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Multi-purchasing in the linear city

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Abstract

This paper allows for the purchase of both goods in the Hotelling model with linear transport costs. Price competition disappears as a result, and we have a stable linear model with the Principle of Minimum Differentiation intact. Stability is dependent on equal marginal costs for the two producers.

The new model lends itself well to bundling. Treating the multi-purchase Hotelling framework under a monopoly, we find that mixed bundling leads to an intermediate level of differentiation with at least half the line between the two goods. Variety is thus greater under a monopoly than under the ordinary duopoly.

Media markets, popularly modelled using two-sided markets, are an example of when purchases of multiple brands are common. This paper fills a gap in the literature on multi-homing in two-sided markets.

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Håkon Sæberg

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1 Introduction

"there are two basic types of competitive advantage a firm can possess: low cost or differentiation"

Michael Porter (1985, p. 11)

Firms face a fundamental pull between wanting to reach more customers and avoiding the price competition that may follow from that tendency. How much to differentiate is accordingly a central strategic question. This is doubly so in markets such as newspapers where customers often buy from several suppliers at the same time. If this behaviour is due to a preference for variety, would that induce firms to move farther away from each other because they believe they will sell to most broadminded customers anyway or will it make them go completely generic in order to become everyone's second choice?

We employ Hotelling's (1929) linear city model, a classic in the duel between differentiation and price competition, and adapt it to accept purchases of two goods with as few changes to the core setup as possible. This yields results that can easily be compared to the existing literature and allows for quick adoption into new models. Our main result is simple: if the willingness to pay for the second good is above a certain level, it will be sold, and firms will concentrate in the middle of the market.

Although linear Hotelling models are notorious for their stability issues (d'Aspremont, Gabszewicz, & Thisse, 1979), we argue that this model stands steady as long as multi-purchasing occurs and the competing firms have equal marginal costs. All other papers in our review of the literature have assumed equal or zero costs, so this condition does not hamper traditional models. Rather, it points out the crucial role this overlooked parameter plays.

After the model has been constructed and tested for its effects on competition we apply it to a monopoly. In order to see how profit may be extracted from the consumers' heterogeneity we make the monopolist keep both the horizontally differentiated goods and open for the possibility of price discrimination through bundling. Differentiation will then be intermediate, with at least half the line between the two goods. Compared with the competitive model, variety is greater under a monopoly.

1.1 Media models need multi-purchasing

Information goods such as newspapers or music are examples of where purchasing multiple brands is common. No one has any use for a second CD of the same album, but many a teenager's room is filled with the sounds of near identical boy bands or rock groups. Similarly, there's little use in reading the same article twice, but two papers covering the same story from different angles may still have value enough for a reader to buy both. This model will map the conditions for when multiple papers will be bought and the consequences for the publishers' positioning.

Media models are also a popular topic in the literature of two-sided markets (Armstrong, 2006; Rochet & Tirole, 2006). This field considers the actions of platforms (the publishers) when they receive income from two sides (readers and advertisers) and one side is dependent on the other (advertisers pay more to reach more readers). Hotelling's (1929) linear city is a widespread way to give the retail side in these models homogenous preferences. Yet no known paper considers how platforms and advertisers would react if readers started buying more than one paper.

Only one study, by Kim and Serfes (2006), has looked at multi-purchasing in a Hotelling setting, and although that paper is ground-breaking, it is a mathematically dense exercise due to its quadratic transportation costs. In order to make a model that is easy to implement in a two-sided structure, we use linear transport and aim for minimal changes to the ordinary Hotelling setup. At the expense of certain known stability issues – which will not be a problem as long as multi-purchasing occurs – the result is a simple puzzle piece for use in future media models.

The paper is structured as follows. Chapter 2 gives an overview of the literature on Hotelling models and on bundling. Chapter 3 establishes the competitive multi-purchase model and debates its merits. Chapter 4 tests the same model under a monopoly, which with multiple goods leads to bundling. Chapter 5 concludes.

2 Literature review

We will concentrate on goods where the consumers differ in their opinion on which version is best. Such goods are said to be horizontally differentiated. Vertical differentiation, where all consumers agree on the ranking of the goods, will not be discussed.[1] Diversity among consumers' preferences is also a common starting point in the theory of bundling, which we will cover in the second part of this chapter.

Part one of this chapter covers the legacy of Hotelling's (1929) paper "Stability in competition", which inspired a branch of horizontal differentiation models called address models or models of spatial competition. These are systems where firms compete by choosing locations in some n-dimensional plane, n representing the competitive attributes in the product space, in addition to the usual parameters of price or quantity. Consumers traditionally buy only one option based on what gives the highest utility. Other forms of horizontal differentiation, such as monopolistic competition[2], will not be covered.

Spatial competition was most actively developed around the 1980s. Some might argue that it is a field of research where the cost of finding additional insights is no longer worth the time it takes. However, address models are still popular as building blocks in other fields. In particular, the topic of two-sided markets has several papers using some form of the Hotelling model to form consumer demand (Armstrong, 2006; Choi, 2010; Hagiu, 2009; Peitz & Valletti, 2008). A common example of a two-sided market is that of newspapers; a paper earns income from both readers and advertisers, and the latter is dependent on the former. Yet news is also an instance where one should expect customers to buy more than one brand. In order to develop a good model of a two-sided market for newspapers, our understanding of multi-purchasing under Hotelling therefore needs to be improved.

Part two of this chapter covers the topic of bundling. Monopoly behaviour with multi-purchasing, which we cover in chapter 4, will necessarily involve attempts to exploit the spread in consumers' preferences for the two goods by package selling. A brief introduction to bundling is therefore useful. Tying and bundling will in this paper be taken to mean the same thing.

[1] These are common definitions, but see Gabszewicz and Thisse (1992) for one that also interprets vertical differentiation in a spatial setting.

[2] Monopolistic competition is distinguished by an assumption that a change in price by one firm does not alter the demand faced by other firms by much (Kreps, 1990; Tirole, 1988). This fits poorly with what we want to discuss.

2.1 Hotelling's linear city

Not everyone buys the cheapest gas; some people just stop at whatever pump is nearest. Even with the simplest and seemingly most homogeneous goods, price is not the only competitive factor.[3] Hotelling (1929) used this recognition to make the smallest possible model of differentiation: two goods equal in every way except for one aspect, their placement along a straight line. The line may represent a town's main road, the age profile of a radio station's listeners or the political spectrum from left to right. In terms of a main road, which is why the model is often called the linear city model, we can think of placement as the distance consumers have to travel to buy gas. Regardless of interpretation, it is the only feature other than price to separate the products in the eyes of the consumers, whose preferences are evenly spread along the line. Despite his wish to explain competition in terms of more than just price, Hotelling concludes that the firms will tend to imitate one another. This has come to be known as the Principle of Minimum Differentiation. However, if the firms' locations are the same, then price will be left as the only difference between the products, and the cheapest gas should win.

Transport costs are central to the concept of the linear city. These are the costs from buying a good located at another point than where a given consumer is. As a consumer has to "travel" farther along the line, his utility falls somewhat. Transport costs were literal in our example with a main street, but may just as well be a general expression of dislike where one simply holds one's nose and buys something other than the ideal. Depending on the interpretation, different functions may be natural. Starting with the simplest case, Hotelling made these costs a linear function of the distance between each consumer and the good.

Fifty years after Hotelling's classic, albeit paradoxical paper, d'Aspremont, Gabszewicz and Thisse (1979) proved him wrong. They show that the incentive to undercut the opponent's price means that there will not be a tendency to gather at the midpoint, as Hotelling claimed, but a troubling instability instead.[4] Hotelling's conclusion was based on observing that the firms' profit functions were increasing as they approached the middle, but d'Aspremont et al. show that the only candidate for a price equilibrium has to be some distance away from the middle. A placement in the centre by both firms would inevitably lead to price competition with the zero-profit Bertrand outcome. These two observations conflict and therefore create instability. Furthermore, d'Aspremont et al. demonstrate that with quadratic transportation

[3] The history of competition analysis starts with quantity, not price. Martin (2002) covers this well.
[4] Which is ironic given the title of Hotelling's paper

costs instead of the original linear model, the varying degree of price competition along the line will give a strategic incentive to differentiate as much as possible. This again contradicts Hotelling's model but also restores meaning to differentiation. Based on these findings, every time one of Hotelling's assumptions is changed to form a new model, two questions must therefore be answered:

1) Does an equilibrium exist?
2) If so, will we find minimal or maximal differentiation – or something in between?

2.1.1 The purchase of both goods

Kim and Serfes (2006) were the first to properly contest the assumption that the consumers only bought one of the goods. Using quadratic costs they find that under certain conditions the Principle of Minimum Differentiation is restored. They motivate their model by saying that keeping the single purchase assumption would imply that "consumers do not care for diversity" (2006, p. 569) and point out several cases where this is not true. Among the examples they use are newspapers, where it is not uncommon for people to read both a tabloid and a broadsheet, and two academic journals, where differentiation between a theoretical concentration and an empirical periodical does not imply that no one reads both. Indeed, this argument seems especially suited to information goods where a second unit of the same good has no value but a competing product may still be of interest.[5] Minimal differentiation is consequently due to an incentive to increase the group of consumers who buy both products. Kim and Serfes name this the *aggregate demand creation* effect.

The challenge in Kim and Serfes (2006) is that their results rely on complicated conditions that make the model hard to interpret or to apply in another theoretical structure. As we will see, linear costs solve this problem. The trade-off is that in the cases where no consumer is better off by purchasing both goods we are back in the unstable world of Hotelling's original model.

[5] Kim and Serfes also use credit cards as an example, but this is harder to justify. What, after all, would be the competitive attribute that consumers disagreed about? Card colour? Surely, the value of a card is determined more by its credit limit and by the number of stores that accept them. Both of these attributes are such that all customers can be expected to have a reasonably identical preference of more being better. Hence, a two-sided interpretation as in Rochet and Tirole (2002) or at least a model that includes a vertical quality component such as Anderson, Foros and Kind (2012) should be more suitable.

Our model is also closely related to Choi (2010). His setup starts with Hotelling, has linear transport costs and allows the purchase of both goods but does not stop to consider the locational incentives of the structure he has created. Instead, he assumes maximal differentiation where we show that firms will tend towards none. Choi moves on to set up a two-sided market and to test tying strategies. These are interesting applications, but when the foundation falters, the setup should be revisited. Still, there is no difference between the first building blocks in Choi's model and ours. Indeed, the choice of the letter λ for a key parameter in this paper was directly inspired by Choi. In all, this paper answers the questions from Kim and Serfes using Choi's model.

Another related work is Anderson, Foros and Kind (2012). Like us, they use a linear Hotelling model and allow the purchase of both goods. Unlike us, they include quality as an element of vertical differentiation that governs the incremental utility of the second purchase. This allows for a more sophisticated model, but also complicates interpretation. Just as we will do, they find that prices are strategically independent when both goods are bought. However, they study changes in profit in terms of quality instead of the traditional discussion of location. This paper adds that piece of the puzzle.

Every paper on the possibility of buying several goods has its own name for the concept. Choi's (2010) paper calls it multi-homing. Associated with the literature on two-sided markets (Armstrong, 2006), applying that label here would risk confusing the reader. Anderson et al. (2012) use the term multi-purchasing to mean the same. Kim and Serfes (2006) also talk of "multiple purchases". We will take the terms as synonymous but choose to use multi-purchasing.

Variety is also a term related to buying several brands. Sajeesh and Raju (2010, p. 949) define variety seeking as "a relative reduction in the willingness to pay of the previously purchased brand" and use a Hotelling model with three stages: one for locations and two market stages with separate prices. They find that firms differentiate less when there are consumers who seek variety, much like we will discover. All consumers have to buy some good twice in their model, however, and the question becomes not who participates in multi-purchase but who changes his mind and buys a different good in the last stage. Sajeesh and Raju explain variety seeking as satiation with the consumers' first purchase and cite an empirical study which finds that the purchase of multiple brands is more likely in markets with small perceived differences between the brands. While the three-stage model dictates a behaviour that does not fit our intentions (not all readers buy two newspapers), the motivation behind Sajeesh and Raju's variety seeking is very compatible with ours. Having a preference for variety will in this paper therefore refer to a positive utility from buying another brand after one's most preferred option.

Finally, Anderson and Neven (1989) construct a Hotelling model that only allows the purchase of one unit and permits fractions of both in any combination that sums to one. Their transport costs are quadratic. They too see a midsection of the line where both goods are bought. All consumers who lie between the locations of the firms in Anderson and Neven's model will buy some portion of each good. In contrast, our multi-purchase section can span both longer and shorter than the distance between the firms' locations; even with placements at each end of the line we will have consumers who buy only one good. Since mixing means obtaining a consumer's first best configuration, Anderson and Neven find that the social as well as the competitive optimum is maximum differentiation.

