Does Stochasticity matter? Dynamic Pigouvian Taxation in an Uncertain Environment

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Abstract

The e¤ects of discounting, stochasticity, non-linearities and maximum decay upon an optimal corrective tax are analyzed using stochastic dynamic optimization. Optimal corrective taxes are derived as explicit feedback control laws in the presence of both ‡ow and stock externalities when the decay of aggregated pollution is subject to a general stochastic process. This represents an adaptive approach to regulation of the environment. The problem has been solved using a non-linear Hamilton-Jacobi-Bellman equation.

The model applied is quite general in the state variable, accumulated pollution, and in the control variable, production. The objective function is to maximize expected social welfare de…ned as the sum of consumers' and producers' surplus adjusted for externalities. Social welfare is not assumed to be separable in production and accumulated pollution.

The main result is that the optimal tax is more sensitive to discounting and non-linearities than to stochasticity.

Keywords: Global warming; ‡ow and stock externalities; dynamic corrective taxes; stochastic dynamic optimisation.

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INTRODUCTION

During the last few decades there has been an increasing concern with, and interest in, environmental matters. This can be exemplified through the concern about the so-called 'greenhouse effect', which refers to global warming through emission of CO_2 into the atmosphere, the thinning of the ozone layer and pollution in general.

During the last few years there has emerged an entire literature on the economics of global warming alone. Nordhaus (1991a and 1991b) has formulated an economic model that links emissions of greenhouse gases and climate changes and analyses the trade-off between the cost of reducing emissions and the damage from global warming.

Sinclair (1992 and 1994) and Ulph and Ulph (1994) consider the optimal time path of a carbon tax linked to the extraction of fossil fuels as nonrenewable resources and discuss whether this tax should increase or decrease over time. In most cases the rule is to let the tax decrease over time in order to postpone extraction. Wirl (1994a) uses a Nordhaus model to study the dynamic and strategic interactions between producers and consumers of fossil energy in the presence of carbon taxes. Wirl (1994b) discusses the more general problem of finding a time path for taxation of energy in the presence of flow and stock externalities. Sandal and Steinshamn (1998) derive an explicit feedback rule for an optimal corrective tax in the presence of both flow and stock externalities and use this to analyze the time path of the tax.

In this paper the effects of discounting, stochasticity, nonlinearities in the decay function and maximum decay upon the optimal tax are analyzed. The optimal tax is derived as a feedback control law.

To find a closed form solution to the feedback model, is not trivial even in the deterministic case. This problem has, however, been solved by Sandal and Steinshamn (1998). In contrast to most of the existing literature the model is defined in terms of market parameters which can be estimated as opposed to utility parameters which are much harder to assess. Another paper that uses market parameters is Chakravorty et al. (1997).

In this paper both the objective function and the dynamic constraint may be fairly general both in the control variable and in the state variable. This makes the model distinct from the literature on dynamic programming that uses so-called linear quadratic models with quadratic objective function and linear constraints.

THE GENERAL MODEL

The objective function is to maximize social welfare, defined as the sum of consumers' and producers' surplus corrected for externalities, when the decay of the pollution causing externalities is subject to stochasticity.

There are two types of externalities. Flow externality is defined as the externality associated with production, and is hence the difference between social and private marginal costs. Stock externality is defined as the externality associated with the aggregated level of the pollutant. There is a fixed level of emissions associated with each unit produced.

A list of the major symbols and definitions in this article can be found in Appendix. Let x denote production and a denote the aggregated level of pollution. The maximization problem is given by

$$\max_{x \geq 0} E \int_0^{\infty} e^{-rt} W(x(t), a(t)) dt \tag{1}$$

where E is the expectation operator, and W represents welfare. For each unit produced there is a fixed level of emission, δx, and the decay of pollution is a general function, $f(a)$. The time change in the aggregated level of pollution is then

$$da = [\delta x - f(a)]\, dt + \sigma(a) dw \tag{2}$$

where σ is a general volatility function and the term dw is a standard Wiener process (independent and identically distributed) with variance dt and zero mean. In the rest of this paper it is assumed that $\delta \equiv 1$ which simply is to say that production and pollution are measured in the same units.

The welfare function is defined as

$$W(x,a) = \int_0^x [P(z,a) - C^s(z,a)]\,dz - D(a) \tag{3}$$

where P is the inverse demand function, C^s is the social marginal cost of production and D represents the stock externality.[1] In this paper we assume that $P(x,a) - C^s(x,a)$ is a decreasing function of production.

The possible a-dependence in the demand function may, for example, reflect consumers' concern for the environment. If it is present, we expect the derivative $P_a < 0$ as a higher level of accumulated pollution, if anything, will shift the demand curve for the polluting product down. Possible a-dependence in the marginal cost function simply reflects that the costs of production may increase in a polluted environment.[2] The a-dependence in P and C^s may also be skipped completely.

