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Abstract

A new model is presented which produces credit spreads that do not converge to zero for short maturities. Our set-up includes incomplete, i.e., *delayed* and *asymmetric* information. When the financial market observes the company's earnings with a delay, the effect on both default policy and credit spreads is negligible, compared to the Leland (1994) model. When information is asymmetrically distributed

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between the management of the company and the financial market, short credit spreads do not converge to zero. This result is similar to the Duffie and Lando (2001) model, although our simpler model improves some limitations in their set-up.. Short interest rates from our model are used to illustrate effects similar to the dry-up in the interbank market experienced after the summer of 2007.

Keywords and phrases: Credit risk, credit spreads, delayed information, asymmetric information.

JEL-classification: G12, G33

We analyze the effects of *delayed* and *asymmetric* information on credit spreads and bankruptcy policy. In the case of delayed information the company's earnings process is revealed to the agents in the economy with a time delay. This time delay reflects the fact that it takes time for the management of a company to gather, structure, analyze, and report information. Disregarding all potential agency problems we assume that the management always acts on the best behalf of the equity holders. Furthermore, management has access to information no later, but possibly earlier than the participants in the financial market. In the special case where the two groups have access to the same information at the same time, we show that a time delay has a minor effect on credit spreads for realistic parameter values. On the other hand, any information asymmetry between the management and the financial market turns out to be of crucial importance for the credit spreads. Delayed and asymmetric information have a small impact on bankruptcy policy.

The risk that a debtor will not honor his contractual obligations with the creditor is called credit risk. This topic has received attention in both the academic literature and among practitioners. There are two dominating approaches to credit risk in the finance literature; *structural models* and *reduced form models*. The first was pioneered by Merton (1974) and essentially consists of modeling the value of a company's assets by a stochastic process. Debt and equity are then considered as contingent claims on the total asset value. Some of the papers in this tradition include Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Leland (1994), and Duffie and Lando (2001). The second approach assumes the existence of a default arrival intensity. This approach was pioneered by Jarrow and Turnbull (1992), for extensions see e.g., Jarrow and Turnbull (1995), Jarrow,

Lando, and Turnbull (1997), and Schönbucher (1998).¹

In traditional structural models, as a bond's time to maturity approaches zero, the credit spread approaches zero. This property is not in line with observations in financial markets and is considered a problem with the structural models. The reduced form approach is typically able to produce strictly positive credit spreads also for short term maturities, but is not founded on economic models of the company. However, they seem to be more useful than structural models when it comes to practical use and calibration to market data.

Our paper is related to Duffie and Lando (2001) who present the first example of a model where, in the presence of incomplete accounting information, there is equivalence between the structural and the reduced form approach. In their model there is an information asymmetry between equity and bond holders. In particular, equity holders have access to more information than the bond holders. To keep these two groups of financial agents separated, equity is by assumption not traded, eliminating bond holders' access to the equity market. Equity holders are by assumption precluded from buying corporate debt. The latter assumption is justified by insider-trading regulation.

In our model the financial market participants, i.e., both equity and bond holders, have access to the same information at any point in time. We do not need to restrict each of these groups' access to investments in the other asset class. This is a reasonable and tractable property of our model and corresponds nicely with what is observed in real markets. We claim that this assumption is superior to Duffie and Lando's for the following reasons:

1. Tradability of equity extends the model's applicability to a much wider set of companies. There are relatively few companies with a secondary

market for corporate debt whose equity is not traded.²

2. When the company needs more capital to run its operations, non-tradability of equity can cause equity holders to file for bankruptcy due to liquidity problems, not because it is economically optimal. The reasons are:

- The equity holders cannot finance infusion of capital by selling or diluting their stocks.
- Equity holders might have problems borrowing money to finance any infusion of capital by using the equity as collateral since the true value of equity is only known by themselves and not by the lender. Moreover, this information can not be revealed to the lender.

3. We show that asymmetric information leads to wider credit spreads, and thereby a higher cost of debt financing. This result implies a lower debt ratio than the optimal ratio in the case of fully informed bond holders. This again leads to a lower total value of the firm, and, thus, asymmetric information also reduces the value of the equity. There is therefore no economic rationale for keeping information away from bond holders in these types of models.

The classical structural models use the value of the company as the state variable. In real life the value of a company is typically not observable. According to Jarrow and Protter (2004) the implications of this are not well understood. Our model increases the understanding of this problem because all decisions are endogenously modeled and take the lack of complete information into account.

In the first version of our model the information is distributed to the financial market from the management without delay. The true value of the earnings process is assumed to be observed with a time delay m by all agents. In this case the time of bankruptcy, seen from the bond holders perspective, is not *totally inaccessible*³, but the payoff received in case of bankruptcy is random. This is also the case for the equity holders, and they can in some cases “wrongly” choose to file for bankruptcy in the sense that the value of the company may more than cover the debt holders’ claim and bankruptcy costs. We label this possibility *the bankruptcy lottery*. This situation has sometimes been observed in practice⁴.

In the second version of our model the management of the company has a shorter delay of information than the financial market, i.e., $m \leq l$, where l represents the time lag of information to the financial market. In this case the time of bankruptcy is totally inaccessible for the bond holders (and the equity holders), and this information asymmetry makes the model equivalent to an intensity based model and the model produces strictly positive credit spreads as the time to maturity vanishes.

The paper is organized as follows: In section I we present the economic model and set-up. The classical case with full information is reviewed in section II. The case with delayed information is analyzed in section III, while the case with delayed and asymmetric information is analyzed in section IV. In section V we show that our model produces short-term credit spreads that can possibly explain the recent dry-up in the interbank market. Section VI contains some concluding remarks. Technical results and proofs are collected in 5 appendices.

I The Economic Model

We use the EBIT (earnings before interest and taxes) version of the economic model of Leland (1994), see e.g., Goldstein, Ju, and Leland (2001). The EBIT process of the company is given by the stochastic differential equation

$$d\delta_t = \mu\delta_t dt + \sigma\delta_t dB_t, \quad (1)$$

where the drift and volatility parameters μ and σ are constants. We assume that $\mu \leq r$, where r represents the constant risk free interest rate and that the initial value of the process δ_0 is a positive constant. Here B_t is a standard Brownian motion that is defined on a fixed, filtered probability space (Ω, \mathcal{F}, P) . Furthermore, P represents the original probability measure, and all agents are risk neutral.

The information filtration \mathcal{F}_t is generated by the process $\{\delta_s, 0 \leq s \leq t\}$ (augmented with all sets of measure zero). To incorporate delayed information into our model we assume that the management's and the financial market's information is given by the filtrations \mathcal{F}_t^m and \mathcal{F}_t^l , respectively, where

$$\mathcal{F}_t^m = \mathcal{F}_{t-m}, \text{ for all } t \geq m,$$

$$\mathcal{F}_t^l = \mathcal{F}_{t-l}, \text{ for all } t \geq l,$$

and $0 \leq m \leq l$. Clearly, from this specification

$$\mathcal{F}_t^l \subseteq \mathcal{F}_t^m \subseteq \mathcal{F}_t.$$

The time lags are illustrated in Figure 1. In addition to the information contained in \mathcal{F}_t^l , the financial market also observes whether the company is bankrupt or not.

