What is the optimal allocation towards real estate in the portfolio of the Global Pension Fund Global?

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Abstract

In this master thesis we evaluate the optimal future investment allocation towards real estate for the Norwegian Government Pension Fund Global (GPFG). Based on an assessment of the relative risk and return attributes of equities, bonds and real estate - and using a mean-variance optimization – we have found that the fund should allocate a full 11,2 % of its capital towards real estate (59,4 % to equities and 29,4 % to bonds). This is twice the current target level, and would represent an additional 235,6 BNOK (42 BUSD) of GPFG funds being allocated to investments in the global real estate markets.

In performing the above analysis we have been able to rely on a fairly well documented analysis based on long term global data for the performance and volatility of bonds and equities. Our key focus has been to assess and derive the appropriate performance characteristics of real estate. By doing looking at different property data, we have been able to develop a well-founded view of the historic performance of real estate over the last 25 years. In addition to this we have made a qualitative assessment of the asset class and have used this to develop what we feel are robust and reasonably conservative estimates for the expected future performance characteristics of a global property portfolio.

Because of several specific characteristics of real estate it has been argued that it cannot be analyzed in a simple mean variance framework. We have therefore tested the robustness of our findings by applying additional perspectives and approaches. On this basis we remain convinced that no substantial additional adjustments need to be done to the application of a mean variance framework to account for real estate specific risk and cost aspects.

Based on our analysis, we are confident that the GPFG over time would benefit from increasing its allocation towards real estate to approximately at least 10 %. This could contribute to improving the risk return relationship of the portfolio, as measured through the Sharpe ratio. We have quantified the likely effect from an improvement in the risk reward ratio to 250 million NOK (45 mill USD) in additional return per year, with the current market capitalization of the fund.

We have tested our findings by applying the expectations of the Ministry of Finance in our mean-variance framework. With their input data, we find that the exposure to the world property markets should increase to 9 %, and that the fund would be able to realize significant benefits by shifting their allocations more towards real estate at the expense of bonds.

A comparison with the allocation strategies of similar funds also demonstrates that the targeted GPFG allocation of 5 % to real estate is clearly below the average. We see this as a further validation of our clear findings that the GPFG should increase its exposure to the global real estate market.

Preface

This thesis was written as part of my Master's degree in Financial Economics at NHH. It is worth 30 ECTS credits, which corresponds to a full semester of studies.

It has been very exciting to apply what I´ve learned as a student to an interesting, important and "real life problem". The amount the GPFG ultimately decides to allocate towards real estate investments, will indeed affect the welfare of generations of Norwegians to come.

I want to thank my supervisor Thore Johnsen for all his contributions during my work with this thesis. His insightful challenges, kind support and willingness to be an active speaking partner – have been tremendously helpful and made the process of researching the thesis both fun and rewarding.

Håvard Bjørå also deserves a special thanks. He helped me get access to the International Property Data (ipd.com), which proved to be critically important to gaining a deep insight into the history and performance of the worldwide property markets.

I am as always grateful for the continuous support, interest and patience received from my family, and my girlfriend.

Bergen, December 14th

Jon Anders Matre

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Abbreviations:

CAL: Capital Allocation Line

CAPM: Capital Asset Prising Model

CPI: Consumption Price Index

DMS: Dimson, Marsh and Staunton (the name of the three authors behind Triumph of the optimist)

EMH: Efficient Market Hypothesis

GDP: Gross Domestic Product

GPFG: Global Pension Fund Global

IPD: International Property Data

MPT: Modern portfolio theory

MSCI: Morgan Stanley Capital International

NAREIT: North American Real Estate Investment Trust

NASDAQ: National Association of Securities Dealers Automated Quotations

NBIM: Norges Bank Investment Management

NCREIF: National Counsel of Real Estate Fiduciaries

NOK: Norwegian Kroner

REITs: Real Estate Investment Trusts

TBI: Transaction Based Index

WACC: Weighted Average Cost of Capital

Problem and restrictions

In this thesis we will respond to the following question: "What is the ideal capital allocation towards real estate for the GPFG?"

We define a real estate investment as: *the purchase of a property or land with the right to build properties, either directly or indirectly*. It is also possible to gain exposure to real estate through debt instruments, but this we have excluded from our analysis, since the risk and return attributes would be closer to bonds than to real estate. The overall purpose and long-term goals of the GPFG are taken as given, and will not be challenged.

Introduction

The Government Pension Fund Global (GPFG) was founded in 1990 as a vehicle for the long-term investment of Norway's excess petroleum revenue. Today the fund has a capital of more than 3,7 trillion NOK (roughly 0,67 trillion USD), and is thus the largest investment fund in the world. The fund will continue to receive very significant additional capital infusions in the years to come. As the primary purpose of the GPFG is to safeguard the Norwegian welfare model for future generations, the management of the fund has been hotly debated and to some extent criticized in the media. However, we feel that much of the recent criticism appears poorly founded in economic theory, and taking a too short-term view.

In 2008 the Ministry of Finance decided that Real Estate should be included as part of the portfolio of the GPFG. They directed Norges Bank Investment Management (NBIM) to gradually phase in real estate as part of the portfolio, and to reduce the allocation to bonds until Real Estate would constitute up to 5 % of the capital of the fund. Interestingly this recommendation was not based on a clearly defined optimization exercise. The move was described as a way to start getting exposure to a third asset class, and thereby achieve additional diversification benefits. The ideal long-term allocation towards real estate is yet to be determined, which is also why we wanted to look at exactly this aspect of the management of the GPFG in our thesis.

We will mainly discuss the ideal allocation towards real estate based on a Markowitz mean-variance framework. The result of this analysis will depend on the input variables in the model, which in this case are the expected returns and volatilities for equities, bonds and real estate, as well as the correlation between the asset classes. Firstly, we have used the estimates of the Norwegian Ministry of Finance to see what the ideal allocation towards real estate would look like using mean-variance optimization with their expectations. Secondly, we have carefully developed our own view related to the risk and return attributes of the different asset classes, based on an assessment of historic return data combined with more qualitative assumptions. Based on these expectations we have done the Markowitz optimization again.

To further validate our findings we have also taken a high level look at some aspects that fall outside a traditional Markowitz optimization, to understand whether any of these elements are likely to significantly alter or challenge our conclusions. Finally, we have compared our findings with the allocations of other similar large funds. Relevant theory will be explained early on to facilitate the understanding of the analysis for the reader.

Briefly about NBIM and the GPFG

The Government Pension Fund Global (GPFG) was established in 1990. Norges Bank Investment Management (NBIM), which is part of the Norwegian central bank manages the fund[1]. The Ministry of Finance provides the fund with the investment mandate, which is primarily based on recommendations from NBIM and discussions in the Parliament. Petroleum revenues are regularly transferred to the fund from the Ministry of Finance. The capital is invested abroad to avoid overheating of the Norwegian economy and to make it less vulnerable to oil price fluctuations.

There are two purposes behind the fund. Firstly, it is intended to provide the government with a fiscal policy tool versus "the mainland economy". Secondly, it is designed to support the government in confronting the challenges related to keeping up with future pension liabilities. It is important to state that no decision has been made concerning when the money should be withdrawn. Therefore, the fund is managed with a long-term perspective, but one should be able to draw on it when needed.

Goals, timeframe and restrictions

The goal of the GPFG "is to maximize the international purchasing power of the fund capital, given a moderate level of risk"[2]. The fund is unique in that it has no clearly defined liabilities and because of its size. As of October 23, 2012 the fund manages more than 3 726 billion NOK (equivalent to more than half a trillion USD).

Real estate and the GPFG

NBIMs first investment in real estate happened in 2011, more than 20 years after the fund was established. This goes to show that it wasn´t an easy decision to include real estate as part of the GPFG portfolio. In October 2006 NBIM recommended to the Norwegian Ministry of Finance to include real estate investments as part of the

[1] (Norges Bank Investment Management, 2011)
[2] (Ministry of Finance)
[3] (Norges Bank , 2006)

investment mandate[3]. NBIM made this recommendation, asserting that it would enable them to realize more diversification benefits in the portfolio[4]. They recommended that real estate over time should ideally constitute 10 % of the invested capital.

In 2008 the Ministry of Finance decided that real estate should be included as part of the GPFG portfolio. However, the current mandate only allows for 5 % of the fund's assets to be allocated towards real estate. It has also been decided that the real estate investments will be made through equity instruments.

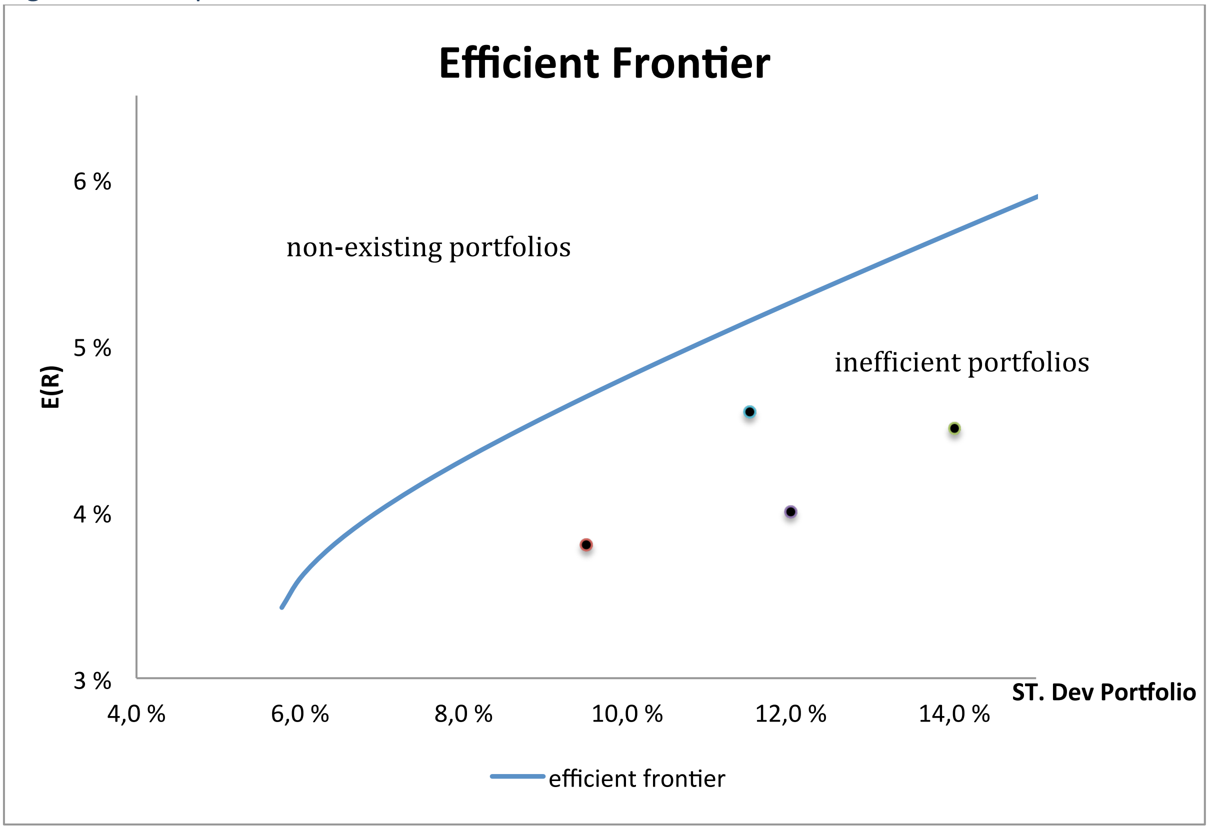
[3] (Norges Bank , 2006)
[4] (Norges Bank , 2006)

Theory

Modern Portfolio Theory

Modern portfolio theory (MPT) is largely based on a framework developed by Harry Markowitz, called mean-variance portfolio optimization[5]. His key point was that through proper diversification the investor could increase his overall expected return without increasing the volatility. He claimed that this optimization represented “a free lunch”. The framework he proposes can be used to optimize the allocation towards different assets or securities, given the simplification that the investor only is concerned with the expected return and volatility of the portfolio. By minimizing the volatility for given expected returns the investor is left with a set of rational asset allocations, called “the efficient frontier”. This frontier is displayed graphically in the figure below. Here the expected arithmetic return is illustrated on the y-axis, while the volatility (measured in terms of standard deviations) can be read from the x-axis. The blue line represents the efficient frontier, while the dots to the right and below are examples of inefficient portfolios, since you could increase the expected return without affecting the volatility.

