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Measuring Asymmetries in Financial Returns: An Empirical Investigation Using Local Gaussian Correlation

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by

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Measuring Asymmetries in Financial Returns: An Empirical Investigation Using Local Gaussian Correlation

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Abstract

A number of studies have provided evidence that financial returns exhibit asymmetric dependence, such as increased dependence during bear markets, but there seems to be no agreement as to how such asymmetries should be measured. We introduce the use of a new measure of local dependence to study this asymmetry. The central idea of the new approach is to approximate an arbitrary bivariate return distribution by a family of Gaussian bivariate distributions. At each point of the return distribution there is a Gaussian distribution that gives a good approximation at that point. The correlation of the approximating Gaussian distribution is taken as the local correlation in that neighbourhood. The new measure does not suffer from the selection bias of the conditional correlation for Gaussian data, and is able to capture nonlinear dependence. Analysing several financial returns from the US, UK, German and French market, we confirm and are able to explicitly quantify the asymmetry. Finally, we discuss a risk management application, and point out a number of possible extensions.

1 Introduction

It is well documented that there are asymmetries in the distribution of financial returns (Silvapulle and Granger 2001; Okimoto 2008; Ang and Chen 2002; Hong et al. 2007; Chollete et al. 2009; Garcia and Tsafack 2011). For example, there are often stronger dependence between returns of financial objects when the market is going down than when it is stable or going up. It does not seem to be consensus, however, as to how the asymmetries should be measured, quantitatively interpreted and tested for. In this paper we present a new approach for analysing these issues.

One popular approach to analysing asymmetry has been to use the conditional correlation. For time series $\{X_t\}$ and $\{Y_t\}$ of two (log) returns, the *conditional correlation* between X and Y when they are restricted to a set A is

$$\rho_A = \mathrm{corr}(X_t, Y_t \mid (X_t, Y_t) \in A), \tag{1}$$

where we assume at least stationarity for the series.

We define the *exceedance correlation* as the conditional correlation with $A = (-\infty, -c] \times (-\infty, -c]$ or $A = [c, \infty) \times [c, \infty)$, where $c > 0$ is the exceedance level, and define *truncated correlations* similarly, but where A only restricts one of the returns. An estimate of ρ_A can be obtained by the ordinary correlation estimator applied to truncated variables.

The set A can be, and has been, chosen in many ways, and rather different results are obtained depending on the conditioning strategy used (Campbell et al. 2008). The multitude of conditioning methods used, and corresponding differences in interpretation, have contributed to conditional correlation analysis being considered as quite problematic despite its immediate intuitive appeal.

As an example, let us assume a bivariate Gaussian distribution with correlation $\rho = 0.40$ for some returns data. Using results from Boyer et al. (1999) on truncated correlations, and conditioning on one of the returns being large, for example larger than its 75% quantile, $A = \{(X, Y) \mid X > q_{75}\}$, the conditional correlation is reduced to $\rho_A = 0.21$ (and tending to zero for increasing quantiles), while the conditional correlation on the complement set, $\rho_{A'}$, is reduced to 0.30 (and tending to 0.40 for increasing quantiles). When analysing (Gaussian) returns data and detecting such correlation values, one might be tempted to incorrectly

conclude that the dependence has been lower in periods of large X-returns, and that it goes to zero as $X \to \infty$.

Conditioning on *both* returns being large or small (exceedance correlation), or lying in various finite sets, again leads to different correlation values (see Campbell et al. 2008). As remarked by Longin and Solnik (2001), '[i]t would be wrong to infer from this large difference in conditional correlation that correlation differs between volatile and tranquil periods, as correlation is constant and equal to [ρ] by assumption'.

In spite of the interpretational difficulties the bias effect has been sought adjusted for (e.g., Campbell et al. 2002), and it is still possible to construct useful tests of asymmetry in terms of conditional correlation; see Ang and Chen (2002). The test used in that paper was model-dependent, but Hong et al. (2007) have designed a test based on a model-free approach, and put it to effective use in a number of applications.

There are several alternative methods of studying asymmetry of financial returns. Silvapulle and Granger (2001) have looked at various quantile estimation methods, and Longin and Solnik (2001) have used extreme value theory to show that for monthly data there is a bear effect, but no bull effect. Okimoto (2008) and Rodriguez (2007) have employed regime-switching copulas to study asymmetric dependence for various international stock indices, while Aas et al. (2009) and Nikoloulopoulos et al. (2012) have used vine copulas (also called the pair-copula construction) to model multivariate financial return data. Related work is Ang and Bekaert (2002) and Ang and Chen (2002), who have based themselves on Markov regime structures with ARCH–GARCH modelling.

Also, in the statistical literature there exist methods for modelling and measuring local dependence. Bjerve and Doksum (1993) (see also Blyth 1994) proposed a measure based on localising a regression model, but this has the problem of not being symmetric in (X, Y), so that $\rho_{X,Y} \neq \rho_{Y,X}$. Holland and Wang (1987) suggested a symmetric measure based on limiting arguments starting from the odds ratios in a contingency model, but its range is not from -1 to 1, and it does not reduce to the ordinary correlation – but is a function of it – in the bivariate normal case. Further work on this measure has been done by Jones (1996, 1998) and Jones and Koch (2003). However, none of these measures have had much influence on the finance literature. Other global and local measures of dependence can be found in the books of Drouet Mari and Kotz (2001) and Joe (1997).

A common feature for many of the alternative approaches is that one ends up with one or

more parameters that have a rather indirect interpretation as a measure of dependence. In this respect, correlation has a more natural basis, and in this paper we present an approach based on the correlation concept, but from a very different angle than that of the conditional correlation.

To introduce our new point of view, we first return to the conditional correlation ρ_A in Equation (1), and note that in principle we can compute it for any set A, and if we have sufficient number of observations, we can let the size of A be small, and in this way obtain a conditional correlation in a small neighbourhood of a point (x, y). Such a localisation is a basic feature of nonparametric analysis (see, for instance, Scott 1992), where the size of A would be determined by a pair of bandwidths (b_1, b_2) in the x and y direction, respectively. Asymptotically, as the number of observations increases, b_1 and b_2 can be made smaller, making it possible to describe ever more fine details of the phenomenon under study.

Similar localisation is crucial in our new approach, but we do not use the conditional correlation as defined in (1). The central idea is to approximate the general return density f of (X, Y) at a point (x, y) by a bivariate Gaussian density $\psi_{x,y}$. We take the correlation $\rho(x, y)$ of that Gaussian density as our measure of local dependence, and we call it the *local Gaussian correlation*. Some of its theoretical properties are given in Tjøstheim and Hufthammer (2013), one important property in the present context being that it is constant over all (x, y) for a bivariate Gaussian distribution.

In Section 2 we explain this new approach in more detail in a return distribution context, and demonstrate that it has a number of advantages compared to conditional correlation, while Section 3 presents a brief overview of the theory. In Section 4 we apply it to stock market indices such as S&P 500, FTSE 100, CAC 40 and DAX 30, using both monthly and daily data. We present a risk management application in Section 5, and in Section 6 we conclude, discuss limitations and point out possible extensions.

2 Local Gaussian approximation and local correlation

The correlation ρ is primarily meaningful for a bivariate Gaussian density,

$$\psi\Big(u, v, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho\Big) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\Bigg\{-\frac{1}{2\Big(1-\rho^2\Big)}\Bigg[\left(\frac{u-\mu_1}{\sigma_1}\right)^2 + \left(\frac{v-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{u-\mu_1}{\sigma_1}\right)\left(\frac{v-\mu_2}{\sigma_2}\right)\Bigg]\Bigg\}. \quad (2)$$

There are several precise interpretations of the correlation ρ or its estimate (see, for example, Rodgers and Nicewander 1988; Rovine and von Eye 1997), but, most importantly, it completely describes the dependence structure of a pair of random variables (U, V) having ψ as its density, and it is invariant to linear transformations.

The bivariate return density f for two returns, X and Y, is never Gaussian, and in the conditional correlation approach one tries to take care of this by computing the ordinary (sample) correlation restricted to a set A, as described in the introduction, but we believe that it is better to start with the density f itself, and approximate *it*, not the correlation, locally. This local approximation is done with a family of Gaussian distributions such that at each point (x, y), the density $f(x, y)$ is approximated (in a sense which will be made precise below) by a Gaussian bivariate density,

$$\psi_{x,y} = \psi\Big(u, v, \mu_1(x,y), \mu_2(x,y), \sigma_1(x,y), \sigma_2(x,y), \rho(x,y)\Big) = \frac{1}{2\pi\sigma_1(x,y)\sigma_2(x,y)\sqrt{1-\rho(x,y)^2}} \exp\Bigg\{-\frac{1}{2\Big(1-\rho(x,y)^2\Big)} \times \Bigg[\left(\frac{u-\mu_1(x,y)}{\sigma_1(x,y)}\right)^2 + \left(\frac{v-\mu_2(x,y)}{\sigma_2(x,y)}\right)^2 - 2\rho(x,y)\left(\frac{u-\mu_1(x,y)}{\sigma_1(x,y)}\right)\left(\frac{v-\mu_2(x,y)}{\sigma_2(x,y)}\right)\Bigg]\Bigg\}, \quad (3)$$

where the parameters depend on (x, y), and in such a way that $\psi_{x,y}$ is close to f in a

neighbourhood A of (x, y), but not necessarily elsewhere. As we move to another point (x', y') of f, another Gaussian $\psi_{x',y'}$ is required to approximate f in a neighbourhood A' of (x', y'). The correlation $\rho(x, y)$ completely characterises the dependence structure of $\psi_{x,y}$, and since $\psi_{x,y}$ is close to f in A, it approximates the complete dependence structure of f in A (but again not necessarily elsewhere). In this way the dependence in f is described (given the appropriateness of the local fitting) by the family of Gaussian distributions $\{\psi_{x,y}\}$ and the associated correlations $\{\rho(x, y)\}$.

