Estimation of biological and economic parameters of a bioeconomic fisheries model using dynamical data assimilation

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Abstract

A new approach of model parameter estimation is used with simulated measurements to recover both biological and economic input parameters of a natural resource model. The procedure efficiently combines time series of observations with a simple bioeconomic fisheries model to optimally estimate the model parameters. Using identical twin experiments, it is shown that the parameters of the model can be retrieved. The procedure provides an efficient way of calculating poorly known model parameters by fitting model results to observations. In separate experiments with exact and noisy data, we have demonstrated that the adjoint technique of parameter estimation can be an efficient method of analyzing bioeconomic data. Results of sensitivity analysis of the model parameters show that the environmental carrying capacity (e.g., habitat) is the most important input parameter in the model.

Keywords: Dynamic parameter estimation;adjoint method;twin experiments; fisheries

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1 Introduction

In this paper we use a very simple bioeconomic fisheries model to introduce a new technique of optimally estimating the parameters of a dynamic fisheries model and to demonstrate its potential usefulness. The approach is known as data assimilation. In data assimilation mathematical or numerical models are combined with available measurements in order to improve the model itself or to improve the model forecasts. The former application is known as model fitting (Smedstad and O'Brien, 1991; Yu and O'Brien, 1991; Lawson et al., 1995; Spitz et al., 1997). A variety of these techniques such as the Kalman Filter (Kalman, 1960; Gelb, 1974), the Extended Kalman Filter (Gelb, 1974; Evensen, 1994), Optimal Interpolation (Lorenc, 1986) and the Variational Data Assimilation (Smedstad and O'Brien, 1991) are already in common use. These techniques have extensively been used in areas such as groundwater hydrology and petroleum reservoirs (see for example Carrera and Neuman, 1986(a,b,c); Yeh, 1986) and more recently in ecosystem models (Spitz et al., 1997; Matear, 1995). The technique in this paper is the so called adjoint parameter estimation. This method minimizes a preconstructed loss or criterion function which is defined by the differences between the data and the model forecasts. The optimal or best fit parameters are obtained by minimizing the loss function subject to the dynamical constraints via the so called adjoint equations which map the predefined loss function into the gradient with respect to the control parameters (Lawson et al., 1995; Spitz et al., 1997). The gradients are then used iteratively in a descent or Newton type of algorithm in order to search for the minimum of the loss function.

Application of these techniques is spreading very rapidly to many areas such as physical and biological systems (Spitz et al., 1997). Lawson et al., (1995) applied the technique of data assimilation to the well known and extensively studied predator-prey model in biology. In another application, Gauthier, (1992) applied data assimilation with an adjoint model technique to the Lorenz model. Other techniques of data assimilation are the simulated annealing (Kirkpatrick et al., 1983; Kruger, 1992) and the hybrid Monte Carlo techniques (Harmon and Challenor, 1996). These are stochastic in nature and may be very costly to implement. Data assimilation is also widely used in meteorology and oceanography to estimate initial and boundary conditions. For reviews of these methods in meteorology and physical oceanography refer to (Bengtsson, 1981; Navon, (1986, 1997)).

Research in bioeconomics of fish stocks has been based on simple aggregated biological models (Schaefer, 1954). Classical growth models such as the logistic and the exponential functions are the commonly employed models in the qualitative analysis of fish stocks (Sandal and Steinshamn, 1997(a,b,c); Clark, 1990).

The reality of these models has not yet been rigorously tested. Data assimilation gives us the opportunity to test these models using the available data which is assumed to be more realistic than the models themselves. More realistic bioeconomic models of renewable resource stock may be multidimensional and highly complex. They may contain many parameters such us the intrinsic growth rate, the catchability coefficient, the environmental carrying capacity, etc, whose values are extremely difficult to measure. Parameterization of the models become mathematically untractable and impractical. As a consequence, biologists and bioeconomists have found it necessary to introduce simpler models. The issue of identification, i.e., the problem of estimating parameters so that the model predictions are more realistic and useful, has been raised by many natural resource economists (Clark, 1990). However, adequate attention has not been given to the problem. Instead, economists have focused on the analytical considerations leading to a neglect of most of the vital questions resource managers/fishers are mostly concerned about, e.g., what is the safe biomass level (Deacon et al., 1998), etc.

Due to the progress in data collection and processing in recent times from both the fisheries biologists and economists and the advances in computer technology, techniques of data assimilation which are both data and computer intensive have good future prospects. The assimilation technique introduced in this paper has some advantages compared to the conventional techniques. First it is very attractive and computationally efficient to implement. Second, it can be used for more realistic and complex dynamical models of bioeconomic systems. Third, it can be used to adjust both the initial conditions and the parameters of the model. Fourth, it can also be used to estimate the variables of the dynamical model.

An adjoint variational formulation will be used in this paper as an alternative approach to the widely used regression analysis in economics. It is a more general formulation than the traditional regression. It can be used for linear and nonlinear models and is highly suitable for more realistic models where closed form solutions are unattainable. The methods can be used to simultaneously estimate a large number of parameters. It can also be used to perform sensitivity analysis of the input parameters of the model.

Most of the optimization algorithms used in adjoint parameter estimation are iterative. Parameter estimation in dynamical systems is generally nonlinear even if the model is linear (Evensen et al., 1998). Consequently, the loss function may contain multiple extrema. Notice that as with all iterative techniques, one should expect problems with convergence to the global optimum due to flat regions in the domain of search.

The technique introduced in this paper provides a more general and novel approach of incorporating information from field measurements into bioeconomic models. Primarily, the focus of the paper is to demonstrate the applicability of the methods in natural resource economics. The plan of this paper is as follows. First, the technique of data assimilation will be introduced and discussed. Second, the adjoint "strong constraint" (Sasaki, 1970) method will be presented and the solution algorithm outlined. Third, the bioeconomic fisheries model will be presented and a sensitivity analysis performed to investigate the importance of the input parameters to the response. Fourth, identical twin experiments with clean and noisy data will be performed and the result discussed. Finally, we discuss and summarize the work. The mathematical details are relegated to the Appendix.

