

# Long-Term Sustainable and Optimal Management of Multispecies Stochastic Fisheries

Diwakar Poudel  
Leif K. Sandal  
Sturla F. Kvamsdal  
Stein Ivar Steinshamn

SNF





**SNF Working Paper No. 01/14**

**Long-Term Sustainable and Optimal  
Management of Multispecies  
Stochastic Fisheries**

by

**Diwakar Poudel  
Leif K. Sandal  
Sturla F. Kvamsdal  
Stein Ivar Steinshamn**

**SNF Project No. 5187  
A General Age-structured Model for Ecosystem Management**

**The project is financed by the Research Council of Norway**

**INSTITUTE FOR RESEARCH IN ECONOMICS AND BUSINESS  
ADMINISTRATION  
BERGEN, February 2014  
ISSN 1503-2140**

© Dette eksemplar er fremstilt etter avtale med KOPINOR, Stenergate 1, 0050 Oslo. Ytterligere eksemplarfremstilling uten avtale og i strid med åndsverkloven er straffbart og kan medføre erstatningsansvar.



## Long-Term Sustainable and Optimal Management of Multispecies Stochastic Fisheries

Diwakar Poudel<sup>a\*</sup>, Leif K Sandal<sup>a</sup>, Sturla F Kvamsdal<sup>a</sup> and Stein I Steinshamn<sup>a</sup>

<sup>a</sup>NHH Norwegian School of Economics,  
Department of Business and Management Science, Helleveien 30, 5045,  
Bergen, Norway

\*Corresponding author, email: [Diwakar.poudel@nhh.no](mailto:Diwakar.poudel@nhh.no)

### Abstract

*Multispecies fisheries management is the first step towards ecosystem-based fisheries management. Multispecies management accounts for a number of species and their physical, biological, and economic interactions. These interactions increase complexity in understanding stock dynamics and optimal catch. To address the issue of identifying optimal catch of stochastically growing multi stocks, we have formulated and applied a time-continuous stochastic model. The model, applied in prey-predator ecosystem, contributes to sustainable yet optimal management of multispecies marine fisheries. The findings suggest that the optimal catch for stochastically growing stocks in a multispecies interaction ecosystem is different from the deterministic catch.*

### Keywords

Bioeconomic model, ecosystem, multispecies, stochasticity

JEL Classification C61, C63, Q21, Q22, Q57



## Introduction

Marine fisheries are vital resources for the ecology and economy. They play a crucial role in ensuring food security for the growing population ([FAO, 2008](#)). To render these fisheries as a sustainable source of food for the world requires an effort to address overexploitation in the fishing industry and to improve fisheries management ([Speer, 1995](#)). However, there exist management problems for the species that involve interactions between species at different trophic levels ([May, Beddington, Clark, Holt, & Laws, 1979](#)). The most common approach, single species management in multispecies fisheries ignores the ecological relationships among species as well as the technological and economic relationships between species ([Kasperski, 2010](#)). This may lead to misleading results and incorrect policy decisions causing an over or under exploitation of the stocks ([Fleming & Alexander, 2003](#); [Hoff et al., 2010](#)). Therefore, multispecies management is central for sustainable management of most marine fisheries. Multispecies fishery management looks at the bigger picture and addresses the long-term consequences of present decisions. It implies an analysis and management of marine ecological system that includes a number of species and their biological and physical interactions, in contrast to managing different species individually. The economic interaction plays a crucial role in creating the overall harvesting pressure on commercially important species.

