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Five essays on risk analysis in agriculture

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Abstract

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Agricultural production is typically a risky business. For many decades governments around the world have intervened in order to try to help farmers cope more effectively with risk. Both national and international developments have led many countries to reorientate their agricultural policies towards deregulation and a more market-oriented approach. Much of the protection that farmers have had from the vagaries of the market may therefore be removed. Thus, it can be expected that in the future risk management in agriculture will receive increased attention from farmers, agricultural advisers, commercial firms, agricultural researchers, and policy makers. The objective of the three first essays in the dissertation is to contribute to the available formal methods of farm planning under uncertainty. Such methods are usually based on the propositions, not always made explicit, that farmers are risk averse and that the opportunities for them to trade away the risks they face in markets are constrained. The last two essays are studies of risk in the markets for agricultural commodities, and the objective is to improve the understanding of how the agricultural derivative markets work and to develop an option pricing model for commodity futures options.

Essay 1 outlines an alternative method for estimating decision maker's risk aversion. The method uses the expected value-variance ($E-V$) framework and quadratic programming. An empirical illustration is given using Norwegian farm-level data.

Essay 2 provides a two-stage utility-efficient programming approach to modelling integrated dairy and cash crop farming in a whole-farm context that includes both embedded and non-embedded risk. The model is used to provide insight into the impacts of degree of risk aversion, subsidy schemes and the choice of utility function on optimal farm plans in Norwegian agriculture.

In essay 3 a stochastic budgeting model that simulate the business and financial risk and the performance over a medium term planning horizon is presented. Some methods to account for stochastic dependencies are outlined. In contrast with earlier studies with stochastic farm budgeting, the option aspect is included in the analysis.

The objective in essay 4 is to model the spot-price process for an agricultural product, where we find that adding a jump component to a diffusion process contributes to a better fit on monthly spot wheat data from 1952 to 1998 in Atlanta.

Essay 5 investigates implication that price jumps and the volatility term-structure have for option pricing of agricultural futures commodities. We extend a jump-diffusion model to include both seasonal and maturity effects in volatility. An in-sample fit to market option prices of Chicago Board of Trade wheat futures from 1989 to 1999 shows that our model outperforms models previously described in the literature.

Keywords: Risk analysis; Risk aversion; Utility function; Mathematical programming; Stochastic budgeting; Simulation; Derivative pricing; Jumps; GMM; MLE; Term structure of volatility; Agricultural markets; NLS.

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Preface

My dissertation was financed through a four-year grant at the Norwegian Agricultural Economics Research Institute (Norsk Institutt for Landbruksøkonomisk Forskning (NILF)), from 1997 to 2001. The dissertation work is a part of the 'AGROMANAGEMENT' strategic research program at NILF financed by a grant from the Research Council of Norway. I gratefully acknowledge NILF for excellent and stimulating working environment and for financial support.

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Gudbrand Lien

Introduction

1 Background and objectives

Agricultural production is typically a risky business. The profitability of farming depends on many uncertain factors such as weather conditions, biological variability in the performance of crops or livestock, prices of farm inputs and outputs, government policies and regulations that affect agriculture, fluctuations in inflation and interest rates, ecological risk, and human risk. Because agriculture is often carried out in the open air, and always entails the management of inherently variable living plants and animals, it is exposed to particular risk. In many cases, farmers are also confronted by the risk of catastrophe. For example, crops may be completely destroyed by hurricane, fire, drought, pest or diseases, and product prices may plummet because of sudden and unexpected adjustment in world markets. The people who operate the farm may themselves be a source of risk for the profitability of the farm business. Major personal crises such as death, serious illness and break-up of relationships can seriously threaten the viability of a family farm business.

Given these concerns, it is hardly surprising that governments around the world have intervened in order to try to help farmers cope more effectively with risk. Governments' interventions in agricultural markets have varied much over time and between countries. During recent decades, at least some sources of risk have been eliminated by various government regulations and price support schemes, such as the Common Agricultural Policy in the EU, the farm support programmes in the USA, as well as the Norwegian Agricultural Policy.

Both national and international developments have prompted many countries to start to reorientate their agricultural policies towards deregulation and a more market-based approach. Compared to farmers of the past, tomorrow's farmers will have to be much more flexible and adapt to changing policies, increased influence of the market on price development, increased competition, new environmental considerations, and regulations and new consumer patterns and demands. With a shift towards less government intervention and less regulation, a more sophisticated understanding of risk, risk management and the markets

will be needed to help producers to make better decisions in risky situations and to assist analysts, advisers and policy makers in assessing the effectiveness of different types of risk management tools.

What strategies can farmers (and other agricultural businesses) employ to deal with risk? Based on Hardaker et al. (1997) and Harwood et al. (1999), farmers' available strategies to manage agricultural risks can be divided into two broad categories: on-farm risk-management strategies, and strategies to share risk with others.

Strategies in the first category are: *collecting information*, a process of refining subjective prior distributions in the light of accumulating information; *avoiding or reducing exposure to risk*, such as 'wait and see' strategies and precautionary principles; *selecting less risky technologies/production activities*, e.g., selecting production activities with more or less guaranteed prices by the government before those for which output prices are determined in a fluctuating world market, and the use of production contracts that give the farmer an assured market, often in return for the buyer of the commodity having considerable control over the production process; *diversification*, such as selecting a mixture of farm activities that have net returns with low or negative correlation. Off-farm income is also a form of diversification, and diversifying into financial assets may yield important gains in risk efficiency for farm households. Another opportunity to spread risk is spatial diversification, meaning owning farms in several locations sufficiently widely scattered to reduce positive correlation due to weather effects; *flexibility*, meaning selecting farm production activities that can be adjusted to changing circumstances. Farmers can enhance flexibility by such choices as investing in assets that have multiple uses, maintaining financial reserves in the form of liquidity and solvency to carry the business through low-income or loss periods, producing products that have more than one end use or enterprises that yield more than one product, selling product in different markets, leasing inputs and hiring custom labour, choosing activities with short production cycles, etc.

Strategies to share risk with other groups include: *farm financing*, such as the financial leverage impacts of variability of farm returns, and the dynamics of financing; *insurance*, such as fire and theft cover for assets, yield insurance, revenue insurance, etc.; *contract marketing and derivative trading*, e.g. cooperative marketing with price pooling, forward contracts for commodity sales or input delivery, price risk management by the use of futures price contracts and futures options contracts, market-based instruments for managing yield

risk, revenue risk management by combining yield contracts with futures price contracts. Most producers combine the use of many different strategies and tools.

What strategies and tools farmers, analysts, advisers and policy makers can and should use or advocate depend on how deregulated the market is and to what extent derivative markets exist. Less government intervention and less regulation will imply more uncertainty about farmer's input and output prices and marketing possibilities for the products, and this will in turn have implication for farm organisation and the farmer's needs for decision supports.

In addition, in the agricultural commodity derivative markets are expected to become more and more important in the coming years with government deregulation and liberalised international trade (Carter, 1999). The economic functions of derivative markets are to reallocate risk (hedging), and provide valuable information for the farmer's (or in general the decision maker's) management and adjustment.

An assumption about whether or not the Separation Theorem¹ holds is necessary in farm planning under uncertainty. The Separation Theorem states, in the context of farming that if markets are efficient then the investment and production decision is not influenced by risk preferences.

However, there are many reasons to believe that efficient markets for risk are normally an unrealistic assumption for farmers today. First, even if efficient markets for risk exist, it seems clear they are not good enough to reallocate all risk on a farm. Farmers normally have a large part of their assets placed in agriculture, so their portfolios are not well diversified. Poorly diversified investment portfolios imply that the farmers require a return premium for the unsystematic risk that in principle could be eliminated by diversification. Second, agricultural assets are less easily traded than stock market assets, implying investors will require a return premium for illiquidity (Bjornson and Innes, 1992). Third, trades with agricultural assets are often regulated, and that violates the assumption of perfect capital markets.

In farm planning under uncertainty where the market for risk is not good enough to reallocate all risk so that the Separation Theorem holds, it is necessary to account for the individual decision maker's risk preferences. The objective of the three first essays in the dissertation is to outline formal methods of farm planning under uncertainty without any requirement of

¹ Fisher's Separation Theorem is described, for example, by Copeland and Weston (1988).

perfect capital markets or efficient markets for risk. The methods used in these essays can then be applied as decision support in existing regulated regimes, such as exist in Norway.

A farm plan for a person who is risk averse should normally be quite different from a plan for a risk neutral person. The decision maker's attitude to risk is a necessary input in many models of farm planning under uncertainty. *Essay 1* presents an alternative method for estimating decision maker's risk aversion.

When searching for the optimal (risk-efficient) portfolio of farm production activities and technologies, programming models accounting for risk are appropriate. These models are suitable, for example, for product-mix and factor-mix under different policy scenarios. *Essay 2* provides an approach to modelling integrated dairy and cash crop farming in a whole-farm context that includes both embedded and non-embedded risk. This model is used to provide insight into the impacts of degree of risk aversion, subsidy scheme and the choice of utility function, on optimal farm plans in Norwegian agriculture.

In assessing any business investment, particularly for a family business such as a farm, there are two aspects to consider. One is the profitability of the investment, which is often a fairly long-run matter. The future is shrouded in uncertainty so such decisions often involve a high degree of intuition or strategic thinking. The other aspect is financial feasibility. Usually large investments involve borrowing substantial amounts of money, implying a significant increase in the financial risk of the business. For example, a couple of bad years in production and an unexpected rise in the interest rates can bankrupt the business. This risk is most severe in the first years after the investment when the debt is at a peak. *Essay 3* presents a whole-farm stochastic budgeting model of the business and financial risk of the farm over such a shorter time horizon.

The last two essays in the dissertation reflect a switch away from the farm planning aspect to studies of risk in the markets for agricultural commodities. These two essays are on modelling the uncertainty in market prices and investigate implication for pricing of derivatives. Essays 4 and 5 are based on the assumption of perfect capital and derivative markets. The results are useful for advisers, analysts and policy makers (and perhaps farmers) in deregulated regimes.

A critical factor for correct pricing of derivatives is the description of the stochastic process governing the behaviour of the basic asset. The objective in *essay 4* is to model the spot-price process for an agricultural product. In particular, we investigate whether adding a jump

component to a diffusion process contributes to a better fit on monthly spot wheat data from 1952 to 1998 in Atlanta.

In *essay 5* the focus is on the forward curve dynamics of wheat futures prices and their implications for option pricing. We present an option pricing model that incorporates several stylised facts reported in the literature on agricultural commodity futures price dynamics. In our option pricing model futures prices are allowed to make sudden discontinuous jumps and both seasonal and maturity effects are included in the volatility function. Wheat data on futures and futures options from Chicago Board of Trade for eleven years are used in an in-sample performance fit.

This introduction proceeds as follows. For each of the essays, existing literature, methodology, and results found are summarised in section two. In section three some opportunities for further research is discussed. Section four contains concluding remarks.

2 Methodology and results

Essay 1: Non-parametric estimation of decision makers' risk aversion

A survey of different approaches to specifying decision maker's risk attitudes is given in Robison et al. (1984). The following approaches have been utilised to assess risk attitudes: (1) direct elicitation of utility functions (see Anderson et al., 1977; or Hardaker et al., 1997 for details); (2) experimental procedures in which individuals are presented with hypothetical questionnaires regarding risky alternatives with or without real payments (e.g. Dillon and Scandizzo, 1978; Binswanger, 1980); and (3) inference from observation of economic behaviour, based on an assumed relationship between the actual behaviour of a decision maker and the behaviour predicted from empirically specified models. Empirical inference of risk attitudes from observed economic behaviour can be divided into mathematical programming (e.g. Simmons and Pomareda, 1975; Brink and McCarl, 1978; Hazell et al., 1983; Wiens, 1976) and econometric approaches (e.g. Moscardi and de Janvry, 1977; Antle, 1987; Bar-Shira et al. 1997).

All of these approaches have pros and cons. To find the decision maker's 'real' risk attitude is very difficult (and may be impossible). It will either require much work and experience with

interviews and problems of inconsistency of the farmer's risk attitude over time etc.² (direct elicitation of utility functions, experimental procedures) or you have to deal with decision maker's beliefs in the past (inference from observation of economic behaviour). Compared with the programming approach, the econometric approach has the advantage of straightforward hypotheses testing. On the other hand, non-parametric methods offer greater flexibility in modelling the firm/farm situation.

My contribution within the field of estimating decision maker's risk attitude is within the mathematical programming approach in an expected value-variance ($E-V$) framework. By combining solutions from the $E-V$ formulation of Markowitz (1952) and Freund (1956) to derive the efficient frontiers using historical data for a decision maker or a group, I am able to approximate the coefficient of absolute risk aversion. In more detail, my approach is as follows: First, formulate the Quadratic Risk Programming (QRP) model to represent the farm's resource base, activities, expected activity net revenues per unit level and fixed costs. The model also includes a variance-covariance matrix of activity net revenues derived to reflect the decision maker's beliefs. The model is there designed to represent the farmer's circumstances and perceived decision options as closely as possible. Second, for an observed farm plan presumed to reflect a farmer's risk-averse behaviour, calculate expected net farm income and variance. Third, solve the QRP problem setting expected net farm income equal to the farm's observed net farm income and minimise variance. Fourth, solve the QRP problem again with variance set equal to the farm's actual variance and find maximal expected net farm income. Ideally, these two points will coincide and the degree of risk aversion of the farmer could be derived from the gradient of the $E-V$ frontier at this point, since this may be presumed to be tangential to the farmer's $E-V$ indifference curve. In practice, the two points diverge because of imperfections in the model or because of inconsistency in the farmer's choice. Therefore, the fifth step, having ascertained two points on the efficient frontier, is to use the gradient of the line in $E-V$ space between these two solutions to approximate the relevant gradient and hence to estimate the coefficient of absolute risk aversion. To my knowledge, no one has used this approach before.

² Huirne et al. (1997) confirms the strong suspicion that eliciting utility functions from farmers is at best a risky business. They found that a significant proportion of farmer respondents revealed a preference of risk, which could be regarded as unrealistic, and they have shown that elicited risk attitudes are very unstable over time.

Simmons and Pomareda (1975), Brink and McCarl (1978) and Hazell et al. (1983) also used the *E-V* framework but they used linear programming. The approach of Wiens (1976) was to match the primal QRP solution with the actual land patterns and the dual solution (shadow prices) with the market prices of the farm resources, and from these results to derive the decision maker's coefficient of risk aversion.

As an example of its application, my approach outlined was applied on Norwegians farm-level data (NILF, 1994-1999). Two methods to compare the estimated coefficient of absolute risk aversion between farmers are also illustrated. Some confidence in the validity of the model may be deduced from the fact that the observed expected net farm incomes and the estimated optimal net farm incomes generally proved to be rather close to each other. Moreover, the approximated coefficients of relative risk aversion were mostly within the range of 0.5 to about 4 (proposed by Anderson and Dillon, 1992 as the range to be expected).

The main advantage with my model is simplicity. It is easy to understand and implement. I think this model is a real alternative to existing programming models used to approximate decision maker's risk aversion. However, some basic weaknesses have to be mentioned: (1) the model is sensitive to mis-specification; (2) the model assumes a normal distribution of total net revenue if the set of solutions are to be equivalent to maximising expected utility. It can be argued that to measure risk only by the mean and variance of income is a problem, but I think the normal distribution assumption is sufficient for this kind of analysis.³ Of course, any model will only approximate a decision maker's risk attitude; no model will calculate it exactly; (3) the model as formulated does not account for farmers' responses to non-business risk. Business-risk may affect the farmers' decisions about financial risk taking (Gabriel and Baker, 1980).⁴

³ A thorough comparison of *E-V* and expected utility (*EU*) models for ranking distributions and for theoretical analysis is given in Robison and Hanson (1997). They conclude that both models will continue to dominate risk analysis, but more complicated risk models will increasingly rely on *E-V* models.

⁴ However, it is easy to extend the model to account for aspects of financial risk such as purchase of insurance, hedging, etc.

Essay 2: Whole-farm planning under uncertainty: impacts of subsidy scheme and utility function on portfolio choice in Norwegian agriculture

Earlier studies in programming models for whole-farm planning under risk have either considered non-embedded risk (e.g. Nanseki and Morooka, 1991; Bhende and Venkataram, 1994) or considered embedded risk using a two-stage programming model (e.g. Kaiser and Apland, 1989; Kingwell, 1994; Pannell and Nordblom, 1998). Our⁵ study provides an approach to modelling integrated dairy and cash crop farming in a whole-farm context that includes both embedded and non-embedded risk. The modelling procedure utilises two alternative utility functions, the negative exponential function with constant absolute risk aversion, CARA, and the power function with constant relative risk aversion, CRRA. In the paper we account for the complexity of making the move from utility of wealth to the utility of income (Hardaker, 2000). Earlier risk analysis studies have overlooked this complexity. The move implies that the handling of the coefficient of risk aversion is more precise than in earlier studies within the field. Data from the Farm Business Survey (NILF, 1992-1998) from 1991 to 1997 are combined with subjective judgements to formulate a two-stage utility-efficient programming model.

Under existing policy and market condition in Norway, the ex ante expectation was that farmer' risk attitudes are unlikely to have a large effect on choice of enterprise mix. The results tended to confirm this view, which indicates that farmer's risk aversion and shape of the utility function are not very important in farm planning in a regulated regime. These results are in contrast with many earlier studies within this field. Other studies have found risk aversion to have an important influence on the choice of the whole-farm management plan (e.g. Kaiser and Apland, 1989; Nanseki and Morooka, 1991; Kingwell, 1994; Pannell and Nordblom, 1998). However, political intervention to stabilise prices is not as strong in regimes analysed in these studies (United States, Indonesia, Western Australia and Syria, respectively) as in Norway. Another reason may be that they have used a larger range of risk aversion. On the other hand, even within a free market Pannell et al. (2000) found that the extra value of representing risk aversion (compared to a model based on risk neutrality) is commonly very small, which is in line with our conclusion. Other factors on the farm are often more important determinants of the optimal farm plan than the farmers' attitude to risk. Our results are consistent with Kallberg and Ziemba's (1983) study of the functional form of

⁵ J. Brian Hardaker, University of New England, Australia is co-author on this paper.

utility functions. But since Kallberg and Ziemba investigate utility of wealth, and we study the utility of income, a direct comparison of results is difficult. Our results are important for future research in farm planning, at least in a regulated regimes, since they imply that more focus should be directed to obtaining good specifications of the probability distributions of outcomes rather than worrying about how risk averse farmers may be.

Essay 3: Assisting whole-farm decision-making through stochastic budgeting

When making a decision about a business investment or future strategy farmers have to account for many, often uncertain aspects. Yet whole-farm budgeting is still quite often based on fixed-point estimates of production, prices and financial variables to derive point estimates of financial results. In reality, the events and conditions planned will not occur as assumed. A common response to this problem is to conduct sensitivity analysis as part of the planning exercise in order to determine the range of possible results. Pannell (1997) argues that sensitivity analysis can be theoretically respectable in decision support if applied and interpreted consistent with Bayesian decision theory (i.e. adjustment of strategies and decisions as new information is obtained). Sensitivity analysis is easy to understand, easy to communicate, and easy to apply to many types of model. However, in a sensitivity analysis it is common to consider changes in only one variable at time. The effects on the performance measure of combinations of errors in different variables are, therefore, largely ignored (Hull, 1980). When many variables are uncertain, a sensitivity analysis of the effect on financial performance for more than just a few variables becomes tedious and difficult to interpret. Moreover, the sensitivity analysis gives no indication of the likelihood of a particular result being achieved (Little and Mirrlees, 1974).

To overcome these problems an alternative approach is stochastic budgeting, which accounts for some of the main uncertainties in the evaluation and then gives an indication of the distribution of outcomes. In this framework uncertain variables can be expressed in stochastic terms, and many combinations of variable values can be analysed to provide a full range of expected outcomes. There is not much work published within the fields of whole-farm stochastic budgeting, and furthermore the method is not widely used in practice. Richardson and Nixon (1986) developed the stochastic whole-farm budgeting model FLIPSIM (Farm level income and policy simulator). Milham et al. (1993) developed a stochastic whole-farm budgeting system, called RISKFARM. Compared to FLIPSIM, RISKFARM had more stochastic variables and the stochastic dependency was specified in another way (multivariate

empirical probability distribution in FLIPSIM vs. hierarchy of variables approach in RISKFARM).

In essay 3 a whole-farm stochastic budgeting model is used which includes stochastic gross margins, interest rates, fixed costs, labour requirements for activities, and milk quota price. The model simulates the farm performance and the business and financial risk over a six-year planning horizon. Risky strategies are evaluated by cumulative distribution functions and by stochastic dominance. In concept, the model draws on the work of Milham et al. (1993). In contrast with earlier studies using stochastic farm budgeting, the option value of a 'wait and see' strategy is included in the analysis.

Experiences gained in my study reported in this essay suggest some principles for similar work. First, the model should be kept as simple as is judged reasonable. It is important to be critical in the choice of stochastic variables in the model - too many make it complicated to account for stochastic dependencies between variables. The intention of budgeting models is not to give exact answers, but to highlight consequences of different strategies. Second, it is critical to make good estimates of the distributions of the key uncertain variables. Unrealistic estimates make the analysis a waste of time. Third, it is important to identify and measure stochastic dependencies between variables satisfactorily, at least if this is thought to be important. Both intratemporal (across activities) and intertemporal (across time) stochastic dependency have to be incorporated in a stochastic dynamic farm-level analysis (Richardson et al., 2000). This paper illustrates three methods to build in these dependencies, namely the hierarchy of variables approach, the autoregressive model, and a method that combine subjective probabilities, estimates of historical correlation between activities and a simulation of stochastic trends combined with the hierarchy of variables approach.

The main advantage of stochastic budgeting is that greater flexibility in planning can be represented. A pitfall is that the large volume of numbers produced by a simulation study can create a tendency to place greater confidence in a study's results than justified. Models that are not valid will provide little useful information about the actual system (Law and Kelton, 1991). Another drawback with stochastic budgeting for practical use is the complexity. An analysis which is not understood is unlikely to be believed (Pannell, 1997).

Essay 4: Modelling jumps in commodity prices

A critical factor for correct pricing of derivatives (and any contingent claims) is the description of the stochastic process governing the behaviour of the underlying asset (Cox and Ross, 1976). The objective of our⁶ paper is to model the spot-price process for an agricultural product. This paper employs methods from modern finance to analyse the behaviour of wheat prices.

Three main models are examined: Vasicek's (1977) mean reverting model, Vasicek plus jumps and Aït-Sahalia (1996) models incorporating non-linear drift. Other simpler model specifications, such as Brownian motion with jumps are also investigated. Models investigated without jumps are one-factor models⁷, while models combining a diffusion and a jump term are three-factor models⁸. The models are applied to monthly wheat price data from 1952 to 1998. The estimation is also broken down into sub periods to see whether any shifts in parameters are evident. The higher moments of the mean reverting and the jump model are developed, following Das (1999). These models are tested with the General Method of Moments and Maximum Likelihood Estimation.

Ex ante, one would expect that three-factor models would have a better fit to the data than one-factor models. The results tend to confirm this view. Jump behaviour is clearly present in the data. When the period was divided into two, with 1973 chosen as the dividing year, the jump diffusion model did not perform better than the mean reverting model in the first period. However, in the later period the jump diffusion model clearly outperformed the mean reverting model. Non-linear drift is rejected. Although we have looked into the price behaviour of only one commodity, wheat, it seems unlikely that our method would be limited to wheat only. Our main conclusion is that investigators of derivatives pricing as well as the pricing of real options ought to take the jumpiness of commodity prices into account.

⁶ This paper is written together with Øystein Strøm, Østfold College, Norway.

⁷ In these one-factor models a Brownian motion generates the uncertainty.

⁸ In these three-factor models the uncertainty is generated by a diffusion component plus a component where a Poisson process decides when the jumps occur, and a normal distributed component that determines the jump size.

Essay 5: Term structure of volatility and price jumps in agricultural markets - evidence from option data

Empirical evidence suggests that agricultural futures prices exhibit sudden and unexpected price jumps (Hall et al., 1989; Hilliard and Reis, 1999). There is also evidence that the volatility of futures prices contains a term structure depending on both the calendar-time and time to maturity (usually referred to as the "Samuelson hypothesis") (e.g. Anderson, 1985; Bessembinder et al., 1996; Galloway and Kolb, 1996).

Commodity futures option pricing models, e.g., Black's (1976) model, typically assume that the logs of futures price relatives are normally distributed with constant variance. Hilliard and Reis (1999) used the jump-diffusion model developed by Bates (1991) on transaction data on soybean futures and futures options and found this performs considerably better than Black's model. Still, any regular pattern in the volatility is inconsistent with the underlying assumptions of both Black's (1976) and Bates' (1991) option pricing models.

Some studies have developed option pricing models for agricultural commodities that incorporate regular patterns in the volatility (e.g. Choi and Longstaff, 1985; Myers and Hanson, 1993), but nobody has yet included both jumps and time-varying volatilities. In our⁹ paper we assume that the futures price follows a jump-diffusion process. In addition, the diffusion term includes time dependent volatility that captures (possibly) both a seasonal and a maturity effect. This model therefore incorporates several stylised facts reported in the literature relating to commodity futures dynamics.

We derive a futures option pricing model given our specified futures price dynamics, and we test our model empirically on American futures option prices from the Chicago Board of Trade. We estimate the parameters of the futures price dynamics using non-linear least squares to fit our model to eleven years of wheat options data. Several models suggested in the literature are nested within our model (Black, 1976; one-factor model of Schwartz, 1997; Bates, 1991; special cases of our general model), and they all gave a significantly poorer fit than our more complete model formulation. The maturity effect is especially strong in this market. In a numerical example we show that ignoring the term structure and jump effects in futures prices may lead to severe mispricing of options.

⁹ This paper is written jointly with Steen Koekebakker, Agder University College, Norway.

Essay 5 investigates the wheat market only. Since other crop commodities and many other agricultural commodities have a seasonal pattern and/or a maturity effect (see, e.g., Galloway and Kolb, 1996) it seems likely that our model will also be applicable to these markets.

3 Opportunities for further research

Implications of an efficient derivative market

A realistic planning model should account for each decision maker's subjective probabilities about the chances of occurrence of uncertain consequences and for her/his preferences for those consequences, reflecting her/his attitude to risk. Most economists assume that the subjective expected utility (SEU) hypothesis is the most appropriate framework for structuring these two components into a workable model of risky choice (Hardaker et al., 1997). At least in regulated regimes it seems that farmers' risk attitude are of little importance in affecting the choice of farm plan (Lien and Hardaker, 2001). However, one important area for improvements within farm planning under uncertainty in future research is technologies that give better estimates of expected returns. In cases where abundant, reliable and obviously relevant data exist for some uncertainty quantity of interest, such abundant evidence will swamp any prior subjective beliefs, and there will be no practical difference between decision maker's subjective beliefs and objective probabilities.

Unfortunately, relevant data are rarely available to provide an objective basis for assessing the probabilities required for making some decision. Product prices from past time periods are often not relevant for the future outcomes. In these cases it is important to obtain as reliable subjective probabilities as possible. Some rules to derive probabilities based on careful thought and debate about what is reasonable in various types of situations are given in Hardaker et al. (1997). Kenyon (2001) found that producers' subjective probability distributions (i.e. not experts' distributions) about output prices have smaller variance than the market.¹⁰

¹⁰ In Kenyon's (2001) analysis producers were asked in January and February each year from 1991-1998 to estimate harvest prices that reflect only a 10% probability of going below or above these prices at harvest. To compare subjective probabilities and the market he reported the percentage of time the actual harvest price exceeds the 10% lower or upper bound price each year.

As mentioned, derivative markets are expected to become more used and more important in the coming years with government deregulation and liberalised international trade. Given an efficient market, futures and forward prices are forward looking and provide useful information of spot prices in the future (Fama and French, 1987; Sick, 1995).¹¹ Futures and forward prices represent the markets certainty equivalents. Given that long-term contracts exist, they can give valuable public information about future expectations. Gardner (1976) states that futures prices represent rational expectations. He therefore argued that use of futures prices is useful in supply analysis. Yet Kenyon's (2001) results indicate that the futures market estimates of harvest prices for corn and soybeans were not substantially better than producers' price expectations. However, more derivative price data should be useful data to guide subjective probability judgement. An interesting aspect is to investigate to what extent derivative prices are useful to make better specifications of price probability distributions for input in programming and simulation models in farm planning.

Risk-attitude assessment

In the field of estimating decision maker's risk aversion at least two aspects are interesting for future research. One is to develop an alternative to the non-parametric estimation method developed in essay 1, where a parametric (econometric) method is used to estimate V^* and E^* . The idea is to use a stochastic frontier model on panel data¹² in an E - V framework. There are several econometric studies purporting to derive estimates of farmers' degree of risk aversion (see essay 1). So far as I know, none has used stochastic frontier methods. However, it is not at all obvious that frontier methods are likely to be better than other econometric methods to estimate decision makers' risk attitude, but it is an interesting aspect to investigate.

A second possibility for further work on the method outlined in essay 1 could be to compare different programming methods on the same dataset. At least one problem is that we do not have any benchmark, since we do not know the analysed farmers' "real" risk attitudes.¹³

¹¹ The motive to deal with futures and options is that the dealer has subjective probabilities that deviate from the probabilities implied by the market behaviour.

¹² Stochastic frontier models are described by, for example, Coelli et al., (1998).

¹³ One possibility to validate the results is to reverse the normal method and go back to the individual farmers and tell them, based on the results from the model, what they would or should prefer in various hypothetical simplified choice situations, and then ask whether they agree.

Modelling agricultural spot- and derivative prices

In essay 4 we mainly tested whether adding a jump component to a diffusion process contributed to a better fit of monthly spot wheat data from 1952 to 1998. Surprisingly little empirical work has been done on jump behaviour for agricultural commodities. More empirical work is needed on jump behaviour for both spot and futures prices for a number of commodities and frequencies of data. Relatively much more work has been published on documenting any term structure in the volatility of the futures prices (e.g. Anderson, 1985; Bessembinder et al., 1996; Galloway and Kolb, 1996), but not many of these investigations are done on spot prices. Yang and Brorsen (1992) find that the discrete-time GARCH model best represents the stochastic properties of agricultural and precious metal commodity prices, using daily cash prices. Baillie and Myers (1991) found that a GARCH specification described cash commodity prices reasonably well (Beef, Coffee, Corn, Cotton, Gold, and Soybeans). Other (also continuous-time) stochastic volatility models are also of interest. Stochastic volatility models are widely used within finance (e.g. see Bates, 1995 for a survey). Future research could then extend in various ways in a nested model including for example jumps, seasonal variability, maturity effect and stochastic volatility in the spot and futures price process.

Further extensions of option pricing models for agricultural commodities

For future research, an actual extension of our option pricing model on agricultural futures contract is to incorporate stochastic volatility. Many (stochastic) factors other than the season and/or the maturity, as assumed in our model in essay 5, can affect the volatility function. It may be that a more general jump and seasonal stochastic volatility model for pricing of agricultural commodity options will give a better explanation of the empirical evidence.

4 Concluding remarks

As agriculture becomes more deregulated so that farmers are more exposed to risk, risk management will become more important for them to succeed. Better risk management is likely to entail better management of on-farm risk, employing such methods as investment analysis and careful selection of a portfolio of production alternatives. This will also entail farmers and others in agriculture exploiting more fully the markets for risk such as insurance and agricultural commodity derivative markets. The contribution of this dissertation is to

expand upon existing work in these areas that are seen as of growing importance for the future of agriculture in general and Norwegian agriculture in particular.

