

Working Paper No. 72/04

**Taxation of uncertain business profits, private
risk markets and optimal allocation of risk**

by

**Kåre P. Hagen
Jan Gaute Sannarnes**

SNF-project No. 2410
Skatt, usikkerhet og ufullstendige risikomarkeder

The project is financed by the Research Council of Norway

INSTITUTE FOR RESEARCH IN ECONOMICS AND BUSINESS ADMINISTRATION
BERGEN, December 2004
ISSN 1503-2140

© Dette eksemplar er fremstilt etter avtale
med KOPINOR, Stenergate 1, 0050 Oslo.
Ytterligere eksemplarfremstilling uten avtale
og i strid med åndsverkloven er straffbart
og kan medføre erstatningsansvar.

Taxation of uncertain business profits, private risk markets and optimal allocation of risk

Kåre P. Hagen

Norwegian School of Economics and Business Administration, Helleveien 30,
5045 Bergen, Norway¹

Jan Gaute Sannarnes

Norwegian School of Economics and Business Administration, Helleveien 30,
5045 Bergen, Norway²

JEL classification: D82; G24; H25

Keywords: Public policy; Moral hazard; Venture capital

Abstract:

In this paper we explore what happens if the government bears some of the risk through a profit tax when the risk sharing in the venture capital market is incomplete due to non-observability of effort and moral hazard. If the external equity investors can enforce exclusive contracts with the entrepreneurs, the risk relief through a profit tax will give too much insurance and too low effort compared with a second best optimal solution. Bond & Devereux (1995) show that a proportional profit tax would be actuarially neutral in the absence of moral hazard. In the presence of moral hazard we demonstrate that the tax may affect the risk shifting through the market, in which case the premise for the neutrality result will no longer hold. We also find that in contrast to exclusive risk sharing contracts non-exclusive contracts may in conjunction with a proportional profit tax lead to too little provision of effort.

¹Tel.: +4755959270; fax: +4755959543. E-mail address: kare.hagen@nhh.no

²Corresponding author. Tel.: +4755959433; fax: +4755959543. E-mail address: jan.sannarnes@nhh.no

1 Introduction

Many investment projects, for example within the IT area, are highly risky. In many cases there is a small probability for the project to succeed, in which case it is expected to earn a sizeable profit, and a high probability of project failure. If the project requires specialized equipment and competence, the investment is not recoverable if the project fails. The intangible nature of investment in specialized knowledge entails that it cannot be used as collateral for loans. Hence, such investment must mainly be financed by equity, either in the external equity market by seeking equity finance from outside equity investors or by resorting to internal equity finance, which is limited by the entrepreneur's own resources. Borrowing may be possible if the entrepreneur has tangible assets to pledge as collateral, but as to risk exposure this is similar to internal finance as the entrepreneur in either case bears the risk.

Thus, the risk market that we consider in this paper is the market for external equity. It may be difficult or costly for external equity investors to observe the effort that the entrepreneur invests in his project in order to make the project succeed. This problem may be aggravated by the fact that the entrepreneur's own effort may be vital for a successful result. However, as effort is costly, financing by outside equity weakens the incentives for the entrepreneur as the outside equity holders share in the returns due to extraordinarily high efforts by the entrepreneur and cover part of the losses due to low effort. Thus, providing risk relief to the entrepreneur gives rise to moral hazard which increases the probability that the project will fail. This problem could be overcome if the outside equity investors could deduce the effort from the outcome of the project. In that case the economic terms for outside equity could be conditioned on outcome, which would be equivalent to conditioning on effort. In order to preclude that possibility we therefore assume that entrepreneurial effort only affects the probability of the outcomes and not the outcome size.

The present paper will explore to what extent public policy, and in particular tax policy, will affect risk taking and effort choice by an entrepreneur in the presence of moral hazard. Of particular interest will be the resulting risk allocation when a risk absorbing profit tax interacts with the risk allocation in the financial market. We demonstrate that if private risk markets coexist with governmental measures for reallocating risk, implemented

through a profit tax, this will generally alter the equilibrium compared with the no tax situation. This result is in contrast to Buchholz and Konrad (2000) and Keuschnigg and Nielsen (2003), who confirm the neutrality of a non-redistributive proportional tax. The non-neutrality result in our paper is due to the fact that external equity investors do not fully internalize the moral hazard cost from increased insurance as part of the cost is borne by the government through the tax system. Our results also relates to the discussion on the design of a neutral tax on uncertain profits. As shown by Fane (1987) and Bond and Devereux (1995), a cash flow tax³ will be neutral also in the presence of risky profits if the market allocation of risk is unaffected by the cash flow tax. However, when risk markets are incomplete because of moral hazard, entrepreneurs have to bear some idiosyncratic risk that would otherwise be washed away in complete risk markets.

There is a related literature on the effects from moral hazard in the insurance market with contributions by a.o. Stiglitz (1983), Arnott and Stiglitz (1988) and (1991a, b), Kaplow (1991), Kahn and Mookherjee (1998) and Bisin & Guaitoli (2000). Except for Kaplow (1991) these papers do not address explicitly the interaction between market and public provision of insurance. Kahn and Mookherjee (1998) show in a setting with non-exclusive contracts between the entrepreneur and the external investor that a profit tax would only crowd out risk relief from the private market. We argue that a proportional profit tax might alter the equilibrium risk allocation compared with the no tax situation.

The paper is organized as follows. In Section 2 we use a simple model to discuss an entrepreneur's optimal supply of effort in a situation with a private risk market and without taxation. In Section 3 we assume that the financial investors can enforce exclusive risk-sharing contracts. We establish the central result that risk shifting through the tax system in conjunction with a private risk market will lead to too much insurance and too little effort. In section 4 we discuss a proportional profit tax in conjunction with a private risk market with non-exclusive risk sharing contracts. Section 5 discusses some related results in the literature, while Section 6 concludes.

³In our context a profit tax is equivalent to a cash flow tax.

2 The model

We employ a simple model for discussing these issues. We consider a sector where there is a large number of entrepreneurs facing identical investment opportunities with uncertain returns. Thus, we abstract from adverse selection problems altogether. Each project requires a fixed investment equal to I . Hence, the investment scale is exogenous so that we concentrate on the problem of allocating project risk. For simplicity we assume that there are only two economic outcomes from the investment. One outcome denoted success, is a state in which the investment generates a net surplus equal to $\Pi - I$. In the other state the investment turns out to be a failure and in which case the amount invested is lost altogether. The state dependent final wealth is then given by

$$W_i = \begin{cases} W + \Pi - I, & i = s(\text{uccess}) \\ W - I, & i = f(\text{ailure}), \end{cases}$$

where $W > I$ is initial wealth. The probability of success depends on the effort that the investor provides in order to make the project succeed. Denoting the amount of effort e , the probability of success is $p(e)$, with $p'(e) > 0$ and $p''(e) < 0$. Thus, we have positive and decreasing marginal returns to effort. The probabilities are independent across entrepreneurs so that the project risk is unsystematic. We assume that the sector is sufficiently large so that the expected outcome per entrepreneur equals the average outcome. Also, the analysis is partial in that we only consider the entrepreneurial sector and disregard interaction with the rest of the economy. As the entrepreneurs are all identical, we can focus on the situation for a representative entrepreneur. As the model is timeless there is no discounting.

The entrepreneur is maximizing the expected net utility from the project with respect to effort. Letting $U(W_i)$ denote the utility of the outcome W_i , the entrepreneur is maximizing

$$EU = p(e)U(W + \Pi - I) + (1 - p(e))U(W - I) - V(e),$$

where we have assumed that the disutility of effort, $V(e)$, is separable. We further assume $U'(W) > 0$ and $U''(W) < 0$, so that entrepreneurs have risk aversion, and that the disutility of effort is increasing and convex in e , $V'(e) > 0$ and $V''(e) > 0$, and $V(0) = 0$. As to the expected utility, the

indifference curves will be convex over state dependent outcomes for a given effort level⁴.

