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FAIR VALUE MEASURING IN THE PRIVATE EQUITY INDUSTRY

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Abstract

The objective of this thesis is to address the question of how fair value reporting is implemented in the private equity industry, and how good fair value estimates (*FV*) are compared to the intrinsic value of the investments measured by the transaction price. Further, the value relevance of book values (*BV*) and *FV* has been examined across type of investment and valuation method.

My data sample shows that the industry's *FV* are underestimated in 75 % of the cases. The average deviation from the transaction price achieved in the market is -25 %. Multiples appear to be the most accurate method to use when assessing fair value. On average, multiples have a -14 % deviation between *FV* and transaction price.

The regression analysis has shown that both *BV* and *FV* are value relevant, but *FV* are more value relevant than *BV*. In addition, *BV* is more value relevant for venture companies than other companies, while non recognized goodwill is less value relevant for venture companies.

Preface

This master thesis is written as a part of my master degree at the Norwegian School of Economics and Business Administration (NHH). The thesis amounts to 30 credits, which corresponds to one semester full-time studies.

My interest of valuation and accounting issues is a result of different courses taken at NHH. Both *BUS401 – Financial Accounting* and *BUS425 - Valuation and Strategic Financial Analysis* have been of great inspiration when writing this thesis. When I got the opportunity of combining these two fields, I knew that fair value measuring in the private equity industry was the perfect topic for me.

Writing a master thesis has been exciting, interesting, demanding and, last but not least, very informative. I would like to give appreciation to Kjell Henry Knivsflå, who has been my thesis advisor and has helped me with guidance and insight. In addition, I want to give gratitude to PwC who contributed with the research question and support during the process. A special thanks to Thomas Samdal Rasmussen for indispensable feedback and advice during the last moths.

Finally, I want to give appreciation to the private equity companies who provided me with information and data to the analysis. This master thesis could not have been written without their contribution.

Bergen, 17.12.2010

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Kristoffer Heimland Wist

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Abbreviations

CAPM – Capital Asset Pricing Model

CF – Cash flow

DCF – Discounted cash flow

EBIT – Earnings before interest and tax

EBITDA – Earnings before interest, tax, depreciation and amortization

EEA – European Economic Area

EV – Enterprise value

EVCA - European Private Equity and Venture Capital Association

FAS – Financial Accounting Standards

FASB – Financial Accounting Standards Board

GP – General partner

IAS – International Accounting Standards

IASB – International Accounting Standards Board

IASC – International Accounting Standards Committee

IFRIC – International Financial Reporting Interpretations Committee

IFRS – International Financial Reporting Standard

IPEV Guidelines - International Private Equity and Venture Capital Valuation Guidelines

IRR – Internal rate of return

LP – Limited partner

NAA – The Norwegian Accounting Act

NAV – Net asset value

NGAAP – Norwegian generally accepted accounting principles

NVCA – Norwegian Venture Capital and Private Equity Association

USGAAP – U.S. generally accepted accounting principles

WACC – Weighted Average Cost of Capital

1 Introduction

Today, fair value is increasingly required due to accounting requirements of International Financial Reporting Standards (IFRS) and U.S Generally Accepted Accounting Principles (USGAAP). The requirements applied in IFRS and USGAAP hold for all entities legally bound to maintain accounting records in accordance with the two standards. Basically, it means that American companies report in accordance with USGAAP, and European companies use IFRS as the accounting standard.

Before I go any further it is necessary to define fair value. In this thesis I am going to use the definition in IFRS:

"Fair value is the amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arm's length transaction."(IASB, 2009c)

As you will see throughout the thesis, fair value is defined somewhat different in IFRS and USGAAP. However, it is possible to argue that the definitions are not fundamental different from each other despite the use of different wording. A common feature is to apply fair value as an exit price, and thereby creating a hypothetical selling price. I will come back to the consequences later on when discussing accounting models and requirements.

For some industries fair value measurements are more challenging. One example is the alternative investment sector, and especially the private equity industry, where increasing investor and accounting pressure have made fair value one of the hottest topics. In addition, the introduction of the International Private Equity and Venture Capital Guidelines (IPEV Guidelines) in 2005, has contributed to make fair value the most relevant measurement attribute for the industry. The old principle of keeping investments at cost provided fewer challenges than today's practice where lack of transactions and volatile markets make the valuation process more difficult (PwC, 2008).

You might ask why fair value is important in an industry like private equity? Some would argue that what matters in private equity is how much cash is generated when realizing an investment. Fair value is basically an accounting issue, and has nothing to do with the investment itself (PwC, 2008). On the other hand, fair value can contribute with

information to investors in private equity funds. Investment managers can communicate to investors where value is being added or lost over time by reference to specific portfolio companies. The managers are more capable of making informed decisions on entry and exit by understanding upside and downside scenarios. In addition, they are able to meet the requirements of IFRS and USGAAP accounting standards, and thereby satisfy auditor review (PwC, 2008). These arguments are in favour of fair value measurement and might be one of the reasons why fair value reporting has become important for the private equity industry.

The nature of private equity investments bring along rapid changes from period to period, which means that one year's results are often vastly different from results reported a year earlier. In addition, private equity firm specialize in different types of investments, ranging from seed capital to buyouts, and multi-national to domestic investments to mention a few (PwC, 2008). This diversity makes it hard to exercise judgment to arrive at a robust fair value estimate, even though IFRS, USGAAP and IPEV Guidelines have published guidelines where the objective is actually that.

1.1 Research question

Increasing demand for fair value reporting in the private equity industry, in combination with lack of market transactions and high volatility, makes the requirement difficult to achieve. Thus, this master thesis will address the question of how fair value reporting is implemented in the industry, and how good the fair value estimates are compared to the intrinsic values of the investments measured by the transaction price. Further, the value relevance of *BV* and *FV* will be examined across type of investment and valuation method.

1.2 Approach

To understand challenges that can arise when estimating fair value at private equity portfolios, we need to gain insight in the industry and the different methods of financial accounting. To give a theoretical context, I will start by giving a presentation of the private equity industry and its attributes in chapter 2. Then, I will present the concepts behind different accounting methods in chapter 3, before I am going to look more detailed into accounting regulations of fair value in the Norwegian Accounting Act (NAA), IFRS and USGAAP in chapter 4. I will not discuss the concept of fair value

accounting from a critical point of view, but rather pinpoint where there are problems regarding the private equity industry. In chapter 5, I will give a presentation on how fair value measuring is practiced in the private equity industry. An important part of chapter 5 is the IPEV Guidelines. The Guidelines have an important position in the private equity industry, and are used actively in the work of estimating fair value by private equity companies. In light of that, the Guidelines are published to make the practical part of valuation easier and more transparent for both private equity managers and fund investors.

Once the theoretical framework is set, I will in chapter 6 look at the Norwegian private equity industry and see how estimated values correspond to the actual market value of investments. In order to do so, I have gathered observations from Norwegian private equity firms related to fair value estimates and transactions prices. These observations will function as the fundament of my analysis in order to decide whether the private equity firms' estimates are over- or under valuated. Hopefully, the data set can help decide which valuation methodology has the best accuracy when estimating fair value, and help us understand the value relevance of different explanatory variables for the transaction price. The analysis will probably not give an exhaustive answer, but we should be able to decide upon some trends in the industry.

1.3 Scope limitations

The Norwegian private equity industry will be the main focus, due to access of information. The industry has traditionally been very reserved in matters concerning distribution of internal information to external parties. From a strategic point of view, I considered it easier to achieve the understanding from Norwegian companies, due to the fact that I am a student at a Norwegian business school.

In addition, Norwegian private equity companies have historically kept investments at cost due to requirements in the Norwegian Accounting Act (NAA). The introduction of IPEV Guidelines in 2005, and influence of IFRS have made fair value accounting more relevant for Norwegian firms today. Thus, I thought it would be interesting to see how Norwegian firms managed the work of estimating fair value compared to the investment's market value.

2 The private equity industry

For those who have studied economic and business the private equity industry is probably familiar. However, the industry is complex and the development has been significant over the last decades. In addition, the industry contains many terms that are essential to understand. Thus, we are going to take a closer look at the private equity industry to grasp the extent and get an understanding of the industry.

2.1 Definition

If you search the Internet for a definition of the term "private equity" you get approximately 16 300 000 results[1]. In this master thesis, we are going to understand private equity in accordance with the definition used by the European Private Equity and Venture Capital Association (EVCA):

"Private equity provides equity capital to enterprises not quoted on a stock market. Private equity can be used to develop new products and technologies, to expand working capital, to make acquisitions, or to strengthen a company's balance sheet. It can also resolve ownership and management issues. A succession in family-owned companies, or the buyout and buyin of a business by experienced managers may be achieved using private equity funding. Venture capital is, strictly speaking, a subset of private equity and refers to equity investments made for the launch, early development, or expansion of a business." (EVCA, 2010)

2.2 Type of private equity

The definition is comprehensive and helps us understand all the basics of the industry. The definition also describes what is meant by the term venture capital. While Europeans usually refer to venture capital meaning the same as private equity, we are going to make a distinction between the terms in accordance with the definition. Basically it means that venture capital is a subset of private equity where the investment is done earlier on the firm's business cycle. In addition to venture capital, private equity also consists of seed capital and buy-out investments. Thus, private equity can be understood as an umbrella term that includes different types of investments carried through in various phases on a firm's business cycle. Table 1, on the next page, gives a brief account of the different types of private equity and their respectively definition.

[1] The search was conducted Des 7, 2010 through www.google.no.

Business cycle	Type of private equity	
Entry	Seed capital	Investments made in companies where the risk for success is low. The companies use much resources on research and development and the turnover is low.
Growth	Venture capital	Investments made in companies with potential for growth. The investment is typically done before the company enters the global stage.
Mature	Buy out	Investments made in companies where there is potential for value increasing.
Decline		

Table 1 – Different types of private equity (NVCA, 2010a)

Figure 1 gives a graphic context of the cycles a business usually goes through during its economical life. As you can see from table 1, the business cycle is decisive in order to classify what type of private equity is applied to the different investments.

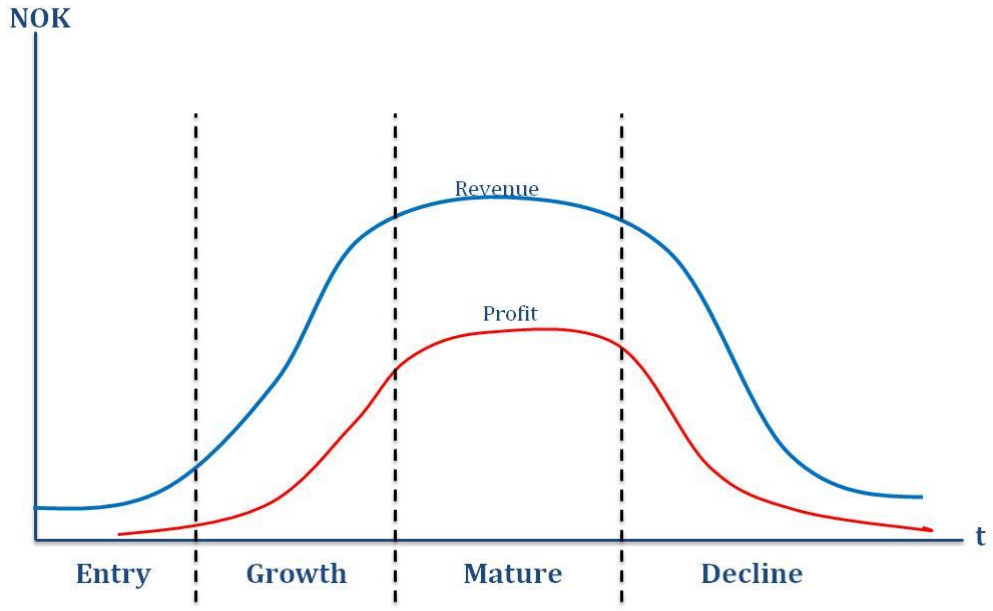

Figure 1 - The business cycle (Damodaran, 2002)[2]

2.3 The structure of private equity investments

The structure of private equity investments is very different compared to the majority of other investments. One reason is because private equity provides capital to companies not quoted on stock markets. Thus, the investment deal and price is a result of negotiation rather than a purchase of listed stocks.

[2] Figure 1 is an adaption from Damodaran's (2002) figure on page 639.

According to Argentum[3] (2010a) the structure of private equity investments can be divided in three tiers, where the particular type of organization or contract is referred to a limited partnership.

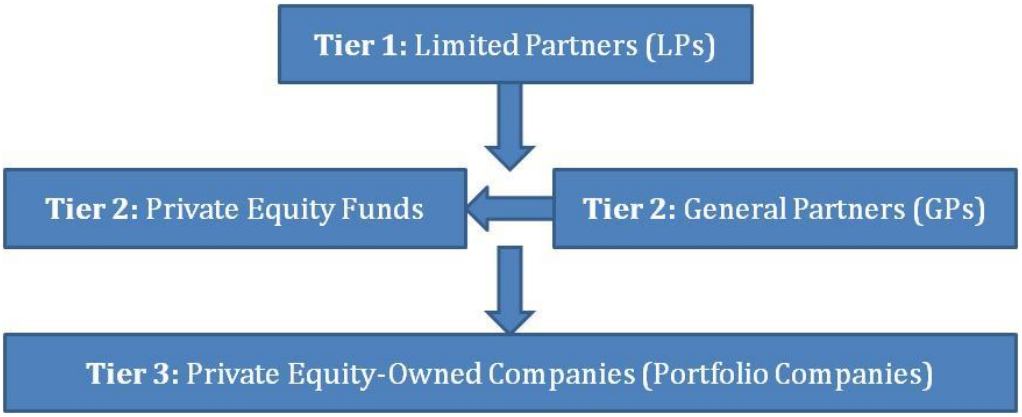

Figure 2 - The structure of a private equity investment (Argentum, 2010a)

Tier 1: Institutional Investors

Tier 1 represents those who invest in the private equity funds. The investors are referred to as limited partners (LPs) because they only invest into the fund and do not play an active part in managing the fund (Argentum, 2010a).

LPs are usually institutional investors, meaning insurance companies, pension funds, banks and funds-of-funds. However, sometimes you will also find that wealthy individuals act as LPs. The reasons why there are so little private investors are due to the considerable size an investment in private equity funds call for (Argentum, 2010a).

Tier 2: Fund Managers

The second tier represents the fund managers, who actively manage the private equity funds. These are commonly referred to as General Partners (GPs). The GPs are teams of experienced investment professionals with a wide range of specialist skills. The GPs select, invest, consult, and exit the private equity funds' portfolio companies. Thus, the GPs are the ones who provide the portfolio companies with active ownership. The GPs will also invest in the fund they manage so that they are more personally bound to the funds' performance and profitability (Argentum, 2010a).

[3] Argentum is an asset manager specializing in Nordic private equity funds (fund-of-funds).

A GP can manage several funds. When a new fund is being established, a target size and investment strategy is defined. Private equity funds have a limited time horizon, usually 8-12 years. During their time frame the capital should have been invested in portfolio companies, the companies should have been exited and the capital realized (Argentum, 2010a).

Tier 3: The private equity-owned companies

Tier 3 represents the companies that receive capital, knowledge and expertise from the private equity funds and make up the private equity fund's portfolio. These companies represent a broad spectrum of industries. Because these firms are mostly unquoted and may be start-up or early stage businesses, funds tend to specialize in certain types of investments in accordance with knowledge, experience and the strength of their team. Due to high asymmetries of information in the industry, the team's specialist knowledge is crucial to select the firms which they can improve, develop, expand, restructure and resell (Argentum, 2010a).

2.4 Private equity in Norway

The private equity industry in Norway is small compared to countries like Sweden, Great Britain, Finland and the Netherlands (Baygan, 2003). Private equity companies in Sweden manage about 10 times as much capital compared to Norwegian fund managers (Grünfeld & Jakobsen, 2006). However, the Norwegian private equity industry grows rapidly, and according to NVCA (2009), total funds under management was about NOK 60 billion by the end of 2009, compared to NOK 26 billion by the end of 2005.

In Norway there are 51 different fund managers who manage 107 different private equity funds. In total the 107 funds have invested in 735 portfolio companies. 69.8 % of the portfolio companies are registered in Norway. Foreign private equity funds have, on the other hand, ownership interests in 52 Norwegian companies. In total, 48 000 are employed in Norwegian companies where either a Norwegian or a foreign private equity fund has ownership interest. This is approximately 1.9 % of the total Norwegian workforce (NVCA, 2010c).

2.5 Return in the industry

Due to much secrecy in the industry, finding information about return on investments in the Norwegian private equity industry is difficult. Usually, such information is only given to investors of the fund and other stakeholder like external auditors.

In general, the return of the private equity fund and its portfolio companies follows a "J-curve". The "J-curve" effect occurs because, in the first years, the fund's invested capital is used to analyze and discover good investment objects. The process of finding potential investments costs money. In addition, negative return is usually a result of write-downs caused by unprofitable venture investments which are recognized at an early stage (Kleven, 2006).

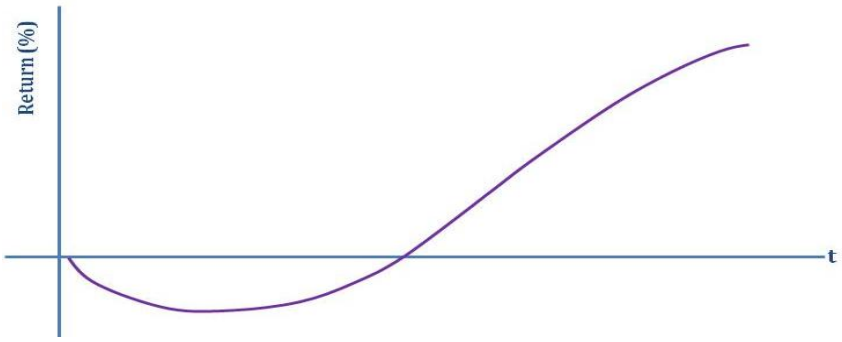

Figure 3 – The "J-curve" effect (Kleven, 2006)

The "J-curve" effect has gained its name due to the curve's form when plotted in a diagram. A period with negative return is, usually, followed with years of positive return for the private equity fund. Notice that the curve shows the aggregated return in the private equity fund.

Some research has been done on the subject, but I have not found any published figures concerning just Norwegian private equity return. According to EVCA (2006), the annual average return by European private equity funds, established in the years 1980 to 2005, were 10.3 %. Looking at the 25 % best performing funds the return is as high as 23 %. However, an important question is whether the return in private equity funds is higher than in other assets. According to Kaplan and Schoar (2005), there are only small differences between the return of American private equity funds and stock investments in S&P500. In addition, Kaplan and Schoar (2005), Lerner et al (2004) and Kehoe and Heel (2005) show that the return spread in private equity funds is more systematically than in other investments. In other words, the funds that have high return one year,

usually perform good the following year. The continuing ability to outperform return in other investments is said to be a result of good managing skills by the fund managers (Kehoe & Heel, 2005) (Alemany & Marti, 2006).

In general, it is difficult to explain why the private equity industry manages to attract big investments when the expected return is not higher than stock investments. One explanation could be that since some fund managers manage to systematically outperform stock investments, the inflow of capital is a natural result. Thus, these management environments grow large and create a concentration of management competence which continues to exist (Grünfeld & Jakobsen, 2007).

2.6 Prospects of the private equity industry

The Economist proclaimed in 2004 that private equity firms were the new kings of capitalism. To what extent private equity firms can continue to rule depends basically on three different factors (Grünfeld & Jakobsen, 2006):

- Access to interesting new portfolio companies
- Good flow of knowledge and competence to the industry
- Inflow of capital to the private equity funds

The different Norwegian private equity participants emphasize that lack of funding is not the major reason for the industry's development, but rather access to knowledgeable and competent employees. To build a specialized private equity environment in Norway takes time. To succeed you have to build step by step and continuously (Grünfeld & Jakobsen, 2006).

By the end of second quarter of 2010 the Norwegian private equity funds managed NOK 62.4 billion, which is the highest level ever. The investment rate is also at its highest since before the financial crises. In fact, it has been invested more in the first six months of 2010 compared to 2009 as a whole (NVCA, 2010c). The prospects of the Norwegian industry are hard to forecast, but based on figure facts the industry has potential for further development and expansion.

3 Financial accounting models

In order to understand the development of financial accounting, from historical cost to fair value, we need to understand the fundamental differences between the accounting models. Thus, chapter 3 is a presentation of different accounting models and their use in financial accounting today. Notice that this is only a presentation of the models' fundamentals. I will come back to the different accounting standards and their requirements later on.

It is essential to understand to whom we are reporting when we talk about financial accounting. The development of financial accounting is to a large extent a result of stakeholder's different information requirements (Penman, 2007a).

Roughly, we can identify two different models of financial accounting:

- Historical cost accounting
- Fair value accounting

The different models will be explained separately in the paragraphs below. In addition, each model will be put in a valuation context to see how it can be used for estimating fair value in a generally context.

3.1 The concept of historical cost accounting

Historical cost is defined as a measure of value used in accounting, in which the price of an asset on the balance sheet is based on its nominal or original cost when acquired by the company (Investopedia, 2010a). Historical cost is the main measure of value used under Norwegian Generally Accepted Accounting Principles (NGAAP).

Using historical cost accounting implies that fixed assets on a firm's balance sheet are reported at the original cost adjusted for depreciation. However, under NGAAP there is a principle called the "prudence principle" which states that if an asset has decreased in value and it is not temporary, the value has to be written down to the lowest value of historical cost or fair value (Johnsen & Kvaale, 2007). The prudence principle implies that it is not allowed, within NGAAP, to report historical cost values that are higher than the asset's fair value.

3.1.1 The concept of the equity method

Within historical cost accounting, there is technique called the equity method. The equity method of accounting is used to assess a firms' profit earned by their investments

in other companies. The reported profit is equal to the size of the equity investment, and the firm reports the profit on its income statement. The equity method is a standard technique to use when a company has significant influence over another company (Investopedia, 2010b).

The method can be exemplified by imagine that firm A owns 20 % of firm B that has NOK 1 million in net income. Firm A will then report earnings of NOK 200 000[4].

When the method is used to account for ownership in a company the investor firm reports the initial investment in the stock at cost. The value is then periodically adjusted to reflect the changes in value due to the investor's share in the company's income or losses. Thus, the equity method is as a technique within the historical cost framework used to adjust the value. By applying the technique it is possible to adjust historical values and thereby converge it closer to the investment's fair value.

3.1.2 Historical cost and valuation

Historical cost accounting reports a balance sheet with historical cost rather than fair values. An important question is how we can use a historical cost financial statement in our valuation of a firm. Penman (2007a) makes some good points in his article when he states that under historical cost accounting:

- The income statement is the primary source for conveying information about value to shareholders and not the balance sheet. This is true because, like Penman (2007a) writes, historical cost earnings reports the value-added buying inputs at one price, transforming them according to a business model, and selling them at another price.
- Current income, as he says, forecast future income on which a valuation can be made.
- The book value of equity does not report the value of the equity, which means that the Price/Book ratio is typically not equal to 1.
- Penman argues that earnings do not report shocks to value, but shocks to trading in input and output markets.
- Finally, he says that earnings measure the stewardship of management in arbitraging input and output markets, that is, in adding value in markets.

[4] To estimate reported earnings you calculate: 20 % x 1 000 000 = 200 000

Based on Penman's argumentation, the income statement comes to the fore with a matching of revenues with costs. Thus, the balance sheet is not a statement of values but rather a by-product of this matching. The historical cost accounting reports a history of transactions with the market, and this history can be used for valuations.

3.2 The concept of fair value accounting

Fair value accounting has become one of the most debated accounting principles the resent years. Many still favour historical cost accounting, but the supporters of fair value argue that fair value is a superior economic measurement compared to historical cost. One argument is that investors are concerned with value and not cost. Another is that as time passes by, historical prices become irrelevant when assessing a firm's current financial position (Penman, 2007a).

What is fair value? To understand the concept behind fair value we need to clarify three different notions of fair value accounting (Penman, 2007a).

1. **Fair value applied in a mixed attribute model[5]:**
    - The accounting is primarily based on historical cost, but fair value is used when certain conditions are satisfied.
2. **Fair value continually applied as entry value:**
    - Assets are re-valued at their replacement cost.
    - Current costs are recorded in the income statement.
    - Unrealised gains or losses are also recognised in the income statement.
3. **Fair value continually applied as an exit value[6]:**
    - Assets and liabilities are assessed each period to current exit price.
    - Unrealised gains or losses are recorded as part of comprehensive income.

According to Penman (2007a), fair value applied in a mixed attribute model and as entry value is basically modified historical cost accounting. What makes fair value accounting different from historical cost is the recognition of exit values without an historical exit transaction.

[5] NGAAP is an example of a mixed attribute model.
[6] IFRS is an example where fair value is continually applied as exit value.

Financial Accounting Standards Board's (FASB) definition of fair value is described in FASB's Statement 157, and is in conformity with applying fair value as an exit value in the account:

"*Fair value is the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date*."

The definition has not the same wording compared to the definition of IASB. However, as I pointed out earlier in the introduction, the interpretation of the two definitions is basically identical.

3.2.1 Fair value and valuation

Fair value accounting at its best uses information about equity value to transform all assets and liabilities on the balance sheet as their value to the shareholder (Penman, 2007a). Some important factors are:

- The balance sheet becomes the primary source of information to shareholders.
- With a balance sheet recorded at fair value, the book value equals fair value and the Price/Book ratio is 1.
- The income statement reports what Penman (2007a) calls "economic income", because it is simply the change in value over time.
- Unlike historical cost accounting, earnings cannot forecast future earnings because changes in values today do not predict future changes. Instead the balance sheet provides the valuation.