References

- Aït-Sahalia, Y., 1996. Testing continuous-time models of the spot interest rate. *Review of Financial Studies* 9: 385-426.
- Anderson, J.R., Dillon, J., 1992. *Risk analysis in dryland farming systems*. Farming Systems Management Series No. 2, FAO, Rome.
- Anderson, J.R., Dillon, J., Hardaker, J.B., 1977. *Agricultural Decision Analysis*. Iowa State University Press, Ames, Iowa, 344 pp.
- Anderson, R.W., 1985. Some determinants of the volatility of futures prices. *Journal of Futures Markets*. 5: 331-348.
- Antle, J.M., 1987. Econometric estimation of producers' risk attitudes. *American Journal of Agricultural Economics* 69: 509-522.
- Baillie, R.T., Myers, R.J., 1991. Bivariate GARCH estimation of the optimal commodity futures hedge. *Journal of Applied Econometrics* 6: 109-124.
- Bar-Shira, Z., Just R.E., Zilberman, D., 1997. Estimation of farmers' risk attitude: an econometric approach. *Agricultural Economics* 17: 211-222.
- Bates, D.S., 1991. The crash of '87: was it expected? The evidence from options markets. *Journal of Finance* 46: 1009-1044.
- Bates, D.S., 1995. Testing option pricing models. Working Paper 5129, National Bureau of Economic Research, Cambridge, MA.
- Bessembinder, H, Coughenour, J.F., Seguin, P.J., Smoller, M.M., 1996. Is there a term structure of futures volatilities? Reevaluating the samuelson hypothesis" *Journal of Derivatives*. Winter: 45-58.
- Bhende, M.J., Venkataram, J.V., 1994. Impact of diversification on household income and risk: a whole-farm modelling approach. *Agricultural Systems* 44: 301-312.
- Binswanger, H.P., 1980. Attitudes toward risk: experimental measurement in rural India. *American Journal of Agricultural Economics* 62: 395-407.
- Bjornson, B., Innes, R., 1992. Another look at returns to agricultural and nonagricultural assets. *American Journal of Agricultural Economics* 74: 109-119.
- Black, F., 1976. The pricing of commodity contracts. *Journal of Financial Economics* 3: 167-179.
- Brink, L., McCarl, B., 1978. The tradeoff between expected return and risk among cornbelt farmers. *American Journal of Agricultural Economics* 60: 259-263.
- Carter, C.A., 1999. Commodity futures markets: a survey. *Australian Journal of Agricultural and Resource Economics* 43: 209-247.
- Choi, J.W., Longstaff, F.A., 1985. Pricing options on agricultural futures: an application of the constant elasticity of variance option pricing model. *Journal of Futures Markets* 5: 247-258.

- Coelli, T., Rao, D.S.P, Battese, G.E., 1998. *An introduction to efficiency and productivity analysis*. Kluwer Academic, Dordrecht, 275 pp.
- Copeland, T.E., Weston, J.F., 1988. *Financial Theory and Corporate Policy*, Third Edition. Addison-Wesley, Int.
- Cox, J.C., Ross, S.A., 1976. The valuation of options for alternative stochastic processes. *Journal of Financial Economics* 3: 145-166.
- Das, S.R., 1999. The surprise element: jumps in interest rate diffusions. Working Paper, Harvard University
- Dillon, J.L., Scandizzo, P.L., 1978. Risk attitudes of subsistence farmers in northeast Brazil: a sampling approach. *American Journal of Agricultural Economics* 60: 425-434.
- Fama, E., French, K.R., 1987. Commodity futures prices: some evidence on forecast power, premiums, and the theory of storage. *Journal of Business* 60: 55-73.
- Freund, R.J., 1956. The introduction of risk into a programming model. *Econometrica* 24: 253-263.
- Gabriel, S.C., Baker, C.B., 1980. Concepts of business and financial risk. *American Journal of Agricultural Economics* 62: 60-564.
- Galloway, T.M., Kolb, R.W., 1996. Futures prices and the maturity effect. *Journal of Futures Markets*. 16: 809-828.
- Gardner, B.L., 1976. Futures prices in supply analysis. *American Journal of Agricultural Economics* 58: 81-84.
- Hall, J.A., Brorsen, B.W., Irwin, S.H., 1989. The distribution of futures prices: a test of the stable paretian and mixture of normal hypothesis. *Journal of Finance and Quantitative Analysis* 24: 105-116.
- Hardaker, J.B., 2000. Some issues in dealing with risk in agriculture. Working Papers in Agricultural and Resource Economics. School of Economic Studies, University of New England.
- Hardaker, J.B., Huirne, R.B.M., Anderson, J.R., 1997. *Coping with Risk in Agriculture*. CAB International, Wallingsford.
- Harwood, J., Heifner, R., Coble, K., Perry, J., Somwaru, A., 1999. *Managing risk in farming: concepts, research, and analysis*. Agricultural Economics Report No. 774. U.S. Department of Agriculture, Washington.
- Hazell, P.B.R., Norton, R.D., Parthasarathy, M., Pomareda, C., 1983. The importance of risk in agriculture planning models. In: Norton, R.D., Solis, M.L. (Eds.), *The book of CHAC: programming studies for Mexican agriculture*. Johns Hopkins University Press, Baltimore, pp. 225-249.
- Hilliard, J.E., Reis, J.A., 1999. Jump processes in commodity futures prices and option pricing. *American Journal of Agricultural Economics* 81: 273-286.
- Huirne, R.B.M., Harsh, S.B., Dijkhuizen, A.A., Bezemer, S., 1997. Assessing the risk attitude of dairy farms with respect to income and sire selection. In: Huirne, R.B.M., Hardaker, J.B., Dijkhuizen, A.A. (Eds.), *Risk Management Strategies in Agriculture – State of the Art and Future Perspectives*. Backhuys Publishers, Leden, The Netherlands, pp. 113-120.
- Hull, J.C., 1980. *The Evaluation of Risk in Business Investment*. Pergamon, London.

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- Johnson, R.W.M., 1992. Risk and the farm firm: a corporate finance view. *Review of Marketing and Agricultural Economics* 60: 9-21.
- Kaiser, H.M., Apland, J., 1989. DSSP: a model of production and marketing decisions on a midwestern crop farm. *North Central Journal of Agricultural Economics* 11: 157-170.
- Kallberg, J.G., Ziemba, W.T., 1983. Comparison of alternative utility functions in portfolio selection problems. *Management Science* 29: 1257-1276.
- Kenyon, D.E., 2001. Producer ability to forecast harvest corn and soybean prices. *Review of Agricultural Economics* 23: 151-162.
- Kingwell, R.S., 1994. Risk attitude and dryland farm management. *Agricultural Systems* 45: 191-202.
- Law, A.M., Kelton, W.D., 1991. *Simulation Modelling and Analysis*, 2nd edn. McGraw-Hill, New York.
- Lien, G., Hardaker, J.B., 2001. Whole-farm planning under uncertainty: impacts of subsidy scheme and utility function on portfolio choice in Norwegian agriculture. *European Review of Agricultural Economics* 28: 17-36.
- Little, I.M.D., Mirrlees, J.A., 1974. *Project Appraisal and Planning for Developing Countries*. Heinemann, London.
- Markowitz, H., 1952. Portfolio selection. *Journal of Finance* 7: 77-91.
- Milham, N., Hardaker, J.B., Powell, R., 1993. RISKFARM: a pc-based stochastic whole-farm budgeting system. Centre for Agriculture and Resource Economics, University of New England, Armidale (unpublished paper).
- Moscardi, E., de Janvry, A., 1977. Attitudes toward risk among peasants: an econometric approach. *American Journal of Agricultural Economics* 59: 710-716.
- Myers, R.J., Hanson, S.D., 1993. Pricing commodity options when underlying futures price exhibits time-varying volatility. *American Journal of Agricultural Economics* 75: 121-130.
- Nanseeki, T., Morooka, Y., 1991. Risk preference and optimal crop combinations in upland Java, Indonesia: an application of stochastic programming. *Agricultural Economics* 5: 39-58.
- NILF., 1992-99. *Driftsgranskinger i jord- og skogbruk* (Account Results in Agriculture and Forestry). Norwegian Agriculture Economics Research Institute, Oslo. Annual publication.
- Pannell, D.J., 1997. Sensitivity analysis of normative economic models: theoretical framework and practical strategies. *Agricultural Economics* 16: 139-152.
- Pannell, D.J., Malcolm, B., Kingwell, R.S., 2000. Are we risking too much? Perspectives on risk in farm modelling. *Agricultural Economics* 23: 69-78.
- Pannell, D.J., Nordblom, T.J., 1998. Impacts of risk aversion on whole-farm management in Syria. *Australian Journal of Agricultural and Resource Economics* 42: 227-247.
- Richardson, J.W., Klose, S.L., Gray A.W., 2000. An applied procedure for estimating and simulating multivariate empirical (MVE) probability distributions in farm-level risk assessment and policy analysis. *Journal of Agriculture and Applied Economics* 32: 299-315.

- Richardson, J.W., Nixon, C.J., 1986. Description of FLIPSIM V: a general firm level policy simulation model. Texas Agricultural Experiment Station. Bulletin B-1528.
- Robison, L.J., Barry, P.J., Kliebenstein, J.B., Patrick, G.F., 1984. Risk attitudes: concepts and measurements approaches. In: Barry, P.J. (Ed.), *Risk management in agriculture*. Iowa State University Press, Ames, Iowa, pp 11-30.
- Robison, L.J., Hanson, S.D., 1997. Analysing firm response to risk using mean-variance models. In: Huirne, R.B.M., Hardaker, J.B, Dijkhuizen, A.A. (Eds.), *Risk Management Strategies in Agriculture – State of the Art and Future Perspectives*. Backhuys Publishers, Leden, The Netherlands, pp. 121-137.
- Schwartz, E.S., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. *Journal of Finance* 52: 923-973.
- Sick, G., 1995. Real Options. In: R. Jarrow et al.: *Handbook in OR and MS, Vol. 9*. Elsevier Science B.V.
- Simmons, R.J., Pomareda, C., 1975. Equilibrium quantity and timing of Mexican vegetable exports. *American Journal of Agricultural Economics* 57: 472-479.
- Vasicek, O.A., 1977. An equilibrium characterization of the term structure. *Journal of Financial Economics* 5: 457-470.
- Wiens, T.B., 1976. Peasant risk aversion and allocative behaviour: a quadratic programming experiment. *American Journal of Agricultural Economics* 58: 629-635.
- Yang, S.-R., Brorsen, B.W., 1992. Nonlinear dynamics of daily cash prices. *American Journal of Agricultural Economics* 74: 706-715.

Essay 1:

Non-parametric estimation of decision makers' risk aversion*

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Abstract

A new non-parametric method to estimate a decision maker's coefficient of absolute risk aversion from observed economic behaviour is explained. The method uses the expected value-variance ($E-V$) framework and quadratic programming. An empirical illustration is given using Norwegian farm-level data.

Keywords: Risk analysis; Risk aversion; Quadratic programming; Norwegian agriculture

1 Introduction

In much risk-related work it is necessary to have some measure of the decision maker's attitude to risk. Risk attitudes may be measured by either the coefficient of absolute or the coefficient of relative risk aversion. This paper describes a non-parametric method to estimate the coefficient of absolute risk aversion from observed economic behaviour.

A survey of different approaches to specifying decision maker's risk attitudes is given in Robison et al. (1984). The following approaches have been utilised to assess risk attitudes: (1) direct elicitation of utility functions (see Anderson et al., 1977; or Hardaker et al., 1997 for details; an example on a new empirical study within this approach is presented by Abadi Ghadim and Pannell, 2000); (2) experimental procedures in which individuals are presented

* *Forthcoming Agricultural Economics.*

with hypothetical questionnaires regarding risky alternatives with or without real payments (e.g. Dillon and Scandizzo, 1978; Binswanger, 1980); and (3) inference from observation of economic behaviour. In this paper I focus on approach (3): inference from observation of economic behaviour, based on an assumed relationship between the actual behaviour of a decision maker and the behaviour predicted from empirically specified models. Empirical inference of risk attitudes from observed economic behaviour can be divided into non-parametric (mathematical programming) and parametric (econometric) approaches. The pioneering work with econometric applications was that of Moscardi and de Janvry (1977). Antle (1987) estimated producer risk attitudes by applying econometric techniques to cross-sectional data from individual farms. Bar-Shira et al. (1997) used an econometric approach to examine the effect of wealth changes on the measure of absolute, relative, and partial risk aversion.¹ Compared with the programming approach, the econometric approach has the advantage of straightforward hypotheses testing. On the other hand, non-parametric methods offer greater flexibility in modelling the farm situation.

Applications with mathematical programming have usually been used in an expected value-variance ($E-V$) framework.² Simmons and Pomareda (1975) used linear programming in an $E-V$ framework to compute optimal input choices at different levels of risk aversion. Each solution (in hectares (ha)) was compared with actual choices to determine the level of risk aversion that gave the solution most closely corresponding to actual choice. Brink and McCarl (1978) and Hazell et al. (1983) derived farmers' coefficient of risk aversion as that value of estimated coefficient which minimised the difference between the farmer's actual behaviour and the results of a linear programming model. The difference was measured in terms of summed total absolute deviation of areas for all crops. The approach of Wiens (1976) was to match the primal Quadratic Risk Programming (QRP) solution with the actual land patterns and the dual solution (shadow prices) with the market prices of the farm resources, and from these results derive the decision maker's coefficient of risk aversion.

¹ The econometric approach to inference of risk attitudes is related to stochastic specification and estimation of the production function. Asche and Tveterås (1999) model the production risk with a two-step procedure, where they estimate the mean and risk function separately.

² The study of Amador et al. (1998) is somewhat related to the mathematical programming approach used to estimate decision maker's risk attitudes. Amador et al. use goal programming to elicit farmers' multi-criteria utility function.

The $E-V$ framework and QRP are also used in this paper but in a different way. The approach is as follows. First, formulate the QRP model to represent the farm's resource base, activities, expected activity net revenues per unit level, fixed costs, variance and covariance of expected net revenues to reflect the decision maker's beliefs and circumstances as closely as possible. Second, for an observed farm plan presumed to reflect a farmer's risk-averse behaviour, calculate expected net farm income and variance. Third, solve the QRP problem setting expected net farm income equal to the farm's observed net farm income and minimise variance. Fourth, solve the QRP problem again with variance set equal to the farm's actual variance and find maximal expected net farm income. Fifth, having ascertained two points on the efficient frontier, the gradient of the line in $E-V$ space between these two solutions is used to approximate the coefficient of absolute risk aversion. To my knowledge, no one has used this approach before.

This paper is structured as follows. Section 2 describes the model. An application of the model is presented in Section 3. Section 4 contains some concluding comments.

2 The model

Given (approximately) normally distributed total net revenue³ and assuming that the decision maker's utility function is represented by a negative exponential utility function, we maximise the decision maker's expected utility with the following $E-V$ formulation (Freund, 1956):

$$\max U = E - \frac{r_a}{2} V = cx - f - \frac{r_a}{2} x' Q x \quad (1)$$

subject to:

$$Ax \leq b$$

$$x \geq 0$$

where U is expected utility, $E = cx - f$ is expected net farm income, c is a 1 by n vector of expected activity net revenues per unit level, r_a is the absolute risk aversion coefficient, x is an n by 1 vector of activity levels, Q is a n by n variance-covariance matrix so $V = x' Q x$ is the

³ Since total net revenue is the sum of several random variables, appeal to the Central Limit Theorem suggest approximate normality (Anderson et al., 1977, p. 193; Hardaker et al., 1997, p. 187).

variance of expected net farm income, f is fixed costs, A is an m by n matrix of technical coefficients, and b is an m by 1 vector of resource stocks.

Solving this problem for various values of r_a gives points exhibiting minimum variance for a given expected net farm income, and/or maximum expected net farm income for a given variance of income. The frontier ACB in Figure 1 is the E-V efficient set.

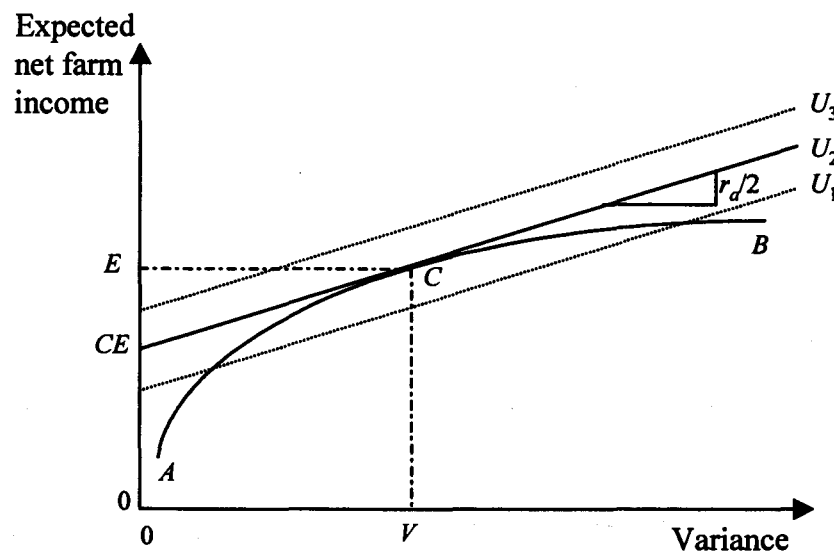


Figure 1 Optimal portfolio choice illustrated in E - V space

Consider a decision maker with indifference curve U , which is linear in the E - V space given normally distributed total net revenue (Freund, 1956). Assuming the decision maker's absolute risk aversion coefficient is r_a , his or her indifference lines are given by equation (1) for various values of U . As illustrated in Figure 1, the tangent between the decision maker's indifference line, U_2 , and the efficient frontier is at point C which corresponds to the optimal production mix with expected net farm income E and variance of expected net farm income V . Since point CE has zero variance it is called the certainty equivalent (CE) to the risky expected net farm income E . The indifference line's slope coefficient is $r_a/2$ and the decision maker's coefficient of absolute risk aversion to this constructed problem is r_a .

Freund's E - V formulation may also be formulated as (Hardaker et al., 1997):

$$\max E = cx - f \quad (2)$$

subject to:

$$x'Qx = V, V \text{ varied}$$

$$Ax \leq b$$

$$x \geq 0.$$

Likewise, Markowitz's (1952) original formulation of the E - V problem set up to minimise variance subject to a given level of expected net income is formulated as:

$$\min V = x'Qx \quad (3)$$

subject to:

$$cx - f = E, E \text{ varied}$$

$$Ax \leq b$$

$$x \geq 0$$

with the same notation as in equation (1). Freund and Markowitz's formulations yield identical efficient frontiers. The differences between the formulations are the way the frontier is derived. In equation (1) r_a is parameterised, in equation (2) V is parameterised and in equation (3) E is parameterised.

The framework described above is used to estimate a decision maker's coefficient of absolute risk aversion, as illustrated in Figure 2. Formulate the QRP model to represent the farm's resource base, activities, expected activity net revenue per unit level (in this paper expected gross margin (GM) per unit level is used), fixed costs, variance and covariance of expected GMs which are assumed to reflect the farmer's beliefs and circumstances. Further, for a current farm situation (the farm we want to analyse) calculate from observed economic behaviour net farm income E_a (a for actual) and variance V_a . Then, using Markowitz's formulation solve the QRP problem setting expected net farm income E to E_a and minimise variance V at $V_{min}=V^*$. Next, using Freund's formulation (equation 2) solve again with V set to V_a to find $E_{max}=E^*$. We have then two points on the efficient frontier, (E_a, V^*) and (E^*, V_a) . The gradient of the line in E - V space between these two solutions is used to approximate the coefficient of absolute risk aversion,

$$r_a = \frac{(E^* - E_a)}{2(V_a - V^*)} \quad (4)$$

The point (E_a, V_a) is inefficient, since the farmer can increase the expected net farm income to E^* and still have the same variance V_a , or the farmer can have the same expected net farm

income E_a with lower variance V^* . The farmer can get these efficient portfolios if she or he choose the optimal combination of activities.⁴

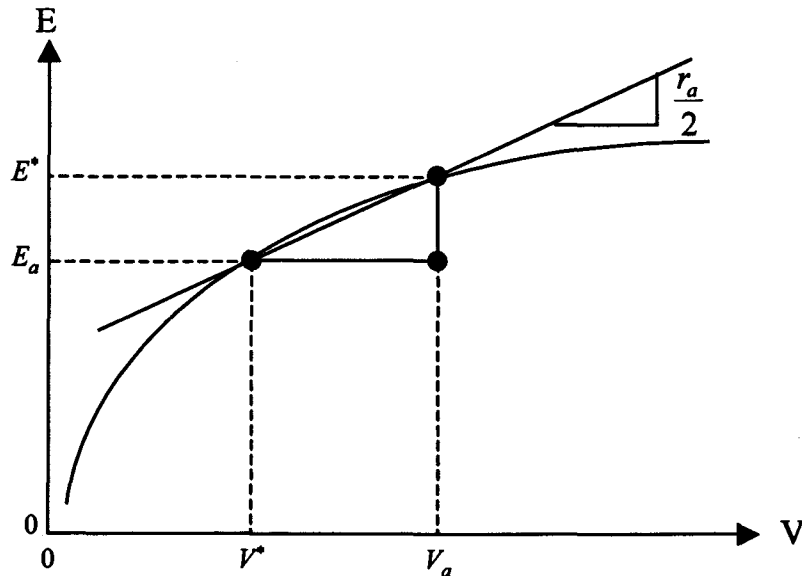


Figure 2 Approximation of a decision marker's coefficient of absolute risk aversion

In the model it is also possible to get a solution where the actual farm plan (E_a, V_a) is north-west of the frontier. One reason for this is a mis-specified variance-covariance matrix, Q , for the analysed farm, e.g. that the analysed farm has a smaller variances for some activities and/or different covariances between activities than assumed in the QRP model. Alternatively, the vector of net revenue per unit level, c , may be mis-specified. A third possible reason is that the constraints, A , are less restrictive than assumed in the specified QRP model. For all these cases, equation (4) is still assumed to be valid to approximate the coefficient of absolute risk aversion.

One thing, which is important to consider, is which r_a we are estimating. In the model outlined in this section the payoffs are expressed in terms of annual income. Following Hardaker (2000) we have to distinguish whether transitory income or permanent income is the argument of the utility function. Permanent income is where the uncertainty is about the long-run level of income. Transitory income is where the income in some future year, say next

⁴ The efficient and inefficient portfolios are somewhat related to technical efficiency in the efficiency and productivity literature. Technical efficiency reflects the ability of a firm to obtain maximum output from a given set of inputs (Coelli et al., 1998). The vertical difference between E^* and E_a in Figure 2 can be interpreted as an output-oriented measure of 'technical efficiency', and reflects the farm's ability to select proportions of activities which give maximal expected net farm income for given variance.

year, is uncertain. The approximate relationship between coefficient of absolute and relative risk aversion with respect on both permanent and transitory income is given by Hardaker (2000).

3 Application

In this section, as an example of its application, the approach outlined above is used to estimate the coefficient of absolute risk aversion for some case-study farmers in Norway. Two methods to compare the estimated coefficient of absolute risk aversion between farms are also illustrated.

3.1 The farm system and data

Ideally, in constructing a QRP model the variance-covariance matrix should be formed for each individual farmer. In practise, the required historical data may not be available from the analysed farm. In particular, of course, there will be no data for activities not previously included on that farm that are nevertheless of interest for the programming analysis. Therefore calculation of a variance-covariance matrix that reflects GM interaction between activities for a particular farm normally requires data for combinations of activities from many similar farms over several years.

In this analysis the data used came from the Farm Business Survey (*driftsgranskingsdata*), collected by the Norwegian Agricultural Economics Research Institute. Information used relates to unbalanced panel data consisting of a total of 2136 observations from the Norwegian lowlands⁵ over the six-year period 1993 to 1998 (NILF, 1994-99a). The number of observations on each activity varied from 1472 for barley to 70 for vegetables. The lowlands of Norway were used since within this area production possibilities are rather homogeneous. The growth season is about 180 days from April/May to September/October. Subsidies and production regulation are important factors influencing farmers' choice of mix of farm activities. Apart from production regulations, farmers in the Norwegian lowlands

⁵ The Norwegian Farm Business Survey (NILF 1994-99a) sample is subdivided into lowlands and other parts. Parts of Eastern Norway, parts of Trøndelag and Jæren are categorised as lowlands. The production basis is substantially better in lowland regions than elsewhere.

region can choose between many activities: cereals, potatoes, oilseed, grass seed, vegetables, and pig, dairy, beef and sheep farming chiefly.

The model used in this analysis finds the optimal farm plan given a planning horizon of one year. At the beginning of the season the farmer chooses a cropping and stocking pattern conditional on his or her expectation of output at the end of the season. In principle, the expected GM vector and variance-covariance matrix should be represented by the farmer's subjective beliefs about returns from the production. Obtaining such data is generally very demanding and difficult if not infeasible. Thus, the historical mean GM vector and variance-covariance matrix were assumed to represent farmers' beliefs.

Expected net farm income, E , on a specific farm in a specific year is given by:

$$E = \sum_q \left(c_q x_q + \sum_p sub_{qp} x_{qp} \right) - f \quad (5)$$

where E is expected net farm income including subsidies, c_q is expected GM for enterprise q (without subsidies), sub_{qp} is subsidy for enterprise q at activity level p , and f is fixed costs. The subsidies are not proportional to production area/herd size but are partially differentiated according to headage and area-size. The variance including subsidies is calculated in the model depending on activity levels, rather than a simple historical trace of subsidy payments. The average subsidy level for the periods 1993 to 1998 is used and assumed to reflect as closely as possible farmers' expectation for the range of years for which the actual farm plan is applied. One part of the subsidy scheme in dairy production ('driftstilskudd i melkeproduksjonen') is product-specific. This product-specific support is included in the historical GM for dairy cows and then incorporated into the variance-covariance matrix.

Annual per ha GMs are developed for activities over a six-year period (1993-1998) in the following manner. First, nominal gross returns are developed from the Farm Business Survey (NILF, 1994-99a). Second, the individual activity nominal gross returns are converted to a real 1998 Norwegian kroner (NOK) basis using the consumer price index (CPI). Third, the individual activity GMs are developed by subtracting 1998 budgeted variable cost (NILF, 1994-99b) from real 1998 NOK gross returns. Budgeted variable costs are used, since the survey only has aggregated variable costs, not specific costs for each activity. These measures from the unbalanced panel data are used to calculate the variance-covariance matrix for GM used in the QRP model. Budgeted variable costs can remove some of the real variation in GM

per unit. It is therefore important to realise that this approach can underestimate the variation in activity GMs.

Although almost all activities in the analysis have administered prices, the GM per unit for each activity within a farm may vary greatly from year to year. This variability is caused by factors such as weather, plant and animal diseases, which induce yield and product quality variation. In other words, activity expected GMs are uncertain, and this is accounted for in the variance-covariance matrix. The following model was used to measure variation within farms between years and calculate the GM variance-covariance and correlation matrix within farms:

$$c_{qit} = \alpha_{qi} + \beta T + w, \quad w \approx N(0, \sigma^2) \quad (6)$$

$$s_q^2 = \frac{\sum_{i=1}^n \sum_{t=c_i}^{d_i} (c_{qit} - \hat{c}_{qit})^2}{N - n - 1} \quad (7)$$

$$Q(q, p) = \frac{\sum_{i=1}^n \sum_{t=c_i}^{d_i} (c_{qit} - \hat{c}_{qit})(c_{pit} - \hat{c}_{pit})}{N - n - 1} \quad (8)$$

$$\rho_{qp} = \frac{Q(q, p)}{s_q \times s_p} \quad (9)$$

where c_{qit} is activity q 's GM per unit on farm i in year t , α_{qi} is the regression constant for activity q on farm i , T is time ($T=1, \dots, 6$), β is the systematic change in income over the period (this component adjusts for an equal trend on all farms, caused by technological change among other things), w is a random error, \hat{c}_{qit} is activity q 's predicted regression value for mean GM per unit on farm i in year t , N is total number of observations on all farms in the sample, n is number of farms in the sample, c_i is the first year with observation on farm i , d_i is last year with observation on farm i , s_q^2 is activity q 's variance of GM per unit, $Q(q, p)$ and ρ_{qp} are covariance and correlation between activity q and p , respectively. Degrees of freedom are $(N-n-1)$ in equations (7) and (8), where n is lost degrees of freedom caused by calculation of average for each farm and 1 is lost degrees of freedom caused of the estimation of the time trend.

The data in the Farm Business Survey of Norway lowlands for the period 1993 to 1998 restrict the analysis to include only the following activities in calculation of the variance-covariance matrix in the model: barley, oats, wheat, potatoes, oilseed, carrots, grass seed and dairy cows. Activity average GMs, standard deviations (SDs) and coefficients of variations (CVs) are given in Table 1. The correlation matrix of activity GMs is shown in Table 2. Note the low correlation between some of the activities, which implies opportunities for income stabilisation through diversification.

Table 1 Activity mean gross margins (GMs) per unit exclusive of subsidies in Norwegian kroner (NOK), average standard deviation (SD) within farms, and coefficient of variation (CV) for the Norwegian lowlands 1993-98

Activity	Unit	Mean GM	SD	CV
Barley	ha	5 499	1 947	0.35
Oats	ha	5 127	2 295	0.45
Wheat	ha	8 781	3 389	0.39
Potatoes	ha	20 401	11 375	0.56
Oilseed	ha	5 816	2 049	0.35
Carrot	ha	49 990	26 791	0.54
Grass seed	ha	10 226	5 242	0.51
Dairy	no	14 743	2 295	0.16

Table 2 Correlation matrix of expected activity gross margins within farms for the Norwegian lowlands 1993-98

Activity	Barley	Oats	Wheat	Potatoes	Oilseed	Carrots	Grass seed	Dairy
Barley	1.00							
Oats	0.38	1.00						
Wheat	0.28	0.47	1.00					
Potatoes	-0.17	0.07	-0.23	1.00				
Oilseed	0.28	0.40	0.22	0.09	1.00			
Carrots	0.17	0.34	0.21	-0.05	0.03	1.00		
Grass seed	0.05	0.16	-0.01	-0.23	0.62	-0.28	1.00	
Dairy	-0.07	0.04	0.03	0.06	0.55	0.46	-0.43	1.00

3.2 Results

Historical average net farm income in NOK, E_a , and variance, V_a , was calculated for nine farms from the Farm Business Survey of Norwegian lowlands for the period 1993 to 1998. In

addition the same calculations were made for the average of a subsample of 28 farms. This subsample was also divided into two subsamples with above or below average wealth levels.⁶

A common variance-covariance matrix, Q , was used for all farms, c.f. subsection 3.1. As far as possible farm-specific GMs were used in the QRP model. One problem is data for each activity on each farm. For activities without farm-specific data the mean c from Table 1 was used.

The case farms used in the model have the following constraints: (1) actual farm area of arable land; (2) with respect to rotational considerations, no more than two-thirds of the area of agricultural land on the actual holding can be cereals, no more than one-quarter of the area can be potatoes, and a maximum of one-sixth of the area can be carrots; (3) because of contract constraints on grass-seed production, the area of this crop is restricted to three hectares for farms without grass-seed production in the period 1993 to 1998. On farms with grass-seed in the same period, the average of actual grass-seed area in this period is used; (4) the farm's milk quota is set to the average actual milk production on the farm in the period 1993 to 1998. Farms without milk production in the period 1993 to 1998 are assumed to have zero milk quotas; (5) farms without carrots in the analysed period do not have carrots as a possible activity in the QRP model. These restrictions are used since carrot production requires special soil that not all farms have; (6) one constraint on labour family availability in each of the four periods of the year: spring (April-May), summer (June-July), autumn (August-October), and winter (November-March). Average registered hours of family labour available in the period 1993 to 1998 are distributed as one-sixth of the hours in the spring and summer seasons, one-quarter of the hours in the autumn season and three-seventh of the hours in the winter season. Technical input-output coefficients for seasonal labour requirements are assumed fixed and are based on data from NILF (1994-99b); (7) hired labour use is restricted to the actual average registered hired labour for the period 1993 to 1998; (8) subsidies constraints are set according to the average subsidies prevailing for the years 1993 to 1998 (NILF, 1994-99b); (9) actual fixed cost for the case farms are used in the model.

⁶ Occasionally (not for the results presented here) when I tried to estimate the coefficient of absolute risk aversion I found no feasible solution. The apparent reason for the infeasible solutions was either that the technical input-output constraints, A , and/or the expected activity GMs per unit level, c , and/or the variance-covariance matrix, Q , was misspecified and not representative for the analysed farm. For these reasons the

Solutions from Freund's $E-V$ formulation (equation 2) and Markowitz's $E-V$ formulation (equation 3) were used in equation (4) to estimate the coefficient of absolute risk aversion, r_a (Table 3). Observe that observed expected net farm income, E_a , and estimated optimal net farm income, E^* , are rather close each other, which may indicate a quite valid model.