The first order condition for the entrepreneur's optimal supply of effort is given by

$$p'(e)(U_s - U_f) = V'(e), \quad (1)$$

where U_s and U_f are total utility in states s and f , respectively.

We assume that there exists a financial sector providing risk bearing capital to the entrepreneurial sector. In order to fix ideas we may think of financial equity investment as venture capital⁵. That means that agents in the financial sector invest equity capital in the entrepreneurial sector. We abstract from borrowing so that the external equity is replacing part of the entrepreneur's equity in the project. External equity investors will have a claim to a share of the net surplus in the successful state. On the other hand the external equity investment will be totally lost if the project fails. Hence, the external equity investment will reduce the loss for the entrepreneur by the same amount so that external equity investors will bear some of the project risk. Financial investors are assumed to be risk neutral, possibly because they have perfectly diversified portfolios. Hence, they will maximize the expected return on their portfolios. As the model is static we may disregard opportunity costs of financial equity investment.

⁴There is however a complication arising from the fact that the incentives for providing effort depend on the difference between the expected utility in the two states, and the indifference curves corresponding to an optimized level of effort need not be convex. The reason for this is that the marginal rate of substitution between final wealth in the successful and unsuccessful state will depend on the probability of succeeding which is increasing in effort. If not explicitly stated otherwise, we disregard for the present analysis non-convexity problems and assume that at the optimum the indifference curves are locally convex.

⁵In some settings a venture capitalist not only supplies equity finance but also provides valuable business advice. In our paper this is not the case and the venture capitalist may be seen as a financial intermediary.

3 Exclusive private risk sharing contracts

3.1 Introduction

In this section we assume that the financial investors can enforce exclusive risk sharing contracts. An exclusive risk sharing contract is defined as a contract where the financial investor can control the entrepreneur's total risk relief through the external risk market. This presupposes that each entrepreneur enters into a risk sharing contract with only one financial investor. We let E denote the amount of external equity investment in each project and π the profits accruing to external stake holders, i.e., the amount net of the equity investment accruing to the financial investor in the successful state. It may be natural to assume that the share of the net surplus accruing to the external equity holder is given by the share of external equity to total equity. We shall, however, allow for the possibility that π and E are determined independently subject to the constraint that the external financial equity investment at least breaks even.

The expected profit for external investors is then given by

$$R(\pi, E) = p(e)\pi - (1 - p(e))E, \quad (2)$$

where π/E is the rate of return in the successful state on the external equity investment.

The state dependent final wealth for the entrepreneur is then given by

$$W + \Pi - I - \pi, \quad i = s(\text{uccess})$$
$$W_i =$$
$$W - I + E, \quad i = f(\text{ailure}).$$

The financial investor will choose π and E in order to maximize expected profits subject to the constraint that the entrepreneur at least obtains his reservation utility. The solution can however be characterized more conveniently by assuming that the optimal choice of financial contract maximizes the expected utility of the entrepreneur subject to a constraint on expected profit for the financial investor. As we have abstracted from financial opportunity costs we assume that the expected returns on the external equity investment must be non-negative.

Exclusivity means that the external equity investors can fully control the total amount of external equity capital in each project. Hence, the investor can figure out the incentive effects of the financial contract from condition (1). Therefore, the financial investor may as well be thought of as choosing the

effort level subject to the incentive constraint (1) governing the entrepreneur's choice of effort.

3.2 Observability of effort

As a point of reference we first assume that entrepreneurial effort can be observed by the financial investors. In that case the optimum effort level can be enforced as part of the financial contract. The optimal contract is then determined by

$$\underset{(\pi, E, e)}{\text{Max}} Eu = p(e)U(W + \Pi - I - \pi) + (1 - p(e))U(W - I + E) - V(e)$$

subject to,

$$p(e)\pi - (1 - p(e))E \geq 0$$

The first order condition for optimal risk sharing implies

$$U'_s = U'_f = \lambda \tag{3}$$

where λ is the shadow price of the zero profit constraint which will be equal to the marginal utility of certain income. With state independent utilities condition (3) implies equal total utilities in the two states. Hence, $W_s = W_f$ so that the optimal contract gives the entrepreneurs full insurance against failure. Hence, $\Pi - \pi = E$ so that $\pi + E = \Pi$ ⁶.

The first order condition for optimal effort is

$$p'(e)(U_s - U_f) + \lambda[p'(e)\pi + p'(e)E] - V'(e) = 0. \tag{4}$$

For later reference it will be useful to rewrite the first order condition for optimal effort (4) as

$$\frac{p'(e)(U_s - U_f)}{\lambda} + p'(e)(\pi + E) = \frac{V'(e)}{\lambda} \tag{5}$$

The first term on the left hand side of (5) is the expected gain in monetary terms accruing to the entrepreneur as a result of increased effort. The second

⁶In this context full insurance must be interpreted as a situation where the financial investor takes over the project and hires the entrepreneur at a fixed wage.

term is the increase in the expected profit on the financial contract from increased effort as $\pi + E$ is the total value at risk for the external investor. A first best contract implies full insurance for the entrepreneur. In that case the entrepreneur does not gain from increased effort, so that one is only left with the expected gain accruing to the financial investor. With full risk shifting the value at risk in the financial contract equals the total value at risk, i.e., $\pi + E = \Pi$. Hence the total social gain from increased effort is fully internalized by the financial market, and the effort level determined by the financial external investors is the first best level as given by condition (5).

3.3 Unobservability of effort

We now turn to the case where the entrepreneurial effort is not observable by the external investors. They must however take into account the fact that the risk sharing contract will affect the effort incentives for the entrepreneur. Even though the external investors cannot observe effort directly, they can figure out how the degree of risk relief through the risk sharing contract affects the optimum supply of effort determined by condition (1). Hence, the external investors can be seen as indirectly choosing entrepreneurial effort subject to (1). The optimal contract can then be characterized by the following optimization problem:

$$Max_{(\pi, E, e)} EU = p(e)U(W + \Pi - I - \pi) + (1 - p(e))U(W - I + E) - V(e)$$

subject to

$$p(e)\pi - (1 - p(e))E = 0 \quad (\lambda)$$

$$p'(e)(U_s - U_f) - V'(e) = 0 \quad (\mu).$$

This yields the first order conditions

(wrt. π)

$$-p(e)U'_s + \lambda p(e) - \mu p'(e)U'_s = 0 \quad (6)$$

(wrt. E)

$$(1 - p(e))U'_f - \lambda(1 - p(e)) - \mu p'(e)U'_f = 0 \quad (7)$$

(wrt. e)

$$p'(e)(U_s - U_f) + \lambda p'(e)(\pi + E) + \mu(p''(e)(U_s - U_f) - V''(e)) - V'(e) = 0 \quad (8)$$

Conditions (6) and (7) characterize the optimum risk sharing, and condition (8) characterizes the optimum effort level. From (6) and (7) we have

$$U'_f \left[1 - \frac{\mu p'(e)}{1 - p(e)} \right] = \lambda = U'_s \left[1 + \frac{\mu p'(e)}{p(e)} \right] \quad (9)$$

It is easy to show that μ must be positive when the incentive constraint is binding⁷. Assuming $\mu < 0$ implies $U'_f < U'_s$. That implies $W_f > W_s$ in which case the incentive constraint (1) cannot hold. Hence, μ must be positive. Then, when the incentive constraint is binding, $W_f < W_s$ and $\Pi - \pi > E$ so that the risk shifting offered by the optimal contract will be less than complete, which reflects the moral hazard cost from the dilution of the effort incentives. Hence, the value at risk in the financial market will be less than the total value at risk as $\Pi > \pi + E$ so that the financial investors will not internalize fully the total gains from increased effort. Moreover, $\pi + E > 0$ by (8). Then, by the zero profit constraint, $E > 0$ so that in a situation with risk averse entrepreneurs and risk neutral financial investors the efficient solution will involve some risk shifting through external equity investments, but the entrepreneur will have to bear some risk due to the presence of moral hazard⁸.