Figure 1: Example of an efficient frontier



5 (Markowitz 1952)

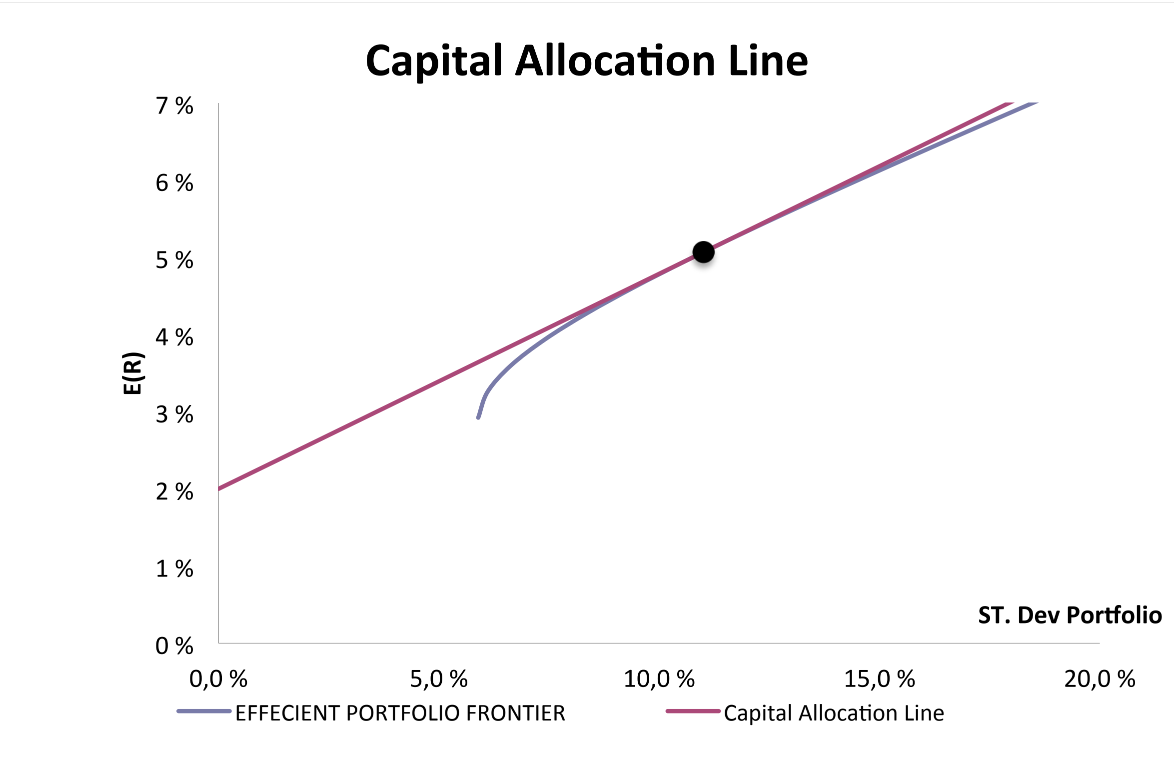
James Tobin further developed this concept when he demonstrated what he called the separation theorem in 1958[6]. He argued that when there is a risk free asset that the investor could use for lending and borrowing, the optimal allocation was independent of the investors risk preferences. Since the risk (in terms of volatility) could be reduced or increased by different combinations of the risk free asset, all investors should hold the same portfolio, which is the one that maximizes the excess return over volatility. This relationship is referred to as the Sharpe ratio, after William Sharpe. It can be seen as a measure of how well the investor is compensated for the risk he is taking on, and is frequently used to compare different portfolios[7]. The formula is illustrated below.

Formula 1: The Sharpe ratio

$$Sharpe\ ratio = \frac{portfolio\ return - \ risk\ free\ rate}{Volatility}$$

All combinations of the optimal risky portfolio and the risk free asset are on the so-called capital allocation line (CAL). An example of this can be found in the figure below where the CAL is represented by the straight red line.

Figure 2: Example of the efficient frontier and the Capital Allocation Line



[6] (Tobin, Liquidity preference as behaviour towards risk, 1958)

[7] (Sharpe, 1966)

An investor unwilling to take on any risk would receive the risk free rate (arbitrarily set at 2 % in figure 2), while others will be proportionally compensated through a higher expected return depending on the level of volatility they accept. The slope of the CAL is the same as the Sharpe ratio, which in this case is about 0,25. This implies that for each percentage point of added volatility the investor accepts, the expected return increases with 25 basis points. In the graph the tangency point represents a portfolio without use of the risk free asset (the black dot in figure 2). For all points on the CAL that are to the left of this lending is used, while all adaptations to the right of this point contain leverage.

CAPM

The capital asset pricing model (CAPM) was developed in the early 1960s, and is still popular among practitioners. It is typically used to find the cost of capital that should be employed when valuing an asset. The model relies on several assumptions such as: perfect capital markets, full divisibility of assets and that the investors are rational mean variance optimizers with homogenous expectations. Empirically testing demonstrates that it has decent prediction power. When other risk elements are added (such as size and value) it performs very well[8].

In CAPM a clear distinction is made between firm-specific risk and market risk of a security. The firm specific risk can be eliminated through diversification, while the second represents a so-called "non-diversifiable risk". It is assumed that the marginal investor (i.e. the price setter in the market) is fully diversified, since she would have the lowest cost of capital and would thereby value securities higher. Hence, market risk is deemed the only relevant risk measure, and is typically denoted with the Greek letter for Beta, where an average asset has a beta of 1. The compensation the investor receives for a positive Beta depends on the risk premium in the market, which typically is around 5 %. The CAPM formula is displayed below:

Formula 2: CAPM

$$Cost\ of\ capital = Risk\ free\ rate + Beta * Risk\ Premium$$

[8] (Eugene F. Fama, 1993)

An interesting implication of the formula is that when an asset has a negative Beta, meaning that it correlates negatively with the market, its cost of capital (which is equivalent to the expected return since investors are assumed to be rational) can be lower than the risk free rate. This is because the asset is valuable in reducing the overall volatility of the portfolio. Furthermore since the CAPM can be applied to all assets, arbitrage will in theory ensure that the model holds. Consequently it should be rational for an investor to hold a market-weighted portfolio of all tradable assets in the economy.

The investment universe

The relevant investment universe basically reaches across the whole world, and is extensive in the range of products. An investor can trade gold, oil and even pork bellies. However, we will focus on the three largest asset classes: equities, bonds and real estate. This is because for the GPFG and similar funds, they traditionally focus primarily on these investment vehicles. It is also usually assumed that an investor can take part in most of the value creation in the world by being exposed through these assets.

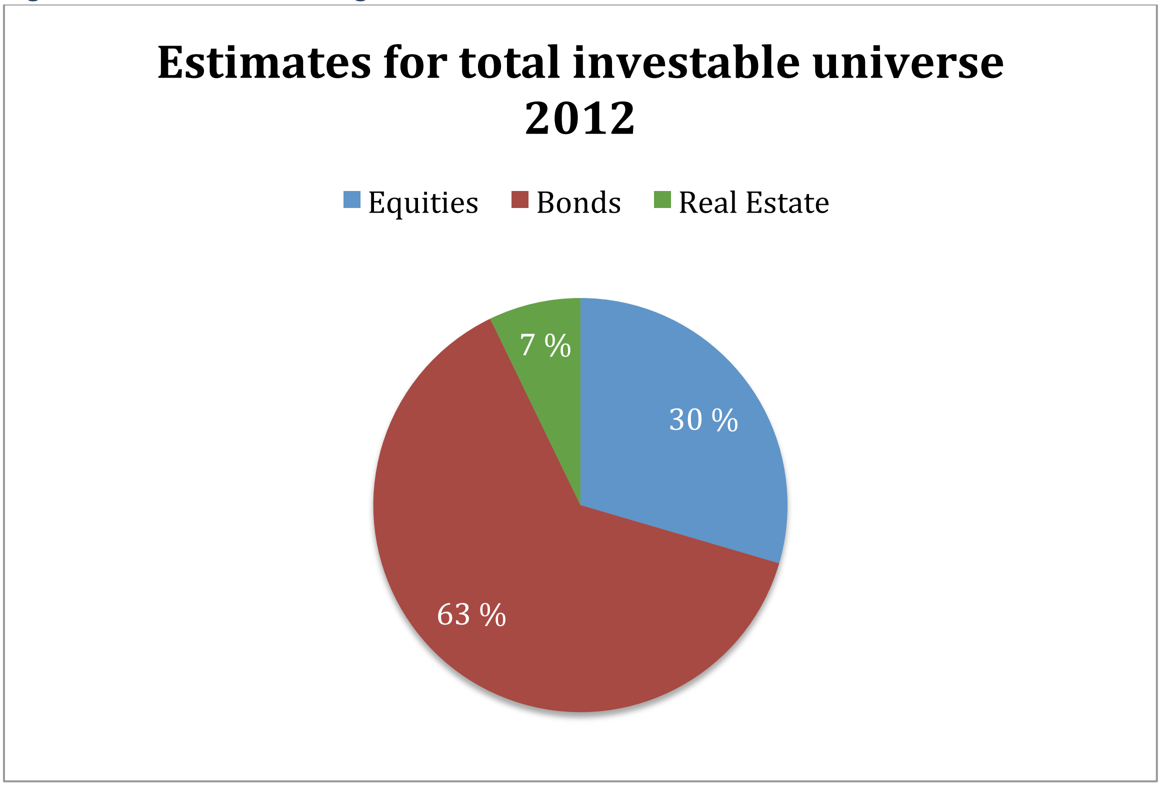
We demonstrated the underlying assumptions hold CAPM implies that it is rational for an investor to hold the market portfolio. If you think of the market portfolio as the global investable universe, it seems obvious that real estate merits a significant share in a globally diversified portfolio. In the following this idea will be pursued.

We estimate that "the investable real estate universe" is worth roughly 8,5 trillion USD in 2012. This is based on numbers that were provided in “Commercial Real Estate: Analysis & Investments” in 2007, and have been adjusted to reflect the nominal appreciation in property values[9]. The global market capitalization of equities is found implicitly by looking at how big of a share NBIM says the GPFG comprises of the total market (see appendix 2)[10]. In the estimation of the aggregate world bond market we have relied on the numbers presented by qvmgroup, who in turn have gotten their numbers from several trustworthy sources (see appendix 3)[11].

9 (Geltner, Miller, & Clayton, 2007)
10 (Norges Banks Investment Management, 2012)
11 (qvmgroup, 2012)

In the graph below (figure 3) one can find how large of a share equities, bonds and real estate comprises of the total global investable universe, based on my computations. Real estate is market in green, while bonds and equities are represented by the red and blue colors respectively.

Figure 3: Estimates for the global investment universe[12]



From figure 3 one can see that bonds constitutes the biggest share of the total investable universe (63 %), followed by equities (30 %). Real estate comprises 7 % of the world's three largest asset classes combined (when infrastructure is not included). However, Real Estate as an investment class has several characteristics that make it different from investments in shares and bonds. Property investments tend to be less liquid, not easily dividable (e.g. it is difficult to buy a small part of a building) and the transaction costs can be high. The notion that the real estate market in general is seen as less efficient makes CAPM somewhat less applicable for this asset class. So how much of a funds capital that should be allocated towards real estate needs to be decided on a broader basis.

[12] (Geltner, Miller, & Clayton, 2007), (qvmgroup, 2012), (Norges Banks Investment Management, 2012)

The efficient market hypothesis (EMH)

The EMH is the hypothesis that markets fully reflect all available information[13]. It is important to develop a view regarding what we think about the efficiency of the market, because it will have implications for the way an investor manages his money. If we believe that the market is efficient then the passive strategy of investing in the market index would be rational. This is called the mutual fund theorem. Since it is so important EMH is probably one of the paradigms in finance that has been most widely tested. In general one has found strong empirical support for the hypothesis when it comes to stocks and bonds. However, it is complicated given that any test of the efficient market hypothesis is dual, as you necessarily test your asset-pricing model at the same time.

The EMH and real estate

Grossman and Stiglitz have argued that for EMH to hold true, both the trading cost and cost of retrieving information would have to be zero[14]. When this is not true, they assert that prices will reflect information up to the point of the marginal cost of acting on it. This can partly explain why real estate tends to be seen as less efficient, since local knowledge can be hard to retrieve and transaction costs are significant.

Yet other economists have gone further than this in claiming that bubbles are frequent in the real estate market[15]. In "a bubble" the prices of a group of assets are so significantly different from their intrinsic value, that it has to be caused by irrational "herd behavior". One of the reasons why this might be more likely to happen in real estate is that it is difficult to go short, so that the view of the most optimistic will reign. Furthermore, the housing market is particularly vulnerable to bubbles as buyers tend to be less professional and are thereby more inclined to make irrational investments during the bubble euphoria[16].

13 (Fama, 1991)
14 (Sanford J. Grossman, 1980)
15 (Shiller, 2009)
16 (Case & Shiller, 1989)

Random walk

Earlier scholars believed that the stock markets and other tradable securities behaved in "a random walk"[17]. This means that the future fluctuations were independent of past price development, something that was seen as support of the EMH. Under this assumption the investor that has constant risk aversion should rebalance his portfolio so that he holds a fixed mix of assets from a risk perspective. It furthermore implies that the risk will be constant with time (i.e. no time diversification).

Mean reversion

Siegel demonstrated that the yearly variance for equities tends to be lower when the holding period increases[18]. All else kept equal, this means that the share of equities in a portfolio should increase with the timeframe of the investment, as the risk is less and the expected return (measured annually) is the same. It could also indicate that stocks exhibit mean reversion. It has been argued that this is a sign of excess volatility of stocks (i.e. that the prices move more than to account for fundamental changes), which would be conflicting with the EMH[19]. Recently, academics are increasingly agreeing that it is the cost of capital, and not the stock prices, that follow a random walk in an efficient market. Random walk in the cost of capital could easily lead to mean reversion, e.g. if the investors demanded a higher expected return due to increased uncertainties, it would lead the stock markets to fall. However, the higher expected return makes it likely that the stock market will rebound to its trend in due course.

The key implication of mean reverting prices would be that rebalancing of the portfolio becomes even more attractive. Independent of whether it forces you to buy dear and sell cheap or whether you are taking advantage of time varying risk premiums, it will be beneficial. This is also partly due to the fact that the volatility gets reduced which is synonymous to less risk[20]. Less fluctuation also improves the geometrical return, as we will see later. Going through with a rebalancing in practice can require a lot of persistence, particularly after dramatic falls in prices of one asset class, as it would be

[17] (Samuelson, 1969)
[18] (Siegel, 2008)
[19] (Shiller, Do Stock Prices Move Too Much to Be Justified by Subsequent Movements in the Dividends, 1981)
[20] (Erb & Harvey, 2005)

easy to redefine the characteristics just after a big drop in values. This can however be the time when it is particularly beneficial to get the asset allocation back to its target. The GPFG were among those who benefitted greatly from sticking to their rebalancing-policy during the financial crisis in 2007-2008, although the general public was concerned.

