5] we start out by considering the case $n = 2$. Straightforward calculations reveal that there are unique equilibria, and moreover that all four elements of S can be equilibrium strategies depending on the values of the parameters (θ, x, y).

Remark 1 *All four pure strategy combinations are possible symmetric BNE of* $\Gamma(2, \theta)$*. Furthermore, if there exists a symmetric pure strategy BNE, then it is unique.*

Proof. See Appendix A.

Recall that x (y) is the probability of a l (h) agent obtaining the highest outcome if she plays r. With both x and y large, (r, r) is the equilibrium, which is natural. In the

[5] For example, an investor can use a professional evaluation firm in order to hire more highly skilled fund managers. Almost all large investors pay professional firms to evaluate fund managers (Heinkel and Stoughton (1994)).

case where both x and y are small, (s,s) is the equilibrium. That seems counterintuitive since in that equilibrium a l agent loses with certainty if the other agent is a h type. The intuition behind the (s,s) equilibrium is that the probability of a l type winning against a h type (by playing r) is sufficiently small for the l type to rather care about her best chance of winning were she to play against another l type agent.[6]

A first guess might be that it is advantageous to improve the expected ability of the contestants (i.e. to increase θ), as long as there are no intrinsic costs associated with doing it. However, Proposition 1 shows that this conjecture can be false if increased ability among the contestants induces more risk-taking.

Proposition 1 *Contestant Quality.*

i) In a low-quality pool a marginal increase in contestants' average quality improves selection efficiency.

ii) In a medium-quality pool a marginal increase in contestants' average quality may have non-monotone effect on selection efficiency.

iii) In a high-quality pool a marginal increase in contestants' average quality improves selection efficiency.

Proof. We first prove i) and iii) and then prove ii). i) From the proof of Remark 1, we have that for $\theta < \min[1-2x, 2-2y]$, (s,s) is a unique equilibrium strategy. As can easily be verified, $\Pi(s,s) = 1-(1-\theta)^2$, increases with θ. iii) From the proof of Remark 1, we have that for $\theta > \frac{1-2x}{y-x}$, (r,r) is a unique equilibrium. As can be easily verified, $\Pi(r,r) = \theta^2 + 2(1-\theta)\theta\left(y(1-x) + \frac{1}{2}xy + \frac{1}{2}(1-x)(1-y)\right)$, increases in θ. ii)We show that if $\min[1-2x, 2-2y] < \theta < \frac{1-2x}{y-x}$, Π can decrease in θ. Consider $\theta = 1-2x < 2-2y$.

[6] Of course, this equilibrium disappears as θ goes to zero.

In this case, (s, s) is the equilibrium. A small increase in θ induces the equilibrium to switch to (s, r), and Π decreases, as can easily be verified. ■

In situations in which risk taking is very attractive (case iii) or very unattractive (case i) quality improvements would only lead a positive statistical effect Π. The contestants does not change their equilibrium strategies. In contrast, in cases in which a quality improvement implies that contestants' increased fear of facing a contestant of the high ability type switch to a the high risk strategy (case ii), quality improvement may imply a significant negative equilibrium effect which exceeds the positive statistical effect.[7] Figure 1 illustrates the results in Proposition 1.

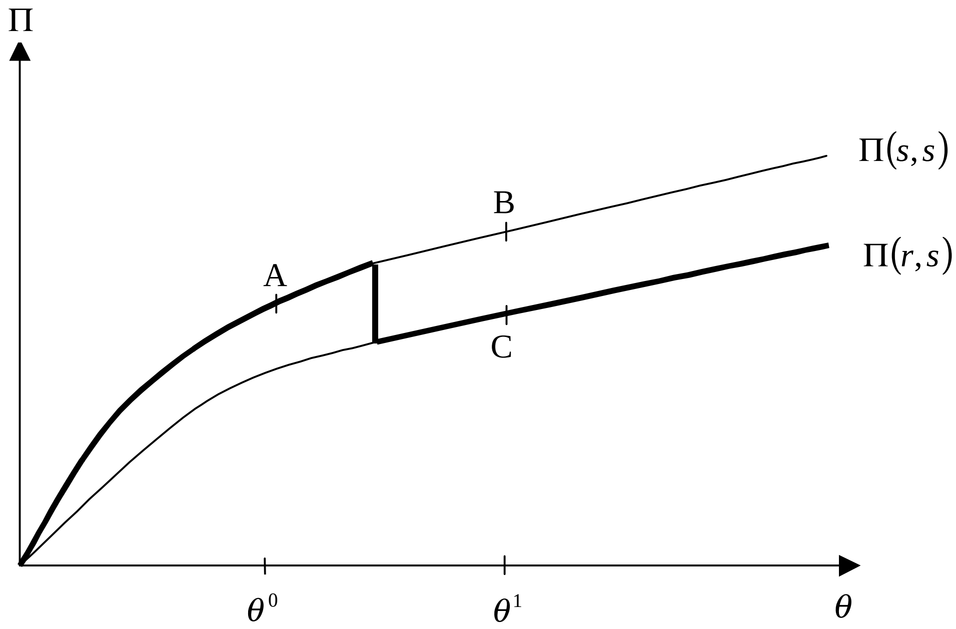

Fig. 1: Improved contestant quality and increased risk taking.

[7] In cases in which expected output depends on the risk of the project (i.e., the non-MPS case), selection efficiency as well as aggregate output may be of importance for a principal. Our analysis can straightforwardly be extended to analyze the trade off between aggregate output and selection efficiency. Examples in which both selection efficiency and aggregate output decrease in θ can easily be constructed.

Figure 1 depicts a typical example of Π as a function of θ, given by the bold line. For a low θ, (s, s) is the equilibrium and increases in θ only induces a statistical effect on Π, implying that Π increases with θ. For a higher θ, (r, s) is the equilibrium, and the same argument applies. In the intermediate range, however, Π can decrease with θ due to a negative equilibrium effect. The movement A$\longrightarrow$C is the total effect on Π from increasing θ from θ^0 to θ^1. The total effect can be decomposed into the statistical effect A$\longrightarrow$B, which is positive, and the equilibrium effect B$\longrightarrow$C, which is negative.

