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Can price reversals on Oslo Stock Exchange be exploited?

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Abstract

We investigate if price reversals on the Oslo Stock Exchange can be exploited using a twofold method. Our method includes stock selection based on variance ratios and simulation of portfolio performance using a contrarian trading strategy. Existing evidence of mean reversion from major stock exchanges motivate our approach. Our results show consistently better performance for portfolios sorted by desirable variance ratios than for portfolios with undesirable variance ratios. However, most of these portfolios fail to produce excess profits after transaction costs.

Keywords: mean reversion, Oslo Stock Exchange, market efficiency, variance ratio, contrarian strategy

*This thesis was written as a part of the Master of Science in Economics and Business Administration at NHH. Please note that neither the institution nor the examiners are responsible through the approval of this thesis – for the theories and methods used, or results and conclusions drawn in this work.

Contents[1]

[1] An overview of used abbrevations and notation can be found in appendix A.3. Part of our results can be reproduced in this shared *R code* https://drive.google.com/folderview?id=0B-VDwXnxUxV1NlJJU3VPLUJDMUk&usp=sharing

1. Introduction

"Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as a result of animal spirits — of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities."

These are the more than 75 year-old words from John Maynard Keynes' famous book from 1936, "The General Theory of Employment, Interest and Money". It states that in desire to do well, individuals tend to take action rather than not. In effect, this could cause irrational overreacting to news and events. Does this cause anomalous behaviour on the Oslo Stock Exchange? This would suggest some degree of mean reversion in stock prices. Could it prove exploitable? These two questions are the main focus for this paper. This way, we shed light on whether the assumed rationality and non-existing arbitrage opportunities assumed in financial theory holds true in practice. It could also help us by getting increased understanding of investor behaviour and its consequences.

Our research is inspired by the works of Andrew W. Lo and A. Craig MacKinlay, who formally introduced the variance ratio in 1988. Why their concept applies to us is best explained in context with our goals for this paper. Aiming to form a concise paper focusing on readability and continuity, models and data are kept as intuitively comprehensible as possible. The variance ratio is a relatively simple measure using well known concepts. When added to the implementation of an exploiting contrarian trading algorithm, our study is one of few taking this practical approach on a smaller exchange like the OSE.

First we investigate the possibilities of mean reverting behaviour by using the variance ratio criterion. We then relate the observed variance ratio and other stock characteristics to the success of a contrarian trading strategy. The practical applicability of our strategy is evaluated by using a trade simulation program. We find clear signs of a positive relationship between the variance ratio and the success of a contrarian strategy. Some degree of predictable persistence in the variance ratio is also found. However this is not sufficient to consider our strategy profitable, as most considered portfolios report negative returns even before transaction costs. Hence overall, we are not able to reject the existence of weak form efficiency for the OSE.

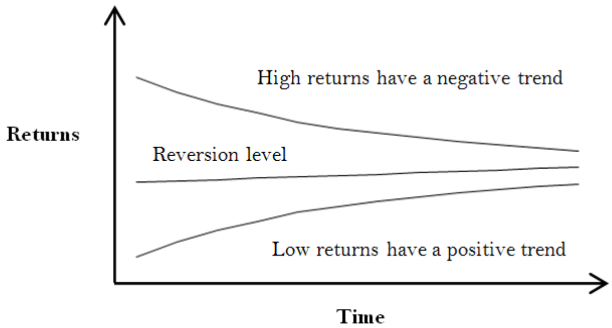
The paper is organised as follows: Section 2 is a review of relevant literature. Section 3 explains the methodology. Section 4 describes the data. Section 5 presents and discusses our results, while the 6^{th} and final section contains our concluding remarks.

2. Literature review

2.1 Mean reversion

In finance, a popular approach in predictions of the future is to investigate possible convergence towards an underlying level or rate over time. The subject is essential to financial market analysts and academics estimating future cash flows and speculating in stocks, as well as more formal testing of equity markets[2]. We apply the following definition of mean reversion in equity markets, as defined by Balvers et al. (2000) "Mean reversion refers to a tendency of asset prices to return to a trend path". Figure 2.1 illustrates the concept. In this paper, we refer to this trend path as the *fundamental value* of the stock price[3]. As noted by Hillebrand (2003) mean reversion in stock prices must necessarily imply the same for stock returns.

Figure 2.1 Mean reversion concept illustrated



The efficient market hypothesis (EMH) is one of the most covered aspects of financial literature. As seen in Malkiel & Fama (1970) this hypothesis supports the existence of a pure random walk (RW) in stock prices. This has to be mutually exclusive with mean reversion. Therefore, much of the existing literature on the latter subject involves challenging the random walk model as the null hypothesis. In practice, this is testing the EMH claim of weak form efficiency in stock prices. This degree of efficiency claims that all past prices of a stock are already reflected in today's price. If so, spending time doing technical analysis would be pointless.

[2] As seen in e.g. Lo and MacKinlay (1988).
[3] Other suitable names could have been *intrinsic value* or *underlying level* of the considered stock price.

Main contributions

"Does the stock market overreact?" by DeBondt and Thaler (1985) was one of the first articles bringing mean reversion (MR) up explicitly. They test whether returns over different time periods are dependent on each other, contradicting the assumption of i.i.d returns. Focusing on stocks having experienced extreme gains ("winners") or losses ("losers") in the past, they check for longer term mean reverting tendencies. On a data set from 1926 to 1982, they find that the losers outperform the winners with a statistical significant difference of 24.6% in returns over three years. These results show that the null hypothesis of equal expected returns between past winners and losers can be rejected.

The variance ratio (VR) test as it is used here was first employed in Lo and MacKinlay (1988). Using data from 1962 to 1985, they find that the behaviour of weekly returns is not consistent with the RW model, especially for smaller cap stocks. They find significant positive serial correlation for weekly and monthly portfolio returns. The weekly first order serial correlation of their return index is as high as 30 percent. As this may sound unpromising for our approach, this is not the case. The estimated autocorrelations of the *individual* securities are generally found to be negative, which would be an attractive property to use in our trading strategy.

Poterba and Summers (1988) analyze whether transitory components account for much of the variance in common stock returns. In turn, this could indicate mean reversion. They conclude that if mean reverting components exist, the stock market may be much less risky than it appears when considering the variance of single-period returns. In their results, stock returns are positively serially correlated for short horizons and negatively serially correlated for long horizons, accounting for more than 50 percent of the variance in monthly returns. This should favour longer term investors, who then should invest more in equity. Supporting these results, Fama & French (1988) use regression methods on data from 1926 to 1985 finding that for a 3-5 year period, 40 percent of the variance in the returns of small firms is predictable, against 25 percent for bigger firms. Like previous studies, they conclude that stock prices of smaller firms are more likely to exhibit mean reversion than that of bigger firms. Mukherji (2011) uses bootstrap methods[4] for both older and newer data (1926–1966 and 1967-2007). He concludes that evidence of mean reversion has weakened in recent decades, but still persists for US stocks.

The availability on research papers regarding efficiency on the OSE is sparse. Older studies by Jennergren & Korsvold (1974) find signs of weak inefficiency. A more

[4] Techniques being used for estimation and re-sampling of sampling distributions.

interesting finding is that Norges Bank Investment Management fund reports of mean reverting tendencies on international stock exchanges. The mentioned works of Poterba & Summers also observe a clear tendency of more mean reversion on smaller stock exchanges. This could very well apply to the OSE.

Main concerns

Studies have also produced evidence against mean reversion. McQueen (1992) states that earlier tests are biased towards old data, and points out the dangers of relying on an asymptotic test[5] like the VR test. In addition, Pástor & Stambaugh (2012) emphasize that variability in non-observables like future expected returns and estimation risk is higher at longer horizons, which could offset the effects of mean reversion in the longer run. Both studies conclude that longer term stock investors face more volatility than their short term companions.

Through Monte-Carlo simulation, Poterba and Summers (1988) conclude that variance ratios are powerful for detecting mean reversion, but has little power against the principal alternatives for the RW hypothesis. They demonstrate the difficulty of distinguishing the RW from its alternatives, and argue that the only way to handle this problem is the collection of more data. Deo and Richardson (2003) show that when the time window for the mean reversion increases together with the length of the sampling period, the VR statistic becomes increasingly inconsistent[6].

2.2 Contrarian strategies

A contrarian strategy sells previous winners and buys previous losers in anticipation of a mean reverting effect. As stated by Forbes (1996), there is an intuitive link between mean reversion and a contrarian strategy trading rule. According to Jensen (1978) the EMH is violated if we can exploit reversion tendencies via a net profiting trading rule. To estimate the profitability of our approach we have applied an automatic trading framework inspired by Faber (2007) and Lo and MacKinlay (1990). Trading rules are based on input parameters and mechanical algorithms. The benefit of this approach is the ability to handle large datasets, as well as denying any form of subjectivity and biases in our trading decisions.

[5] When the limiting distributions of a random variable are unknown.
[6] Using our notation, this corresponds to $k/n \to \delta > 0$.

For producing buy – and sell signals we have made use of Bollinger bands, as presented in Bollinger (1992). Many studies exist on the possible profitability of technical analysis indicators, but most have failed to show their ability of producing consistent net profits. This is often due to excessive trading activity resulting in overwhelming transaction costs. On results being sensitive to parameter values and data-snooping, we notice the warnings of Black (1993) and a recent article by Pavlov and Hurn (2012). The latter found that after smoothing out and employing a general set of values for their considered time window, the strategy producing the "parameter-robust" positive return was in fact a contrarian strategy. More recent techniques of similar technical analysis include the use of for example stochastic discount factors, as seen in Cochrane (2001) or Hansen et.al. (1997).

2.3 Anomalies

The amount of research testing the efficient market hypothesis is extensive. In the words of Malkiel (2003) "(...) stock markets are far more efficient and far less predictable than some recent academic papers would have us believe." He also states that "markets can be efficient even if stock prices exhibit greater volatility than can apparently be explained by fundamentals like earnings and dividends." This is backed up by the belief that markets successfully reflect all new information rapidly and accurately. The problem is that the correct market response is never observable to us, not even in hindsight; the closest we get may be to use *ex-post* values. Even then, this will just be an agreed upon conventional value with no real guarantee for reflecting past true values. We look at two separate cases when considering market anomalies further:

- Irrational investor behaviour contradicting the EMH
- Seemingly irrational investor behaviour when the EMH still holds

Investor anomalies

Economic wisdom tells us to "buy cheap and sell dear". This sounds appealing and straightforward, but it has been shown that judgements are usually made using a representativeness heuristic. As stated by Tversky and Kahneman (1974), many will try to predict by seeking the closest match to past patterns without regarding the probability of matching the pattern. This has also been backed up by experimental evidence, as seen in Andreassen and Kraus (1988) or Marimon and Sunder (1993). Daniel et al. (1998) show

how individuals exhibit self-attribution and overconfidence in themselves. People tend to attribute events that confirm the validity of their actions to their own ability, while less favourable outcomes are attributed to bad luck or possible sabotage. They show how this implies negative long-lag serial correlation and excess volatility. DeBondt and Thaler (1985) also conclude that most people tend to overreact to unexpected and dramatic news events.

There is some evidence for a price-to-price feedback theory. According to Shiller (2003), initial speculation will cause prices to go up, benefitting initial speculators. By attracting attention, word-of-mouth enthusiasm and in-hindsight "new era" theories, expectations for the considered asset are once again heightened. During more rounds of positive feedback, this gives rise to a speculative bubble. We now have high expectations for future price increases, justifying the very high price level of the asset. This expectation-driven rapid increase in the price level cannot be sustainable in the longer run. The bubble eventually bursts, causing prices to fall drastically. A famous example of this is the Dutch tulip mania and following market crash in the 1630s. Following the same psychology we may now see a similar negative spiral, again driving values away from its relevant fundamentals. It then appears that the tendency of relying on empirical data, self-attribution and the return chasing nature of investors may cause the observed anomalies of the stock market.

Observing seemingly irrational behaviour in an efficient market

This part covers a more optimistic view on behalf of the investor. While observing deviations from the EMH, there could still be possible explanations for this behaviour to be rational. Main consensus has been that rational speculators must stabilize stock prices. Buying when prices are relatively low and selling when prices are high puts upwards and downwards pressure on the current price, respectively. In other words, rational speculators cause mean reversion through a negative feedback mechanism.

In the presence of a positive feedback mechanism, rational speculation can be destabilizing. When rational investors receive good news and prepare to trade on this, they anticipate that the price increase from the initial level will trigger positive feedback traders to buy the next day. As a response, the rational investors buy more than the news actually calls for. The next day, positive feedback traders buy in response to the price increase, keeping price above fundamental values even after rational speculators sell out to profit. The forward-looking rational speculators anticipate the trending behaviour of the market, and magnify the overall trading reaction by buying more than warranted for by the initial news. Long et al.(1990) conclude that rational behaviour by investors may increase market volatility. Note that this is only partly able to rationalise the behaviour of investors in the market, as someone (here; the positive feedback traders) has to be on the losing end once the bubble bursts and prices revert back.

Fama and French (1988) argue that the predictability of returns could also be the result of time-varying equilibrium expected returns. These may very well be generated by rational pricing in an efficient market. One example is the estimated risk premium, as seen in the Capital Asset Pricing Model (CAPM)[7]. They conclude that the cumulative effect of shocks in the expected returns must be exactly offset by an opposite adjustment in the current price. This highlights the downside of applying time-series tests of market efficiency; irrational price bubbles are indistinguishable from rational time-varying expected returns.

3. Methodology

3.1 Our approach

Numerous studies have been examining the possibility of imperfect capital markets. In many of these studies, the link between theory and practice may seem unclear. In this paper we take a more practical approach, resulting in a twofold paper. We will first provide examples of some of our data, and then explain how we want to investigate and possibly exploit these. First we use a theoretical approach to indicate possible mean reverting tendencies. Secondly, this approach is tested in practice by simulating its performance.