Earlier studies focused mainly on a predator- prey relationship on different trophic levels in their multispecies management model (for example [Bogstad, Hauge, & Ulltang, 1997](#); [May et al., 1979](#); [Yodzis, 1994](#)). Those studies, however, focused on the biological yields without considering the economic aspects of harvesting. Later authors like [Fleming and Alexander \(2003\)](#) and [Kar and Chaudhuri \(2004\)](#) suggested a deterministic bioeconomic model with an optimal equilibrium solution. However, they also remarked that it is difficult to find the optimal paths, even in the cases with linear objective functions. [Hollowed et al. \(2000\)](#) compared multispecies models with single species models and found that multispecies models provide distinct advantage over the single species models allowing the users to model natural mortality and growth rates more realistically. They also indicated that multispecies models improve the understanding of fish population dynamics. Unfortunately, the multispecies bioeconomic models are very limited due to unavailability of the analytical solutions ([Posch & Trimborn, 2010](#)) and due to the computational difficulties ([Singh, Weninger, & Doyle, 2006](#)), particularly in solving non-linear dynamic models in higher dimensions. Some bioeconomic models by [C. W. Clark \(1990\)](#), [Woodward and Bishop \(1999\)](#), [Iversen \(2006\)](#), [Agnarsson et al. \(2008\)](#) and [Sandal and Steinshamn \(2010\)](#) suggest optimal management of multispecies fishery in deterministic settings. Nevertheless, in reality, most of the decisions are required to take place in an uncertain environment ([Charles & Munro, 1985](#)). The physical

interactions, such as growing environment, different external shocks and diseases in the ecosystem create stochastic growth of the marine resources. The stochastic process is central in explaining the uncertainty in growth and development of natural resources.

Although researchers such as [Reed \(1979\)](#), [Charles \(1983\)](#), [Charles and Munro \(1985\)](#), [C.W. Clark and Kirkwood \(1986\)](#), [Hannesson \(1987\)](#), [Sandal and Steinshamn \(1997a\)](#), [Sethi, Costello, Fisher, Hanemann, and Karp \(2005\)](#), [Singh et al. \(2006\)](#), [Kugarajh, Sandal, and Berge \(2006\)](#) and [McGough, Plantinga, and Costello \(2009\)](#) include stochasticity in their single species models, stochastic multispecies models lack in the bioeconomic literature ([Agnarsson et al., 2008](#)). To address the issue of optimal management of stochastically growing multi-stocks, we have formulated and applied a time-continuous stochastic model that contributes in multispecies bioeconomic management of marine ecosystems. This study advances previous work by [Sandal and Steinshamn \(2010\)](#). In our study, first, we explore how the species interaction and stochasticity affects the optimal management of the multispecies fisheries, and then investigate and compare long-term sustainable state (LSO) levels in the stochastic and deterministic growth models.

This study employs a feedback approach ([Sandal & Steinshamn, 1997b, 2001](#)), where the optimal control (harvest) is a direct function of the state variable (stock). In contrast to the commonly used time paths approach, the feedback approach is superior when faced with uncertainty ([Agnarsson et al., 2008](#)). We apply a dynamic programming (DP) technique to obtain the optimal feedback solution. In the DP technique, value function iteration is carried out to solve for the optimal solution ([Judd, 1998](#)). DP is especially a useful method when considering multispecies management model under stochasticity ([Sanchirico & Springborn, 2011](#)).

## Model

We consider a two species prey-predator ecosystem model and a general biological interdependent deterministic growth model for two interacting predator- prey species is obtained from [C. W. Clark \(1990\)](#), [Agnarsson et al. \(2008\)](#) or [Sandal and Steinshamn \(2010\)](#). Letting  $x$  be prey species and  $y$  be the predator, the deterministic growth increments of the species can be given as:

$$\begin{aligned} dx &= [f(x, y) - h_x] dt \\ dy &= [g(x, y) - h_y] dt \end{aligned} \tag{1}$$

Furthermore, a two-species interaction model with stochastic dynamic can be formulated by adding stochastic terms in equation (1) as:

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} f(x, y) - h_x \\ g(x, y) - h_y \end{pmatrix} dt + \begin{pmatrix} \sigma_{11}(x, y) & \sigma_{12}(x, y) \\ \sigma_{21}(x, y) & \sigma_{22}(x, y) \end{pmatrix} \begin{pmatrix} dB_x \\ dB_y \end{pmatrix} \quad (2)$$

or formally as  $dZ = F(Z, h)dt + \sigma(Z)dB$ .

