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Risk Aversion in the Large and in the Small

BY

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Abstract

Estimates of agents' risk aversion differ between market studies and experimental studies. We demonstrate that the estimates can be reconciled through consistent treatment of agents' tendency for narrow framing, regarding integration of background wealth as well as across risky outcomes: Risk aversion is similar whenever similar degrees of narrow framing is assumed in either setting.

JEL-Classification: G11, G12, D81.

Keywords: Risk aversion, narrow framing, background wealth, laboratory experiments, market studies, equity premium puzzle.

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1 Introduction

The level of risk aversion is a key determinant of economic behavior. It determines for example the degree of insurance households buy, is decisive for their assets allocation decisions, and is a key determinant of firms' cost of capital. Research on the level of risk aversion has therefore been at the center of economics in general and of finance in particular. The question of how to measure the level of risk aversion has been addressed using quite different research methodologies. Laboratory experiments infer the level of risk aversion from individuals' choices between simple lotteries. Empirical studies, on the other hand, infer the average risk aversion of market participants from risk premia of observed stock prices. The two approaches do not lead to similar estimates of the level of risk aversion, however: The degree of risk aversion found in individual data is typically significantly lower than that found in market data.

The purpose of this paper is to reconcile the results of these two research methodologies. For at least the following two reasons this task is not immediate. First, the two research methodologies work with different auxiliary assumptions. Second, the aggregation from individual decisions to market averages has to be addressed.

We argue that the main reason for the different estimates is differences in auxiliary assumptions about how agents apply "narrow framing" when evaluating risky prospects (see for instance Kahneman and Tversky, 1984; Tversky and Kahneman, 1986; Redelmeier and Tversky, 1992; Read et al., 1999). An econometrician must make assumptions about not only the degree to which agents take "background wealth" into account, but also about to what extent they evaluate a given risky prospect independently from other risky prospects. While laboratory studies commonly assume narrow framing, this is typically not done in studies that rely on market data.

The intuition for our main result is as follows. Not integrating background wealth reduces the risk aversion found—both in the interpretation of stock market data and of experimental data. The first claim is true because not integrating background wealth means stock investments are evaluated by their dividends, which are more volatile than consumption. Hence, evaluating stocks according to the dividends they pay out increases the volatility of the stochastic discount factor (SDF), which is an upper bound for the Sharpe ratio of asset returns (Hansen and Jagannathan, 1991).[1] The second claim follows because the size of the stakes of lotteries does not matter in experiments when background wealth is not integrated. For a standard utility function, this reduces the

[1]This is observed by Hagiwara and Herce (1997). They find a coefficient of relative risk aversion of about three in annual data (1889–1994) from the Standard & Poor's U.S. stock price index.

risk aversion needed to explain observed individual behavior.

Table 1 summarizes the main results of our paper. It reports the degree of risk aversion we estimate from stock market data and experiments, with and without narrow framing. It is apparent that the level of risk aversion is higher in the market than in the laboratory when narrow framing is assumed in the interpretation of experiments, but assumed absent in the interpretation of stock market data. When the assumption of narrow framing is applied consistently, the degree of risk aversion coincides. Similarly, if narrow framing is assumed in the analysis of stock market data, but not in experiments, then the level of risk aversion in the laboratory is higher than that found in market data.

Table 1: Estimates of risk aversion from stock market data and experiments, with or without integration of background wealth.

	Market data	Experiments
Integrated	10–20	10–20
Not integrated	2.25	3.22

A second contribution is to establish that assumptions about individuals' tendency for narrow framing carry over to the representative agent in a Rubinstein-Lucas style pure exchange economy (Rubinstein, 1974; Lucas, 1978). Estimates of risk aversion from stock market data are typically carried out using this construct. This result is important, because it implies that, *ceteris paribus*, estimates of risk aversion from stock market data and data from experiments are consistent only if similar assumptions are made about the agents' degree of narrow framing. The qualification *ceteris paribus* refers to the possibility that individuals' behavior is different in the laboratory and in real financial markets—an issue we do not address in this paper.

Our paper contributes to various strands of literature. Focussing on the diagonal (10–20, 3.22) of Table 1, our paper reproduces the explanation of the equity premium puzzle found by Hagiwara and Herce (1997). Moreover, we share with Barberis and Huang (2008) the idea of narrow framing. Barberis and Huang (2008) use narrow framing to argue that investors are loss averse, i.e. that losses affect utility at a higher rate than gains. These ideas lead them to suggest that the standard consumption-based SDF is *augmented* by a term that captures loss aversion. This extra term provides sufficient variation in their SDF to explain the equity premium puzzle with a low coefficient of risk aversion. Note that our conclusion from the same observation is to *restrict* the SDF to consumption proxies that are more specific than the standard notions of *per capita* consumption. Our model does therefore not rely on an extra term that provides more freedom to pick up otherwise unexplained variation.[2]

A novelty of our paper is however also to also point out the diagonal (2.25, 10–20) of Table 1. If narrow framing is applied to stock market data but not to experiments a reverse equity premium puzzle obtains. Hence we claim that the equity premium puzzle may not only be an empirical question but can also be perceived as a conceptual one.

