Finding Core Allocations for Fixed Cost Games in Electricity Networks

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Abstract

We discuss the cost allocation problem faced by a network operator, where the fixed (residual) cost of the network has to be allocated among its users. Usage-based methods, such as the postage stamp rate method and the MW-mile method, are easy to understand and compute, but may yield cost allocations for which some transactions are subsidizing others. Formally, this is equivalent to allocations outside of the core of the corresponding cooperative cost game. Our main contribution is to present a method, similar to a well-known method for computing the nucleolus, by which several usage-based methods may be combined in order to produce allocations that are in, or as close as possible to, the core. The method is illustrated using a model of an AC power network.

Keywords: Power System Fixed Cost Allocation, Cooperative Game Theory, Core

1. Introduction

Since the mid 1980s, the electric power systems of many countries have been subject to deregulation. Prior to the deregulation, the systems were characterized by vertical integration and little competition.

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Typically, a single company controlled the entire production, transmission, and distribution system for a fixed geographic area. After the deregulation competition has been introduced with respect to production, while transmission and distribution are still performed by regional monopolies.

Competition with respect to production means that more than one producer must be allowed to use transmission/distribution networks in order to sell its power to consumers, hence allocation of network costs has become an important topic. A good cost allocation method has to satisfy a number of requirements, such as:

- transparency
- ease of computation
- recovery of total costs
- fairness (e.g., non-discrimination of different users)
- no cross-subsidies (e.g., between short- and long-distance transactions)
- sending correct economic signals to market participants (e.g., with respect to the location of new generator capacity)

Most of the cost allocation methods that have been proposed base their allocations on some measure of network usage. Of course, the choice of usage measure is restricted by the availability of objective data, such as production/consumption quantities, or line flows (these data are available to the network operator). We describe some of these methods, such as the *postage-stamp rate* method and the *MW-mile* method, in Section 3. While these methods are relatively intuitive and does not involve heavy computations, a disadvantage is that they are often inconsistent with economic efficiency. For example, the postage-stamp rate method, which allocates the total network cost in proportion to the production/consumption quantities, does not take into account the distance that the power travels in the network. It therefore leads to cross-subsidization of long-distance transactions by short-distance transactions, which again results in an efficiency loss because some profitable short-distance transactions are not performed. Efficiency losses can also result if an allocation method sends incorrect signals to potential investors with respect to the installation of new production or network capacity. Economists generally advocate the use of *marginal/incremental* cost allocation methods, arguing that a new transaction should pay the extra cost that it causes. The problem with such methods is that they in most cases do not recover the entire network cost, most of which is fixed. Another problem with them is that the cost allocation depends on the ordering of the transactions, and it is not obvious how transactions should be ordered.

Another approach to the cost allocation problems has come from *cooperative game theory*, the subject of Section 2. The literature on application of cooperative game theory to fixed cost allocation in electricity networks has mainly focused on applying solution concepts such as the *Shapley value* and the *nucleolus* in order to produce good cost allocations. This literature includes Tsukamoto and Iyoda (1996), Contreras and Wu (1999), and Zolessi and Rudnick (2002)). The attractiveness of allocations based on methods from cooperative game theory mainly stems from the fact that they in many cases (this is always the case for the nucleolus) belong to the *core*, and hence avoids the problem of cross-subsidization. However, the fact that such methods are relatively complicated and can be computationally demanding, makes their implementation more difficult.

In this paper we analyse the relationship between usage-based methods and the core of a cooperative game. The main idea is that while usage-based methods may yield allocations that are not in the core, it may be possible to obtain core allocations by combining allocations from several methods. The different allocations are combined using a set of nonnegative weights, and are obtained from an LP-based procedure that is similar to the most common procedure for computing the nucleolus of a cooperative game. The idea is, as in the case of the nucleolus, to find a central point in the core, with the additional restriction that the allocation should be a convex combination of a set of given allocations, and we will refer to the resulting allocation as the *restricted nucleolus*. We believe that our procedure could be a valuable tool in the process of designing a cost allocation system, where one of the tasks could be to decide the weighting of different usage-based allocation schemes.

The organization of the paper is as follows: Section 2 describes the fixed cost allocation problem and the related cost game. Section 3 describes some usage-based allocation methods that we will analyse, and relates them, via an example, to the core of the cost game. In Section 4 we explain the procedure for computing the restricted nucleolus, and in Section 5 we investigate the properties of the weighting scheme using numerical examples.

