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The Sole Proprietor's Income Shifting under the Dual Income Tax

by

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The sole proprietor's income shifting under the dual income tax

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Abstract

The dual income tax provides the sole proprietor with large incentives to participate in tax minimizing income shifting to have more of his income taxed as capital income. The Norwegian split model is designed to remove these incentives, but it contains loopholes. The present paper concludes that the split model to some extent counteracts the negative effect of technology risk on the level of real capital in the sole proprietorship. But the split model also induces the sole proprietor to over-invest in less risky real capital. In addition, the widely held corporation serves as a tax shelter for the sole proprietor. The higher the business income and the higher the difference between the marginal tax rates on labor and capital, the larger the incentives to incorporate.

JEL-classifications: H24; H25; H32.

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1 Introduction

In line with the trend in the OECD-area, the Nordic countries carried out base broadening and rate cutting tax reforms in the early nineties. By introducing the dual income tax[1] they went even further and in a different direction than previous reforms in other countries. The dual income tax separates capital income from labor income. In contrast to the global income tax, which levies one tax schedule on the sum of income from all sources, the dual income tax combines a low proportional tax on capital income with a progressive tax on other income, mostly labor income. Later Belgium, France, Italy, and Japan also introduced versions of the dual income tax and have separate tax schedules for labor income and interest income[2]. This constitutes a huge natural experiment from which lessons are to be drawn for future tax reforms.

One weakness of the dual income tax is the distributional implications of the taxation of entrepreneurs and small businesses. Income from self-employment and small businesses stems partially from return to the labor effort put in by the active owner, and partially from the return to capital invested in the firm. For medium and high income classes, there is a large difference in the marginal tax rates on capital and labor income[3], providing large incentives for income shifting from labor income to capital income in order to minimize tax payments. Owners of small businesses can easily do this by reducing their own wage payments and increase dividend payments, in order to maximize net income. In the extreme case, all individuals would start own businesses in order to participate in this tax arbitrage, which would erode the tax base. To prevent this, the Nordic countries have implemented different versions of a "split" system of dual income taxation for sole proprietors and closely held corporations. Under this split system, one part of a firm's profits is taxed as capital income and the remaining profits are taxed as labor income.

The Norwegian split model of dual income taxation applies to sole proprietorships and closely held corporations. A corporation is defined as closely held if 2/3 or more

[1] The dual income tax was introduced in Sweden in 1991, Norway 1992, and Finland 1993. The idea originated in Denmark, and was implemented in their 1985 tax reform. Later they introduced a hybrid system, mostly due to redistributive concerns.

[2] See Fuest and Weichenrieder (2002).

[3] At present, the difference in the top marginal tax rates on labor income and capital income is 37.3 percentage points in Norway, including social security contributions.

of the shares are held by active[4] owners. A corporation is defined as widely held if less than 2/3 of the shares are held by active owners, and it is then taxed according to corporate tax rules. The split model was introduced at the end of a depression, and a period of strong economic expansion followed. In the years after the tax reform, the number of sole proprietors decreased, while the number of corporations increased. Does this mean that the split model discourages entrepreneurship, or does it mean that the activity of the entrepreneurs is unchanged, while their preferred organizational form has changed[5]? Also, the share of corporations being closely held decreased from 52% in 1992, to 32% in 2000. Which factors make this type of behavior rational? The present paper studies the tax induced distortions in a small firm's investment decision and choice of organizational form in a theoretical model, and three questions are asked. First, which are the sole proprietors' incentives to invest in risky real capital under the split model? Second, which are the widely held corporations' incentives to invest in risky real capital? And third, which are the sole proprietor's determinants for incorporating? But before these questions are answered in the specific case considered in this paper, let us take a closer look at the tax literature.

The tax code's effect on the firm's choice between debt and equity, as well as the choice of whether to retain or distribute, earnings are thoroughly discussed in the literature. See for instance Gentry (1994). Different levels of corporate and personal tax rates provide private investors with incentives to use corporations as a tax shelter to save their capital income from high personal tax rates, a point highlighted by Fuest and Weichenrieder (2002).

The combination of low corporate tax rate and high personal income tax rate provides managers with incentives to relabel labor income as capital income, effectively reducing their tax on salaries, an effect identified empirically on Norwegian micro data by Fjærli and Lund (2001)[6]. But this income shifting may not be optimal if the individual has a long-term horizon. By receiving wages, he pays higher taxes, but he

[4] An owner is characterised as active if he works more than 300 hours annually in the firm. Close family members of active owners are not recognized as passive owners by the tax authorities.

[5] Slemrod (2001) states that in many cases, what appear to be real effects of tax changes are in fact only the result of creative re-labelling activity by the individuals, and this needs to be carefully considered when evaluating the effects of a tax reform.

[6] This study utilizes rich micro data from 1991, a year prior to the full implementation of the 1992 tax reform. Hence the split model does not apply here.

also becomes entitled to future pension payments from the public sector. Dividends do not entitle him to future pension. If the individual cares about his retirement, it might be optimal to pay more wages than the short-term tax minimization predicts, and Fjærli and Lund also document the presence of this effect.

There is an endogeneity of a firm's tax system: by changing organizational form the firm can experience a shift in the taxes it faces. Gravelle and Kotlikoff (1989, 1993) started a new strand of the literature on the firm's choice of organizational form following a tax reform that altered the relative tax rates on personal and corporate income. If corporate tax rates increase relative to personal tax rates, this reduces the firm's incentives to incorporate, and vice versa. Empirical support for this is presented by Goolsbee (1998), Gordon and MacKie-Mason (1990, 1994), and MacKie-Mason and Gordon (1997).

Non-tax factors also play an important role in the firm's choice of organizational form, as Ayers et al (1996) thoroughly discuss. Business risk and default risk are factors that work in favor of the corporate organizational form. The sole proprietor carries all risk himself and is personally responsible for all claims. In case of a bankruptcy he may be liable to pay damages beyond the capital he has invested in the firm. In a corporation, the individual shareholder has limited liability and may in case of a bankruptcy lose at most the capital he has invested in the firm. The higher the relative risk of the operation, the more likely the business will be organized as a corporation. Another important factor is the opportunity to raise new capital. A corporation may issue new shares and might more easily raise new capital than the self-employed entrepreneur. Also, size does matter. As firms become large, owners are more likely to hire professional managers and become less directly involved in management decisions. Similarly, the higher the number of owners in a firm, the higher the probability of conflict among them. Then conflicts may be minimized by choosing the corporate form with a more formal ownership structure. The sole proprietor has full control over the activity and strategy of his firm. This might change if he organizes as a corporation with passive shareholders who have strong opinions on how the firm should be run.

The incentives to income shifting under the dual income tax are particularly strong for smaller, often family owned firms. The different Nordic countries have different ways of solving these income shifting problems. Lindhe et al. (2002) analyze the effects of the different Nordic split models on the long-run cost of capital. They find that while in Sweden the cost of capital is the same in closely and widely held corporations, the Finnish system reduces the long-run cost of capital in closely held corporations. The effect of the Norwegian system depends on the size of the imputation rate. Öberg (2003) extends the analysis of Lindhe et al. to find how the cost of capital is affected by the source of finance under the different Nordic split models. Kari (1999) analyzes the effects of mainly the Finnish split model on the splitting of dividend income from a closely held firm into capital and earned income parts. He concludes that the distortions imposed by the split model are very sensitive to the tax system's definition of the capital base of the firm. Risk is not included in any of these three papers. Sannarnes (1995) analyzes how the Norwegian split model in the presence of risk affects the investment behavior of external investors when deciding to invest in a closely or widely held corporation. He concludes that the split model encourages more investments in the closely held corporation.

The analysis in the present paper concludes that the split model counteracts the negative effects of the risk of a technology shock on the sole proprietor's investments in firm specific real capital. It actually induces the sole proprietor to over-invest in less risky real capital. Real capital investment becomes a device for shifting income from the labor income tax base to the capital income tax base and thus reduces total tax payments of the sole proprietor. The incentives to participate in tax minimizing income shifting increase as his income increases. The net risk compensation rate under the split model is higher the higher the labor income tax rate, and thus the incentives to over-invest in firm specific real capital may increase as the labor income tax rate increases.

In addition, the widely held corporation serves as a tax shelter for high income entrepreneurs. The higher his income, and the larger the difference between the tax rates on labor income and capital income, the larger the incentives to become a widely held corporation in order to escape the split model and reduce total tax payments. Only low-income entrepreneurs have incentives to stay under the split model in order to enjoy the forwarding of negative imputed return to labor and deduct this against future positive imputed return to labor. The prediction of the model is supported by actually observed behavior of sole proprietorships after the introduction of the dual income tax and the split model in Norway in 1992.

Section 2 describes the Norwegian version of the split model of dual income taxation in detail. Section 3 presents the model, and sections 4 and 5 analyze the effect of the split model on the self-employed and the incorporated entrepreneur's investment portfolio. Section 6 compares the two organizational forms, and section 7 presents empirical evidence. Section 8 concludes.

2 The Norwegian split model

The Norwegian tax reform of 1992 implemented the dual income tax in a purer form than all the other Nordic countries. When considering how to solve the problems of a consistent tax treatment of small businesses, the split model of dual income taxation was chosen, separating income from different sources. Under the split model, an imputed return to the capital invested in the firm is calculated by multiplying the value of the capital assets[7] by a fixed rate of return on capital[8]. The imputed return to capital is taxed at the corporate rate, which equals the capital income tax rate at the individual level. Business profits net of imputed capital return[9] are the imputed return to labor, which is taxed as labor income whether the wages are actually paid to the owner or not. This reduces the possibility for the sole proprietor to classify all income as capital income in order to reduce taxes. If imputed labor income is negative, the loss does not offset other income, but may be carried forward to be deducted against future imputed labor income.

By exaggerating the capital assets of the firm, the sole proprietor achieves a reduction in the imputed labor income, and reduces his tax payments. This may be done in several ways, for instance by shifting from leased to owned[10] premises and machinery, by increasing stocks at the end of the year, by increasing and extending customers' trade receivables at the end of the year, and by financing private durable goods in the firm. Acquired good-will is very hard for the tax authorities to value, and overstating this and other parts of firm capital reduces the imputed labor in-

[7] These assets include physical business capital, acquired good-will and other intangible assets, business inventories, and credit extended to customers net of debt to the firm's supplyers.

[8] This rate of return on capital is set anually by the Parliament on the basis of the average rate of return on government bonds (5% in 2000) pluss a risk premium (5% in 2000).

[9] If the firm has employees in addition to the owner(s), a salary deduction of 12% of the wage bill from taxable wage payments applies before the return to the owner's labor effort is imputed.

[10] There is an offsetting shift of ownership regarding former owners of leased assets. Presumably there will be a clientele effect where assets are owned by sole proprietors and closely held corporations.

come. Also, by letting the firm invest in durable private consumption goods such as boats, cars, holiday homes, etc. the owner increases his consumption and reduces tax payments. Even if the increased wealth tax due on the value of capital assets is taken into account this strategy is lucrative for the sole proprietor[11]. It can even be profitable to borrow in the financial market to invest in business capital. Such debts are private and entitle the borrower to tax allowances.

But the largest loophole in the split system is probably at the margin, the question of whether a firm is subject to the split model at all. By incorporating and selling more than one-third of the shares to passive investors, firms can avoid being taxed according to the split system. The widely held corporation is free to pay its active owners as little wage and as much dividends as it likes. This technique is especially attractive for individuals in "liberal" professions, such as lawyers, medical doctors and dentists. These are typically professions with little capital required to run a business, and the imputed labor income is accordingly high. As a widely held corporation they may take out all the compensation for their own labor effort as dividends.

3 The model

For simplicity, the following analysis abstracts from many of the details discussed above. Consider a utility maximizing entrepreneurial individual who lives for two periods and who is about to start a business. He needs to decide how much to invest in real capital in the firm, which has a stochastic second period return, as well as which organizational form to choose. As a sole proprietor he is taxed under the split model. As a widely held corporation he is subject to corporate tax rules, but is required to pay a part of dividends to passive shareholders. Individuals differ in their preferences of which is the preferred organizational form. Here consider the marginal

[11] Assume that the sole propriertor increases his investments by NOK 100. At the going rate his imputed return to capital increases with NOK 10, which means that the imputed return to labor income is reduced by the same amount. Assuming that he is in the top wage income bracket, this increased investment reduces his personal taxes by NOK 5.2. The increased return to capital is subject to taxation on firm level at 28 per cent. In addition he is subject to a wealth tax of 1.1 per cent on total wealth. His taxes on firm level hence increase by NOK 3.9. Even when the increased wealth tax is taken into consideration, it still pays off to engage in this kind of income shifting.

entrepreneur who initially has no intrinsic value of either of the two organizational forms, and who chooses the organizational form that maximizes his utility.

