Viewing the Deregulated Electricity Market as a Bilevel Programming Problem

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Abstract

In this paper we present a bilevel programming formulation of a deregulated electricity market. By looking at a deregulated electricity market in this format we achieve two things,

i) the relation between a deregulated electricity market and other economic models that can be formulated as bilevel programming problems becomes clear, (i.e. Stackelberg leader-follower games and principal-agency models)

ii) an explanation of the reason why the so called "folk theorems" in electricity networks can be proven to be false

The interpretation of a deregulated electricity market as a bilevel program also indicates the magnitude of the error that can be made if the electricity market model studied does not take the physical constraints into account or oversimplifies the electricity network to a radial network.

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Introduction

Wu et al. (1996) give counter-examples to a number of propositions regarding the characteristics of optimal nodal prices, which at first sight, without any specific knowledge of power networks, seem quite intuitive. Among the "folk theorems" that are proven false are

1) Uncongested lines do not receive congestion rents (defined through nodal price differences)
2) In an efficient allocation power can only flow from nodes with lower prices to nodes with higher prices, and
3) Strengthening transmission lines or building additional lines increases transmission capacity

It is argued that these assertions stem from the incorrect analogy between power transmission and the transportation of goods. Economic analyses of the transportation of goods can be found already in the classical works on *spatial price equilibrium* by Enke (1951) and Samuleson (1952)[2]. While appealing to economic intuition, this note intends to give one possible explanation of the foundation for the difference between markets that are based on power transmission networks and spatial markets based on simpler models for transportation of goods, like commodity flows or transportation problems.

Let $B_i(S_i^d)$ be the benefit from consuming complex power $S_i^d = P_i^d + jQ_i^d$, and $C_i(S_i^s)$ the cost of producing $S_i^s = P_i^s + jQ_i^s$ in node *i*. A general formulation of the optimal dispatch problem, taking into account thermal capacity limits, is then given by problem (1)-(7) (ref. Wangensteen et al. (1995)). (1) is the objective function, maximizing social surplus while

[2] The spatial price equilibrium model can be phrased as follows. Buyers and sellers of a commodity are located at the nodes of a transportation network and the issue is to determine simultaneously the quantities supplied and demanded at each node, the local (nodal) prices at which the commodity is bought and sold, and the commodity flows between pairs of nodes.

summing benefits and withdrawing cost over all the nodes. (2) defines net injection $S_i = P_i + jQ_i$ in every node, and (3) and (4) relate complex power to complex voltage V_i and the conjugates of complex node and line currents I_i and I_{ik}. Inequalities (5) represent the thermal capacity constraints, which are stated in terms of limits C_{ik} on the magnitude of apparent power, $|S_{ik}| = \sqrt{P_{ik}^2 + Q_{ik}^2}$. Equations (6) represent Kirchhoff's junction rule and (7) Ohm's law with Kirchhoff's loop rule incorporated, Y_{ik} being the admittance of line *ik*.

(1) max $\sum_i [B_i(S_i^d) - C_i(S_i^s)]$

(2) s.t. $S_i = S_i^s - S_i^d \qquad \forall i$

(3) $S_i = V_i \cdot I_i^* \qquad \forall i$

(4) $S_{ik} = V_i \cdot I_{ik}^* \qquad \forall ik$

(5) $|S_{ik}| \le C_{ik} \qquad \forall ik$

(6) $I_i = \sum_{k \ne i} I_{ik} \qquad \forall i$

(7) $I_{ik} = Y_{ik}(V_i - V_k) \qquad \forall ik$

A Bilevel Programming Formulation of the Deregulated Electricity Market

It is well known (since the work of Kirchhoff and Maxwell in the 19th century) that the physical equilibrium of electric networks can be described in terms of minimization of total power-losses, i.e. the electric current follows the path of least resistance. To simplify, consider now a direct current (DC) model, where all power flows, voltages and currents of problem (1)-(7) are real numbers. Given node currents I_i, optimal line currents I_{ik} are obtained by solving the following convex flow problem (see for instance Dembo et al. (1989)):

(8) $$\min \quad \tfrac{1}{2}\sum r_{ik} I_{ik}^2$$

(9) $$\text{s.t.} \quad I_i = \sum_{k \neq i} I_{ik} \qquad \forall i$$

where r_{ik} is the resistance of line *ik*.