Buying fractions of a good is possible for what Anderson and Neven's (1989) title calls "combinable products" such as – citing their own examples – blends of coffee beans to adjust the darkness of the brew or alcohol of different strengths for a medium strength punch. In the latter case, Anderson and Neven's model would have one firm sell pure alcohol and the other plain water and let each consumer buy what he needs to produce his own cocktail. The farther apart the firms move, the greater share of the customers will buy some of both goods – the opposite of the aggregate demand creation effect and of intuition. Information goods such as news cannot be consumed partially or mixed to an optimal blend. Instead of weighing averages, we must sum the full purchases.

2.1.2 Overview of other extensions

A full survey of the ways in which Hotelling's assumptions have been tested would distract from the purpose of this paper. At the same time some of these extensions are related to our topic of multiple purchases and consumers' preference for variety. What follows is therefore a brief look into these fields. Broader overviews can be found in Gabszewicz and Thisse (1992), Brenner (2001) or Martin (2002).

Several papers have allowed for the purchase of many units of the same good in a Hotelling setting. Among these, Anderson and Neven (1991) have the firms compete in quantities instead of price and find that with linear transport costs there is a unique equilibrium in which both firms locate in the middle given a sufficiently high reservation price compared to the transport coefficient.[6] Rath and Zhao (2001) have the firms compete in prices as usual and find a unique equilibrium with quadratic transport costs. The firms locate closer to the centre as the reservation price decreases relative to the transport coefficient and will be placed at the ends of the line if the ratio of reservation price to transport is too high. While we turn the model around by allowing the purchase of both goods but not more than one unit of each, we may still expect the relationship between the willingness to pay and the transport cost to be important.

Not all models of horizontal differentiation involve flat lines. Salop (1979) takes Hotelling's main road and joins the ends together to make a circle. Consequently, he avoids having any "corners" in the form of maximal differentiation and can instead estimate the number of equally-spaced firms the market will sustain. This measures a capacity for variety. Yet such a model is only interesting if differentiation is really the best approach. Indeed, Salop proves that the equilibrium variety in the circular city will be greater than the social optimum.

Instead of making a circle, the line can go on forever if the distribution of consumers changes from uniform to some continuous function. Of all changes that can be made to the model, this is the one most closely related to a potential preference for variety. Neven (1986) finds that sufficiently concentrated (heavy in the centre) concave distributions will make the firms approach each other even with quadratic transportation costs. Anderson, Goeree and Ramer (1997) expand to logconcave distributions with quadratic transportation costs and also find a unique solution where more concentrated distributions give closer locations. The result requires the distribution not to be too asymmetrical or "too concave", but many common distributions including the normal, gamma and beta are covered. As we know from d'Aspremont et al. (1979), quadratic transportation and a uniform distribution of consumers' preferences produce maximal differentiation. From a social welfare point of view, that means the firms are too far away from each other. Echoing Salop (1979), Anderson et al. show that all the distributions they study will indeed have "excess differentiation", although none more than the uniform. Even when we lump the customers together towards the middle, quadratic transportation costs will pull the firms too much apart.

[6] This result holds for *n* firms. Note that Cournot competition means that the firms cover the transport costs and are therefore able to discriminate among consumers. Moreover, without the transport costs to consider, the consumers see the goods as equal. The details of this model are thus far from what we want to configure.

Outside of the world of spatial competition, Dixit and Stiglitz (1977) use the very curvature of common utility functions (convexity of the indifference curves to be precise) to study a preference for variety. This is elegant, but risks missing the insight of game theory that first makes people discuss where gas stations should be placed.[7] Dixit and Stiglitz is one of the classic papers within monopolistic competition and is as such outside the scope of this study. Nevertheless, an insightful bridge between spatial and monopolistic competition can be found in Anderson, de Palma and Thisse (1989; 1992). Advanced studies of the general Hotelling model should start there.

2.2 Bundling

A monopolist will naturally attempt to coordinate the sales of his two goods. Bundling, which is exactly the practice of selling several goods at once to heterogeneous customers, will as such be a part of any monopoly version of the multi-purchase model. Most of the mechanics of the model are still determined by its spatial properties and not by the literature on bundling, however. Below follows only a basic overview of the concepts needed.

Bundling will in this paper be defined in accordance with Adams and Yellen (1976, p. 475) as “the practice of package selling”. A firm can either adopt a pure bundling strategy by offering the goods only as one package or a mixed bundling strategy by also selling the goods separately. Adams and Yellen show that the profitability of bundling is due to its ability to divide consumers by their willingness to pay. If consumers are spread in their valuation of either of two goods, but agree in sum, a bundle could extract more consumer surplus than single sales by simply setting price equal to that sum. If one consumer has a high valuation of one good so that he would buy it individually if he could but a very low valuation of another compared to some “midrange” consumers, pricing the bundle to include that extreme customer might not be worth the loss of income from the midrange group – hence opening for mixed bundling. In Adams and Yellens framework, pure bundling, mixed bundling and single sales can all be the most profitable choice depending on how the goods are related.

Our model breaks one of the assumptions in Adams and Yellen’s (1976) paper because the reservation price of our bundle by definition does not equal the sum of each good’s reservation price. Dubbed an assumption of independence, it has, in the aftermath of Adams

[7] For a thorough critique of the curvature-of-utilities approach, see Lancaster (1971).

and Yellen, been proven to be unnecessary for their claim that any of the bundling strategies may be best (Lewbel, 1985). An address model like ours takes this one step further by allowing the firms to choose their own degree of substitutability.

This paper’s inspiration from Choi (2010) shines through in the use of bundling as well. Both models test the implications of the package selling of goods in a linear Hotelling system. Yet Choi’s bundle does not contain the same goods as ours. Rather, he ties the sale in the (still competitive) Hotelling market to a good in another, monopolized market and finds that tying makes more consumers buy from both suppliers. Tying to a monopolized market belongs to the leverage theory of bundling (Tirole, 1988, pp. 333-335; Schmalensee, 1982). We will stay within the linear city even in our bundling; more than being just a case for exercising monopoly power, bundling is about exploiting consumer diversity (Schmalensee, 1984; Bakos & Brynjolfsson, 1999).

3 Minimal model of multi-purchasing under Hotelling

Our starting point is Hotelling's (1929) traditional linear city model. A group of consumers whose size is normalized to 1 is uniformly distributed along a line of length 1.[8] Two goods exist. They are equal in every feature except in the *x* dimension, where they are placed somewhere along the line corresponding to their choice of differentiation. The goods are produced by separate firms that compete in prices. Each consumer has a reservation price R for a good located exactly where he is placed and incurs a linear cost t per unit of distance when he consumes a good located elsewhere.

In contrast to Hotelling, who required each consumer to buy one and only one good, this model opens for a possible purchase of both goods. We assume that no utility can come from buying a second unit of the same good. The consumers can thus choose to buy one unit of good 1, one of good 2 or one of both. The choice is determined by the option that yields the highest utility. When both goods are bought we simply sum the separate utility expressions but reduce the reservation price for the second good to λR, where λ is some percentage so that $\lambda \in [0,1]$.[9] Consumer surplus under multi-purchasing is always greater than or equal to the surplus in the standard model because the consumers are free to choose from the old alternatives as well.

3.1 Exogenous locations

We begin by studying the case of maximal differentiation. Let good 1 be located at 0 and good 2 at 1. The utility functions for a consumer located at point *x* thus become:

$$u_1 = R - tx - p_1 \tag{3.1}$$

$$u_2 = R - t(1-x) - p_2 \tag{3.2}$$

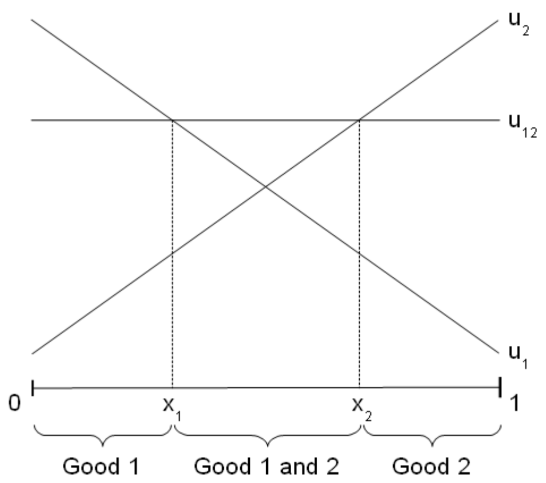
[8] Hotelling assumes a line of length *l* but this is a mere choice of units and does not change anything.
[9] In comparison, Kim and Serfes (2006) go from a reservation price of α to θ, where $\alpha \geq \theta$. Choi (2010) employs a utility of b per participant and has m_A participants for good/platform A and m_B participants for B. He further assumes an overlap between the participants of δ such that the total number of participants is $m = mA + mB - \delta$. Defining exclusive participants as $\lambda = 1 - \delta$ and assuming that $m_A = m_B = 1$, a consumer purchasing both goods receives $bm = b(1 + \lambda)$. Hence, θ in Kim and Serfes corresponds to λb in Choi, which again corresponds to λR in this model. The interpretation of λ as a degree of exclusivity or variety is tempting, but it should be emphasized that we assume the goods to be equal in all other ways than along the line whereas Choi's setup is due to his model's two-sidedness.

$$u_{12} = R + \lambda R - tx - t(1-x) - p_1 - p_2$$
$$= (1+\lambda)R - t - p_1 - p_2 \tag{3.3}$$

We note that u_{12}, the utility from buying both goods, is independent of x when the firms are located at opposite ends because the consumer then has to travel the full distance to buy both.

Plotting the functions in a diagram as in Figure 3.1 gives an immediate overview of when each line is on top. Conditions needed for the market to be covered and for both firms to sell a positive amount are discussed only in the case with endogenous locations.[10]

Figure 3.1: Utilities and location



A consumer buys both goods if u_{12} is the highest utility curve at his location in Figure 3.1. If this is to happen for at least one consumer, u_{12} must be above the intersection of u_1 and u_2 at the midpoint. This is the same as requiring that $x_1 < x_2$. Indeed, when we define $D_1 = x_2$ as the demand for good 1 while $D_2 = 1 - x_1$ is the demand for good 2 we can show that the same requirement corresponds to saying $D_1 + D_2 > 1$, which of course implies that someone is buying both.[11]

Moving on, we see from the diagram that there will be two indifferent consumers; one, located at x_1, will be indifferent between buying only good 1 and both while at x_2 a consumer will be indifferent between consuming good 2 and both. Locating these indifferent consumers determines the demand for each good. We use the intersection of u_1 and u_{12} to find x_1 and the intersection of u_{12} and u_2 to find x_2.

[10] See Tirole (1988, p. 98) for analysis and graphs of the exogenous, end-of-the-line situation.
[11] This last interpretation is due to Choi (2010).

$$x_1 = 1 - \frac{1}{t}(\lambda R - p_2) \tag{3.4}$$

$$x_2 = \frac{1}{t}(\lambda R - p_1) \tag{3.5}$$

3.1.1 Multi-purchase

Multi-purchase only occurs when $x_1 < x_2$. Inserting (3.4) and (3.5) yields

$$\frac{1}{2}(p_1 + p_2) + \frac{1}{2}t < \lambda R \tag{3.6}$$

Intuitively, the benefit of buying a second good must be greater than the private cost of acquiring it for at least the consumer at the midpoint. The left hand side of (3.6) represents expected private cost, using an average price and the transport cost out to one end of the line. If this condition is broken, we revert to the standard single purchase outcome. For now we suppose that it holds.

Demand for the goods is now easily determined.

$$D_1 = x_2 = \frac{1}{t}(\lambda R - p_1)$$

$$D_2 = 1 - x_1 = \frac{1}{t}(\lambda R - p_2)$$

Each good is dependent only on its own price. This is contrary to the single-purchase model. As we change from an either-or approach to allowing the purchase of both goods their competitive nature changes. Now each firm is the sole provider of its sort, giving it a local monopoly. Geometrically, x_2 is determined only by p_1 because that parameter just shifts u_{12}. A change in p_2, on the other hand, would affect u_2 and u_{12} in equal measures, leaving the intersection constant.

As the demand functions follow the same form, we will continue with generic expressions where $i \in \{1,2\}$.

$$D_i = \frac{1}{t}(\lambda R - p_i) \tag{3.7}$$

We assume constant marginal costs c_i. Choi (2010) does the same but with the added assumption that costs are the same for both firms. As Choi inspired the structure of this model, our results will of course be comparable. Kim and Serfes (2006) assume no marginal costs. Our results can be simplified to compare, but we will later find that cost differences do indeed matter and choose for now to keep our own method.