3.2 Number of Contestants

To improve Π it seems natural to increase the number of contestants in order to increase the probability of a good agent participating.[8] Proposition 2 shows that increasing competition, through increasing the number of contestants, can be a two-edged sword, because increased competition increases the equilibrium risk taking.

Proposition 2 *Number of Contestants.* Π *may decrease when the number of contestants increases from 2 to 3.*

Proof. Note that if $n = 2$, $\theta = \frac{1}{2}$, $x = \frac{1}{5}$, $y = \frac{1}{4}$, then from the proof of Remark 1, (s, s) is the unique BNE. That gives $\Pi(2, \frac{1}{2}) = \theta^2 + 2\theta(1-\theta) = \frac{3}{4} = \frac{150}{200}$. Now increase n to 3. In that case, (s, s) is no longer a BNE since

$$U_L(s, s) = \frac{1}{3}(1 - \frac{1}{2})^2 = \frac{1}{12} < U'_L(s, s) = \frac{1}{5}$$

[8] For example, if an investor is uncertain about the investment skill of various potential mutual fund managers, it might be tempting to invite a large number to engage in the management of its investment portfolio.

However, (r, s) is indeed the BNE since a)$U_L(r, s) = \frac{67}{300} > U'_L(r, s) = \frac{48}{300}$.While on the other hand, b) $U_H(s, r) = \frac{532}{1200} > U'_H(s, r) = \frac{319}{1200}$. Thus Π decreases

$$\Pi(3, \frac{1}{2}) = \theta^3 + 2\theta^2(1-\theta)(1-x) + 2\theta(1-\theta)^2(1-x)^2 = \frac{97}{200} < \frac{150}{200}.$$

■

Proposition 2 shows that the increase in noise resulting from increasing n may harm the selection efficiency more than the benefits of the greater likelihood of having at least one h-type agent participating in the contest. The equilibrium effect may dominate the statistical effect.[9]

Note also that if a switch from a safe to a risky strategy yields a sufficiently large reduction in expected output, an increase in the number of contestants (which induce more risk taking) may reduce expected aggregated output.

When the number of agents is already large, then adding a player presumably has no equilibrium effect since both types play risky already. Intuitively, Π may decrease for a small increase in n, but must increase for a large increase in n.[10] But, as Proposition 3 shows, this intuition is false. The proposition builds on a useful result from Dekel and Scotchmer (1999).

[9] Notice that in contrast to the case of increasing θ, the statistical effect on Π of increasing n is ambiguous. To see why, assume that the (r, s) equilibrium is played for some n. Then, keeping the strategies fixed, Π clearly approaches zero as n increases, and thus the statistical effect is negative for the (r, s) equilibrium. On the other hand, the statistical effect on Π of increasing n, given the (s, r) equilibrium, is clearly positive. Thus the statistical effect on Π of increasing n is ambiguous, since it depends on the equilibrium strategies played.

[10] Notice that this intuition holds for the quality of contestants. A very high contestant pool quality (θ close to 1) certainly gives at least as good value of Π as low values of θ.

Proposition 3 Π *may be larger for 2 contestants than for an infinite number of contestants.*

Proof. From Dekel and Scotchmer (1999), Proposition 3, we know that there exists a finite n, denoted n^*, such that for all n larger than n^*, (r, r) is the unique equilibrium. It follows that (r, r) is the only equilibrium for an infinite number of contestants. Consequently, with an infinite number of contestants, the winner has output equal to z_4, with probability 1. By the law of large numbers, the share of h agents that achieve z_4 is just y, and the share of l agents that achieve z_4 is equal to x. Thus $\Pi(\infty) = \dfrac{\theta y}{\theta y + (1-\theta)x}$. Now consider $\theta = \frac{1}{2}, x = \frac{1}{5}, y = \frac{1}{4}$. With those parameter values, we have $\Pi(\infty) = \frac{5}{9} < \frac{3}{4} = \Pi(2)$. ∎

To sum up, we have shown that Π can be non-monotone in n and in θ, due to the equilibrium effect of increases in n or θ. These results were shown for $n = 2$, and where merely examples. Since the non-monotonicity in θ is the most surprising result, we would like to generalize it. In the following we show that non-monotonicity of $\Pi(\theta)$ holds for *all* n.

Proposition 4 *For all* n *and* θ *there exists (x,y) such that* Π *is non-monotonic in* θ*.*

Proof. See Appendix A.

The result generalizes the insight from the examples, by showing that non-monotonicity can occur for all n and θ. This is a rather strong possibility result, but is mute on the magnitude of non-monotonicity. We now use numerical analysis to assess how large the downward movement in Π associated with increases in θ (and consequent shift of equilibrium) can be.[11]

[11] The program generating the numbers in the table is available from the authors.

n	θ^0	θ^1	$y^l(\theta^0)$	$\Pi(\theta^0)$	$\Pi(\theta^1)$	$\Pi(\theta^1) - \Pi(\theta^0)$
2	.25	.25	.87	.44	.39	-.05
3	.25	.28	.77	.58	.51	-.07
4	.25	.30	.68	.68	.60	-.08
5	.25	.33	.61	.76	.67	-.09
10	.25	.45	.38	.94	.84	-.10
20	.25	.65	.20	1.0	.94	-.06

Table 1: Reduction in selection efficiency (Π) when θ increases from θ^0 to θ^1.