The functions  $f(x, y)$  and  $g(x, y)$  are the biological growth functions of prey and predator species respectively, while  $h_i$  represents the harvest rate of species ( $i = x, y$ ). The term  $dt$  is time increment and  $dB_{(i)}$  denotes the incremental basic Brownian motion which is independent and identically distributed with mean zero and variance,  $dt$ . We assume the natural restrictions, stocks and harvest to be non-negative ( $x, y \geq 0, h_x, h_y \geq 0$ ).

The second part of the model consists of economic component. This is the net revenue from the harvesting of the two species, which can be obtained by adding the total revenue from each species<sup>1</sup>. Let  $\pi(x, y, h_x, h_y)$  be the net revenue from the harvesting of two species with  $\pi_x(x, h_x)$  and  $\pi_y(y, h_y)$  as the revenues from species  $x$  and  $y$  respectively. Furthermore, by specifying the individual species revenue functions  $\pi_x(x, h_x)$  and  $\pi_y(y, h_y)$  as:

$$\pi_x(x, h_x) = p_x(h_x)h_x - c_x(x, h_x) \text{ and } \pi_y(y, h_y) = p_y(h_y)h_y - c_y(y, h_y), \text{ the total net revenue can be given as:}$$

$$\pi(x, y, h_x, h_y) = p_x(h_x)h_x - c_x(x, h_x) + p_y(h_y)h_y - c_y(y, h_y) \quad (3)$$

Where, the functions  $p(\cdot)$  and  $c(\cdot)$  are inverse demand functions and cost functions respectively.

Given the biological growth functions and net revenue function, the objective of the Fishery management authority (such as such as Regional Fishery Management Organization, at least theoretically, is the ‘‘sole owner or the largest player’’ of the resource) is to maximize the expected net present value of the return from the harvest schedule over an infinite time horizon from the resources. This can be achieved by maximizing following function:

---

<sup>1</sup> We make an assumption that there is no market interaction between the demand and prices of the two species, therefore, the revenue from both species can be added together.



$$J(x, y, h_x, h_y) = \underset{h_x, h_y}{\text{Max}} E \left[ \int_0^\infty e^{-\delta t} \pi(x, y, h_x, h_y) dt \right] \quad (4)$$

The non-negative parameter  $\delta$  is the discount rate and E is the expectation operator. Along with dynamic constraints and appropriate boundary conditions, the dynamic optimization problem can be written as:

$$\begin{aligned} V(x_0, y_0) &= \underset{h_x, h_y \geq 0}{\text{max}} J(y, h_x, h_y) \\ x(t=0) &= x_0 \\ y(t=0) &= y_0 \end{aligned} \quad (5)$$

The optimal solution in predator-prey model can be obtained by solving following Hamilton-Jacobi-Bellman (HJB) equation ([Kushner & Dupuis, 2001](#)) along with appropriate boundary conditions. This is an equation for any feasible initial condition and hence we replace  $(x_0, y_0)$  with  $(x, y)$ .

$$\begin{aligned} \delta V(x, y) &= \underset{h_x, h_y \geq 0}{\text{Max}} \left\{ \pi(y, h_x, h_y) + V_a^T(x, y) F(x, y, h_x, h_y) \right. \\ &\quad \left. + \frac{1}{2} \text{tr} \left[ \sigma(x, y) \sigma^T(x, y) V_{aa}(x, y) \right] \right\} \end{aligned} \quad (6)$$

$$\text{where, } V_a(x, y) = \begin{pmatrix} V_x(x, y) \\ V_y(x, y) \end{pmatrix}, F(x, y, h_x, h_y) = \begin{pmatrix} f(x, y) - h_x \\ g(x, y) - h_y \end{pmatrix} = \begin{pmatrix} a_1 x^2 - a_2 x^3 - a_3 xy - h_x \\ b_1 y^2 - b_2 y^4 + b_3 xy - h_y \end{pmatrix},$$