In Section 2 we consider experimental evidence on the level of risk aversion and show it depends on the way background wealth is treated. We then develop a simple utility model in Section 3 that allows for various degrees of narrow framing, show our aggregation result, and calibrate the utility model to market data by computing consistent levels of risk aversion for various degrees of narrow framing. We use these results to infer the degree of narrow framing that makes the risk aversion inferred from market data consistent with that from experimental data. Section 4 concludes.

2 Risk Aversion in Experimental Studies

Determining an agent's risk aversion is a well researched topic in experimental economics and the psychology literature. It is well known that questions like the following get more robust answers than asking for certainty equivalents, or for instance using unequal probabilities.[3]

Question 1: *Consider a fair lottery where you have a 50% chance of doubling your income, and a 50% chance of losing a certain percentage, say x% of your income. What is the highest loss x that you would be willing to incur in order to agree taking part in this lottery?*

Barsky et al. (1997) report that the average answer in such an experiment is about $x = 23\%$. What does this answer imply, assuming participants maximize expected utility with constant relative risk aversion?

Suppose first that the decision problem is perceived as: Find an x such that the utility of the lottery and the *status quo* income are the same, that is

$$0.5\frac{(2y)^{1-\gamma}}{1-\gamma} + 0.5\frac{((1-x)y)^{1-\gamma}}{1-\gamma} = \frac{(y)^{1-\gamma}}{1-\gamma},$$

[2]The effect is similar to any approach that introduces a risk factor in the SDF that is more volatile than consumption, for instance as wealth in combination with *per capita* consumption (as in Epstein and Zin, 1991) or wealth as the only risk factor (as in Bakshi and Chen, 1996). It is unclear though, how to project these inherent multi-period models onto the one-period lotteries used in the experiments.

[3]We refer to Harrison and Rutström (2008) for a thorough comparison of different risk aversion elicitation procedures in the laboratory. Similar questions are reported in Barsky et al. (1997), Mankiw and Zeldes (1991), Shefrin (1999) and Gollier (2001).

where y is the income referred to in Question 1. Then $x = 23\%$ reveals a relative risk aversion of $\gamma = 3.22$. This is somewhat higher than levels argued on normative grounds. Arrow (1971) argues for a constant relative risk aversion of about one.[4] A participant with unit risk aversion would, however, have to answer $x = 50\%$, which is clearly an outlier among participants (Barsky et al., 1997, among others). Hence, the experiment seems to reveal that the average decision maker is more risk averse than the ideal Arrow (1971) decision maker. This claim is commonly made in the literature. Samuelson (1991), for example, compares the unit relative risk aversion case to the one with a relative risk aversion of two, and finds the latter to be the "*more realistic case.*" On the other hand, assuming a constant relative risk aversion in the range 10–20, which is necessary to explain the equity premium, as reported in Table 1, implies an x in the range 3–7%, which is also an outlier among laboratory observations. From the latter observation, Shefrin (1999) and many others conclude that the equity premium is "*a puzzle,*" since the risk aversion necessary to explain observed equity returns is far off from observed risk aversion in experimental research.

The validity of the above argument depends, however, on the validity of auxiliary assumptions about participants' tendency for narrow framing. Using the classic interpretation of the expected utility model, one should not separate the lottery payoffs from background wealth and should instead evaluate every lottery based on its effect on total wealth, as pointed out early on by Mossin (1968, p. 215): "*... a formulation of the decision problem [...] in terms of portfolio rate of return tends to obscure an important aspect of the problem, namely, the role of the absolute size of the portfolio.*" If the income y in Question 1 is not understood by participants to refer to their total wealth, and one maintains the classic interpretation of the decision problem participants are really solving the problem

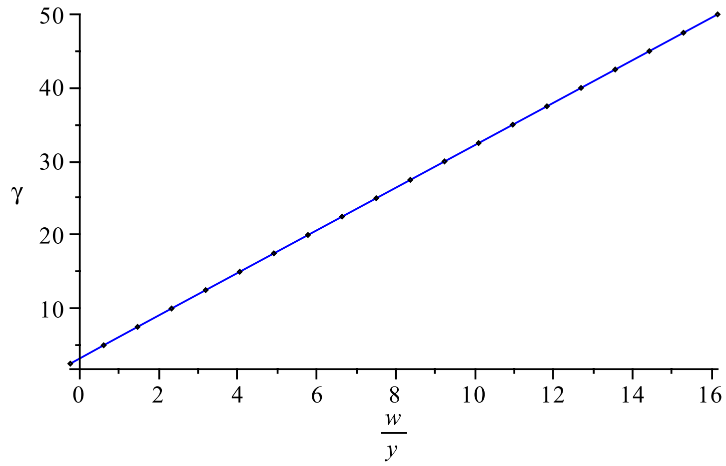
$$0.5\frac{(w+2y)^{1-\gamma}}{1-\gamma} + 0.5\frac{(w+(1-x)y)^{1-\gamma}}{1-\gamma} = \frac{(w+y)^{1-\gamma}}{1-\gamma}, \tag{1}$$

where w is the participant's background wealth, unrelated to the experiment.