In a row, the first two columns are our choices of n and θ^0. $y^l(\theta^0)$ is the highest y that is consistent with (s,s) being an equilibrium given n and θ^0. $\Pi(\theta^0)$ is equal to $\Pi(s,s)$ computed for $(x = 0, y = y^l(\theta^0),\ \theta^0)$. The column θ^1 is the smallest θ that makes (s,r) an equilibrium, given that $(x = 0, y = y^l(\theta^0))$. $\Pi(\theta^1)$ is $\Pi(s,r)$ computed for $(x = 0, y = y^l(\theta^0), \theta^1)$. Notice that $\Pi(\theta^1) - \Pi(\theta^0)$ is negative for all the (n,θ^0) combinations in the table even if θ increases significantly in some of the cases.[12]

4 Design Issues

In this section we discuss whether a principal can modify the contest to solve the selection problem. We consider two possible modifications. First, we consider the case where the principal constructs several different contests, to induce agents of different types to self-select into different contests, and hence solve the selection problem right away.[13] Second we

[12] It can be noted that since we compute $\Pi(\theta^1) - \Pi(\theta^0)$ for an increase in θ that is just sufficient to make (s,r) an equilibrium, the computed $\Pi(\theta^1) - \Pi(\theta^0)$ is an upper bound on the magnitude of the reduction in Π from an increase in θ that induces a switch in pure equilibria.

[13] The classic paper in the tournament literature, Lazear and Rosen (1981), also considers the possibility of multiple contests. Their purpose is to study how multiple contests can generate efficient levels of effort.

consider that the principal limits attention the competition to one contest, but increases the number of prices, in order to reduce the amount of risk taking in the contest.

4.1 Multiple Contests

For illustration, we confer to discuss implementation in the $n = 2$ game. We ask under what conditions constructing two contests can make agents of different types voluntarily choose different contests, and hence solve the self-selection problem directly.[14]

Suppose that the principal lets each agent decide whether to enter either contest 1 or contest 2. The principal sets the prizes in the contests to C_1 and C_2, respectively. After choosing which contest to enter, the agent observes whether there is a rival in the contest and of which type the rival is. Thereafter the agent chooses to play r or s. Since an agent in a contest can observe the rival's type he can also detect a rival's deviation from the "self-selection-equilibrium" before r or s is chosen. By allowing agents to respond to deviations from the "self-selection-equilibrium" we make deviating less attractive and self-selection easier to achieve.[15] Since only the ratio between C_1 and C_2 matters for equilibrium, we can restrict attention to considering ζ, where $\zeta = \dfrac{C_1}{C_2}$ and $\zeta \in [0, 1]$, and consider ζ as the only choice variable of the principal. Consider the case in which h agents self-select into contest 1 and l agents self-select into contest 2.

In a contest in which a h type faces competition from a l type, (s, r) is the equilibrium strategies if $(1 - x) \geq y$ and (r, r) if the converse holds. we focus on the low risk (s, r) equilibrium in case one of the types deviate from intended contest (i.e. $(1 - x) \geq y$).

[14] When there are more types T than two, implementation is generally made more difficult, since there will be $T(T-1)$ incentive restrictions to take care of, instead of only 2. We therefore consider a case when implementation should be relatively simple.

[15] Since deviating from the self-selection equilibrium is more attractive when a rival cannot observe the type of the other agents entering the same contest, it is straightforward to show that also in this case self-selection can be achieved only for a limited set of parameter values.

Remark 2 *For θ sufficiently high, self-selection is not feasible.*

Proof. Self-selection can only be achieved if the following two conditions hold (none of the types will deviate from the "correct" contest)

$$\begin{aligned} h \text{ type} &: \left[\frac{1}{2}\theta + (1-\theta)\right] C_1 \geq [\theta + (1-\theta)(1-x)] C_2 \\ l \text{ type} &: \left[\frac{1}{2}(1-\theta) + \theta\right] C_2 \geq [(1-\theta) + \theta x] C_1 \end{aligned}$$

Both conditions holds only if $\theta \in \left[\frac{1}{2(1-2x)}\left(5 - 2x - \sqrt{(17 - 4x + 4x^2)}\right), 1\right]$ (since the h type chooses strategy s, y does not enter into the condition). ■

The intuition for the result is as follows. For low θ the l type will be tempted to choose the contest for the h type since it is a low probability for facing a h type in that contest. Moreover, the principal cannot reduce the prize in the h contest since this will induce a potential h type to choose the l contest.

This result is related to Lazear and Rosen (1981). They study "effort" contests and show that a principal arranging a contest with homogenous agents can induce efficient effort. They extend their analysis to heterogenous agents and study whether self-selection into different contest can ensure that efficient effort still can be induced. In line with our analysis they show that self-selection cannot be achieved.

4.2 Multiple Prizes

In some cases, it is possible for the principal to increase the number of prizes, to avoid the risk taking problems outlined. The idea behind increasing the number of prizes is to make the prize structure less convex, and hence decrease the amount risk taking and make the ranking of agents more informative. There is a trivial sense in which this can be obtained, as shown in the following remark.

Remark 3 *Suppose there are n participants in the contest, and that the prize structure is (1/n, 1/n, ..., 1/n). Then* $\Pi = 1$ *in equilibrium*

Proof. Since the agents face no incentives at all, it is trivial to see that (s,s) is indeed an equilibrium, and hence the agents reveal their type. ■

A practical problem with increasing the number of prizes is financial constraints. For example, in the race between vice presidents to become the CEO of a firm, it may put a harsh financial strain on the firm to pay the runner up a wage that is close to the CEO wage. Another problem with increasing the number of prizes is that effort may decrease: A central insight from the effort strand of the tournament theory is that equilibrium effort is an increasing function of the prize spread (see Lazear and Rosen (1981)). Hence the direct effects of increasing the number of prizes could be to imply less risk taking, but also less effort, and the optimal number of prizes would involve some trade-off between these two effects.[16] A third problem with increasing the number of prizes arises if the h type agents have higher participation constraints than the l type agents. In the context of a one-prize contest, there is no tension between selection efficiency and participation constraints since in any (symmetric) equilibrium, selection efficiency and the utility of a h type agent will be in a one-to-one relation. However, with more than one prize, this relation breaks down, because although the selection efficiency can increase, the expected payments to an agent of the h type can decrease. This point can easily be seen from the following example.

Example 2

[16] The trade-off is complicated by the fact that the indirect effect of less risk taking on effort would be to *increase* effort (see Lazear and Rosen (1981)).