The firms compete in prices. The optimization problem is therefore:

$$\max_{p_i} \pi_i = \max_{p_i} (p_i - c_i) D_i$$

First order conditions give

$$\Rightarrow p_i = \frac{1}{2}(\lambda R + c_i) \tag{3.8}$$

Second order conditions for a maximum are easily verified.

Inserting (3.8) back into the demand (3.7) gives

$$D_i = \frac{1}{2t}(\lambda R - c_i)$$

Altogether, each firm can expect a profit of

$$\begin{aligned}\pi_i &= \left[\frac{1}{2}(\lambda R + c_i) - c_i\right]\frac{1}{2t}\,(\lambda R - c_i)\\ &= \frac{1}{4t}(\lambda R - c_i)^2\end{aligned}$$

Interestingly, profit is decreasing in *t*. This property is inherited from the demand, which is now sensitive to what Kim and Serfes (2006) have called the aggregate demand creation effect. As *t* falls, demand reaches farther into the line as more consumers will find it worth the transport cost to buy a second good.

Moving back to the condition for multi-homing to hold, (3.6), we now update it with the price expressions.

$$\frac{1}{2}(\lambda R + c_1) < 2\lambda R - t - \frac{1}{2}(\lambda R + c_2)$$

$$t + \frac{1}{2}(c_1 + c_2) < \lambda R \tag{3.9}$$

In this form the left hand side of the condition can be compared to the expected social cost of an additional purchase. Multi-purchasing would imply that at least the consumer at the midpoint, who would incur a transport cost of $\frac{1}{2}t$, buys both goods. The social condition for multi-purchasing is therefore $\frac{1}{2}t + \frac{1}{2}(c_1 + c_2) < \lambda R$ where the second term of the left hand side is the expected cost of producing the second unit whereas the right side is the gain from the sale. Our model requires twice the transport costs of the socially efficient solution.

3.1.2 Single purchase

Next, we find the equilibrium if (3.9) does not hold. In other words, we calculate the single purchase event. This doubles as a reference for the plain Hotelling model with exogenous locations at (0,1). The consumer who is indifferent between the goods is then located at the intersection of u_1 and u_2 in Figure 3.1. We denote the location x^S and let a superscript S signal the single purchase solution for all parameters.

$$x^S = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

All consumers buy one of the goods, so demand is simply a matter of dividing the line.

$$D_1 = x^S = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

$$D_2 = 1 - x^S = \frac{1}{2} - \frac{p_2 - p_1}{2t} = \frac{1}{2} + \frac{p_1 - p_2}{2t}$$

Demand is now dependent on the competitor's price. Again we generalize the expression so that $i, j \in \{1,2\}$ and $i \neq j$.

$$D_i = \frac{1}{2} + \frac{p_j - p_i}{2t}$$

The firms still compete in prices. Their profit's first order condition gives the following reaction function:

$$p_i = \frac{1}{2}(p_j + t + c_i)$$

By knowing p_i we also know p_j and can therefore insert to find the equilibrium price and demand.

$$p_i^S = \frac{3t + 2c_i + c_j}{3}$$
$$D_i^S = \frac{3t + c_j - c_i}{6t}$$

Single purchase profit is thus

$$\pi_i^S = \left(\frac{3t + 2c_i + c_j}{3} - c_i\right)\frac{3t + c_j - c_i}{6t}$$
$$= \frac{1}{18t}\left(3t + c_j - c_i\right)^2$$

In any normal situation, consumers will be free to decide if they want to buy both goods. Hence, condition (3.6), which derives from the consumers' utility, determines whether multi-purchasing will occur. Yet if the consumers lay the framework, the firms have the final say through their pricing decision. We therefore compare profit under single-purchase to multi-purchase profit.

$$\pi_i^S < \pi_i^M$$

$$\frac{1}{18t}\left(3t + c_j - c_i\right)^2 < \frac{1}{4t}(\lambda R - c_i)^2$$

$$\sqrt{2}t + \frac{\sqrt{2}}{3}\left(c_j - c_i\right) + c_i < \lambda R$$

For equal costs this is a stricter condition than the inequality for multi-purchasing, (3.9). Hence, there will be some interval where multi-purchasing will not be the most profitable choice for the firms. Alas, when locations stop being fixed, there will be no corresponding single purchase equilibrium to compare with.

3.2 Endogenous locations

We expand the model to a game with two stages. First the firms choose locations. Then they compete in prices like before. Backwards induction is used to solve the game.

As the naming is arbitrary, let firm 1 be the leftmost of the two. It is then located at a distance a from the left of the line, while firm 2 is located at a distance b from the right.[12] Keeping the names in order implies $a + b \leq 1$.

To allow for the possibility that consumers may be on both sides of the firms' locations we need to use absolute differences in the transport cost term in the utility expressions. Other than that, they are as before.

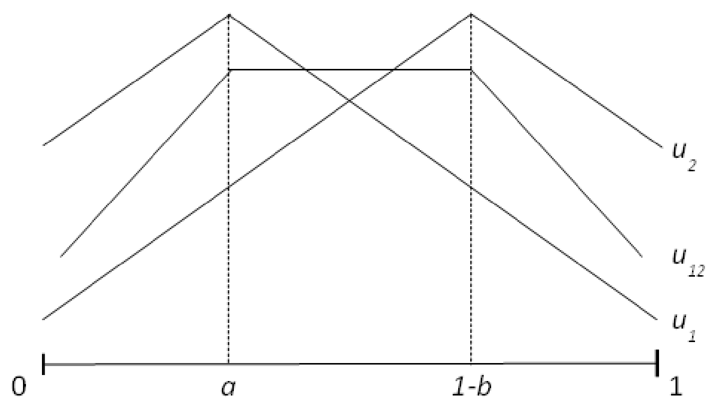
$$u_1 = R - t|x - a| - p_1$$

$$u_2 = R - t|(1 - b) - x| - p_2$$

$$\begin{aligned} u_{12} &= R + \lambda R - t|x - a| - t|(1 - b) - x| - p_1 - p_2 \\ &= (1 + \lambda)R - t(|x - a| + |1 - b - x|) - p_1 - p_2 \end{aligned}$$

The utility of buying both goods is no longer fully independent of x. Only between a and b will the total distance travelled to buy both be constant such that u_{12} is a flat line. Outside of the interval the double transport will make the fall in utility steeper than for the single purchase curves. This is illustrated in Figure 3.2 below, which shows the utility functions over the preference line when the firms have chosen to place themselves at locations a and $1 - b$.

Figure 3.2: Utilities for a given (a,b) location set



The locations of the indifferent consumers are now

$$u_1 = u_{12}$$

[12] The naming of the location variables follows the convention established by Hotelling (1929) and used by d'Aspremont et al. (1979). However, Kim and Serfes (2006) let *b* be the distance from the left. Direct comparisons with that paper can therefore not be made. Note also that although convention shortens the exogenous and endogenous cases to (0,1) and (a,b) respectively, these are not on the same form. Extreme differentiation would imply a = b = 0, which gives (a,b) = (0,0), not the ususal (0,1). Still, this is an inconsistency that we are willing to live with.

$$R - t|x_1 - a| - p_1 = (1+\lambda)R - t(|x_1 - a| + |1 - b - x_1|) - p_1 - p_2$$

$$|1 - b - x_1| = \frac{1}{t}(\lambda R - p_2)$$

$$\Rightarrow x_1 = 1 - b - \frac{1}{t}(\lambda R - p_2) \tag{3.10}$$

$$u_2 = u_{12}$$

$$R - t|1 - b - x_2| - p_2 = (1+\lambda)R - t(|x_2 - a| + |1 - b - x_2|) - p_1 - p_2$$

$$|x_2 - a| = \frac{1}{t}(\lambda R - p_1)$$

$$\Rightarrow x_2 = a + \frac{1}{t}(\lambda R - p_1) \tag{3.11}$$

3.2.1 Firm restrictions

We rule out the possibilities that $1 - b - x_1 < 0$ or $x_2 - a < 0$ because this would imply that firm 1 would locate itself to the right of all its customers (or, for firm 2, to the left). Even if only one of the two were true, so that the naming of the firms might still be correct, that would simply not make sense.

One set of locations can be ruled out as possible Nash equilibria right away. These are the combinations where one single purchase utility curve is strictly above the other so that the lower firm has no customers. Clearly, that firm would then want to change its placement strategy. In other words we want u_1 and u_2 to cross at some point. Let that point be called $\hat{x}$, which we will now find.

$$u_1 = u_2 \Rightarrow |1 - b - \hat{x}| - |\hat{x} - a| = \frac{1}{t}(p_1 - p_2)$$

The intersection does not exist if $1 - b < \hat{x}$ while $\hat{x} \geq a$ or if $1 - b \geq \hat{x}$ while $\hat{x} < a$. Two combinations remain. Later, using Figure 3.3, we will show that second of these is impossible. For now, we conclude that if $1 - b \geq \hat{x}$ while $\hat{x} \geq a$, then

$$\hat{x} = \frac{1}{2}(1 + a - b) - \frac{1}{2t}(p_1 - p_2) \tag{3.12}$$

Whereas if $1 - b < \hat{x}$ while $\hat{x} < a$, then

$$\hat{x} = \frac{1}{2}(1 + a - b) + \frac{1}{2t}(p_1 - p_2) \tag{3.13}$$

3.2.2 Consumer restrictions

In addition, we check that $x_1 > 0$ and $x_2 < 1$ so that they lie within the line we allow the firms to differentiate over. This implies that $D_2 < 1$ and $D_1 < 1$, respectively. In practice this means that we want some consumers in each category: some buy only good 1, some buy both and some buy only good 2. Substituting (3.10) and (3.11) into these assumptions we find the following inequalities:

$$p_2 > \lambda R - t(1 - b)$$

$$p_1 > \lambda R - t(1 - a)$$

Intuitively, both intersections in our model occur within the length of *x* as long as each incremental good is too expensive for at least the person farthest away. Without these requirements we could see all customers buying both goods.

d'Aspremont et al. (1979) prove that any single-purchase equilibrium must satisfy equivalent conditions of intersection within the line. Crucially, their proof of instability relies on the resulting inequalities to select the price with which the firms will try to undercut the other. Yet the prices do not interact in our model.

We know from d'Aspremont et al. (1979) that the model with linear transport costs breaks down under single purchase conditions. Studying the effects of multi-purchase thus becomes all the more interesting. We begin with the alignment of intersections in Figure 3.2 that multi-homing requires.

$$x_1 < x_2$$

$$1 - b - \frac{1}{t}(\lambda R - p_2) < a + \frac{1}{t}(\lambda R - p_1)$$

$$\frac{1}{2}(p_1 + p_2) + \frac{1}{2}(1 - a - b)t < \lambda R \tag{3.14}$$

All together we can sum up the conditions needed for multi-purchasing in Table 3.1 below.

Table 3.1: Consumer conditions required for multi-purchasing

Consumer behaviour	Constraint	Geometric interpretation	Condition
Some buy both goods	$D_1 + D_2 > 1$	$x_1 < x_2$	$\frac{1}{2}(p_1 + p_2) + \frac{1}{2}(1 - a - b)t < \lambda R$
Not all buy good 1	$D_1 < 1$	$x_2 < 1$	$p_1 + t(1 - a) > \lambda R$
Not all buy good 2	$D_2 < 1$	$x_1 > 0$	$p_2 + t(1 - b) > \lambda R$

3.2.3 Multi-purchase

Assuming that the required inequalities hold, we continue with the expressions for demand.

$$D_1 = x_2 = a + \frac{1}{t}(\lambda R - p_1)$$

$$D_2 = 1 - x_1 = b + \frac{1}{t}(\lambda R - p_2)$$

Not only is demand still independent of the competitor's price, we find that the competitor's location does not matter either. This continues the argument about local monopolies from the exogenous case.

Moving on, we solve the general case where $i \in \{1,2\}$, $a_1 = a$ and $a_2 = b$.

$$D_i = a_i + \frac{1}{t}(\lambda R - p_i) \tag{3.15}$$

First order conditions yield the reaction function.

$$\frac{\partial \pi_i}{\partial p_i} = D_i + (p_i - c_i)\frac{\partial D_i}{\partial p_i}$$

$$= a_i + \frac{1}{t}(\lambda R - p_i) - (p_i - c_i)\frac{1}{t} = 0$$

$$\Rightarrow p_i = \frac{1}{2}(a_i t + \lambda R + c_i) \tag{3.16}$$

Again, second order conditions for a maximum are easily verified. (3.16) shows that price is increasing in a_i. Mathematically this is inherited from the demand, which we would indeed expect to increase as the firm got closer to the middle. When the firm moves closer to more customers, the consumers' transport costs fall. With the local monopolies intact the firms are able to capture some of this benefit. (3.16) solves step two of the game.