Figure 1 shows risk aversion as a function of background wealth relative to the size of the stake, using the average answer to Question 1. The graph shows that $x = 23\%$ is consistent with a risk aversion of 10–20 if the background wealth is in the range of

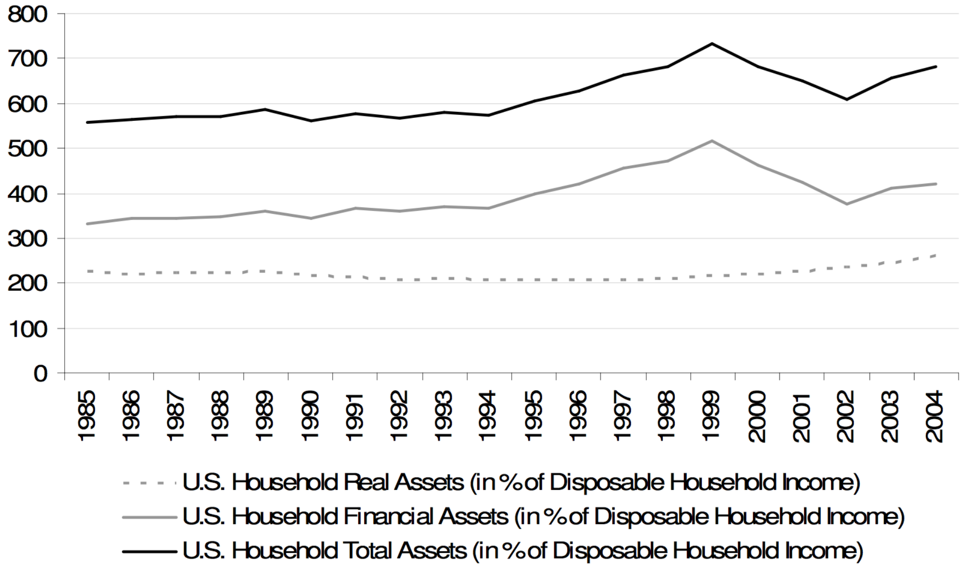
[4] Empirical studies on the equity risk premium do not seem to take the theoretical point of Arrow too seriously. Maybe this is because, in any applied study, the range of payoffs is bounded so that going to the limit of extreme payoffs, as Arrow does in his argument, is out of the range in which the observations are made. Luenberger (1996) and Hellwig (1995) suggest consistency with Arrow's argument can be maintained by postulating a unit relative risk aversion for extreme payoffs only.

Figure 1: Risk Aversion and Relative Background Wealth



three to six times the participants' perceived income, when interpreting Question 1 in the way of (1). This size of background wealth is reasonable, as it should include the participant's income, the value of her house, land and human capital. Figure 2 shows real and financial assets of U.S. households in percentage of their disposable income (source: OECD, taken from Datastream). The representative household's total assets constituted between 550% and 750% of its income during 1985–2004. Notice that these figures serve only as a lower bound for w in a Rubinstein-Lucas type economy, since w should represent the price at which the representative agent can buy the entire economy.

Figure 2: Household Data on Real and Financial Assets



We have so far considered narrow framing by the degree of integration of background wealth. Classic finance theory implies that optimizing 'rational' agents should not consider a risky prospect in isolation from other risky prospects, as pointed out by for example Campbell and Viceira (2002, page 10): "*More specifically, we believe that any normative model should judge a portfolio by its total value, rather than by the values of the individual assets it contains, and should ultimately be based on the standard of living that the portfolio supports.*" Kahneman and Tversky (1979), and many others, have observed from laboratory experiments, however, that individuals typically use 'narrow frames' and separate different lotteries from each other. Hence from a prospect theory perspective the first interpretation of the answer to Question 1 is the correct one and the implied relative risk aversion is in the range of two to four.

If individuals' tendency for narrow framing aggregates, however, and one assumes (no) narrow framing at the level of individuals, one should also interpret stock market data in the context of (no) narrow framing. As we show below, the degree of risk aversion implied by stock market data is in the same range of small (large) numbers when one assumes (no) narrow framing. This is our main point.