The individual has a given time endowment in both periods, which he spends working in his firm and enjoying leisure. In order to study the individual's investment decision and the choice of organizational form separately from his labor supply decision, assume that total time spent working in the firm is given. The remaining leisure is hence also given. A change of organizational form in order to reduce tax payments is only a re-labelling of the existing nature of the sole proprietor's activity, and he puts in the same amount of labor in the two cases. But the change of organizational form could nevertheless change the return to working, since it affects the net return to entrepreneurial activity in the presence of taxes.

Expected utility. The individual's expected utility function is represented by

$$EU = u(C_1) + E\left[v\left(C_2\right)\right], \tag{1}$$

which has positive and decreasing marginal utilities of both first period consumption, C_1, and second period consumption, C_2, such that $u'(C_1) > 0$, $u''(C_1) < 0$, $v'\left(C_2\right) > 0$, and $v''\left(C_2\right) < 0$ Assume that the individual has a decreasing absolute risk aversion, such that $u'''\left(C_1\right) < 0$ and $v'''\left(C_2\right) < 0$.

The individual chooses the investment portfolio and organizational form that maximize his lifetime utility.

Investments and income. In the first period he has initial wealth Y, which he allocates to investing in risky real capital K in the firm, and saving B in the financial market. Investments in the financial market yield the exogenously given real rate of return r. Savings may be negative, and then the individual borrows in the financial market. Loans are repaid in full in the second period. The gross return to real capital investments is the sales income net of the real capital depreciation, where the depreciation rate is given by δ. In addition, there is a possibility of a technological shock that makes the firm specific real capital obsolete and reduces its second period market value. Let this be represented by the shock-related depreciation rate $\widetilde{\gamma}$, which is discussed more closely below. The net of taxes sales income depends on the tax regime and thus on the chosen organizational form. It will be specified separately in the two following sections, as will the expressions for first and second period consumption.

The entrepreneur is the only person employed in the firm, and thus labor as a production factor is fixed. The firm produces one type of product, which is sold in the second period at a given price[12] set to unity, $p = 1$. The production level X varies according to the amount of capital, K, invested in the firm, and sales income is thus given by the production function

$$X = F(K).$$

The production function has a positive and decreasing marginal product of capital; $F_K > 0$ and $F_{KK} < 0$.

Risk. There is a possibility of a technology shock that will reduce the second period sales value of the firm specific real capital. Assume that the shock never increases the value of the real capital. A technology shock will always reduce the firm's profits, and the shock-related deprecation is thus positive, $\widetilde{\gamma} > 0$. The expected value of the shock-related depreciation is also positive and given as:

$$E[\widetilde{\gamma}] = \overline{\gamma} > 0. \tag{2}$$

The individual demands a risk premium in order to invest in risky real capital in the firm. Define this risk premium as λ, and let it be the income required to compensate the individual for the relative expected second marginal utility reduction caused by the shock:

$$\lambda \equiv \frac{E[v'(C_2) \cdot \widetilde{\gamma}]}{E[v'(C_2)]} > 0. \tag{3}$$

The size of the risk premium is decided by two factors, the individual's risk aversion and the probability of a shock. This is better seen by rewriting expression (3):

$$\lambda = \frac{cov[v'(C_2), \widetilde{\gamma}]}{E[v'(C_2)]} + \overline{\gamma}. \tag{4}$$

A higher probability of a technology shock increases the expected shock-related depreciation rate, $\overline{\gamma}$, which in turn induces the individual to demand a higher risk premium in order to invest in firm specific real capital. Also, the technology shock

[12] The market demand for this good is nevertheless not given.

reduces his second period consumption. His marginal utility is positive and decreasing in the level of second period consumption. The more risk averse the individual is, the larger is the effect on the marginal utility of second period consumption. Thus the covariance of the shock parameter and the second period marginal utility is positive and higher the more risk averse the individual is, $\frac{cov[v'(C_2),\widetilde{\gamma}]}{E[v'(C_2)]} > 0$. The risk averse individual demands a risk premium higher than the expected shock-related depreciation ate, $\lambda > \overline{\gamma}$. Only if the individual were risk neutral would the risk premium be identical to the shock-related depreciation rate. In that case the marginal utility of second period consumption would be constant and the covariance with the shock-related depreciation rate would be zero.

An example can illustrate how the risk premium depends on the degree of risk aversion. Assume for a moment that the individual's utility function is quadratic in second period consumption, such that $v(C_2) = C_2 - \frac{\alpha}{2} \cdot C_2^2$. The parameter α represents the degree of risk aversion, such that the higher α is, the more risk averse is the individual. In this case the risk premium is given by $\lambda = \frac{\overline{\gamma}-\alpha \cdot E[C_2 \cdot \widetilde{\gamma}]}{1-\alpha \cdot E[C_2]}$, which depends positively on the degree of risk aversion: $\frac{\partial \lambda}{\partial \alpha} = -\frac{cov[C_2,\widetilde{\gamma}]}{(1-\alpha \cdot E[C_2])^2} > 0$.

Taxes. Let t_w be the proportional tax rate on labor income and t_k the proportional tax rate on capital income. We simplify by assuming that the tax on labor income is proportional, when it in fact is progressive in most countries, including the countries with a dual income tax. But one might think of this tax as the top marginal tax rate on labor income. The progressive labor income tax schedule is then in fact "flat on the top". Assume that the tax rate on labor income is higher than that on capital income, $t_w > t_k$. Total tax payments are given by T. No wealth tax is present in the model.

4 Sole proprietorship

Let the subscript "s" denote the previously described variables when the entrepreneur is a sole proprietor. First period consumption is given as the initial wealth net of investments:

$$C_{1,s} = Y - K_s - B_s. \tag{5}$$

The sole proprietor owns the firm and has full disposal over total sales income. His gross second period income consists of the return to his entrepreneurial investments, which are the sales income $F(K_s)$, as well as the return to his investments in the financial market and the invested capital, $[1+r]\cdot B_s$. Also, the real capital is capitalized in the second period, and the market value is reduced by both ordinary and shock-related depreciation: $[1-\delta-\widetilde{\gamma}]\cdot K_s$. Thus the net of taxes second period income is given by

$$C_{2,s} = F(K_s) + [1-\delta-\widetilde{\gamma}]\cdot K_s + [1+r]\cdot B_s - T_s.$$

The imputation rate. The sole proprietor would, if he could and ceteris paribus, have all income taxed as capital income. The tax authorities assign a part of the income as a return to the capital invested, and the residual as a return to labor, which is taxed as labor income. When assigning the part of the income to be taxed at the capital income tax rate, a return to real capital in the firm is imputed at a fixed imputation rate r_i of the total value of the firm real capital at the beginning of the period[13]. The subscript "i" refers to "imputed".

The imputation rate is set by the parliament, and it is the sum of the average return to government bonds, r, and a risk compensation factor, μ, such that $r_i = r+\mu$. The risk compensation factor acknowledges the fact that the entrepreneur takes a risk by investing in real capital in the firm and hence loses the possibility of risk diversification in the financial market. The government's risk compensation is the same for all types of firms and all types of real capital.

Tax payments and the individual's budget constraint Capital income tax is paid on the imputed return to invested capital, $[r+\mu]\cdot K_s$. Labor income tax is paid on the imputed return to labor, which is the value of the production net of production costs (which are here the ordinary and shock-related depreciation rates) and the imputed return to invested capital[14]. In addition, capital income tax

[13] When the split model was first introduced, the self-employed individual could choose whether the value at the beginning or at the end of the period should be used in the imputation of the return to firm capital. Later this changed, and at the present, the average of the values of firm capital at the beginning and at the end of the period should be used to impute the return to firm capital. The first specification is chosen for this paper.

[14] If the imputed labor income exceeds a given threshold, which in 1993 was NOK 1.25 Million, the remainder is taxed as capital income. Assume in this analysis that the imputed labor income is always below this threshold.

is paid on interest income from the investments in bonds. Total taxes due for the self-employed are thus given by

$$T_s = t_k \cdot [r+\mu] \cdot K_s + t_w \cdot \{F(K_s) - (\delta+\widetilde{\gamma}) \cdot K_s - (r+\mu) \cdot K_s\} + t_k \cdot r \cdot B_s.$$

The second period income of the sole proprietor, $C_{2,s}$, can then be written as:

$$\begin{aligned} C_{2,s} &= [1-t_w] \cdot F(K_s) - (\delta+\widetilde{\gamma}) \cdot K_s \\ &\quad + \{1+(t_w-t_k) \cdot (r+\mu)\} \cdot K_s + [1+(1-t_k) \cdot r] \cdot B_s \end{aligned} \tag{6}$$

The first part of the right hand side of (6) represents the individual's net of taxes income from his firm if all income were taxed as labor income. But the imputed return to capital is actually taxed as capital income, which increases his net income by a fraction $(t_w - t_k)$ of total imputed return to capital. The larger the difference between the marginal tax rates on labor income and capital income, the more attractive it is to participate in income shifting activities in order to have more of his income taxed as capital income. But this is only relevant if he in fact pays labor income taxes. Thus assume that the sole proprietor at least expects to have positive profits in the firm, such that

$$[F(K_s) - (\delta+\overline{\gamma}) \cdot K_s] > 0. \tag{7}$$

The individual chooses the investment portfolio that maximizes his expected utility.

4.1 The investment portfolio.

The sole proprietor's optimization problem is given by

$$\max_{K_s, B_s} EU_s = u(C_{1,s}) + E\left[v\left(C_{2,s}\right)\right]$$

where $C_{1,s}$ and $C_{2,s}$ are given by equations (5) and (6). The resulting first order conditions are given by

$$FOC_{B_s} \quad : \quad -u'(C_{1,s}) + E\left[v'(C_{2,s}) \cdot \{1+(1-t_k) \cdot r\}\right] = 0 \tag{8}$$

$$FOC_{K_s} \quad : \quad -u'(C_{1,s}) + E\left[v'(C_{2,s}) \cdot \left\{ \begin{array}{c} [1-t_w] \cdot [F_{K_s} - (\delta+\widetilde{\gamma})] \\ +[t_w-t_k] \cdot [r+\mu] + 1 \end{array} \right\}\right] = 0. \tag{9}$$

The optimal investment condition is found by combining the two first order conditions, as well as applying the definition of the risk premium λ_s :

$$F_{K_s} = r + \delta + \lambda_s - \frac{t_w - t_k}{1 - t_w} \cdot \mu. \tag{10}$$

The sole proprietor invests in real capital in the firm until the value of the marginal product equals the risk adjusted user cost of capital. The higher the risk premium the individual demands, the higher is the user cost of capital, and the lower is the optimal level of real capital investments in the firm. This effect is counteracted by the risk compensation factor, μ, which isolated considered works as a governmental subsidy on real capital investments. The total risk compensation under the split model is the relative after tax risk compensation rate, $\frac{t_w - t_k}{1-t_w} \cdot \mu$. Thus even if the risk compensation factor μ is constant over a period of time, a tax change will change the net risk compensation, and thus also the investment incentives of the sole proprietor. The net risk compensation is larger the higher the difference between the two marginal tax rates, and the higher the tax rate on labor income.