Suppose $n = 2$, and it is common knowledge between the contestants and the principal that only one of the agents is the h type. Each contestant knows his own type. Furthermore assume that $x = 1/2$ and $y = 3/4$. With the price structure $(1, 0)$, the unique equilibrium induces both agents to play r, and $\Pi = U_H = 11/16$. Suppose now that the prize structure is altered to $(1/2, 1/2)$. Then, by indifference, (s, s) is an equilibrium and $\Pi = 1$. However, with the latter prize structure, $U_H = 1/2 < 11/16$. Hence if the participation constraint is between 1/2 and 11/16, the h agent will not participate.

5 Conclusion

Contests are used both to induce agents to work hard and to solve selection problems. It is therefore surprising that the tournament literature has almost exclusively considered the former function. In this paper, however, we have mainly considered how well contests select talented agents when risk taking is the decision variable of the agents.

We have used promotion decisions in firms and the selection of mutual fund managers as examples of situations where fiercer competition may lead to more risk taking and reduced selection efficiency. However, the insights from our analysis can be applied to other contexts also. For instance, governments and private firms often sponsor tournaments to induce research on specific topics. The reward structure and selection issues of these tournaments is close to what we have discussed in this paper: there is usually only one large prize and selection of a high-quality firm is essential since the winner is going to take care of prospective production. Our results indicate that an organizer of a research tournament may want to restrict the number and quality of contestants in a research tournament.

We have two main results. We show that although increasing the number of firms participating in a contest makes it more likely that the pool of contestants includes a high-quality firm, it might make it less likely that a high-quality firm will be awarded the prize. We also show that an increase in the expected ability or quality of the contestants may make it less likely that a high-quality firm will be selected. The intuition behind the results is that a more competitive tournament – more contestants or higher expected abilities among the contestants – induces firms to adopt riskier strategies, which may harm the selection of high-quality firms. Riskier projects create more noise in the selection contest, and thereby reduce the informativeness of the rank.

6 Appendix A: Proofs

Proof of Remark 1: We use the following convention: $U_i(j,k)$ denotes the win probability of an agent of type i when agents of her own type (including herself) play strategy j and agents of the other type play strategy k. For example, $U_H(s,r)$ denotes the win probability of an h agent when all h agents (including herself) play s, and all l agents play r. The individual payoffs in the symmetric tuples (when all agents of the same type choose the same strategies) are:

$$
\begin{array}{ll}
U_H(r,r) = \frac{1}{2}(1+(1-\theta)(y-x)) & U_L(r,r) = \frac{1}{2}(1+\theta x-\theta y) \\
U_H(s,r) = 1-\frac{1}{2}\theta - x + \theta x & U_L(s,r) = \frac{1}{2}(1+\theta)-\theta y \\
U_H(r,s) = \frac{1}{2}\theta + (1-\theta)y & U_L(r,s) = \frac{1}{2}(1-\theta)+x\theta \\
U_H(s,s) = 1-\frac{1}{2}\theta & U_L(s,s) = \frac{1}{2}(1-\theta)
\end{array}
$$

For individual deviations, we use the following convention: $U_i'(j,k)$ denotes the win probability of an agent of type i when she plays strategy $-j$, other agents of her own type play strategy j, and agents of the other type play strategy k. Since the payoff from letting $-j$ be a mixed strategy is a convex combination of playing s and playing r, we only need to consider pure strategy deviations . For example, $U'_H(s,r)$ denotes the win probability of an h agent playing r, when all other h agents play s, and all l agents play r. The individual payoffs from individual deviation are:

$$U'_H(r,r) = \theta(1-y) + (1-\theta)(1-x)$$

$$U'_H(s,r) = \theta y + (1-\theta)(\tfrac{1}{2}xy + y(1-x) + \tfrac{1}{2}(1-x)(1-y))$$

$$U'_H(r,s) = \theta(1-y) + (1-\theta)$$

$$U'_H(s,s) = y$$

$$U'_L(r,r) = \theta(1-y) + (1-\theta)(1-x)$$

$$U'_L(s,r) = \theta(\tfrac{1}{2}xy + x(1-y) + \tfrac{1}{2}(1-x)(1-y)) + (1-\theta)x$$

$$U'_L(r,s) = (1-\theta)(1-x)$$

$$U'_L(s,s) = x$$

First consider equilibrium (r,r). Notice that the payoff from individual deviation is the same for an h agent and an l agent, and moreover that $U_H(r,r) > U_L(r,r)$. Thus we only have to check a deviation from an l agent. An l agent follows the supposed equilibrium strategy if $\frac{1}{2}(1-\theta y+\theta x) > \theta(1-y) + (1-\theta)(1-x)$, which implies that $y > \frac{1+\theta x-2x}{\theta}$. Now consider equilibrium (s,s). An l agent follows the supposed equilibrium strategy if $x < \frac{1}{2}(1-\theta)$. The condition for an h agent is $y < 1-\frac{1}{2}\theta$. Now consider equilibrium (r,s). An l agent follows the supposed equilibrium strategy if $\frac{1}{2}(1-\theta)+x\theta > (1-\theta)(1-x)$, which implies that $x > \frac{1}{2}(1-\theta)$. The condition for the h type is $\frac{1}{2}(1+(1-\theta)(y-x)) > \theta y + (1-\theta)(\frac{1}{2}xy + y(1-x) + \frac{1}{2}(1-x)(1-y))$, which implies that $y < \frac{1}{2}$. Finally, consider equilibrium (s,r). An l agent sticks if $\frac{1}{2}(1+\theta) - \theta y > \theta(\frac{1}{2}xy + x(1-y) + \frac{1}{2}(1-x)(1-y)) + (1-\theta)x$, which implies that $x < \frac{1-\theta y}{2-\theta}$. The condition for the h type is $\frac{1}{2}\theta + (1-\theta)y > \theta(1-y) + (1-\theta)$, which implies that $y > 1 - \frac{1}{2}\theta$. The uniqueness of BNE, given (x, y, θ), follows directly from the argument. ■