Inserting (3.16) back into (3.15) produces the equilibrium demand, which increases as the firm advances along the line.

$$D_i = \frac{1}{2t}(a_i t + \lambda R - c_i) \tag{3.17}$$

Step one is the firms' choice of location. To find this we combine (3.16) and (3.17) and complete the profit function.

$$\begin{aligned} \pi_i &= (p_i - c_i)D_i \\ &= \left[\frac{1}{2}(a_i t + \lambda R + c_i) - c_i\right]\frac{1}{2t}(a_i t + \lambda R - c_i) \\ &= \frac{1}{4t}(a_i t + \lambda R - c_i)^2 \end{aligned}$$

The profit is clearly increasing in a_i. Differentiation tends towards the minimal. Summing up we have:

Proposition 1: *When multi-purchasing occurs in a covered market, the firms will profit from moving closer together* $\left(\frac{\partial \pi_i}{\partial a_i} > 0\right)$.

3.2.4 Equilibrium location

The footsteps of Kim and Serfes (2006), who we set out to follow, stop here. Yet the hasty conclusion of Hotelling (1929) himself and d'Aspremont et al.'s (1979) subsequent refutation have shown that we cannot simply be satisfied by a positive first derivative. Indeed, since we have used linear costs just like Hotelling, an increasing tendency was to be expected. The conclusion to the preceding subchapter answered the second question that d'Aspremont et al. taught us to ask, but we still need to establish whether an equilibrium exists.

Returning to the table of conditions necessary for multi-purchasing gives us Table 3.2 after the price function has been substituted into the previous expressions. An interval of values of λ for which multi-purchasing will occur is beginning to take shape.

Table 3.2: Consumer conditions required for multi-purchasing in price equilibrium

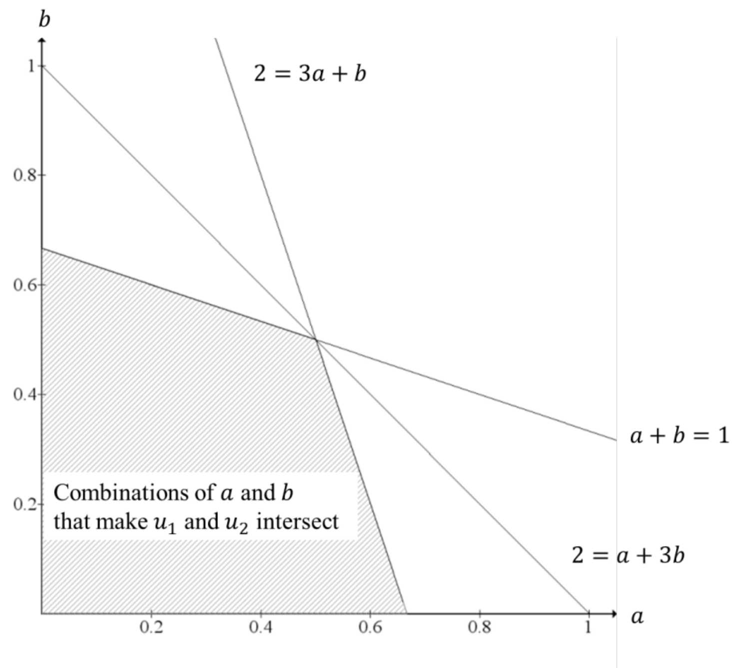
Consumer behaviour	Constraint	Geometric interpretation	Equilibrium condition
Some buy both goods	$D_1 + D_2 > 1$	$x_1 < x_2$	$\left(1 - \frac{1}{2}(a+b)\right)t + \frac{1}{2}(c_1 + c_2) < \lambda R$
Not all buy good 1	$D_1 < 1$	$x_2 < 1$	$(2-a)t + c_1 > \lambda R$
Not all buy good 2	$D_2 < 1$	$x_1 > 0$	$(2-b)t + c_2 > \lambda R$

Compared to that for exogenous (0,1) locations, the condition needed for at least some consumers to buy both goods is weaker for all other values of a and b under endogenous locations. Multi-purchasing is therefore more likely to occur when the firms are able to choose their degree of differentiation.

Indeed, if λ is big enough to break the last two conditions in Table 3.2, then all consumers will buy both goods. These requirements are the same as asking that the single purchase utility curves are on top at each end of the line, meaning $u_{12}(0) < u_1(0)$ and $u_{12}(1) < u_2(1)$.

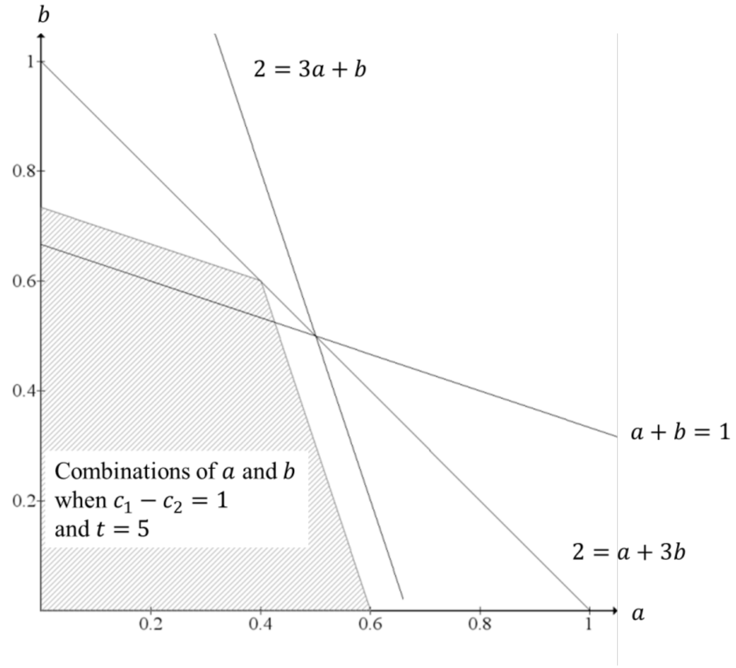
Going back to the conditions necessary for the single purchase utility curves to intersect, we can use our price function to get another look at the limits of what combinations of a and b are reasonable. Recall that we required either $1 - b \geq \hat{x}$ while $\hat{x} \geq a$, or $1 - b < \hat{x}$ while $\hat{x} < a$. For the first case, substituting (3.16) into (3.12) gives $2 + \frac{1}{t}(c_1 - c_2) \geq a + 3b$ while $2 - \frac{1}{t}(c_1 - c_2) \geq 3a + b$. In the second case, substituting (3.16) into (3.13) gives $2 + \frac{1}{t}(c_1 - c_2) < a + 3b$ while $2 - \frac{1}{t}(c_1 - c_2) < 3a + b$. These lines are plotted in Figure 3.3 together with the old restriction that $a + b \leq 1$ and an assumption of equal marginal costs. As can easily be seen geometrically, the second case is incompatible with $a + b \leq 1$ in all other points than $a = b = \frac{1}{2}$.

Figure 3.3: Permissible combinations of a and b



If marginal costs are not equal, the disadvantaged firm's possible locations will be limited and its competitor's set expanded. Below, Figure 3.4 shows the same lines as before but with the permissible area shifted by a cost difference of one in favour of firm 2. The difference is softened by the effect of transport costs, which makes consumers hesitant to change goods even in the face of price differences. The kink now comes at (0.4, 0.6). Note that the midpoint is only reachable if costs are equal.

Figure 3.4: Permissible combinations of a and b with a cost difference



Along the edges of the areas in Figure 3.3 and Figure 3.4 the two single purchase utility curves will overlap for some portion of the *x*-line. Consumers will be indifferent between the goods in the case of an overlap and should be assumed to randomize between them. Expected demand for one firm across an overlap is therefore half of the mass of consumers covered. Proposition 1 tells us that any location inside the area cannot be optimal because it would be more profitable to move out towards the edges. A move beyond the line $a + b = 1$ would be the same as swapping names for the two firms, and the incentives would then change direction, pulling back towards the line.

Overlapping utility curves have to affect at least one firm's side of the market entirely since there is no change in the utility curves' slope to break them apart. Panel a) in Figure 3.5 illustrates a partial overlap where firm 1 has twice the marginal cost of firm 2 and they have chosen symmetric locations at $a = b = 0.45$.[13] Partial overlaps are also possible without a cost difference. Randomizing demand for as long as the consumers are indifferent between the goods means that firm 2 still dominates on the right side whereas firm 1 "shares" his side. An infinitesimally small change in location backwards by firm 1, as shown in panel b) in Figure 3.5, will allow it to capture the other half of that demand. Since the move will only cause a very small change in price, the increased demand should result in greater profits. This suggests that partial overlaps cannot be Nash equilibria.

[13] All in all $R = 10, \lambda = 0.5, t = 5, c_1 = 2, c_2 = 1$. In panel a) $a = b = 0.45$. In panel b) $a = 0.4$. The size of the move backwards has been chosen for graphic effect and would in reality be expected to be much smaller.

Figure 3.5: a) Partial overlap b) Firm 1 moves back slightly

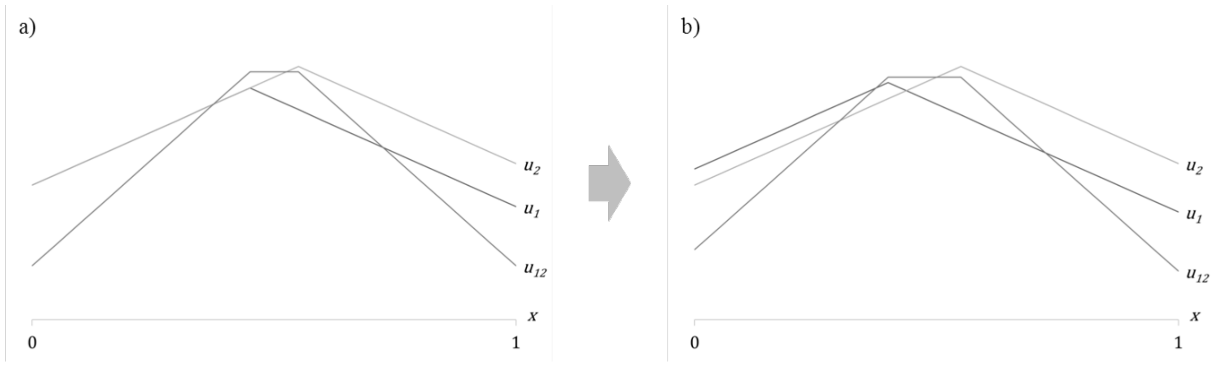
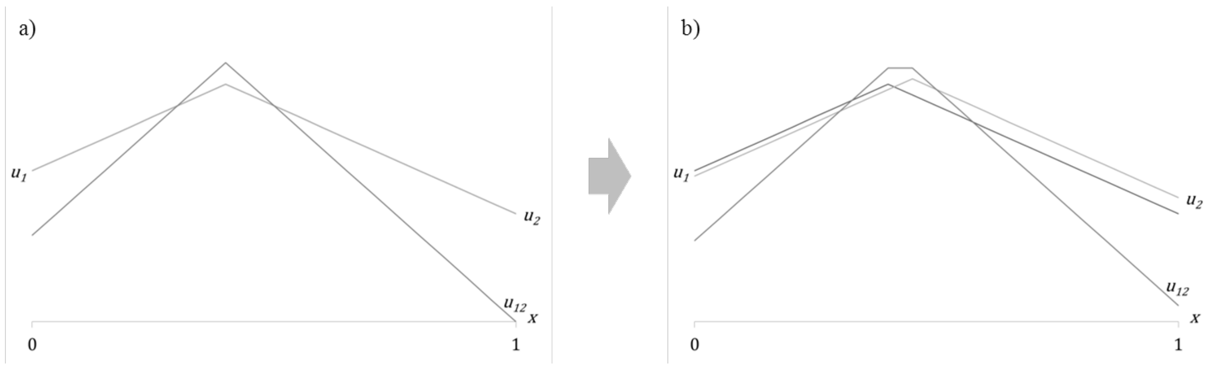


Figure 3.6: a) Full overlap b) Firm 2 moves back slightly



A full overlap occurs in the kinks of Figure 3.3 and Figure 3.4. This time demand is shared on both sides. Panel a) of Figure 3.6 shows a full overlap when firm 1 again has twice the marginal cost of firm 2 and they select the same location.[14] The share of consumers lying to the right of the firms' locations is larger than that to the left due to the shift in the kink's location coming from firm 2's cost advantage. Consequently, firm 2 would gain more demand on the right hand side by retreating somewhat than it would lose on the left hand side. Once more, overlaps seem unstable.