The relationship between the variance ratio and the success of a contrarian trading strategy is of small practical value if there is no predictability in the variance ratio. If VR patterns persist, we could form portfolios out of stocks that have showed mean reverting patterns in the previous period. To test this we will investigate the contrarian success of portfolios that showed significantly low VRs in the previous period. We will also investigate whether the market cap and liquidity of a stock possess predicative power of exploitable patterns. More details on these characteristics are described in section 3.4.

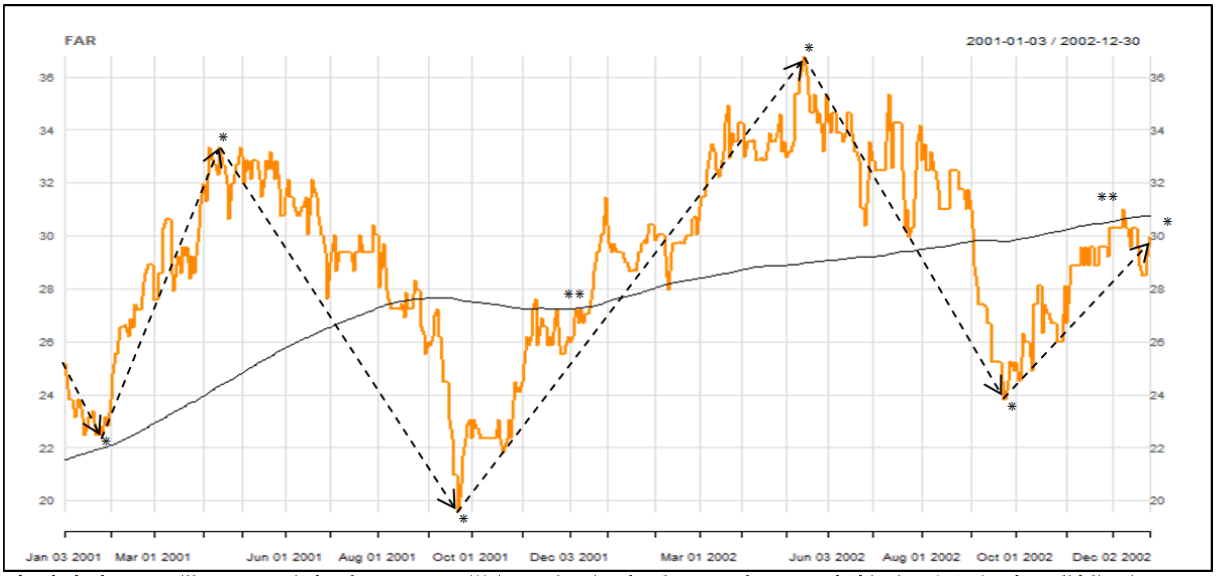
[7] $E[r_i] = r_f + \beta_{i,m} * (E[r_m] - r_f)$, where $(E[r_m] - r_f)$ is the time-varying estimated risk premium.

3.2 Mean reversion

Besides the random walk model, two alternatives are considered for explaining the development pattern of a stock price. If the price increases and the stock price exhibits mean reversion, the price is expected to revert back towards its fundamental value. This corresponds to the stock exhibiting negative serial correlation. On the other hand, the price pattern can be based on *momentum*; the price of the stock is likely to keep moving in the same direction rather than to change direction. This corresponds to exhibiting positive serial correlation. The term *fundamental value* might appear a bit vague. Here it proxies the k-period *moving average*. This will be explained in more detail in the next sections.

A stock price can exhibit different development patterns for different time horizons. For example, the price of a stock may overreact to short-term shocks, causing momentum for shorter time windows. It may also display mean reversion in the longer run if these short term shocks tend to wear off after a certain time period. Thus, momentum and mean reversion are only mutually exclusive over the same time windows. This is demonstrated in Figure 3.2.1.

Figure 3.2.1 Shorter term momentum and mean reversion in the longer term



The dashed arrows illustrate periods of momentum (*) in stock price development for Farstad Shipping (FAR). The solid line is a one year simple moving average which the stock price reverts back to both year-end 2001 and 2002 (**). In several shorter term periods FAR stock price exhibit momentum properties, while it continues to revert towards its one-year moving average in the longer run.

The random walk model

The efficient market hypothesis assumes that prices follow random walks. If this holds true, it is not possible to obtain excess profits by modelling future stock price movements. We challenge this debated theory with the objective to profit from a contrarian strategy. If an empirical approach proves beneficial, it could imply that the weak form efficiency does not hold for the OSE. One model describing efficient pricing is the *geometric Brownian motion (gBm)*[8]. We will here treat the *gBm* as a continuous-time variant of the random walk[9].

$$\frac{dS_t}{S_t} = \mu_t * dt + \sigma_t * dW_t, \qquad S(0) = S_0$$

S_t is the stock price, μ_t is its percentage drift rate and σ_t is the volatility over the considered time interval dt. dW_t is a standard Brownian motion $\varepsilon\sqrt{dt}$ where $\varepsilon \sim N(0, \sigma^2)$. By solving the equation for S_t we get

$$S_t = S_0 * e^{\hat{\mu}_t * t + \sigma_t * W_t}, where\ \hat{\mu}_t = \mu_t - \frac{1}{2} * \sigma_t{}^2$$

using logs we get

$$\ln(S_t) - \ln(S_0) = \hat{\mu}_t * t + \sigma_t * W_t$$

which yields

$$Var[\ln(S_t) - \ln(S_0)] = \sigma_t{}^2 * dt,$$

The variance of the logged stock price movements must be linearly increasing in the time interval. In other words, the variance in returns over t days should not be different from t times the one-day variance. From now on, we will use k as notation for the (daily) length of the mean reverting cycle.

[8] A stochastic differential equation used in mathematical finance, e.g. as in the Black-Scholes option pricing formula. See Wilmott (2007) for an introduction to the topic.
[9] As showed by Sottinen (2001), the discrete-time RW model converges to a Brownian model as dt approaches zero.

The Variance Ratio test

The variance ratio test investigates if this property holds true. It provides a simple specification test based on the variance in returns, testing the above stated assumption of linearity in returns. If consistently able to reject the null hypothesis of a random walk, there could still be room for earning profits by modelling stock price patterns. The main strengths of this test is that is intuitively comprehensible, it requires little computational power, and has through Monte-Carlo simulation proved to be more reliable than other comparable statistic tests like the Dickey-Fuller t-test and the Box-Pierce Q test[10]. As there exist general consensus that stock market volatility changes over time, it is also worth noting that the VR test under certain assumptions is robust to heteroscedasticity. Defining $r_t = \ln(S_t) - \ln(S_{t-1})$, the variance ratio can be formulated as

$$VR(k) = \frac{Var(r_t + r_{t-1} + \cdots + r_{t-k+1})}{k * Var(r_t)} = 1 + 2\sum_{j=1}^{k-1} \frac{(k-j)}{k} * \rho_j$$

The first term compares the *total period* (numerator) and *k* times the *daily* (denominator) volatility of individual stocks. When the daily return volatility is high compared to that of the total period[11], it would seem that the shorter term variance is overstated. This is what we want to exploit with our mean reverting strategy, and corresponds to a low $VR(k)$ statistic. For the second term ρ_j is the *j*th lag serial correlation coefficient of the returns. We want to test the null hypothesis that the log stock price and its first difference, the returns, is a collection of i.i.d. observations. When returns are uncorrelated over time, we should have $VR(k) = 1$. A variance ratio significantly less than 1 reveals possible mean reverting tendencies for the stock. A variance ratio significantly greater than 1 could indicate momentum behaviour. As mentioned earlier, focus will be on findings of the former case.

One should also be aware of some shortcomings of the VR test as a tool for predictive purposes. The calculated variance ratios are positively skewed, as the variances cannot be negative. This causes the variance ratios to have a lower bound of zero, while all positive values are theoretically possible. Both Poterba and Summers (1987) and Deo and Richardson (2003) point out that this has implications for the power of the test when k increases relative to the number of observations n. The rejection of a random walk does not offer any explicit guidance towards a more credible model. For example, the alternative of

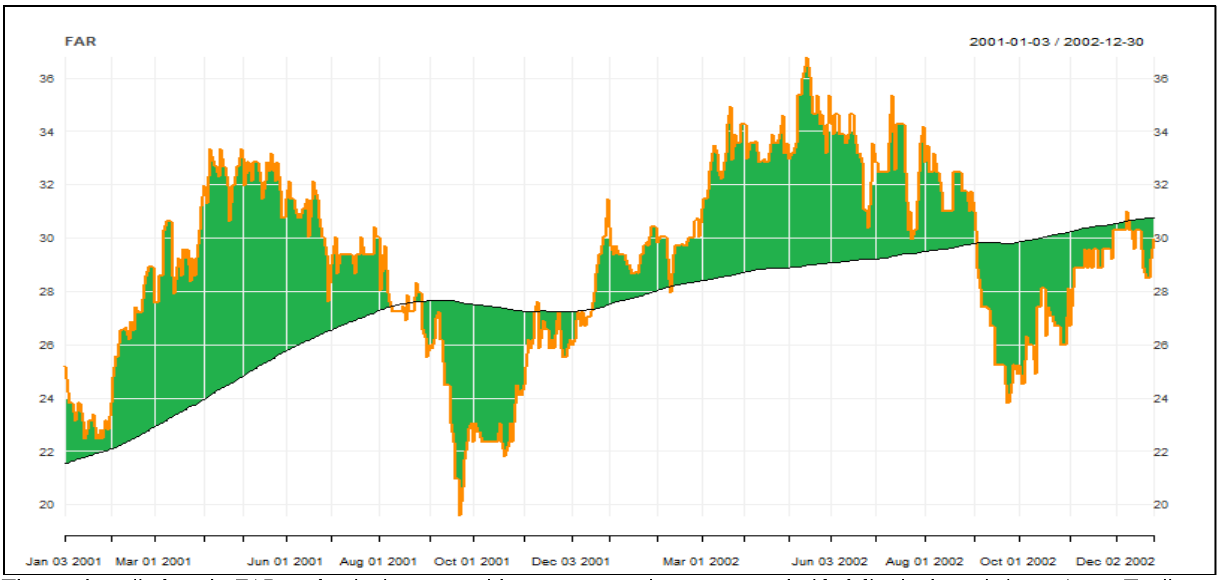
[10] Lo and MacKinlay (1989)
[11] The variance ratio in terms of prices can be written as $Var(\ln(S_t) - \ln(S_{t-k}))/(k * Var(r_t))$

an Ornstein-Uhlenbeck process states that the speed of reversion depends on the deviation from the mean[12]. If this would be a more precise formulation, its properties would be a valuable attribute in our trading strategy. But for the time being, we leave this alternative as a subject for future research.

3.3 Contrarian strategies

Having an idea of which stocks and for which time windows we have mean reverting tendencies, we test whether an exploiting trading algorithm makes excess profits. This is done to confirm or disconfirm that the variance ratio measure indicates success of a contrarian strategy. The profits we try to obtain can be illustrated as in Figure 3.3.1.

Figure 3.3.1 Illustrating scope for profits using a contrarian strategy



The top chart displays the FAR stock price in orange with a one year moving average as the black line in the period 2001/2002. Trading positions that are taken in our contrarian strategy are set to be closed when the stock price crosses the moving average which is the middle band. The distance between the price and the moving average therefore indicates scope for profits at the point of time considered.

We make use of what we refer to as *modified Bollinger bands*. The modification is due to the inclusion of estimated transaction costs as part of the trading band, trying to avoid undertaking trades where these costs are expected to outweigh that of the expected return on the trade. The strategy itself is made out of a trading algorithm, for which the framework is rigid yet simple[13]. The trading algorithm is a precise recipe that specifies the exact sequence of steps required to simulate the trading strategy. It provides for simulation of large datasets while keeping behavioural biases and possible suspected data mining to a minimum. The algorithm opens positions using buy/sell indicators that are triggered by

[12] $dS_t = \alpha(\mu_t - S_t)dt + \sigma_t dW_t$
[13] See Appendix A.5 for the formulations of the trading signals in the quantitative model framework.

trading signals. The weight of each position will be dependent on the available number of tradable shares and the total current value of the portfolio. In other words, all initiated trades are given equal weights in terms of current portfolio value[14].

By using this approach, we are assuming that the movements in the stock price are "noise" around the fundamental value. Otherwise they are outcomes of changing market conditions for the stock, in which case the market would be correct to make a price correction. This could lead to lacking or inappropriate responses from our side.

Transaction costs

An important feature of real-world trading is transaction costs. Many strategies could have potential of achieving excess profits, but these strategies seldom survive after accounting for imposed costs. Often this comes from over extensive trading. According to Ødegaard (2008), most of the direct trading costs arise from the relative bid-ask spread[15]. On average, the percentage cost of a round-trip (one open and one close) is equal to this measure. The transaction cost c is therefore equal to the average relative bid-ask spread in addition to a minor brokerage fee of 0.1%. These costs are incorporated into our trading algorithm, aiming to account for the issue of over-trading. Each stock does *not* face its own real-time individual transaction costs in our simulations. Due to unstable bid-ask data and programming issues, transaction cost is calculated as the broad daily average of all considered stocks.

Bollinger bands

Existence of a mean reverting component in the stock price is in itself not enough to develop a complete trading strategy. We also need a framework for producing trading signals. Bollinger bands construct trading bands around the price path of the stock, creating upper and lower bands indicating whether prices are high or low on a relative basis. When the stock price crosses outside of the band, a trading signal is made. A signal of a high price will initiate a short position, and a signal of a low price will initiate a long position. A position is closed when the price reverts back to its moving average. The size of the band range is determined by the time-varying volatility of the stock.

The technique makes use of basic measures like the simple moving average and standard deviations, and is applicable to any market or security. The standard value for

[14] Formally, the size of each position is given by $(500\,000 + available\ equity)/Number\ of\ stocks\ in\ portfolio$

[15] The relative bid – ask spread (rBAs) is $(Ask - Bid)/(Ask + Bid)/\ 2$, so total transaction costs $c = rBAs + 0.1\%$.

triggering a trade signal is two standard deviations, which will also be used here[16]. As even stated by its proponents, it must be emphasized that these indicators in themselves do not make absolute buy or sell signals. Combined with the proposed tendency of overstated volatility, they may contain exploitable information. Transaction costs are attached into the bands. As mentioned previously, this is a way to prevent excessive trading and transaction costs. For a higher transaction cost, there must also be percentagewise higher expected return to take the trade. This can be seen in Figure 3.3.2.