Table 3 Approximated coefficient of absolute risk aversion, $r_a(c_i)$, for case farmers and subsamples, Norway lowlands 1993-98

Case farmer	E_a	V_a	E^*	V^*	$r_a(c_i)$
1	357 974	33 401 217 600	387 493	8 674 138 549	0.00000060
2	26 933	946 362 169	46 482	462 899 657	0.00002022
3	224 919	7 905 213 933	225 323	4 503 820 138	0.00000006
4	237 693	8 705 266 936	236 368	8 993 218 790	0.00000230
5	267 012	18 215 011 369	321 153	11 115 334 991	0.00000381
6	92 600	3 379 110 851	126 987	480 662 822	0.00000593
7	249 988	14 615 731 592	257 543	5 917 899 470	0.00000043
8	303 147	4 495 367 256	186 836	10 471 231 970	0.00000973
9	233 304	13 498 257 124	233 251	13 517 354 484	0.00000140
Subsample	284 950	22 631 591 844	341 005	3 109 608 536	0.00000144
'Wealthy'	367 533	28 337 682 244	381 667	14 901 283 442	0.00000053
'Non-wealthy'	231 510	11 842 880 625	280 282	1 993 850 986	0.00000248

In the single-year farm plan used in this model, income can be considered as transitory income, and the absolute risk aversion coefficient estimated is with respect to transitory income, c_t (Hardaker, 2000). For the individual case farms the results show the estimated coefficient of absolute risk aversion with respect to transitory income, $r_a(c_t)$, vary from 0.00000006 to 0.0000202. The estimated $r_a(c_t)$ values vary considerably from farm to farm. The results show that the estimated $r_a(c_t)$ for the subsample existing of 28 farmers was 0.0000014. The subsample with 13 farmers in the 'wealthy' group had an absolute risk aversion of 0.00000053, which is lower than for the subsample existing of 15 farmers in the 'non-wealthy' group of 0.00000248. That the absolute risk aversion is a decreasing function of wealth is in accordance with Arrow's (1970) expectation.

For case farmers 4 and 8 the actual farm plan (E_a, V_a) is to the north-west of the frontier. The reason may be that these farmers have smaller variance for some activities, and/or different covariance between activities than assumed in the QRP model, and/or that the constraints are less restrictive than assumed in the QRP model.

QRP problem may sometimes be infeasible when expected net farm income is set to E_a and variance V is minimised or variance is set to V_a and E is maximised.

It is not straightforward to compare $r_a(c_i)$ between case farmers in Table 3. Among many possibilities, I used some approximate quantitative indication of whether risk aversion matters. The method used is to calculate the proportional risk premium, PRP , representing the proportion of the expected payoff of a risky prospect that the farmers would be willing to pay to trade away all the risk for a sure thing, proposed by Hardaker (2000). Following Freund (1956), if the net revenue for each activity is normally distributed and assuming a negative exponential utility function, we have the following relationship: $U = CE = E - 0.5r_a(c_i)V$, cf. equation (1). The risk premium, RP , is given by $RP = E - CE = 0.5r_a(c_i)V$. The PRP is defined as $PRP = RP/E$ so that here:

$$PRP = 0.5r_a(c_i)V_a/E_a \quad (10)$$

The more risk averse the farmer is, the higher will the PRP be. In Table 4 we observe that case farmer 2 is willing to pay a rather large proportion of the expected net farm income of the risky prospect for the sure thing. Case farmer 3 is willing to pay almost none of the expected net farm income for the sure thing. Note also that the 'non-wealthy' group has a larger PRP than the 'wealthy' group.

Table 4 Approximated proportional risk premium, PRP , and coefficient of relative risk aversion with respect to wealth, $r_r(W)$, for case farmers and subsamples, Norway lowlands 1993-98

Case farmer	PRP	Wealth (in NOK)	$r_r(W)$
1	0.028	2 937 787	1.75
2	0.355	433 484	8.76
3	0.001	1 296 224	0.08
4	0.042	2 534 455	5.83
5	0.130	717 518	2.74
6	0.108	455 811	2.70
7	0.013	1 505 067	0.65
8	0.072	1 109 625	10.80
9	0.040	2 583 345	3.61
Subsample	0.057	1 540 770	2.21
'Wealthy'	0.020	2 753 189	1.45
'Non-wealthy'	0.063	756 263	1.87

An alternative way to compare estimated absolute risk aversion, $r_a(c_i)$ values between case farms is to calculate the corresponding coefficient of relative risk aversion, $r_r(W)$, with respect to wealth, W . The approximate relationship between these two measures of risk aversion is shown by Hardaker (2000) as:

$$r_r(W) = r_a(c_i)W \quad (11)$$

The relationship in equation (11) requires a rational farmer, i.e., asset integration where a farmer shows consistent risk attitude to risky prospects whether they are presented in terms of wealth, income or losses and gains. Anderson and Dillon (1992) have proposed a rough and ready classification of degrees of risk aversion, based on the relative risk aversion with respect of wealth, $r_r(W)$, in the range 0.5 (hardly risk averse at all) to about 4 (very risk averse). The results in Table 4 display $r_r(W)$ mostly within this range. That case farmers 2, 4 and 8 show a large $r_r(W)$ may be caused by failure of asset integration, i.e., these farmers may be more risk averse when they contemplate transitory income than they would be if the same risky prospects were presented to them in terms of wealth.

Note also that $r_r(W)$ decreases with increasing wealth. This result is not in accordance with Arrow (1970), who argued on theoretical and empirical grounds that $r_r(W)$ would generally be an increasing function of W . However, Hamal and Anderson (1982) found that, in extremely resource-poor farming situations, relative risk aversion could reach values as extreme as four or more, contrary to what Arrow had hypothesised. Binswanger (1980) found that wealth appeared to have little influence on risk-taking behaviour.

Saha et al. (1994, pp. 175) present an overview of the principal findings of earlier studies. But it is important to remember that the coefficient of absolute risk aversion, r_a , is not constant for change in currency units. That makes it meaningless to compare coefficients of absolute risk aversion in different countries with different units (Hardaker, 2000).

4 Concluding comments

The main advantage with the approach outlined in this paper is simplicity. If you have a farm or a group of farms with data on activity GMs and fixed costs over some years, the method can easily be implemented in a standard software program that solves non-linear programming problems. If the coefficient of relative risk aversion is needed it is, following Hardaker (2000), possible to derive the approximate relationship between the coefficients of absolute and relative risk aversion.

Some basic weaknesses with this approach to approximating a farmer's risk aversion have to be mentioned. First, estimation of the risk aversion parameter will pick up errors in model specification and data, and it is difficult to know how serious these errors might be (Hazell et

al., 1983). Good model specification is essential to get trustworthy estimates of the absolute risk aversion coefficient. One approach that may reduce possibilities for actual farm plans above the frontier is to estimate and use pooled variance-covariance matrix for different groups of e.g. type of farming or farm size in the programming model.

Second, a feature of this approach is that the risk parameter estimates are conditional (Saha et al., 1994). That is, the coefficient of absolute risk aversion is estimated conditional upon a specific risk preference structure implied by the assumed negative exponential utility function form. The negative exponential utility function imposes constant absolute risk aversion, usually not regarded as a desirable property.

Third, this approach requires normally distributed total net revenue if the set of solutions are to be equivalent to maximising expected utility (Freund, 1956). Hardaker et al. (1997, pp. 187) write 'The distribution of total net revenue varies from case to case and may not be normal ...[but], at least for a mixed farming system, appeal to the Central Limit Theorem suggests that the distributions of total net revenue may be approximately normal'.

Fourth, this model does not account for farmers' responses to non-business risk (not explicitly considered in the model). Introduction or modification of business risk in the production process may affect the farmers' decisions about leverage and financial risk-taking (Gabriel and Baker, 1980).

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References

- Abadi Ghadim, A.K., Pannell, D.J., 2000. Risk attitudes and risk perceptions of crop producers in Western Australia. Paper presented at Australian Agriculture and Resource Economics Society 44th Annual Conference, 23-25 January, 2000, Sydney.
- Amador, F., Sumpsi, J.M., Romero, C., 1998. A non-interactive methodology to assess farmers' utility functions: An application to large farms in Andalusia, Spain. *European Review of Agricultural Economics* 25: 92-109.
- Anderson, J.R., Dillon, J., 1992. *Risk analysis in dryland farming systems*. Farming Systems Management Series No. 2, FAO, Rome.
- Anderson, J.R., Dillon, J., Hardaker, J.B., 1977. *Agricultural decision analysis*. Iowa State University Press, Ames, Iowa, 344 pp.
- Antle, J.M., 1987. Econometric estimation of producers' risk attitudes. *American Journal of Agricultural Economics* 69: 509-522.
- Arrow, K.J., 1970. *Essays in the theory of risk-bearing*. North-Holland, Amsterdam, 278 pp.
- Asche, F., Tveterås, R., 1999. Modeling Production Risk with a Two-Step Procedure. *Journal of Agriculture and Resource Economics* 24: 424-439.
- Bar-Shira, Z., Just R.E., Zilberman, D., 1997. Estimation of farmers' risk attitude: an econometric approach. *Agriculture Economics* 17: 211-222.
- Binswanger, H.P., 1980. Attitudes toward risk: experimental measurement in rural India. *American Journal of Agricultural Economics* 62: 395-407.
- Brink, L., McCarl, B., 1978. The tradeoff between expected return and risk among cornbelt farmers. *American Journal of Agricultural Economics* 60: 259-263.
- Coelli, T., Rao, D.S.P, Battese, G.E., 1998. *An introduction to efficiency and productivity analysis*. Kluwer Academic, Dordrecht, 275 pp.
- Dillon, J.L., Scandizzo, P.L., 1978. Risk attitudes of subsistence farmers in northeast Brazil: a sampling approach. *American Journal of Agricultural Economics* 60: 425-434.
- Freund, R.J., 1956. The introduction of risk into a programming model. *Econometrica* 24: 253-263.
- Gabriel, S.C., Baker, C.B., 1980. Concepts of business and financial risk. *American Journal of Agricultural Economics* 62: 560-564.
- Hamal, K.B., Anderson, J.R., 1982. A note on decreasing absolute risk aversion among farmers in Nepal. *Australian Journal of Agricultural Economics* 26: 220-225.
- Hardaker, J.B., 2000. Some issues in dealing with risk in agriculture. Working Papers in Agricultural and Resource Economics. School of Economic Studies, University of New England, Armidale.
- Hardaker, J.B., Huirne, R.B.M., Anderson, J.R., 1997. *Coping with risk in agriculture*. CAB International, Wallingford, 274 pp.
- Hazell, P.B.R., Norton, R.D., Parthasarathy, M., Pomareda, C., 1983. The importance of risk in agriculture planning models. In: Norton R.D., Solis M.L. (Eds.), *The book of CHAC:*

- programming studies for Mexican agriculture*. Johns Hopkins University Press, Baltimore, pp. 225-249.
- Markowitz, H., 1952. Portfolio selection. *Journal of Finance* 7: 77-91.
- Moscardi, E., de Janvry, A., 1977. Attitudes toward risk among peasants: an econometric approach. *American Journal of Agricultural Economics* 59: 710-716.
- NILF., 1994-99a. *Driftsgranskinger i jord- og skogbruk* (Account results in agriculture and forestry). Norwegian Agriculture Economics Research Institute, Oslo, Annual publication.
- NILF., 1994-99b. *Håndbok for driftsplanlegging* (Handbook in farm business planning). Norwegian Agricultural Economics Research Institute, Oslo, Annual publication.
- Robison, L.J., Barry, P.J., Kliebenstein, J.B., Patrick, G.F., 1984. Risk attitudes: concepts and measurements approaches. In: Barry, P.J. (Ed.), *Risk management in agriculture*. Iowa State University Press, Ames, Iowa, pp 11-30.
- Saha, A., Shumway, C.R., Talpaz, H., 1994. Joint estimation of risk preference structure and technology using expo-power utility. *American Journal of Agricultural Economics* 76: 173-184.
- Simmons, R.J., Pomareda, C., 1975. Equilibrium quantity and timing of Mexican vegetable exports. *American Journal of Agricultural Economics* 57: 472-479.
- Wiens, T.B., 1976. Peasant risk aversion and allocative behaviour: a quadratic programming experiment. *American Journal of Agricultural Economics* 58: 629-635.

Essay 2:

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Whole-farm planning under uncertainty: impacts of subsidy scheme and utility function on portfolio choice in Norwegian agriculture

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Summary

This paper addresses the impacts of degree of risk aversion, subsidy scheme and choice of utility function on optimal farm plans in Norwegian agriculture. Data from a farm business survey (1991–1997) are combined with subjective judgements to formulate a two-stage utility-efficient programming model. Under existing policy and market conditions, the *ex ante* expectation was that farmers' risk attitudes are unlikely to have a large effect on choice of enterprise mix. The results tend to confirm this view, and a farmer who is hardly risk averse at all would choose the same farm plan as a very risk averse farmer. Factors such as subsidy schemes, market conditions for the products and available labour on the farm are found to be more important determinants of the optimal plans than farmers' risk attitude or the form of the utility function.

Keywords: discrete stochastic utility-efficient programming, risk aversion, utility function, subsidy schemes, Norwegian agriculture

JEL classification: Q12, D81

1. Introduction

Compared with other countries, Norway's natural resources are not very favourable for agriculture. In this country, which lies furthest north of any country in Europe, the climate significantly limits agriculture. Moreover, the topography means that fields are often scattered and steep. Recognising these conditions and in pursuit of the goals of encouraging people to stay in rural areas, maintaining cultural landscapes and ensuring food security in times of crisis, the Norwegian government has assigned relatively large subsidies to the agriculture sector compared with other countries. Almost all product prices are administered. Nevertheless, there is large variability between years in activity gross margins (GMs). This is caused by large yield

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and quality variations. In whole-farm planning it may be important to take account of this source of risk.

In a planning model accounting for uncertainty it is usually important to take account of the farmers' risk attitude. Earlier studies have often assumed complete certainty or have overlooked farmers' risk aversion. Others who have incorporated farmers' risk attitudes have found risk aversion to have an important influence on the choice of the whole-farm management plan (e.g. Kaiser and Apland, 1989; Nanseki and Morooka, 1991; Kingwell, 1994; Pannell and Nordblom, 1998). However, political intervention to stabilise prices is not as strong in regimes analysed in those studies (United States, Indonesia, Western Australia and Syria) as in Norway.

Norway has had (and still has) a problem of agricultural surpluses, mainly of milk and meat. In an attempt to reduce overproduction, there has been a shift in agricultural policy over the past 10 years away from price supports and towards forms of support that are not linked to production volume. Our aim, therefore, is to analyse farmers' responses to different forms of subsidy and to examine how these responses are affected by their attitudes to risk.

In the economic literature, it is argued that the utility function should exhibit positive but decreasing absolute risk aversion (Pratt, 1964; Arrow, 1970). However, empirical work shows no universal consensus (Saha *et al.*, 1994). Kallberg and Ziemba (1983) show that 'similar' absolute risk aversion values yield 'similar' portfolios, regardless of the functional forms of the utility functions concerned. However, Kallberg and Ziemba's study was based on utility of wealth, whereas in this paper utility of income is investigated. In this paper, we examine the effect on risk aversion that this difference might have. It is also possible that the form of utility function chosen to reflect farmers' risk aversion will affect the implied response to different forms of subsidy. As the actual forms of farmers' utility functions are difficult to establish, we also investigate this aspect in this paper.

Among recent empirical applications of risk and stochastic programming models in whole-farm planning problems are studies where risk is captured only in the objective function coefficients (e.g. Nanseki and Morooka, 1991; Bhende and Venkataram, 1994). Other applications of stochastic programming that allow for stochastic elements in the objective function, right-hand side and/or input-output coefficients include those by Rae (1971b), Kaiser and Apland (1989), Kingwell (1994) and Pannell and Nordblom (1998).

In this paper, a two-stage stochastic utility-efficient programming model is developed. Compared with earlier studies this model incorporates the following advances in analytical methods:

- (i) the study provides an approach to modelling integrated dairy and cash crop farming in a whole-farm context that includes both embedded and non-embedded risk;
- (ii) the modelling procedure utilises two alternative utility functions;
- (iii) a consistent method for adjusting the risk aversion parameter is used.

The model developed is for a case farm that reflects the conditions of a typical farm in the eastern Norwegian lowlands.¹ The lowlands of eastern Norway enjoy geographical, soil and climatic conditions that are more favourable for agriculture than those of other regions of the country. The growing season lasts about 180 days from April–May to September–October. Production possibilities on the case farm are livestock and crops. Quotas regulate many of the enterprises.

Our empirical objectives are to examine the effect on the optimal farm plan of differences in (i) subsidy system, (ii) farmers' risk aversion, and (iii) the form of the utility function. The paper is structured as follows. The model is presented in Section 2, where farmers' behaviour and utility, activities and constraints, data and the matrix structure are presented. The empirical results are presented in Section 3. Finally, Section 4 contains some concluding comments.

2. The model

Our model incorporates both non-embedded crop risk and embedded livestock risk. This is shown in Figure 1, where the upper branch on the first decision fork denotes non-embedded risk and the lower branch outlines the embedded risk. Early in the year, the farmer must decide how many suckler cows (for beef production) and sheep to keep. Taking account of land already established with coarse fodder or pasture in an earlier year, the farmer must decide by early spring which crop to sow on the rest of the arable area. The unstable weather in the region implies yield uncertainty, with the actual yield being known only after harvest. For simplicity, decisions such as split versus single fertilising, herbicide use, etc., which may depend to some extent on the weather, are not included in the model. Therefore, once the crops are sown, it is assumed that there are no more important crop management decisions to be made. The risk associated with cropping we therefore call non-embedded risk (see upper branch of Figure 1).

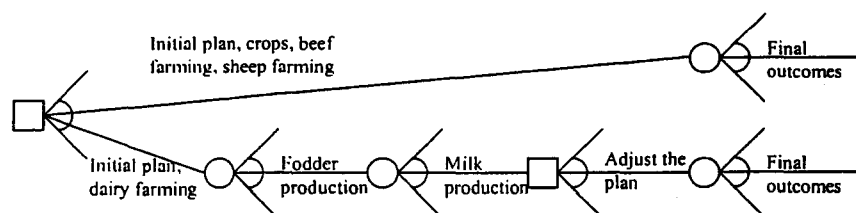


Figure 1. Outline decision tree for our problem.

The milk quota year starts in January, when the farmer must decide the number of cows to keep in production next year. Milk production per cow is uncertain, depending on disease incidence, fodder yields, fodder quality,

¹ Agriculture in eastern Norway is subdivided into lowlands and other parts (highlands). The production basis is substantially better in lowland regions than elsewhere.

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calving intervals, etc. Hence, at the beginning of the year the farmer is uncertain about the number of cows needed to produce the farm's annual milk quota. The farmer can sell cows, buy more cows or bring more heifers into the herd at any stage during the quota year. Although adjustments can be made at any stage, we assume for simplicity that the farmer will adjust cow numbers only once during the year, in early October. At that time, the type of season and the level of milk production to date will be known.

In addition to the uncertain milk production per cow, fodder yield is also uncertain. Following any adjustment of the cattle numbers, the farmer must take steps to meet any shortfall in winter feed availability. The adjustment possibilities for cows and feed depend on both earlier decisions and uncertain seasonal conditions. The need to adjust the farm plan in response to uncertain intermediate outcomes of fodder and milk production creates a case of embedded risk, as illustrated in the lower branch of Figure 1. Embedded risk is modelled using discrete stochastic programming (Cocks, 1968; Rae, 1971a).

In a multi-stage decision problem, the later strategies need to be present in sufficient detail to ensure 'correct' first-stage decisions. Actual later-stage decisions can be resolved by running further more refined models incorporating the outcomes of uncertain events as they unfold (Kaiser and Apland, 1989). With this in mind, it was decided to model fodder yield uncertainty with only two outcomes of high and low yields whereas three possible levels of milk production are represented.

2.1. Farmers' behaviour and utility

We assume that farmers are risk averse and that beliefs and preferences vary between farmers. A realistic planning model should then account for each farmer's subjective probabilities about the chances of occurrence of uncertain consequences and for the preferences regarding those consequences, reflecting the farmer's degree of aversion to risk. We assume that the subjective expected utility (SEU) hypothesis is the best framework for structuring these two components into a workable model of risky choice (Hardaker *et al.*, 1997).

Many alternative programming models for whole-farm system planning under risk have been developed.² For our problem we use the utility-efficient programming (UEP) approach (Patten *et al.*, 1988). Given a programming problem with non-risk neutrality and some knowledge about the relevant form of utility function and risk attitudes, Hardaker *et al.* (1991) recommend UEP. UEP can be used when advice to a group of decision-makers is being formulated, and we obtain an efficient set of farm plans using a method somewhat similar to the stochastic dominance with respect to a function rule developed by Meyer (1977).

In the UEP, any convenient form of utility function can be used. Because we assume that farmers are risk averse, we are restricted to using any concave

² Reviews of alternative programming models are given by Hardaker *et al.* (1991), and Hardaker *et al.* (1997: Ch. 9).

form of the utility function, i.e. $U''(z) < 0$. A utility function with many intuitively plausible properties is the power function. This function has constant relative risk aversion (CRRA), which means that if we start with indifference between a certain sum and a risky prospect and multiply all payoffs by a positive constant, indifference is not disturbed. In this analysis we use a special form of the CRRA power function:

$$U = \left(\frac{1}{1-a} \right) z^{(1-a)} \quad (1)$$

where z is net income per year, a is the coefficient of relative risk aversion, and $U(z)$ is positive ($U'(z) > 0$) but decreasing ($U''(z) < 0$). This function has decreasing absolute risk aversion, $r_a(z) = -U''(z)/U'(z) = a/z$ and constant relative risk aversion $r_r(z) = zr_a(z) = a$. When $a = 1$, the CRRA power function reduces to the logarithmic function, $U = \ln z$. When $a = 0$, $U = z$ and we find the solutions for a risk neutral farmer (Pratt, 1964; Arrow, 1970).³

To investigate whether different utility functions make much difference to the optimal solutions, in addition to the CRRA power function we also used the negative exponential function:

$$U = 1 - \exp(-cz) \quad (2)$$

where c is a non-negative parameter representing the coefficient of absolute risk aversion, $U'(z) > 0$, and $U''(z) < 0$. This function exhibits constant absolute risk aversion (CARA). CARA means that if we start with indifference between a certain sum and a risky prospect then add a (positive or negative) constant sum to all payoffs, indifference is not disturbed.

Anderson and Dillon (1992) have proposed a rough and ready classification of degrees of risk aversion, based on the relative risk aversion with respect to wealth $r_r(W)$ in the range 0.5 (hardly risk averse at all) to about four (very risk averse), typically about one (somewhat risk averse). If the coefficient of absolute risk aversion with respect to wealth $r_a(W)$ is needed, we can use $r_a(W) = r_r(W)/W$.

In this paper, we are not considering utility and risk aversion in terms of wealth, but in terms of income. As we want to analyse in a range of $r_r(W)$ from 0.5 to 4.0 and to compare CRRA power function and negative exponential function in terms of income, we need relations between $r_r(W)$, $r_a(W)$, $r_r(z)$ and $r_a(z)$. At least two types of risky choice affecting farm income can be imagined (Hardaker, 2000). One is where the uncertainty is about the long-run level of income. The other type is where the uncertainty relates to transitory income, such as when income next year is uncertain. The latter is the typical situation in annual farm planning, as in this paper. We assume a rational farmer makes the same choice whether the risky outcomes are expressed in terms of wealth, income or losses and gains, i.e. we assume asset integration. We define W as uncertain wealth, W_0 as initial wealth and z as uncertain transitory income, and let $W = W_0 + z$. Then the choice

3 The CRRA power function may be difficult to solve and unsatisfactory for values of a close to one or zero.

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problem can equivalently be expressed in terms of W and z , given W_0 is non-stochastic or z is stochastically independent of W_0 , and we should expect no change in preference as a result. Therefore, if we do not want preferences to change whether we express outcomes in terms of W or z , we can assume that $r_a(W) \approx r_a(z)$. Then, it follows that $r_a(z) = r_r(W)/W$. Moreover, because $r_a(z) = r_r(z)/z$ by definition, we obtain the following relationship (Hardaker, 2000):

$$r_r(z) = zr_a(z) = zr_a(W) = (z/W)r_r(W). \quad (3)$$

In other words, in assessing risky choices expressed in terms of transitory income, it is not correct to apply the same relative risk aversion coefficient as for wealth.

2.2. Activities and constraints

The main groups of activities in the model are as follows:

- (i) Cash crop activities: barley, oats, wheat, potatoes, oilseed, grass seed and carrots.
- (ii) Livestock activities: dairy, beef and sheep activities. In beef production, both intensive (slaughtered at 18 months) and extensive production (slaughtered at 24 months) are included. In stage two, dairy cow numbers may be adjusted depending on the level of production to date. Milk yield per cow is assumed to be high (M_1), normal (M_2) or low (M_3) at 7,500, 6,500 or 5,500 kg per cow per year, respectively.
- (iii) Fodder crop activities: root crops, green fodder and grassland. Straw from the farm's cereal production is also included as alternative fodder for beef cattle.
- (iv) Concentrate feed activities. Three types of concentrate feed, with different levels of protein, are included in the model. The animals' requirements are assumed fixed per head, but choice of feed types is possible.
- (v) Hire labour and rent land activities. Provision is made in the model to hire labour at the current wage rate of NOK (Norwegian kroner) 116 per hour. It is assumed to be possible to hire labour at any time of year. There is also provision to rent in land at NOK 3720 per ha, which is the present average cost of renting land in eastern Norway.
- (vi) Subsidies adjustment. In the first run of the model, the prevailing subsidy arrangements are included. Apart from one subsidy scheme for dairy milk production, all are headage or area-based production subsidies. The level of the product-specific subsidy for dairy production is influenced by both stage 1 and stage 2 decisions. The other subsidies are influenced by decisions at stage 1 only.

The main constraints are as follows:

- (i) Land constraint. A farm size of 20 ha is assumed, which is close to the average farm size in the lowlands of eastern Norway.

- (ii) Rotational limits. To avoid the build-up of pests and diseases we assume that no more than one-third of the area can be potatoes, and a maximum of one-sixth of the area can be carrots.
- (iii) Marketing limit. Grass seed is regulated by production contracts. Many requirements must be satisfied to obtain a contract. In this analysis, we restrict grass seed production to 3 ha.
- (iv) Milk quota constraint. The farm's milk quota is set at 100,000 litres, which is the average for lowland dairy farms in eastern Norway. Since 1983, production quotas have regulated the production of cow's milk.⁴ Production above the quota has no commercial value; and of course, it is not necessarily profitable to produce milk at all.
- (v) Root crop limit. Root crops are limited so as to constitute no more than 25 per cent of the coarse fodder produced, measured in terms of live-stock feed units. The basis for this constraint is that, as ruminants, cows and sheep need a minimum proportion of coarse fodder in their feed.
- (vi) Seasonal labour constraints. There is one constraint on labour availability in each of the four seasons of spring, summer, autumn and winter. The spring season covers April and May (spring work period). The summer season is June and July, and the autumn period covers August, September and October (harvesting period). The winter season is from November to March. It is assumed that the maximum amount of family labour available is 3,600 h per year, distributed as 600 h in the spring and summer seasons, 900 h in the autumn season and 1,500 h in the winter season. Labour availability is calculated on the basis of one full-time owner operator and one part-time family worker. Technical input–output coefficients for seasonal labour requirements per unit of the activities are assumed fixed and are based on data from the Norwegian Agricultural Economics Research Institute (NILF, 1999).
- (vii) Hire labour constraint. Family labour may be supplemented with hired labour at times of peak need in the model. The maximum amount of hired labour per year is set at 300 h, as it is sometimes difficult to find qualified farm workers in this area.
- (viii) Rent land constraint. We set a limit of 15 ha on the amount of land that can be rented in. Because many fields are scattered, transport costs tend to increase rapidly with increased hired area. Therefore we estimate that 35 ha, comprising 20 ha of existing land and 15 ha rented, is the maximum area that could be cultivated with the farm's existing machinery.
- (ix) Subsidy constraints. Subsidy constraints are set initially according to the subsidy system that prevailed in 1999 (NILF, 1999).

4 From 1996, the government introduced a system for the redistribution of milk quotas using regulated quota sales. As the regulations are very restrictive, little redistribution has occurred. Farmers who have no milk quota cannot start production of milk.

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- (x) Insert heifers and buy and sell cow constraint. In the model in stage 2, we suppose it is not possible to insert heifers to an extent greater than 20 per cent of the existing number of cows. Furthermore, in stage 2 we assume that it is not possible to buy or sell cows to an extent of more than 40 per cent of the existing number of cows. These limits are due to limits on quota adjustment in the following years.

2.3. Data

To represent the uncertainty in activity GMs, we mainly used the method described in Hardaker *et al.* (1997: 53–55). We used the Farm Business Survey (driftsgranskingsdata) from NILF (1992–1998) to estimate the historical variation in enterprise GMs within farms between years. Individual enterprise performance, measured as GM per unit of activity, was calculated from historical data from 1991 to 1997. This period includes years with the full range of weather types. The GMs of the livestock enterprises exclude fodder costs, as the least-cost supply of feed is decided in the model. To bring the individual enterprises to 1997 money values we used the consumer price index (CPI).

In the panel data used, the number of observations for each enterprise varied from 1,187 for barley to 68 observations for sheep. The number of farms with each enterprise varied from 215 with barley to 27 farms with sheep. We used the unbalanced panel data to find the parameters that describe the variation in the individual enterprise GMs per unit within farms between years. For activity j we estimated the following two-way fixed effects model:⁵

$$x_{it} = \mu + \alpha_i + \beta_t + e_{it} \quad (4)$$

where x_{it} is GM per unit of activity j on farm i in year t ($t = 1, \dots, 7$), μ is general mean, α_i is the effect on GM of activity j as a result of farm i (variation between farms caused by different management practice, soil, etc.), β_t is the effect on GM as a result of year t (variation within farms caused by different weather, prices, yield, etc. between years), and the residual e_{it} is a random variable with mean zero. This model can be estimated with a least-squares dummy variable approach.

The estimated individual enterprise GM per unit for a representative farm for year t is

$$\hat{x}_{.t} = \hat{\mu} + \hat{\beta}_t \quad (5)$$

We then removed from the panel data the within-farm effect caused by different management practices, soil, etc., $\hat{\alpha}_i$, and unexplained white noise, \hat{e}_{it} . We adjusted for trend by regressing the estimated $\hat{x}_{.t}$ from equation (5) against time, t , for each enterprise. We then added this regression's residual for each year to our regression's predicted trend value for the planning year (in our case 2001), to construct detrended series.

⁵ Fixed effects and variance component models are described by, for example, Searle *et al.* (1992).

Table 1. Distribution of activity GMs in NOK per unit^a by state of nature (with probabilities given in parentheses)

Activity	State							Mean	SD
	1 (0.14)	2 (0.11)	3 (0.11)	4 (0.18)	5 (0.18)	6 (0.18)	7 (0.11)		
Barley	8,500	5,210	6,964	5,816	6,059	7,024	6,904	6,633	1,068
Oats	7,578	4,605	6,496	5,128	6,162	6,254	6,176	6,067	961
Wheat	9,745	6,207	9,255	5,943	7,778	7,683	8,047	7,733	1,414
Potatoes	19,815	28,672	15,503	27,081	24,703	18,291	21,060	22,333	4,818
Oilseed	6,442	4,568	7,067	5,262	5,531	6,356	5,638	5,833	839
Grass seed	13,422	9,259	6,709	13,439	10,200	9,599	12,161	10,867	2,475
Carrots	70,310	53,019	68,875	68,580	66,749	54,612	74,458	65,000	8,165
Cow, 5500 kg	18,166	19,274	16,603	15,489	16,814	19,932	16,869	17,574	1,590
Cow, 6500 kg	21,971	23,199	20,239	19,005	20,473	23,928	20,534	21,315	1,762
Cow, 7500 kg	25,578	26,913	23,696	22,354	23,950	27,706	24,016	24,865	1,915
Beef, intensive	11,664	12,278	11,020	11,571	10,095	11,150	12,460	11,358	812
Beef, extensive	9,051	9,692	8,379	8,954	7,414	8,515	9,882	8,731	847
Sheep	1,080	1,107	1,096	1,250	1,037	1,112	1,064	1,111	69

^aBarley, oats, wheat, potatoes, oilseed, grass seed and carrots are per hectare. Dairy cow, beef cow and sheep are per head.

