Optimal Fishing Policy for Two Species in a Three-Species Predator-Prey Model
The case of Capelin, Cod and Juvenile Herring in the Barents Sea

Sigurd Aanestad[*], Leif Sandal[†] and Arne Eide[‡]

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Abstract

This paper presents a management model for the Barents Sea capelin and cod fisheries including juvenile herring in the biological model as the young herring influences the cod-capelin system. The objective of the study is to balance model-complexity of biology and economics when investigating possible optimal catch strategies given that one aims to maximize economic rent in the fishery. The three species constitute a highly dynamic system, also because prey-predation relations are functions of ages within each stock. A top-down approach is employed and the biological growth equations relate to stock biomass estimates. Economic relations are based on empirical data and previous studies. Optimal fishing strategies are identified by employing a numerical feedback rule for optimal fishing through dynamic programming. The feedback rule suggests that previous TAC (total allowable catch) levels on average have been too large for both capelin and cod over the past 30 years, according to the management objectives assumed in the study. Moreover, presence of some herring in the system is important for the economic yield although the herring fishery is closed. This indicates that a focus only on the capelin-predator role of herring is too narrow, as herring is also an important prey for cod.

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[*] Corresponding author. Phd. candidate at Norwegian College of Fishery Science. Email: Sigurd.Aanestad@imr.no

[†] Professor, Dr.Scient. Norwegian School of Economics and Business Administration. Email:Leif.Sandal@nhh.no

[‡] senior scientific officer, Norwegian College of Fishery Science. Email: Arne.Eide@nfh.uit.no

1 Introduction

The Barents Sea is one of the most productive ocean areas in the world [5]. It represents a highly diverse arctic ecosystem [25], whereof three fish species are of major commercial importance. The key species are the plankton feeders capelin (*Mallotus villosus*) and herring (*Clupea harengus*), as well as their main fish predator, Northeast Arctic Cod (*Gadus morhua*)[1]. The relationships between the three species are highly dynamic, and the cod-capelin relationship is particularly essential in the Barents Sea ecosystem [2]. Therefore we choose a growth model consisting of capelin, cod and herring as foundation for the decision making of total allowable catches (TACs) for capelin and cod.

The herring stock has shown substantial natural fluctuations during the last century [20],[21], and a high harvest pressure and unfavorable climatic conditions led to stock depletion around the 1970s [7], [22]. Although herring is mainly a Norwegian Sea species, the juvenile part of the stock occasionally enters the Barents Sea area. This inflow of young herring into the Barents Sea basin has a substantial impact on the system; the cod-capelin relation is particularly affected. Strong year classes of herring moves into the Barents Sea by larvae drift and feeds in that area until it moves back into the Norwegian Sea at age 3-4 years [6]. There is no herring fishery during this period.

The presence of herring in the Barents Sea causes mass death of capelin fry,which has major impacts on other capelin dependent species, as in the end of the 1980s when there was a rather large cod stock [19] and herring had recovered from the collapse around 1970. A strong year class of herring occurred in the south of the Barents Sea 1983-1986, causing a capelin stock collapse in 1985-1986. This collapse led to cod starvation and had a negative impact also on other fish predators like sea birds and marine mammals [12]. The capelin stock is the largest pelagic fish stock in the Barents Sea and also potentially the largest capelin stock in the world [8] and [9], and is of crucial importance for the growth of juvenile cod [4].

The strong dependence between the biomasses of the three species in our model calls out for a management regime aware of it. However, it is not an easy task to include capelin, cod and Barents Sea herring in a limited multispecies growth model and get good statistical results. The picture is complicated by other factors such as primary production and predators not included in our model. Especially when it comes to herring, stock size and catch of the mature population further south is of great importance for the extent of new recruits in the Barents Sea. Nevertheless, limitation is essential for solvability of the bioeconomic model within the frames of numerical dynamic programming, which is the instrument of use in this work. It is very computer demanding, and we have therefore limited ourselves to work with a three-dimensional state space (capelin, cod and herring). Although higher dimensions are possible according to theory [18], such a rise will result in a dramatic growth of troughput, and is therefore beyond the scope of this work. However,

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[1] From now on Northeast Arctic Cod will be denoted cod.

it would be interesting as a future extension to include time dependence as a cyclic driving force. The time dependence could be represented by a fourth dimension of the growth model. (Capelin, cod, herring and time).

The results of this work indicates that moderate presence of herring has a clear positive effect on the sum of yield from the capelin and cod fishery of the Barents Sea, and this finding contradicts the one-track interpretation of herring as juvenile capelin predator and a tragedy to the capelin-cod system.

2 Model description and parameter estimation

2.1 Management model

Our bioeconomic management model is based on a biologic multi-species growth model created for this work and on an economic profit model used in a previous work [23]. The system of the two models constitutes an optimal control problem, where the objective is to find optimal total Norwegian and Russian harvest levels from a Norwegian point of view when the Norwegian and Russian fraction of total harvest is pre-decided. If we assume $\underline{x}$ is stock biomasses and $\underline{u}$ is corresponding harvest, the continuous time Hamilton-Jacobi-Bellman equation with discount rate δ can be written (see Kushner [10])

$$\delta V(\underline{x}) = \max_{\underline{u}} \left\{ \Pi(\underline{x}, \underline{u}) + \nabla V(\underline{x})(\underline{F}(\underline{x}) - \underline{u}) \right\}, \tag{1}$$

where the optimal value function $V(\underline{x})$ represents total future discounted profit from optimal harvest. This equation describes the optimal balance between immediate harvest and saving of stocks for future exploitation, and gives sufficient conditions for optimum (See Kamien and Schwarz [16] or Bertsekas [1]). $\Pi(\underline{x}, \underline{u})$ represents immediate profit (economic model) and $(\underline{F}(\underline{x}) - \underline{u})$ represents biologic growth of the stocks. From equation (1) we see that as far as optimality is concerned, equal emphasis should be placed on the economic and the biologic sub-models. In other words, it is not worthwhile to be extremely detailed and accurate in one of the sub-models if it is not followed up in the other.

2.2 Biologic growth model

The complexity of the real nature can not be fully covered in a model used for management purposes, but a good model should be consistent with some of the main features of the real marine life. In this work predation and multi-species interaction are given most priority.

The three stocks in our growth model; namely capelin, cod and juvenile herring age one-three, have historically shown a cyclic behavior, which includes several collapses in the capelin stock due to high inflow of herring and good growth conditions for both cod and herring [12]. It is reasonable to assume that there exists for all the stocks a saturation level affected by food supply, predation pressure and other external effects like marine temperatures and ocean currents. We focus on saturation and predation, and choose logistic growth modified by predation as functional form. Furthermore, the predation reflects the well-documented fact that cod seems to prefer capelin as prey, but turns over to herring and cannibalism in years of capelin scarcity [13].

Since only juvenile herring is found overlapping capelin and cod in the Barents Sea, it is somewhat difficult to implement herring growth in a closed model for the Barents Sea. Historical stock sizes of the Barents Sea fraction of the population are not available. However, there are assessment estimates available for each year class of herring, and it is known that the three to four first year classes are sometimes found in the Barents Sea, and that age one to age three predate heavily on capelin when overlapping between the two species takes place [3]. Therefore, in the growth of capelin and cod, these age classes represent herring influences, and the growth of juvenile herring itself is explained by its own stock size, external inflow and predation relationships with capelin and cod.

Assume stock biomasses are given by the vector $\underline{x} = [x_1, x_2, x_3]$ and harvest by $\underline{u} = [u_1, u_2, u_3]$, where the indexes 1, 2 and 3 represent capelin, cod and herring respectively. Assuming a closed herring fishery in the Barents Sea we could in principle remove herring from the model and instead let $\dot{x}_3 = 1$, which means that instead of being herring, x_3 is a direct measure of time. Another possibility is to let $\dot{x}_3 = F_3(x_3)$ be cyclic and measure time dependent inflow of herring. These are good alternative forms if cyclic time dependence is more important to the model results than predation-prey relationships with herring, and a lot of literature claims that long term biomass cycles play an important role in the Barents Sea ecosystem (See e.g. [26] and [11]). Unfortunately we did not succeed in finding such forms that fitted well with our data.

The forms of the growth functions were selected based on assumptions about the overall structure of the growth dynamics and on numerous statistical fitness-testing with relative growth for stock data $((\underline{x}(t+1) - \underline{x}(t)) + \underline{u}(t))/\underline{x(t)} = \underline{F}(\underline{x}(t))/\underline{x} + \underline{R}_t$, where R_t is a vector of error terms). The growth functions are

$$
\begin{aligned}
F_1(\underline{x}) &= r_1 x_1 \big(1 - \frac{x_1}{k_1} + b_1 g(x_2, x_3) - b_2 \big(1 + x_2\big)(1 + x_3^2)\big) \\
F_2(\underline{x}) &= r_2 x_2 \big(1 - \frac{x_2}{k_2} + b_3 (1 + x_1^2)(1 + x_3)\big) \qquad (2) \\
F_3(\underline{x}) &= \tilde{r}_3(x_1, x_2)(1 + x_3)\big(1 - \frac{x_3}{\tilde{k}_3(x_1, x_2)} + b_4 \frac{x_1(1 + x_1 x_2)}{1 + x_3^3}\big), \text{ where}
\end{aligned}
$$

$$g(x_2, x_3) = \min\left\{\frac{1+x_3}{1+x_2^4}, \frac{0.95}{b_1}\right\} \tag{3}$$

$$\tilde{r}_3(x_1, x_2) = \min\left\{\frac{r_3}{1+x_1x_2}, 0.23\right\}, \tag{4}$$

$$\tilde{k}_3(x_1, x_2) = k_3\frac{1+x_1}{1+x_1x_2} \tag{5}$$

$$r_i, b_i, k_i > 0 \text{ for all } i.$$

Notice that the growth of herring is somewhat different from that of capelin and cod. Whereas r_1 and r_2 represent intrinsic growth-rate of respectively capelin and herring, the interpretation of $\tilde{r}_3$ is more double-faced, as it also represents positive inflow of biomass when the stock-size is zero. In fact

$$F_3\big([x_1, x_2, 0]\big) = \tilde{r}_3(x_1, x_2) + r_3\, b_4\, x_1 \quad > 0 \quad \text{for all } x_1 \text{ and } x_2. \tag{6}$$

This reflects the positive inflow of herring from the Norwegian Sea to the Barents Sea, which in this model only depends on year class 1-3. There are two main reasons for this. First of all we want to focus on the joint Norwegian and Russian resources of the Barents Sea, and in such a perspective management of the herring stock in the North Sea limits the possibility of a closed model since the herring resource is shared with other partners than Russia. Secondly, it is difficult to find the connection between the size of the mature fraction in the south and the juvenile fraction in the north. Biomass of spawners is only a weak indicator of egg production in some populations. ([24] and [27]).