Proof of Proposition 4: The idea of the proof is to consider a small increase in θ that induces equilibrium to switch from (s, s) to a mixed strategy equilibrium, and to show that the total effect on Π from increasing θ is negative. To limit the equilibrium effects to changes in the strategy of h agents, we assume for convenience that $x = 0$, so that s is a dominating strategy for the l agents. Suppose that the h agents play r with probability α, and s with probability (1-α). If α is played,

$$\Pi(\theta, \alpha) = 1 - \sum_{m=0}^{n-1} (\alpha(1-y))^m \theta^m (1-\theta)^{n-m} \binom{n}{m} \tag{1}$$

where the second expression on the right is the probability that an l agent wins the contest given that α is played, and where y is fixed. The event that an l agent wins occurs only when all m agents of the h type obtain z_1 (which occurs with probability $(1-y)^m\alpha^m$). Equation (1) is clearly differentiable, and provided that $\frac{\partial\alpha(\theta,y)}{\partial\theta}$ exists, increasing θ has a direct and an indirect effect on Π,

$$\frac{d\Pi}{d\theta} = \frac{\partial\Pi}{\partial\theta} + \frac{\partial\Pi}{\partial\alpha}\frac{\partial\alpha}{\partial\theta} \tag{2}$$

The first term on the right side is the statistical effect of increasing θ and the second term is the equilibrium effect. We wish to show that that $\frac{d\Pi}{d\theta} < 0$ for a suitably defined value of y, which implies that Π is non-monotone on some interval of θ.

Denote by $U_H^i(s, \alpha)$ $[U_H^i(r, \alpha)]$ the utility of an h agent playing s $[r]$, given that the other h agents play r with probability α, and s with probability (1-α). Let the probability of there being m agents of the h type given that there are $n-1$ agents participating be denoted by Ψ, where $\Psi = \binom{n-1}{m}\theta^m (1-\theta)^{n-1-m}$. Then,

$$U_H^i(s,\alpha) = \sum_{m=0}^{n-1} \left(\sum_{k=0}^{m} \frac{\binom{m}{k}(1-y)^{m-k}(1-\alpha)^k \alpha^{m-k}}{k+1} \right) \Psi \tag{3}$$

By playing s, the agent will beat all l agents, but can only win if all the h agents playing r obtain z_1. Moreover, k is the number of other h agents that play s and $(m-k)$ is the number of h agents that play r.

$$U_H^i(r,\alpha) = (1-y)\frac{\theta^{n-1}(\alpha(1-y))^{n-1}}{n} + y\sum_{m=0}^{n-1}(\sum_{k=0}^{m} \frac{\binom{m}{k}(\alpha y)^k (1-\alpha y)^{m-k}}{k+1})\Psi \tag{4}$$

The first term on the right side is the win probability conditional on obtaining z_1 and the second term is the win probability conditional on obtaining z_4. Moreover, k is the number of other h agents that obtain z_4. The mixed strategy α is an equilibrium if

$$F \equiv U_H^i(s,\alpha) - U_H^i(r,\alpha) = 0 \tag{5}$$

Equation (5) determines implicitly a function $\alpha(\theta, y(\theta))$. Now define y^l as the value of y that makes an h agent indifferent between playing s and r, given that all the other h agents play s. In other words, $y^l := \sup(y : (s,s) \in NE)$. We suppress notation by simply writing α instead of $\alpha(\theta, y(\theta))$.

$$\begin{aligned} y^l \quad &= \quad \{y : U_H^i(s,\alpha)_{\alpha=0} = U_H^i(r,\alpha)_{\alpha=0}\} = \{y : \sum_{m=0}^{n-1}\frac{\Psi}{m+1} = y\} = \\ &\sum_{m=0}^{n-1}\frac{\Psi}{m+1} \end{aligned} \tag{6}$$

Notice that by construction, $\alpha(\theta, y^l(\theta)) = 0$, which will be several times later. The strategy now is to find the derivatives on the right hand side of equation (2), and to evaluate at $y = y^l$, where the α-terms drop. We begin by deriving $\frac{\partial\alpha(\theta,y)}{\partial\theta}_{y=y^l}$ and then derive $\frac{\partial\Pi(\theta,\alpha)}{\partial\theta}_{y=y^l}$ and $\frac{\partial\Pi(\theta,\alpha)}{\partial\alpha}_{y=y^l}$.

$$\begin{aligned} F &= U_H^i(s,\alpha) - U_H^i(r,\alpha) \\ &= \sum_{m=0}^{n-1}\left(\sum_{k=0}^{m}\frac{\binom{m}{k}(1-y)^{m-k}(1-\alpha)^k\alpha^{m-k}}{k+1}\right)\Psi \\ &\quad -\frac{(1-y)\,\theta^{n-1}(\alpha(1-y))^{n-1}}{n} \\ &\quad -y\sum_{m=0}^{n-1}(\sum_{k=0}^{m}\frac{\binom{m}{k}(\alpha y)^k(1-\alpha y)^{m-k}}{k+1})\Psi \end{aligned} \tag{7}$$

To find $\frac{\partial\alpha(\theta,y)}{\partial\theta}_{y=y^l}$ we use the implicit function theorem, i.e., $\frac{\partial\alpha}{\partial\theta} = -\frac{F_\theta}{F_\alpha}$, and then evaluate at $y = y^l$. First find $F_\alpha = \frac{\partial U_H^i(s,\alpha)}{\partial\alpha} - \frac{\partial U_H^i(r,\alpha)}{\partial\alpha}$.

$$\frac{\partial U_H^i(s,\alpha)}{\partial\alpha} = \sum_{m=0}^{n-1}\left(\sum_{k=0}^{m}\frac{\binom{m}{k}(1-y)^{m-k}\left((m-k)\,\alpha^{m-k-1}(1-\alpha)^k - k(1-\alpha)^{k-1}\alpha^{m-k}\right)}{k+1}\right)\Psi \tag{8}$$