2. The cost game

We consider an AC power network consisting of a set of lines, and where the network users are identified as transactions. The total amount of active power of transaction i is given by P_i, and the active power flow caused by transaction i over line l is denoted by $P_{i.l}$. The power flow caused by a transaction is computed under the assumption that no other transactions are using the network. This is the most common way to define the stand-alone usage of the network. Similarly, let $P_{S,l}$ denote the flow over line l if a group S (coalition) of transactions uses the network. Although the definition given above is consistent with the economic definition of stand-alone usage, it may result in significant problems by the power flow calculations in large networks. First of all, if an AC program is used, regardless of whether it is OPF or just a simple load flow program, it may be impossible to find a solution in large networks. This can happen in real interconnected networks where the AC will not converge if the player's load is just a small part of the total network capacity**.** In the case of DC load flow this convergence problem will never occur, but, unfortunately, this algorithm provides only an approximation of the real network situation. Future work should focus on a definition of stand-alone usage which will be more compatible with the reality of electrical networks. We refer to (Bergen and Vittal, 1986) for a more detailed description of the network, as well as the computation of the line flows.

The cost game is given by the pair *(N,c)*, where $N = \{1,...,n\}$ is the set of players (transactions), and c is a function that assigns a real number to each subgroup (coalition) of N. For some nonempty coalition S Í N, the stand-alone cost is

$$c(S) = \sum_{\ell} \left| P_{S,\ell} \right| \cdot C_{\ell} \,, \tag{1}$$

where C_l is the cost to transfer 1 MW over line ℓ. Hence $P_{S,l} \cdot C_l$ is the cost that is caused by coalition S with respect to line l, and the total stand-alone cost is then obtained by summing over all the lines in the network.

A solution to a cost game is a cost allocation, i.e., a vector $\mathbf{x} \in \mathbf{R}^n$. The most prominent solution concept is the *core*, which consists of all the cost allocations that satisfies the following rationality requirements:

Global rationality: $\sum_{i \in N} x_i = c(N)$ (2)

Group/individual rationality: $\sum_{i \in S} x_i \leq c(S)$ for all $S \subseteq N$ (3)

The core may consist of more than one point, but it may also be empty. According to the *Shapley value*, player *i* is allocated the following cost:

$$\mathrm{SV_i} = \sum_{\mathrm{S} \subset \mathrm{N}} \frac{(|\mathrm{S}|-1)!(|\mathrm{N}|-|\mathrm{S}|)!}{|\mathrm{N}|!}[\mathrm{c(S)} - \mathrm{c}(\mathrm{S} \setminus \{\mathrm{i}\})] \quad (4)$$

The Shapley value does not, in general, belong to the core, except for the class of concave (cost) games. The *nucleolus* is defined using the coalition excesses, given by $e(S,\mathbf{x}) = c(S) - x(S)$. Let the vector

$$?(\mathbf{x}) = \left(e(S_1, \mathbf{x}), e(S_2, \mathbf{x}), \ldots, e(S_{2^n-2}, \mathbf{x})\right) \quad (5)$$

consist of the excess values for all coalitions arranged in non-decreasing order, i.e., such that $i < j \Rightarrow e(S_i, \mathbf{x}) \leq e(S_j, \mathbf{x})$. Then the (pre)nucleolus (**NU**) is defined as the unique allocation vector such that $\mathbf{x} = \mathbf{NU}$ implies that $?(\mathbf{x})$ is lexicographically maximal. An appealing property of the nucleolus is that if the core is nonempty, then we must have $?(\mathbf{NU}) \geq \mathbf{0}$, i.e., the nucleolus always belongs to the core. An introduction to cost games, including the three solution concepts described above, can be found in (Young, 1985).

Game-theoretic solution concepts such as the Shapely value and the nucleolus possess some appealing properties, one of which is their relationship to the core, but their rather complicated structure makes it difficult to motivate market participants to accept the resulting allocations. It should also be noted that the implementation of these solution concepts requires that the value of *c* be computed for each of the 2^n-1 nonempty coalitions, and the computational costs may therefore become excessive. We will in the next section present some of the most common (usage-based) cost allocation methods. In contrast to the game-theoretic solution concepts, the usage-based methods have relatively simple and intuitive structures. Via an example we will demonstrate that these methods yield allocations that are not in the core. Then, in Section 4, we will show how core allocations may be obtained by combining several of the usage-based allocation methods.