When we subtract harvest from growth, the state equations are summarized with

$$\dot{x}_i = F_i(\underline{x}) - u_i, \quad i = 1, 2, 3, \tag{7}$$

but herring is not exploited in the Barents Sea, so $u_3 = 0$.

All the growth functions are concave with respect to the species. Notice that $b_1 > 0$ to some extent yields positive influence from herring on capelin. Yet, this term is only meant as an adjustment to weaken the negative influence from herring when both the cod and herring stock are high. Negative influence on capelin from herring is already accounted for in the last term with the coefficient b_2. Therefore we set an upper limit to $b_1 g(x_2, x_3)$. (See eq. (3)). We also adjust the intrinsic growth rate of herring to lower growth when small biomasses of capelin interfere with small biomasses of cod. Without this limitation, the herring growth would become too extreme in such cases.

2.3 Economic model

The optimal value function represents the maximum discounted future profit from the fishery. If $\mathcal{U}$ is the set of admissible controls, the optimal value function is given by

$$V(\underline{x}) \equiv \max_{\underline{u}\in\mathcal{U}} \int_0^\infty \Pi\big(\underline{x}, \underline{u}, \underline{\alpha u}\big)e^{-\delta\, t}\, dt, \tag{8}$$

under the conditions in (7) with $u_3 = 0$.

Here δ is the discount rate, $\underline{u}$ is total landings, $\underline{\alpha}$ is the Norwegian fraction of total landings and Π is the current profit of the fishery.

It is assumed that there are no interactions in the market of fish, meaning that profit from cod and capelin can be added together. Further, it is assumed that there is no economic protection on capelin; that is: Landing costs are considered so little density-dependent that we neglect finding costs. Beyond that, capelin is not price elastic, but Russian and Norwegian landings of cod are large enough to influence the price level, $P_2(u_2)$. The Norwegian profit function can be written

$$\begin{aligned}
\Pi(\underline{x}, \underline{u}, \underline{\alpha u}) &= CaP(\alpha_1 u_1) + CoP(x_2, u_2, \alpha_2 u_2), \text{ where} \\
CaP(\alpha_1 u_1) &= p_1\alpha_1 u_1 - C_1(\alpha_1 u_1) = p_1\alpha_1 u_1 - c_1(\alpha_1 u_1)^{c_2}, \\
CoP(x_2, u_2, \alpha_2 u_2) &= P_2(u_2)\alpha_2 u_2 - C_2(x_2, \alpha_2 u_2) = (p_2 - p_3 u_2)\alpha_2 u_2 - \frac{c_3}{x_2}(\alpha_2 u_2)^{c_4},
\end{aligned} \tag{9}$$

and all the coeffisients $\underline{\alpha}, p_1, p_2, p_3, c_1, c_2, c_3 > 0$.

2.4 Solution procedure

The optimal control problem defined in eq. (7) and (8) is solved by numerical dynamic programming. The solution is found on feedback-form, which implies that optimal catches of capelin and cod are functions of stock sizes only, and we can write the optimal harvest policy as $\underline{u}^\star \equiv \tilde{u}^\star(\underline{x})$. The problem, however, is not algebraically solvable, and the solution must be obtained by numerical techniques.

The main procedure used to find the optimal policy is described in Grüne and Semmler [18], but our approach differs a bit. A discrete representation of the continuous-time problem in (7) and (8) could be expressed by a discrete first order approximation given by

$$V_h(\underline{x}) \approx \max_{u} h \sum_{i=0}^{\infty} \beta^i \Pi(\underline{x}_h(i), u_i) \tag{10}$$

$$\underline{x}_h(0) = \underline{x}, \quad \underline{x}_h(i+1) = \underline{\varphi}(\underline{x}, \underline{u}) \equiv \underline{x}_h(i) + \big(\underline{F}\big(\underline{x}_h(i)\big) - \underline{u}_i\big)h, \tag{11}$$

where h is the discrete time step and $\beta = 1 - \delta h$ is the discrete discount rate.

By inserting (11) into (10) and extracting profit of the first period from the summation sign, we get the discrete version Hamilton-Jacobi-Bellman equation

$$V_h(\underline{x}) = \max_{\underline{u}} \Big\{ h\Pi(\underline{x}, \underline{u}) + \beta V_h\big(\varphi(\underline{x}, \underline{u})\big) \Big\}. \tag{12}$$

Optimal value is obtained by solving this equation for every possible stock combinations on a selected stock-grid.

Now we define the linear operator

$$L(u)(V_h) \equiv h\Pi(\underline{x},\underline{u}) + \beta V_h\big(\varphi(\underline{x},\underline{u})\big), \tag{13}$$

and the dynamic programming operator

$$T_h(V_h)(\underline{x}) \equiv \max_{u\in} \big\{L(u)(V_h)\big\}. \tag{14}$$

V_h being the solution to the fixed point equation

$$V_h(\underline{x}) = T_h(V_h)(\underline{x}). \tag{15}$$

We use the dynamic programming operator (14) to solve the Hamilton-Jacobi-Bellman equation (12) with fixed point iteration starting with the initial value $V(\underline{x}_0) = 0$, where $\underline{x}_0$ is the $\underline{x}-$grid chosen. To speed up convergence we switch from this policy-iteration to less numeric expensive value-iterations. In the value-iterations we fix the policy, $\underline{u}^f(\underline{x})$, and therefore also the profit $\Pi(\underline{x},\underline{u}^f)$ when using the linear value-iteration operator (13). When the value-iterations

$$V_h^{n+1} = L(u^f)V_h^n \tag{16}$$

stabilize, we shift to policy-iterations in accordance with equation (14), before returning again to value-iteration. The alternating between value- and policy-iteration continues until convergence.

2.4.1 Utilization of first order conditions

The Hamilton-Jacobi-Belmann equation (12) can be solved in various ways, but in three dimensional space through-put and computer-time becomes a critical point. When going through the iterative process there is only two limitations in policy-space. The feasible policy, that is the harvest on each species, is non-negative and not higher than the size available. Instead of working with a discrete control space, $\tilde{u} = (\underline{\tilde{u}}_0, \underline{\tilde{u}}_2, \cdots, \underline{\tilde{u}}_n)$ and for each fix-point iteration do a crude search for the "argmax" to (14) among all of the $\underline{\tilde{u}}_i$'s, we check only boundary values and the inner point solution to equation (14) when $V_h\varphi(\underline{x},\underline{u})$ is replaced by its first order Taylor approximation to reduce number of terms.

$$V_h\big(\varphi(\underline{x},\underline{u})\big) \approx V_h(\underline{x}) + (\nabla V_h)^T(\underline{x})(\underline{F}(\underline{x}) - \underline{u})h. \tag{17}$$

That is; inserting (17) into (14) reduces the problem to

$$V_h(\underline{x}) = \frac{h}{1-\beta}\max_{\underline{u}}\Big\{\Pi(\underline{x},\underline{u}) + \beta(\nabla V_h)^T(\underline{x})(\underline{F}(\underline{x}) - \underline{u})\Big\}, \tag{18}$$

which may be tested for three control values only, namely the lower bound $\underline{u} = \underline{0}$, the upper bound and the interior solution solving

$$\frac{\partial\Pi(\underline{x},\underline{u})}{\partial\underline{u}} - \beta\nabla V = \underline{0} \tag{19}$$

with respect to $\underline{u}$. Optimal $\underline{u}$-value in each fixed point iteration step will be the one that gives the highest value to (14).

2.5 Parameter Estimation

The econometrics software *Eviews* is used to fit the parameters of functions (2-4) to the discrete relative growt $\big(\underline{x}(t+1)-\underline{x}(t)-\underline{u}(t)\big)/\underline{x}(t)$ for the years 1973-2004. The statistical results for each of the species are listed in tab. 2 and the final parameter values found in tab. 3. However, the growth functions in equation (2) are factorized versions of the growth used in the regression analysis. The value of the growth function coefficients and their connections with the regression analysis coefficients are listed in tab. 1, where $C(i), \quad i \in [1,10]$, refers to the coefficients in tab. 2.

All coefficients, except from b_1, have signs corresponding with the fact that cod predates on both capelin and herring whereas herring predates on capelin. The coefficient b_1 should, as already mentioned in sec. 2.2, be looked upon as a correctional term preventing underestimation of capelin growth when the cod stock is low. We can therefore deduce that predation between these species is probably a more important factor for the total biomass than exogenous factors working in other directions, e.g. primary production.

It is rather surprising that the fit for herring is quite good although catch of herring is not a part of the model and mature herring, in best case, could be said to be involved only indirectly through new mature recruitment from the surviving fraction of juvenile herring.

Table 1: Coefficients for the growth functions. $C(\cdot)$ is found in the regression analysis results summarized in tab. 2 and 3.

	r_1	r_2	r_3	k_1	k_2	k_3
Formula	$r_1 = C(1)$	$r_2 = C(5)$	$r_3 = C(9)$	$k_1 = \frac{-r_1}{C(2)}$	$k_2 = \frac{-r_2}{C(6)}$	$k_3 = -\frac{C(9)}{C(10)}$
Value	0.7962	0.1448	$4.97 \cdot 10^5$	3419.6	984.11	$4.31 \cdot 10^6$

	b_1	b_2	b_3	b_4
Formula	$b_1 = \frac{C(3)}{r_1}$	$b_2 = \frac{-C(4)}{r_1}$	$b_3 = \frac{C(7)}{r_2}$	$b_4 = \frac{C(8)}{C(9)}$
Value	$2.51 \cdot 10^9$	$4.88 \cdot 10^{-11}$	$7.01 \cdot 10^{-11}$	$3.01 \cdot 10^{-7}$

All the economic parameters apart from the discount rate $\delta = 0.05$ are collected from [23]. They are summarized in tab. 4, but for further information about the economic model we refer to the original source [23].