2 Data Assimilation

Variational adjoint technique determines an optimal solution by minimizing a loss function that measures the discrepancy between the model counterparts to the data and the available measurements. The method leads to the solution of the model equations that best fits the available measurements throughout the assimilation interval in a least squares sense. Data assimilation systems consist of three components: the forward model with a criterion function, the adjoint or backward model and an optimization procedure

(Lawson et al., 1995). The forward model is our mathematical representation of the system we are interested in studying, e.g., an open access, a regulated open access or a sole owner fishery.

The adjoint model consists of equations obtained by enforcing the dynamical constraints through Lagrange multipliers and provide a method of calculating the gradient of the loss function with respect to the control variables. The gradients are then used in a line search using standard optimization packages to find the minimum of the loss function. Most optimization routines are based on iterative schemes which require the correct computation of the gradient of the loss function with respect to the control variables. In the adjoint formulation, computation of the gradient is achieved through the adjoint equations forced by the model-data misfits. The model equations are run forward in time while the adjoint equations are run backward in time which is then used to calculate the gradient of the loss function.

An important step in data assimilation is the choice of the criterion function for the goodness of fit. The commonly used criterion is the generalized least squares criterion. It can be defined with no a priori information about the parameters or with prior information about the parameters incorporated as a penalty term in the criterion function. Some researchers argue that since some information about the parameters and their uncertainties are always available, adding the information is a plausible thing to do (Harmon and Challenor, 1997; Evensen et al., 1998).

3 The Penalty Function

In adjoint parameter estimation a loss functional which measures the difference between the data and the model equivalent of the data is minimized by tuning the control variables of the dynamical system. The goal is to find the parameters of the model that lead to model predictions that are as close as possible to the data. A typical penalty functional takes the more general form

$$\mathcal{J}[\mathbf{X},\alpha] = \frac{1}{2}\int_0^{T_f} (\mathbf{X}-\hat{\mathbf{X}})^T \mathbf{W} (\mathbf{X}-\hat{\mathbf{X}}) dt + \frac{1}{2T_f}\int_0^{T_f} (\alpha-\alpha_0)^T \mathbf{W}_\alpha (\alpha-\alpha_0) dt \tag{1}$$

where $\hat{\mathbf{X}}$ is the observation vector, $\mathbf{X}$ is the model equivalent of the data, α_0 is first or best guess of the parameter vector and α is the parameter vector to be estimated. The period of assimilation is denoted by T_f and T is the transpose operator. The $\mathbf{W}'\mathbf{s}$ are the weight matrices which are optimally the inverses of the error covariances of the observations. They are assumed to be positive definite and symmetric. The second term in the penalty functional represents our prior knowledge of the parameters and ensures that the estimated values are not too far away from the first guess. It may also enhance the curvature of the criterion function by contributing a positive term $\mathbf{W}_\alpha$ to the Hessian (second order derivatives) of $\mathcal{J}$ (Smedstad and O'Brien, 1991).

For this analysis, it will be convenient to assume that the errors are not serially correlated. This implies that the covariance matrices are now diagonal matrices with the variances along the diagonal. We further assume that the variances are constant. Then the weight matrices can be written as $\mathbf{W} = w\mathbf{I}$ and $\mathbf{W}_\alpha = w_\alpha \mathbf{I}_\alpha$ where the $\mathbf{I}'s$ denote unit matrices and the small lettered $w's$ are the weights. The discretized loss function becomes

$$\mathcal{J} = w \sum_{n=1}^{N} (\mathbf{X}_n - \hat{\mathbf{X}}_n)^T (\mathbf{X}_n - \hat{\mathbf{X}}_n) + w_\alpha \sum_{p=1}^{P} (\alpha_p - \alpha_{0p})^T (\alpha_p - \alpha_{0p}) \tag{2}$$

where P is the number of parameters and N is the number of observations. One special case is the choice of the weights equal to unity which leads to the least-squares procedure. The frequently made statistical assumption about the errors in the literature is that of normality. If this assumption is satisfied, then the optimal least squares estimators are the maximum likelihood estimators.

3.1 The adjoint method

Construction of the adjoint code is identified as the most difficult aspect of the data assimilation technique (Spitz et al., 1997). One approach consists of deriving the continuous adjoint equations and then discretizing them (Smedstad and O'Brien, 1991). Another approach is to derive the adjoint code directly from the model code (Lawson et al., 1995; Spitz et al., 1997). For this analysis, it might be instructive to derive the adjoint equations using the first approach. Consider the forward model equation

$$\begin{aligned} \frac{\partial \mathbf{X}}{\partial t} &= F(\mathbf{X}, \alpha) \\ \mathbf{X}(0) &= \mathbf{X}_0 \\ \alpha &= \alpha_0 + \hat{\alpha} \end{aligned} \tag{3}$$

where $\mathbf{X}$ is a scalar or vector of model variables, $\hat{\alpha}$ represents vector of model parameter errors, F is a linear or nonlinear operator, and $\mathbf{X_0}$ is the vector of initial conditions.

Formulating the Lagrange function by appending the model dynamics as a strong constraint

$$\mathcal{L}[\mathbf{X},\alpha] = \mathcal{J} + \frac{1}{2}\int_0^{T_f} \mathbf{M}(\frac{\partial \mathbf{X}}{\partial t} - F(\mathbf{X},\alpha))dt \tag{4}$$

where $\mathbf{M}$ is a vector of Lagrange multipliers which are computed in determining the best fit. The original constrained problem is thus reformulated as an unconstrained problem. At the unconstrained minimum the first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \mathbf{X}} = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{M}} = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \tag{7}$$

It is observed that equation (5) results in the adjoint or backward model, equation (6) recovers the model equations while (7) gives the gradients with respect to the control variables. Using calculus of variations or optimal control theory (see Appendix), the adjoint equation is

$$\begin{aligned} \frac{\partial \mathbf{M}}{\partial t} + [\frac{\partial F}{\partial \mathbf{X}}]^T \mathbf{M} &= \mathbf{W}(\mathbf{X} - \hat{\mathbf{X}}) \\ \mathbf{M}(T_f) &= \mathbf{0} \end{aligned} \tag{8}$$

and the gradient relation is

$$\Delta_{\alpha\mathcal{J}} = -\int_0^{T_f} [\frac{\partial F}{\partial \alpha}]^T \mathbf{M} dt + \mathbf{W}_\alpha(\alpha - \alpha_0) \tag{9}$$

The term on the RHS of (8) is the weighted misfit which acts as forcing term for the adjoint equations. It is worth noting here that, we have implicitly assumed that data is continuously available throughout the integration interval. Equations (3) and (8) above constitute the Euler-Lagrange (E-L) system and form a two-point boundary value problem. For details of the derivation see the Appendix.