Condition (8) can be rewritten as

$$\frac{p'(e)(U_s - U_f)}{\lambda} + p'(e)(\pi + E) = \frac{V'(e)}{\lambda} - \frac{\mu}{\lambda}(p''(e)(U_s - U_f) - V''(e)) \quad (10)$$

The terms on the left hand side of (10) are the gains from increased entrepreneurial effort accruing to the entrepreneur and the external financial investors respectively. The first term on the right hand side is marginal effort cost and the second term represents marginal moral hazard cost due to the fact that increasing the incentives for effort means that more of the risk has to be born by the entrepreneur. When the level of effort is privately optimal in the sense of condition (1), the expected marginal gain to the entrepreneur equals marginal effort cost. Hence, the condition for optimal effort simplifies to

$$p'(e)(\pi + E) = -\frac{\mu}{\lambda}(p''(e)(U_s - U_f) - V''(e)) \quad (11)$$

⁷If $\mu = 0$, then the constraint is not binding, $U'_f = U$, and the entrepreneur bears no risk.

⁸This is a well known property of optimal insurance with moral hazard, see Shavell (1979).

Thus, the second best optimal effort is characterized by the expected gain to the external financial investors from increased effort being equal to moral hazard cost, e.g. risk shifting through the profit tax.

Risk shifting means redistributing income from good states to bad states. In the present context this redistribution is constrained by the adverse effect on the profitability of the external equity investments resulting from the dilution effect on effort incentives.

3.4 The role of taxes

One might ask whether this risk shifting could have been obtained through the tax system as well. Risk relief through the tax system would mean shifting some of the risk from the entrepreneurial sector to the government. The government will have a stake in all taxable activities. The authorities are in effect perfectly diversified so that all unsystematic risk will wash out in the net tax revenue. Hence, expected tax revenue would equal average revenue. In the present model it is clear that an actuarially neutral profit tax would provide the same opportunities for redistribution of risk as the financial market for external equity.

We consider a situation where there is initially a given tax system, and we then examine how the presence of such a tax affects the risk allocation at a financial market equilibrium. It is assumed that in the market for venture capital the financial investors take the tax rates of the profit tax as given parameters when choosing the optimum financial contract. The government taxes both the entrepreneurial profit and the profit accruing to the external investor. The tax system in our model may be seen as a tax on the net present value of a project. Supra-normal profit is taxed, while profit lower than normal is deductible⁹. We assume that any tax revenue from the profit tax does not affect the utility of the entrepreneurs and the financial investors.¹⁰ Our model is partial also in the fact that we ignore that the tax

⁹The tax system is a differential tax system, which taxes the differential between the actual profit and a normal rate of return, see Sandmo (1989). For simplicity, the normal rate of return is set to zero. An external investor will not require a risk premium to invest in the project.

¹⁰Alternatively we could assume that the profit tax may finance a lump sum subsidy. The financial investors and entrepreneurs would then take the profit tax and the lump sum subsidy as given parameters when choosing the optimum financial contract. That could be because they do not see through the government budget.

might affect prices elsewhere in the economy.

The redistribution of risk through the tax system can in the present model be implemented by an entrepreneurial tax rate t in the good state and a corresponding tax rate αt in the bad state, with $\alpha \geq 0$. $\alpha > 1$ means more than full loss offset in case of failure.

After tax final wealth for the entrepreneur is then

$$W + (\Pi - I - \pi)(1 - t), \quad i = s$$

$$W_i = W - (I - E)(1 - \alpha t), \quad i = f.$$

We may note that government risk relief would not improve on the market allocation of project risk provided the government is subject to the same informational constraint as the financial market, see Kaplow (1991). In that sense the risk allocation in the financial market equilibrium will be second best optimal.¹¹

With private information with respect to effort, the Pareto optimal risk sharing contract would offer less than full insurance and is given by the first order condition for the following optimization problem:

$$\underset{(\pi, E, e)}{\text{Max}} EU = p(e)U(W + (\Pi - I - \pi)(1 - t)) + (1 - p(e))U(W - (I - E)(1 - \alpha t)) - V(e)$$

subject to

$$((1 - t)p(e)\pi - (1 - \alpha t)(1 - p(e))E) = 0$$

$$p'(e)(U_s - U_f) - V'(e) = 0.$$

Pareto optimal risk sharing is given by

$$U'_f \left[1 - \frac{\mu p'(e)}{1 - p(e)} \right] = \lambda = U'_s \left[1 + \frac{\mu p'(e)}{p(e)} \right] \quad (12)$$

For given effort the first order condition for the optimal risk relief is the same as in the no tax case and given as in (9), confirming that the optimal risk relief will imply partial risk coverage. However, in determining the socially optimal effort level it must be taken into account that the supply of effort

¹¹However, even though the government cannot observe effort, it has the right to tax and it can stimulate entrepreneurial effort indirectly by taxing substitutes and subsidizing complementary activities to entrepreneurial effort. This argument is however valid regardless of whether or not there is a private market for reallocating risk.

has to be privately optimal, given the insurance embodied in the risk sharing contracts.

3.5 Graphical illustration of the second best optimum

To gain some more insight into how a profit tax distorts the allocation of risk in the market for risk bearing capital, we illustrate the problem graphically. The supply side of venture capital is characterized by feasible combinations of π and E such that $R(\pi, E) = p(e)\pi - (1 - p(e))E \geq 0$. In a competitive financial market financial portfolios will break even in equilibrium. Hence $R(\pi, E) = 0$. When deriving the zero profit frontier of the supply set it must be taken into account that the effort level will be a function of the terms of risk coverage as given by π and E . From (1) we have in the no tax situation that

$$\frac{\partial e}{\partial E} = \frac{p'(e)U'_f}{p''(e)(U_s - U_f) - V''(e)} < 0$$

and

$$\frac{\partial e}{\partial \pi} = \frac{p'(e)U'_s}{p''(e)(U_s - U_f) - V''(e)} < 0$$

such that effort decreases with both the amount of external equity which bears part of the risk in the unsuccessful state, and with the surplus that goes to the external equity holders, which reduces the entrepreneurial profit in the successful state. Hence, effort decreases along any positively sloped line in the (π, E) -plane.

Along the zero profit frontier π must be some function of E as E affects the privately optimal effort and hence the probability of success. We let $\pi = \pi(E)$ denote this relationship. Hence, $p(e) = p(e(\pi, E)) = p(e(\pi(E), E)) \equiv p(E)$ which is assumed to be locally monotone. Thus, along the zero profit frontier the supply of external equity determines the effort level and the success probability. Along $R(\pi, E) = 0$ and we then have $\pi(E) = E \frac{1-p(E)}{p(E)}$.

The slope of the zero profit frontier is given by¹²

$$\frac{\partial \pi}{\partial E} = \frac{1 - p(E)}{p(E)} - \frac{E}{(p(E))^2} \frac{\partial p(E)}{\partial E}. \quad (13)$$

¹²This approach parallels Stiglitz (1983).

Equation (13) may be thought of as the slope of the transformation curve in the financial market for transferring income from the good to the bad state. The transformation curve is an increasing function of E with slope larger than $(1-p(E))/p(E)$ since $\partial p/\partial E < 0$. It will, however, not necessarily be convex everywhere.