Briefly, the balance sheet is used for valuation while the income statement is used for information concerning risk exposure and management performance.

3.3 Historical cost versus fair value in valuation

We are now going to compare historical cost accounting and fair value accounting, and see how the different methods can be used in valuation. In order to do so, I assume that the methods are implemented in their ideal form. Consequently, ideal fair value accounting reports a book value that is sufficient to value a firm, but earnings that that are useless. On the contrary, ideal historical cost accounting reports a book value that does not report the market value. Instead we have to use the earnings in our valuation (Penman, 2007a).

To illustrate the differences, we can use a discounted cash flow model presented in Penman's article (2007a),

$$(A) \quad Value_t = \frac{Expected\ Earnings_{t+1}}{r}$$

where *r* is the required return for the equity holder.

When using ideal fair value accounting we know that the balance sheet provides a good basis for our valuation. Thus, we can forecast earnings using the current book value:

$$(B) \quad Expected\ Earnings_{t+1} = r\ x\ Book\ Value_t$$

The reason for doing so is because the current book value, using ideal fair value accounting, is per definition the real value or market value.

Using ideal historical cost accounting, the balance sheet is not a sufficient basis for our valuation. Instead we need to use the income statement to forecast future earnings. We can simplify and assume that:

$$(C) \quad Expected\ Earnings_{t+1} = Earnings_t$$

Some might question this simplification, but if we assume that a firm is in steady state, today's earnings are the best estimate for future earnings. Ideal historical cost accounting then determines equity value by capitalising current earnings:

$$(D) \quad Value_t = \frac{Earnings_t}{r}$$

According to Penman (2007a) the lessons are:

1. We can use the historical cost income statement to determine the equity value. It is not necessary to report the balance sheet at fair value.
2. If we assume that we know required return of equity there is no fundamental reason to say that fair value accounting is better than historical cost accounting. Which method is better depends on how measurement in practise differs from the ideal.
3. Usually we do not know the required return and in those occasions fair value accounting has a distinct advantage. Historical cost valuation requires a required

return to convert a cash flow to a stock of value. Fair value accounting, on the other hand, delivers the value directly from the balance sheet. Thus, the forecast of earnings is not necessary when book value already reports the value. Instead, using faire value accounting we can assume:

$$Value_t = Book\ Value_t$$

Briefly, fair value accounting has its advantages when implementation issues are put aside. Nevertheless, historical cost accounting has features that provide an alternative, should faire value accounting not be attainable (Penman, 2007a). These three lessons are interesting because they tell us that regardless of fair value or historical cost accounting the valuation can be achieved.

It is worth noticing that fair value accounting requires that the bookkeepers have good valuation expertise. If the valuation expertise is poor, the balance sheet would not report fair values, and the consequence would be a less credible balance sheet.

4 Accounting requirements

So far, we have looked at the different models of accounting from a general point of view. The next step, in order to understand the different challenges in determining fair value, is to get knowledge about accounting rules and accounting standards. Financial accounting is important information when assessing a firm's value. The different accounting rules and standards contribute to third-parties being able to relay on the information being reported.

The scope of this chapter will be how the private equity firms have to report investments carried out by the fund. In other words, how to report values in the private equity fund. The financial reporting carried out in the different portfolio companies is irrelevant for my question of discussion. It is worth noticing, that the reporting by portfolio companies and the funds can be different if accounting requirements are not similar.

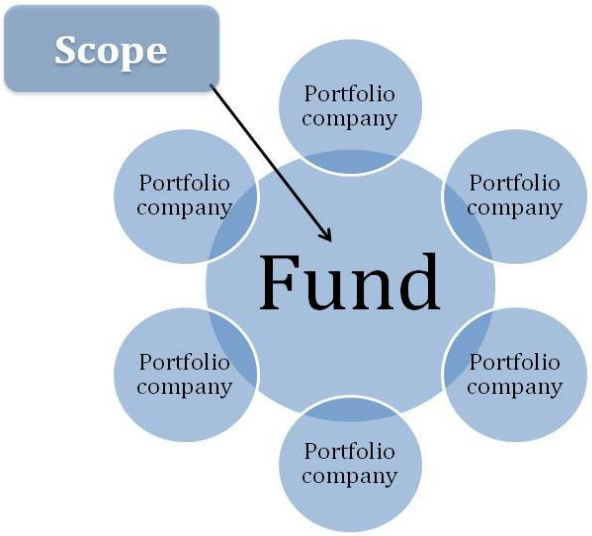

Figure 4 – Accounting scope

In recent years the financial accounting rules have been harmonized across borders due to globalization and urge of more transparency. Since this thesis has a Norwegian approach to the private equity industry, I will start by looking at the Norwegian Accounting Act (NAA). Then, I will look at the International Financial Reporting Standards (IFRS) and the U.S. Generally Accepted Accounting Principles (USGAAP). These two accounting standards are important when discussing fair value requirements, because fair value is an essential measuring attribute in both IFRS and USGAAP.

4.1 The Conceptual Framework

Before I look into detail at the different accounting rules, we need to get an understanding of what point of departure the different rules are based on. In other words, we need to understand what a conceptual framework is. To answer that question we can turn to Kvifte and Johnsen's (2008) explanation. They say that a conceptual framework can briefly be explained as a normative accounting theory or a platform for derivation of solutions relating to practical accounting issues, which in turn limits the possible range of acceptable solutions. Another definition could be that the framework should be a theoretical structure of assumptions, principles, and rules that holds together the ideas comprising a broad concept (Business Dictionary, 2010). The motivation for developing a framework is the objective to indicate the important characteristics regarding information reported in the financial statement.

A normative theory is different from philosophy of science because preferences from individuals, or groups of individuals, have influence on the contents of the theory. The conceptual frameworks from the leading standard boards (IASB and FASB) are deductive. It means that solutions related to accounting issues are derived from the normative objective. The counterpart to normative framework is descriptive framework where the solution to an accounting issue is derived from approved and well established accounting practise (Kvifte & Johnsen, 2008). The Norwegian framework is an example of a descriptive framework.

International Accounting Standards Board (IASB), Financial Accounting Standards Board (FASB) and Norwegian Accounting Standards Board (NASB) are all primary users of their respective frameworks. These Boards use the frameworks in their work trying to develop new and improved accounting standards. The frameworks of IASB and FASB have an asset-liability view, while the framework of NASB is revenue-expense orientated (Kvifte & Johnsen, 2008). Introducing the asset-liability view represented a departure from the traditional view that accounting should focus on the measurement of income through the matching of costs with revenue. Instead, the asset-liability view focuses on defining and measuring assets and liabilities. Thereby, income is recognized based on changes in the balance sheet accounts (Gore & Zimmerman, 2007).

4.2 The Norwegian Accounting Act

The Norwegian Accounting Act (NAA) is best characterized as a frame-law without detailed regulation, which means that the Act implies professional judgment when used. However, one of the main principles is that the financial statement is supposed to be prepared in accordance with generally accepted accounting principles (GAAP). GAAP is a dynamical term meaning that it can be adjusted to follow the development of the accounting practices (Kvifte & Johnsen, 2008).

4.3 Accounting requirements in NGAAP

The Norwegian GAAP (NGAAP) is based on fundamental accounting principles and a transaction based model. These fundamental principles are legally established in the NAA. Thus, NGAAP can only be used if they are in accordance with the NAA and they have practical utilization (Kvifte & Johnsen, 2008).

Because NGAAP is a transaction based model and revenue-expense orientated, the financial statement is prepared based on historical cost values. For Norwegian private equity funds who reports in accordance with NGAAP, it implies that the fund's investments are reported at their historical costs values, adjusted for impairments.

4.3.1 Developments in NGAAP

The development of NGAAP has traditionally been orientated towards USGAAP. Even though IASB was established in 1973, the influence of IFRS was of little importance until the late 80s and early 90s. IFRS' position is today much more important, and when the new NAA was passed in 1998 the Norwegian Parliament expressed that NGAAP had to be harmonized with IFRS. In addition, the EU-commission decided in 2002 that all listed companies had to report in accordance with IFRS. Due to Norway's membership in the European Economic Area (EEA) the decision also include Norwegian listed companies (Kvifte & Johnsen, 2008).

Due to the decision made by the EU-commission, the Norwegian legislative authorities wanted to revise the NAA in order to be harmonized with IFRS. The attempt to combine two different accounting theories was difficult, and would lead to fundamental conflicts because the NAA framework has a revenue-expense view while IFRS' conceptual framework has an asset-liability view. These two frameworks are different in its nature and the attempt to combine the NAA and the IFRS were a dead end.

To overcome the problem regarding a revenue-expense view, on one hand, and an asset-liability view on the other hand, the Norwegian Ministry of Finance decided upon a "two tier system". The "two tier system" means that Norwegian private equity firms can choose between NGAAP and IFRS as long as they are not listed.

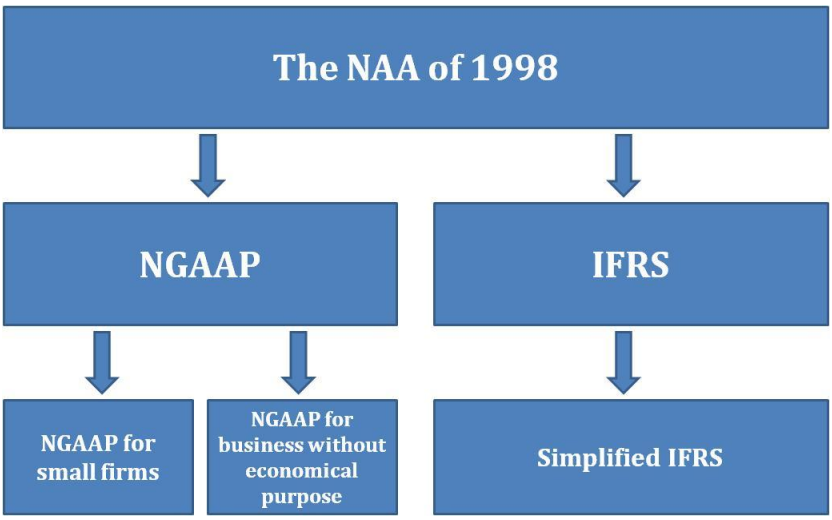

Figure 5 - The two tier system (Kvifte & Johnsen, 2008)

The "two tier system" implies that private equity firms who choose to follow the IFRS path have to report in accordance with IFRS regulations. Compared to NGAAP, the companies then report the fair value of the fund's investments instead of historical costs. I will come back to IFRS regulations in paragraph 4.5. Notice that simplified IFRS is a distinctively Norwegian regulation without any foundation in the IFRS framework.

4.3.2 Fair value measuring in NGAAP

As we have seen, NGAAP is basically an historical cost account where fixed assets on the balance sheet are reported at their nominal value adjusted for depreciation. When that is said, NGAAP is better characterized as a mixed attribute model because there is some fair value reporting when certain conditions are satisfied. One example is § 5-8 concerning financial instruments and commodity derivatives which states that (RSKL, 2007):

Financial instruments and commodity derivatives shall be recognized at fair value if these

1. *are classified as current assets;*
2. *form part of a trading portfolio intended for resale;*

3. *are traded on a securities exchange, an authorized market place or a corresponding regulated marked abroad; and*
4. *have a diversified ownership structure and favorable liquidity.*

If a private equity firm has some financial instruments or commodity derivatives, and they satisfy the requirements, they are supposed to be recognized at fair value. Another example is § 5-2 dealing with current assets, which states that current assets should be valued at the lower of acquisition cost and fair value. Similar is there in § 5-3 a statutory provision which says that fixed assets should be written down to fair value in occasions where the decrease in value is not temporary. Commonly, these two statutory provisions are referred to as the prudence principle (RSKL, 2007).

An important question when discussing fair value is what guidelines you should use when estimating the fair value. In contrast to financial current assets, many current and fixed assets are not traded on a stock exchange or in other marketplaces. One big and important challenge is then to estimate the correct fair value.

To overcome this problem, NASB has issued a preliminary accounting standard regarding write-downs of fixed assets. This standard can be used when assessing the fair value of a fixed asset when the impairment is not temporary.

According to NASB, ideal fair value of a fixed asset is the price an independent third party would be willing to pay in an arm's length transaction between well informed and voluntary participants. The ideal situation is not necessarily possible to achieve. Thus, fair value is due to practical reasons referred to as recoverable amount, meaning the highest of net market value and value in use. The process of assessing recoverable amount is similar to the requirements in IAS 36, which I will present in paragraph 4.5.

Net market value

Net market value is defined as the asset's transaction price between independent participants adjusted for possible costs relating to the sale. If an equal asset is being sold in a market, the net market value should be estimated based on observable market prices. In the contrary case, the net market value has to be estimated using professional judgment.

Value in use

Value in use is estimated in two steps:

a) The expected cash flow belonging to the asset has to be estimated, including cash flows regarding disposal.

b) The present value of the cash flow is estimated using a discount rate.

Box 1 - Net market value and value in use (NRS(F), 2009)

The NAA states that during uncertainty you have to use the best estimate based on available information. Best estimate of a period's cash flow is usually the expected value. The expected value is equal to the weighed sum of the different outcomes multiplied with the different probabilities of that outcome. It is also possible to use the individual most likely outcome. Cash flow estimates are supposed to be based on reasonable and valid assumptions, and thus reflect the best judgment of the management. External information is usually more creditable and should be emphasized more than internal information (NRS(F), 2009)

Further, NASB states that cash flow estimates should be based on the newest budgets and prognosis approved by the management. In addition, the forecast period should be no longer than five years, unless you can give ground for a longer period. Estimates for the following period are supposed to be projected using budgets or forecasts as a base, and a constant or decreasing growth rate. However, if you have good reasons a higher growth rate can be used. The most important thing is to make sure to not use a growth rate that exceeds the long term average growth rate for the economy, unless there are good and justified reasons.

What discount rate are you suppose to use when estimating the present value? Ideally the discount rate should reflect the required market return for investments in the same type of industry as the fair value estimate is being calculated. Because almost all investments are financed with both equity and debt, the Weighted Average Cost of Capital (WACC) is usually a good approximation to the investments required return. Thus, when firms can estimate WACC, they are supposed to use it for the calculation of the present value (NRS(F), 2009). The WACC is explained in detail on page 41.

The cost of equity is more difficult to estimate when the company is not listed on a stock exchange. For companies not listed, NASB has announced that it is possible to use an alternative borrowing rate as discount rate. However, the requirement for doing so is that the alternative borrowing rate cannot be totally misguiding compared to WACC. The alternative borrowing rate is, according to NASB, the borrowing rate the firm has to pay to the lender to completely finance the investment during the investment's economical life (NRS(F), 2009).

4.4 International Financial Reporting Standard (IFRS)

Briefly, IFRS can be described as a standard for preparation of financial statements. IFRS is a very complex framework according to organization and the level of detailed accounting requirements. The accounting standards are issued by International Accounting Standards Board (IASB), earlier named International Accounting Standards Committee (IASC). IASC issued accounting standards named International Accounting Standards (IAS). IASB has not changed the name of accounting standards inherited after IASC. Thus, IFRSs consist of both IFRSs and IASs (DnR, 2009).

In addition to the different accounting standards issued by IASB, the International Financial Reporting Interpretations Committee (IFRIC) publishes interpretations concerning accounting issues. IFRIC is a body within IASB, and its main concern is to develop guidelines on accounting issues not specified in the different IFRSs (DnR, 2009).

4.5 Accounting requirements in IFRS

As already mentioned, IFRS is based upon a framework which is asset-liability orientated. IFRS uses fair value as a central measuring attribute. Thus, the accounting is more value-driven compared to transaction based accounting. This paragraph explains the requirements private equity firms have to comply when following the IFRS.

IASB does not have an own IFRS or IAS that regulates how fair value is supposed to be estimated. Private equity firms reporting in accordance with IFRS have to use many different IFRSs when deciding upon the fair value. We have already defined fair value in accordance with IFRS, but we can afford to look at it again:

"Fair value is the amount for which an asset could be exchanged, or a liability settled, between knowledgeable, willing parties in an arm's length transaction."(IASB, 2009c)

For private equity firms especially one standard is very important – namely IAS 39. The object of IAS 39 is to establish principles for recognizing and measuring financial assets, financial liabilities and some contracts to buy or sell non-financial items (IASB, 2009c). The standard is essential because private equity investments are classified as financial assets. In addition, it is possible to use the valuation principles in IAS 39 for estimating fair value of investments not quoted in active markets.

A financial instrument is defined as *a contract that gives rise to a financial asset of one entity and a financial liability or equity instrument of another entity* (IASB, 2010a). The definition is stated in IAS 32, but the same definition is applied for IAS 39.

As a rule, the initial recognition of financial assets should be reported at fair value. For the purpose of measuring a financial asset after initial recognition, IAS 39.45 classifies financial assets into four categories. These categories have different measuring rules which can be viewed in table 2:

Financial assets	
Category:	**Measuring:**
Financial assets at fair value through profit or loss	Fair value
Held-to-maturity investments	Amortised cost using the effective interest method
Loans and receivables	Amortised cost using the effective interest method
Available-for-sale financial assets	Fair value

Table 2 – Overview: measuring of financial assets (IASB, 2009c)

As we can see from the table above, fair value measuring is used for two of the categories. When fair value is used, IAS 39 has its own guideline on how to report the most correct value. These guidelines are enclosed in IAS 39 appendix A, article AG69 – AG82.

Entities that report fair value do not have any problems measuring fair value if the financial asset is regarded as quoted in an active market. If that is the case, the firm can easily get access to market information. In IAS 39, a financial asset is regarded quoted in an active market if quoted prices are readily and regularly available from an exchange, dealer, broker, industry group, pricing service or regulatory agency, and those prices represent actual and regularly occurring market transactions on an arm's length basis. Further, IAS 39 states that the existence of published price quotations in an active

market is the best evidence of fair value and when they exist, they are used to measure the financial asset or financial liability (IASB, 2009c).

Unfortunately, very few financial assets satisfy these conditions. Especially, the private equity industry faces a challenge estimating fair value because many of its investments are in private firms not listed on a stock exchange. To overcome this challenge, IAS 39 has guidelines on how to estimate fair value when an active market is missing.

"If the market for a financial instrument is not active, an entity establishes fair value by using a valuation technique. Valuation techniques include using recent arm's length market transactions between knowledgeable, willing parties, if available, reference to the current fair value of another instrument that is substantially the same, discounted cash flow analysis and option pricing models. If there is a valuation technique commonly used by market participants to price the instrument and that technique has been demonstrated to provide reliable estimates of prices obtained in actual market transactions, the entity uses that technique." (IASB, 2009c)

According to IAS 39, the objective of using a valuation technique is to establish what the transaction price would have been at the measurement date – in other words the exit price. One important factor to consider is that fair value is suppose to be estimated on the basis of a valuation technique that makes maximum use of market inputs, and relies as little as possible on entity-specific inputs (IASB, 2009c).

IAS 39 states that "*an appropriate technique for estimating the fair value of a particular financial instrument would incorporate observable market data about the market conditions and other factors that are likely to affect the instrument's fair value"* (IASB, 2009c). These market data and other factors are:

a) The time value of money (i.e. interest at the basic or risk-free rate)
b) Credit risk
c) Foreign currency exchange prices
d) Commodity prices
e) Equity prices
f) Volatility (i.e. magnitude of future changes in price of the financial instrument or other item)
g) Prepayment risk and surrender risk
h) Servicing costs for a financial asset or a financial liability

The fair value of a financial instrument is supposed to be based on one or several of these factors, and even other factors if that is more appropriate (IASB, 2009c).

As we can see, the regulation of fair value measuring is very detailed, and entities reporting in accordance with IFRS have many factors to consider.

In addition to the different guidelines in IAS 39, IASB Expert Advisory Panel has issued guidance on valuing financial instruments when markets are no longer active. The report provides useful information and educational guidance for measuring and disclosing fair values for entities applying IFRS. It has not been approved by the IASB and does not establish new requirements. The report provides guidance about the processes used and the judgments made when measuring and disclosing fair value (IASB, 2008).

4.5.1 Fair value development in IFRS

Even though IAS 39 is the most important accounting standard for private equity firms, the accounting requirements also consists of IAS 32 and IFRS 7. As a result, some IFRSs contain limited measurement guidance, whereas others contain extensive guidance and that guidance is not always consistent. Inconsistencies in the guidance have added to the complexity of financial reporting and have contributed to diversity in practice (IASB, 2010). To overcome these problems, IASB has started a fair value measurement project where the goal is to develop common requirements for measuring and disclosing of fair value. The project is part of IASB and FASBs *Memorandum of Understanding*, where the convergence between IFRS and USGAAP is an important objective (IASB, 2010b).

The work is based on FAS 157, and according to the schedule issued by IASB, common IFRS and USGAAP faire value measurement standards are suppose to be published during the first quarter of 2011 (IASB, 2010b).

4.6 U.S. Generally Accepted Accounting Principles (USGAAP)

The Financial Accounting Standards Board (FASB) has existed since 1973, and according to themselves their mission is to establish and improve standards of financial accounting and reporting that foster financial reporting by non-governmental entities that provides decision-useful information to investors and other uses of financial reports (FASB, 2010). Basically, this means that FASB is in charge of establishing GAAPs for the private sector in the U.S.

The accounting standards issued by FASB are called Financial Accounting Standards (FAS). In addition to issue FAS, the Board is also responsible for developing the conceptual framework (FASB, 2010). As we can see, many parallels can be drawn between the organizational structure of FASB and IASB.

4.7 Accounting requirements in USGAAP

So far we have looked at accounting requirements in the NAA and IFRS. As we have seen that the two accounting standards are based upon two fundamental different frameworks. USGAAP have the same asset-liability view as IFRS. Thus, we can find many similarities when comparing these two accounting standards. Next, is to see what accounting requirements private equity firms need to follow when reporting in accordance with USGAAP.

Just as investments by private equity funds are reported at fair value under IFRS, the same holds for investments reported in accordance with USGAAP. The requirements are stated in an own standard called FAS 157, which was published in 2006. The background for the statement was different definitions of fair value and limited guidance for applying these definitions in USGAAP. Moreover, that guidance was dispersed among the many accounting pronouncements that require fair value measurements. Differences in the guidance created inconsistencies that added to the complexity in applying USGAAP. As we can understand, FAS 157 was a result of the inconsistency that appears in IFRS today.

In developing FAS 157, FASB considered the need for increased consistency and comparability in fair value measurements and for expanded disclosures about fair value measurements. FAS 157 defines fair value, establishes a framework for measuring fair value in USGAAP and expands disclosures about fair value measurements (FASB, 2006).

Fair value in FAS 157 is defined as "*the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date*" (FASB, 2006). In paragraph 3.2 we discussed three different notions of fair value which Penman (2007a) clarified. One of the notions were fair value continually applied as an exit value, and we concluded that fair value in accordance with USGAAP is to be understood as en exit value. This conclusion is based on FAS 157 where we can read the following:

"*The transaction to sell the asset or transfer the liability is a hypothetical transaction at the measurement date, considered from the perspective of a market participant that holds the asset or owes the liability. Therefore, the objective of a fair value measurement is to determine the price that would be received to sell the asset or paid to transfer the liability at the measurement date (an exit price)*" (FASB, 2006).

FASB uses the term exit price that can be understood as a hypothetical selling price. The argument from FASB in doing so is based on an assumption that the owners of a firm is interested in the selling value of an asset, not the historical cost value (FASB, 2006).

Like IFRS, USGAAP has its own valuation techniques to estimate fair value when prices are not quoted in an active market. These techniques are published in FAS 157 and consist of:

- **The market approach**
  "The market approach uses prices and other relevant information generated by market transactions involving identical or comparable assets or liabilities (including a business)" (FASB, 2006).
- **The income approach**
  "*The income approach uses valuation techniques to convert future amounts (for example, cash flows or earnings) to a single present amount (discounted). The measurement is based on the value indicated by current market expectations about those future amounts*" (FASB, 2006).
- **The cost approach**
  "*The cost approach is based on the amount that currently would be required to replace the service capacity of an asset (often referred to as current replacement cost)*" (FASB, 2006).

Choosing an appropriate valuation technique depends on the circumstances. In some cases, a single valuation technique is enough. In other cases, multiple techniques will be appropriate. If multiples techniques are used to estimate fair value, the result is supposed to be evaluated and weighed to consider the reasonableness of the estimate (FASB, 2006).

In FAS 157 inputs used in the valuation can be either observable or non-observable:

- *"Observable inputs are inputs that reflect the assumptions market participants would use in pricing the asset or liability developed based on market data obtained from sources independent of the reporting entity"* (FASB, 2006).
- *"Unobservable inputs are inputs that reflect the reporting entity's own assumptions about the assumptions market participants would use in pricing the asset or liability developed based on the best information available in the circumstances*" (FASB, 2006).

As a rule, valuation techniques used to measure fair value is supposed to maximize the use of observable inputs and minimize the use of unobservable inputs (FASB, 2006). We see the same in NGAAP and IFRS, where external information is superior to internal information when estimating fair value.