One may argue that the above argument is invalid because of inappropriate designs of experiments. For instance Kandel and Stambaugh (1991) claims that "*It seems possible in such experiments to choose the size of the gamble so that any value of [risk aversion] seems unreasonable.*" Their claim is based on the observation that the average answer to Question 1 does not depend on the size of the gamble, and that changing background wealth changes risk aversion. The argument assumes, however, a specific form of participants' utility functions. Rabin (2000) makes a rigorous argument for the observation of Kandel and Stambaugh, but arrives at the opposite conclusion. While risk aversion estimates in the standard expected utility model vary wildly between low and high stake lotteries, the experimental data yields reasonable estimates when background wealth is not integrated. Recent experimental results support Rabin's point of view. Post et al. (2008) analyze participation in simple TV lotteries that are played in 140 countries across the world.[5] Lottery stakes can reach as high as 1.5 million U.S. Dollars. They find that the average degree of risk aversion is about three, and is independent of the size of the stakes.

[5] See also Bombardini and Trebbi (2005) for an analysis of Italian TV game shows.

3 Risk Aversion in Stock Market Studies

Based on the observations in Section 2, the challenge is to confirm if the low risk aversion observed in experimental data matches the risk aversion obtained from stock market data when the latter is analyzed using comparable auxiliary assumptions. We therefore proceed to set up a model of individuals with various degree of narrow framing, demonstrate that it aggregates, and calibrate the representative agent's risk aversion to observed equity returns.

Consider a discrete time model, $t = 1, 2, \ldots, T$, with $k = 1, .., K$ assets. Let p_t^k be the price and D_t^k the dividends paid by asset k in period t. Investors are denoted $i = 1, 2, ..., I$, have an initial endowment of assets $\theta_0^i \in \mathbb{R}_+^K$, and an exogenous flow of income y_t^i.[6] The aggregate endowment is normalized, so that $\sum_{i=1}^I \theta_0^{i,k} = 1$ for all k.

To allow different degrees of narrow framing while maintaining tractable notation we introduce the following consumption 'goods' that individuals may derive utility from: The income good $c_t^{i,0} = y_t^{i,0}$, the investment k good $c_t^{i,k} = D_t^k \theta_{t-1}^{i,k} + p_t^k \left(\theta_{t-1}^{i,k} - \theta_t^{i,k}\right)$, and the portfolio good $c_t^{i,d} := \sum_{k=1}^K c_t^{i,k}$—the latter two including both dividend and capital gains. Similarly, the aggregate consumption good is denoted by $c_t^{i,a} := \sum_{k=0}^K c_t^{i,k}$, while $c_t^i = (c_t^{i,0}, c_t^{i,1}, \ldots, c_t^{i,K})$ denotes the vector of the consumption goods.

Investors are homogeneous with respect to their utility functions $u : \mathbb{R}_+^{K+1} \to \mathbb{R}$,[7] respectively $u : \mathbb{R}_{++}^{K+1} \to \mathbb{R}$,[8] and discount factors β, such that

$$U_t(c^i) = \mathrm{E}_t \left\{ \sum_{\tau=t}^T \beta_\tau u(c_\tau^i) \right\}, \text{ for all } c^i \in \mathbb{R}_+^n \text{ (respectively } c^i \in \mathbb{R}_{++}^n). \qquad (2)$$

We consider the standard model with no narrow framing, as well as two sequentially more severe cases of narrow framing.

(i) Integration of all payoffs:

$$u(c_\tau^{i,0}, ..., c_\tau^{i,K}) = v(c_\tau^{i,a})$$

[6] We use standard assumptions about measurability and information filtrations throughout.

[7] We make this assumption for simplicity of exposition. It is possible to extend our aggregation results to heterogeneous time and risk preference, as for example in Theorem 14.1 of Shefrin (2008).

[8] As we see below, the restriction of the domain to strictly positive numbers is important if the agent has unit relative risk aversion.

(ii) Separation of portfolio payoffs:

$$u(c_\tau^{i,0}, ..., c_\tau^{i,K}) = v(c_\tau^{i,0}) + v(c_\tau^{i,d})$$

(iii) Separation of all payoffs:

$$u(c_\tau^{i,0}, ..., c_\tau^{i,K}) = \sum_{k=0}^{K} v(c_\tau^{i,k})$$

Agents with preferences described in (i) take background wealth into account (as well as possibly other factors) because their decision problem—described below—can be reformulated in terms of a Bellman equation with current wealth as a state variable. One may want to specify a separate Bernoulli utility for $c^{i,0} = y^i$ in (ii) and (iii) to stay closer to the analysis in Section 2. How we specify utility from $c^{i,0} = y^i$ is not important for pricing purposes and thereby estimation of risk aversion, however, as will become clear shortly.

One can motivate the extreme case of asset specific mental accounts, as in (iii), by the robust finding that investors underdiversify. Friend and Blume (1975) study a sample of 17 056 accounts of individual investors. They find that 34.1% hold only one stock, 50% hold at most two stocks, and only 10% hold more than 10 stocks (See also Kelly, 1995; Odean, 1999; Polkovnichenko, 2005; Goetzman and Kumar, 2004).