To reflect the chance that similar conditions to those in each of the data years will prevail in the planning period, we assigned differential probabilities to the historical years or 'states of nature' 1991–1997. There are many possible ways of assigning these probabilities. We asked an expert group (a group of regional agricultural research workers) about their subjective relative weights with respect to yield and revenue conditions for the specific years 1991–1997. These assessed probabilities are reported in the upper part of Table 1.

Both national and international (WTO and European Union) developments imply that Norwegian agricultural policy will change in the future. In that case, historical data are not relevant in our decision model. We therefore elicited from an expert⁶ (a national agricultural economics adviser) the subjective marginal distributions of the individual activity GMs. From this expert we obtained judgements of the lowest, highest and most likely values of individual GM in the future (for the next 2–3 years). Then, assuming

6 When taking expert advice, it is normally recommended to use more than one expert to obtain different insights to determine the probability assessment. However, sometimes one expert may be preferred given this expert has good knowledge of the sample space, i.e. the set of all possible outcomes. By using the judgement of one expert we also avoid the problem of how to pool different views.

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that the individual subjective GMs per unit were approximately triangularly distributed,⁷ we calculated means and standard deviations, as shown in the last two columns of Table 1.

Finally, the historical GM series was reconstructed, using the formula (Hardaker *et al.*, 1997: 55)

$$x(n)_{ij} = E[x(s)_j] + \{x(h)_{ij} - E[x(h)_j]\} \frac{\sigma(s)_j}{\sigma(h)_j} \quad (6)$$

where $x(n)_{ij}$ is the synthesised GM for activity j in state i , $E[x(s)_j]$ is the subjective mean of the GM of activity j , $x(h)_{ij}$ is the corrected historical GM of activity j in state i , $E[x(h)_j]$ is the mean GM from the corrected historical data for activity j , $\sigma(s)_j$ is the subjective standard deviation of the GM for activity j , and $\sigma(h)_j$ is the standard deviation of the GM for activity j from the corrected historical data. The reconstructed series has the subjectively elicited means and standard deviations although preserving the correlation and other stochastic dependences embodied in the historical data (see Table 1).

In stage 2 of the model, the level of milk production is conditional on fodder level. If there is a correlation between fodder level per hectare and milk yield per cow, this should be reflected in the probability of milk yield per cow. The relation between milk yield and fodder yield can be treated as an empirical question. In our detrended⁸ historical data we found a significant correlation between fodder yield and milk yield of 0.17, implying a rather weak positive correlation. We used data from the Farm Business Survey to derive the joint distributions of fodder yield and milk yield. From the detrended historical data, we obtained probabilities for milk yield above 7000 kg, $P(M_1)$, between 6000 and 7000 kg, $P(M_2)$, and below 6000 kg, $P(M_3)$, per cow per year at 0.11, 0.54 and 0.35, respectively. The detrended historical fodder yield was divided into two intervals, with the mean as the dividing quantity, and on this basis we found marginal probabilities of high and low fodder yield of $P(F_1) = 0.52$ and $P(F_2) = 0.48$. The detrended historical milk yield was divided into the three above-mentioned intervals and the detrended historical fodder yield was divided into the two above-mentioned intervals. Then we simply counted the numbers of data points in each cell to estimate the joint probability distribution between fodder and milk yields given in Table 2.

If there is a correlation between the state of nature of the enterprise GMs in Table 1 and fodder and milk yield reported in Table 2, this should also be accounted for in the model. We would need to make the fodder and milk probabilities conditional on the state of nature of the enterprise GMs. However, from our historical Farm Business Survey we found a low and insignificant correlation between fodder and milk yield and the GM of

⁷ The triangular distribution is described by, for example, Hardaker *et al.* (1997: 44–45).

⁸ We adjusted for trend by regressing milk yield against time for the whole sample. Then, the regression residual for each observation was added to the predicted milk yield for the planning year 2001. Fodder yield was detrended in the same way. With this approach we assume an equal trend for every farm in the sample. An alternative approach is to detrend individually for each farm.

Table 2. Joint probability distribution for fodder yields and milk yields

Fodder yield	Milk yield			
	Low	Normal	High	Total
Low	0.23	0.25	0.04	0.52
High	0.12	0.29	0.07	0.48
Total	0.35	0.54	0.11	1.00

enterprises, so in the model we assumed no correlation between enterprises' state of nature in Table 1 and fodder and milk yield.

2.4. Matrix structure

The utility-efficient programming model for the case farm was formulated as follows:

$$\max E[U] = p_t U(z_{2t}, r), \quad r \text{ varied} \quad (7)$$

subject to

$$\begin{aligned} A_{11}x_{11} &\leq b_{11} \\ L_{1t}x_{11} + A_{2t}x_{2t} &\leq b_{2t} \\ C_{11}x_{11} + C_{2t}x_{2t} - I_{2t}z_{2t} &\leq f_1 + f_{2t} \\ x_{kt} &\geq 0 \end{aligned}$$

where:

$E[U]$ is expected utility;

t is the state of nature with respect to fodder yield i and milk yield j ($t = 1, \dots, 6$); in our model, $t = 1$ is high fodder yield and high milk yield (HH), $t = 2$ is high fodder yield and normal milk yield (HN), etc.;

k is stage ($k = 1, 2$);

p_t are $1 \times s$ vectors of joint probabilities of activity GM per unit outcome given that a particular fodder and milk yield state of nature and a particular season state of nature have occurred; for example, p_1 is the probability vector of state of season given high fodder yield and high milk yield (HH);

$U(z_{2t}, r)$ is an $s \times 1$ vector of utilities of net income z_{2t} by state t , where the utility function is defined for a measure of risk aversion, r , which is varied;

z_{2t} is an $s \times 1$ matrix of net income;

A_{kt} is an $m_{kt} \times n_{kt}$ matrix of technical coefficients in stage k and state t ;

x_{kt} is an $n_{kt} \times 1$ vector of activity levels in stage k and state t ;

b_{kt} is an $m_{kt} \times 1$ vector of resource stocks in stage k and state t ;

L_{1t} is a set of t matrices linking first- and second-stage activities;

C_{kt} is an $s \times n_{kt}$ matrix of activity GMs by state s and activity n_{kt} in stage k and state t ; it should be noted that, with this formulation, there is no need to assume any standard form of distribution;

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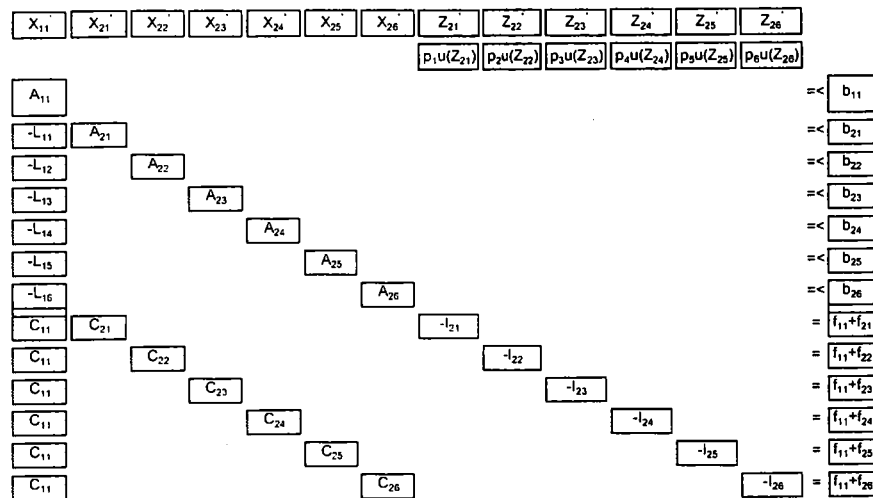


Figure 2. Overview of the matrix structure.

f_{kt} is an $s \times 1$ matrix of fixed costs in stage k and state t ; in this analysis fixed costs are assumed equal in all states; the fixed costs are set at NOK 300,000 for stages 1 and 2 combined, which is approximately the average fixed cost for the farms in the survey;

I_{kt} is a set of $s \times s$ identity matrices in stage k and state t .

Figure 2 shows a diagrammatic overview of the matrix structure. The matrix developed comprised 151 activities and 166 constraints. It was solved using GAMS/MINOS5/CONOPT2. Because this software does not include a parametric programming option, we obtained solutions for stepwise variation in a and c (see equations (1) and (2), respectively).

3. Results and discussion

In this section we present results for three cases. Case 1 in Section 3.1 comprises results under the prevailing Norwegian subsidy system. In Section 3.2 we present results for case 2, where the farmers receive support as a fixed amount. Case 3 is a scenario in which we assume that, in addition to subsidies as a fixed amount, Norwegian agriculture has undergone structural change with more effective production and lower seasonal labour requirements. Furthermore, we assume a reduction in the individual activity GMs and a higher variation in the GMs. These results are presented in Section 3.3.

As noted above, we assumed a range of relative risk aversion with respect to wealth, $r_r(W)$, between 0.5 and 4. The ranges for $r_r(z)$ and $r_w(z)$ are approximated by use of equation (3), with wealth W equal to NOK 1,350,000 and transitory income z equal to NOK 300,000. Farm equity used for wealth and net farm income used for transitory income are approximately the average from the Farm Business Survey (NILF, 1992–1998).

3.1. Case 1: existing Norwegian subsidy schemes

First, we present results from optimisation with the CRRA power utility function. Table 3 shows a brief summary of the main activities in stage 1 for our model by degree of risk aversion. Our main observation is that the degree of risk aversion has no effect on optimal activity choice. In our model, a farmer who is hardly risk averse at all ($r_r(W) \approx 0.5$) would choose the same farm plan as a very risk averse farmer ($r_r(W) \approx 4$).

In stage 2, the degree of risk aversion has no effect at all on the optimal farm plan. The tactical decisions at stage 2 are given in Table 4.

Table 3. Summary of optimal farm activities in stage 1 under existing Norwegian subsidy schemes, calculations with CRRA power utility function

	Unit	Coefficient of risk aversion		
		$r_r(z)$: $r_r(W)$:	0.111 ≈ 0.5	0.444 ≈ 2
CE	NOK $\times 1000$	388	386	385
Activity				
Wheat	ha	24.1	24.1	24.1
Potatoes	ha	0.7	0.7	0.7
Grass seed	ha	3.0	3.0	3.0
Carrots	ha	4.1	4.1	4.1
Grass fodder	ha	3.1	3.1	3.1
Keep cows	number	5.3	5.3	5.3
Hire land	ha	15	15	15
Non-prod.-spec. sup.	NOK $\times 1000$	146	146	146

Table 4. Summary of optimal farm plan activities in stage 2 under existing Norwegian subsidy schemes, calculation with CRRA power utility function

Activities	Unit	HH	HN	HL	LH	LN	LL
Keep cows	number	5.3	5.3	5.3	5.3	5.3	5.3
Buy cows	number		2.1	2.1		2.1	2.1
Insert heifers	number		1.1	1.1		1.1	1.1
Concentrates	-NOK $\times 1000$	40	33	26	40	33	26
Buy feed	-NOK $\times 1000$		8	8	9	13	13
Milk production	litres	36,033	35,738	30,000	36,066	35,738	30,000
Prod. spec. support	NOK $\times 1000$	60	60	60	60	60	60

HH. High fodder yield and high milk yield in stage 2; HN. high fodder yield and normal milk yield in stage 2; HL. high fodder yield and low milk yield in stage 2; LH. low fodder yield and high milk yield in stage 2; LN. low fodder yield and normal milk yield in stage 2; LL. low fodder yield and low milk yield in stage 2.

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Inspection of the solutions obtained shows that the main solution determinants in our analysis are not the farmer's risk aversion, but availability of labour, subsidies and a contract constraint on grass seed. Independent of farmers' risk attitudes, dairy farming seems to be a preferred portfolio choice compared with beef and sheep production. With low milk yield per cow, 5.3 cows kept from the start of the quota year, supplemented with 2.1 purchased cows and 1.1 heifers from own herd in stage 2, produce 30,000 litres to earn product-specific support of NOK 60,000 (see Table 4). It should be noted that the optimal solutions imply production of less than the farm's annual milk quota.

In a second run of the model, we used the negative exponential function with approximately the same range of risk aversion as used for the CRRA power function. An approximate range for $r_u(z)$, which corresponds to the chosen range used for $r_r(z)$, gave almost the same optimal solutions over the range of risk aversion as obtained with CRRA power function. Hence we do not present these results.

Our results indicate that, under existing policy and market conditions in Norwegian agriculture, the degree of risk aversion and the type of utility function make little or no difference to the optimal farm plans for a plausible range in the degree of risk aversion. Factors such as subsidy schemes (which reduce the downside risk), market conditions for the products and availability of labour on the farm seem to be more important in Norwegian farm planning than farmers' risk aversion and the form of utility function.

3.2. Case 2: subsidies as a fixed amount

An alternative to the existing support scheme is a production-neutral support scheme. In this case, farmers receive a fixed amount of subsidy as direct income support, independent of activities and produced quantities. A direct income support scheme could be expected to have little or no distorting effect on farmers' production decisions. Although there are many problems in deciding how this form of support should be paid, we ignore them here. We assume that the fixed amount of support (NOK 206,000) is exactly equal to the existing direct support in the optimal solution for case 1 (NOK 146,000 + NOK 60,000). In this way we illustrate some effects that the existing Norwegian support system has on choice of enterprises.

Evidently, for our case farm, a neutral subsidy scheme would induce the farmer to exclude livestock production and use the labour released for potatoes and carrot production. Comparing the certainty equivalent⁹ (CE) of net income in Tables 3 and 5 we observe a higher CE in the case with a fixed amount of support. This result may indicate that, if government wants to change the basis of subsidy payments while maintaining the same level of welfare (i.e. utility, hence CE), it could do so more cheaply than under existing arrangements. This result is surprising, because a change to

⁹ The certainty equivalent of a risky prospect is the sure amount that would make a decision-maker indifferent between the sure sum and the risky prospect (Hardaker *et al.*, 1997).

a fixed amount of support would be expected to cause a shift to less intensive methods of production; lower product prices imply that lower levels of input use would be optimal. But this form of adjustment is not allowed for in the linear production function implicit in the model. To do so would require further refinement of the model as well as detailed information on production function that is not available.

Table 5. Summary of optimal farm plans for the case with a fixed amount of support, calculation with CRRA power utility function

	Unit	Coefficient of risk aversion		
		$r_r(z)$: $r_r(W)$:	0.111 ≈ 0.5	0.444 ≈ 2
CE	NOK $\times 1000$	417	415	414
Activity				
Wheat	ha	25.6	25.6	25.6
Potatoes	ha	1.1	1.1	1.1
Grass seed	ha	3.0	3.0	3.0
Carrots	ha	5.3	5.3	5.3
Hire land	ha	15.0	15.0	15.0

The results in Table 5 again indicate the unimportance of risk aversion in the determination of the optimal farm plan. Here too, other factors are more important than the farmer's degree of risk aversion.

3.3. Case 3: more liberal policy

This is a hypothetical future case, not intended to be fully realistic. We still assume the subsidies are given to farmers as a fixed amount. In addition, we assume that a structural change occurs in agriculture, with more efficient production where the labour requirements for the individual enterprises are reduced towards Danish levels (to the lower Norwegian level stated by NILF (1999)). Furthermore, we assume that the subjective expected modal GMs per unit would be reduced by 30 per cent for cereals, 20 per cent for dairy and sheep farming, and 40 per cent for beef production. The lowest and highest expected GMs per unit are further assumed to decrease and increase by 20 per cent, respectively. Increased variability is assumed, as we expect more price volatility under deregulation. The triangular distributions then show lower expected means and higher standard deviations, compared with case 1. We used historical data from Denmark to estimate the correlation matrix and stochastic dependence. The Danish data we used, obtained from the Danish Institute of Agriculture and Fisheries Economics via the Internet, were not panel data at farm level but aggregated average GMs for individual enterprises for the accounting years 1991–1992 to 1997–1998. Grouped data may show less dependence (lower correlation) because, for example, local

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rainfall or frost events will be smoothed out. It is therefore important to realise that this approach can underestimate the stochastic dependence between enterprises.

Table 6. Summary of optimal farm plan in stage 1 under a more liberal policy, calculations with CRRA power function

Unit		Coefficient of risk aversion				
		$r_r(z)$: 0.111	0.222	0.444	0.667	0.889
		$r_r(W)$: 0.5	≈ 1	≈ 2	≈ 3	≈ 4
CE	NOK $\times 1000$	446	445	443	441	439
Activity						
Wheat	ha	19.0	19.0	19.0	19.0	20.4
Potatoes	ha	13.3	13.3	13.3	13.3	11.4
Grass seed	ha	3.0	3.0	3.0	3.0	3.0
Carrots	ha	4.7	4.7	4.7	4.7	5.2

As a result of the structural change, we further assume either that one farmer rents another's farm of the same size, or that two farmers have a joint operation, where both have part-time work outside the farm. Therefore, we assume that in this case the 'farm' size is 40 ha, with possibilities to hire 30 ha more. We still assume that the total amount of subsidy is NOK 206,000 per farm, the same level as in case 2. In other words, we assume the level of public subsidies is reduced by 50 per cent per farm but is held constant per full-time farmer.

In case 3, it seems that risk aversion is more important than in cases 1 and 2 in determining the optimal farm plan (see Table 6). A farmer who is weakly risk averse would mainly choose wheat, potatoes, grass seed and carrots in

Table 7. Summary of optimal farm plan in stage 1 under a more liberal policy, calculations with negative exponential function

Activity		Unit	Coefficient of risk aversion				
			$r_u(z) \times 10^5$: 0.037	0.074	0.148	0.222	0.296
			0.5	1	2	3	4
CE	NOK $\times 1000$		445	443	440	438	435
Activity							
Wheat	ha		19.0	19.1	19.2	19.3	19.8
Potatoes	ha		13.3	13.2	13.0	12.9	12.2
Grass seed	ha		3.0	3.0	3.0	3.0	3.0
Carrots	ha		4.7	4.7	4.8	4.8	5.0

his portfolio, but would not rent extra land. More risk aversion implies more wheat, and less potato.

Finally, we compared results using the negative exponential and CRRA power utility function. For $r_r(W)$ in the range 0–4 and with $W = \text{NOK } 1,350,000$ and $z = \text{NOK } 300,000$, applying equation (3) gives a range of $r_a(z)$ of 0.00000037 to 0.00000296. Results calculated with a negative exponential function within this range are reported in Table 7.

Comparing results in Table 7 with the results in Table 6 reveals that the negative exponential function and CRRA power function lead to very similar farm plans over the whole range of risk aversion. Again, it seems that the choice of utility function of income is not important for the farming cases modelled.

4. Concluding comments

Norwegian farmers have limited flexibility in choice of enterprises, caused by both relatively adverse geographical and climatic conditions, and policy and market regulations. In these circumstances, it seems that farmers' risk attitudes are of little importance in affecting the choice of farm plan. The results also indicate that the form of the farmer's utility function for income makes little difference to the optimal farm plan. Factors such as subsidy scheme, market conditions for the products, and available labour on the farm seem more important on these Norwegian farms than the farmer's risk aversion. These factors may reduce the farmer's incentive to let diversification considerations affect choice of enterprise combination. Moreover, having only two or three enterprises, which is normal, can often capture the majority of risk-reducing benefits from diversification (Hardaker *et al.*, 1997).

Our results are in accordance with Kallberg and Ziemba's (1983) study of functional form of utility functions. But because Kallberg and Ziemba investigated the utility of wealth and we look at utility of income, it is difficult to compare the results directly.

Under existing Norwegian agricultural policy we did not find any shift in resource use with increased risk aversion. This result is in contrast to the findings of many earlier studies of farm planning and risk aversion (e.g. Kingwell, 1994; Pannell and Nordblom, 1998). As Kingwell used an absolute risk aversion measure and it is meaningless to compare coefficients in different units, it is difficult to compare the results. Pannell and Nordblom used the same relative risk aversion measure as us, but a larger range of relative risk aversion.

If subsidy arrangements are assumed to be changed to give farmers a fixed subsidy independent of production area and volume, the optimal farm plans do not include any livestock production. In this case the impacts of different risk preferences or form of utility function again remain unimportant.

In this analysis we have assumed a wholly rational farmer, to explore what he might want to do. Rationality in this case includes the asset integration assumption and is in contrast to some empirical evidence showing, for example, that people assess losses and gains differently from how they view

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income and wealth (e.g. Camerer *et al.*, 1997; Thaler, 1981). Our conclusions should be interpreted with this assumption of rationality in mind.

In what types of farming would the results in this paper matter? It is obvious that, in a regulated regime such as currently exists in Norway, the constraints relating to production possibilities and policy and market regulations will strongly affect farmers' choices. This implies that the portfolio options and therefore the impacts of forms of utility function are limited, as our results show. Nevertheless, these findings may suggest that similar conclusions could apply, at least to some extent, for farms in countries with fewer market regulations than in Norway. We suggest that, provided the degree of risk aversion is appropriately estimated and reflected in the analysis, the form of the utility function will often be found to be unimportant.

The study leads to a number of ideas for further research. First, we have not included in our model any financial management options. Fisher's separation theorem (described, for example, by Copeland and Weston, 1988) implies that it is better to diversify through capital markets than through combinations of enterprises. In Norway, financial markets for agricultural commodities are not well developed, for price or for volume. Still, a possible extension of the model would be to include some finance activities such as 'risk-free' loans and private insurance arrangements. A second possible development of the model would be to include off-farm income opportunities as activities in the model. Third, the model used in this analysis finds an optimal farm plan given a planning horizon of 1 year, which may be satisfactory if the production activities for one year do not affect the optimal activities for the following year. However, comprehensive changes in activities will often need investments that have impacts many years into the future. This problem needs techniques that simultaneously determine optimal investments and annual production decisions. One possible approach is multi-period UEP. Fourth, another possible extension is also to include non-linear production functions in the model which, as mentioned in Section 3.2, would allow for changes in intensity of production as prices change. Finally, subsidies and production regulation are very important factors for a farmer's choice of farm enterprises. However, as a result of both national and international developments, Norway is experiencing a reorientation of its agricultural policy towards increased deregulation and a more market-oriented approach. This implies that farm subsidy schemes and production regulations will change in the future, and so are uncertain. More work could then be carried out to model the political uncertainty more explicitly and completely than in this study.

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References

- Anderson, J. R. and Dillon, J. L. (1992). Risk Analysis in Dryland Farming Systems. Farming Systems Management Series No. 2. Rome: FAO.
- Arrow, K. J. (1970). *Essays in the Theory of Risk-Bearing*. Amsterdam: North-Holland.
- Bhende, M. J. and Venkataram, J. V. (1994). Impact of diversification on household income and risk: a whole-farm modelling approach. *Agricultural Systems* 44: 301–312.
- Camerer, C., Babcock, L., Loewenstein, G. and Thaler, R. (1997). Labour supply of New York city cabdrivers: one day at a time. *Quarterly Journal of Economics* 112: 407–441.
- Cocks, K. D. (1968). Discrete stochastic programming. *Management Science* 15: 72–81.
- Copeland, T. E. and Weston, J. F. (1988). *Financial Theory and Corporate Policy*, 3rd edn. Reading, MA: Addison Wesley.
- Hardaker, J. B. (2000). Some Issues in Dealing with Risk in Agriculture. Working Papers in Agricultural and Resource Economics. Armidale, N.S.W.: School of Economic Studies, University of New England.
- Hardaker, J. B., Huirne, R. B. M. and Anderson, J. R. (1997). *Coping with Risk in Agriculture*. Wallingford: CAB International.
- Hardaker, J. B., Pandey, S. and Patten, L. H. (1991). Farm planning under uncertainty: a review of alternative programming models. *Review of Marketing and Agricultural Economics* 59: 9–22.
- Kaiser, H. M. and Apland, J. (1989). DSSP: a model of production and marketing decisions on a Midwestern crop farm. *North Central Journal of Agricultural Economics* 11: 157–170.
- Kallberg, J. G. and Ziemba, W. T. (1983). Comparison of alternative utility functions in portfolio selection problems. *Management Science* 29: 1257–1276.
- Kingwell, R. S. (1994). Risk attitude and dryland farm management. *Agricultural Systems* 45: 191–202.
- Meyer, J. (1977). Second degree stochastic dominance with respect to a function. *International Economic Review* 18: 477–487.
- Nanseki, T. and Morooka, Y. (1991). Risk preference and optimal crop combinations in upland Java, Indonesia: an application of stochastic programming. *Agricultural Economics* 5: 39–58.
- NILF (1992–1998). *Driftsgranskinger i jord-og skogbruk (Account Results in Agriculture and Forestry)*. Annual publication. Oslo: Norwegian Agricultural Economics Research Institute.
- NILF (1999). *Handbok for driftsplanlegging (Handbook in Farm Business Planning) 1999/2000*. Oslo: Norwegian Agricultural Economics Research Institute.
- Pannell, D. J. and Nordblom, T. J. (1998). Impacts of risk aversion on whole-farm management in Syria. *Australian Journal of Agricultural and Resource Economics* 42: 227–247.
- Patten, L. H., Hardaker, J. B. and Pannell, D. J. (1988). Utility-efficient programming for whole-farm planning. *Australian Journal of Agricultural Economics* 32: 88–97.
- Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica* 32: 122–136.
- Rae, A. N. (1971a). Stochastic programming, utility, and sequential decision problems in farm management. *American Journal of Agricultural Economics* 53: 448–460.
- Rae, A. N. (1971b). An empirical application and evaluation of discrete stochastic programming in farm management. *American Journal of Agricultural Economics* 53: 625–638.

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- Saha, A., Shumway, C. R. and Talpaz, H. (1994). Joint estimation of risk preference structure and technology using expo-power utility. *American Journal of Agricultural Economics* 76: 173–184.
- Searle, S. R., Casella, G. and McCulloch, C. E. (1992). *Variance Components*. New York: Wiley.
- Thaler, R. (1981). Some empirical evidence on dynamic inconsistency. *Economics Letters* 8: 201–207.

Essay 3:

Assisting whole-farm decision-making through stochastic budgeting*

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Abstract

Stochastic budgeting is used to simulate the business and financial risk and the performance over a six-year planning horizon on a Norwegian dairy farm. A major difficulty with stochastic whole-farm budgeting lies in identifying and measuring dependency relationships between stochastic variables. Some methods to account for these stochastic dependencies are illustrated.

The financial feasibility of different investment and management strategies is evaluated. In contrast with earlier studies with stochastic farm budgeting, the option aspect is included in the analysis.

Keywords: Decision analysis; Whole-farm stochastic budgeting; Monte Carlo simulation; Real option

1 Introduction

In assessing any business investment, particularly for a family business such as a farm, there are two aspects to consider. One is the profitability of the investment, which is often a fairly long-run matter. The future is shrouded in uncertainty so such decisions often involve a high degree of intuition or strategic thinking. The other aspect is financial feasibility. Usually large

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investments involve borrowing substantial amounts of money, implying a significant increase in financial risk of the business. For example, a couple of bad years in production and an unexpected rise in the interest rates can send the business bankrupt. This risk is most severe in the first years after the investment when the debt is at a peak. In this paper a model of the business and financial risk of the farm over such a shorter time horizon is presented.

A typical farm in Eastern Norway is used as a case study. In the planning year the farm has dairy and some beef production, cereal crops and some forestry. Quotas regulate the milk production. The (male) farmer is thinking about five alternative investment and management strategies, but is very uncertain which he should choose.

In making a decision about a business investment or future strategic choice farmers have to account for many aspects. Among other things, they have to make up their minds about the following questions: What future activity gross margins (GMs) are realistic to use in farm planning? Will the present subsidy scheme change in the future, and if so how? When borrowing money, will there be any changes in the interest rates over the next few years? What about the labour requirement for different activities - how many hours will be required per unit? Will there be a need to hire labour, and if so, how much? What price might be obtainable if milk quota could be sold in the future? These and other similar uncertainties imply use of stochastic budgeting.

Richardson and Nixon (1986) developed the stochastic whole-farm budgeting model FLIPSIM (Farm level income and policy simulator). FLIPSIM simulates, under price and yield risk, the annual economic activities of a representative farm over a multiple-year-planning period. The model uses equations that are either identities or probability distributions. It has been used for policy analysis (e.g. Knutson et al., 1997), comparing risk management strategies (e.g. Knutson et al., 1998), technology assessment (e.g. Nyangito et al., 1996), and financial analysis (e.g. Hughes et al., 1985) etc.

Milham et al. (1993) developed a stochastic whole-farm budgeting system, called RISKFARM. RISKFARM was originally developed to enable the appraisal of the financial performance and risk effects of alternative farm and non-farm investments and potential changes in the financial structure of Australian farms (Milham, 1992). Compared with FLIPSIM, RISKFARM has several stochastic variables and the stochastic dependency is specified in another way (multivariate empirical probability distribution in FLIPSIM vs. hierarchy of variables approach in RISKFARM).

In this analysis a whole-farm stochastic budgeting model is used which includes stochastic GMs, interest rates, fixed costs, labour requirements for activities and milk quota price. The model simulates the farm performance and the business and financial risk over a six-year planning horizon. Risky strategies are evaluated by cumulative distribution functions (CDFs) and by stochastic dominance. In concept, the model draws on the work of Milham et al. (1993). In contrast with earlier studies using stochastic farm budgeting, the option value of a 'wait and see' strategy is included in the analysis.

This paper is structured as follows. First, an overview is given of the farm system and the investment strategies investigated. Then the model is described in the third section. The empirical results are thereafter presented, and the last section contains some concluding comments.

2 Overview of the farm system and investment strategies

The case farm used in this study is in the lowlands of Eastern Norway. Winters are long in this area, normally with snow and temperatures many degrees Celsius below zero. The climate gives high farm business costs compared to most other countries. Farm size is 33 ha of arable land and 50 ha of forest. The main activity on the farm in the planning year 1999 was milk production, with a milk quota of 100 000 litres. There were also a few beef cows on the farm. The area not used to grow fodder crops was used for cereal production, mainly wheat and barley.

For the past several years the prices of farm products in Norway have mainly been decided through annual negotiations between the two farmers' unions and the Government. As a result, prices for almost all enterprise have been administrated. Despite this price regulation, the GM per unit for each enterprise within a farm is uncertain. This uncertainty is caused by factors such as weather and plant and animal diseases causing yield and product quality uncertainty. With increased deregulation more price volatility is expected in future causing still higher GM volatility. The prices of forest products largely follow the world market prices and also vary between years.

The Norwegian government has assigned relatively large subsidies to the agriculture sector compared with other countries. Even if both the national and international agricultural policy environments change in the future, it seems almost certain that the Government will be obliged to continue making high transfer payments to the Norwegian agricultural sector so

long as it is considered desirable to retain a substantial number of people in agriculture. Hence, it was assumed in this paper that the subsidy per farmer will be at the same level in the planning period as in the planning year 1999.

Since 1983, production quotas have regulated the production of cow milk. From 1996, the Government introduced a system for administrative redistribution of milk quotas. Farmers can apply to purchase quotas up to 20% of the total quota they had the previous year, although not more than the farm area allows. The farmer only gets an offer if other farmers are selling their quotas. If a farmer wishes to sell quota, she or he must sell the whole quota.

The floating interest rate on borrowed funds is rather uncertain. A farmer with large investments and high debt is normally rather dependent on the interest rate level over the next few years. It is possible to get a loan at a fixed interest rate to avoid some risk, but in the long run the cost is naturally higher.