Let the *assessment* of the value of the company at time t by an agent

with information delay $k \in \{0, m, l\}$ be denoted by V_t^k . Thus

$$V_t^k = E \left[\int_t^\infty e^{-r(s-t)} \delta_{s-k} ds \middle| \mathcal{F}_t^k \right] = \frac{\delta_{t-k}}{r - \mu}, \quad (2)$$

where $t \geq k$. We remark that V_t^k is simply δ_{t-k} multiplied by a constant, and is therefore also a geometric Brownian motion.

As $k \rightarrow 0$ the value converges to the value where there is no delay in the flow of information. Expression (2) clearly reflects the delayed information through the presence of δ_{t-k} instead of δ_t that is present under complete information. Note that $V_t^k = V_{t-k}^0$, i.e., the value assessed at time t by an agent with information lag k is identical to the value assessed at time $t - k$ by an agent with complete information. In particular the market's assessed asset value, V_t^l is based on the filtration \mathcal{F}_t^l .

As in Leland (1994) we assume that the company has issued perpetual debt with face value D . The debt is serviced by a constant rate of coupon payments C . These payments are tax deductible (only interest is paid on perpetual debt). The tax benefit rate is θC , where θ is the tax rate.

The equity holders decide when to default. In our model they delegate the daily operations of the company to the management. We completely disregard all potential agency problems between the management and the equity holders and assume that the management acts in the best interest of the owners. However, we assume in the second version of our model that management is better informed than the equity holders, and we consider a potential bankruptcy as such a severe event that the management immediately informs the owners if filing for bankruptcy is optimal. Even though the equity holders formally decide when to default, such a decision is based on the information of the management in our model.

We define the stopping time τ with respect to the filtration \mathcal{F}_t^m for fixed

$t \geq m$ as

$$\tau = \inf\{u \geq t : V_u^m \leq V_B^m\}, \quad (3)$$

where from expression (2) $V_t^m = \frac{\delta_{t-m}}{r-\mu}$ and V_B^m is a given constant. In this model the company is bankrupt and liquidated the first time $V_t^m = V_B^m$, i.e., τ represents the time of bankruptcy. We denote the complete-information-value of the company upon bankruptcy by V_τ , i.e., from equation (2),

$$V_\tau \equiv V_\tau^0 = \frac{\delta_\tau}{r-\mu}.$$

Upon bankruptcy a cost of αV_τ occurs. Here α is assumed constant, and the bankruptcy cost is therefore proportional to the complete-information-value. Also, in case of bankruptcy, the debt holders require the face value of the debt D to be repaid. In general V_τ is different from V_τ^m , i.e., the complete-information value of the assets at time τ is different from the value upon which the bankruptcy decision is made. In particular, there is a positive probability that $V_\tau - D - \alpha V_\tau > 0$. In this case the time τ value of the company is sufficient to cover debt and bankruptcy costs, and any proceeds are paid to the equity holders.

II The Classical Case with Complete Information

Let us start by looking at the classical case where there is no delay in the flow of information. In this case $l = m = 0$, $\mathcal{F}_t^l = \mathcal{F}_t^m = \mathcal{F}_t$, and $V_B^m = V_B$, where $V_B = V_B^0$. In this section we denote the initial time by t , where $t \geq 0$.

II.1 Equity Holders' Optimization Problem

The equity holders are faced with the following optimal stopping problem (see e.g., Duffie (2001), chapter 11.C)

$$\phi(v) = \sup_{\tau \in \mathcal{T}} E \left[\int_t^\tau e^{-r(s-t)} (\delta_s - (1-\theta)C) ds \middle| \mathcal{F}_t \right], \quad (4)$$

where \mathcal{T} is the set of \mathcal{F}_t -adapted stopping times. The value function satisfies the ordinary differential equation (ODE)

$$\mu v \phi_v + \frac{1}{2} \sigma^2 v^2 \phi_{vv} - r \phi + (r - \mu)v - (1 - \theta)C = 0, \quad (5)$$

where subscripts denote partial derivatives. The general solution to this equation is

$$\phi(v) = A_1 v^{\gamma_1} + A_2 v^{\gamma_2} + v - (1 - \theta) \frac{C}{r},$$

where A_i , $i = 1, 2$, are constants to be determined from boundary conditions and

$$\gamma_i = \frac{\frac{1}{2} \sigma^2 - \mu \pm \sqrt{(\mu - \frac{1}{2} \sigma^2)^2 + 2r \sigma^2}}{\sigma^2},$$

with $\gamma_1 < 0$ and $\gamma_2 \geq 1$. Differentiating ϕ with respect to v yields

$$\phi'(v) = \gamma_1 A_1 v^{\gamma_1 - 1} + \gamma_2 A_2 v^{\gamma_2 - 1} + 1.$$

When the value of the company approaches infinity, only equity holders benefit from any further increase in asset value, thus

$$\lim_{v \rightarrow \infty} \phi'(v) = 1.$$

As $\gamma_2 \geq 1$, this condition implies that $A_2 = 0$, i.e.,

$$\phi(v) = A_1 v^{\gamma_1} + v - (1 - \theta) \frac{C}{r}. \quad (6)$$

The value matching and the high contact conditions require that

$$\phi(V_B) = 0 \quad (7)$$

and

$$\phi_v(V_B) = 0, \quad (8)$$

respectively. Equations (7) and (8) can be solved for A_1 and V_B . The solution for V_B is⁵

$$V_B = \frac{\gamma_1}{\gamma_1 - 1} \frac{(1 - \theta)C}{r}. \quad (9)$$

II.2 Corporate Bond Pricing

As Duffie and Lando (2001) we analyze a zero coupon bond maturing at time s with recovery rate $R(\tau, s)$ in the case of default at time τ . Our original model is based on one perpetual bond. Duffie and Lando (2001) explain the connection between the original model and the following unit discount bond by e.g., assuming that the perpetual debt is stripped into a continuum of zero-coupon bonds maturing at time s , $s \in (t, \infty)$. The time t bond price, for each dollar of principal, is given by

$$\varphi(t, s) = e^{-r(s-t)}P(s) + \int_t^s R(\tau, u)e^{-r(u-t)}f_\tau(u)du, \quad (10)$$

where $P(s) = P(\tau > s | \mathcal{F}_t)$ is given in expression (23) in appendix A. Here $f_\tau(u)$ is the probability density of the stopping time τ .