Liquidity premium

We define liquidity as the ease with which an investment can be turned into cash. Illiquid investments can also be difficult to buy at a chosen point in time, and large investors can easily move prices when large transactions are done. Less liquid assets also tend to fall particularly hard when the market is contracting. Most academics agree that there tends to be a liquidity premium in the market, meaning that investors are compensated for taking on illiquid investments through a higher expected return.

Special capabilities

It is important for an investor to be aware of his capabilities within the different asset classes, as special competences could shift the optimal allocation. A management that has the rare ability to generate excess return, often referred to as "alfa", within an asset class should overweight this relative to the market portfolio. Under efficient markets this would be impossible, and even when the market is inefficient it is a zero-sum game, meaning that in aggregate it is impossible to generate alfa, but that a few skillful investors can expect to "beat the market".

Risk

NBIM analyzes risk along four different dimensions: operational, market, credit and counterparty risk[21]. We will only look at operational risk and market risk, since these in our view are the most relevant when making real estate investments.

Operational risk

From Basel 2 we have the following definition of operational risk: "the risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events"[22]. When we talk of operational risk we will include both legal and reputational risk as part of the concept, which is in line with NBIMs view. Norges Bank`s Executive Board has decided that there must be less than a 20 % chance that "unwanted events" will have financial consequences of half a billion NOK or more in any given year.

Market risk

Market risk is usually seen as the risk that cannot be diversified away (see explanation of CAPM above). NBIM has identified the most important aspects of such risk as: fluctuation in stock prices, interest rates, exchange rate and credit risk changes for the bond investments. The most critical determinant of market risk exposure is considered to be the choice of benchmark portfolio, including the weighting of the different asset classes. Fluctuations in property values and the income it produces can be seen as the most important elements of market risk for real estate investments. The problem is that it is hard to measure the true volatility of the return. As we will discuss and show, the volatility tends to be perceived as too low, making the investor feel that the investment is safer than what it really is.

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[21] (Norges Bank Investment Management, 2012)

[22] (Basel Committee , 2001)

Interest rates and exchange rate risk will not be discussed in more depth, as the exposure to these risk factors probably won´t be altered significantly by including real estate as part of the portfolio.

For several reasons we expect it to be difficult to separate diversifiable risk from market risk when it comes to real estate. This is partly because the standard quantitative approach used when estimating the beta of stocks (and hence the exposure to non-diversifiable risk) cannot be used for real estate, as the fluctuations in returns aren't measured regularly enough to give precise estimates to the underlying volatility for particular markets, and even less for specific properties. Therefore, a more qualitative approach will have to be taken when estimating the increasing market risk from adding a given property investment to the portfolio.

Investment vehicles in real estate

We will define real estate investments as buying land or buildings attached to land either directly or indirectly. Thereby we are excluding debt instruments, like mortgage-backed securities, from our analysis. Infrastructure investments are not included as part of our definition either.

Indirect property investments

Examples of indirect real estate vehicles would be: real estate mutual funds (usually closed end), listed real estate companies, real estate private equity funds and REITs. We will only focus on REITs, as this is the largest and most relevant way to indirectly invest in real estate for a large global investor.

REITS

Real Estate Investment Trusts (REITS) are companies that have most of their income and assets tied up to real estate investments[23]. Most of them are listed on the stock exchange and can be traded as a common stock. "Private" REITS also exist, but these will not be our focus, since the volume is not big enough to make them an interesting investment vehicle for a large global investor.

There are tax benefits for REITS, stemming from when amendments to the US Internal Revenue Code were made in 1961. As a consequence there are several requirements that the companies have to meet. Firstly, real estate assets, cash and government securities must constitute at least 75 % of the firm`s assets. 95 % or more of the firm´s income must come from interest, rents, dividends or capital gains related to real estate. The REITS are also required to distribute minimum 90 % of their taxable income. However, since REIT companies are allowed to deduct shareholder dividends from their corporate taxable income, it has become most common to distribute everything to the

[23] (National Association of Real Estate Investment Trusts, 2012)

investors. Finally, there are some additional stock and ownership requirements to ensure liquidity.

The market cap of the publicly traded Equity REITS in the USA was 584 billion dollars as of 31 of October 2012[24], and the volume has been increasing rapidly since 1990 (partly due to a tax reform act in 1986). The average daily trading volume in February 2011 was 4,5 billion dollars[25]. We can find the annual turnover by applying the following formula:

Formula 3: Annual turnover

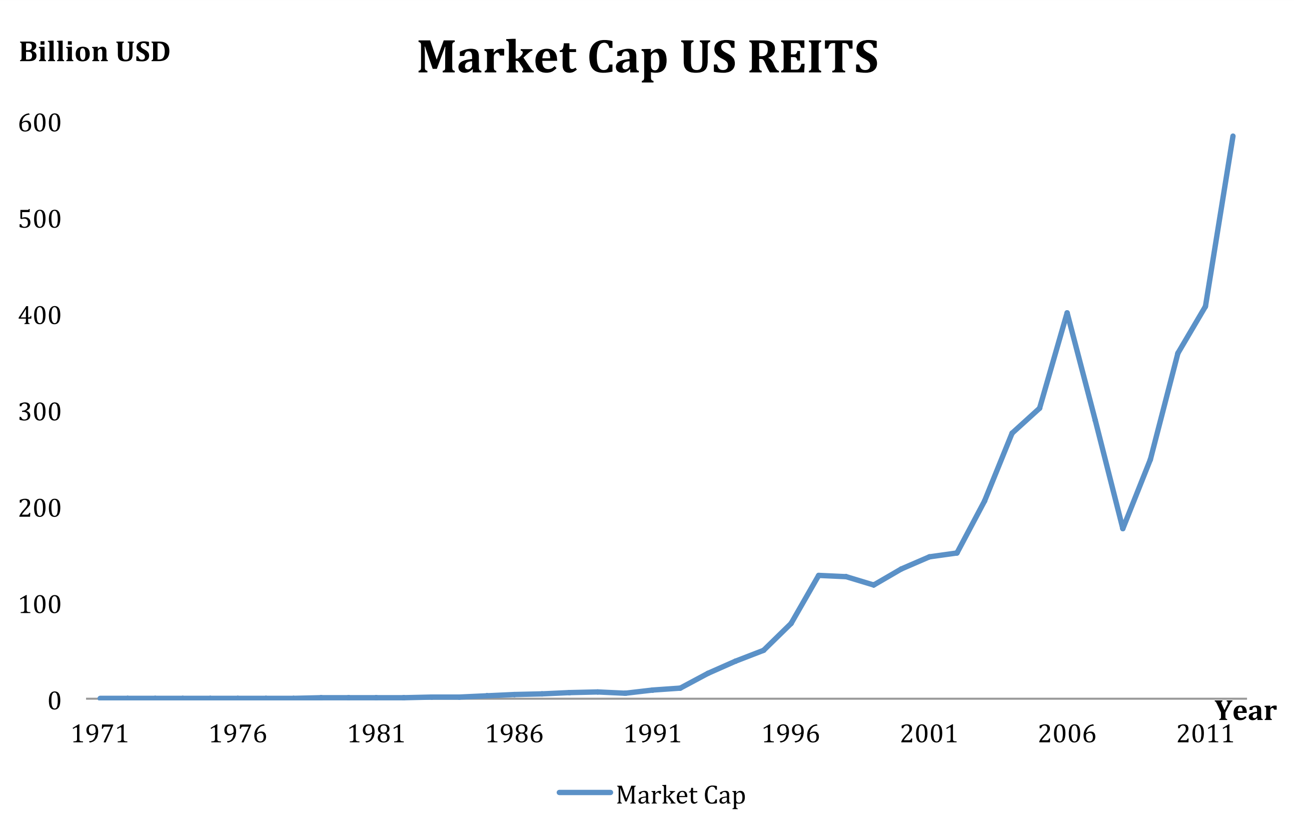
$$Annual\ turnover = \frac{Market\ cap}{daily\ trading\ volume} * \ number\ of\ trading\ days$$

By setting the number of trading days equal to 252 days, we find that the annual turnover is roughly 195 %. This means that an average share in an American REIT changes hands close to two times a year. Hence the number tells us that REITs is a very liquid way of making property investments.

The graph on the following page (figure 4) demonstrates how the market value of the US equity REIT market has developed since 1970 until 2011. The y-axis shows the market cap in billion USD, while the x-axis displays the year.

[24] (National Association of Real Estate Investment Trusts, 2012)
[25] (National Association of Real Estate Investment Trusts, 2012)

Figure 4: Development of the market capitalization of US REITs



The graph shows how the equity REIT market increased rapidly in size from 1990 to 2006. After that we can see that the market cap declined sharply for two years (due to the subprime crisis), only to recover almost as fast. The graph stops in 2011, but the market cap has continued to rise since.

REITs are becoming more and more popular also outside the US, making it a viable option for global investors. We think that the increasing popularity is stemming from more investors realizing the attractiveness of this investment vehicle. REITS provide the investors with advantages related to the low costs associated with buying and selling while they can lever on local competences. However, it is important to mention that REITs themselves have transaction costs related to buying and selling of properties. The fund management fee tends to be relatively low. In 2011 it averaged 0,75 % of the capital invested.

Interestingly, one could expect that as REITS become more common, the average cost of capital of investors should go down, which in turn should lead to higher property prices.

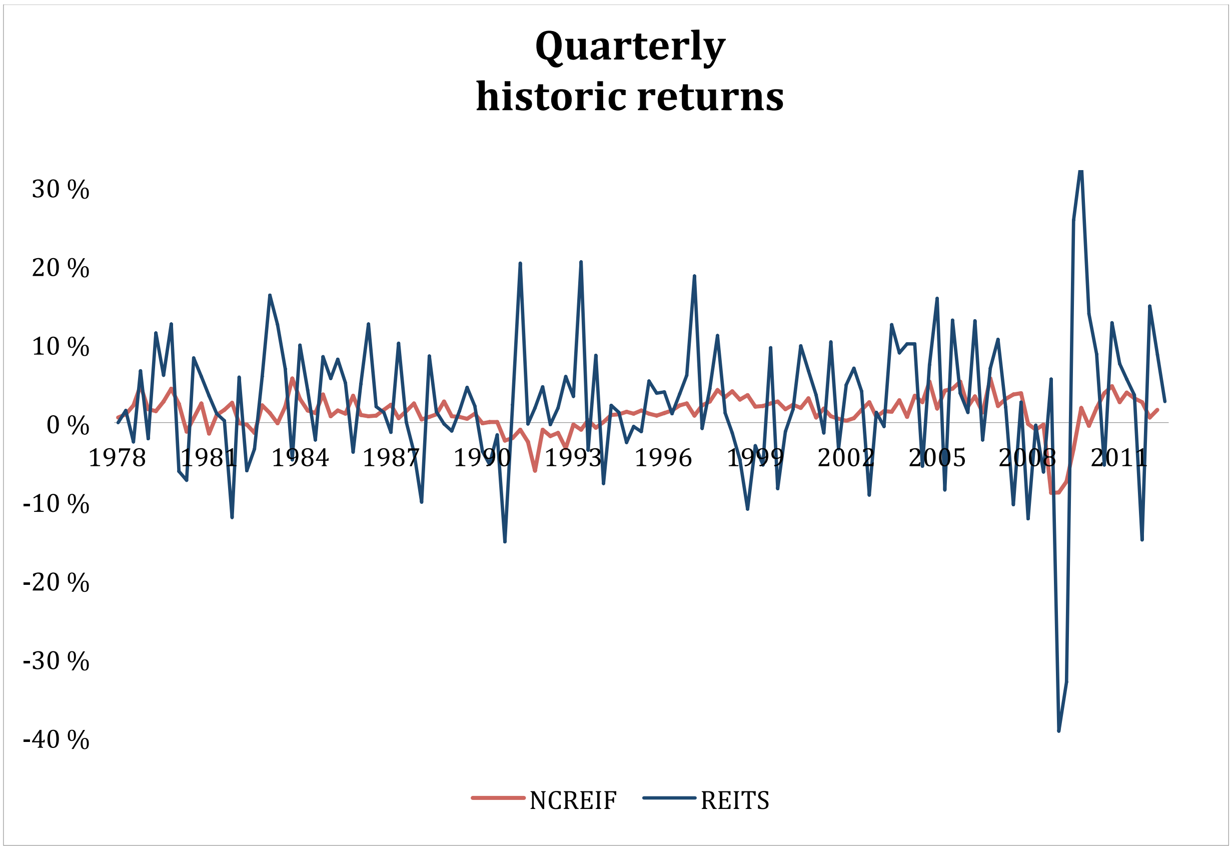
The explanation behind this belief is that REITs will allow the marginal real estate investor to be increasingly better diversified (he can now buy a part of many different properties across the world), making the prices only reflect the market risk, which would lead to a lower cost of capital. If this is correct it could mean that investors would benefit from entering into the property market sooner rather than later.

Indices

We have quarterly historic returns for real estate in the USA from two different sources dating back to 1978: NCREIF[26] and the NAREIT equity index[27]. NCREIF is the most commonly quoted property index. The values are found by adding the operating income (rent) and the increase in the property value, which is based on quarterly appraisals by authorized valuators. The NAREIT equity index includes all real estate investment trusts (REITs) currently trading on the New York Stock Exchange, the NASDAQ and the American Stock Exchange that owns and operates income-producing real estate. The index reflects the total return, i.e. dividends and capital appreciation.

From the graph below (figure 5) we can see how the total returns of the two indices have developed since 1978. The annual returns are depicted by the y-axis, while the years are shown through the x-axis.