Maximum family labour available on the farm is 2600 h per year, on the basis of one full-time owner operator. If the labour requirement on the farm exceeds this limit, the farmer must hire labour at a fixed cost per hour. The main problem with labour demand coefficients for different activities is lack of good and certain data for planning. This is especially a problem for new production methods to be taken up on a farm.

The total value of assets on the farm at December 1999 was NOK (Norwegian kroner) 3.03 million, valued at market prices, of which NOK 2.45 million was equity. All the debt capital was borrowed at floating interest rates.

The plan was prepared in 1999 for the planning period 2000 to 2005. In 1999 the farmer was concerned that existing level of production was too low to return an adequate level of profit in future, but he was very uncertain what strategy he should then choose. The farmer wished to investigate a range of alternative investment and management strategies that can help him decide which to adopt starting next year (2000). The choice was among the following five strategies:

1. Continue as today. This choice implies continuing to produce milk to the level of the quota of 100 000 litres and use the arable land not under fodder crops for cereals.
2. Continue as today, but invest in a new farm building for chicken production. The new building would be for 80 000 chickens per year and was estimated to cost NOK 1 440 000.
3. Invest in improvements of the present farm building and combine milk production with beef production in addition to cereal production. A new cowshed would reduce the labour

needed for milk production. This released time would be used for beef production. In addition to producing the milk quota of 100 000 litres, the improved building would make it possible to keep 30 beef cows. The total investment cost was estimated to be NOK 2 700 000.¹

4. Abandon the milk production, sell the milk quota for NOK 5.50/litre² today and only produce cereals. It was assumed that 50% of the available family labour per year (1300 h) would be devoted to half-time paid off-farm work at a fixed wage of NOK 125 000 per year. If the labour requirement on the farm were to exceed 1300 h, labour would be hired at a fixed cost. No investment cost was required.
5. Same as strategy 4, except wait to sell the milk quota until the quota price eventually get above NOK 7.00/litre.

If the farmer does not invest in farm improvements, 300 m³ of forestry will be felled every second year. If the farmer does invest, 1000 m³ of forestry will be felled in the investment year and 500 m³ the first year after the investment.

3 The model

Traditional whole-farm budgeting is done on the basis of fixed-point estimates of production, prices and financial variables to predict point estimates of financial results. In reality, the events and conditions planned for will not turn out as assumed. A common response to this problem is to conduct sensitivity analysis as part of the planning exercise in order to determine the range of possible results. In a sensitivity analysis it is customary to consider changes in only one variable at time. The effects on the performance measure of combinations of errors in different variables are, therefore, largely ignored (Hull, 1980). And, when many

¹ For strategy 2 no specific investment cost for livestock was accounted for since these were included in the livestock GMs. For the beef production in strategy 3 it was assumed that the farmer can partly recruit from own herd (since he already had some beef cows) and that he would buy more beef cows, the costs of which were included in the estimated investment cost.

² In the existing system for redistribution of milk quotas, quota sellers are offered a price of NOK 5.50/litre for the first 100 000 litres, NOK 2.75/litre for the second 100 000 litres, and no additional compensation for quotas exceeding 200 000 litres. The autumn 1999, the quota price was increased by NOK 2.00/litre as a special measure designed to reduce total milk production (NILF, 2000).

variables are uncertain, sensitivity analysis of the effect on financial performance for more than just few variables becomes tedious and difficult to interpret. Moreover, the sensitivity analysis gives no indication of the likelihood of a particular result being achieved (Little and Mirrlees, 1974).

To overcome these problems an alternative approach is stochastic budgeting, which accounts for some of the main uncertainties in the evaluation and then gives an indication of the distribution of outcomes. In this framework uncertain variables can be expressed in stochastic terms, and many combinations of variable values can be analysed to provide a full range of expected outcomes (Milham et al., 1993).

The model in this paper was built up from a deterministic whole-farm budgeting model, formulated in an Excel spreadsheet. The model operates over a year-to-year strategic level, and produces annual financial reports over a six-year time horizon. The financial reports are derived from functional equations linking the farm production activities, subsidy schemes, capital transactions, consumption activities and financing and tax obligations.

Stochastic features were introduced into the budget by specifying probability distributions for variables assumed to be most important in affecting the riskiness of the selected measure of financial performance. Note that, to keep the model practicable and reasonably transparent, only those stochastic variables assumed to be most important for the decision were modelled using probability distributions.

Objective probabilities based on historical data alone can seldom reflect the uncertainty about future situations in stochastic analysis (Hull, 1980; Hardaker et al., 1997; Milham, 1998). The subjective expected utility theorem leads to the conclusion that the right probabilities to use for decision analyses are the decision maker's subjective probabilities (Savage, 1954). The probability distributions used in the model in this paper were partially based on historical data (objective frequencies) and partially based on elicited subjective judgments.

One aspect that is important to consider in stochastic budgeting is the question of the stochastic dependency between variables (Hull, 1980; Hardaker et al., 1997). The distribution of performance variables will be seriously compromised if important stochastic dependencies are ignored. For example, if yield and price are positively correlated, an analysis that assumes zero correlation will under-estimate variance of revenue, and will over-estimate it if they are negatively correlated. Stochastic dependency between variables was built in to the model either by use of the stochastic dependency embodied in the discrete historical data matrix or

by use of the 'hierarchy of variables approach' (Hardaker et al., 1997). Description and specification of the stochastic variables and specification of their dependency are further described in the subsection below.

A Monte Carlo sampling procedure with Palisade's @Risk add-in software was used to evaluate the budget for a large number of iterations. In the simulation, values of parameters entering into the model were chosen from their respective probability distributions by Monte Carlo sampling, and were combined according to functional relationships in the model to determine an outcome. The process was repeated a large number of times to give estimates of the distributions of the performance measures which can be expressed as cumulative distribution functions (CDFs), or summarised in terms of moments of the distributions. The appropriate number of samples to draw in the Monte Carlo sampling exercise depends on the required degree of stability of the simulation results. In this analysis, adequate stability in the output distribution was assumed when the average percentage change in 5% fractiles of the probability distribution, the mean and the standard deviation of output were each below 1.5% for an increase of one hundred iterations. Experiments showed that some of the strategies required very large numbers of sample points before this degree of stability of the results was attained. To ensure stability, 1500 sample simulation experiments were used. The random generator used in the simulation process was seeded to ensure that the same set of random samples would be sampled for each strategy evaluated.

In financial analyses such as this it is not always obvious which performance measures one should use; the choice depends on the purpose of the analysis. Milham (1992) used net worth and net cash flow at the end of the planning period as objectives in appraisal of financial performance of alternative farm and non-farm investments on Australian farms. The purpose of this analysis is to compare different investment and production strategies with respect to financial feasibility, and the measure of performance used is equity at the end of the last (sixth) planning year. Equity is a measure of financial solidity, and a large equity promises the ability to survive losses in the future. A farmer is technically bankrupt if the equity is negative. One problem with this measure is in case when the equity is positive at the end of the planning period yet in some of the years between the start and end of the period the equity was negative, and the farmer was therefore insolvent. To prevent this scenario an extra high interest rate on loans was built in to apply if the equity became negative at any year during the planning period. In practice, banks also require a higher interest rate for loans with high risk.

Private consumption was assumed fixed every year in the planning period, independent of bad or good years.

3.1 Specification of stochastic variables in the model

As already noted, the stochastic variables in the model include fixed costs, activity GMs³, interest rates, labour requirement for activities and milk quota price.

The fixed costs are assumed normally distributed around a stochastic time trend, and the hierarchy of variables approach (Hardaker et al., 1997; Milham, 1998) was used to account for this. The hierarchy of variables approach is a means of avoiding the need to directly determine the relationship between each pair of co-related variables. The approach requires selection of a macro-level variable to which all types of fixed costs can be expected to be correlated. The macro-level variable used was the price index of agricultural means of production and production services, PC , maintained by Statistics Norway (1986-99) over the period 1985 to 1998. The hierarchy of variables approach involved the following steps. First, the time trend was derived by regressing the price index of agricultural means of production and production services, PC , against time, t :

$$PC_t = \gamma + \delta t + e_{PCt} \quad e_{PCt} \sim N(0, \sigma_{PC}^2), \quad t = (1, \dots, 14) \quad (1)$$

Second, equation (1) was used to predict the price index agricultural means of production and production services, PC , for every year in the farm plan period. The predicted means from equation (1) were assumed to be the means of a normal distribution, with the standard deviation of error component, σ_{PC} , used as the standard deviation of the normal distribution:

$$PC_t = \hat{\gamma} + \hat{\delta}t + N(0, \sigma_{PC}^2) \quad t = (16, \dots, 21) \text{ for the planning years 2000 to 2005} \quad (2)$$

Third, each price index for farm buildings, FC_1 , machinery and equipment, FC_2 , hired labour, FC_3 , and other fixed costs, FC_4 , was regressed against PC :

$$FC_{it} = g_i + h_i PC_t + e_{FC_{it}} \quad e_{FC_{it}} \sim N(0, \sigma_{FC_i}^2), \quad t = (1, \dots, 14), \quad i = (1, \dots, 4) \quad (3)$$

where i is type of fixed costs index, FC_i . Fourth, the predicted stochastic time trend in equation (2) was used in equation (3) to forecast price indexes of future fixed costs for each i . The error component from equation (3) with mean zero and standard deviation, σ_{FC_i} , was included to account for normally distributed fixed costs for each i :

$$FC_{it} = \hat{g}_i + \hat{h}_i PC_t + N(0, \sigma_{FC_i}^2) = (\hat{g}_i + \hat{h}_i \hat{\gamma}) + \hat{h}_i \hat{\delta} t + N(0, \hat{h}_i^2 \sigma_{PC}^2 + \sigma_{FC_i}^2), \quad t = (16, \dots, 21) \quad (4)$$

From equation (4) observe that the predicted price index of each fixed cost i has: a different constant term, a different drift term and different variance but the constant term, drift term and variance for each price index of fixed cost depend partly on the predicted trend in the macro variable PC . An implicit simplifying assumption is that all stochastic effects derived from national costs data are applicable to the individual case farm. For this analysis the standard deviation of the error component, σ_{FC_i} , was assumed to increase linearly by 2.5% a year over the planning period.

The estimation of parameters of the probability distributions for the stochastic GM variables and their stochastic dependency was partially empirically based and partially based on elicited subjective distributions. Since no suitable data for the case farm exist, the Farm Business Survey (driftsgranskingsdata) from the Norwegian Agricultural Economics Research Institute (NILF, 1992-99) was used to estimate historical GM variation of activities within farms between years. Individual activity performance, measured as GM per unit, was calculated from historical data from 1991 to 1998. This period covers the different year types w.r.t. weather. To bring the individual activities to 1997-money value the consumer price index (CPI) was used. From the unbalanced panel data the parameters that describe the variation in the individual activity GMs within farms between years for each activity j was estimated using the following two-way fixed effects model⁴:

$$x_{it} = \mu + \alpha_i + \beta_t + e_{it} \quad (5)$$

³ For simplicity, uncertainty in activity costs and returns was represented at GM level. A more refined model might include stochastic variables for prices, yields, and variable costs separately. However, high levels of disaggregation lead to an increase in the number of 'messy' stochastic dependencies (Hull, 1980).

⁴ Fixed effects and variance component models are described in, for example, Searle et al. (1997, Ch. 9).

where x_{it} is GM of activity j on farm i in year t ($t=1, \dots, 8$), μ is general mean, α_i is the effect on GM of activity j due to farm i (variation between farms caused by different management practice, soil etc.), β_t is the effect on GM due to year t (variation within farms caused by different weather, prices, yield etc. between years), and the residual e_{it} is a random variable with mean zero. This model was estimated with a least squares dummy variable approach. The estimated individual activity GM for a representative farm for year t was:

$$\hat{x}_{.t} = \hat{\mu} + \hat{\beta}_t \quad (6)$$

From the panel data the within farm effect caused by different management practice, soil etc., $\hat{\alpha}_i$, and unexplained white noise, \hat{e}_{it} , were then removed. Trend was adjusted for by regressing the estimated $\hat{x}_{.t}$ from equation (6) against time, t , for each activity. The residual for each year was added to the predicted trend value for the first planning year (in this case year 2000) in order to construct de-trended series.

To reflect the chance that similar conditions to those in each of the data years will prevail in the planning period, I assigned differential probabilities to the historical years or 'states of nature' 1991 to 1998. An expert group (a regional agricultural research workers group) was asked about their subjective relative weights w.r.t. yield and revenue conditions for the specific years 1991 to 1998. These assessed probabilities are reported on the upper part of Table 1.

Table 1 Distribution of activity GMs in NOK per unit^a by state for the first planning year in the model

State	1	2	3	4	5	6	7	8	Mean	Std.dev.
Prob.	0.13	0.09	0.09	0.16	0.16	0.16	0.09	0.13		
Barley	8392	5164	6885	5759	5997	6943	6826	7145	6633	1068
Wheat	9540	6127	9068	5873	7643	7551	7902	8669	7733	1414
Milk cow	13056	13795	12015	11273	12156	14233	12192	13124	12720	1051
Beef cow	5659	6013	5288	5606	4755	5363	6118	5674	5507	398
Chicken	2974	3072	2822	2809	2750	2880	2966	3033	2900	122
Forestry	207	200	185	194	209	202	199	198	200	8

^a Barley and wheat are per hectare. Milk and beef cows are per head. Chicken is per 1000 head. Forestry is per m³ sold spruce roundwood.

Both national and international developments (WTO and European Union) suggest that Norwegian agricultural policy will be changed in the future. In that case historical data are not

relevant in our decision model. I therefore also elicited from an expert⁵ (a national agricultural economics adviser) his subjective marginal distributions of the individual activity GMs. This expert gave judgements of the lowest, highest and most likely values of individual GM for farms in the Eastern Norway region in the first planning year. Then, assuming that the individual subjective GMs were approximately triangularly distributed, means and standard deviations were calculated.

Finally, the historical GM series was reconstructed, using the formula (Hardaker et al., 1997):

$$x(n)_{ij} = E(x(s)_j) + \{x(h)_{ij} - E(x(h)_j)\} \frac{\sigma(s)_j}{\sigma(h)_j} \quad (7)$$

where $x(n)_{ij}$ is the synthesised GM for activity j in state i , $E(x(s)_j)$ is the subjective mean of the GM of activity j , $x(h)_{ij}$ is the corrected historical GM of activity j in state i , $E(x(h)_j)$ is the mean GM from the corrected historical data for activity j , $\sigma(s)_j$ is the subjective standard deviation of the GM for activity j , and $\sigma(h)_j$ is the standard deviation of the GM for activity j from the corrected historical data. The reconstructed series have the subjectively elicited means and standard deviations while preserving the cross-section stochastic dependencies embodied in the historical data. Then, the 'state of nature'-matrix in Table 1 is a discrete distribution of expected activity GMs for the first year in the planning model.

As with fixed costs, stochastic trend in the different activity GMs (except forestry) in the state of nature-matrix (Table 1) were also accommodated using the hierarchy of variables approach. The macro-level variable used was the price index of total farm products for the period 1985 to 1998, provided by Statistics Norway (1986-99). The hierarchy of variables approach used for the stochastic trend in activity GMs follows the same steps as described for fixed cost earlier. The only difference was that the stochastic noise term from step 3 was not included in step 4, since the stochastic noise in the activity GMs was described by the state of nature matrix. The predicted stochastic trend index for each year from the hierarchy of variables approach was multiplied by the corresponding activity GM in the state of nature

⁵ When taking expert advice, it is normally recommended to use more than one expert to get different insights to determine the probability assessment. However, sometimes one expert may be preferred given this expert has good knowledge of the sample space, i.e., the set of all possible outcomes. To derive judgement from one expert we then avoid the problem with pooling different views.

matrix. This procedure implies an assumption that the stochastic time trend in the total farm products experienced between 1985 to 1998 will continue. It also assumes that the time trend, which was derived from national price data, was applicable to activity GMs on the farm analysed.

The forestry GM per unit was assumed to be independent of the other activities. The forestry prices for the period 1990 to 1998 were regressed against time, and the predicted prices from this equation are assumed to represent trend in the forestry GM in the model. The trend was assumed stochastic and normally distributed, with the predicted value assumed to be the means of normal distribution and the standard deviation of the error component from the regression being the standard deviation of the normal distribution.

It was assumed that the uncertainty increases with the planning horizon.⁶ A linear increase in the subjective standard deviation of the activity GMs with \pm a specified percentage (2.5% used in this paper) for each year represents increased uncertainty. For every year, equation (7) implies that there will be increased variation between states of nature in Table 1. This adjustment, in addition to the stochastic trend adjustment, gives a different state of the nature matrix for every year in the plan. To account for the cross-section stochastic dependency, in each iteration of the simulation the sampling procedure was programmed so that the same state of nature was used for all activities.

In the year 1999, when the plan was done, the following levels of interest per year were assumed: short-term loan interest rates 9%, long-term loan interest rate 7.5%, deposit interest rate 6%. The probability distributions and trends over the planning horizon in the stochastic interest rate on financial assets and liabilities were forecasted with an autoregressive model. The reason for using an autoregressive model and not a simple regression model is that interest rate often has a mean reversion trend, i.e. the interest rate normally reverts to a long-run trend. Time-series forecasting is described in, e.g., Griffiths et al. (1993, Ch. 20). The forecasting model was estimated using annual average rates on Governments bonds of ten years maturity for the period 1985 to 1999. Interest on Governments bonds was assumed to be the macro-level variable affecting all interest rates. It was assumed that the interest rates on short- and long-term loans and deposit are all perfectly correlated. After identification, estimation and diagnostic checking, a simple first-order autoregressive model, AR(1) was

⁶ Increased uncertainty in activity GMs was used because the uncertainty increases with time in the planning period, partly due to expected increased volatility under possible increased deregulation.

identified. In this model interest rate this year depends only on interest rate last year plus a random disturbance, which was assumed normally distributed. The forecast values and their standard deviations from the estimated AR(1) equations were used as indexes for the stochastic distribution and stochastic trends of all interest rates used in the budgeting model.

Labour requirements of activities were assumed stochastically independent of the other groups of variables. The uncertainty about the labour requirements per unit was specified by triangular probability distributions. An expert (a national agricultural economics adviser) specified the minimum, maximum and most likely labour requirements for each activity on the farm. It was assumed that these probability distributions remain the same over the six years modelled.

The milk quota price was assumed fixed for the year 2000 (NOK 5.50/litre) and for the years 2001 to 2005 was assumed to follow a discrete distribution, stochastically independent of the other groups of variables. The lowest assumed quota price was zero (the case when the redistribution of milk quota is removed) and the highest assumed price were NOK 9.00/litre (NOK 1.50/litre higher than quota price under the extraordinary redistribution round autumn 1999).

In this subsection some approaches to dealing with stochastic specification are illustrated. Which method should be chosen in a particular application will depend on the nature and causes of the dependency between the stochastic variables and data and information available. The hierarchy of variables approach and the autoregressive model require relevant historical data. In cases where historical data not are relevant, as for the GMs in this paper, some combination of subjective probabilities, estimates of historical correlation between activities and simulation of stochastic trend combined with the hierarchy of variables approach may be a suitable method.

3.2 Ranking risky strategies

The term risk is used in different ways. Three common interpretations are the chance of bad outcomes, the variability of outcomes and uncertainty of outcomes. Following Hardaker (2000) risk is best formalised as uncertainty of outcomes, e.g., as the whole distributions of outcomes.

To present the financial feasibility of alternative strategies CDFs of the performance measure are informative. For example, from the CDF for equity we can find the likelihood for each of the analysed strategies that the farmer will be insolvent at the planning horizon.

Stochastic dominance analysis is often used to order risky prospects for which whole distributions of outcomes are available (e.g. Milham, 1992; Nyangito et al., 1996). A stochastic dominance criterion is a decision rule that provides a partial ordering of risky prospects for decision-makers whose preferences conform to a specified set of conditions. First- and second-degree stochastic dominance (Hadar and Russell, 1969) are often not discriminating enough in empirical work (King and Robison, 1981).⁷ A more powerful criterion, stochastic dominance with respect to a function (SDRF), was introduced by Meyer (1977), and was used in this analysis. The decision making class is defined by upper and lower bounds on the absolute risk aversion coefficient, r_a . In this paper the software computer program developed by Goh et al. (1989) was used for the computational task of ranking the prospects using the SDRF-approach.

4 Results

Figure 1 show the graphs of CDFs generated for equity for each of the five strategies, while Table 2 contains a summary of the final results of the stochastic dominance analysis.

Figure 1 show that strategy 3 has about 25% chance that the farmer will be insolvent by the end of the planning period. The lower tails of the CDFs for strategies 1, 2, 4 and 5 all lie to the right of the point representing zero equity, implying zero probability of insolvency at the planning horizon. Note that accounting for the wait and see option value of milk quota sale increases the equity measure at the end of the planning horizon considerably (strategy 5 c.f. strategy 4).

The relation between absolute and relative risk aversion is $r_a(w) = r_r(w)/w$ where w is wealth. Anderson and Dillon (1992) have proposed a rough and ready classification of degrees of risk aversion, based on the relative risk aversion with respect of wealth, $r_r(W)$, in the range 0.5 (hardly risk averse at all) to about 4 (very risk averse). With an equity of NOK 2 450 000 (the farmers equity at the beginning of the planning period) a value of $r_a(w)$ in the range 0.0000002 to 0.0000016 correspond to $r_r(w)$ in the range 0.5 to 4. These bounds on $r_a(w)$ were

⁷ There is third to t-th degree stochastic dominance criterion but they are usually not useful.

used in the SDRF analysis. The main results from Goh et al.'s (1989) SDRF program ranked the 5 strategies as follows: strategy 5 dominates strategy 1 dominates strategy 4 dominates strategy 2 dominates strategy 3 (Table 2). In other words, SDRF analysis, in this case, leads to a risk-efficient set with only one member - strategy 5 - and this was the option recommended to the farmer.

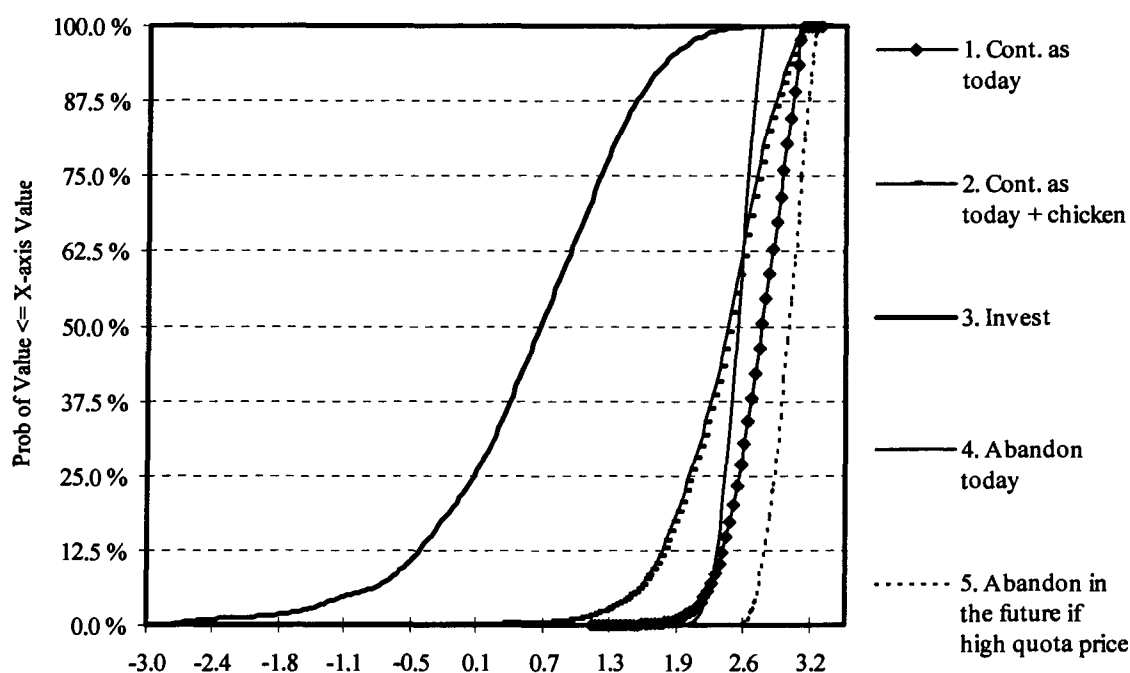


Figure 1 Cumulative distribution of equity in millions NOK for different investment and management strategies

Table 2 Pairwise comparison matrix^a to investigate SDRF for a set of bounds for the investment and management problem

Range	$0.0000002 \leq r_a(w) \leq 0.0000016$				
	Strategy				
Strategy	1	2	3	4	5
1. Cont. As today	-	1	1	1	0
2. Cont. As today + chicken	0	-	1	0	0
3. Invest	0	0	-	0	0
4. Abandon today	0	1	1	-	0
5. Abandon in the future if quota price is high	1	1	1	1	-

^a 1 = win, 0 = loss, - not compared

5 Concluding comments

Since farming is a risky business it is important in planning to account for risk. Information from an ordinary deterministic budgeting model done on the basis of point estimates of uncertain variables may not tell the whole story for future investment and management decisions on a farm. A stochastic budgeting approach may give more realistic and more useful information about alternative decision strategies.

Great flexibility in planning can be represented using stochastic budgeting. In this paper business risk, financial risk and the option aspect are integrated, and different investment and management strategies are evaluated. Many other applications are possible. Available special-purpose software (e.g. @Risk) allows stochastic budget models to be constructed and used much more easily than in the past.

Experiences gained in this study suggest some principles for similar work. First, the model should be kept as simple as is judged reasonable. It is important to be critical in choice of stochastic variables in the model - too many make it complicated to account for stochastic dependencies between variables. The intention with budgeting models is not to give exact answers, but to highlight consequences of different strategies. Second, it is critical to make good estimates of the distributions of key uncertain variables. Unrealistic estimates make the analysis a waste of time. Third, it is important to identify and measure stochastic dependencies between variables satisfactorily, at least if this is thought to be important. Some methods to build in these dependencies are illustrated in the paper.

One issue in financial feasibility studies such as this to explore in further research is whether different terminal performance measure (e.g. equity, equity ratio, net cash flow (working capital), return/equity, return/asset) rank alternatives differently, and if so why.

The farmer's decision problem was to choose between continuing farming as today, making some investment, or abandoning the milk production, becoming a part-time farmer and part-time wage earner. With respect to financial feasibility, this paper shows that investment in an improved cowshed (strategy 3) is very risky. It seems that the best he can do with respect to financial feasibility is to keep going producing milk and sell the milk quota in the future if the quota price goes up and then become a part-time farmer.

As explained at the start of the paper, which strategy the farmer should choose with respect to profitability was not investigated in this paper.

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References

- Anderson, J.R., Dillon, J.L., 1992. *Risk analysis in dryland farming systems*. Farming Systems Management Series No. 2, FAO, Rome.
- Goh, S., Shih, C-C., Cochran, M.J., Raskin, R., 1989. A generalised stochastic dominance program for the IBM pc. *Southern Journal of Agricultural Economics* 21: 175-182.
- Griffiths, W.E., Hill, R.C., Judge G.G., 1993. *Learning and Practicing Econometrics*. Wiley, New York.
- Hadar, J., Russell, W.R., 1969. Rules for ordering uncertain prospects. *American Economic Review* 49: 25-34.
- Hardaker, J.B., 2000. Some issues in dealing with risk in agriculture. Working papers in Agricultural and Resource Economics, School of Economic Studies, University of New England, Armidale.
- Hardaker, J.B., Huirne, R.B.M., Anderson, J.R., 1997. *Coping with Risk in Agriculture*. CAB International, Wallingford.
- Hughes, D.W., Richardson, J.W., Rister, M.E., 1985. Effects of sustained financial stress on the financial structure and performance of the farm sector. *American Journal of Agricultural Economics* 67: 1116-1122.
- Hull, J.C., 1980. *The Evaluation of Risk in Business Investment*. Pergamon, London.
- King, R.P., Robison L.J., 1981. An interval approach to measuring decision maker preferences. *American Journal of Agricultural Economics* 63: 510-520.
- Knutson, R.D., Romain, R., Anderson, D.P., Richardson, J.W., 1997. Farm-level consequences of Canadian and U.S dairy policies. *American Journal of Agricultural Economics* 79: 1563-1772.
- Knutson, R.D., Smith, E.G., Anderson, D.P., Richardson, J.W., 1998. Southern farmers' exposure to income risk under the 1996 farm bill. *Journal of Agricultural and Applied Economics* 30: 35-46.
- Little, I.M.D., Mirrlees, J.A., 1974. *Project Appraisal and Planning for Developing Countries*. Heinemann, London.
- Meyer, J., 1977. Choice among distributions. *Journal of Economic Theory* 14: 326-336.
- Milham, N., 1992. Financial structure and risk management of woolgrowing farms: a dynamic stochastic budgeting approach. Unpublished MEd dissertation, University of New England, Armidale.

- Milham, N., 1998. Technical report on the practice of whole-farm stochastic budgeting. NSW Agriculture, Armidale.
- Milham, N., Hardaker, J.B., Powell, R., 1993. RISKFARM: a pc-based stochastic whole-farm budgeting system. Centre for Agriculture and Resource Economics, University of New England, Armidale (unpublished paper).
- NILF, 1992-99. *Driftsgranskinger i jord- og skogbruk* (Account results in agriculture and forestry). Norwegian Agriculture Economics Research Institute, Oslo. Annual publication.
- NILF, 2000. *Utsyn over Norsk landbruk - tilstand og utviklingstrekk 2000* (Norwegian agriculture - status and trends 2000). Norwegian Agricultural Economics Research Institute, Oslo.
- Nyangito, H.O., Richardson, J.E., Mundy, D.S., Mukhebi, A.W., Zimmer, P., Namkan, J., 1996. Economic impacts of East Coast Fever immunization on smallholder farms, Kenya: a simulation analysis. *Agricultural Economics* 13: 163-177.
- Richardson, J.W., Nixon, C.J., 1986. Description of FLIPSIM V: a general firm level policy simulation model. Texas Agricultural Experiment Station. Bulletin B-1528.
- Savage, L.J., 1954. *The Foundations of Statistics*. Wiley, New York.
- Searle, S.R., Casella, G., McCulloch, C.E., 1992. *Variance Components*. Wiley, New York.
- Statistics Norway, 1986-99. Statistisk årbok (Statistical yearbook). Statistics Norway, Oslo-Kongsvinger, Annual publication.

Essay 4:

Modelling jumps in commodity prices*

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Abstract

Models that have been in use in the study of the short term interest rates and foreign exchange rate markets are employed in the study of the jumpiness in commodity prices. In particular, a mean reverting model, a mean reverting with jump and models incorporating non-linear drift are investigated. The higher moments of the mean reverting and the jump model are developed, following Das (1999). These models are tested with the generalised method of moments and maximum likelihood. Monthly wheat prices from 1952 to 1998 constitute the data to which the models are applied. Jump behaviour is clearly present in the data. Dividing the period in two sub-periods with 1973 as the dividing year, the jumpiness is strong in the latter period, while mean reversion fits the data before 1973. Non-linear drift is rejected. Even other model specifications, such as Brownian motion with jumps, are rejected. The results are promising in that commodity pricing may benefit from models developed in the finance field.

Key words: Commodity pricing; Jump diffusion; Stochastic models; GMM and ML estimation

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1 Introduction

Is a jump process suitable for studying the price path of a commodity such as wheat? Many commodity markets seem to exhibit occasional spikes and sharp turns, as experienced in the "oil shock" of the seventies. In this paper, we test whether jump processes are present in wheat pricing. The advantage of using wheat is that it is a widely traded commodity in world markets, and jumps may occur for natural reasons, that is, as a pure random process. Indeed this uncertainty of the weather is the basic assumption in studies of storable agricultural products by Deaton and Laroque (1992) and Williams and Wright (1991) who have discussed the jumpiness of commodity prices. At the same time, these writers have noted that storage from one period to the next may smoothen the prices somewhat, at least not allowing the prices to fall to a very low level. Furthermore, since storage may smooth the prices, the prices may exhibit autocorrelation. For Deaton and Laroque this aspect has been the main object of interest. However, Deaton and Laroque were unable to find a statistical fit to the observations.

