Tournament Rewards and Risk Taking*

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Abstract

In a tournament, a principal sets a prize, and several agents then compete to attain the highest observed output, and win the prize. This paper departs from the existing literature on tournaments by assuming that agents can influence the spread of their distribution of output, in addition to the mean. We ask in which way risk taking and effort interacts in equilibrium. First, under standard tournament rewards, the unique equilibrium will have a low level of effort and a high level of risk taking. Second, by modifying the tournament scheme to give the prize to the agent with the 'most moderate' output, a high level of effort can be implemented. We argue that the first result can be useful to understand the RPE puzzle of executive compensation, and the second result can be useful to understand puzzling workplace norms promoting mediocrity.

Keywords: Personell Economics, RPE Puzzle.

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1 Introduction

To the extent that real-world rewards are based on measures of performance, they often depend on relative performance. For example, promotion is awarded to the most productive member of a level in an organization; the CEO of the least profitable firm in an industry gets fired, and the mutual fund with the highest return one year gets a higher investor inflow the next year.

The main theoretical rationale for rewarding relative performance stems from the Informativeness Principle (Holmstrom, 1982), which, informally, states that an optimal compensation contract conditions rewards on any variable that is (incrementally) informative about work intensity (effort). Recently, a corollary of the Informativeness Principle known as the relative performance evaluation (RPE) hypothesis has been extensively tested in the large empirical literature on CEO compensation (see Murphy, 1999, or Prendergast, 1998, for overviews). The idea behind the RPE hypothesis is that if firms in the same industry face some common random shock, like changes in industry demand, an optimal compensation contract for a CEO makes his payment conditional on the relative performance of the firm (in addition to its absolute performance); the higher the profit of the other firms, the lower the reward of the CEO. In the empirical literature, researchers tend to be puzzled by the lack of evidence for RPE in the CEO compensation data. For example, Aggarwal & Samwick (1999a) 'suggest that relative performance evaluation considerations are not incorporated into executive compensation contracts' (p. 104, ibid.). And, Murphy (1999, page 40) states that: 'The paucity of RPE in options and other components of executive compensation remains a puzzle worth understanding'.

A seemingly unrelated puzzle to the RPE Puzzle is the 'Mediocrity Puzzle'; sometimes there are stronger incentives for delivering a mediocre performance than for a higher performance. The mechanism underlying such non-monotonic rewards is sometimes informal, through peer pressure, and sometimes formal, through explicit working contracts. For peer pressure towards mediocrity, Levine (1992) reports of several illustrating cases. For example, Frederick Taylor, the creator of Scientific Management, was threatened with shooting by his co-workers for being too productive and 'innovative'. And Mui (1995) reports of a well-publicized case in China where a successful village entrepreneur was haunted by several 'misfortunes' after becoming rich; the timber of his house were stolen, a pregnant cow was stabbed to death, several other animals were poisoned. There are also examples where formal rewards work in favor of mediocrity. For example, fund manager compensation schemes sometimes have an outlier effect: a very low return *and* a very high return yields a lower reward from the principal than performances in the middle.[1]

The purpose of the paper is to provide explanations of the RPE Puzzle and of the Mediocrity Puzzle, based on the concepts of tournaments and risk taking. In a tournament, a principal sets a prize, and several agents then compete to attain the highest observed output, and win the prize. The paper departs from the existing literature on tournaments by assuming that agents can influence the spread of distribution of output, e.g., through the choice of projects, and the project mean through the choice of effort. For example, CEOs can choose whether the firm should pursue a safe or a risky R&D profile, and a variety of other decisions that also affects the risk profile of a firm (e.g., which type of workers to employ, whether to enter emerging markets or not), in addition to deciding how hard to work. And fund managers can choose the riskiness of their portfolio, in addition to choosing how much resources to spend on providing and analyzing relevant stock information.[2]

Intuitively, there are two types of combinations of risk and effort that are consistent with equilibrium. If the equilibrium risk taking is high, then the marginal increase in the probability of winning from increasing effort is low, and equilibrium effort must be low. And conversely, if the equilibrium risk taking is low, then the equilibrium effort is high, since the marginal increase in the probability of winning from increasing effort is high. Hence for a given prize structure, equilibrium must either have a high level of risk and a low effort level, or a low level of risk and a high level of effort. Notice, however, that is not obvious which of these configurations will be consistent with equilibrium, since both risk taking and effort are treated as endogenous variables.

[1] To illustrate this point, Skandia Fund Managment (SFM), which manages approximately $50 billion in the Scandinavian Market, first selects an initial pool of fund managers, and then gradually terminates the relationship with the managers whose return are too high or too low compared to an index return. SFM engages in a long(er) term relationship with the remaining managers. I am grateful to the CEO of SFM, Harald Troye, for providing this information.

[2] Other examples include employees aspiring for promotion or tenure.

In Proposition 1, it is shown that with no limits to possible risk taking, agents exert zero effort and choose an infinite risk in equilibrium. Since the expected production is zero in this case, the tournament breaks down as a reward scheme. This result is somewhat modified in Proposition 2, where possible risk taking is limited, but still the moral hazard problem is grave. Proposition 1 and Proposition 2 together indicate that a reason why CEO compensation to a small extent depends on the relative performance of the firm is that putting weight to the rank of the CEO in the industry may induce a manager preference for risky projects, that moreover are not properly cared for.

Given this negative result, we ask whether the tournament reward scheme can be modified to avoid the risky-lazy 'trap' of the standard tournament. To this end, a scheme where agents are ranked according to the relative closeness of their output to a benchmark k is considered. The idea behind this scheme, labeled k-contracts, is that excessive risk can be avoided, which in turn can provide incentives for working hard.