To increase consistency and comparability in fair value measurements, in addition to related disclosures, FASB has developed a fair value hierarchy that prioritizes the inputs used in valuation techniques. The fair value hierarchy is divided into three different levels, where level 1 inputs have the highest propriety and level 3 inputs the lowest (FASB, 2006):

- **Level 1 inputs:**
  "*Level 1 inputs are quoted prices (unadjusted) in active markets for identical assets or liabilities that the reporting entity has the ability to access at the measurement date. An active market for the asset or liability is a market in which transactions for the asset or liability occur with sufficient frequency and volume to provide pricing information on an ongoing basis. A quoted price in an active market provides the most reliable evidence of fair value….*" (FASB, 2006).
- **Level 2 inputs:**
  "*Level 2 inputs are inputs other than quoted prices included within Level 1 that are observable for the asset or liability, either directly or indirectly*" (FASB, 2006).
- **Level 3 inputs:**
  "Level 3 inputs are unobservable inputs for the asset or liability. Unobservable inputs shall be used to measure fair value to the extent that observable inputs are not available, thereby allowing for situations in which there is little, if any, market activity for the asset or liability at the measurement date….. In developing

unobservable inputs, the reporting entity need not undertake all possible efforts to obtain information about market participant assumptions…." (FASB, 2006).

Inputs on level 1 and 2 are relatively easy to understand and accessible. Level 3 inputs, on the other hand, which are relevant for many assets and liabilities, are not self explanatory. When using level 3 inputs the firm itself has to estimate what price the different market players are willing to pay. Thus, the reliability requirements become important. However, when FASB chooses to publish FAS 157 with a level 3 regulation they have to assume that a valuation based on these inputs can be reliable (Kvifte & Johnsen, 2008).

4.8 Important discoveries

During the survey of accounting requirements, both similarities and differences have been discovered in the NAA, IFRS and USGAAP. While IFRS and USGAAP have many similarities, the differences are bigger between the NAA and IFRS and USGAAP.

We have looked at the different conceptual frameworks and learned that the frameworks of IASB and FASB have an asset-liability view while the framework of NASB is revenue-expense orientated. Thus, faire value accounting is more common in IFRS and USGAAP compared to the NAA with a historical cost accounting basis. However, because NAA is classified as a mixed attribute model elements of fair value measuring occurs.

Due to elements of fair value accounting in all standards NASB, IASB and FASB has published guidelines on how to estimate faire value when there is lack of active markets with quoted prices. These guidelines are surprisingly similar, yet convergence has still more potential.

5 Measuring fair value in the private equity industry

So far, I have looked at different accounting models and explained how historical cost and fair value accounting work in their ideal form. In addition, we have gained insight to the different accounting requirements, and discovered that fair value is a central measurement attribute in IFRS and USGAAP. The next step is to see how the valuation of fair value is implemented in the private equity industry. Private equity firms need to valuate fair value on all their investments which make up their investment portfolio. For private equity funds located in Norway, the accounting requirements are basically historical cost accounting due to NGAAP. However, the private equity managers are required to carry out periodic valuation as part of the reporting process to investors in the funds they manage. Thus, fair value measuring also becomes an important measuring attribute for Norwegian companies as well.

As we have seen through the review of accounting requirements, fair value measuring can be challenging in lack of an active market. The different accounting standards give guidance on how to overcome these challenges, but the technical description is missing. Thus, we will now go more into detail and look at different methods used in the private equity industry to estimate the fair value of portfolio investments. In other words, the objective of this chapter is to gain insight of valuation methodologies actually used in the industry.

I will start by looking at more general valuation techniques that can be used to determine the value of any asset. The reason of such an approach is because the specific valuation techniques used by private equity fund managers are based on the same principles.

5.1 Measuring fair value in general

When estimating fair value it is possible to use a wide range of models. The models often make different assumptions, but some common characteristics can be classified in broader terms (Damodaran, 2002).

Damodaran (2002) divides valuation into three different approaches. The first is discounted cash flow (DCF) valuation. The second approach is called relative valuations, where you estimate the value of an asset by looking at the pricing of comparable assets relative to a common variable. The third approach is called contingent claim valuation.

This approach uses option pricing models to measure the value of assets that share option characteristics (Damodaran, 2002). Next, DCF valuation and relative valuation will be presented in more detail because the techniques are common in the private equity industry.

5.1.1 Discounted cash flow valuation

The discounted cash flow approach has its foundation in the present value rule, where the value of any asset is the present value of expected future cash flows generated by the asset (Damodaran, 2002):

$$Value = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t}$$

where n = Life of the asset

CF_t = Cash flow in period t

r = Discount rate reflecting the riskiness of the estimated cash flows

Box 2 – Present value of CF (Damodaran, 2002)

As we can see from the relationship in the formula above the discount rate will be a function of the risk associated with the estimated cash flows, with higher rates for riskier assets and lower rate for safer projects. In addition, it is important to be aware that the cash flows will vary from asset to asset. Damodaran (2002) thinks of discounted cash flow valuation on a continuum. At one end of the spectre, you have the default-free zero coupon bond with a guaranteed cash flow in the future. When discounting a cash flow at the riskless rate, the yield should be the value of the bond. Further up the risk spectrum, you find corporate bonds where the cash flows take form of coupons and there is risk default. These bonds can be valued by discounting the cash flows with a discount rate reflecting the default risk. Even more risk is related to equities where we find expected cash flows with substantial uncertainty regarding the expectations. The value in such a situation should be the present value of the expected cash flows at a discount rate that reflects the uncertainty (Damodaran, 2002).

Using discounted cash flows in valuation, the basic idea is to estimate the intrinsic value of an asset based on its fundamentals.

"What is intrinsic value? In lack of a better definition, consider it the value that would be attached to the firm by an all-knowing analyst, who not only estimates the expected cash flows for the firm correctly but also attaches the right discount rate to these cash flows and valuates them with absolute precision." (Damodaran, 2002, p. 12)

As we can understand from the definition by Damodaran (2002), it might seem impossible to estimate the intrinsic value. Especially when valuing companies with substantial uncertainty regarding the future fundamentals. Thus, estimates on some companies differ from the market price attached to these companies. It is not necessarily a sign that markets are inefficient, because even though market prices can differ from intrinsic value it is expected that these values will converge over time (Damodaran, 2002).

Discounted cash flow valuation can be divided into three different paths. The first is to value the equity stake in the business. Second, it is possible to value the entire firm. In that case the valuation includes the equity and other claimholders in the firm such as bondholders and preferred stockholders. The third path is to value the firm in pieces, beginning with its operations and then adding the effects on value of debt and other non-equity claims. Similar for all three approaches are discounting expected cash flows. However, the cash flows and discount rates used are different under each approach (Damodaran, 2002).

Value of equity

The value of equity is estimated by discounting expected cash flows to equity. The cash flows to equity are the residual cash flows after meeting all expenses, reinvestments, tax obligations, and interest and principal payments. These cash flows are discounted at the cost of equity (Damodaran, 2002).

$$Value\ of\ equity = \sum_{t=1}^{t=n} \frac{CF\ to\ equity_t}{(1 + k_e)^t}$$

where
- n = Life of the asset
- $CF\ to\ equity_t$ = Expected cash flow to equity in period t
- k_e = Cost of equity

Box 3 – Value of equity (Damodaran, 2002)

To estimate the cost of equity[7] we must determine the expected rate of return of the company's stock. This is quite challenging because expected rates of return are unobservable. However, we can rely on asset-pricing models that translate risk into expected return. The most common asset-pricing model is the capital asset pricing model (CAPM), which defines a stock's risk as its sensitivity to the stock market (Koller et. al, 2005).

The CAPM is a model where expected rate of return of any security equals the risk-free rate plus the security's beta multiplied with the market risk premium.

$$E(R_i) = r_f + \beta_i \left[E(R_m) - r_f\right]$$

where $E(R_i)$ = Security i's expected return

r_f = Risk-free rate

β_i = Stock's sensitivity to the market

$E(R_m)$ = Expected return of the market

Box 4 – CAPM (Koller et. al, 2005)

The risk-free rate and the market risk premium, defined as the difference between $E(R_m)$ and r_f, are common to all companies. That is possible because the risk free-rate is an interest rate everyone can obtain from the bank when depositing money. Likewise, the expected return of the market is the same for all investors and companies. The beta however, varies across industries and companies. The beta represents a stock's incremental risk to diversified investors, where risk is defined by how much the stock covaries with the aggregated stock market (Koller et. al, 2005).

Understanding beta can be difficult. Thus, we can use an example[8] to see how it works in practice. Consider two companies A and B. Company A produces and sells a product where its demand is relatively independent of the stock market's value. Thus company A's beta is low, and estimated at 0.4. Based on a risk-free rate of 4.3 % and a market premium of 5 %, the cost of equity for company A is 6.3 %. Company B operates in the technology industry which tends to have higher beta values. When the economy struggles, the stock market drops, and companies stop buying new technology. The

[7] Meaning the rate of return required by equity investors in the firm.
[8] The example is fetched from Koller et. al (2005) on page 301.

value of company B is said to be highly correlated with the market's value. The estimated beta for company B is 1.4, which indicates that the expected rate of return is 11.3 %. Since company A offers more protection against market downturns than company B, the investors are willing to pay a premium for the stock resulting in a lower expected return. On the contrary, company B offers little diversification[9] to the market portfolio and the company must earn higher returns to entice investor (Koller et. al, 2005).

Value of firm

The value of the firm is estimated by discounting expected cash flows to the firm. The cash flows to the firm are the residual cash flows after meeting all operating expenses, reinvestment needs, and taxes. Notice that the cash flows are calculated prior to any payments to either debt or equity holders. The cash flows are then discounted with the weighted average cost of capital (WACC), which is the cost of the different components of financing used by the firm weighed by their market value proportions (Koller et. al, 2009).

$$Value\ of\ firm = \sum_{t=1}^{t=n} \frac{CF\ to\ firm_t}{(1+WACC)^t}$$

where
- n = Life of asset
- $CT\ to\ firm_t$ = Expected cash flow to firm in period t
- $WACC$ = Weighted average cost of capital

Box 5 – Value to firm (Damodaran, 2002)

The WACC represents the opportunity cost that investors face for investing their funds in one particular business instead of other businesses with similar risk. It is important to remember to maintain consistency between the components of WACC and free cash flow when implementing such a method. For most companies, discounting cash flows at the WACC is a simple, accurate and a robust method of corporate valuation (Koller et. al, 2005).

[9] A perfect diversified investor is only exposed to systematic risk, also called market risk. On the contrary, an undiversified investor is also exposed to firm-specific risk, which makes the investor more sensitive towards value changes in the investment (Damodaran, 2002).

The WACC, in its simplest form, is the market based weighted average of the after-tax cost of debt and cost of equity.

$$WACC = \frac{D}{V} k_d (1 - T_m) + \frac{E}{V} k_e$$

where
- D/V = Target level of debt to enterprise value[10] using market values
- E/V = Target level of equity to enterprise using market values
- k_d = Cost of debt
- k_e = Cost of equity
- T_m = Company's marginal income tax rate

Box 6 – WACC (Koller et. al, 2005)

To determine the cost of equity we rely on the CAPM model introduced earlier. When approximate the cost of debt of an investment firm it is possible to use the company's yield to maturity on its long-term debt. If a company has public traded debt, calculate the yield to maturity directly from the bond's price and promised cash flow. In situations where companies have illiquid debt, use the company's debt rating to estimate yield to maturity. Because free cash flow is measured without interest tax shields it is important to measure the cost of debt on an after tax basis to avoid mix-ups. To complete the calculation, the after-tax cost of debt and cost of equity should be weighted using target levels of debt to value and equity to value. It might be difficult to figure out the target capital structure, but for mature companies a good approximation is to use the current debt-to-value ratio with market-values (Koller et. al, 2005).

5.1.2 Relative valuation

Multiples are a type of relative valuation where the objective is to value assets based on how similar assets are currently priced in the market. On one hand, multiples are easy to use and intuitive. On the other hand, they are also easy to misuse (Damodaran, 2002).

Relative valuation consists of two components. The first component is to make sure that prices are standardized by converting prices into multiples of earnings, book values or sales. The second component, which is the most difficult one, is to find similar firms. Similar firms are hard to find because even firms in the same business can differ on risk, growth potential and cash flows (Damodaran, 2002).

[10] Enterprise value is defined on page 45.

Deciding upon an appropriate multiple can be difficult. Many different multiples are used, among others:

$$(A)\ \frac{P}{B} = \frac{Price}{Book}$$

$$(B)\ \frac{P}{E} = \frac{Price}{Earnings}$$

$$(C)\ \frac{EV}{EBIT} = \frac{Enterprise\ value}{Earnings\ before\ interest\ and\ tax}$$

$$(D)\ \frac{EV}{EBITDA} = \frac{Enterprise\ value^{11}}{Earnings\ before\ interest, tax, depreciation\ and\ amortisation}$$

Box 7 – Different multiples

Why are multiples much used when they are easy to misuse? To get an understanding we can start by looking at the advantages relating to the use of multiples. One advantage is that the valuation can be completed with fewer assumptions and more quickly than for instance a discounted cash flow valuation. In addition, a relative valuation is easier to understand and easier to present to clients and customers. Finally, the method is more likely to reflect the current mood of the market, since it is an attempt to measure relative value and not intrinsic value (Damodaran, 2002).

"In fact, relative valuations will generally yield values that are closer to the market price than discounted cash flow valuations." (Damodaran, 2002, p. 453-454)

However, the strengths of relative valuation are also its weaknesses. Because multiples are easy to put together it can result in inconsistent estimates of value where risk, growth or cash flow potential are ignored. Using multiples to estimate fair value can result in values that are too high if the market over-valuates comparable firms, and vice versa. The multiples are also vulnerable for manipulation due to lack of transparency in the assumptions. A biased valuer[12] who chooses the multiple can easily ensure that almost any value can be justified (Damodaran, 2002). To overcome the weaknesses, it is

[11] Enterprise value = Market value of equity + Market value of debt – Cash. The cash is netted out because interest income is not part of EBITDA. Not netting out cash will result in an overstatement of the true value of the EBITDA multiple (Damodaran, 2002)

[12] The valuer is the person with direct responsibility for valuing one or more on the investments of the private equity fund or funds-in-funds (IPEV, 2009).

important to be presented the underlying assumptions in an attempt to question the ruggedness of the estimate.

5.2 Specific valuation techniques used in the private equity industry

We know that one of the biggest challenges in the private equity industry is a volatile market with few transactions. This, in combination with lack of quoted prices, give increasing demand for transparency over fair value reporting in the industry (PwC, 2008). As we have seen from the general presentation of DCF and relative valuation the challenges can be many. Next in line is to understand how the industry itself has tried to overcome these challenges.

5.2.1 IPEV Guidelines

To cope with demand for transparency, The International Private Equity and Venture Capital Valuation Guidelines (IPEV Guidelines) were developed in 2005 by the Association Française des Investisseurs en Capital (AFIC), the British Venture Capital Association (BVCA) and the European Private Equity and Venture Capital Association (EVCA). There was also a need for consistency with IFRS and USGAAP accounting principles, thus the Guidelines are based on the overall principle of fair value in accordance with IFRS and USGAAP.

The Guidelines provide a framework for fund managers and investors to monitor the value of existing investments (IPEV, 2009). The Norwegian Venture Capital and Private Equity Association (NVCA) is one of many associations that have endorsed the IPEV Guidelines. Thus, Norwegian private equity fund managers should follow these guidelines. The fund managers I have been in contact with have confirmed that they do so.

The IPEV Board confirms fair value as the best measurement of valuing private equity portfolio companies and investments in private equity funds. Further, the support for fair value is underpinned by the transparency it offers investors in funds. The investors use fair value as an indicator of the interim performance of a portfolio, and thereby get an opportunity to monitor the fund managers (IPEV, 2009).

Enterprise value

An important term in the IPEV Guidelines is the term enterprise value. Regardless of method used when estimating fair value of an investment, the valuer estimates the fair value based on the enterprise value. Enterprise value is the value of the financial instrument representing ownership interests in an entity plus the net financial debt of the entity (IPEV, 2009).

IPEV Guidelines (2009) uses the following steps when estimating fair value through enterprise value:

1. *Determine the enterprise value of the investee company[13] using the valuation methodologies;*
2. *Adjust the enterprise value for surplus assets or excess liabilities and other contingencies and relevant factors to derive an adjusted enterprise value for the investee company;*
3. *Deduct from this amount any financial instruments ranking ahead of the highest ranking instrument of the fund in a liquidation scenario (e.g. the fund amount that would be paid) and taking into account the effect of any instruments that may dilute the fund's investment to derive the attributable enterprise value;*
4. *Apportion the attributable enterprise value between the company's relevant financial instruments according to their ranking;*
5. *Allocate the amounts derived according to the fund's holding in each financial instrument, representing their fair value.*

In the private equity industry, value is generally realized through sale or flotation of the entire underlying business. Thus, the value of the business as a whole at the reporting date will provide insight into the value of investments stakes in that business. For this reason, a number of the methods described below involve estimating the enterprise value as an initial step. Even though enterprise value is an important attribute, there will be situations where fair value is derived mainly from expected cash flows and risk of the relevant financial instrument. It is important that valuation methods used in these situations reflect this fact (IPEV, 2009).

[13] The term investee company refers to a single business or group of business in which the private equity fund is directly invested (IPEV, 2009).

Valuation methodologies

In general, when estimating fair value for an investment, it is important to apply a method that is appropriate in light of the nature, facts and circumstances of the investment and its materiality in the context of the total investment portfolio. In addition, it is important to use reasonable data and market inputs, assumptions and estimates (IPEV, 2009).

If there is conflict between a recommendation in the guidelines and requirements of any applicable laws, regulation, accounting standards or GAAP the latter requirements should take precedence (IPEV, 2009). In other words "law before guidelines" is the principle in use when conflict appears.

We know from our study of accounting requirements that external and market information is superior to internal information based on assumptions. Thus, the IPEV Guidelines (2009) emphasize that fair value estimates based entirely on observable market data should be of greater reliability than estimates based on assumptions.

I will now present the different valuation methodologies used in the IPEV Guidelines. In total, there are 6 different methodologies (A-F) to use in lack of quoted prices, while one presents practice when quoted prices are available (G).

A. Price of recent investment

A transaction price between two market participants is by definition fair value of that specific investment. In situations where the investment being valued was made recently, the transaction cost may provide a good indicator for the fair value. The same holds if there has been any recent investment in the investee company. The price of that recent investment will then provide a basis for the valuation. Briefly, this is the basic idea when price of recent investment is used for estimating the fair value (IPEV, 2009).

Price of recent investment is likely to be appropriate for all private equity investments, but with a limited time horizon, meaning a limited period after the date of the relevant transaction. Sometimes the method is not representative due to different rights attached to the new and existing investments. In some situations the transaction might be considered to be a forced sale or rescue package not representing the true fair value (IPEV, 2009).

The method is much used in seed, start-up or early-stage situations where the lack of positive earnings and cash flows are the reality. Estimating cash flows is difficult and the price of recent investments can then contribute to a reasonable fair value estimate (IPEV, 2009).

If the method is no longer relevant to use, it might be appropriate to apply an enhanced assessment based on analysis of the industry, the sector or milestones. In that case, the valuer has to decide whether there has been a change in the milestones which could indicate a change in the fair value. For investments in early stages, different milestones can serve as indicators, and vary across financial, technical and marketing and sales measures (IPEV, 2009).

Financial measures:	Technical measures:	Marketing and sales measures:
•Revenue growth •Profitability expectations •Cash burn rate •Covenant compliance	•Phases of development •Testing cycles •Patent approvals	•Customer surveys •Testing phases •Market introduction •Market share

Table 3 - Different milestones (IPEV, 2009)

If the valuer, based on changes in the milestones, conclude that the fair value is changed, it is necessary to estimate the amount of any adjustment from the last price of recent investment. Adjustments of this kind are likely to be subjective and based on professional judgment (IPEV, 2009). Thus, the method looses one of its advantages when the assessment is no longer based entirely on market inputs.

It is also important to stress that the recent turbulence in financial markets has made a price of recent investment approach more difficult to apply. A volatile market, with fewer transactions and distressed sales, makes it harder to find good reference transactions (PwC, 2008).

B. Multiples

Multiples are likely to be appropriate for investments in businesses with an identifiable stream of continuing earnings that are considered to be maintainable (IPEV, 2009). In other words, the method is most appropriate to use in mature businesses.

The method involves the application of an earnings multiple to the earnings of the business being valued to attain a value for the business. Usually, a revenue multiple is a result of an assumption predicting a normalized level of earnings from the current situation. When using earnings multiples it is important to apply a multiple that is appropriate and reasonable to the maintainable earnings of the company. Thus, the valuer has to consider the risk profile and growth prospects of the underlying business (IPEV, 2009).

When using this methodology, it is important to be aware of different financial structure and level of borrowing when comparing multiples across companies. Ideally, for a P/E multiple to be comparable, the two companies should have similar financial structure and borrowing level (IPEV, 2009). However, when using P/E multiples caution is important because earnings can vary much from period to period causing variation in the multiple. A better alternative is to use EBITDA multiples when available. Then you do not need to consider differences in financial structure and debt to the same degree. In addition, the EBITDA multiple is independent of tax, depreciation schedule, goodwill, write-downs, write-ups and occasional financial income. However, EBITDA multiples need to be used with care as well. If not, the valuer can fail to recognise that business decisions to spend heavily on fixed assets or to grow by acquisition are associated with costs (IPEV, 2009).

Since P/E multiples are more commonly reported, the IPEV Guidelines states that these multiples can be used, but emphasises that it should generally be applied to an EBIT figure:

"*....it should generally be applied to an EBIT figure which is adjusted for finance costs relating to operations, working capital and tax. These adjustments are designed to eliminate the effect on the earnings of the acquisition finance on the enterprise value since this is subsequently adjusted"* (IPEV, 2009).

A much used approach is to derive a multiple by reference to current market-based multiples. These multiples are based on the market valuations of quoted companies or the price these companies have changed ownership at. The assumption behind this type of market-based approach is that the market correctly valuates the comparator companies. Some might argue that the market capitalisation of a quoted company does

not reflect the value of the company, but instead the value a small fraction of the company is exchanged at. The IPEV Guidelines believe in market efficiency, and that market based multiples are indicative of the value of a company as a whole (IPEV, 2009).

When estimating fair value of an investment using multiples, it is important to consider the risk aspect of the investment. Risk usually occurs in different forms, including the nature of the company's operations, the market in which it operates, its competitive position, management and employees. For the private equity industry the company's capital structure and the ability of the private equity fund to effect changes in the company is a significant risk factor. In addition, there is a risk associated with a lack of liquidity and marketability which is potential bigger for private equity firms due to investments in unquoted companies (IPEV, 2009).

Even though IPEV Guidelines recommends using multiples when estimating fair value, it is important to be critical and not have a blind faith in the estimate. We have seen that multiples are mostly based on market information. However, adjustments are based on assumptions from the valuer where lack of transparency can occur. Thus, two primary questions need to be answered before using a multiple. What are the fundamentals that determine at what multiple a firm should be traded? How do changes in the fundamentals affect the multiple (Damodaran, 2002)? These questions should be applied both to the valuer and the user of the valuer's estimate.

C. Net assets

Third in the line of methodologies published in the IPEV Guidelines is the use of net assets. Briefly explained, the method involves deriving the value of a business by reference to the value of its net assets (IPEV, 2009).

This method is not suitable to all businesses. Mainly, the net asset methodology has its advantages in businesses where the value is derived mainly from the fair value of its assets rather than earnings. Examples can be property holding companies and investment businesses. Another situation where the method can be applied is when a business is not making an adequate return on assets and greater value can be realized by liquidation. Thus, the net assets method can be used by private equity companies when valuing investments in loss-making and low profit-making companies (IPEV, 2009).

Fund-of-funds[14] and investors in private equity funds need to value their interest in an underlying fund at regular intervals to support their financial reporting. The most common method used estimating the fair value of an interest in a fund, is the net asset value (NAV) based on the underlying fair value of the investments (IPEV, 2009).

"In estimating the fair value of an interest in a fund, the valuer should base their estimate on their attributable proportion of the reported fund NAV." (IPEV, 2009, p. 24)

Basically, the value of an underlying fund interest is equal to the summation of the estimated value of the underlying investment as if realized on the reporting date. The realization is expected to flow through to the investor in an amount equal to NAV (IPEV, 2009).

"This concept makes particular sense for closed-end fund investors who realize cash returns for their investment when realization events occur through the sale of the underlying portfolio companies." (IPEV, 2009, p. 24)

Assumed that NAV is estimated based on principles of fair value and the guidelines provided by IPEV, the method is the best way to valuing fund interests (IPEV, 2009).

D. Discounted cash flows or earnings of underlying business

The discounted cash flows technique is flexible because it can be applied to any stream of cash flows or earnings. For private equity valuation the flexibility enables the method to be used in situations where other methods might be incapable of using. The method can be used for businesses going through a period of change, such as rescue financing, turnaround, strategic reposition, loss making or in start-up phases. However, there is risk associated with applying the method. Especially centred around the requirements for detailed cash flow forecasts and the needs for estimating a terminal or continuing value. These inputs call for needs of subjective judgment, and the derived present value estimated is sensitive to small changes in the inputs (IPEV, 2009).

Market inputs are superior to internal and subjective judgments both in the IPEV Guidelines and the accounting standards. Thus, IPEV Guidelines (2009) is not the biggest supporter of the method. They recommend discounted cash flow based valuations to be

[14] Fund-of-funds is the generic term used in IPEV Guidelines to refer to any designated pool of investment capital targeted at investment in underlying private equity funds (IPEV, 2009).

used as a cross-check of values estimated under market-based methodologies, and should only be used in isolation of other methodologies under extreme caution.

"In assessing the appropriateness of this methodology, the valuer should consider whether its disadvantages and sensitivities are such, in the particular circumstances, as to render the resulting fair value insufficiently reliable." (IPEV, 2009, p. 21)

While the IPEV Guidelines explicitly state that this method should be avoided, IFRS is more supportive of discounted cash flows as an approach. How is that possible? One partial explanation can be that IAS 39 needs to cover all types of investments, and not only private equity investments (PwC, 2008). Generally, cash flows are more reliable when observed in the market which is possible for many financial investments in contrast to the majority of private equity investments.