The exception to this conclusion is at the midpoint because a full overlap would be of equal length on both sides. Here, the demand gained by one side by moving slightly is nothing more than the demand lost on the other side. Expected demand is then equal to what we get using our old, deterministic approach. Crucially, the midpoint location is only possible with equal costs.

[14] As before $R = 10, \lambda = 0.5, t = 5, c_1 = 2, c_2 = 1$. This gives $a = 0.4$ and $b = 0.6$ in panel a). Panel b) sees $b = 0.55$. Again, the size of the move is chosen with visibility in mind, not reality.

Proving that overlaps are not viable strategies has turned out to be a mathematically intractable problem. The theory is therefore left in the following form:

Conjecture 1: *Given that multi-purchasing occurs, a unique equilibrium exists at the midpoint if the firms have equal costs. No equilibrium exists with unequal costs.*

From now on, we assume that the firms have equal costs. That is, $c_1 = c_2 = c$.

If Conjecture 1 is true, the Principle of Minimal Differentiation is restored. The midpoint equilibrium results in Figure 3.7. Updating the table of conditions to reflect the positions and costs gives us Table 3.3.

Table 3.3: Consumer conditions required for multi-purchasing in placement equilibrium

Consumer behaviour	Constraint	Geometric interpretation	Condition when $a = b = \frac{1}{2}$ and $c_1 = c_2 = c$
Some buy both goods	$D_1 + D_2 > 1$	$x_1 < x_2$	$\frac{1}{2}t + c < \lambda R$
Not all buy good 1	$D_1 < 1$	$x_2 < 1$	$\frac{3}{2}t + c > \lambda R$
Not all buy good 2	$D_2 < 1$	$x_1 > 0$	$\frac{3}{2}t + c > \lambda R$

Figure 3.7: Utility with equilibrium locations

In Figure 3.7 above, u_1 and u_2 overlap completely.[15] This does not mean that both firms will serve all the customers. Any multi-purchase is still represented by u_{12}. What it does mean is that the consumers who only buy a single good are indifferent between the goods. Expected demand for either firm is half of the overlap plus the multi-purchasing group. We could randomize the sales over the line, but the symmetry means that our old definitions of D_1 and D_2 ensure a proportional allocation already.

3.2.5 Market coverage

Another potential problem with our definitions of demand would occur if the market were not covered. That is, if the focus on the pull towards the centre made us forget to check if the consumers at the ends of the line still wanted to buy anything. They will do so as long as their utility is positive in at least one case. Our previous assumption that there will be demand for each good sold separately means that the single purchase utility curves will be on top at the ends. Algebraically, we should verify that $u_1(0) \geq 0$ and $u_2(1) \geq 0$ in our proposed equilibrium. The former solves to

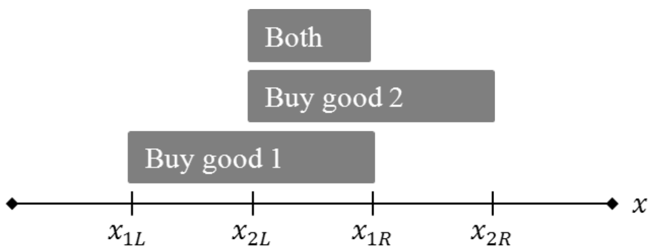
$$R \geq at + p_1$$

$$2R - \frac{3}{2}t - c \geq \lambda R \tag{3.18}$$

Solving $u_2(1) \geq 0$ gives an equivalent condition.

If (3.18) fails and the market is not covered, the system becomes even less dependent on location because it does not matter which part of the line is left unserved. Suppose the firms have simply placed themselves somewhere along the line, illustrated in Figure 3.8 below, in such a way that not all consumers choose to buy anything. That is, the market is not covered.

Figure 3.8: Demand when the market is not covered



[15] The graph was made using the following parameters: $R = 10, \lambda = 0.5, t = 5, c = 1$

Each of the four points in Figure 3.8 lies where a utility curve binds. For x_{1L} we solve $u_1(x_{1L}) = 0$. For x_{1R} we solve $u_{12}(x_{1R}) - u_2(x_{1R}) = 0 \Rightarrow u_{12}(x_{1R}) = u_2(x_{1R})$. Naturally, the model only works if $x_{1L} > 0$ and $x_{2R} < 1$, the opposite of the previous system. Assume that $a \in (x_{1L}, x_{1R})$ and $(1-b) \in (x_{2L}, x_{2R})$. This is the only necessary restriction on the firms' placement, but to say that they locate somewhere within the interval they serve is hardly controversial. Calculating for firm 1 gives

$$x_{1L} = a - \frac{1}{t}(R - p_1)$$

$$x_{1R} = a + \frac{1}{t}(\lambda R - p_1)$$

Note that this model takes multi-purchasing over some interval for granted. To test when that is correct, let $x_{2L} < x_{1R}$. Simple substitution shows that this solves to the same inequality, (3.14), as in the full coverage model where we test $x_1 < x_2$.

$$\begin{aligned} D_1 &= x_{1R} - x_{1L} \\ &= \frac{1}{t}[(1+\lambda)R - 2p_1] \end{aligned}$$

Demand is independent of location and of the competitor. Firm 2 has an equivalent demand function. Kim and Serfes' (2006) aggregate demand creation effect is gone since the new expression of demand lacks any dependence on a. The effect is therefore a consequence of the one-sidedness of the covered model where each firm is sure to keep its hinterland and the only customers worth chasing are in the middle. There is no aggregate demand creation until all consumers buy at least one good.

Moving on, the solution is generalized for firm $i \in \{1,2\}$.

First order conditions for the basic profit function, $\pi_i = (p_i - c)D_i$, are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{t}[(1+\lambda)R - 2p_i] + (p_i - c)\left(-2\frac{1}{t}\right) = 0$$

$$\Rightarrow p_i = \frac{1}{4}[(1+\lambda)R + 2c]$$

Second order conditions are easily verified. We substitute price into demand and find total profit.

$$D_i = \frac{1}{2t}[(1+\lambda)R - 2c]$$

$$\pi_i = \frac{1}{8t}[(1+\lambda)R - 2c]^2$$

Comparing the uncovered profit to the covered model will not tell us when either option is better than the other. The price reaction functions differ between the two models such that the assumptions behind each model will not be true at the same time. For any combination of parameters the assumptions of at most one model will hold. A proof can be found in the appendix. To compare profit is futile since one does not have the opportunity to select between the models.

To test that this model really is not fully covered we let the length l equal $x_{2R} - x_{1L}$ and require $l < 1$. This solves to

$$2R < t(a+b) + p_1 + p_2 \tag{3.19}$$

Compared to the condition for market coverage that introduced this subchapter, we see that the inequality is nothing more than a combination of $u_1(0) < 0$ and $u_2(1) < 0$. Substituting the new price reaction function gives

$$(3-\lambda)R < 2t(a+b) + 2c \tag{3.20}$$

In addition, we still require multi-purchasing. Hence, we substitute the price into $x_{2L} < x_{1R}$, which we know equals $\frac{1}{2}(p_1 + p_2) + \frac{1}{2}t(1-a-b) < \lambda R$ just like in the covered case. Doing so results in the following expression:

$$2t(1-a-b) + 2c < (3\lambda - 1)R \tag{3.21}$$

Both (3.20) and (3.21) are needed if the model for an uncovered market is to work. Using the fact that $\lambda \in [0,1]$ because λ is defined as a percentage gives the following relation between the inequalities:

$$2t(1-a-b) + 2c < (3\lambda - 1)R \leq 2R \leq (3-\lambda)R < 2t(a+b) + 2c$$

$$2t < 4t(a+b)$$

$$\frac{1}{2} < a + b$$

Even though the firms are free to decide their precise placement, we can see that they have to stay relatively close together if some consumers are to buy both goods while others buy none. Whether the firms will be able to coordinate that placement is beyond the topic of this paper.

3.3 Discussion of results

Hotelling's (1929) assumption that only one of the two goods could be bought was removed because in practice it is hard to restrict consumers from buying what they want. Consumers should be able to add their two utility expressions when that outcome is better for them. Multi-purchasing is not just an expansion to fit other models but an adjustment to reality.

We set out to test Kim and Serfes' (2006) arguments using Choi's (2010) linear model. Similar to their quadratic version, we find that multi-purchasing will support the Principle of Minimum Differentiation if certain conditions are met. We have provided easy geometric and economic interpretations of these conditions, and this may be considered one of the strengths of the linear approach. Still, should the conditions not hold, the linear model would be thrown back into instability.

3.3.1 The Principle of Minimum Differentiation

The Principle of Minimum Differentiation does not fit well with Hotelling's intentions.[16] Where we want to model differentiation, we get imitation. Depending on one's expectations, this may be a more interesting conclusion than the opposite. Political competition (Downs, 1957) is an example of when it is as important to explain imitation as polarization. Clustering (Porter, 2000) is another example of when (geographic) differentiation is in fact the hypothesis we want to reject. On the other hand, we are still at a loss when it comes to explaining any incentives to differentiate.

Furthermore, if the goods are located in the same spot, they have become identical. Then there is no longer a reason to allow multi-purchasing but not a second purchase of one good. Such equality should, as in d'Aspremont et al. (1979), lead to pure price competition, which points

[16] And to complicate matters, the Principle was not true to Hotelling's own model – regardless of his intentions. Martin (2002, p. 116) said it well: "Hotelling failed in the same way Columbus failed. Columbus did not discover a westward sea route to India, and Hotelling did not demonstrate a tendency for firms to minimize differentiation"

to a Bertrand solution where all profit is competed away. This “same-but-different” result is an unavoidable weakness in any Hotelling model with minimal differentiation.

3.3.2 Stability

Hotelling’s original model is unstable because the only candidate for a price equilibrium requires that the two firms are located somewhat apart at the same time as their profit functions tell them to move closer. The condition that requires the firms to stay apart comes from a comparison of the possible equilibrium with the complete sales to be had by undercutting the competitor’s price by just more than total transport costs, which would make all customers come to that firm. This incentive to undercut is removed when multi-purchasing is allowed. Pricing is now independent of what the competitor’s does, as seen in (3.16). All that is left is Kim and Serfes’ (2006) aggregate demand creation effect. As long as the conditions for multi-purchasing to occur hold, our model is stable.

The conjecture about instability in the face of cost differences is new to the literature. All comparable papers assume equal (Choi, 2010; Anderson & Neven, 1989) or even zero (Kim & Serfes, 2006; Anderson, Foros, & Kind, 2012; Sajeesh & Raju, 2010) marginal costs. It turns out that this is a knife-edge assumption; any deviation from the traditional practice could potentially throw the system off. In addition to the need for a proof of Conjecture 1, an investigation of cost differences in a multi-purchase model with quadratic transportation costs is therefore a useful path for future study.

3.3.3 Second reservation price

Multi-purchasing is supported solely by the second reservation price, λR, which may in the end seem arbitrary. When the goods are supposed to be equal in every other way than along the line it is admittedly strange to allow the consumer to be willing to pay for a second version. Problems in interpreting the second reservation price are still essential in comparing our results to those in the papers that motivated our exercise. This model attempts to simplify the problem by relating the second reservation price to the first. Whereas Kim and Serfes (2006) introduce a completely new variable for the second reservation price, stating only that it is less than for the first, this model lets some degree λ of the original willingness to pay remain.[17] This interpretation underlines how the second good is simply lower ranked and at the same time opens for the possibility of making λ a function of decisions taken in an expanded system.

Kim and Serfes (2006, p. 587) suggest that their own incremental utility should be a function of the location and will "depend on a given market". The latter is obviously true. But without any other motivation or assumptions, to turn that precaution into an analysis of derivatives like they do becomes a purely academic exercise. Indeed, to make λ dependent on a and b is in effect to say that transport costs are not the only difference between the goods. This disturbs a central element in how we think about the Hotelling model and is therefore not part of what we call our "minimal" model. Rather, future research might let λ depend on the shared characteristics of the goods.

Turning back to Choi (2010) – whose model we have built on and whose use of reservation prices is in fact dependent on other layers of the model – reveals a weakness in his assumptions as well. In his setup the reservation price is based on the amount of content available with each purchase option, and he assumes that the two alternative platforms are based at the extremes of Hotelling's line. Buying both options gives a reservation price equal to the sum of content available minus any overlap. We now know that the platforms will have an incentive to move closer. This will make the content providers, whose product is delivered through the intermediary platforms, less likely to distinguish between the alternatives. An assumption of placement at the ends of the line occurs often in the literature on two-sided markets (Armstrong, 2006), and Proposition 1 shows that it is not sustainable.