The flow externality, which is associated with production, x, is $C^s - C^p$. Here C^p denotes the private marginal costs. The a-dependence in the demand, if it exists, represents possible a-dependence in the flow externality and must not be confused with the stock externality, D, which comes in addition. Even though the flow externality associated with the a-dependence in the inverse demand and possibly in marginal cost may be weak dependence, it may have a significant effect on the optimal feedback policy.

A competitive economy is assumed throughout this paper. Therefore private marginal costs represent the supply curve. In market equilibrium the supply curve rep-

[1] Functional dependence of the variables is often skipped in order to make the visual appearance of the equations clearer. The functional dependence is always indicated when new variables are introduced and in the symbol list in the appendix.

[2] It is straightforward to let marginal costs depend on a as well, but this does not seem relevant in the present global-warming context.

resents the producer price and the demand curve represents the consumer price. As we shall soon see, these two may deviate due to the corrective tax.

The approach taken here is that the problem is solved for optimal production in the stochastic environment. The Pigouvian tax needed to achieve this production is then calculated through the market parameters, and it is assumed that there is no uncertainty in the market. In this paper an ad valorem tax is applied, defined as

$$\theta = \frac{P(x,a) - C^p(x,a)}{C^p(x,a)}. \tag{4}$$

Dynamic programming (in continuous time and state space is used to solve the stochastic optimal control problem (SOC) given by (1) and (2); see, for example, Kamien and Schwartz (1991). The Hamilton-Jacobi-Bellman (HJB) equation associated with our SOC for the value function

$$V(t,a) = \max_{x \geq 0} E \int_t^{\infty} e^{-r(s-t)} W(x(s), a(s)) ds$$

is given by

$$V_t + \max_{x \geq 0} \left\{ e^{-rt} W(x,a) + V_a \cdot (x - f(a)) \right\} + \frac{1}{2}\sigma^2(a) V_{aa} = 0. \tag{5}$$

It will now be useful to define a new variable $\Lambda(a)$ implicitly through

$$V(t,a) = e^{-rt}\Lambda(a) - \frac{1 - e^{-rt}}{r} K \tag{6}$$

where K is a constant. The constant K depends on r such that $\lim_{r \to 0} K = K_0$, and $\lim_{r \to 0} \Lambda(a) = \Lambda_0(a)$. Hence (6) is valid also when $r \to 0$ as $V \to +\Lambda_0(a) + tK_0$. It is seen that $\Lambda(a) = V(0,a)$. An alternative interpretation of Λ is therefore the initial value. The definition in (6) inserted into (5) implies

$$K = -r\Lambda(a) + \max_{x \geq 0} \left[W + \Lambda' \cdot (x - f) + \frac{1}{2}\sigma^2 \Lambda'' \right]. \tag{7}$$

All terms in this expression have interpretation. The term $r\Lambda$ is the return on the initial value. The first term in square brackets, W, is the return on current activity and $\Lambda' \cdot (x - f)$ is the loss/gain associated with changes in a. The last term $\frac{1}{2}\sigma^2\Lambda''$ is the change in welfare due to the stochasticity. From this the constant K can be interpreted as the net benefit from the total optimal activity. One should not be surprised to find that $K = 0$ can be chosen for our problem. The optimal value may be set to zero for extremely high stock of pollution. In this region we would expect the value function to be flat. Eq. (7) then becomes of principle form for the viscosity solution concept which represents the unique economic solution for many autonomous SOC formulated problems. We may always choose $K = 0$. This is just the following formal transformation $\Lambda - \rightarrow -\frac{1}{r}K + \Lambda$. It reflects that the HJB-equation stated in eq(5) is invariant under a constant change in the value function (it only contains the derivatives of the value function).

$$r\Lambda(a) = \max_{x \geq 0} \left[W + \Lambda' \cdot (x - f) + \frac{1}{2}\sigma^2\Lambda'' \right] \tag{8}$$

The optimal feedback control, $x = G(a, \Lambda'(a)) = X(a)$, is given partly by positive parts of the unique solution of the algebraic equation $P + \Lambda' = C^s$ (inner optimum) and partly by $x = X(a) = 0$ (corner solution).

Equation (8) is the tool needed for finding the optimal corrective tax both in special and more general cases. It is solved and the optimal production $x = X(a)$ is determined. This feedback production is then substituted into the ad valorem tax expression given by eq(4). Our numerical procedure follow closely the procedure outlined in Kushner and Dupois (2001). The eq(8) is formally discretized in order to get the local consist probabilities in a controlled Markov Process that converges to the viscosity solution of our problem. A thorough exposition of the concept of viscosity solution and controlled Markov Processes can be found in Fleming and Soner (1991).

NUMERICAL EXAMPLE

In this section we look at a specific numerical example based on the problem of global warming due to accumulation of CO_2 in the atmosphere. Meteorological data have been provided by the "Nansen Environmental and Remote Sensing Center" in Bergen, Norway. CO_2 can be measured in parts per million carbon (p.p.m.) or gigatonnes CO_2 (Gt-CO_2) which will be used here. The relationship is approximately 1 p.p.m. $\simeq$ 7.8 Gt-CO_2. The numerical specification of the meteorological model is based on data from 1997.

