Nordea Kraftobligasjon Index Bond

Value and Expected Return Estimated by a Retail Investor

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NORGES HANDELSHØYSKOLE

Abstract

Index Bonds are a particular form of a structured investment product that consists of a bond element and a return element. The bond element is based on the guaranteed amount repaid to the investor at maturity. The return element is typically a bundle of options on an index or financial asset, scaled by a participation factor. Nordea Kraftobligasjon was the first Norwegian index bond with electricity forward contracts as underlying.

This thesis demonstrates the analysis that an investor should perform to be able to make an independent and prudent decision on whether or not to invest in Nordea Kraftobligasjon XV. The results indicate that an investor could pay as much as 3,5% to 4,5% in commission fee in addition to up to 3% in subscription fee. The expected return on the investment is estimated at approx. 3,5% p.a., compared with the risk-free rate of 4,5% p.a.

The thesis illustrates what methodological and practical challenges an investor can meet analysing an index bond on electricity contracts.

Preface and Acknowledgements

This paper is the final part of the siviløkonom-program at Norges Handelshøyskole.

The main goal of this thesis is to show what sort of analysis a non-professional investor should and can perform to be able to make an independent and prudent decision on whether or not to invest in an index bond, here exemplified by Nordea Kraftobligasjon XV. To reach this goal, a twofold objective for this thesis has been set. The normative or quantitative objective is to make a quantitative judgment of the value and the expected return and risk associated with the bond. Equally important is to illustrate the non-professional investor's calculation process, required body of knowledge, methodological and practical challenges met. This last part constitutes a descriptive objective for meeting the goal of the thesis.

There are theoretical and practical parts of the thesis. For the calculations performed, the theoretical part is substantial, and the discussions go beyond what is absolutely necessary to support the calculations. Nevertheless, I chose to present the theory and derivations partly to illustrate the thinking progress toward the goal and partly to demonstrate what sort of theoretical basis and solution knowledge an investor may require in order to build realistic and effective pricing and simulation models.

As the work with this thesis draws to a close, I would like to extend my thanks to my academic advisor, Professor Dr. oecon. Petter Bjerksund with the Department of Finance and Management Science. He has, through his flexibility and co-operation as well as his high level of expertise in the area of financial economics and derivatives, provided invaluable comments and constructive feedback. More importantly, his interest in structured financial products and his involvement in the public debate on the topic have contributed to a deeper understanding on the part of investors, regulators as well as general public. A special thanks to Pontus Ripstrand at Nord Pool AB Market Data Services for providing the market data.

Throughout the work on this thesis I have had a chance to revisit my theoretical knowledge of the option pricing and, most importantly, gained invaluable first-hand practical skills in building Monte Carlo simulation models.

Oslo, June 2008

Valeri Andreev

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1. Introduction

1.1 Background

The subject of this thesis, Nordea Kraftobligasjon, came to my attention almost five years ago, in the fall of 2003. At the time, this product was truly innovative, as only few other financial institutions were offering structured products, even fewer electricity-related instruments. Since that time the market for structured products has gone through its top and is now somewhat resembling a decline. The products' historical development is presented in the figure below:

Figure 1: Historical development of structured financial products
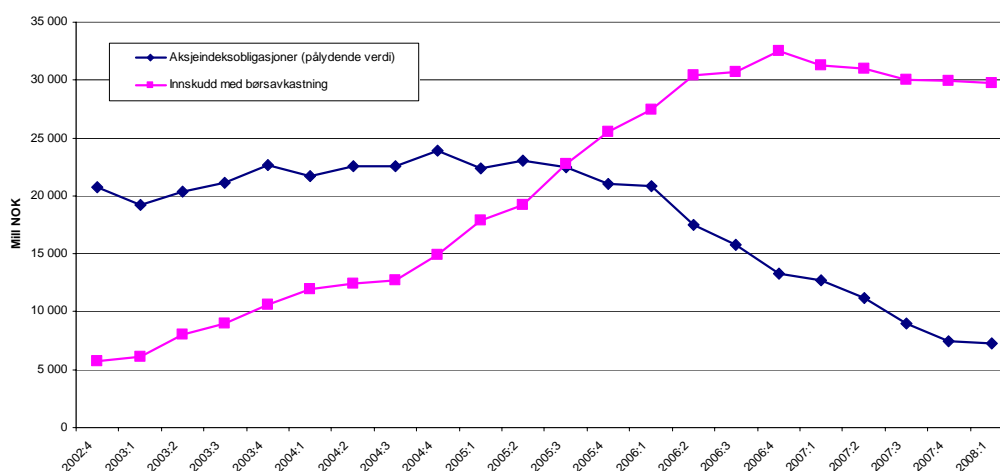
Source: SSB (table 43 - Banks. Deposits. Specified on different kinds of deposits; table 201 - Banks. Balance sheet statistics with specifications)

The products were first introduced in Norway in 1992 and went under different names: index bonds, bank deposits with stock return *etc*. They proved to be very popular with the private investors. However, the products also met wide criticism from both academics and consumer-protection organisations. Dine Penger magazine was among others active in the debate, several research papers and reports were published. Customer complaints and court cases came up.

In September 2006 a Directive from The Financial Supervisory Authority of Norway (*Kredittilsynet*) has passed regulating the offering of the structured products. Earlier in 2008, *Kredittilsynet* issued a new and more stringent regulation in the wake of its review report,

and the sale of the index bonds fell. New and simpler products, warrants, have entered the market.

Today, in summer 2008, arguing for unnecessary complexity of index bonds may seem as a "late-to-the-battle" appearance. However, from the analytical point of view index bonds are as intriguing now as they were in the fall of 2003.

1.2 Objectives

The main goal of this thesis is to show what sort of analysis a non-professional investor should and can perform to be able to make an independent and prudent decision on whether or not to invest in an issue of the Nordea Kraftobligasjon Index Bond on the terms offered. In order to reach this goal, a twofold objective for this thesis has been set.

The normative objective will be to make a quantitative judgment of the value and the expected return and risk associated with the bond. In doing so an investor will have to:

1. Understand the product and its fee structure and perform the component analysis.

2. Make value estimates for each of the bond components and the bond as a whole and perform reasonability assessment, if possible.

3. Calculate the uncertainty of the estimates above, perform sensitivity analysis and compare it with the relevant benchmarks, *e.g.* estimate provided by the issuer or comparable warrant pricing.

4. Assess the expected return on the investment, quantify the return distribution.

The quantitative analysis listed above will be performed on one of the Nordea Kraftobligasjon issues on its respective settlement date[1]. The normative objective is to conclude on value and expected return of the issue and compare the results with Nordea's internal estimates. It is expected that the results will be highly dependent on ones assumptions about the underlying price process and its volatility.

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[1] Investment decision is made no later than by the end of the subscription period. However, the settlement date is normally so close to the last subscription date (four trading days for the last issue), that for simplicity all analyses are performed on the settlement date.

Equally important is to illustrate the non-professional investor's calculation process, required body of knowledge, methodological and practical challenges met. This defines a descriptive objective for meeting the goal of the thesis.

What constitutes an independent and prudent investment decision process will vary from investor to investor. While one will be satisfied with unaudited estimates provided by the issuer or at most use third-party assumptions in a simple closed-form model, the other will not invest until he/she understands and independently estimates the product's value and expected return and risk. The descriptive objective is to identify the methodological and practical challenges that a non-professional investor might meet making a decision on Nordea Kraftobligasjon Index Bond.

1.3 Scope of Work and Delimitations

Meeting the descriptive objective of the thesis requires a rather detailed description of the steps and choices taken in order to arrive at the quantitative results, as well as methodological reasoning behind those choices. The scope of this thesis does not leave the opportunity to analyse more than one issue of the index bond. Therefore general conclusions about pricing and return levels of all Nordea Kraftobligasjon Index Bonds or other structured products are out of scope. For broader analyses covering several products and issuers, I refer to Koekerbakker and Zakamouline (2007), Bøe (2007) and Kredittilsynet (2008).

Investments in structured products, including the NKIB, are often accompanied by debt financing. According to *Kredittilsynet* approx. three quarters of the amount invested is financed by loan. The potential effects of such leveraging on the expected equity returns for investors are outside the scope of this analysis. While enhancing the upside potential for an investor, the leveraging also "converts" investor's alternative cost of capital into fixed interest expenses, increasing total possible losses beyond the difference between the face value and the guaranteed amount. Bøe (2007) shows that expected return with debt financing is always lower than the unleveraged return. *Kredittilsynet* (2008) confirmed this relationship for historical returns in structured products.

Only solutions that are methodologically acceptable, but easy to implement, do not require expensive parameter estimation and modelling, and are customary used by the investors, *e.g.* the Black 76 model, will be used in the calculations. It does not, however, mean that the

chosen model gives the best theoretically available description of the underlying price process (e.g. stochastic volatility) or takes into account all characteristic of the product (e.g. calculation period averaging).

Only publically available information has been used in the thesis. This particular concerns OTC quotes for longer-maturity options and estimation models for implicit volatility that could probably be obtained from electricity brokers.

The descriptive objective of this thesis is to arrive at a list of challenges that a non-professional investor might meet when evaluating an electricity-linked index bond. It does not purport to conclude on whether or not Nordea Kraftobligasjon Index Bond is suited for a retail investor or whether the issuer meets the information requirements in its offer document. The later is a legal or regulatory question which this thesis does not concern with. The term "non-professional investor" is used here is a broader sense, and does not necessary coincide fully with the same term used in § 10-2 of the Regulation to the Norwegian Security Trading Act ("*verdipapirforskriften*").

1.4 Methodology and Structure

The methodology chosen to achieve the normative objective stated above is to describe the steps taken to arrive at the quantitative value and return estimates. At the very end a brief summary of the methodological and practical challenges met in route will serve as an answer to the descriptive part of the thesis.

In the next section a short introduction to the structured products in Norway will be given. The structure of the Nordea Kraftobligasjon Index Bonds is described, and my first qualitative observations given, identifying early on some bond features that may require special attention. Upon choosing one of issues of the NKIB, I perform a component analysis of this particular issue, and finally conclude on the value of one of the components, namely the Certain Element.

General theoretical basis for valuing the other bond component, the Return element, is presented in section three. The first part of this section is built on a top-down approach and briefly covers the general theory of price movements and an option pricing framework, both in their general forms and when applied to underlying forward contracts or underlying that

exhibit some form of implicit dividend yield. The second part is bottom-up built, and treats the NKIB characteristics of multiple underlying, averaging over several observations and foreign currency exposure, applying the general theory relayed above. The section finishes with a choice made for calculation methods that shall be applied to valuation of the Return element and total expected return analysis.

The issue of volatility is treated in section four. Section five revisits the fundamental price process assumption made in the previous sections, and questions whether it is actually applicable for electricity forwards as underlying. This section concludes on a simplest closed-form solution that can prudently be used for valuing the Return element.

Section six is dedicated to the numerical techniques, particularly Monte Carlo simulation applied for valuation and return analysis. The Monte Carlo approach is conceptually simple, and given description of the underlying process one could perform simple simulation without knowing much about the theoretical basis of the procedure.

Section seven contains the estimated input to and actual results of the value and expected return calculations, assesses them for reasonability, quantifies estimation uncertainty and compares with relevant benchmarks, *e.g.* internal estimates provided by Nordea or comparable warrant pricing.

Final section closes this thesis with a conclusion on the value of the chosen issue, the expected return and risk profile. Here I also offer a summary of the challenges that in my view a retail investor would meet when analysing one of the NKIB issues or similar products. Further work on the topic is suggested.

2. Nordea Kraftobligasjon Index Bonds

To begin with, I would like to emphasize that my understanding of Nordea Kraftobligasjon Index Bond is based solely on the information made publicly available on the Nordea's web site[2], and that a prudent investor would always discuss an offered product directly with the issuer before performing his/her independent analysis and drawing an investment decision. In fact, in light of the current regulation changes[3], a non-professional retail investor would probably not be offered this product at all without his/her investment objectives and knowledge being assessed and the product explained.

2.1 Description of the Nordea Kraftobligasjon Index Bond

Nordea Kraftobligasjon Index Bond investment product was first offered to the market in spring 2001. The electricity-linked return element was designed by Tafjord Kraft utility company. As far as I know, it was the first product offering Norwegian retail investors direct exposure to electricity markets. Some of the electricity-linked index bonds and warrants offered to the Norwegian market are presented in the table below:

Table 1: Selection of electricity-linked structured investment products

Issuer	Index Bonds	Warrants
Nordea	Kraftobligasjon II 2001-2004 to Kraftobligasjon XV 2008-2011, Kraft Privat I to III, Tysk Kraft, Trippel Kraft 2007-09	Kraft XV Gearing 2008-2011
DnB NOR	Kraftobligasjon 2007-2009	Warrant Kraft & Kraft II 2007-2009, 2007-2010, 2007-2011, 2008-2011
Orkla Finans	Kraft, Kraft (BMK), Kraft II, Kraft II (BMK), Kraft III, Kraft IV	

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[2] http://www.nordea.no/Privat/Sparing%2bog%2binvestering/Verdipapirer/Kurser%2bog%2bprospekter/612402.html

[3] Ref. Directive 4/2008 of 12.02.2008 from The Financial Supervisory Authority of Norway (*Kredittilsynet*)

The NKIB is Nordea's "general financing" credit instrument with a tenor of between 2,5 and 3,5 years. The bond is structured as a bullet loan, with no amortization of the principal nor any coupon payments during its life. As any index bond, also the NKIB has a bond part built into it and a derivative part. The derivative part is usually an option on some indices or single contracts in the equity, foreign exchange, fixed income or commodities' market. The NKIB's option is written on a bundle of forward contracts traded on the Nord Pool Power Exchange. At settlement the NKIB consists of the following main elements:

1. Certain element ("*CE*"[4]) in a form of Guaranteed amount ("*garantert investering*" or "*GA*") at maturity T, where historically 95% to 100% of face value (excluding any premium) were guaranteed by the bank. The proportion guaranteed may be called a Guarantee Factor ("*GF*").

2. Return element ("*RE*") in the form of Additional amount ("*tilleggsbeløp*" or "*AA*") at maturity T, which is based solely on the development of prices for a bundle (two to four) of yearly, base-load forward contracts traded on Nord Pool Power Exchange[5]. The periods taken into calculations range from the issue date until almost at expiration of each contract (mid-December). Each contract's return is usually equally weighted in the resulting average. By design, the Return element cannot be negative, representing in effect a European Call Option on the average of these contracts written by Nordea. To arrive at the Additional Amount, any positive appreciation of the average is finally multiplied by a Return Factor ("*RF*"), since 2005 varying between 0,95 and 1,25, which depends on Nordea's subsequent hedging cost. As the underlying forward contracts are quoted in euro currency starting from FWYR-06 while the loan is in Norwegian kronas, the investor may or may not be exposed to the currency risk in addition to the market risk. An explicit "currency cross"-adjustment introduces currency exposure while its absence effectively hedges it. Starting from NKIB-IV, the currency-cross adjustment has been taken out of the return formula.

3. Commission fee ("*tilretteleggerprovisjon*" or "*CF*") retained by the bank, per definition representing the difference between the face value of the loan (including

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[4] "Certain element" ("*CE*") is not to be confused with Certainty Equivalent for which the "CE" abbreviation is often used.

[5] Only NKIB-X's Return elements were based on forward contracts quoted at the European Energy Exchange (EEX)

any premium as the case may be) and the total market value of the Certain element and the Return element; and

4. Subscription fee ("*tegningsomkostninger*" or "*SF*") paid to and retained by the bank, between 3,0% on the first NOK 0,5 mill for unaffiliated small investors and 0,35% on amounts above NOK 5 mill for the bank's Private Banking clients and employees.

The bond may be offered at par (face) value (as the most NKIB issues were), or at a premium (as was the case for the NKIB-XIII in February 2007). In this case, percent-wise values of all elements should be re-calculated to take the premium into account. For simplicity, I will use NOK 100 as a practical expedient expression for the face value of the bond. The lowest investment amount for the NKIB-XV is NOK 10.000, but all the valuation results for NOK 100 can easily be scaled up to whatever investment amount is relevant for the investor.

A summary of characteristics and pricing information given in the offer documents for the NKIB-II to NKIB-XV issues is presented in Appendix A.

2.2 Initial Observations

Exposure to Forward Prices As Opposed to Electricity Spot Prices

The *AA* of the NKIB is based on the return on the forward contracts, not on the development of the spot prices. This fact is clearly stated in the offer documents. However, since many of the arguments in the offer brochure make reference to the expected increase in electricity prices (high regional demand growth, CO_2-costs and integration with Continental Europe's electricity markets), I believe it is important to make this critical distinction. The electricity traded on the spot market is, for all material purposes, a non-storable commodity exhibiting to some degree predictable cycle and trend behaviour. On the other hand, forward contracts are investment assets with cost of carry equal to zero, which if efficiently priced should fully reflect the market expectations for the future spot prices (adjusted for cost of risk). To test this notion I have plotted the historical development in already elapsed forward contracts (FWYR-01 to FWYR-08) against spot price development and realised spot annual averages,

as presented in Appendix B[6]. The diagram shows that the forward prices have followed the upward trend found in the spot prices since 2001 (with exception of the second half of 2006). It seems that investments in these forward contracts (and options based on them) should have yielded positive, possibly abnormal returns. However, for contract bundles that have already elapsed (until NKIB-IX) and where I have all start and stop quotes available (see Appendix A), some issues such as NKIB-IV and -IX have been highly profitable for investors, while others ended with a zero *AA* (*e.g.* NKIB-VI running briefly in 2003-2004). Interestingly, since 2003 the first and the shortest forward contracts within the bundles have almost never contributed with any material positive returns to the average. Even taking into consideration the apparent historical upward trend in the forward prices coinciding with rising spot prices prior to 2006, there is still no theoretical basis to claim that an investment in the electricity forwards would guarantee any abnormal returns even if the spot prices are certainly expected to rise. This is supported by the theory presented in section three.

Adverse Historical Trend in the Return Element Structure

The Return element is, in essence, an option on the average of two to four forward contracts. The structure elements that influence this option's value will also affect Nordea's total expected funding cost. Apart from the market-given interest rates and the strike value (set equal to current forward price), the nature, trend and volatility of the underlying forward price process, the lives of the options and the Return Factor will all be significant. In this light it is interesting to note that

- The tenor of the bonds and the weighted average life of the options have increased since 2001 (from NKIB-VIII: approx. 3 years and 1,7-1,9 years, respectively).

- The bank has chosen yearly forward contracts, the longest available.

- The number of contracts in the bundle is now almost never less than three.

- Averaging over five days was introduced starting from NKIB-XII[7].

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[6] In this thesis historical time series for daily closing spot and forward prices quoted at Nord Pool for the period 01.01.1999 through 29.05.2006 have been used.

[7] May be the five-days averaging was not the banks initiative, but that the options it hedges by on the OTC are so defined.

- When Nord Pool went over from quoting in NOK to EUR (from FWYR-06 onwards), the bank chose not to adjust for the development in the exchange rate in its Return element, effectively hedging the currency exposure. Only the NKIB-XIII had the "currency cross"-adjustment in its formula.

- Tthe guaranteed amount set at 100% for the first issues, has been reduced to 90%-98% since NKIB-V. Again, the NKIB-XIII is an exception.

Exclusion of "Market Disruptions" from the Return Volatility

The spot prices' behaviour clearly exhibits jumps and spikes. Yearly forward contracts can be expected to be less "jumpy", still some abnormal short-term movements in the forward prices cannot be ruled out, particularly on days when the Nord Pool financial market does not function properly or when the contracts approach their expiry in December. I have plotted historical weekly returns on FWYR-01 to ENOYR-08 forward contracts presented in Appendix C. I observe material outliers (several above +/- 6% or +/-40% p.a.), and anecdotal evidence suggest that the traders use models with jump diffusion when forecasting future volatility, Deyna and Hulström (2007). These possible "fat tails" in the return distribution can prove to be very valuable for the value of the options built into the Return element. However, by contract Nordea is protected from "market disruptions" ("*markedsforstyrrelser*") that should befall on the electricity or currency markets on start or stop dates. Until NKIB-XIII the brief definition of what constitutes "market disruption" was taken into the offer document, from NKIB-XIV a reference is made to appropriate ISDA-regulation[8].

Use of Traded Options Or Warrants To Price the Return Element

There are Nord Pool-traded options that are written on the same yearly base-load forward contracts that underlie the Return element in the NKIB. On the start dates Nordea hedges its exposure on its written options by buying necessary amount of electricity options on the Nord Pool, and thus setting its final Return Factor as a proportion of the amount "available" for hedging to the market hedging cost. I believe that the main challenge in understanding and valuing the NKIB is to price the options built into the Return element. Since there exist

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[8] 2005 SIDA Commodity Definitions published by the International Swaps and Derivatives Association (www.isda.org).

traded options on the same underlying assets and Nordea is actually using them, could it be that the easiest way to price the Return element is to look up the Nord Pool option prices? It depends on availability of traded options for all underlying contracts, the comparability of dates, of credit margin and of course on liquidity and efficiency of the option market. I also observe that almost all NKIB issues start on Fridays.

I note that in September 2007 DnB NOR issued a Warrant Kraft 2007/2010. A warrant is in essence the Return element without a bond attached. DnB NOR Warrant Kraft 2007/2010 is built on the same forward contracts as the NKIB-XV, with the same participation factor of 1,0. It was settled on 12.10.2007 (three months prior to the KNIB-XV) and is to expire on 23.12.2010 (again 3 months before the NKIB-XV expires). Although the warrant is not actively traded, if quantitative analysis is to be done on the NKIV-XV, the value estimated and information provided by DnB NOR for Warrant Kraft 2007/2011 may be indicative.

2.3 Choice of Issue for Further Quantitative Analysis

Based on the review of the offering documents for NKIB-II to NKIB-XV, as summarised in Appendix A, it is evident that the main structure of the NKIB has not materially changed since the spring 2001. Thus the general theory, the methods and the market data available for valuing the bond will probably be the same whatever issue one chooses to analyse. At the same time, some of the key elements of the bond structure have been altered during the course of the years, so that the practical implementation will be somewhat different.

For purposes of this thesis the last issue offered in February 2008, the NKIB-XV, is analysed. The offer documents were dated January 11, 2008, the subscription period lasted from January 14 to February 11, and the settlement date was set on February 15 this year. Some of the reasons for the choice are the following:

- Although one of the oldest electricity markets in the world, the Nord Pool is still comparatively young, with historical data available for a not very long period. The underlying structure of the market, the market drivers, as well as products' definition and liquidity have been changing constantly: *e.g.* Denmark joined only in 2000, contracts have been redefined several times (lately in September 2003), the Nord Pool went over to quoting in EUR in 2003, new cables to the neighbouring electricity markets were commissioned (*i.e.* the NorNed link), the fuel prices have surged, *etc.*

All this may and should change the price dynamics over the years; therefore it would be preferable to work with market data which is as "fresh" as possible.

- Taking into account the recent changes in the regulation for structured products, the NKIB offer documents provide more relevant information. In addition, it can be interesting to compare the results with DnB NOR's valuation of their Warrant Kraft 2007/2010.

- NKIB-XIII was extensively analysed in Bjerksund (2007) and Bjerksund (2008). Although this thesis draws heavily on these two works, for numerical calculations it is only proper to choose another, preferably later issue.

2.4 Component Analysis of the NKIB-XV

For simplicity, it can be thought that the sum of the cash flows received by the bank at the settlement time t_0 should be equal to the total present value of the obligations incurred and the total profit earned. It can be represented as:

$$FV + P + SF = (CE_0 + RE_0) + (CF_0 + SF) \tag{2.1}$$

where FV – the face value of the loan, P – any premium, SF – the Subscription fee, CE_0 – value of the Certain element at t_0, RE_0 – value of the Return element at t_0, and CF_0 is the implicit value of the Commission fee at t_0. The bank earns the SF which is an explicit fee plus the CF_0 which is sometime called a "hidden fee". The value of the product for the investor at t_0 is of course $CE_0 + RE_0$. As mentioned above, any effects of leveraging are out of scope of this thesis.

Since the NKIB-XV was offered at par, there is no need for adjusting for premium P. The face value FV and the Subscription fee SF (expressed as a percentage of the FV) are known with certainty at t_0.

Assuming that the Value Additivity Principal ("VAP") holds[9]. It is therefore possible to calculate the value of the bond for the investor as a sum of its components, the CE_0 and the RE_0.

The valuation of the CE, which for a bullet loan is just the present value of the Guaranteed amount GA at maturity T, should not cause any major misunderstandings (more on this in the next subsection).

$$CE_0 = V_0[GA_T] = V_0[GF \cdot FV] \tag{2.2}$$

Here $V_0[.]$ is the present value of a future cash flow, FV – the face value of the bond and the GF – the Guarantee Factor (0,98 for the NKIB-XV).