The modelling of the random shocks to the commodity pricing is undertaken in this paper. The shocks may, of course, be due to shifts of either supply or demand schedules. Here, we use models derived from finance, in particular from the research on the term structure of interest, to model the price process of wheat, that is, we bring in models from a different field and test its applicability in a commodity market. Models suggested by Vasicek (1977) with and without jumps are studied, giving us models of mean reverting and of jump diffusion. Also a model showing a non-linear drift term due to Aït-Sahalia (1996) is tested as an alternative to the mean reverting and jump diffusion models. Jump processes have been studied in the literature on term structures on interest rate and also in the foreign exchange literature, e.g. Ball and Roma (1993, 1994); Ball and Torous (1983); Nieuwland, Verschoor and Wolf (1994); Das (1999); Das and Foresi (1996); Jorion (1988); Ahn and Thompson (1988); Bates (1996). Merton (1976) was perhaps the founding father of this modelling approach.

Correct pricing of derivatives is the motive behind the modelling of the underlying price process in the finance literature, as underlined by Cox and Ross (1976):

The critical factor in this argument and in any contingent claims valuation model is the precise description of the stochastic process governing the behavior of the basic asset. (Cox and Ross, 1976, p. 146).

Thus, when we know the value of the stock we can value the derivative. Therefore, our paper may contribute to the correct pricing of derivatives written on wheat.

An issue of further interest is whether the volatility of the commodity market, in this case wheat, has increased over the years. This issue is not clear cut. In step with increasing integration of the world economy, the diversification of the weather risk should lead to less volatility. Important wheat growing areas are the North and South America, Australia and Europe. On the other hand, the escalation of prices is evident from Figure 1. Also, it seems, the volatility has increased. Learn (1986) has found a number of reasons for the 1974 price hike: "... a shift in Soviet foreign policy to accommodate crop failure through imports of grain rather than through forced reduction in domestic consumption; OPEC-induced increases in energy costs; reduced harvest of anchovies off the coast of Peru; drought in portions of the U.S. Midwest; and devaluation of the dollar...". In passing, this statement illustrates the many complexities that in the end make up the supply and demand schedules for a specific year. An important issue in this paper is to study whether this seemingly higher volatility is captured by jump models of the price process.

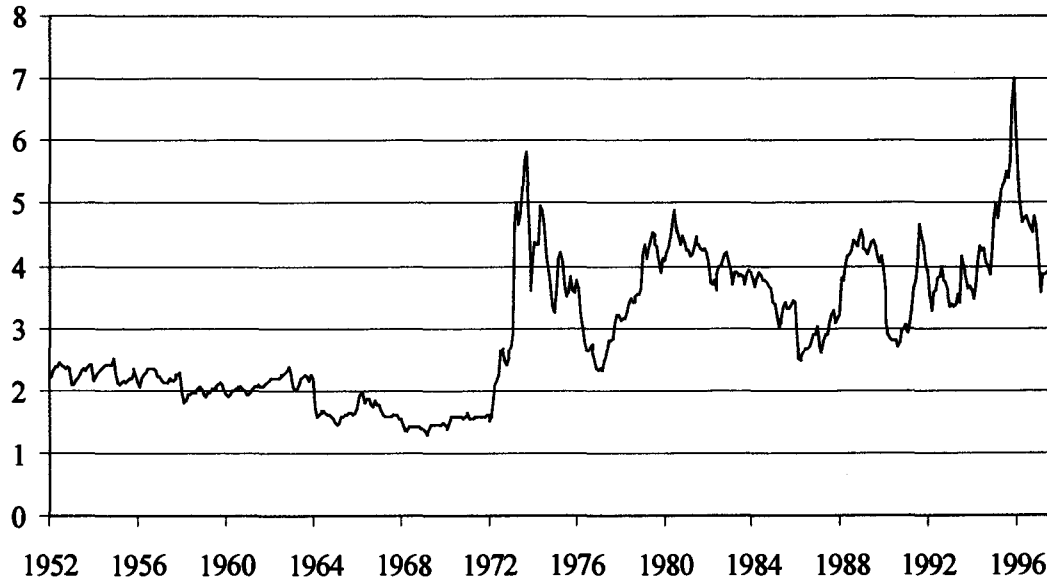


Figure 1 Nominal hard red winter wheat prices, 1952–1998, USD per bushel

We present data extending from 1952 to the present, and we are able to study whether a change in the level of volatility has taken place. "Hard red winter" wheat is used. Instead of

trying one model over a number of commodities, we try a number of models on this one commodity.

Our estimation strategy is to use the generalised method of moments approach of Hansen (1982) together with the maximum likelihood approach. The moments are generated from the moments of the models of price processes directly. We present the higher moments of the Vasicek model with and without jumps. An advantage of this modelling strategy is that conclusions are stronger, if the results pull in the same direction.

We find that jump diffusion models perform well in the wheat market, while the non-linear drift model of Aït-Sahalia (1996) is discarded. Furthermore, the mean reverting models do not pick up the dynamics of the price process. The coefficients for the jump variable are high in all models studied. Also, the analysis clearly shows a higher volatility in the wheat prices in the latter half of the period.

The paper proceeds as follows. In section two theories of price processes in the literature on commodities is presented. Part three presents the models to be used for estimation, while part four take up estimation issues. Then in part five our data is discussed, before results are presented in parts six and seven, followed by final conclusions.

2 Theories of price processes of agricultural commodities

Dynamic theories of the price process of storable agricultural commodities, typically wheat, were investigated for the first time it seems by Scheinkman and Schechtman (1983). They assumed identical and independent returns distribution. Later contributions have been proposed in the 90's by Williams and Wright (1991), Deaton and Laroque (1992, 1996) and Chambers and Bailey (1996). These works try to model the dynamics of the price process by taking into account the non-normality of the returns distribution, in particular the leptocurtosis, skewness as well as serial correlation. They share the observations that the prices of the agricultural commodities show serial correlation, heteroskedasticity and sudden jumps in the series.

A common feature of their models is that the variability of the prices is driven by random fluctuations of the harvest. At the same time, the sudden jumps may have repercussions in later periods as for instance a bad harvest may induce producers to plant more in the next season, as this will depend on farmer's expectations. A good harvest may, on the other hand, lead to greater speculative storage of the commodity, thus depressing prices in later periods.

However, this depressing of prices is counteracted by producers who plant less. Thus, the theories may be interpreted as models of storage as much as theories of pricing. Rosen et al. (1994) discuss similar models for studying cycles in cattle stocks. In the finance literature Schwartz (1997), Miltersen and Schwartz (1998), Hilliard and Reis (1998) et al. have modelled commodity prices. This literature also uses the theory of storage, by exogenous estimates of convenience yield.

On the other hand, noting these features of the price process, it is surprising that none of the authors mentioned take account of the higher moments of the distribution. If leptocurtosis prevails, it should be brought explicitly into the analysis. In this paper, we suggest ways to do so for two classes of return processes, the mean reverting and the jump diffusion.

3 Models of pricing processes

In this section we present the three main classes of models that are being tested, a jump diffusion model, a mean reversion model, as well as models incorporating non-linear drift. Jump diffusion models have turned out to be relevant in the short term studies of the term structure of interest rates. Our justification for studying jump models stems from observation of the price process of agricultural products, where spikes and sharp turns are often evident. There seems to be a lack of studies utilizing some of the more recent models developed in the finance literature for agricultural commodities, although the markets in many ways exhibit the same characteristics.

The specifications suggested here are as follows. One is the mean reverting model in which the price is pulled towards some long-term mean. Next, a jump component is added to the mean reverting model, giving the jump diffusion model. Furthermore, we also consider the model proposed by Aït-Sahalia (1996), where the parameters are time varying. This will be called the non-linear drift model. Lastly, these three models are contrasted to simpler representations, such as Brownian motion with jump. These stand as a check to the other models specified. The models are described below.

The three basic models may be nested within a general framework. If $\{x_t; t \geq 0\}$ may be defined as the unique, time-homogeneous Markov process that solves a stochastic differential equation of the form

$$dx_t = \mu(x_t)dt + \sigma(x_t)dz \quad (1)$$

The models used here are then specifications of this general differential equation. z is a standard Brownian Motion process, μ is the drift function and σ is the diffusion function. Here, we take x_t to be the price of wheat in period t ($t = 1, \dots, T$).

3.1 The mean reverting model

Our point of departure is the specification suggested by Vasicek (1977). Letting $\mu(x_t) = \kappa(\theta - x_t)$ and $\sigma(x_t) = \sigma$ the Vasicek model emerges:

$$dx_t = \kappa(\theta - x_t)dt + \sigma dz \quad (2)$$

κ , θ and σ are constants to be determined, θ being the long term mean to which the process is pulled. For $\kappa, \theta > 0$, the process corresponds to a continuous time first-order autoregressive process where the randomly moving price is elastically pulled toward a central location or long-term value, θ . The parameter κ determines the speed of adjustment. The model exhibits mean reversion, and follows what is sometimes denoted as an Ornstein-Uhlenbeck process. As can be seen, the Vasicek model does not make the volatility dependent on the level of the price, and is thus simpler to handle than the Cox, Ingersoll and Ross (1985) model, where $\sigma(x_t) = \sigma\sqrt{x_t}$.

A model very reminiscent of the Vasicek model was employed by Deaton and Laroque (1996) to illustrate one of their cases:

$$y_{t+1} - \mu = \rho(y_t - \mu) + \sigma\varepsilon_{t+1}$$

where y_t is the harvest at time t , μ is the mean of the harvests, ρ is the autoregressive parameter, and ε_t is the harvest shock in time t . ρ may be interpreted as the speed of adjustment in the Vasicek model, while ε_t may be likened to the standard Brownian motion process above. Unfortunately, Deaton and Laroque do not make relation between their and Vasicek model to the central focus of their study.

Moreover, their model does not contain a jump element. This implies that the jumps and eventual reversion are picked up by the mean reversion parameter κ . An issue of the testing is then whether the mean reversion coefficient is able to pick up both the "big" and the

"small" jumps of the price series into one measure, or whether it is necessary to model the jumps explicitly.

3.2 The jump diffusion model

A jump diffusion version of the Vasicek model may be easily specified. The introduction of a jump element in the diffusions means that the state variable x will follow a discontinuous sample path. We shall employ the formulation of the jump diffusion model of Das (1999), although the model is very similar to the one proposed by Merton (1976). The model's price process may be written:

$$dx_t = \kappa(\theta - x_t)dt + \sigma dz + Jd\pi(h) \quad (3)$$

Two new elements have been added, J and $d\pi(h)$. J is a random jump having a Poisson distribution, and the arrival of jumps is governed by a Poisson process π with arrival frequency parameter h , which denotes the number of jumps per year. The diffusion and Poisson processes are independent of each other, and independent of J as well. The returns evolve with a mean-reverting drift and two random terms, one a diffusion and the other a Poisson process involving the random jump J .

Ball and Roma (1993) studied the Vasicek model with jump diffusion for the European monetary system. For the purposes of their study, the jump size was made a function of the displacement from the central parity.

Das (1999) showed that the first four moments of the jump-diffusion process are obtained by differentiating the characteristic function with respect to s and then finding the derivative when $s = 0$, s being the characteristic function parameter. The moments are:

$$\mu_1 = \left(\theta + \frac{hE[J]}{\kappa} \right) (1 - e^{-\kappa T}) + x e^{-\kappa T} \quad (4)$$

$$\mu_2 = \frac{\sigma^2 + hE[J^2]}{2\kappa} (1 - e^{-2\kappa T}) + \mu_1^2 \quad (5)$$

$$\mu_3 = hE[J^3] \left(\frac{1 - e^{-3\kappa T}}{3\kappa} \right) + 3\mu_1 \left(\sigma^2 + hE[J^2] \right) \left(\frac{1 - e^{-2\kappa T}}{2\kappa} \right) + \mu_1^3 \quad (6)$$

$$\begin{aligned} \mu_4 = & hE\left[J^4\left(\frac{1-e^{-4\kappa T}}{4\kappa}\right)\right] + 3\left(\sigma^2 + hE\left[J^2\right]\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right)\right)^2 + 4\mu_1 hE\left[J^3\left(\frac{1-e^{-3\kappa T}}{3\kappa}\right)\right] \\ & + 6\mu_1^2\left(\sigma^2 + hE\left[J^2\right]\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right)\right) + \mu_1^4 \end{aligned} \quad (7)$$

These moments may be used when the moment conditions of the Generalised Method of Moments is being specified. We return to the issue below.

3.3 Non-linear drift

As an alternative to models with linear parameters, $\mu(x_t) = \kappa(\theta - x_t)$ and $\sigma(x_t) = \sigma$ suggested by Vasicek (1977), we may consider non-linear functions as in Aït-Sahalia (1996). Effectively, this comes down to choosing flexible functional forms that are capable of nesting a variety of possible shapes. The functional forms specified by Aït-Sahalia (1996) was:

$$\mu(x_t) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^{-1}$$

and

$$\sigma^2(x_t) = \beta_0 + \beta_1 x + \beta_2 x^{\beta_3}$$

From this, Aït-Sahalia was able to nest a large number of models by specifying values for the parameters. For instance, by including α_0 and α_1 in the drift function and β_0 in the diffusion, the Vasicek (1977) model emerges.

We choose to work in a simple framework here, stressing the testing of non-linearity of the drift term. Accordingly, $\alpha_0 + \alpha_1 x_t$ is set equal to $\kappa(\theta) - x_t$, while $\beta_1 = \beta_2 = 0$. Thus, the two models to be tested reduce to the following.

$$dx_t = \left[\kappa(\theta - x_t) + \alpha_2 x_t^2 + \frac{\alpha_3}{x_t} \right] dt + \sigma_t dz_t \quad (8)$$

This model is the mean reverting diffusion with non-linear drift, and

$$dx_t = \left[\kappa(\theta - x_t) + \alpha_2 x_t^2 + \frac{\alpha_3}{x_t} \right] dt + \sigma_t dz_t + Jd\pi(h) \quad (9)$$

is the jump mean reverting diffusion with non-linear drift. If there is any non-linearity present, the α 's will be significant.

It should be added that Aït-Sahalia used a different estimation procedure. However, in order to compare within an estimation procedure that is common to all specifications in this paper, we will utilise maximum likelihood estimation for these models.

4 Estimation

Our estimation strategy will be to calculate parameter values by utilising the Generalised Method of Moments (GMM) as well as maximum likelihood (ML). The empirical specifications are set out below. Our strategy is to use the GMM as the main estimation vehicle, and then check the results of the models by the ML method. If the results point in the same direction, our conclusions will be strengthened.

4.1 The GMM

In this section, the empirical specifications to be tested are given. The mean reverting models and the jump diffusion models will be estimated using the GMM. The method of moments is well suited for estimating jumps. Furthermore, the method accommodates both conditional heteroskedasticity and serial correlation. These features seem to be prevalent in most commodity markets.

The moments in (4)–(7) above may be used to construct the needed orthogonality conditions. The orthogonality conditions may be summed up in the following:

$$E[f_t(\psi)] = E \left[\begin{pmatrix} \mu_1 - x_t \\ \mu_2 - x_t^2 \\ \mu_3 - x_t^3 \\ \mu_4 - x_t^4 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ x_{t-1} \end{pmatrix} \right] = 0 \quad (10)$$

where ψ represents the parameters to be estimated. This gives eight orthogonality conditions in all, using the simple instruments 1 and x_{t-1} . Notice that the exact time moments are used, and not some discretisation of the price process.

The GMM procedure involves replacing $E[f_i(\psi)]$ with its sample counterpart, $g_T(\psi)$, using the T observations. $g_T(\psi)$ is defined by:

$$g_T(\psi) = \frac{1}{T} \sum_{i=1}^T f_i(\psi) \quad (11)$$

and then choosing the parameter estimates that minimise the quadratic form,

$$Q_T(\psi) = g_T'(\psi) W_T(\psi) g_T(\psi) \quad (12)$$

where $W_T(\psi)$ is the positive definite symmetric weighting matrix. Next, Hansen (1982) showed that choosing $W_T(\psi) = S^{-1}(\psi)$, where

$$S(\psi) = E[f_i(\psi) f_i'(\psi)] \quad (13)$$

results in the GMM estimator of ψ with the smallest asymptotic covariance matrix. Denoting an estimator of this matrix by $S_0(\psi)$, the asymptotic covariance matrix for the GMM estimate of ψ is

$$\frac{1}{T} (D_0'(\psi) S_0^{-1}(\psi) D_0(\psi))^{-1} \quad (14)$$

where $D_0(\psi)$ is the Jacobian evaluated at the estimated parameters.

We specify conditions for two models, one is the mean reverting model, the other is a diffusion model with jumps, the jump-diffusion model. The mean reverting model is obtained by letting the jump intensity parameter $h = 0$ in equation (4)–(7). This gives the moments for the mean reverting model

$$\mu_1 = \theta(1 - e^{-\kappa T}) + x e^{-\kappa T} \quad (15)$$

$$\mu_2 = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa T}) + \mu_1^2 \quad (16)$$

$$\mu_3 = 3\mu_1\sigma^2\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right) + \mu_1^3 \quad (17)$$

$$\mu_4 = 3\left(\sigma^2\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right)\right)^2 + 6\mu_1^2\left(\sigma^2\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right)\right) + \mu_1^4 \quad (18)$$

Combined with the instruments above, this gives the eight orthogonality conditions.

Now, let us turn to the jump diffusion model. Compared to the model in (4)–(7) some simplifications have to be made. We comment on these below. The moments of the jump diffusion model may be written:

$$\mu_1 = \theta'(1 - e^{-\kappa T}) + xe^{-\kappa T} \quad (19)$$

$$\mu_2 = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa T}) + \mu_1^2 \quad (20)$$

$$\mu_3 = hE\left[J^3\left(\frac{1-e^{-3\kappa T}}{3\kappa}\right) + 3\mu_1\sigma^2\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right) + \mu_1^3\right] \quad (21)$$

$$\mu_4 = hE\left[J^4\left(\frac{1-e^{-4\kappa T}}{4\kappa}\right) + 3\left(\sigma^2\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right)\right)^2 + 4\mu_1hE\left[J^3\left(\frac{1-e^{-3\kappa T}}{3\kappa}\right)\right] \quad (22)$$

$$+ 6\mu_1^2\left(\sigma^2\left(\frac{1-e^{-2\kappa T}}{2\kappa}\right)\right) + \mu_1^4$$

A difficulty of the GMM when obtaining the jump diffusion model is that some parameters are not identifiable. In the case of the jump model, the first jump moment $E[J]$ enters only as a sum with θ in the first moment. Apart from this, the second jump moment $E[J^2]$ always enters as a sum with σ^2 in the second, third and fourth moments. The values of these two parameters are subsumed under $\theta' = \theta + \frac{hE(J)}{\kappa}$ and $\sigma'^2 = \sigma^2 + hE(J^2)$ respectively. Furthermore, the composites $hE(J^3)$ and $hE(J^4)$ may not be separated, since $E(J^3)$ and $E(J^4)$ do not appear except as multiplied by h .

The parameters to estimate are thus κ , the speed of mean reverting; θ' , the long term mean together with a jump component; σ' , the long term volatility together with a jump component; and lastly the two jump composites $hE(J^3)$ and $hE(J^4)$. That is, a total of five parameters are to be estimated.

Again, combining these moments with the instruments gives the necessary orthogonality conditions for estimation. In fact, we have five parameters to estimate and eight moment conditions, indicating that the system is overidentified.

4.2 GMM diagnostic tests

The number of relations and parameters limit the diagnostic tests that may be performed. The minimised value of the quadratic form $Q_T(\psi)$ is distributed χ^2 under the null hypothesis that the model is true with degrees of freedom equal to the number of orthogonality conditions net of the numbers of parameters to be estimated. That is, the χ^2 measure provides a test statistic for the overall fitness of the model. A high value of this measure means that the model is misspecified.

The χ^2 measure is used as well in the testing of two models against each other. This is the Newey and West (1987) procedure test, and shows whether a restricted model has different parameters from an unrestricted. The χ^2 is computed in the absence of restriction and then again under the added constraints imposed on the model, but this time using the weighting matrix W of the unconstrained model. The difference in the two χ^2 statistics is itself χ^2 distributed, while the degrees of freedom are equal to the number of new restrictions. The Newey-West statistic is analogous to the likelihood ratio test, see also Green (1997) and Ogaki (1993).

The estimated parameters in GMM are asymptotically normal, implying that a simple t -test may be used.

4.3 The ML method

Estimation methods based on maximum likelihood for the estimation of the mean reverting and the jump diffusion models are presented here. Estimation of the model using continuous time data is a very demanding process. Instead, a Bernoulli approximation first introduced by

Ball and Torous (1983) relying upon discrete data, is utilised here. The assumption in the model is that in each time interval either only one jump occurs or no jump occurs. The drawback using this method is that discretisation of continuous time stochastic differential equations for estimation introduces an estimation bias, which may be significant for data sampled on a monthly basis.

The discrete version of (3) is:

$$\Delta x = \kappa(\theta - x)\Delta t + \sigma\Delta z + J(\mu, \gamma^2)\Delta\pi(q) \quad (23)$$

where σ^2 is the annualised variance of the Gaussian shock, and Δz is a standard normal shock term. $J(\mu, \gamma^2)$ is the jump shock, being normally distributed with mean μ and variance γ^2 . $\Delta\pi(q)$ is the discrete-time Poisson increment, approximated by a Bernoulli distribution with parameter $q = h\Delta t + O(\Delta t)$. $O(\Delta t)$ is the asymptotic order symbol used to denote a function ζ such that $\lim_{\Delta t \rightarrow 0} \zeta(\Delta t)/\Delta t = 0$. Then, the transition probabilities for the price following a jump diffusion process are written as (for $s > t$):

$$f[x(s)|x(t)] = q \exp\left(\frac{-(x(s) - x(t) - \kappa(\theta - x(t))\Delta t - \mu)^2}{2(\sigma^2\Delta t + \gamma^2)}\right) \frac{1}{\sqrt{2\pi(\sigma^2\Delta t + \gamma^2)}} \quad (24)$$

$$+ (1 - q) \exp\left(\frac{-(x(s) - x(t) - \kappa(\theta - x(t))\Delta t)^2}{2\sigma^2\Delta t}\right) \frac{1}{\sqrt{2\pi\sigma^2\Delta t}}$$

this approximates the true Poisson-Gaussian, jump diffusion density with a mixture of normal distributions. The following maximisation is involved:

$$\max_{[\kappa, \theta, \sigma, \mu, \gamma^2, q]} \sum_{t=1}^T (\log(f[x(s)|x(t)])) \quad (25)$$

The model with non-linear drift is estimated using maximum likelihood. Restricting exposition to the jump version, the model is specified in a discrete version as:

$$\Delta x = (\kappa(\theta - x) + \alpha_2 x^2 + \alpha_3 / x)\Delta t + \sigma\Delta z + J(\mu, \gamma^2)\Delta\pi(q) \quad (26)$$

Here, the new parameters are (α_2, α_3) . They examine whether the drift is a function of squared wheat prices or inversely related to wheat price levels. If any of these parameters is

significantly different from zero, the drift term is non-linear. The results are given in Table 6 below.

5 Data

We use monthly observations of wheat prices from June 1952 to January 1998 collected from the USDA via the web. Prices are quoted in USD per bushel. The wheat is hard red winter, and we have used the quotes in Atlanta. To motivate our choice of models, some descriptive statistics is given in Table 1, see also Figure 1.

Table 1 Means, standard deviations, skewness, kurtosis, minimum, maximum, autocorrelation and time varying conditional heteroskedasticity of monthly US wheat prices (hard red winter) from June 1952 through January 1998 and for the periods 1952 to July 1973 and August 1973 to 1998. The variable x denotes wheat spot price and dx is the associated monthly change, measured as $dx = x_{t+1} - x_t$. ρ_k denotes the autocorrelation coefficient of order k

Statistic	x	$dx(52-98)$	$dx(52-73)$	$dx(73-98)$
Mean	2.9538	0.0024	0.0025	0.0024
Stand. Dev.	1.1114	0.1899	0.0772	0.2491
Skewness	0.4617	1.1891	-1.1247	1.0135
Kurtosis	-0.5874	18.3148	11.4026	10.3998
Minimum	1.2800	-0.9400	-0.5100	-0.9400
Maximum	7.0200	1.7700	0.3700	1.7700
T	548	547	253	294
ρ_1	0.9848*	0.3306*	0.3327*	0.3103*
ρ_2	0.9597*	-0.0879*	-0.1027	-0.0961
ρ_3	0.9372*	-0.1321*	-0.0856	-0.1445*
ρ_4	0.9186*	-0.0125	-0.0097	-0.0235
ρ_5	0.9003*	0.0423	-0.0381	0.0502
ρ_6	0.8809*	-0.0413	-0.0438	-0.0269
ρ_7	0.8630*	-0.1466*	-0.0140	-0.1685*
ρ_8	0.8487*	-0.0876*	-0.0362	-0.1280*
ρ_9	0.8365*	0.0150	0.0009	0.0137
ρ_{10}	0.8237*	0.1686*	0.0664	0.1966*
ρ_{11}	0.8061*	0.1801*	0.1866*	0.1619*
ρ_{12}	0.7833*	0.1004*	0.2393*	0.0523
$Q(12)$	5175.67*	132.13*	60.03*	73.12*
$Q(1)$		90.85*	50.01*	66.62*
$Q(12)$		144.93*	58.23*	66.91*

* Significant at 5% level

T is the number of observations

A short account of the statistical measures in Table 1 follows, see also Campbell, Lo and MacKinley (1997). In a "White Noise" process the k th autocorrelation ρ_k is normally distributed with $E(\rho_k)=0$ and $Var(\rho_k)=1/\sqrt{T}$, and a test at the 5% level is then approximately $\pm 2/\sqrt{T}$. In our case the critical values are ± 0.0854 , ± 0.0854 , ± 0.1257 , ± 0.1166 , respectively.

The Ljung-Box Q statistic is distributed as $\chi^2(k)$ under the null hypothesis of no autocorrelation. In our case is the critical value 21.03 at 95% level. k is the number of lags.

ARCH effects are captured by the test statistic $Q^2(k)=TR^2$, and under the null hypothesis $H_0 = \alpha_2 = \alpha_3 \dots = \alpha_k = 0$, $Q^2(k)$ is asymptotically distributed as $\chi^2(k)$. In our case the critical values are $\chi_{0.95}^2(1)=3.84$ and $\chi_{0.95}^2(12)=21.03$. This is done by following the test devised by Engle (1982). He derived the following test, $Q^2(k)$, based on Lagrange multiplier principle. First the regression of:

$$dx_t = B_0 + B_1 dx_{t-1} + B_2 dx_{t-2} + \dots + B_k dx_{t-k} + u_t$$

is estimated by OLS for observations $t=1+k, \dots, T$ and the OLS sample residuals \hat{u}_t are saved. Next, \hat{u}_t^2 is regressed on a constant and k of its own lagged values:

$$\hat{u}_t^2 = \alpha_0 + \alpha_2 \hat{u}_{t-1}^2 + \dots + \alpha_k \hat{u}_{t-k}^2 + e_t$$

for $t=1+k, \dots, T$. The sample size T times the uncentered R_u^2 from the regression on \hat{u}_t^2 then converges in distribution to a χ^2 variable with k degrees of freedom under the null hypothesis that u_t is actually i.i.d. $N(0, \sigma^q)$.

Table 1 shows that the returns on the wheat spot prices have a very high degree of kurtosis. This fact alone motivates the use of a jump model (Das, 1999). Furthermore, the minimum and maximum values give evidence to the same effect. These values of the changes in the price process show very high values compared to the mean. The table shows high values for skewness as well. Moreover, we find significant autoregressive conditional heteroskedasticity (ARCH) effects in the data.

Figure 1 indicate that the price followed different processes in two sub-periods, the one extending from the beginning of our data series and up to 1973, while the other period is

made up by the rest of the series. The descriptive statistics are shown in the two last columns of Table 1. It turns out that the skewness has opposite signs for the two periods, more autocorrelations are significant in the later period, and that the ARCH effects are stronger in this later period.

All of these characteristics point to the use of models incorporating non-linearity, that is, models that take jumps or non-linear drift terms into account, but also, that the parameter values and the fit of the models may be different in the two sub-periods identified in the sample.

Both our models of pricing processes and GMM assume stationary time series. We test for stationarity by using Augmented Dickey-Fuller (ADF) test, which focusses on finding a "unit root" in the time series of the commodity prices. The ADF-tests are based on

$$\Delta x_t = \mu + \eta x_{t-1} + \sum_{i=1}^k \Delta x_{t-i} + \varepsilon$$

$$\Delta x_t = \mu + \gamma t + \eta x_{t-1} + \sum_{i=1}^k \Delta x_{t-i} + \varepsilon$$

where x_t is the series of prices analysed, $\Delta x_t = x_t - x_{t-1}$, and $\varepsilon_t \sim i.i.d(0, \sigma^2)$. The first equation includes a constant, and the second a constant and a trend. In both cases the null hypothesis $\eta = 0$, i.e. that the variable contains a unit root, is tested against the alternative hypothesis $\eta < 0$.

Table 2 summarises the results of the ADF-tests. The results of the ADF-tests provide evidence that hard red winter wheat prices in the period 1952–98 and period 1973–98 should be classified as series integrated of order 0, i.e. stationary series. The ADF-test does not give support to the classification of wheat prices in the period 1952–73 as stationary series.¹ Even if stationarity not is an absolute requirement in practice, our estimation results from the period before 1973 should be evaluated with caution.

¹ The alternative Phillips-Perron unit root tests gave analogous results. The power of ADF unit root tests to detect stationarity is, however, under discussion (Campbell and Perron, 1991). Lo and MacKinlay (1989) suggest the variance ratio test.

Table 2 Unit Roots test of nominal I(0) US wheat prices (hard red winter)

Period	Lag	Constant, trend	Constant, no trend
		ADF-value	ADF-value
1952–98	18	-3.655**	-2.292*
1952–73	15	-1.059	-1.654
1973–98	16	-3.403**	-3.167**

Lag-length by the Schwert-criterion, i.e. $k_{12} = \lceil 12(T/100)^{1/4} \rceil$.

* Reject unit root null hypothesis at 15% level of significance.

** Reject unit root null hypothesis at 5% level of significance.

6 Results – the overall period

In this section we report results of the estimation of the mean reverting and the jump diffusion models. Taking the whole period as the object of our study, estimation is performed using GMM estimation and ML estimation. Furthermore, the Aït-Sahalia (1996) model of non-linear drift is tested on the period as a whole. As was apparant, the parameters of the jump diffusion model were not clearly specified under the GMM methodology. However, should the results of the GMM and the ML estimation point in the same direction, we should have added faith in the results. Also, the Aït-Sahalia (1996) may be seen as an alternative to the specifications made, thus allowing a test against a different specification.

6.1 GMM estimation

The results of the GMM estimation are presented in Table 3. The table shows the results of both the mean reverting and the jump diffusion models. In addition, the value of the H , showing the overall function value of minimising the objective, is given in the last line.

Estimation results of the mean reverting and the jump diffusion models may be summed up in the following points. First, the jump diffusion model fits the wheat data better (smaller objective function H) than the mean reverting model. A second point is that the jump parameters are significant. This implies that a pricing process of wheat characterised by jumps is evident. Furthermore, the coefficient of mean reversion, κ drops from 0.1711 to 0.1099 when jumps are added to the mean reverting model. Also, the significance of this factor disappears in the jump diffusion model. This may imply that jumps provide a source of mean reversion, and that models of mean reversion and of jump diffusion are alternatives. The Newey-West asymptotic chi-square statistic is used to test whether the parameters of the jump diffusion process are significant, see Table 4.