For each trade, we must also be able to formulate what price path (and hence; what returns) we expect to develop over the time window of k. The solution of the random walk-modelling *gBm* earlier gives us

$$E\left[ln\left(S_{i,t=k}\right)\right]_{RW} = ln\left(S_{i,0}\right) + \hat{\mu}_{daily} * k_i$$

which again yields[17]

$$E\left[S_{i,k}\right]_{RW} = S_{i,0} * \left(1 + \hat{\mu}_{daily} * k_i\right)$$

As the simple moving average is a proxy for the true fundamental value of the price, we get

$$E\left[S_{i,k}\right]_{MR} = MA_{i,0} * \left(1 + \hat{\mu}_{daily} * k_i\right)$$

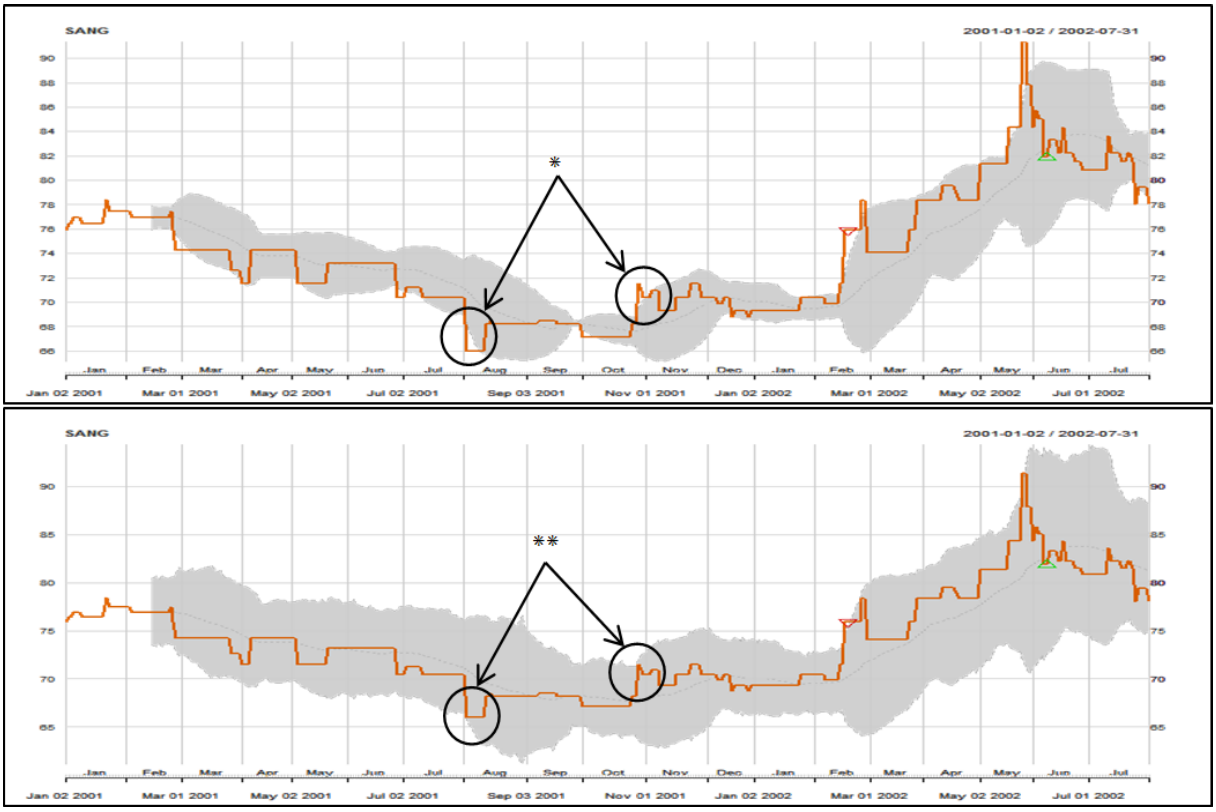
In addition to returning towards its moving average, we also expect the price to increase by a drift term over k days. It is important to note that the length of the mean reverting time window k for each stock will be equal to the time window for which we found MR tendencies in the VR test. E.g. if results are indicating MR tendencies for a k-value of 32 days for a stock, Bollinger bands will also be calculated on a basis of 32-day moving averages and standard deviations. These will be denoted as $MA_{t,k}$ and $\sigma_{t,k}$ respectively.

$$Upper\ band: \theta_U = (MA_{t,k} + 2\sigma_{t,k}) * (1 + c + \hat{\mu}_{daily} * k_i)$$

$$Lower\ band: \theta_L = \frac{MA_{t,k} - 2\sigma_{t,k}}{(1 + c + \hat{\mu}_{daily} * k_i)}$$

[16] Some increases the no. of standard deviations as k increases, but there seems to lack consensus for k-values over 50.
[17] Note that μ_t is already defined as the percentage drift rate of the stock price.

Figure 3.3.2 The effect of modification to standard Bollinger bands[18]



The two charts show SANG stock price development in 2001/2002-07 coloured in orange. The top chart has standard Bollinger bands shaded around the stock price while the bottom chart has modified Bollinger bands that are wider. In the top chart we see that the stock price cross the outer bands at several times (*). This would have produced more than the single pair of one trading and close signal indicated as red and green triangles, respectively a short signal in Feb 2002 and a close position signal in June 2002. In the bottom chart we see that the modified Bollinger bands isolate those potential signals above, allowing only for the single trade mentioned. (**)

Riding the bands

A longer-lasting persistent price shock may induce what is called "Riding the bands", if the momentum is strong enough. This will cause volatility to increase, and the price may stay outside of its bands during the upturn (fall). A contrarian strategy will react by rapidly trying to short (buy) the stock. If this process goes on unhindered, it may cause significant losses as we initiate increasingly larger positions in losing investments.

As a way to mitigate this problem, we do not allow for subsequent buying/shorting of the same stock before the position is closed. In this way we help to control the downside of our investments. Our investments still have a limited upside but a larger downside, as the price always reacts faster than the moving average curve. For the upside, the reverting price must sooner or later cross the moving average, and the profit is realised. For the downside, this does not need to be the case. If the price keeps moving rapidly in the same direction, the moving average may not catch up. A short position will then have an infinite

[18] When the band is crossed we have $S_t \approx (MA_{t,k} \mp 2\sigma_{t,k})$

downside (as a price has no upper bound), while a long position will have a downside equal to the size of the initial investment[19]. An infinite downside is not realistic, but the general concept of potential great losses still applies. One way to mitigate some of this risk is to use stop-loss rules. But following our mean reverting train of thought and to avoid further complexity, no restrictions will be considered here.

Portfolio details

It is assumed ability to trade on all days. All trades are initiated at the end of the day of the trading signal. The trading strategies employed consider each stock separately. This means that even as the securities are tested collectively as a portfolio, they are all objects to their own independent trading algorithm. The portfolios have no initial positions, and can be characterized as *null portfolios*. We still regard the trading as if we have initial long positions in addition to actively adjusting the positions a proportion up (buy) or down (sell) based on trading signals. This allows us to consider our null portfolio performance as net performance over a passive long portfolio performance. A net negative position does not necessarily constitute a short-sale; it indicates the deviation from a passive holding portfolio. As a result, a net short/selling position imposes no additional cost over taking a long position. Long positions are financed by borrowing money and cash flows from "short" positions are put into a liquid risk free investment[20].

3.4 Portfolio selection

It is interesting to check if any particular stock characteristics seem to influence its performance in a contrarian strategy. We will rank the suggested stock characteristics into two different portfolios, one *top* and one *bottom* portfolio. We then use the difference in results from these portfolios to explore possible systematic patterns that could indicate a relation to mean-reverting behaviour.

VR sorted portfolios

Applying the trading algorithm on stock price series with the most desirable VR test statistics, does the success of the contrarian strategy increase with the observed statistic? It would then appear that the variance ratio measure is appropriate for trading purposes.

[19] A practical example of this is illustrated in section 5.4.
[20] The risk free rate will be equal the 3-month NIBOR rate, and is the same for both borrowing and lending.

Short vs. long term mean reversion

From earlier literature, mean reversion has shown to be more present over the long term rather than the shorter term. One of the important features of a longer investment horizon is the increase in absolute risk[21]. The scope for profits will be higher, but the same will also be the case for losses. A longer time horizon gives the market more time to return back to its fundamental value if the price pattern is dependent on underlying (but hopefully mean reverting) components. Common examples could be market interest rates or business cycles.

Market cap sorted portfolios

Earlier studies have found small cap stocks to exhibit mean reversion. One explanation for this is that these stocks do not receive as much attention as others in the market, and the probability of erratic and seemingly irrational behaviour in prices could increase relative to others.

Liquidity sorted portfolios

The liquidity of a stock could relate to mean reversion tendencies. It is known that the liquidity of a stock is reflected in the trading cost[22]. As the bid-ask spread is a large component of the trading cost, it provides us with a good estimator for the liquidity of a stock.

[21] To see this, see for example the last equation on p.9.
[22] E.g. as stated by Foster and Viswanathan (1993).

4. Data

4.1 Børsprosjektet

All of the empirical data used in our dataset are collected from NHH's database, *Børsprosjektet*. The database contains daily Norwegian equity price data from 1984 and onwards. All prices have been adjusted for dividends and splits. Taxes and possible slippage are not included.

Oslo Axess

The opening of Oslo Axess in May 2007 gave smaller and medium size companies with growth ambitions opportunities to access an authorised and fully regulated marketplace with venture capital. The requirements for admission to listing on Oslo Axess are less detailed than for Oslo Børs. For our data period this marketplace has a short history with generally less mature and infrequently traded securities. We exclude securities listed on Oslo Axess from the dataset and focus on the larger and more stable history from Oslo Børs. Therefore, we disregard Oslo Axess when we refer to the OSE.

4.2 Data overview

The data set ranges from the January 1993 to December 2012. We consider this period appropriate as markets had settled after introduction of the electronic order book system in 1988, and it also includes the turbulences of the 1990s and 2000s. We will look at blocks of four year sub periods, leaving us with five sub periods in total.

If a stock enters or exits the exchange during a sub period, it is not included[23] – only full datasets for a sub period are considered. All stocks must have a price of at least NOK 10 exiting the previous sub period[24]. It is very challenging to produce net profits from trading high transaction cost stocks. As a result, stocks having an average relative bid-ask spread of over five percent over the previous sub period are also excluded. The remaining dataset contains daily observations of 144 unique stocks. A brief summary of these can be seen in Table 4.2.

[23] The requirement of complete time-series is a programming issue. See explanation in section 4.4.
[24] These are criterias also used in Ødegaard (2012).

Table 4.2 Sample statistics

Period	Number of trading days, n	Number of stocks listed excluding Oslo Axess	Number of stocks listed throughout period	Number of stocks listed throughout with - price above 10	- relative bid-ask < 5%	- all criterias met: our selection
1993-1996	1006	254	59	46	32	21
1997-2000	1004	314	129	98	83	72
2001-2004	1001	278	144	119	80	71
2005-2008	1006	311	143	98	118	78
2009-2012	1007	251	172	84	119	72

The table presents sample statistics from the dataset obtained from Børsprosjektet. We have divided the sample into 5 sub periods of 4 years length, with *n* number of trading days. The starting point for our sample selection is the total number of stocks that appear in each sub period excluding Oslo Axess. First we require that stocks must be listed throughout the whole sub period. The last observed price for the previous sub period must be above 10 NOK and the average relative bid-ask spread cannot exceed 5%.

A time window of four years is well suited for VR tests for up to one year, yielding a maximum k/n ratio of 1/4[25]. This is below the recommendation of a ratio not exceeding 1/3 to ensure adequate sample size for the VR test. The use of different sub periods allows us to check for development patterns by comparing these periods up against each other.

On performance sensitivity, a sizeable number of considered stocks and trading signals should provide a robust number of trading observations. For simplicity, the total performances of whole portfolios rather than for individual stocks are reported.

4.3 Data biases

The removal of incomplete time-series leads to forward-looking and possible survivorship bias in our data. Withdrawals from the exchange could be due to mergers and bankruptcy[26]. Excluding several smaller and less liquid stocks may cause some lack of external validity for the OSE as a whole. Ødegaard (2007) states that OSE is influenced by the positive "January effect". This could inflict some minor bias in our trade simulations.

4.4 Implementation and programming

Originally we planned to use Excel as our main tool to organise and prepare our data for testing and simulation. While planning research methods our supervisor introduced us to the VR test package in *R*. *R* is a programming language and environment for statistical computing and graphics. Its free software is of growing popularity and turned out to be the primary workhorse for all the tasks we have performed on our dataset. Neither of us had any previous experience with *R*, and at first the learning process was demanding. However, we soon realised Excel's limitations to processing of larger data frames. After making a considerable time investment we harvested great benefits from power in data preparation, analysis and computational efficiency.

[25] For $k = 256 \; and \; n \approx 1000, k/n \approx 1/4$

[26] In our original dataset, bankruptcies are rare. We do however expect a stock price to reflect this risk if present.

Besides the 'vrtest' package (Kim, 2010) we have used a quantitative strategy model framework found within the 'quantstrat' package (Carl et. al., 2013). This package has enabled us to implement the trading strategy and produce evaluation results. The model still has a couple of shortcomings that we were unable to program around. It cannot handle incomplete time-series and we were therefore bound to be forward looking and exclude price series that were unlisted during a period. A smaller programming issue is the inability to implement the individual transaction costs for each stock. As a compromise, we used a daily average for all stocks.

5. Results

5.1 The variance ratio test

The variance ratio test is applied for all sub periods using k-values of 32, 64, 128 and 256. The complete results for the standardized test statistics are supplied in Appendix A.1, and a summary can be seen in Table 5.1.