The second main result, Proposition 3, states that there exists intermediate values of the benchmark k such that first best level of effort can be implemented under risk neutrality. The empirical value of Proposition 3 is that it sheds light on why sometimes higher rewards are given to agents with a modest performance than to agents with a very high performance. A norm that gives the highest informal status to agents that have a moderately high relative performance can be more efficient than a norm that gives highest informal status to agents with the highest relative status. Or, a fund management company that wants to reward their fund managers according to their relative output to e.g., protect the managers against common risk factors, may consider to give the highest reward to the manager with a portfolio return that comes closest to some benchmark or index, rather than giving the highest reward to the manager with the highest portfolio return.

The paper is structured as follows. In the next section, related literature is discussed. Section 2 sets up the model and contains the analysis, while Section 3 concludes. Some of the proofs are relegated to appendices A and B.

1.1 Related literature

As outlined by e.g., Lazear (1995, 1999), tournament theory is one of the cornerstones of personell economics. While there is a growing empirical literature on tournaments (Ehrenberg and Bognanno, 1990, Brown et al. 1996, Chevalier and Ellison, 1997, and Eriksson, 1999), this literature is limited to overview the theoretical literature on tournaments, before it briefly discusses received explanations of the RPE Puzzle and the Mediocrity Puzzle.

Tournaments were first studied by the classic Lazear & Rosen (1981), who in a model with effort as the only choice variable showed that individualistic schemes and tournament schemes under certain conditions are equivalent. Later contributions to the 'effort' strand of the tournament literature includes Nalebuff and Stiglitz (1983) on correlated output, Rosen (1988) on knock-out tournaments, Clark and Riis (1998) and Moldovanu and Sela (2000) on the case with multiple prizes, and Fullerton and McAffee (1999) on tournaments with a fee for entering. On the other hand, tournaments with risk taking as the only choice variable was first considered by Bronars (1987), who studied the differences for incentives to take risk between leaders and followers in sequential tournaments. Other papers in the risk taking strand of the literature include Cabral (1997), on the endogenous choice of covariance, Dekel & Scotchmer (1999) on tail dominance, and Hvide & Kristiansen (1999) on the selection properties of tournaments.

Importantly, while the received literature on tournaments consider effort *or* risk taking as choice variables for the agents, the present paper considers the *interaction* between effort and risk taking. By including both variables we can highlight how the equilibrium choice of effort depends on the equilibrium choice of risk taking, and vice versa, a topic that has not been treated before by the literature.[3]

On the RPE Puzzle, Aggarwal and Samwick (1999b) argue that the RPE effect on compensation schemes can be neutralized by a delegation effect stemming from imperfect competition in the product markets. However, their delegation argument is ambiguous; if Cournot competition, rather than Bertrand competition, prevails in the product markets, their model *strengthens* the prediction of RPE hypothesis. In contrast, we point out

[3] This also holds true of the (non-tournament) agency literature, with the exception of Diamond (1998).

harmful effects of RPE in compensation schemes also even when product markets are competitive.[4]

On the Mediocrity Puzzle, Heinkel and Stoughton (1994) derives an optimal reward scheme for fund managers, where managers with a 'too' high return will be replaced. However, this result refers to the solution of an adverse selection problem, while the scheme proposed in the present paper solves an unrelated moral hazard problem. Moreover, there is no notion of risk taking in the model of Heinkel and Stoughton (1994). Gibbons (1987) argue that piece rate schemes are necessarily non-monotonic, when taking into account dynamic effects; a high performance will indicate to the principal that the job is simple, and hence that a less lucrative piece rate is sufficient to keep the worker in the firm. However, since Gibbons (1987) considers environments where the payoff of an agent is independent of the productivity of other agents, it is mute on the link between relative performance and non-monotonic rewards, and hence unable to explain the Mediocrity Puzzle, where relative performance is an essential ingredient. Levine (1992), building on Jones (1984), argues that a norm to punish 'ratebusters' may be efficient for the workgroup, from the same type of argument as in Gibbons (1987). However, since the firm realizes a low level of profit under this norm, the overall level of welfare is suboptimal, and it is therefore not clear why a norm for mediocrity should survive in their setting. In contrast, we show that a norm for mediocrity can realize high levels of overall welfare.

2 Analysis

Section 2.1. sets up a standard tournament model with effort as the only choice variable; Section 2.2 adds a notion of risk taking to that model, and Section 2.3 introduces k-contracts.

[4] An older argument against RPE is that compensation schemes that put too much weight on relative performance are sensitive to collusion between the agents that are compared. For illustration, if the sum of compensation for two workers is constant, then both workers would be better off if they could collude in slacking their effort. However, since collusion typically requires a long-term relationship, such arguments seems more applicable to explain lack of intra-firm RPE than lack of inter-firm RPE.

2.1 The Tournament Model

In this section we review the standard tournament model of Lazear and Rosen (1981). There is one risk-neutral principal and several risk neutral agents, for convenience assumed to be only two.[5] The value of agent i's output equals $Y_i = \mu_i + \varepsilon_i$, where μ_i is agent i's choice of effort, and where ε_i is an iid shock with $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = \sigma^2$. Y_i and Y_j are the only contractible variables. The cost of effort is symmetric, $V_i(\mu_i) = V_j(\mu_j)$, and $V_i(\mu_i)$ is assumed to satisfy $V_i(0) = V_i'(0) = 0$, and V_i', $V_i'' > 0$ for $\mu_i > 0$. The first-best level of effort, denoted μ_i^*, is the μ_i that solves $V'(\mu_i) = 1$. The cost of effort is symmetric, with $V_1(..) = V_2(..) = V(..)$. For clarity of exposition, we deviate from Lazear and Rosen (1981) by considering the special case when ε_i is normally distributed.