3. Usage-based cost allocation methods and the core

There are several usage-based methods that have been developed in order to deal with the task of allocating the fixed cost of a power system among the market participants (Pan et all, 2000; Marangon Lima, 1996). One of the traditional methods is the *postage stamp rate* method, also known as the rolled-in method. According to this method the network usage of a transaction is measured by the magnitude of the transaction P_i, not taking into account how the transaction affects the power flows over various lines in the network, and the amount to be paid by transaction *i* is

$$PS_i = K \frac{P_i}{\sum_{j \in N} P_j} \,. \tag{6}$$

Here, *K* is the total cost to be covered by the market participants. In our examples we will assume that $K = c(N)$. Obviously, since the postage stamp rate method does not take distances into account, it may lead to cross-subsidization of long-distance transactions by short-distance transactions. In order to cope with this disadvantage, a category of methods based on power flow data has emerged. The MW-mile method (Shirmohammadi, 1989) was the first such method to be introduced. In order to compute the allocation, the network operator runs a power flow program for each single transaction and calculates the power flow due to this transaction over each system line. The line flows are then weighted with a factor that reflects the cost characteristics of each line, e.g. line lengths and construction costs per unit length. The weight of line ℓ is denoted by C_ℓ. The usage of line ℓ by transaction *i* is

$$f_{i,\ell} = C_\ell \left| P_{i,\ell} \right| \tag{7}$$

The absolute value in (7) means that the power flow direction is disregarded. The total system usage by transaction *i* is given by summing over all lines:

$$f_i = \sum_{\ell} f_{i,\ell} \tag{8}$$

By allocating the total system cost in proportion to usage, the contribution of transaction *i* will be

$$MWM_i = K \frac{f_i}{\sum_{j \in N} f_j} . \tag{9}$$

As has already been mentioned the MW-mile method does not consider the direction of power flow which each transaction causes. However, it is often argued that power flows having opposite direction from the net flow, which is the power flow due to all transactions, contribute positively in the system situation by relieving congestion and increasing the available transfer capacity. In order to take this into account, we replace (7) by

$$f_{i,\ell} = C_{\ell} P_{i,\ell} . \tag{10}$$

We then use (10) instead of (7) as a basis for (8) and (9). This allocation procedure is called the *counter flow* (*CF*) method. Since (10) may be positive as well as negative the denominator in (9) could be zero, although in a realistic network the probability that this would occur is very small.

According to the counter flow method, the contribution of a transaction may be negative, i.e., the network operator has to pay agent *i* for carrying out his transaction. For various reasons this may not be acceptable to the network owner and/or the market participants. A compromise that avoids negative contributions is the *zero counter flow* (*ZCF*) method. According to this method, the usage of a line by a particular transaction is set to zero if the power flow due to the transaction goes in the opposite direction of the net flow for the line. Thus, instead of (7) we use

$$f_{i,\ell} = \begin{cases} C_{\ell} P_{i,\ell} & P_{i,\ell} \geq 0 \\ 0 & P_{i,\ell} < 0 \end{cases} \tag{11}$$

The amount to be paid by transaction *i* is then found by using (11) as a basis for (8) and (9).

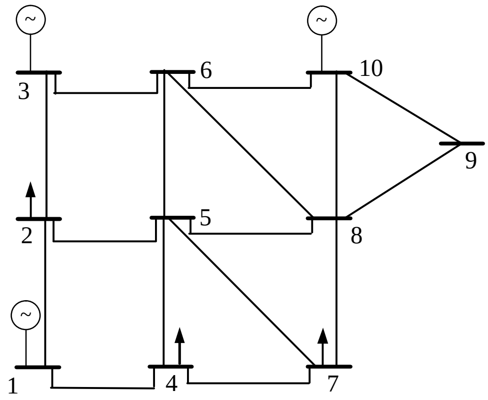

Figure 1: 10-bus network

Line	r(p.u.)	x(p.u.)	b(p.u.)
1-2, 1-4, 2-3, 2-5, 3-6, 4-5, 5-7, 6-8	0.0017	0.0180	0.6348
4-7, 5-6, 8-10	0.0014	0.0144	0.5078
5-8, 7-8, 8-9	0.00085	0.0090	0.3174
6-10, 9-10	0.0012	0.0126	0.4444

Table 1: 10-bus network data (S_b = 100MVA, V_b = 380kV).