The implementation of the adjoint technique on a computer is straightforward. The algorithm is outlined below.

- Choose the first guess for the control parameters.
- Integrate the forward model over the assimilation interval.
- Calculate the misfits and hence the loss function.
- Integrate the adjoint equation backward in time forced by the data misfits.
- Calculate the gradient of $\mathcal{L}$ with respect to the control variables.
- Use the gradient in a descent algorithm to find an improved estimate of the control parameters which make the loss function move towards a minimum.
- Check if the solution is found based on a certain criterion.
- If the criterion is not met repeat the procedure until a satisfactory solution is found.

The optimization step is performed using standard optimization procedures. In this paper, a limited memory quasi-Newton procedure (Gilbert and Lemarechal, 1991) is used. The success of the optimization depends crucially on the accuracy of the computed gradients. Any errors introduced while calculating the gradients can be detrimental and the results misleading. To avoid this incidence from occurring, it is always advisable to verify the correctness of the gradients (see, Smedstad and O'Brien, 1991; Spitz et al., 1997). Verification of the adjoint code is performed by a simple Taylor series expansion of the penalty functional about a certain vector of the control variable $\mathbf{x}$, i.e.,

$$\mathcal{J}(\mathbf{x} + \gamma\mathbf{u}) = \mathcal{J}(\mathbf{x}) + \gamma\mathbf{u}^T\Delta_{\mathbf{x}}\mathcal{J}(\mathbf{x}) + O(\gamma^2) \tag{10}$$

where γ is a small scalar, $\mathbf{u}$ is an arbitrary vector (see, Smestad and O'Brien, 1991; Lawson et al., 1995). Defining a function $\Phi(\gamma)$ and rewriting (10) we have

$$\Phi(\gamma) = \frac{\mathcal{J}(\mathbf{x} + \gamma\mathbf{u}) - \mathcal{J}(\mathbf{x})}{\gamma\mathbf{u}^T\Delta_{\mathbf{x}}\mathcal{J}(\mathbf{x})} = 1 + O(\gamma) \tag{11}$$

Hence, in the limit as γ approaches zero, the value of $\Phi(\gamma)$ approaches unity. If the scalar γ is small, the values of the function $\Phi(\gamma)$ will be approximately equal to one.

4 The Bioeconomic Model

This section presents the bioeconomic fisheries model. It is an aggregated or lumped parameter model of a single cohort or year-class (Clark, 1990). The population dynamics of the fishery will be modeled as

$$\frac{dx}{dt} = f(x) - h \tag{12}$$

where $h(t)$ is the harvest rate from the stock, $x(t)$ is the stock biomass, $f(x)$ is the growth function which for this analysis we will use the simple logistic model. That is

$f(x) = rx(1 - x/K)$ where r and K are positive constants called the intrinsic growth rate and the environmental carrying capacity respectively. The biological model specified above depends on two parameters (r, K). These parameters are little known in the scientific community for most fish stocks around the world. Accurate measurements of their values are difficult if not impossible. Statistical estimation methods were devised by (Schaefer, 1967). This paper has similar goals of devising mathematical methods of estimating the parameters of the model equations (differential equations), testing the realism of the models against the observational data and subsequently improving the models.

The economics is also simple in this analysis. A hypothetical sole owner is envisaged. The goal of the management is to maximize a given discounted net economic function

$$max \int_0^{T_f} e^{-\delta t} \pi(h) dt \tag{13}$$

where $h(t)$ is the control variable, T_f is the time horizon which may be finite or infinite and $\pi(h)$ is a rescaled or normalized economic function given by

$$\pi = ah - h^2 \tag{14}$$

where a is an economic parameter to be estimated. The objective function is assumed to depend explicitly only on the harvest rate from the stock. This could be commercial, social or even political. For a commercially managed fishery, the objective may be maximization of discounted rents accruing from the exploitation of the biological species. A socially oriented manager may optimize the total utility or aim at conserving the stock. As oppose to these two objectives, a politically motivated goal may be to protect peoples jobs.

In the above formulation, δ is the discount rate which has been commonly set to the current interest rate. It may however be plausible to estimate the value of the discount rate from available data. That is instead of pre-setting the value, we allow the data to tell us what its value is for the fishery we are modeling. Theoretically, the optimum equilibrium conditions are met if the marginal productivity of the resource in question is equal to the discount factor or the social time-preference. However, this may not be the case in a real-world scenario. There are many convincing reasons why one will think that the managers time-preference is significantly different than the market interest rate. Another important input in the analysis of resource models is the time horizon. In this paper, the time horizon is a free parameter. It can be preset exogenously or it can be determined as an endogenous parameter in the problem. Hence, in spite of the quite restrictive form of the performance index in this paper the control problem is to some extent quite general. Various special cases can be formulated and discussed. The case considered here is a fixed horizon control problem.

Maximizing the above problem (13) subject to the natural constraint is a nonlinear optimal control problem (Clark and Munro, 1970). The technique has been discussed by several authors (e.g., Kamien and Schwartz, 1981)

Application of maximum principles to the above problem yields the following two system of coupled nonlinear ODEs (see Clark, 1990; Appendix).