In the special case that $\lambda_s = \frac{t_w - t_k}{1-t_w} \cdot \mu$ the individual is fully compensated for the risk of investing in real capital in the firm, and he invests in real capital as he would in the absence of both risk and taxes. Then the optimal investment condition reduces to the Fisher condition, $F_{K_s} = r + \delta$. On the other hand, if $\lambda_s > \frac{t_w - t_k}{1-t_w} \cdot \mu$, the risk compensation under the split model is too small to compensate the individual for the risk he is exposed to by investing in risky real capital. But the split model still counteracts the negative effect on the level of entrepreneurial investments in the society from the risk of technology shock, and the sole proprietor invests more in real capital in the firm than in the absence of taxes. As is seen from the definition of the risk premium, equation (4), if the expected shock-related depreciation, $\overline{\gamma}$, is low, or if the individual is not very risk averse, such that $\frac{cov[v'(C_2), \widetilde{\gamma}]}{E[v'(C_2)]}$ is low, then the risk premium is lower than the net risk compensation, such that $\lambda_s < \frac{t_w - t_k}{1-t_w} \cdot \mu$. In that case the sole proprietor will use real capital investments as a means to shifting income from labor income to capital income. The split model induces the sole proprietor to over-invest in less risky types of real capital, in order to minimize tax payments. This effect is larger the less risk averse the sole proprietor is and the smaller the expected shock-related depreciation rate is.

In the absence of risk, then $\lambda_s = 0$, and the split model induces the sole proprietor to over-invest in real capital. The more capital he has, the higher is the imputed return to capital, and the larger share of his income is taxed as capital income. This tax induced over-investment is larger the higher the difference between the two tax rates, as well as the higher the risk compensation rate under the split model.

In the present model, the net risk compensation rate is constant, as long as none of the parameters are changed. This is due to the simplifying assumption of the labor income tax rate being constant. But under the dual income tax, the marginal tax rate on labor income increases as the income increases, while the capital income tax rate is constant. Thus the net risk compensation rate under the split model increases as the imputed labor income of the sole proprietor increases. This means that high income sole proprietors have higher incentives to participate in this tax minimizing income shifting by increasing real capital investments. In the context of this model, though, only one individual is considered, and the labor income tax rate is assumed to be independent of income level.

4.2 The effect of tax changes on the investment behavior.

Tax reforms change the investment incentives of the sole proprietor. Below, the effects of changes in both the labor income tax and the capital income tax rate are analyzed through comparative static analysis of the first order conditions (8) and (9). The effects of tax changes in the sole proprietor's real capital investments can be expressed as a twofold effect, both an income effect and a substitution effect. It can be shown[15] that from the assumption of decreasing absolute risk aversion it follows that the income effect is positive:

$$\frac{\partial K_s}{\partial Y} > 0.$$

4.2.1 Labor income tax.

The effect of a labor income tax increase on the level of real capital in the sole proprietorship is given by

$$\frac{\partial K_s}{\partial t_w} = -Z_{inc} \cdot \frac{\partial K_s}{\partial Y} - Z_{bus} \cdot Z_{sub} \tag{11}$$

[15] See the appendix for the formal deduction.

where

$$Z_{inc} = \frac{[F(K_s) - (\delta + r + \mu) \cdot K_s]}{[1 + (1 - t_k) \cdot r]} + [1 + (1 - t_k) \cdot r] \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{u''(C_{1,s})} \cdot K_s$$

$$Z_{bus} = \frac{u''(C_{1,s}) - [1 + (1 - t_k) \cdot r]^2 \cdot E[v''(C_{2,s})]}{D \cdot E[v'(C_{2,s})]}$$

$$Z_{sub} = \frac{1 - t_k}{1 - t_w} \cdot \mu \cdot E[v'(C_{2,s})]^2 - [1 - t_w] \cdot K_s \cdot \left\{ \begin{array}{c} E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \cdot E[v'(C_{2,s})] \\ -E[v'(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right\}$$

It is not clear what the total effect is here. Given that the imputed return to labor is positive, then $Z_{inc} > 0$, and the total income effect is negative. The full expression defined as D is found in the mathematical appendix. D is positive, and thus $Z_{bus} < 0$. The total effect of the tax change depends on whether Z_{sub} is positive or negative. It can be shown[16] that from the assumption of decreasing absolute risk aversion it follows that

$$\left\{ \begin{array}{c} E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \cdot E[v'(C_{2,s})] \\ -E[v'(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right\} > 0.$$

Thus the tax increase induces the individual to increase his real capital investments, $\frac{\partial K_s}{\partial t_w} > 0$, if

$$\frac{1 - t_k}{1 - t_w} \cdot \mu \cdot E[v'(C_{2,s})]^2 > \left\{ \begin{array}{l} [1 - t_w] \cdot K_s \\ \cdot \left[\begin{array}{c} E[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \cdot E[v'(C_{2,s})] \\ -E[v'(C_{2,s}) \cdot \widetilde{\gamma}] \cdot E[v''(C_{2,s}) \cdot \widetilde{\gamma}] \end{array} \right] \\ \\ -\frac{Z_{inc}}{Z_{bus}} \cdot \frac{\partial K_s}{\partial Y} \end{array} \right\}, \tag{12}$$

otherwise the tax increase reduces the optimal level of real capital in the sole proprietorship. The reason why the sole proprietor might want to respond to the tax increase by increasing his real capital investments is the increased private return to this tax minimizing income shifting. Everything else equal the tax change increases the net risk compensation factor under the split model, as defined in equation (10). It is more likely that condition (12) holds the larger the difference between the marginal tax rates, that is, the higher $\frac{1-t_k}{1-t_w}$, and the higher the gross risk compensation rate, μ, under the split model.

[16] See the appendix for the proof.

In the absence of risk[17] it can be shown that condition (11) reduces to

$$\frac{\partial K_s}{\partial t_w}|_{\widetilde{\gamma}=0} = -\frac{[1-t_k]\cdot\mu}{[1-t_w]^2\cdot F_{K_sK_s}} > 0,$$

and the effect of a tax increase on the level of real capital in the firm is unambiguously positive. The higher the labor income tax rate, the larger the extent of the tax induced over-investment in real capital by the sole proprietor under the split model. The risk compensation factor μ partly determines how large share of the sole proprietor's income is taxed as capital income, and the larger this factor, the stronger is the substitution effect of the tax increase.

All real capital is owned by the firm in this model, and in order to benefit from the possibility to reduce tax payments through increased investments, the entrepreneur must increase the total level of real capital in the firm. On the other hand, if parts of the real capital were leased, the entrepreneur could purchase this real capital and still have the same level of expenses, just switching from having to pay lease to paying interest on a loan. This manoeuvre would leave the level of firm real capital unchanged, and it would reduce the entrepreneur's tax payments. No wealth tax is present in this model, and in this framework the presence of a wealth tax would not alter the split-model's distortions to the investment portfolio of the entrepreneur. Increased investments in real capital mean reduced investments in financial capital and do not increase the wealth tax liability.

4.2.2 Capital income tax.

The effect of an increase in the capital income tax rate on the level of real capital in the sole proprietorship is unambiguously negative:

$$\begin{aligned}\frac{\partial K_s}{\partial t_k} &= -\left\{\frac{[r+\mu]\cdot K_s + r\cdot B_s}{1+(1-t_k)\cdot r} - r\cdot\frac{E\left[v'(C_{2,s})\right]}{u''(C_{1,s})}\right\}\cdot\frac{\partial K_s}{\partial Y} \\ &\quad +\mu\cdot E\left[v'(C_{2,s})\right]\cdot\frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]}{D}.\end{aligned} \tag{13}$$

When the capital income tax rate increases, the incentives to participate in any kind of income shifting decrease, since the difference $(t_w - t_k)$ decreases, as does the

[17] In the absence of risk $\widetilde{\gamma}=0$, $E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]=0$, and $E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]=0$.

private gain from income shifting. Also, increased capital income tax rate means a decreased net risk compensation rate under the split model. Both factors induce the sole proprietor to invest less in risky real capital in the firm. The higher the sole proprietor's capital income is, the larger share of his total income is affected by the tax increase, and the more is his net income reduced. Thus the negative effect of the tax increase on the real capital investments in the firm is stronger the higher the imputation rate $r_i = r + \mu$.

In the absence of risk condition (13) reduces to

$$\frac{\partial K_s}{\partial t_k}|_{\widetilde{\gamma}=0} = \frac{\mu}{[1-t_w] \cdot F_{K_s K_s}},$$

which is also negative.

4.3 The indirect utility function.

The investment portfolio $\left[\widehat{K}_s, \widehat{B}_s\right]$ maximizes the sole proprietor's expected utility. Thus his maximal achievable level of expected utility, $\widehat{EU}_s$, is given by the indirect utility function:

$$\widehat{EU}_s = u(\widehat{C}_{1,s}) + E\left[v'(\widehat{C}_{2,s})\right] \tag{14}$$

where

$$\widehat{C}_{1,s} = Y - \widehat{K}_s - \widehat{B}_s \tag{15}$$

and

$$\begin{aligned}\widehat{C}_{2,s} = {} & [1-t_w] \cdot \left[F(\widehat{K}_s) - (\delta + \widetilde{\gamma}) \cdot \widehat{K}_s\right] + \{1 + [t_w - t_k] \cdot [r + \mu]\} \cdot \widehat{K}_s \\ & + [1 + (1-t_k) \cdot r] \cdot \widehat{B}_s.\end{aligned} \tag{16}$$

This will be applied in the analysis of the entrepreneur's choice of organizational form.

5 The widely held corporation

The only reason for the individual to incorporate is to reduce his total tax burden by escaping the split model. A closely held corporation would still be subject to the split model, so in this context he has no incentive for choosing that organizational form. Assume thus that the alternative to being a sole proprietor is to organize as a widely held corporation with the minimum required number of passive[18] shareholders, namely 1/3. The entrepreneurial individual receives revenue from selling 1/3 of the shares in his firm to external investors. This can be modelled as a corresponding reduction in the amount of real capital investment required by the individual. The entrepreneurial individual invests 2/3 of total real capital. Assume that the passive shareholder is not more risk averse than the active shareholder, such that the risk premium required by the passive investor is equal to or less than that of the active shareholder.

All shareholders receive dividend payments as a return to their invested capital. The shareholder majority, which here means the entrepreneur, decides what wage to pay the active shareholder as a compensation for his labor effort, as well as how much to pay in dividends. Since an additional pay-roll tax applies on all wage payments made by the corporation, the total tax burden on labor income is higher under the corporate tax than under the split model. Hence it is irrational for the tax minimizing entrepreneur to receive any wages as compensation for his own labor efforts. All firm profits are paid as dividends in the second period, of which the entrepreneurial individual receives 2/3 and the passive shareholders 1/3.

The widely held corporation considered here is typically a smaller, often family owned corporation, whose objective it is to maximize the utility of the active shareholder. This is in contrast to the larger corporations listed on the stock exchange that usually are described in the optimal tax literature, whose goal it is to maximize the stock value of the corporation.

In the following, use the same variables as previously described in the paper, with the subscript "l" denoting the variables when the entrepreneur organizes as a widely held corporation.

First and second period consumption. First period consumption is given by

$$C_{1,l} = Y - \frac{2}{3} \cdot K_l - B_l \tag{17}$$

No wages are paid, and thus the net sales income is defined as firm profits, which are taxed at the corporate tax rate t_k at firm level. Then all net profits are distributed

[18] In this model all shareholders are passive, except for the entrepreneur.

tax free to the owners, of which the active shareholder receives 2/3. The firm specific real capital is capitalized in the second period, and the sales value depends on the ordinary and shock-related depreciation. In addition, the entrepreneurial individual receives the net of taxes return to his investments in the financial market. The second period consumption is given by

$$C_{2,l} = \frac{2}{3} \cdot [1 - t_k] \cdot [F(K_l) - (\delta + \widetilde{\gamma}) \cdot K_l] + \frac{2}{3} \cdot K_l + [1 + (1 - t_k) \cdot r] \cdot B_l. \quad (18)$$

5.1 The optimal investment condition.

The entrepreneur's optimization problem is given by

$$\max_{K_l, B_l} EU_l = u(C_{1,l}) + E\left[v\left(C_{2,l}\right)\right]$$

where $C_{1,l}$ and $C_{2,l}$ are given by equations (17) and (18). The resulting first order conditions are given by

$$FOC_{B_l} \ : \ -u'(C_{1,l}) + [1 + (1 - t_k) \cdot r] \cdot E\left[v'(C_{2,l})\right] = 0$$

$$FOC_{K_l} \ : \ -\frac{2}{3} \cdot u'(C_{1,l}) + E\left[v'(C_{2,l}) \cdot \left\{\frac{2}{3} \cdot [1 - t_k] \cdot [F_{K_l} - (\delta + \widetilde{\gamma})] + \frac{2}{3}\right\}\right] = 0$$

Combining the first order conditions yields the optimal investment condition:

$$F_{K_l} = r + \delta + \lambda_l \quad (19)$$

Real capital is invested in the firm until the value of the marginal product equals the risk adjusted cost of capital. As long as external investors are not more risk averse than the active shareholder, and as long as their alternative return is the interest rate r, then there will always be sufficient passive shareholders that want to invest in the firm. Everything else equal, the optimal level of real capital in the widely held corporation is lower than in the sole proprietorship. This is due to the fact that the corporation does not experience any risk compensation through the tax system, as the sole proprietor does.