Table 5.1 Summary of VR test results. No. of stocks with a significantly low test statistic

Period	k=32	k=64	k=128	k=256	Universe of stocks in sub period
1993-1996	3 (14%)	1 (05%)	4 (19%)	0 (00%)	21
1997-2000	17 (24%)	16 (22%)	17 (24%)	19 (26%	72
2001-2004	15 (21%)	10 (14%)	6 (08%)	11 (15%)	71
2005-2008	7 (09%)	7 (09%)	5 (06%)	5 (06%)	78
2009-2012	27 (38%)	19 (26%)	12 (17%)	21 (29%)	72

We calculate log returns of stock prices and apply the VR test to all stocks in each sub periods. When testing k-period=32 the critical value is -1.5 and stocks with lower test-statistics than -1.5 are then considered significant over 32-day periods. I.e. 3 stocks had a significant test-statistic for k=32 in sub period 1993-1996 (equalling 14% of the total number of stocks). The rest of the numbers are found in the same manner for the other k-values. Individual test statistics for all stocks and the critical values are reported in Appendix A.1.

Price patterns showing a significantly low variance ratio deviates from a random walk, possibly in favour of a mean reverting alternative. These will be our main candidates for the contrarian trading algorithm. The number of significant findings does not seem to decrease with the value of k. This indicates that our proposed k/n-ratio of maximum 1/4 is sufficient to ensure decent power of the variance ratio test. The efficient market hypothesis assumption of i.i.d. returns can be rejected for at least one stock price for all sub periods and k-values but one. In the small dataset from 1993–1996, only a few stocks show a significantly low variance ratio. For the later sub periods, the number of deviating stock price patterns varies. Almost 25% of the stocks in 1997–2000 are rejected to follow a random walk. This number falls to below 15% for 2001-2004, and as low as 6% during 2005–2008. It increases sharply in the most recent sub period from 2009 to 2012.

5.2 Portfolio performance

Portfolios are split into *top* and *bottom* segments. First, the average net daily profit & loss and daily standard deviations are reported. The Sharpe ratio is calculated by dividing the daily net portfolio returns by its standard deviations. A higher Sharpe ratio means a higher reward-to-risk ratio. We also report the percentage maximum drawdown and transaction costs for each portfolio[27]. The approximate gross profit before transaction cost is found by adding the transaction costs back to the net profit[28].

Portfolios sorted by hindsight variance ratios

The results of this section answer whether the variance ratio relates to the success of a contrarian trading strategy. The performances of the *top* and *bottom* portfolios in terms of the observed variance ratio are reported in Table 5.2.1.

Table 5.2.1 Performance of portfolios sorted by VR in hindsight

Period	Daily Net P&L NOK			Daily σ NOK			Sharpe NOK			Max Drawdown %			Daily trading cost NOK		
k	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ
1993-1996															
32	-217	-298	81	2147	2487	-340	-0.101	-0.120	0.019	0.247	0.334	-0.088	116	137	-21
64	-111	-311	199	2196	2717	-521	-0.051	-0.114	0.064	0.155	0.437	-0.283	72	62	10
128	-56	-307	251	2268	3041	-773	-0.025	-0.101	0.076	0.201	0.510	-0.309	35	32	3
256	-31	-237	206	1460	2204	-744	-0.021	-0.107	0.086	0.141	0.428	-0.287	8	5	3
1997-2000															
32	-123	-293	170	1684	2775	-1091	-0.073	-0.105	0.032	0.148	0.367	-0.218	154	186	-32
64	-48	-332	285	1675	3330	-1655	-0.028	-0.100	0.071	0.121	0.472	-0.351	87	103	-16
128	14	-316	330	1994	3129	-1135	0.007	-0.101	0.108	0.103	0.441	-0.338	49	100	-51
256	24	1	24	1405	3145	-1740	0.017	0.000	0.017	0.063	0.153	-0.090	16	25	-9
2001-2004															
32	-217	-559	343	2076	3579	-1503	-0.104	-0.156	0.052	0.225	0.531	-0.306	198	238	-40
64	-147	-704	557	2141	5075	-2934	-0.068	-0.139	0.070	0.162	0.784	-0.622	120	83	37
128	-17	-694	678	2258	5726	-3468	-0.007	-0.121	0.114	0.148	0.770	-0.622	74	70	4
256	-39	-297	258	1659	4776	-3117	-0.024	-0.062	0.039	0.190	0.493	-0.303	20	28	-8
2005-2008															
32	-76	-435	359	2910	3837	-927	-0.026	-0.113	0.087	0.178	0.516	-0.338	87	132	-45
64	-107	-466	359	2839	4203	-1364	-0.038	-0.111	0.073	0.162	0.601	-0.439	68	70	-2
128	-96	-382	286	3148	4837	-1689	-0.030	-0.079	0.048	0.167	0.562	-0.395	33	37	-4
256	-60	-226	165	2158	3693	-1535	-0.028	-0.061	0.033	0.156	0.415	-0.259	13	13	0
2009-2012															
32	-31	-348	316	2097	2854	-757	-0.015	-0.122	0.107	0.090	0.421	-0.331	135	143	-8
64	-54	-359	305	2238	3045	-807	-0.024	-0.118	0.094	0.128	0.504	-0.376	72	79	-7
128	37	-136	173	1764	3144	-1380	0.021	-0.043	0.064	0.070	0.309	-0.239	30	39	-9
256	47	-9	56	922	1779	-857	0.051	-0.005	0.056	0.035	0.131	-0.096	8	12	-4

All portfolios are actively managed by an automatic trading algorithm. For each *k*-value in all sub periods we simulate performance for portfolios sorted by VR in hindsight. This includes 40 simulations (4 k-values x 5 sub periods x 2 sorted portfolios). The stocks with the lowest VR statistics are included in the *Top* portfolio. The other half of the stocks forms the *Bottom* portfolio. The *Sharpe ratio NOK* is calculated as the average *Daily Net Profit* & *Loss* relative to the *Daily standard deviation NOK*. *Maximum drawdown %* is calculated as the largest drawdown from peak equity attained throughout the trading period. The delta column (Δ) is the difference between results in *top* and *bottom* portfolios.

[27] The percentage is calculated from an initial available capital of one million, but its nominal value is not too relevant in our approach. It shows the maximum peak-to-trough decline for the portfolio during the period.
[28] This approximation ignores the risk free returns, as these are rather low on a daily basis. The net profits include the returns from the risk free investments minus transaction costs.

Top portfolios are the half of stocks with the desirable low variance ratios, while stocks with higher variance ratios are in the *bottom* segment. The results are not promising in terms of beating the market. Only in four out of the twenty sub periods, the *top* portfolio is able to obtain a relatively small positive profit net of transaction costs. We see that the impact of transaction costs do affect the returns by a substantial amount. Many of the experienced losses are showing a negative gross return even before these costs. This could be an effect of including as many as half of the stocks for each strategy. Testing this by only allowing stocks with statistically low significant variance ratios to be traded increases profitability, but still falls short of making net profits in 12 out of the 19 sub periods.

Results show indications of a relationship between the variance ratios and the success of contrarian strategies. Out of four different time horizons over five different time periods, the *top* portfolios outperform their *bottom* counterparts in all sub periods. The average daily return is consistently higher for these portfolios than for the *bottom* portfolios. There is also a tendency regarding the volatilities of the portfolios. The standard deviations are lower for the *top* portfolios than for their counterparts. Combined with the higher daily return, the calculated Sharpe ratios must also be higher. The Sharpe ratios range between 0.05 for the best and almost -0.2 for the worst performing portfolios. Considering the maximum drawdown of the portfolios, the same pattern is observed. The maximum percentage drawdowns are lower for the *top* portfolios than for the *bottom* portfolios. This might not come as a surprise due to the overall superior returns. In addition, there seems to exist a weak tendency for net profits to increase as the variance ratio k-value increases. This is described in more detail in the next section.

Short vs. long term mean reversion

The results of this section answer whether the length of the mean reverting time window k relates to the success of a contrarian trading strategy. Table 5.2.2 reports performances in terms of the considered k value for each time period.

Table 5.2.2 Performance of stock universe portfolio

Period *k*	Daily Net P&L NOK	Daily σ NOK	Sharpe NOK	Max Draw-down %	Daily Transaction Cost	Daily Gross P&L NOK	Number of trades per stock	β to passive portfolio
1993-1996								
32	-276	1893	-0.146	0.295	136	-140	18.0	-0.27
64	-215	1970	-0.109	0.303	66	-149	10.1	-0.26
128	-182	2154	-0.084	0.343	33	-149	5.0	-0.32
256	-139	1470	-0.094	0.288	6	-133	1.0	-0.17
1997-2000								
32	-151	1998	-0.075	0.271	114	-37	26.5	-0.01
64	-197	2154	-0.091	0.295	92	-105	12.2	-0.02
128	-103	2332	-0.044	0.172	50	-53	6.5	-0.02
256	12	1923	0.006	0.090	20	32	2.5	0.03
2001-2004								
32	-396	2454	-0.161	0.380	221	-175	19.4	0.14
64	-447	3170	-0.141	0.544	129	-318	11.4	0.10
128	-375	3754	-0.100	0.547	73	-302	6.1	-0.04
256	-170	2871	-0.059	0.322	24	-146	2.0	-0.11
2005-2008								
32	-269	2990	-0.090	0.332	125	-144	24.5	0.18
64	-291	3197	-0.091	0.406	67	-224	13.1	0.16
128	-237	3557	-0.067	0.384	34	-203	6.4	0.06
256	-142	2673	-0.053	0.284	13	-129	2.4	0.10
2009-2012								
32	-195	2295	-0.085	0.231	141	-54	22.5	0.02
64	-212	2430	-0.087	0.346	76	-136	12.4	0.02
128	-52	2198	-0.023	0.167	35	-17	5.5	0.02
256	19	1175	0.016	0.063	10	29	1.6	0.03

All portfolios are actively managed by an automatic trading algorithm. For each *k*-value in all sub periods we simulate portfolio performance .This includes 20 simulations (4k-values x 5sub periods x 1portfolio). The *Sharpe ratio NOK* is calculated as the average *Daily Net Profit* & *Loss* relative to the *Daily standard deviation NOK. Maximum drawdown %* is calculated as the largest drawdown from peak equity attained throughout the trading period. The column of *Daily Gross Profit* & *Loss NOK* is the sum of *Daily Net Profit & Loss* and *Daily Transaction cost.* The beta measure is the systematic risk of the active portfolio compared to a passive buy and hold portfolio.

The table is not divided into *top* and *bottom* portfolios, as we want to observe the isolated effect of increasing the mean reverting time windows. All available stocks are therefore included. There is a tendency of Sharpe ratios to rise as k increases. Much of the increase in the Sharpe ratio as k increases is due to reduced transaction costs. This comes from more frequent trading per stock for smaller k values. Looking at the standard deviations and the maximum drawdowns, it does not exist tendencies that are strong enough to be considered as a describable pattern. Overall, our results suggest that trading on longer term MR tendencies looks more efficient than for the shorter term due to improved accuracy and lower trading activity.

Another interesting measure is the systematic risk, to which degree the returns from the active portfolios respond to swings in the market. Beta coefficients are calculated against a passive buy and hold strategy picked from the same stock universe[29]. Our results show patterns of some systematic risk. For the small number of stocks in the first period, coefficients are as low as -0.26 on average. This indicates that we carry substantial negative systematic risk. Our portfolios should then be expected to do well in a bearish market, and accordingly bad in a bullish market. This is well in line with our negative results for this period, as OSE was in a bullish phase at the time. For 1997-2004, the results are varying but relatively weak. Portfolios move in direction with the market in the period of 2005-2008. In the last period of 2009-2012 beta values never rise above 0.03. Low beta values are also the case for our more specific portfolios, showing relatively small signs of systematic risk. We conclude that only a small part of the success or shortfalls of our strategy can be attributed to swings in the market.

Portfolios sorted by market capitalisation

The results of this section answer whether the market cap of the stocks relates to the success of a contrarian trading strategy. The performances of our market cap sorted portfolios are presented in Table 5.2.3.

Table 5.2.3 Performance of portfolios sorted by market cap

Period	Daily Net P&L NOK			Daily σ NOK			Sharpe NOK			Max Drawdown %			Daily trading cost NOK		
k	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ
1997-2000															
32	-268	-161	-107	3231	2620	611	-0.083	-0.061	-0.021	0.340	0.290	0.050	162	146	16
64	-195	-223	28	3614	3165	449	-0.054	-0.071	0.017	0.320	0.368	-0.048	102	77	25
128	-61	-99	38	3761	2945	816	-0.016	-0.034	0.017	0.219	0.283	-0.064	58	41	17
256	108	-16	124	3706	2144	1562	0.029	-0.007	0.036	0.147	0.110	0.036	30	14	16
2001-2004															
32	-398	-255	-142	3345	1907	1438	-0.119	-0.134	0.015	0.389	0.269	0.120	216	143	73
64	-350	-335	-15	3310	2773	537	-0.106	-0.121	0.015	0.423	0.445	-0.022	126	110	16
128	-303	-294	-9	3387	2967	420	-0.089	-0.099	0.010	0.419	0.451	-0.033	64	55	9
256	-86	-157	71	2559	1945	614	-0.034	-0.081	0.047	0.173	0.312	-0.139	24	16	8
2005-2008															
32	-175	-204	29	3756	2723	1033	-0.046	-0.075	0.029	0.261	0.269	-0.008	129	121	8
64	-192	-247	55	3892	2827	1065	-0.049	-0.087	0.038	0.323	0.360	-0.037	68	62	6
128	-141	-187	47	4186	3012	1174	-0.034	-0.062	0.029	0.301	0.341	-0.039	34	31	3
256	-169	-143	-26	3402	2356	1046	-0.050	-0.061	0.011	0.336	0.297	0.038	12	10	2
2009-2012															
32	-96	-233	138	3010	2078	932	-0.032	-0.112	0.081	0.162	0.239	-0.076	126	143	-17
64	-88	-205	117	3195	2186	1009	-0.028	-0.094	0.066	0.233	0.290	-0.058	75	74	1
128	-8	-35	28	2744	1860	884	-0.003	-0.019	0.016	0.141	0.105	0.036	32	33	-1
256	21	24	-3	1461	947	514	0.014	0.025	-0.011	0.075	0.062	0.014	8	9	-1

All portfolios are actively managed by an automatic trading algorithm. For each *k*-value in the 4 out of sample sub periods we simulate performance for portfolios sorted by market cap. This includes 32 simulations (4*k*-values x 4sub periods x 2sorted portfolios). The stocks with the highest market caps are included in the *Top* portfolio. The other half of the stocks forms the *Bottom* portfolio. The *Sharpe ratio NOK* is calculated as the *Daily Net Profit & Loss* relative to the *Daily standard deviation NOK. Maximum drawdown %* is calculated as the largest drawdown from peak equity attained throughout the trading period. The delta column (Δ) is the difference between results in *top* and *bottom* portfolios.