Table 1. Meteorological data.

	p.p.m.	Gt-CO_2
Pre-industrial a	280	2187
Current a	360	2812
Current $\underline{a}$	1.5	11.7
Current x	2.8	21.9
Current $f(a)$	1.3	10.2

The specification of the economic model and the damage and decay function is assumed to be as follows:

$$P(x) = 15 - 0.64 \cdot x,$$
$$C^p(x) = 1 + 0.05 \cdot x,$$
$$C^s(x) = 1 + 0.12 \cdot x,$$
$$D(a) = 0.000005 \cdot a^2,$$
$$f(a) = \frac{\beta \cdot a}{a + \gamma}$$

where x is measured in a-units; that is, δ in (2) is set equal to one. These parameters are calibrated based on the assumption that the private marginal cost is normalized to one. A price 15 times this level will choke all demand, and the market equilibrium without any policy measures corresponds to the present emission level.

The specification of demand and supply functions are based on some assumptions about elasticities. If current production is 22, this implies a demand elasticity of -0.16 and a supply elasticity of 1.9. This is in accordance with Jorgenson and Wilcoxen (1990) who state that short-term demand elasticities are about one tenth of the long-term, and probably lie between -0.1 and -0.2. As this model is dynamic and adaptive, only short-term elasticities are of interest. The supply elasticity is in accordance with Burniaux et al. (1992) who state that supply elasticities from countries within OPEC vary between one and three.

The level of CO_2 is measured from the estimated pre-industrial level 2187. In other words, the current level is 625 ($= 2812 - 2187$). This makes it possible to make the natural assumptions that $f(0) = 0$ and $D(0) = 0$.

From Table 1 it is seen that the physical change is slow; $f(a)/a$ today is 10.2/625 = 0.0163. Optimal policies will be quite sensitive to discount rates higher than this, that is higher than 1.6 %. In this example however we have chosen to concentrate on zero discounting as almost any discount rate will imply that the future is neglected compared to the present. In the case of global warming, where the harmful effects will take place some time in the future, it is important that these effects are not ruled out by a high discount rate.

The parameters in the f-function, β and γ, have been calibrated such that $f(a)$ goes through $f(0) = 0$ and the present situation $f(625) = 10.2$. The functional form of f makes it monotonically increasing, concave and $f(a) \to \beta$ as $a \to \infty$. The parameter β is interpreted as the maximum level of decay and can also be used to determine the degree of nonlinearity. Small values of β, that is β close to the present observed decay, implies a highly non-linear function. On the other hand, large values of β makes the decay function almost linear for realistic values of a. Hence the parameter β can be used to test how sensitive the optimal tax is to different degrees of non-linearity.

Two σ-functions have been used: the linear $\sigma(a) = \sigma_0 a$ and the non-linear $\sigma(a) = \sigma_0 f(a)$ which mimics the decay-function, and all uncertainty is in the magnitude of the decay. Further, it has been assumed that the stock externality is $D(a) = 0.000005\, a^2$. The results are illustrated in Figures 1 - 6.

Figure 1 illustrates the deterministic part of the decay-function and how the degree if non-linearity varies with different values of β. Figure 2 illustrates how these different values of β affect the optimal ad valorem tax. It is seen that the optimal tax is quite sensitive to the degree of non-linearity in the decay function.

Figure 3 illustrates the effect of discounting. Here two values of β have been used: $\beta = 12$ (highly non-linear) and β ! 1 (almost linear). Then tree values for the discount rate have been used: δ = 0.5 %, 1.5 % and 3 %. The important thing to note is that the optimal tax is quite sensitive to changes in the discount rate, and more so with non-linear decay than with linear.

Next we look at how the results are affected by stochasticity. Figure 4 illustrates this with linear decay (β ! 1). In this case we only have a linear σ-function. It is seen that the resulting optimal tax is not very sensitive to different values of σ_0. Stochasticity will only have any effect when it is significantly larger than the deterministic part, and this supposition is not supported by data. Figure 5 looks at the effect of stochasticity with non-linear decay and linear $\sigma(a)$ whereas Figure 6 looks at the situation with non-linear decay and non-linear $\sigma(a)$. In both cases the effects of stochasticity are almost negligible for all reasonable combinations of σ_0 and a.

These examples are only meant for illustrative purposes. However, noting how the expected range for the optimal tax varies with changes in the volatility function and in the decay function emphasizes the importance of performing quantitative studies in order to estimate the input parameters in this model.

SUMMARY

The primary purpose of this paper has been to investigate the effects of discounting, stochasticity, nonlinearities in the decay function and maximum decay upon and optimal corrective tax in the presence of both flow and stock externalities. Using stochastic dynamic optimization the optimal ad valorem tax is given as a feedback control law. Sensitivity analysis has been performed with respect to stochasticity, maximum decay and discounting.

This paper brings in ideas used in the real option theory to environmental issues without using the limitations of the discrete choice found in the applications of real option theory that all boil down to more or less simple stopping problems. A stochastic, dynamic optimization problem is solved which addresses directly the often raised issues of different kinds of options related to reducing greenhouse gas emissions. As far as we know, this has so far largely been addressed in much simpler models, e.g. two periods instead of continuous time and simple stopping problems instead of continuous decisions.