As noted in Tjøstheim and Hufthammer (2013) the representation in (3) is not well-defined unless it is the result of a minimization of a penalty function that measures the distance between $\psi_{x,y}$ and f in a neighbourhood of (x, y). Such a neighbourhood can be introduced by a kernel function K and two bandwidths b_1, b_2 and a local penalty function

$$q = \int K_{b_1}(u - x) K_{b_2}(v - y) \Big[\psi_{x,y}(u, v, \theta(x, y)) - \log \psi_{x,y}(u, v, \theta(x, y)) f(u, v) \Big] \mathrm{d}u \mathrm{d}v, \qquad (4)$$

where $K_{b_1}(u - x) = b_1^{-1} K(b_1^{-1}(u - x))$ and similarly for K_{b_2}. Moreover, $\theta(x, y) = [\mu_1(x, y), \mu_2(x, y), \sigma_1(x, y), \sigma_2(x, y), \rho(x, y)]$. The penalty function q is measuring a sort of Kullback-Leibler distance between $\psi(\cdot, \theta(x, y))$ and $f(\cdot)$, and the population parameter $\theta = \theta_b(x, y)$ depending on b is defined as the minimizer of q. A population parameter $\theta = \theta(x, y)$ can subsequently be defined by letting $b \to 0$, this giving a mathematical interpretation to the representation in (3). For more details we refer to Tjøstheim and Hufthammer (2013) and Hjort and Jones (1996) who have considered the minimization of q in another context.

To find an estimate of θ (or θ_b), we require a method for fitting a Gaussian $\psi_{x,y}$ to f in a neighbourhood of (x, y) given observed data, and one such method is local likelihood, as described in Hjort and Jones (1996), and which was applied to local Gaussian approximations in Tjøstheim and Hufthammer (2013). To keep this paper reasonably self contained, we give a brief outline of this method in the next section, and refer to the cited publications for further details. The chief objective in the present paper is to demonstrate the usefulness of the local correlation through a series of empirical examples using financial returns from major markets.

Before embarking on the technicalities of Section 3, note the following advantages of applying the local Gaussian correlation model as a description of asymmetries in financial

data.

1. The dependence measure is based on a family of Gaussian distributions, and describes the dependence relation for $\psi_{x,y}$ and hence for f at the point (x, y), since $\psi_{x,y}$ approximates f at that point. Moreover, properties that are true for global Gaussian dependence can be transferred locally in a neighbourhood of (x, y).

2. Using local Gaussian likelihood theory (summarised in Section 3) we can construct asymptotic confidence intervals for $\rho(x, y)$, allowing us to judge whether an observed asymmetry for financial returns measured by $\hat{\rho}(x, y)$ is statistically significant.

3. Unlike the conditional correlation and similar local dependence measures, in the case that f itself is Gaussian, $\rho(x, y) \equiv \rho$ everywhere, where ρ is the ordinary correlation of f. This follows from the definition of ρ (and ρ_b) via the minimization of q and it is demonstrated for one realisation of 3500 bivariate Gaussian observations with standard normal marginals and global correlation $\rho = 0.5$ in Figure 1. Here ρ has been estimated by the local likelihood procedure of Section 3. Thus ρ does not suffer from the bias problem of the conditional correlation described in the introduction. There is some boundary bias close to the edges of the data set, though. This bias is similar to boundary bias in ordinary kernel estimation, see e.g. Jones (1993), and its size and direction can be explained in an analogous fashion.

4. As will be seen in Section 4, the local Gaussian correlation $\rho(x, y)$ is capable of detecting and quantifying asymmetries in financial returns such as bull and bear effects. Moreover, a quantitative interpretation can be given in terms of the strength of the correlation of the approximating local Gaussian distribution. Also, $\rho(x, y)$ can be applied to obtain generalisations of classical portfolio theory, and to study contagion effects of financial markets, see Støve et al. (2012).

5. It is possible to generalise the local Gaussian approach to a set of d variables, financial returns $(X_1, \ldots, X_d)$, having a joint density function f. The localised correlation $\rho(x, y)$ is then replaced by a local covariance matrix $\Sigma = [\sigma_{ij}(x_i, x_j)]$ for $i, j = 1, \ldots, d$. Note that to avoid the curse of dimensionality in the estimation procedure, for each pair of variables (x_i, x_j) the local covariance at the point $(X_1 = x_1, \ldots, X_d = x_d)$ have been restricted to depend only on (x_i, x_j). This is analogous to the simplification obtained

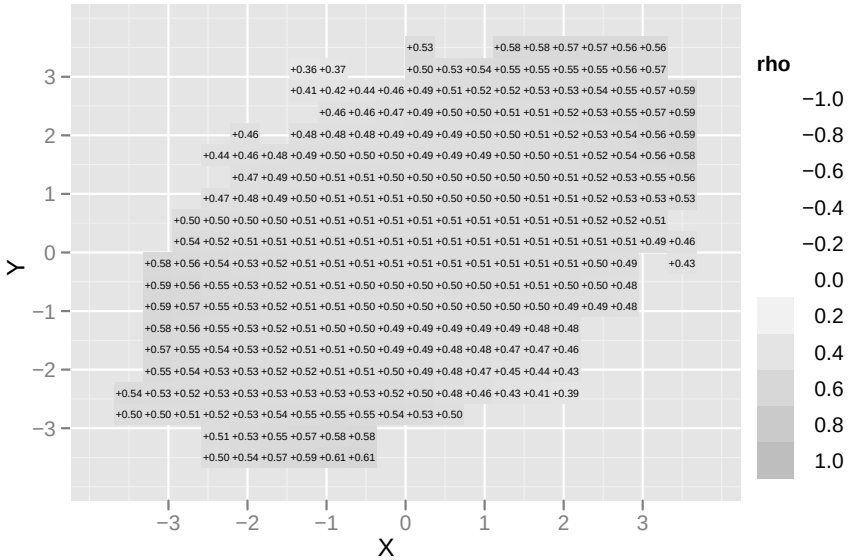


Figure 1: Local Gaussian correlation map for a bivariate Gaussian distribution with correlation $\rho = 0.5$, based on 3500 observations.

by using additive models for regression problems of high dimensions (cf. Hastie and Tibshirani 1990). See also Mammen et al. (2009) and Teräsvirta et al. (2010). Further properties of local covariance matrix modelling are currently being investigated.

3 A brief summary of local likelihood theory

Given the observations $\{(X_1, Y_1), \ldots, (X_T, Y_T)\}$, the ordinary (standardised) log likelihood for a density f is given by

$$L^\star = \frac{1}{T} \sum_{t=1}^{T} \log f(X_t, Y_t).$$

With ρ as in Equation (2), the maximum likelihood estimate is

$$\widehat{\rho} = \frac{\sum (X_t - \overline{X})(Y_t - \overline{Y})}{\left(\sum (X_t - \overline{X})^2 \sum (Y_t - \overline{Y})^2\right)^{1/2}}.$$

As in the preceding section we introduce kernel functions $K_{b_1}(X_i - x)$ and $K_{b_2}(Y_i - y)$ to describe a neighbourhood A around (x, y). One might think that the appropriate local likelihood associated with (3) would be given by

$$L' = \frac{1}{T}\sum_{t=1}^{T} K_{b_1}(X_t - x)K_{b_2}(Y_t - y)\log\psi_{x,y}(X_t, Y_t),$$

using the kernel function to localise the log likelihood, but it turns out (cf. Hjort and Jones (1996)) that an adjustment is needed, resulting in the local log likelihood

$$\begin{aligned} L = \frac{1}{T}\sum_{t=1}^{T} & K_{b_1}(X_t - x)K_{b_2}(Y_t - y)\log\psi_{x,y}(X_t, Y_t) \\ & - \int K_{b_1}(u - x)K_{b_2}(v - y)\psi_{x,y}(u, v)\,\mathrm{d}u\,\mathrm{d}v. \end{aligned}$$

It is seen that by letting $T \to \infty$ and using the law of large numbers or the ergodic theorem, in case (X_t, Y_t) is ergodic, then $-L$ will converge towards the penalty function q defined in Equation (4).

Again, letting $\theta(x, y)$ be the 5-dimensional parameter vector of $\psi_{x,y}$, and letting $w_j(u, v, \theta)$ denote the derivative $\partial\log\psi_{x,y}(u, v, \theta)/\partial\theta_j$, it is easily seen that

$$\frac{\partial L}{\partial\theta_j} = \frac{1}{T}\sum_{t=1}^{T} K_{b_1}(X_t - x)K_{b_2}(Y_t - y)w_j(X_t, Y_t, \theta) \tag{5}$$

$$- \int K_{b_1}(u - x)K_{b_2}(v - y)w_j(u, v, \theta)\psi_{x,y}(u, v, \theta)\,\mathrm{d}u\,\mathrm{d}v. \tag{6}$$

Letting $T \to \infty$ and again using the law of large numbers (or the ergodic theorem) on the term on the right hand side of Equation (5) the expression for $\partial L/\partial\theta_j$ converges towards

$$\int K_{b_1}(u - x)K_{b_2}(v - y)w_j(u, v, \theta)[f(u, v) - \psi_{x,y}(u, v, \theta)]\,\mathrm{d}u\,\mathrm{d}v.$$

For small bandwidths, under appropriate smoothness conditions, and requiring $\partial L/\partial\theta_j = 0$ for all j, we have

$$w_j\Big(x, y, \theta(x, y)\Big)[f(x, y) - \psi_{x,y}\Big(x, y, \theta(x, y)\Big)] + O(b^{\mathrm{T}}b) = 0,$$

and the local likelihood estimates, satisfying $\partial L/\partial\theta_j = 0$, constrains $\psi\Big(u, v, \theta(x, y)\Big)$ to be close to $f(x, y)$ when (u, v) is close to (x, y). This is the sense in which the family $\psi_{x,y}$ approximates f as the neighbourhood defined by the bandwidth $b = [b_1, b_2]$ shrinks, which is also obtained by differentiating the penalty q of Equation (4).

In practice we obtain the estimates $\widehat{\theta}_{b,T}(x, y)$ by requiring Equation (6) to be zero, and solving the resulting 5-dimensional set of equations numerically. Note that we then obtain not only a local correlation estimate $\widehat{\rho}_b(x, y)$, but also local mean estimates $\widehat{\mu}_{1,b}(x, y)$, $\widehat{\mu}_{2,b}(x, y)$, and local variances $\widehat{\sigma}^2_{1,b}(x, y)$ and $\widehat{\sigma}^2_{2,b}(x, y)$, where the latter can be used to obtain local covariance estimates.