3 Results

3.1 The optimal value function

The optimal value function, $V(\underline{x})$, is a theoretical function used to deduce optimal TAC-levels in the management model. However, the estimated size of the function being a measure of the total future value of the fishery as a function of stock sizes, also gives some

Table 2: Regression analysis of relative growth for capelin, cod and herring. The analysis is based on official stock and catch data from ices.
Estimation Method: Weighted Least Squares
Sample: 1974 2004
Included observations: 31
Total system (balanced) observations 93
Linear estimation after one-step weighting matrix

Determinant residual covariance 0.008417

Eq.: $\frac{x_1(t+1)-x_1(t)-u_1}{x_1(t)} = C(1)+C(2)x_1(t)+C(3)\frac{x_3(t)+1}{1+x_2^4}+C(4)x_2(x_3(t)^2+1)$
Obs.: 31

R-squared	0.585491	Mean dependent var	0.320644
Adjusted R-squared	0.539434	S.D. dependent var	1.303849
S.E. of regression	0.884857	Sum squared resid	21.14026
Durbin-Watson stat	2.328894		

Eq.: $\frac{x_2(t+1)-x_2(t)-u_2}{x_2(t)} = C(5)+C(6)x_2(t)+C(7)x_1(t)^2(x_3(t)+1)$
Obs.: 31

R-squared	0.570388	Mean dependent var	0.004749
Adjusted R-squared	0.539702	S.D. dependent var	0.190148
S.E. of regression	0.129006	Sum squared resid	0.465993
Durbin-Watson stat	1.508753		

Eq.: $\frac{x_3(t+1)-x_3(t)}{x_3(t)} = \left(1+\frac{1}{x_3(t)}\right)\left(C(8)\frac{x_1(t)}{1+x_3(t)^3}+C(9)\frac{1}{1+x_1(t)x_2(t)}+C(10)\frac{x_3(t)}{1+x_1(t)}\right)$
Obs.: 31

R-squared	0.914086	Mean dependent var	1.050224
Adjusted R-squared	0.907949	S.D. dependent var	3.456490
S.E. of regression	1.048695	Sum squared resid	30.79331
Durbin-Watson stat	1.859404		

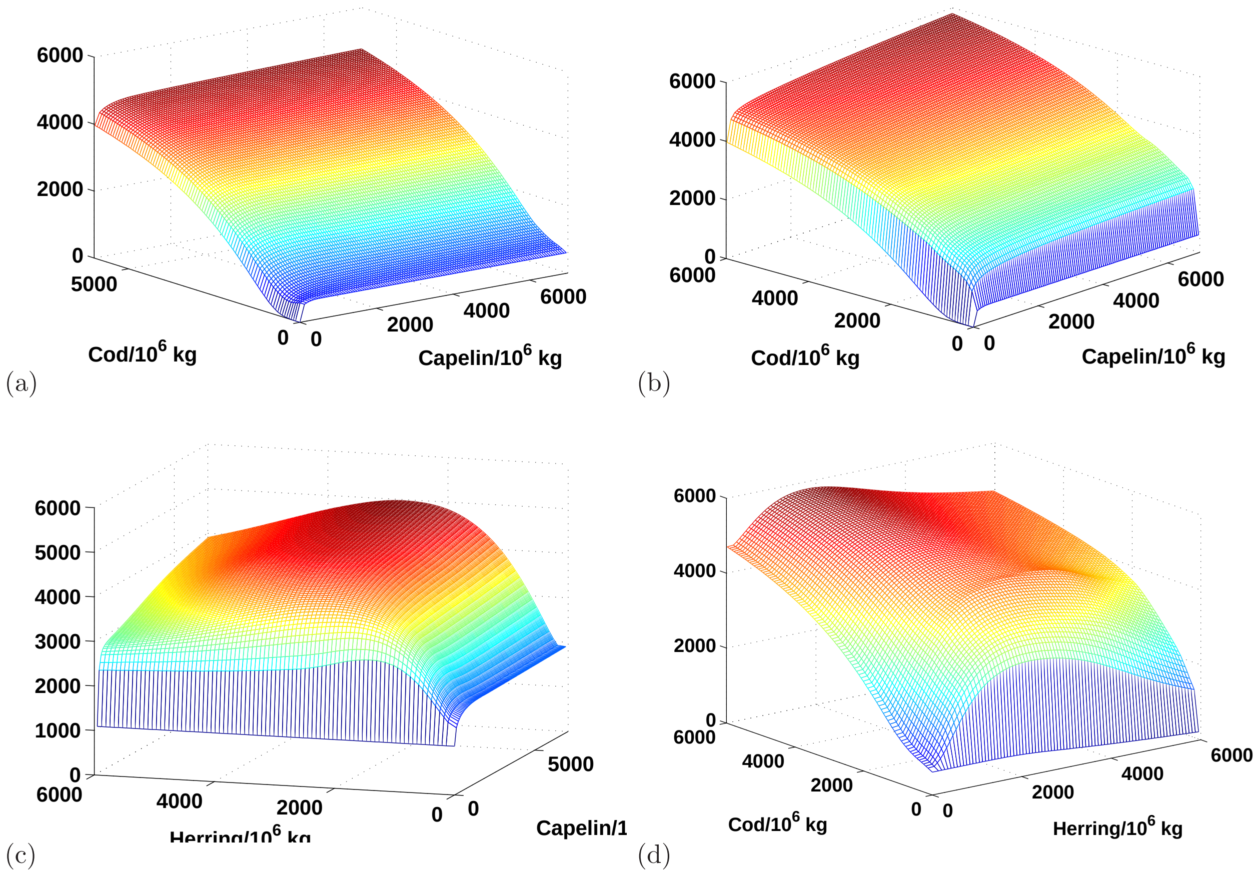


Figure 1: The optimal value function (a) in absence of herring, (b) with 800 thousands tons of juvenile herring, (c) with 1.5 million tons of cod and (d) with 3 million tons of capelin present.

Table 3: Coefficients from regression analysis. These coefficients are used to determine the coefficients in the growth function according to the rules in tab: 1

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.796199	0.293846	2.709582	0.0082
C(2)	-0.000233	6.62E-05	-3.518541	0.0007
C(3)	2.00E+09	3.87E+08	5.168128	0.0000
C(4)	-3.88E-11	1.02E-11	-3.803060	0.0003
C(5)	0.144812	0.070738	2.047168	0.0438
C(6)	-0.000147	4.44E-05	-3.317194	0.0014
C(7)	1.01E-11	1.75E-12	5.784812	0.0000
C(8)	0.149400	0.008308	17.98239	0.0000
C(9)	4.97E+05	88086.94	5.643437	0.0000
C(10)	-0.115302	0.048778	-2.363823	0.0204

Table 4: Economic parameters.

	demand parameters	cost parameters
capelin	$p_1 = 1$	$c_1 = 0.07, \quad c_2 = 1.4$
cod	$p_2 = 12.65, \quad p_3 = 0.00839$	$c_3 = 5848.1, \quad c_4 = 1.1$

interesting interpretations. Since a simultaneous graphical presentation of this function on the whole capelin, cod and herring stock range would imply a four-dimensional figure, which is hard to depict, it is presented with one of the stocks fixed in fig. (1(a)- 1(d)). The shapes of the figures verify rather intuitive results. Absence of both the harvested stocks capelin and cod means zero profit, and a rise in one or both of them means a rise in future profit and therefore also rises in the value function. However, the value function flattens out for extreme values of the stocks.

If either capelin or cod is depleted, there is a dramatic fall in the value function. On the surfaces in fig. 1(a) and 1(b) we observe this clearly as the edges are very steep from zero stock value to the first non-zero value along both the capelin-axis and the cod-axis. It is also interesting to observe that absence of juvenile herring (see fig. 1(a)) yields lower profit than presence (fig. 1(b)). In fact, in fig. (1(c)) and (1(d)) we see that for fixed cod and capelin stocks of respectively 1.5 million and 3 million tons, moderate presence of herring adds value to the optimal value function. Actually, a small rise in the herring biomass seems to be very positive when the biomass is low, but for large stocks the influence becomes negative.

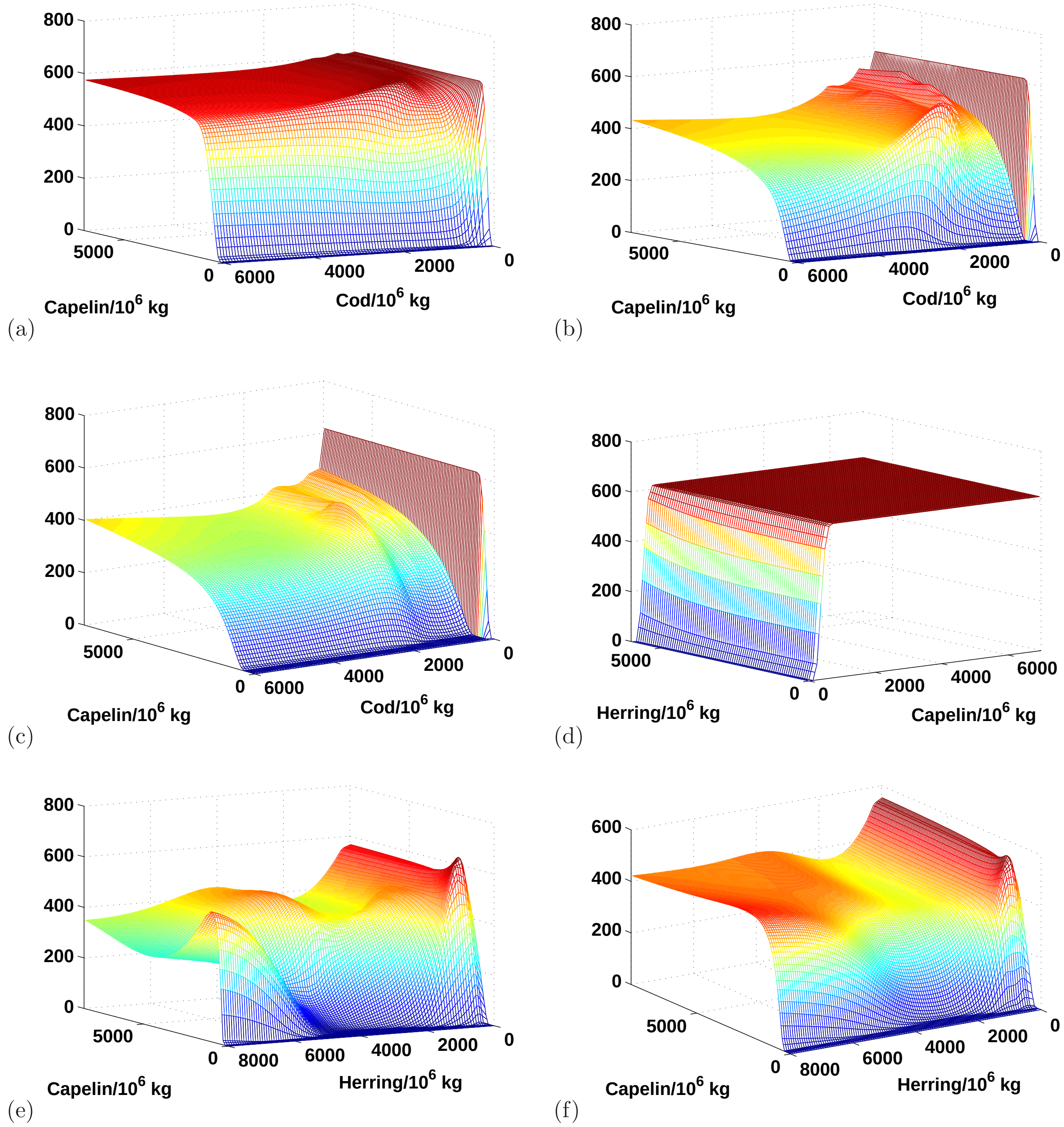


Figure 2: Optimal TACs (thousand tons) of capelin. The stock of juvenile herring is fixed in fig. (a)-(c) and in fig. (e)-(f) cod is fixed. (a) absence of herring, (b) 800 thousands tons of juvenile herring present, (c) 1.5 million tons of juvenile herring present. (d) absence of cod, (e) 1.5 million tons of cod present and (f) 2.5 million tons of cod present.