Figure 5: Quarterly historic return of US properties since 1978 (two different indices)



[26] (National Council of Real Estate Fiduciaries , 2012)
[27] (National Association of Real Estate Investment Trusts, 2012)

It is apparent that the REITs returns (drawn with a blue line in figure 5) demonstrate a much higher volatility than NCREIF (marked in red). Another interesting point is that the NCREIF index demonstrates sign of autocorrelation. One can see that from 1994 to 2007 the NCREIF index didn´t have one quarter with negative returns, while the REITs index had several. Overall there are few shifts from negative to positive returns in the NCREIF index, which to us indicates that there is autocorrelation in the time series.

Both indices have drawbacks that are important to be aware of. The valuations of the NCREIF are only conducted once a year, and not at the same time. This has two effects. Firstly the index will lag behind the true development of the market, as the prices are updated too slowly, causing autocorrelation. Secondly the low adjustment frequency reduces the volatility in the returns. This happens since on average only one fourth of the properties are reappraised each quarter. Appraisals also have a tendency to exhibit autocorrelation, as valuations tend to be derived partly from past appraisals (together with recent transactions)[28]. It has also been argued that the appraisals in general underestimate the market values[29]. This would make some sense, as it could be in the client´s interest to keep the appraised value low to receive tax benefits and attract more people when the property is put for sale.

REITs returns also demonstrate several imperfections when used in our analysis. Firstly they tend to contain leverage. In 2012 the average debt ratio was 35,1 % (total debt/market capitalization of equity) for listed REITS in the US, while historically it has tended to be a bit higher. This obviously has the effect of increasing the volatility. Furthermore the properties held by REITS don´t need to be representative for the real estate market in general. It is also sometimes argued that the REITs become less attractive since they correlate more with the stock market than direct property investments.

There are several reasons, as we see it, for a relatively high correlation between shares and property. Firstly, both asset classes are dependent on the same economic environment. Secondly, and probably more importantly, the weighted average cost of

[28] (Brown & Matysiak, 2000)
[29] (Kenneth M. Lusht, 1988)

capital (WACC), used when estimating both asset classes should be highly correlated, as both the borrowing rates and risk premiums move in the same direction. A final point is that a crucial part of listed companies´ assets are in fact real estate (approximately 25 %)[30]. As the value of a company largely depends on the value of its assets, the property prices should also affect listed companies that are not primarily real estate firms, more directly. We would therefore argue that the correlation between REITs and stocks is reasonable, and may give a more correct picture than if we compare the stock return with the NCREIF index.

It has been demonstrated that the housing market is more predictable than the stock market[31]. Robert Shiller argues that the high transaction costs in the property market prevent "the smart money" from exploiting the predictability[32]. This is something that might make the actual volatility in the direct property market lower than what it otherwise would have been, and might help explain why REITS can drift significantly off from the Net Asset Value of the underlying properties. One can argue that REITs therefore are displaying a more "truthful" volatility than the actual transactions in the direct property market demonstrate.

Deleveraging of REITs

To make the REITs return a better proxy for the return of the underlying real estate market, the REITs index must be deleveraged. From Damodarans homepage we see that US property companies have tended to have around 40 % debt compared to their market capitalization of equity[33]. Even though the debt level has been substantial, the relative low risk of the real estate sector should make the bonds issued by REITs secure. Therefore we assume that the industry average has been a BBB- rating. The observed credit premium over long US Treasuries has been roughly 1,7 % since 1987 for US companies with the aforementioned rating. Since the REITs hardly pay corporate tax we find it reasonable to assume they don´t benefit from any debt tax shield. As a proxy for the inflation we have relied on the

[30] (Zeckhauser & Silverman, 1983)

[31] (Case & Shiller, The Efficiency of the Market for Single Family Homes, 1989)
[32] (Shiller, Irrational Exuberance, second edition, 2009)
[33] (Damodaran, 2012)

Consumer Price Index (CPI) provided by the U.S. Department of Labor Bureau of Labor Statistic[34].

To deleverage the REITs index we solve for the following equation:

Formula 4: From deleveraging of REITs:

$$0{,}6 * Actual\ real\ return + 0{,}4 * \left(\frac{US\ Bonds\ long\ rate + 1{,}7\ \%}{1 + inflation}\right) = Deleveraged\ return$$

The TBI index

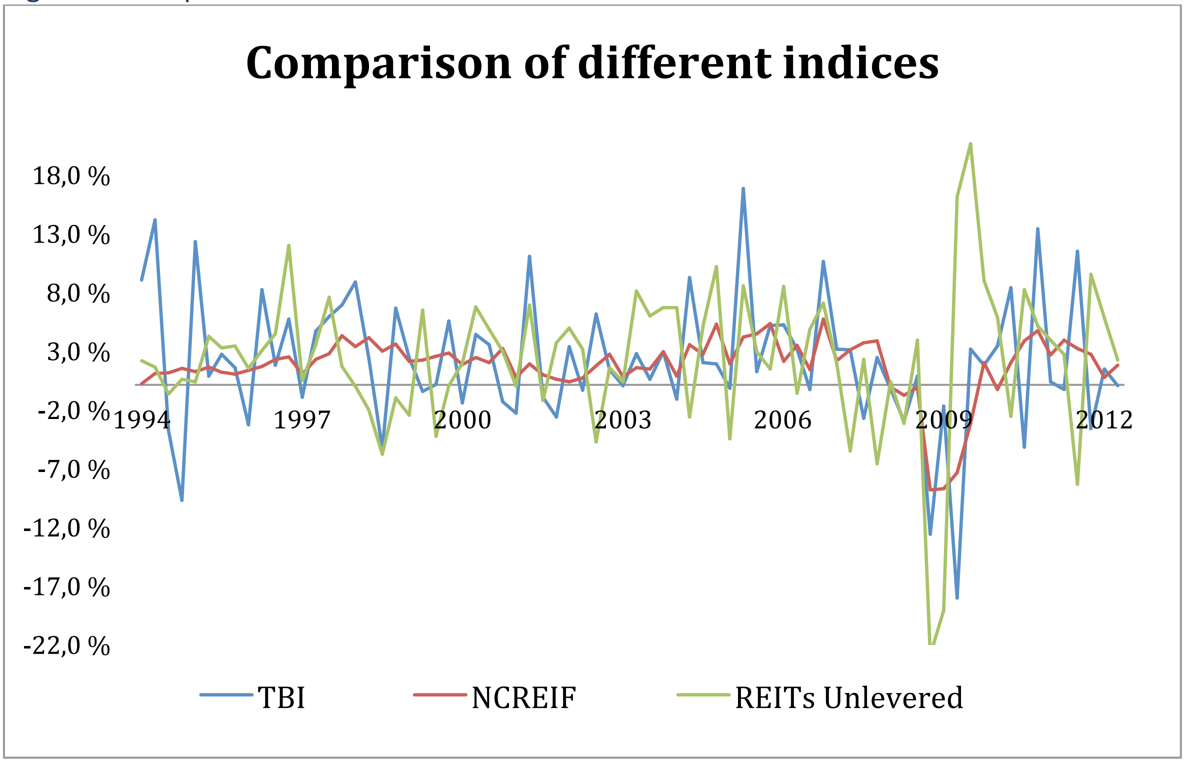
As mentioned, we are critical of the NCREIF index as it is appraisal based, hence the returns lag and the volatility is not fully reflected. However, from 1994 and onwards, the NCREIF[35] has also maintained a transaction based index (TBI). This index only includes properties that were sold in a given quarter. The fact that the index is transaction based should make it more comparable to stock and bond indices.

The most apparent remaining drawback of this index is that it only focuses on real estate that has been sold during the last quarter. One might expect that this will cause a bias in the statistics towards new properties, as these come on the market. The TBI index will also have seasonality in it, making some quarters consistently better than others. For a long-term investor market fluctuations stemming from seasonality is unimportant, since it doesn´t represent changed valuation of real estate[36].

34 (Federal Reserve Bank of St.Louis , 2012)
35 (National Council of Real Estate Fiduciaries , 2012)
36 This we can assume since there is no reason why investors systematically should demand a higher return for investments done in a given time of the year

In the graph below (figure 6) the total return of the TBI index has been compared with that of the unlevered NAREIT index and the NCREIF. The quarterly real return can be read on the y-axis while the years are displayed on the x-axis.

Figure 6: Comparison of different US real estate indices



We can see that the TBI index (represented by the blue line) is more similar to the unlevered REIT index than the NCREIF index in that it fluctuates more (standard deviation of 11,4 % compared to 5,1 % for the NCREIF index). However the TBI and the REITs are not highly correlated. We believe that the most important reasons for this are that the two indices are representing different kinds of properties and that the TBI index includes seasonality, while the NAREIT does not.

Measuring returns

To address the optimal allocation towards real estate in the GPFG, we find it is important to measure the return in real terms. This is because the pension liabilities can be expected to increase together with inflation. Deflation of the returns are done by the following formula:

Formula 5: From nominal to real return:

$$\frac{1 + nominal\ return}{1 + inflation} - 1 = real\ return$$

Since all our data is in USD we simply deflate using the US inflation. To do this we have relied on the Consumer Price Index (CPI) provided by the U.S. Department of Labor Bureau of Labor Statistic[37]. We use real returns throughout the whole thesis.

When doing the Markowitz optimization we are relying on arithmetic returns, while we other times make reflections regarding the geometric returns. Therefore it is important to be able to go from one to the other. To calculate the geometric average we use the following formula throughout:

Formula 6: From arithmetic to geometric return

$$Arithmetic\ average - 0{,}5 * variance = geometric\ average$$

This formula can also be reversed, when we want to find the arithmetic average from the geometric one. A closer look at the graph will make us understand why it is sometimes considered more cautious to use geometric return, as it will always be lower than the arithmetic return as long as there are fluctuations in returns.

[37] (U.S. Department of Labor Bureau of Labor Statistic, 2012)

Recommended real estate exposure in the GPFG

By applying the Markowitz framework to an international portfolio consisting of real estate, stocks and bonds we intend to optimize the relationship between variance and expected return, for a given level of risk. To do this we need to make assumptions related to the expected arithmetic return and volatility for the three asset classes. Secondly, and equally important, we have to develop a view related to the correlation between the different assets. This will result in an efficient frontier that we will use to define a proxy benchmark portfolio for the GPFG, using the mean-variance optimizing model.

Efficient frontier with the expectations of the ministry of finance

We will first go through the optimization process using the Ministry of Finance´s view of expected returns and volatility for the different asset classes and the correlation between them. Based on these estimates we find an efficient frontier, using solver in excel. From the efficient frontier we can determine the optimal portfolio for a given level of risk.

In the table on the following page (table 1) we can find the estimates for volatility and expected arithmetic return of the Norwegian Ministry of Finance[38]. One can see that they expect equities to provide the highest return (6,1 %), while being the most risky, with a standard deviation of 15 percent per year. Bonds are expected to be the least risky asset class with an annual volatility of only 6 %. It is, however also, the asset class that is expected to provide a global investor with the lowest real return (less than 3 % annually). Real estate lies, as one might expect, between equities and bonds both in risk and return attributes. The risk free rate is expected to provide the investor with an annual real return of 2 % a year.

[38] (Døskeland, 2012)

Table 1: The expectations of the Ministry of Finance regarding risk and return

	Risk free rate	Equities	Bonds	Real estate
Expected Arithmetic return	2 %	6,125 %	2,88 %	4,22 %
Expected volatility	0 %	15 %	6 %	12 %

In the table below (table 2) we can find the expectations of the Ministry of Finance related to the long-term correlation between the three asset classes. One can observe that equities and bonds are expected to correlate 40 %. Real estate is expected to correlate 60 % towards equity and 30 % towards bonds. This means that the potential to gain diversification benefits through adding real estate to the portfolio should be relatively large.

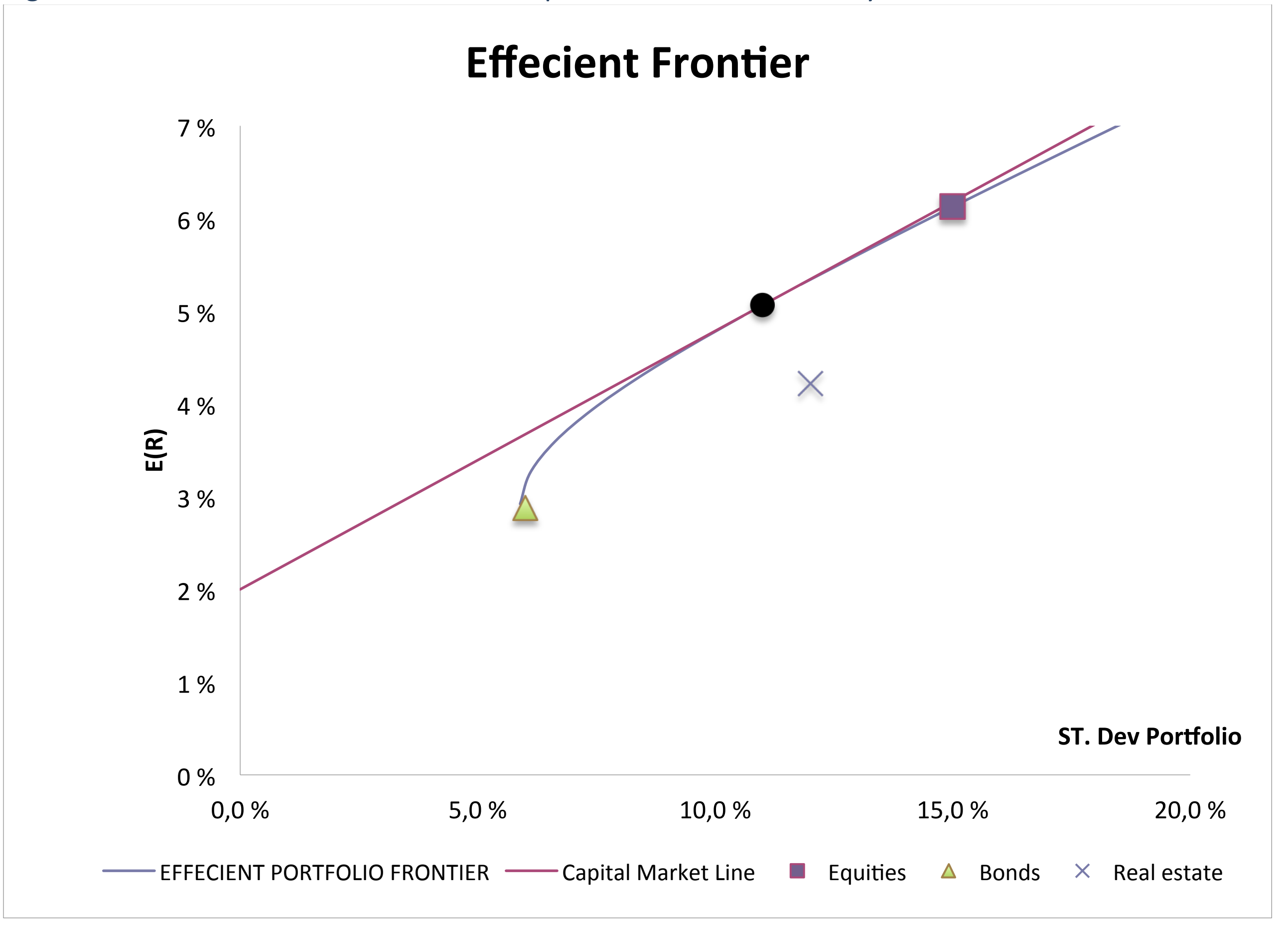
Table 2: The expectations of the Ministry of Finance regarding correlations