3.3.4 Transport function

Transport can also be thought of as a unit cost of substitution and consequently as a competitive barrier. In the standard, single purchase quadratic case, which leads to maximum differentiation, profit is increasing in *t*. The higher the consumers' cost of choosing another product, the higher profits the firms make.

With linear transport, multi-purchasing and a covered market, profit is increasing in *t* only if $t > \frac{\lambda R - c}{a_i}$, which turns into $\frac{1}{2}t + c > \lambda R$ for the optimal location $a_i = \frac{1}{2}$. Table 3.3 tells us that the inequality has to turn the other way in order for multi-purchasing to occur. Multi-

[17] In terms of Kim and Serfes (2006), where the first reservation price is α and the second is θ, $\lambda = \theta/\alpha$.

purchasing ensures that the aggregate demand creation effect is stronger than *t*'s role as a competitive barrier. In an uncovered market, profit is always decreasing in *t* because firms can no longer use transport to push up price at all. Again, multi-purchasing turns the nature of competition around.

Proposition 2: *Under multi-purchasing, increasing unit costs of transport decreases profit* $\left(\frac{\partial \pi_i}{\partial t} < 0\right)$.

The choice of transport cost function is the final determinant of the system's stability and direction. It should not be modelled based only on mathematical elegance. Rather, the choice between linear, quadratic and any other cost function must be made with an eye to the real world. The interpretations of *t* are many. Literal transport (Hotelling, 1929) is one which may suit a linear form, as may distance in terms of time (Salvanes, Steen, & Sørgard, 2005), but differentiation can also be political (Downs, 1957) with *t* representing a dislike of other ideologies, which for some would certainly come at a higher order. The function must follow reality, not the other way around. If instability haunts us, that does not call for a change of transport costs but of business model.

3.3.5 Interpretation in media markets

The motivation to test multi-purchasing comes from media markets. While the model we have seen is meant for use in greater systems and is far too abstract to be put into a business context, it is still important that the results have real interpretations. What follows is an indication of how the linear multi-purchasing model can be applied to the market for news in future research.

An immediate prediction is that newspapers will tend towards the same, generic product. Two popular interpretations of location in media markets are the political spectrum and a tabloid-broadsheet scale. Many markets have papers that are traditionally thought to speak with either a conservative or a liberal viewpoint, but our results add fuel to the popular claim that such differences are in name only and that all mainstream media is the same.

Newspapers, especially online, are unlikely to look for a competitive advantage in marginal costs. Indeed, the costs are probably relatively small. Assuming that our first, competitive model's two firms have zero variable costs gives us Table 3.4.

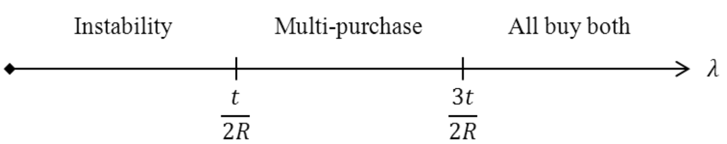
Table 3.4: Consumer conditions required for multi-purchasing in placement equilibrium with equal costs

Consumer behaviour	Constraint	Geometric interpretation	Condition when $a = b = \frac{1}{2}$ and $c_1 = c_2 = 0$
Some buy both goods	$D_1 + D_2 > 1$	$x_1 < x_2$	$\frac{1}{2}t < \lambda R$
Not all buy good 1	$D_1 < 1$	$x_2 < 1$	$\frac{3}{2}t > \lambda R$
Not all buy good 2	$D_2 < 1$	$x_1 > 0$	$\frac{3}{2}t > \lambda R$

While Table 3.4 is good for consistency with the other chapters, Figure 3.9 gives a better overview of how the range of the consumers' willingness to pay for the second paper affects the outcome of the model. Only two issues remain to be answered by a practitioner:

(1) How is the consumers' disutility of picking something other than their favourite compared to the price they're willing to pay for that same first-best?
(2) What degree of the initial willingness to pay remains when considering the second purchase?

Figure 3.9: Range of values for λ and corresponding system results



The difference between an outcome with no equilibrium and one in which all consumers buy both turns out to be exactly the ratio of t to R. Take therefore the consumer at one end of the line and divide his loss in utility from reading a paper at the other end by his maximum willingness to pay. Half of this ratio is the minimum required degree of remaining reservation price for the second purchase if the model is to be stable. Add to that one unit of the ratio and everyone buys two newspapers. Utility is notoriously abstract and hard to quantify. Using relative sizes will hopefully make rough estimates easier.

Advertising is a central feature in two-sided media models that is missing here. Assuming that advertisers only want to reach as large an audience as possible, minimal differentiation makes the firms perfect substitutes. Consequently, one could suspect that price competition in this end of the market would pull down profit as the firms locate further towards the middle. This is the opposite of the effect on the consumer end. Which effect would dominate is hard to say without a complete model and is therefore an obvious candidate for further study.

Coming from the other, single purchase, two-sided end of the field, Peitz and Valletti (2008) requested exactly this multi-purchase puzzle piece when they compared a model where advertising was the only source of funding for television ("free-to-air") with one including viewer payment ("pay-tv"). In their system, which uses quadratic transportation costs, the model that included viewer fees gives maximal differentiation. If some of the consumers were willing to pay for a second channel, Kim and Serfes (2006), who also use quadratic transport, indicate that the incentive to differentiate would be weakened. This paper confirms the same tendency in linear systems.

Admittedly the idea of a second good works better with newspapers than television. Peitz and Valletti (2008) suggest the model constructed by Anderson and Neven (1989) as a way to implement multi-purchasing, but we have already argued that their model of fractional consumption is a poor fit for information goods. This underlines the difference between television, where one cannot consume two shows that air at the same time, and papers, where the publishing date is irrelevant to how many one can read. Still, Gal-Or and Dukes (2003) use Anderson and Neven's model to find minimal differentiation between TV channels because of the effect of substitutability of the channels on competition between advertisers. Their result depends on a fixed difference between the goods being advertised but points towards the need to look at the effect of advertising combined with multi-purchasing.

4 Bundling in the linear city

Now that the main multi-purchasing incentives are established, we investigate the model when the firms act as one monopolist with two products. With variety still as our motivation, we let the monopolist offer both goods as one bundle. Even without selling the bundle, a monopolist will want to exploit the diversity in preferences among the consumers by placing the goods apart. Package selling is designed explicitly to divide consumers by their willingness to pay and thus adds a new layer to our study of differentiation.

Bundling is also inspired by media markets. For example, cable TV can come in packages with hundreds of channels. Newspapers have not traditionally been bundled, but the challenges to their business model introduced by the Internet have brought new vigour to the debate on how best to pay for access to news. Another market where electronic publishing is taking over is that for academic journals, Kim and Serfes' (2006) second example of when multi-purchasing can be expected to occur. Journals are commonly sold to universities in large bundles while actors outside of academia pay rather expensive fees for single purchases.[18] This chapter aims to predict what package selling does to information diversity.

This chapter is structured as follows. We begin by establishing the workings of a basic monopoly without multi-purchasing. Then we turn Hotelling's starting point inside out; his model forced each consumer to buy one, and only one, good. We will make them buy both. Finally, we will discuss mixed bundling. To keep the discussion of mixed bundling tractable we require that the market is covered. No resale will be allowed. This chapter assumes equal marginal costs: $c_1 = c_2 = c$.

4.1 Single purchase monopoly

Given that no fixed costs have been introduced, a monopolist would still prefer to produce two goods even if no customer bought more than one of them. He would do so because spreading the goods out would mean that less utility was lost in transport and could therefore be taken up in a higher price instead.

As long as the monopolist wants to cover the whole market, meaning total demand is fixed to one, price is the only element to worry about. The logic is the same for both goods, so their

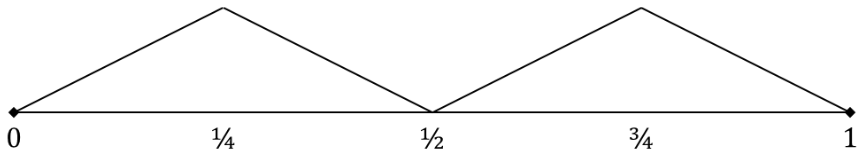
[18] Both cable TV and journals in universities are also cases where access is so seamless (just turn on your TV or log on to the Internet through a campus computer) that transaction costs are not a problem. This lets us exclude another potential motivation for bundling and leaves us focused on price discrimination.

locations will be symmetrical. Optimal locations should be such that the marginal consumer on both sides has zero utility, and the outer side is clearly at the end of the line. The utility curves of buying either good should meet at the midpoint and equal zero. In short, to maximize price means to minimize transport for the marginal consumer, and that is done at $a = b = \frac{1}{4}$. This is illustrated in Figure 4.1 below. This location is also socially optimal. Mathematically, the results are:

$$p_1 = p_2 = R - \frac{1}{4}t$$

$$\pi_{1+2} = R - \frac{1}{4}t - c$$

Figure 4.1: Utility under optimal locations for a single purchase monopoly when the market is covered

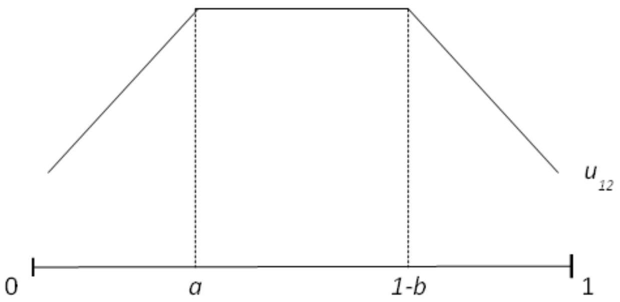


An uncovered market has the same symmetrical margins, but a higher price. Just like in the competitive model it does not matter which part of the market is left untouched. Let $l > 0$ be the percentage share of the line left with positive utility – which equals demand – when the price is increased to $p_1 = p_2 = R - \frac{1}{4}tl$. Profit is maximized at $l = \frac{2}{t}(R - c)$. To want to supply the whole market the monopolist would need to see $\frac{1}{2}t + c < R$. Marginal social costs would only be $\frac{1}{4}t + c$. Hence, we observe double transport costs compared with what maximizes welfare. Recalling Table 3.3, we expect multi-purchasing to occur if $\frac{1}{2}t + c < \lambda R$ is true, so as soon as we allow consumers to buy what they want, a covered single-purchase monopoly seems unlikely.

4.2 Pure bundling

The utility of buying both goods, u_{12}, is the only new element in our model compared to the standard Hotelling version. We proceed to look at it in isolation. That is, the consumers can no longer buy either good separately. This gives us a model of pure bundling. Price becomes a single parameter p_B and the production cost of one bundle is $2c$. For a set of locations a and b, the consumers' utility over x is illustrated in Figure 4.2.

Figure 4.2: Utility with pure bundling

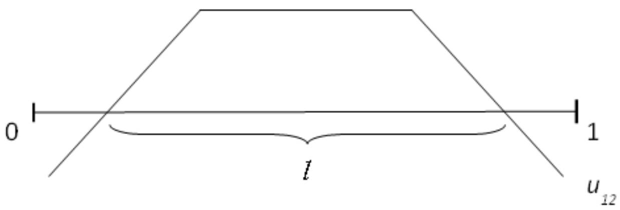


Of course, only consumers with a positive utility will choose to buy the bundle. Consumers at the extremes of the line will clearly be the first to reach zero utility for any choice of (*a*,*b*). These will have transport costs totaling *t* if locations are symmetric, more in one end if not. Symmetric locations will therefore let the monopolist extract the greatest profit when the consumers are uniformly distributed. As a result, the highest price the monopolist can charge and still sell to all customers is $p_B^0 = (1+\lambda)R - t$. Profit becomes

$$\pi_{PB} = (1+\lambda)R - t - 2c$$

Maximizing profit is a question of what share of the consumers the monopolist wants to serve. Again, let $l > 0$ be the percentage share of the line left with positive utility when the price is increased to $p_B^1 = (1+\lambda)R - lt$ as in Figure 4.3. Demand is then $D_B = l$. These expressions are independent of the firm's placement decision as long as $a + b \leq l$ because only the marginal consumer counts.

Figure 4.3: Utility with pure bundling when not all consumers are served



If we combine price and demand together in a profit function, first order conditions will show that the optimal share of consumers to serve is

$$l = \frac{1}{2t}[(1+\lambda)R - 2c]$$

All consumers are included if $l \geq 1$. That is, if

$$(1 + \lambda)R \geq 2t + 2c \tag{4.1}$$

From a social point of view, the share of the line to service should be determined by marginal revenue, $(1 + \lambda)R$, and marginal social cost, $t + 2c$. This is a less restrictive condition than (4.1). In other words, a pure bundling monopoly is inefficient in the same double-transport way that we have seen earlier. Moreover, the firm's decision of where along the line to place the goods is only a choice of leaving surplus for those consumers between the extremes. A social optimum is to locate in the middle of the served line. In comparison, Hotelling's standard model has a socially optimal placement at a quarter of the line's length inwards from each side.