3.2 Optimal Catches

In this section we focus on total Norwegian and Russian landings of capelin and cod. The optimality condition maximizes discounted Norwegian profit from the fishery when the Norwegian fraction of the total quota is $\alpha_1 = 0.6$ and $\alpha_2 = 0.5$ for capelin and cod respectively. New TACs are decided once a year.

Optimal TAC levels for capelin are presented for several different initial conditions in Fig. 2(a)-2(f). The main trend is that optimal TAC of capelin grows with the capelin stock, but the surfaces also show clear dependence on the levels of cod and juvenile herring. In fig. 2(a) we observe that TACs of capelin fall slightly with growing cod stock. There is even a small discontinuity between zero-level of cod and the first non-zero level. This is a discontinuity between ”bliss”, which is the TAC level that maximizes current profit, and lower TAC-levels in presence of cod. The discontinuity becomes more explicit with more herring present as in fig. 2(b) and 2(c). The trend that more cod means lower capelin TACs is also confirmed in these figures, but for high levels of capelin there is a bump in the TACs interrupting this trend. Presence of herring has a similar impact as presence of cod; it reduces the capelin TACs, though not as much as the cod. Fig. 2(d)) shows capelin TACs on ”bliss-level” for all stock of herring in absence of cod.

With some cod and herring present (fig. 2(e) and 2(f)), optimal TACs of capelin are very different. Their presence leads to lower TACs. We can however observe some interesting disturbances for high stocks of both capelin and juvenile herring (fig. 2(e) and 2(f)). In fig. 2(e) it seems like optimal catch of capelin grows with the herring levels when the herring stock is between three and 4.5 million tons and there is much capelin present. In addition there is a peak in the optimal catch of capelin for extreme levels of herring (7 million tons) when the capelin stock is moderate (see (2(e)).

In fig. 2(e) and 2(f) we can see that there is a peak in the optimal TACs of capelin for low biomasses of herring. In a specific area (0-0.5 million tons) and moderate capelin biomasses (0-1.9 million tons) optimal capelin TACs grows with the herring biomass. Also, for capelin biomasses between 1.5 and 1.9 million tons, optimal TACs of capelin fall with the capelin stock. This behavior is unexpected and counterintuitive. (See section 4.3)

Optimal catch of cod can be viewed in fig. 3(a)-3(d). The surfaces mainly visualize the intuitive positive correlation between the size of the cod stock and the optimal catch of it. The sizes of the capelin and herring stocks do not seem to be very important for optimal catch of cod. Only in absence of capelin do we notice interesting changes as this yields lower optimal harvest of cod (see fig. 3(b)).

3.3 Paths towards equilibrium

Our deterministic growth model with optimal management implicates that the stocks will move towards equilibrium levels. These theoretical levels have not been observed in the

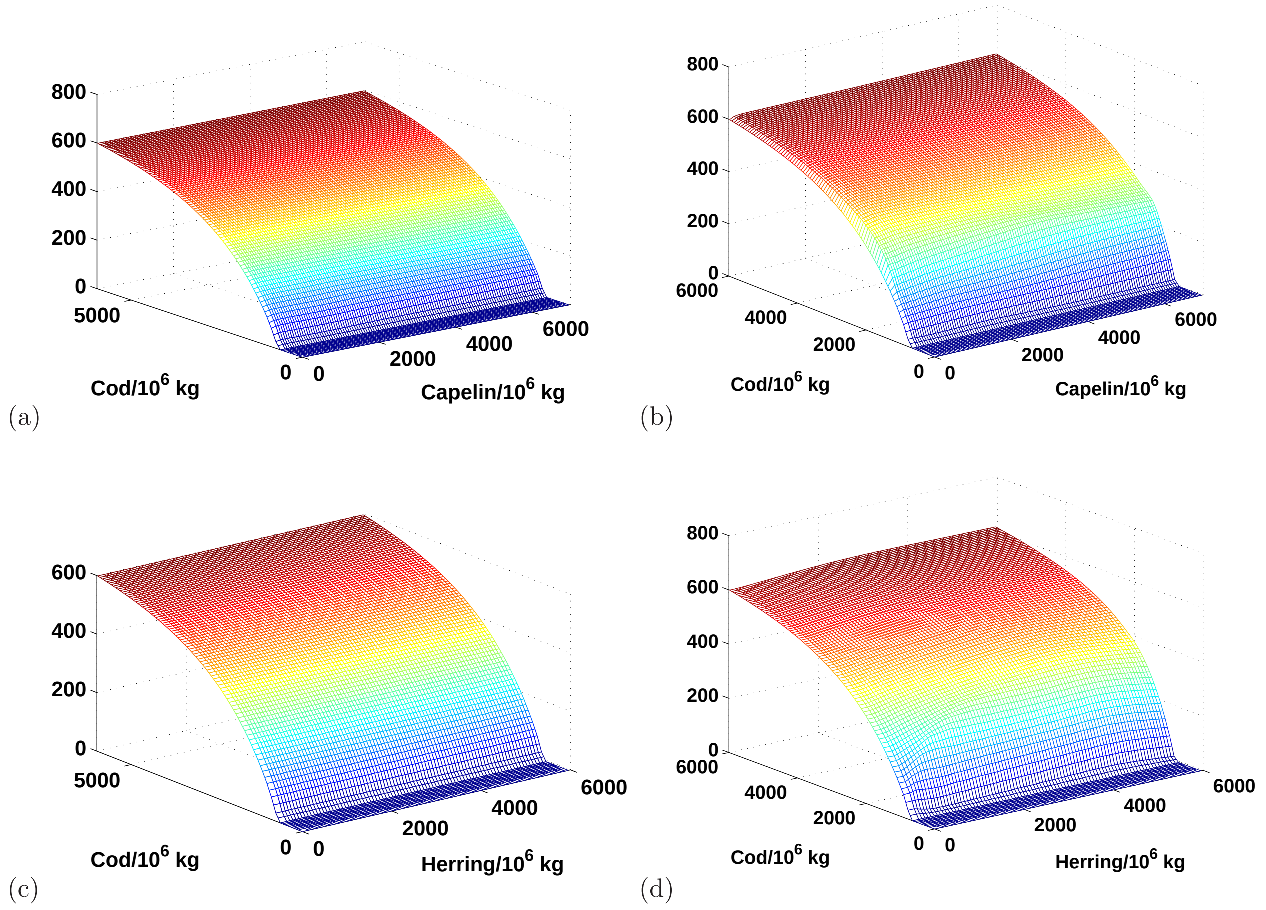
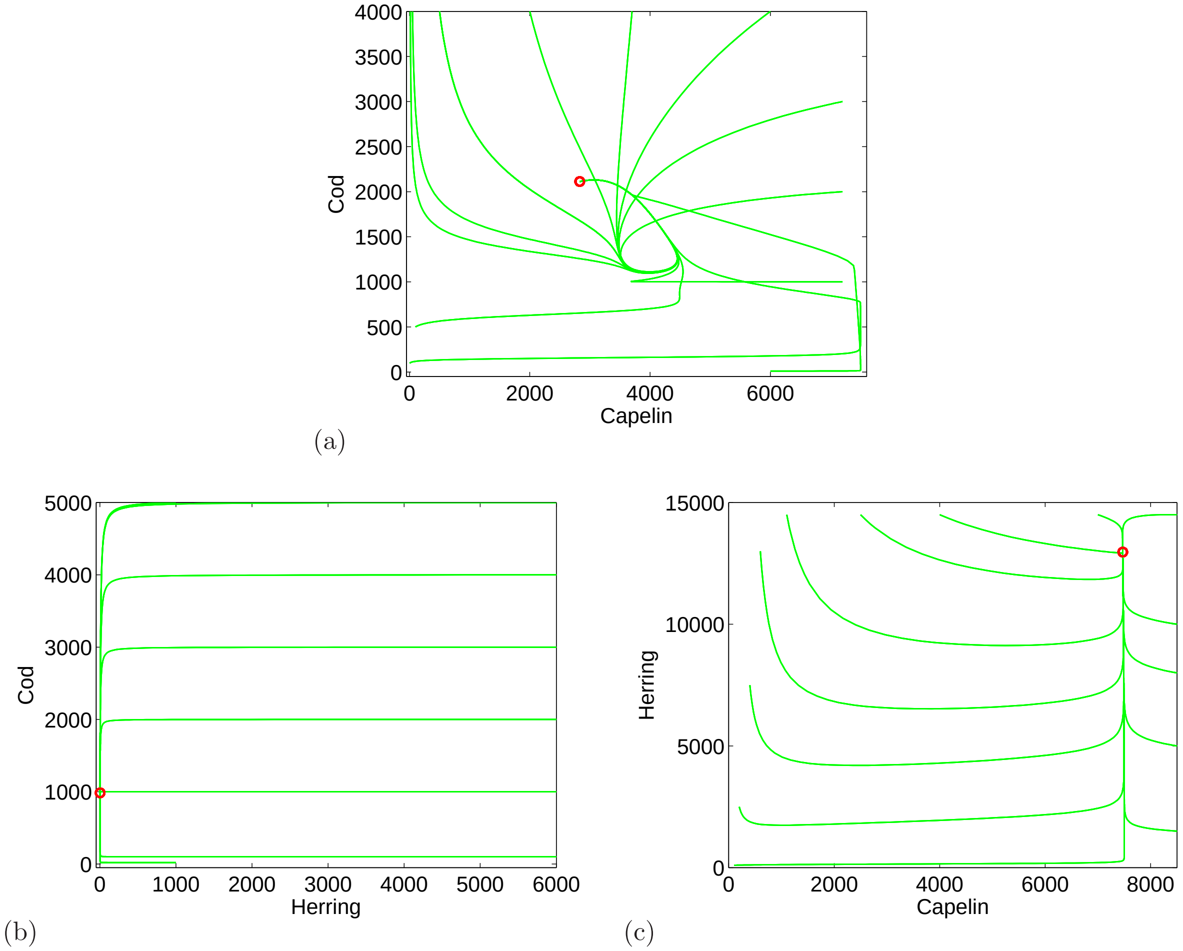


Figure 3: Optimal TACs (thousand tons) of cod as a function of capelin and cod under (a) absence of herring, with (b) 1.5 million tons of juvenile herring present, and optimal catch of cod as a function of cod and herring under (c) absence of capelin and with (d) 6 million tons of capelin present.