For the entrepreneur the marginal rate of substitution between π and E is given by

$$\frac{d\pi}{dE} \Big|_{EU=given} = \frac{(1-p(e))U'_f}{p(e)U'_s}$$

so that for given effort, indifference curves in the (π, E) -plane will be concave¹³ with positive slope. However, since E affects effort, the indifference curve may have convex parts. Assuming that the second best optimum can be characterized by a tangential solution it is given by equating the marginal rate of transformation on the supply side with the marginal rate of substitution on the demand side:

$$\frac{(1-p(e))U'_f}{p(e)U'_s} = \frac{1-p(e)}{p(e)} - \frac{p'(e)}{(p(e))^2} \frac{\partial p(E)}{\partial E}. \quad (14)$$

This is illustrated in figure 1.

¹³The indifference curves for given effort are concave because the second derivative is negative;

$$\frac{d^2\pi}{dE^2} \Big|_{EU=given} = -\frac{(1-p(e))U''_f}{p(e)U''_s} < 0.$$

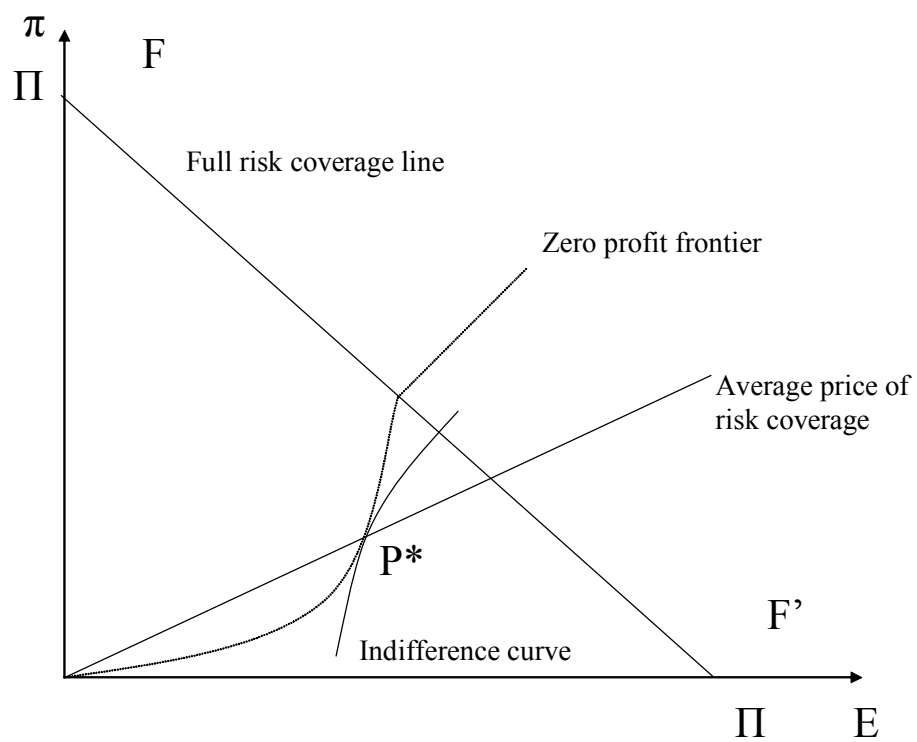


Figure 1: Second best optimum with exclusive contracts

The line $F - F'$ given by $\pi + E = \Pi$, implies that $W_s = W_f$, and represents combinations of π and E yielding full risk coverage for the entrepreneur, while $R(\pi, E) = 0$ is the zero profit frontier. Any combination of π and E above the $F - F'$ -line implies minimum effort so that the zero profit frontier above the $F - F'$ line must be a straight line with slope given by the probabilities corresponding to minimum effort, $(1-p(e_0))/p(e_0)$, where e_0 is minimal effort. The optimum is at point P^* , where the indifference curve is tangential to the zero profit frontier.

The slope of the ray through P^* determines the average price of risk coverage as it shows how much the entrepreneur has to sacrifice of the surplus in the successful state per unit of external equity. This is given by $(1 - p(e^*))/p(e^*)$ where e^* is optimal effort at P^* . Thus we see that the average price for external equity is less than the marginal price. The reason for this is of course that effort is decreasing as one moves upwards the zero profit frontier. We may note that if the entrepreneur can obtain additional equity finance from other sources at this average cost of coverage, P^* would not represent a privately optimal risk coverage for the entrepreneur. The optimal risk coverage would then be where an indifference curve is tangential to the average price line.

3.6 Taxes and optimal effort provision

To examine how the tax effects the optimal effort provision, we differentiate condition (1) for optimum effort with respect to the tax rate. We have

$$\frac{\partial e}{\partial t} = -\frac{-p'(e)U'_s(\Pi - I - \pi) + p'(e)U'_f\alpha(-I + E)}{p''(e)(U_s - U_f) - V''(e)} < 0, \quad (15)$$

since $\Pi - I - \pi > 0$ and $-I + E < 0$ with partial risk sharing. Hence, as long as $t < 1$, increasing the tax rate leads to lower effort. Notice that this result is independent of α as long as $\alpha \geq 0$.

The profit tax may be compared with an insurance contract as illustrated in Figure 2. With taxation, the combined risk shifting by the profit tax in conjunction with the supply of external equity is given by the points $(\pi + (\Pi - I - \pi)t, E + (I - E)\alpha t)$ in the (π, E) - plane. The full insurance line is now given by $\pi + (\Pi - I - \pi)t + E + (I - E)\alpha t = \Pi$, where Π is the same as in the no-tax case. Note that zero private insurance now corresponds to the point $((\Pi - I)t, I\alpha t)$.

Redistribution of risk through the financial market could alternatively be implemented through a non-linear profit tax such that $p(e)t(\Pi - I) - (1 - p(e))\alpha tI \equiv p(e)g_s - (1 - p(e))g_f = 0$, where g_s and $-g_f$ are the government's stake in the good and bad state, respectively. The parameters g_s and $-g_f$ could be set equal to π and E in the no tax case. The tax rates would then have to satisfy $\alpha t/t = p(e)(\Pi - I)/(1 - p(e))I$ so that $\alpha t \geq t$ ¹⁴. A risk averse entrepreneur needs higher expected profit in the good state than in the bad state to invest in the project. Thus, if the tax system should replicate the risk relief through the financial market, $\alpha \geq 1$. In the specific case where the government bears all risk, the tax rates would be equal, $\alpha t = t$, or $\alpha = 1$. Then there is no moral hazard because the entrepreneur bears no risk and provides minimum effort.

In the following we discuss three different tax schedules, differing only with respect to the actual notion of actuarial neutrality. The first two schedules allow for more than full loss offset, i.e. $\alpha > 1$. One of them is actuarially neutral in the absence of an external equity market, while the other offers risk relief at the same implicit average price as an external equity investor would do in a no tax situation. The third tax schedule is a proportional tax system ($\alpha = 1$), which is actuarially neutral in the absence of moral hazard, see Bond & Devereux (1995).

In Figure 2 we illustrate an actuarially neutral profit tax absent an external equity market. Thus, the risk sharing mechanism through the profit tax has to lie on the zero profit frontier for an external equity investor in the no-tax case. The profit tax is represented by the point on the zero profit frontier in the no tax case. The zero profit frontier for an external investor, given the tax system, must be located to the right of the zero profit curve in the no tax case, as illustrated in the figure. The reason for this is that the external investor will not internalize the government's increased moral hazard cost, due to the entrepreneur's increased total risk relief. The optimal risk sharing is now given by P^{**} , rather than P^* in the no tax case. P^{**} is the second best optimum in the tax situation in the sense that it is not possible to increase the expected utility for the entrepreneur without violating the zero profit constraint for the external equity investor. The average cost of risk coverage from an external investor is higher¹⁵, which reflects the fact that the after

¹⁴If investment expenditures were fully tax deductible the government would absorb all the risk through the tax system if the tax rates satisfy $(\alpha t - t)I = (1 - t)\Pi$.

¹⁵This can be seen in the figure because the line for average cost of risk coverage from an external investor is steeper in the tax case.

tax solution involves less effort and a higher probability of project failure.