The RE is contractually structured as follows:

$$RE_0 = V_0[AA_T] = V_0\left[FV \cdot \sum_{i=1}^{N} \max\left(\frac{\widetilde{F}_{t_i}^i - F_{t_0}^i}{F_{t_0}^i} \cdot w_i; 0 \right) \cdot RF \right] =$$
$$= RF \cdot \sum_{i=1}^{N} w_i \frac{FV}{F_{t_0}^i} V_0\left[\max\left(\widetilde{F}_{t_i}^i - F_{t_0}^i; 0 \right) \right] = RF \cdot \sum_{i=1}^{N} w_i \frac{FV}{F_{t_0}^i} e^{-r(T-t_i)} c_i \tag{2.3}$$

where AA_T is the Additional amount paid at maturity T, N – number of forward power contracts in the bundle (three for the NKIB-XV), F_{t0}^i – quoted price for forward power contract i at start date t_0, $\widetilde{F}_{t_i}^i$ – arithmetic average of five consecutive quoted prices for forward power contract i at respective stop dates[7] t_1 to t_3, RF – Return Factor (sometime also called "participation factor"), w_i – the contracts' corresponding weights, c_i – value of a call option on the underlying contract F^i with a remaining life of t_1 to t_3, and $e^{-r(T-t)}$ takes into account delayed payment of options proceeds until bond maturity T. The options used in structured products can be very exotic, but in the NKIB's case this is a plain European call on an average. To hedge the exposure the bank has to buy w_i*FV/F_{t0}^i options on each contract.

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[9] If the VAP does not hold, there will be opportunities for instant arbitrage profits by taking contrary positions on the whole and on the parts.

Table 2: The time progress of the underlying contracts

Contract (i)	Start date (15.02.08)	Stop date (08.12.08[10])	Stop date (08.12.09[7])	Stop date (08.12.10[7])	Maturity (15.02.11)
time	$t_0 = 0$	$t_1 = 0,817$	$t_2 = 1,810$	$t_3 = 2,818$	$T = 3,008$
Exchange rate	X_{t_0}	$\widetilde{X}_{t_1}$	$\widetilde{X}_{t_2}$	$\widetilde{X}_{t_3}$	
Interest rate					r_T
1 ENOYR-09	$F_{t_0}^1$	$\widetilde{F}_{t_1}^1(\sigma_{01})$			
2 ENOYR-10	$F_{t_0}^2$		$\widetilde{F}_{t_2}^2(\sigma_{02})$		
3 ENOYR-11	$F_{t_0}^3$			$\widetilde{F}_{t_3}^3(\sigma_{03})$	

The "hat" over the forward price quotes for t_1 to t_3 denotes that these variables are uncertain or stochastic. Nor $F^i{}_{t0}$ is actually known at the latest investment decision date (11.02.2008), however for simplicity I assume that $F^i{}_{t0}$ is known.

The Return Factor indicated in the offer document dated 11.01.2008 was 1,00. The final *RF* was to be set on the start date 15.02.2008 at 1,10[11]. However, at the point of time for investment decision, the best *RF* estimate available for the investor was still 1,00.

2.5 Pricing of the Certain Element

The *CE* is a zero-coupon bond with known date and a payout amount. However, since the *GA* represents the major part of the payout and the maturity *T* is the longest life of all NKIB elements, the estimated discount rate will materially influence the total value of the bond.

$$CE_0 = V_0[GF \cdot FV] = e^{-r_T T} \cdot GF \cdot FV \tag{2.4}$$

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[10] This is the median (third) date of the five-days period which the closing average is based on.

[11] To my knowledge, such an increase in the *RF* from the indicated to the final has happen only once before, in the NKIB-XIII.

where r is zero-coupon continuously compounded interest rate for NOK-denominated debt with T years to maturity. Counting actual trading days, the time to maturity from 15.02.2008 until 15.02.2011 is $T = 3,008$ years.

Directly Estimated Interest Rate

Nordea's appropriate market borrowing rate with the currency, the duration and the seniority corresponding to the *CE* would be estimated as follows:

- Use risk-free market rate with corresponding currency and duration (*e.g.* 3-years Norwegian government bonds) and add estimated appropriate credit margin; or

- Use commonly-used interest rate base (*e.g.* 3 month NIBOR) and add credit margin known to be applied on this base for Nordea's borrowing with corresponding duration and seniority; and finally for the both above

- Convert the resulting rate into a continuously compounded rate.

Hull (2000) shows that one can convert to continuous compounding as:

$$r = m \cdot \ln\left(1 + \frac{R}{m}\right) \tag{2.5}$$

where R is a rate compounded m times a year and r - continuously compounded.

For retail investor it may pose some challenges to construct and smooth the zero-coupon yield curve. Further one has to estimate the margin based on Nordea's rating. In the offering documents Nordea Bank Finland Abp (which is the formal counterpart in the transaction) informs that it enjoys "AA-" rating at the S&P and "Aa3" at the Moody's. The Index Bond has ordinary priority and carries no pledge.

In its offer document, Nordea informs that it borrows at commensurable terms at 3 month NIBOR minus 3 p.b. On 15.02.2008 the effective 3M NIBOR was 6,23%, so Nordea's estimated rate is 6,20%[12] (or 5,99% continuously). Compared with 3-year government rate of 4,49%, it represents a margin of 170 b.p. which is historically high. The market was

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[12] Nordea's borrowing rate information is dated 10.01.2008, I choose to apply the same margin on the settlement date.

clearly in backwardation at this time, in addition the NIBOR margin has increased significantly since summer 2007 from its historical level of 20-40 b.p. to the range between 60 b.p. and 160 b.p.

Interest Rate Implied in Nordea's Own Valuation

In its offer document for the NKIB-XV, Nordea also offers its own valuation of the *CE*. According to the bank, on 10.01.2008 ($T' = 3,099$ years) CE_0 on was equal to 83,75%. Based on the equation (2.5):

$$r = \frac{\ln\left(\dfrac{GF \cdot FV}{CE_0}\right)}{T'} = \frac{\ln\left(\dfrac{0,98 \cdot 100}{83,75}\right)}{3,099} = 5,07\% \quad \text{or} \quad R = 5,20\%^{[13]} \tag{2.6}$$

Since the main focus of this thesis is analysis of the derivative component of the NKIB, I find it practical to accept this *CE* valuation as fair, and use the rate $r = 5,07\%$ implied therein in the further analysis. This implied rate is much lower than the rate Nordea itself indicated as 3M NIBOR – 3 p.b. However, it represents a 70 b.p. margin on the 3-year government bond, which in other times would not be unreasonable.

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[13] The result, 5,20%, is supported by my direct calculation using Excel formula "XIRR".

3. Option Pricing Theory and Its Application

The following two subsections concisely describing the general theory of price process and derivative pricing are not strictly necessary for an investor to be able to use one of the closed-form option pricing formulas. Nevertheless, I see it as important for an investor to understand the price process assumptions behind a formula in order to judge whether the formula fits the observed electricity forward price behaviour. In addition, the understanding of the underlying price process is critical for investor's ability to perform a numerical procedure, e.g. Monte Carlo simulation.

3.1 General Theory of Price Movement and Its Application

Prices for financial and consumption assets (including commodities) move in a non-deterministic (unpredictable, uncertain) manner, and are said therefore to follow one or another stochastic process. The application of the stochastic processes within the financial markets was first done for the financial assets like equities, fixed income instruments and currencies.

A pure stochastic process is the one where each step is completely independent of all the previous ones. However, the process is still dependent on its pervious state. If a process is only dependent of its last state, it is called the Markov stochastic process. The Markov property is consistent with the weak-form hypothesis of the market efficiency (the Weak EMH"), see Bodie *et al* (1996). Numerous empirical studies of, among others, Kendall (1953), Roberts (1959), Fama (1965) and Fama (1970), Sharp (1966), McDonald (1974), Conrad and Kaul (1988) and Lo and McKinlay (1998) indeed showed that the equity market exhibits at least the weak-form of the EMH. That the Markov process can therefore be used to describe the movement of stock does not automatically mean that it is the best representation for all commodities, including spot electricity prices.

A Wiener process is a form of the Markov process, with no drift and an annual variance (measure of volatility[14]) equal to one. This is a pure noise process or a Brownian motion. According to Hull (2000) this process can be described as

$$dz = \varepsilon \cdot \sqrt{dt} \qquad (3.1)$$

where dz is a Wiener process, ε is a normally distributed value with a mean equal to zero and a variance of one, and dt is a small time shift. The Wiener process's noise is normally distributed and every dz is independent of all pervious movements (no autocorrelation).

The Wiener process has per definition no drift term. Such a process would not describe movements of investment assets' prices well since the expected return would be zero, and such an asset would be unattractive for investors. A generalized Wiener process, or an Arithmetic Brownian Motion ("ABM"), does not have restrictions on drift or volatility:

$$dS = \mu dt + \sigma dz = \mu dt + \sigma \varepsilon \sqrt{dt} \qquad (3.2)$$

where dS is a ABM process for price S, μ is the instantaneous drift term of the process, σ is the magnitude of the volatility, and dz is the Wiener noise process described above. The drift and the volatility can be constant or time-varying, deterministic or stochastic. In its simplest form the ABM process assumes both to be constant deterministic. While the Wiener process is expected to wander around its starting price level S_0, the ABM will "swing" around its drift term. The drift represents return to investor, so it can be thought of consisting of a risk-free rate of return and a risk premium: $\mu = r + \lambda$.

However, with a strong negative drift or high volatility, the ABM can result in negative prices. This is not compatible with the notion of prices for investment assets. A Geometric Brownian Motion ("GBM") introduces the price S as a scaling measure for the drift and the volatility, meaning that the noise generated by the process is proportional to the price:

$$dS = \mu S dt + \sigma S dz = \mu S dt + \sigma S \varepsilon \sqrt{dt} \qquad (3.3)$$

From (3.3) one arrives at the instantaneous return on the movement over the time period dt:

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[14] By "volatility" here is meant a standardized measure of the standard deviation.

$$\frac{dS}{S} = \mu dt + \sigma dz = \mu dt + \sigma \varepsilon \sqrt{dt} \qquad (3.4)$$

The Itô's process is the same as a Wiener process, only with the drift and the volatility (the standard deviation) not deterministic, but expressed by functions of time and price $f(S,t)$.

$$dS = \mu(S,t)Sdt + \sigma(S,t)Sdz = \mu(S,t)Sdt + \sigma(S,t)S\varepsilon\sqrt{dt} \qquad (3.5)$$

Itô (1951) proposed a lemma that allows us to price derivatives which values are based on the stochastic variables underlying those derivatives and time. The Itô's lemma converts the assumed stochastic process of the underlying into a new stochastic process for the derivative.

The Itô's method simplifies the resulting derivative expression by applying the the first two Taylor expansions for the underlying and the first expansion for the time. If one assumes a derivative function G of the underlying x and the time t, $G(x,t)$, than a change in this function can be expressed as:

$$dG = \frac{\partial G}{\partial x}dx + \frac{\partial G}{\partial t}dt + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}dx^2 \qquad (3.6)$$

If the underlying process is expressed by the Itô process (3.5), than the derivative process takes a form of (3.7):

$$dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial G}{\partial S}\sigma Sdz \qquad (3.7)$$

Forward contract is an investment asset, an agreement to buy or sell an asset at a certain future time for a certain price (Hull 2000). It can be shown that for an asset providing no income or dividend yield until the maturity, the spot and the forward prices at time 0 are linked thus:

$$F_{0,T} = S_0 e^{rT} \quad \text{and} \quad S_0 = F_{0,T} e^{-rT} \qquad (3.8)$$

where $F_{0,T}$ and S_0 are the forward and spot prices at time 0, respectively, r – appropriate continuous interest rate and T – maturity point of time. For an asset with a constant dividend yield q (or a convenience yield δ), the expression (3.15) takes form:

$$F_{0,T} = S_0 e^{(r-q)T} \quad \text{and} \quad S_0 = F_{0,T} e^{-(r-q)T}$$
(3.9)

The term $(r - q)$ is called "cost of carry" and q – "rate of return shortfall". The term q can be dividend yield, but it can also incorporate convenience yield or interest rate differential.

If we now assume S to follow the Itô's process (3.5) with a trend μ and volatility σ, it can be proved that (3.7) for a forward contract F on S becomes:

$$dG = dF = \left[e^{r(T-t)} \mu S - rS e^{r(T-t)} \right] dt + e^{r(T-t)} \sigma S dz = (\mu - r)F dt + \sigma F dz$$
(3.10)

This means that also the forward prices follow the GBM with the same volatility, but with a drift $(\mu - r)$ reduced by the interest rate, compared with the underlying process.

If we now define $G = \ln(F)$, dG representing the logarithmic return, and if F follows the Itô's process (3.5) with a trend $(\mu - r)$ and volatility σ, it can be shown that (3.7) now becomes:

$$dG = d\ln(F) = \left(\mu - r - \frac{\sigma^2}{2} \right) dt + \sigma dz = \left(\lambda - \frac{\sigma^2}{2} \right) dt + \sigma dz$$
(3.11)

It appears that $\ln(F)$ follows the ABM or the generalized Wiener process with a constant drift rate of $(\lambda - \sigma^2/2)$. From (3.11) we see that the change in $\ln(F)$ and the logarithm of return between two points of time t and T will be normally distributed at $\varphi(\lambda, \sigma)$:

$$\Delta G = \Delta \ln(F) = \ln(F_T) - \ln(F_0) = \ln\left(\frac{F_T}{F_0} \right) \sim \phi\left[\left(\lambda - \frac{\sigma^2}{2} \right) T, \sigma\sqrt{T} \right]$$
(3.12)

This is confirmed by a number of empirical studies for stock price returns. If this is the case, the GBM forward price process is lognormally distributed.

The forward price return movement follows the ABM as shown (3.5). According to Back (2005), the equations (3.5) and (3.11) are equivalent. If we solve (3.11) for F, we can express the price following the GBM at any future point of time T as (3.13) in continuous and discrete forms:

$$F_T = F_0 e^{\left(\lambda - \frac{\sigma^2}{2} \right) t + \sigma dz} \quad \text{or} \quad F_{t+\Delta t} = F_t e^{\left(\lambda - \frac{\sigma^2}{2} \right) \Delta t + \sigma \varepsilon \sqrt{\Delta t}}$$
(3.13)

The equation above can be used to model forward price movements in a Monte Carlo simulation.

3.2 Pricing of European Call Options Under Risk Neutrality

Let's assume one has an underlying price process *S* that follows the GBM as described in (3.3) and a derivative *G* of this price that follows its own process presented in (3.7). One can then construct a risk-free portfolio consisting of a short position in one unit of the derivative and a long position in $\partial G / \partial S$ units on the underlying. The positions are funded at the risk-free rate. The value of the portfolio and the change in such a portfolio are shown in (3.14) and (3.15), respectively. The value of and the change in such a portfolio will than be:

$$\Pi = -G + \frac{\partial G}{\partial S} S \qquad (3.14)$$

$$d\Pi = -dG + \frac{\partial G}{\partial S} dS \qquad (3.15)$$

Substituting (3.3) and (3.7) into (3.15), it can be shown that the change will be:

$$d\Pi = \left(-\frac{\partial G}{\partial t} - \frac{1}{2} \frac{\partial^2 G}{\partial S^2} \sigma^2 S^2 \right) dt \qquad (3.16)$$

Since the portfolio Π is instantaneously[15] risk-free, it may only earn the risk-free rate *r*:

$$d\Pi = r \cdot \Pi dt \quad \text{or} \quad \left(-\frac{\partial G}{\partial t} - \frac{1}{2} \frac{\partial^2 G}{\partial S^2} \sigma^2 S^2 \right) dt = r \cdot \left(-G + \frac{\partial G}{\partial S} S \right) dt \qquad (3.17)$$

$$\left(\frac{\partial G}{\partial t} + r \cdot S \frac{\partial G}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} \right) = r \cdot G \qquad (3.18)$$

---

[15] The portfolio Π is not permanently riskless though, but only for a infinitely short period of time *dt*. As S and t changes, so changes *dG/dS*. To keep such a portfolio riskless it is therefore necessary to continuously adjust the underlying-to-derivative proportion.

The equation (3.18) is the Black-Scholes-Merton ("BSM") differential equation. It can be solved for various derivatives according to their respective boundary conditions. The assumptions used to arrive at this equation are:

- The underlying S asset follows the BSM with constant trend μ and volatility σ;

- The short selling of the underlying or debt financing is permitted;

- There are no transaction costs and taxes;

- All securities are perfectly divisible, and the trading is continuous;

- The risk-free rate is constant deterministic and the same for all maturities;

- There are no riskless arbitrage opportunities; and

- The underlying is paying no income or dividends during the life of the derivative, and offers no convenience yield.

As shown in (2.3), in order to price the *RE* of the NKIB-XV, one has to arrive at a solution to value the at-the-money European call options on forwards built into it. A call option is a right, but not an obligation as the forward is, to buy the underlying asset by or on a certain date (called "expiration date" or "maturity") for a certain amount (called "exercise price" or "strike". For the NKIB-XV the underlying are yearly base-load forward contracts traded at the Nord Pool. The term "European" indicates that the holder can exercise his/her right only at maturity itself, while "at-the-money" means that the strike is set equal to the price of the forward at the settlement date.

In its general form, a call option c as a derivative can be valued using the BSM differential equation, as presented in (3.19), with boundary condition as in (3.20):

$$\left(\frac{\partial c}{\partial t}+r\cdot S\frac{\partial c}{\partial S}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 c}{\partial S^2}\right)=r\cdot c \tag{3.19}$$

$$c=\max(S_T-X;0) \tag{3.20}$$

One important feature of the BSM is that it does not include the drift term μ, the expected return on the underlying, which would be dependent of the risk preferences. Therefore the

BSM should to hold for all sets of risk preferences, and we can simplify the analysis a lot by assuming that all investors are risk neutral. However, the closed-form solutions based on this assumption do hold not only in the risk-neutral world, but also in the risk-averse one.

For a European call option on an underlying spot price process which meets the conditions set for the BSM solution (3.18), Black and Scholes (1973) offered their famous closed-form solution (the B-S formula):

$$c_{0,T} = S_0 N(d_1) - X e^{-rT} N(d_2) = F_{0,T} e^{-rT} N(d_1) - X e^{-rT} N(d_2) \tag{3.21}$$

$$d_1 = \frac{\ln\left(S_0 / X\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(F_{0,T} / X\right) + \left(\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} ; \quad d_2 = d_1 - \sigma\sqrt{T}$$

where $c_{0,T}$ is the value of the option at time 0, and $N(x)$ is the cumulative probability distribution function for a variable that is normally distributed with a mean of zero and a standard deviation of one.

Equation (3.13) showed how a forward price process that follows the GBM can be modelled from starting point of time 0 to the maturity T. However, we see that the expressions contain the trend λ, the risk premium which of course depends on the risk preferences. Øksendal (2003) refers to the Girsanov's Theorem for solution. Simply put, the Theorem says that by changing between equivalent probability measures, we can change the drift of an Itô process into anything we like, but we cannot change to volatility. So we can go from the original process in (3.3) over to a new process below, introducing the new Wiener process $d\tilde{z}$ with new risk-neutral probabilities Q instead of the original real-world probabilities P, but keeping the volatility[16]:

$$dS = (r - \delta)S dt + \sigma S dz = (r - \delta)S dt + \sigma S \varepsilon \sqrt{dt} \tag{3.22}$$

For a forward process the expression (3.13) then becomes as shown below, including a dividend yield term. One can now use the expression (3.23) in a risk-neutral simulation.

$$F_{t+\Delta t} = F_t e^{\left(-\delta - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}} \tag{3.23}$$

---

[16] Here a convenience yield is introduced as well.

3.3 Option Pricing Solutions for Certain Situations

When the Underlying Pays Dividends or Carries Convenience Yield

If the underlying asset pays constant continuous proportional dividend q (investment asset) or carries convenience yield δ (consumption assets such as electricity), or exhibits some other form of return shortfall (e.g. unhedged currency exposure), the last of the standard BSM assumptions has to be relaxed. Generating δ as dividend or convenience yield, the underlying has to grow with only $\mu = r - \delta$. Such process can be presented as follows:

$$dS = (r - \delta)Sdt + \sigma Sdz \tag{3.24}$$

The change in the hedged portfolio we constructed to arrive at the BMS in (3.16) now has to be adjusted for the dividend/yield that the holder earns on the position in dG/dS underlying. The BSM expression (3.18) can then be re-written as follows:

$$dW = \left(-\frac{\partial G}{\partial t} - \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2 + \delta S \frac{\partial G}{\partial S} \right)dt \tag{3.25}$$

$$\left(\frac{\partial G}{\partial t} + (r - \delta)\cdot S\frac{\partial G}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 G}{\partial S^2} \right) = r \cdot G \tag{3.26}$$

Here G is the derivative, *e.g.* an option c. The expression (3.26) still does not have any variable affected by the risk preferences. Therefore, under risk-neutrality assumption one still may argue that the total return has to be equal the risk-free rate r.

Merton (1973) showed that for European call options on dividend-paying assets the B-S formula (3.21) can be modified into the following:

$$c_{0,T} = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2) \tag{3.27}$$

$$d_1 = \frac{\ln\left(S_0 / X \right) + \left(r - \delta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

When the Underlying Is a Forward Contract

If the underlying investment asset is a forward contract F on another asset S with a delivery price K at maturity T, such contract can be priced under the risk neutrality as follows:

$$\hat{E}(S_T) = S_0 e^{rT} \tag{3.28}$$

$$F = \hat{E}(S_T - K)e^{-rT} = \hat{E}(S_T)e^{-rT} - Ke^{-rT} = S_0 e^{rT}e^{-rT} - Ke^{-rT} = S_0 - Ke^{-rT} \tag{3.29}$$

For the underlying forward the BSM equation (3.18) takes the form of (3.30). Term G in the equation represents the derivative function, *e.g.* a call option $c = \max(S_T - X, 0)$.

$$\left(\frac{\partial G}{\partial t} + \frac{1}{2}\sigma^2 F^2 \frac{\partial^2 G}{\partial F^2} \right) = r \cdot G \tag{3.30}$$

If we compare the equation above with that in (3.26), we see that the forward prices development can be equated with the development of a stock paying a dividend of $\delta = r$. Therefore the expected growth of the forward prices in a risk-neutral world is $r - \delta = 0$ and no drift term should be included in this differential equation. This applies to both deterministic and stochastic interest rates. Thus:

$$F_0 = \hat{E}(\tilde{F}_T) \tag{3.31}$$

As mentioned in subsection 2.2 there is uncertainty about whether an expected growth of the spot electricity prices should automatically apply to forwards. The expression (3.31) now supports this theoretically.

Black (1976) proposed a closed-form solution for European call options on forwards, which is a version of the B-S formula. This solution is popularly called "Black 76", and its assumptions are the same as for the BSM differential equation listed above.

$$c_{0,T,M} = e^{-rT}\left[F_{0,M} N(d_1) - X \cdot N(d_2) \right] \tag{3.32}$$

$$d_1 = \frac{\ln\left(F_{0,M} / X \right) + \left(\frac{1}{2}\sigma^2 \right)T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

Here $F_{0,M}$ is the forward price at time 0 with delivery at time M, X – the option's strike, T – time to the option's maturity ($T < M$), and σ – cumulative volatility of the forward. In its strict interpretation, Black's forward price $F_{0,M}$ should be derived from the spot price S_0 as $F(t,T) = S(t)e^{(r+w-\delta)(T-t)}$ (where w is a cost of storage and δ – convenience yield). In practice, it can be replaced by the market forward price in effect making the convenience yield a general function of time.

The risk-neutral forward price process is described in expression (3.23). It can be shown that the volatilities of the forward prices are constant and equal to the spot volatility:

$$\sigma_F(t,T) = \sigma \tag{3.33}$$

3.4 Conceptual Challenges in Pricing the Return Element

In the previous two subsections the general theory of price movement and derivative pricing was recapped, including special cases for underlying being a forward or paying a dividend yield. This subsection will be devoted to more stringent quantitative analyses of the NKIB issues related to averaging and currency exposure that was briefly noted in subsection 2.3. From the equations (3.27) and (3.32) it is easy to see that the value of an option is directly linked to (a) any dividend yield that may "bleed" the total return from holding the underlying, and (b) the volatility of the underlying. Anything that increases the yield and/or reduces the volatility will be to the investor's disadvantage.