(a)

(b)

(c)

Figure 4: Paths towards the biologic equilibriums (red rings) resulting from a total closure of the capelin and cod fishery. (a) Initial absence of herring, (b) absence of capelin and (c) absence of cod.

real marine ecosystem, and there are several reasons for this discrepancy. A lot of varying factors and stochasticity cause huge fluctuations in the stocks. Moreover, the management system in place has not managed the stocks according to our model and hence it has created a different path. Yet, paths towards equilibrium may be interesting from a theoretical point of view, and it is possible that the real biomasses would have fluctuated around these paths with a management governed by our prescribed harvest policy.

The paths towards equilibrium resulting from a total closure of the capelin and cod fishery may tell us much about the biologic growth model used. In the 2D-curves of fig. 4(a) - 4(c), where in each figure one of the stocks is suppressed, we can see the three equilibriums of the biologic model. Notice that the external inflow of young herring from the Norwegian Sea means that there is no equilibrium with absence of herring. Therefore, with initial absence of herring as in fig. 4(a), the phase-plot shows evolution towards an interior equilibrium (all model stocks present) of $(2.832, 2.113, 2.040)$. With initial absence of capelin (fig. 4(b)) or cod (fig. 4(c)) these species will never return.

Capelin seems to be a key-species as its absence leads to a very poor equilibrium of $(0, 984.1, 1.7)$ hundred thousand tons. In absence of cod the biologic equilibrium is $(7.472, 0, 12.962)$ million tons.

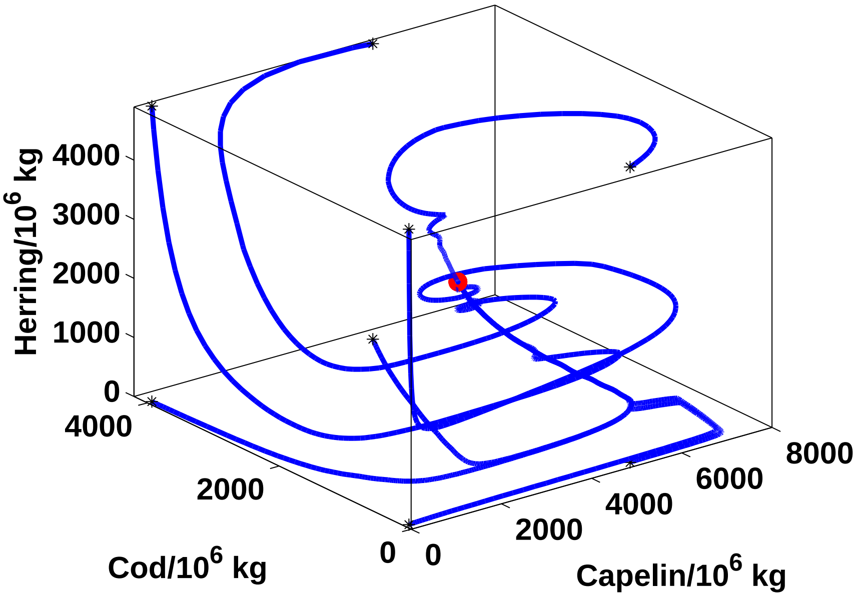
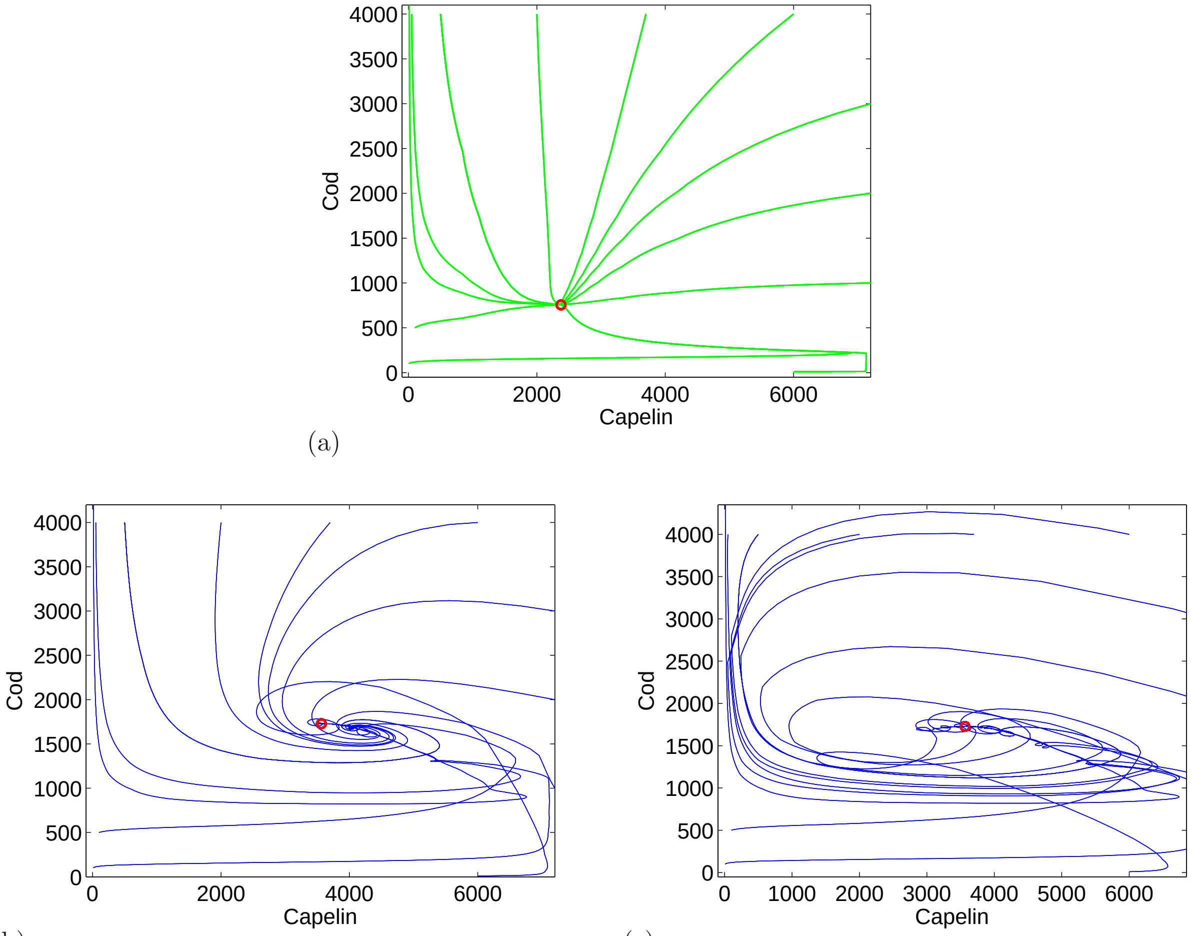


Figure 5: Paths towards equilibrium in the 3D capelin-cod-herring plane for different initial stock sizes of capelin, cod and herring. Equilibrium marked with a red ring, and initial positions marked with "$*$".

The management optimizing the model gives of course different equilibriums and different paths towards them. Fig. 5 is a 3D phase-plot showing paths towards equilibrium from different initial conditions when new TACs are set ones each year according to the optimal management strategy. Although all the initial stock sizes in fig. 5 are at the edge, and also outside the edge, of a stock space that is reasonable based on historical biomass estimates, they all end up in the same equilibrium. The single equilibrium marked with a red ring, $(3.564, 1.728, 2.492)$ million tons, appears to be the only interior equilibrium point resulting from optimal policy in the management model. In addition we know there are equilibriums when either capelin, cod or both of these stocks are depleted, but seemingly none of these equilibriums will ever be reached when optimal management is followed.



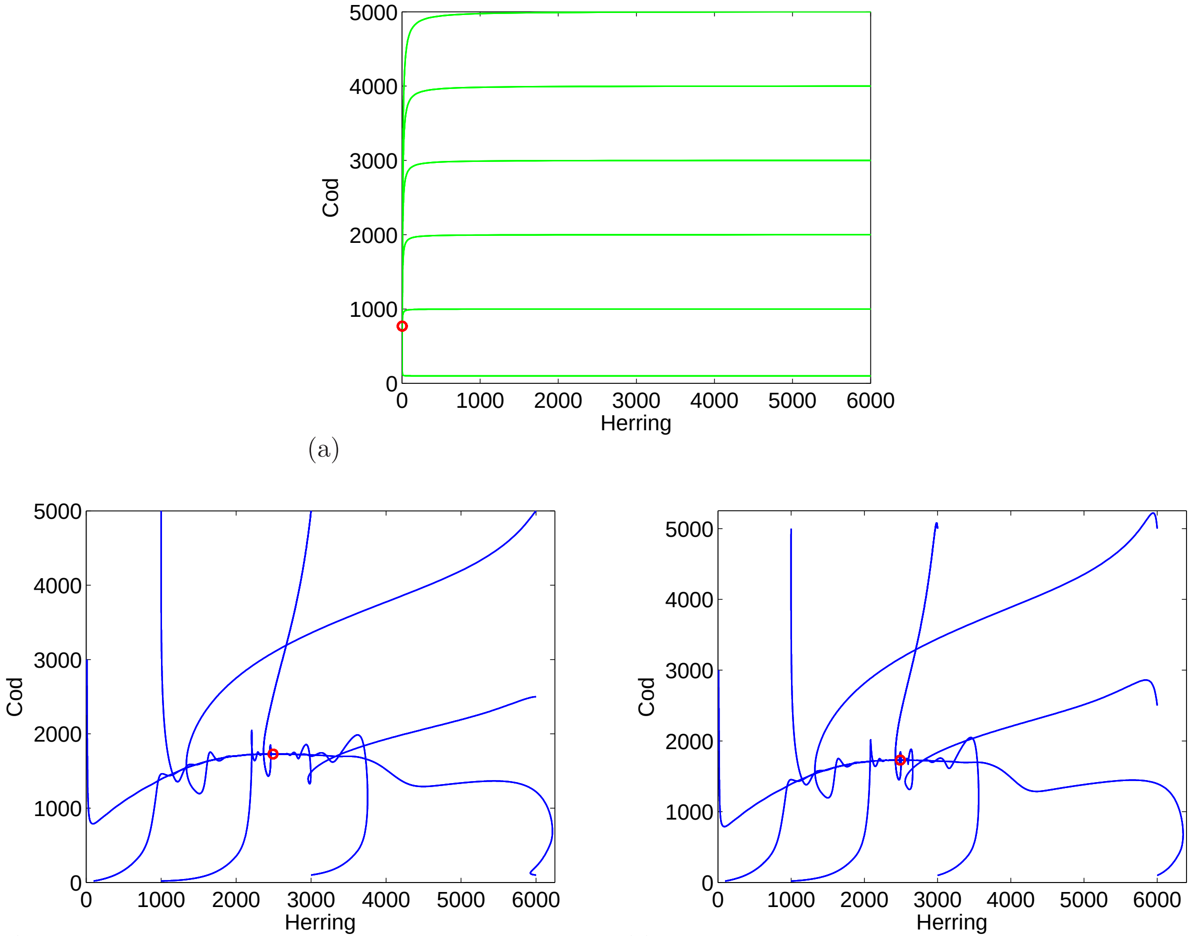
(a)

(b)

(c)

Figure 6: Paths towards equilibrium stock sizes on the capelin-cod plane for different initial conditions of capelin, cod and herring. (a) Absence of herring and initial herring stock of (b) 1.5 million tons and (c) 6 million tons.