When $y = y^l$, the inner sum is zero except for at $k = m-1$ and at $k = m$. Therefore,

$$\begin{aligned} \frac{\partial U_H^i(s,\alpha)}{\partial\alpha}_{y=y^l} &= \sum_{m=0}^{n-1}\left[\frac{\binom{m}{m-1}\left(1-y^l\right)(1)}{m} + \frac{\binom{m}{m}(-m)}{m+1}\right]\Psi \\ &= \sum_{m=1}^{n-1}\frac{m\left(1-y^l\right)}{m}\Psi - \sum_{m=0}^{n-1}\frac{m\Psi}{m+1} \\ &= \sum_{m=1}^{n-1}\left(1-y^l\right)\Psi - \sum_{m=0}^{n-1}\frac{m\Psi}{m+1} \end{aligned} \tag{9}$$

Moreover,

$$
\begin{aligned}
\frac{\partial U_H^i\left(r,\alpha\right)}{\partial \alpha} &= \frac{\left(1-y\right)\theta^{n-1}(n-1)(1-y)\left(\alpha\left(1-y\right)\right)^{n-2}}{n} + \\
& y\sum_{m=0}^{n-1}\left(\sum_{k=0}^{m}\frac{\binom{m}{k}ky\left(\alpha y\right)^{k-1}\left(1-\alpha y\right)^{m-k}}{k+1} - \right. \\
& \left. \frac{\binom{m}{k}\left(\alpha y\right)^{k}(m-k)y\left(1-\alpha y\right)^{m-k-1}}{k+1}\right)\Psi \qquad (10)
\end{aligned}
$$

For $y = y^l$, the first term drops, since $n > 2$. The inside sum in the second term drops except for at $k = 1$ and at $k = 0$. Hence we have,

$$
\begin{aligned}
\frac{\partial U_H^i\left(r,\alpha\right)}{\partial \alpha}_{y=y^l} &= y^l\sum_{m=0}^{n-1}\left(\frac{\binom{m}{1}y^l}{2} - \frac{\binom{m}{0}my^l}{1}\right)\Psi \\
= y^l\sum_{m=0}^{n-1}\left(\frac{my^l}{2} - my^l\right)\Psi &= -(y^l)^2\sum_{m=0}^{n-1}\frac{m\Psi}{2} = -\frac{(n-1)}{2}\theta(y^l)^2 \qquad (11)
\end{aligned}
$$

And moreover,

$$
F_{\alpha|y=y^l} = \sum_{m=1}^{n-1}\left(1-y^l\right)\Psi - \sum_{m=1}^{n-1}\frac{m\Psi}{m+1} + \frac{n-1}{2}\theta(y^l)^2 \qquad (12)
$$

Now find $F_\theta = \frac{\partial U_H^i(s,\alpha)}{\partial\theta} - \frac{\partial U_H^i(r,\alpha)}{\partial\theta}$. Define $\Phi = \sum_{k=0}^{n}\frac{\binom{m}{k}(1-y)^{m-k}(1-\alpha)^k\alpha^{m-k}}{k+1}$, which is independent of θ, and notice that $\Phi_{y=y^l} = \frac{1}{m+1}$. Hence,

$$
\frac{\partial U_H^i\left(s,\alpha\right)}{\partial \theta}_{y=y^l} = \sum_{m=0}^{n-1}\left(\Phi\frac{\partial\Psi}{\partial\theta}\right)_{y=y^l} = \sum_{m=0}^{n-1}\frac{1}{m+1}\frac{\partial\Psi}{\partial\theta}_{y=y^l} \qquad (13)
$$

Furthermore, we have that,

$$\frac{\partial U_H^i\left(r,\alpha\right)}{\partial\theta}=\frac{\left(1-y\right)\left(n-1\right)\theta^{n-2}\left(\alpha\left(1-y\right)\right)^{n-1}}{n}-y\sum_{m=0}^{n-1}\left(\sum_{k=0}^{m}\binom{m}{k}\left(\alpha y\right)^k\left(1-\alpha y\right)^{m-k}\right)\frac{\partial\Psi}{\partial\theta} \tag{14}$$

Notice that for $y=y^l$, the first term on the right hand side is zero for $n>1$. Furthermore $\sum_{m=0}^{n-1}\frac{\partial\Psi}{\partial\theta}=0$ since $\sum_{m=0}^{n-1}\Psi=1$, which implies that also the second term drops, since the term inside the brackets just equals 1. Hence we have that,

$$\frac{\partial U_H^i\left(r,\alpha\right)}{\partial\theta}_{y=y^l}=0 \tag{15}$$

It follows that $F_{\theta|y=y^l}=\sum_{m=0}^{n-1}\frac{\frac{\partial\Psi}{\partial\theta}}{m+1}$. Hence we get,

$$\frac{\partial\alpha}{\partial\theta}_{y=y^l}=-\frac{F_\theta}{F_\alpha}_{y=y^l}=-\frac{\sum_{m=0}^{n-1}\frac{\frac{\partial\Psi}{\partial\theta}}{m+1}}{\sum_{m=1}^{n-1}\left(1-y^l\right)\Psi-\sum_{m=1}^{n-1}\frac{m\Psi}{m+1}+\frac{n-1}{2}\theta(y^l)^2} \tag{16}$$

where $F_{\alpha|y=y^l}>0$. Now find $\frac{\partial\Pi}{\partial\theta}_{y=y^l}$.

$$\frac{\partial\Pi}{\partial\theta}=-\sum_{m=0}^{n-1}\binom{n}{m}\left(\alpha\left(1-y\right)\right)^m\theta^m\left(1-\theta\right)^{n-m} \tag{17}$$

Substituting for $y=y^l$, we have that $\alpha=0$ and all terms drop except for at $m=0$,

$$\frac{\partial\Pi}{\partial\theta}_{y=y^l}=-\binom{n}{0}\left(-n\theta\left(1-\theta\right)^{n-1-0}\right)=n\left(1-\theta\right)^{n-1} \tag{18}$$

Now find $\frac{\partial \Pi}{\partial \alpha}_{y=y^l}$.

$$\frac{\partial \Pi}{\partial \alpha} = -\sum_{m=0}^{n-1} \binom{n}{m} \theta^m (1-\theta)^{n-m} m (\alpha(1-y))^{m-1} (1-y) \tag{19}$$