Quotation of Forward Contracts in Foreign Currency

All the general option pricing theory presented in the pervious subsections was built for single-currency situations. We saw that for the derivatives on forward contracts no drift term and no cost of carry ($r - \delta$) should be included in the single-currency differential equation. However, from the component analysis it was clear that while the borrowing and payout on the NKIB-XV happens in NOK, the underlying forward contracts are quoted in EUR.

If an index bond's *RE* is built on an underlying quoted in a foreign currency, the treatment of the "currency issue" in the bond structure can be threefold. Let us denote currency rates (NOK/EUR) as X_{t0} and X_{ti} in times t_0 and t_i (the last one is uncertain at the settlement date):

$$
\text{(A)} \qquad \text{(B)} \qquad \text{(C)}
$$

$$
\max\!\left(\frac{\widetilde{F}_{t_i}^{i}\widetilde{X}_{t_i}-F_{t_0}^{i}X_{t_0}}{F_{t_0}^{i}X_{t_0}};0\right) \qquad \max\!\left(\frac{\widetilde{F}_{t_i}^{i}\widetilde{X}_{t_i}-F_{t_0}^{i}\widetilde{X}_{t_i}}{F_{t_0}^{i}X_{t_0}};0\right) \qquad \max\!\left(\frac{\widetilde{F}_{t_i}^{i}-F_{t_0}^{i}}{F_{t_0}^{i}};0\right) \qquad (3.34)
$$

In the case (A) the issuer offers full adjustment for the changes in the exchange rates from t_0 to t_i, meaning that the Return component is fully exposed to the expected currency appreciation/depreciation. In the case (C), as in the NKIB-XV or in DnB NOR's Kraft warrants, no adjustment of any kind happens, meaning that the return is effectively hedged at the settlement (the issuer has in fact hedge its currency exposure). The case (B) is a mixture which was built into the NKIB-XIII.

Bjerksund, Carlsen and Stensland (1999) offer quantitative explanation for a situation such as in cases (A) and (C). The only difference is that they analyse options on foreign indices, while here we have options on "foreign" forwards. This fact is very important for answering whether one should include interest rate differential as implied dividend yield for forward price process.

The starting point is substitution of the B-S formula into the *RE* of an index bond with one option maturing at T on one underlying foreign index q^{*} paying a dividend yield δ^{*}:

$$
V_0[AA_T] = RF\cdot\frac{FV}{q_0^{*}}\cdot c_{0,T} = RF\cdot\frac{FV}{q_0^{*}}\cdot\left[q_0^{*}e^{-\delta T}N(d_1)-q_0^{*}e^{-rT}N(d_2)\right] =
$$
$$
= RF\cdot FV\cdot\left[e^{-\delta T}N(d_1)-e^{-rT}N(d_2)\right] \qquad (3.35)
$$

For the case (A) with full exchange rate adjustment, Bjerksund, Carlsen and Stensland (1999) show that the implied dividend yield and the volatility will be:

$$
\delta = \delta^{*} \quad \textit{(as before)} \qquad (3.36)
$$

$$
\sigma^2 T \equiv Var_0\!\left(\ln\!\left(\frac{\widetilde{q}_T^{*}\widetilde{X}_T}{q_0^{*}X_0}\right)\right) = Var_0\!\left(\ln\!\left(\frac{\widetilde{q}_T^{*}}{q_0^{*}}\right)+\ln\!\left(\frac{\widetilde{X}_T}{X_0}\right)\right) \qquad (3.37)
$$

For the case (C) with no exchange rate adjustment, it can by analogy be shown that the yield and the volatility take a form:

$$
\delta \equiv \delta^{*}+\left(r-r^{*}\right)+\frac{1}{T}Cov\!\left(\ln\!\left(\frac{\widetilde{q}_T^{*}}{q_0^{*}}\right),\ln\!\left(\frac{\widetilde{X}_T}{X_0}\right)\right) \qquad (3.38)
$$

$$\sigma^2 T \equiv Var_0\left(\ln\left(\frac{\tilde{q}_T^*}{q_0^*}\right)\right) \quad (\textit{as before}) \tag{3.39}$$

It is now evident that by not adjusting for the currency changes, the bank both increases the implied dividend yield, while not introducing "new" volatility element. Norway usually has a positive rate difference with the Euro-area[17], therefore the underlying quoted in EUR will probably not be to the Norwegian investor's advantage.

How it affects the valuation of the *RE* in the NKIB-XV where the underlying are foreign currency-quoted forwards? Assuming no correlation between the forward electricity prices and the exchange rates ($Cov^{F,X} = 0$), the Cost of Carry goes from $(r - \delta)$[18] which was zero in the standard Black 76 to $(r - \delta^* - (r - r^*))$ for "foreign" forwards where there is no adjustment for currency appreciation in the calculation of the *AA*. Since $\delta^* = r^*$ for forwards, the "new" Cost of Carry is again zero.

Bjerksund (2008) showed that for the NKIB-XIII which did have exchange rate adjustment mechanism similar to the case (B) as presented in (3.40), the value of an option may be calculated by (3.44) below. Here we assume F_i price dynamics as in (3.41) and F_i return as in (3.42) – see also (3.23) above – and X exchange rate dynamics as in and (3.43), F_i and X independent of each other:

$$\max\{\tilde{R}^i; 0\} = \max\left\{\frac{\tilde{F}_{t_i}^i - F_{t_0}}{F_{t_0}}; 0\right\} \cdot \frac{X_{t_1}}{X_{t_0}} \tag{3.40}$$

$$\tilde{F}_{t_i}^i = F_{t_0}^i e^{\left(\lambda - \frac{1}{2}\sigma^2\right)(t_i - t_0) + \sigma\sqrt{t_i - t_0}\tilde{\varepsilon}_i} = F_{t_0}^i e^{\frac{1}{2}\sigma^2(t_i - t_0) + \sigma\sqrt{t_i - t_0}\tilde{\varepsilon}_i} \quad \text{[19]} \tag{3.41}$$

$$\tilde{R}^i = \frac{\tilde{F}_{t_i}^i - F_{t_0}}{F_{t_0}} = e^{\left(\lambda - \frac{1}{2}\sigma^2\right)(t_i - t_0) + \sigma\sqrt{t_i - t_0}\tilde{\varepsilon}_i} - 1 = e^{\frac{1}{2}\sigma^2(t_i - t_0) + \sigma\sqrt{t_i - t_0}\tilde{\varepsilon}_i} - 1 \quad \text{[15]} \tag{3.42}$$

---

[17] On 15.02.2008 the interest difference on 3-years government bond's yield was: $r^{NOK} - r^{EUR} = 4,49\% - 3,22\% = 1,27\%$.

[18] Here the storage cost term w is of course zero for the forwards.

[19] Forward price risk premium $\lambda = (\mu - r)$ equal to zero under the risk-neutral Q.

$$\widetilde{X}_{t_1} = X_{t_0} e^{\left(\lambda - \frac{1}{2}s^2\right)(t_i - t_0) + s\sqrt{t_i - t_0}\widetilde{\xi}} = X_{t_0} e^{\left(\left(r - r^*\right) - \frac{1}{2}s^2\right)(t_i - t_0) + s\sqrt{t_i - t_0}\widetilde{\xi}} \qquad (3.43)$$

$$
\begin{aligned}
c_i &= E_{t_0}\left[e^{-r(T-t_0)} \max\left\{\frac{\widetilde{F}_{t_i}^i - F_{t_0}}{F_{t_0}}; 0\right\} \cdot \frac{\widetilde{X}_{t_1}}{X_{t_0}}\right] = \\
&= E_{t_0}\left[\frac{\widetilde{X}_{t_1}}{X_{t_0}}\right] + E_{t_0}\left[e^{-r(T-t_0)} \max\left\{\frac{\widetilde{F}_{t_i}^i - F_{t_0}}{F_{t_0}}; 0\right\}\right] = \\
&= e^{(r-r^*)(t_i - t_0)} \cdot e^{-r(T-t_i)} \cdot E_{t_0}\left[e^{-r(t_i - t_0)} \max\left\{\frac{\widetilde{F}_{t_i}^i - F_{t_0}}{F_{t_0}}; 0\right\}\right]
\end{aligned}
\qquad (3.44)
$$

We see that the expected appreciation term $e^{(r-r^*)t}$ comes from the currency cross adjustment included in the NKIB-XIII. Since the NKIB-XV does not have such an adjustment, the reasoning above about zero Cost of Carry seems to be justified. The conclusion is confirmed by DnB NOR in their offer document for Warrant Kraft 2007/2010.

Thus we can substitute Black 76 as in (3.32) into the NKIB-XV Return element expression as in (2.3), keeping in mind that the strikes X^i are equal to the forward prices $F^i_{0,M}$ at time 0.

$$
\begin{aligned}
RE_0 &= V_0\left[AA_T\right] = RF \cdot \sum_{i=1}^{N} w_i \frac{FV}{F_{t_0}^i} e^{-r(T-t_i)} c_i = \\
&= RF \cdot \sum_{i=1}^{N} w_i \cdot FV \, e^{-r(T-t_i)} \cdot e^{-rt_i} \cdot \left[N\left(d_1^i\right) - N\left(d_2^i\right)\right]
\end{aligned}
\qquad (3.45)
$$

$$d_1^i = \frac{\left(\frac{1}{2}\sigma_i^2\right)t_i}{\sigma_i\sqrt{t_i}} = \frac{1}{2}\sigma_i\sqrt{t_i}; \quad d_2^i = d_1^i - \sigma_i\sqrt{t_i}$$

where $e^{-r(T-t)}$ takes into account the delays of the option proceeds until the bond's maturity T.

Averaging Over Three Forward Contracts

The *AA* is calculated based on the equally weighted arithmetic average of the positive returns on three forwards contracts, ENOYR-09, ENOYR-10 and ENOYR-11, as shown in (2.3). The options' lives are, respectively, 0,8 years, 1,8 years and 2,8 years. The forward contracts expire some 2-3 weeks after the respective options on them expire.

What does such diversification mean for the value of the *RE*? Since this is an average of options, and not an option on an average, based on the Value Additivity Principal, the

investor receives the full 1/3 value of each call option in the portfolio, with unchanged volatilities of each option.

As Bjerksund (2007) showed, it would be much more unfortunate for an investor if the *AA* would be based on one basket option, as illustrated below:

$$c = \sum_{i=1}^{N} c_i \cdot w_i = \sum_{i=1}^{N} \max\left(\frac{\tilde{F}_{t_i}^i - F_{t_0}^i}{F_{t_0}^i} \cdot w_i ; 0 \right) \quad \text{Vs.} \quad c = \max\left[\sum_{i=1}^{N} \left(\frac{\tilde{F}_{t_i}^i - F_{t_0}^i}{F_{t_0}^i} ; 0 \right) \right] \cdot 1 \qquad (3.46)$$

To explain this, let's assume that an option is written on the return on a basket index *I*, which is an average of returns on the forwards F_1, F_2 and F_3. The total return is then:

$$\frac{\tilde{I}_T - I_0}{I_0} = \frac{1}{3}\left(\frac{\tilde{F}_T^1 - F_0^1}{F_0^1} \right) + \frac{1}{3}\left(\frac{\tilde{F}_T^2 - F_0^2}{F_0^2} \right) + \frac{1}{3}\left(\frac{\tilde{F}_T^3 - F_0^3}{F_0^3} \right) = \frac{1}{3}\frac{\tilde{F}_T^1}{F_0^1} + \frac{1}{3}\frac{\tilde{F}_T^2}{F_0^2} + \frac{1}{3}\frac{\tilde{F}_T^3}{F_0^3} - 1 \qquad (3.47)$$

The volatility var_I or σ_I^2 of the index *I* can then be estimated as:

$$\sigma_I^2 T \equiv \text{var}\left[\ln\left(\frac{\tilde{I}_T}{I_0} \right) \right] \approx \text{var}\left[\frac{1}{3}\ln\left(\frac{\tilde{F}_T^1}{F_0^1} \right) + \frac{1}{3}\ln\left(\frac{\tilde{F}_T^2}{F_0^2} \right) + \frac{1}{3}\ln\left(\frac{\tilde{F}_T^3}{F_0^3} \right) \right] = \left(\frac{1}{3} \right)^2 \sum_{i=1}^{3}\sum_{j=1}^{3} \sigma_i \sigma_j \rho_{ij} T \qquad (3.48)$$

$$\sigma_I^2 = \left(\frac{1}{3} \right)^2 \sum_{i=1}^{3}\sum_{j=1}^{3} \sigma_i \sigma_j \rho_{ij} \qquad \text{where} \qquad \rho_{ij} = \frac{Cov_0\left(\ln\left(\frac{F_T^i}{F_0^i} \right), \ln\left(\frac{F_T^j}{F_0^j} \right) \right)}{\sigma_i \sigma_j T} \qquad (3.49)$$

Clearly, as long as some correlation coefficients ρ_{ij} are different from 1,0, the resulting index volatility will be reduced. One can still value such basket options using Black 76, but then one has to use the implicit volatility above. For the adjustment of the dividend yield, see Bjerksund, Carlsen and Stensland (1999).

Average Return on Three Forward Contracts

Another issue that may impact the average return is that the underlying forward contracts are highly correlated while they run in parallel, with coefficients between 0,8 and 0,9. It is observable from the historical development, as presented in Appendix B. I have also calculated correlation coefficients for elapsed forward contracts, the results are shown in the table below:

Table 3: Historical correlation between forward contracts

	YR-03	YR-04	YR-05	YR-06	YR-07	YR-08	YR-09	YR-10
FWYR-02	0,975	0,922						
FWYR-03		0,809	0,457					
FWYR-04			0,802	0,718				
FWYR-05				0,751	0,727			
ENOYR-06					0,933	0,862		
ENOYR-07						0,871	0,814	
ENOYR-08							0,929	0,798

A closed-form solution such as Black 76 applied to each of the three options in the Return element, will not be able to take this correlation into account. However, Bjerksund (2008) showed how it can be implemented with Monte Carlo simulation, applying the same Wiener process *dz* for all contracts, but with their respective spot volatilities, while they run in parallel. This approach is implemented in the Monte Carlo simulation used in this thesis.

Averaging Over Five Trading Days

Another challenge which is relevant for valuing the NKIB-XV is that the closing values, F^i_t, are based on an average over five trading days prior and on 10.12. of each year. Such options are called "Asian" or options with an "Asian" tail. Since returns in those five days are not perfectly correlated, the resulting volatility of the average will be less (probably much less) that the average volatility of these five days. However, since the five observations have only one trading day between them, and not one or several months as many other structured products, the volatility-reducing effect is not as strong as for other Asian options.

Let's assume again that an option is written on a forward F, but that the closing value index I_T is actually an average of M observations of F over the period of time from T_1 to T_2, see (3.47). The time between observations, if evenly spaced, is $\Delta t = (T_2 - T_1)/M$ (note that the first observation is not T_1, but $T_1 + \Delta t$). The total return on such Asian option is shown in (3.51).

$$\widetilde{I}_T = \frac{1}{M} \sum_{k=1}^{M} \widetilde{F}_{T_1 + k \cdot \Delta t} \qquad (3.50)$$

$$\frac{\widetilde{I}_T - F_0}{F_0} = \frac{1}{M} \sum_{k=1}^{M} \frac{\widetilde{F}_{T_1 + k\Delta t} - F_0}{F_0} \qquad (3.51)$$

Kemna and Vorst (1990) showed that we can adjust volatility σ^2_I of the index I according to the expression below to be able to value the option with the Black 76 formula. In this equation the volatility of a geometric average is used, however, it is a good approximation for the volatility of an arithmetic one.

$$\sigma^2_I T_2 = \sigma^2 \left[T_1 + \frac{1}{6} \frac{(T_2 - T_1 + \Delta t)(2(T_2 - T_1) + \Delta t)}{T_2 - T_1} \right] \qquad (3.52)$$

If one puts $T_1 = 0$, $T_2 = T$, and $\Delta t = 1/3T$ into the equation (3.49), the result, $\sigma^2_I T = 14/27\sigma^2 T$, corresponds to the results presented in Bjerksund (2007). In general, if we assume $T_2 = T$ (final day of the calculation period), n – number of consecutive days in the Asian tail so that $T_1 = T - n/251$, and $\Delta t = 1/251$, the volatility adjustment can be calculated as:

$$\sigma_I = \sigma \sqrt{1 - \frac{4n^2 - 3n - 1}{1506n} \frac{1}{T_2}} \qquad (3.53)$$

In the case of the NKIB-XV, with $n = 5$, T_2 – between 0,8 and 2,8 years and underlying cumulative volatilities σ between 19% and 27%, the volatility reductions are approx. 0,04% to 0,18%. The details are shown in the following table:

Table 4: Estimated volatility adjustments for the averaging effect

Remaining life T_2 (years)	Underlying volatility σ (% p.a.)	Index volatility σ_I (% p.a.)
0,821	26,6%	26,4%
1,817	21,6%	21,5%
2,810	19,0%	19,0%

3.5 Alternatives for Valuing the Return Element

There are in principle two ways to value European call options c_i in the RE-equation (2.3): (a) an indirect approach – *i.e.* to use reliable external valuations of similar options (or the

Return element as a whole), and/or (b) a direct approach – *i.e.* independently chose and model valuation solution(s), make necessary assumptions, and calculate the results.

Indirect Valuation – Use of Quotes for Traded Options

Since Nordea actually buys options with matching underlying and maturities on the settlement date to hedge its market exposure on the NKIB[20], the most effective and reliable way to value the *RE* would be to use the market quotes for these options. Options on Nord Pool electricity forwards are traded on Nord Pool and in the OTC market.

Before using any market quotes, one should confirm whether these quotes come from a sufficiently efficient and liquid market. According to Prof. Bjerksund, Put-Call Parity and Concavity-in-Strikes tests on the option quotes did not indicate that the options quoted at Nord Pool are obviously mispriced.

Options on the nearest two yearly contracts were quoted at Nord Pool on 15.02.2008: ENOC*kk*YR-09 and ENOC*kk*YR-10 (where *kk* is the strike). To be able to value the entire *RE* a retail investor will have to obtain "sharp" quotes for the last contract, ENOYR-11, from the OTC electricity brokers. One would still have to adjust for five-days averaging effect, covariation of the contracts, somewhat different maturities, EUR interest rate and different credit risk.

Due to the scope constrain, no option quote for ENOYR-11 contract has been obtained. Therefore, the indirect approach cannot be used. However, the available market quotes for ENOYR-09 and ENOYR-10 will be used in reasonability analysis of the results from the direct approach. They will also be used to estimate implied cumulated volatilities on the first two contracts.

Direct Valuation – Application of Close-form Solutions

For valuation of European call options on forwards, the Black 76 solution adjusted for delayed payment has been presented in equation (3.45). This solution is appropriate if the assumptions behind the BSM-equation as met, particularly the assumption about the GBM-type underlying process.

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[20] That is how the final Return Factor is set.

There is a simpler form of the B-S formula for at-the-money options called "The 0,4-rule":

$$c_i = e^{-rt_i} \cdot 0{,}4 \cdot \sigma_i \sqrt{t_i} \qquad\qquad (3.54)$$

The derivation of the formula is outside the scope of this analysis. An investor could use this formula to make a quick "back-of-the-envelope" estimate for the *RE*, however one does not need it with Microsoft Excel spreadsheet available. The results calculated by the 0,4-rule are used as a calculation error control to a more complex Black 76 calculation.

In the next section closed-form solutions for other underlying price processes are presented. As a rule, they are more complicated to model and require several input parameters that have to be estimated form the historical data.

Direct Valuation – Application of Numerical Techniques

Valuation procedures by numerical techniques are more transparent in that the techniques primarily concern themselves with modelling underlying price process as realistic as possible. The application of the numerical techniques does not therefore depend on the particular underlying process, such as the GBM for the B-S closed-form solution. If the price at time *t* does not depend on the previous values (*e.g.* random walk GBM) and the derivative does not require several measurement points (e.g. plain-vanilla European option), one does not have to simulate a complete time series, but just the value at maturity.

The numerical techniques are more flexible than the analytical closed-form solutions in that they can accommodate exotic structures with complicated underlying processes with several stochastic factors, several correlated underlying, Asian tails etc. For such exotic derivatives only analytical approximation may exist, if any.

Numerical techniques consist mainly of Tree Building Procedures and Monte Carlo (or Quasi-Monte Carlo) simulations. Their comparative description and application for the *RE* analysis are presented in section five.

If the GBM assumption for forward price process is acceptable and the five-days averaging and correlation issue in the NKIB-XV are not expected to materially disturb the results, then the options in the *RE* can be easily valued by Black 76 solution. In this situation Monte Carlo will not be absolutely necessary for an investor. However, the Monte Carlo simulation is the only option to perform the expected return analysis.

3.6 Alternatives for Expected Return Analysis

The forward price GBM process in its general form with a drift term $\lambda = \mu - r$ is presented in equation (3.13). For a risk-neutral valuation, we have substituted the drift of the underlying spot process with the risk-free rate, leaving its risk premium and the drift of the forward price equal to zero.

The expected return calculation is based on the real-world probabilities P, and not on the risk-neutral probabilities Q. Therefore we have to abandon the risk-neutral world of derivative valuation and the risk premium has to be re-introduced and estimated.

The only alternative to perform such expected return calculation is a numerical procedure simulating the development of the forward with implied risk premium.

4. Seasonality, Volatility and Covariation

4.1 Seasonality

Seasonal patterns in their broader meaning (intra-year, intra-week, intra-day predictability) are one of the particular characteristics of the electricity spot prices. However, the seasonality should not be a major concern for forward price process. Hjalmarsson (2003) argues that the forward prices do not exhibit strong seasonality, the reason being that they specify the average price for electricity for a fixed future period of time. In this analysis I therefore treats the underlying forward process as if it has no material seasonal component.

4.2 Volatility

Volatility Definition and Models

The concept of volatility of the underlying process, both volatility structure and volatility level, is paramount to all option pricing. It is even more so for pricing electricity-related options, as in the NKIB-XV, where volatility estimation is fairly complex.

According to Hull (2000), volatility of a price process can be defined as the annualised standard deviation of the return on the underlying, expressed using continuous compounding. In expression (3.12) it is shown the volatility is also the standard deviation of the natural logarithm of the return at the end of the year.

If the volatility of a GBM process F_i over a time interval of T is assumed to be non-autoregressive, it is empirically calculated from $n - 1$ observations as follows[21]:

$$u_i = \ln\left(\frac{F_i}{F_{i-1}}\right) = \left(\mu - r - \frac{\sigma_i^2}{2}\right)dt + \sigma_i\sqrt{dt}\,\varepsilon_i \qquad (4.1)$$

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i \qquad (4.2)$$

---

[21] Volatility is calculated a bit different for Value-At-Risk (VAR) purposes.

$$s_u = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2} \qquad (4.3)$$

$$\sigma_u = \frac{s_u}{\sqrt{T}} \quad \text{(with a standard error} = \frac{\sigma_u}{\sqrt{2n}}\text{)} \quad \text{and} \quad Var(u) = \sigma_u^2 \qquad (4.4)$$

This way of calculating return as a natural logarithm of the last price divided by the previous price was used by Koekebakker and Ollmar (2001). Another alternative is to divide the price change by the previous price.

Engle (1982) suggested a way to weigh returns observations differently within the sample, giving more recent observations more weight or introducing and giving weight to a known long-run average volatility V. This is known as an Autoregressive Conditional Heteroscedasticity model or ARCH(n)[22]:

$$\sigma^2 = \gamma \cdot V + \sum_{i=1}^{n}\alpha_i u_i^2 = \varpi + \sum_{i=1}^{n}\alpha_i u_i^2 \qquad (4.5)$$

where α_i – weights given to single observations, and γ – the weight given to the long-run average volatility V ($\omega = \gamma * V$). The weights have to sum up to one: $\gamma + \sum_{i=1}^{n}\alpha_i = 1$.

A particular case of the weighting approach is the Exponentially Weighted Moving Average (EWMA) model , where weights α_i decrease exponentially as one moves back through time: $\alpha_{i+1} = \lambda\,\alpha_i$ (λ is a constant between zero and one). The parameter λ governs how responsive the current estimate to the most recent observation. With high λ the volatility estimate responses slow to new information.

$$\sigma_n^2 = (1 - \lambda)u_{n-1}^2 + \lambda\sigma_{n-1}^2 \qquad (4.6)$$

The EWMA approach was used by J.P. Morgan to update daily volatilities in their RiskMetrics database.

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[22] The ARCH(n) model was first proposed for inflation and later extended to other areas.