The main conclusion is that optimal corrective taxes are quite sensitive to the assumptions made about discounting and nonlinearities in the decay of pollution but not very sensitive to the degree of stochasticity for reasonable combinations of pollution level and stochasticity. The implication of this is that future effort is better put on empirical studies of the decay function than on advanced stochastic models as long as the stochastic process itself is poorly known.

APPENDIX. LIST OF MAJOR SYMBOLS AND DEFINITIONS.

This appendix gives brief definitions of some of the major symbols. For more thorough definitions, see the text.

x : Some product causing emissions

a : The stock pollutant measured in physical units

θ : A corrective ad valorem tax

$D(a)$: Disutility caused by the presence of the stock pollutant; that is the stock externality

$f(a)$: The deterministic part of the decay function for pollution

$\sigma(a)$: Volatility function

dw : Standard Wiener process increment

$P(x, a)$: Inverse demand function for x

$C^P(x)$: Private cost of producing x

$C^S(x)$: Social cost of producing x

$W(x, a) = \int_0^x [P(z, a) - C^s(z)]\, dz - D(a)$: Social welfare from the production and consumption of x

$V(t, a)$: Value function

¤(a) : Variable defined through $V(t, a) = e^{-rt}$¤$(a) - \frac{1-e^{-rt}}{r} K$

$K,\ K_0$: Constants

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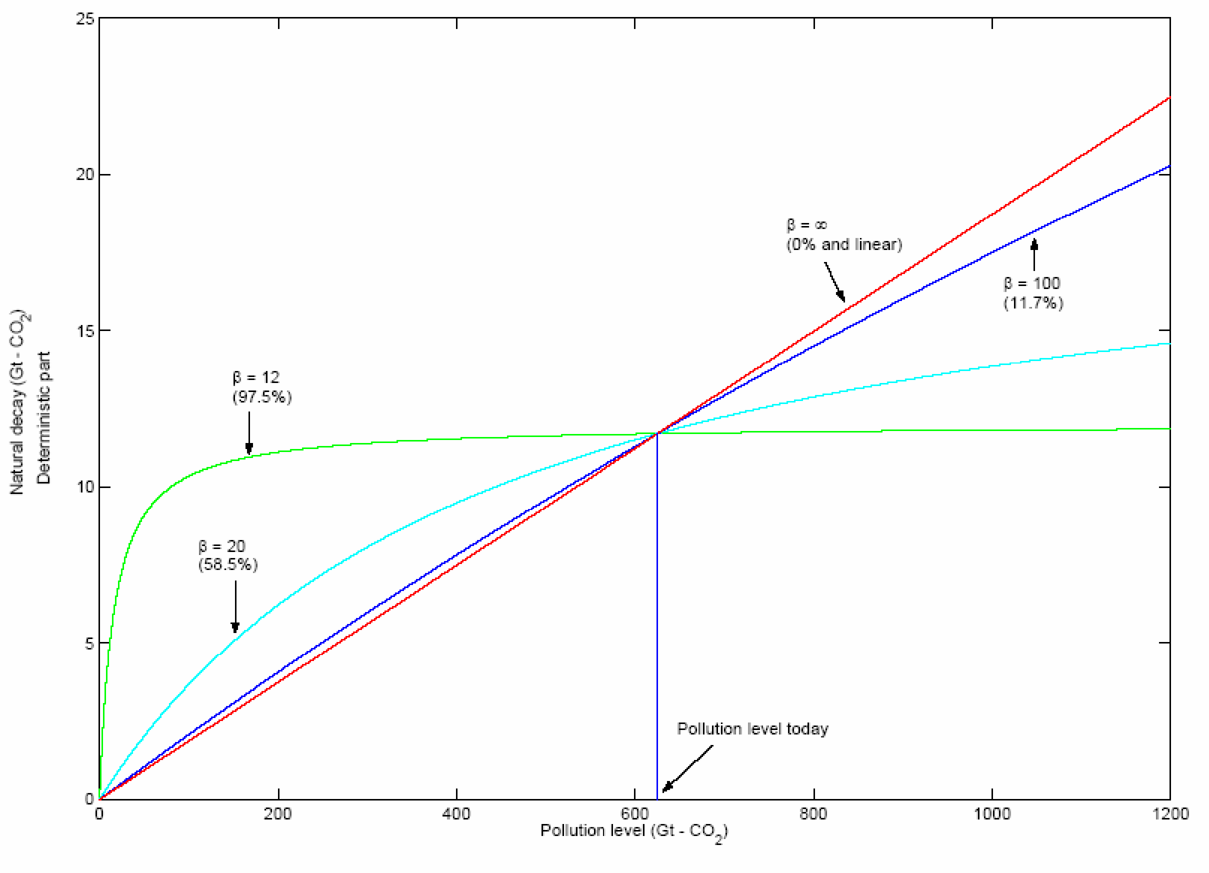

Figure 1.
The decay model given by $f(a) = ba/(a - a_0 + bk)$ where $k = a_0 / f(a_0)$ for different values of b. Decay today in percentage of the maximum decay level b.