Under a rank-order scheme, the principal fixes the prizes W_1 and W_2 [where $W_1 > W_2$], and the agents then compete in winning the first prize W_1, which is awarded to the agent with the highest Y_i. Expected utility for agent i, U_i, equals,

$$U_i = P_i W_1 + (1 - P_i) W_2 - V(\mu_i) = P_i \Delta W + W_2 - V(\mu_i) \tag{1}$$

where $\Delta W = W_1 - W_2$, and $P_i = Prob(Y_i > Y_j) = Prob(\mu_i - \mu_j > \varepsilon_j - \varepsilon_i)$. For agent 1 we get, $P_1 = Prob(Y_1 > Y_2) = Prob(\mu_i - \mu_j > \varepsilon) = G(\mu_1 - \mu_2)$, where $G(..)$ is the cdf of ε [$\varepsilon \equiv \varepsilon_2 - \varepsilon_1$]. Clearly ε is normally distributed with $E(\varepsilon) = 0$ and $E(\varepsilon^2) = 2\sigma^2$. The first order condition for optimal provision of effort becomes,

$$\frac{\partial U_i}{\partial \mu_i} = \frac{\partial P_i}{\partial \mu_i}\Delta W - \frac{\partial V}{\partial \mu_i} = 0, \; i = 1, 2. \tag{2}$$

Notice that due to the option-like structure of the prizes, only the difference between the first and the second prize, ΔW, enters the first order conditions. By symmetry, if there exists an equilibrium, then in equilibrium $\mu_1 = \mu_2$, and the outcome is purely random, i.e., $P = \frac{1}{2}$, since $G(0) = \frac{1}{2}$. By substituting $\mu_1 = \mu_2$ in (2), equilibrium effort, μ_i^*, can be

[5] All results can easily be generalized to hold for an arbitrary number of agents.

characterized by,

$$\frac{\partial V}{\partial \mu_i} = \Delta W g(0), i = 1, 2. \tag{3}$$

Inserting for the normal density,

$$\frac{\partial V}{\partial \mu_i} = \frac{\Delta W}{2\sqrt{\sigma^2 \pi}}, \; i = 1, 2 \tag{4}$$

From inspecting (4), it can easily be seen that μ_i^* is implementable with an appropriate choice of ΔW. Since ΔW can be made independent of the total outlays, $W_1 + W_2$, the positive implementation result is consistent with zero profits. Notice also that it follows from (4) that the equilibrium effort is decreasing in σ. Intuitively, a higher σ makes the outcome of the tournament more noisy, which decreases the marginal gain of increasing effort, and hence reduces equilibrium effort.

2.2 Risk Taking in the Tournament Model

By 'increasing risk' it is meant that the agent induces a mean-preserving spread of Y_i, through increasing the variance of ε_i. The way the agent induces this spread may either be through his choice of projects or through manipulating the principal's measurement error of Y_i. In particular, the assumptions of the previous section are adhered to, except that the shock ε_i is now an endogenous variable. The variance of ε_i equals η_i^2, where $\eta_i^2 = \sigma^2 + s_i^2$, with $\sigma > 0$ and $s_i \in \Re_+$. The interpretation of σ is the level of non-diversifiable, background, noise, and s_i is the degree of voluntary spread in the output distribution. Thus s_i is a choice variable for agent i, while σ is, as before, a parameter. The cost of adjusting s_i is assumed to be uniformly zero. As before, output is assumed to be the only contractible variable.

Notice that risk taking added to the agents' choice set in this manner has the convenient property that increased risk has no direct effect either on utility or on profits (expected output), first best levels of effort is identical to in the previous section.[6] Al-

[6] We are well aware of cases where it can be difficult to separate the expectation of Y_i and its variance. For example, in the classic CAPM model of financial asset pricing, a portfolio with a higher risk will also

though there is no direct link between risk taking and welfare, the following result shows that the indirect effect can be dramatic.

Proposition 1 *The unique equilibrium induces infinite variance and zero effort from both agents.*

Proof. We first show that $X^* = \{s_i = s_j = \infty \text{ and } \mu_i = \mu_j = 0\}$ is a Nash Equilibrium, and then show uniqueness. Suppose $\mu_i = 0$ and $s_i = \infty$. Then agent j wins with probability $\frac{1}{2}$ irrespectively of his choice of μ_j and s_j. Therefore, $\mu_j = 0$ and $s_j = \infty$ is a best reply to $\mu_i = 0$ and $s_i = \infty$, and hence X^* is a NE. To prove that X^* is a unique NE, first consider tuples with (i)$\mu_i < \mu_j$. For (i) to be a Nash equilibrium, clearly $s_i = \infty$, since that choice of s_i maximizes P_i. That implies $\mu_i = 0$. But, in that case, $\mu_j = 0$ is a best reply from agent j, which contradicts (i). So in any Nash equilibrium we have that $\mu_i = \mu_j$. Tuples with (ii)$\mu_i = \mu_j > 0$ are now excluded. If $\mu_i = \mu_j$ then $P = 1/2$. But since both players have positive cost of effort, player i can gain by changing μ_i (one obvious improvement is to set $\mu_i = 0$ and $s_i = \infty$). But then we are in case (i). Hence neither (i) nor (ii) is consistent with Nash behavior, and X^* is a unique NE. ■

Thus if agents can choose their level of risk taking, in addition to their effort, a tournament induces extremely risky and lazy behavior from workers. The intuition for the result is that the agents have a common incentive to increase the level of noise in the tournament, to thereby lessen the importance of differences in means (effort) to the win probability. And, in turn, when effort becomes less detrimental to the win probability, the agents have less incentive to expend effort, which makes the equilibrium levels of effort more comfortable to them. It should be emphasized that the intuition for the result is *not* that tournament rewards are convex in performance, which in turn gives incentives for an extreme degree of risk taking. The reason why this intuition is false is that whether an agent has incentives for risk taking or not depends on whether he exerts more effort than the other agents. If an agent exerts more effort than the other agent, he has an incentive to choose a low level of risk rather than a high level of risk. So this intuition does not take into account that effort is an endogenous variable.

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generate a higher expected return. To treat such a case, we would need to have a risk averse principal, which would harden the computations significantly.