Damodaran (2002) states that the biggest problem when applying discounted cash flow valuation to private firms are the measurement of risk used in the discount rate. Most risk models require that the risk parameter can be estimated from historical prices of the asset being analyzed. For private firms that is difficult because they are not traded. One solution is to use comparable firms that are public traded, or to relate the measure of risk to accounting variables which are available for private firms (Damodaran, 2002).

E. Discounted cash flows from the investment

This methodology is very similar to discounted cash flows of underlying business. The method applies the discounted cash flow concept and technique to the expected cash flows from the investment itself. When realization of an investment or a flotation of the underlying business is imminent, and the pricing of the relevant transaction has been substantially agreed, IPEV (2009) believes that this method is likely to be the most appropriate one.

Due to its flexibility, the method is capable of being applied to all private equity investment situations. The method is especially suitable for valuing non-equity investments in instruments such as debt. The reason is that the value of that kind of instruments mainly derives from instrument-specific cash flows and risk rates, rather than value from the underlying business (IPEV, 2009).

Similar to discounted cash flows from underlying business, this method also relies on subjective judgments. Thus, the valuer has to be very cautious of using this method as the main basis of estimating fair value. It is better to use the method as a sense-check of values attained using other methods, like multiples (IPEV, 2009).

F. Industry valuation benchmarks

The basis behind industry benchmarks in valuation is similar to the idea behind multiples. A number of industries have industry-specific valuation benchmarks, such as price per subscriber for cable television companies. In some financial services, information technology sectors and service sectors where long-term contracts are a key feature, multiples of revenues are a valuation benchmark much used (IPEV, 2009).

An important assumption applied to these industry norms, are that investors are willing to pay for turnover or market share, in addition that normal profitability of business in the industry does not vary much (IPEV, 2009).

"The use of such industry benchmark is only likely to be reliable and therefore appropriate as the main basis of estimating fair value in limited situations, and is more likely to be useful as a sense-check of values produced using other methodologies." (IPEV, 2009)

G. Methods for valuing quoted instruments

In some situations private equity funds also holds quoted instruments where it is possible to observe available prices. Such instruments should be valued using the bid price on reporting date. If a bid price is not possible to obtain, the most representative point estimate in the bid/ask spread can be used. It is important to be consistent, thus the valuer should use either the bid price or the bid/ask spread and not mix between the estimates (IPEV, 2009).

The IPEV Guidelines (2009) also have a statement where it says that the mid-market price can be used as an alternative to the bid price when bid prices are not required by regulation. The mid-market price is defined as the average of the bid and ask price.

As a main rule, discounts should not be applied to prices quoted in an active market. However, if there is some contractual, governmental or other legally restriction that would impact the value realized at the reporting date, then it is admission to do so (IPEV, 2009).

5.2.2 Practical difficulties in IPEV Guidelines

We know that one of the objectives when developing the IPEV Guidelines was to ensure consistency with the reporting standards. Taking a closer look at the Guidelines you actually discover that the definition of fair value is not fully complied with the definition of fair value within USGAAP and IASB's fair value measurement exposure draft (PwC, 2009).

Fair value definition	
FASB	**IPEV Guidelines**
Fair value is the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date.	The fair value is the price at which an orderly transaction would take place between market participants at the reporting date.

Table 4 – Fair value definition

Why the definitions are not completely converged is a big question. However, the definitions are not fundamental different from each other and would probably not lead to any confusion. If there is any uncertainty, the definitions of FASB and IASB are the one to be complied.

The Guidelines are mainly focused on the market multiple approach, and there are a variety of ways the approach can be performed. One reason could be that multiples are easily collected and calculated, in addition to be based on market information in most occasions. PwC (2009) has written an article on fair value challenges in the private equity industry where they discuss the different methodologies published by IPEV Guidelines. In the article PwC supports the use of multiples as a valuation approach.

"We support the consideration of multiple valuation approaches but taking a straight average of the approaches should be avoided." (PwC, 2009)

As mentioned earlier, the Guidelines appear biased against discounted cash flow (DCF) analysis, even though the technique is recognized by the reporting standards and represent standard market practice for certain assets (PwC, 2009). This might be understood as the technique should not be a primary valuation methodology for a private equity firm. Instead, the method is said to be useful as a cross-check, especially for material investments, early-stage investments or in situations where no comparable

company exists for multiple valuation. To make the use of DCF analysis transparent, it is important to calibrate the underlying assumptions to market data such as analyst reports or other industry analysis. The same holds for assumptions regarding the discount rate (PwC, 2009). In addition, industry valuation benchmarks are stated to be useful as a sense-check. One possible interpretation using methodologies as sense-checks is that at least two methodologies should be used when estimating fair value. This is also supported by PwC (2008) who writes that they support the use of several approaches, normally at least two, to arrive at a robust view.

As we have seen, when valuing private firms estimating valuation inputs is one of the biggest challenges. When applying DCF valuation, the discount rate is one input that is difficult to estimate. The cost of equity, for instance, needs information about the market beta. Usually, we can find information about market beta by using historical stock prices. However, for private firms historical stock prices do not exist. We have to estimate it by applying other methods and professional judgment. In addition, we need information about the cost of debt which also has to be estimated.

Estimating growth can also be challenging because we cannot really on analyst estimates. Instead, we have to use own judgment based on historical developments in fundamental inputs. In other word, the subjective assumptions increase (Damodaran, 2002).

In addition, private firms are associated with illiquidity discounts. Such discounts appear when liquidation is difficult and the liquidation costs as percent of the value is substantial. Thus, the value of equity in a private firm has to be discounted for the potential illiquidity. These illiquidity discounts are usually difficult to estimate (Damodaran, 2002).

6 Analysis of fair value measuring

The goal so far has been to gain a theoretical fundament and understanding of fair value as a measurement in financial reporting in the private equity industry. As we have seen the accounting requirements are relatively complex, especially in IFRS and USGAAP. To overcome some of the challenges regarding complex accounting requirements, international guidelines have been published to achieve best practice, and make the estimating and reporting of fair value in the private equity industry easier and more transparent.

To give a complete answer to the research question, we need to take a closer look at the actual numbers and estimates being calculated and reported to investors and the public by the private equity industry. Due to scope limitations, the private equity industry in Norway will be the main focus of this analysis. The goal is to look at the value relevance between reported book values and fair value estimates, compared to the private equity firms' achieved transaction prices of their portfolio investments. In order to do so, I need to make assumptions regarding what is the correct fair value. Because the transaction price is achieved in the market, and agreed between knowledgeable and willing parties, the transaction price will act as measure of correct fair value. In other words, I will regard the transaction price as the intrinsic value of the portfolio company.

6.1 Hypotheses

Before starting my analysis, I am going to define some hypotheses, which I later will test to evaluate whether I can reject they or not. Because my point of view is the Norwegian private equity industry, the majority of private equity companies report historical cost values. However, the fund managers also report fair value due to demands from stakeholders, in particular from limited partners.

Because the data sample I have contains observations with reported book value and estimated fair value, it is possible to test which of the two independent variables explain the transaction price best. Thus, my first hypothesis will be:

1. **Fair value estimates predict the transaction price better than historical cost values.**

The first hypothesis is motivated based on an expectation where I believe that reported historical cost values have little influential force on the transaction price. The economical logic behind this statement is derived from the accounting requirements where we have seen that historical cost has few requirements of fair value adjustment. From that point of departure, fair value accounting, and thus fair value estimates should have more explanatory power than historical cost values towards the transaction price.

In addition, the hypothesis is motivated based on an article by Landsman (2006). In his article Landsman asks whether fair values are useful to investors in the bank sectors. To answer that question, he points to research done by Bart, Beaver and Landsman (1996), Eccher, Ramesh and Thiagarajan (1996), Nelson (1996) and Barth (1994). These studies find that investment securities' fair value is incrementally informative relative to their book values in explaining bank share prices (Landsman, 2006). Even though, the studies have focus on the bank sector, I will expect to find the same pattern in the private equity industry.

Since I also have information about what investment type the observations are (e.g. seed, venture or buy-out), I would like to test and see whether investment type has any influence on the transaction price:

2. **The type of investment affects the transaction price, and how well the fair value estimate explains the transaction price.**

The second hypothesis is motivated based on the differences between a seed/venture investment and a buy-out investment. In paragraph 2.2 I discussed the different types of private equity investments. From that discussion we learned that seed and venture investments were carried out in companies with potential for growth. Buy-out investments were made in companies with potential for value increasing. Independent of investment type, they are all motivated based on expectations of positive return. However, the economical logic behind the second hypothesis is based on an expectation that assessing the transaction price for a venture investment might be more difficult than for a buy-out investment. Valuating a company in growth brings along more challenges concerning assumptions regarding the valuation inputs. This is a well known fact, and pinpointed by both Koller et al (2005) and Damodaran (2002). Consequently, I

believe that there is a possibility that some of the deviation between transaction price and fair value estimate could be explained by the type of investment.

In light of hypothesis 1 and 2, it would be interesting to test whether there is possible to find any evidence that suggests a relationship between reported book values, fair value estimates and type of investment. Thus, the third hypothesis I will test is:

3. **The transaction price is best explained by the reported book value, goodwill in terms of fair value exceeding the book value, and how book value and goodwill interact with type of investment.**

From a theoretical point of view the valuer usually makes use of historical values when assessing fair value of an investment. This suggests that historical values affect fair values and thereby also the transaction price. In addition, type of investment might also affect the transaction price, which suggests that three variables have some influence on the transaction price.

6.2 Regression analysis

In order to test the different hypothesis, I need to apply a statistical technique to arrive upon an answer. As we can see from the three hypotheses in the previous paragraph, they have one variable in common, which is the transaction price. However, each hypothesis has different variables that are meant to explain the transaction price. Thus, I am going to apply regression analysis as statistical technique to test the different hypothesis. Regression analysis is used to predict the value of one variable on the basis of other variables. The technique is probably the most commonly used statistical procedure because it is easy to adopt (Keller, 2005).

Regression analysis involves developing a mathematical equation that describes the relationship between the variable to be forecasted (called dependent variable), and variables (independent variables) you believe are related to the dependent variable (Keller, 2005).

When applying regression analysis, it is possible to use either the simple linear regression or a multiple regression. The difference between the regressions is how many independent variables the model uses to forecast the dependent variable (Keller, 2005). To test my three hypotheses, I need to apply both models.

Because regression analysis is a well known statistical technique I will not use time to present the theory behind the technique. Instead, I will pinpoint important theoretical factors during the testing when I find it necessary. For those who would like to read more about regression analysis both Keller (2005) and Green (2008) have written books where the subject is described in detail. Keller's book has a more practical point of view, while Green's book offers a more theoretical view on the subject. Both books are enclosed in the thesis' bibliography if the reader would like to investigate further.

6.3 Data collection

When collecting data for my analysis, I started by getting an overview of the different private equity companies in Norway. NVCA has published a list of its members on their home page which became my starting point. NVCA has a total of 90 members, where 37 are primary members and 53 associated members (NVCA, 2010d). I decided to use the primary members as a starting point for my data collocation. The reason for doing so was based on NVCA's endorsement of the IPEV Guidelines, and an assumption that the primary members thereby follow these guidelines. Some of the primary members are not Norwegian companies but registered in Sweden, Finland, Denmark and even New Jersey. These companies have not been contacted when requesting information.

As mentioned earlier, the private equity industry is known to be very secretive regarding figures on return of investments, and company-specific estimates in general. Thus, I was very anxious whether the companies would be willing to release the information I was requesting. To increase the chances of getting access to requested information I decided to look at fair value estimates on realized investments[15].

After contacting 25 different private equity companies, six decided to provide me with the information I requested, which means a 24 % response. Many more have been positive regarding the thesis' research question, but due to internal guidelines and little time available they have not been able to participate.

Due to questions of confidentiality, the respondents have not been forced to provide the name of the former portfolio company which constitutes each observation.

[15] Realized investments are investments where private equity companies no longer have any ownership interests – the investments are exited.

6.4 The data sample

Based on figures provided by the six respondents, the whole data set consists of 55 observations which make up the analysis sample. Each observation gives information about different variables related to the former portfolio company. These variables are:

Variable	Notation	Definition
Book value (mill NOK)	**BV**	BV is the equity value reported in the annual/quarterly report by the portfolio company. Due to NGAAP reporting BV is usually historical cost values.
Fair value estimate (mill NOK)	**FV**	FV are the private equity fund's own estimate of the portfolio company's reported equity. In other words, the value adjusted equity in the portfolio company. These estimates are figures available as close up to the transaction date as possible.
Transaction price (mill NOK)	**TP**	The TP is the achieved selling price when the portfolio company was sold in the market. Thus, I regard TP as the intrinsic value of the portfolio company.
Goodwill (mill NOK)	**GW**	GW is the portfolio company's excess value calculated by the fund manager. We can also define it as goodwill not recognized in the portfolio company's balance sheet. GW is defined as the difference between FV and BV.
Type of investment	**V**	V is an indicator variable (dummy), where the value of V=1 if the portfolio company is a venture investment or 0 if the company is something else.
Valuation methodology	**-**	According to IPEV it is recommended to use, at least, two methodologies when estimating fair value. Thus, the respondents have been able to report all methodologies used during valuation. The methodologies are preferred to be in accordance with the Guidelines, but if the methodologies do not correspond to those in the Guideline the company-specific[16] valuation method is reported.

Table 5 – Definition of data sample variables

The following table[17] gives information about the variables' mean, standard deviation (StDev), first quartile (Q1), median and third quartile (Q3). In addition, the relationship between *BV, FV* and *TP* is enclosed expressed as two different multiples in order to give an impression of the relative difference between the mentioned variables:

Variable	N	Mean	StDev	Q1	Median	Q3
BV	55	49,40	102,50	9,80	28,40	41,80
FV	55	113,80	229,50	24,40	41,80	112,80
TP	55	160,40	247,30	34,50	81,50	230,00
GW	55	64,40	137,10	0,00	11,90	72,30

[16] By company-specific I mean the actual method used that is not in accordance with the IPEV Guidelines.
[17] All figures are denoted in million NOK except for *V* which is a dummy variable.

V	55	0,67	0,47	0,00	1,00	1,00
FV/BV	55	3,57	5,46	1,00	2,00	4,00
TP/BV	55	5,79	7,60	1,69	2,99	5,34

Table 6 – Descriptive statistics of sample variables

As we can see from the table above, the mean of *BV, FV* and *TP* is increasing. The logical reason behind this pattern is that *TP* values are, on average, larger than *FV*, while *FV* values are larger than *BV*. The standard deviation shows that variation from the mean, for these three variables, is larger for *TP* than *FV*. *BV* has the lowest standard deviation. Notice that *V* is applied as a dummy variable which can only be 1 or 0. Thus, the mean states that 67 % of the observations are classified as venture investments. The remaining 33 % is classified as buy-out investments. I would like to pinpoint that this is only a simplification I have done in order to reduce the number of dummy variables in the regression analysis. The complete distribution of investment types will be presented on page 61.

We can also compute the correlation between the sample's variables. The correlation matrix shows that *BV, FV, TP* and *GW* are highly positive correlated with each other.

	BV	FV	TP	GW	V
BV	1,00	-	-	-	-
FV	0,94	1,00	-	-	-
TP	0,89	0,94	1,00	-	-
GW	0,83	0,97	0,91	1,00	-
V	-0,32	-0,31	-0,26	-0,29	1,00

Table 7 – Correlation matrix

These correlations are important to have in mind during the testing of the hypotheses I defined earlier, because it can potentially cause some complications in the regression analysis. I will come back to possible complications if the problem arises.

An interesting relationship is to see in what occasions *TP* is greater, equal or lower than *FV* depending on the relationship between *BV* and *FV*. The whole distribution of *TP* versus *FV*, given the relationship between *BV* and *FV*, is summarized in table 8 on the next page. Table 8 tells us that 4 observations have a reported book value greater than estimated fair value. Of these, 2 observations achieved a transaction price greater than the estimate, while 2 were below the estimate.

		TP > FV	2
BV > FV	4	**TP < FV**	2
		TP = FV	0
		TP > FV	11
BV = FV	11	**TP < FV**	0
		TP = FV	0
		TP > FV	28
BV < FV	40	**TP < FV**	12
		TP = FV	0

Table 8 – Distribution between BV, FV and TP

According to NGAAP and the prudence principle, the investment is supposed to be written down when fair value is lower than historical cost values. Based on that principle it is somewhat strange that we can observe *BV* greater than *FV*. However, the data sample consists of the last reported *BV*, and the latest *FV*. Thus, one explanation why we observe *BV* greater than *FV* might be due to events affecting the value between reporting dates. The remaining 51 observations have *BVs* equal, or greater than *FVs*. Independent of the relationship between *BV* and *FV*, the fund managers never compute a *FV* equal to the *TP*.

Due to different numbers of realizations between the private equity respondents, there is not an even distribution of observations in the sample. The distribution between the different private equity companies denoted A-F, is as followed:

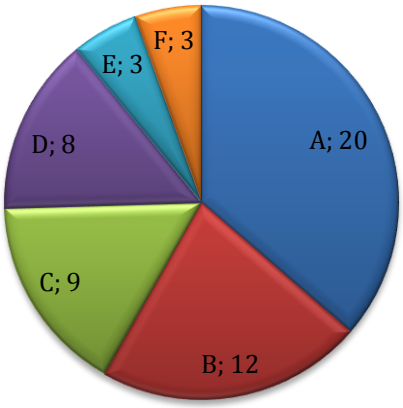

Diagram 1 – Share of observations between respondents

Diagram 1 is important to have in mind when we interpret discoveries later on, because the different practice and skill level of each fund manager affects their respective fair value estimates.

We can also sort the different observations according to type of investment, which yields the following distribution:

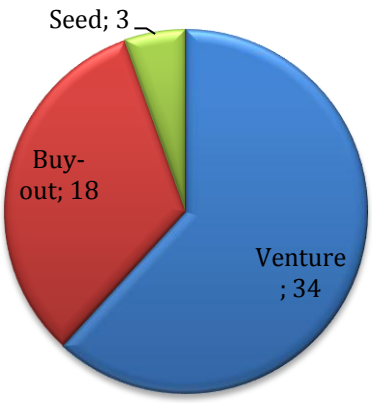

Diagram 2 – Distribution of investment types

Notice that venture investments also include expansion investments. These investments are carried out in firms with considerable turnover, but where the fund managers consider the potential for growth increasing as high. I choose to characterize these investments as venture in order to use the same categorizing as presented in chapter 2. In addition, these investments have more common features with venture investments than buy-outs due to growth potential rather than potential for value increasing.

6.5 How good are the estimates?

When estimating fair value, the different private equity companies have a wide range of guidelines to support their work. As we have seen, IFRS, USGAAP and IPEV have developed guidelines to make fair value estimating manageable. Based on the variety of guidelines available, estimating fair value should not be too difficult in a theoretical setting. However, one thing is what the theory tells us, something different is how easy it is to implement theory in practical situations.

My calculation of differences between fair value estimates and transaction prices is done relative to each other. In other words, I would like to measure how big the difference is relatively to the transaction price. The calculation then yields the difference measured in percent, rather than the absolute value. By using a relative measuring term it is possible to overcome issues regarding different size of total investment in the portfolio company. The example below gives an illustration behind the choice of calculation. All NOK values are in millions.

Investment	Estimated fair value	Transaction price	Relative difference	Absolute difference
1	NOK 4	NOK 8	-50 %	NOK 4
2	NOK 150	NOK 162	-7,4 %	NOK 12

As we can see from this example, the absolute calculation yields a difference equal NOK 12 mill for investment 2. The same difference for investment 1 is just NOK 4 mill. Based on the absolute difference, it is tempting to say that the estimated fair value of investment 1 is the better compared to investment 2. However, if we calculate the relative difference, investment 2 has a fair value estimate closer to the achieved transaction price. In other words, the fair value estimate for investment 2 is a more correct value than the fair value estimate of investment 1. The calculation is based on the following formula:

$$Relative\ difference = \frac{Estimated\ fair\ value - Transaction\ price}{Transaction\ price}$$

By applying transaction price as the denominator, we can calculate by how many percent the fair value estimate is either over- or underestimated. Based on the example above, investment 2's fair value estimate is underestimated with 7.4 % compared to the actual transaction price. We can also say that the fair value estimate, of investment 2, amounts for 92.6 % of the transaction price. If we only know the estimate and the relative difference, we could calculate the transaction price by dividing the estimated fair value with (1 + the relative difference):

$$Transaction\ price = \frac{Estimated\ fair\ value}{1 + Relative\ difference} = \frac{150}{1 + (-0{,}074)} = 162$$

I have completed this calculation on the whole data sample, and it shows that only 25.5 % of the observations have a fair value estimate greater than the transaction price. In other words, the fair value is overestimated. Consequently, the remaining 74.5 % of the observations are underestimated, meaning that the transaction price is greater than the fair value estimate.

Both the under- and overestimated observations vary in terms of deviations. In one range of the scale some estimates are more than 50 % below the transaction price, while others are 50 % above the transaction price. I have prepared a histogram where the distribution is divided into 20 percent increasing intervals.

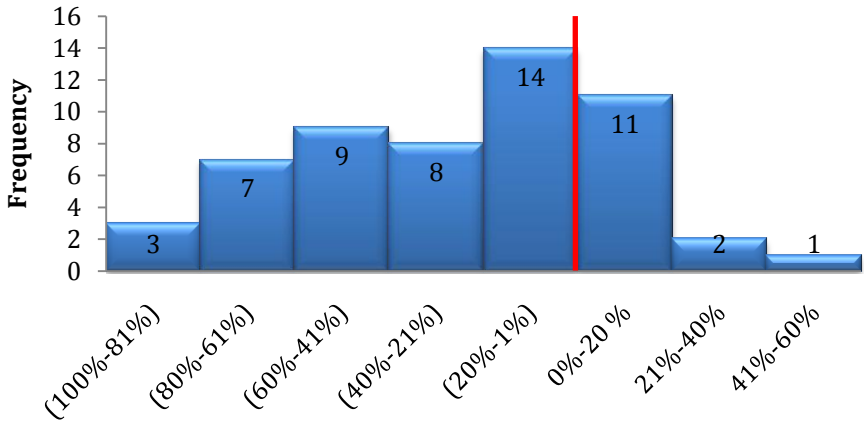

Diagram 3 – Histogram of relative differences

The red line distinguishes positive intervals from negative ones. Negative intervals are denoted with brackets to avoid any misunderstandings. In addition, the red line indicates which observations that are over- or underestimated. Consequently, observations to the left of the red line are underestimated, while observations to the right are overestimated.

What is interesting to see is how the estimated fair value deviates from the transaction price if we look at different types of private equity investments. I will use the same classification of investments as in diagram 2, which are seed, venture and buy-out investments. Because seed and venture investments are associated with more risk, I would expect that these investments have higher standard deviation than buy-out investments. The complete calculation of descriptive statistics, according to the different investment types, can be seen in the table below. The different figures are calculated based on the relative different between fair value estimates and transaction prices using the formula on page 62.

Investment type	N	Mean	StDev	Q1	Median	Q3
Seed	3	-0,3880	0,4100	-0,8330	-0,3030	-0,0270
Venture	34	-0,2788	0,3514	-0,6255	-0,2102	0,0238
Buy-out	18	-0,1577	0,3330	-0,4544	-0,1305	0,1418
All together	55	-0,2451	0,3478	-0,5510	-0,2000	0,0194

Table 9 – Investment type: Descriptive statistics of relative difference

As we can see, buy-out investments have the lowest mean and standard deviation compared to seed and venture investments. This could indicate that it is easier to value a buy-out investment due to less uncertainty associated with different valuation inputs.

None of the investment types have a positive mean, indicating that regardless of investment type, there seems to be a trend of underestimation in the industry.

6.5.1 Underestimated observations

The data sample shows that 1 out of 4 estimates deviate more than 50 % compared to the transaction price. In diagram 4, I have plotted the 14 most extreme underestimates.

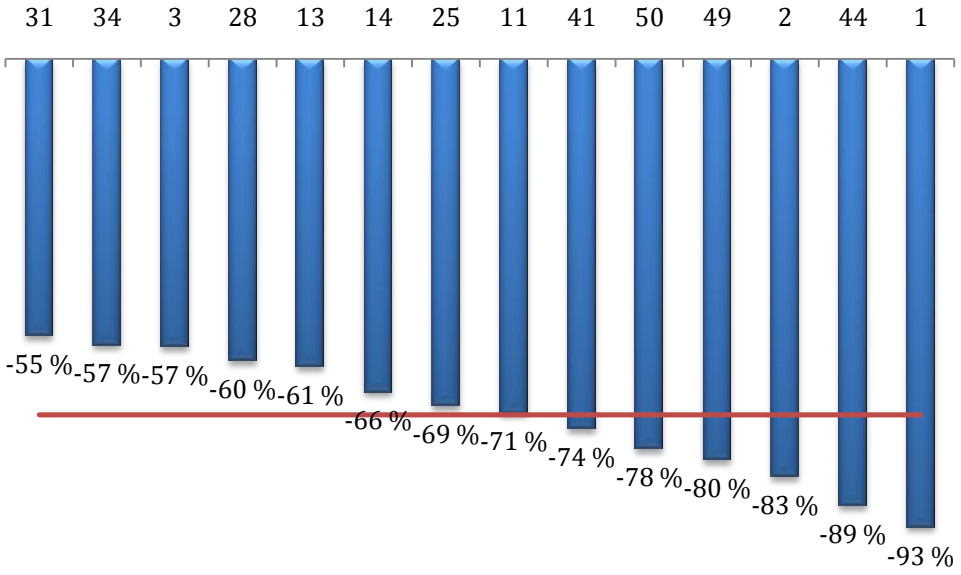

Diagram 4 – 14 most extreme underestimates

The observations are sorted from the lowest deviation to the highest. The number indicating which observation the deviation belongs to, is randomly assigned based on the order the observations are plotted in the sample spreadsheet. The red line indicates that the average underestimation, by the 14 most extreme observations, is -71 %. If we include the remaining 27 observations that are underestimated, and calculate the average deviation we end up with an average equal to -39 %.