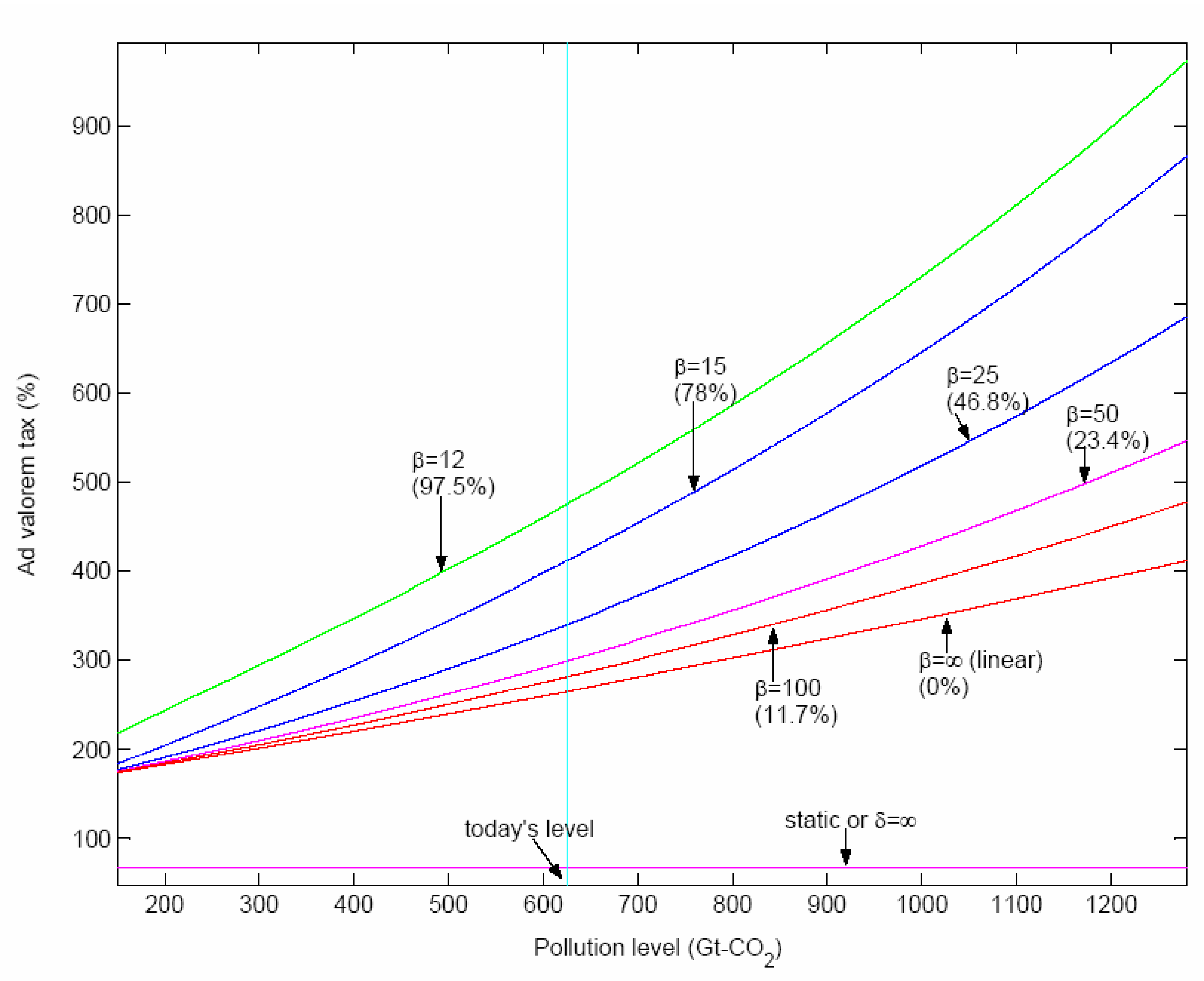

Figure 2.
Effects of different values of b upon the optimal tax for the deterministic case with 1.5 % discount rate. Decay today in percentage of the maximum decay level b .