In order to illustrate how the usage-based methods may be related to the core, we use the 10-bus power system illustrated in Figure 1. The data of the system lines are given in Table 1. There are three power transactions in the system. The first one injects 15 MW at node 1 and withdraws them at node 2. The second transaction is a transfer of 275 MW from node 3 to node 7 while the third one injects 15 MW at node 10 and takes them out at node 4. The area enclosed by the triangle in Figure 2 represents the set of allocations for which all three players are allocated nonnegative amounts, and the respective vertices of the triangle represents the allocations where one player pays the entire cost. The characteristic function values of all the one or two player coalitions are indicated by lines, and the core corresponds to the area confined by all these lines. As it can be seen only the allocation corresponding to the zero counter flow method is located inside the core. The allocation corresponding to the counter flow method is located outside of the triangle, since player 1 here is allocated a negative amount.

In many cases, even though simple rules like the usage-based methods in this section do not yield core allocations, such allocations may be obtained by combining several allocation methods. If we, for the example illustrated by Figure 2, e.g. decide to combine the postage stamp rate method and the counter flow method by taking a convex combination of the corresponding allocations, we may obtain all the allocations lying on the straight line between *PS* and *CF* in the figure.

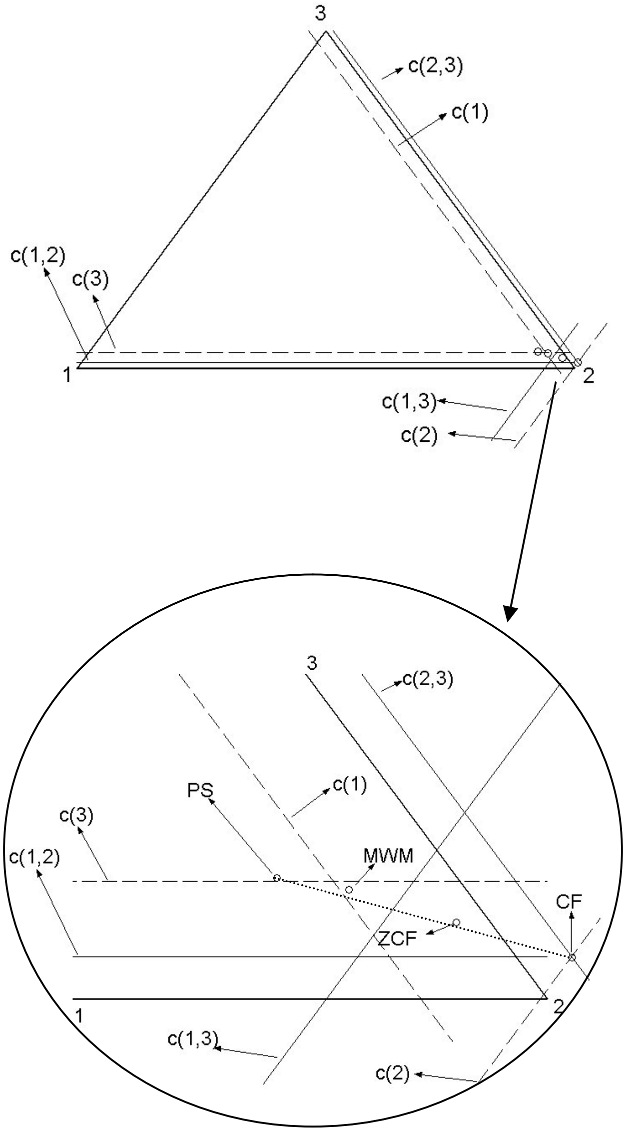

Figure 2: Core of game with three players/transactions.

4. A method to obtain core points from usage-based allocations

In this section we will describe a formal approach that can be used to combine the usage-based methods. Let M be the set of candidate allocations, where a_i^j is the cost allocated to player $i \in N$ given allocation method $j \in M$. We require the cost allocation vector x to be a convex combination of the candidate allocations, i.e.,

$$x_i = \sum_{j \in M} w_j a_i^j \qquad \text{for } i \in N \qquad (12)$$

$$\sum_{j \in M} w_j = 1 \qquad (13)$$

$$w_j \geq 0 \qquad \text{for } j \in M \qquad (14)$$

In order to find allocations that are as central in the core as possible, we also require that x should be chosen such that the excess vector $\mathbf{?}(\mathbf{x})$ is lexicographically maximal. Since this definition is similar to the definition of the nucleolus, except that we have added the requirements (12)-(14). Note that (2), used in the definition of the nucleolus, is implied by (12) and (13).