The more risk averse the entrepreneur, and the higher the shock-related depreciation rate, the less real capital is invested in the firm. Taxes have an indirect effect on the level of real capital in the widely held corporation since only the risk premium is affected through taxes. The extent to which the capital income tax affects the investment level in the firm is studied in detail below. Labor income tax changes have no effect on the investment behavior of the firm, since no wages are paid.

5.2 Effect of increased capital income tax rate

Again, it can be shown that from the assumption on decreasing absolute risk aversion it follows that the income effect is positive:

$$\frac{\partial K_l}{\partial Y} > 0.$$

Now the effect of the level of real capital investments in the widely held corporation can be expressed as the sum of an income and a substitution effect:

$$\frac{\partial K_l}{\partial t_k} = -H_{inc} \cdot \frac{\partial K_l}{\partial Y} - H_{sub}$$

where

$$\begin{aligned} H_{inc} &= [1 + (1 - t_k) \cdot r] \cdot \frac{2}{3} \cdot K_l \cdot \frac{E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right]}{u''(C_{1,l})} \\ &\quad + \frac{\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l}{1 + (1 - t_k) \cdot r} - r \cdot \frac{E\left[v'(C_{2,l})\right]}{u''(C_{1,l})} \end{aligned}$$

$$\begin{aligned} H_{sub} &= \left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ -E\left[v'(C_{2,l})\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ &\quad \cdot \left(\frac{2}{3}\right)^2 \cdot \frac{[1 - t_k] \cdot K_l}{E\left[v'(C_{2,l})\right]} \cdot \frac{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]}{F} \end{aligned}$$

Clearly, H_{inc} is positive, and the total income effect of a tax change is negative. It can be shown that from the assumption of decreasing relative risk aversion it follows that

$$\left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ -E\left[v'(C_{2,l})\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} < 0.$$

The expression defined as F is positive, which means that H_{sub} is negative. Thus an increase in the capital income tax rate reduces the optimal level of firm specific real capital in the widely held corporation.

In the absence of risk, the optimal investment condition reduces to the Fisher condition, and tax changes have no effect on the investment decision in the widely held corporation.

5.3 The indirect utility function.

The indirect utility function of the individual when his firm is organized as a widely held corporation is given by

$$\widehat{EU}_l = u(\widehat{C}_{1,l}) + E\left[v(\widehat{C}_{2,l})\right] \tag{22}$$

where

$$\widehat{C}_{1,l} = Y - \frac{2}{3} \cdot \widehat{K}_l - \widehat{B}_l \tag{23}$$

$$\widehat{C}_{2,l} = \frac{2}{3} \cdot [1 - t_k] \cdot \left[F(\widehat{K}_l) - (\delta + \widetilde{\gamma}) \cdot \widehat{K}_l\right] + \frac{2}{3} \cdot \widehat{K}_l + [1 + (1 - t_k) \cdot r] \cdot \widehat{B}_l \tag{24}$$

This will be used in the analysis of which organizational form to choose.

6 When to incorporate?

The only reason for the sole proprietor to incorporate is assumed to be to reduce tax payments. Under the split model, a part of the firm's income is taxed as labor income, whereas all firm income is taxed as capital income under the corporate tax schedule. But in order to be taxed as a widely held corporation, at least 1/3 of profits must be paid as dividends to passive shareholders. Only if the sole proprietor has positive imputed personal income has he incentives to incorporate. Thus assume that the expected imputed personal income of the sole proprietor is positive after he has exhausted the income shifting possibilities inherent in the split model through real capital investments:

$$F(\widehat{K}_s) - (\delta + r + \mu + \overline{\gamma}) \cdot \widehat{K}_s > 0. \tag{25}$$

For simplicity, let the costs[19] of incorporating be zero. The sole proprietor incorporates if he achieves the higher maximum achievable expected utility as a widely held corporation:

$$\text{Incorporate if} \quad \widehat{EU}_l - \widehat{EU}_s > 0,$$

where $\widehat{EU}_l$ is defined by the equations (22)-(24) and $\widehat{EU}_l$ is defined by the equations (14)-(16). The larger this difference, the higher the incentives to incorporate in order to reduce total tax payments.

[19] This is a simplifying assumption. Still, the actual costs of organizing as a corporation are moderate, and the process is also not that complicated. But corporations are subject to stricter regulations than sole proprietors. For instance, they are obliged to have an accountant.

By applying the envelope theorem, let us now study how policy changes affect the incentives to incorporate.

The labor income tax rate. The effect on the incentives to incorporate by an increase in the labor income tax rate is given by:

$$\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial t_w}=E\left[v'(\widehat{C}_{2,s})\right]\cdot\left\{F(\widehat{K}_s)-\left(\delta+r+\mu+\widehat{\lambda}_s\right)\cdot\widehat{K}_s\right\}. \tag{26}$$

It is already assumed that the individual must expect to have a positive imputed labor income in order to even consider incorporating. From the definition of the risk compensating factor, condition (3), it follows that (26) is positive if the expected imputed return to labor income at least covers the shock part of the risk compensation. That is, the tax change increases the incentives to incorporate if $\left[F(\widehat{K}_s)-(\delta+r+\mu+\overline{\gamma})\cdot\widehat{K}_s>\frac{cov\left[v'(\widehat{C}_2),\widetilde{\gamma}\right]}{E\left[v'(\widehat{C}_2)\right]}\cdot\widehat{K}_s\right]$. The higher the expected imputed return to labor income, the larger are the incentives to incorporate. The factor working against this is the fact that the net risk compensation factor under the split model actually increases when the labor income tax rate increases.

The capital income tax rate. The effect of an increase in the capital income tax rate on the incentives to incorporate is given by

$$\begin{aligned}\frac{\partial\left(\widehat{EU}_l-\widehat{EU}_s\right)}{\partial t_k} &= E\left[v'(\widehat{C}_{2,s})\right]\cdot\left\{(r+\mu)\cdot\widehat{K}_s+r\cdot\widehat{B}_s\right\}\\ &\quad -E\left[v'(\widehat{C}_{2,l})\right]\cdot\left\{\frac{2}{3}\cdot\left[F(\widehat{K}_l)-\left(\delta+\widehat{\lambda}_l\right)\cdot\widehat{K}_l\right]+r\cdot\widehat{B}_l\right\},\end{aligned}$$

which most likely is negative. The reason for this is twofold. First, the overall incentives for participating in tax minimizing income shifting decrease when the difference between the marginal tax rates on labor and capital decrease. Second, all income of the entrepreneur is affected by the tax increase when he is organized as a widely held corporation, while only part of the sole proprietor's income is affected by the tax increase.

The risk compensation factor. An increase in the risk compensation factor under the split model actually reduces the incentives for the sole proprietor to incorporate, as is seen in the below expression:

$$\frac{\partial\left(\widehat{EU}_l - \widehat{EU}_s\right)}{\partial\mu} = -E\left[v'(\widehat{C}_{2,s})\right]\cdot[t_w - t_k]\cdot\widehat{K}_s < 0$$

The higher the risk compensation factor, the more of the sole proprietor's income is taxed as capital income, and it is less attractive to incorporate in order to avoid the split model.

7 Empirical observations.

High-income self-employed entrepreneurs are subject to the top marginal tax rate on the imputed return to labor, and these are expected to take advantage of the income shifting possibilities through increased real capital stock. And in fact the Norwegian sole proprietors in the top decile of the income distribution more than doubled the value of their real capital from 1992 to 2000[20], as figure 1 shows. These are aggregate data, and it is not possible to see whether there has been a shift in the type of real capital investments. Unfortunately, there are no available data prior to the 1992-tax reform. Still, it ought to take the firm some time to adjust its investment decision to the new tax rules. As new sole proprietors reach the top marginal tax bracket on labor income, they adapt to the tax minimizing incentives inherent in the split model. Hence one would expect a development towards more real capital in this group over time, rather than a shift to a new investment level directly after the tax reform.

1992 marked the end of an economic depression in Norway and was followed by a period of strong economic expansion. This would spur increased investments independent of the tax regime. But then the rate of real capital per unit of business income ought to be more or less constant. As seen in figure 2, this is not the case. The high-income entrepreneurs still increased their share of real capital per unit of business income more than the average in non-primary sectors.

The number of sole proprietors decreased during the 1990's, while the total number of corporations increased by more than the same amount, as is seen in figure

[20] Calculations made on combined survey and register data from Statistics Norway. Annual sample of ca. 4000, but weighted for representability. The primary sector is heavily regulated and subsidized, self-employed in this sector are excluded from the sample.

Figure 1: *The value of sole proprietorships' real capital in 1998-prices.*

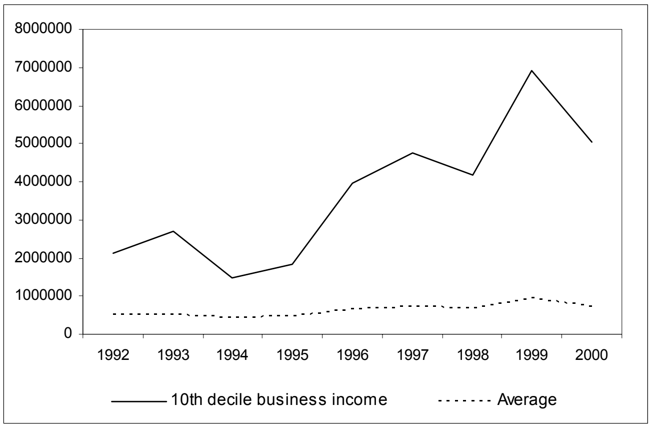


Figure 2: *Units of firm capital per unit of business income of the sole proprietorships in 1998-prices.*

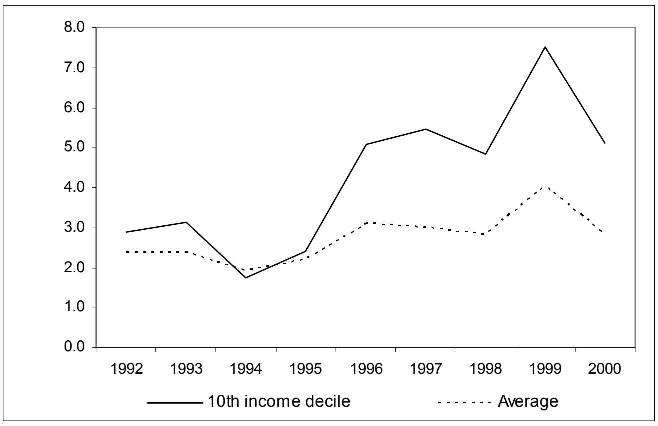
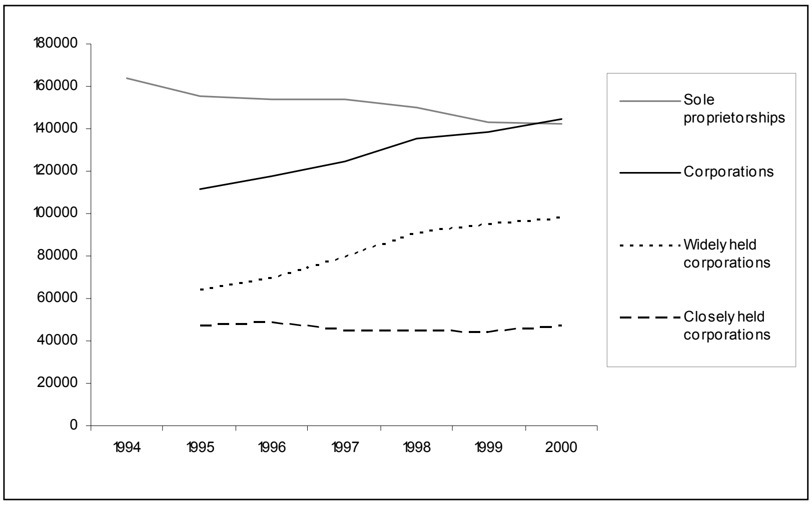


Figure 3: *Number of self-employed individuals and corporations.*



3[21]. Even if part of the decline of sole proprietors is due to a reduction of the primary sector, mostly farming, there was also a reduction in other sectors. At the same time there was a reduction in the number of closely held corporations, as well as an increase in widely held corporations. A strong selection also took place. The closely held corporations mostly have negative imputed return to labor, and their active owners hence do not pay labor income taxes. In 1992, 65% of the closely held corporations had negative imputed return to labor, while this share had increased to 80% in 2000. Only 3.5 % of all closely held corporations had positive imputed return to labor in 2000. Also, in 1995, 28% of all one-man corporations were closely held, and already two years later this share had fallen to 20%.