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) - h \tag{15}$$

$$\frac{dh}{dt} = -0.5(a - 2h)(\delta - r(1 - \frac{2x}{K})) \tag{16}$$

Analytical solutions of the system of nonlinear ODEs are in general unobtainable. However, approximate numerical solutions of these equations are easy to obtain if the necessary initial and boundary conditions are specified. In reality however, the initial conditions that lead to the separatrix solution are not precisely known a priori. Given a set of data, a solution could be found by either manually or automatically tuning the poorly known initial and/or boundary conditions as well as the free parameters of the model until a satisfactory solution is obtained. This could easily and efficiently be achieved by the use of the data assimilation technique introduced in this paper.

Identical twin experiments will be used throughout the analysis. That is data will be generated from the model itself using Monte Carlo simulations. This guarantees that the model and the data are consistent and serves as a good test of the adjoint algorithm. A moderate objective of the paper is to demonstrate the usefulness of this approach in resource modeling, hence the use of this simple model which is fairly known and used in bioeconomic fisheries analysis. To carry out the experiments, reasonable parameter values will be chosen for the model and an adjoint parameter estimation technique will be used to recover the parameters.

4.1 Equilibrium analysis

The equilibrium behavior of the simple fishery model in this paper is studied. For an infinite horizon problem, the optimal path for the dynamics is the separatix leading to the equilibrium point. At the optimal equilibrium, the following conditions hold: $\frac{dx}{dt} = 0$ and $\frac{dh}{dt} = 0$ which implies that $rx^*(1 - \frac{x^*}{K}) - h^* = 0$ and $-0.5(a - 2h^*)(\delta - r(1 - \frac{2x^*}{K})) = 0$. This gives rise to these equilibrium points: ($h^* = a/2$, $h^* = rx^*(1 - x^*/K)$) and (

$\delta = r(1-2x^*/K)$, $h^* = rx^*(1-x^*/K)$). Notice that the first equilibrium is a bliss point (i.e., point at which the benefits are maximum) and most rewarding from the economic viewpoint if achievable. It is the theoretical optimum for the quadratic benefit function defined above.

- Case 1. For the case of the bliss point the optimal equilibrium harvest and stock are $h^* = a/2$ and $x^* = \frac{2r \pm \sqrt{4r^2 - \frac{8ar}{K}}}{4r/K}$, where the term under the square root sign is restricted to be nonnegative. Many interesting observations are made here. Notice here that when the term under the square root sign is zero, we recover the maximum sustainable yield (MSY) optimum $x^* = K/2$. If the square root term is large compared to $2r$, we have a single equilibrium. However, if the square root term is small compared to $2r$ two different values exist for the equilibrium stock and the larger may be preferred. A larger standing equilibrium stock leads to higher sustained yield for the capital asset.

- Case 2. This case leads to the equilibrium biomass $x^* = K(1-\delta/r)/2$. The term δ/r, i.e., the ratio of the discount factor to the growth rate, is what is referred to as the bionomic growth ratio (Clark, 1990). Setting $\delta = 0$ yields the MSY policy as usual. The optimal equilibrium harvest h^* is obtained by simply substituting the x^* into the equilibrium stock-harvest relation above. The second case seems to be embedded in case one if $\delta = \sqrt{r^2 - 2ar/K}$.

To illustrate the technique, we use the population dynamic model with the popular logistic growth function as in Sandal and Steinshamn (1997) and Homans and Wilen

(1997). We choose the biological parameters based on the views of some scientists for the Norwegian cod fisheries (NCF) stock. For this biological species a value between 0.2 and 0.5 per year for the growth rate and between 5000 and 7000 Kilo-tons for the carrying capacity are considered reasonable. Immediately after World War II, biologists have estimated the stock biomass for NCF to be at about 4230 Kilo-tons which is believed to be greater than the x_{MSY}.

For the ensuing analysis, we assume that $r = .35$ per year and $K = 6000$ Kilo-tons. We also assume that the management objective is the maximum sustainable yield (MSY) policy in most parts of the paper. Using these values the $x_{MSY} = K/2 = 3000$ Kilo-tons and the corresponding $y_{MSY} = rK/4 = 525$ Kilo-tons. From case one, $a = 2y$ implies $a = 1050$ Kilo-tons. To get simulation results which are of the same order of magnitude as in the case of Norwegian cod fishery stock values, we use the hypothetical stock values of 2300 Kilo-tons and 10% of the stock for the harvest rate as the initial conditions of our model.

4.2 Scaling

The performance of an optimization algorithm generally depends on the particular choice of the variables of the problem. Change of variables is usually recommended in practical applications (Luenberger, 1984; Yu and O'Brien, 1995). To study the effect of different choices of the parameter scales we rescale the variables by introducing the following transformation: Let $x = \rho z$ and substituting in

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) - h$$
$$\frac{dh}{dt} = -0.5(a - 2h)(\delta - r(1 - \frac{2x}{K}))$$

yields :

$$\frac{dz}{dt} = rz(1 - \frac{z}{k}) - y \quad (17)$$
$$\frac{dy}{dt} = -0.5(a^{'} - 2y)(\delta - r(1 - \frac{2z}{k})) \quad (18)$$

where: $z = x/\rho$, $y = h/\rho$ and $a' = a/\rho$. The scaling factor ρ will be allowed to vary and the effect studied later in the paper. The small k is the normalized carrying capacity given by $k = K/\rho$. It must be noticed that the quantities are now nondimensional. It may easily be checked by dimensional analysis.

4.3 Sensitivity analysis

Models of physical, biological and economic systems have input parameters on which their predictions depend. Some of these parameters are more important than others. Sensitivity is a measure of the effect of changes in the given input parameter on a model solution. It quantifies the extent that uncertainties in parameters contribute to uncertainties in the model results (Navon, 1997). Several analytical techniques of sensitivity analysis exist. The approach here is numerical method of calculating sensitivity of model response to input parameters. Let the quantity X depend on a parameter α. Then the absolute S_a and relative S_r sensitivities with respect to α can be defined respectively as

$$S_a = \frac{\partial X(\alpha)}{\partial \alpha} \tag{19}$$

and

$$S_r = \frac{\partial X(\alpha)}{\partial \alpha} \frac{\alpha}{X(\alpha)} \tag{20}$$

S_r is a nondimensional quantity which has the interpretation of percentage change in the output due to 1 percent change in the input parameter. To economists, equation (20) is an equivalent measure of elasticity. Input parameters will be perturbed from their true values and the model integrated over a given time horizon. Sensitivity of the output to the discount factor has not been studied in this paper. Its effect has previously been studied by, among others, (Sandal and Steinshamn, 1997c). The results are graphed and discussed.