In the special case considered in Duffie and Lando (2001), where $R(u, s) = (1 - \alpha)e^{-r(s-u)}$, $u \in (t, s]$, the pricing expression (10) simplifies to

$$\varphi(t, s) = e^{-r(s-t)}P(s) + (1 - \alpha)e^{-r(s-t)}(1 - P(s)). \quad (11)$$

We use this recovery rate throughout the paper because it leads to tractable analytical expressions. Other, possibly more realistic recovery functions have much of the same qualitative properties as the one above, but they may require numerical solutions.

Example 1. Assume that $\delta_t = 3.5$, $r = 0.08$, and $\mu = 0.045$. From expression (2) these parameters give $V_t = 100$. Furthermore, assume that $\sigma = 0.3$, $\theta = 0.3$, $\alpha = 0.3$, and $C = 13$. From expression (9) $V_B = 65$. With the recovery policy in expression (11) the credit spreads for zero-coupon bonds with maturities of up to three years are plotted in Figure 2.

[Figure 2 should be placed about here.]

From Figure 2 we clearly see that the credit spread approaches zero as the time to maturity approaches zero, a typical property of structural models of credit risk.

II.3 Credit Default Swap (CDS)

CDSs are the most common form of credit derivatives. A CDS is a default insurance contract. For each unit of face value of debt it pays the amount

$$X = 1 - \frac{(1 - \alpha)V_B}{D},$$

at the time of default, if default happens before the CDS matures at some time T . The cost of the insurance is covered by coupon payments to the issuer of the CDS, known as the CDS spread. The CDS spread is the annualized coupon rate $c(t, T)$ that implies a total market value of the swap of zero at the time of issue. Assuming semi-annually coupons and that $n = 2T$,

$$\begin{aligned} c(t, T) &= \frac{2XE[e^{-r(\tau-t)}1\{\tau \leq T\}|\mathcal{F}_t]}{\sum_{i=1}^n e^{-0.5ri}E[1\{\tau > t + 0.5i\}|\mathcal{F}_t]} \\ &= \frac{2X\Upsilon(T-t, V_t, V_B)}{\sum_{i=1}^n e^{-0.5ri}P(t+0.5i)}, \end{aligned}$$

where $\Upsilon(T-t, V_t, V_B)$ and $P(t+0.5i)$ are given in the appendices in expressions (25) and (23), respectively.

We present a numerical example illustrating the CDS spreads in section IV.3 where we also include delayed and asymmetric information.

III The Case with Delayed Information

III.1 The Equity Holders' Optimization Problem

In the case with delayed information equity holders, debt holders, and management have access to the same information, but they receive the information with a time lag, i.e., $l = m > 0$. Also in this section the initial point in time is denoted by $t \geq l$. Thus, at time t agents observe the state variable (i.e., the EBIT process) at time $t - m$. This assumption changes the optimization problem for the equity holders. From standard properties of geometric Brownian motions follow that the complete-information-value of the assets at the bankruptcy time τ is given by the lognormally distributed random variable

$$V_\tau = V_\tau^m e^{(\mu - \frac{1}{2}\sigma^2)m + \sigma(B_\tau - B_{\tau-m})}. \quad (12)$$

From the definition of the barrier V_B^m in expression (3), the time τ value of the assets in expression (12) can also be written as

$$V_\tau = V_B^m e^{(\mu - \frac{1}{2}\sigma^2)m + \sigma(B_\tau - B_{\tau-m})}.$$

In the case where the value of the assets is sufficiently high to cover both repayment of the debt and bankruptcy costs, i.e., $V_\tau - D - \alpha V_\tau > 0$, the equity holders get the payoff $V_\tau - D - \alpha V_\tau$. By deciding to file for bankruptcy at time τ , the equity holders enter a bankruptcy lottery⁶ with payoff $((1 - \alpha)V_\tau - D)^+$. The time τ value of this lottery is

$$\begin{aligned} \pi(V_B^m) &= E[((1 - \alpha)V_\tau - D)^+ | \mathcal{F}_\tau^m] \\ &= (1 - \alpha)e^{\mu m} V_B^m N(z) - DN(z - \sigma\sqrt{m}), \end{aligned} \quad (13)$$

where

$$z = \frac{\ln\left(\frac{(1-\alpha)V_B^m}{D}\right) + (\mu + \frac{1}{2}\sigma^2)m}{\sigma\sqrt{m}}$$

and $N(\cdot)$ is the cumulative normal probability distribution function.

Proof. The result follows from the standard Black-Scholes-Merton formula for a European call option, but without discounting since the payoff is received instantaneously when $V_\tau^m = V_B^m$. \square

The equity holders' optimization problem is now given by

$$\phi(v) = \sup_{\tau \in \mathcal{T}_m} E \left[\int_t^\tau e^{-r(s-t)} (\delta_{s-m} - (1-\theta)C) ds + e^{-r(\tau-t)} \pi(V_B^m) \middle| \mathcal{F}_t^m \right], \quad (14)$$

where \mathcal{T}_m denotes the set of \mathcal{F}_t^m -adapted stopping times. There are three differences between expressions (14) and (4). The first is that the bankruptcy lottery enters the optimization problem. Second, the state variable δ enters with a lagged value, and third, the information set at time t is lagged.

In appendix C we show that the equity holders' optimization problem is equivalent to a standard stopping problem. Thus, also in the case with delayed information the value function $\phi(v)$ is the solution to the ODE (5) and has general solution given by expression (6). However, the value matching and the high contact conditions now change to

$$\phi(V_B^m) = \pi(V_B^m) \quad (15)$$

and

$$\phi_v(V_B^m) = \pi_v(V_B^m) = (1 - \alpha) e^{\mu m} N(z), \quad (16)$$

respectively.

Using expressions (15) and (16), we are not able to find analytical expressions for A_1 and V_B^m . However, equations (15) and (16) can easily be solved numerically.

In Table I we illustrate the effect of different delays on the default barrier and the value of the bankruptcy lottery. The effect is relatively small for delays less than one year. For instance, when $m = 0.5$, the value of V_B^m is only changed at the second decimal place and the value of the bankruptcy

lottery is zero with two digits accuracy. For longer delays, the value of the bankruptcy lottery is larger and, thus, more important for the equity holders' bankruptcy decision. The value function ϕ is increasing in the assessed value of the assets (V_t^m). Combining this observation with expression (15), it is clear that the assessed value V_t^m that makes (15) hold (i.e., V_B^m) must be higher for higher values on the righthand side. Thus, longer delays increase the default barrier.

From Table II it is clear that increasing the volatility has virtually no effect on the value of the bankruptcy lottery, except for extremely high values of the volatility. Note that we have taken into account that a more risky firm typically has less debt than a less risky firm.

III.2 Corporate Bond Pricing

The bond holders still price corporate bonds according to expressions (10) and (11). The only way in which delayed information can change the credit spreads, compared to the classical case, is if the default barrier $V_B^m \neq V_B$. As we saw in Table I, this can happen for large delays in the flow of information and/or very high asset volatilities, c.f. Table II. In Table III we report credit spreads for different delays in information and different levels of the volatility. The corresponding default barriers are taken from Tables I and II. As Table III shows, for reasonable delays (typically less than one year) and levels of the volatility, any changes in the credit spreads are negligible.