Investors are price takers, and are faced with the following problem at date t:

$$\max_{\theta^i} U_t(c) \text{ s.t. } c_\tau^{i,a} \le y_\tau^i + \sum_{k=1}^{K} D_\tau^k \theta_{\tau-1}^{i,k} + \sum_{k=1}^{K} p_\tau^k \left(\theta_{\tau-1}^{i,k} - \theta_\tau^{i,k}\right), \ \tau = t, \ldots, T-1. \quad (3)$$

A *financial market equilibrium* for this economy consists of a process of asset prices p and I consumption processes $c^i, i = 1, \ldots, I$ such that every consumption process solves (3) and all markets clear, i.e. $\sum_i \theta_t^{i,k} = 1$ and $\sum_i c_t^{i,a} = \sum_i y_t^i + \sum_{k=1}^K D_t^k$, $t = 1, 2, \ldots, T$. We show in Appendix B that the equilibrium prices can also be obtained from the decision problem of a representative agent who inherits the degree of narrow framing in (i)–(iii). Because the representative agent has to hold all assets and consume all income the possible first order conditions become:

(i) Integration of all payoffs—utility from *per capita* consumption:

$$p_t^k = v'(c_t^a)^{-1}\beta_t \mathrm{E}_t \left\{v'(c_{t+1}^a)(D_{t+1}^k + p_{t+1}^k)\right\} \text{ where } c_t^a = \sum_{i=1}^{I} y_t^i + \sum_{k=1}^{K} D_t^k \ \forall k, t. \quad (4)$$

(ii) Separation of asset payoffs from income:

$$p_t^k = v'(c_t^d)^{-1}\beta_t \mathrm{E}_t \left\{v'(c_{t+1}^d)(D_{t+1}^k + p_{t+1}^k)\right\} \text{ where } c_t^d = \sum_{k=1}^{K} D_t^k \ \forall k, t. \quad (5)$$

(iii) Separation of all payoffs:

$$p_t^k = v'(c_t^k)^{-1}\beta_t \mathrm{E}_t \left\{v'(c_{t+1}^k)(D_{t+1}^k + p_{t+1}^k)\right\} \text{ where } c_t^k = D_t^k \ \forall k, t. \quad (6)$$

It follows that the SDF in the case of constant relative risk aversion can be expressed for all three cases as

$$\beta_t \left(\frac{c_{t+1}^*}{c_t^*}\right)^{-\gamma}, \text{ for } * = a, d, k. \quad (7)$$

Table 2 shows that Hansen-Jagannathan bounds (Hansen and Jagannathan, 1991) for model (ii) are in line with the observed Sharpe ratio of 0.3459 for a risk aversion in the interval 2.25–2.50 when agents derive utility from aggregate dividends, compared to the less volatile *per capita* consumption in model (i).[9] Narrow framing in the sense of (ii) thus makes the market inferred risk aversion consistent with the experimental data.

The extent or nature of narrow framing is not known, and it may be that agents keep separate mental accounts for each asset payoff, as in (iii). The asset specific SDFs are more volatile than the SDF based on aggregate dividends because dividends are not perfectly correlated. Consequently, the corresponding Hansen-Jagannathan bounds are in line with the observed Sharpe ratio of 0.3459 for an even lower risk aversion—not higher than $\gamma = 0.5$ (Table 3, in Appendix A).

Recall that the laboratory evidence on risk aversion indicate a value of γ of about three. This means that narrow framing that evaluate aggregate risky payoffs (model (ii), with γ about 2.5) fits the laboratory evidence better than narrow framing at the level of individual payoffs (model (iii), with $\gamma \leq 0.5$).

[9] This is essentially the same effect as that observed by Bakshi and Chen (1996) and Epstein and Zin (1991), when the SDF is a function of wealth—which is also more volatile than consumption.

Table 2: Hansen-Jagannathan Bounds Based on Annual Returns Since 1889

Risk aversion	Consumption SDF	Aggregate Dividend SDF
1.00	0.0358	0.1260
1.50	0.0540	0.2014
2.00	0.0723	0.2893
2.25	0.0816	0.3391
2.50	0.9090	0.3935
2.75	0.1002	0.4530
3.00	0.1096	0.5181
3.22	0.1180	0.5805
4.00	0.1479	0.8640
5.00	0.1873	1.3082
10.00	0.4084	5.2337
15.00	0.6889	7.4788
20.00	1.0560	8.1135

The SDF in column two is based on *per capita* consumption, while the SDF in column three is based on aggregate dividends accruing to the S&P 500. The S&P 500 Sharpe ratio is 0.3459, when using the one year riskless rate (source: Online data provided by Robert Shiller)

4 Conclusion

Estimates of risk aversion from experiments are typically lower than those based on observed market prices. We demonstrate that the estimates become consistent if the data is interpreted using consistent assumptions about narrow framing: either no narrow framing in both settings, or some degree of narrow framing in both settings. If one postulates a high level of risk aversion, to make the standard consumption-based model consistent with the observed Hansen-Jagannathan bounds (typically referred to as "the equity premium puzzle," starting with Mehra and Prescott, 1985), then interpreting the experimental data within the same framework leads to similarly high levels of risk aversion. If one postulate instead that individuals have narrow frames, as typically assumed in laboratory experiments, then we find that a similar extent of narrow framing by a representative agent implies Hansen-Jagannathan bounds that are consistent with observed market prices at low levels of risk aversion. Thus, with a consistent application of mental accounting in both settings there is no disagreement between market- and experiment-based estimates of risk aversion.