	Bonds	Real estate
Equities	0,4	0,6
Bonds		0,3

Using the aforementioned estimates as input data we can create the efficient frontier. This is done in excel, where we use solver to minimize the volatility for a given expected return by changing the weights of the different asset classes. We use a macro so that solver can find solutions for different expected returns (see appendix 1). After this we compute the maximum Sharpe ratio, which is later used together with the risk free rate to draw the CAL. We have not added any short-restrictions, meaning that we implicitly assume that the GPFG can go short (i.e. have negative exposure) to the different asset classes. This would probably be difficult due to its size and so on, but it is not a relevant problem in our case, which is why we will not further discuss this issue.

The result can be seen in the graph below (figure 7), where the expected arithmetic return is shown on the y-axis and the volatility on the x-axis. The efficient frontier is the blue, curved line and the CAL is red and straight. One can also see where the different asset classes would be if you chose to invest exclusively in one or the other.

Figure 7: The efficient frontier with the expectations of the Ministry of Finance



Bonds are represented by the green triangle and are close to the minimum variance portfolio both in terms of risk and return. Equities are also positioned close to the efficient frontier, but with significantly higher risk and return. In the figure it is represented by the blue square. Real estate has been marked with a cross, and is showing between equities and real estate. That it is so far away from the efficient frontier implies that it is a particular bad option to focus exclusively on real estate, independent of your risk preferences. Be aware that all points are on the inside of the efficient frontier, as they per definition have to be. The angle of the CAL represents the reward the GPFG could expect to get for taking on risk (same as the Sharpe ratio as we earlier have discussed). When this is 27,9 %, as in our case, it indicates that the investor would expect to get a little more than 1 percentage point more in return, for every 4 percentage points of additional volatility he takes on. It is interesting to note that the efficient frontier is close to a straight line around the tangency point (represented with a black dot in figure 7), as this implies that the allocation choice is robust.

Each of the infinite number of points on the efficient frontier is represented by a unique allocation towards the different asset classes. Some of these points can be seen in the table below (table 2). Certain expected returns have been marked in the blue column to the left. For each of these one can see the respective allocations that minimize the volatility in the three columns in orange (equities to the left, followed by bonds and real estate)[39]. The standard deviation can be seen in the green column to the right.

Table 3: Portfolio return matrix for the efficient frontier with the expectations of the Ministry of Finance

	Weights			
Expected return	Equities	Bonds	Real Estate	Standard Deviation
2,9 %	-7 %	92 %	15 %	5,8 %
3,2 %	4 %	82 %	14 %	6,0 %
3,6 %	17 %	70 %	13 %	6,6 %
4,0 %	30 %	59 %	12 %	7,6 %
4,4 %	42 %	47 %	11 %	8,7 %
4,8 %	55 %	35 %	10 %	10,1 %
5,1 %	64 %	27 %	9 %	11,0 %
5,2 %	68 %	23 %	9 %	11,5 %
5,6 %	81 %	12 %	8 %	13,0 %
6,0 %	93 %	0 %	7 %	14,5 %

It is worth pointing out that the minimum variance portfolio (the first line in the table above) actually involves a negative position in equities (- 7 %). In this point most capital would of course be allocated towards bonds (92 %) as this is considered the least risky asset class, while real estate would constitute 15 % of the portfolio. The allocations that have been marked in green represent the positions that maximize the Sharpe ratio (the tangency between the CAL and the efficient frontier seen in figure 7). Here we can see that equities would account for 64 %, bonds 28 % and **real estate 9 %** of the total portfolio. With these weights the expected arithmetic return would be 5,1 % and the volatility 11 % (geometric return of 4,5 %). NBIMs current mandate allows for holding 5 % real estate, 60 % equities and 35 % bonds. With this the expected arithmetic return is

[39] The percentages does not necessarily sum up to a 100 % due to approximations

4,6% while the volatility lies at 10,4 % per year, indicating that both the risk and return increases with the new allocation. The Sharpe ratio increases marginally from 27,83 % to 27,87 %, implying that the GPFG could expect to get better paid for the volatility in a portfolio with the new allocations. We find the increased annual expected return from the improvement of the Sharpe ratio by applying the following formula (sharpe ratio is denoted SR):

Formula 7: Increase in expected return from an improvement of the Sharpe ratio

$$(New\ SR - Old\ SR) * Old\ volatility * Capital\ =\ Increase\ in\ expected\ return$$

With the current market capitalization of approximately 3,8 trillion NOK, the increase in the expected return from the improved risk reward relationship is roughly 200 million kroner a year. This means that when exclusively seen in a mean-variance framework, there seems to be substantial room for improvement of the portfolio characteristics by increasing the amount allocated towards real estate.

Conclusion

We have found real estate should ideally account for 9 % of the portfolio of the GPFG when the expectations provided by the Ministry of Finance are used as input for our analysis. This would however only improve the Sharpe ratio incrementally, and would come at the cost of a higher volatility, if not combined with the risk free rate. Bear in mind that due to the large size of the fund a small increase in the Sharpe ratio can lead to a large effect on the expected return of the fund. In our case, and with the current 4,8 trillion NOK under management, the increase in the capital appreciation that could be expected from an improvement of the risk reward relationship would amount to 200 million kroner annually. Therefore, it appears beneficial to alter the current allocation-goals, when seen in a mean variance framework exclusively.

Developing our own view of the attributes of the different asset classes

In the following we will go through the historic performance of real estate, bonds and equities. Based on a discussion around these numbers we develop a view related to the expected return, volatility and correlation of the different asset classes. To get estimates for expected returns, we have mostly tried to go as far back as we have found trustworthy data. We develop our assumptions related to volatility and correlations based on more recent data, as economists have suggested that this is a more reasonable approach.

Real estate estimates

Real estate is the most difficult asset class to give proper estimates for. The characteristics in terms of expected return; volatility and correlation towards the rest of the portfolio will largely depend on location, geographical diversity, property and what kinds of contracts are entered into. Furthermore, total return property data only exist a few decades back, and not for the whole world. To get past this problem we will combine our quantitative historic analysis with a more qualitative approach. For our numerical analysis we rely chiefly on data from the International Property Databank (IPD), which is considered the most trustworthy source for global real estate data[40]. When other sources are used this will be explicitly mentioned.

[40] (International Property Data, 2012)

Input data

For this part of the analysis we will assume that the investor is able to diversify globally in the strongest developed countries across the retail, office, residential and industrial sectors. This has enabled us to use IPDs global data that date back to 1987[41]. The index consists of 24 countries[42], that have been weighted according to market cap, it includes around 15 000 different properties, with a capital value of more than one trillion USD[43]. We look at the total return, which we define as: The annual rate of capital appreciation, net of capital expenditure, plus net income. The locations have not been specified, but are wherever they are held in professionally managed portfolios. Whether the index is reflecting an optimal geographical distribution, within the given countries is doubtful. If not, it would imply that the overall volatility of the portfolio could be reduced. The capital appreciation is found by comparing current with past appraisals (and transaction values when this exist), which is done at least once a year for all properties by independent firms.

As previously discussed appraisal-based property indices tends to exhibit falsely low volatility. The correlation towards stocks will also be smaller than what one should expect. To account for these problems we have looked at REIT data from the US, which gives us an indication of how the dataset should be manipulated.

A closer look at the historic performance of real estate

We have global real estate data (from international property data) dating back to 1987, where 24 well-developed countries are represented. The total return is expressed in USD. Since the returns are nominated in USD we can use the same approach as earlier to deleverage the returns, with the US CPI index. On the following page the results have been displayed graphically (figure 8). The blue line represents the arithmetic real return on a per-year basis. On the vertical axis you can read the real arithmetic return, while the year is displayed horizontally.

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[41] (International Property Data, 2012)
[42] Australia, Austria, Belgium, Canada, Denmark, Czech Republic, Finland, France, Germany, Hungary, Republic of Ireland,Italy, Japan, South Korea, Netherlands, New Zealand, Norway, Poland, Portugal, South Africa, Spain, Sweden, Switzerland, UK, USA
[43](International Property Data, 2012)

Figure 8: Historic return of global real estate since 1987

One can observe that the two first years in our dataset where particularly attractive for the globally diversified real estate investor. The following couple of years the returns turned negative, probably due to the bursting of the Japanese asset bubble. From 1993 until 2008 the global real estate investor realized a relatively stable return, where losses only had to be taken in one of the 16 years (1997). After this the subprime crisis in the US dragged down the returns significantly (-10 % in 2009). The last couple of years we can see signs of a recovery within the world property markets with high single digits returns in both 2010 and 2011. Our dataset reveals an average arithmetic real return of 6,43 % while the standard deviation has been 13,47 % per year, over a 25-year period starting in 1987. This implies that the geometric real return has been 5,52 % annually, which is high in comparison to bonds and equities in the same timeframe.

The sensitivity of starting point

In general there is a high sensitivity to the starting point of a time series. In our case the numbers only date back to 1987, making this problem particularly acute. One can observe that both this year and the following were particularly attractive years to hold a global real estate portfolio. In fact they were by far the two strongest years in our time series, with a total return of 46,7 % and 27,5 % respectively. To reduce the sensitivity in our analysis to two so such anomalous years, we have decided to make 1988 our starting point. One could argue that an overall increase of 27,5 % for the global real estate sector seems too high. In general there´s the tendency that older data are more uncertain, but we make no attempt to correct for this beyond starting our analysis in 1988 instead of one year earlier. The result is that the average arithmetic return gets reduced to 4,7% while the standard deviation becomes 10,76 %.

The currency exposure

It is important to be aware that since all the returns are nominated in USD the volatility might increase due to currency risk. Therefore we conducted an analysis of the global real estate return in local currency as well. To do this we once again use the data facilitated to us by IPD. The reason why we didn´t rely on this data initially was that we didn´t know how to proceed to deleverage the data.

We found that the nominal annual arithmetic return, when expressed in local currency, amounted to 4,8 %, while the standard deviation only was 7,81 %. Since we don´t know how much the USD appreciated towards the other currencies, we will not manipulate the realized return. Furthermore, we have that the equity and bond returns also are nominated in USD, which implies that although the mean might be affected, there will not be a systematic bias towards any particular asset class. Since real estate typically is seen as an inflation hedge, we assume that the volatility would not increase if one unlevered the nominal return by a weighted CPI index based on the relative exposure to each of the currencies. We find support in this assumption when we notice that the nominal USD returns actually exhibit a higher volatility than the deflated returns. When this argument seen in isolation it points towards significantly reducing the volatility estimate from the 10,76 % we saw for the USD investor.

Adjusting the volatility

All appraisal-based indices, such as the IPD-data, will tend to underestimate the volatility. We argue that the returns of REITS give a better picture of the volatility in real estate, once you have adjusted for the leverage. However, we do not have access to a fully global REITS index that goes far back, so we use global IPD data as a starting point. By comparing the volatility from US REITS (unlevered) with US IPD data (both from 1988), we find that the REITs returns fluctuate 10 % more. On this basis we think it is reasonable to increase the volatility estimate for the world index with 10 %. This will be in line with what we observed in the US, which is the largest constituent of the IPD global index.