The computation of the restricted nucleolus may be implemented using the LP-problem given by

$$\text{maximize } r \qquad (15)$$

$$\text{subject to } x(S) + r \leq c(S) \qquad \text{for } S \in \Sigma^0 \qquad (16)$$

in addition to (12)-(14). The set Σ^0 consists of all the $2^n\text{-}2$ nonempty coalitions except the grand coalition. If the solution of (12)-(16) is unique, we stop, and this solution is our restricted nucleolus. If there are an infinite number of imputations that lead to the maximum r, one of which is the restricted nucleolus, we must perform additional calculations in order to eliminate some imputations. This is done by identifying the inequality constraints of (16) that are binding for *all* the optimal solutions. Denote the set of coalitions corresponding to these constraints by Σ_1, the remaining coalitions by $\Sigma^1 = \Sigma^0 \setminus \Sigma_1$, and the optimal value of (12)-(16) by r_1. Then a new LP-problem is formed by replacing (16) by

$$x(S) + r \leq c(S) \qquad \text{for } S \in \Sigma^1 \qquad (17)$$

$$x(S) = c(S) - r_1 \qquad \text{for } S \in \Sigma_1 \qquad (18)$$

If the solution to the second LP-problem also consists of an infinite number of imputations, we continue the process by identifying the set Σ_2 of binding constraints, and we form a new set of equations similar to (18) by using Σ_2 and the optimal value r_2 of the second problem. Also, we update (17) by defining $\Sigma^2 = \Sigma^1 \setminus \Sigma_2$. This process is continued until the LP-problem has a unique solution. Our procedure is similar to a procedure, first used by (Kopelowitz, 1967), for computing the nucleolus. The procedures are also discussed in chapter 6/7 of (Bjørndal, 2002).The *k*-th problem consists of (12)-(15) and

$$x(S) + r \leq c(S) \qquad \text{for } S \in \Sigma^{k-1} \tag{19}$$

$$x(S) = c(S) - r_j \qquad \text{for } S \in \Sigma_j,\ j = 1, \ldots, k-1 \tag{20}$$

For the three-player example from Section 1, we compute the restricted nucleolus (*RN*) for the case where $M = \{PS, MWM, CF\}$. The resulting allocation, illustrated in Figure 3, is a combination of the MWM allocation and the CF allocation, where $w_{MWM} = 0.573$ and $w_{CF} = 0.427$.