The above observations may shed light on two important aspects:

1. The effect of not being able to observe the process for the value of the assets in a structural model has a minor effect on the optimal default policy, c.f., the discussion in Jarrow and Protter (2004). This effect becomes noticeable for larger delays and extremely high levels

of volatility.

2. Delayed information has a negligible effect on credit spreads.

[Tables I, II, and III should be placed about here.]

IV The Case with Asymmetric Information

We now analyze the case with asymmetric and delayed information. Here $0 \leq m < l$. As before, the initial point in time $t \geq l$. The asymmetry occurs because there is a difference in the information delay between the financial market and the management.

In a financial market where both stocks and corporate bonds are traded, both equity and bond holders typically have the same information. If this is not the case, and information is considered valuable, bond holders can buy one share of stock each if equity holders have more information. Similarly, if bond holders have more information than the equity holders, the equity holders can each buy one share of bond. Both strategies eliminate any information asymmetry.

Both equity and bond holders observe the value of the assets with a delay l . The management of the company observes the value of the company with a delay $m \leq l$. As argued in the introduction, and as we show appendix D and further illustrate in Example 2, there are no reasons for keeping information away from the capital market in these kinds of models. The difference in time lag, $l - m$, therefore reflects the time it takes to inform the financial market. Reporting information right away may be costly. In practice, there can be strategic reasons (outside of our model) for management to keep information away from the financial market for some time. Collin-Dufresne, Goldstein, and Helwege (2003) find that since 1937 only four companies

have defaulted on bonds with an investment grade rating from Moody's. This suggests that the time lag $m - l$ cannot be too long. With a sufficiently large time lag, even a company issuing bonds rated investment grade may have enough time to move into default.⁷

IV.1 The Equity Holders' Optimization Problem

The management of the company are now better informed than the equity holders. Because they act in the best interest of the owners, they will now solve the equity holders' optimization problem *as if* the owners had the same information as the management, i.e.,

$$\phi(v) = \sup_{\tau \in \mathcal{T}_m} E \left[\int_t^\tau e^{-r(s-t)} (\delta_{s-m} - (1 - \theta)C) ds + e^{-r(\tau-t)} \pi(V_B^m) \Big| \mathcal{F}_t^m \right]. \quad (17)$$

This problem is identical to the problem in expression (14), and thus have the same solution. In many countries it is illegal for the management to run the company on the debt holders' expense if they know that the company should have been declared bankrupt, partially justifying this assumption.

Financial distress and bankruptcy are characteristics of a highly extraordinary situation for a company. It is therefore reasonable to assume that such an event leads to an increased speed in the flow of information between the management and the equity holders.⁸ In our model when the management observes that the value hits the (optimal) default barrier, the equity holders are informed and immediately file for bankruptcy, c.f., equation (17).

IV.2 Corporate Bond Pricing

In section II (section III) the bond holders observe that the value of the assets (lagged asset value) approaches and eventually hits the default barrier V_B (V_B^m). In contrast to this situation, under asymmetric information they may observe that the assessed asset value V_t^l approaches V_B^m , but they never

observe that it hits V_B^m because the bankruptcy decision is based on the value V_t^m with $l > m$. Formally, the stopping time τ is *totally inaccessible*, as also is the case in the model of Duffie and Lando (2001). Thus, also our model is an example of a structural model that is equivalent to an intensity based model.

For the bond holders to calculate the bond prices, they still need to estimate the default probability. To this end, let $P^m(s) = P(\tau > s | V_t^m > V_B^m; V_t^l)$. Here $\{\tau > s\}$ represents the event ‘no default before time s ’ observed by the management. Thus $P^m(s)$ is the probability for no default before time s observed by the management, conditional on the information available for the bond holders. The bond holders’ information set contains two relevant pieces of information:

1. the lagged asset value observed by the management is greater than the default barrier V_B^m (otherwise the company would have been declared bankrupt), and
2. the assessed asset value V_t^l .

Using Baye’s rule, we have that

$$\begin{aligned} P^m(s) &= \frac{P(\tau > s | V_t^l) \cdot P(V_t^m > V_B^m | \tau > s; V_t^l)}{P(V_t^m > V_B^m | V_t^l)} \\ &= \frac{P(\tau > s | V_t^l)}{P(V_t^m > V_B^m | V_t^l)}. \end{aligned}$$

From expression (22) in appendix A, we can write this as

$$P^m(s) = \frac{\Psi(s - t + l - m, V_t^l, V_B^m)}{N\left(\frac{-\ln y + \nu(l-m)}{\sigma\sqrt{l-m}}\right)}, \quad (18)$$

where $y = V_B^m/V_t^l$ and $\nu = \mu - \frac{1}{2}\sigma^2$.

Assuming the same recovery rates as in subsection II.2 and replacing $P(s)$ by $P^m(s)$, the time t value of a zero-coupon bond maturing at time s

is similar to expression (11) and is given by

$$\varphi^m(t, s) = e^{-r(s-t)} P^m(s) + (1 - \alpha) e^{-r(s-t)} (1 - P^m(s)). \quad (19)$$

Based on the definition of the credit spread η^m and expression (19) we have that

$$e^{-(r+\eta^m)(s-t)} = e^{-r(s-t)} P^m(s) + (1 - \alpha) e^{-r(s-t)} (1 - P^m(s)),$$

so

$$\eta^m = \frac{-\ln(\alpha P^m(s) + (1 - \alpha))}{s - t}. \quad (20)$$

First we notice that the credit spread vanishes as $\alpha \rightarrow 0$. If there is no economic loss in case of bankruptcy, there is of course no credit risk. In appendix D we show that the spread under asymmetric information is greater than the spread under symmetrically distributed information.

Taking the limit as $s \rightarrow t$, we have that

$$\lim_{s \rightarrow t} P^m(s) = 1 - y^{2\nu\sigma^{-2}} \frac{N\left(\frac{\ln y + \nu(l-m)}{\sigma\sqrt{l-m}}\right)}{N\left(\frac{-\ln y + \nu(t-m)}{\sigma\sqrt{t-m}}\right)}. \quad (21)$$

The limit in (21) is strictly positive and less than 1, thus it must be the case that

$$\lim_{s \rightarrow t} \eta = \infty.$$

The intuition for this result is that if the defaultable zero-coupon bond has a price less than the default-free zero-coupon bond when the time to maturity vanishes, this can only be achieved for a “very” high credit spread (i.e., an infinite credit spread).

Example 2. Assume that $\delta_t = 3.5$, $\sigma = 0.3$, $\mu = 0.045$, $r = 0.08$, $C = 13$, $\theta = 0.3$, $\alpha = 0.3$, $m = 0.1$, and $l = 0.25$. These parameter values give

$V_B^m = 65$. With the recovery policy in expression (11) the credit spreads for zero-coupon bonds with maturities of up to three years are plotted in Figure 3. For comparison, also the credit spreads from Example 1 are plotted, demonstrating that asymmetric information leads to higher credit spreads.