As diagram 4 shows, the deviations are in a range of -55 % to -93 %. Seen from an accounting view, it implies that a lot of value is missing from the company's balance sheet, given that the company where to report in accordance with a value driven accounting standard. Worth noticing is that the book value for these observations are either lower or equal to the fair value estimate, except for observation 25 where book value is higher than estimated fair value.

6.5.2 Overestimated observations

What could be interesting is to take a closer look at the observations that are overestimated. For Norwegian private equity firms this is probably not a real problem, because NGAAP and historical cost is the most widespread practise. Anyhow, overestimation is interesting when we are looking on how good fair value estimates actually correspond to the transaction price.

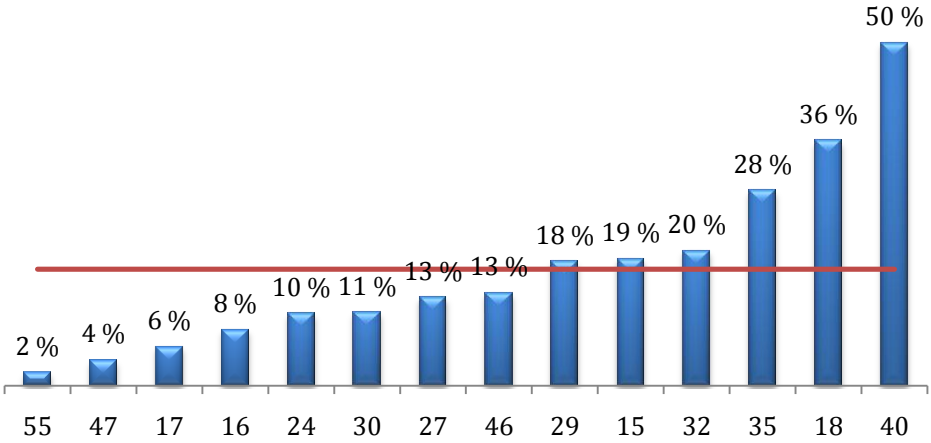

Diagram 5 – Observations that are overestimated

Overestimated observations are in the range of 2 % in one end of the scale, to 50 % in the most extreme case. Indicated by the red line, the data sample shows that private equity firms, on average, overestimate the fair value with 17 %. This is much lower compared to the average underestimation, which was -39 % when calculated based on all 41 underestimated observations. Consequently, on average the underestimated observations deviate more from the transaction price than overestimated observations. This could suggest that fund managers, in general, are relative more defensive than offensive when it comes to estimating the intrinsic value of the portfolio companies. Notice that the average deviation of the whole sample is a weighed sum based on the probability of under- or overestimation.

6.5.3 Are some methods better than others?

Based on accounting requirements and IPEV Guidelines it would be interesting to see whether some of the methodologies used for estimating fair value outperforms others. Because the data sample contain information of method used when estimating fair value, some considerations can be drawn. Unfortunately, not all the 55 observations include

information about what methodology is used. Thus, I have removed the observations that lack the information which gives me 36 observations left.

Based on these 36 observations, the methods used are distributed between four different techniques:

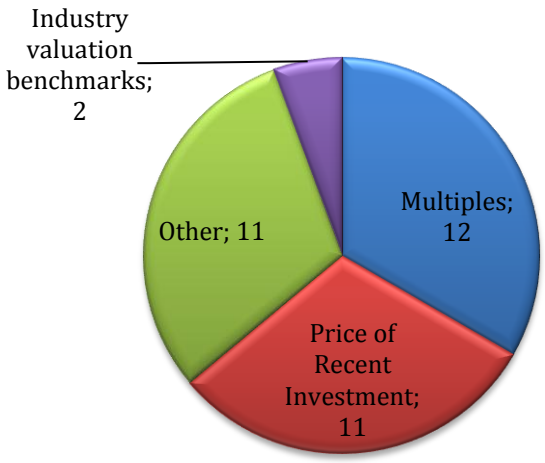

Diagram 6 – Distribution of methodology

Both multiples, price of recent investment and industry valuation benchmarks are methodologies presented in the IPEV Guidelines. The category "other" is a company-specific technique used to valuate investments. The method is best characterized as a normal profit method used in a simplified DCF analysis. As we can see, multiples are a much used technique which is not surprising in light of the IPEV Guidelines' recommendation. Notice that none of the fund managers in my data sample uses fundamental DCF valuation.

I have calculated the average deviation between the transaction price and the fair value estimate for each methodology. The calculation shows that using multiples as valuation technique minimizes the deviation between transaction price and fair value estimate.

Methodology	N	Mean	StDev	Q1	Median	Q3
Multiples	12	-0,1373	0,2129	-0,2642	-0,0760	0,0078
Other	11	-0,1970	0,3830	-0,4770	-0,1720	0,0810
Price of recent investment	11	-0,3739	0,3112	-0,6632	-0,3013	-0,0693
Industry valuation benchmark	2	-0,8833	0,0707	NA	-0,8833	NA

Table 10 – Methodology: Descriptive statistics of relative difference

When using multiples the average deviation is only - 14 %. For industry valuation benchmarks (IVB) the deviation is as much as - 88 %. However, we should not emphasize the IVB deviation too much because the average value is calculated based on just 2 observations. In addition, these 2 observations are part of the 3 most extreme deviations in the sample.

Generally, we should be careful with drawing substantial conclusions based on the diagram above. At best, the diagram helps us explain what could be the truth. However, it is positive to see that the most popular technique also predicts the transaction price on average best. Notice that regardless of valuation technique the result is average underestimation of fair value compared to *TP*.

6.5.4 Possible explanations

Based on my data sample, estimating fair value seems to be quite a challenge. You might ask how it is possible to miscalculate fair value of a portfolio investment to such a degree? To find an answer to that question we have to look behind the figures.

- First, it is vital to get an understanding of what kind of investments these observations are related to. If we look at the 14 most extreme deviations, the majority of observations are all related to seed and venture investments. These types of investments usually contain a great deal of uncertainty regarding the future of the company. Even when a private equity company decides to realize such an investment, it might be difficult to agree upon the future prospects of the investment. Thus, the investor selling the investment and the one buying it can have a different understanding of the company's future potential in terms of growth and profitability. In addition, the underlying assumptions in relation to the fair value estimate can be different and thereby play a critical role.
- For some investors a company can contribute to synergies in terms of economies of scale. If that is the case, one company can have different value for different investors. While some fund managers do not see any more potential for value increasing, the buyer might expect cost-efficiency effects in terms of economies of scale when acquiring the same company.

- Another factor that might influence the sample is illiquidity discounts. These discounts are difficult to estimate, and fund managers might overestimate it, and thereby underestimate the fair value of the investment.
- It is important to notice that *FV* and *TP* are measured at different points in time. Thus, there is a possibility that *TP* is influenced by new information after *FV* is estimated, and thereby contributes to an increased deviation between *FV* and *TP*.
- Finally, some of the fund managers I have been in contact with also emphasize the fact that fair value measuring has been of little importance previous to the introduction of IPEV Guidelines. Prior to the IPEV Guidelines, the fund managers did not use much time estimating *FV* in periods between investment and realization. Instead, the value of the portfolio company was reported at the cost value, adjusted for impairments.

6.6 Testing hypothesis 1

To figure out whether *FV* or *BV* predicts the transaction price best, I need to test the first hypothesis I defined earlier:

1. **Fair value estimates predict the transaction price better than historical cost values.**

Hypothesis 1 will be answered based on two simple linear regressions with two different independent variables. The general form of a simple regression equation can be expressed as:

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where
- y = Dependent variable
- x = Independent variable
- β_0 = y-intercept
- β_1 = Slope of the line
- ε = Error variable

Box 8 – Simple linear regression model (Keller, 2005)

The first simple linear regression I will assess examines the relationship between the dependent variable *TP*, and the independent variable *BV*. Expressed mathematically the equation can be written as:

$$TP_1 = \beta_0 + \beta_1 BV + \varepsilon$$

The second simple linear regression will estimate the relationship between *TP* and *FV*. Thus, the equation can be expressed as:

$$TP_2 = \beta_0 + \beta_1 FV + \varepsilon$$

The objective is to see which of the two independent variables that explain the transaction price best. In other words, I would like to compare the coefficient of determination between the two equations. I let R_1^2 denote the coefficient of determination where *x* is *BV,* while R_2^2 denotes the coefficient of determination where *x* is *FV.* Thus, the following hypotheses can be formulated:

$$H_0: R_1^2 = R_2^2$$

$$H_1: R_1^2 < R_2^2$$

6.6.1 Simple linear regression with x = BV

Before I determine the regression equation, I have to draw a scatter diagram to determine whether a linear model appears to be appropriate to use.

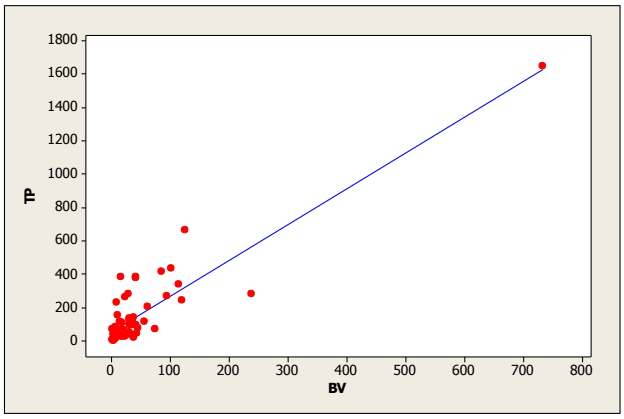

Diagram 7 – Scatter plot of TP vs BV

As we can see from the plot, there is definitely one influence observation in addition to two possible outliers. Regarding the outliers I have no reason to believe that there is something wrong with the observations. The data sample has been handed to me by professionals. Thus, there is no evidence to suggest that the data are wrong. On the other hand, I have not been able to double check each observation because the fund managers had the possibility to anonymize the data. In addition, we have to remember that we are looking into the private equity industry where the variety regarding investments

amounts are large. Some investments are occasionally larger than other, which implies that there should not be any suspicious by observing some large transactions. Over all, I consider it to be nothing wrong with the observations.

Based on the discussion above, I believe that removing the influential observation will not affect the linear relationship. To be sure, I can draw a scatter plot without the influential observation.

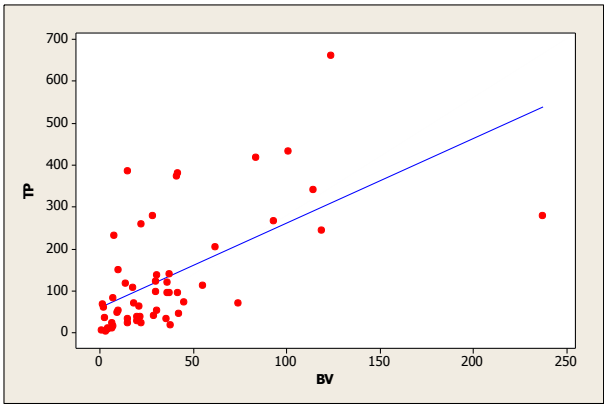

Diagram 8 – Scatter plot of TP vs BV without influential observation

As we can see the linear relationship did not change much. In fact, the line seems to be fitted in the same position, which makes me believe that the observation is actually another outlier. Thus, I will keep the observation as a part of the data sample.

Based on the scatter plot, I would say that a linear relationship does exist. However, the relationship is not very strong because the plots are spread somewhat uneven across the regression line. A strong linear relationship should result in plots centred more round the regression line.

The regression equation[18]

Calculating the regression line yields the following coefficients:

$$\beta_0 = 54{,}2$$
$$\beta_1 = 2{,}15$$

Thus, the regression equation is given by:

$$TP_1 = 54{,}2 + 2{,}15\ BV\ +\ \varepsilon$$

[18] Complete regression printout is enclosed in appendix A.

In other words, if we assume $BV = 2$ mill, the equation yields;

$$54{,}2 + 2{,}15 * 2 = 58{,}5$$

which implies a transaction price equal to 58.5 mill. Notice that all values in the data sample are denoted in NOK millions. For every one million increase in the reported book value, the transaction price will increase with 2.15 million.

Regression diagnostics

For both simple and multiple regression models to be valid, four[19] requirements involving the probability distribution of the error variable must be satisfied (Keller, 2005):

1. The probability distribution of ε is normal.
2. The mean of the distribution is 0, meaning $E(\varepsilon) = 0$.
3. The standard deviation of ε is σ_ε, which is a constant regardless of the value of x.
4. The value of ε associated with any particular value of y is independent of ε associated with any value of y.

Box 9 – Required conditions for the error variable (Keller, 2005)

To evaluate the requirements, Minitab produces a residual plot for the dependent variable that we can use.

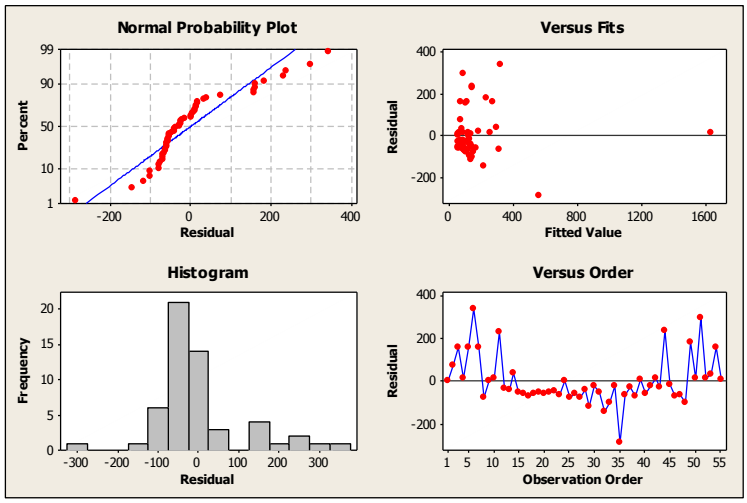

Diagram 9 – Residual plot for TP with x = BV

[19] In addition to the four required conditions Keller (2005) presents, there is a fifth requirement which states that any particular value of x is independent of ε, cov(ε, x) = 0. If this requirement is not fulfilled the estimates will be biased. However, the least squares method can still be used when the purpose it predictions (Møen, 2009).

Normality

Both diagrams to the left in the residual plot test for normality. The histogram is supposed to be bell shaped. In this case, the histogram is bell shaped but not completely centred round zero. From a theoretical point of view, the requirement could be violated, but when applying statistics the "perfect" histogram if difficult to obtain and small deviations are usually accepted.

The normal probability plot is supposed to follow a straight line. As you can see, the plots are a bit of the blue line. However, some variation has to be expected. Thus, I would say that the normal probability plot does not abandon the assumption that the error variable is normally distributed.

Notice, that in addition to the residual plot I could have used more formal tests to determine whether the requirements of normality were violated, e.g. the Jarque-Bera test. The test is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness. For now, I choose not to apply any more tests, because normality is not a required condition for least squares method to be an "efficient" estimator. If we experience non-normality, inference is not valid with small samples, thus hypothesis tests and confidential intervals might be wrong. If the deviation from normal distribution is not too strong, inference would be valid in large samples (Møen, 2009). For those who would like to read more about the Jarque-Bera test you can read the article by Bera and Jarque (1981).

Heteroscedasticity

Looking at the plot up to the right, we can determine whether σ_ε is constant or not. Ideally, the plots should be evenly distributed around zero. The plots in our diagram are not totally even distributed around zero, but seem to have more plots below zero. However, it does not seem to be any severe change in the spread of the plotted points. If we have heteroscedasticity, ordinary least squares method is not the best estimator and inference is not valid. However, the estimates are still unbiased (Møen, 2009).

To determine heteroscedasticity I could have applied White's general test, where we test the hypotheses:

$$H_0: \sigma_i^2 = \sigma^2$$

$$H_1: Not\ H_1$$

However, the test is extremely general. If we reject the null hypothesis the test gives no indication of what to do next (Green, 2008). Thus, I will not use time to conduct the test. The important thing is to be aware of the results if heteroscedasticity exists. More about White's general test can be read in Green (2008).

Non-independence of the error variable

Because my sample consists of cross-sectional data[20], error dependency should not be a problem. Dependency of the error variable is more common when dealing with time-series data[21]. As we can see from the diagram 9 at the bottom to the right, there is no pattern that should suggest autocorrelation. However, it is difficult to determine whether autocorrelation exists based on the diagram. Thus, it is better to use the Durbin-Watson statistic to test for autocorrelation. The test allows us to determine whether there is evidence of first-order autocorrelation. In other words, whether there is a relationship between consecutive residuals e_i and e_{i-1} where i is the time period. The Durbin-Watson statistic is defined by the equation (Keller, 2005):

$$d = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$$

The range of the values of d is $0 \leq d \leq 4$, where values of d less than 2 indicate a positive first-order autocorrelation. Large values of d, consequently larger than 2, indicate a negative first-order autocorrelation. Positive first-order autocorrelation occurs when the consecutive residuals tend to be similar, and the term $(e_i - e_{i-1})^2$ is small, leading to a small value of d. Not surprisingly, negative first-order autocorrelation occurs when consecutive residuals differ widely and the value of d is large (Keller, 2005).

When testing for autocorrelation, we can use tables[22] that provide us with values regarding d_L and d_U for a variety of values of n, k and for α = 0,01 and 0,05. d_L and d_U are the critical lower and upper values we use in order to decide whether we should reject the null hypothesis or not.

[20] Cross-sectional data are observations made at approximately the same time (Keller, 2005)
[21] Time-series data are observations taken at successive points of time (Keller, 2005).
[22] I will use tables 8(a) and 8(b) reproduced in appendix B in Keller (2005) to determine d_L and d_U.

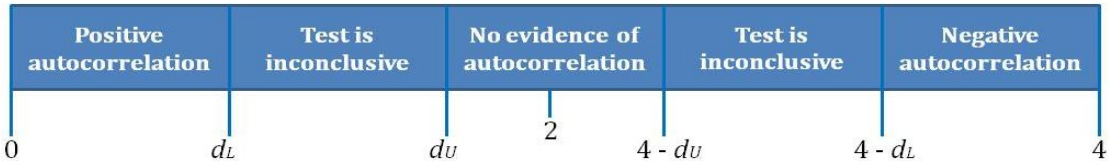

Figure 6 – Test for autocorrelation (Keller, 2005)

To test for positive first-order autocorrelation with $n = 55$, $k = 1$ and level of significant $\alpha = 0.05$, we test the following hypotheses:

$$H_0: There\ is\ no\ first - order\ autocorrelation$$
$$H_1: \ There\ is\ positive\ first - order\ autocorrelation$$

The decision is made as followed:

If $d < d_L = 1.53$, reject the null hypothesis in favour of the alternative hypothesis
If $d > d_U = 1.60$, keep the null hypothesis
If $1.53 \leq d \leq 1.60$, the test is inclusive

Calculation the Durbin-Watson statistic we get $d = 1.39$, which indicates positive first-order autocorrelation. How is it possible that the errors are correlated with cross-sectional data? One possible explanation is that the observations are given by different fund managers, where each fund managers' observations could be dependent of each other and thus indicate autocorrelation.

If we conduct the same the same test, but instead uses $\alpha = 0.01$, the decision is made on the following values:

If $d < d_L = 1.36$, reject the null hypothesis in favour of the alternative hypothesis
If $d > d_U = 1.43$, keep the null hypothesis
If $1.36 \leq d \leq 1.43$, the test is inclusive

The Durbin-Watson test is with $\alpha = 0.01$ is inclusive, and we cannot determine whether there is autocorrelation or not.

In occasions with autocorrelation ordinary least squares method is not the best estimator and inference is not valid. However, the estimates are still unbiased (Møen, 2009).

As we have seen so far, there is a chance that the requirements involving the probability distribution of the error variable are not satisfied. For now, I assume that the requirements are satisfied and proceed with the evaluation. There are several methods that can be used to evaluate the model. I am going to use (Keller, 2005):

I. The standard error of estimate
II. The *t* test of the slope
III. The coefficient of determination

I. Standard Error of Estimate

Minitab calculates the standard error of estimate (s_ε) automatically when computing the regression equation. s_ε is useful because it helps us determine whether the model's fit is good or poor. Formally, s_ε is a by-product of the sum of squared errors (SSE), which minimizes the sum of the squared differences between the points and the line given by the formula (Keller, 2005):

$$SSE = \sum (y_i - \hat{y}_i)^2$$

The standard error of estimate can then be calculated by using the following formula (Keller, 2005):

$$s_\varepsilon = \sqrt{\frac{SSE}{n-2}}$$

The smallest value the s_ε can assume is zero, and will occur when SSE equals 0. When s_ε equals zero all the points fall on the regression line, and the model fits perfectly. When s_ε is large the model is poor, and should either be improved or rejected (Keller, 2005).

When judging the value of s_ε, we have to compare it to the value of the dependent variable. More specifically, we compare it to the sample mean $\bar{y}$. However, comparing s_ε and $\bar{y}$ is not necessarily that easy, because there is no predefined upper limit of s_ε. Thus, the standard error of estimate cannot be used as an absolute measure of the model's validity. Usually, s_ε is instead used when comparing different models. In that case, the model with the smallest value of s_ε should generally be used (Keller, 2005).

In our case s_ε equals 112.988 which suggests that the model fits somewhat poorly. If we can compare s_ε to the sample mean, $\bar{y} = 160.4$, we can see that s_ε appears to be large.

II. Testing the Coefficient

When assessing a linear model, it is important to test whether there is linear relationship between the value of *y* and the value of *x.* Thus, we have to test whether there is a linear relationship between the value of *TP* and the value of *BV.* To do so, I have to test the hypotheses:

$$H_0: \quad \beta_1 = 0$$
$$H_1: \quad \beta_1 \neq 0$$

If the null hypothesis is true, no linear relationship exists. If the alternative hypothesis is true, some linear relationship exists.

Minitab computes the test, and I can use the regression printout to determine the linear relationship.

Predictor	Coef	SE Coef	T	P
Constant	54,18	16,94	3,20	0,002
BV	2,1513	0,1500	14,35	0,000

As the printout shows the test statistic *t* is = 14.34 with a *p*-value of 0. A large *t*-value and a low *p*-value result in overwhelming evidence to infer that a linear relationship exists between the transaction price and the reported book value. The printout also includes a test for β_o. However, interpreting the value of the *y*-intercept can lead to erroneous and sometimes ridiculous conclusions. Thus, we usually ignore the test of β_o (Keller, 2005).

III. Coefficient of Determination

Since I have determined that there exist a linear relationship I have to measure the strength of the relationship. In order to do so, we can use the coefficient of determination which is denoted R^2 (Keller, 2005).

The variation of *y* can be split into two parts, which is the sum of squares of error (SSE) and sum of squares for regression (SSR). SSE measures the amount of variation in *y* that remains unexplained, while SSR measures the amount of variation in *y* that is explained by the variation in the independent variable *x* (Keller, 2005).

$$Variation\ i\ y = SSE + SSR$$

If we incorporate the relationship into the definition of R^2, we can show that:

$$R^2 = 1 - \frac{SSE}{\sum(y_i - \bar{y})^2} = \frac{\sum(y_i - \bar{y})^2 - SSE}{\sum(y_i - \bar{y})^2} = \frac{Explained\ variation}{Variation\ in\ y}$$

In other words, the coefficient of determination measures the proportion of the variation in *y* that is explained by the variation in *x* (Keller, 2005).

The coefficient of determination does not have a critical value that enables us to draw conclusions whether the model is good or poor. The value of R^2 has to be evaluated based on professional judgment, and in context to other statistical test. However, the higher value of R^2, the better the model fits the data sample (Keller, 2005).

Looking at the regression printout we can find the coefficient of determination:

```
R-Sq = 79,5%
```

As we can see, R^2 tells us that 79.5 % of the variation in the dependent variable, *TP*, is explained by the variation in the independent variable *BV*.

Brief summary

The standard error of estimate is somewhat large compared to the sample mean, which suggest that the model fits poorly. On the other hand, a linear relationship exists between the depended and the independent variable. In addition, the coefficient of determination implies that 79.5 % of the variation in *TP* is explained by *BV*.

We have seen that the requirements for the error variable might not be satisfied. The standard error of estimate is also large. Thus, I would say that the model does not fit the data very good. On the other hand, I have managed to prove that a linear relationship exists and the explanation power is surprisingly good.