Introducing dual variables V_i of equations (9), the Lagrangian can be written

(10) $$\Phi = \tfrac{1}{2}\sum_{ik} r_{ik} I_{ik}^2 + \sum_i (I_i - \sum_{k \neq i} I_{ik}) \cdot V_i$$

with first order conditions

(11) $$\frac{\partial \Phi}{\partial I_{ik}} = r_{ik} I_{ik} - V_i + V_k = 0 \qquad \forall ik$$

and

(12) $$\frac{\partial \Phi}{\partial V_i} = I_i - \sum_{k \neq i} I_{ik} = 0 \qquad \forall i\,.$$

Condition (11) implies

(13) $$I_{ik} = \frac{V_i - V_k}{r_{ik}} = Y_{ik}(V_i - V_k) \qquad \forall ik$$

since admittance $Y_{ik} = 1/r_{ik}$ in a DC network. I.e. the first order conditions of problem (8)-(9) correspond to equations (6) and (7). This means that we can reformulate the optimal dispatch problem (assuming a DC network with real power only, i.e. $S_i = P_i$) to:

P1 $$\max_{P_i^s, P_i^d, I_i} \sum_i [B_i(P_i^d) - C_i(P_i^s)]$$

s.t. $$P_i = P_i^s - P_i^d \qquad \forall i$$

$$P_i = V_i I_i \qquad \forall i$$

$$P_{ik} = V_i I_{ik} \qquad \forall ik$$

$$P_{ik} \leq C_{ik} \qquad \forall ik$$

and given I_i $\forall i$, I_{ik} is implicitly defined by,

P2 min $$\tfrac{1}{2} \sum r_{ik} I_{ik}^2$$

s.t. $$I_i = \sum_{k \neq i} I_{ik} \qquad \forall i$$

which provides also the dual variables V_i. In this formulation it is evident that the first level, P1, sets the node currents, and the agents, the electrons, react on this by following the path of least resistance. Hence, in economic modeling terms this, represented by P2, is the behavioral assumption made upon the agents.

Problem P1-P2 fits into the framework of bilevel programs that are discussed in Kolstad (1985). Thus, the optimal dispatch problem can be seen as a bilevel program consisting of an *upper level* program, which is the social maximization problem P1, and a *lower level* program or *behavioral* problem P2, which determines line currents and, as a byproduct, voltages. The intention of this bilevel construction is to reveal the structure of the problem, not to indicate how it should be solved. In general, the problem is highly nonlinear and non-convex with interdependencies between the variables. However, according to the classification of Kolstad (1985), formulation (1)-(7) can be understood to arise after applying a *Kuhn-Tucker-Karush-*method to P1-P2, transforming the behavioral problem P2 into Kuhn-Tucker-Karush necessary conditions for optimality, and solving (1)-(7) is equivalent to solving P1-P2.

A number of economic problems can be interpreted as bilevel programs. For instance, a *Stackelberg leader-follower* game can be viewed as a bilevel program with the leader's problem corresponding to P1 and the follower's problem corresponding to P2 (Kolstad (1985), Migdalas and Pardalos (1993), and Vicente and Calamai (1994)). In this type of model, the follower chooses his strategy in full knowledge of the leader's decision, a fact that the leader takes into consideration when determining his own actions. Similarly, *principal-agent* problems can be interpreted in the same manner, as the principal takes into account the behavior of the agent acting in his own self interest (modeled through P2) when solving the upper level program P1.

Returning to the optimal dispatch problem of electrical networks, and the discussion of Wu et al. (1996) concerning the incorrect analogy between power transmission and transportation of goods, constraint (6), which is Kirchhoff's junction rule, is normally accounted for in most transportation models. However, if one is to disregard Kirchhoff's loop rule in the analysis, thus assuming power is routable, the error made may be of the same order as ignoring the behavior of the followers in a Stackelberg leader-follower game or the behavior of the agents in a principal-agent setting. Despite obvious similarities between the operation of the power market and spatial price equilibrium models, focusing on the physical equilibrium of a power network leads to the awareness that one should rather have in mind something similar to *traffic equilibrium* problems as the underlying network model when investigating power markets. In power networks, strengthening a line may lead to reduced transmission capacity and/or reduced social surplus in optimal dispatch, which is an analogy to the famous Braess' paradox (1968) in traffic equilibrium networks. Also the same *non-cooperative* phenomenon is recognized in communication networks, as is evident from the works of for instance MacKie-Mason and Varian (1995), Shenker (1995), Shenker et al. (1996), Korilis et al. (1997a, 1997b) and Gupta et al. (1997).

Conclusions

Viewing the optimal dispatch problem as a bilevel mathematical program with interacting physical and economic equilibria may make it easier for an economist to understand the difference between classical spatial equilibrium models and equilibria in power networks. It also provides a way to get an understanding of the magnitude of the error that can be made by simplifying the network description by disregarding Kirchhoff‘s loop rule or by simplifying the network description to a radial network. As a by-effect the formulation can lead to new ideas regarding optimal transmission pricing in a decentralized electricity market. For instance, instead of (or additional to) checking if a market equilibrium is physically feasible, one could check whether a physical equilibrium is economically viable. It can also be fruitful to have this formulation in mind when simplifying an electricity network into a virtual radial network to be used in aggregate electricity market models. Whether these are interesting approaches, and how they could be used in a practical procedure, is a topic for future investigation.

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