Letting b be fixed and T tend to infinity, $\widehat{\theta}_b(x, y)$ converges in distribution to $\theta_b(x, y)$, satisfying $\frac{\partial q}{\partial\theta} = 0$, with q as in Equation (4), that is θ_b is defined by

$$\int K_{b_1}(u - x)K_{b_2}(v - y)w_j\Big(u, v, \theta_b(x, y)\Big) \times [f(u, v) - \psi\Big(u, v, \theta_b(x, y)\Big)]\,\mathrm{d}u\,\mathrm{d}v = 0.$$

In practice we can check the quality of the Gaussian approximation by comparing $\psi(x, y, \widehat{\theta}_b(x, y))$ to $\widetilde{f}(x, y)$, the kernel estimate of f. (In fact in Hjort and Jones (1996) the main point of the local likelihood analysis is to derive alternatives to the kernel estimate of f.) Arguments in Hjort and Jones (1996), Tjøstheim and Hufthammer (2013) demonstrate that $\widehat{\theta}_b(x, y)$ is asymptotically normal such that

$$(Tb_1b_2)^{1/2}[\widehat{\theta}_{T,b}(x, y) - \theta_b(x, y)] \xrightarrow{\mathrm{d}} \mathcal{N}(0, J_b^{-1}M_b(J_b^{-1})^{\mathrm{T}}),$$

where

$$\begin{aligned} J_b = &\int K_{b_1}(u - x)K_{b_2}(v - y)w\Big(u, v, \theta_h(x, y)\Big) \\ &\quad \times w^{\mathrm{T}}\Big(u, v, \theta_b(x, y)\Big)\psi\Big(u, v, \theta_b(x, y)\Big)\,\mathrm{d}u\,\mathrm{d}v \\ &- \int K_{b_1}(u - x)K_{b_2}(v - y)\nabla w\Big(u, v, \theta_b(x, y)\Big) \\ &\quad \times \Big[f(u, v) - \psi\Big(u, v, \theta_b(x, y)\Big)\Big]\,\mathrm{d}u\,\mathrm{d}v \end{aligned} \tag{7}$$

and

$$
\begin{aligned}
M_b = b_1 b_2 \int & K_{b_1}^2(u-x) K_{b_2}^2(v-y) w\Big(u,v,\theta_b(x,y)\Big) \\
& \times w^{\mathrm{T}}\Big(u,v,\theta_b(x,y)\Big) f(u,v)\,\mathrm{d}u\,\mathrm{d}v \\
& - b_1 b_2 \int K_{b_1}^2(u-x) K_{b_2}^2(v-y) w\Big(u,v,\theta_b(x,y)\Big) f(u,v)\,\mathrm{d}u\,\mathrm{d}v \\
& \times \int K_{b_1}^2(u-x) K_{b_2}^2(v-y) w^{\mathrm{T}}\Big(u,v,\theta_b(x,y)\Big) f(u,v)\,\mathrm{d}u\,\mathrm{d}v. \qquad (8)
\end{aligned}
$$

These expressions are somewhat deceptive, since $J_b^{-1} M_b J_b^{-1}$ is of order $(b_1 b_2)^{-2}$, so that $\mathrm{Var}[\widehat{\theta}_{T,b}(x,y)]$ is of order $(T b_1^3 b_2^3)^{-1}$, a considerably slower convergence rate than the traditional nonparametric rate of $(T b_1 b_2)^{-1}$, when $b_1, b_2 \to 0$. This limiting process is needed when the asymptotic distribution of $\widehat{\theta}_{T,b}$ is considered as $T \to \infty$ and $b_1, b_2 \to 0$. We refer to Tjøstheim and Hufthammer (2013) for more details.

The leading term in the covariance expression $J_b^{-1} M_b J_b^{-1}$ is quite problematic to evaluate, and in practice we have used two alternative methods. The obvious alternative is to use the bootstrap, but then we have to assume iid observations. The other alternative is to estimate (7) and (8) directly using a mixture of numerical integration and empirical averages, with estimates of the parameters inserted. The latter method has the advantage that it can be extended to the case of stationary observations. The block bootstrap would be another alternative for handling the stationary non-iid case. Simulations in Berentsen et al. (2012), Berentsen and Tjøstheim (2012) and Tjøstheim and Hufthammer (2013) demonstrate that for a reasonable smooth f good results can be obtained for T in the range going down to 250-500 observations.

4 Empirical analysis of dependence of financial returns

In this section we apply the local Gaussian correlation to describe daily and monthly returns from financial markets. Confidence intervals are computed using the bootstrap and this assumes that the return pairs are independent and identically distributed with an absolute continuous multivariate distribution. Empirical facts suggest that the variance of the returns

may be dependent in time (see, for instance, the references listed in Bollerslev et al. 1992), and we therefore filter some of the return series by estimating GARCH(1,1) models, and then analyse the standardised residuals. But as will be demonstrated, for some of the data in this section, the filtering has only a moderate impact on the main results.

4.1 Daily data

We use daily international equity price index data for the United States (i.e. S&P 500), the United Kingdom (i.e. FTSE 100), France (i.e. CAC 40) and Germany (i.e. DAX 30) in local currency. The data are obtained from Datastream, and the returns are defined as 100 times the change in the natural logarithm of each market's price index, ie. as $r_t = 100 \times \Big(\log(p_t) - \log(p_{t-1})\Big)$, where p_t is the index from Datastream.

These markets have earlier been studied by Longin and Solnik (2001) (only monthly data), Campbell et al. (2002) and Campbell et al. (2008), but we now use the observation span from July 7th 1987 to July 28th 2011, a total of 6275 daily observations. As a first step we treat these return data as coming from the same bivariate density f, i.e. stationarity. Stationarity, of course, depend on the nature of the time scale and the length of the time period involved. We will show towards the end of this subsection, that when the time preiod is divided into 5 periods, significant differences in the shape of $\rho(x, y)$ will arise.

The price indices, where we normalise the indices to 100 on 7th July 1987, and the daily returns are shown in Figure 2 and 3, respectively. Summary statistics for the returns are shown in Table 1.

Statistic	S&P 500	FTSE 100	CAC 40	DAX 30
Mean	0.02	0.01	0.01	0.03
Variance	1.41	1.29	1.92	2.09
Skewness	−1.36	−0.52	−0.14	−0.33
Kurtosis	33.19	13.55	8.91	9.87
Maximum	10.96	9.38	10.59	10.80
Minimum	−22.83	−13.03	−10.14	−13.71

Table 1: Summary index statistics for daily data from July 1987 to July 2011.

The daily returns range from −22.8% to almost 11%. All series have a small negative skewness, and the kurtosis is generally high, between 8 and 33 (we use the kurtosis definition

Figure 2: Market indices July 1987 to July 2011 for France, Germany, the United Kingdom and the United States normalised to 100 at the start date.

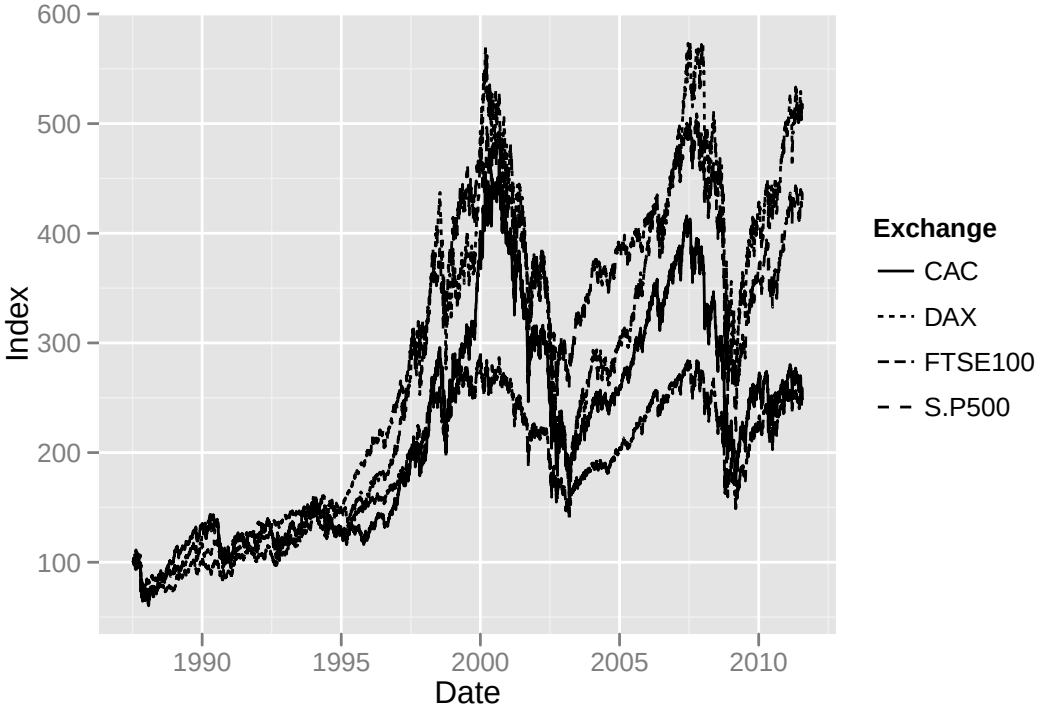
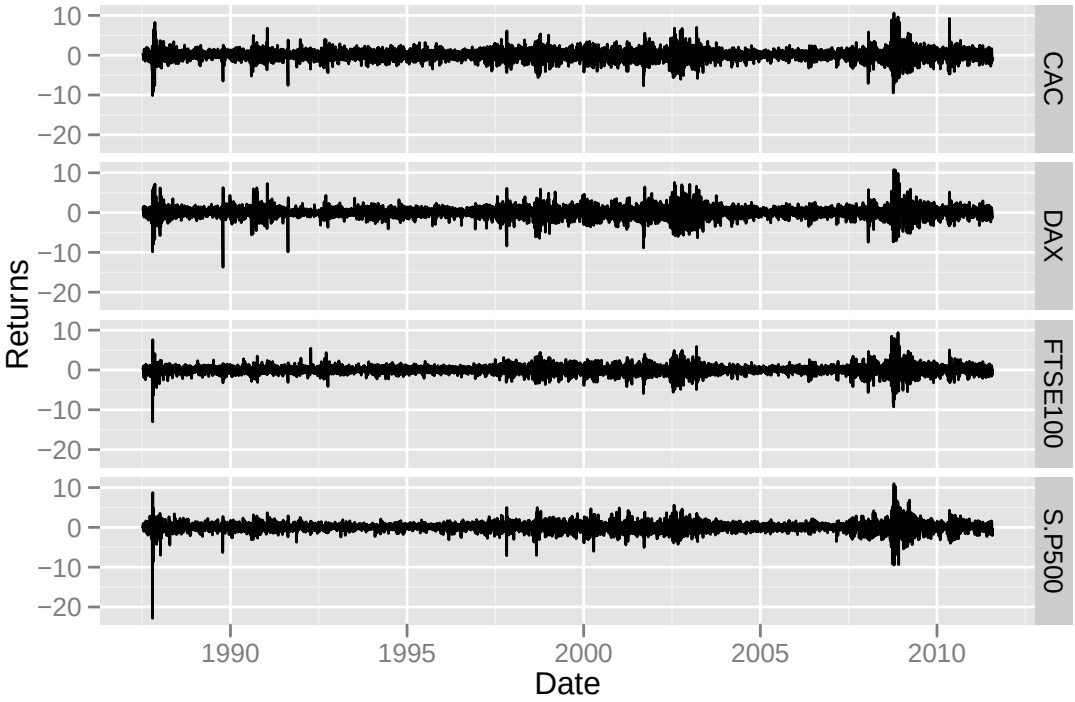


Figure 3: Non-standardised returns for the market indices for France, Germany, the United Kingdom and the United States (in this order, from top to bottom).