Table 3 The table present results for generalised method of moments estimation, using four moment conditions. The instruments used are a constant and once lagged values of the wheat price. The table presents estimates for a mean reverting model and a jump diffusion model. T-statistics in parenthesis

Parameter	Model	
	Mean reverting	Jump diffusion
κ	0.1711 (2.33)	0.1099 (1.45)
θ	2.2121 (7.71)	
$\theta' = \theta + \frac{hE(J)}{k}$		3.3425 (3.06)
σ	0.2919 (10.56)	
$\sigma' = \sigma + \sqrt{hE(J^2)}$		0.6452 (10.42)
$hE(J^3)$		0.7826 (4.21)
$hE(J^4)$		1.3633 (2.71)
H	17.71	0.54

Table 4 Specification test of the hypothesis that the parameters of the mean reverting and jump diffusion process are the same

Assumptions	H_0	χ^2	$\chi^2_{0.95}$	Decision
No jumps effects				
Mean rev vs. Jump-diff	$h=0$	17.17	5.99	Reject H_0

Clearly, the evidence does not support the hypothesis that the mean reverting and the jump diffusion are the same. In all, the GMM estimation results are that the wheat prices in the post-war era follow a jump diffusion process.

6.2 ML estimation

Let us see whether the results obtained above are supported by maximum likelihood estimation on the same data. Our motivation for doing so is twofold. On the one hand, supporting evidence will strengthen the conclusions above. A second motivation is that the parameters identifying the jump process are better specified here than in the GMM model. The results for the estimation of the mean reverting and the jump diffusion models using ML estimation are presented in Table 5.

Table 5 The table presents ML estimates for a mean reverting model and a jump diffusion model on monthly US wheat prices covering the period June 1952 to January 1998. Estimation is carried out using maximum likelihood incorporating the transition density function in equation (24). T-statistic in parenthesis

Parameter	Mean reverting	Jump diffusion
κ	0.1752 (2.00)	0.1252 (1.96)
θ	3.1203 (5.56)	2.7262 (5.68)
σ	0.6547 (32.98)	0.2424 (9.90)
μ		0.0171 (0.54)
γ		0.3310 (3.82)
q		0.2808 (5.96)
Log-Likelihood	135.11	291.62

The results of the ML estimation confirm the estimation results of the GMM. Some of the noteworthy points on this estimation are:

- There is a drop in the volatility parameter when jumps are introduced into mean reverting models, suggesting that jumps account for a substantial component of volatility.
- The coefficient of mean reversion drops from 0.1752 to 0.1252 when jumps are added to the diffusion process. Again, this is in line with the results of the GMM estimation.
- The jump diffusion model shows a better fit than the mean reverting model, as is evident from the larger log-likelihood value.

Furthermore, the mean and variance terms in the jump process are identified by the ML estimation. Table 5 shows that the variance term is significant, while the mean term is not. Also, the parameter of the Bernoulli distribution, used for parametrisation of the jump process in the ML estimation, is significant. These results may be interpreted as showing that jumps are present in the process, but that the size of the jumps vary to such an extent that a significant value for the mean does not appear.

Again, it is interesting to note that the results for our wheat price data are parallel to the results obtained by Das (1999) for interest rate data.

6.3 Non-linear drift

Aït-Sahalia (1996) has argued that the drift term itself should be modelled as a variable. Below we present evidence of two versions of this model, see Table 6. The results give an important check on the specification of the models presented earlier.

The parameters α_2 , α_3 examine whether the drift is a function of squared spot prices or inversely related to spot price levels. If any of these parameters are different from zero, it means that its drift term is non-linear. Our results in Table 6 show that no parameters are significantly different from zero. This indicates that the drift term of the wheat prices is linear. Table 6 shows that the size of the non-linear coefficients α_2 , α_3 diminishes when jumps are added to the mean reverting model. There is also a reduction in the level of significance. This indicates that the drift term in the stochastic process does not appear to be non-linear. Perhaps the problem is not the linear drift term, but an incomplete specification of the random variation in the stochastic process.

Table 6 This table presents the results of the estimation when the drift term is non-linear. Estimation is carried out using maximum likelihood on a mean reverting model and a jump diffusion model on monthly US wheat prices covering the period June 1952 to January 1998. T-statistic in parenthesis

Parameter	Mean reverting	Jump diffusion
κ	-2.8345 (1.82)	-2.0446 (1.00)
θ	2.1978 (1.84)	1.8381 (3.12)
σ	0.6477 (32.92)	0.2123 (7.18)
α_2	-0.3826 (1.34)	-0.3241 (1.20)
α_3	4.3308 (1.02)	2.1451 (0.58)
μ		0.0344 (1.02)
γ		0.3077 (5.19)
q		0.3286 (5.80)
Log-Likelihood	140.96	297.81

The addition of a jump process possibly diminishes the extent of non-linearity. However, the coefficients are not significant in the mean reverting model either. That is, even without jumps the non-linearity is not present. Therefore, the jump diffusion model stands out well against the non-linear drift model too.

7 End of the tranquil post-war era?

Next, we investigate whether these results change when the models are studied for the two sub-periods separately. Figure 1 indicates that 1973 was a watershed in that a relatively calm post-war era was followed by greater uncertainty. Would investors have to revise their models for the pricing process after 1973 accordingly? Table 7 gives an overview of the results. In the same manner as used in testing the parameters above, an asymptotic χ^2 statistic is used to test whether the parameters of the jump diffusion process for the period to July 1973 and after July 1973 are the same, see Table 8.

Table 7 Results for generalised method of moments estimation with four moment conditions. The instruments used are a constant and once lagged values of the wheat price. The table presents estimates for a mean reverting model and a jump diffusion model, for the periods June 1952 to July 1973 and August 1973 to January 1998. T-statistic in parenthesis

Parameter	June 52–July 73		August 73–January 98	
	I Mean reverting	II Jump diffusion	III Mean reverting	IV Jump diffusion
κ	0.2858 (1.83)	0.1460 (0.88)	0.3900 (2.16)	0.5143 (2.76)
θ	1.9855 (9.37)		3.7323 (10.35)	
$\theta' = \theta + \frac{hE(J)}{k}$		2.2298 (3.59)		3.7454 (13.50)
σ	0.2552 (8.78)		0.6248 (15.04)	
$\sigma' = \sigma + \sqrt{hE(J^2)}$		0.2598 (8.50)		0.7891 (14.80)
$hE(J^3)$		0.0141 (2.18)		0.5792 (2.93)
$hE(J^4)$		-0.0002 (0.06)		1.1301 (2.10)
H	2.92	4.07	13.90	0.10

The results are quite different for the two sub-periods. For the period before July 1973 the jump diffusion model and the mean reverting model have almost the same value of the objective function, H . The difference in the objective functions (tested by χ^2 statistics) between the two models is not significant, and the jump diffusion model does not show a better fit than the mean reverting model. On the other hand, from August 1973 to January 1998 the jump diffusion model has a significantly smaller objective function, H , than the mean reverting model. This is shown in Table 8. All of the parameters are significant,

including the jump parameters. In short, the jump diffusion model performs better in the latter period, in the first period we cannot tell which model has the better fit.

Table 8 Specification test of the hypothesis that the parameters of the jump diffusion and the mean reverting process for the period to July 1973 and after July 1973 are the same

Assumptions	H_0	χ^2	$\chi_{0.95}^2$	Decision
No jumps effects				
Model II vs. I	$h = 0$	1.15	5.99	Accept H_0
Model IV vs. III	$h = 0$	13.80	5.99	Reject H_0
Parameters before and after July 1973 are the same				
Model IV vs. II	Equal parameters	3.97	11.07	Accept H_0
Model III vs. I	Equal parameters	10.98	7.82	Reject H_0

From this analysis the observation that there is a higher volatility in the wheat prices in the latter half of the period is confirmed, and for this period the jump diffusion model outperforms the mean reverting model.

7.1 Experiments with other models

The lessons of the last section motivate an investigation into other model specifications. Even though the fit is satisfactory, and the coefficients are sharply determined, models more economical in terms of parameters to be determined may be found. Specifically, the mean reversion element seems to be superfluous in some models. Also, the highly significant jump parameters inspire the question of the usefulness of a drift term in the model. We also have to consider the possibility of non-linearity of the drift term in wheat prices.

First, the following specifications are considered:

Models:

- (V) $dx = \kappa(\theta - x)dt + \sigma dz$ Mean reverting diffusion
- (VI) $dx = \kappa(\theta - x)dt + \sigma dz + Jd\pi(h)$ Jump mean reverting diffusion
- (VII) $dx = \alpha_0 dt + \sigma dz + Jd\pi(h)$ Jump Brownian motion with drift
- (VIII) $dx = \sigma dz + Jd\pi(h)$ Jump Brownian motion.

Our strategy is simply to run these models and consider their merits against one another. For simplicity, we use ML estimation. Given the agreement of the results in GMM and ML

estimation, this may be advisable. Model V and VI have been specified earlier, in model VII the mean reversion component has been replaced by a deterministic drift term, while even this drift term has been removed in model VIII, leaving only diffusion and jumps in the price process. Table 9 gives an overview of the results.

Table 9 Estimates of mean reverting diffusion, jump – mean reverting diffusion, jump – Brownian motion with drift and jump – Brownian motion model on monthly US wheat prices covering the period July 1973 to January 1998. Estimation is carried out using maximum-likelihood incorporating modification of the transition density function in equation (24). T-statistics are presented below the parameter estimates

Parameter	Model			
	V	VI	VII	VIII
κ	0.6801 (2.97)	0.4478 (2.41)		
θ	3.8628 (15.21)	3.5499 (12.26)		
α_0			-0.0364 (0.28)	
σ	0.8487 (24.12)	0.4629 (11.63)	0.4842 (12.15)	0.4843 (12.13)
μ		0.0574 (0.87)	0.0281 (0.38)	0.0215 (0.31)
γ		0.4381 (3.58)	0.4664 (3.44)	0.4670 (3.44)
q		0.2178 (3.69)	0.1939 (3.54)	0.1938 (3.53)
L	-3.65	42.31	39.43	39.39

The table shows that model VI, jump diffusion with mean reversion, performs best, judging from the log likelihood value. On the other hand, model V, the mean reverting diffusion, has a very low log likelihood value. Clearly, this model does not fit the price process after 1973. Looking at the alternative models, we discover that the log likelihood values for these models (VII and VIII) are slightly lower than for the jump mean reversion model, and besides, the two models have an almost identical log likelihood value.

Let us look at the formal tests. The negative of twice the logarithm of the generalised-likelihood ratio, $\lambda = -2[L(H_0) - L(H_A)]$, for this problem has approximately χ^2 distribution with parameter equal to the number of parameters assumed to be zero in the null hypothesis, H_0 , provided H_0 is true (Green, 1997).

Table 10 shows that the hypothesis of no jumps effects is clearly rejected. But, on the evidence in the table, we cannot confirm the hypothesis that there is no mean reversion in the models. That is, we cannot use the simpler models using only drift or even dropping the drift

term, together with the jump parameter to describe the price process after 1973. It should be noted as well that the mean reversion parameter is significant. This confirms results from the GMM estimation using the entire period.

Table 10 Specification test of mean reverting diffusion, jump mean reverting diffusion, jump Brownian motion with drift, jump Brownian motion models for the period July 1973 to January 1998

Assumptions	H_0	χ^2	$\chi_{0.95}^2$	Decision
No jumps effects Model V vs. VI	$q = 0$	91.92	5.99	Reject H_0
No mean reversion Model VII vs. VI	$\kappa(\theta - r) = \alpha_0$	5.76	3.84	Reject H_0
Model VIII vs. VI	$\kappa(\theta - r) = 0$	5.84	3.84	Reject H_0
No drift Model VIII vs. VII	$\alpha_0 = 0$	0.08	3.84	Accept H_0

The $\chi_{0.99}^2$ -statistics are 9.21, 6.63, 6.63, 6.63, respectively

The results of Table 10 indicates that more economical models of the price process do not give better statistical fit. Also, it should be noted that the mean reversion parameter is significant.

Now let us turn to models employing non-linear drift terms. Two such models are specified in section 3.3. In the first model, the mean reverting diffusion is used, with non-linear terms added to the mean reverting term. The second model incorporates jump. Table 11 gives an overview of the results. The model with diffusion and non-linear drift term will be called model (IX) here, the model also incorporating jumps is given the name (X).

Table 12 shows that the hypothesis that no jumps are present, is rejected, while a hypothesis specifying that there is no linear drift in the process is accepted. Again, the presence of a jumps effect is accepted, while models relying upon linear drift does not fit the facts. As for the results for the entire period, the α_2 and α_3 do not have significant values. Furthermore, we notice the very large difference in the likelihood values of the two specifications, and how clearly the model with a jump specifications performs so much better. It seems as if in whatever manner we look upon the pricing process, the jump models keep coming back.

Table 11 The results of the estimation of models where the drift term is non-linear, cf. equation (20). Estimation is carried out using maximum-likelihood on a mean reverting model and a jump diffusion model on monthly US wheat prices covering the period July 1973 to January 1998. T-statistics are presented below the parameter estimates

Parameter	Mean reverting non-linear drift	Jump diffusion non-linear drift
	IX	X
κ	0.8262 (0.32)	0.8324 (0.20)
θ	7.4656 (0.69)	4.2017 (1.05)
σ	0.8471 (24.03)	0.4621 (11.40)
α_2	-0.0524 (0.25)	0.0212 (0.06)
α_3	-8.0371 (0.51)	-2.8099 (0.14)
μ		0.0584 (0.86)
γ		0.4681 (3.48)
q		0.2192 (3.60)
L	-3.11	42.33

Table 12 Specification test of non-linear drift for the period July 1973 to January 1998

Assumptions	H_0	χ^2	$\chi_{0.95}^2$	Decision
No jumps effects Model IX vs. X	$hE(J^3) = hE(J^4) = 0$	90.88	5.99	Reject H_0
No linear drift Model VI vs. X	$\alpha_2 = \alpha_3 = 0$	5.76	0.04	Accept H_0

8 Conclusions

In this paper, the price process of wheat has been tried in several model specifications, that is, the Vasicek model specified as a mean reverting and a jump diffusion process, together with a model with non-linear drift due to Aït-Sahalia. The models have been tested using both GMM and maximum likelihood.

The results may be summed up as follows:

- The presence of jumps in the price process was clearly evident.

- When the period was divided into two, with 1973 chosen as the dividing year, the jump diffusion model did not perform better than the mean reverting model in the first period. However, in the later period the jump diffusion model clearly outperformed the mean reverting model.
- The jump diffusion model incorporated mean reversion. This feature stood up well in tests against other specification, notably no mean reversion and non-linear drift terms.

Although we have looked into the price behaviour of only one commodity, wheat, it seems unlikely that our method would be limited to wheat only. On this basis, we venture two other conclusions. One upshot of our study is that models of price processes developed in the finance literature may have a wide applicability in the pricing of commodities. May be they are not very different. This points to a unified research agenda for commodities as well as for assets such as equity and bonds. Another consequence of our study is that investigators into derivatives pricing as well as the pricing of real options should take the jumpiness of commodity prices into account. We plan to return to these issues on a later occasion.

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References

- Ahn, C.M., Thompson, H.E., 1988. Jump-diffusion processes and the term structure of interest rates. *Journal of Finance*, 43(1): 155–174.
- Aït-Sahalia, Y., 1996. Testing continuous-time models of the spot interest rate. *Review of Financial Studies*, 9: 385–426.
- Ball, C.A., Roma, A., 1993. A jump diffusion model for the European monetary system. *Journal of International Money and Finance*, 12(5): 475–492.
- Ball, C.A., Roma, A., 1994. Target zone modelling and estimation for European monetary system exchange rates. *Journal of Empirical Finance*, 1: 385–420.
- Ball, C.A., Torous, W.N., 1983. A simplified jump process for common stock returns. *Journal of Financial and Quantitative Analysis*, 18(1): 53–61.
- Bates, D.S., 1996. Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *The Review of Financial Studies*, 9(1): 69–107.
- Campbell, J.Y., Lo, A.W., MacKinlay, A.C., 1997. *The Econometrics of Financial Markets*. Princeton University Press, Princeton, New Jersey.

- Campbell, J.Y., Perron, P., 1991. Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots, volume 6 of *NBER Macroeconomics Annual*, pages 141–201. Cambridge, MIT Press.
- Chambers, M.J., Bailey, R.E., 1996. A theory of commodity price fluctuations. *Journal of Political Economy*, 104(5): 924–957.
- Cox, J.C., Ingersoll, J.E. Jr., Ross, S.A., 1985. A theory of the term structure of interest rates. *Econometrica*, 53(2): 385–407.
- Cox, J.C., Ross, S.A., 1976. The valuation of options for alternative stochastic processes. *Journal of Financial Economics*, 3: 145–166.
- Das, S.R., 1999. The surprise element: Jumps in interest rate diffusions. Working Paper, Harvard University.
- Das, S.R., Foresi, S., 1996. Exact solutions for bond and option prices with systematic jump risk. *Review of Derivatives Research*, 1: 7–24.
- Deaton, A., Laroque, G., 1992. On the behaviour of commodity prices. *Review of Economic Studies*, 59(2): 1–23.
- Deaton, A., Laroque, G., 1996. Competitive storage and commodity price dynamics. *Journal of Political Economy*, 104(5): 896–923.
- Engle, R.F., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation. *Econometrica*, 50: 987–1008.
- Green, W.H., 1997. *Econometric Analysis*, 3rd edition. Prentice Hall, New York.
- Hansen, L.P., 1982. Large sample properties of generalized method of moments estimators. *Econometrica*, 50(4): 1029–1054.
- Hilliard, J.E., Reis, J., 1998. Valuation of commodity futures and options under stochastic convenience yields, interest rates and jump diffusions in the spot. *Journal of Financial and Quantitative Analysis*, 33(1): 61–86.
- Jorion, P., 1988. On jump processes in the foreign exchange and stock markets. *The Review of Financial Studies*, 1(4): 427–445.
- Learn, E.W., 1986. Agricultural price and income policy: A need for change. *Contemporary Policy Issues*, 4(1): 49–61.
- Lo, A.W., MacKinley, C., 1989. The size and power of the variance ratio test in finite samples: A Monte Carlo investigation. *Journal of Econometrics*, 40: 203–238.
- Merton, R.C., 1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, 3: 125–144.
- Miltersen, K.R., Schwartz, E.S., 1998. Pricing of options on commodity futures with stochastic term structures of convenience yields and interest rates. *Journal of Financial and Quantitative Analysis*, 33(1): 33–59.
- Newey, W.K., West, K.D., 1987. Hypothesis testing with efficient method of moments estimation. *International Economic Review*, 28(3): 777–787.
- Nieuwland, F.G.M.C., Verschoor, W.F.C., Wolf, C.C.P., 1994. Stochastic trends and jumps in ems exchange rates. *Journal of International Money and Finance*, 13(6): 699–728.

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- Ogaki, M., 1993. Generalized method of moments: Econometric applications, volume 11 of *Handbook of Statistics*, chapter 17, pages 455–488. Elsevier Science Publishers, Amsterdam.
- Rosen, S., Murphy, K.M., Scheinkman, J.A., 1994. Cattle cycles. *Journal of Political Economy*, 102(3): 468–492.
- Scheinkman, J.A., Schechtman, J., 1983. A simple competitive model with production and storage. *Review of Economic Studies*, 50: 427–441.
- Schwartz, E.S., 1997. The stochastic behavior of commodity prices: Implications for valuation and hedging. *The Journal of Finance*, 52(3): 923–973.
- Vasicek, O.A., 1977. An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5: 457–470.
- Williams, J.C., Wright, B.D., 1991. *Storage and Commodity Markets*. Cambridge University Press, Cambridge.

Essay 5:

Term structure of volatility and price jumps in agricultural markets - evidence from option data*

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Abstract

Empirical evidence suggests that agricultural futures price movements have fat-tailed distributions and exhibit sudden and unexpected price jumps. There is also evidence that the volatility of futures prices contains a term structure depending on both calendar-time and time to maturity. This paper extends Bates (1991) jump-diffusion option pricing model by including both seasonal and maturity effects in volatility. An in-sample fit to market option prices on wheat futures shows that our model outperforms previous models considered in the literature. A numerical example illustrates the economic significance of our results for option valuation.

Keywords: Option pricing; Futures; Term structure of volatility; Jump-diffusion; Agricultural markets

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1 Introduction

Black (1976) derives a pricing model for European puts and calls on a commodity futures contract, assuming that the futures price follows a geometric Brownian motion (GBM). In the literature on agricultural futures markets (as in many other markets) however, several empirical regularities have been documented, indicating that the GBM assumption may be too simplistic. Research on futures prices has found distributions that are leptokurtic relative to the normal distributions (e.g. Hudson et al., 1987; Hall et al., 1989) and the prices often exhibit sudden, unexpected and discontinuous changes. Jump behaviour of this sort will typically occur due to abrupt changes in supply and demand conditions, and naturally it will affect option pricing. Hilliard and Reis (1999) used transactions data on soybean futures and futures options to test American versions of Black's (1976) diffusion and Bates' (1991) jump-diffusion option pricing models. Their results show that Bates' model performs considerably better than Black's model.

A number of studies have demonstrated the presence of a term structure of volatility in agricultural futures prices. Samuelson (1965) stated that the volatility of futures price changes per unit of time increases as the time to maturity decreases. This maturity effect is usually referred to as the "Samuelson hypothesis". Another view, the "state variable hypothesis" is that the variance of futures prices depends on the distribution of underlying state variables. For crop commodities with annual harvest, seasonality in the volatility of futures prices is typically expected. Empirical research on the former approach has produced mixed evidence on the maturity effect (Rutledge, 1976). Milonas (1986) found strong support for the maturity effect after controlling for the year effect, seasonality effect and the contract-month effect. Galloway and Kolb (1996) concluded that the maturity effect is an important source of volatility in futures prices for commodities that experience seasonal demand or supply, but not for commodities where the cost-of-carry model works well. Anderson (1985) found support for the maturity effect, but concluded it is secondary to the effect of seasonality. Anderson also concluded that the pricing of options on futures contracts should be made for the regular pattern to the volatility of futures. Bessembinder et al. (1996) have reconciled much of the early evidence on the "Samuelson hypothesis". They have shown that in markets where spot price changes include a temporary component so investors expect some portion of a typical price change to revert in the future, the "Samuelson hypothesis" will hold. Mean reversion is more likely to occur in agricultural commodity markets than in markets for

precious metals or financial assets (Bessembinder et al., 1995), so we expect to see maturity effects in agricultural commodity markets.

Any regular pattern in the volatility is inconsistent with the underlying assumptions of the Black's (1976) and Bates' (1991) option pricing models. Choi and Longstaff (1985) applied the formula of Cox and Ross' (1976) for constant elasticity of variance option pricing in the presence of seasonal volatility. They found this superior to Black's model for pricing options on soybeans futures. Myers and Hanson (1993) present option-pricing models when time-varying volatility and excess kurtosis in the underlying futures price are modelled as a GARCH process. Empirical results suggest that the GARCH option-pricing model outperforms the standard Black model. Fackler and Tian (1999) proposed a simple one-factor spot price model with mean reversion (in the log price) and seasonal volatility. They show that futures prices consistent with this spot price model have a volatility term structure exhibiting both seasonality and maturity effects. Their empirical results indicate that both phenomena are present in the soybean futures and option markets.

In this paper we assume that the futures price follows a jump-diffusion process. The diffusion term includes time dependent volatility that captures (possibly) both a seasonal and a maturity effect. We derive a futures option pricing model given our specified futures price dynamics, and we test our model empirically using eleven years of data on American futures option prices on wheat from Chicago Board of Trade (CBOT). We find that our model does a better job in explaining the option prices than the models previously suggested in the literature. The maturity effect is especially strong in this market. A numerical example illustrates the economic significance of our results. This paper is organised as follows: In the next section we present the model and derive the option pricing model. Thereafter the data are described and preliminary evidence on volatility term structure and jump effects is given, then the empirical results are presented. Finally, we illustrate the economic significance of volatility term structure and jump parameters and a numerical example is given. The paper ends with a summary and concluding comments.

2 The model

We shall present a jump-diffusion model for the futures price dynamics and derive an option pricing model for a European futures option. Fundamental to the pricing of contingent claims is the derivation from the real world distribution of the asset price, to the equivalent "risk-

neutral" distribution, or the equivalent martingale measure (EMM) in modern terminology. The value of a contingent claim is the expected value under the EMM discounted by the risk free rate. In the paper by Merton (1976), jumps are assumed to be symmetric (zero mean) and nonsystematic. In a stock market model, this means that jumps are of no concern to an investor with a well-diversified portfolio, since jumps on average cancel out. Given such assumptions of firm specific jump risk, parameters concerning the jump part are equal under both the real world probability measure and the EMM. In our setting, focusing on wheat futures prices, the assumption of non-systematic jump risk may be inappropriate. If, for example, bad weather results in a poor harvest, futures prices may jump. However, the occurrence of such an event is likely to move all the commodity futures prices in the same direction, and so diversifying the jump risk is impossible. In other words, jump risk is systematic. To derive the EMM when jump risk is systematic, we have to make assumptions about the price of jump risk. In this paper we follow Bates (1991) closely.¹ Bates assumed frictionless markets, optimally invested wealth follows a jump-diffusion, and a representative consumer with time-separable power utility. He then derived the EMM from the real world probability measure. Under the assumptions on preferences and technology, he showed that jump parameters under the EMM need to be adjusted according to the preferences of the representative consumer. In case of risk neutrality, the jump parameters are equal under both measures. The only difference between our model and that of Bates is that we impose time dependence in the diffusion term of the GBM. It is well known that the diffusion term is unchanged, going from one probability measure to an equivalent probability measure. Hence, the results in Bates apply to our model as well. We shall set up the model directly under the EMM. Denote the price of a futures contract as $F(t, T^*)$, where t is today's date and T^* is the maturity date of the contract. The futures price is assumed to follow the following dynamics under the EMM:

$$\frac{dF(t, T^*)}{F(t, T^*)} = -\lambda \kappa dt + \sigma(t, T^*) dB(t) + \kappa dq \quad (1)$$

where $B(t)$ is standard Brownian motion under the EMM and κ is the random percentage jump conditional upon a Poisson distributed event, q , occurring. We assume that $(1 + \kappa)$ is a lognormal random variable with mean $(\gamma - 1/2v^2)$ and variance v^2 . Consequently, the

¹ A full derivation of the EMM in an equilibrium setting is given in the appendix in Bates (1991).

expected percentage jump size is $E[\kappa] \equiv \bar{\kappa} = e^\gamma - 1$. The frequency of Poisson events is λ and q is the Poisson counter with intensity λ . Note that the jump parameters are independent of time to maturity. This means that if a jump occurs, a parallel shift in the term structure of futures prices will occur. If we observe several futures contracts with time to maturity spanning several years into the future, the jump structure described above may seem inadequate. If, for example, exceptional bad weather (such as a hurricane) partly destroys a harvest, then futures prices are likely to jump. But we would expect contracts with maturity before the next harvest to experience a greater price change than contracts with maturity preceding the next harvest, since the next harvest is likely to turn out better than the previous one. This behaviour can easily be incorporated in our model by imposing a term structure on the jump amplitude. Such an extension is ignored in this paper since the maturity of the futures contracts analysed in this paper never exceed one year. Hence, in our data set, imposing parallel jumps may be a satisfactory assumption. The function $\sigma(t, T^*)$ represents the instantaneous volatility of the futures price conditional on no jumps. We want to capture two possible effects in the specification of the volatility function; periodic seasonality and maturity effect. We shall concentrate on the following candidate

$$\sigma(t, T^*) = \sigma(t) \sum_{i=1}^I \sigma_i(T^* - t) \quad (2)$$

The first term represents the time t dependent seasonal volatility pattern. We model the periodic function as a truncated Fourier series

$$\sigma(t) = \bar{\sigma} + \sum_{j=1}^P (\alpha_j \sin 2\pi t + \beta_j \cos 2\pi t)$$

The maturity effect is modelled by negative exponentials

$$\sigma_i(T^* - t) = e^{-\delta_i(T^* - t)}$$

This model provides a fairly rich volatility term structure, and as we shall see below, a straightforward closed-form pricing formula for vanilla European options can be derived.

2.1 Relation to other models in the commodity literature

This model nests several models proposed for commodities in the literature. The seminal Black's (1976) model is given by $\lambda = \delta_i = \alpha_j = \beta_j = 0$. The one-factor model of Schwartz (1997), that captures the maturity effect, appears if we set $\lambda = \alpha_j = \beta_j = 0$. The jump-diffusion model of Bates (1991) is $\delta_i = \alpha_j = \beta_j = 0$. Bates (1991) extended with maturity effect is $\alpha_j = \beta_j = 0$, and Bates (1991) extended with seasonal effects is given by $\delta_i = 0$.

2.2 Valuation of futures options

Valuation of both European and American futures options in this model are slight generalisations of the formula given in Bates (1991) and Merton (1976). Let n be the number of jumps occurring in the interval $[t, T]$. Then the solution to equation (1) is

$$F(T, T^*) = F(t, T) \exp\left(-\lambda \bar{\kappa}(T-t) - \frac{1}{2} \int_t^T \sigma(s, t)^2 ds + \int_t^T \sigma(s, T) dB(s)\right) \prod_{j=0}^n (1 + \kappa_j) \quad (3)$$

The value of a European futures call option written on the contract $F(t, T^*)$ where $T \leq T^*$ with strike price K and maturity at time T , is given by

$$\begin{aligned} c(F(t, T^*), T) &= e^{-r(T-t)} \sum_{n=0}^{\infty} (\text{Pr}_- n \text{ jumps}) (F(t, T^*) e^{b(n)(T-t)} N(d_{1n}) - KN(d_{2n})) \\ &= e^{-r(T-t)} \sum_{n=0}^{\infty} \left(\frac{e^{-\lambda T} (\lambda T)^n}{n!} \right) (F(t, T^*) e^{b(n)(T-t)} N(d_{1n}) - KN(d_{2n})) \end{aligned}$$

where

$$b(n) = -\lambda \bar{\kappa}(T-t) + \frac{n\gamma}{(T-t)}$$

$$d_{1n} = \frac{\ln\left(\frac{F(t, T^*)}{K}\right) + b(n)(T-t) + \frac{1}{2}\omega^2 + nv^2}{\sqrt{\omega^2 + nv^2}}$$

$$d_{2n} = d_{1n} - \sqrt{\omega^2 + nv^2}$$

$$\omega = \sqrt{\int_t^T \sigma(s, T^*)^2 ds}$$

Put options can be calculated explicitly, or they can be found via the futures option put-call parity. In the empirical part of this paper, we use data on American futures options, consequently some modification of the above model is required. Bates (1991) derives an approximation for an American option in the jump-diffusion framework. His approximation follows the work of Barone-Adesi and Whaley (1987) in the standard case where the underlying asset follows a GBM. We use the same approximation as described by Bates (1991), replacing the constant volatility in his setting with the time-dependent volatility given by ω above.

3 Preliminary analysis and data description

Weekly data were obtained for call options on wheat futures and for the underlying futures contract traded on the CBOT from January 1989 until December 1999. Wheat futures contracts are available with expiration in March, May, July, September, and December. We first present a simple regression model to illustrate the term structure of volatility present in our eleven years sample of futures data.

3.1 Term structure effects in futures price volatility

We ran the following regression for each of the five contracts:

$$V_t = \eta_1 + \sum_{k=2}^{12} \eta_k D_{kt} + e_t \quad (4)$$

where V_t is estimated standard deviation of the log changes of wheat futures prices for month t based on daily data, D_{kt} are seasonal dummy variables for month t : $k=2$, February, ...,

$k=12$, December, and e_t is an error term assumed to follow an AR(1) process. The regression model was estimated by Hildreth and Lu (1960) grid search method.²

In Table 1 the results from the regression are reported in the following way; January is the constant term, η_1 , February is $\eta_1+\eta_2$ etc. From the results in Table 1 we see a very pronounced maturity effect, and weak evidence of seasonality for each contract. Looking for example at the March contract we see that volatility starts to rise in December. The volatility in January, February and March is approximately six times the volatility in April.³ We also see that the volatilities of the remaining months of the March contract are significantly different from volatility in January. Note also that the summer months have slightly higher volatilities than April and the autumn months. We find this pattern for the other contracts as well. In this paper we shall investigate whether this term structure effect is priced in the option market.