Figure 1. is a plot of the unperturbed solution and the solutions with each of the parameters reduced by 1% of their values. Results of the sensitivity analysis show that the model solution is sensitive to all the biological and economic parameters. The growth rate appears to have least effect on the model outputs while the carrying capacity has most influence (see Figure 1.).

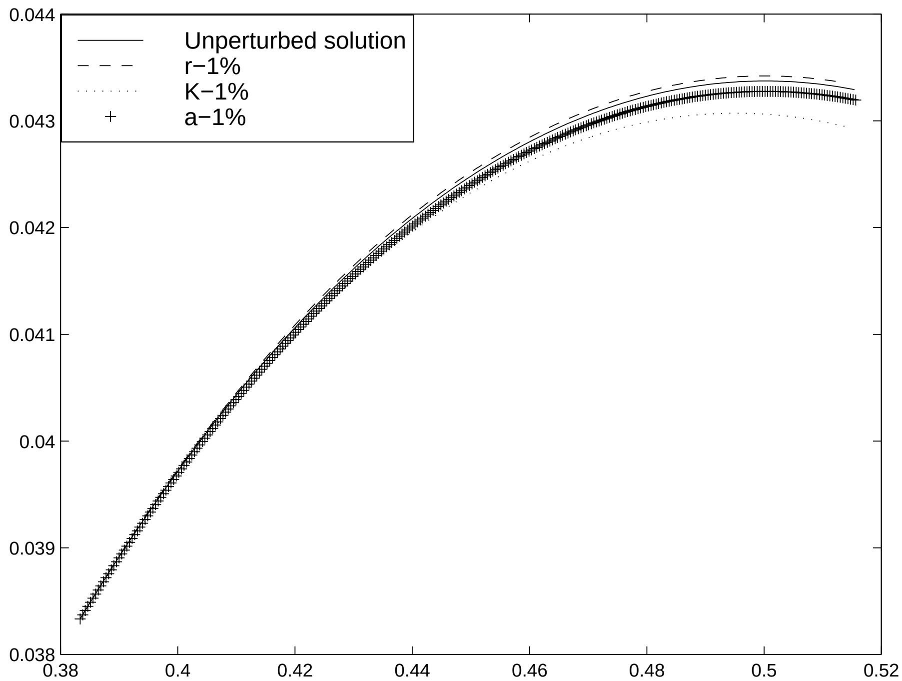

Figure 1: Phase plane plots of the sensitivity analysis.

It is observed that perturbing the parameter by larger values (10%) results in large distortion of the solution. Hence, care must be taken when choosing the initial guesses in a parameter estimation.

5 The Twin Experiments

As a first step we use artificially generated data (twin experiments) to test the performance of the data assimilation method. This will avoid any inconsistencies between the data and the model. First clean or exact data, i.e., data without measurement errors, will be simulated by integrating the model equations from known initial conditions and the parameters whose values are to be recovered. Second, stochastic errors will be introduced using random number generators from standard packages. The model will be solved and a certain level of noise added to the "true" solution. The data is assumed to be related to the model in a linear fashion, i.e, $\hat{x}_n = x_n + \epsilon_n$ where ϵ_n denotes the stochastic error term. In all the experiments to be carried out, we assume that data is available at every grid point. This might not be the case however in practice as data is sparse in time. For an actual fishery, biological and economic time series of observations may be available only on an annual basis.

5.1 Experiment with clean data

Several experiments were performed using data without any type of errors. That is, the exact solution of the model is used as the measurements. We investigate the effects of adding a priori knowledge of the parameters in the formulation of the penalty functional. Two results are shown here. Figure 2. is the plot of the parameters against the number of simulations for the case where we assume no a priori information, i.e., complete ignorance about the parameters. In Figure 3., we incorporated some information about the parameters and their uncertainties. Recovery of the model parameters was achieved in a few iterations depending on the initial guess. The convergence of the minimization procedure did not seem to depend very much on the initial guess of the parameters as long as they are fairly reasonable guesses. The graph of the normalized loss function shows a sharp decrease in its value. In about 2 iterations the value of the loss function fell to about 5% of the initial value. Not surprisingly, its value vanished at the optimum. This is expected in a twin experiment with a perfect data.

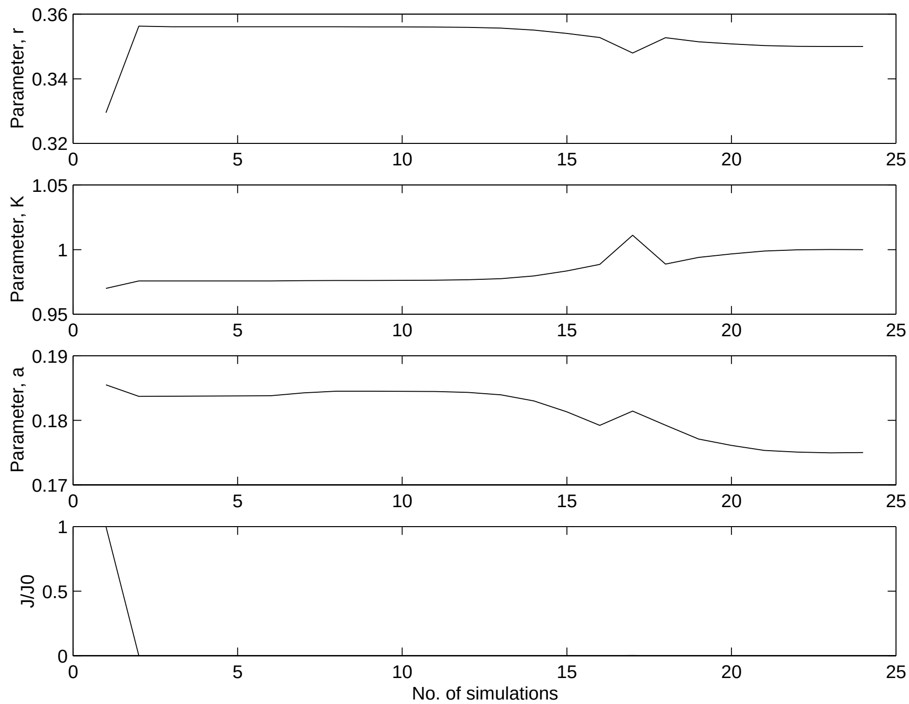

Figure 2: Plot of the parameters and the normalized loss function vs. no. of simulations: No penalty on parameters.