	S&P 500	FTSE 100	CAC 40	DAX 30
S&P 500	1.00			
FTSE 100	0.48	1.00		
CAC 40	0.47	0.78	1.00	
DAX 30	0.48	0.70	0.79	1.00

Table 2: Correlation of index returns for daily data from July 1987 to July 2011.

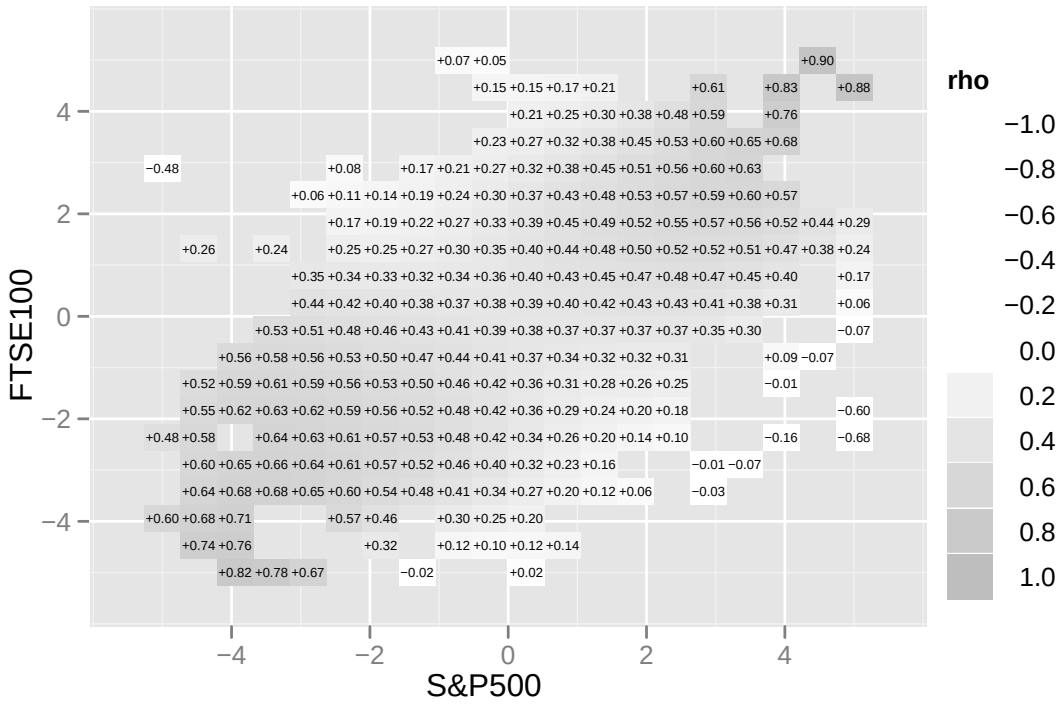
where a normal distribution has kurtosis 0). The high kurtosis values indicate a departure from the Gaussian distribution, and in these cases the global correlation may not be a good measure of dependence ((see Campbell et al. 2008)). Table 2 shows the correlation estimates for the returns. The correlations between the European indices are between 0.7 and 0.8, while the correlations between the European indices and S&P 500 are much lower, around 0.5. These estimates are in agreement with results from the previously mentioned studies.

We next turn to estimates of the local Gaussian correlation. As mentioned in Section 2, the estimator of local correlation depends on two smoothing parameters (bandwidths). They are for now choosen by a simple method (i.e. the standard deviation times a constant), and using a visual check to ensure that the bandwidths should be chosen such that $\psi(x, y, \hat{\theta}_b(x, y))$ does not deviate too much from the ordinary kernel estimate $\widetilde{f}(x, y)$ of $f(x, y)$. More formal methods for choosing the bandwidth have been described in Tjøstheim and Hufthammer (2013) and Berentsen and Tjøstheim (2012), but our experience with the present data is that the informal method used here is reliable, and the kind of results presented do not depend critically on a more advanced choice of bandwidth. In fact, we believe that our main conclusions do not depend on the choice of bandwidth as long as it is within a reasonable range. Note that if the bandwidhts are very large, the local correlation estimates can be shown to converge to the global correlation in all gridpoints.

Figure 4 shows the estimated local Gaussian correlation between the returns from S&P 500 and FTSE 100. Comparing to Figure 1 and to the error limits of Figure 5, it is seen that the bivariate return distribution is not Gaussian. In particular, there are large local correlations for both large negative and large positive returns. The large local correlations for negative returns, imply that diversification opportunities would erode in bear markets (e.g. a downward price trend), which is when they are needed the most; see e.g. Butler and Joaquin (2002).

To construct confidence intervals, we use bootstrap simulations (Efron and Tibshirani 1993), and assume iid observation pairs. The actual asymptotics of the local correlation estimator is complex; see Tjøstheim and Hufthammer (2013). The approximate 5% lower and 95% confidence limits from 1000 bootstrap replications are shown in Figure 5, top and bottom plot, respectively. Since we are mainly interested in data for fixed quantiles of the two series, and returns data are basically measured on the same scale, for greater clarity we may restrict ourselves to looking at the local Gaussian correlation $\rho(x, y)$ at values at the diagonal, i.e. for $x = y$, and effectively turn the three plots into one, see Figure 6. We note that the local Gaussian correlation properties in fact seems to be symmetric as far as bear and bull (e.g. upward price trend) market is concerned.

Figure 4: Local Gaussian correlation map for FTSE 100 against S&P 500 returns.



Much of the variation in local Gaussian correlation in Figure 4 and in Figure 6 could be expected to be due to volatility, since we have daily data, see Forbes and Rigobon (2002) for a corresponding reasoning for the conditional correlation. As our next step we therefore filter the log returns with univariate GARCH(1,1) models with a skewed Student t error

Figure 5: Approximate lower (top plot) and upper (bottom plot) 5% confidence limits for the local Gaussian correlation map of US against UK returns shown in Figure 4, based on 1000 bootstrap replications.

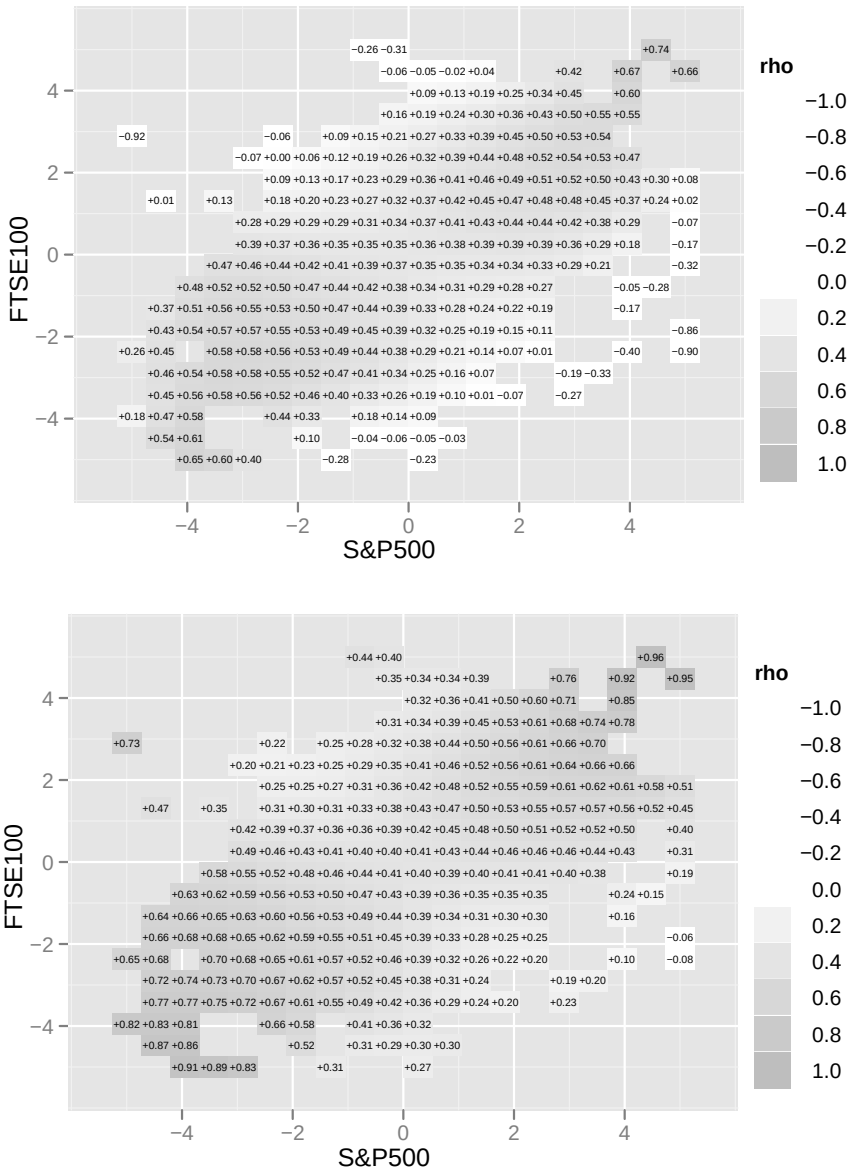
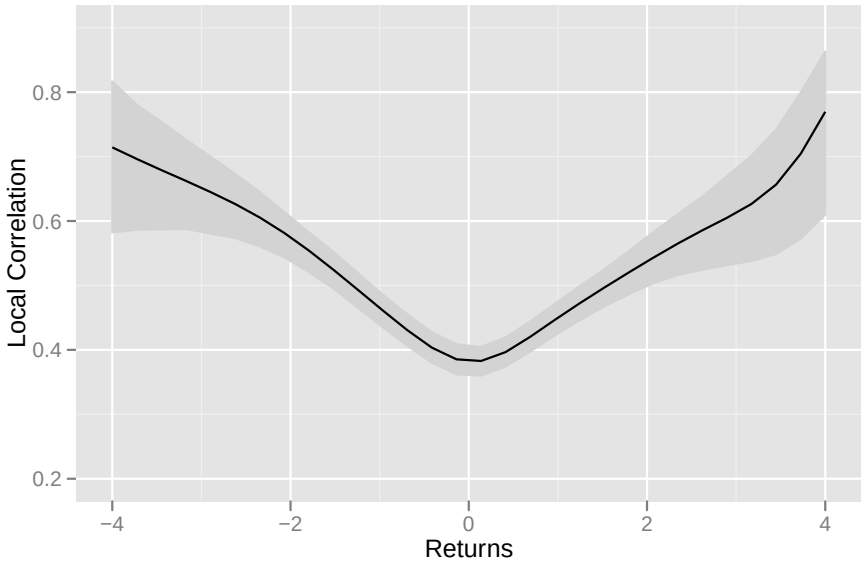