Bollerslev (1986) proposed another model with generalised autoregressive conditional heteroscedasticity or GARCH(1,1). The GARCH introduces a long-run average variance V into the EWMA-framework:

$$\sigma_n^2 = \gamma \cdot V + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 = \varpi + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad [23]$$

(4.7)

Thus the EWMA is a particular case of the GARCH(1,1), where $\gamma = 0$, $\alpha = 1 - \lambda$ and $\beta = \lambda$. The connotation "(1,1)" means that the most recent observations of u_i and σ_i is used, a more general form is a GARCH(p,q).

Deterministic Volatilities

Price and return processes are either homoskedastic, meaning having a constant volatility, or heteroskedastic, where volatility changes deterministically or stochastically. For a contract with an expiry date, the deterministic future volatility can be assumed to have a predictable term structure.

The volatility term structure is the relationship between the future volatility of the underlying and their time to maturity. In Appendix C the historical weekly returns on elapsed forward contracts are plotted along the time axis – it is clear that they start to "fan out" as contracts approach maturity. I Appendix F 26-weeks rolling average volatility is shown: again, one can see that volatility level picks up sharply as contracts pass half way through their lives. The same picture is seen in Appendix G, where 60-days rolling average volatility is plotted against time to maturity. Finally, in Appendix H, the historical cumulative volatility to maturity is presented – the same volatility term structure phenomenon is observed. From these observations it is clear that the forward contracts exhibit a term structure. The time-varying volatility assumption is supported by other researchers, *e.g.* Koekebakker and Ollmar (2001). Since simple closed-form option pricing solutions such as Black 76 assume deterministic constant volatility, one way to incorporate the term structure is to use cumulative volatility until maturity for each option. The cumulative volatility represents all volatilities within the interval it "coves" or "accumulates over", given there is no autoregressiveness, as in Pilipović (1998):

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[23] Again, the weights have to sum up to one: $\gamma + \alpha + \beta = 1$.

$$\sigma_{0,t_N} = \sqrt{\frac{\sum_{n=0}^{N-1} \sigma_{t_n t_{n+1}}^2 (t_{n+1} - t_n)}{t_N}} \qquad (4.8)$$

There have been proposed several analytical models for the volatility term structure. For a process that mean-reverts in price or return, a volatility approaching zero as maturity increases was used for Nord Pool forwards by *inter alia* Lucia and Schwartz (2000) and Clwelow and Strickland (2000):

$$\sigma(t,T) = \sigma e^{-\kappa(T-t)} \qquad (4.9)$$

where k is the mean-reversion factor and σ is constant. As maturity T increases, so the volatility approaches zero.

Another deterministic volatility model was proposed by Bjerksund *et al.* (2000). In order to keep the long-term at a realistic level, they proposed an empirical, data-fitted model:

$$\sigma(t,T) = \frac{a}{(T - t + b)} + c \qquad (4.10)$$

where a, b and c are constants, and as T increases, the volatility converges to c.

Skogen and Bjørdal (2002) tested both volatility models above and found that the empirical model produced the best results.

Koekebakker and Ollmar (2001) and (2005) proposed using multiple volatility fractions for their multi-factor GBM forward price model.

In this analysis the models (both the closed-form Black 76 and the Monte Carlo simulation) are built under the assumption of a homoskedastic underlying process. This means that there is no time-dependent formula for calculation volatilities used in the calculations. The observed term structure is attempted incorporated through using accumulated-until-maturity rather than instantaneous volatility measures.

Stochastic Volatilities

In order to accommodate the observed fat-tail and spike behaviour of the electricity spot and, to a less degree, forward prices, there have been proposed several models with stochastic or stochastic and autoregressive volatility.

One type is exemplified by a model proposed by Derman and Kani (1994) and Dupire (1994):

$$dS = \mu S dt + \sigma(t, S) dz \tag{4.11}$$

where $\sigma(t,S)$ is a general non-linear function of the spot price. One simple example of such function would be $\sigma(t,S) = \beta_1 S^{\beta 2}$, where $\beta_1 > 1$ and $\beta_2 > 1$ are constants. This is also known as the Constant Elasticity of Variance Model or CEVM, see Cox and Ross (1976).

Another type of model was proposed by Hull and White (1988), subsequently extended by *e.g.* Heston (1993), Bates (1996) and Scott (1997). Here, a GBM process for the volatility is introduced (correlated or independent of the price GBM process). The volatility variance $V = \sigma^2$ mean-reverts toward $\overline{V}$ at a rate a and has a random driver dw:

$$dS = \mu S dt + \sigma S dz \tag{4.12}$$

$$V = \sigma^2, \rightarrow dV = a(\overline{V} - V)dt + \xi \sqrt{V} dw \tag{4.13}$$

If the variance is both stochastic and autoregressive, exhibiting mean-reversion as it often does, the GARCH(1,1) model described above will be able to incorporate this, while the EWMA model will not. Thus the GARCH(1,1) is more theoretically appealing to use, unless the long-run term ω turns out to be negative making the GARCH model unstable.

Practical Estimation of Volatility – Historical Volatility

The purpose of any volatility estimation procedure is to predict as realistically as possible the behaviour and level of future volatility for a given period of time or for a given time to maturity. The estimation basis can be either (a) historical time series (historical volatility) or (b) market prices for derivative instruments such as options where volatility is one on the input parameters and where pricing formula is assumed to be known, *e.g.* Black 76 (implied

volatility). As long as one can trust that the model used to back out the implied volatility is "correct", the implied volatility procedure is to be preferred.

In this analysis homoskedastic historical volatility is estimated for the historical return time series in a manner described in equations (4.1) through (4.4). An often used rule of thumb is to set the measurement interval T equal to the time period over which the volatility will be applied.

Should one assume more advanced volatility structures, *i.e.* a time-varying deterministic model (e.g. the Bjerksund *et al.* empirical model), or a stochastic volatility model with or without some form of autoregression (ARCH, EWMA or GARCH), several more parameters would have to be estimated form the historical series. The stability of these parameters may be uncertain. As mentioned, in this thesis no such volatility models were applied.

More sophisticated techniques for measuring historical volatility make use of intra-day high-low as well as daily closing quotes.

Practical Estimation of Volatility – Implied Volatility

Another, market-based and therefore more preferable way to estimate expected volatilities is to back them out of *e.g.* the Black 76 model given market option quotes. Black 76, as any known algorithm, works well for quoting options in terms of "implied" volatility. The term "implied" here only means that if everybody knows the formula and there is no uncertainty about other input factors than volatility, then everybody can convert option premium in "EUR/MWh" into a measure of volatility in "% p.a.". However, if Black 76 is in fact not used by the market to price the options, then the volatility level backed out of the formula will be different from the actual volatility implied in the market quotes.

Due to the scope constraints, only standard Black 76 is used to estimate implied volatility.

An interesting phenomena that illustrates the point made above is so-called "volatility smile" or "smirk". When volatilities implied in the standard B-S (or Black 76) formulas are plotted against the strike value, one would expect no relationship for a log-normal underlying process. In fact the implied volatility increases as options goes "out-of-the-money". I illustrate this in the graph below for Nord Pool options on ENOYR-09 and ENOYR-10 quoted on 15.02.2008.

Figure 2: Volatility "smirk" for options on ENOYR-09 and -10 on 15.02.2008
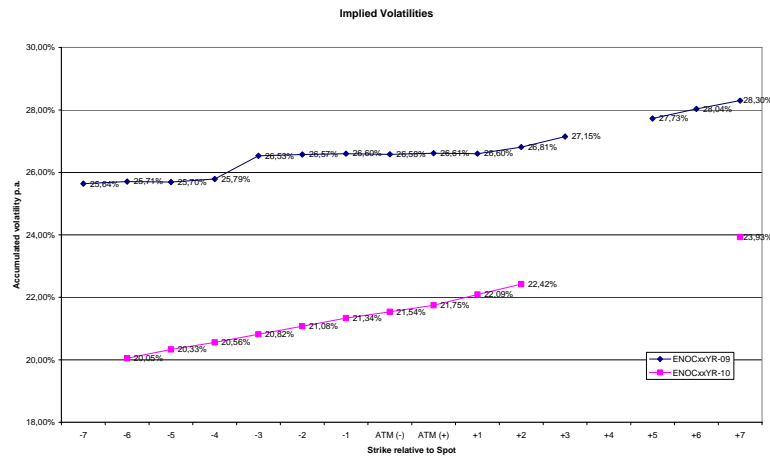
Source: Nord Pool, analysis by Andreev (standard Black 76, EUR continuous risk-free rate 3,47% and 3,21%)

As the options built into the *RE* of the NKIB-XV are per design at-the-money, closing market quotes for at-the-money options on the settlement date are used.

4.3 Covariation

Covariance is a measure of covariation between two stochastic processes. If volatility is calculated as in (4.1) through (4.4), then the correlation coefficient can be expressed as:

$$\rho_{uv} = \frac{\text{cov}(u,v)}{\sigma_u \sigma_v} = \frac{\frac{1}{n}\sum_{i=1}^{n}(u_i - \bar{u})(v_i - \bar{v})}{\frac{1}{n-1}\sqrt{\sum_{i=1}^{n}(u_i - \bar{u})^2 \sum_{i=1}^{n}(v_i - \bar{v})^2}} \tag{4.14}$$

where *u* and *v* are two correlated return processes. As above, the GARCH approach can also be used for updating autocorrelative covariance estimate.

In this thesis, there are two correlations that an investor has to make his/her judgement about: correlation between the three underlying forward contracts and the correlation between electricity forward prices and exchange rates. The last one is assumed to be zero. The covariation between log-returns on the forward contract is material, as shown in Table 2 above. This correlation is treated in the simulation code.

5.   Pricing with Close-form Solutions

A closed-form solution is a solution to a differential equation that expresses the change in the option relative to all key variables, subject to hedging assumptions and end conditions. The previous sections identified one acceptable closed-form analytical solution, Black 76, for an underlying forward price process following the GBM. The supporting argument was that the forward will follow the GBM if its own underlying, the spot, follows it. The equation (3.12) showed that the log-returns on a GBM process will be normally distributed. This section concerns with whether normality assumption in fact holds for the forwards traded on Nord Pool, and what other closed-form solutions are available if it does not.

5.1 Description on the Underlying Forward Contracts

The forward contracts underlying the *RE* of the NKIB-XV as yearly base-load forward contracts traded at Nord Pool: ENOYR-09, ENOYR-10 and ENOYR-11.

Nord Pool ASA financial market was established in 1993. Currently it is one of the leading and liquid power derivative exchanges in Europe[24], counting more than 400 members from over 20 countries. In addition to facilitating and regulating trading, Nord Pool Clearing (the clearinghouse) is the contractual counterparty in all exchange traded contracts and the OTC volumes reported for clearing. Future contracts are quoted for daily and weekly base-load underlying, while forward contracts "cover" monthly, quarterly and yearly base-load blocks. Futures and forwards can be priced similarly if future interest rates are deterministic. However, detailed discussion of differences between forwards and futures is outside the scope of this analysis.

Product definitions and horizons were last changed in 2003-2005. Historically forward contracts run for three years, but from ENOYR-11 lanched in 2006 there are contracts for up to five years to maturity. Contracts are sized in 1 MWs and since FWYR-06 are quoted in EUR. The reference price is the official Nordic underlying day-ahead spot price. The market is open 250-252 days a year, the 252-days convention is used in this thesis.

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[24] The other European electricity exchanges where futures and forwards are traded are European Power Exchange EEX, Amsterdam Power Exchange APX and Paris Power Exchange POWERNEXT.

There are European call options on the shortest two forward contracts quoted on Nord Pool. One option is on 1 MWh volume of the underlying. The strikes are set at a 1 EUR/MWh tick, with premium quoted at a 0,01 EUR/MWh. Options on yearly forwards expire on the third Thursday in December.

Forward contracts' settlement structure (with no mark-to-market) is in Figure 3 below:

Figure 3: Nord Pool forward contract settlement structure
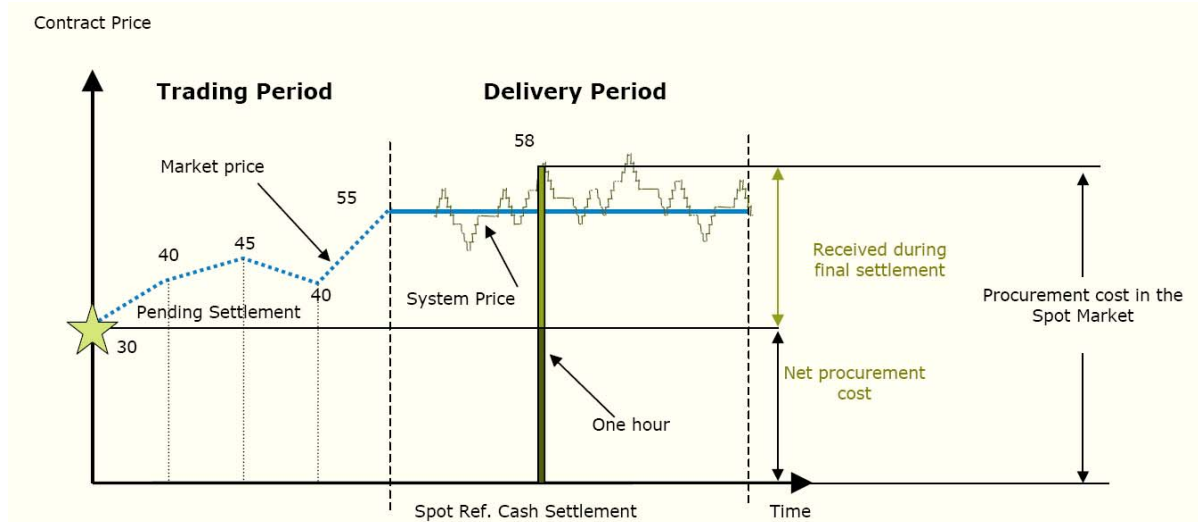
Source: Nord Pool (2007)

Prices and other relevant information for forwards and related options as quoted on the NKIB-XV settlement date 15.02.2008 are presented in the table below.

Table 5: Market information on forward and option contracts on 15.02.2008

Forward contract	Expire	Time to maturity	Closing 15.02.08	ATM option contracts	Expire	Closing 15.02.08	c/F rate
ENOYR-09	23.12.08	218 days	53,10	ENOC53YR-09	18.12.08	5,09	9,6%
				ENOC54YR-09	18.12.08	4,67	8,8%
ENOYR-10	28.12.09	469 days	52,50	ENOC52YR-10	17.12.09	5,97	11,4%
				ENOC53YR-10	17.12.09	5,61	10,7%
ENOYR-11	28.12.10	721 days	52,15				

5.2 Distinctive Characteristics of Electricity Spot and Forward Prices

Characteristics of the Electricity Markets

The following is a list of characteristics where electricity markets may differ from the stock and other financial markets for which the GBM framework was originally developed:

Lack of storability	Electricity as a consumption asset cannot be effectively stored[25]. This has implication on spot volatility/jump behaviour and spot-forward relationship, as traders cannot set up arbitrage portfolios using spot prices, see Eydeland and Geman (1999). Lesser relevance of storage cost and convenience yield.
Limitations on transportability and delivery problems	Introduces spot price basis risk due to location and time of delivery.
Seasonality and cycles	Strong seasonality and cycle factors built in both regional demand and, as in the case of Nord Pool, supply. Introduces predictable "seasonal" patterns for intrayear, intramonth, intraweek and intraday prices.
Negative spot prices	Relatively rare, more relevant for systems dominated by base-load thermopower generators. Nord Pool's System Price cannot be negative by design.
Mean reversion (autocorrelation)	In the longer-term at least, electricity prices tend to gravitate toward the marginal cost of production. Demonstrated by Gibson and Schwartz (1990), Brennan (1991), Cortazar and Schwartz (1994), Schwartz (1997) and Ross (1995)
Non-continuous process – price jumps and spikes	Due to supply shortages or demand shocks, spot prices show sudden, relatively large, unexpected and discontinuous changes that rapidly revert to their long-term (normal) level.
Stochastic volatility	GBM-based models are homoskedastic, which is clearly not the case for spot and forward prices. Solved by introducing heteroskedastic models, with deterministically changing or stochastic volatility. For autoregressive stochastic volatility exhibiting clusters, ARCH models could be used.

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[25] Hydropower can be stored as water in reservoirs, something called "pump power" ("*pumpekraft*") in Norway. See *inter alia* Gjolberg and Johnsen (2002). In addition, electricity can of course be stored in batteries on minor scale.

Volatility term structure	Rapid decay of volatility as time-to-maturity increases is very typical for electricity (and other energy) derivatives. Can be explained by *inter alia* the Samuelson's hypothesis.
Non-normal and non-stable distributions of returns	Distribution of electricity price returns exhibits fat tails (leptokurtic distributions) and skews, ref. Clowlow and Strickland (2000). Abnormally high kurtosis leads to volatility "smiles". Fat tails can be introduced by price jumps and stochastic volatility, as proposed by Knittel and Roberts (2001) as well as Eydeland and Geman (1998). Skews are more difficult to take into account.
Dual nature of the spot and forward prices	Forward prices are based on financial contracts which are easily storable and transferable. The standard assumptions of the effective financial markets apply to forwards to a much larger extent than to spot prices.
Forward contracts on flow delivery	Forwards are not on a point delivery, but on a flow delivery over a period T_1 to T_2. In addition, there are no point forward prices quoted as they are traded in blocks.
Liquidity of emerging electricity markets	Organised electricity markets can still be described as emerging, with their microstructure still changing. The Nord Pool spot and forward markets are the oldest and sufficiently efficient and liquid. On the other hand, OTC forward market for longer maturities and option market can lack in liquidity.

Empirical Evidence for Forward Prices

The observations above indicate that the general option pricing solution for the GBM process drawn up the previous section may not fit the electricity forwards. At the same time, there are some arguments and anecdotal evidences to that while spot prices definitively cannot be described by a GBM, longer-term forwards probably may. In order to conclude on the suitability of Black 76, one needs to perform normality tests on the historical return data.

One can either perform statistical normality tests or "graphical tests" comparing the actual distribution histogram with a normal probability curve or constructing a Q-Q Plot[26]. There have been proposed a number of normality tests: *inter alia* the Kolmogorov-Smirnov test, D'Agostino's K-squared test, the Jarque-Bera test, the Anderson-Darling test, the Cramér-

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[26] A Quantile-to-Quantile (or Q-Q) test or plot compares actual probabilities of the variable with the expected probabilities if the variable was normally distributed. A diagonal line indicates normality while an "S"-shaped plot indicates the contrary.

von-Mises criterion, the Lilliefors test for normality, the Shapiro-Wilk test, the Pearson's chi-square test, and the Shapiro-Francia test for normality. D'Agostino and Stephens (1986) recommend the Shapiro-Wilk test for moderate sample sizes and the Anderson-Darling test is recommended for sample sizes larger than 4000 observations. Conover (1999), Shapiro and Wilk (1965), Royston (1982) and Royston (1995) also agree that the Shapiro-Wilk test is the most reliable normality test for small to medium sized samples. Judge et al. (1988) and Gujarati (2003) recommend the Jarque-Bera test. Another indirect normality test is a test for autocorrelation.

Deyna and Hulström (2007) assessed return distributions on spot prices and three yearly contracts, ENOYR-06, ENOYR-07 and ENOYR-08 for the period from 01.10.2003 until 01.04.2007. They calculated skewness and excess kurtosis of the return distributions and applied the Jarque-Bera (1980) normality test:

$$T_{JB} = N \left[\frac{Skew^2}{6} + \frac{ExcessKurtosis^2}{24} \right] \tag{5.1}$$

The null-hypothesis is that the forward prices are log-normally distributed. The null-hypothesis is rejected is to be rejected if T_{JB}-value is larger than the critical value of X^2, which is on the 5%-level is equal to approx. 6.

Deyna and Hulström (2007) found that although none of the forward contracts met the tests with their T_{JB} statistics at between 890 and 3.054, the abnormality of forward returns are much smaller than that for the spot prices ($T_{JB} = 13.936$) or for the shorter futures.

Koekebakker and Ollmar (2001) also concluded that the models they used to describe the volatility of the forward prices both failed the normality tests.

In this thesis both the Shapiro-Wilk test and the Jarque-Bera test have been performed on the log of returns for forward contracts from FWYR-01 though ENOYR-08. The results are presented in Appendix D and in the table below.

Table 6: Results of normality tests on historical returns on forward contracts

Forward contract	Observations N	Skew	Kurtosis	Jarque-Bera T_{JB}	Shapiro-Wilk W_{SW} (p_{SW})
FWYR-01	102	0,27	3,21	45	0,95 (0,0004)
FWYR-02	146	0,39	2,96	57	0,93 (0,0000)
FWYR-03	152	3,90	27,75	5.262	0,71 (0,0000)
FWYR-04	154	0,30	2,50	42	0,95 (0,0001)
FWYR-05	154	-0,41	1,47	18	0,98 (0,0114)
ENOYR-06	154	-0,09	1,25	10	0,98 (0,0256)
ENOYR-07	155	-3,23	22,90	3.656	0,78 (0,0000)
ENOYR-08	154	-1,70	11,24	885	0,88 (0,0000)

These tests confirm that none of the historical forward contracts were log-normally distributed over their entire lives. However, compared with the spot, some of the contracts were very close to the stringent criteria ($T_{JB} < 6$ and $W_{SW} \rightarrow 1,0$ / $p_{SW} < 0,05$).

5.3 Can the GBM Be Acceptable for Forward Prices?

In contrast to the spot prices, forward prices are based on financial contracts which are easily storable and transferable. Therefore, the standard assumptions of the effective financial markets may apply to forwards to a much larger extent than to spot prices.

The long-run contracts such as yearly forwards with several years to maturity show materially lower volatility and a less jumpy behaviour than the spot prices. There can be several reasons for that. Firstly, long-term equilibrium prices that forward relate to, is unaffected by temporary shortages that cause spot jumps. Secondly, lower liquidity of the longer maturities would, *ceteris paribus*, reduce volatility, see Lo and MacKinley (1999).

Although several empirical studies have been carried out on the spot prices[27], there has been relatively few academic works empirically examining electricity forward prices. There are several recent works authored by Koekebakker, Ollmar and/or Benth on the electricity term

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[27] E.g. Knittel and Roberts (2001) for traditional linear models and models with jumps and time variation, Johnson and Barx (1999) for models with jumps.

structure modelling, including the recent "Stochastic Modelling of Electricity and Related Markets" by Benth, Benth and Koekebakker (2008). According to Koekebakker (2006), a multi-factor GBM model could be a good alternative for forward price modelling. However, a detailed and complete overview over current status of the body of knowledge on the topic of forward price dynamics is outside the scope of this thesis.

Koekebakker and Ollmar (2001) performed Principal Component Analysis (PCA) on the smoothed forward prices from 1995 to 2001. They found that there were two factors influencing volatility and explaining approx. 75% of the total variation. The first factor was shifting all forward prices in the same direction, the second caused short- and long-term forward prices to move in the opposite directions. "*The main sources of uncertainty affecting the movements in the long end of the curve, have virtually no influence on variation in the short end of the curve.*" wrote Koekebakker and Ollmar.

Hjalmarsson (2003) looked at whether the B-S formula works for the electricity markets, and compared it with a nonparametric approach. He concluded that while more accurate option prices can be obtained from the process based the nonparametric estimate, "*given the shape of the nonparametric estimates, it is difficult to think of any parametric model that would give a better approximation than the linear geometric Brownian motion… Therefore, from the practical viewpoint, the Black-Scholes option prices might be the best achievable.*"

Several experts believe that traditional models like GBM (or multi-factor GBM) and pure mean-reversion can be appropriate for modelling forward prices. According to Deyna and Hulström (2007), large Scandinavian electricity traders use standard version of Black 76 to quote options. However, Black 76 is not used for forecasting purposes, and the traders utilise jump-diffusion models.

Based on the arguments above and in order to stay within the scope of the analysis, it is chosen to retain the GBM assumption for the forward prices made and elaborated upon in the previous sections. Therefore standard Black 76 model is used as the only closed-form analytical solution for pricing the options built into the NKIB-XV's Return element.

6. Pricing and Return Analysis with Numerical Techniques

This section looks closer at the simulation numerical procedures. Although the technical side of the Monte Carlo simulation is not the primarily focus of this thesis, to be able to independently model and value the *RE* of the NKIB-XV by way of simulation, any investor has to acquire basic understanding and technical skills of the subject.

6.1 Brief Comparative Analysis of Numerical Techniques

A closed-form solution is relatively easy to model and use, and provides the implementation flexibility. However, the complicated the underlying process and the more sophisticated the structure of the option, the more difficult, if not impossible, it becomes to arrive at a closed-form solution. Sometimes there exist sufficiently good approximations, sometime there are none.