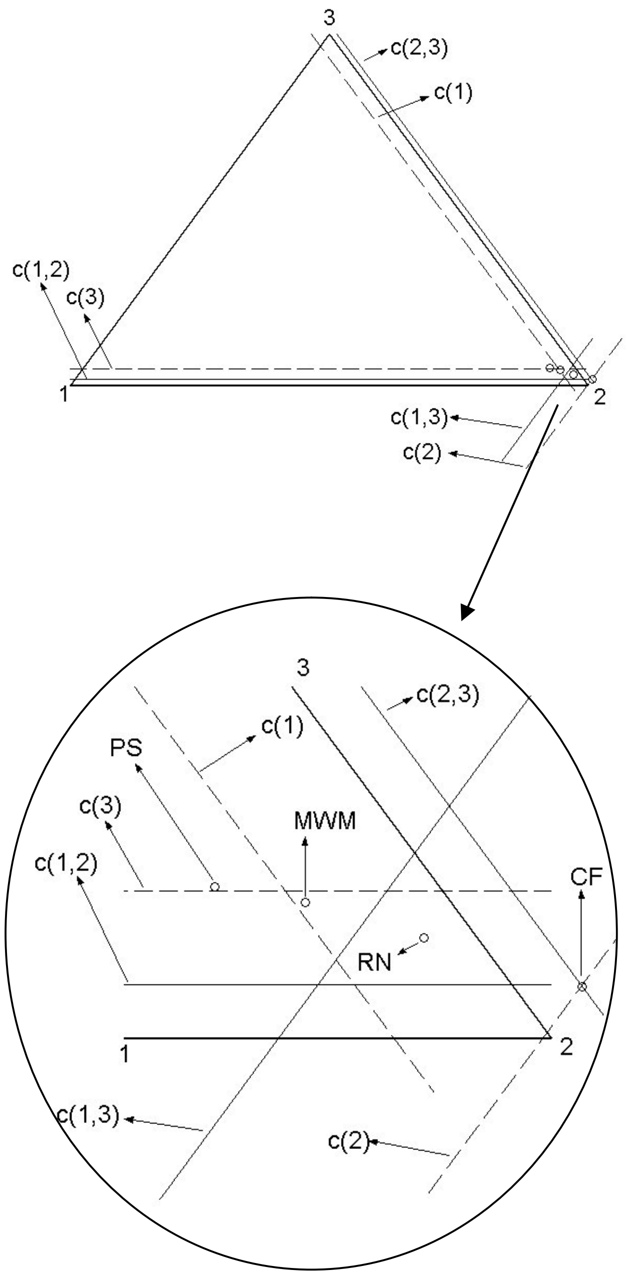

Figure 3: Core of the three-transactions case and the restricted nucleolus (*RN*).

The procedure described above locates a core point (if possible) by maximizing, in a lexicographical sense, the vector of excesses. In other words, we maximize the dissatisfaction, as measured by the excess value, of the least satisfied coalition. In the LP-problem given by (12)-(14), (19), and (20), this excess value is given by the number r_1. We will in the following use this number in order to compare different core points, e.g., resulting from different choices with respect to the set *M* of candidate allocations. In order to compare different games, we normalize the excesses by computing $r_1/c(N)$.

The solution of (12)-(14) can only be unique if the vectors $a^1,\ldots,a^m$ are linearly independent. Moreover, we must have $|N|\geq|M|$, i.e., there must be at least as many players as candidate allocation vectors. An example of a violation of the first of these requirements is found in the case of the usage-based allocation methods of Section 3. Assuming, without loss of generality, that $K=C_\ell=1$, the usage measures for the three flow-based methods can be rewritten as

$$f_i^{MWM} = \sum_\ell \max\left(P_{i,\ell};-P_{i,\ell}\right),\quad f_i^{CF} = \sum_\ell P_{i,\ell}\text{, and } f_i^{ZCF} = \sum_\ell \max\left(P_{i,\ell};0\right). \tag{21}$$

From these expressions we easily see that $f_i^{ZCF} = \frac{1}{2}\left(f_i^{MWM} + f_i^{CF}\right)$, hence

$$ZCF_i = MWM_i \frac{\sum_j f_j^{MWM}}{2\cdot\sum_j f_j^{ZCF}} + CF_i \frac{\sum_j f_j^{CF}}{2\cdot\sum_j f_j^{ZCF}}, \tag{22}$$

i.e., the three allocation vectors are linearly dependent. Hence, our method will fail to produce a unique solution if we try to include all three allocation. Since the ZCF allocation is a convex combination of the MWM and the CF allocations, choosing the set *M* such that it includes both of the latter allocations will yield a value of r_1 that is at least as low as if the ZCF allocation was substituted for either one of MWM or CF.

The method described above could be used as a tool in the process of designing a fixed cost allocation system, where one has to decide which set of usage-based cost allocation methods to use, and how to combine them. In practice load patterns change over time, whereas the cost allocation system must be designed in advance and held constant. As we will see in the numerical examples in the next section, the optimal weights are typically not stable as the load patterns of the transactions are changed, hence a weight system computed for one particular load profile is of little use when the load pattern changes. However, by combining weight systems that are optimal with respect to a number of different load profiles, e.g., by taking the arithmetic average of the weights, we get a more robust cost allocation system.

5. A numerical example

We illustrate the method proposed in Section 4 with the 14-bus network shown in Figure 4, and where the line characteristics are given in Table 2. The seven transactions to be performed are listed in Table 3. Eight different transaction profiles were considered, where the amounts of power transacted are shown in Table 4 below. As can be seen from the minimal excess values in Table 4, the *CF* allocation belonged to the core (non-negative excess values) for all transaction profiles, whereas none of the other three usage-based allocation methods produced core points for any of the transaction profiles.