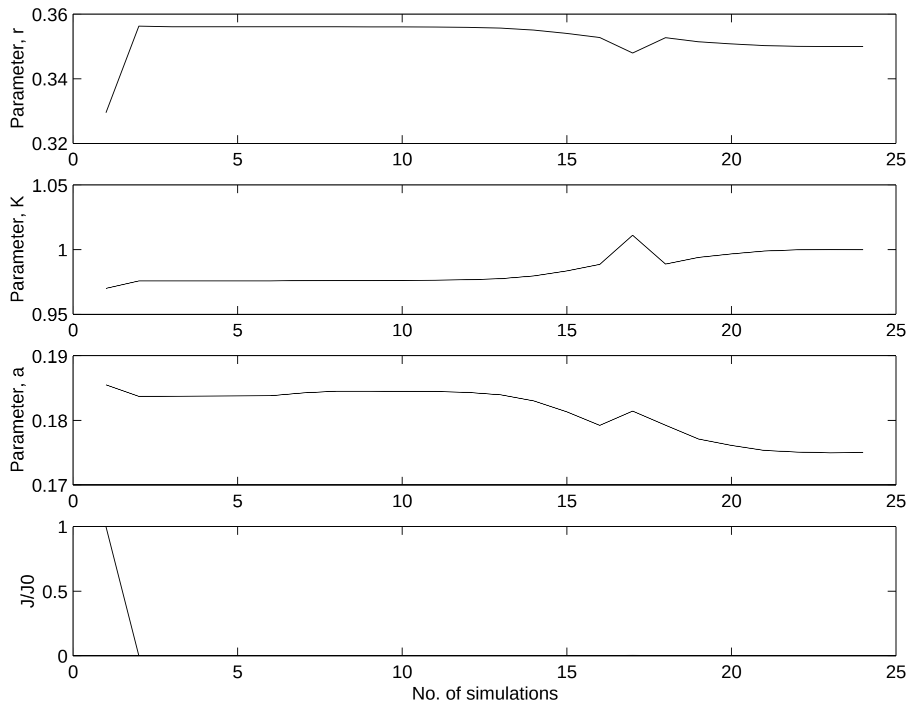

Figure 3: Plot of the parameters and the normalized loss function vs. no. of simulations: With penalty on parameters.

The conclusion drawn from the two cases with penalty and without is that, once the penalty is included, it becomes incumbent to have reasonably good prior (best) guesses of the parameters or their uncertainties or both. In fact, in this experiment, we encountered problems with recovering the parameters if our best guesses were a bit away and the weights relatively close to the data weights. If the weights were correctly chosen, the gains compared to the first case with no information on penalty were not very substantial. An initial best guess of 10% from the true parameters required data-to-parameter weight ratio of at least 10^6 for convergence.

5.2 Experiment with noisy data

For this experiment, the data is contaminated with normally or Gaussian mean zero and constant σ^2 variance $N(0, \sigma^2)$ distributed random errors. Similar experiments as in the case with perfect data were performed. We have made several runs to study various effects of adding different level of random noise to the solution of the model equations. With no penalty on the parameters, estimates were relatively further away from the "true" ones. Convergence to the "true" solution was quite difficult signifying the existence of local minima as the level of noise becomes significant. Very accurate initial guesses were required. This does not, however, indicate any serious setback in the adjoint parameter estimation. The aim of the assimilation is to adjust the free parameters such that the model predictions are as close as possible to the data. What happens is that the data is given large weight if there is no penalty on the parameters. Including prior information on the parameters results in better recovery of the parameters if the best guesses were quite good. The most troubling aspects of the experiment with a penalty on the parameter is the choice of the weights. Plots of the experiments are shown in the figures below. Figure 6. shows the simulated data and the best fit solutions. The level of noise added to the model solution to some extent affects the rate at which the parameters are recovered as well as the accuracy of the estimates.

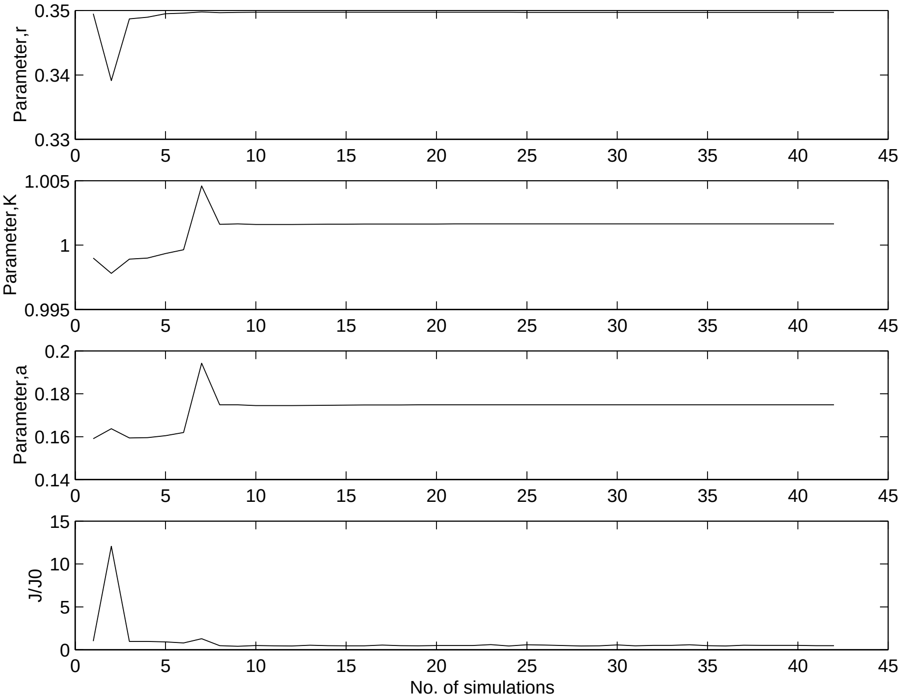

Figure 4: Plot of the parameters and the normalized loss function vs. no. of simulations: No penalty on parameters.