[Figure 3 should be placed about here]

In contrast to the credit spreads under complete information, the spreads under asymmetric information in the short end of Figure 3 do not approach zero. The credit spread under asymmetric information to the far left (2.4%) is for a bond maturing in half a day. In practice soon to mature corporate bonds are not analyzed in terms of their credit spread because “the discount” mostly reflects the probability for immediately bankruptcy, adjusted for the recovery rate (see e.g., the discussion on page 14-15 in Lando (2004)). Although not commented by the authors, in similar figures in Duffie and Lando (2001) it seems like the x -axes are truncated, possibly to exclude such “high” short-term credit spreads. However, as we argue in section V, this property of the model can be used for explaining high short-term interest rates as, e.g., recently observed in the interbank market.

IV.3 Credit Default Swap

In the case of delayed and asymmetric information the coupon rate is given by

$$\begin{aligned} c(t, T) &= \frac{2XE[e^{-r(\tau-t)}]1\{\tau \leq T\} \Big| V_t^m > V_B; V_t^l}{\sum_{i=1}^n e^{-0.5ri} E[1\{\tau > t + 0.5i\} \Big| V_t^m > V_B; V_t^l]} \\ &= \frac{2X\Gamma(T-t, V_t^m, V_B^m)}{\sum_{i=1}^n e^{-0.5ri} P^m(t+0.5i)}, \end{aligned}$$

where $\Gamma(T-t, V_t^m, V_B^m)$ is given in expression (29) in appendix E and $P^m(\cdot)$ is defined in expression (18). Note that X now depends on V_B^m (instead of

V_B).

Example 3. Assume that $\delta_t = 3.5$, $\sigma = 0.3$, $\mu = 0.045$, $r = 0.08$, $C = 13$, $\theta = 0.3$, $\alpha = 0.3$, $m = 0.1$, $l = 0.25$, and the coupon payments on the CDS are paid semiannually. These parameter values give $V_B^m = 65$. For these parameters, the CDS rates for the classical case and the case with delayed and asymmetric information are plotted in Figure 4. The highest rates are for the case with delayed and asymmetric information.

[Figure 4 should be placed about here]

V Implications for Credit Spreads in the Inter-bank Market

In this section we demonstrate how our model can help explain the effect the sub-prime crises had on credit spreads in the interbank market, i.e., the market for over-night borrowing and lending between banks. As the sub-prime crises started to evolve during the summer of 2007, the inter-bank market dried-out. Banks ceased to lend money to each other and central banks supplied short-term liquidity. The dry-up of liquidity in this typically high-liquid market, was caused by concerns about in which financial state different banks were. Any bank had sound concerns about other banks exposure to the sub-prime market. This was (and still is) a situation where information about the quality of a given bank's balance sheet (and thereby also its earnings potential) is definitively asymmetrically distributed between this particular bank and other banks. Our model shows that short-term credit spreads can be highly sensitive to asset volatility and assessed asset values. In Figure 5 we show how, *ceteris paribus*, the overnight interest rate changes with the volatility of the assets. Similarly, Figure 6 shows

how, *ceteris paribus*, the spread varies for different levels of the assessed asset value V_t^l . Thus, if the asset value assessed by other banks drop because of exposure to the sub-prime market, the overnight borrowing rate for this bank can increase quite dramatically. Therefore, the combination of increased risk (i.e., higher volatility) and lower assessed asset values due to exposure to the sub-prime market, could easily lead to very high credit spreads. This description corresponds to recent market observations, and it also fits well into our simple model with asymmetric information.

[Figures 5 and 6 should be placed about here.]

VI Conclusions

This paper analyzes the effect of asymmetric and delayed information on credit spreads on corporate bonds traded in a secondary market and on CDS spreads. In the case where the bankruptcy decision is based on delayed information we identified a potential gain for equity holders as a lottery with non-negative payoff. This payoff may be strictly positive in the case the actual market value of the company is significantly higher than the value of the company on which the bankruptcy decision was based. For realistic parameter values, and as long as all market participants, including the management of the company, have the same amount of information, this lottery has a rather small value, and the effect on bankruptcy policy and credit spreads is also small.

Asymmetric information, on the other hand, has a substantial effect on credit spreads. Asymmetric information explains why credit spreads on bonds with short time to maturity do not approach zero. Our results are qualitatively similar to the results of Duffie and Lando (2001), which they de-

rive in a model with incomplete accounting information. Our model relaxes some of their assumptions, e.g., regarding tradability of stocks and bonds, although it does not contain noisy information, only delayed information.

Our analysis shows that it is not the incomplete accounting information per se that causes the equivalence between the structural and reduced form approaches, but rather the information asymmetry between the equity and the bond holders.

As an additional application of our model we illustrate how the overnight interest rate can become very high when assessed asset values fall and/or asset volatility increases. The results from our model are consistent with the dry-up in the interbank market in the wake of the sub-prime crises.

A Survival Probability

Consider a geometric Brownian motion with dynamics as in expression (1) and initial value v . The probability of not crossing the barrier v_b in a time period of length s when $v > v_b$, is

$$\Psi(s, v, v_b) = N\left(\frac{-\ln \frac{v_b}{v} + \nu s}{\sigma\sqrt{s}}\right) - \left(\frac{v_b}{v}\right)^{2\nu\sigma^{-2}} N\left(\frac{\ln \frac{v_b}{v} + \nu s}{\sigma\sqrt{s}}\right), \quad (22)$$

where $\nu = \mu - \frac{1}{2}\sigma^2$, see e.g., Musiela and Rutkowski (1997) Corollary B.3.4.

We define

$$P(s) = \Psi(s - t, V_t, V_B). \quad (23)$$

B Definition of Υ

For $s > t$, let

$$\Upsilon(s - t, v, v_b) = E[e^{-r(\tau-t)} 1_{\{\tau \leq s-t\}} \mid \mathcal{F}_t]. \quad (24)$$

Then

$$\Upsilon(s-t, v, v_b) = e^{b(z-w)} N\left(\frac{b-w(s-t)}{\sqrt{s-t}}\right) + e^{b(z+w)} N\left(\frac{b+w(s-t)}{\sqrt{s-t}}\right), \quad (25)$$

where

$$\begin{aligned} b &= \ln(v_b/v)/\sigma, \\ z &= (\mu - \frac{1}{2}\sigma^2)/\sigma, \end{aligned}$$

and

$$w = \sqrt{z^2 + 2r},$$

see e.g., Lando (2004), appendix B.