Figure 2 is consistent with equation (15) for the special case where $\alpha > 1$. Risk shifting through the tax system in conjunction with a private risk market leads to too much insurance and too little effort. The inefficiency is due to the fact that external equity investors do not fully internalize the moral hazard cost from increased insurance as part of the cost is borne by the government through the tax system. This is a special case of a common agency problem discussed by Bernheim and Whinston (1986), as the entrepreneur may be thought of as having two principals, namely, the financial investor and the government, which in this case do not coordinate their insurance policies. However, in our model the problem is sequential in that the government decides the tax policy before the entrepreneur obtains equity finance from an external investor.

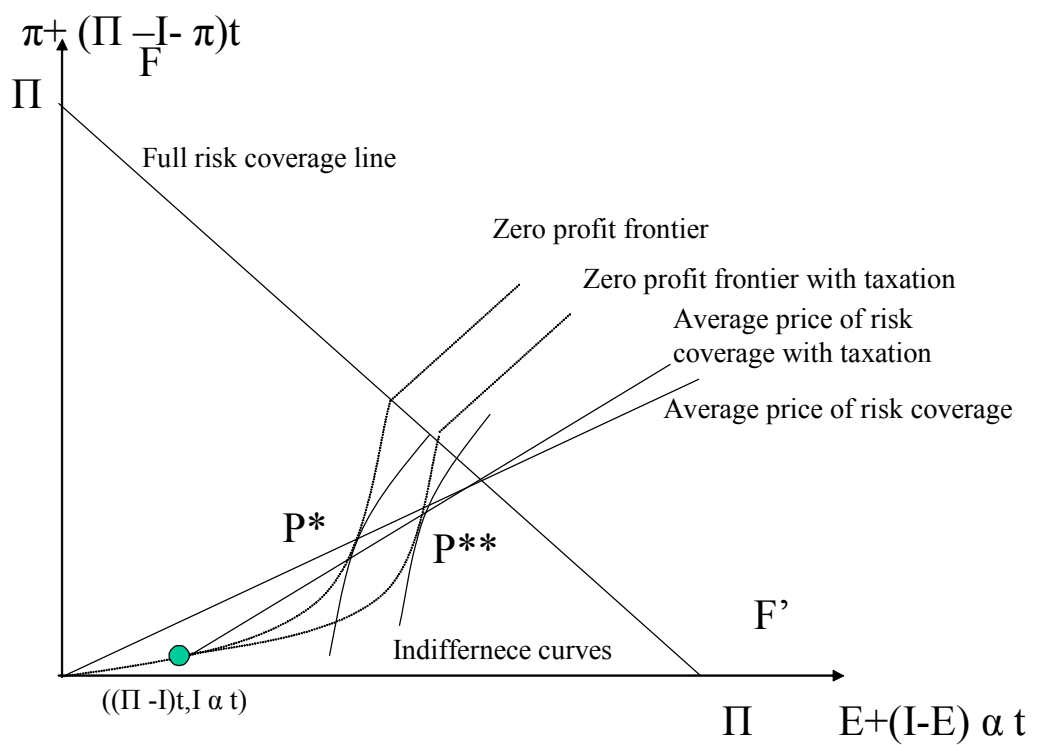


Figure 2: Actuarially neutral profit tax absent an external equity market

In Figure 3 we illustrate a profit tax that provides risk relief at an implicit average price of risk coverage, which equals that of an optimal exclusive contract between the entrepreneur and an external equity investor in the no tax situation. This tax system is illustrated by the point on the average price line for this exclusive contract in the figure. The optimal risk sharing contract is now P^{**} , rather than P^* in the no-tax case. Since the government's risk relief is at the same implicit average price as the external investor, we should perhaps expect that the tax would only crowd out risk relief provided by the external investor. However, this is not the case. Risk shifting through the tax system in conjunction with a private risk market will also in this case lead to too much insurance and too little effort. This will also be the case even if the government designs a tax system which exactly mimics the optimal contract between the entrepreneur and the equity investor in the no-tax case. This is due to the sequential structure of our model. The external investor will provide an additional contract which does not internalize the increased moral hazard cost of the government.

Figure 3 illustrates another special case of condition (15) when $\alpha > 1$. We see from both Figure 2 and Figure 3 that the utility level of the entrepreneur is increased. This is so because the financial investor always breaks even in our model. The government would not break even however because of the additional moral hazard costs. Hence, the government bears the moral hazard cost due to overinsurance.

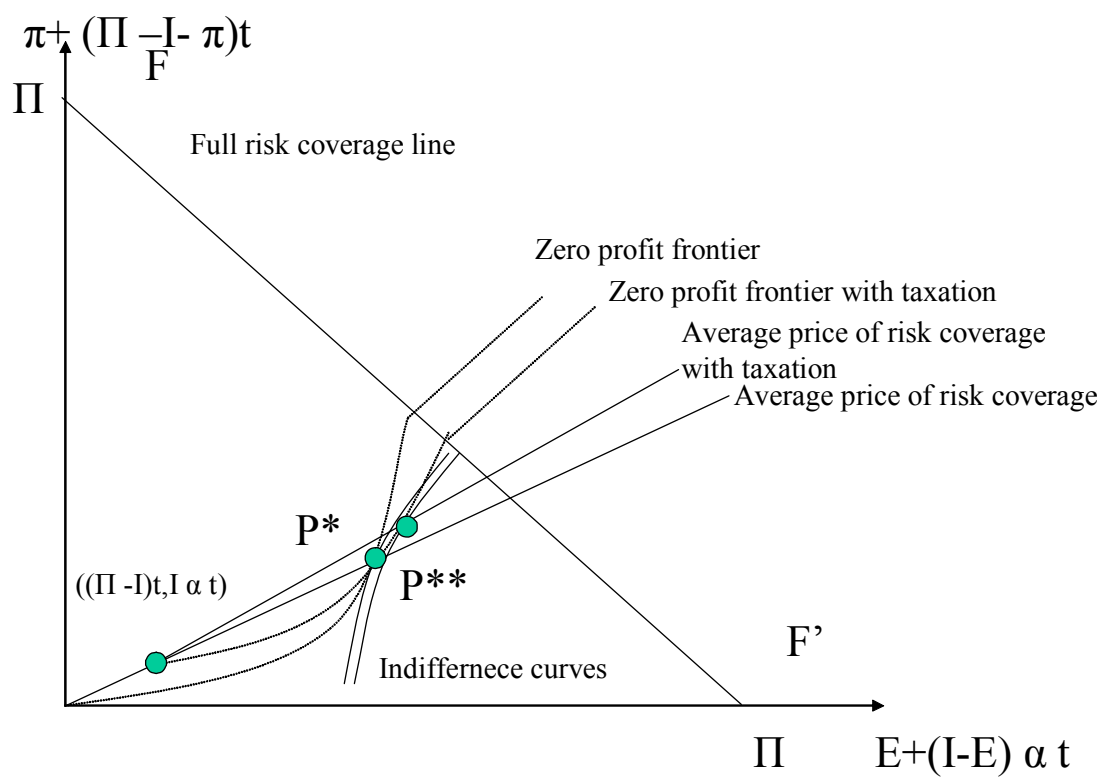


Figure 3:

In Figure 4 we illustrate the profit tax as a risk sharing device at the same implicit price as in a private competitive insurance contract, where the effort level is zero and $\alpha = 1$. If the effort level is zero, the external investor owns the whole company. Because of the zero profit constraint for a risk neutral external investor the expected profit in the good state has to equal expected profit in the bad state. To mimic the private competitive insurance contract the government's expected tax revenue in the good state has to equal expected tax revenue in the bad state. Thus, $\alpha = 1$.