Figure 6: Diagonal local Gaussian correlation values and approximate 90% pointwise confidence intervals for FTSE 100 against S&P 500 returns, based on 1000 bootstrap replications.



distribution (Bollerslev 1987), and then compute the standardised residuals. To perform the actual calculations, we have used the software package `fGarch` (Wuertz and Chalabi 2008) in the `R` environment (R Development Core Team 2008). The model equations are

$$
\begin{aligned}
r_t &= \mu + a_t, \\
a_t &= \sigma_t \epsilon_t, \\
\sigma_t^2 &= \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2,
\end{aligned}
$$

where r_t is the log returns and where the remaining notation is self explanatory. The GARCH(1,1) parameters are statistically significant for the return series, and the Ljung-Box statistics calculated for squared and non-squared residuals indicate that the fitted models are adequate. We do not report these filtrations, but details are available from the authors upon request. The standardised residuals are calculated as $\hat{a}_t = (r_t - \hat{\mu})/\hat{\sigma}_t$.

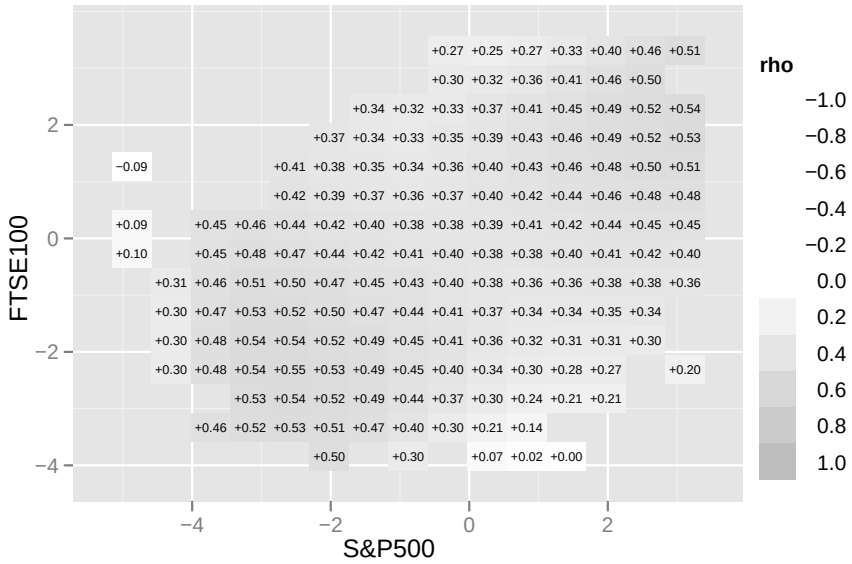
Table 3 shows the full correlation matrix for the standardised returns. Note that the coefficient is 0.44 between the S&P 500 and FTSE 100, marginally smaller than for the non-standardised returns. The estimated local Gaussian correlations between the standardised

returns is shown in Figure 7.

	S&P 500	FTSE 100	CAC 40	DAX 30
S&P 500	1.00			
FTSE 100	0.44	1.00		
CAC 40	0.42	0.72	1.00	
DAX 30	0.40	0.63	0.74	1.00

Table 3: Correlation of standardised index returns for daily data from July 1987 to July 2011.

Figure 7: Local Gaussian correlation map for FTSE 100 against S&P 500 standardised returns.



The general pattern is the same as for the non-standardised returns, but as expected, removing volatility reduce the local correlation effects, although, as seen from Figure 8 (upper left plot) taking confidence intarvals into account, they are still statistically significant.

In the rest of the paper we will mainly look at GARCH filtered returns and diagonal local correlation plots. The local Gaussian correlation on the diagonals for the pairwise market returns are given in Figure 8. These plots suggest asymmetric local dependence among the European markets CAC 40, DAX 30 and FTSE 100, specifically a somewhat higher local

dependence when market returns are negative (bear market). But, in general, the local correlations are very high between the European markets. Contrast this to the S&P index, which has a more symmetric local Gaussian correlation against the European indices, except for large negative returns, where there are large errors, though. Note, however, that in areas with relatively few observations, the local correlation estimate has a large uncertainty.

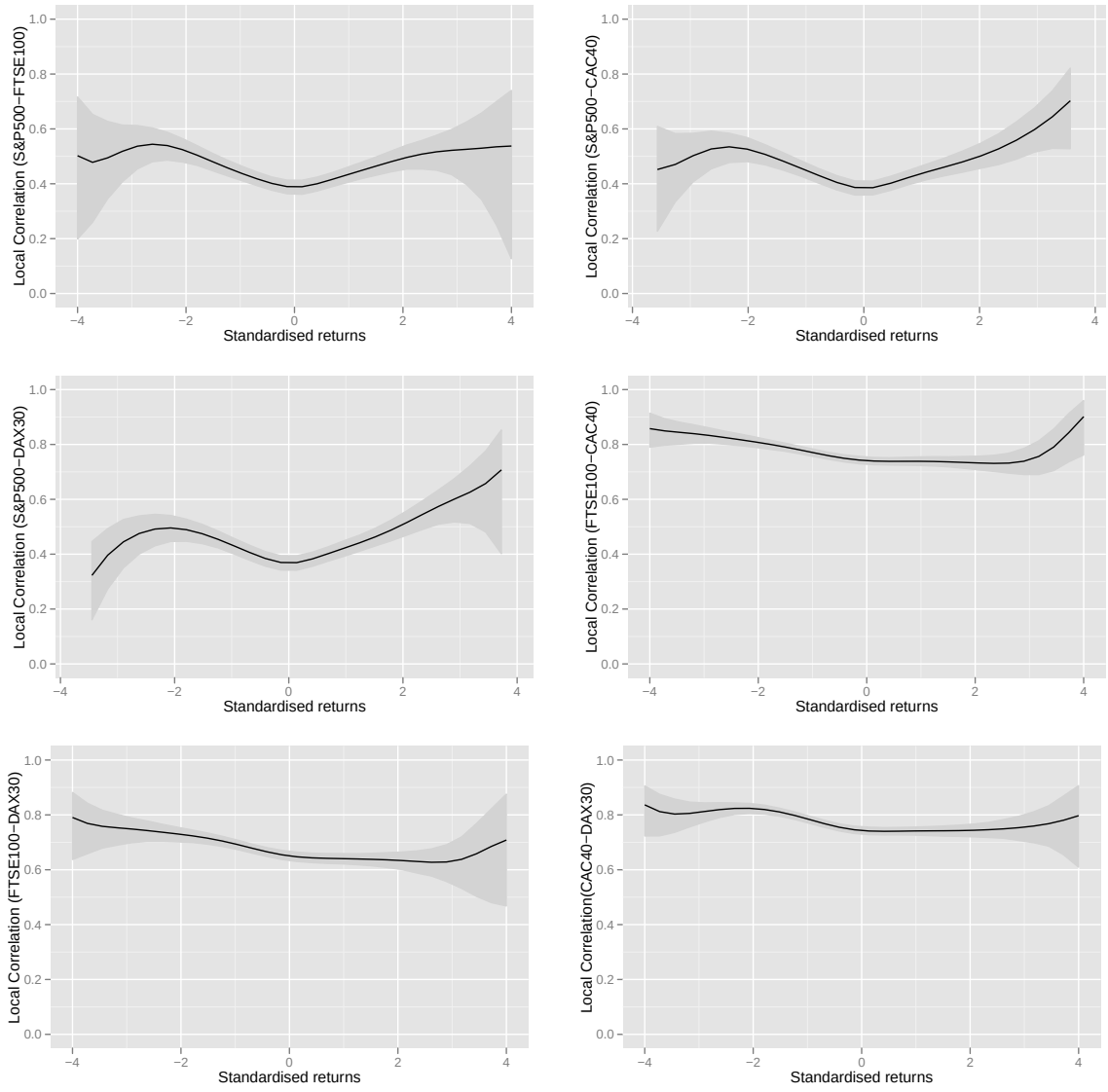
Note also that the latter correlation plots are somewhat difficult to interpret, due to the difference in opening hours for these markets. In order to control for this effect, we have estimated the local Gaussian correlation for the the average of two-days rolling returns, and similar dependence patterns occurs.