In fig. 5 we also observe over- and under-shooting for both capelin and cod before settlement in equilibrium, whereas the herring stock is monotonic increasing or decreasing depending on the size of the initial stock. Fig. 6(a)-6(c), 7(a)-7(c), and 8(a)-8(d) illustrate more detailed phase plots in two dimensions. In each of these plots one of the stocks are

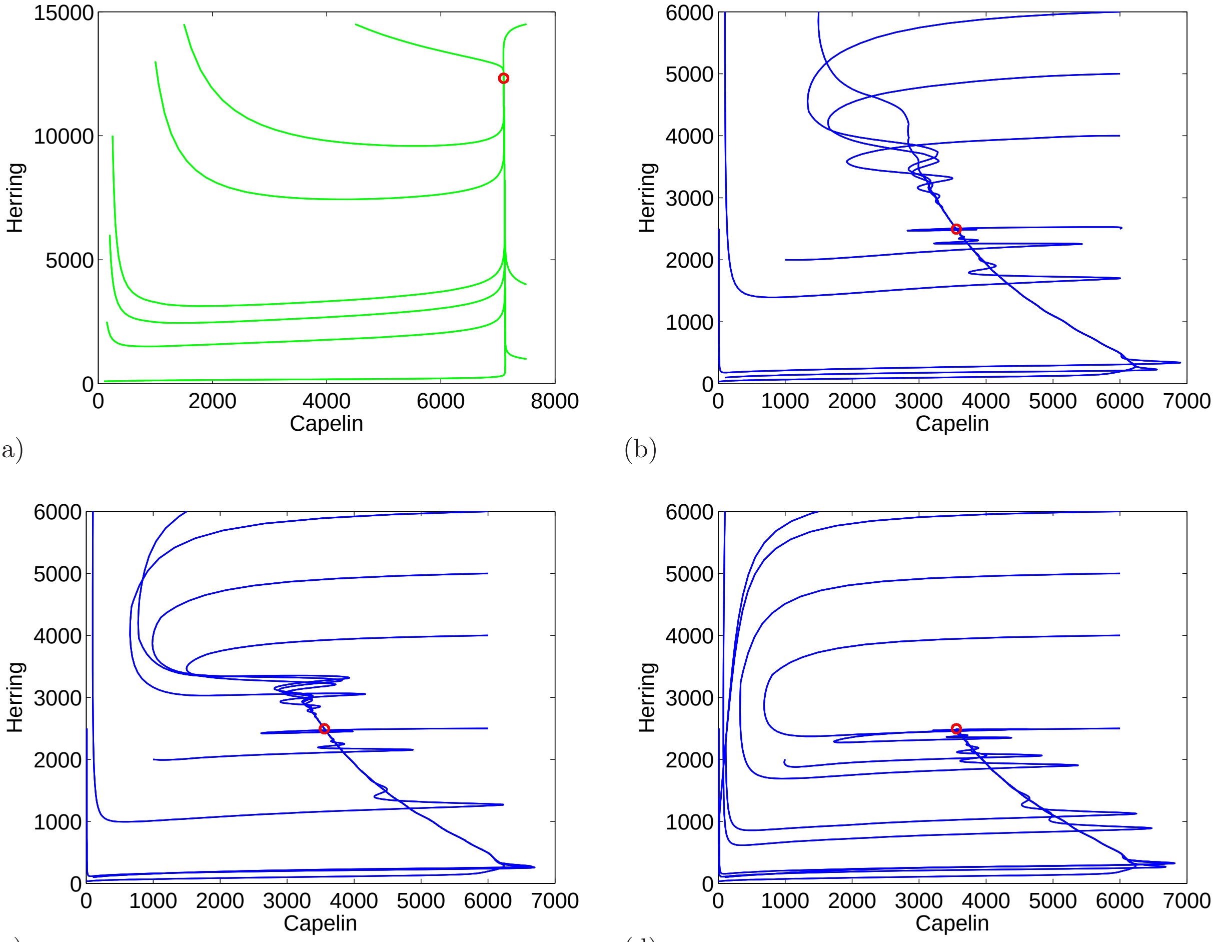


(a)

(b)

(c)

Figure 7: Paths towards equilibrium on the herring/cod plane for different initial stock sizes of capelin, cod and herring. (a) Absence of capelin, (b) initial capelin stock of 2.5 million tons and (c) initial capelin stock of 6 million tons.



(a) (b) (c) (d)

Figure 8: Paths towards equilibrium on the capelin/herring plane for different initial stock sizes of capelin, cod and herring. (a) Absence of cod, initial cod stock of (b) 1 million tons, (c) 2 million tons and (d) 4 million tons.

suppressed from the figure because of the dimension reduction. However, in each picture this stock works indirectly in the development of the other two stocks.

Other equilibriums than the interior one can be viewed in fig. 7(a) and 8(a). Here respectively capelin and cod are absent, which leads to totally different stock behavior as balance between the three stocks are destroyed. The equilibriums and the paths towards them are dramatically changed. With absence of capelin, both cod and herring suffer from poor growth conditions leading to an equilibrium of $(0, 769.2, 1.7)$ thousand tons. In absence of cod, on the other hand, the growth is really good for both capelin and herring, and the equilibrium is $(7.100, 0, 12.318)$ million tons.

Fig. 6(b)-6(c) confirm the over- and under-shooting of both capelin and cod observed in fig. 5, and for capelin the under-shooting rises with high initial levels of herring as do also the over-shooting of cod. This is also confirmed for cod in fig. 7(b) and 7(c) and for capelin in fig. 8(b)-8(d).

The spiraling behavior of the paths towards the interior equilibrium (see e.g. fig. 5), means that the stocks are fluctuating before settlement in equilibrium. This fluctuation is confirmed in fig. 9(a)-9(f). These figures show evolution towards equilibrium on the time-stock plane. The figures to the left, (a) (c) and (e), show the first ten years before settlement close to the equilibrium, and the figures to the right, (b), (d) and (f), show the evolution towards equilibrium over a 110 years period.

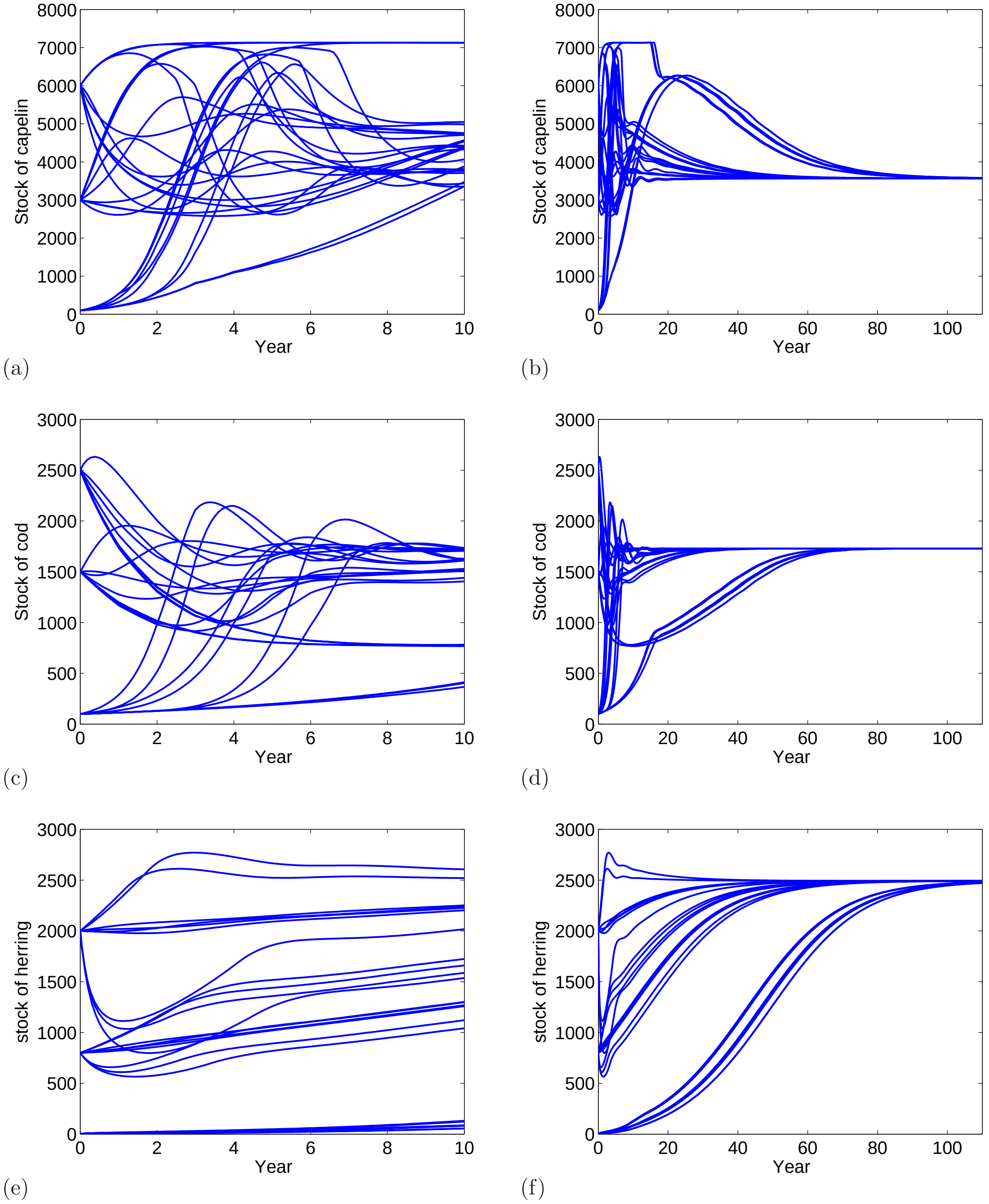
Fig. 10(a)-10(d) show optimal catch scenarios for four different initial stock sizes. These initial sizes are (a) $(500, 1000, 200)$, (b) $(500, 1000, 2000)$, (c) $(6000, 1000, 200)$ and (d) $(6000, 1000, 2000)$ thousand tons of capelin, cod and herring, respectively. Notice that TACs according to all these figs. leads to an equilibrium stock size of $(3.564, 1.729, 2.492)$ million tons, and the entire catch curves yield catches substantially lower for both capelin and cod than official average yearly catches the last 30 years (906 thousand tons for capelin and 549 thousand tons for cod). In addition the interior equilibrium of all the stocks is substantially higher than average stocks the last 30 years; especially for capelin and herring they are substantially bigger.