The entrepreneur is risk averse and requires a risk premium to bear some of the idiosyncratic risk in the risk sharing equilibrium. The profit tax reduces this risk premium. The zero profit frontier in the (π, E) - plane must be located to the left of the zero profit curve in the no-tax case, as illustrated in the figure. The utility level of the entrepreneur decreases in the tax case compared to the no-tax case. In the figure the average price of private risk sharing is higher¹⁶ and the entrepreneur's effort level is lower in equilibrium in the tax case. In the exclusive contract case the proportional profit tax will distort the second best allocation of risk in an economy where markets for risk shifting are constrained by the presence of moral hazard. In conjunction with the insurance provided by the financial market and short of complete risk coverage, the proportional profit tax will shift income from the good to the bad state at a higher average implicit price than the financial market. This will lower the entrepreneur's utility level and will lead to too much risk relief and too little effort from an efficiency point of view.

A proportional profit tax, which is actuarially neutral in the absence of moral hazard, see Bond and Devereux (1995), will distort the second best optimum allocation of risk when moral hazard is present. This result is consistent with condition (15) and will lead to too much risk relief and too little effort from an efficiency point of view.

¹⁶The slope of the average price line is higher in the tax case.

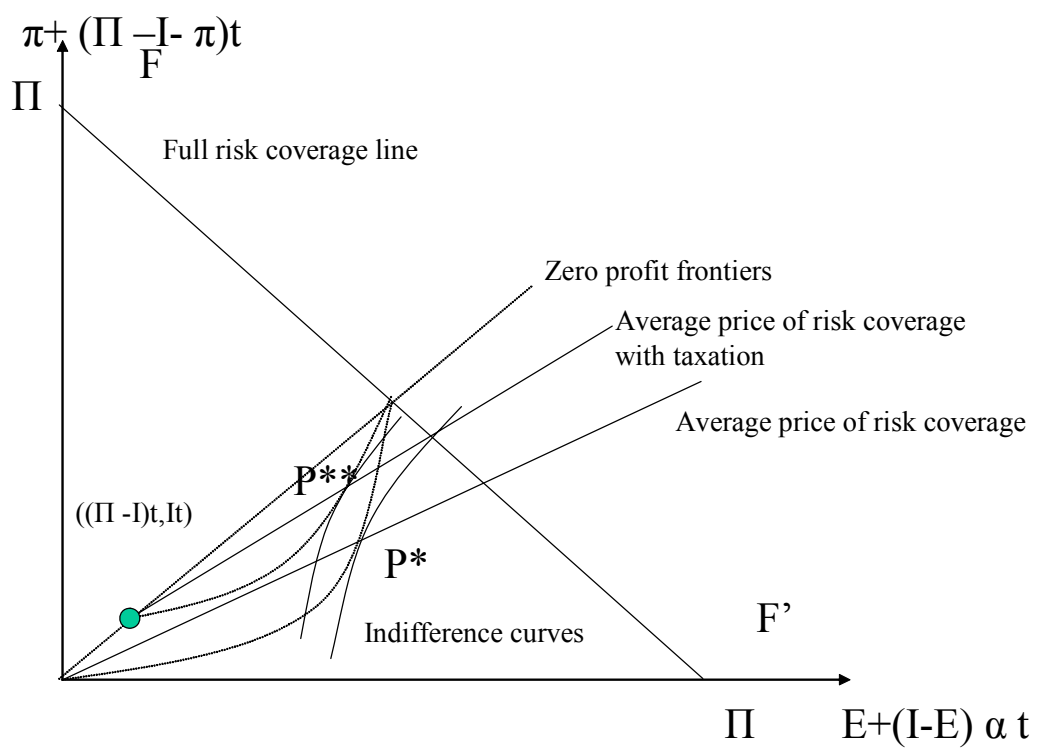


Figure 4:

4 Non-exclusive risk sharing contracts

To the extent that a profit tax acts as a risk sharing device shifting risk from entrepreneurs to the tax payers, the presence of a private market for risk bearing capital in conjunction with the tax effectively means that the exclusivity of the risk sharing relation between the firm and the government cannot be enforced. Thus, as shown in the exclusive contract case, even if the risk sharing through the tax were actually optimal in the first place, i. e. equal to the optimal private contract in the no tax case, a private equity investor would still find it profitable to provide additional risk relief for the entrepreneur as he would only consider the risk pertaining to the after tax profit and disregard the moral hazard costs that fall on the government. Similarly, if external equity investors could not enforce exclusive contracts, the moral hazard costs of the new contract that fall on the previous equity investor are not taken into account when recontracting and will therefore be in the nature of external costs. If the total amount of external equity finance cannot be observed, the possibility of providing supplementary risk relief would tend to lower effort incentives. The general problem is that an optimal contract between two parties is eroded by the possibility of recontracting by a third party, see Bernheim and Whinston (1986).

Kahn & Mookherjee (1998) and Bisin & Guaitoli (2000) discuss moral hazard and non-exclusive contracts in a discrete effort model, while Arnott & Stiglitz (1991b) use a continuous model¹⁷. Kahn & Mookherjee show that if an equilibrium is defined as a state where no profitable supplementary contract exists regardless of the choice of effort, then rationing risk bearing capital might be enforceable even in a situation with non-exclusive contracts.

Following Kahn & Mookherjee (1998) we demonstrate by means of a simple example the interaction between publicly provided risk relief in conjunction with a private market based on this notion of equilibrium. We assume that entrepreneurial effort can be either low or high. The entrepreneur's choice of effort level depends on how much there is to be gained by increasing the probability of success. The greater the risk relief, the smaller the incentive for choosing high effort. The indifference curves showing the trade off between wealth in the success and failure states corresponding to the reduced form utility function being maximized with respect to effort, will

¹⁷There may be a problem with non-existence in a model with continuous effort. Here we only discuss the two effort case.

typically be non-convex.

The FF' line represents full insurance for the entrepreneur and in the area close to this line the entrepreneur will choose low effort. The marginal rate of substitution between wealth in the failure and success state will be less when effort is low compared to when it is high. The reason for this is that high effort is maximizing the probability of the more favorable outcome. Hence, where the indifference curves cross, the entrepreneur will switch from high to low effort. The reduced form indifference curve when effort is optimized, will therefore have a kink where effort switches from high to low. Connecting the switching points gives the borderline SS' where effort is low above this line and high in the area below. In the case of exclusivity the equilibrium risk sharing contract would be one of two types. One possibility would be maximum insurance compatible with high effort, i. e., on the SS' line. The other possibility would be full insurance and minimum effort, i. e., on the FF' line.

In the non-exclusive case the equilibrium is a third best allocation and can take either of two forms in the two effort case. One possibility is full insurance and minimum effort, i.e., on the FF' line. The other possibility is a third best equilibrium compatible with high effort. The latter case is illustrated in Figure 5 for the no tax case at point E. Kahn & Mookherjee (1998) define the third best equilibrium as an equilibrium which is immune to supplementary risk shifting contracts. In the two effort case the high effort equilibrium will be immune to a risk sharing contract at a low effort equilibrium price. We see that the equilibrium at point E in Figure 5 is a third best equilibrium with high effort because this equilibrium is immune to a risk sharing contract at a low effort equilibrium price. The equilibrium at point E is also immune to an additional risk sharing contract at a high effort equilibrium price. If the investors provide higher risk relief than point E, the entrepreneur would prefer an additional risk sharing contract at a low effort equilibrium price. For proof, see Kahn & Mookherjee. A second best equilibrium would be on the borderline for shift of effort, SS'. We see from Figure 5 that the third best equilibrium at point E would give less risk relief than a second best equilibrium on the SS' line.