For $y = y^l$, all terms drop except for at $m = 1$,

$$\frac{\partial \Pi}{\partial \alpha}_{y=y^l} = -\binom{n}{1} \theta (1-\theta)^{n-1} (1-y^l) = -n\theta (1-\theta)^{n-1} \left(1-y^l\right) \tag{20}$$

Hence for $y = y^l$, we get that,

$$\frac{d\Pi}{d\theta}_{y=y^l} = n(1-\theta)^{n-1} + \frac{n\theta(1-\theta)^{n-1}(1-y^l)\sum_{m=0}^{n-1}\frac{\frac{\partial \Psi}{\partial \theta}}{m+1}}{\sum_{m=1}^{n-1}(1-y^l)\Psi - \sum_{m=1}^{n-1}\frac{m\Psi}{m+1} + \frac{n-1}{2}\theta(y^l)^2} \tag{21}$$

To show that non-monotonicity is possible for all n and θ, we need to show that this expression is negative. Hence $\frac{d\Pi}{d\theta}_{y=y^l} < 0$ if,

$$\Lambda = (1-y^l)\theta \sum_{m=0}^{n-1} \frac{\frac{\partial \Psi}{\partial \theta}}{m+1} + \sum_{m=1}^{n-1} \left(1-y^l\right)\Psi - \sum_{m=1}^{n-1} \frac{m\Psi}{m+1} + \frac{n-1}{2}\theta(y^l)^2 < 0 \tag{22}$$

By standard summation rules, we can simplify this expression, observing that,

$$\begin{aligned}
y^l &= \sum_{m=0}^{n-1} \frac{\Psi}{m+1} = \frac{1-(1-\theta)^n}{n\theta} \\
\sum_{m=1}^{n-1} \frac{m\Psi}{m+1} &= \frac{\theta n - 1 + (1-\theta)^n}{n\theta} = 1 - y^l \\
\frac{\partial y^l}{\partial \theta} &= \frac{(1-\theta)^{n-1}(n\theta + 1 - \theta) - 1}{\theta^2 n}
\end{aligned} \tag{23}$$

Using eq. (23) and simplifying we obtain,

$$\Lambda = \frac{2 - \theta(1+n) + (1-\theta)^n[(1-\theta)^n(n\theta + 2 - \theta) - 2(2-\theta)]}{(n\theta)^2} \tag{24}$$

It suffices to show that the top of this expression, labeled Λ_1, is always negative. Suppose that $\frac{\partial \Lambda_1}{\partial n} < 0$. It then suffices to show that $\Lambda_1(n = 2) < 0$. But, as can easily be verified, $\Lambda_1(n = 2) = -\theta^4(2 - \theta) < 0$. It then only remains to show that is negative. As can easily be verified, $\frac{\partial \Lambda_1}{\partial n}$, is negative

$$\frac{1}{\theta}\frac{\partial \Lambda_1}{\partial n} = -1 + (1-\theta)^{2n}[\ln(1-\theta) + 1] \tag{25}$$

Notice that if the expression on the right should be positive for any value of n, it must be positive for $n = 1$ (since if the second term on the right hand side is positive, it is decreasingly so in n). However, substituting in for $n = 1$ it can easily be verified that the right hand side of (25) is negative. Hence $\frac{\partial \Lambda_1}{\partial n} < 0$. That completes the proof. ■

7 Appendix B: Numerical Analysis

The discrete output space, $\{z_1, z_2, z_3, z_4\}$, places tight restrictions on the type of risk taking allowed. Specifically, the only way for an agent to increase risk is by putting more probability weight on the endpoints z_1 and z_4. With a continuous output space, say the interval $[z_1, z_4]$, increased risk does not necessarily imply more weight at the endpoints. In this appendix we use simulation techniques to consider the case with a continuous output space and three different ability levels: High (H), Medium (M), and Low (L). The results of this section show that our main results also hold when the output space is continuous.[17]

As before, the agents maximize the probability of being selected by choosing between safe and risky projects. To conduct the simulation analysis we make the following assumptions.

1. The outcomes of the agents' projects are normally distributed with expected outcomes $L = 0$, $M = 3$ or $H = 6$.

2. The agents choose between a safe and a risky project with the same expected outcome. The safe project is assumed to have a standard deviation of 1. The risky project has a standard deviation of σ, where $\sigma \in [3, 7]$.

3. The probability of being of a particular type is:

	L	M	H
Probability	$\frac{1}{2} - \theta$	$\frac{1}{2}$	θ

   An increase in θ implies that it is more likely for any agent to meet an opponent with high ability.

7.1 Quality of the Contestants

In this section we show that Π may decrease with an increase in the quality of the contestants (θ).

Consider the case with two contestants. It is simple to verify that there exists an equilibrium in dominant strategies where the H type always chooses a safe strategy and

[17] The MapleV programs used in this section can be obtained from the authors. We have experimented with different parameter values and obtained similar results, so the results seem robust.

the L type always chooses a risky strategy.[18] Let us now focus on the M type. If θ is small, then the likelihood of facing a better contestant is small and the M type behaves as if she is best and, hence, chooses the safe strategy. But if θ is high then the M type is more likely to face a better contestant and, hence, chooses the risky strategy. In Figure B-1, the curve G shows the critical values for θ, such that the M type is indifferent between choosing a safe and a risky strategy.