An alternative to closed-form solution is simply to forecast the underlying prices process and make option pay-out calculation at maturity, and finally discount it to the present. In case of a stochastic underlying price process, such simulation has to be run sufficiently many times, and the average taken. This alternative is called numerical techniques.

There are three main types of numerical techniques:

- Binominal or trinominal tree building[28].

- Monte Carlo or Quasi-Monte Carlo simulation.

- Other methods (Finite Difference methods, implicit or explicit, numerical integration, finite element methods, *etc.*).

The numerical methods are more flexible than the analytical solutions in that they can accommodate multiple assets, complex price processes (*e.g.* multiple stochastic factors,

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[28] The binominal method was first introduced by Cox, Ross and Rubinstein (1979) and Rendleman and Bartter (1979). The method was first applied for the GBM process, but later was extended to multidimensional trees and implied trees with changing volatilities/probabilities.

stochastic volatility, jumps and spikes, mean-reversion) and advanced pay-out structures (e.g. average rate, barrier, lookback).

The Monte Carlo simulation technique is best suitable for path-dependant derivatives, *e.g.* Asian options or options with Asian tails. The simulation can also handle a multifactor approach and a discrete volatility term structure. One drawback of this approach is that it captures the probability through the sheer number of simulations which for several underlying and timepoints may require massive calculation capacity. Since computing power has became cheaper and more readily available in the recent years, this drawback is not as critical as it was before. Also, the simulation cannot be used for valuing American options.

The Trees and the Finite Difference methods are the best alternative for American options with earlier exercise (optional or triggered) and for convertible features. According to Pilipović (1998) they can also handle multifactor approach up to two factors.

It is then clear that in order to take into account the Asian tail in the NKIB options, one should use the Monte Carlo technique. In addition, it is required for expected return analysis.

6.2 Introduction to Monte Carlo Technique

The Monte Carlo ("MC") procedures have been used since 1930th for calculations in Physics, Mathematics and Chemistry. Boyle applied the MC to valuing options first in 1977, he also introduced the Variance Reduction Procedures.

The pricing process is fairly simple: having a price process model. *e.g* as in (3.41), with a stochastic variable(s) built into it, for each simulation one generates this stochastic variable for each time step (or for the whole time period, if possible) determines the resulting price and calculates the payout from the derivative (*e.g.* an option). Such simulation is then repeated sufficient amount of times to make the averaging result statistically significant. The result is the expected value under risk-neutral probabilities, so it is then discounted back to the valuation point of time by the risk-free (or credit risk-adjusted) interest rate with appropriate maturity. One can also estimate the statistical error and the distribution of the payout. The error is normally distributed with the mean of zero and the variance of σ^2/N, where σ is the standard deviation of one simulation and N – number of runs. The error will

therefore diminish with increasing number of simulation, but very slowly, with a rate of $1/\sqrt{N}$.

For pricing European options without Asian elements, not path-dependent (almost what we have in the KNIB-XV if one disregards the five-days averaging), and assuming that the underlying follows the GBM, it is not necessary to simulate the whole path, but only the final price at maturity which the payout is based upon.

However, the strongest argument in favour of the MC procedure, even above the tree-building methods, is its ability to efficiently price the path-dependent, multi-underlying derivatives. Jäckel (2002) explains that the MC procedure is equivalent to calculating a multidimentional integral with $d = i * k$ dimensions, where i is a number of underlying and k – amount of time steps. Estimation error in an MC run is independent of the number of dimensions, while the error in the tree-procedures increases as the number of dimensions rises. This makes the MC the preferred choice for calculation of multidimensional derivatives.

Solutions for drawing normally-distributed variables, as described in the following subsection, constitute a necessary part of any Monte Carlo simulation, including that which is performed in this analysis. The following subsection discusses Monte Carlo simulations for correlated assets. Although not implemented in this thesis through the Cholesky decomposition, it is an alternative that can be utilised given more time to estimate variance-covariance matrix and model the solution. The last two subsections dwell on effectiveness and efficiency improvements and Quasi-Monte Carlo sequences. For purposes of this analysis, these are not-necessary elements. However, they are presented here for the sake of completeness.

6.3 Drawing Normally-Distributed Variables

The most important calculation part of the MC procedure is to draw series of independent normally distributed ε^i (or ξ^i). It is usually done by first drawing a random uniformly-distributed number between 0 and 1, and then transforming it to a normally-distributed variable.

One alternative for the uniformly-distributed number $x^i = Uni(0,1)$ is to use "random number generator" built into Excel, *RAND*(), or the *Rnd*-function in the VBA. Such numbers are however not completely random, but "pseudo-random". They "cluster", and this means that it takes longer time for a MC model using such generator to converge on the true solution.

Should one accept the use of a pseudo-random generator, a better alternative may be the Mersenne Twister developed by Matsumoto and Nishimura (1997). The authors claim that this is one of the fastest pseudo-random generators available, and that it uniform distribution is the best approximation. Bøe (2007) tested the Mersenne Twister against the standard VBA generator and found the differences to be small. There are of course other alternatives, offered as add-ins for Microsoft® Excel. Following Bøe (2007), the VBA/Excel generator as pseudo-random generator in the MC procedures has been chosen for this thesis.

When it comes to converting the drawn uniform number x^i into a normally-distributed variable ε^i, the default choice is again the standard Excel function *NORMINV*(*rand*(),*mu*, *sigma*), or the *Application.NormSInv*-function in VBA. This function works slowly. Other alternatives available are (a) the Central Limit Theorem (uses lots of points), (b) the Box-Muller (1958) transformation (takes two points) and (c) Moro (1995) transformation from his article "The Full Monte" in the Risk magazine (requires one point).

The Box-Muller method takes two uniform variables *u* and *v*, and converts them to two normally-distributed variables *x* and *y*:

$$x = \sqrt{-2\ln(u)}\sin(2\pi v) \quad \text{and} \quad y = \sqrt{-2\ln(u)}\cos(2\pi v) \tag{6.1}$$

The Moro's code is presented in Appendix D.

Bøe (2007) compared the standard VBA NormSInv-function with the Box-Muller and the Moro transformations, and found that although the Box-Muller code runs somewhat faster than the Moro's, the both are vastly superior to the standard Excel alternative. Further, Jäckel (2002) pointed out that the Box-Muller method results in somewhat imprecise tails of the distribution. In his MC analyses Bøe (2007) chose therefore to use the Moro's procedure, and here we follow his example.

6.4 The Monte Carlo Procedures for Correlated Underlying

What if several underlying assets in a derivative are correlated, as a case may be for a basket option? This means that for each time step (or for the whole period) we have to simulate i correlated normal distributed variables. The explanation below is based on Koekebakker and Zakamouline (2006). For simplicity, $i = 1, 2$ and 3 is used.

For notation, let us assume three assets that each follows the continuous GBM as in (6.2) with bilateral correlations as in (6.3). Here t_k are points in time, μ^i is the expected total return on asset i, δ^i – dividend yield, and dz^i – a Wiener process $d\tilde{z}^i = \tilde{\varepsilon}^i \cdot \sqrt{dt}$ as in (2.1). The future prices can than be modelled according to (6.4), see also (3.13), with returns calculated as in (6.5).

$$d\tilde{S}_t^i = \left(\mu^i - \delta^i\right)S_t^i dt + \sigma S_t^i d\tilde{z}^i \tag{6.2}$$

$$d\tilde{z}^i d\tilde{z}^j = \rho^{ij} dt \tag{6.3}$$

$$\tilde{S}_{t+\Delta t}^i = S_t^i e^{\left(\left(\mu^i - \delta^i\right) - \frac{1}{2}\sigma^{i2}\right)\Delta t + \sigma^i \tilde{\varepsilon}_t^i \sqrt{\Delta t}} \tag{6.4}$$

$$\ln\left(\frac{\tilde{S}_{t+\Delta t}^i}{S_t^i}\right) = \left(\left(\mu^i - \delta^i\right) - \frac{1}{2}\sigma^{i2}\right)\Delta t + \sigma^i \tilde{\varepsilon}_t^i \sqrt{\Delta t} \tag{6.5}$$

As we cannot directly simulate three correlated normally distributed variables, we first simulate three independent ones, and than convert those by applying the Cholesky Decomposition of covariance martix. First step is to estimate assets' correlations (and if necessary variance) from the historical logarithmic return data, filling the variance-covariance matrix Σ. The matrix Σ is converted into an upper triangle matrix C so that $C^T C = \Sigma$. Finally, we draw a vector of three independent variables $\tilde{\xi}^i$, and then arrive at three dependent variables ε^i by way of $\tilde{\varepsilon}_t^i = \sum_{j=1}^{3} c^{ij} \xi_t^i$. The variables ε^i are used in the MC simulations. This procedure is presented in (6.6) trough (6.8):

$$\Sigma = \begin{bmatrix} 1 & \rho^{12} & \rho^{13} \\ \rho^{21} & 1 & \rho^{23} \\ \rho^{31} & \rho^{32} & 1 \end{bmatrix} \tag{6.6}$$

$$C^T \cdot C = \Sigma \quad \text{or} \quad \begin{bmatrix} c^{11} & 0 & 0 \\ c^{21} & c^{22} & 0 \\ c^{31} & c^{32} & c^{33} \end{bmatrix} \cdot \begin{bmatrix} c^{11} & c^{21} & c^{31} \\ 0 & c^{22} & c^{32} \\ 0 & 0 & c^{33} \end{bmatrix} = \begin{bmatrix} 1 & \rho^{12} & \rho^{13} \\ \rho^{21} & 1 & \rho^{23} \\ \rho^{31} & \rho^{32} & 1 \end{bmatrix} \tag{6.7}$$

The calculations in (6.7) can be demanding, solving for one argument at a time, however, there is a quite efficient Cholesky VBA code available. The solution (6.7) is only possible if the Σ matrix is a positive-semidefinite Hermitian matrix. It may not be the case when we have many underlying assets. In such case one can apply the Singular Value Decomposition ("SVD") instead of the Cholesky factorisation. For details on the SVD, please see *e.g.* Dahl and Benth (2001).

$$\begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^3 \end{bmatrix} \cdot \begin{bmatrix} c^{11} & 0 & 0 \\ c^{21} & c^{22} & 0 \\ c^{31} & c^{32} & c^{33} \end{bmatrix} = \begin{bmatrix} \xi_t^1 \\ \xi_t^2 \\ \xi_t^3 \end{bmatrix} \tag{6.8}$$

The VBA code for the Cholesky decomposition is presented in Appendix D.

6.5 Effectiveness and Efficiency Improvements

For calculations covering many underlying and time points, the MC calculation may require excessive computation power. In addition, it is desirable to reduce the error of the final estimate. Several methods have been proposed to achieve these goals. For an extensive discussion on the subject, see Jäckel (2002). Here, two variance reduction solutions are briefly illustrated: the Antithetic Variable Technique and the Control Variate Technique.

In the Antithetic Variable Technique, one simulation trial produces two normally-distributed values. The first one, ε^1, is calculated in the usual way, see above, while the second one is equal the first with an opposite sign: $\varepsilon^2 = -\varepsilon^1$. The two resulting derivative estimates are then averaged, $G^* = (G^1 + G^2)/2$, and the final derivative estimate G is usually the average of all G^* from all runs. Quantitatively it can be expressed as

$$Var(G^*) = Var\left(\frac{G^1 + G^2}{2}\right) < Var(G) \quad \text{when} \quad Cov(G^1, G^2) < 0 \tag{6.9}$$

Since the condition in (6.9) often holds, the result arrived at by way of the Antithetic Variable Technique usually approaches the true value of the derivative faster (has lower

standard deviation) than if one would run corresponding number of single-variable simulations. Another advantage of the technique is that it requires only half of the random number generations. For derivatives where complete time series have to be simulated, the technique may represent both lower error and faster execution.

Another variance reduction procedure is the Control Variate Technique. It can be readily illustrated on the challenge of valuing an arithmetic Asian option. Although there exist a number of closed-form approximations, *e.g.* Kemna and Vorst (1990), Turnbull and Wakeman (1991), Levy (1992), Curran (1992), Geman and Yor (1993), etc., there are no true closed-form solution for an arithmetic Asian option. However, Kemna and Vorst (1990), offered a formula for valuing a geometric Asian option, see the expression (3.62). The Control Variate Technique suggests calculating both arithmetic and geometric values for each trial (they will be strongly correlated), and then uses the difference between the estimated and the true values of the geometric Asian option to "correct" the arithmetic estimate. Let us denote option values thus: f_A – true arithmetic (unknown), f_G – true geometric (knowable), f_A^* and f_G^* - estimates simulated by each MC trial. Using f_G^* as a control variate, we can estimate f_A more effectively:

$$f_A = f_G + \left(f_A^* - f_G^*\right) \tag{6.10}$$

The MC estimates are unbiased expectations, $E(f_G^*) = f_G$ and $E(f_A^*) = f_A$, therefore the variance for f_A estimate will be as in (6.11). And as long as f_A^* and f_G^* are strongly positively correlated, $Cov(f_A^*, f_G^*) >> 0$, $Var(f_A) < Var(f_A^*)$ should hold.

$$Var\left(f_A\right) = Var\left(f_A^*\right) + Var\left(f_G^*\right) - 2Cov\left(f_A^*, f_G^*\right) \tag{6.11}$$

Bøe (2007) tested the Control Variate Technique on a spread option, and found its variance-reducing effect very powerful.

Other variance reduction procedures include the Importance Sampling, the Stratified Sampling (*e.g.* Curran (1994) and Moro (1995)), and the Moment Matching.

6.6 Quasi-Monte Carlo Sequences

As mentioned above, one of the weakest points of the standard Monte Carlo procedure is its difficulty to draw truly uniformly-distributed numbers to convert those later to normally-distributed ones for use in a Wiener process.

We can instead choose to draw representative samples from known probability distributions using deterministic algorithms. Such methods are called quasi-random sequences or low-discrepancy sequences (or Quasi-Monte Carlo, "QMC"). The Halton sequence is an example of such algorithm. The sequence is unlimited and operates in such a manner that each new number lies farthest possible from all the already drawn numbers, reducing "clamping". While the conversion rate of the standard MC is $1/\sqrt{N}$, the rate for a QMC sequence is potentially $1/N$.

Many exotic options will require simulations along several dimensions, e.g. number of underlying and number of Asian tail observations. The Halton sequence (its VBA code is shown in Appendix D) will then use different bases for each dimension, each base built on a prime number. For every trial, a number will be drawn from each base, for each dimension. These numbers can then be transformed into normally-distributed variables (by *e.g.* the Moro transformation), and used in the Monte Carlo simulation of derivative's value. However, the Halton procedure distributes less and less uniform as the number of dimensions and therefore bases increase. According to Marco A.G. Dias, many practitioners limit therefore their application of the Halton procedure to 6 or 8 dimensions.

The notion that the Halton sequence does not perform well in higher dimensions has been conformed by the studies of Paskov and Traub (1995) and Boyle, Broadie and Glasserman (1997). Other low-discrepancy sequences that perform better the Halton are *inter alia* the modified, "leaped" Halton sequence, the Sobol sequence and the Faure sequence.

The QMC methods also run faster than the standard MC, and they may with advantage be combined with the variance reduction techniques.

7. Practical Implementation and Results

Based on the theoretical fundaments and valuation solutions for the electricity forward price process and European call options on such forwards, as described in the previous sections, the closed-form and the simulation-based valuations of the Return element of the NKIB-XV have been performed. The simulation-based expected return analysis has also been done.

7.1 Estimation of Volatility and Covariation

The general definitions of volatility, its characteristics in the electricity market and estimation procedures are described above. Since the closed-form Black 76 model assumes deterministic constant volatilities, one has to take into account the existing volatility term structure by using cumulative volatilities. This is done by applying the cumulative volatilities corresponding to each option's remaining life[29]: $\sigma_{01} = \sigma_{t0\text{-}t1}$ for the option c_1 on ENOYR-09 (t_1 – approx. 0,8 years or 43 weeks or 218 trading days), $\sigma_{02} = \sigma_{t0\text{-}t2}$ for the option c_2 on ENOYR-10 (t_2 – approx. 1,8 years or 94 weeks or 469 days) and $\sigma_{03} = \sigma_{t0\text{-}t3}$ for the option c_3 on ENOYR-11 (t_3 – approx. 2,8 years or 147 weeks or 721 days). For the simulation-based analysis the subperiod volatilities (σ_{01} , σ_{12} and σ_{23}) as explained in (4.8) were used. Although the three underlying contracts are different, and may have some year-specific volatility drivers, a single volatility term structure for all three contracts has been used. The contacts' historical volatilities up to the settlement date do not indicate that any of the contracts stand out and require separate treatment.

Indirect Estimation Method – Publicly Available Information

Before doing any calculations, one has to explore whether there are reliable and relevant future volatility estimates publicly available. For a professional investor or a player on the financial electricity market, both advanced in-house and/or third-party provided models and analytics will be available, for historical as well as for implied or forecasted volatility. One would expect electricity brokers to quote expected volatilities for their customers. This analysis relies on the volatility assumptions disclosed in the offering documents for the

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[29] For closed-form calculations, the period is measured until the mid-date of the five-days calculation periods, while for simulation purposes, periods until each calculation date are measured precisely.

NKIB or other structured products. In addition, the term structures published in the research papers have been gathered.

The review of the offer documents from Nordea, DnB NOR and Orkla Finans for the electricity-linked index bonds and warrants yielded the results presented in the table below. One notes that Nordea only once has disclosed its volatility assumptions, for the NKIB-XIII in January 2007.

Table 7: Volatility assumptions disclosed by Nordea, DnB NOR and Orkla

Contract	Nordea Dec. 2006	DnB NOR Jul. 2007	DnB NOR Sept. 2007	DnB NOR Nov. 2007	DnB NOR Jan. 2008
ENOYR-08	26,0%: 1,0yr				
ENOYR-09	22,5%: 2,0yr	24,9%: 1,5yr	24,6%: 1,3yr		
ENOYR-10	20,0%: 3,0yr	22,8%: 2,5yr	23,7%: 2,3yr		
ENOYR-11			22,8%: 3,3yr	26,8%: 3,1yr	22,4%: 3,0yr
ENOYR-12				27,0%: 4,1yr	23,9%: 4,0yr

For its Orkla Finans Kraft IV instrument, Orkla informs in December 2007 that annual volatilities of between 26% and 30% were used for internal simulations for yearly forwards 2009 to 2012.

Volatility term structures for the Nord Pool-traded forward were also obtained from, *inter alia*, Koekebakker and Ollmar (2001) for a 1995-2001 sample, in Skogen and Bjørdal (2002) for 2000 to 2001, as well as in Bjerksund, Rasmussen and Stensland (2000). This is certainly not a complete list of all published research that may contain forward volatility estimates. However, the numbers in these papers indicate a level between 10% and 20% one to two years to maturity (40 to 90 weeks). In 2002 Skogen and Bjørdal it is stated that options maturing four years into the future exhibit approx. 15% volatility. These numbers are materially lower than the ones the banks disclose. May be it is so because the recent contracts are more volatile than the ones used as references by the researchers in 2001-2002? Appendix F shows calculations of 26-week rolling annualised historical volatilities. The recent contracts do not seem to be more volatile that the older ones in the first two-thirds of their lives, with a half-year rolling of 10% to 15%.

When one refers to the historic volatilities, one runs the risk of using outdated data. Among others Koekebakker and Ollmar (2001) and Deyna and Hulström (2007) showed that the volatility term structure is not only uncertain at any given point of time, but is also constantly changing. Nevertheless, the levels estimated earlier by the issuers as well as by the researchers, are in my opinion informative.

Direct Method – Historical Volatility Level and Structure

In accordance with the methods described in the previous sections, the historical forward price volatilities were calculated along the time-to-maturity axis as (a) 60-days rolling average (for both expired and currently running contracts) as well as (b) cumulative until maturity (for expired contracts only). The resulting graphs are presented in the Appendices G and H. The cumulative-until-maturity numbers are presented in the table below:

Table 8: Historical cumulative volatilities to maturity

Contract	0,817 yrs to maturity		1,810 yrs to maturity		2,817 yrs to maturity[30]	
	60-d roll	cumul.	60-d roll	cumul.	60-d roll	cumul.
FWYR-02	17,7%	22,1%	11,5%	17,1%	n/a	15,8%
FWYR-03	16,3%	41,5%	15,4%	31,0%	n/a	25,7%
FWYR-04	35,8%	30,6%	13,5%	25,6%	n/a	22,3%
FWYR-05	18,6%	17,5%	20,8%	16,3%	n/a	15,7%
ENOYR-06	17,2%	22,9%	9,8%	17,6%	n/a	15,8%
ENOYR-07	18,0%	33,1%	13,8%	26,8%	n/a	22,4%
ENOYR-08	21,9%	18,2%	12,4%	19,8%	n/a	18,4%
Median		**22,1%**		**19,8%**		**18,4%**

Direct Method – Implied Volatility Level and Structure

The standard Black 76 model was used to back out implied volatilities from the quotes for options ENOC*kk*YR-09 and ENOC*kk*YR-10 per 15.02.2008. The options are nominated in EUR and cleared at Nord Pool Clearing, so the EUR risk-free rates for corresponding

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[30] For contracts that run for a shorter period than 721 trading day, cumulative volatility for the longest period (from start).

maturities were taken as input. The assumptions and the results are presented in the table below. The implied volatility "smirk" is shown in figure 2 above.

Table 9: Input and implied volatilities from option market prices

| | | ENOYR-09 | | ENOYR-10 | |
		ATM(-)	ATM(+)	ATM(-)	ATM(+)
Forward price	EUR/MWh	53,10		52,50	
Opt. fix. date		18.12.08 (0,857 yrs)		17.12.09 (1,849 yrs)	
Risk-free rate	EUR, cont.	3,47%		3,21%	
Option strike	EUR/MWh	53,00	54,00	52,00	53,00
Option price	EUR/MWh	5,09	4,67	5,97	5,61
Implied vol.		**26,6%**	**26,6%**	**21,5%**	**21,8%**

Interestingly, the implied volatility levels are very close to the ones used by Nordea in January 2007 for its NKIB-XIII options with 1,0 and 2,0 years to maturity, namely 26,0% and 22,5%, respectively. The implied volatility for the NKIB-XIII input was tested, and the volatility results were 24,8% and 22,6%, sufficiently close since the date of calculation of the Nordea's volatilities are unknown (probably December 2006).

Taking into account historical cumulative volatility levels, implied volatilities disclosed in Nordea and DnB NOR's offer documents as well as volatilities implied in option prices on the settlement dates, and taking into account an adjustment necessary with regard to the five-days averaging, as in (3.53), the following volatilities were used $\sigma_{01} = \sigma_{t0-t1} = $ **26,4%** for the shortest contract, $\sigma_{02} = \sigma_{t0-t2} = $ **21,5%** for 2010-contract and $\sigma_{03} = \sigma_{t0-t3} = $ **20,0%** for the longest one in the base-case analysis. The sub-period volatilities are then $\sigma_{12} = $ **16,4**% and $\sigma_{23} = $ **17,0**%.

For sensitivity analysis option values are simulated with +/-20% changes to the base-case volatility levels, up to **31,7%**, **25,8%** and **24,0%**, respectively.

7.2 Estimation of Other Parameters

Remaining Option Lives

Remaining option lives are estimated as follows:

- For closed-form solution: from the settlement date 15.02.2008 until the mid-day of the respective calculation periods (08.12.2008, 08.12.2009 and 08.12.2010).

- For MC simulation: from the settlement date 15.02.2008 until the first day of the respective calculation periods (04.12.2008, 04.12.2009 and 06.12.2010). Simulations are then made for each of the remaining four days at timestep $\Delta t = 1/252$.

The 52-weeks and 252-trading days conventions were controlled against actual trading days from the calendar for the Nord Pool financial market, and used for conversion. Non-trading days in 2008, 2009 and 2010 were taken into account.

Interest Rates

Interest rates are assumed to be deterministic. There is no need to take into account the actual term structure (both NOK and EUR rates were in backwardation in the shorter end). The Nordea's relevant borrowing NOK-nominated continuous rate is set to 5,07%, equal to the rate implied in Nordea's valuation of the Certain element as explained in subsection 2.6.

Dividend Yield and Cost of Carry

As argued in section 3, there is no implicit dividend yield to be applied for the NKIB-XV options, neither in the Black 76 nor in the simulation procedure. For reference, on 15.02.2008 the interest rate differential between NOK and EUR on three-years government bond was 4,39% – 3,22% = 1,17%, continuously compounded.