This can be interpreted as an indication of a tax induced shift in organizational form and choice of tax regime. Sole proprietors incorporate in order to escape the split model, and corporations choose to be widely held in order to escape the split model. Only corporations with low profits and thus also low or negative imputed return to labor stay under the split model.

[21] Source: Statistics Norway.
Data are unfortunatelly not available for the whole time period in question.

8 Conclusions.

The above analysis concludes that the split model counteracts the negative effects of the risk of a technology shock on the sole proprietor's investments in firm specific real capital, and it encourages more real capital investments than in the absence of taxes. The split model might actually induce the sole proprietor to over-invest in less risky real capital. Real capital investments are a device for shifting income from the labor income tax base to the capital income tax base in order to reduce the sole proprietor's total tax payments. The incentives to participate in tax minimizing income shifting increase as his income increases. The net risk compensation rate under the split model is higher the higher the labor income tax rate, and thus the incentives to over-invest in firm specific real capital may increase as the labor income tax rate increases.

In addition, the widely held corporation serves as a tax shelter for high income sole proprietors. The higher his income, and the larger the difference between the tax rates on labor income and capital income, the larger the incentives to become a widely held corporation in order to escape the split model and reduce total tax payments. Only low-income entrepreneurs have incentives to stay under the split model in order to deduct the negative imputed labor income against future positive imputed return to labor.

The predictions of the model are supported by actually observed behavior of sole proprietorships after the introduction of the dual income tax and the split model in Norway in 1992.

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10 Mathematical appendix

10.1 Conditions for the existence of a local maximum for the sole proprietor:

$$\begin{aligned} 1) &: EU_{BB} < 0 \\ 2) &: EU_{KK} < 0 \\ 3) &: EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 > 0 \end{aligned}$$

From equation (8) it follows that

$$EU_{BB} = u''(C_{1,s}) + \{1 + (1 - t_k) \cdot r\}^2 \cdot E\left[v''(C_{2,s})\right] < 0.$$

From equation (9) it follows that

$$\begin{aligned} EU_{KK} &= u''(C_{1,s}) + [1 - t_w] \cdot F_{K_s K_s} \cdot E\left[v'(C_{2,s})\right] + A^2 \cdot E\left[v''(C_{2,s})\right] \\ &\quad -2 \cdot A \cdot [1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] + [1 - t_w]^2 \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \\ &< 0 \end{aligned}$$

where

$$A \equiv [1 - t_w] \cdot [F_{K_s} - \delta] + 1 + [t_w - t_k] \cdot [r + \mu]$$

Also, from equation (8) it follows that

$$EU_{BK} = u''(C_{1,s}) + [1 + (1 - t_k) \cdot r] \cdot \{A \cdot E\left[v''(C_{2,s})\right] - [1 - t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right]\}$$

Define

$$\begin{aligned} D &\equiv EU_{BB} \cdot EU_{KK} - (EU_{BK})^2 > 0 \\ &\Downarrow \end{aligned}$$

$$
\begin{aligned}
D &= \left\{u''(C_{1,s}) + \{1+(1-t_k)\cdot r\}^2 \cdot E\left[v''(C_{2,s})\right]\right\} \qquad (27)\\
&\cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] + A^2\cdot E\left[v''(C_{2,s})\right] \\ -2\cdot A\cdot[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] + [1-t_w]^2\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array}\right\}\\
&- \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\}\right\}^2\\
&> 0
\end{aligned}
$$

10.2 The income effect of the sole proprietor.

From the first order condition (9) it follows that

$$
\begin{aligned}
& u''(C_{1,s})\cdot\left[\frac{\partial K_s}{\partial Y}+\frac{\partial B_s}{\partial Y}-1\right] + [1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right]\\
& +A\cdot E\left[v''(C_{2,s})\cdot\left\{\begin{array}{c} [1-t_w]\cdot(F_{K_s}-\delta-\widetilde{\gamma})\cdot\frac{\partial K_s}{\partial Y} \\ +(1+[t_w-t_k]\cdot[r+\mu])\cdot\frac{\partial K_s}{\partial Y} \\ +[1+(1-t_k)\cdot r]\cdot\frac{\partial B_s}{\partial Y}\end{array}\right\}\right]\\
& -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\left\{\begin{array}{c} [1-t_w]\cdot(F_{K_s}-\delta-\widetilde{\gamma})\cdot\frac{\partial K_s}{\partial Y} \\ +(1+[t_w-t_k]\cdot[r+\mu])\cdot\frac{\partial K_s}{\partial Y} \\ +[1+(1-t_k)\cdot r]\cdot\frac{\partial B_s}{\partial Y}\end{array}\right\}\right]\\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
&\Downarrow \qquad (28)\\
&\frac{\partial K_s}{\partial Y}\cdot\left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] + A^2\cdot E\left[v''(C_{2,s})\right] \\ -2\cdot A\cdot[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] + [1-t_w]^2\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array}\right\}\\
&+\frac{\partial B_s}{\partial Y}\cdot\left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\}\right\}\\
&= u''(C_{1,s})
\end{aligned}
$$

And from the first order condition (8) it follows that

$$
\begin{aligned}
& u''(C_{1,s})\cdot\left[\frac{\partial K_s}{\partial Y}+\frac{\partial B_s}{\partial Y}-1\right]\\
& +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,s})\cdot\left\{\begin{array}{c} [1-t_w]\cdot(F_{K_s}-\delta-\widetilde{\gamma})\cdot\frac{\partial K_s}{\partial Y} \\ +(1+[t_w-t_k]\cdot[r+\mu])\cdot\frac{\partial K_s}{\partial Y} \\ +[1+(1-t_k)\cdot r]\cdot\frac{\partial B_s}{\partial Y}\end{array}\right\}\right]\\
&= 0
\end{aligned}
$$

$$\Downarrow \tag{29}$$

$$\begin{aligned}
&\frac{\partial K_s}{\partial Y}\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\} \\
&+\frac{\partial B_s}{\partial Y}\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\} \\
= \;& u''(C_{1,s})
\end{aligned}$$

Apply the investment condition (10) to find an alternative expression for A and use this below:

$$\begin{aligned}
A &= [1+(1-t_k)\cdot r]+[1-t_w]\cdot\lambda_s \\
&= [1+(1-t_k)\cdot r]+[1-t_w]\cdot\frac{E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v'(C_{2,s})\right]}
\end{aligned}$$

By Cramer's rule, equations (28) and (29) yield:

$$\frac{\partial K_s}{\partial Y} = -\frac{u''(C_{1,s})}{D}\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1+(1-t_k)\cdot r] \\ \cdot\left\{\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\} \\ -\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\}\end{array}\right\}$$

$$\Downarrow$$

$$\frac{\partial K_s}{\partial Y} = -\frac{u''(C_{1,s})}{D}\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1+(1-t_k)\cdot r] \\ \cdot\left\{\begin{array}{c} [1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,s})\right] \\ +[1-t_w]\cdot\frac{E[v'(C_{2,s})\cdot\widetilde{\gamma}]}{E[v'(C_{2,s})]}\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\} \\ -u''(C_{1,s})-[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\end{array}\right\}$$

$$\Downarrow$$

$$\frac{\partial K_s}{\partial Y} = \frac{u''(C_{1,s})\cdot[1+(1-t_k)\cdot r]\cdot[1-t_w]}{D\cdot E\left[v'(C_{2,s})\right]}\cdot\left\{\begin{array}{c} E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ \\ -E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]\end{array}\right\}$$

As $\frac{u''(C_{1,s})\cdot[1+(1-t_k)\cdot r]\cdot[1-t_w]}{D\cdot E[v'(C_{2,s})]} < 0$, the sign of the income effect is determined by the covariance-expressions in the parenthesis. Thus

$$\frac{\partial K_s}{\partial Y} > 0 \;\text{ if }\; E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] < E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right] \tag{30}$$

By using the the following two covariances:

$$cov\left\{v'(C_{2,s})\cdot\widetilde{\gamma},\quad v''(C_{2,s})\right\}=E\left[v'(C_{2,s})\cdot v''(C_{2,s})\cdot\widetilde{\gamma}\right]-E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\right]$$

and

$$cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma},\quad v'(C_{2,s})\right\}=E\left[v'(C_{2,s})\cdot v''(C_{2,s})\cdot\widetilde{\gamma}\right]-E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right].$$

condition (30) can be rewritten as

$$\frac{\partial K_s}{\partial Y}>0\text{ if }cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma},\quad v'(C_{2,s})\right\}>cov\left\{v'(C_{2,s})\cdot\widetilde{\gamma},\quad v''(C_{2,s})\right\}$$

The individual is assumed to have decreasing absolute risk aversion, such that $v'''(C_{2,s})>0$. Then $cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma},\quad v'(C_{2,s})\right\}>0$ and $cov\left\{v'(C_{2,s})\cdot\widetilde{\gamma},\quad v''(C_{2,s})\right\}<0$. Hence the above condition is met, and the income effect is positive:

$$\frac{\partial K_s}{\partial Y}>0.$$

10.2.1 The effect on the investment portfolio and risk profile of the SP by changed tax on labor income.

From equation (8) we find that:

$$\begin{aligned}&-u''(C_{1,s})\cdot\{-K'(t_w)-B'(t_w)\}\\&+\{1+(1-t_k)\cdot r\}\cdot E\left[v''(C_{2,s})\cdot\frac{\partial C_{2,s}}{\partial t_w}\right]\\=&\ 0\end{aligned}$$

$$\Downarrow \tag{31}$$

$$\begin{aligned}&K'(t_w)\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]\cdot\left[\begin{array}{c}A\cdot E\left[v''(C_{2,s})\right]\\-[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right]\right\}\\&+B'(t_w)\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\}\\=&\ [1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c}[F(K_s)-(\delta+r+\mu)\cdot K_s]\cdot E\left[v''(C_{2,s})\right]\\-K_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{aligned}$$

Next, condition (9) is differentiated:

$$
\begin{aligned}
& u''(C_{1,s}) \cdot \{K'(t_w) + B'(t_w)\} \\
& + \{-[F_{K_s} - \delta] + [1-t_w] \cdot F_{K_sK_s} \cdot K'(t_w) + r + \mu\} \cdot E\left[v'(C_{2,s})\right] \\
& + A \cdot E\left[v''(C_{2,s}) \cdot \frac{\partial C_{2,s}}{\partial t_w}\right] \\
& + E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \\
& - [1-t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \frac{\partial C_{2,s}}{\partial t_w}\right] \\
= \; & 0
\end{aligned}
$$

$$
\Downarrow \tag{32}
$$

$$
\begin{aligned}
& K'(t_w) \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1-t_w] \cdot F_{K_sK_s} \cdot E\left[v'(C_{2,s})\right] \\ +A^2 \cdot E\left[v''(C_{2,s})\right] - 2 \cdot [1-t_w] \cdot A \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ + [1-t_w]^2 \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\} \\
& + B'(t_w) \cdot \left\{ u''(C_{1,s}) + [1 + (1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} A \cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \right\} \\
= & \left\{ \begin{array}{c} A \cdot [F(K_s) - (\delta + r + \mu) \cdot K_s] \cdot E\left[v''(C_{2,s})\right] \\ + [F_{K_s} - (\delta + r + \mu)] \cdot E\left[v'(C_{2,s})\right] \\ -A \cdot K_s \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] - E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ -[1-t_w] \cdot [F(K_s) - (\delta + r + \mu) \cdot K_s] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \\ + [1-t_w] \cdot K_s \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \end{array} \right\}
\end{aligned}
$$