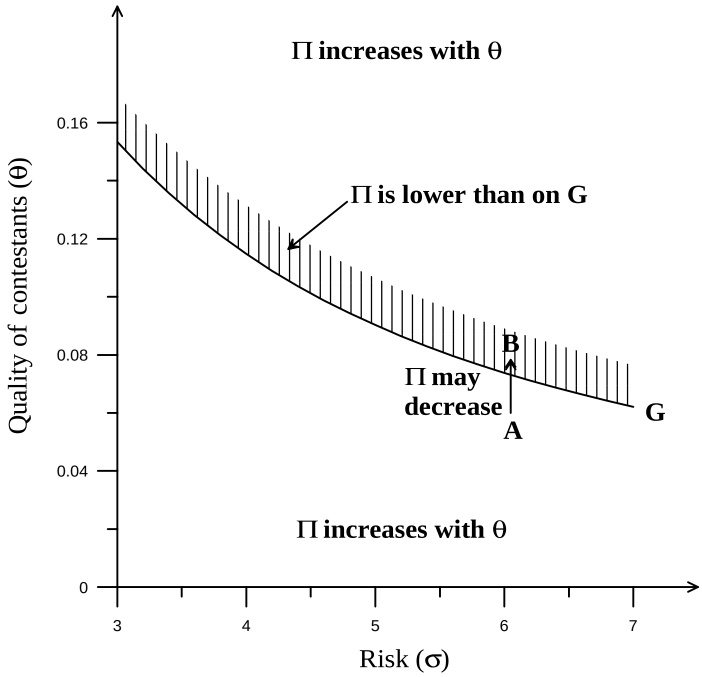

Figure B-1: Higher quality (θ) of contestants

The shaded area represents the possibility that an increase in θ reduces Π. Moving northwards from a point on the G line into the shaded area, causes a decrease in Π. To illustrate further, take two points on the diagram and label them A and B. Then Π

[18] To see why, first note that for type L the high risk strategy dominates the low risk strategy. If she is facing a better type, she will always increase her probability of winning by choosing the riskier strategy. If she is facing another L type she is indifferent about the choice between a high and low risk strategy. Hence, a high risk strategy is a dominant strategy for the L type. Second, note that the low risk strategy is the dominant strategy for the H type. A high risk strategy will increase the probability of low outputs and hence increase the likelihood of less able contestants achieving a higher output. Furthermore, the H type will be indifferent to the choice between low and high risk strategy facing another H type.

increases from A to B if B lies further north than A, as long as we do not cross the G line. If A is below the G line and B is above, as illustrated in Figure 2, then Π may decrease.

An increase in the quality of the contestants makes it more likely that one of the contestants is a H type. But higher quality induce the M types to choose a risky strategy, which may decrease Π.

7.2 Number of Contestants

In this section we illustrate that Π may decrease as a result of adding one contestant to a group of two contestants. For simplicity, we focus on the case in which adding a contestant induces the M type to change strategy, but not the L type or the H type. It is straightforward to show that $(risky, risky, safe)_{n=3}$ is a unique equilibrium for $\theta < \frac{1}{5}$, which is the case we consider in the following figure.

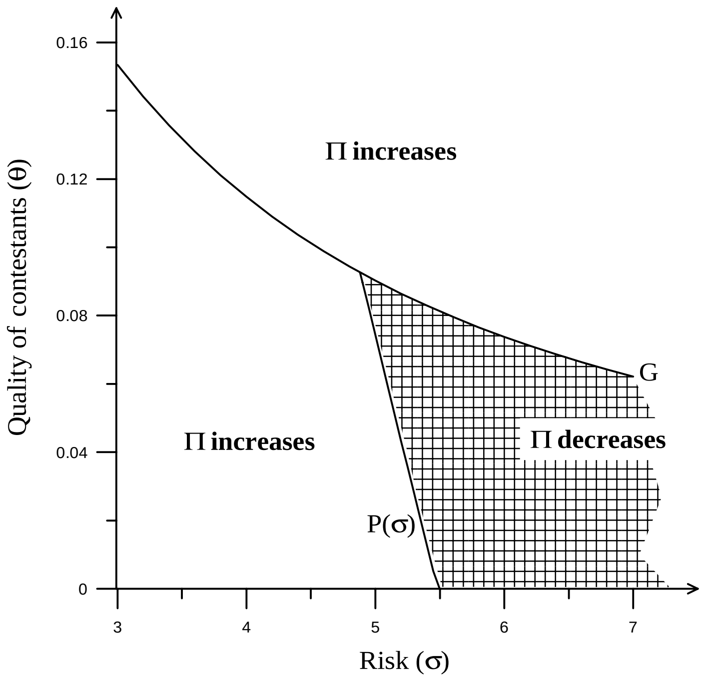

Figure B-2: Adding one more contestant

In Figure B-2, the line P gives the points where Π is identical for $n = 2$ and $n = 3$.

In the shaded area of Figure B-2, Π decreases when the number of contestants increases from two to three.

References

Baye, M. R., Kovenock, D. and de Vries, C. G. (1993), Rigging the lobbying process: An Application of the all-pay auction, *American Economic Review*, 83, 289–94.

Chevalier, J. and Ellison, G. (1997), Risk taking by mutual funds as a response to incentives, *Journal of Political Economy*, 105, 1167–1200.

Dekel, E. and Scotchmer, S. (1999), On the Evolution of Attitudes Towards Risk in Winner-Take-All Games, *Journal of Economic Theory*, 87, 125–143.

Gibbons, J. D., Olkin, I. and Sobel, M. (1977), *Selecting and ordering populations: A new statistical methodology*, New York: Wiley.

Harrington, J. J. E. (1998), The social selection of flexible and rigid agents, *American Economic Review*, 88, 63–82.

Harrington, J. J. E. (1999), The equilibrium level of rigidity in a hierarchy, *Games and Economic Behavior*, 28, 189–202.

Heinkel, R. and Stoughton, N. M. (1994), The dynamics of portfolio management contracts, *Review of Financial Studies*, 7, 351–387.

Lazear, E. and Rosen, S. (1981), Rank-order tournaments as optimum labor contracts, *Journal of Political Economy*, 89, 841–864.

Malcomson, J. (1984), Work incentives, hierarchy, and internal labor markets, *Journal of Political Economy*, 92, 486–507.

McLaughlin, K. J. (1988), Aspects of Tournament Models: A Survey, *Research in Labor Economics*, 9, 225–256.

Rosen, S. (1986), Prizes and incentives in elimination tournaments, *American Economic Review*, 76, 701–715.

Schlicht, E. (1988), Promotions, Elections, and other contests: A Comment, *Journal of Institutional and Theoretical Economics*, 144, 94–99.