We hope that this note encourages further work to bridge experimental analysis and market analysis. For example, other data sets should be considered and also more elaborate utility models that have recently been developed. In particular, this note does not offer an answer to what level of narrow framing investors actually apply, and to what extent subjects' behavior in the laboratory carries mirrors behavior by real investors.

Appendix

A Evidence from individual stocks

Table 3: Sharpe ratios and Hansen-Jagannathan bounds for cash-dividend paying single stocks in the S&P 100, based on annual data ranging from 1973 to 2005. Column one lists the firms that pay cash dividends to their investors. Column two reports the Sharpe ratio with respect to the one year interest rate from Robert Shiller. Column three gives the bounds on the Sharpe ratio imposed by the SDF (7), when consumption equals the individual firm's dividends accruing to the single stocks. The risk aversion is $\gamma = 0.5$ for the Hansen-Jagannathan bounds column. Data is taken from Datastream.

Firm	Sharpe ratio	H-J bound ($\gamma = 0.5$)
Alcoa	0.1496	1.4748
Allegheny Technologies	0.2577	1.3468
Altria Group	0.0717	1.3447
American Electric Power	0.2606	2.7210
American Express	-0.0518	1.6604
American International Group	0.1667	2.0281
Anheuser-Busch	0.3113	1.5272
AT&T (delisted on 21/11/05)	0.1741	1.8193
Avon Products	-0.1091	1.6548
Baker Hughes	0.0912	2.1958
Bank of America	-0.1685	1.7244
Baxter International	0.3532	1.8626
Black & Decker	0.1959	1.5114
Boeing	0.0460	1.5100
Bristol Myers Squibb	0.2461	1.6350
Burlington Northern Santa Fe	0.3017	1.4064
Campbell Soup	0.2102	1.6531
Cigna	0.1400	1.4744
Citigroup	0.3687	1.4225
Coca Cola	0.3850	1.5730
Colgate-Palm	0.2436	1.7702
Delta Air Lines	0.2819	1.5904
Dow Chemical	0.0329	1.3124
Du Pont de Nemours	0.1199	1.4771
Eastman Kodak	0.1851	2.1841
Entergy	0.2935	2.4352
Exelon	-0.1547	1.6073
Exxon Mobil	0.1314	1.7538
Ford Motor	0.3205	1.3830

General Dynamics	0.1822	1.4273
General Electric	0.1896	1.5351
General Motors	0.2663	1.6609
Gilette (delisted on 20/10/05)	-0.2211	1.6583
Halliburton	0.2434	1.4235
Heinz	0.1532	1.8242
Hewlett-Packard	0.2116	1.4290
Home Depot	0.3549	1.5758
Honeywell International	0.4737	1.6266
Intel	0.0082	1.4191
International Business Machines	0.5629	1.2927
International Paper	0.1310	1.7509
Johnson & Johnson	0.1143	1.4640
J.P. Morgan Chase	0.2596	1.5224
Limited Brands	0.3611	1.4288
May Department Stores (delisted on 14/09/05)	0.4110	1.4893
McDonalds	0.1697	1.8132
Medtronic	0.1740	1.6205
Merck	0.3922	1.2845
Merrill Lynch	0.2123	1.7123
OfficeMax	0.0913	2.0470
PepsiCo	0.4637	1.3637
Pfizer	0.2672	1.5403
Procter & Gamble	0.3217	1.7954
Radioshack	0.1651	1.8107
Raytheon	0.1772	1.6750
Rockwell Auto,mation	0.2357	1.8199
Sara Lee	0.0651	1.6433
AT&T	0.2318	1.5135
Schlumberger	0.2257	1.7056
Sears Roebuck	0.1735	1.9559
Southern	-0.2514	2.2301
Texas Instruments	0.0680	1.3499
Tyco International	0.2321	1.7204
Unysis	0.2480	1.2928
United Technologies	0.1540	1.7664
Verizon Communications	0.2749	1.7346
Wal-Mart Stores	0.1932	1.4135
Wells Fargo	0.2480	1.6679
Weyerhaeuser	0.3638	1.5376
Xerox	0.2684	1.4323
3M	0.1081	1.7750

B Pareto Efficiency and Aggregation

In this section we show that financial market equilibria are Pareto efficient, that asset prices must satisfy the first order condition of the decision problem of a so called ”representative investor,” and that the utility function of the representative investor inherits the degree of narrow framing from individual investors. The results assume complete markets, and is based on arguments that goes back to Negishi (1960), later introduced into finance by Constantinides (1982). Our derivation is based on Magill and Quinzii (1996), and does not reproduce parts that are invariant to our formulation. We rather point out the necessary departures caused by narrow framing.