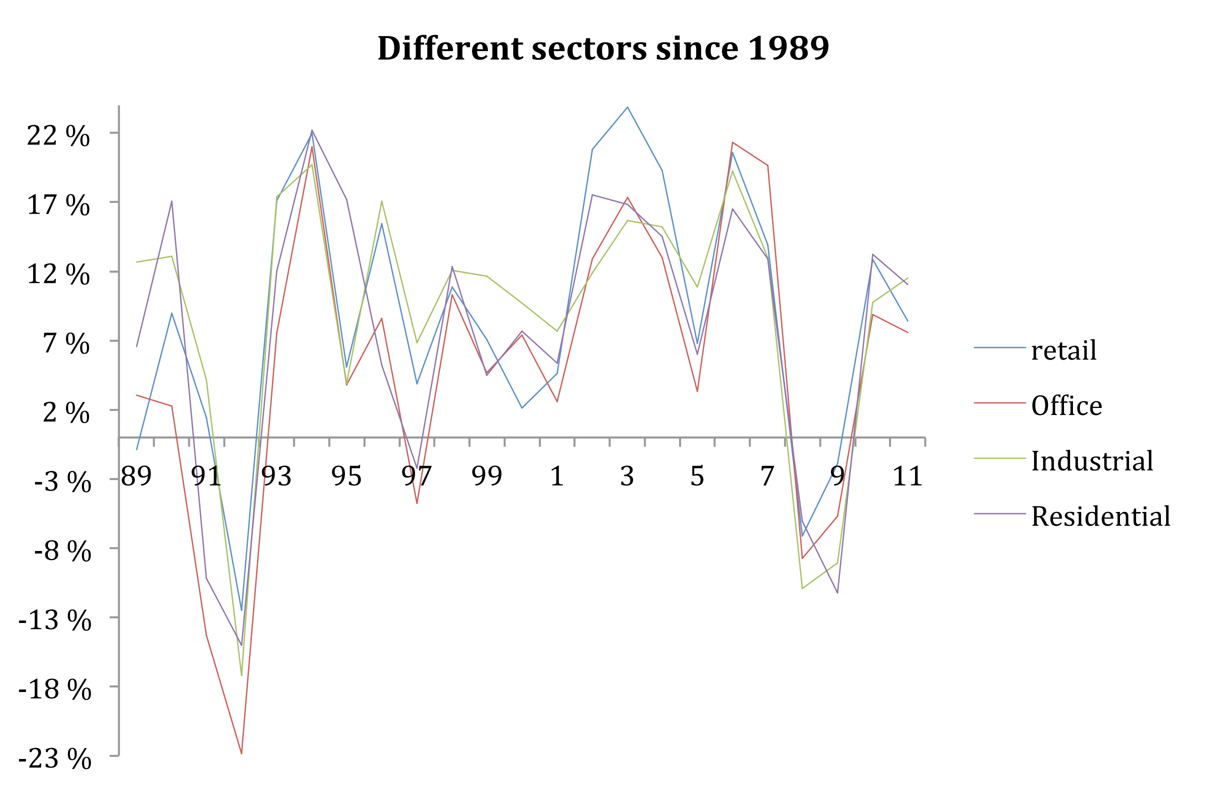
Geographical diversity and location

It is reasonable to expect that the country/region in which the property is located will play a significant role in determining the risk and return characteristics of a real estate investment. It is problematic to quantify these effects, as our data material doesn´t go very far back and we do not have trustworthy data for less developed countries.
In due course, we expect NBIM will have to refine/adapt its geographic focus when it comes to real estate investments (as they have done for equities), in a way that reflects both the historic track record and a dynamic perspective on the attributes of the various Real Estate markets across the world.

Property type

The graph on the following page (figure 9) demonstrates the total real return of different property sectors on a global basis since 1989. All the returns are expressed in USD, which allowed us to use the US CPI index to deleverage the numbers. The blue line represents retail, the red office, the green industrial, while the return of residential properties is drawn in purple. The annual total return can be read of the y-axis while the year, spanning from 1989 to 2011, can be found on the x-axis.

Figure 9: Comparison of the return of different real estate sectors since 1989



We see that there is a clear tendency for the returns to move in tandem (so much that the graph has become difficult to read). Interestingly, our data indicate that offices are the most volatile, while exhibiting the lowest real return. Retail property has on the other hand had the lowest standard deviation. Our interpretation is that companies are more inclined to move from their office buildings when the economy is contracting, than to change their retail locations, as being near to the customer always is important.

The standard deviation and the realized return for each property type can be found in the table below (table 4). I is based on yearly data from 1989.

Table 4: Historic volatility and return of different property sectors (globally since 1989)

	Retail	Office	Industrial	Residential
Standard deviation	9,6 %	11 %	9,5 %	10,2 %
Arithmetic mean	8,8 %	5,2 %	9,0 %	7,6 %
Geometric mean	8,4 %	4,6 %	8,5 %	7,0 %

From the table below (table 5) one can see that the correlation of returns between the different sectors all have been around 80-90 % (from 1989 until 2011), when our input data is used. We find this correlation to be too high, and see it as a sign of smoothed data (due to the use of appraisals). For the US REITs market, a similar analysis indicates correlations between the different asset classes from 60-70%, which we find to be a more credible result. The high correlation between the different real estate sectors suggests that there is a limited possibility to receive diversification benefits by investing in different property types.

Table 5: Correlation between global portfolios of different property sectors

	Office	Industrial	Residential
Retail	88 %	85 %	82 %
Office		84 %	84 %
Industrial			81 %

Contract type

Two "buy and let" investments next to each other might have significantly different return characteristics. In general, the more financially robust the tenants of the property are, and the longer the contracts are, the safer the investment. When the investor has to find new tenants immediately after buying a property, or in the medium term for properties under construction, he is severely exposed to the economic environment at that moment, and the present value of the investment will mostly come from a more insecure cash flow. When a property is bought with a long term contract with a good tenant, the risk and return attributes of the investment should be quite similar to bonds, except for two things[44]. Firstly it is normal for the property owner to receive a rent that is adjusted for inflation. This reduces the volatility of the real return of the investment. Secondly, since property investments are less liquid than bonds, the investor would normally be compensated through a higher expected return (including the appreciation of the underlying asset.). If no lease exists, the risk and return attributes are clearly different.

[44] (Swensen, 2009)

Lack of long time series

We see that over the last 24 years real estate have been very attractive in terms of both risk and return. A global real estate investor could have expected to realize 4,7 % annual arithmetic return since 1988, while the volatility have been around 10 %. Since the time series is so short we think it is important to express cautiousness. Since the returns realized have been above our expectations we therefore see it as prudent to lower our expectations towards the future returns of the global property market somewhat. We also assume that the fund will realize most of its investments in prime locations and with solid tenants, as this could lower the risk and the correlations towards equities (which will be discussed later). This means that the risk and return attributes would be more similar to bonds than to equities. Paradoxically it does not imply that real estate will correlate closer to bonds than equities, as we will see later. Keeping this in mind, together with our previous discussions, we have come to the following estimates: arithmetic mean of 3,5 %, geometric mean of 3 % and volatility of 10 %

Conclusion

Based on a combination of the historic returns and a discussion around qualitative aspects related to the management of the real estate portfolio of the GPFG we have come up with the risk and return estimates for real estate presented in the table below. One can see that we expect the real arithmetic return to be 3,5 %, the geometric real return to be 3 % and the volatility to be 10 %. All measures are on a per year basis. The correlations towards the other asset classes will be discussed later on.

Table 6: Risk and return expectations for real estate

Real estate estimates (in real terms)	
Expected arithmetic return	3,5 %
Expected geometric return	3,0 %
Standard deviation	10 %

Equity estimates

For equities we have indices going back more than 200 years. Except for the first decades where the indices only comprised a few shares (mostly railway companies), the data can be said to be reliable and useful. The quality and long history of stock returns makes us comfortable in chiefly relying on past performance when determining the expectations of the asset class for the future. However, we will make some adjustments based on a qualitative assessment.

The dataset

For the expected returns of equities we mainly rely on an analysis done by Dimson, Marsh and Staunton (DMS)[45]. They base their research on global data from 1900 to 2011, were all returns are computed as arithmetic averages. Their index comprises 85 % of the world market cap today and about 90 % of the market when the index started. The index consists of 19 countries[46] and includes mostly strong western economies. It has been weighted based on the respective country´s GDPs in 1900, with rebalancing being done at the start of every decade. From 1968 the index were rebalanced based on the market cap within each country. The weighting is done based on starting point, so as to avoid a "success bias", where countries that has done well, like the US, end up contributing too much to the overall returns. Sharpe has claimed that future correlations and volatility tend to be more similar to recent past than the distant past[47]. This is why we will compute estimates of the volatility (and later the correlations) ourselves from the global Morgan Stanley Capital International (MSCI) equity index over the last 22 years[48]. The index is denominated in USD and has been accessed through Bloomberg. It

[45] (Dimson, Marsh, & Staunton, 2011)
[46] List of countries in the DMS index: Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, The Netherlands, South Africa, Spain, Sweden, Switzerland, United Kingdom, United States.
, Portfolio Theory and Capital Markets, 2000)
[48] (MSCI, 2012)

is free float adjusted, which means that exposure is reduced towards illiquid stocks, but still includes approximately 99 % of the global equity universe.

The historic return of equities

From 1900 to 2011 the real geometric return of the global portfolio of stocks of DMS has been 5,5 % annually, while the standard deviation has been 17,7 %. This means that the arithmetic real return has been 7,07 %. Since the time series stretches over more than a century the sensitivity of the starting point becomes smaller, making the numbers more reliable. However, we have found several reasons for scaling down the expected return to some extent.

Survivorship bias

Survivorship bias is the tendency of excluding poor performers from a dataset, making the result appear better than what it should be. Although DMS has made a conscious effort to avoid this, they have still been criticized for basing their analysis on a dataset with survivorship bias. The argument is that by not including economies like the Russian and the Indian in the dataset (that were pretty large in 1900), the performance of equities becomes exaggerated. However, we believe this effect to be small. Furthermore, we will have the same effect for bonds, so there want be a systematic bias towards equities.

Volatility under mean reversion

We have earlier pointed to the tendency of mean reversion in the returns of equities. By looking at the MSCI index for the last 22 years, we find support for this belief. Since 1980 the MSCI index have had an average arithmetic real return of 4,4 %, while the volatility has been 19 % when measured annually. When we compute the volatility based on a rolling average of 10-year returns we find that the standard deviation becomes reduced to 14,1 %. When the volatility gets reduced with the timeframe this is an indication of mean reversion. Since the GPFG has a long time horizon for its investments (with no clearly defined liabilities) we consider the fluctuations over larger periods to be important. Therefore we have reduced our volatility estimate to 16,5 % (from the 17,7 % in the DMS analysis)

Qualitative considerations

As several smart people have pointed out: "we have but one history". Therefore one should always remain skeptical when interpreting historic return data. The last 110 years can be seen as a golden age of capitalism, especially for the free market economies represented in the DMS dataset. There has been a major productivity increase together with improvements of the political and legal frameworks around the world. The population has been rising steadily, and there haven't been any major shortages of production factors (except OPEC 1 and 2). To expect this development to continue over the next hundred years might seem optimistic, especially since the population is growing extremely fast and we already have problems related to the climate and lack of natural resources such as water. Therefore we find it reasonable to reduce our estimate for the expected arithmetic real return to 6,85 % (down from 7,07%).

Conclusion

By examining an analysis of past return data for a global portfolio of stocks we see that the asset class has performed extraordinary from 1900 until 2011. We are uncertain whether this success story is repeatable over the next century, so we have been cautious and lowered our expectations somewhat compared to the historical mean. Because of the timeframe of the investments of the fund, we feel that it is better to look at volatility over longer time horizons. As we believe in the mean reversion of stock return, and found support for this belief by examining a world stock index, we have lowered our expectations related to the future volatility somewhat.

The result is in the table below (table 7). Here one can see that we expect an arithmetic return of 6,85 %, a geometric return of 5,49 % and a volatility of 16,5 % for equities. All estimates are given in real terms on a per year basis.

Table 7: Risk and return expectations for equities

Equity estimates (in real terms)	
Expected arithmetic return	6,85 %
Expected geometric return	5,49 %
Standard deviation	16,50 %

Bond estimates

For bond returns we have long time series, but when you go far back one has to be cautious with the interpretation. Especially related to the world wars, several countries were struggling. We will base our adjusted estimates on analysis going back to 1900 in combination with qualitative assessments.

The input data

We use the analysis from Dimson, Marsh and Staunton (DMS) from 1900 to 2011 as a basis for our estimates of the future expected returns[49]. They have mainly focused on long-term Government bonds in their analysis. The index consists of 19 countries[50] and includes mostly strong western economies. It has been weighted based on the respective country´s GDP throughout. We have also used Barclays Global Aggregate total return bond index, which has been accessed through Datastream. The returns are expressed in local currency, where USD, EUR, JPY and GBP are the most important ones. The index includes both Government and corporate bonds of investment grade. One can argue that this index is more relevant for the globally diversified investor as it constitutes a larger specter of bonds. The problem is however that the data only goes back to 1990.

A look at the historic return

The DMS data reveals that the arithmetic mean return for a portfolio of long Government bonds has been 2,21 %, while the volatility has been 14,2 % - when measured annually since 1900. This implies that the geometric return has only been 1,2 %. The high volatility can to a large extent be explained by the long duration (average maturity of discounted cash flows) of the bond portfolio of DMS (about 8 years). A further complication is that the DMS analysis has been done with a US investor in mind. That means that the volatility it measures, is the one that would have been realized by a globally diversified US dollar investor, meaning that it involves currency risk. The currency exposure has had insignificant importance for the mean, while the effect has

[49] (Dimson, Marsh, & Staunton, 2011)
[50] List of countries in the DMS index: Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, The Netherlands, South Africa, Spain, Sweden, Switzerland, United Kingdom, United States.

been hard to quantify in terms of added volatility. Furthermore, the return becomes lower since corporate bonds are not included. When we use Barclays Global Aggregate Bond Index, we find that the yearly volatility of the nominal returns has been 6,3 % since 1990 (measured yearly). This is significantly lower than what has used to be the case. The first reason is that this index does not include any currency risk, and thereby gives a more truthful picture of the underlying volatility of the asset class. The second reason is that the period has been characterized by relatively low inflation, and this typically goes hand in hand with lower volatility as well[51]. Finally you have that the returns have not been deflated. Since bonds perform poorly when there´s unexpected inflation the volatility of the real bond returns will tend to be higher than that of nominal bond returns. However, as the last two decades have been characterized by a relative stable and low inflation in the major currencies, we don´t expect that the volatility would be impacted significantly by a deleveraging of the returns.