A possible reason for the U-shaped local Gaussian correlation curves between the US market and the European markets is that during time periods with a bear market, even though the price trend is falling, for some days we actually will see quite large positive daily returns, e.g. a bear market rally (also known as "sucker's rally"). This may explain why we observe the high local correlation also for positive returns, and not just for negative returns. However, this explanation is rather tentative, since it is not always present when we look at sub-periods (see Figure 9, which in fact indicates that there may be a strong and increasing dependence bewteen S&P 500 and FTSE 100 also in a bull market situation), and it is not present for interelations between the European markets. In the latter situation the absence of this effect could be due to the generally higher correlations. Moreover, it is seen that the plots where S&P is involved is almost completely symmetric between -2 and $+2$, while in the same interval the local correlation between the European markets is almost linear with a negative slope. To the right of $+2$ there are indication of a market rally for all market relationships.

We would expect the bear market rally effect to vanish when looking at returns over a longer period, say weekly or monthly, since the weekly or monthly returns in a bear market would per definition be negative. This is confirmed by the analysis of Section 4.2.

We have also computed the local Gaussian correlations between the standardised returns from S&P 500 and FTSE 100 on different sub-periods of five year intervals, shown in Figure 9. These indicate that the local correlations are time-varying, consistent with similar results for classical global correlation; see, for example, Longin and Solnik (1995). There are also statistically significant differences in curve shape. In the first time period from 1987 to 1991, the market crash of October 1987 may explain the high local correlations we observe

Figure 8: Local Gaussian correlation curves with approximate 90% confidence intervals based on 1000 bootstrap replications, estimated between the US and European equity indices and between pairwise European equity indices.



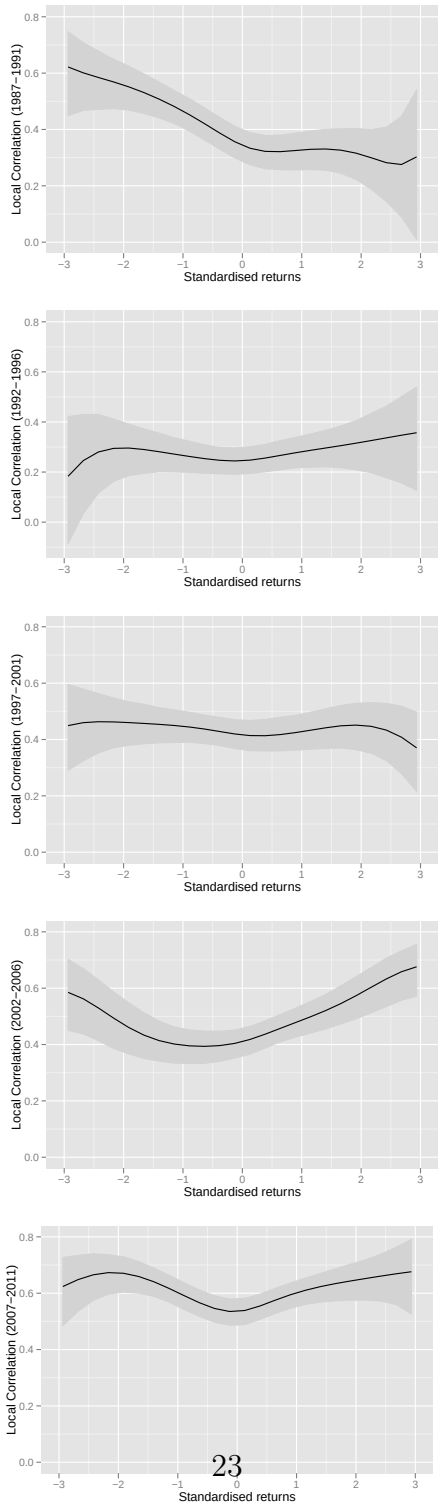
for negative returns. All major world markets declined sharply, many over 20% in just one month. See e.g. Roll (1988) for an account of these events. A period of relatively calm US and UK markets then followed, with an overall low local correlation 1992-1996, but the burst of the dotcom-bubble beginning early 2000 and lasting to 2002, can explain the increased local correlations observed from 1997 and onwards. See e.g. Phillips et al. (2011) for some background information of the dotcom-bubble. The years from late 2002 until mid-2007, was characterized by a booming US and UK market, that may explain the large local correlations for positive returns in the plot. The last period from 2007-2011 has been dominated by the Financial crisis, and has caused ever more increased local correlations. Putting all of these periods together, it will be seen that the symmetry patters of the upper left plot of Figure 8 could easily emerge. There do exists alternative methods for modelling time-varying correlations, and one can be found in Engle (2002).

4.2 Monthly data

In this section, we study monthly returns from the US, UK and Germany (DE) in local currency, from February 1973 to September 2009, in all 440 observations. The data are total market indices from Datastream, and we calculate the returns as for the daily data, i.e. as 100 times the change in the natural logarithm of each market's price index. Descriptive statistics are given in Table 4. In particular, we see that the maximum and minimum returns are larger than for the daily returns, and again, the kurtosis for all series is high, indicating a departure from the Gaussian distribution. The global correlations between the returns are shown in Table 5, and ranges from 0.59 to 0.66. Note the difference from Tables 2 and 3. For monthly data there are slightly larger correlations between the US and UK, DE than between UK and DE.

The estimate of the local Gaussian correlation (on the diagonal) between the returns from the three markets are shown in Figure 10. Clearly, we have evidence of asymmetric dependence, in particular larger local correlations in bear markets. Further, the local correlations decrease from the left of the plot to the right, implying that diversification opportunities would be largest in a bull market – or rather, erode in bear markets, which is when they are needed the most. The results thus support our intuition from the previous section, that the U-shaped local correlation patterns disappears when looking at returns over a longer period

Figure 9: Diagonal local Gaussian correlations (with approximate 90% confidence intervals) between standardised returns from S&P 500 and FTSE 100, divided into five-year long intervals from 1987 to 2011.



Statistic	US	UK	DE
Mean	0.524	0.645	0.401
Standard deviation	4.590	5.687	5.259
Skewness	−0.731	0.155	−0.835
Kurtosis	5.625	11.452	5.721
Maximum	15.730	41.672	15.173
Minimum	−23.545	−29.938	−24.613

Table 4: Summary index statistics for monthly data from February 1973 to September 2009.

	US	UK	DE
US	1.00		
UK	0.66	1.00	
DE	0.59	0.55	1.00

Table 5: Correlation of the returns for monthly data from February 1973 to September 2009.

than one day, i.e. since the high local correlations for large positive returns, may occur due to bear markets rallies. Further, these findings are in line with Okimoto (2008), where the author find, using monthly return data, that the US–UK market bear dependence can be modelled by an asymmetric copula with lower tail dependence. Using local correlation to formally recognize copulas has been treated in Berentsen et al. (2012).

Further, GARCH(1,1) filtrations with skewed t-distributed error terms are performed on the returns, and standardised residuals calculated. (Tables of these filtrations are not shown, but all GARCH-parameters are significant and the models are reasonable). The corresponding diagonal plot is shown in Figure 11. We see the same pattern as before, i.e. asymmetric dependence. Although these estimates are subject to relatively large estimation errors due to the limited number of observations, especially for extreme values (see Figure 12 for approximate 90% confidence intervals for the local correlation between the US-UK standardised returns), we do see that the local correlation estimates decrease in normal and bull markets, compared to the bear market correlations, as we observed for the unfiltered log returns.

Figure 10: Local Gaussian correlation of the monthly returns, from January 1973 to September 2009.

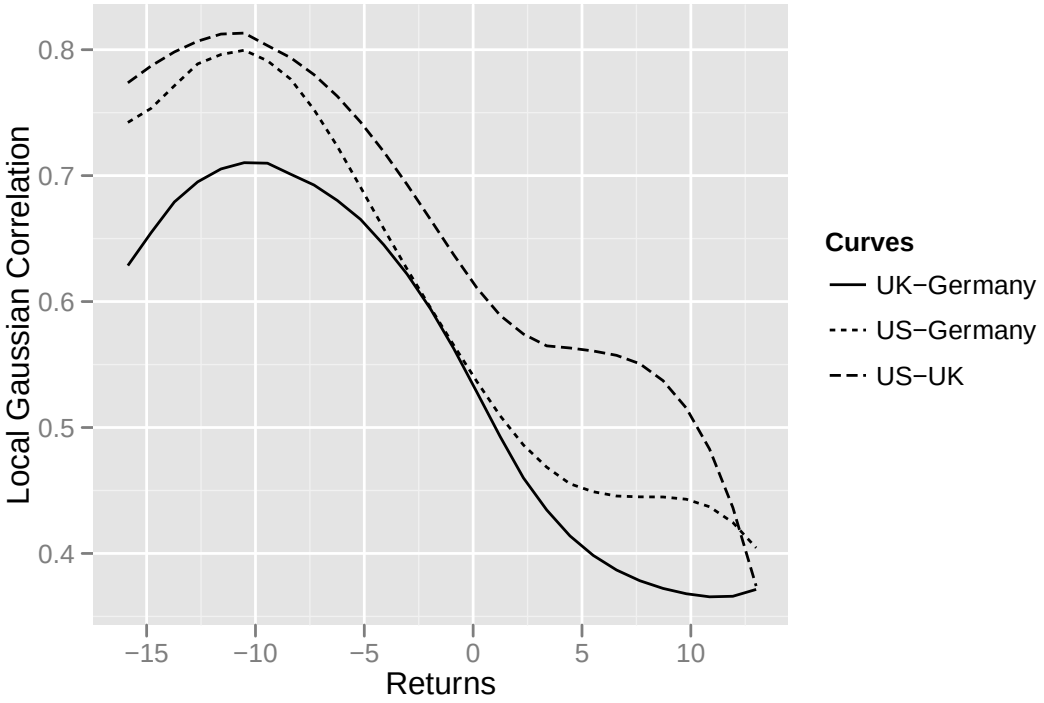


Figure 11: Local Gaussian correlation of the monthly standardised returns, from January 1973 to September 2009.