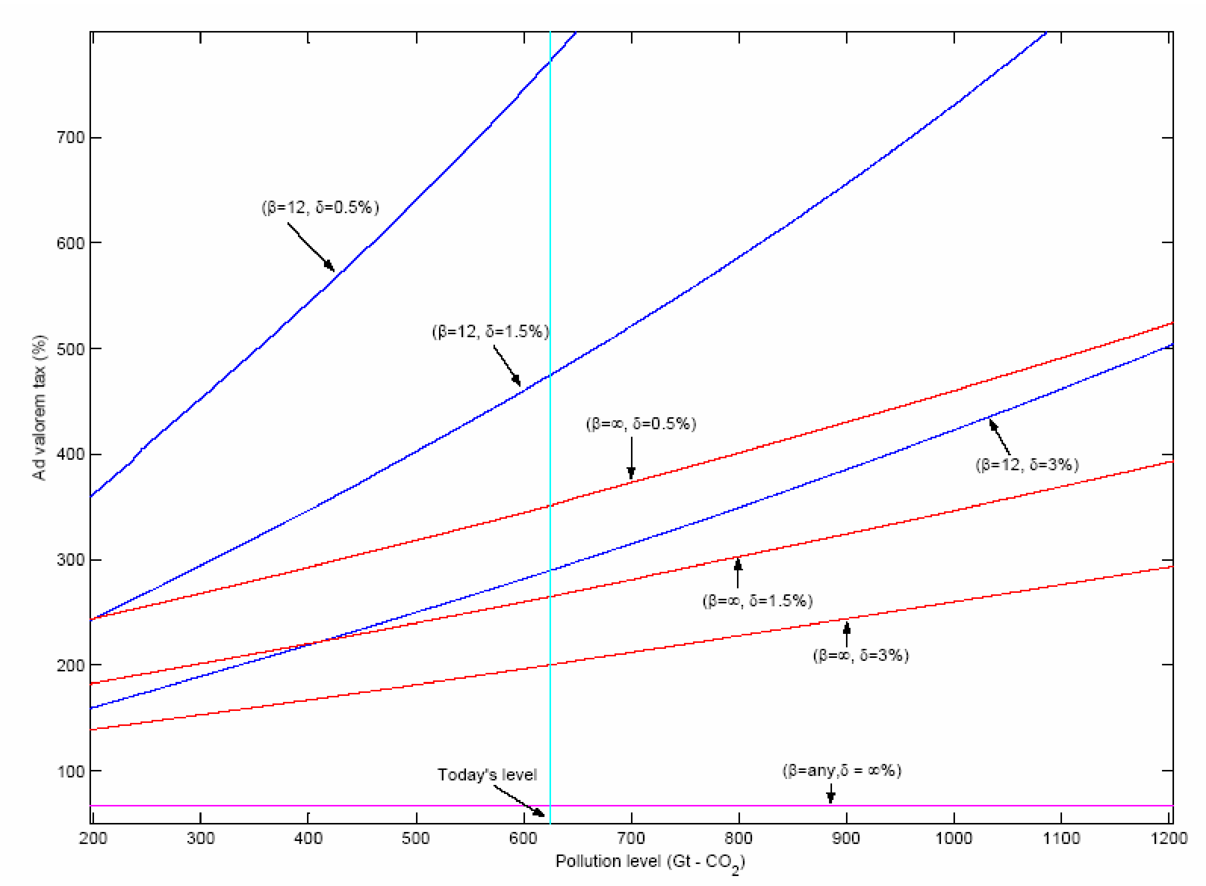

Figure 3.
Effects of different values of the discount rate upon the optimal tax in the deterministic case with two different values of *ß* (*ß* = 12 and $b \rightarrow \infty$).