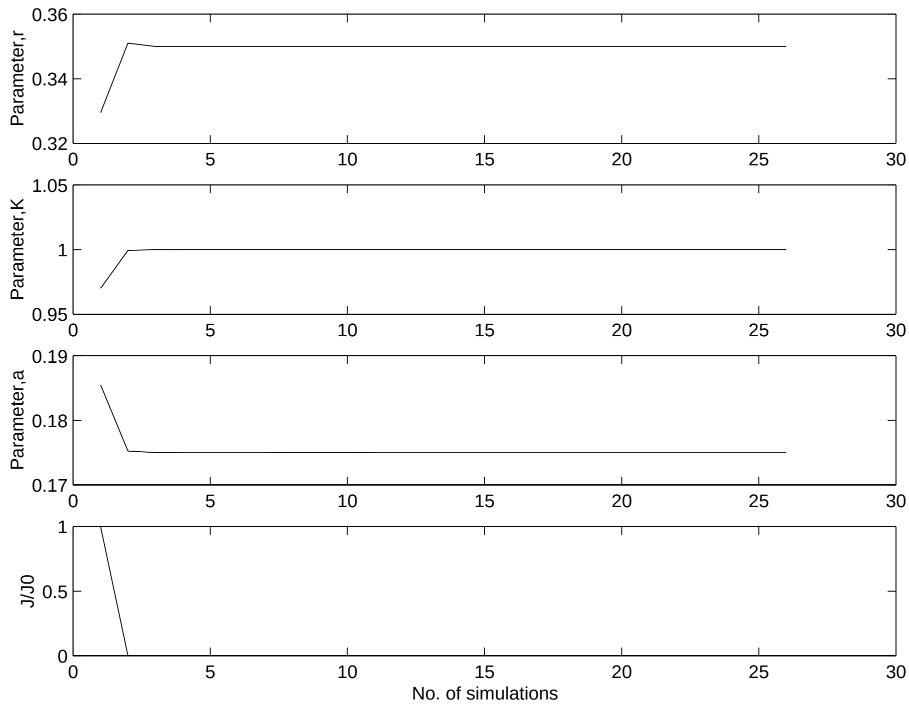

Figure 5: Plot of the parameters and the normalized loss function vs. no. of simulations: With penalty on parameters.

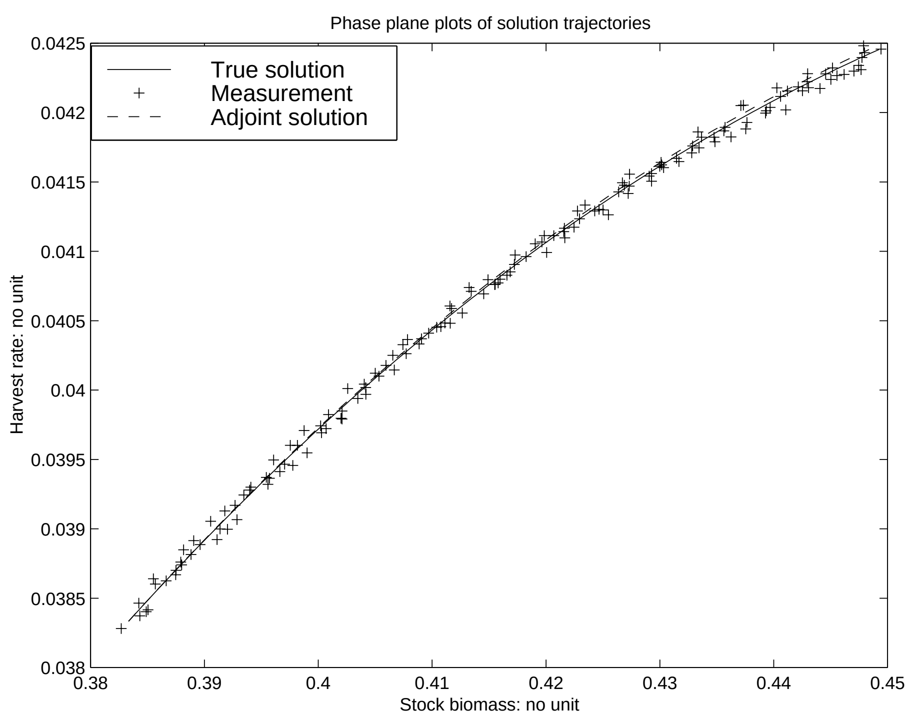

Figure 6: Plot of the harvest rate vs. biomass.

6 Summary and Conclusion

This work to our best knowledge represents the first attempt to explore the highly advanced and attractive method of parameter estimation in resource economics. It serves as a link between data and the model formulation. The basic idea is to find parameters of a dynamic fisheries model which yield model results that are as close as possible to the observed quantities. To achieve this goal a loss function measuring the distance between the model results and the data was predefined and the parameters estimated through an iterative process using the adjoint method. In the adjoint technique the model is assumed to hold exactly "strong constraint formalism" and the free parameters of the model are adjusted until the model predictions are as close as possible to the observations. A gradient search technique was used to iteratively explore the parameter space in order to find the minimum of the loss function.