By Cramer's rule equations (31) and (32) yield:

$$
K'(t_w) = \frac{1}{-D} \cdot (b_B \cdot a_{KB} - b_k \cdot a_{BB}) \tag{33}
$$

where

$$
\begin{aligned}
b_B \cdot a_{KB} = & \left\{ u''(C_{1,s}) + \left[\begin{array}{c} [1 + (1-t_k) \cdot r] \cdot \left\{ A - [1-t_w] \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v''(C_{2,s})]} \right\} \\ \cdot E\left[v''(C_{2,s})\right] \end{array} \right] \right\} \\
& \cdot [1 + (1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} [F(K_s) - (\delta + r + \mu) \cdot K_s] \\ -K_s \cdot \frac{E[v''(C_{2,s}) \cdot \widetilde{\gamma}]}{E[v''(C_{2,s})]} \end{array} \right\} \cdot E\left[v''(C_{2,s})\right]
\end{aligned}
$$

$$\Downarrow \tag{34}$$

$$\begin{aligned} b_B \cdot a_{KB} &= \left\{ \frac{u''(C_{1,s})}{E\left[v''(C_{2,s})\right]} + \left[1+(1-t_k)\cdot r\right]^2 \right\} \cdot \left[1+(1-t_k)\cdot r\right] \\ &\cdot \left[F(K_s) - \left(\delta + r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \cdot E\left[v''(C_{2,s})\right]^2 \\ &- \frac{D\cdot E\left[v''(C_{2,s})\right]}{u''(C_{1,s})} \cdot \frac{\partial K_s}{\partial Y} \cdot \left[1+(1-t_k)\cdot r\right] \\ &\cdot \left[F(K_s) - \left(\delta + r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \end{aligned}$$

Next,

$$\begin{aligned} b_k \cdot a_{BB} &= \\ & \left\{ \begin{array}{c} A\cdot\left[F(K_s) - (\delta+r+\mu)\cdot K_s\right]\cdot E\left[v''(C_{2,s})\right] \\ +\left[F_{K_s} - (\delta+r+\mu)\right]\cdot E\left[v'(C_{2,s})\right] \\ -A\cdot K_s \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E\left[v''(C_{2,s})\right] \\ -\frac{E[v'(C_{2,s})\cdot\widetilde{\gamma}]}{E[v'(C_{2,s})]} \cdot E\left[v'(C_{2,s})\right] \\ -\left[1-t_w\right]\cdot\left[F(K_s) - (\delta+r+\mu)\cdot K_s\right] \\ \cdot\frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \cdot E\left[v''(C_{2,s})\right] \\ \\ +\left[1-t_w\right]\cdot K_s \cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array} \right\} \\ & \cdot \left\{ u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \end{aligned}$$

$$\Downarrow \qquad F_{K_s} - \delta - r - \mu - \lambda_s = -\mu - \frac{t_w - t_k}{1-t_w}\cdot\mu = -\frac{1-t_k}{1-t_w}\cdot\mu$$

$$\begin{aligned} b_k \cdot a_{BB} &= \left\{ u''(C_{1,s}) + \left[1+(1-t_k)\cdot r\right]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \\ &\cdot \left\{ \begin{array}{c} -\frac{D}{u''(C_{1,s})\cdot[1+(1-t_k)\cdot r]} \cdot \frac{\partial K_s}{\partial Y} \cdot \left[F(K_s) - (\delta+r+\mu)\cdot K_s\right] \\ +\left[1+(1-t_k)\cdot r\right]\cdot\left[F(K_s) - \left(\delta+r+\mu+\frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]}\right)\cdot K_s\right] \\ \cdot E\left[v''(C_{2,s})\right] \\ \\ -\frac{1-t_k}{1-t_w}\cdot\mu\cdot E\left[v'(C_{2,s})\right] \\ +\left[1-t_w\right]\cdot K_s\cdot\left\{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] - \lambda_s\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\right\} \end{array} \right\} \end{aligned} \tag{35}$$

This yields

$$
\begin{aligned}
K'(t_w) &= \frac{1}{-D} \cdot (b_B \cdot a_{KB} - b_k \cdot a_{BB}) \\
&= -\frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{u''(C_{1,s}) \cdot [1+(1-t_k)\cdot r]} \\
&\quad \cdot \frac{\partial K_s}{\partial Y} \cdot [F(K_s) - (\delta + r + \mu)\cdot K_s] \\
&\quad + \frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{D} \\
&\quad \cdot \left\{ \begin{array}{l} [1+(1-t_k)\cdot r] \cdot E\left[v''(C_{2,s})\right] \\ \cdot \left[F(K_s) - \left(\begin{array}{c} \delta + r + \mu \\ + \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{E[v''(C_{2,s})]} \end{array} \right) \cdot K_s \right] \\ -\frac{1-t_k}{1-t_w} \cdot \mu \cdot E\left[v'(C_{2,s})\right] \\ + [1-t_w] \cdot K_s \cdot \left\{ E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] - \lambda_s \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \right\} \end{array} \right\} \\
&\quad - \frac{1}{D} \cdot \left\{ u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right] \right\} \cdot E\left[v''(C_{2,s})\right] \\
&\quad \cdot [1+(1-t_k)\cdot r] \cdot \left[F(K_s) - \left(\delta + r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right] \\
&\quad + \frac{E\left[v''(C_{2,s})\right]}{u''(C_{1,s})} \cdot \frac{\partial K_s}{\partial Y} \cdot [1+(1-t_k)\cdot r] \\
&\quad \cdot \left[F(K_s) - \left(\delta + r + \mu + \frac{E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,s})\right]} \right) \cdot K_s \right]
\end{aligned}
$$

$$
\begin{aligned}
K'(t_w) &= -\left\{ \begin{array}{c} \frac{F(K_s) - (\delta + r + \mu)\cdot K_s}{[1+(1-t_k)\cdot r]} \\ + [1+(1-t_k)\cdot r] \cdot \frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{u''(C_{1,s})} \cdot K_s \end{array} \right\} \cdot \frac{\partial K_s}{\partial Y} \\
&\quad - \frac{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]}{[1-t_w] \cdot D \cdot E\left[v'(C_{2,s})\right]} \\
&\quad \cdot \left\{ \begin{array}{l} [1-t_k] \cdot \mu \cdot E\left[v'(C_{2,s})\right]^2 \\ -[1-t_w]^2 \cdot K_s \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,s}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,s})\right] \\ -E\left[v'(C_{2,s}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,s}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}
\end{aligned}
$$

As $\frac{\partial K_s}{\partial Y} > 0$, $\frac{E[v''(C_{2,s})\cdot\widetilde{\gamma}]}{u''(C_{1,s})} > 0$, and it from the assumption follows that $F(K_s) - (\delta + r + \mu) \cdot K_s > 0$,then the total income effect of the tax increase is negative. Also,

$\frac{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E[v''(C_{2,s})]}{[1-t_w]\cdot D\cdot E[v'(C_{2,s})]} < 0$, such that the total effect of the tax change depends on whether the last parenthesis is positive or negative. First, consider the following covariances.

$$\begin{aligned} cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma},\;\; v'(C_{2,s})\right\} &= E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\cdot v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ &\quad -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \end{aligned}$$

and

$$\begin{aligned} cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma},\;\; v'(C_{2,s})\cdot\widetilde{\gamma}\right\} &= E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\cdot v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ &\quad -E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]. \end{aligned}$$

Apply these to rewrite the following:

$$\begin{aligned} & E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right]-E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \qquad (36) \\ = \;& cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma},\;\; v'(C_{2,s})\cdot\widetilde{\gamma}\right\}-cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma},\;\; v'(C_{2,s})\right\} \end{aligned}$$

The individual is assumed to have decreasing absolute risk aversion, such that $v'''(C_{2,s}) > 0$. Then $cov\left\{v''(C_{2,s})\cdot\widetilde{\gamma},\;\; v'(C_{2,s})\cdot\widetilde{\gamma}\right\} > 0$ and $cov\left\{v'(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma},\;\; v''(C_{2,s})\right\} < 0$. Thus the expression (36) is positive, and the substitution effect is positive if

$$[1-t_k]\cdot\mu\cdot E\left[v'(C_{2,s})\right]^2 > [1-t_w]^2\cdot K_s\cdot\left\{\begin{array}{c} E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,s})\right] \\ -E\left[v'(C_{2,s})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\}$$

Effect of increased labor income tax rate in the absence of risk, $\widetilde{\gamma}=0$, and $E\left[v(C_{2,s})\right]=v(C_{2,s})$.

$$K'(t_w)|_{\widetilde{\gamma}=0} = -\left\{u''(C_{1,s})+v''(C_{2,s})\cdot[1+(1-t_k)\cdot r]^2\right\}\cdot\frac{\frac{1-t_k}{1-t_w}\cdot\mu\cdot v'(C_{2,s})}{D|_{\widetilde{\gamma}=0}}$$

where

$$D|_{\widetilde{\gamma}=0} = \left\{u''(C_{1,s})+\{1+(1-t_k)\cdot r\}^2\cdot v''(C_{2,s})\right\}\cdot[1-t_w]\cdot F_{K_sK_s}\cdot v'(C_{2,s})$$

$$\Downarrow$$

$$K'(t_w)|_{\widetilde{\gamma}=0} = -\frac{[1-t_k]\cdot\mu}{[1-t_w]^2\cdot F_{K_sK_s}} > 0$$

10.2.2 The effect on the investment portfolio and risk profile of the SP by changed tax on capital income.

Differentiating the first order condition (8) yields

$$\begin{aligned} & -u''(C_{1,s})\cdot\{-K'(t_k)-B'(t_k)\}-r\cdot E\left[v'(C_{2,s})\right] \\ & +\{1+(1-t_k)\cdot r\}\cdot E\left[v''(C_{2,s})\cdot\frac{\partial C_{2,s}}{\partial t_k}\right] \\ = & \ 0 \end{aligned}$$

$$\Downarrow \tag{37}$$

$$\begin{aligned} & K'(t_k)\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]\cdot\left[\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right]\right\} \\ & +B'(t_k)\cdot\left\{u''(C_{1,s})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,s})\right]\right\} \\ = & \ r\cdot E\left[v'(C_{2,s})\right]+[1+(1-t_k)\cdot r]\cdot\{[r+\mu]\cdot K_s+r\cdot B_s\}\cdot E\left[v''(C_{2,s})\right] \end{aligned}$$

Next, condition (9) is differentiated:

$$\begin{aligned} & -u''(C_{1,s})\cdot\{-K'(t_k)-B'(t_k)\} \\ & +\frac{\partial A}{\partial t_k}\cdot E\left[v'(C_{2,s})\right]+A\cdot E\left[v''(C_{2,s})\cdot\frac{\partial C_{2,s}}{\partial t_k}\right] \\ & -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\frac{\partial C_{2,s}}{\partial t_k}\right] \\ = & \ 0 \end{aligned}$$

$$\Downarrow \tag{38}$$

$$\begin{aligned} & K'(t_k)\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1-t_w]\cdot F_{K_sK_s}\cdot E\left[v'(C_{2,s})\right] \\ +A^2\cdot E\left[v''(C_{2,s})\right]-2\cdot A\cdot[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \\ +[1-t_w]^2\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array}\right\} \\ & B'(t_k)\cdot\left\{\begin{array}{c} u''(C_{1,s})+[1+(1-t_k)\cdot r] \\ \cdot\left\{\begin{array}{c} A\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\} \end{array}\right\} \\ = & \left\{\begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right]+A\cdot([r+\mu]\cdot K_s+r\cdot B_s)\cdot E\left[v''(C_{2,s})\right] \\ -[1-t_w]\cdot([r+\mu]\cdot K_s+r\cdot B_s)\cdot E\left[v''(C_{2,s})\cdot\widetilde{\gamma}\right] \end{array}\right\} \end{aligned}$$