Proposition 1 contradicts the intuition of Lazear & Rosen (1981), which state: "In this paper the worker has no choice over [the variance of individual output]. This does not affect the risk neutral solution but does have an effect if workers are risk averse, since they tend to favor overly cautious strategies ... ." (footnote 1, page 843).[7]

Since Proposition 1 is obtained under rather special assumptions, let us discuss its robustness. First notice that exactly the same argument, and the same negative result, goes through if risk-averse agents play the tournament. Hence agents choose infinite variance and zero effort in equilibrium even if they are risk averse. In fact, the only requirement for the result to go through is that $U(..)$ is monotonic. This can be seen by replacing ΔW by ΔU in equation (4), where $\Delta U \equiv U(W_1) - U(W_2)$. Second, although normality of the shocks is convenient for illustration, weaker distributional assumptions can be made. In Appendix A, we generalize Proposition 1 to hold for ε_i unimodal and symmetric. Third, since lack of independence in the sense of a positively correlated shocks is one of the main justifications for applying tournaments (see e.g., Nalebuff and Stiglitz 1983), it is worth noticing that Proposition 1 holds for any degree of correlation between the shocks. Recall that for the normal distribution, the coefficient of correlation, ρ, can be determined independently of the variances. When the variances go to infinity, P goes to $1/2$ independently of μ_i and ρ. Hence Proposition 1 is robust to introducing risk averse or risk loving preferences, and to having a more general stochastic structure.

However, since the meaning of 'infinite variance' is somewhat unclear, it is useful to consider the case where there are limits to risk taking. It is now assumed that $s_i \in [s^{\mathsf{min}}, s^{\mathsf{max}}]$, $\forall i$, where $0 < s^{\mathsf{min}} < s^{\mathsf{max}}$, with s^{max} finite. Hence risk taking is bounded by a lower limit s^{min} and an upper limit s^{max}. To avoid non-existence problems, we consider the game where the agents first choose level of risk taking and then, after observing each others choice of risk, decide how hard to work. We consider the subgame perfect equilibrium of this two stage game.

Proposition 2 *In the subgame perfect equilibrium, both agents choose* $s_i = s^{\mathsf{max}}$ *in the*

[7] Also Murphy (1999) seems to be overly optimistic with respect to the optimality properties of RPE when the agent has additional choice variables to effort (page 41, ibid.): 'RPE remains a strong prediction of the model after expanding the managerial action set, since paying based on relative performance provides essentially the same incentives as paying based on absolute performance, while insulating risk-averse managers from common shocks.'

first stage, and the corresponding low effort in the second stage.

Proof. In equilibrium at stage 2, $P = \frac{1}{2}$ independently of level of risk taken at stage 1. Since the equilibrium effort is a decreasing function in the sum of s_1 and s_2, both agents choose $s_i = s^{\max}$ at stage 1, in dominant strategies. Since the equilibrium risk taking at stage 1 is high, the equilibrium effort at stage 2 is consequently low. ■

Proposition 2 shows that even when there are limits to risk taking, the moral hazard problem induced by a tournament reward structure is serious: equilibrium behavior by the agents is risky and lazy. As with Proposition 1, Proposition 2 can be generalized to a situation with risk averse agents and a more general stochastic structure (see Appendix A). Notice that a comparative statics exercise on $s^{\max}$ yields a simple result; the equilibrium effort is monotonically decreasing in $s^{\max}$. This can be interpreted as the greater opportunity of taking risk, the less efficient is a tournament reward structure.

However, it is true that given risk neutrality, first best can be implemented for any finite $s^{\max}$, with an appropriate choice of ΔW. Therefore, to make the efficiency argument more clear, we need to move to risk averse agents. In the following, we compare the relative efficiency of piece rates schemes and tournament schemes under risk aversion. Suppose that the principal sets the piece rate, and that the agent then responds by choosing a level of risk and a level of effort. First notice that the efficiency of piece-rates schemes is independent of $s^{\max}$, since the agents will choose $s_i = s^{\min}$ in dominant strategies. This is a consequence of the well-known fact that a risk-averse agent prefers a lower variance to a higher variance, for a given coefficient of the incentive scheme. On the other hand, in tournaments with risk averse agents, an increase in $s^{\max}$ implies that ΔW to induce the same level of effort. But then welfare of the agents is reduced, since the variability of payment increases. The following remark follows

Remark 1 *Under risk-aversion, the relative efficiency of tournaments versus piece-rates is decreasing in $s^{\max}$.*

Hence we have shown that under tournament rewards, the level of risk taking will be high, and the equilibrium effort will be high, compared to a situation where risk taking not taken into consideration as a choice variable. Moreover, under risk aversion the relative efficiency of tournaments versus piece rates is decreasing in the level of possible risk taking.

An application of these results is that they shed light on the RPE Puzzle, why relative performance evaluation is used less in CEO compensation than what standard agency theory suggests. Specifically, if risk taking is a choice variable for a CEO then the principal (e.g., the board) should be careful in conditioning rewards on the performance of other CEO's, since such schemes induce risky and lazy behavior from CEOs.

For example, one potentially important component of CEO pay is the use of relative performance evaluation in annual bonus plans (Murphy, 1999). Such plans could specify a bonus for the CEO if the performance of the firm exceeds that of the competitors. Potentially, there is a gain in such plans, since it insulates the CEO for common risk factors like market demand. For simplicity, assume that there are only two firms in the industry, and that the annual bonus of the CEO can take two values; 0 if the firm has a worse performance than the competitor (e.g., with respect to return on capital), and 1 if the firm is more successful than the competitor. If the CEO of the competing firms also have such a bonus package as an important ingredient of the compensation scheme, then the CEOs in the industry can be viewed as competing in a tournament, where the winner is the CEO whose performance is the highest. We can predict that in equilibrium, the CEO will choose risky projects and work less ardently than if he would have chosen less risky projects, since increasing the mean profit through hard work pays less. The board can offset this effect by increasing the bonus size, but such a move would add risk to the CEOs compensation, and reduce his welfare under managerial risk aversion. In view this argument, the board should be cautious with conditioning the CEO compensation on relative performance. And caution with basing pay on relative performance is exactly what the findings behind the RPE Puzzle tell us is the case in real life executive compensation.