$$\sigma(x, y) = \begin{pmatrix} \sigma_{11}(x, y) & \sigma_{12}(x, y) \\ \sigma_{21}(x, y) & \sigma_{22}(x, y) \end{pmatrix} \text{ and } V_{aa}(x, y) = \begin{pmatrix} V_{xx}(x, y) & \\ & V_{yy}(x, y) \end{pmatrix} \text{ are matrices.}$$

Optimal policy can be derived by solving the HJB equation (6). While it is difficult to solve the HJB equation analytically together with boundary conditions, we solve it using numerical approximation methods.

### Numerical approximation approach

Our problem is nonlinear in the control. Analytic solutions to such problems are extremely rare and it is difficult to solve the HJB equation together with nonlinearity and boundary conditions. Numerical methods are the only viable alternatives. The Markov chain approximation approach is one of the most effective methods, which is based on probability theory ([Song, 2008](#)). General numerical algorithms for optimal stochastic control problems of this kind can be found in [Kushner and Dupuis \(2001\)](#).

The numerical technique entails discretizing the state space for the HJB control problem (6), constructing transition probabilities for the controlled Markov chain by applying finite difference

techniques and then iterating on the resulting discrete value transition equation with the initial guess  $V_0$  for the value function. It results in a contraction operator problem that has a unique solution, which approaches the continuous problem as the discretizing step of the state space approaches zero. With the transition probabilities and the interpolation intervals, the optimal value function in the policy space is obtained and updated by value iteration and policy iteration on controlled approximating Markov chain. We carried out value function iteration with initial guess,  $V_0(x)$  for a given policy for example  $u = u_0$  using the value iteration and policy iteration  $V_{n+1}^k(x) = L(u_k)[V_n^k(x)]$  and  $u_{k+1,n} = \operatorname{argmax}\{L(u)[V_n^k(x)]\}$  respectively. Here,  $L(u)$  is representing the Markov chain operator. The policy iterations (given  $u_k(x)$ ) converges to the true value  $\tilde{V}^k(x)$  for the given policy and the sequence  $\{\tilde{V}^k(x)\}$  converges to the value function  $V(x) = \tilde{V}(x)$  of our problem. A mixture of value and policy iterations was carried out until the value function converged to the optimal value function. The function to which this procedure converges is the unique value function for the infinite horizon problem and the policy associated with the optimal value function is the optimal policy. The uniqueness is due to the contraction operator nature of our approach ([Kushner & Dupuis, 2001](#)). The iteration is stopped by employing stopping criteria for policy and value functions. We carried out the policy and value iteration until the largest absolute difference in the value function was smaller than  $1e^{-6}$  from one iteration to the next to ensure the convergence of the value function.

Furthermore, we conducted Monte Carlo simulation with the optimal policy to find long-term sustainable optimal (LSO) policy for the management multispecies. We simulated the system forward in time for a range of initial stock levels. The simulation was performed for 1000 realizations for 100 years.

### **Application in cod and capelin species in Barents Sea**

The model is applied in a prey predator ecosystem of the Barents Sea, which is one of the most productive ocean areas in the world ([O'Brien, Tompkins, Eriksen, & Prestrud, 2004](#)). Among many, it harbors two key fish species namely capelin (*Mallotus villosus*), the plankton feeder and Northeast Arctic cod (*Gadus morhua*), the main predator of capelin. Cod is considered to be the main basis of the Norwegian commercial white fish industry ([Kugarajh et al., 2006](#)), while capelin is the largest pelagic fish species in the Barents Sea. Capelin is also potentially the largest stock in the world ([Gjøsæter, 1998](#); [Gjøsæter & Bogstad, 1998](#)) and is of crucial importance as a prey for the growth of juvenile cod ([Dalpadado & Bogstad, 2004](#); [Hamre, 2003](#)). The relationship between these two species is highly dynamic and is essential in the Barents Sea ecosystem ([Bogstad et al., 1997](#)).