By applying Cramer's rule, equations (37) and (38) yield:

$$K'(t_k) = \frac{h_B \cdot x_{KB} - h_K \cdot x_{BB}}{-D}$$

where

$$\begin{aligned}
h_B \cdot x_{KB} &= \{r \cdot E[v'(C_{2,s})] + [1+(1-t_k)\cdot r] \cdot \{[r+\mu]\cdot K_s + r \cdot B_s\} \cdot E[v''(C_{2,s})]\} \\
&\quad \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1+(1-t_k)\cdot r] \cdot E[v''(C_{2,s})] \\ \cdot \{[1+(1-t_k)\cdot r] + [1-t_w]\cdot[\lambda_s - \sigma_s]\} \end{array} \right\} \\
&= \{r \cdot E[v'(C_{2,s})] + [1+(1-t_k)\cdot r] \cdot \{[r+\mu]\cdot K_s + r \cdot B_s\} \cdot E[v''(C_{2,s})]\} \\
&\quad \cdot \left\{ \begin{array}{c} u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E[v''(C_{2,s})] \\ +[1+(1-t_k)\cdot r]\cdot[1-t_w]\cdot[\lambda_s - \sigma_s]\cdot E[v''(C_{2,s})] \end{array} \right\}
\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}
h_B \cdot x_{KB} &= \{r \cdot E[v'(C_{2,s})] + [1+(1-t_k)\cdot r] \cdot \{[r+\mu]\cdot K_s + r \cdot B_s\} \cdot E[v''(C_{2,s})]\} \\
&\quad \cdot \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E[v''(C_{2,s})]\right\} \\
&\quad - \frac{r \cdot E[v'(C_{2,s})]}{u''(C_{1,s})} \cdot D \cdot \frac{\partial K_s}{\partial Y} \\
&\quad \cdot \frac{[1+(1-t_k)\cdot r] \cdot \{[r+\mu]\cdot K_s + r\cdot B_s\} \cdot E[v''(C_{2,s})]}{u''(C_{1,s})}
\end{aligned}$$

and

$$\begin{aligned}
h_K \cdot x_{BB} &= \left\{ \begin{array}{c} (r+\mu)\cdot E[v'(C_{2,s})] \\ +([r+\mu]\cdot K_s + r\cdot B_s)\cdot E[v''(C_{2,s})] \\ \cdot[[1+(1-t_k)\cdot r] + [1-t_w]\cdot[\lambda_s-\sigma_s]] \end{array} \right\} \\
&\quad \cdot \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E[v''(C_{2,s})]\right\} \\
&= \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E[v''(C_{2,s})]\right\} \\
&\quad \cdot \left\{ \begin{array}{c} (r+\mu)\cdot E[v'(C_{2,s})] + ([r+\mu]\cdot K_s + r\cdot B_s) \\ \cdot E[v''(C_{2,s})]\cdot[1+(1-t_k)\cdot r] \end{array} \right\} \\
&\quad + \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E[v''(C_{2,s})]\right\} \\
&\quad \cdot ([r+\mu]\cdot K_s + r\cdot B_s)\cdot E[v''(C_{2,s})]\cdot[1-t_w]\cdot[\lambda_s-\sigma_s]
\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} h_K \cdot x_{BB} &= \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]\right\} \\ &\quad \cdot \left\{ \begin{array}{c} (r+\mu)\cdot E\left[v'(C_{2,s})\right] \\ +([r+\mu]\cdot K_s + r\cdot B_s)\cdot E\left[v''(C_{2,s})\right]\cdot[1+(1-t_k)\cdot r] \end{array} \right\} \\ &\quad - \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]\right\} \\ &\quad \cdot \frac{([r+\mu]\cdot K_s + r\cdot B_s)}{[1+(1-t_k)\cdot r]\cdot u''(C_{1,s})} \cdot D \cdot \frac{\partial K_s}{\partial Y} \end{aligned}$$

Thus

$$\begin{aligned} K'(t_k) &= \left\{ r\cdot \frac{E\left[v'(C_{2,s})\right]}{u''(C_{1,s})} - \frac{[r+\mu]\cdot K_s + r\cdot B_s}{1+(1-t_k)\cdot r} \right\} \cdot \frac{\partial K_s}{\partial Y} \\ &\quad + \left\{u''(C_{1,s}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,s})\right]\right\} \cdot \frac{\mu \cdot E\left[v'(C_{2,s})\right]}{D} \end{aligned} \tag{39}$$

10.3 The conditions for the existence of a maximum of the widely held corporation:

$$EU_{BB} = u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right] < 0,$$

$$EU_{KK} = \frac{2}{3} \cdot \left\{ \begin{array}{c} [1-t_k]\cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] + \frac{2}{3}\cdot u''(C_{1,l}) \\ +\frac{2}{3}\cdot G^2 \cdot E\left[v''(C_{2,l})\right] - \frac{4}{3}\cdot G\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\frac{2}{3}\cdot[1-t_k]^2\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array} \right\} < 0$$

$$EU_{BK} = \frac{2}{3}\cdot u''(C_{1,l}) + \frac{2}{3}\cdot[1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} G\cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\}.$$

where

$$\begin{aligned} G &\equiv [1-t_k]\cdot[F_{K_l} - \delta] + 1 \\ &= [1+[1-t_k]\cdot r] + [1-t_k]\cdot\lambda_l \end{aligned}$$

Thus

$$EU_{BB}\cdot EU_{KK} - (EU_{BK})^2 \equiv F > 0$$

$$F = \left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \cdot \frac{2}{3} \cdot \left\{ \begin{array}{c} [1-t_k]\cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right] + \frac{2}{3}\cdot u''(C_{1,l}) \\ +\frac{2}{3}\cdot G^2 \cdot E\left[v''(C_{2,l})\right] - \frac{4}{3}\cdot G\cdot [1-t_k]\cdot E\left[v''(C_{2,l})\cdot \widetilde{\gamma}\right] \\ +\frac{2}{3}\cdot [1-t_k]^2 \cdot E\left[v''(C_{2,l})\cdot \widetilde{\gamma}\cdot\widetilde{\gamma}\right] \end{array} \right\} - \left(\frac{2}{3}\right)^2 \cdot \left\{ u''(C_{1,l}) + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} G\cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\} \right\}^2 \quad (40)$$

10.4 The income effect in the widely held corporation.

From FOC_B :

$$-u''(C_{1,l})\cdot\left\{1-\frac{2}{3}\cdot K_l'(Y) - B_l'(Y)\right\} + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} \frac{2}{3}\cdot G\cdot E\left[v''(C_{2,l})\right]\cdot K_l'(Y) \\ -\frac{2}{3}\cdot [1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot K_l'(Y) \\ +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot B_l'(Y) \end{array} \right\} = 0$$

$$\Downarrow$$

$$K_l'(Y)\cdot\frac{2}{3}\cdot\left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]\cdot \left\{ \begin{array}{c} G\cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\}\right\} + B_l'(Y)\cdot\left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right]\right\} = u''(C_{1,l}) \quad (41)$$

and from FOC_{K_l} :

$$-\frac{2}{3}\cdot u''(C_{1,l})\cdot\left\{1-\frac{2}{3}\cdot K_l'(Y) - B_l'(Y)\right\} + \frac{2}{3}\cdot G\cdot \left\{ \begin{array}{c} \frac{2}{3}\cdot G\cdot E\left[v''(C_{2,l})\right]\cdot K_l'(Y) - \frac{2}{3}\cdot [1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot K_l'(Y) \\ +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot B_l'(Y) \end{array} \right\} - \frac{2}{3}\cdot[1-t_k]\cdot \left\{ \begin{array}{c} \frac{2}{3}\cdot G\cdot E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot K_l'(Y) \\ -\frac{2}{3}\cdot [1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot K_l'(Y) \\ +[1+(1-t_k)\cdot r]\cdot E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot B_l'(Y) \end{array} \right\} = 0$$

$$\Downarrow \tag{42}$$

$$K_l'(Y)\cdot\frac{2}{3}\cdot\left\{\begin{array}{c}\frac{2}{3}\cdot u''(C_{1,l})+\frac{2}{3}\cdot G^2\cdot E\left[v''(C_{2,l})\right]\\ -\frac{4}{3}\cdot G\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\\ +\frac{2}{3}\cdot[1-t_k]^2\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\end{array}\right\}$$

$$+B_l'(Y)\cdot\frac{2}{3}\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c}G\cdot E\left[v''(C_{2,l})\right]\\ -[1-t_k]\cdot E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\}$$

$$=\ \frac{2}{3}\cdot\{u''(C_{1,l})\}$$

By Cramer's rule, equations (41) and (42) yield:

$$\Downarrow$$

$$\frac{\partial K_l}{\partial Y}\ =\ -\frac{2}{3}\cdot\frac{u''(C_{1,l})\cdot[1+(1-t_k)\cdot r]\cdot[1-t_k]}{F\cdot E\left[v'(C_{2,l})\right]}\cdot\left[\begin{array}{c}E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right]\\ -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right]\end{array}\right]$$

$$\left\{\begin{array}{c}E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right]\\ -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right]\end{array}\right\}\ =\ \left\{\begin{array}{c}cov\left\{v''(C_{2,l})\cdot\widetilde{\gamma},\ \ v'(C_{2,l})\right\}\\ -cov\left\{v'(C_{2,l})\cdot\widetilde{\gamma},\ \ v''(C_{2,l})\right\}\end{array}\right\}>0,$$

$$\Downarrow$$

$$\frac{\partial K_l}{\partial Y}\ >\ 0$$

which follows from the assumption on the risk aversion.

10.4.1 The effect on real capital investments from increased tax on capital income.

Use that

$$\begin{aligned}\frac{\partial C_{2,l}}{\partial t_k}\ =\ & -\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l]+\frac{2}{3}\cdot\widetilde{\gamma}\cdot K_l+\frac{2}{3}\cdot G\cdot K_l'(t_k)\\ & -\frac{2}{3}\cdot[1-t_k]\cdot\widetilde{\gamma}\cdot K_l'(t_k)-r\cdot B_l+[1+(1-t_k)\cdot r]\cdot B_l'(t_k)\end{aligned}$$

From FOC_B :

$$
\begin{aligned}
& K_l'(t_k) \cdot \frac{2}{3} \cdot \left\{ u''(C_{1,l}) + [1 + (1-t_k) \cdot r] \cdot \left\{ \begin{array}{c} +G \cdot E[v''(C_{2,l})] \\ -[1-t_k] \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}] \end{array} \right\} \right\} \\
& + B_l'(t_k) \cdot \left\{ u''(C_{1,l}) + [1 + (1-t_k) \cdot r]^2 \cdot E[v''(C_{2,l})] \right\} \\
= \; & r \cdot E[v'(C_{2,l})] \\
& + [1 + (1-t_k) \cdot r] \cdot E[v''(C_{2,l})] \cdot \left\{ \frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l \right\} \\
& - \frac{2}{3} \cdot K_l \cdot [1 + (1-t_k) \cdot r] \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}]
\end{aligned} \tag{43}
$$