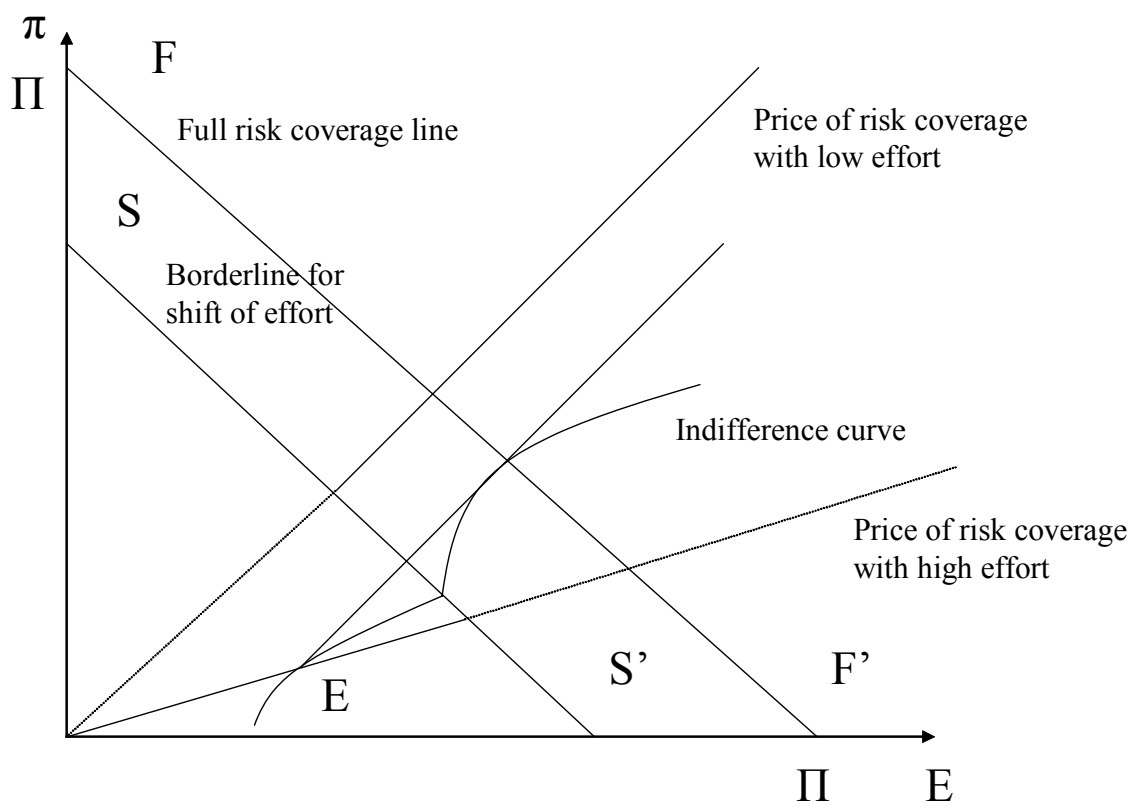


Figure 5:

A proportional profit tax may alter the equilibrium. This is illustrated in Figure 6. The proportional profit tax has to lie on the price line of risk coverage with low effort. Thus, the government provides tax relief at a higher price than the external investor in the high effort case. The entrepreneur's utility from providing high effort will be reduced. Thus, the entrepreneur has to own a larger share of the company to elicit high effort. The tax will reduce the entrepreneur's utility level and may reduce the total risk relief for the entrepreneur from E to E^* . If tax rates are sufficiently high, the tax may shift the equilibrium to a low effort and full insurance equilibrium, in which case there exists no third best equilibrium with high effort. A proportional profit tax will not be neutral with respect to risk exposure in a risk market where external equity investors cannot enforce exclusive contracts. It is also interesting to note that while a proportional profit tax may lead to too much risk relief with exclusive risk sharing contracts, the result may be the opposite with non-exclusive contracts. This is due to the fact that the entrepreneur's owner share of the company has to be increased to ensure that an additional risk sharing contract would not be profitable for the entrepreneur.

4.1 Discussion of the results

In our settings we have shown that a proportional profit tax will not satisfy the criteria for a neutral tax in the presence of moral hazard. This is in contrast to Buchholz and Konrad (2000) and Keuschnigg and Nielsen (2003), who in a model with two effort levels find that a non-redistributive profit tax will be neutral. The analysis of Buchholz and Konrad is within a partial equilibrium model, while Keuschnigg and Nielsen extend and strengthen their result by showing that it also holds in a general equilibrium context. Keuschnigg and Nielsen assume that the tax revenue from workers and entrepreneurs is refunded to the same population group. They assume that the government sets the fiscal policy before the external investor proposes a financial contract to the entrepreneur. They then show that the external investors, having found an optimal risk sharing solution in the first place, will simply undo the extra insurance provided by the government. They prove that it is always possible to replicate the initial situation before taxes. In their proof they implicitly assume that the optimal private risk sharing contract is replication of the initial situation before taxes. This may however not be true. The entrepreneur may have incentives to behave opportunisti-

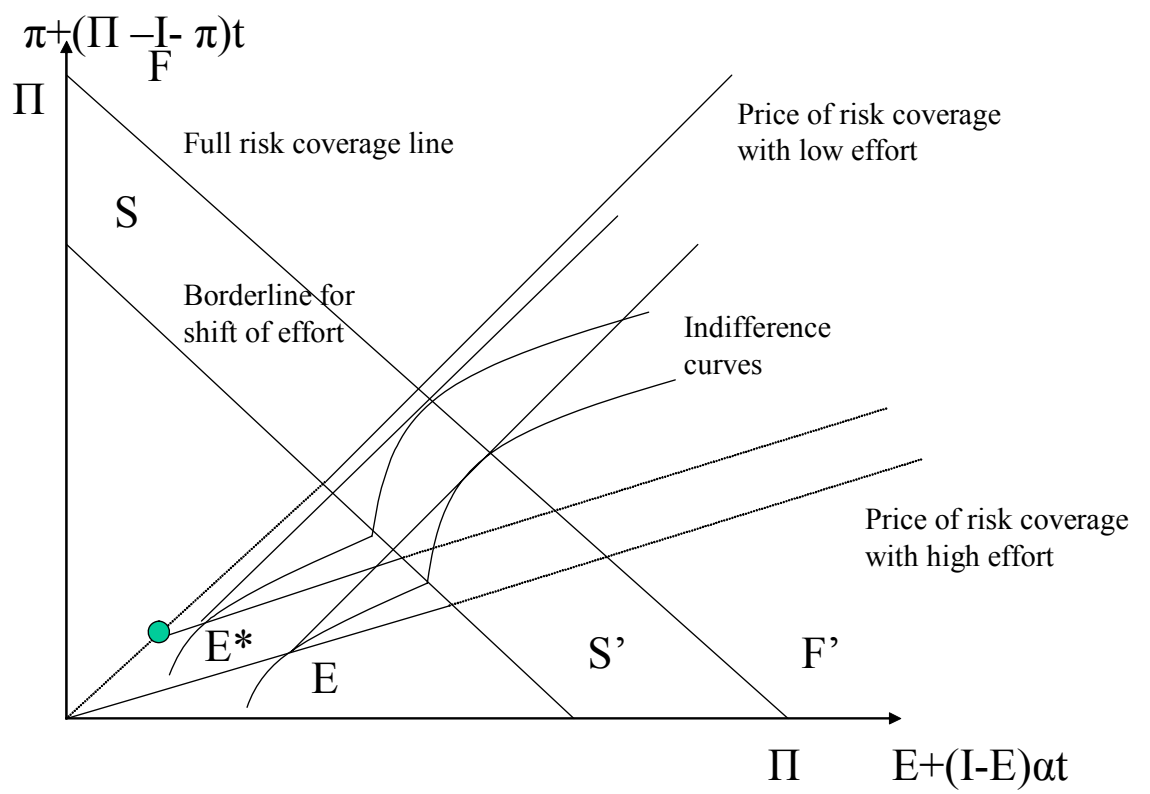


Figure 6:

cally as a "free rider" and try to increase his utility. If he is shirking and the other entrepreneurs are not, he will only reduce his lump sum transfer with T^E/n^E , where n^E is the number of entrepreneurs in the model and T^E is the lump sum transfer. In their model they assume that the entrepreneur will lose the whole lump sum transfer T^E if he is shirking.