6.6.2 Simple linear regression with x = FV

Like I did in paragraph 6.6.1, when assessing the regression equation with *BV* as independent variable, I have to start by drawing a scatter diagram to determine whether a linear model appears to be appropriate. Notice, that the procedure is the same as before. The only different is *FV* instead of *BV* on the *x*-axis in diagram 10 below:

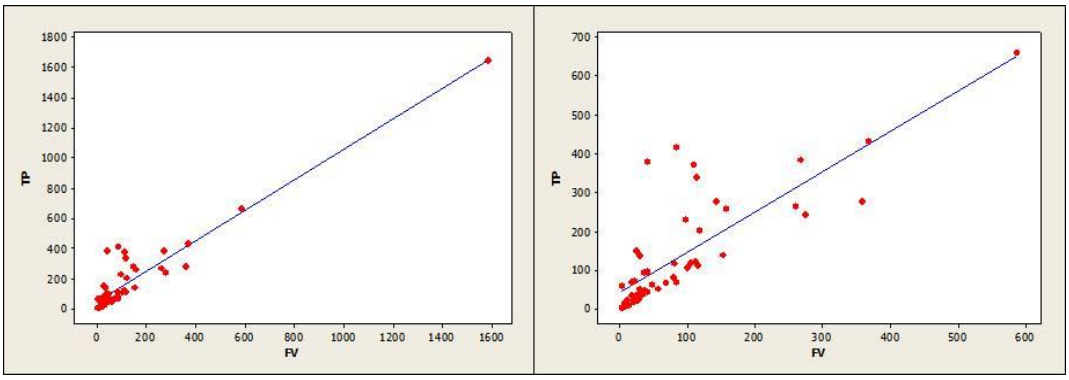

Diagram 10 – Scatter plot of TP vs FV

To the left in diagram 10 all observations are plotted against each other. To the right, the most extreme observation is removed to check for influential observations. However, the linear relationship does not seem to change, which implies that the observation is an outlier. During the discussion of outlier in the provisos regression, I had no evidence to suggest that the data were wrong. Even though it is possible to define some of the observations in diagram 10 as outliers, the same conclusion as before is valid. Some of the observations are simply very large, which lies in the nature of private equity investments. Thus, I will keep the observation as a part of the data sample.

Based on the scatter plot, I would say that a linear relationship does exist. However, as before, the relationship is not very strong.

The regression equation[23]

Calculating the regression line yields the following equation:

$$TP_2 = 44{,}7 + 1{,}02\,FV\ + \varepsilon$$

In other words, if we assume FV = 4 mill, the equation yields;

$$44{,}7 + 1{,}02 * 4 = 48{,}78$$

which implies a transaction price equal to 48.78 mill. For every one million increase in the fair value estimate, the transaction price will increase with 1.02 million.

Regression diagnostics

To evaluate the requirements involving the probability distribution of the error variable, I am going to use a similar residual plot for the dependent variable as previous:

[23] Complete regression printout is enclosed in appendix B.

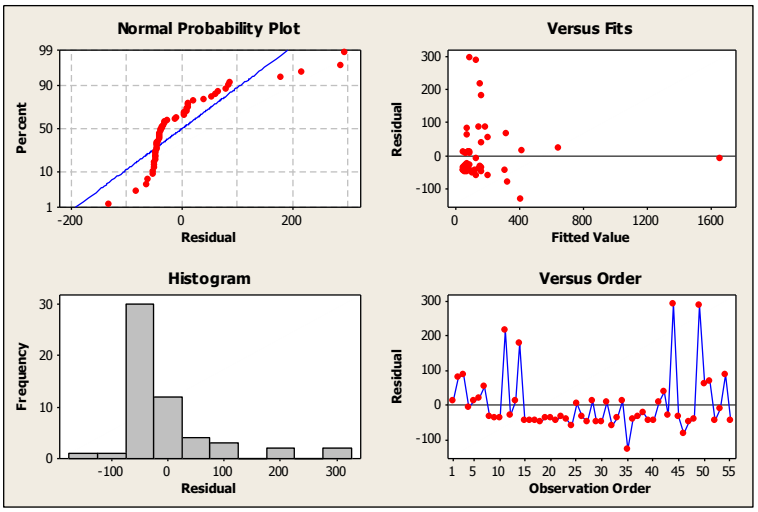

Diagram 11 – Residual plot for TP with x = FV

Normality

Changing the independent variable from *BV* to *FV* has not improved the histogram or the normal probability plot. In fact, the distribution seems to be quite similar as before. In this case, the histogram is still bell shaped but not completely centred round zero. The normal probability plot is supposed to follow a straight line. As you can see, some of the plots are off the blue line. As I concluded previously, some variation has to be expected. Thus, I would say that the normal probability plot does not abandon the assumption that the error variable is normally distributed.

Heteroscedasticity

The plot up to the right has basically the same pattern as before. Ideally, the plots should have been evenly distributed around zero. The plots in our diagram are, as before, not evenly distributed around zero, but seem to have more plots below zero. Still, there is no severe change in the spread of the plotted points. Thus, I would assume that the variance is close to constant.

Non-independence of the error variable

Changing the independent variable has not changed the classification of data. Because the sample consists of cross-sectional data, I expect error dependency to be absent. To be sure, I will use the same Durbin-Watson statistic to test for autocorrelation. Since sample size ($n=55$), number of independent variables ($k=1$) and level of significant ($\alpha=0.05$) is the same as before I can use the same values of d (page 74) to answer H_0 and H_1.

The calculation of the Durbin-Watson statistic shows d = 1.94. Unlike the test statistic where $x = BV$, we do not have to reject the null hypotheses. Thus, the sample does not consist of autocorrelation which means that the errors are independent of each other.

The biggest change when applying *FV* as the independent variable, compared to *BV*, is the Durbin-Watson statistic that shows no evidence of autocorrelation. The requirements regarding non-normality and constant variance are still wage in direction of assuming that the requirements are fulfilled. Thus, we have to consider that the least square method is not the best estimator, and that inference is not valid. Regardless, the estimates are still unbiased.

I. Standard Error of Estimate

When assessing the regression equation with *FV*, the standard error of estimate decreases from 112,998 to 82.8. The sample mean is $\bar{y} = 160.4$, which still makes the s_ε to appear large. However, as Keller (2005) wrote, there is no predefined upper limit for s_ε. The best we can do is to compare the values of s_ε with each other:

$$s_\varepsilon^1 = 112{,}988 > s_\varepsilon^2 = 82{,}8$$

Because $s_\varepsilon^1 > s_\varepsilon^2$ the regression model with *FV* as independent variable fits the observations best. Thus, if the decision was to be made entirely based on s_ε we should choose the model with *FV* as independent variable. Notice that the squared figure only denotes which equation the standard error of estimate belongs to.

II. Testing the Coefficient

I have to test whether there is a linear relationship between the value of y (*TP*) and the value of x (*FV*). As before, I have to test the following hypotheses:

$$H_0: \quad \beta_1 = 0$$
$$H_1: \quad \beta_1 \neq 0$$

If the null hypothesis is true, no linear relationship exists. If the alternative hypothesis is true, some linear relationship exists.

Predictor	Coef	SE Coef	T	P
Constant	44,70	12,49	3,58	0,001
FV	1,01686	0,04911	20,71	0,000

As the printout shows, the test statistic is $t = 20.71$ with p-value = 0. A large t-value and a low p-value result in overwhelming evidence to infer that a linear relationship exists between the transaction price and the fair value estimate.

III. Coefficient of Determination

Since I have determined that a linear relationship exists next is to measure the strength of the relationship. R^2 is given by the regression printout:

```
R-Sq = 89,0%
```

As we can see, R^2 tells us that 89.0 % of the variation in the dependent variable, y, is explained by the variation in the independent variable x. Comparing R_1^2 with R_2^2 shows that R_2^2 is larger than R_1^2:

$$R_1^2 = 79{,}5\ \% < R_2^2 = 89{,}0\ \%$$

Consequently, *FV* is a better predictor of the transaction price than *BV*.

If the difference between the two coefficients of determination had been less, it had been necessary to test whether the difference was statistical significant by applying e.g. bootstrapping. Briefly explained bootstrapping is the practice of estimating properties of an estimator by measuring these properties when sampling from an approximating distribution. One standard choice for an approximating distribution is the empirical distribution of the observed data. In the case where a set of observations can be assumed to be from an independent and identically distributed population, this can be implemented by constructing a number of "resamples" of the observed data set. Each "resample" is obtained by random sampling with replacement from the original data set (Wikipedia, 2010). One possible test statistic[24] could have been:

$$T = \frac{R_2^2 - R_1^2}{\sqrt{\overline{\sigma_{R_1^2}^2} + \overline{\sigma_{R_2^2}^2}}}$$

The numerator is simply the difference between the calculated coefficients of determination. The denominator, on the other hand, is the averaged standard deviation of each coefficient of determination computed by bootstrapping. If you would like to

[24] The test statistic is a result of conversations with my thesis advisor Knivsflå.

read more about bootstrapping, Efron and Tibshirani (1986) have written an article that explains the method in more detail.

6.6.3 Conclusion

To answer hypothesis 1 I had to test the following hypotheses:

$$H_0\colon\ R_1^2 = R_2^2$$

$$H_1\colon\ R_1^2 < R_2^2$$

Based on the two simple regressions I have computed, I reject the null hypothesis. Due to the approximately 10 % difference between R_1^2 and R_2^2 there is reason to believe that the difference is statistical significant.

Notice that the calculated s_ε^1 and s_ε^2 is large compared to their respectively sample means. Thus, the fit of the models is not very good. You can see the same in appendix A and B, where regression printouts are enclosed with the standard deviation of each residual. Thus, either of the equation is able to predict the transaction price good. However, the objective of assessing the equations where not to develop models that could predict fair value. Instead, I wanted to test which of the two independent variables that were better to explain the transaction price.

I would like to pinpoint that the result does not suggest that book values have little relevance when assessing the fair value of a portfolio company. Most valuations are based on historical cost values and historical return on equity. Thus, historical book values and return figures are essential information when arriving upon a fair value estimate. To the point, the test results of hypothesis 1 only emphasizes that fair value estimates are a closer measure of the intrinsic value of a private equity portfolio company than the same book values are. The result is not very surprising due to historical cost requirements in NGAAP.

6.7 Testing hypothesis 2

So far I have looked on the relationship between the transaction price and one independent variable. By testing hypothesis 1, I have established that it exists a linear relationship between the *TP* and both *BV* and *FV*. According to my analysis, *FV* is the independent variable that explains the *TP* best out of the two variables and is most value relevant. However, I believe that adding more variables to the model could result in

better explanation power. First, I will start by including the investment type in the equation. The hypothesis I would like to answer is:

2. **The type of investment affects the transaction price, and how well the fair value estimate explains the transaction price.**

Because regression equation TP_2, with *FV* as independent variable, explained the variation in *y* best, I will try to improve that model. Thus, I have to add another independent variable to the equation. The new equation is denoted TP_3:

$$TP_3 = \beta_0 + \beta_1 FV + \beta_2 V + \varepsilon$$

Here, *V* is a dummy variable indicating whether the investment is a venture or buy-out investment. If the investment is classified as venture you multiple the coefficient with one. On the other hand, if the investment is classified as buy-out you multiple with zero and the term disappears. To simplify the model, and thereby adding only one dummy variable, I have chosen to include seed and expansion investments in the categorization of venture capital. Both seed and expansion investments have potential for growth as common feature with venture investments. Thus, to categorize seed and expansion as venture investments should not influence the regression equation too much.

Notice, that when applying investment type as a dummy variable we can only test whether *V* has any fixed effect on the dependent variable. In other words, a dummy variable will affect the dependent variable in the same way as $\beta_0$does. However, we can also test whether there are any coefficient effects, or interaction effects between *TP* and *V* (Preacher et al, 2006). Consequently, when testing hypothesis 2 it is necessary to add a third variable to the equation:

$$TP_4 = \beta_0 + \beta_1 FV + \beta_2 V + \beta_3 (FV * V) + \varepsilon$$

I choose not to draw a scatter diagrams. We have already seen the scatter diagram with *FV* as the independent variable. Since *V* is used as a dummy variable, there is no reason to draw another diagram because we will not receive any new information.

The regression equations[25]

The regression analysis yields the following equations:

[25] Complete regression printouts are enclosed in appendix C and D.

$$TP_3 = 27{,}7 + 1{,}03\,FV + 22{,}8\,V + \varepsilon$$
$$TP_4 = 29{,}9 + 1{,}02\,FV + 10{,}4\,V + 0{,}168(FV * V) + \varepsilon$$

For equation TP_3 β_2 = 22.8 which means that the transaction price will increase with 22,8 million if the investment is classified as venture. Clearly, type of investment has some influence on the transaction price. However, β_0 has decreased compared to β_0 when we only had *FV* as independent variable in equation TP_2. Thus, this new equation does not necessarily predict the transaction price better.

If we look at equation TP_4 the coefficient β_2 equals 10.4 which is only half the size compared to the same coefficient in TP_3. However, because I have included an interaction effect we can see that coefficient β_4 equals 0,168 and is positive. The interpretation of this coefficient suggests that venture investments have a transaction price premium close to 17 %. This premium is in addition to the 10.4 million already added to the *TP* by the dummy variable. Consequently, it seems that type of investment has both a fixed and an interaction effect on the transaction price.

Regression diagnostics

The residual plots for equation TP_3 and TP_4 are basically the same as in diagram 11. Thus, I will not use time to interpret the different plots. The plots are enclosed in appendix C and D. The only thing that has changed, and worth noticing when adding more independent variables, is the Durbin-Watson statistic. Since we for equation TP_3 has two independent variables, the hypotheses for autocorrelation are now tested with *n = 55, k = 2* and *α = 0.05*.

The decision is then based on the following critical values:

If $d < d_L$ = 1.49, reject the null hypothesis in favour of the alternative hypothesis
If $d > d_U$ = 1.64, keep the null hypothesis
If 1.49 ≤ *d* ≤ 1.64, the test is inclusive

By adding another variable to the equation the Durbin-Watson statistics equals 1.98. Compared to equation TP_2, the statistics has increase some and we cannot reject the null hypotheses.

Equation TP_4 has three independent variables. Thus, the same hypotheses have to be tested based on $n = 55$, $k = 3$ and $\alpha = 0.05$. The critical values for TP_4 when testing for autocorrelation are:

If $d < d_L = 1.45$, reject the null hypothesis in favour of the alternative hypothesis
If $d > d_U = 1.68$, keep the null hypothesis
If $1.45 \leq d \leq 1.68$, the test is inclusive

The Durbin-Watson statistic for TP_4 equals 1.93. Consequently, the test shows no evidence of autocorrelation, and we cannot reject the null hypothesis.

I. Standard Error of Estimate

What is more interesting, is to see how s_ε has changed when adding more variables to the regression equation. When applying multiple regression the formula for s_ε is;

$$s_\varepsilon = \sqrt{\frac{SSE}{n-k-1}}$$

where k is the number of independent variables.

Equation	Sε	Variables (x_i)
TP2	82,80	FV
TP3	82,95	FV; V
TP4	83,27	FV; V; (FV*V)

Table 11 – Sε for equation 2, 3 and 4

For the time being, equation TP_2has the lowest standard error of estimate, followed by $TP_3$and TP_4. Based on the standard error of estimate, adding more variables to the equation have not increased the model's fit. However, the change is not dramatically, and probably not statistical significant.

II. Testing the Coefficients

When testing the slope of the coefficients of a multiple regression, we have to do some adjustments to the hypotheses:

$$H_0: \quad \beta_i = 0$$
$$H_1: \quad \beta i \neq 0 \qquad \text{(for } i = 1,2\ldots..,\ k\text{)}$$

The regression printout for equation TP_3 shows:

Predictor	Coef	SE Coef	T	P
Constant	27,70	22,52	1,23	0,224
FV	1,03158	0,05180	19,92	0,000
V	22,77	25,10	0,91	0,368

As we can see from the printout only *FV* seems to be linear related to *TP*. Both the constant term and *V* have low values of *t* and high *p*-values. The *p*-value of the constant coefficient is meaningless to interpret, because the *TP* will never be calculated if *FV* is equal to zero. One possible interpretation of the *p*-values could be that adding *V* to the equation does not increase the model's explanation power. In other words, type of investment when applied as a fixed effect is not crucial when estimating fair value. The reason is probably because fair value estimate itself stands for a large part of the explanation power.

If we look at the correlation matrix in table 7 (page 59) we see that *FV* and *V* is negative correlated with approximately 0.31. Thus, another possible explanation of the high *p*-value could be multicollinearity[26], because if we look at the analysis of variance the *F* test is high while the *p*-value is zero. The *F* test is useful to determine whether the model itself is valid or not. Unlike the *t* statistic, the *F* statistic tests the whole model and not each independent variable.

The same printout for equation TP_4 is:

Predictor	Coef	SE Coef	T	P
Constant	29,90	22,79	1,31	0,195
FV	1,02141	0,05364	19,04	0,000
V	10,44	29,84	0,35	0,728
FV*V	0,1683	0,2183	0,77	0,444

Similar to equation TP_3, the printout for TP_4 also indicates a high *p*-value for the constant term and *V*. In addition, the *p*-value for (*FV*V*) is also high. Based on the discussion for equation TP_3it would be tempting to say that type of investment has either a fixed or an interaction effect on the transaction price. However, the analysis of variance shows that the model is valid:

[26] Multicollinearity is a condition where the independent variables are highly correlated, and distorts the *t* test of the coefficients, making it difficult to determine whether any of the independent variables are linearly related to the dependent variable. The effect of multicollinearity is that the coefficients of the independent variables that are correlated tend to have large sampling errors (Keller, 2005)

Source	DF	SS	MS	F	P
Regression	3	2950133	983378	141,81	0,000

We know that *FV* and *V* is negative correlated. The correlation matrix in table 7 does not include correlation for *FV*, (*FV*V*) and *V*. The table below shows these correlation figures:

	FV	V	(FV*V)
FV	1,00	-	-
V	-0,31	1,00	-
(FV*V)	0,05	0,49	1,00

Table 12 – Correlation matrix for *FV, V* and (*FV*V*)

As we can see, the correlation between *FV* and (*FV*V*) is low. However, the correlation between *V* and (*FV*V*) is high. This should not come as any surprise due to the way the independent variable is constructed. Due to correlation between independent variables, it is natural to believe that the model faces problems of multicollinearity.

III. Coefficient of Determination

Because I have to compare R^2 for different regressions with different independent variables the numbering of R^2 will denotes which equation each R^2 belongs to. When adding more variables to the equation, we need to use the adjusted coefficient of determination. Adjusted R^2 shows on the printouts and equals:

$$R_3^2 = 88{,}8\ \%$$

$$R_4^2 = 88{,}7\ \%$$

As we can see, the coefficient of determination is basically the same for both equations. Consequently, investment type applied as a fixed effect or an interaction effect does not increase the explanation power of the model. In fact, type of investment contributes to make the model slightly poorer. There seems to be evidence to suggest that type of investment does not contribute to explain the transaction price better than *FV* does standing alone. The table below shows the development in the coefficient of determination based on numbers of independent variables. Notice that R^2 for TP_2 is unadjusted. This is common for all tables in this thesis where R^2 is compared between the different equations, and the equation has just one independent variable (x).

Equation	R^2 adj	Variables (x_i)
TP_2	89,00 %	FV
TP_3	88,80 %	FV; V
TP_4	88,70 %	FV; V; (FV*V)

Table 13 – R^2 for equation 2, 3, and 4

As you can see from the table, equation 2 still has the best explanation power out of the equations I have conducted so far. Notice that the differences between the equations are very small, which makes it difficult to say that the differences are statistical significant.

Alternative equation

There is a final test I can conduct in order to test whether the relationship between *TP* and *V* is statistical significant. Since we know that the correlation between *FV* and (*FV*V*) is low, it is possible to conduct an equation where I only test for interaction effects between *TP* and *V*. The printout[27] for this equation shows that the coefficient (*FV*V*) is not statistical significant. In fact, the printout suggests that there is no linear relationship between *TP* and (*FV*V*). This also helps us understand why (*FV*V*) is not statistical significant when applied as independent variable in equation TP_4. Notice however, that the *F*-test of both equations where *V* is applied as an interaction effect suggests that the models are valid. Thus, the linear relationship between *TP* and (*FV*V*) is not completely absent – only very weak.

6.7.1 Conclusion

The evaluation of equation TP_3 and TP_4 has shown that there is little evidence to suggest that investment type contributes to explain the variation in *TP* better than *FV* does alone. Surly, the differences between the coefficient of determination and the standard error of estimate are small making it difficult to draw sturdy conclusions.

Equation	Sε	R^2 adj	Variables (x_i)
TP_2	82,80	89,00 %	FV
TP_3	82,95	88,80 %	FV; V
TP_4	83,27	88,70 %	FV; V; (FV*V)

Table 14 – Sε and R^2 for equation 2, 3, and 4

The testing of the second hypothesis has shown some spread in the test result depending on whether *V* was applied as a fixed effect or a combination of a fixed and an interaction effect. In equation TP_3 where *V* was included as a fixed effect the coefficient

[27] Printout is enclosed in appendix E.

had a large *p*-value. My findings suggest that the large *p*-value is a result of multicollinearity between *FV* and *V*. When *V* was included as both an interaction and fixed effect in equation TP_4, the test statistic of the coefficients showed evidence of no linear relationship between *TP* and (*FV*V*). This is also supported by the test statistic of the coefficients when computing a regression where *V* is only applied as an interaction effect (*FV*V*).

Summarized, I would say that there is not enough evidence to suggest that type of investment has no influence on the transaction price. Thus, I cannot reject hypothesis 2. However, it is not given that including type of investment has increased the model's explanation power. This is supported by the development in the coefficient of determination and the standard error of estimate for each equation in table 14. In other words, adding *V* to the model has not contributed to explain more of the variation in *TP*.

6.8 Testing hypothesis 3

So far, I have discovered that fair value estimates predict the transaction price better than reported book values. As we saw during the test of hypothesis 2, adding investment type to the regression equation did not improve the explanation power of the model. However, we could not find any strong evidence that suggested that type of invested did not explain any of the variation in the dependent variable. Thus, the next thing I would like to do is to test the relationship between *TP, BV, FV* and *V*.

3. **The transaction price is best explained by the reported book value, goodwill in terms of fair value exceeding the book value, and how book value and goodwill interact with type of investment.**

To test this hypothesis I will apply the same interaction technique, regarding the variable *V,* as I did during the testing of equation TP_4. Since hypothesis 3 requires me to test the relationship between both *BV* and *FV*, I need to make a little adjustment. Instead of using *FV* in the regression analysis, I am going to use the difference between the two variables (*FV-BV*) which I have denoted *GW*. In reality, *GW* is the excess value of the portfolio company calculated by the fund manager. As mentioned earlier this different can be regarded as goodwill not recognized in the portfolio company's balance sheet.

Due to possible interaction effects I need to compute two equations:

$$TP_5 = \beta_0 + \beta_1 BV + \beta_2 GW + \beta_3 V + \varepsilon$$
$$TP_6 = \beta_0 + \beta_1 BV + \beta_2 GW + \beta_3 V + \beta_4 (BV * V) + \beta_5 (GW * V) + \varepsilon$$

Equation TP_5 only tests for fixed effect between *TP* and *V*, while equation TP_6also includes interaction effects between *BV* and *GW*.

Except from *GW*, all other variables are known from before. I have already drawn scatter plots for *BV* and *FV*, and the results would be the same. In addition, I similar scatter plot (not enclosed) for *GW* indicates that a linear relationship exists between *GW* and *TP*.

The regression equations[28]

The regression analysis yields the following equations:

$$TP_5 = 27{,}3 + 1{,}07\, BV + 1{,}00\, GW + 23{,}1\, V + \varepsilon$$
$$TP_6 = 31{,}7 + 0{,}662\, BV + 1{,}29\, GW - 11{,}2\, V + 1{,}81(BV * V) - 0{,}493(GW * V) + \varepsilon$$

Notice, that in equation TP_6 coefficients β_3 and β_5 have negative signs. Based on economical logic, I would expect these coefficients to be positive related to the dependent variable. However, it is difficult to conclude before we have investigated the regression diagnostics.

Regression diagnostics

The residual plots for both equations show no dramatically changes compared to the plot in diagram 11. Testing for autocorrelation shows no sign of a dependent error variable. The plots and the Watson-Durbin statistic are enclosed in appendix G and H for the reader to investigate further if wanted.

I. Standard Error of Estimate

Table 15, on the next page, shows the calculated standard error of estimate for equation $TP_5$and TP_6:

Equation	Sε	Variables (x_i)
TP_5	83,72	BV; GW; V
TP_6	76,12	BV; GW; V; (BV*V); (GW*V)

Table 15 – Sε for equation 5 and 6

Comparing the different standard error of estimates yields the following ranking:

[28] Complete regression printout is enclosed in appendix G and H.

$$s_\varepsilon^5 < s_\varepsilon^4 < s_\varepsilon^3 < s_\varepsilon^2 < s_\varepsilon^6$$

For equation TP_6 s_ε has improved substantially, while the effect on s_ε for equation TP_5 goes in the wrong direction. However, the effect on s_ε^5 is small, and probably not statistical significant.

II. Testing the Coefficients

The regression printout for equation TP_5 shows:

Predictor	Coef	SE Coef	T	P
Constant	27,32	22,80	1,20	0,236
BV	1,0729	0,2014	5,33	0,000
GW	1,0019	0,1491	6,72	0,000
V	23,15	25,39	0,91	0,366

Both *BV* and *GW* have a low *p*-value indicating a linear relationship with the dependent variable. For *V* the same pattern as for equation TP_3seems to be a possible explanation. The results are not very surprisingly, because if you think about it, assessing a regression equation where *BV* and *GW* replace *FV* is basically the same as using just *FV*. This relationship can be clarified if you look at the definition of *FV* used in this analysis:

$$FV = BV + GW$$

In light of that argumentation, it is only reassuring to see that the test statistics are basically the same for TP_5 as for TP_3. Nevertheless, the coefficient effects are in force indicating that *BV* and *GW* explain some of the variation in *TP*. This is also in accordance with theoretical expectations because *BV* usually acts as a starting point when assessing the fair value of basically any asset or investment.