Finally fig. 11(a) and 11(b) shows optimal TACs when respectively the capelin and cod stock are fully depleted. Notice that the optimal catch of cod is very low and that the constant capelin TAC coincides with the ”bliss-level” in accordance with fig. 2(d).

4 Discussions

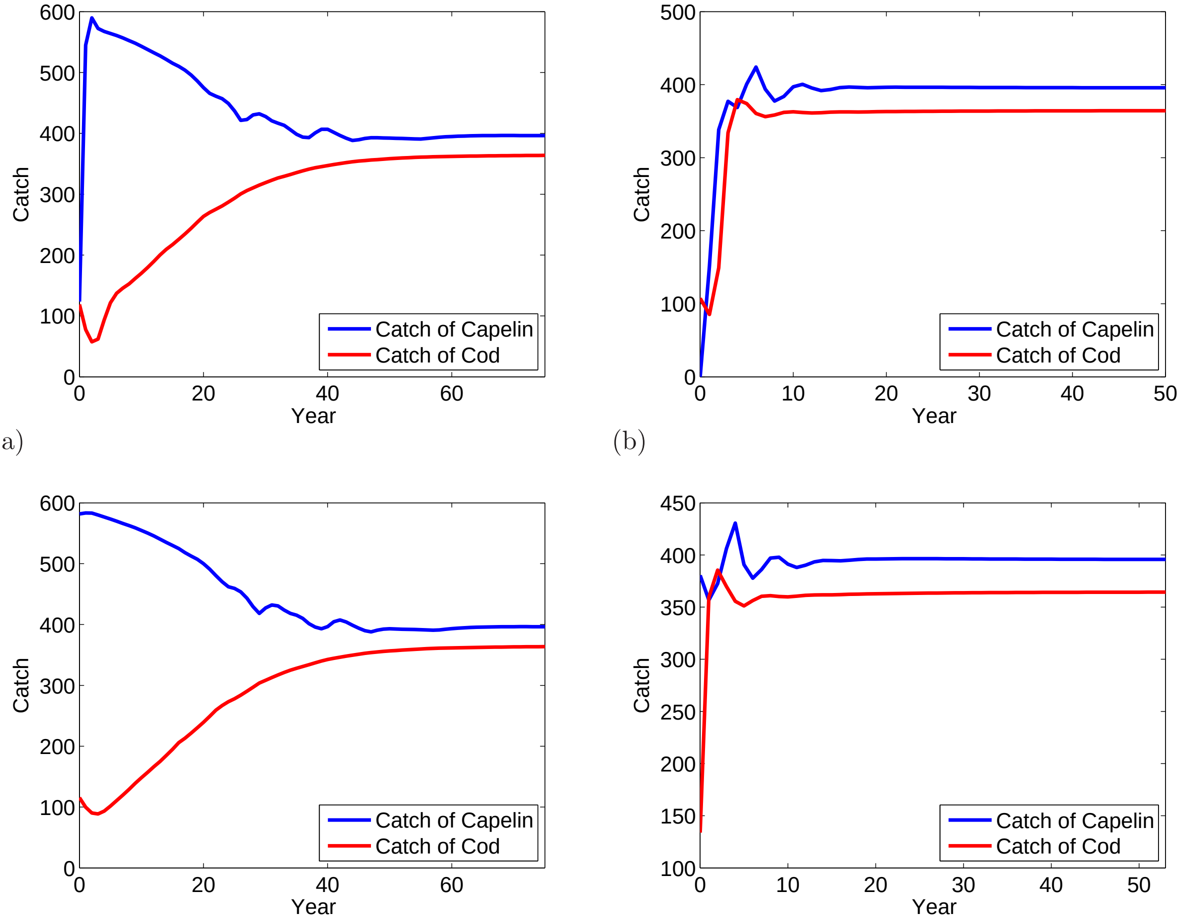
4.1 Biologic model

A limitation in our model is the fact that we have to choose between an autonomous (time independent) growth model with three species (capelin, cod and herring) or a time



(a) (b) (c) (d) (e) (f)

Figure 9: Stocks of capelin, herring and cod approaching equilibrium from several different initial conditions. Fig. (a), (c) and (d) show the first 10 years with fluctuating behavior, whereas (b), (d) and (f) have a time span of 85 years and illustrates the evolution towards equilibrium.



(a) (b) (c) (d)

Figure 10: TACs (thousand tons) approaching equilibrium. Initial capelin stock: 500 thousand tons in (a) and (b), and 6 million tons in (c) and (d). Initial cod stock is 1 million tons in all the figures, and initial levels of herring is 200 thousand tons in (a) and (c), and 2 million tons in (b) and (d).

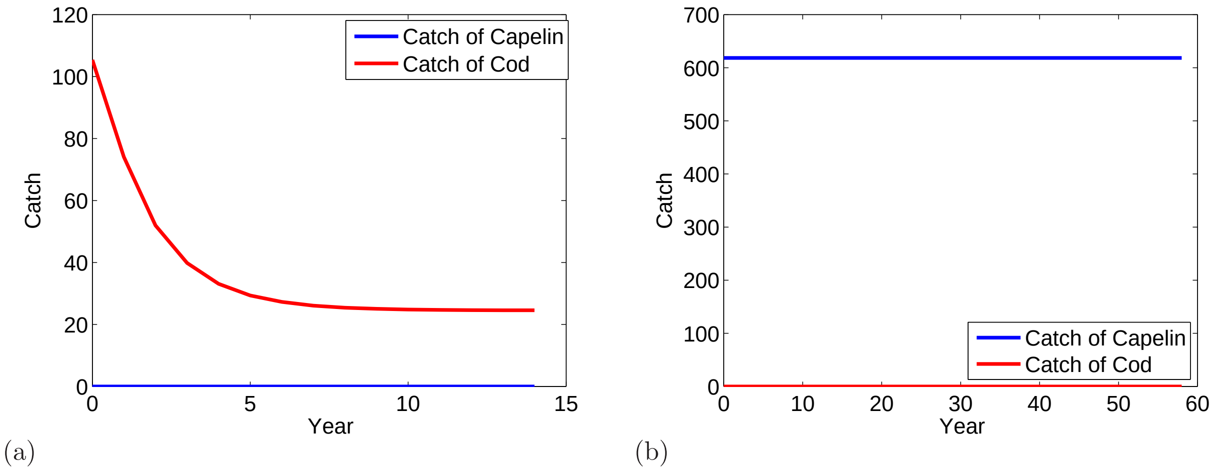


Figure 11: TACs (thousand tons) in (a) absence of capelin and (b) absence of cod. In both figures initial herring stock is 2 million tons.

dependent growth model with only two species (capelin and cod). A series of testing with regression analysis for a number of different growth functions, indicated that the data were best represented when herring was included instead of cyclic time dependent fluctuations. Still, it will be interesting to further investigate time dependent fluctuations in future works, but then it ought to be done in a more general stochastic setting.

If we have a look at the predation terms in the capelin growth function, we see that there is both a term with positive coefficient (b_1) and a term with negative coefficient (b_2). This does not contradict the fact that capelin serves as prey for cod as well as for herring. The positive term is just a correction of the strong influence from the negative term. All in all the influence on capelin from both cod and herring is mainly negative, and this is also obvious from the data. Nevertheless, we shall have in mind that the parameter estimation and the functional forms chosen are based on real data. The growth functions may not give a good match for extrapolation of growth for stock sizes very different from historical ones, especially not for cod stocks close to zero. With this in mind we modified the growth functions in equation (2) slightly for such stock levels before doing the economic optimizing. (See eq. (2) and (3)). Also, the upper limit on the intrinsic growth rate for herring in eq. (4) prevents extreme changes in the herring biomass when there is little of both capelin and cod. Such a biomass situation is extremely rare in historical data, and we choose to damp down extrapolated growth effects we do not trust. However, we should not totally reject the idea that growth of capelin and herring, which are both plankton feeders, could be positively correlated in some periods due to effects beyond the borders of our model, e.g. in periods with very high primary production.

The choice of the specific growth has some model-technical basis as a closed system is essential for optimizing when sizes of external factors beyond control are unknown. (Catch quotas of mature capelin, ocean currents determining inflow of herring fry to the Barents area etc...). Such factors are very difficult to quantify and recognize, and are for the time being not ready for use in this model. However, the growth used in this work is consistent with expected predation relationships and gives a better representation than a pure stochastic inflow, which cannot explain the forces of importance. At least our model aims to explain some of the variability in the herring stock, and independently of the relevance of the predation terms, the growth functions should cover for some of the stock dynamics. Specifically, for herring growth, the regression analysis performed indicates that the stock of capelin and cod to some extent is a proxy for the stock size of juvenile herring. As far as the growth of capelin and cod are concerned, herring could be interpreted to play the role of a time-dependency in the capelin-cod system representing effects of both herring and other non-autonomities.

One might argue that the growth function for herring is not satisfactory from a biological point of view, as the growth is only passed for and based on the immature part of the stock in addition to predation relationships. It may be a little too much to claim that these factors are the most important for growth of juvenile herring in the marine ecosystem, but it is very difficult to quantify others.