To this end let us first compare the case of separation of income from asset payoffs that are integrated in one account (case (ii)), to the standard case without any separation (case (i)). For the latter case we know from Theorem 25.7 in Magill and Quinzii (1996) that equilibria are Pareto-efficient and that there exist some positive weights ν^i such that the utility of the representative agent can be obtained by (Magill and Quinzii, 1996, p. 275)[10]

$$U^R(W^R) = \max_{c^i} \sum_i \nu^i U(c^i) \text{ s.t. } \sum_i c_t^{i,a} = \sum_i y_t^i + \sum_{k=1}^{K} D_t^k = W_t^R, \ \forall \ t = 1, ..., T.$$

Moreover, the additive separable structure of individual utility functions is inherited by the representative investor. This follows because one can interchange the summation across agents with that across time periods and across states of the information filtration, in the definition of the utility of the representative investor. Still, this result assumes:

(A) U^i are infinitely differentiable, strictly increasing, strictly concave, and satisfy Inada conditions (marginal utility tends to infinity as consumption tends to zero.)

(B) In every node ξ of the information filtration the matrix of subsequent dividends $[D^1(\xi), \ldots, D^K(\xi)]$ has rank equal to the number of subsequent nodes.

Note that with expected utility functions of the constant relative risk aversion type assumption (A) is satisfied. Moreover, given assumption (B), markets are complete for a generic set of exogenous wealth $y^i, i = 1, ..., I$ (Magill and Quinzii, 1996).

[10] The solution to this maximization problem exists since the individual utility functions are assumed to be continuous and the choice variable $c^{i,a}$ are bounded below by 0 and bounded above by the constraint given in the decision problem.

Case (ii) differs from the textbook case (i) by the utility from income being separated from the utility from asset payoffs (dividends and capital gains). Since only assets are traded in the model, the income is exogeneously given and does not affect Pareto efficiency. Hence the utility function of the representative investor is again a weighted average of the individual utility functions.

$$U^R(W^R) = \max_{c^i} \sum_i \nu^i \mathrm{E}_t \left\{ \sum_{\tau=t}^{T} \beta_\tau v(c_\tau^{i,0}) + v\left(\sum_{k=1}^{K} c_\tau^{i,k} \right) \right\}$$

$$s.t. \sum_i c_t^{i,a} = \sum_i y_t^i + \sum_{k=1}^{K} D_t^k = W_t^R \text{ for all } t = 1, ..., T.$$

Clearly one can also in this case interchange the summation across agents with that across time periods and across states of the information filtration. It follows that the representative investor inherits the separability from the individual investors.

This last step of the argument is also true for case (iii). However, in this case the preceeding step is not so immediate: Condition (B) on the assets' dividends is no longer sufficient to argue that any consumption stream $(c^0, c^1, ..., c^K)$ can be attained through a dynamic trading strategy. The set of attainable consumption streams now depend on the asset prices:

$$c_\tau^{i,k} = D_\tau^k \theta_{\tau-1}^{i,k} + p_\tau^k \left(\theta_{\tau-1}^{i,k} - \theta_\tau^{i,k} \right)$$

As Magill and Quinzii (1996) show, in general this difficulty can destroy Pareto efficiency. However in the case of identical and homothetic preferences, which is the case we consider in this paper, one can still show a Pareto efficiency property and the aggregation argument works.[11] The Pareto effiency concept has to be constrained so that the alternative allocations that are compared to the financial market allocations are determined by the same trading possibilities as the asset markets offer.

We conclude that the separability properties given in case (i) and (ii) aggregate for any degree of heterogeneity of the agents while the third case (iii) holds only for investors with identical constant relative risk aversion.

[11] This is case (b) of Magill and Quinzii (1996), page 270.

References

Arrow, Kenneth J., *Essays in the Theory of Risk Bearing*, North-Holland, 1971.

Bakshi, Gurdip S. and Zhiwu Chen, "The Spirit of Capitalism and Stock-Market Prices," *American Economic Review*, March 1996, *86* (1), 133–157.

Barberis, Nicholas C. and Ming Huang, "The Loss Aversion/Narrow Framing Approach to the Equity Premium Puzzle," in Rajnish Mehra, ed., *Handbook of the Equity Risk Premium*, North-Holland, 2008, chapter 1.

Barsky, Robert B., F. Thomas Juster, Miles S. Kimball, and Matthew D. Shapiro, "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *Quarterly Journal of Economics*, May 1997, *111* (2), 537–579.

Bombardini, Matilde and Francesco Trebbi, "Risk Aversion and Expected Utility Theory: A Field Experiment with Large and Small Stakes," 2005. Mimeo, University of British Columbia and Harvard University.

Campbell, John Y. and Luis M. Viceira, *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*, Oxford University Press, 2002.