Table 1 Estimates of seasonality and maturity coefficients, March, May, July, September and December wheat futures contracts, 1989-1999. t -values are in parentheses

	March		May		July		September		December	
η_1	0.062	(7.32)	0.011	(1.25)	0.027	(1.86)	0.009	(0.95)	0.003	(0.18)
η_2	0.061	(0.20)	0.010	(0.05)	0.030	(0.40)	0.009	(0.01)	0.004	(0.11)
η_3	0.060	(0.24)	0.032	(2.11)	0.035	(0.71)	0.013	(0.39)	0.014	(0.93)
η_4	0.009	(4.99)	0.065	(5.04)	0.054	(2.15)	0.016	(0.63)	0.032	(2.11)
η_5	0.010	(4.61)	0.071	(5.33)	0.067	(2.93)	0.015	(0.54)	0.035	(2.17)
η_6	0.011	(4.44)	0.008	(0.24)	0.072	(3.17)	0.017	(0.65)	0.035	(2.08)
η_7	0.012	(4.32)	0.010	(0.10)	0.077	(3.47)	0.048	(3.30)	0.040	(2.37)
η_8	0.012	(4.34)	0.010	(0.02)	0.013	(0.91)	0.073	(5.38)	0.055	(3.31)
η_9	0.010	(4.65)	0.009	(0.16)	0.009	(1.24)	0.077	(5.89)	0.073	(4.65)
η_{10}	0.010	(4.83)	0.009	(0.12)	0.019	(0.56)	0.004	(0.41)	0.084	(5.79)
η_{11}	0.010	(5.31)	0.010	(0.12)	0.019	(0.67)	0.005	(0.36)	0.096	(7.58)
η_{12}	0.032	(3.93)	0.008	(0.30)	0.024	(0.33)	0.006	(0.43)	0.098	(10.03)
Adj R ²	0.58		0.56		0.68		0.65		0.73	

3.2 Indication of jump behaviour from option prices

If wheat futures prices are characterised solely by deterministic time-dependent volatility, they are lognormally distributed. Furthermore, the implied volatility from option prices will be constant across strike prices. However, if jumps are likely to occur, implied volatility will

² OLS generally displayed autocorrelated residuals. The Hildreth and Lu grid search procedure was employed to yield consistent parameter estimates.

³ The low t -statistics in February and March simply imply that the volatilities in those months are indistinguishable from the volatility in January.

be skewed. In Figure 1 we have calculated implied volatility from call futures prices at January 18, 1995. When backing out implied volatilities, we used the formula derived by Black (1976) adjusting for the fact that the options are of American type using the approximation of Barone-Adesi and Whaley (1987). Figure 1 shows no horizontal pattern of implied volatility, but an implied "volatility smile". A jump diffusion model may produce such a pattern. When futures prices are allowed to jump upwards, out-of-the-money (OTM) call options have a higher probability of ending in-the-money (ITM) than otherwise would be the case, and they will trade at a higher price. This in turn creates an upward sloping volatility pattern for call options evident from Figure 1. For a call option ITM, the probability of a negative jump will cause the options to be worth more than would be the case in a lognormal world.

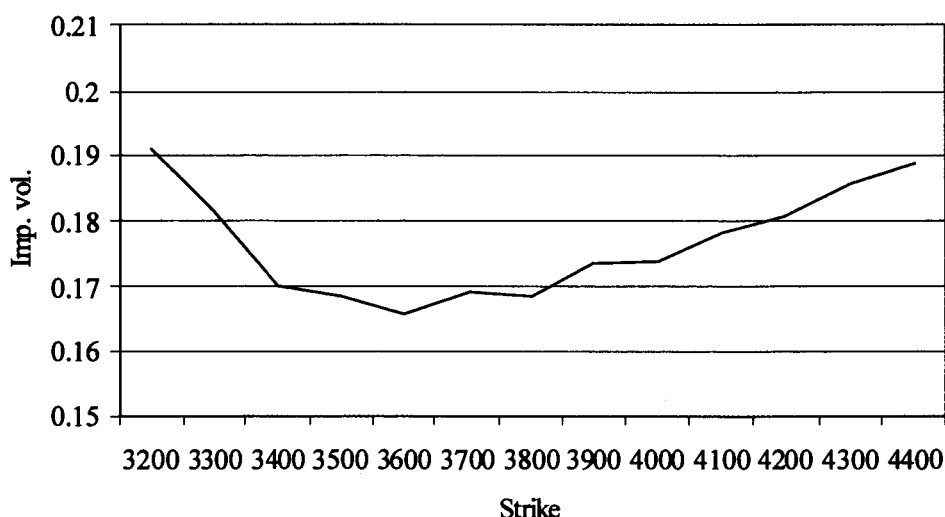


Figure 1 Implicit volatility patterns from CBOT wheat call options with maturity in May 19, 1995 at January 18, 1995. Implied volatility for American options are approximated as in Barone-Adesi and Whaley (1987)

3.3 Constructing the data set

From the preliminary analysis above we have seen evidence suggesting that our model, including both jumps and time dependent volatility, will capture important market characteristics. We have therefore tested our model on wheat futures option prices collected from CBOT. The eleven years of data consist of fifty-five futures contracts. The futures contracts matures in March, May, July, September, and December. At each point in time, there are five contracts traded, meaning that one year is the longest contract an investor can enter into. The options written on the contracts can be exercised prior to maturity, hence they

are of American type. The last trading day for the options is the first Friday preceding the first notice day for the underlying wheat futures contract. The expiration day of a wheat futures option is on the first Saturday following the last day of trading.

We applied several exclusion filters to construct the data sample. First, our sample starts in 1989. We did not use prices prior to 1989 since market prices then were likely to be affected by government programs in the United States (price floor of market prices and government-held stocks). Second, only trades on Wednesdays were considered, yielding a panel data set with weekly frequency. Weekly sampling is simply a matter of convenience. Daily sampling would place extreme demands on computer memory and time. Third, only settlement (closing) prices were considered. Fourth, the last six trading days of each option contract were removed to avoid the expiration related price effects (these contracts may induce liquidity related biases). Fifth, to mitigate the impact of price discreteness on option valuation, price quotes lower than 2.5 cents/bu were deleted. Sixth, assuming that there is no arbitrage in this market, option prices lower or equal to their intrinsic values were removed. Three-month Treasury bill yields were used as a proxy for the risk free discount rate. The exogenous variables for each option in our data set are strike price, K , futures spot price, F , today's date, t , the maturity date of the option contract, T , the maturity date of the futures contract, T^* , the instantaneous risk-free interest rate, r , observed settlement option market price, C_{it} , where i is an index over transactions (calls of assorted strike prices and maturities), and t is an index over the Wednesdays in the sample.

4 Implicit parameter estimation and in-sample performance

4.1 Method

Besides the exogenous variables obtained from the data set, the option pricing formula requires some parameters as inputs. In the full model the following parameters need to be estimated: the season and maturity effect-related parameters $\bar{\sigma}, \alpha_j, \beta_j, \delta_i$ and the jump-related parameters $\bar{\kappa}, \nu, \lambda$. There are two main approaches to estimate these parameters; from time series analysis of the underlying asset price, or by inferring them from option prices (Bates, 1995). There are two main drawbacks of the former approach. First, very long time series are necessary to correctly estimate jump parameters, at least if prices jump rarely. Second, parameters obtained from this procedure correspond to the actual distribution, and

hence the parameters cannot be used in an option pricing formula, since the parameters needed for option pricing are given under the EMM. The latter approach, to infer some or all of the distributional parameters from option prices conditional upon postulated models has been used in, e.g., Bates (1991, 1996, 2000); Bakshi et al. (1997); and Hilliard and Reis (1999). Implicit parameter estimation is based on the fact that options are forward looking assets and therefore contain information on future distributions. Implied estimation delivers the parameters under the EMM.

We infer model-specific parameters from option prices over an eleven years long time period. The model is separately estimated for March, May, July, September and December wheat futures contracts expiring in 1989 through 1999. In previous studies, implicit parameters have been inferred from option prices during a very short time interval, often daily (e.g., Bates (1991, 1996); Hilliard and Reis, 1999). However, this method can be applied to data spanning any interval that has sufficient number of trades (Hilliard and Reis, 1999). Daily recalibrations can fail to pick up longer horizon parameter instabilities (Bates, 2000). In this study, one of the aspects we focus on is the changing volatility during the year. Options written on a specific contract have only one maturity each year. If we were to use daily data, a model with time-dependent volatility would be indistinguishable from a model with constant volatility. Information of changing volatility will be revealed as the option prices change during the course of the year. In other words, we need a long time span, in order to be able to pick up volatility term structure effects in this market.

American option prices, C_{it} , are assumed to consist of model prices plus a random additive disturbance term:

$$C_{it} = C(F_{it}, K_i, t, T, T^*, r, \bar{\kappa}, \nu, \lambda, \bar{\sigma}, \alpha_j, \beta_j, \delta_i) + e_{it} \quad (5)$$

Equation (5) can be estimated using non-linear regression. The unknown implicit parameters $\bar{\kappa}, \nu, \lambda, \bar{\sigma}, \alpha_j, \beta_j, \delta_i$ are estimated by minimising the sum of squared errors (SSE) for all option in the sample given by

$$SSE = \sum_{t=1}^T \sum_{i=1}^N [C_{it} - C(\bullet)]^2 = \sum_{t=1}^T \sum_{i=1}^N [e_{it}]^2 \quad (6)$$

where i is an index over transactions (calls of assorted strike prices and maturities), and t is a time index. The parameters minimising (6) were found using the Quadratic-hill climbing algorithm in GAUSS.

Many alternative criteria could be used to evaluate performance of option pricing models. The overall sum of squared errors (SSE) is used as a broad summary measure to determine how well each alternative option pricing model fits actual market prices. Assuming normality of the error term, nested models can be tested using F -test statistic.⁴ Bates (1996, 2000) points out that the option pricing model is poorly identified. This means that when we minimise the non-linear function (5), quite different parameter values can yield virtually identical results. As a result of this, parameter estimates should be interpreted with care.

4.2 Implied parameters and in-sample pricing fit

The following models were estimated (abbreviations used later in the paper are in parentheses): Black's (1976) diffusion (Black76), Bates's (1991) jump-diffusion (Bates91), Black's model with season and maturity effect (Black SM) and Bates with season and maturity effect (Bates SM). Table 2 shows implicit parameter estimates for March, May, July⁵, September and December wheat options. For the Black SM and Bates SM estimation was done with the maturity effects of order 1, i.e., only one parameter for α , β and δ , respectively.⁶ As a result of forcing eleven years of data into one option pricing model with constant parameters, the SSE is quite large. However, R^2 values are high and vary between 0.967 and 0.988 between contracts and models.

⁴ The F statistic is computed as $F[J, n - K] = \frac{(SSE_R - SSE_U)/J}{SSE_U/n - K}$ where SSE_U and SSE_R are sum squared

errors for unrestricted and restricted models respectively, J is number of restrictions, n is number of observations in the sample, and K is number of parameters in the unrestricted model. In the nonlinear setting, the F distribution is only approximate (Greene, 1993, p. 336).

⁵ For July contracts with the Bates SM model we had a problem in minimising function (6) in one step, so the parameters for this model were estimated in two steps. In step one all parameters except α_1 and β_1 were estimated. The parameters $\bar{\sigma}$ and δ_1 from step one were then used as constants in step two.

⁶ We have also done some estimation of order 2 for both seasonal parameters and maturity parameters. Generally, using SSE as the performance criterion there is little improvement from including seasonal and maturity effects of order 2 compared to the more restrictive order 1 seasonal and maturity effects. Estimations of order 2 for only the seasonal parameters gave almost the same results as estimation of order 2 for both maturity and seasonal parameters, and are not reported here. However, the results are available from the authors upon request.

Table 2 Implicit parameter estimates for various models on March, May, July, September and December contracts on wheat in the period 1989-1999. 4264, 3859, 5074, 3971 and 5231 observations, respectively. *t*-values are in parentheses

	Black76		Black SM		Bates91		Bates SM	
<i>March contracts</i>								
σ	0.21	(514.7)	0.85	(1072)	0.15	(132.1)	1.18	(955.0)
γ					0.04	(51.5)	0.04	(47.9)
$\bar{\kappa}$					0.04		0.04	
ν					0.19	(542.8)	0.19	(215.4)
λ					0.57	(61.3)	0.59	(45.2)
δ_1			2.85	(247.3)			3.98	(812.6)
α_1			-0.11	(-22.6)			-0.11	(-10.2)
β_1			-0.57	(-223.4)			-1.00	(-151.8)
SSE	2 300 600		2 035 600		2 016 600		1 822 600	
<i>May contracts</i>								
σ	0.20	(1388)	0.25	(2897)	0.18	(2146)	0.23	(11.4)
γ					0.08	(6.4)	0.05	(5.9)
$\bar{\kappa}$					0.09		0.06	
ν					0.26	(673.8)	0.17	(466.9)
λ					0.14	(21.4)	0.60	(8.4)
δ_1			0.36	(3935)			0.71	(3.3)
α_1			-0.02	(-74.0)			-0.03	(-1.9)
β_1			-0.02	(-121.3)			-0.05	(-7.0)
SSE	1 514 000		1 458 200		1 399 100		1 299 000	
<i>July contracts</i>								
σ	0.21	(1102)	0.22	(889.7)	0.13	(598.0)	0.39	(183.2)
γ					0.04	(89.4)	0.02	(71.5)
$\bar{\kappa}$					0.04		0.02	
ν					0.05	(206.5)	0.15	(225.2)
λ					6.49	(578.8)	1.52	(93.8)
δ_1			0.01	(0.9)			4.49	(177.0)
α_1			-0.03	(-26.0)			-0.15	(-5.8)
β_1			-0.08	(-76.7)			-0.10	(-6.1)
SSE	4 793 100		3 848 100		4 609 900		3 840 900	
<i>September contracts</i>								
σ	0.24	(330.8)	4.00	(1027)	0.18	(1290)	0.34	(706.9)
γ					0.11	(158.1)	0.14	(21.3)
$\bar{\kappa}$					0.12		0.16	
ν					0.17	(60.8)	0.46	(636.3)
λ					0.56	(60.7)	0.14	(23.7)
δ_1			7.86	(533.8)			1.20	(173.2)
α_1			2.41	(444.3)			-0.15	(421.4)
β_1			2.46	(502.3)			-0.03	(169.8)
SSE	5 591 300		4 664 100		5 335 900		4 242 600	
<i>December contracts</i>								
σ	0.23	(805.3)	0.29	(477.0)	0.15	(156.5)	0.30	(24.5)
γ					0.01	(78.0)	0.05	(271.1)
$\bar{\kappa}$					0.01		0.05	
ν					0.24	(61.3)	0.35	(402.1)
λ					0.65	(442.1)	0.22	(24.4)
δ_1			1.03	(268.1)			1.56	(21.7)
α_1			0.01	(4.7)			0.05	(5.7)
β_1			-0.12	(-144.8)			-0.12	(-11.3)
SSE	4 734 500		4 548 000		4 360 800		4 173 200	

The results provide clear evidence of the importance of the seasonal and maturity effects; Bates SM performed best for all contracts. Furthermore, the inclusion of seasonal and maturity effects in Black76 sometimes gave approximately the same and sometimes better fit than Bates91 jump diffusion model. This indicates that the volatility term structure may be more important, in terms of option pricing, than the possibility of jumps. As Hilliard and Reis (1999) found this analysis also shows that Bates91 performed better than Black76. We have formally tested the models against each other using F -tests. The results given in Table 3, indicate that we can reject the other models proposed in the literature in favour of our model with both jump and time dependent volatility.

Table 3 Model specification tests for March, May, July, September and December contracts

Null hypothesis	Restrictions	F-value	$F_{0.95}$ -critical	Decision
<i>March contracts</i>				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0$	151.0	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	202.1	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0$	187.0	8.5	Reject H0
<i>May contracts</i>				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0$	98.9	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	105.5	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0$	49.2	8.5	Reject H0
<i>July contracts</i>				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0$	338.2	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	67.2	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0$	415.0	8.5	Reject H0
<i>September contracts</i>				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0$	340.5	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	63.3	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0$	262.9	8.5	Reject H0
<i>December contracts</i>				
Bates SM = Bates91	$\delta_1 = \alpha_1 = \beta_1 = 0$	78.3	8.5	Reject H0
Bates91 = Black76	$\bar{\kappa} = \nu = \lambda = 0$	149.3	8.5	Reject H0
Black SM = Black76	$\delta_1 = \alpha_1 = \beta_1 = 0$	71.4	8.5	Reject H0

4.3 A closer look at the volatility term structure

From Table 2 we also see that parameters governing the volatility dynamics differ somewhat across contracts. This may be explained partly by the fact that different parameter values may cause quite similar option prices, as mentioned above. We have plotted the volatility term structure for each contract in Figure 2, using the estimated parameters in Table 2. For each contract, the volatility term structure spans one year, and ends as the futures contract expires.

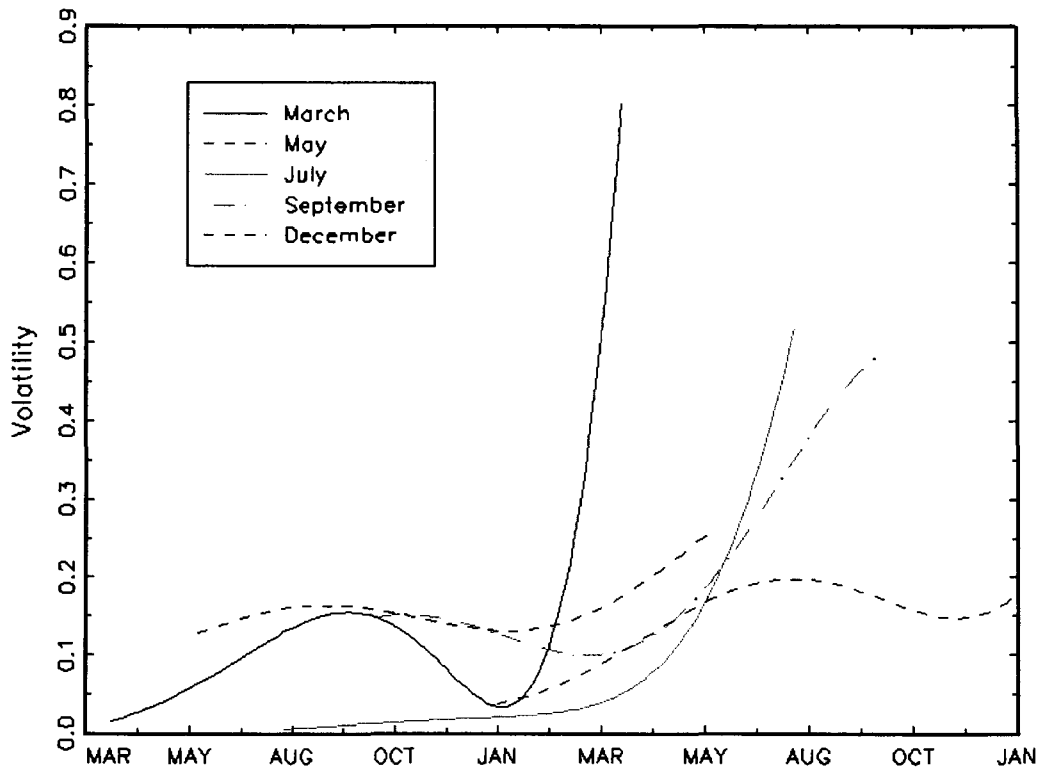


Figure 2 Estimated term-structure of the volatility from option data for March, May, July, September and December futures contracts

We see that March, July and September contracts reveal the most profound maturity effect. The December contract combines high summer volatility and a maturity effect during autumn. In sum, the December contract seems to be more volatile during the second half of the year. The July contract shows few signs of seasonality at all, but from Table 2 we see that the seasonal parameters are significantly different. Again, this illustrates that the maturity effect has a far bigger impact on the term structure of volatility than the seasonal effect.

4.4 A closer look at the jump parameters

As argued elsewhere, implied volatility curves reveal the effects of jumps on option prices. As an illustration of the effect of jumps on implied volatility, we computed theoretical option prices on American calls for different strikes using parameters from the full model (Bates SM) of the May contract in Table 2. The futures price is set to $F(t, T^*) = 3000$, the maturity of the contract $T^* = 7$ months, and the risk free rate $r = 0.05$. We backed out implied volatility curves using 5 strikes ($K = 2400, 2700, 3000, 3300$ and 3600) for three different option maturities ($T = 2, 4$ and 6 months). The results are given in Figure 3.

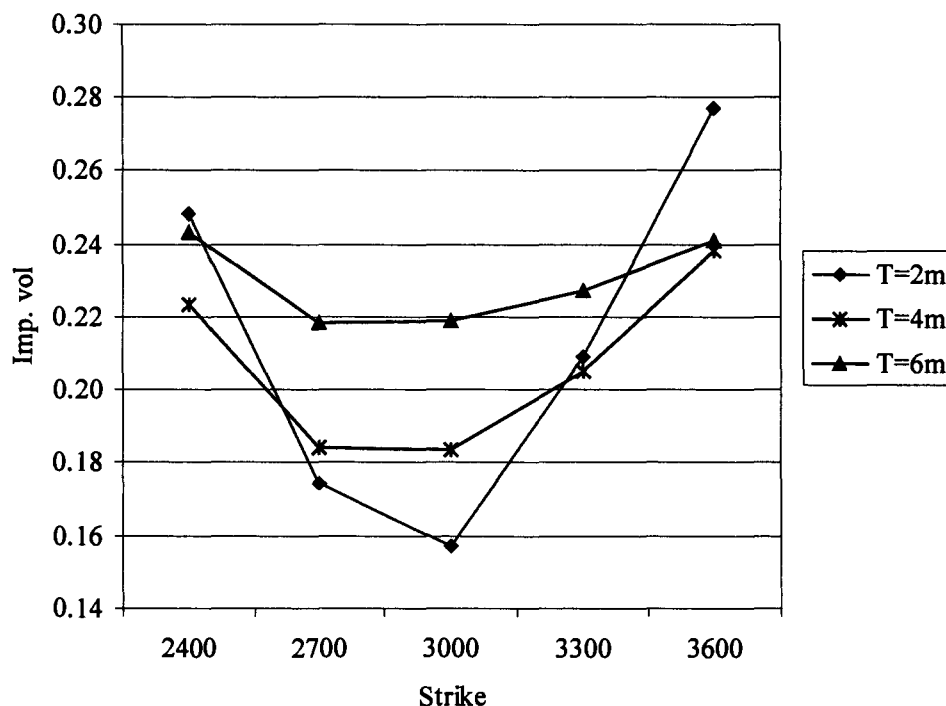


Figure 3 Implicit volatility patterns from CBOT wheat call options where options contracts have 2, 4 and 6 months to maturity, respectively and the underlying futures contract has 7 months to maturity. Implied volatility for American options are approximated as in Barone-Adesi and Whaley (1987)

We recognise the clear "smile" effect from Figure 1, caused by the possibility of both upward and downward jumps. It is also evident that this "smile" gets more pronounced as option expiration gets closer. If there is only a short time to maturity, far OTM options in a lognormal model will be worth relatively little, since an extreme upward price swings is very unlikely. In a jump-diffusion model, these options may end up ITM if a jump occurs, and consequently, these options will be relatively more valuable in a jump-diffusion than in a lognormal world. When there is long time to option maturity, the jump component plays a less prominent part when it comes to moving futures prices upwards or downwards. In the case of OTM options say, the diffusion term alone will be able to move the futures price so that the option will end up ITM.⁷ We also note from Figure 3 that the volatility curve shifts

⁷ In our special case, there is roughly equal chance for the jump to be either positive or negative under the EMM ($\bar{K} \approx 0$). This means that as time to option expiration increases, multiple jumps will have a tendency to cancel each other out. This will enforce the flattening effect on the volatility smile as time to expiration increases.

upwards when option maturity increases. This fact is mainly caused by the maturity effect captured by the volatility term structure.

5 A numerical example

Finally, we provide a numerical example showing the economic significance of our findings. Assume that our model specification is correct; that both the volatility term structure and jumps are present in futures prices, and hence our option pricing formula calculates the true option price. What kind of mispricing will take place if we use the model of Black (1976) or Bates (1991) previously suggested in the literature? We stick to the example above and compute American call option prices based on parameters from the May contract for different option maturities. These prices are compared to Black76 and Bates91 model prices, again picking parameters from Table 2. The results are given in Table 4.

Table 4 Comparison of American wheat futures option prices using Black76, Bates91 and Bates SM for different strikes when the underlying futures contract has 7 months to maturity and the futures price is set to $F(t, T^*) = 3000$, and the risk free rate $r = 0.05$. Parameter estimates for the May contract in Table 2 is used

	K	Black76	Bates91	Bates SM	%Diff.	
					Black76 - Bates SM	Bates91 - Bates SM
$T = 2m$	2600	401.65	402.89	401.38	0.1 %	0.4 %
$T^* = 7m$	3000	96.81	94.07	75.93	27.5 %	23.9 %
	3400	6.97	11.23	13.45	-48.1 %	-16.5 %
$T = 4m$	2600	414.12	414.64	409.74	1.1 %	1.2 %
$T^* = 7m$	3000	136.02	134.96	124.30	9.4 %	8.6 %
	3400	25.45	30.95	31.89	-20.2 %	-2.9 %
$T = 6m$	2600	430.10	432.01	436.09	-1.4 %	-0.9 %
$T^* = 7m$	3000	167.14	168.58	181.44	-7.9 %	-7.1 %
	3400	46.06	53.11	65.47	-29.6 %	-18.9 %

Concentrating on the last two columns, we see that Bates SM produce very different option prices than Black76 and Bates91. We note that the difference between Bates SM and Black76 is as much as 48% for the nearest OTM call. The general results are as follows: The prices from all three models are more or less the same for ITM calls. This is due to the fact that the intrinsic value dominates the value of an option when deep ITM, and hence most models would produce quite similar results. The at-the-money (ATM) price differences are basically

However, jump effects will in general be more visible in terms of implied volatility as time to expiration shortens (see Das and Sundaram (1999) for an investigation of term structure effects in a jump-diffusion model).

influenced by the term structure effect. Both Black76 and Bates91 use an average volatility for the whole period as input. The fact that the volatility of futures contract increases as maturity approaches, means that using an average value for the volatility will produce too high option prices for short maturity options and too low prices for long maturity options. We note that the prices from Black76 and Bates91 are in quite good agreement with each other; however, they differ quite severely from the Bates SM model. Last, the two alternative models produce significantly lower price for OTM calls than Bates SM. For the Black76 model, this fact is not surprising since OTM calls will be more valuable in a jump-diffusion world. The results from the Bates91 model deserve some explanation. We see that the parameters estimated for Bates91 give a less pronounced smile effect than Bates SM. This is because, as the volatility term structure is restricted to be flat, the jump parameters will influence both the prices across strikes, and the overall price level. From the discussion on implied volatility, the jump parameters influence both the "smile" and the level of the implied volatility curve.⁸ In Bates SM, the term structure of volatility can take care of the level, and the jump parameters can "concentrate" on "smile" effects. Hence the parameters in Bates91, through the estimation method, emerge as a compromise of the two effects.

The results provided here may be of great importance in other valuation contexts. For example, Hilliard and Reis (1999) argue that average based Asian options are popular in commodity over-the-counter (OTC) markets. They show that Asian option prices in the Black76 versus Bates91 differ even more than is the case for European/American options prices. Our results indicate, in addition to the jump effect, that Asian option prices will differ quite substantially depending on where in the life of the option the average is calculated. Especially, the relative strong maturity effect will give very different prices on Asian options depending on both the length of averaging period and how close the averaging period is to the maturity of the futures contract.

⁸ This fact may partly explain the observation reported in Hilliard and Reis (1999) that parameter values are not stable over time. In their estimation procedure, they calibrate the model each day. Using their procedure, Bates91 will be able to replicate Bates SM on one given maturity. When either the option or futures maturity changes, the parameters in Bates91 must change to capture the volatility term structure effect. Hence we would expect unstable parameters in the analysis of Hilliard and Reis (1999) if, in fact, there exists volatility term structure effects in the underlying futures data.

6 Summary and concluding comments

In this paper we have developed an option pricing model that incorporates several stylised facts reported in the literature on commodity futures price dynamics. The volatility may depend on both calendar-time and time to maturity. Furthermore, futures prices are allowed to make sudden discontinuous jumps. We estimated the parameters of the futures price dynamics by fitting our model to eleven years of wheat options data using non-linear least squares. Several models suggested in the literature are nested within our model, and they all gave significantly poorer fit compared with our more complete model formulation. In a numerical example we showed that ignoring term structure and jump effects in futures prices may lead to severe mis-pricing of options.

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References

- Anderson, R.W., 1985. Some determinants of the volatility of futures prices. *Journal of Futures Markets* 5: 331-348.
- Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. *Journal of Finance* 52: 2003-2049.
- Barone-Adesi, G., Whaley, R.E., 1987. Efficient analytic approximation of American option values. *Journal of Finance* 42: 301-320.
- Bates, D.S., 1991. The crash of '87: was it expected? the evidence from options markets. *Journal of Finance* 46: 1009-1044.
- Bates, D.S., 1995. Testing option pricing models. Working Paper 5129, National Bureau of Economic Research.
- Bates, D.S., 1996. Jumps and stochastic volatility: exchange rate processes implicit in Deutsche mark options. *Review of Financial Studies* 9: 69-107.
- Bates, D.S., 2000. Post-'87 crash fears in the SP 500 futures option markets. *Journal of Econometrics* 94: 181-238.
- Bessembinder, H., Coughenour, J.F., Seguin, P.J., Smoller, M.M., 1995. Mean reversion in equilibrium asset prices: evidence from the futures term structure. *Journal of Finance* 50: 361-375.
- Bessembinder, H., Coughenour, J.F., Seguin, P.J., Smoller, M.M., 1996. Is there a term structure of futures volatilities? reevaluating the Samuelson hypothesis. *Journal of Derivatives* Winter: 45-58.

- Black, F., 1976. The pricing of commodity contracts. *Journal of Financial Economics* 3: 167-179.
- Choi, J.W., Longstaff, F.A., 1985. Pricing options on agricultural futures: an application of the constant elasticity of variance option pricing model. *Journal of Futures Markets* 5: 247-258.
- Cox, J.C., Ross, S.A., 1976. The valuation of options for alternative stochastic processes. *Journal of Financial Economics* 3: 145-166.
- Das, S.R., Sundaram, R.K., 1999. Of smiles and smirks: a term structure perspective. *Journal of Financial and Quantitative Analysis* 34: 211-239.
- Fackler, P.L., Tian, Y., 1999. Volatility models for commodity markets. Working Paper, North Carolina State University.
- Galloway, T.M., Kolb, R.W., 1996. Futures prices and the maturity effect. *Journal of Futures Markets* 16: 809-828.
- Greene, W., 1993. *Econometric Analysis*, 2nd. ed. NJ, Prentice Hall.
- Hall, J.A., Brorsen, B.W., Irwin, S.H., 1989. The distribution of futures prices: a test of the stable paretian and mixture of normal hypothesis. *Journal of Financial and Quantitative Analysis* 24: 105-116.
- Hildreth, C., Lu, J., 1960. Demand relations with autocorrelated disturbances. *Technical Bulletin* 276, Michigan State University Agricultural Experiment Station.
- Hilliard, J.E., Reis, J.A., 1999. Jump processes in commodity futures prices and option pricing. *American Journal of Agricultural Economics* 81: 273-286.
- Hudson, M.A., Leuthold, R.M., Sarassoro, G.F., 1987. Commodity futures price changes: recent evidence on wheat, soybeans, and live cattle. *Journal of Futures Markets* 7: 287-301.
- Merton, R.C., 1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3: 125-144.
- Milonas, N.T., 1986. Price variability and the maturity effect in future markets. *Journal of Futures Markets* 6: 443-460.
- Myers, R.J., Hanson, S.D., 1993. Pricing commodity options when underlying futures price exhibits time-varying volatility. *American Journal of Agricultural Economics* 75: 121-130.
- Rutledge, D.J.S., 1976. A note on the variability of futures prices. *Review of Economics and Statistics* 58: 118-120.
- Samuelson, P.A., 1965. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review* 6: 41-49.
- Schwartz, E.S., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. *Journal of Finance* 52: 923-973.