From FOC_K :

$$
\begin{aligned}
& -\frac{2}{3} \cdot u''(C_{1,l}) \cdot \left\{ -\frac{2}{3} \cdot K_l'(t_k) - B_l'(t_k) \right\} \\
& -\frac{2}{3} \cdot [F_{K_l} - \delta] \cdot E[v'(C_{2,l})] \\
& +\frac{2}{3} \cdot [1-t_k] \cdot F_{K_l K_l} \cdot E[v'(C_{2,l})] \cdot K_l'(t_k) \\
& +\frac{2}{3} \cdot G \cdot \left\{ \begin{array}{c} -\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] \cdot E[v''(C_{2,l})] + \frac{2}{3} \cdot K_l \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}] \\ +\frac{2}{3} \cdot G \cdot E[v''(C_{2,l})] \cdot K_l'(t_k) \\ -\frac{2}{3} \cdot [1-t_k] \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}] \cdot K_l'(t_k) - r \cdot B_l \cdot E[v''(C_{2,l})] \\ +[1 + (1-t_k) \cdot r] \cdot E[v''(C_{2,l})] \cdot B_l'(t_k) \end{array} \right\} \\
& +\frac{2}{3} \cdot E[v'(C_{2,l}) \cdot \widetilde{\gamma}] \\
& -\frac{2}{3} \cdot [1-t_k] \left\{ \begin{array}{c} -\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}] \\ +\frac{2}{3} \cdot K_l \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \\ +\frac{2}{3} \cdot G \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}] \cdot K_l'(t_k) \\ -\frac{2}{3} \cdot [1-t_k] \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}] \cdot K_l'(t_k) \\ -r \cdot B_l \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}] \\ +[1 + (1-t_k) \cdot r] \cdot E[v''(C_{2,l}) \cdot \widetilde{\gamma}] \cdot B_l'(t_k) \end{array} \right\} \\
= \; & 0
\end{aligned}
$$

$$\Downarrow \tag{44}$$

$$\begin{aligned}
&K_l'(t_k)\cdot\frac{2}{3}\cdot\left\{\begin{array}{c} \frac{2}{3}\cdot u''(C_{1,l})+\frac{2}{3}\cdot G^2\cdot E\left[v''(C_{2,l})\right] \\ -\frac{4}{3}\cdot G\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\frac{2}{3}\cdot[1-t_k]^2\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]+[1-t_k]\cdot F_{K_lK_l}\cdot E\left[v'(C_{2,l})\right]\end{array}\right\} \\
&+B_l'(t_k)\cdot\frac{2}{3}\cdot\left\{u''(C_{1,l})+[1+(1-t_k)\cdot r]\cdot\left\{\begin{array}{c} G\cdot E\left[v''(C_{2,l})\right] \\ -[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\right\} \\
=\;&\frac{2}{3}\cdot\left\{\begin{array}{c} [F_{K_l}-\delta]\cdot E\left[v'(C_{2,l})\right]-E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]-\frac{2}{3}\cdot G\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l]+r\cdot B_l\right\}\cdot E\left[v''(C_{2,l})\right] \\ \cdot\left\{\begin{array}{c} [1+(1-t_k)\cdot r] \\ +[1-t_k]\cdot\left\{\lambda_l-\frac{E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]}{E\left[v''(C_{2,l})\right]}\right\}\end{array}\right\} \\ +\frac{2}{3}\cdot[1-t_k]\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\end{array}\right\}
\end{aligned}$$

$$\Downarrow \quad F_{K_l}-\delta-\lambda_l=r$$

$$\begin{aligned}
&K_l'(t_k)\cdot\frac{2}{3}\cdot\left\{\begin{array}{c} \frac{2}{3}\cdot u''(C_{1,l})+\frac{2}{3}\cdot G^2\cdot E\left[v''(C_{2,l})\right] \\ -\frac{4}{3}\cdot G\cdot[1-t_k]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\frac{2}{3}\cdot[1-t_k]^2\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]+[1-t_k]\cdot F_{K_lK_l}\cdot E\left[v'(C_{2,l})\right]\end{array}\right\} \\
&+B_l'(t_k)\cdot\frac{2}{3}\cdot\left\{\begin{array}{c} u''(C_{1,l})+[1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right] \\ +\frac{[1+(1-t_k)\cdot r]\cdot[1-t_k]}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c} E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{array}\right\} \\
=\;&\frac{2}{3}\cdot\left\{\begin{array}{c} r\cdot E\left[v'(C_{2,l})\right] \\ -\frac{2}{3}\cdot[1+[1-t_k]\cdot r]\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l]+r\cdot B_l\right\}\cdot E\left[v''(C_{2,l})\right]\cdot[1+(1-t_k)\cdot r] \\ +\frac{1-t_k}{E\left[v'(C_{2,l})\right]}\cdot\left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l]+r\cdot B_l\right\} \\ \cdot\left\{\begin{array}{c} E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] \\ -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right]\end{array}\right\} \\ +\frac{2}{3}\cdot\frac{[1-t_k]\cdot K_l}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c} E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\end{array}\right\}\end{array}\right\}
\end{aligned}$$

By Cramer's rule, equations (43) and (44) yield:

$$K_l'(t_k)=\frac{j_B\cdot æ_{KB}-j_K\cdot æ_{BB}}{æ_{BK}\cdot æ_{KB}-æ_{KK}\cdot æ_{BB}}=\frac{j_K\cdot æ_{BB}-j_B\cdot æ_{KB}}{F}$$

$$æ_{BB} = \left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\}$$

$$j_B = \left\{\begin{array}{l} r\cdot E\left[v'(C_{2,l})\right] \\ +[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\right]\cdot\left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l]+r\cdot B_l\right\} \\ -\frac{2}{3}\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array}\right\}$$

$$æ_{KB} = \frac{2}{3}\cdot\left\{\begin{array}{c} u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2\cdot E\left[v''(C_{2,l})\right] \\ +\frac{[1+(1-t_k)\cdot r]\cdot[1-t_k]}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c} E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array}\right\} \end{array}\right\}$$

$$j_K = \frac{2}{3}\cdot\left\{\begin{array}{c} r\cdot E\left[v'(C_{2,l})\right] \\ -\frac{2}{3}\cdot[1+[1-t_k]\cdot r]\cdot K_l\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ +\left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l]+r\cdot B_l\right\}\cdot E\left[v''(C_{2,l})\right]\cdot[1+(1-t_k)\cdot r] \\ +\frac{1-t_k}{E\left[v'(C_{2,l})\right]}\cdot\left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l]+r\cdot B_l\right\} \\ \cdot\left\{\begin{array}{c} E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\right] \\ -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \end{array}\right\} \\ +\frac{2}{3}\cdot\frac{[1-t_k]\cdot K_l}{E\left[v'(C_{2,l})\right]}\cdot\left\{\begin{array}{c} E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array}\right\} \end{array}\right\}$$

$$j_K \cdot æ_{BB} - j_B \cdot æ_{KB}$$

$$= \frac{2}{3} \cdot \left\{ \begin{array}{c} r \cdot E\left[v'(C_{2,l})\right] \\ -\frac{2}{3} \cdot [1 + [1 - t_k] \cdot r] \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ + \left\{\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l\right\} \cdot E\left[v''(C_{2,l})\right] \cdot [1 + (1 - t_k) \cdot r] \\ + \frac{1-t_k}{E\left[v'(C_{2,l})\right]} \cdot \left\{\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l\right\} \\ \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] \\ -E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] \end{array} \right\} \\ + \frac{2}{3} \cdot \frac{[1-t_k] \cdot K_l}{E\left[v'(C_{2,l})\right]} \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}$$

$$\cdot \left\{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\}$$

$$- \left\{ \begin{array}{l} r \cdot E\left[v'(C_{2,l})\right] \\ + [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot \left\{\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l\right\} \\ -\frac{2}{3} \cdot K_l \cdot [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}$$

$$\cdot \frac{2}{3} \cdot \left\{ \begin{array}{c} u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right] \\ + \frac{[1+(1-t_k) \cdot r] \cdot [1-t_k]}{E\left[v'(C_{2,l})\right]} \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}$$

$$j_K \cdot æ_{BB} - j_B \cdot æ_{KB}$$

$$= \frac{2}{3} \cdot \left\{ \begin{array}{c} -\frac{2}{3} \cdot [1 + [1 - t_k] \cdot r] \cdot K_l \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \\ + \left\{\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l\right\} \cdot E\left[v''(C_{2,l})\right] \cdot [1 + (1 - t_k) \cdot r] \\ + \frac{2}{3} \cdot \frac{[1-t_k] \cdot K_l}{E\left[v'(C_{2,l})\right]} \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,l}) \cdot \widetilde{\gamma} \cdot \widetilde{\gamma}\right] \cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \\ - \left\{ \begin{array}{l} [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l})\right] \cdot \left\{\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l\right\} \\ -\frac{2}{3} \cdot K_l \cdot [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\} \end{array} \right\}$$

$$\cdot \left\{u''(C_{1,l}) + [1 + (1 - t_k) \cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\}$$

$$+ \left\{ \begin{array}{l} \left\{\frac{2}{3} \cdot [F(K_l) - \delta \cdot K_l] + r \cdot B_l\right\} \cdot \frac{u''(C_{1,l})}{1+(1-t_k) \cdot r} \\ -r \cdot E\left[v'(C_{2,l})\right] \\ +\frac{2}{3} \cdot K_l \cdot [1 + (1 - t_k) \cdot r] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}$$

$$\cdot \frac{2}{3} \cdot \frac{[1 + (1 - t_k) \cdot r] \cdot [1 - t_k]}{E\left[v'(C_{2,l})\right]} \cdot \left\{ \begin{array}{c} E\left[v'(C_{2,l}) \cdot \widetilde{\gamma}\right] \cdot E\left[v''(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\right] \cdot E\left[v''(C_{2,l}) \cdot \widetilde{\gamma}\right] \end{array} \right\}$$

$$
\begin{aligned}
& j_K \cdot æ_{BB} - j_B \cdot æ_{KB} \\
= \; & \left(\frac{2}{3}\right)^2 \cdot \frac{[1-t_k]\cdot K_l}{E\left[v'(C_{2,l})\right]} \cdot \left\{ \begin{array}{c} E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \\ -E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\} \\
& \cdot \left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \\
& - \left\{ \begin{array}{l} \left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l] + r\cdot B_l\right\} \cdot \frac{u''(C_{1,l})}{1+(1-t_k)\cdot r} \\ +\frac{2}{3}\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ -r\cdot E\left[v'(C_{2,l})\right] \end{array} \right\} \cdot \frac{\partial K_l}{\partial Y} \cdot \frac{F}{u''(C_{1,l})}
\end{aligned}
$$

Thus

$$
\begin{aligned}
K_l'(t_k) \;=\; & -\left(\frac{2}{3}\right)^2 \cdot \frac{[1-t_k]\cdot K_l}{F\cdot E\left[v'(C_{2,l})\right]} \cdot \left\{ \begin{array}{c} -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \\ +E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \end{array} \right\} \\
& \cdot \left\{u''(C_{1,l}) + [1+(1-t_k)\cdot r]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \\
& - \left\{ \begin{array}{l} \left\{\frac{2}{3}\cdot[F(K_l)-\delta\cdot K_l] + r\cdot B_l\right\} \cdot \frac{u''(C_{1,l})}{1+(1-t_k)\cdot r} \\ +\frac{2}{3}\cdot K_l\cdot[1+(1-t_k)\cdot r]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\ -r\cdot E\left[v'(C_{2,l})\right] \end{array} \right\} \cdot \frac{\frac{\partial K_l}{\partial Y}}{u''(C_{1,l})}
\end{aligned}
$$

Consider the following covariances.

$$
\begin{aligned}
cov\left\{v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma},\;\; v'(C_{2,l})\right\} \;=\; & E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\cdot v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\
& -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right]
\end{aligned}
$$

and

$$
\begin{aligned}
cov\left\{v''(C_{2,l})\cdot\widetilde{\gamma},\;\; v'(C_{2,l})\cdot\widetilde{\gamma}\right\} \;=\; & E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\cdot v''(C_{2,l})\cdot\widetilde{\gamma}\right] \\
& -E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right].
\end{aligned}
$$

Apply these to rewrite the following:

$$
\begin{aligned}
& E\left[v'(C_{2,l})\cdot\widetilde{\gamma}\right]\cdot E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\right] - E\left[v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma}\right]\cdot E\left[v'(C_{2,l})\right] \qquad (45) \\
= \; & cov\left\{v''(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma},\;\; v'(C_{2,l})\right\} - cov\left\{v''(C_{2,l})\cdot\widetilde{\gamma},\;\; v'(C_{2,l})\cdot\widetilde{\gamma}\right\}
\end{aligned}
$$

The individual is assumed to have decreasing absolute risk aversion, such that $v'''(C_{2,l}) > 0$. Then $cov\left\{v''(C_{2,l})\cdot\widetilde{\gamma},\;\; v'(C_{2,l})\cdot\widetilde{\gamma}\right\} > 0$ and $cov\left\{v'(C_{2,l})\cdot\widetilde{\gamma}\cdot\widetilde{\gamma},\;\; v''(C_{2,l})\right\} < 0$. Thus the expression (45) is negative.

Effect of increased capital income tax rate in the absence of risk, $\widetilde{\gamma} = 0$, **and** $E\left[v(C_{2,s})\right] = v(C_{2,s})$.

$$F|_{\widetilde{\gamma}=0} = \left\{u''(C_{1,l}) + \left[1 + (1 - t_k) \cdot r\right]^2 \cdot E\left[v''(C_{2,l})\right]\right\} \cdot \frac{2}{3} \cdot \left[1 - t_k\right] \cdot F_{K_l K_l} \cdot E\left[v'(C_{2,l})\right]$$

$$\Downarrow$$

$$K_l'(t_k)|_{\widetilde{\gamma}=0} = 0$$