In our setting we assume that the tax revenue from the profit tax does not affect the entrepreneurs and the financial investors. Our model is also partial, in that we ignore that the tax might affect prices elsewhere in the economy. As we have seen from Figure 3, even if the risk-sharing through the tax were actually mimicking the private risk-sharing in the no-tax situation, a private equity investor would find it profitable to provide additional risk-sharing as he would ignore the moral hazard costs that fall on the government. Only if the private equity investor and the government coordinate their risk bearing, would they replicate the initial situation before taxes.

The proportional profit tax in our model may be compared with a progressive tax in Keushnigg and Nielsen (2003). In their setting a progressive tax is defined as a redistributive tax, where some part of the tax revenue is distributed to a different group of the population. Our proportional profit tax may be seen as a tax where all the tax revenue is given to another sector of the economy. They find that the redistributive tax reduces the optimal total risk relief for the entrepreneur. Our model gives the opposite results as it is privately optimal to increase the total risk relief since the government bears part of the increased moral hazard costs.

The results of this paper are not affected by the presence of financial opportunity costs to the extent that they are fully tax deductible. They do however depend on how the opportunity costs of effort are modelled. In our paper opportunity costs are not included in the tax base, implying that these costs are not tax-deductible, possible because they are not observable. If the entrepreneur chooses between providing effort in his own firm or employment in another firm, the opportunity cost of effort is normally taxed. In that case of deductibility (15) would change to:

$$\frac{\partial e}{\partial t} = -\frac{-p'(e)U'_s(\Pi - I - \pi) + p'(e)U'_f\alpha(-I + E) + \frac{\partial^2 V}{\partial e \partial t}e(1 - t) + V'}{p''(e)(U_s - U_f) - V''(1 - t)^2}. \quad (16)$$

Then it is no longer clear that a small increase in the tax rate will, *cet. par.*, have a negative effect on the provision of effort.

In related models on tax policy and venture capital the results of Keuschnigg and Nielsen (2000, 2002) differ from ours. In Keuschnigg and Nielsen (2000) the opportunity cost of effort is taxed, and postulating constant relative risk aversion utility function they find no welfare effect from a common tax rate for a wage and capital income tax in a situation where the incentive compatibility constraint ensures that the entrepreneur provides high effort. This result depends on the constant relative risk aversion utility function combined with a comprehensive tax on income due to alternative use of effort and the assumption that the entrepreneur has no funds of his own. When alternative uses of effort are taxed equally along with profits, the tax reduces the entrepreneur's risky profits and his opportunity cost of effort equi-proportionally. Then, given the particular utility function and the assumption of no own funds, the tax is neutral.

In Keuschnigg and Nielsen (2002) the opportunity cost of effort is treated as an extra unobserved private benefit. This is equivalent to the treatment of effort in Keuschnigg and Nielsen (2003) and in our model. Thus, the tax system could mimic the contract in the private financial market and given the assumption that the external investor and the entrepreneur will prefer to replicate the initial situation before taxes, the tax system would be neutral.

A natural definition of a non-exclusive contracts equilibrium is one that is immune to recontracting. In such a setting Kahn & Mookherjee (1998) argue that in the two effort level case the tax system will be crowding out private provision of risk relief exactly. However, this hinges on the assumption that the government provides risk relief to the entrepreneur at the same implicit price as the external equity investor does. In our model a proportional profit tax will absorb risk at a higher implicit price than risk sharing through an external investor in the high-effort case. Then the presence of a proportional profit tax may alter the equilibrium since it is affecting private incentives. When risk markets are incomplete because of moral hazard, the entrepreneurs will have to bear some idiosyncratic risk that would otherwise be washed away in complete risk markets. Thus the tax system will no longer be neutral with respect to equilibrium risk exposure.

5 Concluding remarks

This paper analyzes a risk market, which is incomplete because of moral hazard. If private risk markets coexist with governmental measures for real-

locating risk, implemented through a profit tax, this will generally alter the equilibrium compared with the no-tax situation. If the external equity investors can enforce exclusive contracts with the entrepreneurs, the risk relief through a profit tax would lead to too much risk relief and too low effort compared with a second best optimal solution.

Thus, a proportional profit tax will in our model alter the equilibrium solution and lead to a reduced utility level for the entrepreneur. With exclusive risk-sharing contracts this may lead to too low effort provision, while with non-exclusive contracts the results may be the opposite. These results are in contrast with the results from risk market models without moral hazard. Bond & Devereux (1995) rely on the Value Additivity Principle to ensure the neutrality of a business tax. In our model with moral hazard the celebrated Value Additivity Principle does not apply.

This paper discusses three types of equilibria. In the first best equilibrium we assume that entrepreneurial effort can be observed by the financial investors. In that case the optimum effort level can be enforced as part of the financial contract and recontracting is uninteresting. In the second best equilibrium we assume that the entrepreneurial effort is non-observable. However, we further assume that the financial investors can enforce exclusive risk sharing contracts. Thus, recontracting is prohibited. In the third best equilibrium we assume that the total amount of external equity finance cannot be observed. With non-excludable contracts, recontracting has to be non-profitable in the third best equilibrium. In all three types of equilibria the optimal solutions will be immune against recontracting. However, due to different assumptions in the three types of equilibria the entrepreneurs' optimal risk relief will differ and a proportional profit tax may alter the equilibria in different ways.

References

- [1] Arnott, R. J. & Stiglitz, J. E. (1988): The basic analytics of moral hazard. *Scandinavian Journal of Economics* 90(3).
- [2] Arnott, R. J. & Stiglitz, J. E. (1991a): Price equilibrium, efficiency, and decentralizability in insurance markets. NBER Working Paper No. 3642.

- [3] Arnott, R. J. & Stiglitz, J. E. (1991b): Equilibrium in competitive insurance markets with moral hazard. NBER Working Paper No. 3588.
- [4] Bernheim, B. D. & Whinston, M. D. (1986): Common agency. *Econometrica*, Vol 54.
- [5] Bisin, A. & Guaitoli, D. (2000): Moral hazard and non-exclusive contracts. Mimeo.
- [6] Bond, S. R. & Devereux, M. P. (1995): On the design of a neutral business tax under uncertainty. *Journal of Public economics* 58.
- [7] Buchholz, W. & Konrad, K. A. (2000): Risiko und Steuern. In: Andel, Norbert (Ed.), *Probleme der Besteuerung III*, Schriften des Vereins für Socialpolitik, N. F. Band 259/III. Duncker and Humblot, Berlin.
- [8] Fane, G. (1987): Neutral taxation under uncertainty. *Journal of Public Economics* 33.
- [9] Kahn, C. M. & Mookherjee, D. (1998): Competition and incentives with non-exclusive contracts. *Rand Journal of Economics*, Vol. 29, No. 3.
- [10] Kaplow, L. (1991): Incentives and government relief for risk. *Journal of Risk and Uncertainty* 4.
- [11] Keuschnigg, C. & Nielsen, S. B. (2000): Tax policy, venture capital and entrepreneurship. CEPR Discussion Paper No. 2626.
- [12] Keuschnigg, C. & Nielsen, S. B. (2002): Tax policy, venture capital and entrepreneurship. *Journal of Public economics* 87.
- [13] Keuschnigg, C. & Nielsen, S. B. (2003): Progressive taxation, moral hazard, and entrepreneurship, Mimeo.
- [14] Sandmo, A. (1979): Differential taxation and the encouragement of risk-taking, *Economic Letters* 31.
- [15] Shavell, S. (1979): On moral hazard and insurance. *Quarterly Journal of Economics*, 93.
- [16] Stiglitz, J. E. (1983): Risk, incentives and insurance: The pure theory of moral hazard. *The Geneva Papers of Risk and Insurance*, 8.