In a few experiments we have demonstrated the utility of the adjoint technique in recovering model parameters such as the growth rate of a bioeconomic model. These parameters are vital in understanding the dynamics of the exploited species which can lead to a more accurate and realistic management policies. The results of the dual experiments show that model parameter sets can easily be estimated using both the information from measurements and the model formulation. In the experiment with clean data, parameters were recovered to within several orders of magnitudes in a few iterations.

The outcome of the sensitivity studies shows the relative importance of the input parameters. It is observed that the environmental carrying capacity is the most important parameter, i.e., small uncertainty in its value leads to large uncertainty in the output of the model. For an effective MSY policy, it is important to know very precisely the value of K. For most animal species around the world, the carrying capacity has been diminishing overtime. Implementation of MSY policy will require accurate and revised estimates of K regularly. It may thus be tempting to think that one of the reasons why MSY policy has not been very fruitful is due to lack of knowledge of the K, which means our estimate(s) of the sustainable stock biomass is not dependable and hence the MSY. From the little experience gathered, it may be advised that a sensitivity analysis be performed prior to parameter estimation.

To conclude, the paper has accomplished the following. A novel approach to dynamic model parameter estimation method has been introduced with reasonable degree of success using a simple bioeconomic model. The idea here is quite unique in a sense that both biological and economic theory were combined in the modeling of the resource system. In this case the parameter estimates are optimal compared to the situation where either a biological or an economic theory alone is used. Its potential in handling nonlinear models is also demonstrated and thus it is important to explore more of the capabilities of the method in future.

APPENDIX A

A.1 Mathematical Derivations

This section is reserved for the more technical aspects of the paper. First, it will present the mathematical formulation of the bioeconomic model pertaining to the optimal management of the stock in a sole owner context. Second, derivation of the adjoint model code will be presented in detail.

A.2 Derivation of the bioeconomic model

A more general bioeconomic model is formulated. The objective and the growth functions are assumed to depend on the stock biomass, the harvest and explicitly on time. This will allow us to derive several special cases as they apply to different fisheries. Consider the problem

$$max \int_0^{T_f} \pi(x,h,t)dt \tag{A.1}$$

subject to

$$\frac{dx}{dt} = F(x,h,t) \tag{A.2}$$

$$x(0) = x_0 \tag{A.3}$$

Defining the current value Hamiltonian function $H(x,h,m,t)$

$$H = \pi(x,h,t) + mF(x,h,t) \tag{A.4}$$

where $m(t)$ is the current value multiplier. Assuming an inner solution, the first order conditions are:

$$\frac{dH}{dh} = \pi_h + mF_h = 0,$$

$$\frac{dm}{dt} = \delta m - H_x \tag{A.5}$$

From (A.3) we have

$$\frac{dm}{dt} = -\frac{\pi_{hh}\dot{h} + \pi_{hx}\dot{x} + \pi_{ht}}{F_h} + \frac{\pi_h(F_{hh}\dot{h} + F_{hx}\dot{x} + F_{ht})}{F_h^2} \tag{A.6}$$

equating (A.3) and (A.4), and rearranging yields

$$\begin{aligned} \frac{dh}{dt} &= \frac{\delta\pi_h F_h - (\pi_x + (-\frac{\pi_h}{F_h})F_x)F_h^2}{\pi_{hh}F_h - \pi_h F_{hh}} \\ &- \frac{(\pi_{hx}F_h - \pi_h F_{hx})(F-h) + (\pi_{ht}F_h - \pi_h F_{ht})}{\pi_{hh}F_h - \pi_h F_{hh}} \end{aligned} \tag{A.7}$$

Equations (A.2) and (A.7) constitute an n-dimensional dynamical systems.

APPENDIX B

B.1 Derivation of the adjoint model

The adjoint equations are derived by forming the Lagrange functional via the undetermined multipliers. The Lagrange function is

$$\mathcal{L}[\mathbf{X}, \alpha] = \mathcal{J} + \int_0^{T_f} \mathbf{M}(\frac{\partial \mathbf{X}}{\partial t} - F(\mathbf{X}, \alpha))dt \tag{B.1}$$

Perturbing the function $\mathcal{L}$

$$\begin{aligned} \mathcal{L}[\mathbf{X} + \delta\mathbf{X}, \alpha] &= \mathcal{J}[\mathbf{X} + \delta\mathbf{X}, \alpha] \\ &+ \int_0^{T_f} \mathbf{M}(\frac{\partial(\mathbf{X} + \delta\mathbf{X})}{\partial t} - F(\mathbf{X} + \delta\mathbf{X}, \alpha))dt \end{aligned} \tag{B.2}$$

$$\mathcal{L}[\mathbf{X} + \delta\mathbf{X}, \alpha] = \mathcal{J} + \Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T + \int_0^{T_f} \mathbf{M}(\frac{\partial \mathbf{X}}{\partial t} - F(\mathbf{X}, \alpha))dt$$

$$-2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t}-\frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt+O(\delta\mathbf{X}^2) \tag{B.3}$$

Taking the difference $(\mathcal{L}[\mathbf{X}+\delta\mathbf{X},\alpha]-\mathcal{L}[\mathbf{X},\alpha])$

$$\delta\mathcal{L}=\Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T-2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t}-\frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt+O(\delta\mathbf{X}^2) \tag{B.4}$$

Requiring that $\delta\mathcal{L}$ be of order $O(\delta\mathbf{X}^2)$ implies

$$\Delta_{\mathbf{X}}\mathcal{J}\delta\mathbf{X}^T-2\int_0^{T_f}\mathbf{M}(\frac{\partial\delta\mathbf{X}}{\partial t}-\frac{\partial F}{\partial\mathbf{X}}\delta\mathbf{X}^T)dt=0 \tag{B.5}$$

By integrating the second term of the LHS by parts and rearranging, we have

$$\frac{\partial\mathbf{M}}{\partial t}+[\frac{\partial F}{\partial\mathbf{X}}]^T\mathbf{M}=\mathbf{W}(\mathbf{X}-\hat{\mathbf{X}})$$

$$\mathbf{M}(T_f)=\mathbf{0} \tag{B.6}$$

which are the adjoint equations together with the boundary conditions.

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