Interestingly, (4.1) is also the condition for when pure bundling becomes more profitable for the monopolist than to offer the goods separately in the ordinary Hotelling single purchase model with (0,1) placements. Next we combine these two in a model of mixed bundling with fixed locations.

4.3 Mixed bundling with exogenous locations

Mixed bundling means that consumers are free to buy either good separately or a package with both. This will change the utility of buying both goods and consequently gives us new consumer actions as expressed in the intersections and demand functions. If the bundle is going to be better than just buying both separately, we need $p_B < p_1 + p_2$.

Our approach remains the same: demand is determined by the greatest utility, and firms interact with the market through pricing. As before we start with exogenously given locations at the extreme ends of the line. Mixed bundling will be restricted to a covered market.

Given the choice of buying either good 1, good 2 or the bundle, a consumer located at point $x \in [0,1]$ will have the following respective utilities:

$$u_1 = R - tx - p_1$$

$$u_2 = R - t(1 - x) - p_2$$

$$u_B = (1 + \lambda)R - t - p_B$$

Each consumer selects the option that gives him the greatest utility. Two indifferent consumers can be located just like before, and we denote the intersections where they are found in the same way: x_1 is where the utility of buying the bundle starts being greater than that of buying only good 1, whereas x_2 is where the utility of buying good 2 starts being greater than that of buying the bundle.

$$x_1 = 1 - \frac{1}{t}(\lambda R + p_1 - p_B) \tag{4.2}$$

$$x_2 = \frac{1}{t}(\lambda R + p_2 - p_B) \tag{4.3}$$

Demand is defined by the intersections of the utility functions.

$$D_1 = x_1 = 1 - \frac{1}{t}(\lambda R + p_1 - p_B)$$

$$D_2 = 1 - x_2 = 1 - \frac{1}{t}(\lambda R + p_2 - p_B)$$

$$D_B = x_2 - x_1 = \frac{1}{t}(2\lambda R + p_1 + p_2 - 2p_B) - 1$$

If at least some consumers are to buy only one good, we need $D_1 > 0$ and $D_2 > 0$, which means that $x_1 > 0$ and $x_2 < 1$ respectively. Geometrically, the utility functions should intersect within the line. Using (4.2) and (4.3) gives

$$p_1 < t - \lambda R + p_B \tag{4.4}$$

$$p_2 < t - \lambda R + p_B \tag{4.5}$$

In order for at least some consumers to buy the bundle, we require that $D_B > 0$, which means that $x_1 < x_2$, just as multi-purchasing in the competitive model. Using (4.2) and (4.3) gives

$$p_B < \lambda R - \frac{1}{2}t + \frac{1}{2}(p_1 + p_2) \tag{4.6}$$

We will discuss these expressions in detail once prices have been established.

The monopolist wants to maximize the total profit coming from those consumers buying only good 1, those buying only good 2 and those buying the bundle.

$$\pi_{MB} = (p_1 - c)D_1 + (p_2 - c)D_2 + (p_B - 2c)D_B$$

Since the utility of buying the bundle is a flat line, the bundle can be priced to extract all surplus from the consumers who choose that option. That is,

$$p_B = (1 + \lambda)R - t$$

Maximizing profit with respect to the price of each separate good then gives

$$p_1 = p_2 = \frac{1}{2}(2p_B + t - \lambda R - c)$$

$$p_1 = p_2 = \frac{1}{2}((2 + \lambda)R - t - c) \tag{4.7}$$

$$D_1 = D_2 = \frac{1}{2t}(t + c - \lambda R) \tag{4.8}$$

From (4.8) we see that positive demand for either separate good, say $D_1 > 0$, results in the condition $\lambda R < t + c$. An intuitive interpretation is that some will opt to buy only one good if the benefit of a second good to the consumer placed farthest away from it is lower than the product's marginal private cost.

Correspondingly, the bundle's demand, D_B, has to be positive. This requirement was previously expressed in (4.6).

$$D_B = \frac{1}{t}(\lambda R - c)$$

Fully solved the requirement becomes $c < \lambda R$. As long as the consumers' reservation price for the second good is greater than the cost of producing the additional unit, the bundle will be sold.

In combination we thus have a very simple interval in which mixed bundling is possible:

$$c < \lambda R < t + c$$

The willingness to pay for a second good must be greater than the minimal private cost of consuming it and less than the maximal. If it falls below this interval, a single purchase monopoly takes place. Above, pure bundling rules. For as long as the function is defined, mixed bundling is more profitable than pure bundling or single sales.

Altogether profit becomes

$$\pi_{MB} = \frac{1}{2t}\left[(2R + \lambda R - t - 3c)(t + c - \lambda R) + 2\big((1 + \lambda)R - t - 2c\big)(\lambda R - c)\right]$$

4.4 Mixed bundling with endogenous locations

To build a model of bundling when the monopolist is free to choose where to locate his two goods is an optimization problem with many parameters and conditions. Using the lessons from the previous subchapters we can simplify the calculation to one with symmetrical margins. Symmetry will still ensure that the least possible willingness to pay is lost to transport costs. Prices will continue to aim to leave the marginal consumer with no utility left over. Before we come that far, however, we should follow the usual steps and establish the basic properties of the utility curves.

$$u_1 = R - t|x - a| - p_1$$

$$u_2 = R - t|1 - b - x| - p_2$$

$$u_B = (1 + \lambda)R - t(|x - a| + |1 - b - x|) - p_B$$

Next we find where the two indifferent consumers will be, still the same procedure as before.

$$u_1(x_1) = u_B(x_1) \Rightarrow x_1 = 1 - b - \frac{1}{t}(\lambda R + \mathrm{p}_1 - \mathrm{p}_\mathrm{B})$$

$$u_2(x_2) = u_B(x_2) \Rightarrow x_2 = a + \frac{1}{t}(\lambda R + p_2 - p_B)$$

As usual we want the intersections to happen in the right order: $x_1 < x_2$. To test for this in a separate step is redundant, however, because we might as well require that the demand for the bundle is positive. Algebraically, $D_B = x_2 - x_1 > 0 \Leftrightarrow x_1 < x_2$. Demand, then, is:

$$D_1 = x_1 = \frac{1}{t}\big((1 - b)t - \lambda R - p_1 + p_B\big)$$

$$D_2 = 1 - x_2 = \frac{1}{t}\big((1 - a)t - \lambda R - p_2 + p_B\big)$$

$$D_B = x_2 - x_1 = \frac{1}{t}(2\lambda R - (1 - a - b)t + p_1 + p_2 - 2p_B)$$

Symmetrical margins will apply here just as in the previous subchapters on bundling. Crucially, we now ask not only that the buyers of the bundle have binding utility but that buyers of the single good at the ends of the line are also considered. Single purchase prices therefore become

$$u_1(0) = 0 \Rightarrow p_1 = R - at$$

$$u_2(1) = 0 \Rightarrow p_2 = R - bt$$

Like in the competitive model we require $x_1 < 1 - b$ and $x_2 > a$ to achieve the above expressions, but these restrictions will not be a problem.

The utility curve from buying the bundle is a flat line between the two goods' locations because the transport involved in consuming both is constant. Let therefore $u_B(x) = 0$ for $x \in [x_1, x_2]$. The bundle's price is then

$$p_B = (1 + \lambda)R - (1 - a - b)t$$

This expression assumes that no consumers outside of the flat part of the curve should ever buy the bundle. That is, $x_1 > a$ and $x_2 < 1 - b$. Following that assumption, the monopolist can practice perfect price discrimination against the group that buys the bundle because he knows that they will all have the same transport costs of $(1 - a - b)t$ from consuming both goods. From a group with heterogeneous preferences comes thus an identical sum along that part of the curve.

Substituting the price functions into demand gives a set of very simple expressions.

$$D_1 = 2a$$

$$D_2 = 2b$$

$$D_B = 1 - 2a - 2b$$

Consequently, profit is reasonably simple as well.

$$\begin{aligned} \pi &= (p_1 - c)D_1 + (p_2 - c)D_2 + (p_B - 2c)D_B \\ &= 2a(R - at - c) + 2b(R - bt - c) + (1 - 2a - 2b)\big((1 + \lambda)R - (1 - a - b)t - 2c\big) \end{aligned}$$

So far we have only discussed pricing at the margins. Now symmetry is introduced by saying that $a = b$. Note that single purchase demand, which has to be positive, is then defined for $a > 0$, and bundle demand, which also has to be positive in order for the model to make sense, is defined for $a < \frac{1}{4}$. Already we see that maximal differentiation cannot happen.

Profit simplifies to

$$\pi = 4a(R - at - c) + (1 - 4a)\big((1 + \lambda)R - (1 - 2a)t - 2c\big)$$

To find the optimal level of differentiation we take the first order condition of profit with respect to location. Second order conditions can verify that we have a maximum.

$$\frac{\partial \pi}{\partial a} = 0 \Rightarrow a = \frac{1}{12t}(3t + 2c - 2\lambda R) \tag{4.9}$$

Profit is maximized at an intermediate level of differentiation. The location of each good will be closer to the ends of the line for greater reservation prices for the second purchase because it will then pay to have more people buy the bundle. After all, the bundle extracts all surplus from its buyers. Opposing that effect, we also find that the higher the transportation cost, the more important it is to segment away the low-paying consumers at the line's extremes by having them buy only a single good. The latter observation comes from $\frac{\partial a}{\partial t} = \frac{\lambda R - c}{6t}$, which is positive when $\lambda R > c$. As we will soon see, that condition has to be true anyway, so location is always increasing in transportation costs.

Summing up we have the following proposition:

Proposition 3: *Mixed bundling will result in intermediate differentiation. The two goods will always have at least half the line between them as long as mixed bundling is defined. Differentiation increases when*

i) *the willingness to pay for a second good increases* $\left(\frac{\partial a}{\partial \lambda} < 0\right)$

ii) *transport costs decrease* $\left(\frac{\partial a}{\partial t} > 0\right)$

Figure 4.4 shows the resulting utility curves using the same parameters as we have used in previous graphs.[19] The figure makes it clear just how many consumers the monopolist is able to meet with perfect price discrimination; the group buying the bundle is just a long line squeezed flat on the *x*-axis. The customers who only buy one good are still left with some surplus.

[19] Made using $R = 10, \lambda = 0.5, t = 5, c = 1 \Rightarrow a = b = \frac{7}{60} = 0.11\bar{6}$

Figure 4.4: Utility with mixed bundling

Returning to the requirement that demand for the bundle should be positive, $D_B > 0$, which resulted in $a < \frac{1}{4}$, we insert (4.9) into the inequality and find that this is true as long as $c < \lambda R$. Hence, we are left with the claim that if bundling is going to happen, then the willingness to pay for the second good should be greater than the cost of producing it – not an unreasonable demand. If the condition does not hold, we will have single sales only.