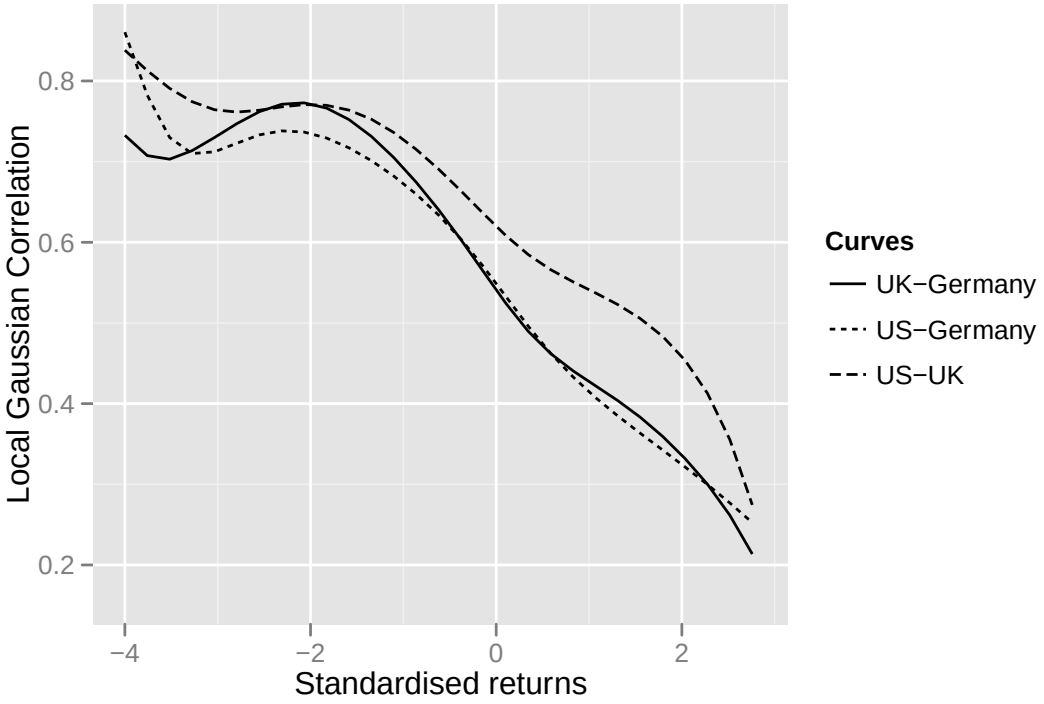
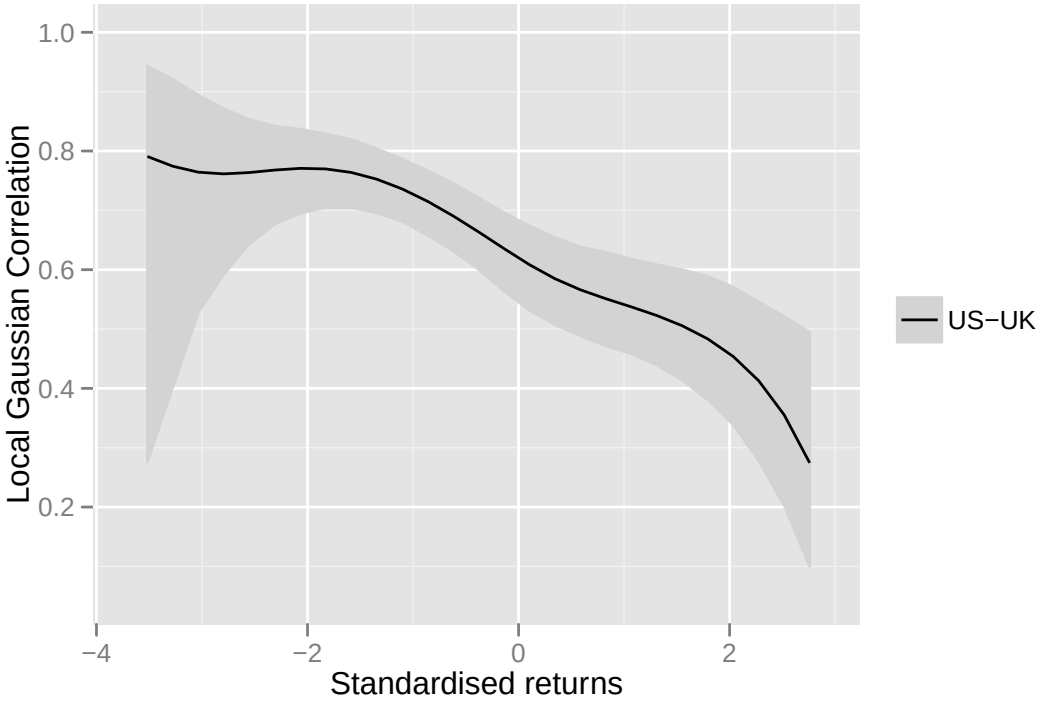


Figure 12: Local Gaussian correlation with approximate 90% confidence intervals based on 1000 bootstrap replications, estimated between the US and UK using monthly standardised returns.



5 A risk management application

In the previous sections we documented asymmetric dependence between asset returns. In particular in bear markets, where for monthly returns, we estimated significantly higher dependence between returns than under normal market conditions. In this section, we will evaluate the economic significance of ignoring this fact from a risk management perspective, using Value-at-Risk.

Value-at-risk (VaR) is a widely used risk measure to assess and manage market risk, for example the risk of a portfolio of assets. The VaR is a bound such that the loss over a chosen time horizon is less than this bound with probability equal to a chosen significance level. There exists a vast variety of methods to calculate VaR, see e.g. Jorion (2001).

As a very simple illustrative example we consider a portfolio of two assets, with weight w_1 on the first asset and weight $w_2 = 1 - w_1$ on the second asset. Let σ_1 and σ_2 be the asset variances, and we assume that for short return horizons, the mean returns are negligible. Based on these assumptions, the portfolio variance is thus

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2. \tag{9}$$

Assume that the asset returns are bivariate Gaussian distributed, the initial investment is I_0, and the significant level is α, then

$$\mathrm{VaR}(\alpha) = I_0\sigma_p z_\alpha, \tag{10}$$

where z_α is the α-quantile in the standard normal distribution. This is a well-known and straightforward method for calculating VaR.

We now use the monthly returns data for the US and UK from the previous section. Consider an investment of 10 million dollars into a portfolio consisting of 50% invested in the US market and the remaining 50% invested in the UK market. From the returns data we calculate the empirical portfolio variance using equation (9), and by equation (10), the VaR(0.01) is calculated to be 1.09 million dollars.

The correlation, ρ, and the asset variances, σ_1 and σ_2, are key quantities for calculating the portfolio variance. Since we document asymmetric dependencies between financial returns data, that is, changes in the local correlations, we propose to use the local correlations

and the local variances to calculate the portfolio variance of the approximating Gaussian. Thus the portfolio variances will be calculated locally, and for now restricting ourselves to gridpoints at the diagonal. Let $x = (x, x)$ denote the points at the diagonal, then the local portfolio variances are,

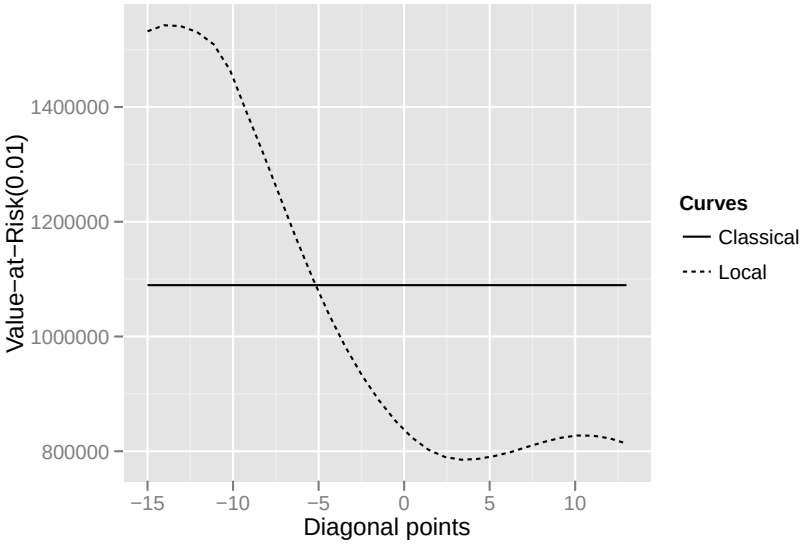
$$\sigma_{p,x}^2 = w_1^2\sigma_{1,x}^2 + w_2^2\sigma_{2,x}^2 + 2w_1w_2\rho_x\sigma_{1,x}\sigma_{2,x}. \tag{11}$$

These local estimates of the portfolio variances will be used as input to equation (10). We will thus end up with several VaR estimates along the diagonal, and in principle we could extend these calculations to all gridpoints. As a simplification, for each x we are then using the tail properties for the approximating Gaussian $\psi_{x,x}$ at x, ignoring the fact that further out in the tail f is approximated by different Gaussians.

Based on the estimation results of the local parameters from the previous section for the monthly data, we can calculate the local VaR for the portfolio above. The local VaRs are shown in Figure 13. The classical VaR is included as a straight line in the figure only for comparison. We clearly see that the increased correlations, for negative returns, give an increased local VaR, the largest local VaR is around 1.5 million dollars. Note that to the far right hand-side of the plot, the local VaR increases, even though the correlation is lower (cfr. Figure 10). This behaviour comes from the fact that the local variances increases in this area. We conclude that a risk manager estimating only the classical VaR will severely underestimate the risk in this portfolio, and the reason for this underestimation is the increased dependencies for negative returns.

We note that there already do exist methods for calculating VaR that take account for increased dependencies, e.g. simulation based techniques using copula and regime-switching models (Okimoto (2008) and the conditional VaR-x (Pownall and Koedijk (1999). A more thorough study is needed for comparing our procedure with these methods. See also Gourieroux and Jasiak (2010), where the authors actually use ideas and methods quite similar to those presented above.

Figure 13: The classical VaR and the local VaR estimated for a portfolio.



6 Discussion, limitations and possible extensions

In this paper we have applied a new measure of dependence, called the local Gaussian correlation, to the study the dependence between international stock market returns. A number of studies have provided evidence that asymmetric dependence in financial returns do exist; however, the methods used for studying this phenomenon must be assessed with care, since the correlation computed conditional on some variables being high or low is a biased estimator of the unconditional correlation, making quantitative statements about the change in dependence difficult. For bivariate normal data, the local Gaussian correlation avoids this bias, and for non-normal data it provides us with a way of describing the changes in dependence and the departure from global normality.