Management of fisheries in the Barents Sea already includes species interactions to some degree. The importance of multispecies management in Barents Sea was realized after the capelin collapse in the mid-1980s and the subsequent dramatic effects on the cod (e.g. cannibalism) and other species, such as the mass migration of harp seals (*Phoca groenlandica*) to the coast of Norway where many of them drowned in fishermen's nets ([Huag & Nilssen, 1995](#)). Following this, a large multispecies research program was initiated. The predation by cod on mature capelin is included into the assessment of capelin, and cod cannibalism; and cod predation on haddock is included in the assessment of cod species ([ICES, 2004](#)). Furthermore, the cod recruitment and survivability is directly affected by climatic environment such as temperature, spawning season and the availability of food, such as prey stocks. Higher temperature at spawning time and more capelin have a positive effect on cod recruitment. However, high temperature results in decrease in capelin biomass through high herring recruitment, which affects the cod recruitment negatively through cannibalism ([Hjermann et al., 2007](#)). Capelin species is very sensitive to the changing hydrobiological conditions of the seas where they feed ([Vilhjálmsson, 2002](#)). Given these uncertainties, we choose a stochastic growth model consisting of capelin and cod as foundation for the decision making in the multispecies management.

### Specification of functional forms and parameters

Both functional forms of biological and economic model and the parameters are based on the results in [Agnarsson et al. \(2008\)](#) and [Sandal and Steinshamn \(2010\)](#). The study by [Agnarsson et al. \(2008\)](#) contains a thorough empirical analysis of cod and capelin species in the Barents Sea. The functional forms were fitted to the data from International Council of the Exploration of the Seas (ICES) during the period of 1972-2005. Therefore, the functional forms and the parameters applied in the study are quite representative.

The biological growth functions of capelin (prey) and cod (predator) species in equation (2) are specified as:

$$\begin{aligned} f(x, y) &= a_1x^2 - a_2x^3 - a_3xy \\ g(x, y) &= b_1y^2 - b_2y^4 + b_3xy \end{aligned} \tag{7}$$

$a_1, a_2, a_3, b_1, b_2, b_3$  are parameters. The first two quadratic terms in prey and predators represent the growth in the absence of the other species, and the term  $xy$  is the interaction between the species where the predator feeds on the prey. The quadratic terms in the growth functions denote that the growth function is right skewed i.e. the growth of the fish species is slow when the stock is small. The growth of the cod

stock is even slower than the capelin species. Similarly, the volatility of each species is assumed to be linear function of its own stock level. This assumption is made for two reasons; for the simplicity in the numerical approximation and there is not much known about more complicated functional volatility dependence. The volatility functions are specified as:

$$\begin{aligned}\sigma_{11}(x, y) &= \sigma_{11}x \\ \sigma_{12}(x, y) &= \sigma_{12}x \\ \sigma_{21}(x, y) &= \sigma_{21}y \\ \sigma_{22}(x, y) &= \sigma_{22}y\end{aligned}\tag{8}$$

The functional forms and the parameters in the economic model in equation (3) are specified as follows.

$$\begin{aligned}p_x(h_x) &= p_1 \\ c_x(x, h_x) &= q_1 h_x^{\alpha_1} \\ p_y(h_y) &= p_2 - p_3 h_y \\ c_y(y, h_y) &= q_2 \frac{h_y^{\alpha_2}}{y}\end{aligned}\tag{9}$$

where  $p_1, q_1, \alpha_1, p_2, q_2, \alpha_2$  and  $p_3$  are economic parameters. We assume that capelin is a schooling species and, therefore, the unit cost of harvest is independent of stock size. Our revenue function depends only on the predator stock and harvest level of prey and predators and the revenue function equation (3) can simply be written as a function of 3 arguments  $\pi(y, h_x, h_y)$ . Furthermore, the revenue equation (3)

can be rewritten after substituting the specific forms from equation (9) as:

$$\pi(y, h_x, h_y) = p_1 h_x - q_1 h_x^{\alpha_1} + p_2 h_y - \frac{q_2}{y} h_y^{\alpha_2} - p_3 h_y^2\tag{10}$$