In general, when risk taking is an option, the choice of risk taking from an agent's standpoint will be a trade off between the reduced positive effect of increasing risk on the relative component of the compensation tournament (decreased effort), and the negative effect of increasing risk on the absolute component of the compensation package (increased variance of payment). With this trade off in mind, a conjecture is that if the CEO is risk averse and faces a mixture of relative and absolute rewards, the optimal contract when risk taking is an option relies *less* on relative factors than when risk taking is not a choice variable. That would be the counterpart of our results to an optimal contract setting.

Since little is known about optimal contracts even when the principal can only condition payment on the agent's own output, to prove this conjecture is unfortunately too difficult given the present state of the literature.

We now turn to discussing whether the tournament reward structure can be modified to avoid the risky-lazy 'trap' of standard tournaments. That will shed light on the Mediocrity Puzzle.

2.3 Extension: k-contracts

The idea behind the contract form proposed in this section is that if agents are motivated to achieve a moderately high output, instead of a very high output, they can get an incentive to choose a moderate level of risk taking, which, in the next turn, can create incentives to work hard.[8]

Consider a modified tournament reward structure, where the winner of the tournament is the agent with output closest to a finite benchmark k. To avoid confusion with standard tournaments, this modified tournament structure is labeled k-contracts.

The distance between k and agent i's observed output, D_i, equals,

$$D_i = |Y_i - k| \tag{5}$$

Denote by $Q_i(..)$ agent i's probability of having an observed output closer to k than agent j, and hence win the tournament. Formally, $Q_i(..) = Prob(D_i < D_j)$. The expected utility for agent i under a k-contract then equals,

$$U_i = Q_i W_1 + (1 - Q_i) W_2 - V(\mu_i) = Q_i \Delta W + W_2 - V(\mu_i) \tag{6}$$

The following remark clarifies the relation between k-contracts and standard tournaments.

Remark 2 *If $k = \infty$ the agents play a standard tournament game. If $k < \infty$, the reward*

[8] An criticism of non-monotonic schemes is that they give incentives to *dispose* with parts of the output (if output falls in the non-monotonic range). However, since disposal is equivalent to theft, this criticism applies to all compensation schemes with marginal reward less than marginal output, which gives the agents incentives to steal the output. Contracts in practice prescribe punishment for theft (or 'disposal'), if detected. Here, I simply assume that disposal is not a choice variable for the agents.

to agent i is non-monotonic in own performance.

Proof. Recall that $P_i(..)$ is the probability of agent i winning in the standard tournament case, where $P_i = Prob(Y_i - Y_j > 0)$. We show that $Q_i(..)$ and $P_i(..)$ converge when k goes to infinity. By definition, $Q_i(..) = Prob(D_i < D_j)$. Since $D_i > 0$, we have that,

$$Q_i(..) = Prob(D_i < D_j) = Prob(D_i^2 < D_j^2) = Prob[(Y_i - Y_j)(Y_i + Y_j - 2k) < 0]. \quad (7)$$

When k tends to infinity, $(Y_i - Y_j)(Y_i + Y_j - 2k) < 0$ occurs if and only if $(Y_i - Y_j) > 0$. Hence, from (7), $Q_i(..) = Prob[(Y_i - Y_j)(Y_i + Y_j - 2k) < 0]$ converges to $Prob(Y_i - Y_j > 0) = P_i(..)$ when k tends to infinity. To see that k-contracts are non-monotonic in own performance, observe that for any Y_j, the rewards to agent i is increasing up to the point $Y_i = Y_j$, and then decreasing. ■

Remark 2 shows that standard tournament reward structure is a special case of k-contracts; when k tends to infinity, a k-contract and a standard tournament, as studied in the previous sections, are identical. However, for finite k, k-contracts differ from standard tournaments in that they give a higher reward to agents with performance in 'the middle' than agents with a top performance.

To solve for equilibrium levels of risk and effort under k-contracts, the following lemma, which we believe is novel, will be very useful. First a standard definition.

Definition 2.1 *(FOSD). Let $G_i(d;..)$ and $H_i(d;..)$ be cdf's of D_i. $G_i(d;..)$ first order stochastic dominate$H_i(d;..)$ if $G_i(d;..) \geq H_i(d;..)$ for all d, with $G_i(d;..) > H_i(d;..)$ for some d.*

Let $F(d;\eta_i)$ be the cdf of D_i, as a function of η_i, holding μ_i and k constant at $\hat{\mu}_i$ and $\hat{k}$, respectively, where $\hat{k} > \hat{\mu}_i$. Furthermore, define $\eta_i^* = \hat{k} - \hat{\mu}_i$. Now choose two values of η_i, denoted η_i^1 and η_i^2, where $\eta_i^1 < \eta_i^2$. Then we have the following.

Lemma 1 *$F(d;\eta_i^1)$ first order stochastic dominates $F(d;\eta_i^2)$, for $\eta_i^* \leq \eta_i^1 < \eta_i^2$.*

Proof. See the appendix. ■

Lemma 1 puts an upper bound on the risk taking of agent i in that any choice of standard deviation η_i larger than η_i^* generates a distribution of D_i that is dominated.

The intuition for Lemma 1 is that η_i^* is the choice of standard deviation that maximizes the probability of hitting very close to the benchmark k. If η_i is set larger than η_i^* then the distribution generated will perform worse with respect to the probability of hitting very close to k, and the potential gains from an increased probability of hitting farther from k does not offset this effect.