C The Equity Holders' Optimization Problem in the Case with Delayed Information

Observe that

$$\begin{aligned} & \sup_{\tau \in \mathcal{T}_m} E \left[\int_t^\tau e^{-r(s-t)} (\delta_{s-m} - (1-\theta)C) ds + e^{-r(\tau-t)} \pi(V_B^m) \Big| \mathcal{F}_t^m \right] \\ &= \sup_{\tau^* \in \mathcal{T}^*} E \left[\int_{t-m}^{\tau^*} e^{-r(s-(t-m))} (\delta_s - (1-\theta)C) ds \right. \\ & \quad \left. + e^{-r(\tau^*-(t-m))} \pi(V_B^m) \Big| \mathcal{F}_{t-m} \right], \end{aligned}$$

where $\tau^* = \tau - m$ and \mathcal{T}^* is the set of all \mathcal{F}_{t-m} -adapted stopping times. The expression in the last line we recognize as a standard optimal stopping problem and its connection to ODEs is known. For details, see Øksendal (2004).

D Proof that Asymmetric Information Increases Credit Spreads

In this appendix we show that, *ceteris paribus*, asymmetric information leads to higher credit spreads. Let η be the credit spread under symmetrically distributed information. We then have that

$$\begin{aligned}
 \eta^m &> \eta \\
 &\Leftrightarrow \\
 \varphi^m(t, s) &< \varphi(t, s) \\
 &\Leftrightarrow \\
 P^m(s) &< P(s). \tag{26}
 \end{aligned}$$

Let \mathcal{A} be the event that no bankruptcy takes place on the time interval $[t, s]$ and \mathcal{B} the event that no bankruptcy takes place on the time interval $[t - (l - m), t]$. We then have that (conditional on $V_t = v$)

$$P(s) = P(\mathcal{A}|\mathcal{B}). \tag{27}$$

Let further $\hat{\mathcal{B}}$ be the event that $V_t^m > V_B^m$. We then have that (conditional on $V_t^l = v^9$)

$$P^m(s) = \frac{P(\mathcal{A} \cap \mathcal{B})}{P(\hat{\mathcal{B}})}. \tag{28}$$

Combining equations (27) and (28) with the inequality in (26), we have that

$$\frac{P(\mathcal{A} \cap \mathcal{B})}{P(\hat{\mathcal{B}})} < P(\mathcal{A}|\mathcal{B}).$$

Using Baye's rule, we get that

$$\begin{aligned}
 P(\mathcal{A}|\mathcal{B})P(\mathcal{B}) &< P(\mathcal{A}|\hat{\mathcal{B}})P(\hat{\mathcal{B}}) \\
 &\Leftrightarrow \\
 P(\mathcal{B}) &< P(\hat{\mathcal{B}}).
 \end{aligned}$$

The last inequality is trivially satisfied, proving that credit spreads are higher under asymmetric information than under symmetrically distributed information.

E Credit Default Swap

In this appendix we derive the expression for $\Gamma(T - t, V_t^m, V_B^m)$.

$$\Gamma(T - t, V_t^m, V_B^m) = E\left[e^{-r(\tau-t)}\mathbf{1}\{\tau \leq T\} \middle| V_t^m > V_B^m; V_t^l\right].$$

Trivially,

$$\begin{aligned} & E\left[e^{-r(\tau-t)}\mathbf{1}\{\tau \leq T\} \middle| V_t^l\right] \\ = & E\left[e^{-r(\tau-t)}\mathbf{1}\{\tau \leq T\}\mathbf{1}\{V_t^m > V_B^m\} \middle| V_t^l\right] \\ & + E\left[e^{-r(\tau-t)}\mathbf{1}\{\tau \leq T\}\mathbf{1}\{V_t^m \leq V_B^m\} \middle| V_t^l\right]. \end{aligned}$$

Observe that

$$\begin{aligned} & E\left[e^{-r(\tau-t)}\mathbf{1}\{\tau \leq T\}\mathbf{1}\{V_t^m \leq V_B^m\} \middle| V_t^l\right] \\ = & E\left[e^{-r(\tau-t)}\mathbf{1}\{V_t^m \leq V_B^m\} \middle| V_t^l\right] \\ = & E\left[\mathbf{1}\{V_t^m \leq V_B^m\} \middle| V_t^l\right] \\ = & P(V_t^m \leq V_B^m \middle| V_t^l). \end{aligned}$$

The second equality follows from the definition of the stopping time τ in (3).

From Baye's formula follows that

$$\begin{aligned} \Gamma(T - t, V_t^m, V_B^m) &= \frac{E\left[e^{-r(\tau-t)}\mathbf{1}\{\tau \leq T\} \middle| V_t^l\right] - P(V_t^m \leq V_B^m \middle| V_t^l)}{P(V_t^m > V_B^m \middle| V_t^l)} \\ &= \frac{\Upsilon(s - t + l - m, V_t^m, V_B^m) - P(V_t^m \leq V_B^m \middle| V_t^l)}{P(V_t^m > V_B^m \middle| V_t^l)} \quad (29) \end{aligned}$$

Notes

¹Comprehensive treatments of these two approaches can be found in the encyclopedic monograph by Bielecki and Rutkowski (2002) or in the more accessible monograph by Duffie and Singleton (2003).

²Examples are Special Purpose Vehicles, mutually owned companies (banks and insurers), foundations, and municipals.

³For a stopping time to have an associated intensity, it must be *totally inaccessible* (in addition to technical requirements). Informally, a stopping time τ is totally inaccessible if any increasing sequence of stopping times converging to τ has probability zero.

⁴For instance when the Norwegian newspaper Oslo Posten was declared bankrupt.

⁵In the special case considered by Leland (1994) where $\mu = r$, $V_B = (1 - \theta)C/(r + 0.5\sigma^2)$.

⁶This lottery has some resemblance to the *wild card play* that is present when trading in the CBOT Treasury bond futures, see e.g., Hull (2006).

⁷This observation also gives some justification for using a diffusion model instead of a jump-diffusion model. Few companies “jump” into default when their bonds are rated investment grade.

⁸The assumption that the equity holders are immediately informed can be relaxed without adding much economic insight, but it will increase the notational burden. It is enough that they are informed before time $\tau + (l - m)$, since the stopping time then still is totally inaccessible for the financial

market.

⁹Note that in the proof we both assume that $V_t = v$ when calculating $P(s)$ and that $V_t^l = v$ when calculating $P^m(s)$, but this is as it should be since we prove the result “*ceteris paribus*”.

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Table I: **Effect of delayed information** The table shows how the default barrier V_B^m and the price of the bankruptcy lottery $\pi(V_B^m)$ varies for different lengths of the information lag m . The parameter values are $\alpha = 0.3$, $r = 0.08$, $\mu = 0.045$, $\sigma = 0.3$, $\theta = 0.3$, $C = 13$, and $D = 90$.

m	V_B^m	V_B	$\pi(V_B^m)$
0.0	65.0000	65.0000	0.0000
0.2	65.0000	65.0000	0.0000
0.5	65.0289	65.0000	0.0036
1.0	65.6098	65.0000	0.1291
1.5	67.0580	65.0000	0.5834
2.0	69.2680	65.0000	1.4737
2.5	72.1945	65.0000	2.8798
3.0	75.8860	65.0000	4.9026
3.5	80.4840	65.0000	7.6939
4.0	86.2511	65.0000	11.4961

Table II: **Effect of volatility under delayed information** The table shows how the default barrier V_B^m and the price of the bankruptcy lottery $\pi(V_B^m)$ varies for different levels of the volatility σ . The parameter values are $\alpha = 0.3$, $r = 0.08$, $\mu = 0.045$, $m = 0.2$, $\theta = 0.3$, and $C = 13$. D is approximately equal to the market value of corporate debt when $\delta_t = 3.5$.