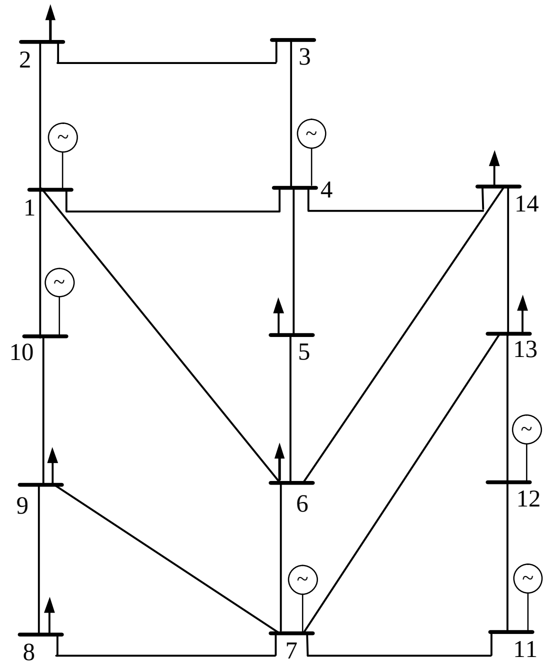

Figure 4: Network for the example.

Line	r(p.u.)	x(p.u.)	b(p.u.)
1-6, 9-7, 6-14, 7-13	0.00425	0.0450	1.5870
all other lines	0.0017	0.0180	0.6348

Table 2: Data for the 14-bus network (S_b=100MVA, V_b=380kV).

Transaction	Injection bus	Delivery bus
T1	1	14
T2	3	2
T3	4	5
T4	7	6
T5	10	13
T6	11	8
T7	12	9

Table 3: Transactions for the example.

T1-T5 (MW)	T6 (MW)	T7 (MW)	PS	MWM	CF	ZCF	A1	RN1	A2	RN2	A3	RN3
100	150	100	-0.0566	-0.0542	0.0020	-0.0188	0.0204	0.0320	-0.0178	-0.0159	0.0201	0.0320
100	200	100	-0.0815	-0.0779	0.0152	-0.0330	0.0049	0.0293	-0.0335	-0.0330	0.0047	0.0293
100	250	100	-0.0994	-0.0953	0.0032	-0.0192	0.0289	0.0325	-0.0200	-0.0192	0.0289	0.0325
100	300	100	-0.1198	-0.1154	0.0054	-0.0353	0.0220	0.0240	-0.0362	-0.0353	0.0220	0.0240
100	300	150	-0.1106	-0.1136	0.0009	-0.0291	0.0186	0.0217	-0.0299	-0.0291	0.0186	0.0217
100	300	200	-0.0880	-0.0968	0.0007	-0.0182	0.0157	0.0188	-0.0190	-0.0182	0.0157	0.0188
100	300	250	-0.0609	-0.0778	0.0021	-0.0251	0.0096	0.0102	-0.0255	-0.0251	0.0096	0.0102
100	300	300	-0.0700	-0.0539	0.0031	-0.0100	0.0067	0.0115	-0.0096	-0.0079	0.0067	0.0115

Table 4: Minimal excesses ($e_{\min}(x)/c(N)$) for various allocations, where

$e_{\min}(\mathbf{x}) = \min\{e(S,\mathbf{x}) : S \notin \{N,\emptyset\}\}$ for a given allocation vector $\mathbf{x} \in \mathbf{R}^n$.

Next, we computed the restricted nucleolus for different combinations of the usage-based allocation methods. Since the allocation vector *ZCF* is a convex combination of *MWM* and *CF*, by (22), including it in the set *M* will not increase the excess vector (in the lexicographical sense) if *MWM* and *CF* are already included. Hence, we tested the following three cases with respect to the choice of *M*, namely *{PS*, *MWM*, *CF}*, *{PS*, *MWM*, *ZCF}*, and *{PS*, *CF*, *ZCF}*. The minimal excess values for the corresponding restricted nucleoli (*RN1*-*RN3*) are shown in Table 4, and we see that *RN1* and *RN3* have higher excess values than any of the four usage-based allocations. Since *ZCF* is a convex combination of *CF* and *MWM*, the allocation *RN1* dominates both *RN2* and *RN3*.