Adjusting for lack of corporate bonds in the DMS data

For a global bond investor it would be reasonable to invest a substantial part of his portfolio in corporate bonds, as this consist 1/3 of the bond market. Over the last 50 years there has been a corporate premium of 80 basis points. Holding this premium constant we feel you can increase the expected arithmetic return of the investor to 2,5 %. The investor should be aware that this comes at the cost of increasing the market risk of the overall portfolio, especially since corporate bonds tend to demonstrate a higher correlation towards the equity market.

A qualitative view of the history through the eyes of a bond investor

As mentioned earlier, one has to be careful when interpreting the bond returns. Various countries went through periods of extremely high inflation, especially related to the two world wars. In Germany there was inflation of more than 200 billion percent in 1922-23, which led the bondholders to be completely wiped out (these two years have been

[51] (Dimson, Marsh, & Staunton, 2011)

excluded from the index). France had similar problems, causing there to be a negative real return on bonds from 1900 until today. Together these two economies comprised roughly 20 % of the index. In the USA you didn´t have a central bank until 1913. Several countries had the Bretton Woods system between 1944 and 1971. This was a system that pegged the dollar to gold, and the other currencies towards the dollar (thereby indirectly to gold). When this system ended a large part of the world had to deal with high and unexpected inflation.

Since we believe that central banks will manage to keep inflation relatively consistent and low in the years to come, we expect the volatility to remain at the current level, around 6,5 %. We also believe that an increased monetary stability would point towards a decent expected return for an investor holding a global bond portfolio for the future. This is why we have altered our estimate for the average arithmetic return up to 2,6 %.

Conclusion

We have looked at data dating back to 1900 together with a Barclay index to assess the historic return of bonds. The resulting estimates can be seen in the table below. We expect holders of a portfolio of global bonds to realize an arithmetic return of 2,6 % a year. Due to central bankers increased ability to control inflation we believe the standard deviation to be best represented by the last 20 years. Therefore we believe it will be around 6,5 % per year. The geometric average return would then be 2,39 % per year.

Table 8: Risk and return expectations for bonds

Bond Estimates (in real terms)	
Expected arithmetic return	2,50 %
Expected geometric return	2,29 %
Standard deviation	6,50 %

Risk free return

The arithmetic real return of US treasury bills (maturity in less than one year) has been 1,18 % a year since 1900. Since the GPFG has a longer time horizon on its investments, we think the relevant risk free rate should be that of AAA- rated Government bonds over longer time horizons. When the time horizon is between 4-5 years the long-term view of the GPFG has been taken into account. Equally important, history has shown us that these bonds have been low risk[52]. Then we get a relevant real return of 1,8 % per year. We use this as our estimate for the future risk free return. As we consider it to be "risk free" we obviously expect the volatility to be negligible.

[52] (Caouette, Altman, Narayanan, & Nimmo, 2008)

Correlations

In a Markowitz framework the expectations and assumptions related to correlations are as important as those with regard to the expected return and the volatility of an asset. Conceptually the lower the expected correlation an asset has with the rest of the portfolio, the more diversification benefits one can expect to harvest, making the overall risk attributes more attractive. The actual management and allocation process is, however, complicated by the fact that correlations are far from constant.

Use of input data

In our computation of correlations we have used the MSCI global index (since 1988) for the development of stock returns, Barclays Global index for bonds and the IPD data for real estate. Since we are somewhat skeptical towards the IPD data for this purpose, we also compare the US REITs returns with the S&P 500 since 1987[53], to give us an indication of whether or not our estimations can be trusted. The S&P 500 is a collection of 500 of the largest publicly held US companies. Since we only have access to the Barclays index expressed in local currency while the MSCI is expressed we have chosen to rely on an analysis performed by DMS when it comes to the correlation between bonds and equities[54].

Correlation between real estate and bonds

For real estate we have total return data from IPD that are expressed in both local currency and in USD dating back to 1987. The Barclays global bond index is expressed in local currency, and dates back to 1990. Therefore we will compare the realized return of the global bond portfolio with the total return of the IPD index nominated in local currency. To avoid the problem of deleveraging of all the different currencies we have used nominal returns. The weights of the different currencies are somewhat different in the two respective indices, but we don´t expect this to affect the perceived correlation significantly.

53 (Federal Reserve Bank of St.Louis , 2012)
54 (Dimson, Marsh, & Staunton, 2011)

What we find is that the correlation between real estate and bonds have been negative. Since 1990, the annual correlation between the two asset classes has been -30,7 %. Because of the different reactions to unexpected inflation one would assume that the correlation between real estate and bonds to be low. We also get a similar result when we compare the 5-year rolling returns of the two asset classes. Specifically, we get that the rolling returns correlate with – 32,9 %.

However, over longer time horizons one should expect that the correlation increase, as both asset classes are dependent on the same economic environment. Therefore we think it is reasonable to expect zero correlation in the future, between global portfolios of properties and bonds.

Correlation between real estate and equities

We have compared a global real estate index (from IPD) denoted in USD with the global MSCI index. Since the returns were expressed in USD in both cases, we felt comfortable deleveraging the returns based on the US CPI. The correlations in the real return of the indices have been 23,9 % when measured annually since 1990. Since we only have returns for the MSCI index dating back to 1990, it becomes futile to compare rolling averages of returns.

Since appraisal based indices tend to lag we would favor using global REITs data to compute the correlations of real estate towards equities and bonds. Since REITs are just starting to become a worldwide phenomenon we don´t have data to go through with this analysis. However, we compared the real return of US REITs with that of the S&P 500, and found that the correlation was 35,2 % since 1990 (when measured annually). This indicates that the "true" correlation between the asset classes is higher than the 23,9 % we computed above.

Furthermore, as we have argued earlier there are several reasons for why real estate and bonds in fact should be expected to correlate significantly over time. Since we have discussed these issues earlier, we will only quickly state the three reasons why we expect a relatively high correlation between real estate and equities. First of all the

WACC for investments made in real estate and stocks should be expected to move in tandem. Secondly, one would expect both the stock market and the real estate market to perform well when the economic environment is strong. In addition to this stocks aren´t affected as adversely as bonds in periods of unexpected inflation. Finally, stock companies typically have a large share of real estate assets, which should make them affected by a drop in the property prices more directly. Based on this discussion we find it reasonable to expect the correlation between real estate and equities to be 50 % over a longer time horizon, which is the most relevant for the GPFG.

Correlation between equities and bonds

When we compare the nominal returns of equities (from the MSCI index) with the nominal return of a global bond portfolio (from the Barclays index) we find that the annual correlation has been 5,6 % since 1990. Unfortunately, our numbers for bonds are nominated in local currency while the MSCI index is nominated in USD. This implies that there will be some "noise" in the dataset, which we expect to influence the correlation we find. Furthermore, we have the problem of deleveraging of the bond returns, which causes us to rely heavily on an analysis done by DMS for our estimates of correlations[55].

DMS conducted a comparison of the real return of a portfolio of bonds and stocks in USA from 1900 to 2011. More specifically they looked at rolling averages of 5-year returns and found that the average correlation has been 19 % with considerable instability (maximum of 68 % and a minimum of -38 %). When the same analysis was done for the UK, they found a tendency towards a higher correlation between the stock and the bond return (31 % on average). In the last decade the correlation between equities and bonds has been sustained at a negative level. This is true even for a global portfolio of bonds and stocks. There might be a tendency for future correlation to be more similar to that of the recent past than that of the distant past, but we expect that the correlation between bonds and equities to increase in the long term. In our opinion we find a decent estimate when averaging the correlation between stocks and bonds that has been experienced in the UK and the US. Than we get the expectation of 25 % correlation.

[55] (Dimson, Marsh, & Staunton, 2011)

Conclusion

Based on the data we have had available and a qualitative assessment we found estimates for how real estate correlates towards bonds and equities respectively. To make an estimate for the correlation between bonds and stocks we had to partly rely on an analysis done by DMS. We have found that we expect zero correlation between real estate and bonds. Real estate and equities we expect will correlate 50 %, while the correlation between bonds and equities should be around 25 %. The results are demonstrated in the table below (table 9).

Table 9: Expectations regarding the correlation between the three different asset classes

	Bonds	Real estate
Equities	0,25	0,50
Bonds		0,0

A repetition of my expectations

All the final estimations with regards to the risk and return attributes of equities, bonds and real estate can be found in the table below (table 10). In table 11 one can see the expected correlations between the assets. Both tables should be straightforward to read.

Table 10: Risk and return expectations for all the asset classes

	Risk free rate	Equities	Bonds	Real estate
Expected Arithmetic return	1,80 %	6,85 %	2,60 %	3,5 %
Expected geometric return	1,80 %	5,49 %	2,39 %	3,0 %
Expected volatility	0	16,5 %	6,5 %	10 %

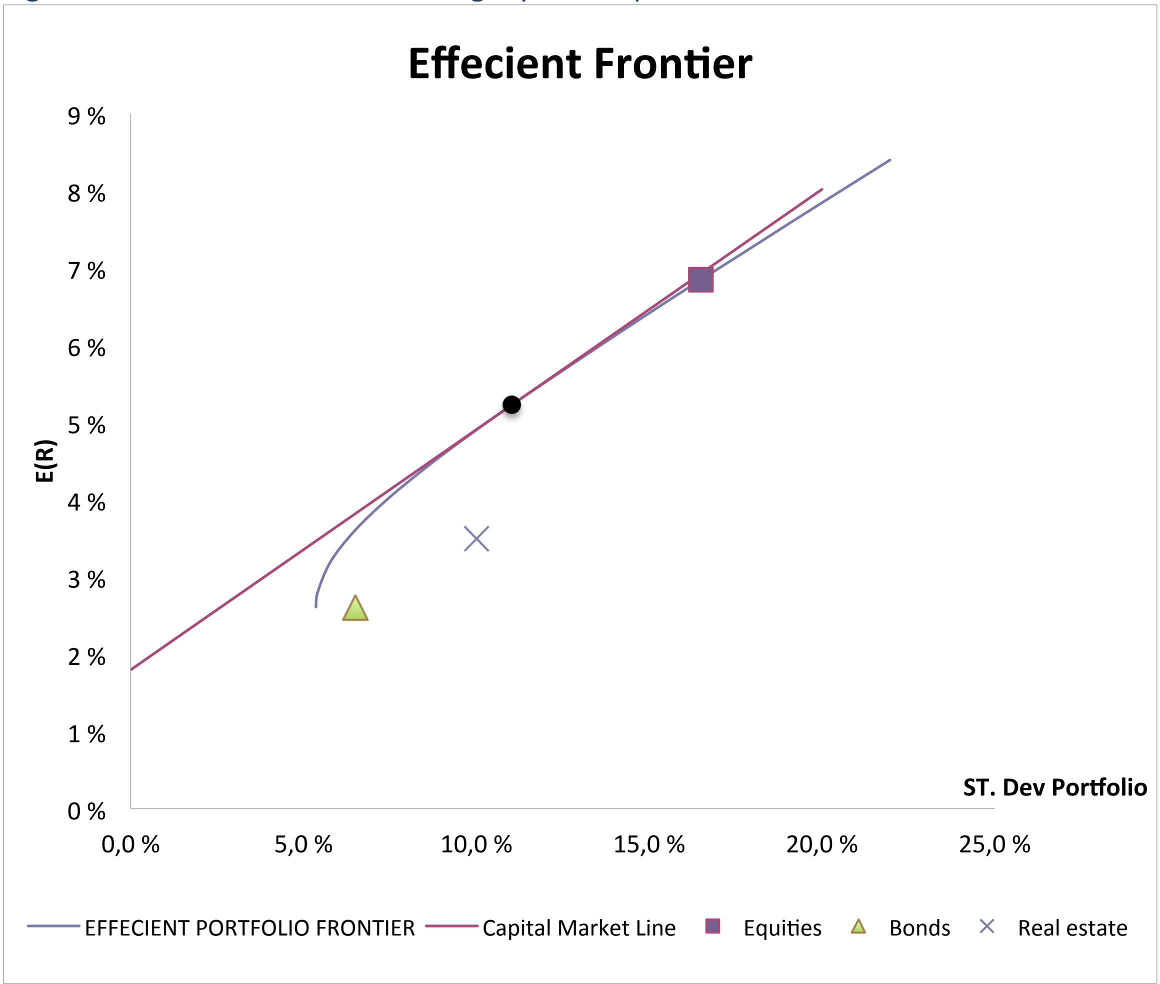
Table 11: Repetition of the correlation table

	Bonds	Real estate
Equities	25 %	50 %
Bonds		0 %

Markowitz framework using my own assumptions

Using the same approach as earlier we compute the efficient frontier based on the new assumptions. One can see the graphical result in the figure below (figure 10), where the efficient frontier is marked in blue. The y-axis shows the expected arithmetic return for a given portfolio, and the x-axis the volatility. The unlevered portfolio that maximizes the Sharpe ratio has been marked with a black dot. This is also the tangency point between the CAL (the red line) and the efficient frontier. The CAL intersects with the y-axis at 1,8 %. This represents a 100 % allocation towards the risk free asset.

Figure 10: The efficient frontier using my own expectations



Compared to when we relied on the estimates of the Ministry of finance the CAL is steeper. This means that the Sharpe ratio has increased. There are two main reasons for the improved risk reward relationship. The first is that we expect the risk free rate to be 1,8 %, instead of 2 % in real terms. Secondly we think the correlation between the different asset classes will prove to be lower than what the Ministry of Finance expect.

As earlier, we have also pinpointed the 100 % allocation to each of the different asset classes in terms of expected return and volatility. We see that bonds, represented by the green triangle, are the least risky, but also produce the lowest expected return. They are close to the mean variance portfolio, which is where the efficient frontier starts. Real estate, marked with a cross in the graph on the previous page, has attributes in between that of equities and bonds. The long distance it has from the efficient frontier indicates that no investors should hold only real estate. Equities have been drawn as a square in the graph, and are characterized by both its high risk and elevated expected return. While it lies close to the efficient frontier there´s a huge risk reduction potential in combining it with real estate and bonds. One can see that the efficient frontier does not curve much around the tangency point. Consequently the allocation that maximizes the Sharpe ratio can be said to be robust. This implies that if we have made a small mistake regarding our expectations of the risk and mean return of the different asset classes, the allocation would still be a good one.