For equation $TP_6$the regression printout gives us the following *p*-values for the coefficients:

Predictor	Coef	SE Coef	T	P
Constant	31,67	20,86	1,52	0,135
BV	0,6620	0,2294	2,89	0,006
GW	1,2922	0,1758	7,35	0,000
V	-11,17	28,05	-0,40	0,692
BV*V	1,8055	0,5122	3,52	0,001
GW*V	-0,4932	0,2906	-1,70	0,096

Once more, *V* seems to show no linear relationship with the dependent variable. The same holds for (*GW*V*) which is not statistical significant within the usual 5 % significant

level. Nevertheless, the *F*-test indicates that the model as a whole is valid. Thus, multicollinearity between two or more independent variables could be a reason worth investigating further by looking at another correlation matrix.

	BV	GW	V	BV*V	GW*V
BV	1,00	-	-	-	-
GW	0,83	1,00	-	-	-
V	-0,32	-0,29	1,00	-	-
BV*V	0,04	-0,10	0,49	1,00	-
GW*V	-0,09	0,20	0,36	0,29	1,00

Table 16 – Correlation matrix between BV, GW, V, BV*V and GW*V

As the correlation matrix in table 16 shows, there is some correlation between the independent variables that could contribute with "noise" in our equation. Especially the correlation between *V* and (*BV*V*) and *V* and (*GW*V*) is worth noticing.

III. Coefficient of Determination

R^2 for each equation is show in the table below:

Equation	R^2 adj	Variables (x_i)
TP_2	89,00 %	FV
TP_3	88,80 %	FV; V
TP_4	88,70 %	FV; V; (FV*V)
TP_5	88,50 %	BV; GW; V
TP_6	90,50 %	BV; GW; V; (BV*V); (GW*V)

Table 17 – R^2 for equation 2, 3, 4, 5 and 6

Comparing the different coefficient of determination yields the following ranking:

$$R_5^2 < R_4^2 < R_3^2 < R_2^2 < R_6^2$$

The ranking of R^2 corresponds to the ranking of standard error of estimate for each equation. This is a natural result, because a high coefficient of determination usual responds to a lower value of s_ε. Notice, that the differences between each of the coefficient of determination is very small, and probably not statistical significant.

Alternative equation

For reasons of curiosity, I have removed the fixed effect of *V* from the regression equation. *V* applied as an interaction effect yields the following equation[29]:

$$TP_7 = 25{,}5 + 0{,}673\,BV + 1{,}30\,GW + 1{,}72(BV * V) - 0{,}52(GW * V) + \varepsilon$$

The test statistics for the slope of the coefficients, R^2 and s_ε shows:

Predictor	Coef	SE Coef	T	P
Constant	25,49	13,83	1,84	0,071
BV	0,6729	0,2259	2,98	0,004
GW	1,2969	0,1739	7,46	0,000
BV*V	1,7226	0,4641	3,71	0,001
GW*V	-0,5203	0,2801	-1,86	0,069

S = 75,4713 R-Sq(adj) = 90,7%

All independent variables, except (*GW*V*), is statistical significant which suggest that a linear relationship is an appropriate assumption. Another interpretation is to say that *GW* is less value relevant for venture companies than the other variables. All variables though, are significant at the 10 %-level. In addition, both R^2 and s_ε have improved compared to equation TP_6 even though it is not much.

6.8.1 Conclusion

When testing the slope of the coefficient of both equation $TP_5$and TP_6there is little evidence that supports a linear relationship between *TP* and *V* when *V* is applied as a fixed effect. However, when *V* is applied as an interaction effect the test statistics suggest that a linear relationship could be an appropriate assumption. Notice that the effect is largest for *BV*, while the effect on *GW* is negative and not statistic significant at 5 % significant level.

Based on the test statistics for equation TP_5, TP_6 and TP_7, I would say that it exists evidence to suggest that type of investment has no fixed effect on the transaction price. However, there seems to be an interaction effect between *BV* and *V* which is statistical significant. Thus, *BV* is more value relevant for venture companies than for other.

The independent variables *BV* and *GW* have been statistical significant for all equations. This was an expected result because *BV* will usually act as a base value for the valuation

[29] The regression printout is enclosed in appendix H.

process. Since *GW* is defined as the difference between *FV* and *BV,* it would be strange, and not economical logic, if *GW* was not statistical significant. Consequently, I would say that there is not enough evidence to reject hypothesis 3.

Finally, I would like to emphasise some important points. Notice that the difference between R^2 for equation TP_2 and TP_7 is only 1.7 %. Thus, by adding more variables to the equation the explained variation in *TP* has not increased much. The most significant change is the decrease in the standard error of estimate which suggest that TP_7 has a better fit thanTP_2. However, if you look at the residuals for each equation the fit is not exceptional good. In light of the residuals, I would not recommend to use the equations for prediction purpose.

6.9 Value relevance of methodology

In paragraph 6.5.3 the descriptive statistics suggested that multiples were the methodology that had the lowest deviation between *FV* and *TP*. In light of that, it would be interesting to assess a multiple regression that investigates the value relevance of multiples as methodology. In order to do so, I have to introduce a new variable *M*. *M* is defined as an indicator variable (dummy) where *M*=1 if multiples are used in the valuation process or 0 otherwise.

To test for both fixed and interaction effects the equation will be expressed similar to TP_6. The reason for choosing $TP_6$is because that equation is the most advanced one, and contains all variables that are expected to be value relevant. The new equation is expressed by:

$$TP_8 = \beta_0 + \beta_1 BV + \beta_2 GW + \beta_3 M + \beta_4(BV * M) + \beta_5(GW * M) + \varepsilon$$

As you can see, I have removed *V* as independent variable and replaced it with *M.* I could have conducted an equation where both *V* and *M* were included, but it would make the regression unreasonable complex.

The regression printout yields the following equation[30]:

$$TP_8 = -1{,}9 + 3{,}84\, BV + 1{,}09\, GW + 20{,}0M - 3{,}10(BV * M) + 0{,}174(GW * M) + \varepsilon$$

The test statistics for the slope of the coefficients, R^2 and s_ε shows:

[30] Complete printout is enclosed in appendix I.

Predictor	Coef	SE Coef	T	P
Constant	-1,88	23,79	-0,08	0,938
BV	3,8388	0,5992	6,41	0,000
GW	1,0863	0,2564	4,24	0,000
M	20,03	38,67	0,52	0,608
BV*M	-3,0983	0,6661	-4,65	0,000
GW*M	0,1744	0,3531	0,49	0,625

S = 75,4341 R-Sq(adj) = 93,4%

According to the printout only the interaction effect between *BV* and *M* seems to be statistical significant. However, once more the *F*-test indicates that the model as a whole is valid. Thus, multicollinearity between two or more independent variables could be a reason why *M* and *GW*M* seems to not be significant. This assumption is confirmed when looking at the correlation (enclosed in appendix I) between the different variables.

Both the coefficient of determination and the standard error of estimate are good compared to earlier values of computed equations. Clearly, multiples are value relevant for *TP*. Due to possible multicollinearity it is difficult to say whether *M* is relevant both as a fixed and interaction effect. Based on the test statistics for the slop of the coefficient, the interaction effect between *BV* and *M* seems to be most significant. This result could suggest that *BV* is the most value relevant figure when using multiples in the valuation process.

Notice that equation TP_8 is assessed with only 36 observations, due to incomplete information regarding the methodology for all observations in the data sample. Thus, we should be cautious when interpreting the test statistics. Since the sample size is less than before, we cannot compare earlier test statistics with statistics of TP_8 without taking this into account. The limited sample size could have contributed to improve both R^2 and s_ε by removing observations which earlier had strong influence on the regression outcome.

6.10 Criticism

The objective of applying regression analysis on the data sample has not been to develop prediction models, but instead perform value relevance analysis with possible variables that could explain the transaction price for former private equity portfolio companies. When testing hypothesis 1, the conclusion was that *FV* outperformed *BV* when it came to explain the variation in *TP*. Trying to include other variables did not significantly increase the coefficient of determination, or the standard error of estimate. However,

there was not enough evidence to suggest that other variables (*V, BV* and *GW*) did not explain some of the variation in the transaction price. Nevertheless, *FV* seems to be the variables that explain the biggest share of the variation in *TP*.

The result is perhaps not surprisingly. The data sample only consists of reported book values in accordance with the firms' last reporting date. Thus, the analysis only checks for linear relationship between transaction price and balance sheet values when assessing *BV* as variable. Since most Norwegian private equity companies reports in accordance with NGAAP, the sample is biased in the sense that historical cost values have a larger deviation from *TP* than *FV*. Consequently, *FV* contribute to explain the variation in *TP* better than *BV*. If the data sample also included information about the investment's profit share, *BV* might have explained more of the variation:

$$TP = \beta_0 + \beta_1 BV + \beta_2 Profit\ share + \varepsilon$$

By including the profit share of the investment, the historical cost values would be adjusted in the same way as the equity method of accounting adjusts historical values. Thus, the value (*BV + Profit share*) might be closer to the fair value estimate. When that is said, I could have used profit shares in combination with *FV* and the conclusion might have been that *FV,* in combination with profit shares, still outperforms *BV* and profit shares. A possible relationship to test could then have been:

$$TP = \beta_0 + \beta_1 FV + \beta_2 (Profit\ share + \Delta GW) + \varepsilon$$

However, what is important to be aware of is that the "biased" sample data could have contributed to favor *FV* on behalf of *BV*.

When interpreting the result of the analysis, we need to have in mind that Norwegian private equity companies reports in accordance with NGAAP. Thus, the investment is recognized by their historical values which contribute to make the deviation between *BV* and *TP* large.

As mentioned earlier, the data sample consists of observations from six private equity companies. Because the companies have not contributed with the same amount of observations, the companies have different influence on the sample. If company A has unsuccessful fair value estimates, due to any reason, this will affect the findings in our analyses. Thus, we should apply caution and not draw too substantial conclusions. In

addition, the sample size of 55 observations is not necessarily a representative selection which could contribute to biasness.

Another source of bias is that the data sample only consists of Norwegian private equity funds. Thus, it is important that all findings are only relevant for the Norwegian private equity industry. However, the IPEV Guidelines and the valuation techniques are international standards, which could suggest that the results would not be fundamental different by including foreign private equity funds in the data sample.

Because *BV* is the last reported book value of the portfolio company, and *FV* is the latest estimate as close up to the point of transaction, it would be desirable to know the exact time when both variables were reported or estimated. Since this information is not part of the data sample, it difficult to check whether expectations of *TP* is included in *FV*.

It would have been desirable to include observations that are not realized in the data sample. Since we need information about an intrinsic value, *TP* when the investment was bought could have been part of the sample together with values of *BV* and *FV* in advance of the transaction. Because the data sample only has information about realized investment this could be a source of bias.

It would be interesting to wait some years and perform a similar analysis with an extended data set to see if the findings then correspond to what I have discovered.

7 How to increase the accuracy of fair value estimates?

Even though my analysis shows that fair value estimates predict the transaction price better than historical cost values, the estimates are not very good from my point of view. Of course, some of the miss-predictions could be explained by lack of comparable market transaction and, in general, a volatile market. In addition, parts of the deviation could be a result of little experience because fair value measuring in the Norwegian industry is a relative new measure attribute. However, the valuation techniques are familiar to the industry, because private equity companies have valuated companies long before IPEV Guidelines was published. Regardless of reasons, I would like to pinpoint some factors that could contribute to raise the credibility of the estimates.

In general, fair value measuring does not have a finite answer. At the end, fair value measuring in the private equity industry is a question of professional judgement. Thus, it is very important that the valuer documents all assumptions used when arriving at an estimate. If the stakeholder is provided necessarily documentation, it is possible to evaluate the estimate and thereby question any disagreements. The documentation should include all relevant information used, such as methodology, assumptions concerning growth rate, discount rate, multiples, risk, profitability and so on.

Another thing to consider is to involve key personnel that possess important insight of the investment (PwC, 2008). Increased involvement by the investment manager can contribute to improve the valuation process, because investments managers are usually involved in the management of the investment. This is in contrast to estimate carried out by personnel in back-office positions, which can be more inaccurate due to lack of the right competence.

PwC (2008) suggests that external advisors can be used as a reference point in the valuation process. From my point of view, I have to agree with that suggestion. By using external advisors the private equity funds obtain an objective view on the process. Securing objectivity in the private equity industry is essential due to the secretiveness that exists among many fund managers. If the fund managers' integrity is being questioned it could result in negative consequences. In addition, external advisors can contribute with knowledge on issues that might be new for the valuer (PwC, 2008). In

general, a win-win situation for the private equity fund which achieve new knowledge and probably increasing credibility concerning their fair value estimates.

You could argue that external advisors are not necessary to use because external auditors audit the financial statement. Thus, the audit process contributes with controlling the different assumptions and fair value estimates. However, an auditor has usually not enough time to check every single assumption in the financial statement. Thus, I believe that more co-operations between investment managers and external advisors could contribute to improve the valuation process, and hopefully reduce the deviation between fair value estimates and market values. A counter argument of using external advisors could be increased costs. Thus, there could be a discussion whether the increased utility outperforms the increased level of costs.

8 Final summary

The objective of this thesis was to see how fair value reporting and measuring is implemented in the private equity industry, and investigate the value relevance of *BV* and *FV* across type of investment and valuation method. To address the different challenges, I have looked at different accounting requirements in accordance with NGAAP, IFRS and USGAAP. The different accounting standards have shown us a complex framework for fair value accounting. Thus, additional guidelines (IPEV) have been published to try to overcome some of the challenges which are especially relevant for the private equity industry.

To test how good the private equity industry is to estimate fair value, I have conducted a study of realized investments by six Norwegian private equity companies. The study reveals that estimating fair value is not necessarily an easy operation. According to my data sample, 3 of 4 investments are underestimated, meaning that the fair value estimate is lower than the transaction price. The averaged deviation for the whole sample is -25 %. Based on the extent of accounting requirements and measuring guidelines, I expected the deviations to be less.

The data sample shows that multiples are the most common valuation technique used to estimate fair value. The findings are not very surprising, because the technique is relatively easy and quick to use and recommended by the IPEV Guidelines. In addition, multiples contribute to the lowest difference between transaction price and fair value estimate with an average deviation of -14 %. Even though I should be careful with drawing fundamental conclusions, it is reassuring to see that the most common method also predicts the transaction price best.

The regression analysis has shown that, when applying only book value (*BV)* or fair value (*FV*) as independent variable, *FV* contributes to explain deviation in the transaction price best. The rational explanation behind this result is that *FV* is the most value relevant figure when arriving upon an intrinsic value of the portfolio company. Further, the analysis shows evidence to support that investment type explains some of variation in the transaction price. However, the explanation power when including type of investment, in addition to fair value estimates, does not increase.

In light of theoretical expectations, *BV* should be able to explain some of the variation in the *TP*. This expectation is motivated by the fact that historical values are used as a base value in valuations. Thus, by expanding the regression equation with more independent variables, and making the model more advanced, it is possible to show that *BV*, together with not recognized goodwill (*GW*) and interaction effects of investment type (*V*), increase the explanation power of the equation. This is an interesting result, because it shows that *BV* is not irrelevant when assessing *FV* of an investment and consequently the belonging *TP*. Even though *FV* is a result of both *BV* and *GW*, the analysis shows evidence to suggest that the coefficient of determination and standard error of estimate improve due to coefficient effects when *FV* is divided between two variables. Notice that *BV* seems to be more value relevant for venture investments than other investments when the intrinsic value is estimated.

When applying multiple (*M*) as a fixed and interaction effect, *BV* is to be the most value relevant figure. However, equation TP_8 is based on only 36 observations. Thus, we should not emphasize the results too much and be careful when comparing test results for TP_8 and the other equations.

The different standard error of estimates and coefficient of determinations for each regression equation is summarized in the table below where they are ranked to each other.

Equation	Sε	R^2 adj	Variables (x_i)
TP_8	75,43	93,40 %	BV; GW; M; (BV*M); (GW*M)
TP_7	75,47	90,70 %	BV; GW; (BV*V); (GW*V)
TP_6	76,12	90,50 %	BV; GW; V; (BV*V); (GW*V)
TP_2	82,80	89,00 %	FV
TP_3	82,95	88,80 %	FV; V
TP_4	83,27	88,70 %	FV; V; (FV*V)
TP_5	83,72	88,50 %	BV; GW; V
TP_1	112,98	79,50 %	BV

Table 18 – Sε and R^2 for all regression equations[31]

Over all, my findings seem to pinpoint the fact that fair value measuring in the private equity is extremely difficult. The lack of quoted prices, in combination with increased demand for transparency and fair value measuring, are factors that are difficult to

[31] The coefficient of determination for equation 1 and 2 is unadjusted due to only one independent variable in the regression equation.

combine. Fair value measuring is relatively new in the Norwegian private equity industry, which could suggest that the accuracy of the estimates should improve in accordance with practical experience. However, estimating fair value might seem as an impossible task when the lack of market information is an essential cause. Thus, minimizing the deviations between the realized transaction price and fair value estimate could be an intermediate aim in order to improve the fair value estimates. From my point of view, I would say that it is important that investors and other stakeholders have faith in the reported figures if the industry should continue to develop in the future. This could be achieved by applying some of the improvements I have suggested in chapter 7.

In general, increased transparency is a key word for the industry. Thus, I would urge the industry to abandon the path of undue secretiveness. Less secretiveness could contribute to put challenges regarding fair value measuring in the private equity industry in the spotlight, and hopefully contribute to improve the valuation process and the quality of fair value estimates.

Focus on fair value measuring is important because both existing investors and potential future investors have a real interest in the fair value of investments in financial statements. This is an important reason to get control of this process and to been seen to be operating at the top of the peer group in terms of the valuation process (PwC, 2008).

9 Bibliography

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10 Appendix

A Regression Analysis: TP versus BV

The regression equation is
TP = 54,2 + 2,15 BV

Predictor	Coef	SE Coef	T	P
Constant	54,18	16,94	3,20	0,002
BV	2,1513	0,1500	14,35	0,000

S = 112,988 R-Sq = 79,5% R-Sq(adj) = 79,1%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	2627170	2627170	205,79	0,000
Residual Error	53	676614	12766		
Total	54	3303784			

Obs	BV	TP	Fit	SE Fit	Residual	St Resid
1				16,8	1,5	0,01
2				16,3	74,3	0,66
3				16,5	158,6	1,42
4				103,6	15,9	0,35 X
5				17,1	160,5	1,44
6				18,9	341,0	3,06R
7				15,8	156,4	1,40
8				15,5	-75,5	-0,67
9				15,5	2,5	0,02
10				15,9	14,5	0,13
11				15,3	229,9	2,05R
12				15,8	-36,6	-0,33
13				15,4	-38,6	-0,35
14				18,1	39,5	0,35
15				16,9	-52,4	-0,47
16				16,7	-55,5	-0,50
17				15,9	-69,4	-0,62
18				16,5	-58,5	-0,52
19				16,1	-53,4	-0,48
20				16,5	-56,2	-0,50
21				16,6	-54,7	-0,49
22				16,5	-46,6	-0,42
23				16,1	-63,0	-0,56
24				15,3	4,8	0,04
25				15,2	-78,6	-0,70
26				15,9	-60,2	-0,54
27				15,8	-79,1	-0,71
28				15,3	-40,8	-0,36
29				15,3	-117,8	-1,05
30				16,4	-23,4	-0,21
31				15,3	-51,0	-0,46
32				15,7	-144,6	-1,29
33				15,4	-99,2	-0,89
34				15,5	-21,3	-0,19
35				32,0	-285,8	-2,64R
36				15,8	-64,3	-0,57
37				16,4	-27,7	-0,25
38				15,5	-68,7	-0,61
39				16,6	11,0	0,10
40				16,7	-59,1	-0,53
41				15,9	-23,9	-0,21
42				15,3	17,3	0,15
43				16,7	-26,1	-0,23
44				15,3	235,9	2,11R
45				15,4	-14,1	-0,13

Due to reasons of confidentiality, these numbers cannot be published.

46	Due to reasons of confidentiality, these numbers cannot be published.	18,5	-68,3	-0,61
47		15,3	-61,7	-0,55
48		15,3	-100,6	-0,90
49		16,1	182,5	1,63
50		15,5	16,5	0,15
51		16,1	297,9	2,66R
52		16,5	12,3	0,11
53		16,1	32,9	0,29
54		15,6	162,1	1,45
55		16,8	8,7	0,08

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,39241

B Regression Analysis: TP versus FV

The regression equation is
TP = 44,7 + 1,02 FV

Predictor	Coef	SE Coef	T	P
Constant	44,70	12,49	3,58	0,001
FV	1,01686	0,04911	20,71	0,000

S = 82,8091 R-Sq = 89,0% R-Sq(adj) = 88,8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	2940345	2940345	428,79	0,000
Residual Error	53	363439	6857		
Total	54	3303784			

Obs	FV	TP	Fit	SE Fit	Residual	St Resid
1	Due to reasons of confidentiality, these numbers cannot be published.			12,4	11,2	0,14
2				12,0	79,9	0,97
3				11,2	85,6	1,04
4				73,0	-8,3	-0,21 X
5				16,7	12,7	0,16
6				25,8	20,6	0,26
7				11,4	53,6	0,65
8				11,9	-33,4	-0,41
9				11,2	-38,2	-0,47
10				11,2	-39,7	-0,48
11				11,2	216,8	2,64R
12				11,6	-30,9	-0,38
13				11,8	11,7	0,14
14				11,2	178,9	2,18R
15				12,4	-45,6	-0,56
16				12,3	-45,5	-0,56
17				11,9	-46,7	-0,57
18				12,2	-48,5	-0,59
19				11,9	-39,4	-0,48
20				12,3	-38,9	-0,48
21				12,3	-44,8	-0,55
22				12,2	-34,3	-0,42
23				12,1	-41,0	-0,50
24				11,3	-61,8	-0,75
25				12,0	5,3	0,06
26				11,9	-36,1	-0,44
27				12,0	-48,1	-0,59
28				11,8	10,9	0,13
29				12,1	-48,3	-0,59
30				11,5	-51,2	-0,62
31				11,7	5,9	0,07

32	Due to reasons of confidentiality, these numbers cannot be published.	11,3	-59,8	-0,73
33		12,0	-38,2	-0,47
34		11,7	10,0	0,12
35		16,4	-130,0	-1,60
36		11,8	-42,9	-0,52
37		11,8	-36,0	-0,44
38		11,9	-24,2	-0,30
39		13,3	-43,8	-0,54
40		12,4	-45,8	-0,56
41		12,1	6,2	0,08
42		11,2	39,5	0,48
43		12,1	-29,2	-0,36
44		11,7	292,6	3,57R
45		11,2	-34,4	-0,42
46		13,7	-82,0	-1,00
47		11,2	-50,7	-0,62
48		11,7	-43,2	-0,53
49		11,3	287,2	3,50R
50		11,9	60,5	0,74
51		13,5	66,6	0,81
52		11,3	-43,8	-0,53
53		11,3	-10,7	-0,13
54		11,3	87,1	1,06
55		11,4	-47,2	-0,57

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,94285

C Regression Analysis: TP versus FV; V

The regression equation is
TP = 27,7 + 1,03 FV + 22,8 V

Predictor	Coef	SE Coef	T	P
Constant	27,70	22,52	1,23	0,224
FV	1,03158	0,05180	19,92	0,000
V	22,77	25,10	0,91	0,368

S = 82,9475 R-Sq = 89,2% R-Sq(adj) = 88,8%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	2946009	1473005	214,09	0,000
Residual Error	52	357775	6880		
Total	54	3303784			

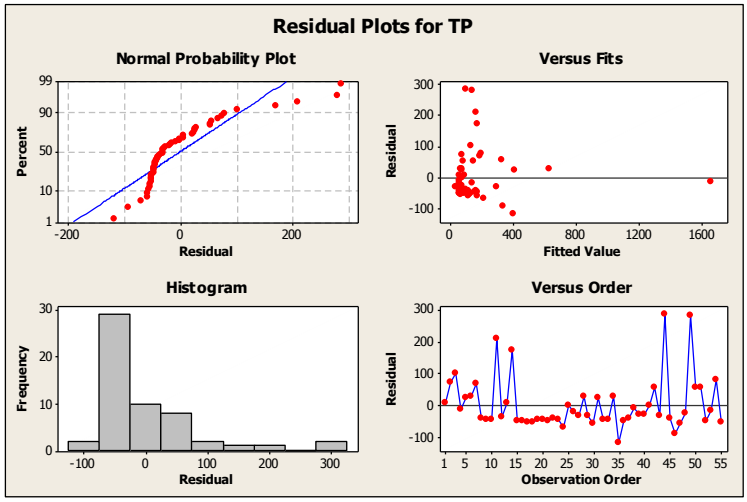
Source	DF	Seq SS
FV	1	2940345
V	1	5664

Obs	FV	TP	Fit	SE Fit	Residual	St Resid
1	Due to reasons of confidentiality, these numbers cannot be published.		14,0	5,4	0,07	
2			13,8	73,7	0,90	
3			20,5	101,2	1,26	
4			73,5	-14,6	-0,38	X
5			21,1	24,3	0,30	
6			27,4	29,0	0,37	
7			19,8	68,3	0,85	
8			13,8	-39,6	-0,48	
9			13,9	-45,6	-0,56	
10			13,8	-47,0	-0,57	
11			13,8	209,4	2,56R	
12			13,7	-37,4	-0,46	