[29] Betas between our passive universes and the Oslo All Share Index are generally calculated to be a little less than unity. This means that our considered stocks can be considered to be representative for the market as a whole.

Top are higher-half market cap stocks, while *bottom* are the lower-halves. Unlike previous studies, no tendency of smaller cap stocks outperforming bigger stocks is seen. This is not too surprising as there are two significant differences between our study and those mentioned earlier. Our maximum time window of one year is shorter than time windows used in earlier studies. For our time windows, neither did they find signs of exploitable mean reverting price patterns. These were also done on other stock exchanges, using older datasets. The different methods, economic climates and the use of more recent data on our part could very well account for the differences in results. Standard deviations for the *top* portfolios are clearly higher than their counterparts, causing many of its (mostly negative) Sharpe ratios to be higher than for the smaller stocks. Differences in the maximum drawdown are varying, without signs of a recognisable pattern.

Portfolios sorted by relative bid-ask spread

The results of this section answer whether the liquidity of the stock (approximated by the rBAs) relates to the success of a contrarian trading strategy. The performances of our liquidity sorted portfolios are presented in Table 5.2.4.

Table 5.2.4 Performance of portfolios sorted by liquidity (relative bid-ask spread)

Period	Daily Net P&L NOK			Daily σ NOK			Sharpe NOK			Max Drawdown %			Daily trading cost NOK		
k	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ	*Top*	*Bottom*	Δ
1997-2000															
32	-224	-207	-17	3179	2566	613	-0.071	-0.081	0.010	0.265	0.363	-0.098	164	145	19
64	-190	-229	40	3708	2956	752	-0.051	-0.078	0.027	0.280	0.417	-0.137	99	83	16
128	-108	-52	-56	3915	2844	1071	-0.028	-0.018	-0.009	0.212	0.253	-0.040	55	44	11
256	35	60	-25	3487	2486	1001	0.010	0.024	-0.014	0.140	0.114	0.026	25	20	5
2001-2004															
32	-426	-222	-204	3464	1632	1832	-0.123	-0.136	0.013	0.433	0.197	0.236	205	155	50
64	-436	-241	-195	3857	2069	1788	-0.113	-0.117	0.004	0.549	0.310	0.239	120	118	2
128	-391	-199	-192	4092	2267	1825	-0.096	-0.088	-0.008	0.580	0.301	0.279	38	60	-22
256	-88	-156	68	2686	1818	868	-0.033	-0.086	0.053	0.220	0.325	-0.105	25	15	10
2005-2008															
32	-158	-222	64	3697	2864	833	-0.042	-0.077	0.035	0.244	0.295	-0.051	127	123	4
64	-191	-249	58	3744	3032	712	-0.051	-0.082	0.031	0.315	0.375	-0.060	67	63	4
128	-134	-194	60	3947	3261	686	-0.034	-0.059	0.025	0.288	0.345	-0.057	34	31	3
256	-165	-150	-15	3325	2516	809	-0.050	-0.059	0.010	0.339	0.290	0.048	12	11	1
2009-2012															
32	-131	-198	67	3337	1771	1566	-0.039	-0.112	0.073	0.225	0.167	0.058	136	135	1
64	-136	-158	22	3528	1923	1605	-0.038	-0.082	0.044	0.295	0.216	0.080	75	74	1
128	-30	-12	-18	3055	1548	1507	-0.010	-0.008	-0.002	0.188	0.087	0.101	34	31	3
256	22	23	-1	1618	811	807	0.013	0.028	-0.015	0.083	0.047	0.036	10	7	3

All portfolios are actively managed by an automatic trading algorithm. For each *k*-value in the 4 out of sample sub periods we simulate the performance for portfolios sorted by relative bid-ask spread (rBAs). This includes 32 simulations (4*k*-values x 4sub periods x 2sorted portfolios). The stocks with the lowest rBAs are included in the *Top* portfolio. The other half of the stocks forms the *Bottom* portfolio. The *Sharpe ratio NOK* is calculated as the average *Daily Net Profit* & *Loss* relative to the *Daily standard deviation NOK. Maximum drawdown %* is calculated as the largest drawdown from peak equity attained throughout the trading period. The *delta* column (Δ) is the difference between results in *top* and *bottom* portfolios.

Top are the stocks with the lowest rBAs, while *bottom* are those of higher rBAs. It is natural to think that this measure is closely linked to the market cap. Larger cap stocks are often more traded than smaller cap stocks, which tightens the spread and provides better liquidity. Results carry some similarity to section 5.2.3. The difference in net returns does not behave very consistently throughout the periods. As described in section 3.3, transaction costs are found from the daily average of *all* spreads. This is also the case for the results in this section. Affecting differences in net profitability between the rBAs-sorted portfolios, the relative real-world performance of the *top* portfolios over the *bottom* portfolios should then increase. As in the previous case, standard deviations are higher for the *top* segment than for the *bottom* segment. This is due to *top* portfolios initiating significantly more trades than their counterparts. As positions tend to be closed most of the time[30], the rise in trading activity increases daily volatility[31].

Predictability and Applicability

The results of this section answer whether there exists persistence in the variance ratios. We here rely on persistence/positive serial correlation between our four year periods. All stocks showing a statistically low variance ratio in one sub period form the portfolio for the subsequent period. The summarizing results can be seen in Table 5.2.5.

Table 5.2.5 Performance of portfolio sorted by historical VR

Period k	Daily Net P&L NOK	Daily σ NOK	Sharpe NOK	Max Draw-down %	Daily Trading Cost	Number of stocks	VR-persistent stocks
1997-2000							
32	-160	4030	-0.040	0.220	154	3	3
64	-3	2835	-0.001	0.134	65	1	1
128	46	2100	0.022	0.088	21	4	3
256	-	-	-	-	-	0	0
2001-2004							
32	-74	1982	-0.037	0.080	152	8	5
64	-43	2801	-0.015	0.110	125	6	2
128	11	2209	0.005	0.083	50	7	2
256	-104	2584	-0.040	0.290	17	10	4
2005-2008							
32	-159	2624	-0.060	0.227	111	10	2
64	-213	3406	-0.063	0.354	53	7	1
128	-169	5221	-0.032	0.418	38	5	1
256	-169	2970	-0.057	0.336	11	10	0
2009-2012							
32	-184	3458	-0.053	0.250	136	4	2
64	-91	4101	-0.022	0.243	79	3	2
128	-13	2857	-0.005	0.135	19	1	1
256	-388	7951	-0.049	0.644	18	1	0

All portfolios are actively managed by an automatic trading algorithm. For each k-value in the 4 out of sample sub periods we simulate the performance of portfolios sorted by historical VR. This includes 16 simulations (4k-values x 4sub periods x 1sorted portfolio) minus one simulation of zero stocks. Portfolios are formed out of stocks that had low VR in sample and also survived through the out of sample period. The *Sharpe ratio NOK* is calculated as the average *Daily Net Profit* & *Loss* relative to the *Daily standard deviation NOK*. *Maximum drawdown %* is calculated as the largest drawdown from peak equity attained throughout the trading period. The last column shows the number of stocks that held on to their significant low VR from the previous period.

[30] Closed positions causes zero net exposure and volatility.
[31] Why trading activity increases for *top* stocks is an interesting observation. Still, we choose not to pursue this further..

As might have been expected due to earlier results, the portfolios are not able earn profits net of transaction costs for most periods. The last column in the table shows signs of persistence in the variance ratios between all periods but 2001-2008. Sharpe ratios are higher than for the more general portfolios in section 5.2.2 in 13 out of the 15 comparable cases. This suggests that it is better to rely on empirical variance ratios to persist than not. One should be aware of the small number of traded stocks, making results relatively sensitive. The main picture is that our proposed strategy does not reap additional returns out from the market.

5.3 Distributions of returns and trades

The distribution tables for portfolio returns are attached in Appendix A.6.1. Most daily observations lie around a zero return, often resulting from holding no active positions. Some outliers are present, but none extreme enough to suspect misbehaving either in the trading algorithm or in the dataset.

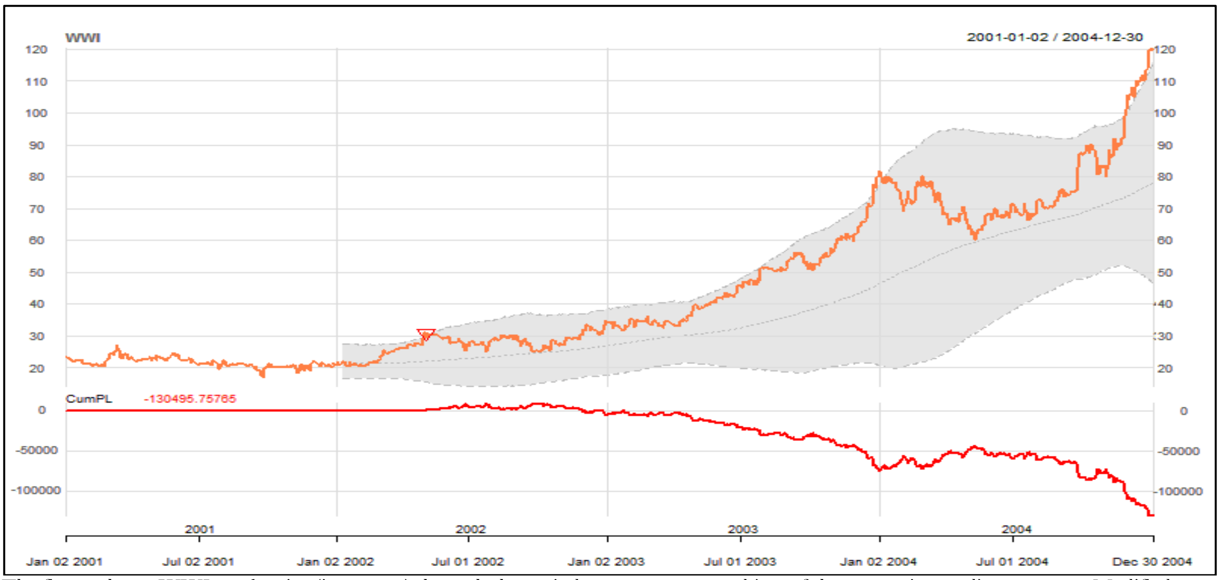
Distribution tables for the daily net value invested are attached in Appendix A.6.2. Many days hold net shorting positions. This is surprising, as we would expect having daily net exposures around a mean of zero due to the symmetry of the Bollinger bands. Net exposure is negative for almost the entire first period, expecting a future fall in prices. This tendency is persistent for all periods but 2001–2004, where we hold a scarce majority of long positions. Across all periods, it is triggered more selling signals than buying signals. This could be due to potentially serially correlated sample distributions of the stock returns. For example, the fall in prices could tend to be less steep than the increases. The Bollinger bands will then be crossed more often in the latter case, causing more selling positions to be initiated.

5.4 The mean reverting alternative: A single case study

Finally, it is interesting to question the variance ratio criterion as an indicator of mean reversion. To provide an example, we look into the behaviour of a stock with a significantly low variance ratio[32]. The stock in question is WWI (Wilh. Wilhelmsen) over the 2001–2004 period, using a k-window of 256. Its simulation history can be seen in Figure 5.4.

[32] Being aware that this chosen example is unlikely to be representable for all others, we still believe it accentuates some relevant points.

Figure 5.4 WWI trading performance



The figure shows WWI stock price (in orange) through the period 2001-2004 as an object of the contrarian trading strategy. Modified Bollinger bands around a 256-day moving average are shaded in gray. The red triangle indicates a short position taken when the price crosses the upper band. From there the price multiply with four without triggering any closing signal and the cumulative profit & loss plunges down.

As can be seen in Figure 5.4 only one trade is being made. This is a shorting position in May 2002, when the price crosses the upper Bollinger band at a price just above 30 NOK. Surprisingly the position is never closed, as the price keeps increasing and never reverts back to its 256-day moving average during the rest of the period. As a result, value equal to three times the size of the initial position is lost[33]. As the variance ratio for the stock over the considered period is confirmed to be significantly low, this seems puzzling. By the expansion of the Bollinger bands, we see that volatility increases together with the price during the period. Just as the moving average closes in on the price in early 2004, another round of increase in the stock price and volatility comes in. As a result, the moving average does not catch up to the price within the period, and the position is never closed. This example shows the threat of large downsides for individual trades, as stated in section 3.3. As the price keeps its momentum behaviour, volatility increases. We conclude that increasing time-changing volatilities can greatly affect and hurt our approach. This also explains why trading strategies based on mean reversion are said to effectively be *shorting* volatility.

[33] Notice that the result would have been even worse if we had allowed for more trades than one at a time, as commented in section 3.3.

5.5 Discussion of results

First there was found promising evidence of irregular stock price patterns on the OSE. Results showed signs of a positive relationship between our test criterion, the variance ratio, and the success of a contrarian trading strategy. When implementing our approach in practice, our portfolios fell short of obtaining excessive profits. Even though we observe irregularities at the OSE with respect to the calculated variance ratios, this seems not to be consistently exploitable using our proposed strategy. A mean reverting strategy is very exposed to changes in volatilities, as sudden momentum in stock prices can cause severe losses. This can be partly avoided by imposing stop-loss rules. When looking at market cap and liquidity, these do not show any signs of predicting mean reversion. We observe a tendency of bigger and more liquid stocks initiating more trades than their counterparts. We want to emphasize that strict criterias and assumptions have been imposed on our data, and our results are relatively sensitive due to small trading samples for some portfolios. Still, the consistency of our results suggests validity for most of the conclusions drawn.