Constantinides, George M., "Intertemporal Asset Pricing with Heterogeneous Consumers and Without Demand Aggregation," *Journal of Business*, 1982, *55*, 253–267.

Epstein, Larry G. and Stanley E. Zin, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, April 1991, *99*, 263–286.

Friend, Irwin and Marshall E. Blume, "The Demand for Risky Assets," *American Economic Review*, December 1975, *65* (5), 900–922.

Goetzman, William N. and Alok Kumar, "Why Do Individual Investors Hold Under-diversified Portfolios?," 2004. Working Paper, Yale University.

Gollier, Christian, *The Economics of Risk and Time*, MIT Press, 2001.

Hagiwara, May and Miguel A. Herce, "Risk Aversion and Stock Price Sensitivity to Dividends," *American Economic Review*, September 1997, *87* (4), 738–475.

Hansen, Lars Peter and Ravi Jagannathan, "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy*, 1991, *99* (2), 225–262.

Harrison, Glenn W. and Elisabet Rutström, "Risk Aversion in the Laboratory," *Reserach in Experimental Economics*, 2008, *12*, 41–196.

Hellwig, Martin F., "The Assessment of Large Compounds of Independent Gambles," *Journal of Economic Theory*, 1995, *67* (2), 299–326.

Kahneman, Daniel and Amos Tversky, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, March 1979, *47* (2), 263–291.

_ **and** _ , "Choices, Values, and Frames," *American Psychologist*, April 1984, *39* (4), 341–350.

Kandel, Shmuel and Robert F. Stambaugh, "Asset Returns and Intertemporal Preferences," *Journal of Monetary Economics*, February 1991, *27*, 39–71.

Kelly, Morgan, "All their Eggs in one Basket: Portfolio Diversification of US Households," *Journal of Economic Behaviour and Organization*, June 1995, *27* (1), 87–96.

Lucas, Robert E. Jr., "Asset Prices in an Exchange Economy," *Econometrica*, November 1978, *46* (6), 1429–1445.

Luenberger, David G., "A Preference Foundation for Log Mean-Variance Criteria in Portfolio Choice Problems," *Journal of Economic Dynamics and Control*, 1996, *17* (5–6), 887–906.

Magill, Michael and Martine Quinzii, *The Theory of Incomplete Markets, Volume 1*, Cambridge, Massachusetts: MIT Press, 1996.

Mankiw, N. Gregory and Stephen P. Zeldes, "The Consumption of Stockholders and Nonstockholders," *Journal of Financial Economics*, March 1991, *29* (1), 97–112.

Mehra, Rajnish and Edward C. Prescott, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, March 1985, *15*, 145–161.

Mossin, Jan, "Optimal Multiperiod Portfolio Policies," *Journal of Business*, April 1968, *41* (2), 215–229.

Negishi, Takashi, "Welfare Economics and Existence of an Equilibrium for a Competitive Economy," *Metroeconomica*, June 1960, *12* (2-3), 92–97.

Odean, Terrance, "Do Investors Trade Too Much?," *American Economic Review*, December 1999, *89* (5), 1279–1298.

Polkovnichenko, Valery, "Household Portfolio Diversification: A Case for Rank-Dependent Preferences," *Review of Financial Studies*, Winter 2005, *18* (4), 1467–1502.

Post, Thierry, Martijn J. van den Assem, Guido Baltussen, and Richard H. Thaler, "Deal or No Deal? Decision Making under Risk in a Large-Payoff Game Show," *American Economic Review*, March 2008, *98* (1), 3871.

Rabin, Matthew, "Risk Aversion and Expected-utility Theory: A Calibration Theorem," *Econometrica*, September 2000, *68* (5), 12811292.

Read, Daniel, George Loewenstein, and Matthew Rabin, "Choice Bracketing," *Journal of Risk and Uncertainty*, 1999, *19* (1), 171–197.

Redelmeier, Donald A. and Amos Tversky, "On the Framing of Multiple Prospects," *Psychological Science*, May 1992, *3* (3), 191–193.

Rubinstein, Mark, "An Aggregation Theorem for Securities Markets," *Journal of Financial Economics*, 1974, *1*, 225–244.

Samuelson, Paul A., "Long-Run Risk Tolerance When Equity Returns Are Mean Regressing: Pseudoparadoxes and Vindication of "Businessman's Risk"," in William C. Brainard, William D. Nordhaus, and Harold W. Watts, eds., *Money, Macroeconomics, and Economic Policy: essays in honor of James Tobin*, MIT Press, 1991, chapter 7, pp. 181–200.

Shefrin, Hersh M., *Beyond Greed and Fear: Understanding Behavioral Finance and the Psychology of Investing*, Oxford, United Kingdom: Oxford University Press, 1999.

_ , *A Behavioral Approach to Asset Pricing*, Academic Press, 2008.

Tversky, Amos and Daniel Kahneman, "Rational Choice and the Framing of Decisions," *Journal of Business*, October 1986, *59* (4), S251–S278.