A global portfolio of equities, bonds and real estate represents each point on the efficient frontier. Only the weights are changing as you move along the line. The start of the efficient frontier (to the left on the blue line in figure 10) is the minimum variance portfolio. It constitutes primarily of bonds (72 %), but real estate also accounts for a significant share (35 %). Equities on the other hand have negative weights in this point (-7 %), implying that the investor is taking a short position in this asset class. As we move along the efficient frontier (towards the right), we are gradually increasing our exposure to shares at the expense of bonds and real estate. This can be seen in the table on the following page (table 12), where the allocations towards the different asset classes can be found for certain points on the efficient frontier. The expected arithmetic return of the given portfolio can be read in the left column (marked in blue). Thereafter one can see the different market weights in the three columns in orange; equities to the left, followed by bonds and real estate. Finally one can observe the standard deviation of each portfolio on the extreme right (in green).

Table 12: Portfolio return matrix for the efficient frontier with properly developed expectations

Expected Return	Weights			Standard Deviation
	Equities	Bonds	Real Estate	
2,6 %	-7 %	72 %	35 %	5,4 %
3,0 %	-2 %	69 %	33 %	5,4 %
3,4 %	8 %	63 %	29 %	5,8 %
3,8 %	18 %	56 %	26 %	6,5 %
4,2 %	28 %	50 %	22 %	7,4 %
4,6 %	38 %	43 %	19 %	8,5 %
5,0 %	49 %	36 %	15 %	9,7 %
5,2 %	59 %	29 %	11 %	11,0 %
5,6 %	70 %	28 %	2 %	12,3 %
6,0 %	80 %	21 %	-2 %	13,6 %
6,4 %	89 %	10 %	1 %	14,9 %

The first portfolio we see in the table above is the mean-variance portfolio that we already have commented on. Subsequently the portfolios have 0,4 percentage points higher expected return, as we move down the table. We can see how the increased return expectations come at the expense of a higher standard deviation. **The allocations that optimize the Sharpe ratio (marked in green in the table 12) are 59,4 % equities, 29,4 % bonds and 11,2 % real estate.** With this, the investor can expect an arithmetic return of 5,2 % and a volatility of 11,0 %, implying a geometric mean of 4,6 %. The Sharpe ratio in this point is 31,10 %, which compares to 31,04 % with the current targeted allocation. To see the increased expected return that comes from an improvement in the Sharpe ratio we apply formula 7, and get the computation seen below:

$$(31{,}1\ \% - 31{,}04) * 10{,}9\ \% * 3800\ billion\ NOK \ = \ 248{,}5\ million\ NOK$$

This imply that with the current market capitalization of the GPFG, the fund can expect to gain 248,5 million NOK extra a year, only from the improvement of the Sharpe ratio. As the fund grows this amount will increase proportionally with the size of the fund.

By shifting towards the new weights the volatility increases marginally from 10,9 % to 11 %. To avoid this added risk one could combine the portfolio with a share in the risk free asset. However, we see no reason to do this, as the risk still is within moderate levels.

Conclusion

Based on a mean-variance analysis, with properly developed estimates we have found new ideal allocation weights. Interestingly we find that the GPFG should over time increase its allocation towards real estate from the targeted 5 % to 11,2 %. This implies that the exposure to this asset class should be more than doubled. The increase should mostly happen at the expense of bonds, which would yield an overall allocation of 59,4 % equities, 29,4 % bonds and 11,2 % real estate.

It might come as a surprise that we should increase the allocation towards real estate compared to when we used the estimates from the Ministry of Finance, as they expect the asset class to realize a higher real return. However, since we expect real estate to have a lower correlation towards bonds and equities, it looks more attractive in a portfolio perspective since more diversification benefits can be realized. In addition to this we expect the property portfolio of the GPFG to be less volatile, than what the estimates of the Ministry of Finance suggests.

By altering the weights we see that the fund could increase its Sharpe ratio, and thereby get better paid for its risk. With the optimal allocation weights the risk is 11,0 %. This is higher than the 10,9 % for the current goal of 60 % equities, 35 % bonds and 5 % real estate, but the investor would expect to be compensated through increased expected return. Theory suggests that we could lower the standard deviation by combining it with an allocation towards the risk free rate. However, we do not see this as necessary as the risk still is moderate.

The analysis so far has been simplistic in that it implicitly assumes that the investor only is concerned with the expected return and standard deviation of the portfolio. In the following we will therefore discuss several issues that goes beyond this, which is sometimes given as reasons for why real estate isn´t suitable to analyze in a mean-variance framework. We will however first look at how our suggested allocation weights compares to that of similar funds.

Comparing our findings with the allocation of similar funds

Increasing the targeted share of real estate to 11,2 % implies more than a doubling compared to the current guidelines. This would be a significant shift, and it might be helpful to test it against what other comparable funds do.

To do this we have looked at the slides in Capital Management at NHH, where the asset allocation of comparable funds has been presented[56]. The class where held in the spring of 2012 by Svein Gjedrem, so we assume that the numbers have been recently updated. He quoted CEM Benchmarking as his source, which is the most acknowledged independent provider of data related to the management of large funds such as the GPFG.

What we find is that an allocation of 11,2 % towards real estate would lie in the high range of what comparable funds do, where the average is about 6 %. Other funds are often present in other asset classes, than the three presented here. If one looks away from these placements the average share of real estate constitute 7 % of the three major asset classes discussed in this thesis.

Interviews has revealed that one of the risk factors institutional investors are most concerned about when making property investments is the lack of trustworthy return data[57]. As the time series become longer and the databases become larger, this problem should get reduced. When this happen, there might be reason to believe that the large institutional funds actually will increase their relative allocation towards the property market.

There were two other risk factors that the institutional investor were concerned about when making property investments. The first were lack of liquidity, and the second the risk of buying a property at a price that deviates from their "fair value". We will examine the relevance of these additional risk factors, when it comes to the management of GPFGs real estate portfolio in the following.

[56] (Gjedrem, 2011)
[57] (Dhar & Goetzmann, 2006)

Elements that fall outside of the analysis we have done

We have already stated that the Markowitz optimization is simplistic, and several elements of risks and costs can fall outside the framework. We will in the following look at some aspects that relate specifically to real estate, and see whether they are likely to significantly affect the optimal allocation. Issues that will be discussed are: transaction costs, liquidity, operational risk, market inefficiencies and inflation. All of the issues will be addressed from the perspective of the GPFG.

Transaction costs

Real estate is different from bonds and stocks in that you have significantly higher transaction costs. Each contract has to be negotiated individually and it can take several months to complete a transaction[58]. Since direct property investments are not listed, the terms of the contracts are not standardized and have to be negotiated between the parties. No matter which of the indices we use to compute our estimates for the expected future return of real estate, they do not account for the transaction costs related to buying a property and negotiating the contracts. However, to give a truthful picture of the attractiveness of real estate several academics claim that there´s a necessity to adjust our estimates to reflect these costs. (The brokerage fees and spreads for bonds and equities, conversely, are too small to significantly alter our return expectations.)

We consider there to be two kinds of transaction costs in real estate: indirect and direct. The first is chiefly the cost related to acquiring information and negotiating contracts, which will depend on the informational efficiency and the standardization of the market[59]. Direct costs are for example registration costs (stamp duty) and sales and transfer taxes. You would also have costs related to insurance, upkeep and repair, but since this is included in the indices we have used we don´t have to account for it again. The problem is that the above costs will be varying from one country to the other, and the GPFG will typically have specific treaties, which causes them to pay less. This is why

[58] (Norges Bank Investment Management, 2012)
[59] (Merton, 1987)

we have used the actual transactions NBIM went through with in 2011 as a basis for the discussion. What we find is that the effect of the transaction costs, were largely dependent of the time horizon. Since we expect most property investments to be held for a very long time the transaction costs would not affect the expected total return of this asset class significantly. This means that we do not need to make adjustments to our mean-variance optimization to account for transaction costs.

Market inefficiencies

Real estate has certain characteristics as an asset class that makes it different from investing in equities and bonds. When constructing a portfolio of equities, theory tells us that the expert and the novice have the same probability of beating the market[60]. The implication is that the investor doesn´t have a significant disadvantage when investing outside of his “home market”.

We have earlier argued that the real estate market is less efficient. A lesser degree of market efficiency will provide some investors with the opportunity to gain excess returns. Yet others can be affected adversely by buying when the prices demonstrate bubble tendencies, or by paying too much due to poor insight into the local market. The latter risk we expect to be avoided by leveraging local knowledge through joint ventures or other forms of partnership. Over time we expect the irrationality of the market (if this exists) to be favorable for the GPFG provided that they have a sufficiently flexible mandate that allows them to “buy when cheap” and even “sell when dear”. Since NBIM hasn´t had sufficient time to demonstrate their ability to generate excess returns in their real estate investments, we deem it prudent to not alter our return expectations at the current point in time. We therefore see no reason why potential market inefficiencies should invalidate the mean variance analysis we have conducted.

[60] This is one of the implications of the efficient market hypothesis

Liquidity

As previously discussed, it is normal for an investor to be compensated for entering into investments with poor liquidity. Since real estate typically is considered as rather illiquid, this could make the asset class look overly attractive when seen in a Markowitz framework (where the only risk the investor is concerned with is the volatility of the returns).

It has been argued that since the GPFG has no clearly defined liabilities it makes them particularly apt to make illiquid investments. We agree that having a long-term investment horizon makes liquidity less important, and one could maybe even use this as an advantage by buying when the liquidity premium in the market is particularly large.

On the other hand we believe that the size of the GPFG easily can work against them when making real estate investments. We earlier estimated the investable global core property market to be 8,5 trillion USD in 2012. However, the transaction volume tends to be low as real estate is primarily held as a long-term investment. In lack of numbers related to the transaction volume, we will assume that there are yearly transactions for 5 % of this amount in a given year. This would imply that properties for more than 400 billion $ worldwide are bought and sold in a given year. The market size of the fund is, as of today, close to 700 billion $. If 5 % of this capital were used to buy real estate in a given year, it would mean that the fund stood for 9 % of the total transaction volume that year, which one could expect to impact the overall market prices to some extent. As we expect the size of the fund to grow faster than the market capitalization of global real estate over the next 50 years, this aspect will increasingly provide a challenge for the fund. Although there have been made considerable approximation in the example above, it clearly demonstrate that the fund should acquire property gradually. If this is done we see no reason why the fund cannot over time reach a target allocation of 11,2 %.

Operational risk

The marketplace for direct real estate is less regulated than for traditional financial investments, making operational risk particularly relevant in the management of a portfolio of properties. Part of the problem is that transparency is lost, making potential corruption more of an issue. Additionally, unique legal agreements have to be put in place for each investment, which increases the chance of making costly mistakes.

Operational risk management is therefore of utmost importance when it comes to real estate investment. By carefully adapting mitigations and control mechanisms, sound legal frameworks in each deal and cooperating with serious partners we expect NBIM to make sure scandals are avoided. Consequently we will not have to make adjustments to the Markowitz analyzes.

Conclusion after looking at additional elements

Above we have gone through various aspects that are not considered in a traditional mean-variance analysis. We do not feel that serious adjustments are warranted to our input parameters, as a result. Expected returns, volatility and correlation for Real Estate appear robust, provided NBIM are aware of the specific characteristics and risks of real estate, and manages to implement strategies to mitigate the risks and turn the specific attributes of the asset class to their advantage. Since we expect this to be possible, we stand by our original analysis. **The GPFG should increase its allocation towards real estate to 11,2 % primarily at the expense of bonds. This implies a recommended allocation of 59,4 % equities, 29,4 % bonds and 11,2 % real estate.**

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Appendix

Appendix 1: Macro used to find the efficient frontier:

```
Sub solverEfficientFrontier()
Dim r As Long
For r = 62 To 83
   SolverReset
   SolverOk SetCell:="$F$" & r, MaxMinVal:=2, ValueOf:="0", ByChange:="$C$" & r &
":$D$" & r & ":$E$" & r, Engine:=1, EngineDesc:="GRG Nonlinear"
   SolverAdd CellRef:="$G$" & r, Relation:=2, FormulaText:="1"
   SolverAdd CellRef:="$H$" & r, Relation:=2, FormulaText:="$B$" & r
   SolverOptions AssumeNonNeg:=False
   SolverSolve UserFinish:=True
Next r
End Sub
```

Appendix 2: Computation of the size of the global equity market

The size of the fund is roughly 3,8 billion NOK the 15th of December 2012. We know that the allocation towards equities is supposed to be 60 %. NBIM displays a graph on their homepage where one can see that the GPFG owns close to 1,25 % of the worlds shares as of the 2nd of November 2012. 1 USD is worth about 5,62 NOK the 15th of December. Using these input variables we are left with the following equation:

$$Size\ of\ global\ equity\ market\ in\ USD = \frac{3{,}8\ billion\ NOK * 0{,}6}{0{,}0125 * 5{,}62} = 32{,}5\ \ Billion\ USD$$

Appendix 3: Computation of the size of the global bond market

We assume that the relevant investment universe for a global bond investor is: the public debt securities outstanding, financial institution bonds outstanding and corporate bonds. From qvmgroup.com we find that these respectively amount to: 41 billion USD, 42 billion USD and 10 billion USD. This means a total of 93 billion USD. Since we assume that not all of this is investable we have taken the number down to 75 billion USD. This is a rough approximation, but sufficiently precise give an indication of the bonds share of the total worldwide investable universe.