Corollary 1 *Suppose $\sigma > k$. Then $s_i = 0$ is a (strictly) dominating choice for agent i.*

Proof. First notice that regardless of η_i, it is dominated for agent i to choose $\mu_i > k$. Now fix μ_i at $\hat{\mu}_i$ and k at $\hat{k}$, where $\hat{\mu}_i \leq \hat{k}$, and recall that $\eta_i^* = \hat{k} - \hat{\mu}_i$. By a simple transformation, it follows that a choice of s_i^2 larger than s_i^{*2} is dominated, where $s_i^{*2} = (\hat{k} - \hat{\mu}_i)^2 - \sigma^2 = (k^2 - \sigma^2) + \mu_i(\mu_i - 2k)$, which is negative for $\sigma > k$. It follows from Lemma 1 that $s_i = 0$ is a dominating choice for agent i. ■

The corollary shows that $\sigma > k$ is a sufficient condition for agents to choose $s_i = 0$ in equilibrium.

Equipped with these results, we have the following.

Proposition 3 *For a sufficiently large σ, the first best provision of effort is implementable with a k-contract.*

Proof. See the appendix. ■

Hence in contrast to the standard tournament scheme, k-contracts and individual schemes are equivalent under risk neutrality: they both implement first best. The intuition behind Proposition 3 is that to avoid excessive risk taking, and hence a low level of effort, the principal rewards the agent with output closest to a positive constant k rather than rewarding the highest output. Reduced risk taking in turn makes it possible to give incentives for effort by increasing the prize spread, ΔW.

Let us make some comments. First, it is sufficient for Proposition 3 that the distribution of the shocks has the FOSD property described in Lemma 1. In addition to the normal, a simple distribution as the uniform also has this property. However, it is unknown whether more general distributions have the FOSD property of Lemma 1, so in this section we rely more on the normality assumption than in the previous section. Second, since linear schemes can also implement first best in the case where agents choose both effort and risk, it is not obvious why k-contracts should be preferred to linear schemes. The downside with individual schemes compared to k-contracts, however, is that they do not exploit commonality of the shocks, which may important e.g., in the market for fund managers. Hence k-contracts can insure risk averse agents as well as linear schemes, and provide stronger incentives. There exist examples with risk averse agents where k-contracts dominate linear schemes, provided that agents are not too risk averse and that the shocks are sufficiently correlated.[9]

The empirical value of Proposition 3 is that it gives an explanation for the Mediocrity Puzzle. If the rewards that accrue to an agent are such that moderately high relative output is more highly rewarded than a very high relative output, that gives agents incentive to choose a low level of risk, and hence gives incentives to work hard.

For example, a fund management company that wants to reward their fund managers according to their relative output to e.g., insulate the managers against common risk factors, may consider to give the highest reward to the manager with an output that comes closest to some benchmark or index. In addition to insulating the managers against common risk factors, such a scheme avoids giving incentives for excessive risk taking, which a scheme rewarding the highest relative performance would. And, giving the managers incentives for low levels of risk will in the next turn give them incentives to hard work on their portfolio, e.g., in collecting and assessing financial data.

As indicated in the Introduction, there are many examples where *informal reward* structures are non-monotonic; agents are encouraged to do not too well compared to a peer group. For such cases, Proposition 3 can be interpreted as saying that a norm for mediocrity, or more precisely a norm for a quite high performance - but not very high - can be more beneficial to a group than a norm for excellence.[10] Informal settings are particularly interesting for our purposes because here the nature of the reward is often such that the reward system is necessarily based on relative features. For example, as

[9] These examples have been obtained with numerical tecniques for utility functions with constant absolute risk aversion.

[10] It is interesting to note that in his satiric description of the Norwegian society, 'The Laws of Jante', Aksel Sandemose, describes a society where excellence is strongly discouraged. For example, two of the ten laws of Jante are 'Thou should not believe you are better than anyone else', 'Thou should not believe you *are* something'. Although the laws of Jante tend to focus on self-beliefs, rather than accomplishment, it seems fair to say that they strongly discourage outlier accomplishments.

emphasized by Frank (1985), social status is a positional good, in fixed supply, and hence the status of an agent must be based on a comparison to other agents.

3 Conclusion

The moral of the paper is that in tournaments where risk taking is an option, the principal gets what he does *not* pay for: rewarding a high relative performance yields a low levels of effort and expected output, while rewarding a 'mediocre' relative performance yields high levels of effort.

We first showed that if a high reward in a group goes to the agent with the highest output, this creates incentives for the agents in the group to take high risks. Although risk taking is not necessarily harmful in itself, high risk taking is associated with *low effort*, which is harmful to expected production. Hence if the rewards to CEOs depends strongly on how well its firm performs compared to other firms in an industry, e.g., through bonus plans, in equilibrium the CEOs in the industry take high risks and put in low work effort. Given this argument, we find it natural that boards in real life are careful with putting too much weight on relative factors in CEO compensation schemes.

Second, we show that if the highest reward in a group goes to an agent with a moderately high output ('mediocre'), instead of to the agent with the highest output, the agents in the group may be provided with an incentive to take a low level of risk *and to work hard*. Hence a norm, or a formal contract, that approves very high relative performances can be self-defeating, while a norm that approves of a 'mediocre' relative performance rather than a very high relative performances, can yield an efficient outcome.

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5 Appendix A

Recall that output of agent i is given by $Y_i = \mu_i + \varepsilon_i$. In Proposition 1 and Proposition 2 it was shown that if ε_i is normally distributed, and agent i controls the variance of ε_i, then in equilibrium agent i chooses to let the variance of ε_i be as high as possible, since that minimizes $g(0)$, and hence equilibrium effort. In this appendix, we generalize these results to a setting where ε_i is only required to be unimodal and symmetric. First notice that the notion of second order stochastic dominance generalizes the notion of increased variance from the normal case. We show that if the agent can choose to induce a second order stochastically dominated distribution of ε_i, he will do so because such an operation reduces $g(0)$. In particular, we show that adding an *iid* (non-degenerate) ε_i will reduce $g(0)$, and hence equilibrium effort. That generalizes Proposition 2. Moreover we show that in the limit, when the agent adds infinitely many *iid* variables to ε_i, then $g(0)$ tends to zero. That generalizes Proposition 1.