Using the local correlation measure, we do find evidence of asymmetric dependence structures between international stock markets, in particular, between the US and European markets for monthly data, where a bear market effect is present. Thus our findings support earlier studies, and, in addition, we have been able to quantify the asymmetry, using a measure that has a characterisation in terms of the ordinary correlation of a local Gaussian approximation. Note that the local Gaussian correlation curves might be quite different for

different pairs of indices. We also demonstrate that both correlation and local correlation between the US and UK markets increase in time, but not uniformly in terms of correlation curves. A very interesting question is of course which economic factors drive such asymmetries. For daily data there are also very significant differences in local Gaussian correlations, but we find significant asymmetries only among the European markets.

Note that for the analysis of US–European markets, we have the problem of different opening hours for the markets, and also of different national holidays. These variables should ideally be taken into consideration when analysing these data, although we have not done so in this paper. Also note that other models than our simple GARCH(1,1) model can be fitted to the data before analysing them, and several of the papers cited describe results from such models. We believe our results to be consistent across various such models that describe the data well, but clearly this should be more closely investigated. It is also of interest whether the local dependence patterns continue to hold for other comparable markets, such as Canada, Italy, China and Japan, and whether they are constant over time, and if not, how they change over time.

Let us briefly note that our approach also gives a new method of assessing the fit of models, by comparing the local Gaussian correlation of data with the the local Gaussian correlation of fitted models. See Berentsen et al. (2012) for such an idea applied to copula theory.

We note that the choice of bandwidths need to be more closely investigated. One possibility is also to use varying bandwidths, in particular larger bandwidths in areas with relatively few observations. We defer these topics to future research.

In the last part of our paper we have studied a risk management application, and we show that increased local correlation will increase risk measures such as VaR.

One additional assumption we have used is that of independent pairs of variables. In the present paper we have used a GARCH model to come closer to this assumption, but the theory of local Gaussian correlation actually holds for stationary ergodic time series case. Moreover, a parametric model for the local Gaussian correlation can be useful for extreme events. With the tools derived in this paper and their extensions, we might study the classical portfolio allocation problem and it will also be possible to study contagion effects, see Støve et al. (2012). Further, we might study portfolio selection for stocks with certain characteristics, local dependence in foreign exchange data, and correlation between

risk factors in credits markets. In principle, every financial or econometric analysis that depends on a covariance matrix can be subject to a local Gaussian covariance analysis. We have only considered bivariate problems here, but a multivariate extension is possible under certain simplifying assumptions, as outlined in Hufthammer and Tjøstheim (2009).

Acknowledgement

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References

Aas, K., Czado, C., Frigessi, A. and Bakken, H. (2009), 'Pair-copula constructions of multiple dependence', *Insurance: Mathematics and Economics* **44**, 182–198.

Ang, A. and Bekaert, G. (2002), 'International asset allocation with regime shifts', *Review of Financial Studies* **15**(4), 1137–1187.

Ang, A. and Chen, J. (2002), 'Asymmetric correlations of equity portfolios', *Journal of Financial Economics* **63**(3), 443–494.

Berentsen, G. and Tjøstheim, D. (2012), Recognizing and visualizing departures from independence in bivariate data using local gaussian correlation, To appear in *Statistics and Computing*.
URL: *http://folk.uib.no/gbe062/local-gaussian-correlation/*

Bjerve, S. and Doksum, K. (1993), 'Correlation curves: measures of association as functions of covariate values', *The Annals of Statistics* **21**(2), 890–902.

Blyth, S. J. (1994), 'Measuring local association: An introduction to the correlation curve', *Sociological Methodology* **24**, 171–197.

Bollerslev, T., Chou, R. Y. and Kroner, K. F. (1992), 'ARCH modeling in finance: A review of the theory and empirical evidence', *Journal of Econometrics* **52**(1-2), 5–59.

Boyer, B. H., Gibson, M. S. and Loretan, M. (1999), Pitfalls in tests for changes in correlations, Discussion paper 597.
URL: *http://www.federalreserve.gov/pubs/ifdp/1997/597/default.htm*

Butler, K. C. and Joaquin, D. C. (2002), 'Are the gains from international portfolio diversification exaggerated? the influence of downside risk in bear markets', *Journal of International Money and Finance* **21**(7), 981 – 1011.

Campbell, R. A., Forbes, C. S., Koedijk, K. G. and Kofman, P. (2008), 'Increasing correlations or just fat tails?', *Journal of Empirical Finance* **15**(2), 287–309.

Campbell, R., Koedijk, K. and Kofman, P. (2002), 'Increased correlation in bear markets', *Financial Analysts Journal* **58**(1), 87–94.

Chollete, L., Heinen, A. and Valdesogo, A. (2009), 'Modeling international financial returns with a multivariate regime switching copula', *Journal of Financial Econometrics* **7**, 437–480.

Drouet Mari, D. and Kotz, S. (2001), *Correlation and dependence*, Imperial College Press, London.

Efron, B. and Tibshirani, R. J. (1993), *An Introduction to the Bootstrap*, Vol. 57 of *Monographs on Statistics and Applied Probability*, Chapman & Hall/CRC, New York.

Engle, R. (2002), 'Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models', *Journal of Business & Economic Statistics* **20**(3), 339–350.

Forbes, K. J. and Rigobon, R. (2002), 'No contagion, only interdependence: Measuring stock market comovements', *The Journal of Finance* **57**(5), 2223–2261.

Garcia, R. and Tsafack, G. (2011), 'Dependence structure and extreme comovements in international equity and bond markets', *Journal of Banking and Finance* **35**, 1954–1970.

Gourieroux, C. and Jasiak, J. (2010), 'Local likelihood density estimation and value-at-risk', *Journal of Probability and Statistics* pp. 1–26.

Hastie, T. J. and Tibshirani, R. J. (1990), *Generalized additive models*, Vol. 43 of *Monographs on Statistics and Applied Probability*, Chapman and Hall Ltd., London.

Hjort, N. L. and Jones, M. C. (1996), 'Locally parametric nonparametric density estimation', *The Annals of Statistics* **24**(4), 1619–1647.

Holland, P. W. and Wang, Y. J. (1987), 'Dependence function for continuous bivariate densities', *Communications in Statistics. Theory and Methods* **16**(3), 863–876.

Hong, Y., Tu, J. and Zhou, G. (2007), 'Asymmetries in stock returns: Statistical tests and economic evaluation', *Review of Financial Studies* **20**(5), 1547–1581.

Joe, H. (1997), *Multivariate models and dependence concepts*, Vol. 73 of *Monographs on Statistics and Applied Probability*, Chapman & Hall, London.

Jones, M. C. (1993), 'Simple boundary correction for kernel density estimation', *Statistics and Computing* **3**, 135–146.

Jones, M. C. (1996), 'The local dependence function', *Biometrika* **83**(4), 899–904.

Jones, M. C. (1998), 'Constant local dependence', *Journal of Multivariate Analysis* **64**(2), 148–155.

Jones, M. C. and Koch, I. (2003), 'Dependence maps: Local dependence in practice', *Statistics and Computing* **13**(3), 241–255.

Longin, F. and Solnik, B. (1995), 'Is the correlation in international equity returns constant: 1960-1990?', *Journal of International Money and Finance* **14**(1), 3–26.

Longin, F. and Solnik, B. (2001), 'Extreme correlation of international equity markets', *The Journal of Finance* **56**(2), 649–676.

Mammen, E., Støve, B. and Tjøstheim, D. (2009), 'Nonparametric additive models for panels of time series', *Econometric Theory* **25**, 442–481.

Nikoloulopoulos, A., Joe, H. and Li, H. (2012), 'Vine copulas with asymmetric tail dependence and applications to financial return data', *Computational Statistics and Data Analysis* **56**, 3659–3673.

Okimoto, T. (2008), 'New evidence of asymmetric dependence structures in international equity markets', *Journal of Financial and Quantitative Analysis* **43**(3), 787–816.

Phillips, P., Wu, Y. and Yu, J. (2011), 'Explosive behavior in the 1990s nasdaq: When did exuberance escalate asset values?', *International Economic Review* **52**, 201–226.

Rodgers, J. L. and Nicewander, W. A. (1988), 'Thirteen ways to look at the correlation coefficient', *The American Statistician* **42**(1), 59–66.

Rodriguez, J. C. (2007), 'Measuring financial contagion: A copula approach', *Journal of Empirical Finance* **14**(3), 401–423.

Roll, R. (1988), 'The international crash of october 1987', *Financial Analysts Journal* **44**, 19–35.

Rovine, M. J. and von Eye, A. (1997), 'A 14th way to look at a correlation coefficient: Correlation as the proportion of matches', *The American Statistician* **51**(1), 42–46.

Scott, D. W. (1992), *Multivariate density estimation*, Wiley Series in Probability and Mathematical Statistics: Applied Probability and Statistics, John Wiley & Sons Inc., New York. Theory, practice, and visualization, A Wiley-Interscience Publication.

Silvapulle, P. and Granger, C. W. J. (2001), 'Large returns, conditional correlation and portfolio diversification: a value-at-risk approach', *Quantitative Finance* **1**(5), 542–551.

Støve, B., Tjøstheim, D. and Hufthammer, K. O. (2012), Using local gaussian correlation in a nonlinear re-examination of financial contagion, Revised manuscript.
URL: *http://folk.uib.no/gbe062/local-gaussian-correlation/*

Teräsvirta, T., Tjøstheim, D. and Granger, C. W. J. (2010), *Modelling Nonlinear Economic Time Series*, Oxford University Press.

Tjøstheim, D. and Hufthammer, K. O. (2013), 'Local Gaussian correlation: A new measure of dependence', *Journal of Econometrics* **172**, 33–48.

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A number of studies have provided evidence that financial returns exhibit asymmetric dependence, such as increased dependence during bear markets, but there seems to be no agreement as to how such asymmetries should be measured. We introduce the use of a new measure of local dependence to study this asymmetry. The central idea of the new approach is to approximate an arbitrary bivariate return distribution by a family of Gaussian bivariate distributions. At each point of the return distribution there is a Gaussian distribution that gives a good approximation at that point. The correlation of the approximating Gaussian distribution is taken as the local correlation in that neighbourhood. The new measure does not suffer from the selection bias of the conditional correlation for Gaussian data, and is able to capture nonlinear dependence. Analyzing several financial returns from the US, UK, German and French markets, we confirm and are able to explicitly quantify the asymmetry. Finally, we discuss a risk management application, and point out a number of possible extensions.

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