The weights corresponding to *RN1*-*RN3* are shown in Tables 5-7 below. As can be seen, the weights vary considerably when the transacted amounts are changed, and the relationship between the amounts of power and the weights is not monotonic. Since the design of a cost allocation system is a long-term decision, the network operator does not have the opportunity to change the weights as transaction profiles change. The long-term choice of weights could e.g. be based on taking the average over a number of transaction profiles, as shown below. We then computed the allocations corresponding to the average weights, and these are denoted *A1-A3* in Table 4. We see that *A1* and *A3* belong to the core for all transaction profiles, and seems to be better (higher minimal excess values) than any of the pure usage-based allocations.

T1-T5 (MW)	T6 (MW)	T7 (MW)	PS	MWM	CF
100	150	100	0.2829	0.0023	0.7148
100	200	100	0.2160	0.0500	0.7340
100	250	100	0.4673	0	0.5327
100	300	100	0.4316	0	0.5684
100	300	150	0.3818	0.1108	0.5074
100	300	200	0.3265	0.0967	0.5768
100	300	250	0.3996	0.0743	0.5261
100	300	300	0.4138	0.1727	0.4135
Average (A1):			0.3649	0.0634	0.5717

Table 5: Weights for $M = \{PS, MWM, CF\}$.

T1-T5 (MW)	T6 (MW)	T7 (MW)	PS	MWM	ZCF
100	150	100	0.0299	0	0.9701
100	200	100	0	0	1.0000
100	250	100	0	0	1.0000
100	300	100	0	0	1.0000
100	300	150	0	0	1.0000
100	300	200	0	0	1.0000
100	300	250	0	0	1.0000
100	300	300	0.0524	0	0.9476
Average (A2):			0.0103	0	0.9897

Table 6: Weights for $M = \{PS, MWM, ZCF\}$.

T1-T5 (MW)	T6 (MW)	T7 (MW)	PS	ZCF	CF
100	150	100	0.2829	0.0032	0.7138
100	200	100	0.2160	0.0720	0.7120
100	250	100	0.4673	0	0.5327
100	300	100	0.4316	0	0.5684
100	300	150	0.3818	0.1629	0.4553
100	300	200	0.3265	0.1422	0.5313
100	300	250	0.3996	0.1097	0.4907
100	300	300	0.4138	0.2571	0.3292
Average (A3):			0.3649	0.0934	0.5417

Table 7: Weights for $M = \{PS, ZCF, CF\}$.

6. Conclusions

In Section 3 we demonstrated how some cost allocations resulting from some simple rules may fail to satisfy all the core constraints. By combining allocations from several methods we may be able to produce core allocations, and we presented an LP-based procedure in Section 4 that finds a good (core) allocation, where "good" means an allocation whose excess vector is as large (lexicographically) as possible. We believe, as is demonstrated using a numerical example in Section 5, that our method can be of help in designing cost allocation systems in practice, where one of the issues is deciding the weights to attach to various measures of network usage.

References

Bergen, A R and Vittal, V (1986): *Power systems analysis*, second edition, Prentice Hall

Bjørndal, E (2002): *Cost Allocation Problems in Network and Production Settings*, Ph.D.-thesis, The Norwegian School of Economics and Business Administration, Bergen, Norway

Contreras, J and Wu, F F (1999): Coalition formation in transmission expansion planning. *IEEE Transactions on Power Systems* **14** (3) 1144-1152

Kopelowitz, A (1967): Computation of the kernels of simple games and the nucleolus of *n*-person games. RM-131, Mathematics Department, The Hebrew University of Jerusalem, Israel

Marangon Lima, J W (1996): Allocation of transmission fixed charges: An overview. *IEEE Transactions on Power Systems* **11** (3) 1409-1418

Pan, J, Teklu, Y, Rahman S and Jun, K (2000): Review of usage-based transmission cost allocation methods under open access. *IEEE Transactions on Power Systems* **15** (4) 1218-1224

Shirmohammadi, D, Gribik, P R, Law, E T K, Malinowski, J H and O'Donnel, R E (1989): Evaluation of transmission network capacity use for wheeling transactions. *IEEE Transactions on Power Systems* **4** (4) 1405-1413

Tsukamoto, Y and Iyoda, I (1996): Allocation of fixed transmission cost to wheeling transactions by cooperative game theory. *IEEE Transactions on Power Systems* **11** (2) 620-629

Young, H P (1985): *Cost allocation: methods, principles, applications*, Amsterdam, North-Holland

Zolessi, J M and Rudnick, H (2002): Transmission Cost Allocation by Cooperative Games and Coalition Formation. *IEEE Transactions on Power Systems* **17** (4) 1008-1015