Electricity Forward Market Risk Premium

In order to perform the expected return analysis, one has to estimate the risk premium built into the forward prices. If one looks from the CAPM point of view, the risk premium on forwards as an individual asset will be proportional to the market risk premium (probably in the order of 4% to 5%) and beta coefficient, measuring the extent of correlation between the investor's reference portfolio and the forwards, Bodie, Kane and Marcus (1996). One could

argue that forward prices are driven by long-term equilibrium expectation and short-term/spot volatility. Both elements in the Scandinavian electricity system are driven by demand factors (economic activity, weather, *etc.*) and supply factors (generation and transportation capacity, fuel costs, precipitation and reservoir situation, electricity prices on the continent, *etc.*). In IntStream E2's market driver analysis for 2009-forward prices in May 2008, the factors were prioritized based on the last six months's correlation: coal production (+0.62), oil prices (+0,35), precipitation (+0,28), reservoir balance (-0,19), System spot price (+0,10) and EEX price (-0,06). Although oil and coal do drive the nearest forward, and one would expect that in the longer run the general economic activity will influence the demand and therefore the prices, the correlation between the reference world stock market index and the forward prices on Nord Pool is probably low. The quantitative analysis of this relation is outside the scope of this Thesis.

Another approach to gauging the risk premium is to analyse which players dominate the forward market and what risk preferences they might have. Bernseter (2003) points out that if risk-averse power producers dominate the market, they would push down forwards and one would observe a positive risk premium. If the demand side is the most risk-averse, the relation is reverse. In addition to these two groups with clear preferences, there are traders and speculators which behaviour may distort the risk premium balance. Both Bernseter (2003) and Ollmar (2004) conclude with a negative risk premium in the shorter run. Bernseter finds statistically significant negative risk premium of one- and two-years forward contracts. However, Bernseter says that there are indications of a small positive premium in the longer end of the market. On the other hand, in the offer documents for their Kraft warrants DnB NOR states that they use a risk premium estimate of 2,0%. Bøe (2007) adopted this estimate in his calculations. In this analysis the same 2,0% p.a. premium is assumed (1,98% continuously). If the risk premium is lower, and even negative for the first two of the three contracts, the expected return should be even lower.

7.3 Valuation by Back 76 model

Based on the expression (3.45), the Black 76 was built and base-case calculation of the *RE* was performed. The resulting value was NOK **9,82** on NOK 100 face value (no premium). The detailed results are presented in the summary table at the end of the section. In order to

control for computation error, a calculation based on the "0,4-rule" was made, resulting in NOK 9,89.

Further, a reasonability analysis was performed comparing the calculated option values for the two first contracts with the market prices for option on the same contracts. Normalising for forward price and adjusting for credit margin and interest rate differential, the market values are NOK **9,38** and NOK **10,66**. It compares well with the option values from the Black 76 calculations (before adjustment for delayed payment) of, respectively, NOK **9,11** and NOK **10,49**. Bøe (2007) valued DnB NOR's Kraft 2007/09 index bond with options on 2008, 2009 and 2010 forwards with approx. the same remaining lives as those in the NKIB-XV. He arrived at approx. NOK 10,5 as his value estimate for the warrant, which is in the range of the results in this analysis.

For comparison, Nordea estimated the value of the *RE* at between NOK 12,75 and NOK 16,75, with their base-case estimate at NOK **14,00**.

A sensitivity analysis was performed to see how responsive the valuation results described above are to the volatility assumptions, and to see how much the volatilities have to be increased in order to arrive at the value range indicated by Nordea. The results are presented in the summary table below. Even with a **+25% adjustment to volatilities,** increasing them to, respectively, 33% for 0,8 years, 27% for 1,8 years and 25% for 2,8 years, the *RE* value lies at approx. NOK **12,30**. In light of the volatilities Nordea and DnB NOR uses in their calculations, ref. Table 7 above, these +25%-volatilities are high. Nevertheless, one does not reach the value range of between NOK 12,75 and NOK 16,75 indicated by Nordea.

Nordea made its estimates from a simulation. Another explanation for the value deviation between their *RE* estimate and the Black 76 results is that the average of Black 76 values does not take into account expected positive correlation between the returns on the single contracts, correlation coefficients being 0,80 and 0,95 as shown in Table 3. However, the MC simulation is modelled in a way that takes it into account, and still the value is lower.

With the *RE* base-case value of NOK 9,82 and the CE value equal to NOK 85,86, the total base-case value of the index bond is approx. NOK **95,70**, compared with NOK 103 in face value plus the subscription fee. The Nordea's implied borrowing cost for three yeas maturity is **2,80% p.a.** (discrete, without the subscription fee) or 1,69% p.a. (discrete, taking into account the subscription fee).

7.4 Valuation by Monte Carlo Simulation

Based on the underlying forward price process as described in the risk-neutral form in equation (3.23), Monte Carlo procedure was coded in VBA and run with 1.000.000 realisations. The simulation is built in accordance with Bjerksund (2008), where the same process drives all contracts while they run in parallel, making their correlation coefficients effectively equal to one. A sensitivity analysis with the same width (+/-20%) on volatilities was built into the calculations. The results are presented in the summary table 10 below. The code is shown in Appendix I. No variance reduction solutions or Quasi-MC solutions were used. Correlation of returns was not modelled directly into the code by way of the Cholesky decomposition, but indirectly, applying the same process for all contracts that run in parallel.

The resulting expected values, volatilities and correlations of the returns were not computed in the code due to the scope constrain of the analysis.

The base-case value of the Return element came at NOK **10,07**, compared with NOK 9,82 resulting from the Black 76 valuation. The difference is NOK 0,25 or approx. 2,5%. The results from the MC simulation support the Black 76-based value conclusion and observations offered in the previous subsection.

The 95%-confidence interval for the procedure with 1.000.000 realisation is +/-NOK 0,014, meaning that the estimate is within the range of NOK **10,04** and NOK **10,10**.

It is interesting to note that not only does the MC procedure deliver a result which deviates from the closed-form solution, but its sensitivity analysis "spread" the estimates a little wider than the Black 76 does. This observation is attributed to the fact that the underlying processes of the average return are not completely the same in the two calculations. The MC calculation indirectly introduces correlations, as described in Bjerksund (2008).

7.5 Expected Return Analysis

An MC simulation with the same assumptions as above, but with a risk premium of 1,98% incorporated now into the drift term, was performed to arrive at the expected return for the whole investment.

The expected annual return (discrete basis) is estimated at **3,55%** p.a. without consideration for the subscription fee, and **2,52%** p.a. taking into account the fee. Nordea does not inform about their estimate of the expected return.

The risk characteristics are as follows: probability of no positive return[31] is **34,4%** (**44,2%** with the fee), probability of return above the risk-free threshold of 4,5% is **31,4%** (**26,4%**), probability of making attractive returns above 6% p.a. is **24,4%** (**20,4%**). The distribution is presented in the figure below.

Figure 4: Expected returns on the NKIB-XV
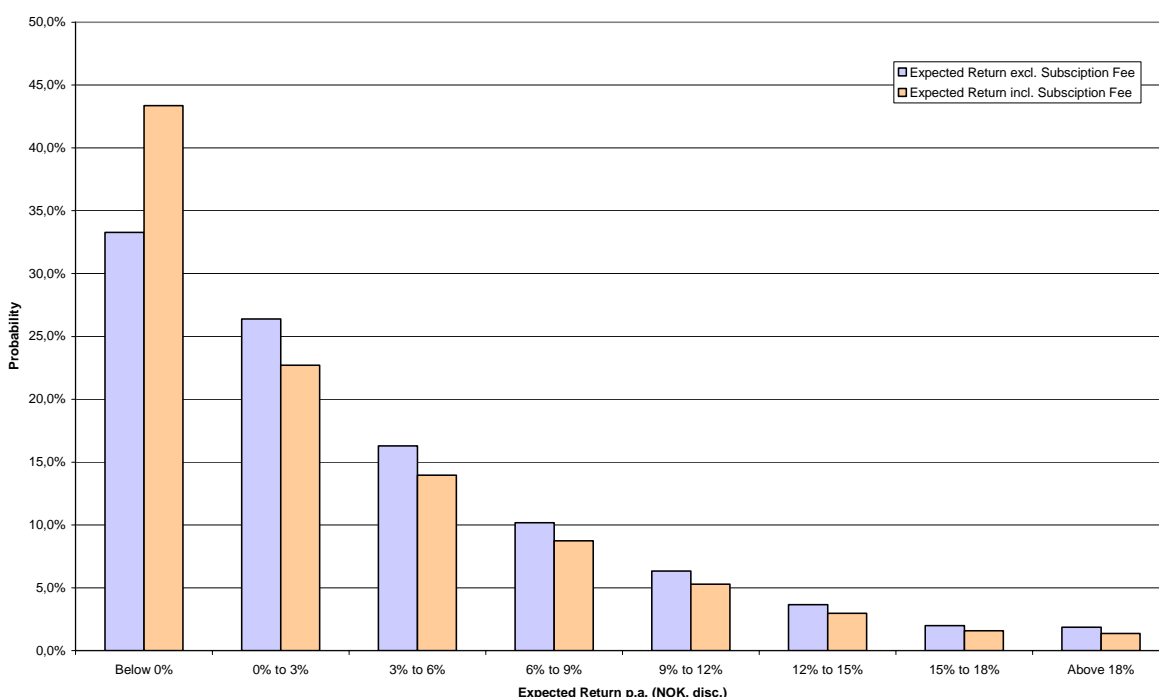
The result above compares well with the general level of expected return for the structured products in Norway, as presented in the recent report from *Kredittilsynet* (2008).

Although one cannot directly compare total return on the whole investment of NOK 100 and the investment of the difference between the face value and the *CE* (NOK 14,4), it is informative to look at the expected return estimate that was made by DnB NOR for their Warrant Kraft 2007/2010 which was issued in October 2007 on the same three underlying forward contracts, with somewhat longer option lives, at NOK 15,50 premium. DnB NOR

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[31] The maximum loss on the investment can be -0,7% p.a. (or -1,6% p.a. with the fee), due to the Guaranteed Amount.

discloses that the average return is simulated at **1,56%** p.a., although the bank adds "*...that the calculations are done on purely theoretical basis, and that in reality the market movements often do not follow the theory. DnB NOR Markets believes in a much better market development than the theoretical expected return indicates...[32]*".

7.6 Putting It All Together

In this subsection results of the calculations described above are presented in a tabular form:

Table 10: Bond valuation – summary of results

	Vol_1	Vol_2	Vol_3	Return element		Total value		Comm. fee[33]	
				Black[76]	MC	Black[76]	MC	Black[76]	MC
High	33,0%	26,7%	25,0%	11,31	12,58	97,17	98,44	2,83	1,56
	30,4%	24,7%	23,0%	10,72	11,56	96,57	97,42	3,43	2,58
	29,0%	23,7%	22,0%	10,42	11,11	96,28	96,97	3,72	3,03
	27,7%	22,6%	21,0%	10,12	10,58	95,98	96,44	4,02	3,56
Base	**26,4%**	**21,5%**	**20,0%**	**9,82**	**10,08**	**95,68**	**95,93**	**4,32**	**4,07**
	25,1%	20,4%	19,0%	9,53	9,56	95,38	95,42	4,62	4,58
	23,8%	19,4%	18,0%	9,23	9,10	85,08	94,95	4,92	5,05
	22,4%	18,3%	17,0%	8,93	8,58	94,78	94,43	5,22	5,57
Low	21,1%	17,2%	16,0%	8,63	8,10	94,48	93,95	5,52	6,05
DnB	**24,6%**	**23,8%**	**22,8%**	**10,52**		**96,37**		**3,63**	

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[32] My translation.

[33] The Subscription fee is not included.

Table 11: Expected return analysis – summary of results

	Ex. subscr.fee	Incl. subscr.fee
Expected return	**3,55% p.a.**	**2,54% p.a.**
Return below 0%	p = 33,4%	p = 44,2%
Return 0% to 3%	p = 25,6%	p = 21,9%
Return 3% to 6%	p = 15,7%	p = 13,5%
Return 6% to 9%	p = 10,0%	p = 8,6%
Return 9% to 12%	p = 6,3%	p = 5,3%
Return 12% to 15%	p = 3,8%	p = 3,1%
Return 15% to 18%	p = 2,2%	p = 1,7%
Return above 18%	p = 2,1%	p = 1,6%
All return	**p = 100%**	**p = 100%**
Return over $r_f = 4,5\%$	**p = 31,4%**	**p = 26,4%**

8. Conclusions and Final Comments

8.1 Conclusion of the Value of and Expected Return

The normative or quantitative objective set forth in this thesis was, based on one of the issues of Nordea Kraftobligasjon Index Bond, to perform component analysis of the product, estimate value of the components including uncertainty level, and finally assess the return an investor can expect to receive from the bond. Nordea Kraftobligasjon XV was chosen for analysis.

The component analysis indicates that at the settlement date the index bond consists of:

- Certain element (the present value of the Guaranteed Amount).

- Return element (the average value of three European call options on electricity forward contracts expiring in 0,8 years, 1,8 years and 2,8 years, respectively, multiplied by the Return or Participation Factor of 1,0, with the pay-outs delayed until the maturity of the bond in 3,0 years).

- Total of Premium (none for Kraftobligasjon XV), Subscription fee and Commission fee (also sometime called "hidden fee").

Based on two calculation methods, the Black 76 closed-form solution and the Monte Carlo simulation, and given volatility levels equal to implied volatility, one can conclude that the value of the Return element lies in a range of NOK 9,00 to NOK 12,00 on a face value of NOK 100. This is significantly lower than the range NOK 12,75 to NOK 16,75 indicated by the issuer in the offer document.

The implicit borrowing rate is 2,8% p.a. or 1,7% p.a., depending on whether or not one takes into account the subscription fee.

The critical uncertainties in the calculation are (a) the nature and parameters of the underlying forward price process, (b) treatment of the correlation between returns on the underlying contracts, and (c) the volatility term structure and level. Sensitivity analysis resulting in the value range mentioned above was done on the volatility levels. Price

processes other than the assumed GBM, *e.g.* mean-reverting forward prices with spikes, may lead to higher option values.

The analysis indicates that the return profile on the product is not attractive: the expected return at approx. 3,6% p.a. (2,5% p.a. if the subscription fee is taken into account). While there is only approx. 25% chance to receive abnormally high return (over 6% p.a.), the probability of return under 3% p.a. is 60%. There is only approx. one in three chance (31,4%) that the product's return will surpass the risk-free rate of 4,5% p.a. (three-year government bond yield).

In its offer document, the issuer does not provide certain critical information such as its borrowing rate, implied market volatilities, expected return range and, where possible, market prices for similar options. Market prices for two of the three options were in fact available, while the third could probably be quoted on the OTC market.

It is imprudent to generalise based only on the valuation of a single issue of only one structured product. However, the valuation and expected analysis conclusions in this thesis fit the broader trend documented in, *inter alia*, Koekebakker and Zakamouline (2007), Bøe (2007) and *Kredittilsynet* (2008).

8.2 Methodological and Practical Challenges for a Non-professional Investor

The descriptive or qualitative objective of this thesis was, based on the example of calculations above, to identify methodological and practical challenges that a non-professional, retail investor may meet attempting to make a prudent and independent investment decision.

 Views on what constitutes a prudent investment process and how independent it should be will vary from investor to investor. As this thesis does not purport to normatively answer this question, the list of challenges below is by nature subjective.

- A non-professional investor may find it difficult to identify and separate the components of a structured product. Although the structure of the Nordea Kraftobligasjon seems clear upon a closer review, understanding it requires use of such financial concepts as value additivity, present value, options and volatility,

guaranteed and expected return, *etc.* The fee structure with thee elements (subscription fee, commission fee and premium) seems unnecessary complicated. One could also question why the bond element and the option element should be bundled together if the purpose was to offer investors exposure to the power prices. The financing rationale of an index bond is even less clear if one takes into account that the issuer offers debt financing of its own bond.

- Several structural elements of the Option part of the bond complicate the investment assessment further. The effects of such features as averaging of return on several underlying indices/contracts, averaging of close and sometimes start values over several days, quoting of indices/contracts in foreign currency and the currency treatment in pay-out calculation are all almost impossible for a non-professional investor to quantify. Simpler and more clear structures, with comparable derivatives traded on the market would be much easier for an investor to price. Even the currency issue with electricity forwards could probably be separated as a swap product.

- The consequences of choice of underlying indices on shares paying dividends or on commodities offering convenience yield may be difficult for a retail investor to understand. Even more so could be to understand the fundamental differences in expected growth in forward prices and the underlying spot prices. At least in theory, forward prices are expected future spot prices adjusted for risk. A negative risk premium on forwards can be particularly challenging for an investor to relate to.

- Any investor who invests in options will have to relate to the notion of volatility. The volatility is notoriously difficult to measure and forecast, particularly in the electricity markets. Access to market-derived measures of relevant volatility would help a retail investor. Without volatility estimates for all underlying for all maturities, one cannot independently price the return element of a index bond.

- All arguments above are relevant for all underlying assets. However, as soon as the underlying are commodity and especially electricity contracts, the degree of complexity increases. If an investor had difficulties pricing options on stock indices, he would have real problems valuing options on electricity derivatives. Forward

contracts are "nicer" in this respect, but this is still probably the most advanced area of quantitative finance.

- One key parameter, namely the Return factor, is unknown at the time of the investment decision. Any change in this parameter may alter the expected return characteristics materially. It is uncertain why the investor should bear the issuer's hedging risks.

- Practical challenges such as spreadsheet modelling of derivative can also hamper the process.

Many of the concerns above can be alleviated if the issuer is to provide valuation, expected return, uncertainty and any other relevant supporting information (such as volatilities and option market quotes) together with the offer. A discussion between the issuer and the non-professional investor about the topics mentioned above would also insure that the investor understands and agrees to the terms and implications. Recent regulatory development from *Kredittilsynet*[34] appears to address the issues above.

8.3 Suggested Further Work

In this thesis the quantitative objective is met from a retail investor point of view, meaning that the conceptually simplest and least modelling- and estimation-demanding solutions are chosen. In addition, only publically available sources of information were used. Thus some interesting and promising angles and routes have remained unexplored. Suggestions for further work related to the topics discussed in this thesis are presented below:

- The product structure and internal value estimates have not been discussed with Nordea. Such discussion could shed a new light.

- It will be interesting to discuss the estimation procedures and models for implied forward volatility with professional players in the power derivative market. Anyhow, the OTC quotes for relevant options would simplify the valuation procedure significantly.

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[34] Directive 4/2008 of 12.02.2008 from The Financial Supervisory Authority of Norway (*Kredittilsynet*).

- It could be interesting to pursue further closed-form solutions for processes with mean reversion, spikes and stochastic volatility.

- The correlation between the three underlying forward contracts was not explored in depth. One could model the estimated correlations explicitly into the Monte Carlo simulation, *e.g.* by way of the Cholesky decomposition.

- Following Bjerksund, it could be interesting to calculate volatilities of each process, correlations and resulting volatility of the average return directly as a part of Monte Carlo simulation. Thereafter one could compare them to the analytical approximation.

- One could complete the analysis presented in this thesis with expected return calculations which take into account debt financing.

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Appendix A:
Information Provided for NKIB-II to NKIB-XV

Table 1

	Name	ISIN nr	Nordea ticker		Offer date	Payment date	Binding date	Maturity date	Years to maturity	Subscription fee	Early retir. fee	Protection against jumps	Priority/pledge	Return factor indicated	Return factor fixed	Return reduction element	Certain element	Ceiling	Contract currency
NKIB-I	Kraftobligasjon I																		
NKIB-II	Kraftobligasjon II	NO 0010009946.8	RC47	2001-04	2001-05-22	2001-07-06	2001-07-05	2004-12-23	3.46	3.0%	<1.0%	Yes	Ordinary/No	1.50	1.50	0%	100%		NOK
NKIB-III	Kraftobligasjon III	NO 0010011934.0	RC52	2001-05	2001-10-15	2001-12-14	2001-12-13	2005-12-23	4.03	4.0%	<1.0%	Yes	Ordinary/No	1.50	1.50	0%	100%		NOK
NKIB-IV	Kraftobligasjon IV	NO 0010017096.3	RC59	2003-05	2002-12-02	2003-01-31	2003-01-30	2005-12-27	2.91	4.0%	<1.0%	Yes	Ordinary/No	1.25	1.25	0%	100%		NOK
NKIB-V	Kraftobligasjon V	NO 0010018764.4	RC62	2003-05	2003-05-26	2003-07-03	2003-07-03	2005-05-19	1.88	4.0%	<1.0%	Yes	Ordinary/No	1.10	1.10	10%	90%		NOK
NKIB-VI	Kraftobligasjon VI	NO 0010019313.9	RC63	2003-05	2003-07-07	2003-07-31	2003-08-05	2005-05-19	1.80	4.0%	<1.0%	Yes	Ordinary/No	1.10	1.10	10%	90%		NOK
NKIB-VII	Kraftobligasjon VII	NO 0010023116.0	RC90->Y777	2004-06	2004-06-10	2004-06-10	2004-07-22	2006-12-28	2.45	4.0%	0.5%	Yes	Ordinary/No	1.60	1.60	10%	90%		NOK/EUR
NKIB-VIII	Kraftobligasjon VIII	NO 0010024873.5		2005-08	2004-12-31	2005-01-12	2005-01-24	2008-04-14	3.26	4.0%	0.5%	Yes	Ordinary/No		1.00	5%	95%		EUR
NKIB-IX	Kraftobligasjon IX	NO 0010028227.0	RC38	2005-09	2005-08-31	2005-09-30	2005-10-14	2009-05-29	3.66	4.0%	0.5%	Yes	Ordinary/No	1.25	1.25	5%	95%	158%	EUR
NKIB-X	Kraftobligasjon X	NO 0010285025	RC38	2006-09	2005-12-12	2005-12-31	2006-01-13	2006-05-29	3.38	4.0%	0.5%	Yes	Ordinary/No	0.95	0.95	5%	100%		EUR
NKIB-XI	Kraftobligasjon XI	NO 0010320898.8	RC104	2006-09	2006-06-06	2006-06-30	2006-07-11	2009-06-30	3.00	4.0%	0.5%	Yes	Ordinary/No	1.00	1.00	5%	95%		EUR
NKIB-XII	Kraftobligasjon XII	NO 0010303228.1	RC110	2006-09	2006-10-06	2006-09-11	2006-10-17	2009-12-30	3.23	4.0%	0.5%	Yes	Ordinary/No	1.15	1.15	5%	95%		EUR
NKIB-XIII	Kraftobligasjon XIII	NO 0010334943.4	RC116	2007-10	2007-01-08	2007-02-09	2007-02-20	2010-01-12	2.93	4.0%	0.5%	Yes	Ordinary/No	1.10	1.36	0%	100%		EUR
NKIB-XIV	Kraftobligasjon XIV	NO 0010398327.6	RC125	2007-10	2007-08-24	2007-10-05	2007-10-16	2010-01-05	2.25	3.0%	0.5%	Yes	Ordinary/No	1.05	1.05	0%	98%		EUR
NKIB-XV	Kraftobligasjon XV	NO 0010040610.1	RC136	2008-11	2008-01-11	2008-02-15	2008-02-26	2011-02-15	3.00	3.0%	0.5%	Yes	Ordinary/No	1.00	1.10	0%	98%		EUR

* FWYR-05 listed on Nord Pool first on 02.01.02. FWYR-06 listed on Nord Pool on 02.01.03 in EUR.

** Subscription fee ("tegningsomkostninger") for ordinary retail investors (not the bank's premium clients) on the first NOK 99.000 for NKIB-I, II; on the first NOK 499.000 for NKIB-V to -VIII;
on the first NOK 999.000 for NKIB-IX, X; on the first NOK 990.000 for NKIB-XI to -XV.