6. Conclusion

Our paper has been examining the possibilities for exploiting over reactive trading at the OSE. This has been done by utilising the variance ratio, which compares the short and the longer run volatilities of individual stocks. More common characteristics like stock market cap and liquidity show no clear predictability for mean reverting tendencies. Our results show that even if this tendency is found and believed to produce better results than other approaches, it may still not be profitable in practice. Empirical data may seem imperfect and exploitable in retrospect. By using an evaluation criterion based purely on historical price data, we show that nature of the real-world environment is neglected[34]. When examining empirical results, one should consider the issues of possible data mining, in-hindsight theory and the inability to separate market noise and fundamental changes. Chances are that present observed deviations are priced into the market in just a fraction of a second. If this is the case, the weak form efficiency hypothesis holds. We conclude that considering the required time invested and the few nuggets to be found, trading on mean reversion may not be too appealing for the common Norwegian investor.

[34] We here refer to entirely relying on the rejection of the variance ratio test to conclude that the EMH hypothesis does not hold.

A. Appendices

A.1 List of VR test results

Table A.1 List of VR test statistics for all stocks per sub period

1993-1996					1997-2000					2001-2004					2005-2008					2009-2012				
k	*32*	*64*	*128*	*256*	*k*	32	64	128	256	*k*	32	64	128	256	*k*	32	64	128	256	*k*	32	64	128	256
ATEA	-1.0	-1.2	-0.9	-0.7	AMA	0.0	0.0	-0.1	-0.7	AIK	-0.6	-0.9	-0.6	-0.3	AFG	**-2.1**	-1.5	-1.3	-1.1	AFG	**-1.5**	-1.0	-1.1	-1.1
BON	0.4	0.6	0.2	0.2	ATEA	0.5	0.5	0.1	-0.2	ASD	0.9	0.9	0.7	-0.2	AIK	**-2.3**	**-1.9**	**-1.5**	-1.1	AFK	**-2.8**	**-2.5**	**-2.1**	**-1.7**
DNB	0.8	0.9	0.5	0.1	AWS	**-2.4**	**-1.9**	**-1.9**	**-1.5**	ATEA	-1.2	-1.1	-0.9	-0.9	AKER	0.0	0.4	0.8	1.0	AKER	**-1.6**	-1.2	-1.0	**-1.4**
FAR	**-1.7**	-1.5	-1.1	-0.8	AWSB	**-2.2**	**-1.8**	**-1.8**	**-1.5**	BNB	-0.6	-0.5	-0.8	**-1.2**	AKSO	0.3	0.3	-0.1	-0.2	AKSO	-1.5	-1.0	-0.8	-1.1
GRO	0.1	0.5	0.2	0.1	BEA	-0.9	-0.3	0.0	-0.2	BON	0.0	1.2	1.5	1.2	ASD	**-2.3**	**-2.0**	**-1.6**	**-1.4**	ALGETA	0.0	-0.2	-0.2	-0.5
HNA	-0.9	-1.2	**-1.5**	-1.1	BEB	-0.7	-0.1	0.2	-0.1	CRU	0.2	0.6	1.1	1.5	ATEA	0.0	0.5	0.1	-0.2	ATEA	-0.5	0.4	0.5	-0.5
HNB	-0.9	-1.1	**-1.4**	-1.2	BLO	-1.0	-0.8	-0.9	-0.8	DNB	-0.8	-0.7	-0.8	-0.8	BLO	0.8	1.4	1.5	0.1	AUSS	2.0	2.4	1.7	1.1
MORG	0.3	-0.1	-0.1	-0.1	BNB	-1.1	-1.0	-1.2	**-1.2**	DOF	-1.1	-0.6	-0.7	-1.1	BON	1.0	1.1	0.3	-0.2	BON	0.1	-0.5	-0.8	-1.2
NHY	0.5	-1.3	**-1.4**	-1.1	BON	0.1	0.4	0.5	-0.2	EKO	**-1.8**	-1.0	-0.6	-1.1	BOR	0.2	-0.1	-0.2	-0.8	BWG	-0.3	0.1	0.4	0.3
ODF	-1.4	-0.7	-0.5	-0.5	BRA	**-1.7**	**-1.6**	**-1.5**	-1.2	ELK	**-2.2**	-1.5	-1.3	-1.1	COV	0.1	0.3	0.7	0.2	CEQ	0.2	0.5	-0.4	-0.7
ODFB	-1.2	-0.5	-0.3	-0.3	CHS	**-2.3**	**-1.6**	-1.1	-0.9	ELT	2.0	1.8	1.5	0.4	CRU	0.8	1.5	1.1	0.4	COP	-0.8	-0.1	0.4	0.2
OLT	0.0	-0.2	0.0	0.0	CKR	-0.6	-0.8	-1.0	-1.0	EVRY	0.6	0.5	0.8	0.6	DIAG	-1.0	-1.2	-1.3	**-1.3**	DNB	-1.1	-0.7	-0.7	-0.7
ORK	0.4	0.0	-0.1	-0.5	COV	-0.9	-0.1	0.3	0.1	EXPERT	**-1.7**	**-1.5**	-1.0	-0.7	DNB	-0.5	-0.1	-0.3	-0.4	DOCK	**-1.5**	-1.1	-1.1	-1.0
PGS	-0.6	-1.0	-1.3	-1.2	DNB	0.5	0.1	-0.2	-0.5	FAR	-0.8	-0.5	-1.1	**-1.4**	DOF	0.0	0.1	-0.4	-0.3	DOF	0.3	0.7	0.5	-0.5
RIE	**-1.9**	-1.3	-0.9	-0.8	EKO	0.2	-1.0	**-1.5**	**-1.4**	FOE	1.8	2.8	2.8	2.8	EKO	**-1.7**	**-1.5**	-1.2	-1.0	EIOF	**-2.8**	**-2.2**	**-1.6**	-1.2
SCH	-0.8	-0.8	-0.7	-0.4	ELK	-0.2	0.4	-0.1	-0.6	FRO	0.7	1.5	1.3	0.9	EMS	0.0	-0.2	-0.8	**-1.2**	EKO	**-2.1**	**-1.8**	-1.3	-0.8
SPOG	**-1.5**	**-1.8**	**-1.6**	-1.2	FAR	**-2.2**	-1.5	-0.9	-0.7	GRE	-0.6	-0.3	-0.5	-0.7	EVRY	0.5	0.5	0.1	-0.2	EMGS	-1.4	**-1.6**	**-1.6**	**-1.5**
TOM	-1.0	-0.7	-0.5	-0.2	GOD	-1.3	-1.1	-1.0	-1.1	GRO	0.0	1.1	1.4	1.3	FAR	-0.5	-0.4	-0.6	-0.5	FAR	**-2.1**	-1.1	-0.9	-1.1
VEI	0.5	0.8	0.9	0.6	GRE	0.0	-0.2	-0.4	-0.7	HNA	**-2.2**	**-1.8**	**-1.5**	**-1.5**	FOE	-1.0	-0.9	-0.7	-0.8	FOE	**-1.5**	-1.2	-1.3	**-1.4**
WWI	-0.2	0.1	-0.4	-0.9	GRO	0.4	0.6	0.7	-0.2	HNB	-1.1	-1.2	-1.3	**-1.3**	FRO	-1.1	-0.9	-1.2	**-1.3**	FRO	-0.6	-0.2	0.1	-0.1
WWIB	0.1	0.5	-0.1	-0.6	HNA	**-3.4**	**-2.7**	**-2.0**	**-1.5**	JIN	-0.8	-0.1	0.4	0.3	GOD	-0.5	-0.4	-0.5	-0.8	GOD	**-2.8**	**-2.1**	**-1.8**	**-1.6**
.	.	.	.	.	HNB	**-2.4**	**-2.0**	**-1.8**	**-1.5**	KOG	**-2.0**	**-1.5**	-1.2	**-1.3**	GOL	-0.4	-0.3	-0.5	-0.5	GRO	-0.4	-0.9	-0.9	**-1.2**
.	.	.	.	.	HYD	-0.2	0.5	-0.1	-0.8	KOM	-0.5	-0.2	-0.1	-0.7	GRO	0.8	0.6	0.1	-0.2	GSF	3.6	4.4	3.2	1.7
.	.	.	.	.	KIT	-0.4	-0.7	**-2.0**	**-2.1**	KVI	0.0	0.0	0.0	-0.2	HELG	-1.1	-0.6	-0.8	-0.9	HELG	**-2.0**	-1.5	-1.1	-0.6
.	.	.	.	.	KLI	**-2.2**	**-1.8**	**-1.5**	**-1.4**	MING	0.1	1.1	2.0	1.1	HNA	0.4	0.5	0.4	0.6	HNA	**-1.6**	**-1.8**	**-1.5**	**-1.4**
.	.	.	.	.	KOG	**-1.5**	-1.4	**-1.4**	**-1.5**	MORG	-0.2	0.0	0.4	-0.5	HNB	0.9	0.9	0.7	0.9	HNB	-1.5	**-1.5**	-1.3	**-1.3**
.	.	.	.	.	KVI	1.2	0.4	-0.3	-0.7	NEC	-0.8	-1.3	**-1.4**	-1.1	IGE	-0.1	-0.1	0.2	0.5	IMSK	-0.2	-0.2	-0.5	-1.0
.	.	.	.	.	LHO	**-2.4**	**-2.0**	**-1.6**	**-1.5**	NER	-0.5	-0.2	0.2	0.6	IMSK	-0.3	-0.3	-0.8	-0.9	KOG	-1.2	-1.0	-0.8	-0.7
.	.	.	.	.	MORG	-1.3	-1.2	-1.0	-0.9	NHY	0.0	0.4	0.3	0.1	JSHIP	0.0	-0.1	-0.6	-0.7	LSG	0.7	1.9	1.4	0.9
.	.	.	.	.	NBK	-0.3	-0.8	-0.8	-1.1	NONG	-0.9	-0.4	0.4	0.1	KOG	-1.3	-1.0	-1.0	-1.0	MING	-0.3	0.2	-0.1	-0.2
.	.	.	.	.	NER	1.6	1.4	1.1	1.4	NSG	-0.8	-0.5	-0.5	-0.6	KOM	0.2	0.6	0.7	0.8	MORG	**-3.2**	**-2.5**	**-1.7**	**-1.3**
.	.	.	.	.	NHY	-0.8	-1.0	-1.0	-1.0	OCR	0.8	1.0	1.1	1.0	LSG	0.3	0.6	0.7	0.4	NAS	1.3	1.0	1.0	0.0
.	.	.	.	.	NONG	-0.5	-1.2	-1.3	-1.1	ODF	-0.2	0.1	-0.1	0.0	MING	-0.7	0.0	0.0	-0.1	NHY	-1.2	-1.0	-0.7	-1.1
.	.	.	.	.	NOV	0.0	0.7	0.2	-0.7	ODFB	0.7	0.8	0.2	0.2	MORG	-1.3	-1.4	-1.1	-1.0	NOD	-0.3	0.5	1.1	1.1
.	.	.	.	.	NSG	0.5	0.3	0.1	-0.8	OLT	**-2.7**	**-2.0**	**-1.4**	**-1.4**	NAS	2.1	2.9	3.9	2.2	NONG	**-1.5**	-0.9	-0.5	-0.5
.	.	.	.	.	NSGB	0.5	0.3	0.0	-0.9	OPC	-1.0	-1.2	-1.1	-1.0	NEC	-0.7	-0.8	-1.0	**-1.2**	NPEL	-0.3	-0.7	-0.5	-0.3
.	.	.	.	.	OCR	1.7	1.3	0.3	-0.4	ORK	-0.4	-0.2	0.1	0.4	NHY	-0.1	0.4	0.1	-0.4	ODF	-1.1	-0.3	-0.4	**-1.3**
.	.	.	.	.	ODF	-0.9	0.2	0.2	0.0	ORO	-0.3	0.1	0.3	0.0	NONG	-0.3	0.2	-0.2	-0.2	ODFB	-0.2	0.5	0.2	-1.1
.	.	.	.	.	ODFB	-0.8	0.4	0.4	0.0	PGS	1.1	1.1	1.4	1.4	NSG	-0.9	-0.9	-0.5	-0.3	OLT	**-2.4**	**-1.7**	-1.2	-1.1
.	.	.	.	.	OLT	-1.0	-0.9	-0.6	-1.0	PHO	0.9	0.8	-0.1	-0.5	ODF	-0.4	-0.6	-0.5	-0.8	OPERA	**-1.7**	-1.5	-1.4	**-1.5**