σ	D	V_B^m	V_B	$\pi(V_B^m)$
0.15	92	93.2899	93.2899	0.0000
0.25	99	73.6273	73.6273	0.0000
0.50	81	39.9644	39.9643	0.0000
0.75	62	23.3913	23.3901	0.0001
1.00	48	14.9151	14.9079	0.0009
2.50	16	3.0370	2.8094	0.0836
5.00	6	1.2562	0.7214	0.3902

Table III: Effect on credit spreads The table shows credit spreads when information is delayed (η^m) and not delayed (η) for different delays m and levels of the volatility σ . The parameter values are $\delta_{t-m} = 3.5$, $\alpha = 0.3$, $r = 0.08$, $\mu = 0.045$, $\theta = 0.3$, and $C = 13$. D is approximately equal to the market value of corporate debt when $\delta_t = 3.5$.

		Time to maturity												
		0.1				1				3				
m	σ	D	V_B^m	$\pi(V_B^m)$	η^m	η	η^m	η	η^m	η	η^m	η	η^m	η
0	0.30	90	65.0000	65.0000	0.0000	0.0000	0.0464	0.0464	0.0464	0.0464	0.0434	0.0434	0.0434	0.0434
0.5	0.30	90	65.0289	65.0000	0.0000	0.0000	0.0465	0.0464	0.0465	0.0464	0.0435	0.0434	0.0435	0.0434
2	0.30	90	69.2680	65.0000	0.0003	0.0000	0.0686	0.0464	0.0686	0.0464	0.0518	0.0434	0.0518	0.0434
4	0.30	90	86.2511	65.0000	0.3650	0.0000	0.2067	0.0464	0.2067	0.0464	0.0884	0.0434	0.0884	0.0434
0.2	0.75	62	23.3913	23.3901	0.0000	0.0000	0.0285	0.0285	0.0285	0.0285	0.0479	0.0479	0.0479	0.0479
0.2	2.50	16	3.0370	2.8094	0.0002	0.0001	0.1868	0.1817	0.1868	0.1817	0.1131	0.1128	0.1131	0.1128

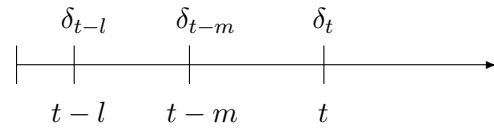


Figure 1: **Observable EBIT process** The figure shows the EBIT process observable at time t for agents with different information delays.

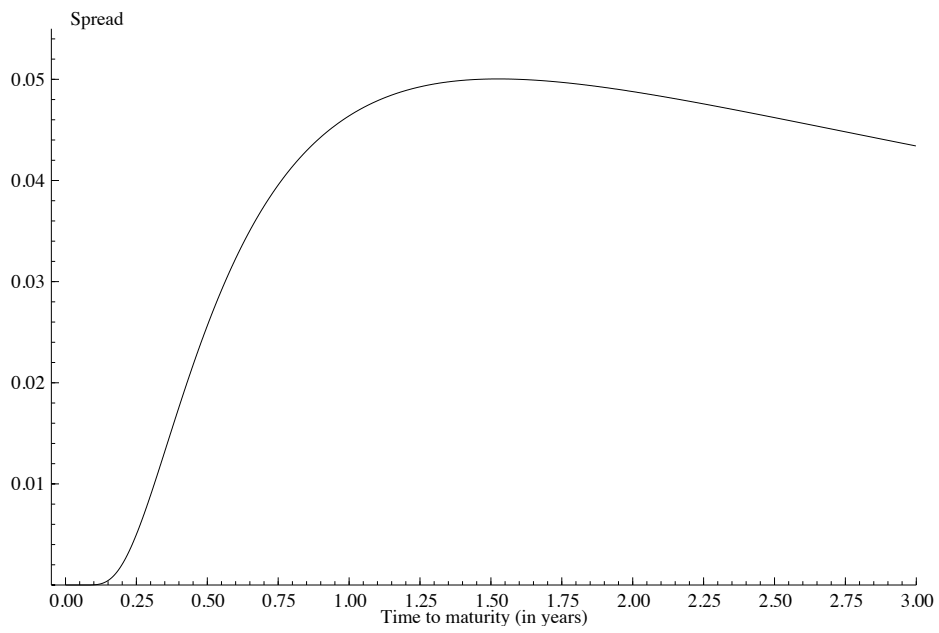


Figure 2: **Credit spreads classical case** The figure shows the credit spreads for zero-coupon bonds with up to three years to maturity.

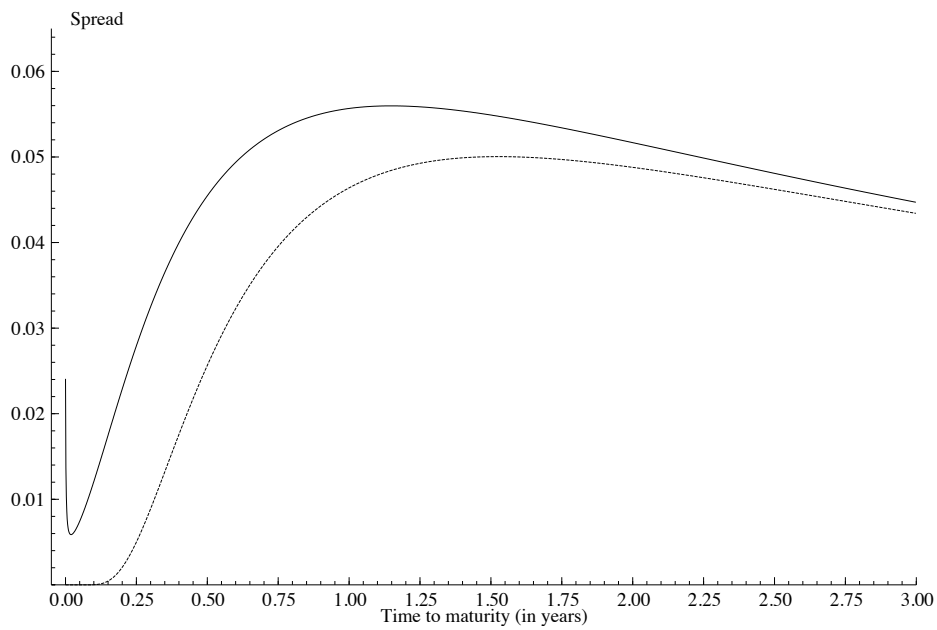


Figure 3: **Effect of asymmetric information on credit spreads** The figure shows credit spreads for zero-coupon bonds with up to three years to maturity. The upper graph represents the case where the management has a delay m and the bond holders a delay l . The lower graph represents the complete information case.