*** Highest commission/fee at early retirement of the loan.

**** Nordea is protected against "market disruption" on the Stop-days. Market disruptions were vaguely defined for NKIB 1-XII, from NKIB-XIV ref. to the ISDA market regulation.
Further, Nordea protected against too high indicated Return factor if the hedge proves to be too expensive (resulting in Return factor under 0,90).
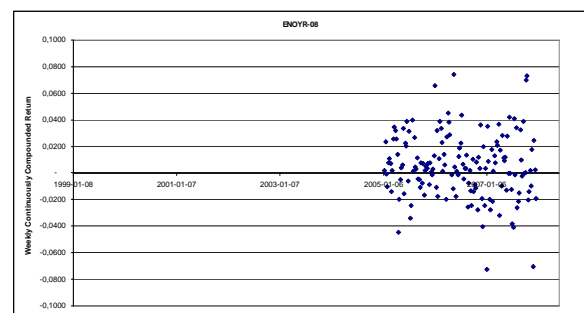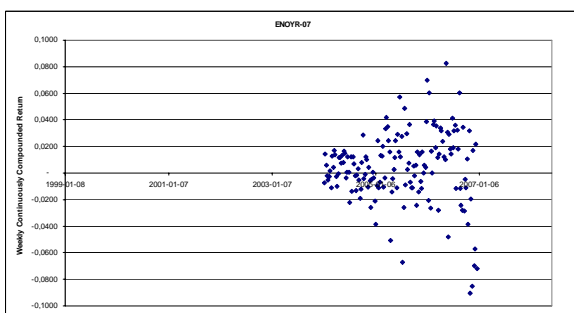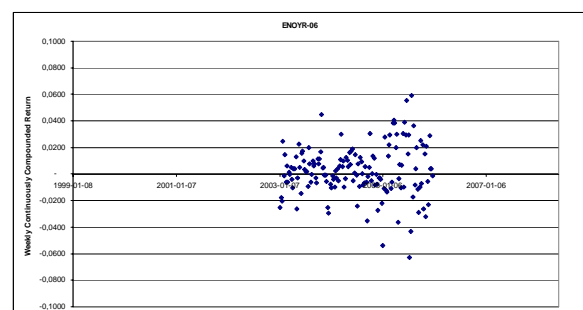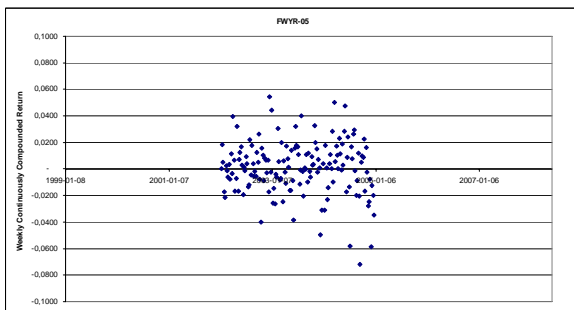
Source: Nordea web site

Table 2

	Name	Exchange	Start	Contract 1	start quote	stop quote	Stop 1	Contract 2	start quote	stop quote	Stop 2	Contract 3	start quote	stop quote	Stop 3	Contract 4	start quote	stop quote	Stop 4
NKIB-I	Kraftobligasjon I																		
NKIB-II	Kraftobligasjon II	NordPool	2001-04	FWYR-03	179.50	316.00	2002-12-19	FWYR-04	183.50	244.00	2003-12-18	FWYR-05	2004-12-16						
NKIB-III	Kraftobligasjon III	NordPool	2001-12-14	FWYR-04	174.75	244.00	2003-12-18	FWYR-05	OTC	222.00	2004-12-16	FWYR-06	2005-12-15						
NKIB-IV	Kraftobligasjon IV	NordPool	2003-01-31	FWYR-04	186.25	244.00	2003-12-18	FWYR-05	182.50	222.00	2004-12-16	FWYR-06	2005-12-15	23.68					
NKIB-V	Kraftobligasjon V	NordPool	2003-07-03	FWV1-04	263.50	281.50	2003-12-18	FWV1-05	233.50	225.98	2004-12-16								
NKIB-VI	Kraftobligasjon VI	NordPool	2003-07-31	FWV1-04	311.25	281.50	2003-12-18	FWV1-05	240.50	225.98	2004-12-15								
NKIB-VII	Kraftobligasjon VII	NordPool	2004-07-15	FWYR-05	266.25	222.00	2004-12-16	FWYR-06	29.52	36.30	2005-12-15	FWYR-07	2006-12-21	29.67	37.40				
NKIB-VIII	Kraftobligasjon VIII	NordPool	2005-01-12	FWYR-06	25.30	36.30	2005-12-15	FWYR-07	26.27	37.40	2006-12-21	FWYR-08	2007-12-21	27.05	50.02				
NKIB-IX	Kraftobligasjon IX	NordPool	2005-09-30	FWYR-06	36.25	36.90	2005-12-11	FWYR-07	35.28	36.30	2006-12-11	FWYR-08	2007-12-11	34.66	51.25	FWYR-09	2008-12-11		
NKIB-X	Kraftobligasjon X	EEX	2006-01-13/16	FWYR-07	51.85	52.74	2006-02-06/07	FWYR-08	49.08	59.80	2007-12-25/06	FWYR-09	2008-12-03/04	48.34	n/a				
NKIB-XI	Kraftobligasjon XI	NordPool	2006-06-30	FWYR-07	46.95	38.88	2006-12-14	FWYR-08	43.70	51.00	2007-12-14	FWYR-09	2008-12-12	42.95	n/a				
NKIB-XII	Kraftobligasjon XII	NordPool	2006-10-06	FWQ1-07	69.45	41.40	2006-09-11	FWYR-08	44.38	50.78	2007-12-04/10	FWYR-09	2008-12-04/10	43.75	n/a	FWYR-09	34.91		
NKIB-XIII	Kraftobligasjon XIII	NordPool	2007-02-09	FWYR-08	39.85	50.78	2007-12-04/10	FWYR-09	40.98	n/a	2008-12-04/10	FWYR-10	2009-12-07/11	41.90	n/a	FWYR-10	44.45		
NKIB-XIV	Kraftobligasjon XIV	NordPool	2007-10-05/11	FWYR-09	48.36	n/a	2008-12-15/19	FWYR-10	48.09	n/a	2009-12-14/18								
NKIB-XV	Kraftobligasjon XV	NordPool	2008-02-15	FWYR-09	53.10	n/a	2008-12-24/10	FWYR-10	52.50	n/a	2009-12-04/10	FWYR-11	2010-12-06/10	52.15	n/a				

Source: Nordea web site

Table 3

	Name	Payment date	Estimate date	Certain el. value	Return el. value (incl. AF) low	best	high	Total value	Premium value	Commission fee	Subscription fee	NIBOR 3M nom	Nordea NIBOR-Mb.p.	Gov. ref.	Contract 1 vol.	Contract 2 vol.	Contract 3 vol.
NKIB-I	Kraftobligasjon I																
NKIB-II	Kraftobligasjon II	2001-04	2001-07-06	100.00%					0.0%	3.0%	7.36%	3.86%					
NKIB-III	Kraftobligasjon III	2001-05	2001-12-14	100.00%					0.0%	4.0%	6.50%	4.36%					
NKIB-IV	Kraftobligasjon IV	2003-05	2003-01-31	100.00%					0.0%	4.0%	5.84%	4.81%					
NKIB-V	Kraftobligasjon V	2003-05	2003-07-03	90.00%					0.0%	4.0%	3.59%	4.74%					
NKIB-VI	Kraftobligasjon VI	2003-05	2003-07-31	90.00%					0.0%	4.0%	3.36%						
NKIB-VII	Kraftobligasjon VII	2004-06	2004-07-15	90.00%	82.30%	15.20%		97.50%	0.0%	4.0%	2.05%					
NKIB-VIII	Kraftobligasjon VIII	2005-08	2005-01-12	95.00%	86.70%	10.90%		97.60%	0.0%	4.0%	1.99%					
NKIB-IX	Kraftobligasjon IX	2005-09	2005-09-30	95.00%	84.45%	12.88%		97.33%	0.0%	4.0%	2.44%					
NKIB-X	Kraftobligasjon X	2006-09	2006-01-13	100.00%	88.00%	9.45%		97.45%	0.0%	4.0%	2.52%					
NKIB-XI	Kraftobligasjon XI	2006-09	2006-06-30	95.00%	84.25%	13.50%		97.75%	0.0%	4.0%	3.05%					
NKIB-XII	Kraftobligasjon XII	2006-09	2006-10-06	95.00%	82.84%	14.66%		97.60%	0.0%	4.0%	3.45%			26.0%		
NKIB-XIII	Kraftobligasjon XIII	2007-10	2007-01-02	100.00%	86.69%	16.06%	17.70%	102.75%	5.0%	3.0%	4.17%	4.14%				
NKIB-XIV	Kraftobligasjon XIV	2007-10	2007-08-17	98.00%	86.95%	11.37%	12.80%	98.32%	0.0%	3.0%	5.72%	5.67%			22.5%	
NKIB-XV	Kraftobligasjon XV	2008-11	2008-01-10	98.00%	83.75%	14.00%	16.75%	97.75%	0.0%	3.0%	6.01%	5.98%				20.0%

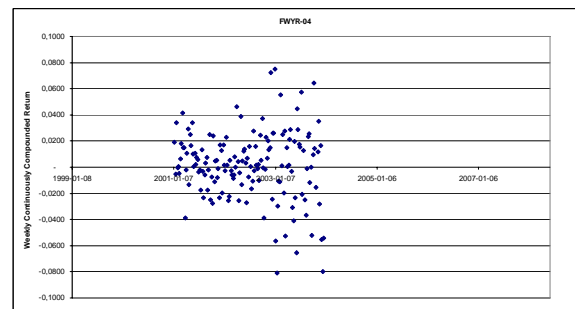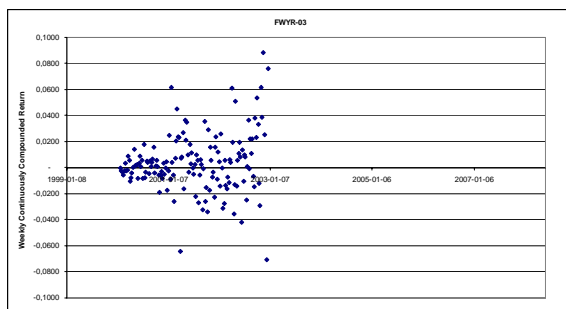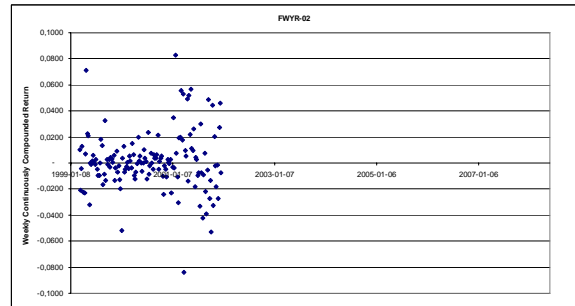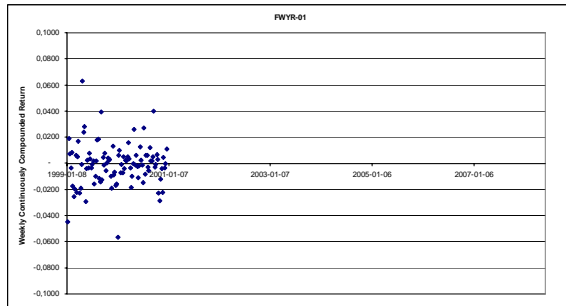| 14.60% | | | | | |
| 10.00% | | | | | |
| 12.75% | | | | | |

Source: Nordea web site

Appendix B:
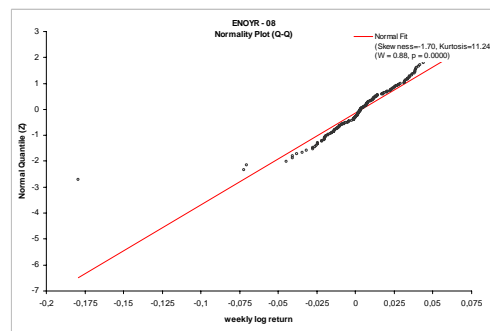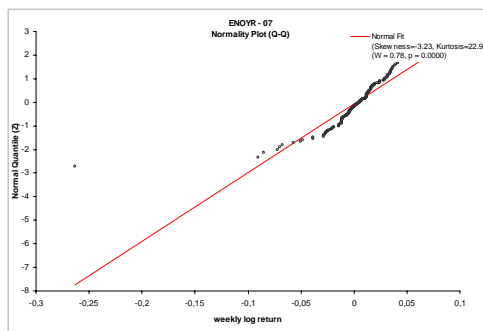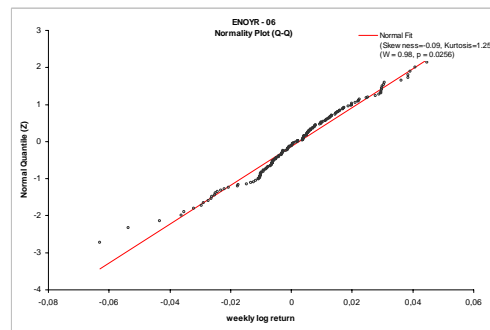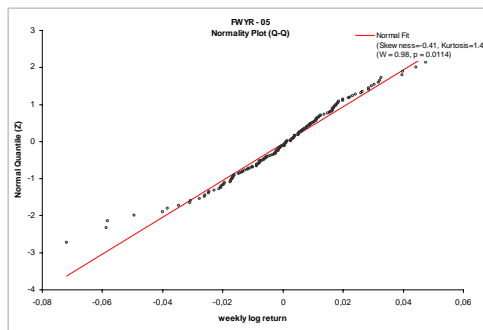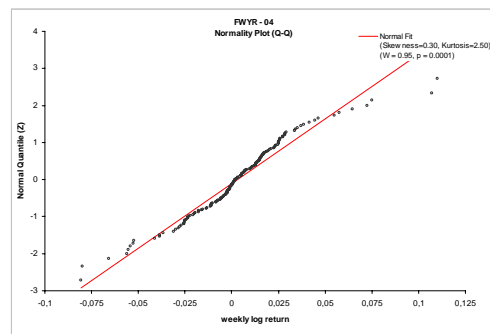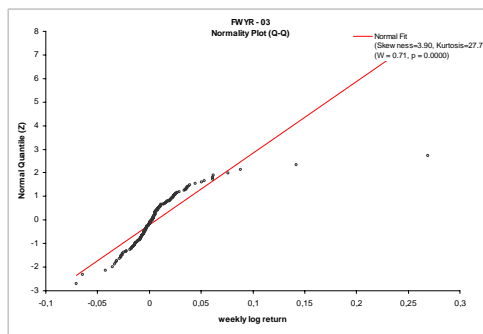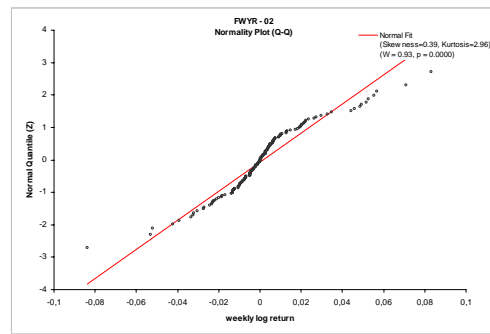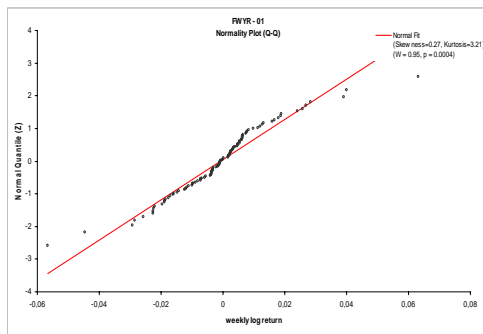# Historical Spot and Forward Prices, by Date

Source: Nord Pool

Appendix C: Weekly Returns on Forwards, by Date

Source: Nord Pool (Few returns above 10% are not shown on the graphs).

Appendix D: Shapiro-Wilk Normality Test

Appendix E: VBA Function Code Useful in MC Simulation

VBA Code for the Cholesky Decomposition[35]

Public Function Cholesky(Mat As Range)
'Function returns a square matrix L which is Cholsky decomposition of input
'matrix. Input matrix must be square, symmetric, positive definite.

```
        Dim A, L() As Double, s As Double
        Dim n As Integer, M As Integer, i As Integer, j As Integer, k As Integer
        A = Mat
        n = Mat.Rows.Count
        M = Mat.Columns.Count
        If n <> M Then
                Cholesky = "?"
                Exit Function
        End If

        ReDim L(1 To n, 1 To n)
        For j = 1 To n
                s = 0
                For k = 1 To j - 1
                        s = s + L(j, k) ^ 2
                Next k
                L(j, j) = A(j, j) - s
                If L(j, j) <= 0 Then Exit For
                L(j, j) = Sqr(L(j, j))

                For i = j + 1 To n
                        s = 0
                        For k = 1 To j - 1
                                s = s + L(i, k) * L(j, k)
                        Next k
                        L(i, j) = (A(i, j) - s) / L(j, j)
                Next i
        Next j
        Cholesky = L
End Function
```

---

[35] The code is provided by Anton Theunissen at http://www.sfu.ca/~rjones/bus864/notes/BUS_864_Computing_Notes.pdf .

VBA Code for Moros Transformation[36]

Option Explicit
Option Base 1
' Option Explicit means that all variables MUST BE declared
' Option Base 1 means that arrays start with 1 instead the default 0
' Option Base 1 is necessary for Moro_NormSInv function and others

Function Moro_NormSInv(u As Double) As Double
'Calculates the Normal Standard numbers given u, the associated uniform number (0, 1)
'VBA version of the Moro's (1995) code in C
'Option Base 1 is necessary to be declared before this function for vector elements positioning to work

> *Dim c1, c2, c3, c4, c5, c6, c7, c8, c9*
> *Dim X As Double*
> *Dim r As Double*
> *Dim a As Variant*
> *Dim b As Variant*
> *a = Array(2.50662823884, -18.61500062529, 41.39119773534, -25.44106049637)*
> *b = Array(-8.4735109309, 23.08336743743, -21.06224101826, 3.13082909833)*
> *c1 = 0.337475482272615*
> *c2 = 0.976169019091719*
> *c3 = 0.160797971491821*
> *c4 = 2.76438810333863E-02*
> *c5 = 3.8405729373609E-03*
> *c6 = 3.951896511919E-04*
> *c7 = 3.21767881768E-05*
> *c8 = 2.888167364E-07*
> *c9 = 3.960315187E-07*
> *X = u - 0.5*
>
> *If Abs(X) < 0.42 Then*
> > *r = X ^ 2*
> > *r = X * (((a(4) * r + a(3)) * r + a(2)) * r + a(1)) / ((((b(4) * r + b(3)) * r + b(2)) * r + b(1)) * r + 1)*
> *Else*
> > *If X > 0 Then r = Log(-Log(1 - u))*
> > *If X <= 0 Then r = Log(-Log(u))*
> > *r = c1 + r * (c2 + r * (c3 + r * (c4 + r * (c5 + r * (c6 + r * (c7 + r * (c8 + r * c9)))))))*
> > *If X <= 0 Then r = -r*
> *End If*
> *Moro_NormSInv = r*
End Function

---

[36] The code is provided by Bøe (2007), published by Jackson and Staunton (2001). The same code published by Marco A.G. Dias at http://www.puc-rio.br/marco.ind/spreads_list.html (QMC_Black_Scholes.xls).

VBA Code for Halton Sequence[37]

Function HaltonBaseb(b As Long, N As Long) As Double
'Returns the equivalent first Halton sequence number

```
        Dim h As Double, ib As Double
        Dim i As Long, n1 As Long, n2 As Long
        n1 = N
        h = 0
        ib = 1 / b
        Do While n1 > 0
                n2 = Int(n1 / b)
                i = n1 - n2 * b
                h = h + ib * i
                ib = ib / b
                n1 = n2
        Loop
        HaltonBaseb = h
End Function
```
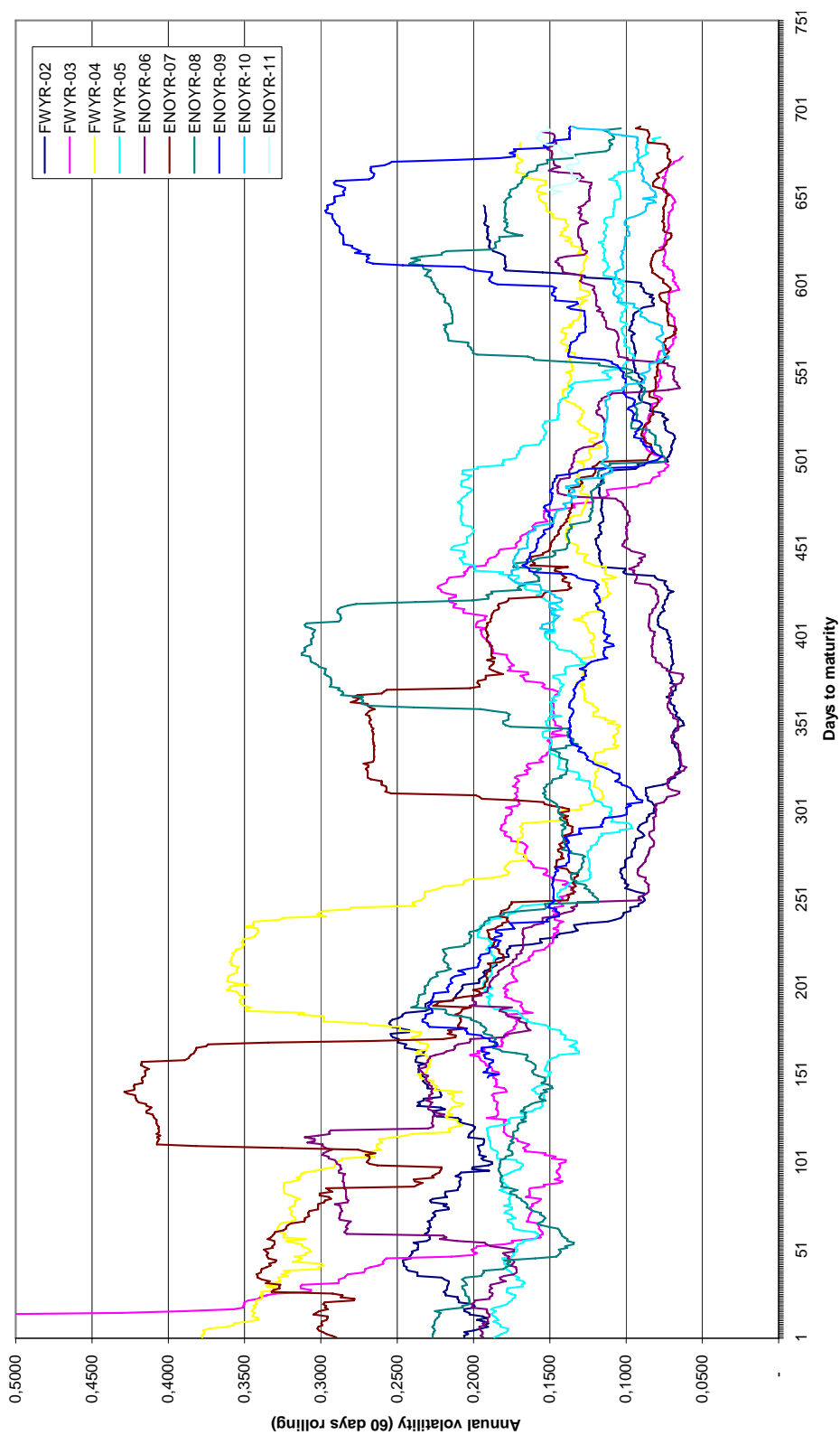
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[37] The code is provided by Marco A.G. Dias at http://www.puc-rio.br/marco.ind/quasi_mc.html .