Although we include inflow of juvenile herring in the herring growth, we disregard dependency between the size of this inflow and the size of the mature fraction of herring in the North Sea and the Norwegian Sea. Such dependency must be expected to be important to herring recruitment [14] in spite the fact that biomass of spawners has turned out to be only a weak indicator of egg production in some populations. ([24] and [27]). However, we use a growth model, not a recruitment model. That is, the aim is not to describe "bottom up" factors in recruitment and mortality, but rather to explain some of the "top down" stock dynamics which is relevant for optimizing. Moreover, the statistical results are quite good although herring growth is fitted with only three parameters. The Durbin-Watson statistics of just about 1.86 shows little signs of auto-correlation, the significance is convincing for all the parameters with an acceptable level of the standard error. Moreover, the R^2 of 0.91 is good. The residues are a little high, but we should remember that herring has shown very large fluctuation and faced a collapse in the beginning of the period our data are built upon. Therefore we cannot expect them to be lower.

As far as the biologic equilibriums are concerned, the interior equilibrium of $(2.832, 2.113, 2.040)$ million tons is the most interesting since this is the only equilibrium resulting from initial stock sizes above zero for all the stocks. One might argue that the cod levels in this equilibrium is too low compared to historical gigantically cod levels reported, but extreme cod levels do not agree very well with the data this analysis is built upon. Also, since the cod stock has a fluctuating nature, the highest sizes reported should probably not be regarded to be close to a natural no-fishing equilibrium level.

4.2 The optimal value function

If we keep in mind that our value function represents the total future discounted profit from harvest of capelin and cod, it is clear that the surfaces must go through the origin between the capelin and cod axes. Absence of both capelin and cod means zero profit. Further on, absence of capelin or cod in one moment of time implies that the stock is exterminated for all future, whereas small positive initial level leads to higher future levels under optimal policy. The difference in economic rent between these two situations explains the discontinuity between zero-levels of capelin and cod and the first non-zero levels.

Generally a rise in one or both stocks means a rise in future profit and therefore also rises in the value function. However, extreme values are not sustainable, and if the stocks get very high, they will fall soon independently of the fishing policy and leave little extra profit. Remember also that the profit function (see eq. (9)) is concave related to harvest. Consequently, extreme harvest in a limited moment of time will not result in extreme profit.

It is interesting to see that presence of some herring has a positive influence on the economics of the model (see fig. 1(c) and 1(d)). This might be a little surprising since the economic model does not include profit from herring catch, but in the biologic model herring is an important growth factor for cod, and this seems to be more crucial for the economics than the negative effect from herring predation on capelin. A moderate presence of herring (800 thousand tons) gives a much higher value function than absence of herring. This does not mean that herring is more important for the cod growth than capelin. Neither does it mean that herring is some kind of catalyzator in the cod predation on capelin, as it does not make sense to say that capelin-herring-cod is more effective for cod growth than the direct linkage capelin-cod. However, there are other species of importance. Herring does not only feed on capelin fry, and for periods of time capelin and herring might be positively correlated. Specifically, in the growth model for low biomasses of both capelin and cod and high biomasses of herring, capelin growth is positively correlated with herring although the impact from the correctional term with coefficient b_1 mentioned earlier in this section is pulled down.

Helstad [15] also concludes that the rebuilding of the herring stock after its collapse in the late 1960s has been optimal from an economic point of view, when considering the three species of relevance in our work.

4.3 Optimal catch policy

In absence of cod and herring there is no pressure on the capelin stock. This explains the discontinuities visible in fig. 2(a)-2(d). Actually, absence of cod alone results in capelin TACs on ”bliss-level” for all levels of herring when the capelin stock is above a certain lower limit(fig. 2(d)). If there is some capelin and no herring in the Barents Sea, the capelin stock will soon grow, but it takes some time before the herring grows. (See e.g. fig. 9(a)-9(b) and 9(e)-9(f)). This results in rather high optimal catch of capelin in absence of herring. TACs on "bliss-level" is only reached when cod is absent as well, but the TACs in presence of cod is not very much lower. (see fig. 2(a)).

The main trend in fig. 2(a)-2(c), that optimal capelin-TACs falls with the number of cod is very intuitive since capelin is more valuable as food for cod than as harvest for humans. The bumps in the optimal catch of capelin for high levels of herring is more difficult to explain (Se fig. (2(e)) and (2(f))). Higher levels of herring puts higher pressure on the capelin stock, so why should it be optimal with higher capelin-TACs when the levels of herring shifts from high to very high? The explanation for this must be that it is better to fish the capelin stock and earn profit on it than letting the herring stock have it for dinner. Without capelin harvest, according to the growth model, the herring stock would have stabilized on a very high level eating greedily from the dinner-table of the cod (capelin) and "stealing" our profit from both capelin and cod. With larger cod stocks, as in fig. 2(f)), cod predation is more important and the profit-damage from high herring stocks is reduced. Therefore the bumps are smoother. For extreme values of herring, however, capelin harvest should be reduced to prevent a collapse. The little bump for low levels of capelin and very high levels of herring tells another story. That is the story about a capelin stock getting totally diminished, regardless of the harvest pressure. Since there is no density dependent costs for the capelin harvest, the stock is not protected from a purely economic point of view. Moreover, the cod stock is held up by the herring abundance. In such circumstances fishing of some capelin is worthwhile. According to the biologic model extinction of capelin is impossible when optimal management is followed. Also, the time of a capelin-rebuilding after a collapse is probably little dependent on the stock size when there are very high levels of herring. The rebuilding will take some time in any cases.

A surprising behavior with the TACs of capelin is that they grow with the stock of herring and fall with the stock of capelin for low levels of capelin ($1.5-1.9$ million tons) and herring ($0-0.5$ million tons). (See fig. 2(e)). This counterintuitive behavior could probably be explained by unexpected characteristics of the growth function. For combinations of low biomasses of both capelin and cod, the intrinsic herring growth rate, $\tilde{r}(x, y)$, grows rapidly with lower biomasses of capelin and cod until it reaches its maximum level of 0.23. (See eq. (2)). When such capelin and cod levels coincide with low herring levels, the herring growth is large, and a growth of herring increases the value of the optimal value function in that case, and this explains the counterintuitive optimal TACs. (See fig. 1(c)). Whether this is credible or not depends on the credibility of the growth functions for capelin, cod and herring for very low biomasses. Since such a combination of biomasses is not found in the data the biologic regression analysis is built upon, one should be careful putting to much trust into this counterintuitive result.

Cod is the economically most important species and the top-predator of this model. Moreover, profit from cod is concave as a function of harvest. One should thus expect stable high, but sustainable, optimal TACs of cod. Furthermore, a rather weak dependence of the other two species should be expected. All of these expectations are confirmed in fig. 3(a)-3(d). The only disturbance to these main findings is that TACs of cod falls clearly in absence of capelin. This can be observed along the cod-axis in fig. 3(b). Harvest of cod should be considerably lower than in presence of capelin. The reason is that growth conditions for cod are poorer in absence of capelin and the situation demands a lower cod harvest.

The sizes of the optimal equilibrium catch curves for capelin and cod is respectively about 40% and 65% of actual average catches the last thirty years. At the same time equilibrium level yields stock sizes considerably larger than average stock sizes, especially for capelin and herring. Average stock size of capelin, cod and juvenile herring year one-three in the period from 1973-2005 is respectively 3.017, 1.522 and 1.084 (million tons), which are considerably lower than the equilibrium of 3.564, 1.729 and 2.492 million tons. This suggests that catches of both capelin and cod should be considerably reduced. However catches of mature capelin after spawning may not harm the capelin stock too much since a high fraction of the capelin population spawns only ones [11]. Nevertheless, the gap between optimal catch of capelin in equilibrium an average actual catch is very big.

Optimal TACs of cod seems to be in agreement with Kugarajh et. al. [17], who claim that cod fishing curves historically are too high, and fits very well with the infinite discounting tracks, i.e., the static optimal solution. Optimal TACs of cod is considerably lower than this solution on most of the possible state-space. The high stocks of capelin and herring in equilibrium have a positive influence on cod growth, but does not affect optimal TAC of cod very much. (See fig. 3(a)-3(d).

Although the equilibrium solution is interesting and may be a good indicator for a sustainable potential of the fishery, we must remember that real growth is by nature fluctuating. In reality, therefore, optimal catch will vary each year and an equilibrium level will never be reached. This calls out for an introduction of stochastic in the growth model. Such an extension is absolutely attainable.

5 Conclusions

This work is a demonstration of optimal dynamic programming as a useful technique in multi-species management modeling. So far multi-species management has been mostly concerned with stock goals, and less with economics and other important objectives utilizing information outside the biological sphere. We have tried to balance model-complexity of biology and economics in the finding of optimal feedback catch-curves for capelin and cod.

Our biologic model is three dimensional in state space (three species), and in spite of rather high through-put, it should be possible to solve similar models in higher dimensions with the techniques we have used. This means that dynamic programming could be used to a greater extent in multi species management modeling. Involving stochastic in the growth model and/or the economic model is both a natural and an attainable extension of this work.

As far as the concrete results are concerned, the principals of the findings are rather intuitive and easy to rely upon. They show that optimal catch of capelin is very dependent on the size of both the cod and herring stock as well as of the capelin stock itself. Furthermore, optimal catch of cod is less dependent on capelin and herring, but a fully depleted capelin stock will result in less cod growth and should therefore also lead to lower cod landings.

When we studied equilibrium points resulting from optimal policy, we found that there was strong attraction to an equilibrium point with larger stocks of all the three species covered by the model than average levels the last 30 years. Moreover, looking at catches, we found that this equilibrium level yields both capelin and cod catches substantially lower than average catches for the same period. This suggests that lower TACs would give higher discounted profit. Another important finding is that presence of some herring in the Barent Sea system apperently gives higher economical yield from capelin and cod than absence of herring would have done.

Our work should not be considered to be the correct answer to what is the best management of the Norwegian and Russian resource of capelin and cod. Rather, it is a first trial of combining complicated biology with economics and giving a clear cut answer to what is the optimal response to the biologic model and the economic model relied upon.

Nomenclature

Variables

$\underline{x}$	Stock levels
$\underline{u}$	Catch levels
δ	Discount rate
$[\alpha_1, \alpha_2]$	Norwegian share of TAC for capelin and cod

Functions

$V(\underline{x})$	The optimal value function
$\Pi(\underline{x}, \underline{u})$	Current profit
$F_i(\underline{x})$	Biological growth function for species i
$CaP(u_1)$	Capelin profit
$CoP(x_2, u_2)$	Cod profit
$P_2(u_2)$	Price for a unit of cod
$C_1(u_1)$	Cost function for catch of capelin
$C_2(x_2, u_2)$	Cost function for catch of cod

Parameters

p_1, p_2 and p_3	Income parameters
c_1, c_2, c_3 and c_4	Cost parameters

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