Suppose ε_i follows the density $f(x)$, with full support. For simplicity, $f(x)$ is assumed to be differentiable. By symmetry, $f(x) = f(-x)$, $\forall x$, and unimodality and symmetry implies that $f\prime(-x) < 0$, $f'(0) = 0$, and $f\prime(x) < 0$. Now construct the variable $\varepsilon = \varepsilon_i + \delta_i$, where ε_i and δ_i are *iid*, and denote the density of ε by $h(y)$, with corresponding cdf $H(y)$. The purpose is to show that $f(0) > h(0)$, from which it follows that equilibrium effort decreases when the agents add a stochastic variable to the noise terms.

First observe that,

$$H(y) = \int_{-\infty}^{\infty} \int_{-\infty}^{y-z} f(x)f(z)dxdz \tag{A1}$$

Differentiating with respect to y we get,

$$h(y) = \int_{-\infty}^{\infty} f(y-z)f(z)dz \tag{A2}$$

Inserting for $y = 0$ into (A2) and by symmetry we get

$$h(0) = \int_{-\infty}^{\infty} f(-z)f(z)dz = \int_{-\infty}^{\infty} f(z)^2dz \tag{A3}$$

It remains to show that $\int_{-\infty}^{\infty} f(z)^2dz < f(0)$. First observe that $\int_{-\infty}^{\infty} f(z)^2dz = 2\int_0^{\infty} f(z)^2dz$ by symmetry. Integrating by parts, we have that,

$$\int_0^{\infty} f(z)^2dz = -\frac{1}{2}f(0) - \int_0^{\infty} F(z)f(z)dz \tag{A4}$$

Using this expression, we get that,

$$f(0) - h(0) = f(0) + f(0) + 2\int_0^{\infty} F(z)f(z)dz \tag{A5}$$

Hence $f(0) - h(0) > 0$ iff $f(0) > -\int_0^{\infty} F(z)f(z)dz$. Substituting for $F(z) = 1 - F(-z)$ and integrating by part once more, we get that,

$$\int_0^{\infty} F(z)f(z)dz = -f(0) - \int_0^{\infty} F(-z)f(z)dz \tag{A6}$$

Simplifying, we get that, $f(0) - (-\int_0^\infty F(z)f(z)dz) = \int_0^\infty F(-z)f(z)dz < 0$, since $f\prime(z) < 0$ for $z > 0$. Hence $f(0) > h(0)$, and we have shown that adding an *iid* random variable to ε_i reduces equilibrium effort. That generalizes Proposition 2. Furthermore, it can easily be shown, and is hence skipped, that in the limit, as the number of added *iid* variables goes to infinity, the density at zero goes to zero. That generalizes Proposition 1.

6 Appendix B

We start out with a remark establishing some distributional properties of the stochastic variable D_i, the distance between agent i's output Y_i and the benchmark k. Then Lemma 1 is proved, and finally Proposition 3. Throughout the appendix, subscripts are skipped when possible.

Remark 3 *D has cdf equal to $F(d; ..) = \frac{1}{\sqrt{\pi}} \int_\alpha^\beta e^{-t^2} dt$, where $\alpha = \frac{\sqrt{2}(\mu-k-d)}{2\eta}$, and $\beta = \frac{\sqrt{2}(\mu-k+d)}{2\eta}$.*

Proof. Recall that $D = |k - Y|$, where Y is normally distributed with mean $k - \mu$ and variance η^2. Hence the cdf of D equals,

$$F(d; ..) = \frac{1}{\sqrt{2\pi\eta^2}} \int_{k-d}^{k+d} e^{-\frac{(d-\mu)^2}{2\eta^2}} \Delta d \tag{B1}$$

where $d \geq 0$. This is just the probability that a single realization of normally distributed variable with expectation μ and variance η^2 falls within a distance d of a benchmark k. By standard procedures, the integral simplifies to,

$$F(d; ..) = \frac{1}{\sqrt{\pi}} \int_\alpha^\beta e^{-t^2} dt, \text{ where } \alpha = \frac{\sqrt{2}(\mu - k - d)}{2\eta}, \text{ and } \beta = \frac{\sqrt{2}(\mu - k + d)}{2\eta} \tag{B2}$$

It is easily checked that $F(d; ..)$ indeed induces a probability distribution, i.e., that $\lim_{d\to\infty} F(d; ..) = \lim_{d\to\infty} \frac{1}{\sqrt{\pi}} \int_\alpha^\beta e^{-t^2} dt = \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty e^{-t^2} dt = 1$. It can be noted that since D_i^2 is χ^2-distributed, D_i is distributed as the square root of a χ^2 variable.

Differentiating $F(d;..)$ with respect to d, we obtain the density $f(d;..)$,

$$f(d;..) = \frac{\partial F(d;..)}{\partial d} = \frac{e^{-\frac{(\mu-k-d)^2}{2\eta^2}} + e^{-\frac{(\mu-k+d)^2}{2\eta^2}}}{\sqrt{2\pi\eta^2}} \tag{B3}$$

■

Proof. of Lemma 1.

Recall that, by definition, $\eta^2 = \sigma^2 + s^2$, and $\eta^* = \hat{k} - \hat{\mu}$. We show that any choice of η greater than η^* is dominated in the sense of FOSD. Substitute $\mu = \hat{\mu}$ and $k = \hat{k}$ into $F(d;..)$ from Remark 1, substitute for η^*, and differentiate with respect to η, to obtain,

$$\frac{\partial F(d;..)}{\partial \eta} = \frac{1}{\sqrt{2\pi\eta^2}}[(\eta^* - d)e^{-\frac{(\eta^*-d)^2}{2\eta^2}} - (\eta^* + d)e^{-\frac{(\eta^*+d)^2}{2\eta^2}}] \tag{B4}$$