The details of the biological and economic parameters are specified as:

Growth of capelin species:  $f(x, y) = 0.00018x^2 - 1.19e^{-8}x^3 - 0.00021xy$  ( $10^6$  Kg/year)

Growth of cod species:  $g(x, y) = 0.00022y^2 - 3.49e^{-11}y^4 + 1.82e^{-5}xy$  ( $10^6$  Kg/year)

Capelin price:  $p_1 = 1$  NOK / kg

Cod price:  $P_2 = 12.65$  NOK/kg /kg and strength of demand  $P_3 = 0.00893$

Cost of capelin catch:  $c_x(h_x) = q_1 h_x^{\alpha_1} = 0.07 h_x^{1.4}$  ( $10^6$  NOK)

$$\text{Cost of cod catch: } c_y(y, h_y) = q_2 \frac{h_y^{\alpha_2}}{y} = 5848.1 \frac{h_y^{1.1}}{y} \text{ (10}^6 \text{ NOK)}$$

It is also worth mentioning that the price of cod (predator) is much higher compared to the capelin (prey) species. We also assume a constant cost of harvesting for capelin, which is independent of the stock size because of the schooling nature of the species ([Aanestad, Sandal, & Eide, 2007](#); [Sandal & Steinshamn, 2010](#)). Fixed costs, that is, the cost independent of stock biomass and harvest are also neglected as they do not affect optimal policy. Moreover, all the optimal feedback solutions are calculated with 5 percent discount rate ( $\delta=0.05$ ) and with different level of stochasticity in the model.

## Results and Discussions

### *Optimal harvest in deterministic growth models*

The deterministic growth model specified in equation (1) is employed to find optimal harvest policy simultaneously for capelin and cod species. The optimal feedback solution for capelin and cod harvest in the two-dimensional cod capelin state space is presented in figure 1 and figure 2 respectively.

It can be observed that in the absence of predator in the ecosystem, it is suboptimal to harvest capelin or prey species if the capelin stock biomass level is below 1.2 million ton. This could be that the gain by conserving the smaller stock will be higher compared to exploitation at low stock level. While, it's harvest can be increased gradually if the stock biomass increases until 2.5 million ton. It is optimal to harvest capelin at a constant rate i.e. 0.6 million ton when the stock exceeds 2.5 million ton suggesting bang-bang policy is optimal even though it is not same at some intermediate stock biomass levels (1.2 to 2.5 million ton).

In the presence of predator species or cod (a high priced species) in the ecosystem, the capelin harvest policy is considerably affected. The harvest pattern of capelin for large cod stocks level is less intuitive (see figure 1). The pattern consists of considerable harvest at low capelin stock levels, then a moratorium over a certain range followed by a gradual approach to the static optimum or constant harvest rate. The intuitive explanation of harvest at low stock levels is that it would go extinct because of cod predation. Sandal and Steinshamn ([2010](#)) have clearly shown that the presence of cod in the model induces critical depensation along the optimal paths. In other words, if the capelin biomass level is below two million ton, it goes extinct even without harvest due to cod predation. Therefore, it is optimal to increase the harvest of capelin because if they are not harvested for human consumption they will be eaten by cod anyway. This occurs only if the cod stock is sufficiently abundant and therefore its commercial

existence (profitability) does not depend too much on its present feeding on capelin. Hence, whether capelin is harvested or not, it does not reduce the value of the cod fishery as it approaches the single species fishery. When the capelin stock increases over two million ton, it should be conserved for cod. At this level, cod can be harvested in large amount so that profit will be increased due to the higher cod price. Sandal and Steinshamn (2010) also show that the ‘valley region’ in figure 1 means capelin stock could go extinct if it is roughly below 4 million ton and could rebuild if it is over 4 million ton. If the stock of capelin increase over 6 million ton, it can be harvested at ‘bliss’ or myopically as in the single species solution.