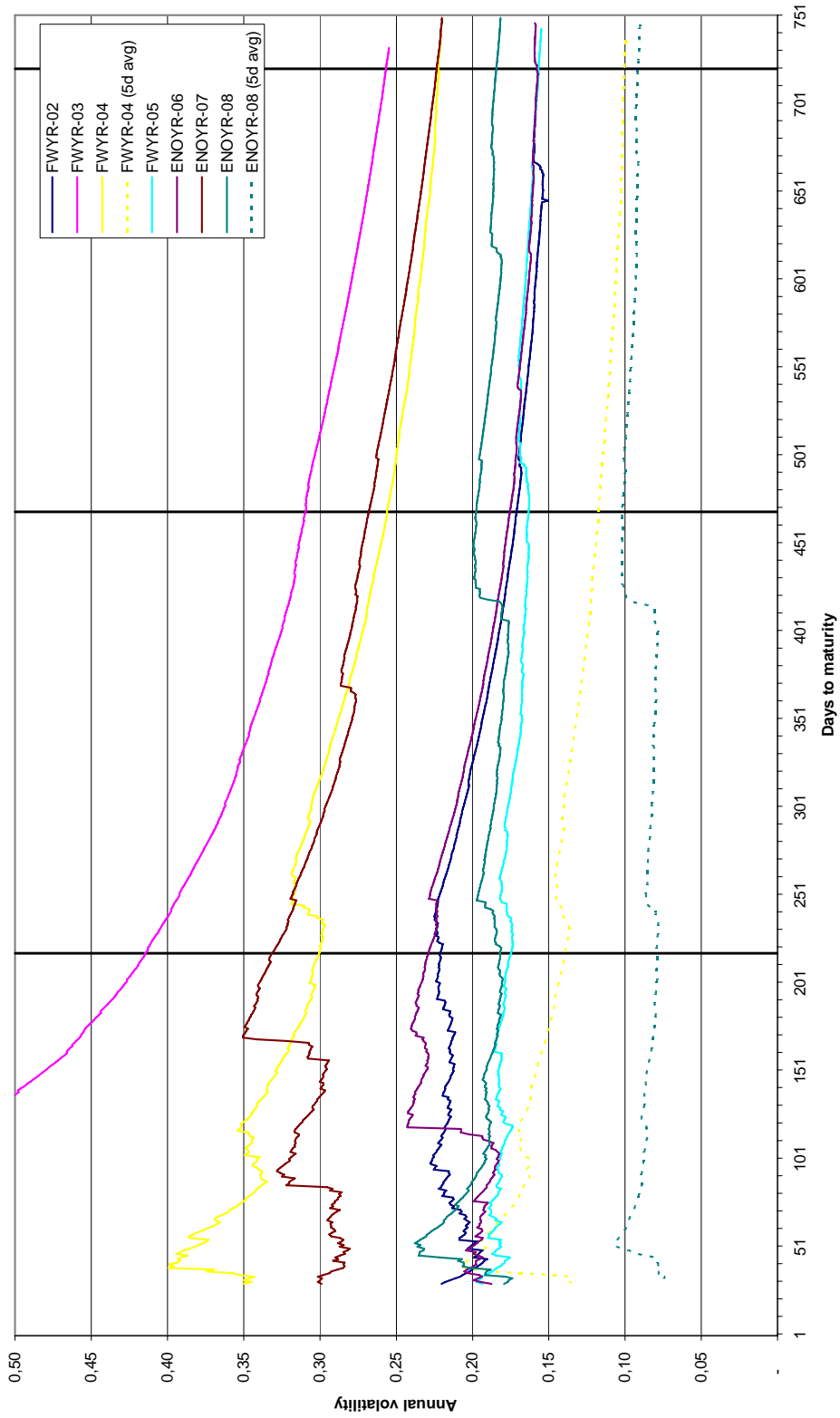# Appendix F: Rolling Volatility, by Date
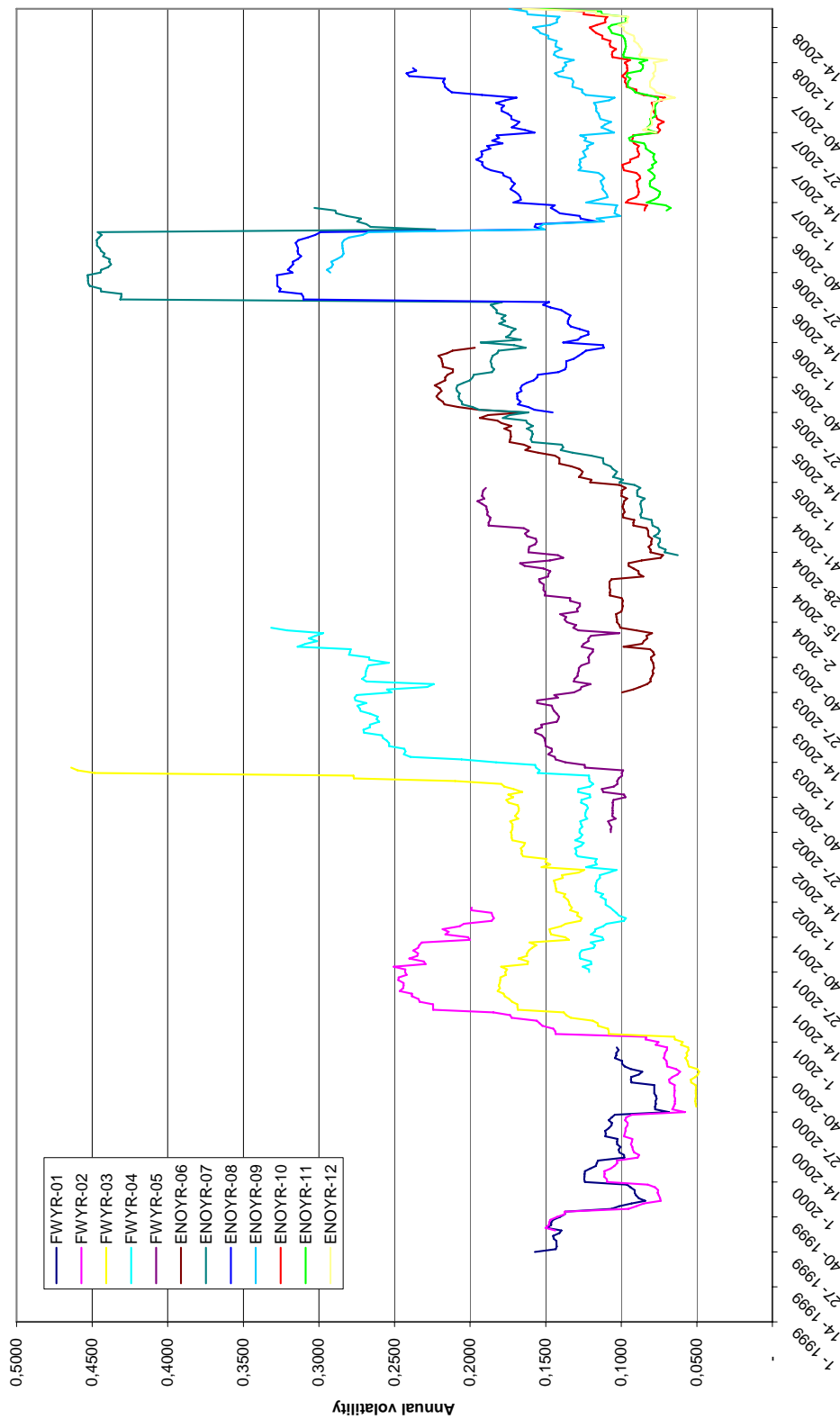
Source: Nord Pool.

Appendix G: Rolling Volatility, by Days to Maturity

Source: Nord Pool.

Appendix H: Cumulative Volatility, by Days to Maturity

Source: Nord Pool.

Appendix I: VBA Code for Simulation-based Analysis

Option Explicit
Option Base 1
' Option Explicit means that all variables MUST BE declared
' Option Base 1 means that arrays start with 1 instead the defaul 0
' Option Base 1 is necessary for Moro_NormSInv function and others

Public Vol_01 As Double, Vol_02 As Double, Vol_03 As Double
Public RiskPrem As Double
Public Cavg As Double
Public SD As Double
Public FV As Double
Public TotInvexSF As Double, TotInvincSF As Double
Public GA As Double
Public RetThres As Double
Public Bound1 As Double, Bound2 As Double, Bound3 As Double, Bound4 As Double, Bound5 As Double, Bound6 As Double, Bound7 As Double
Public Retavg As Double, Retoveravg As Double
Public RetSFavg As Double, RetSFoveravg As Double
Public Retrange1avg As Double, Retrange2avg As Double, Retrange3avg As Double, Retrange4avg As Double, Retrange5avg As Double, Retrange6avg As Double, Retrange7avg As Double, Retrange8avg As Double
Public RetSFrange1avg As Double, RetSFrange2avg As Double, RetSFrange3avg As Double, RetSFrange4avg As Double, RetSFrange5avg As Double, RetSFrange6avg As Double, RetSFrange7avg As Double, RetSFrange8avg As Double

---

Sub MCSimulator()
On Error GoTo ErrorTrap

' External arguments
Dim Sens As String
Dim RetAn As String
Dim CalcTime As Date
Dim FirstRow As Long
Dim Rows As Long
Dim FirstRowRet As Long
Dim RowsRet As Long
Dim Premium As Double, SF As Double
Dim GF As Double

'Internal variables
Dim CalcStartTime As Date, CalcStopTime As Date
Dim k As Long

CalcStartTime = Now()

```
Worksheets("Calculations").Activate
Application.StatusBar = "Calculating......... Please wait"

'Passing input arguments
FV = Range("FV")
Premium = Range("Premium")
SF = Range("SF")
GF = Range("GF")
RetThres = Range("RetThres")
Bound1 = Range("Bound1")
Bound2 = Range("Bound2")
Bound3 = Range("Bound3")
Bound4 = Range("Bound4")
Bound5 = Range("Bound5")
Bound6 = Range("Bound6")
Bound7 = Range("Bound7")

'Global preparatory calculations
TotInvexSF = FV + FV * Premium
TotInvincSF = TotInvexSF + FV * SF
GA = FV * GF

'Runing base-case simulation
RiskPrem = 0
Vol_01 = Range("Vol_01")
Vol_02 = Range("Vol_02")
Vol_03 = Range("Vol_03")

    MonteCarloSimulation

'Transfering base-case output arguments
Range("Cavg").FormulaR1C1 = Cavg
Range("SD").FormulaR1C1 = SD

'Sensitivity analysis ->
Sens = Range("Sens")
Select Case Sens
    Case "No"
        GoTo ReturnAnalysis

    Case "Yes"
'Running sensitivities
        RiskPrem = 0
        FirstRow = Range("FirstRow")
        Rows = Range("Rows")
        For k = 1 To Rows
            Vol_01 = Cells(FirstRow - 1 + k, 3).Value
            Vol_02 = Cells(FirstRow - 1 + k, 4).Value
            Vol_03 = Cells(FirstRow - 1 + k, 5).Value
```

```
        MonteCarloSimulation

'Transfering sensitivities output arguments
        Cells(FirstRow - 1 + k, 7).Value = Cavg
        Cells(FirstRow - 1 + k, 14).Value = SD
    Next k
    Range("SensUpdateTime").FormulaR1C1 = Now()

  Case Else
    GoTo ErrorTrap
End Select

' Return analysis ->
ReturnAnalysis:
RetAn = Range("RetAn")
Select Case RetAn
   Case "No"
     GoTo Final

   Case "Yes"
'Running return analysis
    RiskPrem = Range("RiskPrem")
    FirstRowRet = Range("FirstRowRet")
    RowsRet = Range("RowsRet")
    Vol_01 = Range("Vol_01")
    Vol_02 = Range("Vol_02")
    Vol_03 = Range("Vol_03")

        MonteCarloSimulation

'Transfering return analysis output arguments
    Cells(FirstRowRet + 0, 7).Value = Retrange1avg
    Cells(FirstRowRet + 1, 7).Value = Retrange2avg
    Cells(FirstRowRet + 2, 7).Value = Retrange3avg
    Cells(FirstRowRet + 3, 7).Value = Retrange4avg
    Cells(FirstRowRet + 4, 7).Value = Retrange5avg
    Cells(FirstRowRet + 5, 7).Value = Retrange6avg
    Cells(FirstRowRet + 6, 7).Value = Retrange7avg
    Cells(FirstRowRet + 7, 7).Value = Retrange8avg

    Cells(FirstRowRet + 0, 8).Value = RetSFrange1avg
    Cells(FirstRowRet + 1, 8).Value = RetSFrange2avg
    Cells(FirstRowRet + 2, 8).Value = RetSFrange3avg
    Cells(FirstRowRet + 3, 8).Value = RetSFrange4avg
    Cells(FirstRowRet + 4, 8).Value = RetSFrange5avg
    Cells(FirstRowRet + 5, 8).Value = RetSFrange6avg
    Cells(FirstRowRet + 6, 8).Value = RetSFrange7avg
    Cells(FirstRowRet + 7, 8).Value = RetSFrange8avg

    Range("Retavg").FormulaR1C1 = Retavg
```

```
        Range("RetSFavg").FormulaR1C1 = RetSFavg

        Range("Retoveravg").FormulaR1C1 = Retoveravg
        Range("RetSFoveravg").FormulaR1C1 = RetSFoveravg

        Range("RetAnUpdateTime").FormulaR1C1 = Now()


    Case Else
        GoTo ErrorTrap
End Select

Final:
CalcStopTime = Now()
CalcTime = CalcStopTime - CalcStartTime
Range("CalcTime").FormulaR1C1 = CalcTime
Range("UpdateTime").FormulaR1C1 = CalcStopTime

Application.StatusBar = False
MsgBox "The simulation has been successful!"

Exit Sub

ErrorTrap:
    MsgBox "An error has accurred in the wrap! Please check the code."

End Sub
```

```
Sub MonteCarloSimulation()
' Calculates ...
On Error GoTo ErrorTrap

' External arguments

Dim RF As Double
Dim w_1 As Double, w_2 As Double, w_3 As Double
Dim t_1b As Double, t_2b As Double, t_3b As Double, T As Double
Dim Tail_1 As Integer, Tail_2 As Integer, Tail_3 As Integer
Dim Tailstep As Double
Dim rT As Double
Dim CoC_1 As Double, CoC_2 As Double, CoC_3 As Double
Dim N As Double

'Internal variables
Dim Vol_12 As Double, Vol_23 As Double
Dim t_1b2b As Double, t_2b3b As Double
Dim i As Long, j As Long
Dim F1t1 As Double, F2t1 As Double, F3t1 As Double, F2t2 As Double, F3t2 As Double,
F3t3 As Double
Dim F1 As Double, F2 As Double, F3 As Double
```

```
Dim Sum1 As Double, Sum2 As Double, Sum3 As Double
Dim F1t1avg As Double, F2t2avg As Double, F3t3avg As Double
Dim UN As Double, N01 As Double
Dim R1 As Double, R2 As Double, R3, Ravg As Double
Dim R1opt As Double, R2opt As Double, R3opt As Double, Ravgopt As Double
Dim c As Double, Csum As Double
Dim Sqdiv As Double, Sqdivsum As Double
Dim Ret As Double, Retsum As Double
Dim RetSF As Double, RetSFsum As Double
Dim Retoversum As Double
Dim RetSFoversum As Double
Dim Retrange1sum As Double, Retrange2sum As Double, Retrange3sum As Double,
Retrange4sum As Double, Retrange5sum As Double, Retrange6sum As Double,
Retrange7sum As Double, Retrange8sum As Double
Dim RetSFrange1sum As Double, RetSFrange2sum As Double, RetSFrange3sum As Double,
RetSFrange4sum As Double, RetSFrange5sum As Double, RetSFrange6sum As Double,
RetSFrange7sum As Double, RetSFrange8sum As Double
Dim AA As Double, TAR As Double


'Passing input arguments
RF = Range("RF")
w_1 = Range("w_1")
w_2 = Range("w_2")
w_3 = Range("w_3")
t_1b = Range("t_1b")
t_2b = Range("t_2b")
t_3b = Range("t_3b")
T = Range("T")
Tail_1 = Range("Tail_1")
Tail_2 = Range("Tail_2")
Tail_3 = Range("Tail_3")
Tailstep = Range("Tailstep")
rT = Range("r_T")
CoC_1 = Range("CoC_1")
CoC_2 = Range("CoC_2")
CoC_3 = Range("CoC_3")
N = Range("N")
Cavg = Range("Cavg")

'Local preparatory calculations
Vol_12 = Sqr((Vol_02 ^ 2 * t_2b - Vol_01 ^ 2 * t_1b) / (t_2b - t_1b))
Vol_23 = Sqr((Vol_03 ^ 2 * t_3b - Vol_02 ^ 2 * t_2b) / (t_3b - t_2b))
t_1b2b = t_2b - t_1b
t_2b3b = t_3b - t_2b
Csum = 0
Sqdivsum = 0
Retsum = 0
RetSFsum = 0
Retoversum = 0
```

RetSFoversum = 0
Retrange1sum = 0
Retrange2sum = 0
Retrange3sum = 0
Retrange4sum = 0
Retrange5sum = 0
Retrange6sum = 0
Retrange7sum = 0
Retrange8sum = 0
RetSFrange1sum = 0
RetSFrange2sum = 0
RetSFrange3sum = 0
RetSFrange4sum = 0
RetSFrange5sum = 0
RetSFrange6sum = 0
RetSFrange7sum = 0
RetSFrange8sum = 0

For i = 1 To N
'Preparatory calculations for each realisation
  F1t1 = 0
  F2t1 = 0
  F3t1 = 0
  F2t2 = 0
  F3t2 = 0
  F3t3 = 0
  F1 = 0
  F2 = 0
  F3 = 0
  Sum1 = 0
  Sum2 = 0
  Sum3 = 0
  F1t1avg = 0
  F2t2avg = 0
  F3t3avg = 0

'Calculations at t_1
  UN = Rnd
  N01 = Moro_NormSInv(UN)
  *F1t1 = FV * Exp((CoC_1 + RiskPrem - 0.5 * Vol_01 ^ 2) * t_1b + Vol_01 * Sqr(t_1b) ** N01)*
  Sum1 = F1t1
  *F2t1 = FV * Exp((CoC_2 + RiskPrem - 0.5 * Vol_12 ^ 2) * t_1b + Vol_12 * Sqr(t_1b) ** N01)*
  *F3t1 = FV * Exp((CoC_3 + RiskPrem - 0.5 * Vol_23 ^ 2) * t_1b + Vol_23 * Sqr(t_1b) ** N01)*

  For j = 1 To Tail_1 - 1
      UN = Rnd
      N01 = Moro_NormSInv(UN)

$F1 = F1t1 * Exp((CoC\_1 + RiskPrem - 0.5 * Vol\_01 \wedge 2) * Tailstep + Vol\_01 * Sqr(Tailstep) * N01)$

$Sum1 = Sum1 + F1$

Next j

$F1t1avg = Sum1 / Tail\_1$

'Calculations at t_2

$UN = Rnd$

$N01 = Moro\_NormSInv(UN)$

$F2t2 = F2t1 * Exp((CoC\_2 + RiskPrem - 0.5 * Vol\_01 \wedge 2) * t\_1b2b + Vol\_01 * Sqr(t\_1b2b) * N01)$

$Sum2 = F2t2$

$F3t2 = F3t1 * Exp((CoC\_3 + RiskPrem - 0.5 * Vol\_12 \wedge 2) * t\_1b2b + Vol\_12 * Sqr(t\_1b2b) * N01)$

For j = 1 To Tail_2 - 1

$UN = Rnd$

$N01 = Moro\_NormSInv(UN)$

$F2 = F2t2 * Exp((CoC\_2 + RiskPrem - 0.5 * Vol\_01 \wedge 2) * Tailstep + Vol\_01 * Sqr(Tailstep) * N01)$

$Sum2 = Sum2 + F2$

Next j

$F2t2avg = Sum2 / Tail\_2$

'Calculations at t_3

$UN = Rnd$

$N01 = Moro\_NormSInv(UN)$

$F3t3 = F3t2 * Exp((CoC\_3 + RiskPrem - 0.5 * Vol\_01 \wedge 2) * t\_2b3b + Vol\_01 * Sqr(t\_2b3b) * N01)$

$Sum3 = F3t3$

For j = 1 To Tail_3 - 1

$UN = Rnd$

$N01 = Moro\_NormSInv(UN)$

$F3 = F3t3 * Exp((CoC\_3 + RiskPrem - 0.5 * Vol\_01 \wedge 2) * Tailstep + Vol\_01 * Sqr(Tailstep) * N01)$

$Sum3 = Sum3 + F3$

Next j

$F3t3avg = Sum3 / Tail\_3$

'Calculations of returns and Additional Amounts

$R1 = F1t1avg / FV - 1$

$R2 = F2t2avg / FV - 1$

$R3 = F3t3avg / FV - 1$

$Ravg = R1 * w\_1 + R2 * w\_2 + R3 * w\_3$

If R1 > 0 Then R1opt = R1 Else R1opt = 0

If R2 > 0 Then R2opt = R2 Else R2opt = 0

If R3 > 0 Then R3opt = R3 Else R3opt = 0

$Ravgopt = R1opt * w\_1 + R2opt * w\_2 + R3opt * w\_3$

```
c = FV * Exp(-rT * T) * Ravgopt * RF
Csum = Csum + c
Sqdiv = (c - Cavg) ^ 2
Sqdivsum = Sqdivsum + Sqdiv

AA = FV * Ravgopt * RF
TAR = GA + AA
Ret = Log(TAR / TotInvexSF) / T
Ret = Exp(Ret) - 1
RetSF = Log(TAR / TotInvincSF) / T
RetSF = Exp(RetSF) - 1

Retsum = Retsum + Ret
RetSFsum = RetSFsum + RetSF

If Ret > RetThres Then
   Retoversum = Retoversum + 1
   Else
End If

If RetSF > RetThres Then
   RetSFoversum = RetSFoversum + 1
   Else
End If

Select Case Ret
   Case -1000000 To Bound1
      Retrange1sum = Retrange1sum + 1
   Case Bound1 To Bound2
      Retrange2sum = Retrange2sum + 1
   Case Bound2 To Bound3
      Retrange3sum = Retrange3sum + 1
   Case Bound3 To Bound4
      Retrange4sum = Retrange4sum + 1
   Case Bound4 To Bound5
      Retrange5sum = Retrange5sum + 1
   Case Bound5 To Bound6
      Retrange6sum = Retrange6sum + 1
   Case Bound6 To Bound7
      Retrange7sum = Retrange7sum + 1
   Case Else
      Retrange8sum = Retrange8sum + 1
End Select

Select Case RetSF
   Case -1000000 To Bound1
      RetSFrange1sum = RetSFrange1sum + 1
   Case Bound1 To Bound2
      RetSFrange2sum = RetSFrange2sum + 1
   Case Bound2 To Bound3
```

```
          RetSFrange3sum = RetSFrange3sum + 1
       Case Bound3 To Bound4
          RetSFrange4sum = RetSFrange4sum + 1
       Case Bound4 To Bound5
          RetSFrange5sum = RetSFrange5sum + 1
       Case Bound5 To Bound6
          RetSFrange6sum = RetSFrange6sum + 1
       Case Bound6 To Bound7
          RetSFrange7sum = RetSFrange7sum + 1
       Case Else
          RetSFrange8sum = RetSFrange8sum + 1
    End Select

Next i

Cavg = Csum / N
SD = Sqr(Sqdivsum / (N - 1))

Retavg = Retsum / N
RetSFavg = RetSFsum / N
Retoveravg = Retoversum / N
RetSFoveravg = RetSFoversum / N

Retrange1avg = Retrange1sum / N
Retrange2avg = Retrange2sum / N
Retrange3avg = Retrange3sum / N
Retrange4avg = Retrange4sum / N
Retrange5avg = Retrange5sum / N
Retrange6avg = Retrange6sum / N
Retrange7avg = Retrange7sum / N
Retrange8avg = Retrange8sum / N

RetSFrange1avg = RetSFrange1sum / N
RetSFrange2avg = RetSFrange2sum / N
RetSFrange3avg = RetSFrange3sum / N
RetSFrange4avg = RetSFrange4sum / N
RetSFrange5avg = RetSFrange5sum / N
RetSFrange6avg = RetSFrange6sum / N
RetSFrange7avg = RetSFrange7sum / N
RetSFrange8avg = RetSFrange8sum / N

Exit Sub

ErrorTrap:
   MsgBox "An error has accurred in the core! Please check the code."

End Sub
```

```
Function Moro_NormSInv(u As Double) As Double
```

```vba
' Calculates the Normal Standard numbers given u, the associated uniform number (0, 1)
' VBA version of the Moro's (1995) code in C
'   Option Base 1 is necessary to be declared before this function for vector elements
positioning to work
    Dim c1, c2, c3, c4, c5, c6, c7, c8, c9
    Dim X As Double
    Dim r As Double
    Dim a As Variant
    Dim b As Variant
    a = Array(2.50662823884, -18.61500062529, 41.39119773534, -25.44106049637)
    b = Array(-8.4735109309, 23.08336743743, -21.06224101826, 3.13082909833)
    c1 = 0.337475482272615
    c2 = 0.976169019091719
    c3 = 0.160797971491821
    c4 = 2.76438810333863E-02
    c5 = 3.8405729373609E-03
    c6 = 3.951896511919E-04
    c7 = 3.21767881768E-05
    c8 = 2.888167364E-07
    c9 = 3.960315187E-07
    X = u - 0.5
    If u = 0 Then
       u = 0.000000000001
    Else
    End If

    If Abs(X) < 0.42 Then
       r = X ^ 2
       r = X * (((a(4) * r + a(3)) * r + a(2)) * r + a(1)) / ((((b(4) * r + b(3)) * r + b(2)) * r +
b(1)) * r + 1)
    Else
       If X > 0 Then r = Log(-Log(1 - u))
       If X <= 0 Then r = Log(-Log(u))
       r = c1 + r * (c2 + r * (c3 + r * (c4 + r * (c5 + r * (c6 + r * (c7 + r * (c8 + r * c9)))))))
       If X <= 0 Then r = -r
    End If
    Moro_NormSInv = r
End Function
```