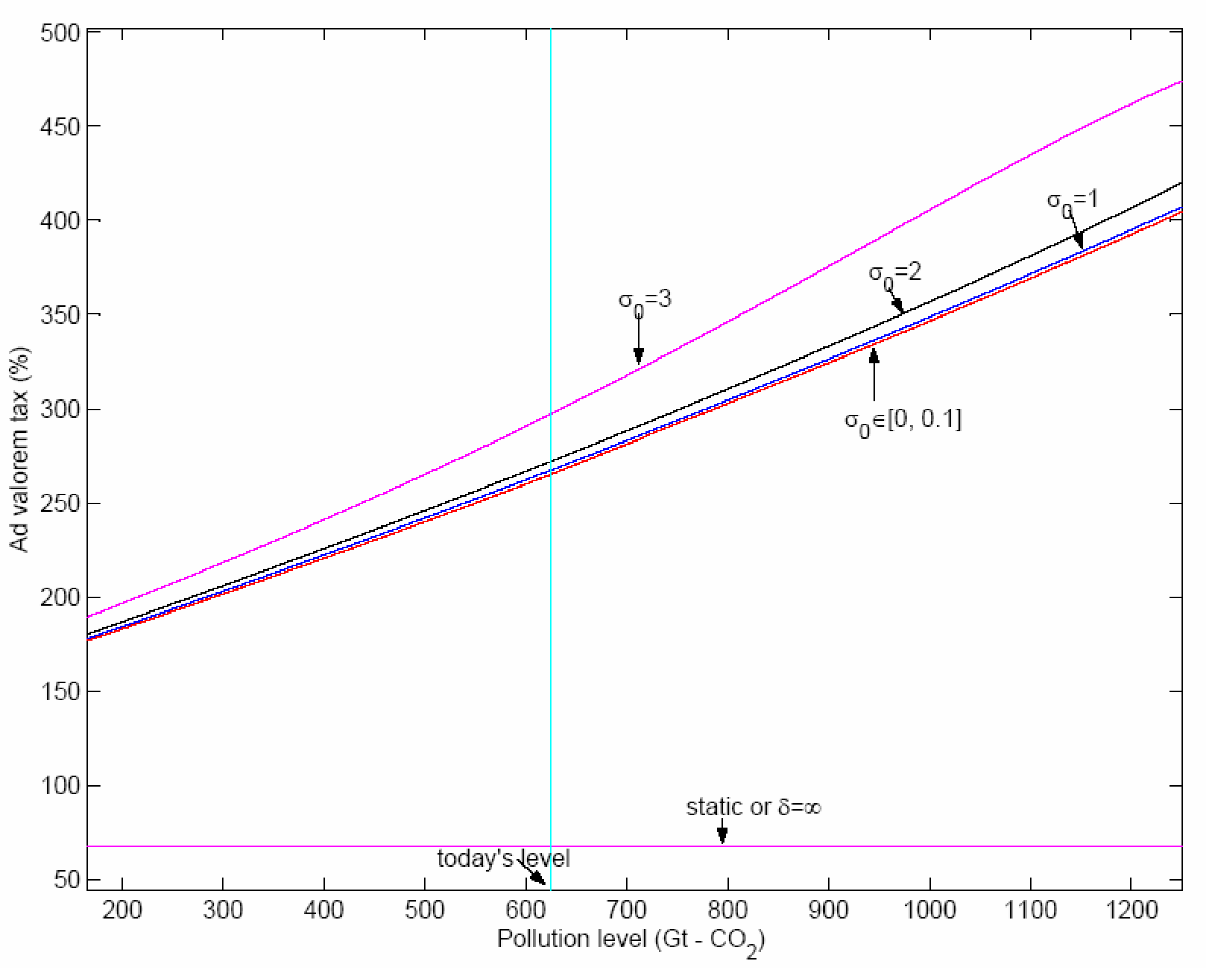

Figure 4.
Effects of the stochasticity upon the optimal tax with 1.5 % discount rate in the case of a linear decay model ($b = \infty$) and linear volatility, $s(a) = s_0 a$.