13	Due to reasons of confidentiality, these numbers cannot be published.	13,7	5,4	0,07
14		13,9	171,4	2,10R
15		14,0	-51,5	-0,63
16		13,9	-51,4	-0,63
17		13,8	-52,9	-0,65
18		13,9	-54,5	-0,67
19		13,8	-45,6	-0,56
20		13,9	-44,8	-0,55
21		13,9	-50,7	-0,62
22		13,9	-40,3	-0,49
23		13,8	-47,1	-0,58
24		14,4	-69,8	-0,85
25		13,8	-0,8	-0,01
26		21,8	-19,5	-0,24
27		21,9	-31,5	-0,39
28		21,6	27,3	0,34
29		22,0	-31,6	-0,39
30		13,6	-57,8	-0,71
31		21,5	22,3	0,28
32		20,7	-44,0	-0,55
33		13,8	-44,3	-0,54
34		21,5	26,4	0,33
35		20,9	-118,3	-1,47
36		13,7	-49,2	-0,60
37		13,7	-42,4	-0,52
38		21,8	-7,7	-0,10
39		19,7	-30,6	-0,38
40		22,4	-28,8	-0,36
41		13,8	0,1	0,00
42		20,2	54,8	0,68
43		13,8	-35,3	-0,43
44		13,7	286,2	3,50R
45		13,8	-41,7	-0,51
46		17,5	-91,9	-1,13
47		13,9	-58,2	-0,71
48		21,5	-26,8	-0,34
49		13,7	280,2	3,42R
50		13,7	54,3	0,66
51		17,3	56,8	0,70
52		13,7	-50,8	-0,62
53		13,7	-17,6	-0,22
54		14,2	79,2	0,97
55		13,6	-53,9	-0,66

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,98130



D Regression Analysis: TP versus FV; V; (FV*V)

The regression equation is
TP = 29,9 + 1,02 FV + 10,4 V + 0,168 (FV*V)

Predictor	Coef	SE Coef	T	P
Constant	29,90	22,79	1,31	0,195
FV	1,02141	0,05364	19,04	0,000
V	10,44	29,84	0,35	0,728
FV*V	0,1683	0,2183	0,77	0,444

S = 83,2727 R-Sq = 89,3% R-Sq(adj) = 88,7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	2950133	983378	141,81	0,000
Residual Error	51	353651	6934		
Total	54	3303784			

Source	DF	Seq SS
FV	1	2940345
V	1	5664
FV*V	1	4124

Obs	FV	TP	Fit	SE Fit	Residual	St Resid
1				18,7	14,9	0,18
2				16,0	79,9	0,98
3				20,6	100,0	1,24
4				76,0	-0,7	-0,02 X
5				21,3	25,8	0,32
6				27,9	32,7	0,42
7				19,9	67,7	0,84
8				15,6	-34,0	-0,42
9				17,1	-53,3	-0,65
10				15,7	-52,7	-0,64
11				16,8	202,2	2,48R
12				14,1	-34,9	-0,43
13				14,9	9,8	0,12
14				17,4	163,4	2,01R
15				18,5	-42,2	-0,52
16				18,0	-42,7	-0,53
17				15,6	-47,4	-0,58
18				17,4	-46,5	-0,57
19				15,8	-39,8	-0,49
20				18,2	-35,8	-0,44
21				17,9	-42,1	-0,52
22				17,6	-32,0	-0,39
23				16,7	-40,0	-0,49
24				23,3	-84,0	-1,05
25				16,3	5,8	0,07
26				22,1	-21,4	-0,27
27				22,1	-33,5	-0,42
28				21,8	25,5	0,32
29				22,2	-33,5	-0,42
30				13,8	-56,7	-0,69
31				21,7	20,5	0,26
32				20,9	-45,4	-0,56
33				16,1	-38,1	-0,47
34				21,7	24,6	0,31
35				21,1	-116,9	-1,45
36				15,0	-44,5	-0,54
37				14,8	-38,1	-0,46
38				22,0	-9,6	-0,12
39				19,8	-30,2	-0,37
40				22,7	-31,0	-0,39
41				16,8	7,4	0,09

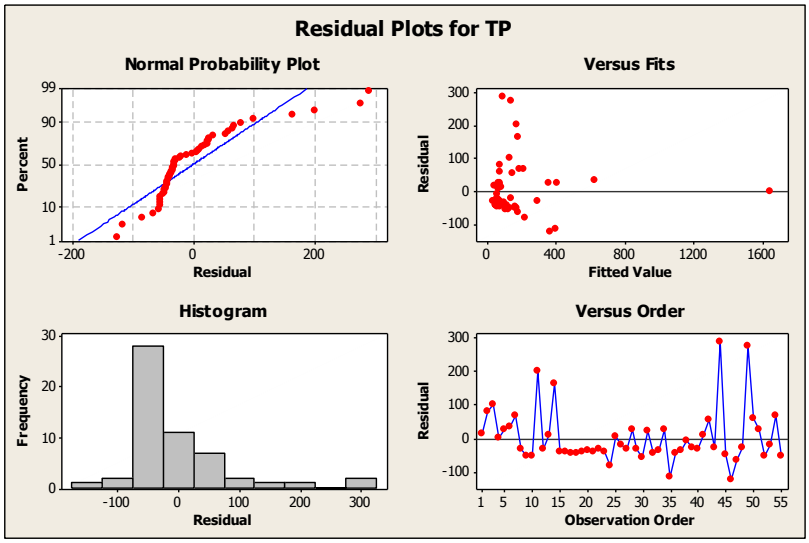
Due to reasons of confidentiality, these numbers cannot be published.

42		20,3	53,8	0,67
43		16,7	-28,1	-0,34
44		14,5	289,8	3,53R
45		16,3	-48,4	-0,59
46		46,7	-125,2	-1,82 X
47		17,5	-66,3	-0,81
48		21,7	-28,6	-0,36
49		14,3	277,0	3,38R
50		15,4	59,6	0,73
51		45,4	24,5	0,35 X
52		14,1	-53,2	-0,65
53		14,2	-20,4	-0,25
54		21,6	66,7	0,83
55		13,7	-54,6	-0,66

Due to reasons of confidentiality, these numbers cannot be published.

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,93363



E Regression Analysis: TP versus FV; (FV*V)

The regression equation is
TP = 36,0 + 1,01 FV + 0,209 (FV*V)

Predictor	Coef	SE Coef	T	P
Constant	35,99	14,59	2,47	0,017
FV	1,01413	0,04902	20,69	0,000
FV*V	0,2092	0,1827	1,15	0,257

S = 82,5670 R-Sq = 89,3% R-Sq(adj) = 88,9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	2949284	1474642	216,31	0,000
Residual Error	52	354500	6817		
Total	54	3303784			

Source	DF	Seq SS
FV	1	2940345
FV*V	1	8939

Obs	FV	TP	Fit	SE Fit	Residual	St Resid
1				14,2	19,1	0,24
2				12,3	83,4	1,02
3				13,6	94,6	1,16
4				73,7	4,7	0,13 X
5				18,7	22,4	0,28
6				27,2	30,9	0,40
7				13,9	62,8	0,77
8				12,1	-30,7	-0,38
9				16,9	-52,8	-0,65
10				15,3	-51,7	-0,64
11				16,5	202,8	2,51R
12				11,6	-32,2	-0,39
13				11,8	13,0	0,16
14				17,1	163,9	2,03R
15				14,0	-38,0	-0,47
16				13,6	-38,7	-0,48
17				12,1	-44,0	-0,54
18				13,2	-42,6	-0,52
19				12,2	-36,3	-0,44
20				13,8	-31,7	-0,39
21				13,6	-38,0	-0,47
22				13,4	-28,0	-0,34
23				12,8	-36,3	-0,44
24	Due to reasons of			23,0	-84,8	-1,07
25	confidentiality,			12,5	9,4	0,11
26	these numbers			14,2	-27,3	-0,34
27	cannot be			14,2	-39,4	-0,48
28	published.			14,0	19,7	0,24
29				14,3	-39,5	-0,49
30				11,8	-54,3	-0,66
31				14,0	14,7	0,18
32				13,7	-50,9	-0,62
33				12,4	-34,5	-0,42
34				14,0	18,8	0,23
35				18,4	-120,3	-1,50
36				11,9	-41,4	-0,51
37				11,8	-35,0	-0,43
38				14,1	-15,4	-0,19
39				15,6	-34,4	-0,42
40				14,5	-37,0	-0,46
41				12,8	11,1	0,14
42				13,6	48,5	0,60
43				12,8	-24,4	-0,30
44				11,7	292,7	3,58R
45				16,0	-47,6	-0,59
46				44,1	-130,1	-1,86 X
47				17,3	-65,8	-0,82
48				14,0	-34,4	-0,42
49				13,5	278,6	3,42R
50				12,0	62,9	0,77
51				43,0	19,8	0,28 X
52				13,1	-51,5	-0,63
53				13,3	-18,8	-0,23
54				21,4	66,2	0,83
55				12,3	-52,5	-0,64

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,91396

F Regression Analysis: TP versus BV; GW; V

The regression equation is
TP = 27,3 + 1,07 BV + 1,00 GW + 23,1 V

Predictor	Coef	SE Coef	T	P
Constant	27,32	22,80	1,20	0,236
BV	1,0729	0,2014	5,33	0,000
GW	1,0019	0,1491	6,72	0,000
V	23,15	25,39	0,91	0,366

S = 83,7196 R-Sq = 89,2% R-Sq(adj) = 88,5%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	2946326	982109	140,12	0,000
Residual Error	51	357458	7009		
Total	54	3303784			

Source	DF	Seq SS
BV	1	2627170
GW	1	313333
V	1	5823

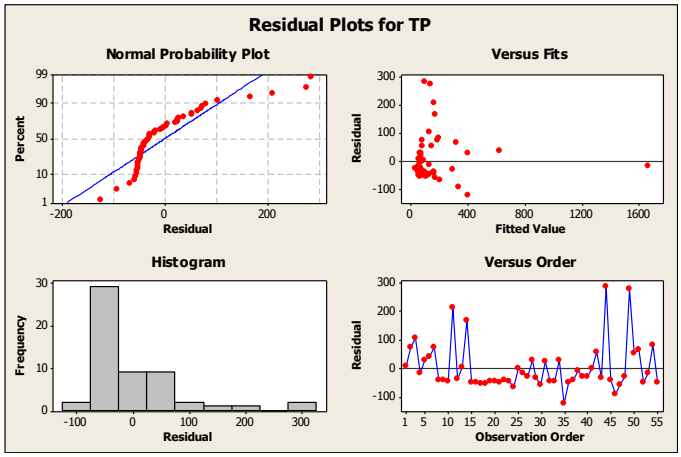
Obs	BV	TP	Fit	SE Fit	Residual	St Resid
1				14,1	5,4	0,07
2				13,9	73,8	0,89
3				24,3	103,9	1,30
4				77,4	-19,3	-0,60 X
5				28,8	28,4	0,36
6				50,5	38,0	0,57 X
7				25,8	71,8	0,90
8				15,0	-40,8	-0,50
9				15,1	-44,4	-0,54
10				16,1	-45,2	-0,55
11				14,1	209,8	2,54R
12				13,8	-37,5	-0,45
13				15,5	3,9	0,05
14				26,3	166,7	2,10R
15				14,1	-51,4	-0,62
16				14,1	-51,5	-0,62
17				14,1	-53,5	-0,65
18				14,0	-54,6	-0,66
19				13,9	-45,8	-0,56
20				14,1	-45,1	-0,55
21				14,1	-50,8	-0,62
22				14,0	-40,4	-0,49
23				14,1	-47,6	-0,58
24				17,1	-67,9	-0,83
25				18,3	-3,3	-0,04
26				22,1	-19,7	-0,24
27				22,2	-32,0	-0,40
28				22,5	26,2	0,32
29				23,6	-33,3	-0,41
30				14,6	-56,8	-0,69
31				22,6	20,9	0,26
32				23,8	-46,5	-0,58
33				16,3	-46,1	-0,56
34				21,9	25,9	0,32
35				34,6	-124,2	-1,63
36				14,1	-49,7	-0,60
37				14,0	-41,9	-0,51
38				22,4	-8,6	-0,11
39				21,1	-29,1	-0,36
40				22,7	-28,6	-0,35
41				14,4	-0,6	-0,01

Due to reasons of confidentiality, these numbers cannot be published.

42	Due to reasons of confidentiality, these numbers cannot be published.	20,5	54,3	0,67
43		14,1	-34,9	-0,42
44		16,0	284,5	3,46R
45		14,2	-41,1	-0,50
46		17,7	-92,2	-1,13
47		14,2	-58,7	-0,71
48		22,7	-28,2	-0,35
49		21,3	276,7	3,42R
50		15,1	53,0	0,64
51		36,9	63,8	0,85
52		16,3	-48,9	-0,60
53		15,4	-16,2	-0,20
54		17,8	81,4	1,00
55		16,4	-52,0	-0,63

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,97434



G Regression Analysis: TP versus BV; GW; V; (BV*V); (GW*V)

The regression equation is
TP = 31,7 + 0,662 BV + 1,29 GW - 11,2 V + 1,81 (BV*V) - 0,493 (GW*V)

Predictor	Coef	SE Coef	T	P
Constant	31,67	20,86	1,52	0,135
BV	0,6620	0,2294	2,89	0,006
GW	1,2922	0,1758	7,35	0,000
V	-11,17	28,05	-0,40	0,692
BV*V	1,8055	0,5122	3,52	0,001
GW*V	-0,4932	0,2906	-1,70	0,096

S = 76,1145 R-Sq = 91,4% R-Sq(adj) = 90,5%

Analysis of Variance

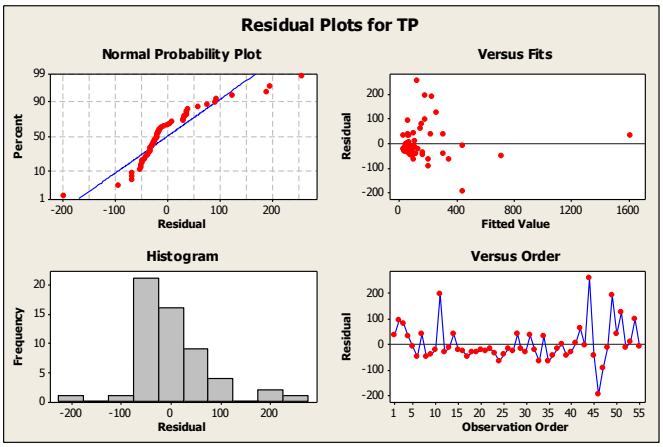
Source	DF	SS	MS	F	P
Regression	5	3019907	603981	104,25	0,000
Residual Error	49	283877	5793		
Total	54	3303784			

Source	DF	Seq SS
BV	1	2627170
GW	1	313333
V	1	5823
BV*V	1	56889
GW*V	1	16691

Obs	BV	TP	Fit	SE Fit	Residual	St Resid
1				18,1	33,0	0,45
2				15,2	92,8	1,24
3				23,8	76,7	1,06
4				72,1	30,6	1,26 X
5				30,6	-12,0	-0,17
6				57,4	-49,7	-1,00 X
7				26,2	37,4	0,52
8				15,3	-50,9	-0,68
9				16,3	-39,5	-0,53
10				17,2	-23,5	-0,32
11				15,5	195,8	2,63R
12				12,9	-31,2	-0,42
13				16,1	-16,3	-0,22
14				44,0	37,0	0,60 X
15				18,1	-22,4	-0,30
16				17,2	-26,9	-0,36
17				14,3	-49,3	-0,66
18				16,5	-32,5	-0,44
19				14,5	-34,3	-0,46
20				17,0	-25,1	-0,34
21				17,2	-26,3	-0,35
22				16,7	-18,9	-0,25
23				15,3	-37,5	-0,50
24	Due to reasons of			22,1	-66,2	-0,91
25	confidentiality,			21,3	-41,1	-0,56
26	these numbers			20,3	-18,1	-0,25
27	cannot be			20,5	-28,2	-0,39
28	published.			21,2	37,1	0,51
29				22,8	-17,2	-0,24
30				15,1	-30,8	-0,41
31				21,5	33,8	0,46
32				23,7	-22,8	-0,32
33				17,6	-67,9	-0,92
34				20,2	30,4	0,41
35				37,1	-66,1	-0,99
36				13,8	-47,8	-0,64
37				14,9	-19,2	-0,26
38				20,9	-0,2	-0,00
39				19,9	-43,7	-0,60
40				20,8	-31,5	-0,43
41				15,4	4,0	0,05
42				18,8	58,9	0,80
43				16,9	-5,9	-0,08
44				17,1	256,5	3,46R
45				14,9	-47,6	-0,64
46				48,6	-196,8	-3,36RX
47				18,2	-93,5	-1,27
48				21,5	-15,2	-0,21
49				31,3	189,7	2,73R
50				15,4	40,6	0,54
51				52,6	124,3	2,26RX
52				18,1	-14,0	-0,19
53				15,9	8,2	0,11
54				21,8	95,1	1,30
55				18,9	-11,2	-0,15

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,73459



H Regression Analysis: TP versus BV; GW; (BV*V); (GW*V)

The regression equation is
TP = 25,5 + 0,673 BV + 1,30 GW + 1,72 (BV*V) - 0,520 (GW*V)

Predictor	Coef	SE Coef	T	P
Constant	25,49	13,83	1,84	0,071
BV	0,6729	0,2259	2,98	0,004
GW	1,2969	0,1739	7,46	0,000
BV*V	1,7226	0,4641	3,71	0,001
GW*V	-0,5203	0,2801	-1,86	0,069

S = 75,4713 R-Sq = 91,4% R-Sq(adj) = 90,7%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	4	3018989	754747	132,51	0,000
Residual Error	50	284796	5696		
Total	54	3303784			

Source	DF	Seq SS
BV	1	2627170
GW	1	313333
BV*V	1	58826
GW*V	1	19659

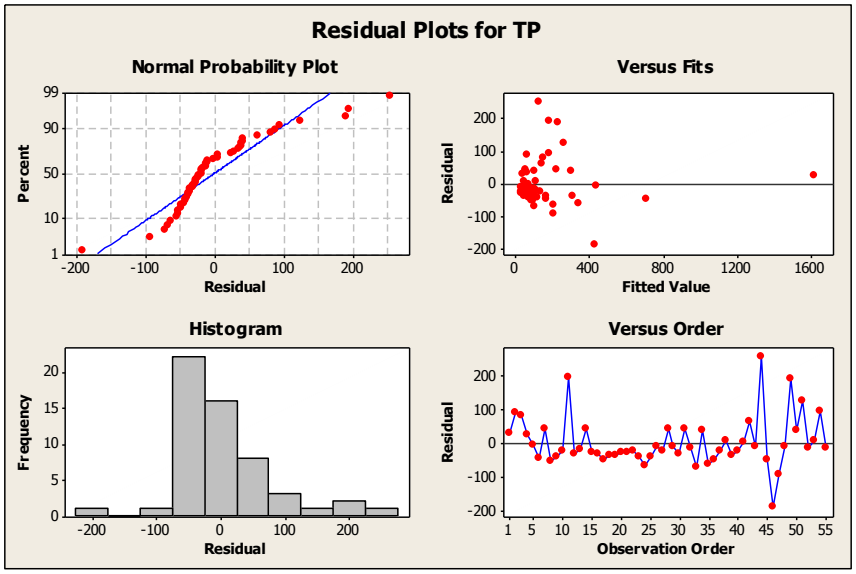
Obs	BV	TP	Fit	SE Fit	Residual	St Resid	
1				13,3	28,2	0,38	
2				11,5	88,9	1,19	
3				18,9	82,4	1,13	
4				70,0	24,8	0,88	X
5				28,8	-8,2	-0,12	
6				56,6	-47,1	-0,94	X
7				22,4	42,6	0,59	
8				13,3	-53,9	-0,72	
9				16,0	-40,5	-0,55	
10				16,4	-25,3	-0,34	
11				15,3	195,3	2,64	R
12				10,7	-34,1	-0,46	
13				14,8	-18,7	-0,25	
14				42,9	40,2	0,65	X
15				13,3	-27,2	-0,37	
16				12,7	-31,4	-0,42	
17				11,4	-52,7	-0,71	
18				12,3	-36,9	-0,50	
19				11,2	-37,9	-0,51	
20				12,6	-29,5	-0,40	
21				12,7	-30,9	-0,41	

Due to reasons of confidentiality, these numbers cannot be published.

22	Due to reasons of confidentiality, these numbers cannot be published.	12,4	-23,3	-0,31
23		11,7	-41,3	-0,55
24		21,9	-65,9	-0,91
25		20,4	-43,4	-0,60
26		13,7	-12,2	-0,16
27		14,0	-22,3	-0,30
28		15,4	42,9	0,58
29		17,2	-11,3	-0,15
30		12,7	-34,0	-0,46
31		15,9	39,5	0,54
32		19,4	-17,5	-0,24
33		16,1	-70,6	-0,96
34		13,9	36,2	0,49
35		36,0	-63,1	-0,95
36		11,2	-50,9	-0,68
37		11,5	-22,9	-0,31
38		14,8	5,6	0,08
39		16,5	-39,3	-0,53
40		13,8	-25,4	-0,34
41		12,2	0,3	0,00
42		13,4	64,2	0,86
43		12,6	-10,4	-0,14
44		16,2	254,5	3,45R
45		14,7	-48,5	-0,65
46		44,8	-189,7	-3,13RX
47		18,1	-93,2	-1,27
48		15,9	-9,5	-0,13
49		30,9	190,7	2,77R
50		13,6	37,8	0,51
51		52,0	126,0	2,30RX
52		16,4	-16,9	-0,23
53		14,6	5,8	0,08
54		21,6	94,7	1,31
55		16,7	-14,6	-0,20

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 1,77415



I Regression Analysis: TP versus BV; GW; M; (BV*M); (GW*M)

The regression equation is
TP = - 1,9 + 3,84 BV + 1,09 GW + 20,0 M - 3,10 (BV*M) + 0,174 (GW*M)

Predictor	Coef	SE Coef	T	P
Constant	-1,88	23,79	-0,08	0,938
BV	3,8388	0,5992	6,41	0,000
GW	1,0863	0,2564	4,24	0,000
M	20,03	38,67	0,52	0,608
BV*M	-3,0983	0,6661	-4,65	0,000
GW*M	0,1744	0,3531	0,49	0,625

S = 75,4341 R-Sq = 94,3% R-Sq(adj) = 93,4%

Analysis of Variance

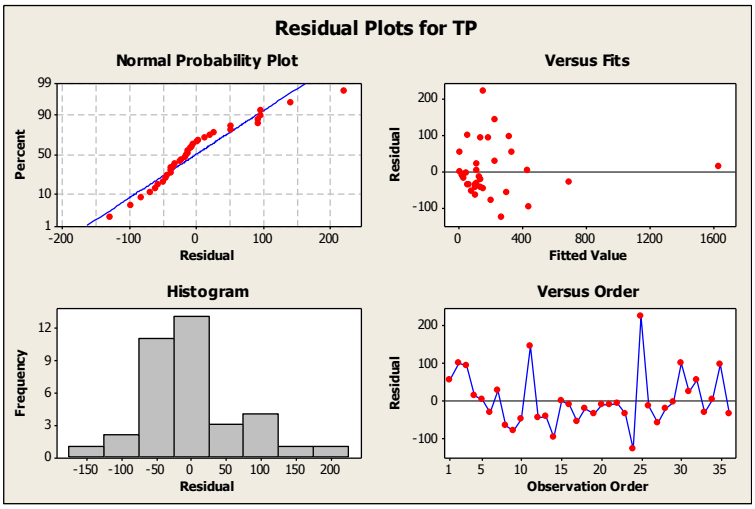
Source	DF	SS	MS	F	P
Regression	5	2845252	569050	100,00	0,000
Residual Error	30	170709	5690		
Total	35	3015961			

Source	DF	Seq SS
BV	1	2547064
GW	1	164473
M	1	8222
BV*M	1	124105
GW*M	1	1388

Obs	BV	TP	Fit	SE Fit	Residual	St Resid
1				22,8	52,0	0,72
2				18,8	97,2	1,33
3				24,5	92,5	1,30
4				74,2	13,4	0,99 X
5				29,7	2,0	0,03
6				65,2	-31,4	-0,83 X
7				30,0	27,3	0,39
8				17,8	-68,0	-0,93
9				20,0	-82,0	-1,13
10				20,1	-49,4	-0,68
11				20,3	141,6	1,95
12				15,8	-45,5	-0,62
13				18,7	-43,3	-0,59
14				55,7	-97,7	-1,92 X
15	Due to reasons of confidentiality, these numbers cannot be published.			22,9	-2,9	-0,04
16				21,6	-12,0	-0,17
17				17,2	-56,7	-0,77
18				20,6	-21,3	-0,29
19				17,7	-36,0	-0,49
20				21,1	-12,7	-0,17
21				21,5	-11,7	-0,16
22				20,8	-7,2	-0,10
23				18,6	-36,8	-0,50
24				27,2	-128,3	-1,82
25				20,0	221,8	3,05R
26				25,4	-14,8	-0,21
27				24,8	-60,5	-0,85
28				28,8	-23,6	-0,34
29				37,0	-5,0	-0,08
30				38,8	97,0	1,50
31				17,9	21,3	0,29
32				58,2	53,2	1,11 X
33				24,5	-33,0	-0,46
34				24,6	3,2	0,05
35				23,6	93,5	1,30
36				24,8	-36,3	-0,51

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large leverage.

Durbin-Watson statistic = 2,06956



Correlations: BV; GW; M; BV*M; GW*M

	BV	GW	M	BV*M
GW	0,859			
	0,000			
M	0,316	0,456		
	0,061	0,005		
BV*M	0,980	0,889	0,407	
	0,000	0,000	0,014	
GW*M	0,890	0,948	0,556	0,927
	0,000	0,000	0,000	0,000

Cell Contents: Pearson correlation
P-Value