Requiring that demand for the individual goods should be positive, $D_1 > 0$ for example, is the same as saying that $a > 0$. Using (4.9) the inequality becomes $\frac{3}{2}t + c > \lambda R$. This is the same condition as we have seen in Table 3.3 for the requirement that not all consumers buy one of the goods in the competitive model and serves the same purpose here: if not everyone buys good 2, then some are buying just good 1. If the willingness to pay for a second good becomes too big and the condition does not hold, we will have pure bundling. Altogether, mixed bundling works over the following interval:

$$c < \lambda R < \frac{3}{2}t + c \tag{4.10}$$

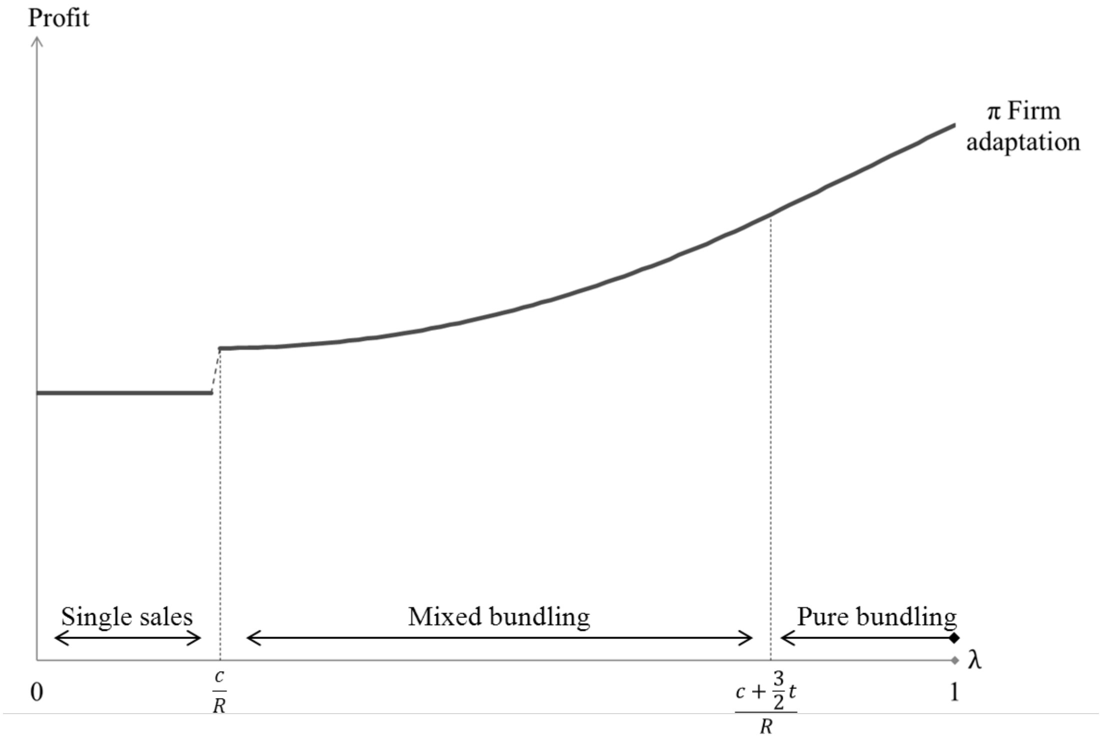
Compared to mixed bundling with fixed locations at the endpoints, the upper limit to the willingness to pay is now higher. The interval is further discussed in the next subchapter.

4.5 Discussion of results

The monopolist adapts to achieve symmetrical margins. Symmetry is needed to minimize utility lost to transport costs. The focus on the marginal consumer is a simple question of what share of the market to cover.

Figure 4.5 below illustrates the monopolist's profit when $R = 10, t = 4, c = 2$. Single sales mean that no consumers buy a second good, so profit is independent of λ and appears as a flat line. Profit under mixed bundling is a quadratic in terms of λ because it appears both in the expressions for price and – through a – for demand. Pure bundling, which takes over from mixed with no discontinuity, is linear in terms of λ.

Figure 4.5: Profit for each demand scenario in mixed bundling with endogenous locations



Compared to Adams and Yellen (1976), who find that pure bundling, mixed bundling and single sales can all be the most profitable choice, the strategic choices are simpler in our model. As long as mixed bundling is possible, it will be better than the other options. The discontinuity in Figure 4.5 between single sales and mixed bundling gives an easy illustration of why this claim must be true in that case. Algebraically, the profit from mixed is greater than that from pure bundling as long as $\lambda R < \frac{3}{2}t + c$, but that has to be true for mixed bundling to be defined anyway. Adams and Yellen required the goods to be independent. In an address model they are anything but. Indeed, firms can choose their own degree of substitutability – the opposite of differentiation – through their locations, which affect the consumers because of transport costs. Although mixed bundling will only happen when the goods are at least somewhat apart, substitutability will increase when transport costs increase in our final model.

Choi (2010) was the direct inspiration for this paper. He also finds that “the number of multi-homing consumers increases with tying” (Choi, 2010, p. 619). Even though his bundle includes a different set of goods than ours and is based on fixed locations at the ends of the line, the conclusion reinforces the link between multi-purchasing when people enjoy both brands and bundling as a tool for harvesting consumers' diversity.

Returning to media markets, where the marginal cost of production is likely negligible, the bundle will be sold for all reservation prices for the second good because the lower limit of (4.10) is then zero. The upper limit is the same as under the competitive duopoly in chapter 3, and even with nonzero costs the monopoly model has a weaker lower limit. Multi-purchasing is therefore more likely to occur under a monopoly than under the ordinary duopoly. More people are likely to read both newspapers if they are made by a monopolist than if they are only offered competitively.

5 Conclusion

This paper has sought to map the differentiation of the linear Hotelling (1929) model's two goods when consumers are permitted to buy both of them. In a competitive market the result is minimum differentiation given that multi-purchasing does indeed occur. As opposed to the ordinary linear Hotelling model, our result is stable but – according to our conjecture – only as long as the goods have the same marginal cost. Differentiation increases under monopolistic bundling but is not maximal.

Multi-purchasing in a competitive market has been modeled before under quadratic transportation costs by Kim and Serfes (2006). They, too, find minimum differentiation and explain the observation by what they call the aggregate demand creation effect. Simply put, approaching the middle of the market no longer steals customers from the other good, but enlarges the group willing to buy both. We confirm this effect in the linear model. Another consequence of the effect is that price competition between the goods disappears. Hence, the old problem of Bertrand competition at the midpoint, which is the source of the linear model's stability issues, is also gone.

Transportation costs, be they linear or quadratic, represent the degree to which consumers dislike having to buy a good that is not perfectly aligned with their preferences. In single purchase models this unwillingness to buy something far away from one's ideal works as a competitive barrier to increase the firms' profits. Yet transportation determines the intervals for which multi-purchasing occurs. Since the aggregate demand creation effect means that both firms want to increase the number buying both goods, transportation costs pull profits down in the competitive multi-purchasing model.

Two-sided markets and other media models need multi-purchasing. Even though buying from both platforms/newspapers is important enough to have a term of its own – multi-homing – in the literature on two-sided markets and the linear city is a common building block on the consumer side, no one has yet tested how advertisers will change the newspapers' incentive to create generic content. This paper provides simple conditions for when multi-purchasing will occur and opens for implementation in an extended system in future research. Furthermore, it also lays the groundwork for a study of bundling in two-sided markets.

Finally, differentiation is greater under a monopoly than a duopoly. Multi-purchasing is also more likely to occur. Variety is therefore greater under a monopoly. Contrary to expectations as that observation may be, anti-trust regulators or other government agencies should not conclude based only on this paper on what a social planner would do. First of all, it may very well be that diversity is not really wanted by the consumers. Any preference for variety would have to involve an interpretation of λ because it is what multi-purchasing depends on. Second, the welfare implications of multi-purchasing in the linear Hotelling model have not been properly discussed in this study. Part of such an exercise would also include calculating whether the two duopolists would want to merge, as chapter 4 of this paper simply takes for granted. All this paper has done in terms of welfare is to determine that a monopolist would provide a more restrictive supply than socially optimal in the cases of single sales only and pure bundling, but that is hardly a surprise. The question of the socially optimal locations for two goods under multi-purchasing remains unanswered. In conclusion, we have not yet established if variety is a good thing, but we now know that a Hotelling duopoly will have none of it.

6 Appendix

Proof that the assumptions behind the covered and uncovered model cannot be true at the same time

Recall that (3.19) gives the condition for an uncovered market:

$$2R < t(a+b) + p_1 + p_2$$

The market is covered if (3.19) has the opposite direction. Using the price reaction functions from the covered model give

$$\lambda R < 2R - \frac{3}{2}(a+b)t - \frac{1}{2}(c_1 + c_2)$$

The price reaction functions from the uncovered model give a rewritten version of (3.20):

$$3R - 2(a+b)t - c_1 - c_2 < \lambda R$$

Suppose there is a combination of parameters such that both models pass all their assumptions. Then both the above inequalities are true and we get

$$3R - 2(a+b)t - c_1 - c_2 < \lambda R < 2R - \frac{3}{2}(a+b)t - \frac{1}{2}(c_1 + c_2)$$

$$R < \frac{1}{2}(a+b)t + \frac{1}{2}(c_1 + c_2)$$

Yet this cannot be true at the same time as the multi-purchasing requirement in the covered model from Table 3.2 when $\lambda \leq 1$. Hence, both models cannot be true at once.

7 Bibliography

Adams, W. J., & Yellen, J. L. (1976). Commodity bundling and the burden of monopoly. *Quarterly Journal of Economics, 90*(3), 475-498.

Anderson, S. P., & Neven, D. J. (1989). Market efficiency with combinable products. *European Economic Review, 33*(4), 707-719.

Anderson, S. P., & Neven, D. J. (1991). Cournot competition yields spatial agglomeration. *International Economic Review, 32*(4), 793-808.

Anderson, S. P., de Palma, A., & Thisse, J.-F. (1989). Demand for differentiated products, discrete choice models, and the characteristics approach. *Review of Economic Studies, 56*(1), 21-35.

Anderson, S. P., de Palma, A., & Thisse, J.-F. (1992). *Discrete choice theory of product differentiation.* Cambridge, MA: MIT Press.

Anderson, S. P., Foros, Ø., & Kind, H. J. (2012). Hotelling competition with multi-purchasing. *Mimeo, Norwegian School of Economics*.

Anderson, S. P., Goeree, J. K., & Ramer, R. (1997). Location, location, location. *Journal of Economic Theory, 77*(1), 102-127.

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics, 37*(3), 668-691.

Brenner, S. (2001). Determinants of product differentiation: a survey. *Mimeo, Humboldt University*.

Choi, J. P. (2010). Tying in two-sided markets with multi-homing. *Journal of Industrial Economics, 58*(3), 607-626.

d'Aspremont, C., Gabszewicz, J., & Thisse, J. F. (1979). On Hotelling's "Stability in competition". *Econometrica, 47*(5), 1145-1150.

Downs, A. (1957). *An economic theory of democracy.* New York: Harper & Row.

Gabszewicz, J., & Thisse, J. F. (1992). Location. In R. J. Aumann, & S. Hart (Eds.), *Handbook of game theory with economic applications* (Vol. 1, pp. 281-303). Amsterdam: North Holland.

Gal-Or, E., & Dukes, A. (2003). Minimum differentiation in commercial media markets. *Journal of Economics & Management Strategy, 12*(3), 291-325.

Hagiu, A. (2009). Two-sided platforms: product variety and pricing structures. *Journal of Economics & Management Strategy, 18*(4), 1011-1043.

Hotelling, H. (1929). Stability in competition. *Economic Journal, 39*(153), 41-57.

Kim, H., & Serfes, K. (2006). A location model with preference for variety. *Journal of Industrial Economics, 54*(4), 569-595.

Kreps, D. M. (1990). *A course in microeconomic theory.* Princeton: Princeton University Press.

Lancaster, K. (1971). *Consumer demand: a new approach.* New York: Columbia University Press.

Lewbel, A. (1985). Bundling of substitutes or complements. *International Journal of Industrial Organization, 3*(1), 101-107.

Martin, S. (2002). *Advanced industrial economics* (2nd ed.). Oxford: Blackwell.

Neven, D. J. (1986). On Hotelling's competition with non-uniform customer distributions. *Economics Letters, 21*, 121-126.

Peitz, M., & Valletti, T. M. (2008). Content and advertising in the media: pay-tv versus free-to-air. *International Journal of Industrial Organization, 26*(4), 949-965.

Porter, M. E. (1985). *Competitive advantage.* New York: The Free Press.

Porter, M. E. (2000). Location, Competition, and Economic Development: Local Clusters in a Global Economy. *Economic Development Quarterly, 14*(1), 15-34.

Rath, K. P., & Zhao, G. (2001). Two stage equilibrium and product choice with elastic demand. *International Journal of Industrial Organization, 19*(9), 1441-1455.

Rochet, J.-C., & Tirole, J. (2002). Cooperation among competitiors: some economics of payment card associations. *RAND Journal of Economics, 33*(4), 549-570.

Rochet, J.-C., & Tirole, J. (2006). Two-sided markets: a progress report. *RAND Journal of Economics, 37*(3), 645-667.

Sajeesh, S., & Raju, J. S. (2010). Positioning and pricing in a variety seeking market. *Management Science, 56*(6), 949-961.

Salop, S. C. (1979). Monopolistic competition with outside goods. *Bell Journal of Economics, 10*(1), 141-156.

Salvanes, K. G., Steen, F., & Sørgard, L. (2005). Hotelling in the air? Flight departures in Norway. *Regional Science and Urban Economics, 35*(2), 193-213.

Schmalensee, R. (1982). Commodity bundling by single-product monopolies. *Journal of Law and Economics, 25*(1), 67-71.

Schmalensee, R. (1984). Gaussian demand and commodity bundling. *Journal of Business, 57*(1), S211-S230.

Stiglitz, J. E., & Dixit, A. K. (1977). Monopolistic competition and optimum product diversity. *American Economic Review, 67*(3), 297-308.

Tirole, J. (1988). *The theory of industrial organization.* Cambridge, MA: MIT Press.