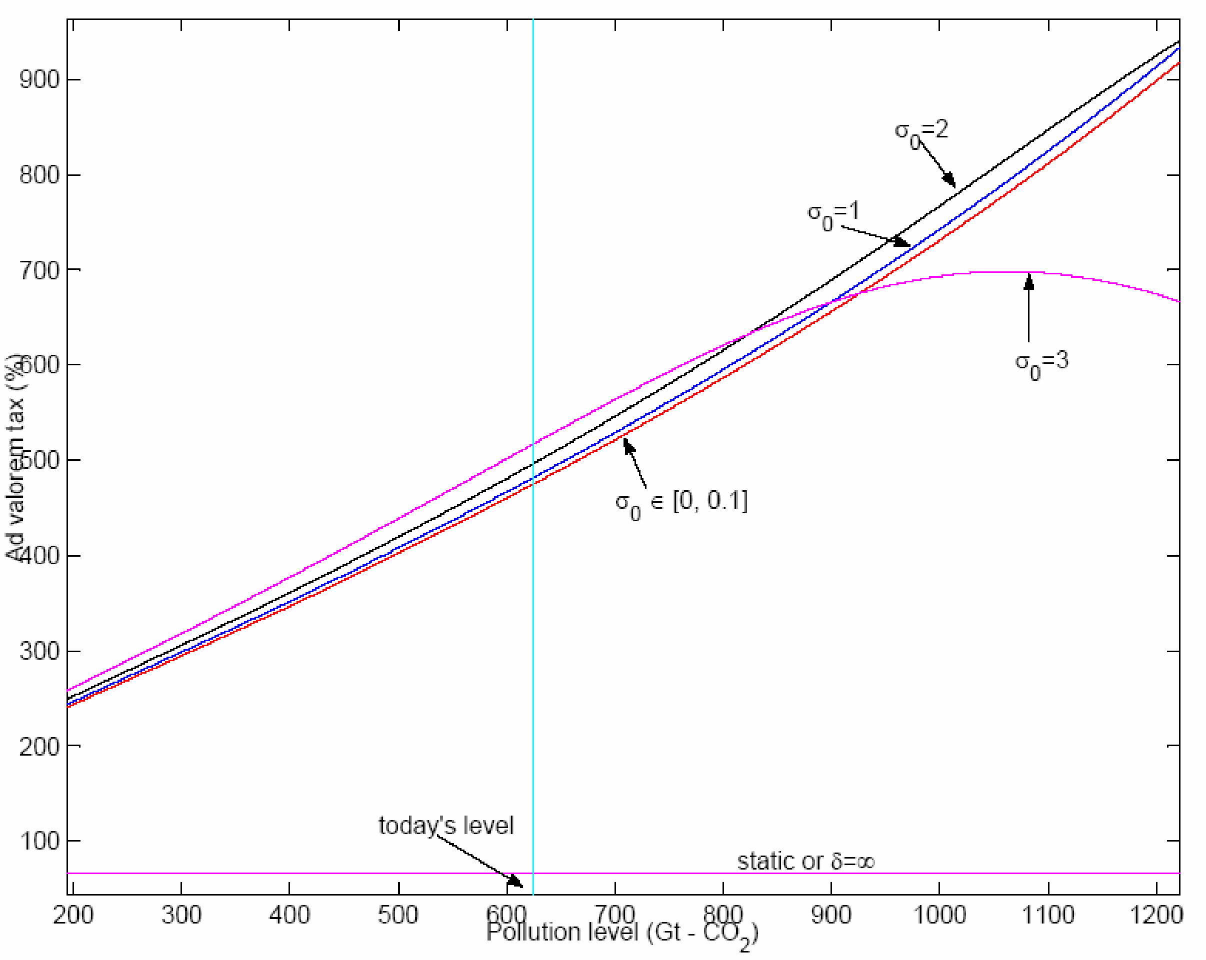

Figure 5

Effects of the stochasticity upon the optimal tax with 1.5 % discount rate in the case of a low level of maximum decay ($b = 12$) and linear volatility, $s(a) = s_0 a$.

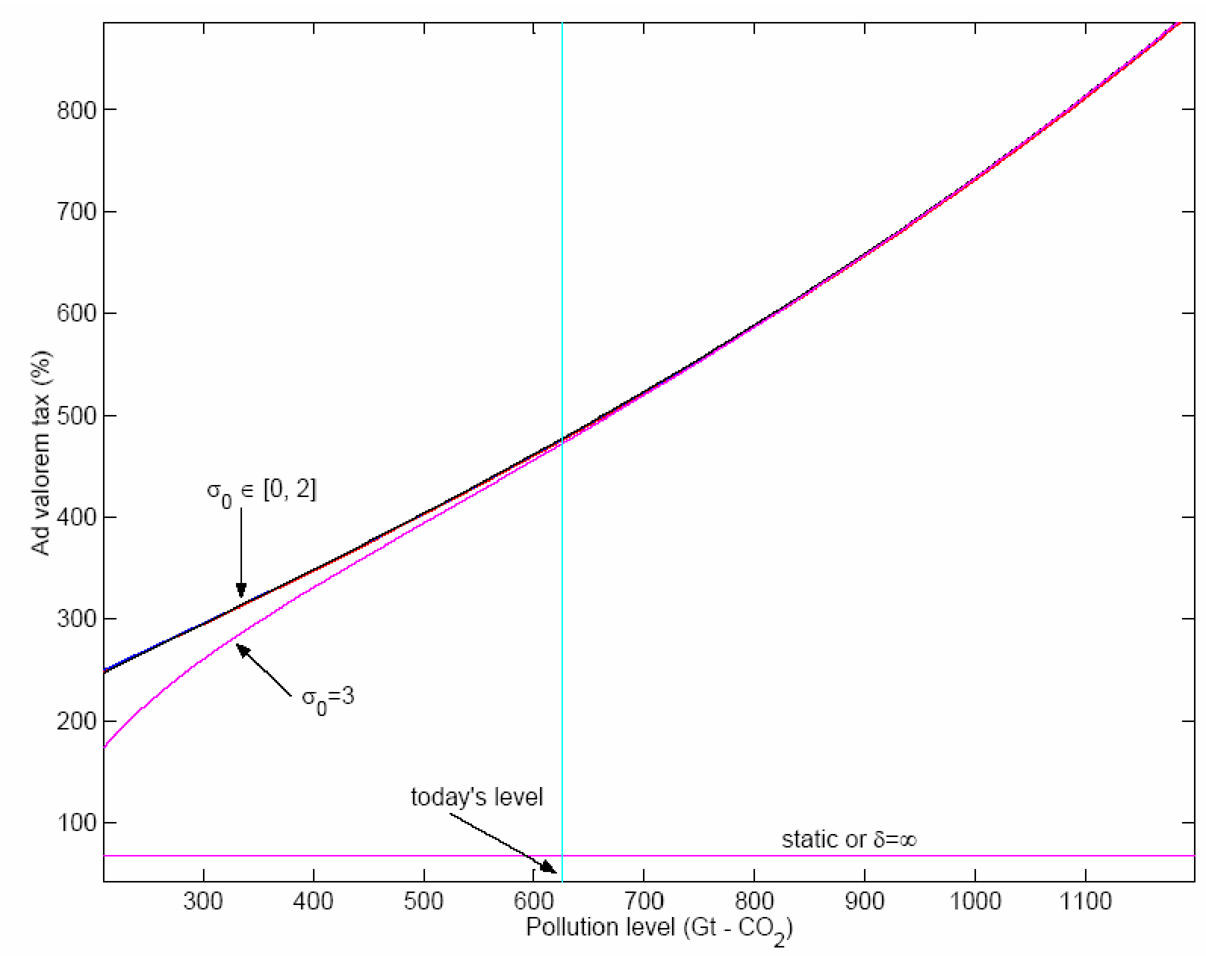

Figure 6.
Effects of the stochasticity upon the optimal tax with 1.5 % discount rate in the case of a low level of maximum decay ($b = 12$) and nonlinear volatility, $s(a) = s_0 f(a)$.