We proceed to show that this expression is negative for $\eta > \eta^*$, and hence Lemma 1 follows. Denote the first term of the right side of (B4) by A_1, and the second term by A_2. Moreover, substitute in $\eta^* + \alpha$ for η, where $\alpha > 0$. Hence $A_1 = (\eta^* - d)e^{-\frac{(\eta^*-d)^2}{2(\eta^*+\alpha)^2}}$ and $A_2 = (\eta^* + d)e^{-\frac{(\eta^*+d)^2}{2(\eta^*+\alpha)^2}}$. Since $A_2 > 0$, $\frac{\partial F(d;..)}{\partial s} < 0$ is equivalent to $\frac{A_1}{A_2} < 1$, for $d > 0$. We finish the proof by showing that $\frac{A_1}{A_2} < 1$, for $d > 0$.

$$\frac{A_1}{A_2} = \frac{(\eta^* - d)e^{-\frac{(\eta^*-d)^2}{2(\eta^*+\alpha)^2}}}{(\eta^* + d)e^{-\frac{(\eta^*+d)^2}{2(\eta^*+\alpha)^2}}} = \frac{\eta^* - d}{\eta^* + d}e^{\frac{2d\eta^*}{(\eta^*+\alpha)^2}} = \frac{\eta^* - d}{\eta^* + d}e^{\frac{2d\eta^*}{(\eta^*+\alpha)^2}} \tag{B5}$$

Notice that from (B5) it follows that $\frac{A_1}{A_2} = 1$ when $d = 0$. We show that $\frac{A_1}{A_2} < 1$ for any $d > 0$. Differentiating (B5) with respect to d yields,

$$\frac{\partial(\frac{A_1}{A_2})}{\partial d} = -2\frac{e^{\frac{2d}{(\eta^*+\alpha)^2}}\eta^*(2\eta^*\alpha + \alpha^2 + d^2)}{(\eta^* + d)^2(\eta^* + \alpha)^2} \tag{B6}$$

which is negative for $d > 0$. Hence $\frac{A_1}{A_2} < 1$, for $d, \alpha > 0$, and consequently $\frac{\partial F(d;..)}{\partial \eta} < 0$ for $\eta > \eta^*$, and $d > 0$, and Lemma 1 follows. ■

Proof. of Proposition 3.

Suppose σ is larger than the first best level of effort, μ_i^*. We show that this condition is sufficient for first best to be implementable. First notice that, for a given k, to choose effort level μ_i larger than k is a dominated choice for agent i. Hence we can restrict attention to $\mu_i \in [0, k]$, $i = 1, 2$. Moreover, choose k such that $\mu_i^* < k < \sigma$. Then, by Corollary 1, $s_i = 0$ is a dominating strategy for agent i, and we can restrict attention to solve for equilibrium in choice of effort. The first order conditions are,

$$\frac{\partial U_i}{\partial \mu_i} = \frac{\partial Q_i}{\partial \mu_i}\Delta W - \frac{\partial V_i}{\partial \mu_i} = 0, \;\; i = 1, 2. \tag{B7}$$

The probability of agent i winning under a k-scheme, $Q_i(..)$, equals,

$$\begin{aligned} Q_i &= \int_0^\infty F_i(d) f_j(d)\Delta d \\ &= \int_0^\infty \frac{e^{-\frac{(\mu_j - k - d)^2}{2\sigma^2}} + e^{-\frac{(\mu_j - k + d)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}} [\frac{1}{\sqrt{\pi}} \int_\alpha^\beta e^{-t^2} dt]\Delta d \end{aligned} \tag{B8}$$

Define $\text{erf}(x) = \int_{-\infty}^x e^{-t^2} dt$, and $erfc(x) = 1 - erf(x)$. Differentiate (B8) by μ_1 and normalize by setting $\sigma = 1$ to obtain,

$$\begin{aligned} \frac{\partial Q_1}{\partial \mu_1}_{\mu_1 < \mu_2} &= -\frac{1}{2\sqrt{\pi}}\{[(e^{\frac{1}{4}(\mu_1 + \mu_2 - 2k)^2})(\text{erf}\, c(k - \frac{1}{2}\mu_1 - \frac{1}{2}\mu_2) - e^{\frac{1}{4}(\mu_1 - \mu_2)^2} + \\ & e^{\frac{1}{4}(\mu_1 - \mu_2)^2}(\text{erf}\, c(\frac{1}{2}\mu_1 - \frac{1}{2}\mu_2)]e^{\mu_1 + \mu_2 - k - \frac{1}{2}\mu_1^2 - \frac{1}{2}\mu_2^2}\} \end{aligned} \tag{B9}$$

while,

$$\begin{aligned} \frac{\partial Q_1}{\partial \mu_1}_{\mu_1 > \mu_2} &= -\frac{1}{2\sqrt{\pi}}\{[(e^{\frac{1}{4}(\mu_1 + \mu_2 - 2k)^2})(\text{erf}\, c(k - \frac{1}{2}\mu_1 - \frac{1}{2}\mu_2) + e^{\frac{1}{4}(\mu_1 - \mu_2)^2} - \\ & e^{\frac{1}{4}(\mu_1 - \mu_2)^2}(\text{erf}\, c(\frac{1}{2}\mu_1 - \frac{1}{2}\mu_2)]e^{\mu_1 + \mu_2 - k - \frac{1}{2}\mu_1^2 - \frac{1}{2}\mu_2^2}\} \end{aligned} \tag{B10}$$

Substitute for $\mu_1 = \mu_2$ to obtain,

$$\frac{\partial Q_i}{\partial \mu_i}_{\mu_i = \mu_j} = \frac{\mathrm{erf}(k - \mu_i)}{2\sqrt{\pi}} \tag{B11}$$

which is continuous and increasing in k. Therefore, since the cost of effort $V(\mu_i^*)$ is convex, the symmetric equilibrium is increasing in k. From equation (2) and equation (B11) it is evident that the symmetric equilibrium is increasing (continuously) in ΔW, where equilibrium effort equals k, in the limit, as ΔW tends to infinity. Hence for $\mu_i^* < \sigma$ and for any k such that $\mu_i^* < k < \sigma$, there exist a ΔW such that μ_i^* is implemented in Nash equilibrium. ■