Table A.1 continued: List of VR test statistics for all stocks per sub period

1993-1996					1997-2000					2001-2004					2005-2008					2009-2012				
k	*32*	*64*	*128*	*256*	*k*	*32*	*64*	*128*	*256*	*k*	*32*	*64*	*128*	*256*	*k*	*32*	*64*	*128*	*256*	*k*	*32*	*64*	*128*	*256*
.	.	.	.	.	ORK	0.7	0.1	-0.1	-0.4	PLUG	**-2.4**	**-1.9**	**-1.6**	**-1.3**	ODFB	-0.2	-0.3	-0.6	-0.7	ORK	**-1.8**	**-1.8**	**-1.5**	**-1.4**
.	.	.	.	.	PDR	-0.8	-0.4	0.1	-0.3	PRS	-1.3	-0.9	-0.7	-1.0	OLT	-1.4	-0.9	-0.9	-0.8	PGS	-1.4	-1.0	-0.8	-1.0
.	.	.	.	.	PGS	-0.3	-0.3	-0.5	-0.8	RCL	2.1	1.0	0.1	-0.4	OPERA	0.6	0.4	-0.2	-0.5	PHO	-1.3	-1.2	-1.0	-1.2
.	.	.	.	.	RANG	**-2.1**	**-2.1**	**-1.8**	**-1.5**	RIE	**-2.8**	**-2.3**	**-1.9**	**-1.5**	ORK	-0.2	-0.1	-0.2	-0.1	PRON	-0.6	-0.7	-0.7	-0.8
.	.	.	.	.	RIE	**-1.5**	-1.1	**-1.4**	**-1.4**	ROGG	0.9	2.3	3.2	1.6	PDR	1.6	1.1	0.9	0.5	PRS	**-1.9**	**-1.5**	**-1.4**	**-1.5**
.	.	.	.	.	RIEB	-1.0	-0.9	-1.2	**-1.3**	SADG	**-1.6**	-1.1	-0.9	-1.0	PGS	0.6	1.0	0.4	0.2	QFR	**-2.1**	**-1.7**	-1.3	**-1.4**
.	.	.	.	.	RING	**-1.9**	**-1.7**	**-1.5**	-1.2	SANG	**-2.0**	-1.1	-0.5	-0.7	PHO	**-2.2**	-1.1	-0.6	-0.7	RCL	0.6	-0.1	-0.2	-0.5
.	.	.	.	.	RNA	-1.0	-1.0	-1.2	**-1.2**	SCH	**-1.5**	-1.0	-0.6	-0.8	PRS	-0.4	-0.3	-0.4	-0.4	RIE	**-2.6**	**-2.4**	**-2.6**	**-2.3**
.	.	.	.	.	ROGG	0.1	-0.4	-0.3	-0.5	SINO	-0.4	-0.4	-0.1	-0.2	RCL	-0.6	-0.7	-1.0	-0.8	SADG	-0.7	-0.9	-0.7	-0.5
.	.	.	.	.	SADG	-0.3	-0.7	-0.5	-0.6	SME	-0.2	0.2	0.4	0.5	REACH	-0.4	0.2	-0.4	-0.4	SALM	**-2.4**	-1.4	-0.8	-0.6
.	.	.	.	.	SANG	**-3.0**	**-2.4**	**-1.8**	**-1.3**	SMEB	-0.5	-0.2	0.0	0.1	RIE	**-2.5**	**-1.8**	**-1.7**	-1.1	SBVG	-1.5	-0.9	-0.8	-0.9
.	.	.	.	.	SASB	-0.4	0.0	0.0	-0.5	SNI	0.8	0.8	0.6	0.7	ROGG	-1.0	-0.6	-0.5	-0.7	SCH	0.7	-0.4	-0.2	-0.6
.	.	.	.	.	SBVG	-1.4	**-1.9**	**-1.7**	**-1.3**	SOFF	-0.8	-0.1	-0.7	-0.9	SADG	0.4	0.9	0.4	0.3	SDRL	-1.3	-1.0	-0.8	-1.1
.	.	.	.	.	SCH	0.4	0.1	0.0	-0.5	SOI	1.2	1.7	2.1	1.4	SBVG	-0.6	-0.4	-0.4	-0.5	SEVAN	1.0	1.0	0.9	0.5
.	.	.	.	.	SFJ	0.1	-0.1	0.2	-0.1	SPOG	**-2.2**	**-1.5**	-0.9	-0.9	SCH	-0.2	0.2	0.1	0.2	SNI	0.1	-0.7	-1.1	-1.1
.	.	.	.	.	SME	-0.1	-0.1	0.3	0.4	STB	0.5	0.5	0.1	0.2	SEVAN	1.0	1.2	0.5	-0.2	SOAG	-1.3	-1.3	-0.8	-0.3
.	.	.	.	.	SMEB	0.4	0.4	0.8	0.8	SUBC	1.0	1.2	1.4	0.9	SFM	-0.8	**-1.5**	**-1.4**	**-1.4**	SOFF	**-2.5**	**-1.7**	-1.4	**-1.3**
.	.	.	.	.	SNIB	1.1	0.8	0.5	-0.1	SUO	**-1.5**	-1.2	-1.0	-0.6	SNI	-1.1	-0.7	-0.8	-0.9	SPOG	2.1	0.8	-0.4	-0.8
.	.	.	.	.	SNOG	0.5	0.1	0.2	-0.1	SVEG	-0.8	-0.5	-0.2	-0.9	SOFF	-0.8	-0.3	-0.4	-0.5	STB	-1.4	-1.1	-0.7	-0.7
.	.	.	.	.	SPOG	**-2.6**	**-2.2**	**-1.7**	**-1.2**	TAA	-0.6	-0.7	-0.3	-0.3	SPOG	-0.7	-0.5	-0.7	-0.8	STL	**-2.3**	**-1.9**	**-1.6**	**-1.5**
.	.	.	.	.	SST	-1.2	-0.6	-0.8	**-1.4**	TAD	1.8	2.9	3.9	2.6	SSI	-0.7	-0.4	-0.8	-0.7	SUBC	**-1.6**	-1.2	-1.2	-1.0
.	.	.	.	.	STB	-1.4	-1.1	-1.3	-1.2	TAT	1.5	1.9	2.9	3.1	STB	0.1	0.1	-0.2	-0.2	SVEG	**-1.8**	**-1.5**	-1.4	-1.2
.	.	.	.	.	SVEG	1.1	0.7	0.4	-0.3	TEL	-1.0	-1.0	-0.6	-0.3	STL	-0.9	-0.8	-0.8	-1.0	TEL	-1.2	-1.3	-1.0	-0.9
.	.	.	.	.	TAA	0.0	-0.1	0.5	0.4	TGS	0.2	0.5	0.2	-0.2	STXEUR	0.1	-0.3	-1.1	-0.8	TELIO	**-2.7**	**-1.7**	-1.0	-0.7
.	.	.	.	.	TAD	1.2	1.8	1.7	-0.3	TOM	-1.3	**-1.6**	**-1.5**	**-1.4**	SUB	0.1	0.3	-0.1	-0.5	TGS	**-1.6**	-1.2	-1.0	**-1.2**
.	.	.	.	.	TAT	-0.4	-0.5	-0.9	**-1.2**	TOTG	-1.5	-0.8	-0.6	**-1.3**	SUBC	-0.1	0.4	0.1	-0.3	TOM	**-2.4**	**-2.0**	**-1.7**	**-1.5**
.	.	.	.	.	TOM	-1.4	-1.3	-1.2	**-1.3**	VEI	-1.3	-0.9	-0.7	-0.9	SVEG	-0.9	-0.7	-0.6	-0.4	TOTG	**-1.7**	-1.4	-1.0	-0.6
.	.	.	.	.	TOTG	**-2.1**	-1.5	-1.1	-1.0	VIS	0.1	-0.2	-0.2	-0.4	TAA	-0.1	-0.2	-0.4	-0.4	VEI	-0.9	-0.3	0.0	-0.7
.	.	.	.	.	UNS	**-2.5**	**-1.8**	-1.4	-1.1	VME	-0.4	-0.4	0.1	0.1	TCO	**-3.1**	**-2.4**	**-1.8**	**-1.3**	VIZ	-0.5	-0.3	-0.9	**-1.4**
.	.	.	.	.	UTO	-1.3	-1.2	-1.1	-1.1	WWI	**-1.8**	-1.3	-1.1	**-1.3**	TEL	0.2	0.4	0.2	0.1	WWI	-0.5	-0.6	-0.8	**-1.3**
.	.	.	.	.	VIS	-0.4	-0.4	-0.8	-1.0	WWIB	**-2.0**	**-1.6**	-1.3	**-1.3**	TGS	-1.1	-0.8	-0.5	-0.4	WWIB	-0.4	-0.5	-0.7	**-1.2**
.	.	.	.	.	WWI	-0.3	-0.3	-0.4	-0.8	.	.	.	.	.	TOM	-1.2	-0.7	-0.5	-0.9	YAR	-1.0	-1.0	-1.3	**-1.2**
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	TOTG	-1.3	-0.9	-0.6	-0.6	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	VEI	-0.2	0.1	0.3	-0.1	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	VIZ	-1.0	-0.2	0.2	0.3	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	WWI	0.3	0.4	-0.1	-0.4	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	WWIB	-0.1	0.2	-0.3	-0.6	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	YAR	0.6	1.2	0.6	-0.6	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
21	3^a	1^b	4^c	0^d	72	17^a	16^b	17^c	19^d	71	15^a	10^b	6^c	11^d	78	7^a	7^b	5^c	5^d	72	27^a	19^b	12^c	21^d

The table lists all VR test statistics on *k*-period values: *32, 64, 128* and *256* for the selected stocks in each of the 5 sub periods from 1993-1996 to 2009-2012. The bottom line shows total number of available stocks and numbers of stocks with significant test-statistic. The critical test-statistic values are collected from Lo & MacKinlay (1988) in their results on the Variance ratio test's empirical quantiles (p. 36). The superscripts *a, b, c* and *d* indicate the actual k-period value (*32, 64,128,* and *256*) and their respective critical values -1.5, -1.46, -1.38 and -1.23. In sub period 1993-1996 for k-period value: *32*, we see that the three stocks FAR, RIE and SPOG have a lower test-statistic (in bold) than the critical value -1.5 and give us the number 3 at the bottom line.

A.2 List of stocks

Table A.2 List of stock tickers and company names for our144 selected stocks

AFG	AF Gruppen	NSG	Norske Skogindustrier
AFK	Arendals Fossekompani	NSGB	Norske Skog B
AIK	Aktiv Kapital	OCR	Ocean Rig
AKER	Aker	ODF	Odfjell ser. A
AKSO	Aker Solutions	ODFB	Odfjell ser. B
ALGETA	Algeta	OLT	Olav Thon Eiendomsselskap
AMA	Aker Maritime	OPC	Opticom
ASD	Axis-Shield	OPERA	Opera Software
ATEA	Atea	ORK	Orkla
AUSS	Austevoll Seafood	ORO	ORIGIO
AWS	Awilco ser. A	PDR	Petrolia E&P Holdings
AWSB	Awilco ser. B	PGS	Petroleum Geo-Services
BEA	Bergesen d.y ser. A	PHO	Photocure
BEB	Bergesen d.y ser. B	PLUG	Sparebanken Pluss
BLO	Blom	PRON	Pronova BioPharma
BNB	Bolig- og Næringsbanken	PRS	Prosafe
BON	Bonheur	QFR	Q-Free
BOR	Borgestad	RANG	Sparebanken Rana
BRA	Braathens	RCL	Royal Caribbean Cruises
BWG	BWG Homes	REACH	Reach Subsea
CEQ	Cermaq	RIE	Rieber & Søn
CHS	Choice Hotels Scandinavia	RIEB	Rieber & Søn B
CKR	Chr. Bank og Kreditkasse	RING	SpareBank 1 Ringerike Hadeland
COP	Copeinca	RNA	Reitan Narvesen
COV	ContextVision	ROGG	SpareBank 1 SR-Bank
CRU	Crew Gold Corporation	SADG	Sandnes Sparebank
DIAG	DiaGenic	SALM	SalMar
DNB	DNB	SANG	Sandsvær Sparebank
DOCK	Dockwise	SASB	SAS Norge B
DOF	DOF	SBVG	SpareBank 1 Buskerud-Vestfold
EIOF	Eidesvik Offshore	SCH	Schibsted
EKO	Ekornes	SDRL	Seadrill
ELK	Elkem	SEVAN	Sevan Marine
ELT	Eltek	SFJ	DSND Subsea
EMGS	Electromagnetic Geoservices	SFM	Synnøve Finden
EMS	EMS Seven Seas	SINO	SinOceanic Shipping
EVRY	EVRY	SME	Smedvig ser. A
EXPERT	Expert	SMEB	Smedvig ser. B
FAR	Farstad Shipping	SNI	Stolt-Nielsen
FOE	Fred. Olsen Energy	SNIB	Stolt-Nielsen B
FRO	Frontline	SNOG	Gjensidige NOR Sparebank
GOD	Goodtech	SOAG	SpareBank 1 Østfold Akershus
GOL	Golar LNG	SOFF	Solstad Offshore
GRE	Gresvig	SOI	Software Innovation
GRO	Ganger Rolf	SPOG	Sparebanken Øst
GSF	Grieg Seafood	SSI	Siem Shipping
HELG	Helgeland Sparebank	SST	Steen & Strøm
HNA	Hafslund ser. A	STB	Storebrand
HNB	Hafslund ser. B	STL	Statoil
HYD	Hydralift	STXEUR	STX Europe
IGE	IGE Resources	SUB	Subsea 7
IMSK	I.M. Skaugen	SUBC	Subsea 7
JIN	Jinhui Shipping and Transportation	SUO	SuperOffice
JSHIP	Jason Shipping	SVEG	Sparebanken Vest
KIT	Kitron	TAA	Tandberg
KLI	Klippen Invest	TAD	Tandberg Data
KOG	Kongsberg Gruppen	TAT	Tandberg Television
KOM	Komplett	TCO	TeleComputing
KVI	Kværner	TEL	Telenor
LHO	Leif Höegh & Co	TELIO	Telio Holding
LSG	Lerøy Seafood Group	TGS	TGS-NOPEC Geophysical Company
MING	SpareBank 1 SMN	TOM	Tomra Systems
MORG	Sparebanken Møre	TOTG	Totens Sparebank
NAS	Norwegian Air Shuttle	UNS	Ugland Nordic Shipping
NBK	Nordlandsbanken	UTO	Unitor
NEC	Norse Energy Corp.	VEI	Veidekke
NER	Nera	VIS	Visma
NHY	Norsk Hydro	VIZ	Vizrt
NOD	Nordic Semiconductor	VME	VMetro
NONG	SpareBank 1 Nord-Norge	WWI	Wilh. Wilhelmsen Holding ser. A
NOV	Norsk Vekst	WWIB	Wilh. Wilhelmsen Holding ser. B
NPEL	Norway Pelagic	YAR	Yara International

A.3 Overview of abbreviations and formula notation

Table A.3.1 Commonly used abbrevations

VR	variance ratio
MR	mean reversion
OSE	Oslo Stock Exchange
EMH	efficient market hypothesis
RW	random walk
rBAs	relative bid-ask spread