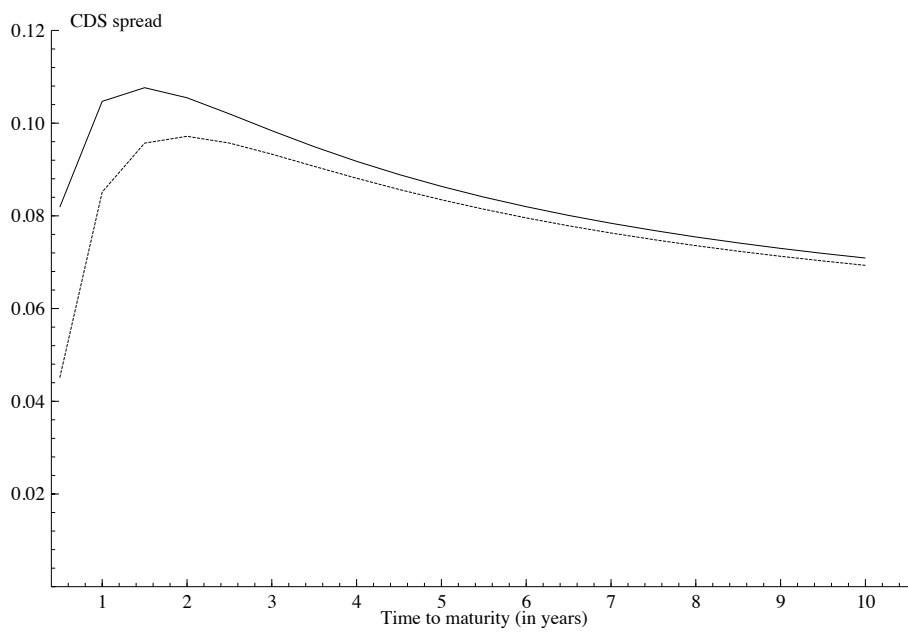


Figure 4: **CDS spreads** The figure shows the CDS spreads for different times to maturity. The top graph represents the case with delayed and asymmetric information.

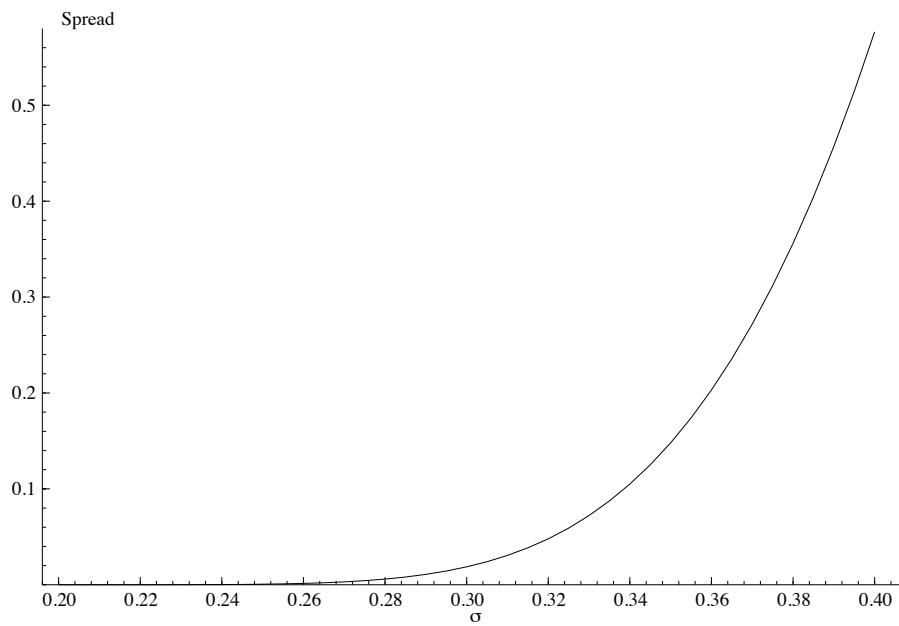


Figure 5: **Effect of volatility on overnight credit spreads** The figure shows the overnight credit spreads for different values of the underlying volatility σ . Other parameter values are: $\delta_{t-l} = 3.5$, $\mu = 0.045$, $r = 0.08$, $\theta = 0.3$, $\alpha = 0.3$, $m = 0.1$, and $l = 0.25$.

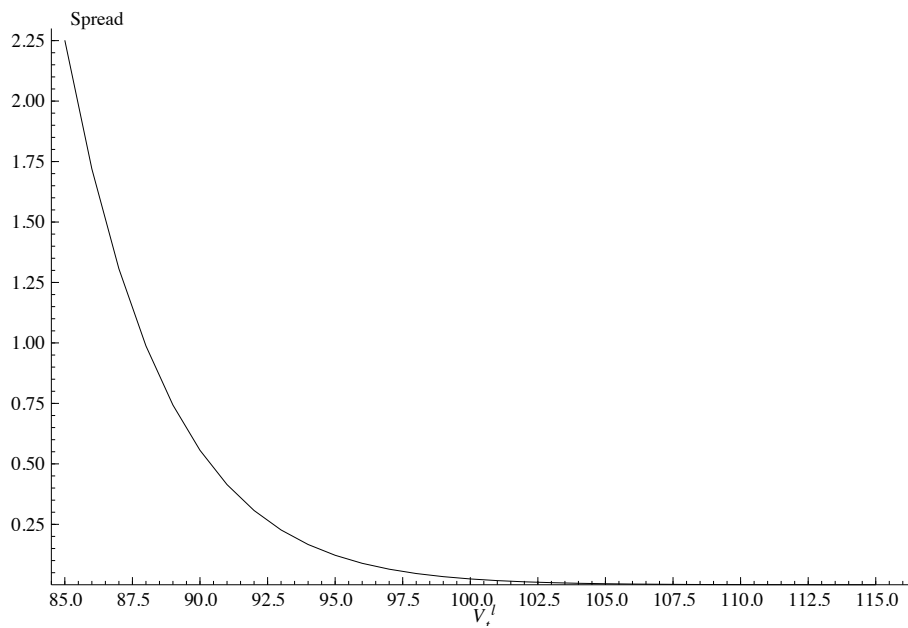


Figure 6: **Effect of assessed asset values on overnight credit spreads**

The figure shows the overnight credit spreads for different values of the assessed asset value V_t^l . Other parameter values are: $\mu = 0.045$, $r = 0.08$, $\sigma = 0.3$, $\theta = 0.3$, $\alpha = 0.3$, $m = 0.1$, and $l = 0.25$.