Table A.3.2 Notation used in section 3.2

t	time (days)
k	daily time window for the mean reverting process
n	number of observations in sub period
S_t	stock price
W_t	standard Brownian motion/wiener process
μ	percentagewise drift rate
σ	standard deviation
dt	time interval
ε	stochastic random variable $\sim N(0, \sigma^2)$
r_t	logarithmic stock returns

Table A.3.3 Notation used in section 3.3

i	notation for stock *i*
c	transaction cost
MA	moving average
θ_U	upper Bollinger band
θ_L	lower Bollinger band
θ_M	middle Bollinger band

Table A.3.4 Widely used formulas

Average	$\bar{X} = \frac{1}{n} * \sum_{i=1}^{n} x_i$
Moving average	$MA(X_{t,k}) = \frac{X_t + X_{t-1} + \cdots + X_{t-k-1}}{k}$
Variance	$Var(X) = \sigma^2 = \frac{1}{n} * \sum_{i=1}^{n} (x_i - \bar{x})^2$
Standard deviation	$SD(X) = \sigma = \sqrt{Var(X)}$
Covariance	$Cov(X,Y) = \frac{1}{n} * \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$
Correlation	$\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$
Beta coefficient	$\beta_{X,Y} = \frac{Cov(X,Y)}{Var(Y)}$

A.4 Derivation of applied formulas

Derivation A.4.1 Solution for the geometric Brownian motion – from section 3.2

The geometric Brownian motion is specified as

$$\frac{dS_t}{S_t} = \mu_t * dt + \sigma_t * dW_t, S(0) = S_0$$

To solve for S_t, we apply $It\bar{o}$ calculus and get

$$d(\ln(S_t)) = \frac{1}{S_t} dS_t - \frac{1}{2} * \frac{1}{{S_t}^2} {dS_t}^2$$

$$= \frac{1}{S_t} * S_t[\mu_t * dt + \sigma_t * dW_t] - \frac{1}{2} * \frac{1}{{S_t}^2} * {S_t}^2[{\sigma_t}^2 {dW_t}^2]$$

$$d(\ln(S_t)) = \mu_t * dt + \sigma_t * dW_t - \frac{1}{2}\sigma^2 dt$$

Integrating and applying the fundamental theorem of calculus we get

$$\ln(S_t) - \ln(S_0) = \left(\mu_t - \frac{1}{2}{\sigma_t}^2\right) t + \sigma_t * dW_t$$

$$S_t = S_0 * e^{\left(\mu_t - \frac{1}{2}{\sigma_t}^2\right)t + \sigma_t * dW_t} = S_0 * e^{\hat{\mu}_t * t + \sigma_t * W(t)}$$

which by taking logarithms gives

$$\ln(S_t) - \ln(S_0) = \hat{\mu}_t * t + \sigma_t * W_t$$

and

$$Var[\ln(S_t) - \ln(S_0)] = {\sigma_t}^2 * dt$$

By applying the formula for the expected value of a Gaussian random variable

$$E[e^x] = E[e^{\mu_t + \frac{1}{2}{\sigma_t}^2}]$$

If quickly follows that

$$E[S_t] = S_0 * e^{\mu_t t}$$

We have set μ equal to 10.7% on a yearly basis, a number used in Johnsen (1996) as a historical average yearly return for the OSE.

By using the definition of returns being equal to the difference in logged prices, we have that

$$VR(k) = \frac{Var(r_t + r_{t-1} + \cdots + r_{t-k+1})}{k * Var(r_t)}$$

$$= \frac{Var\big((\ln(S_t) - \ln(S_{t-1})) + (\ln(S_{t-1}) - \ln(S_{t-2})) + \cdots + (\ln(S_{t-k+1}) - \ln(S_{t-k}))\big)}{k * Var(r_t)}$$

$$= \frac{Var(\ln(S_t) - \ln(S_{t-k}))}{k * Var(r_t)}$$

$$= 1 + 2\sum_{j=1}^{k-1} \frac{(k-j)}{k} * \rho_j$$

Derivation A.4.2 The standardised variance ratio statistic – from section 3.2

The heteroscedasticity-robust standardised test-statistic can then be formulated as:

$$z_2(k) = \frac{VR(k)}{\sqrt{n} * \sqrt{Var(VR(k))}}$$

where

$$Var\big(VR(k)\big) = \sum_{j=1}^{k-1} \left[\frac{2 * (k-j)}{k}\right]^2 * \widehat{\omega}(j)$$

and

$$\widehat{\omega}(j) = \frac{\sum_{k=j+1}^{n}[(S_k - S_{k-1} - \overline{\Delta S})^2 * (S_{k-j} - S_{k-j-1} - \overline{\Delta S})^2]}{\left[\sum_{k=1}^{n}(S_k - S_{k-1} - \overline{\Delta S})^2\right]^2}, \overline{\Delta S} = \frac{1}{n} * (S_n - S_0)$$

For the underlying assumptions on the form of heteroscedasticity, see Lo & MacKinlay (1986) p. 25

A.5 Formulation of the trading signal indicators in the quantitative model

Our model allows no room for emotions and subjective decision making, and can be mathematically formulated as follows. See description of notation above in Table A.3.3.

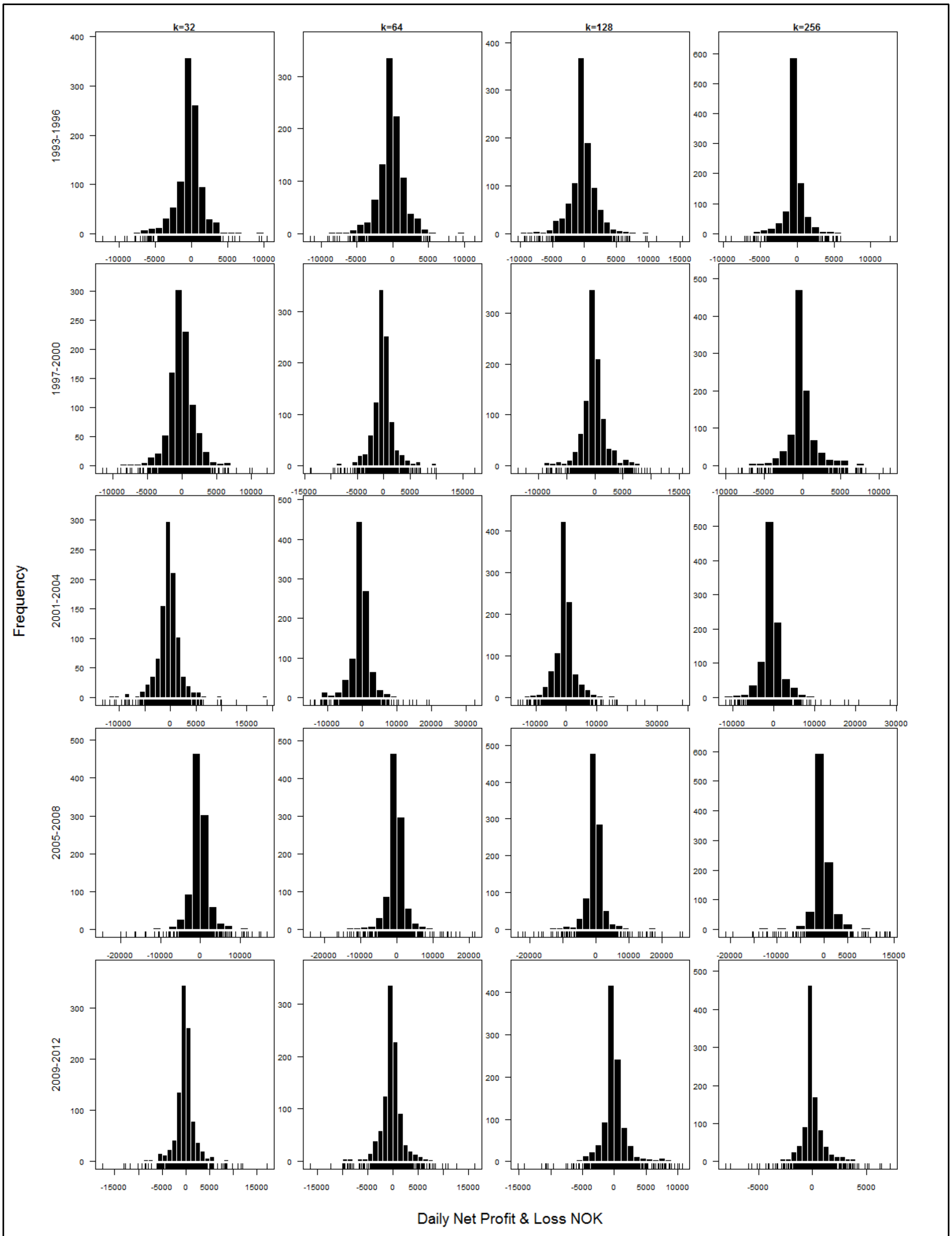
$$B_t\ (Buying\ indicator) = \begin{cases} 1\ if\ S_t < \theta_L \\ 0\ otherwise \end{cases}$$

$$Z_t\ (Selling\ indicator) = \begin{cases} 1\ if\ S_t > \theta_H \\ 0\ otherwise \end{cases}$$

$$C_t\ (Close\ position\ indicator) = \begin{cases} 1\ if\ S_t > MA_{t,k} > S_{t-1} \\ 1\ if\ S_t < MA_{t,k} < S_{t-1} \\ 0\ otherwise \end{cases}$$

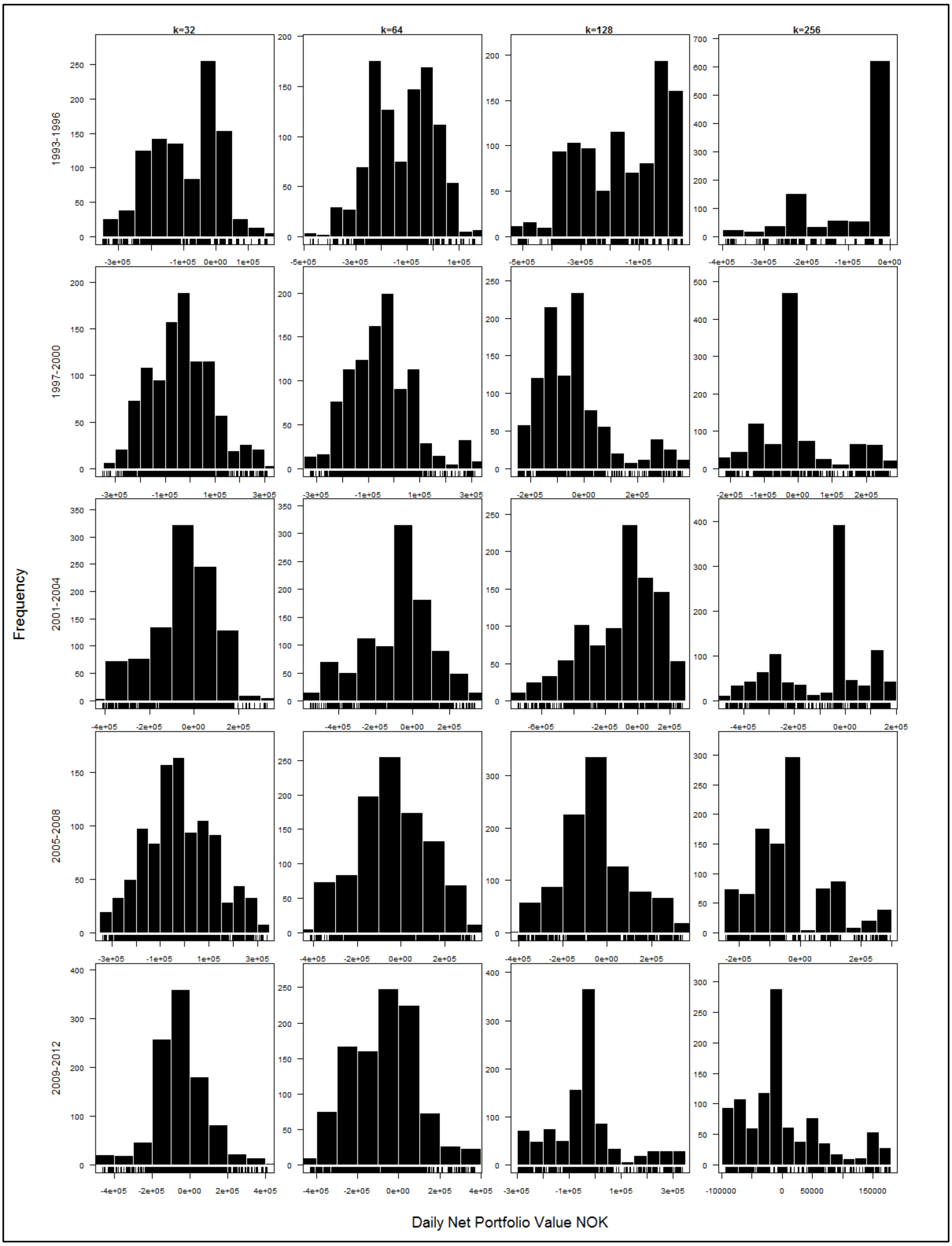
A.6 Sample distributions from the stock universe portfolio

Figure A.6.1 Distributions of Daily Net Profit & Loss



The distributions originate from the results in section 5.2. We retrieve the distribution of average Daily Net Profit & Loss from all the simulations. For each of the 20 simulations (*k*-values x 5sub periods) we chart the distribution and compile them in this overview.

Figure A.6.2 Distributions of Daily Net portfolio value



The distributions originate from the results in section 5.2. We retrieve the distribution of average Daily Net portfolio value from all the simulations. For each of the 20 simulations (4*k*-values x 5sub periods) we chart the distributions and compile them in this overview.

A.7 Price development on Oslo Stock Exchange

Figure A.7 Oslo All Share Index 1993-2012

The chart shows the development of the broad Oslo All Share index from 1993-2012

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