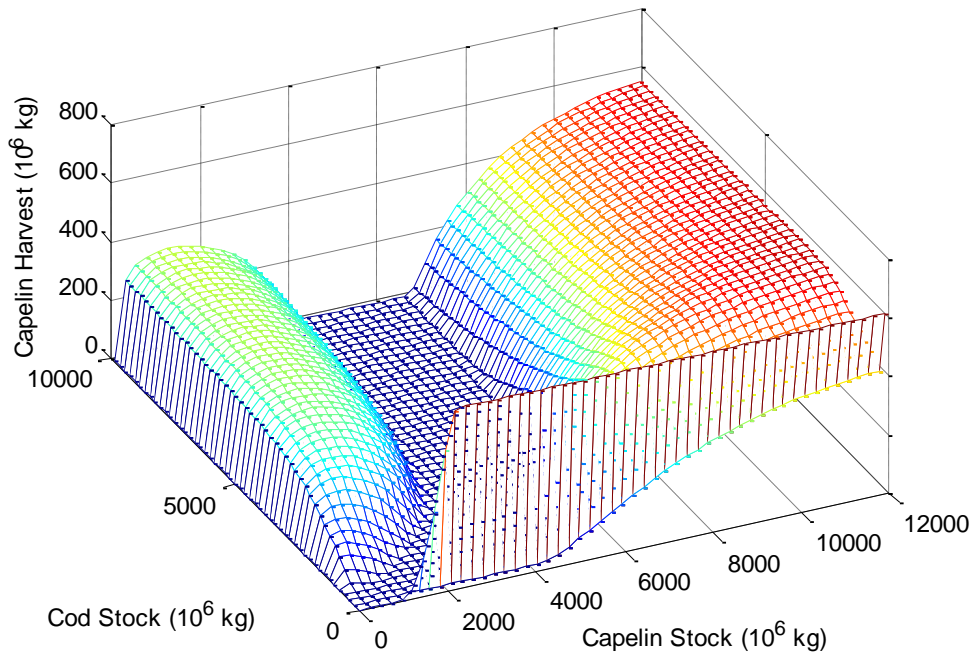


Figure 1: Optimal feedback policy for capelin harvest in cod-capelin space in a deterministic model

The ‘valley region’ reflects at least two properties in the bioeconomic system. It represents an intrinsic precautionary policy trying to avoid capelin from going extinct and at those ranges of stocks it is more profitable to harvest the capelin through the cod. That is, the value created by letting cod eat it a unit of capelin is higher than the value created by landing that unit. For sufficiently large capelin stock biomass, it should be harvested as the gain by conserving all capelin for cod consumption may not increase the value of cod rather the value of fishery can be increased by harvesting both prey and predator species.



The optimal harvest in cod species is more conservative than in capelin species. The moratorium is around 1.2 million ton of biomass near the maximum sustainable yield (1.5 million ton). There are several reasons for a higher moratorium level in cod. First, there could be low net revenue from the harvest due to stock dependent higher harvesting costs if harvested at a low stock level (if fixed costs are assumed in the model, which could also lead to a higher moratorium). Second, there is a possibility of higher future gain from conserving the stock. Although, we don't look at the risk aversion case, the downward sloping demand curve has the same effect as risk aversion. Furthermore, a downward sloping demand curve will give smoother, but continuous harvests. The curve (figure 1b) shows that it is optimal to increase harvest gradually with the increase in the stock size up to two million ton.

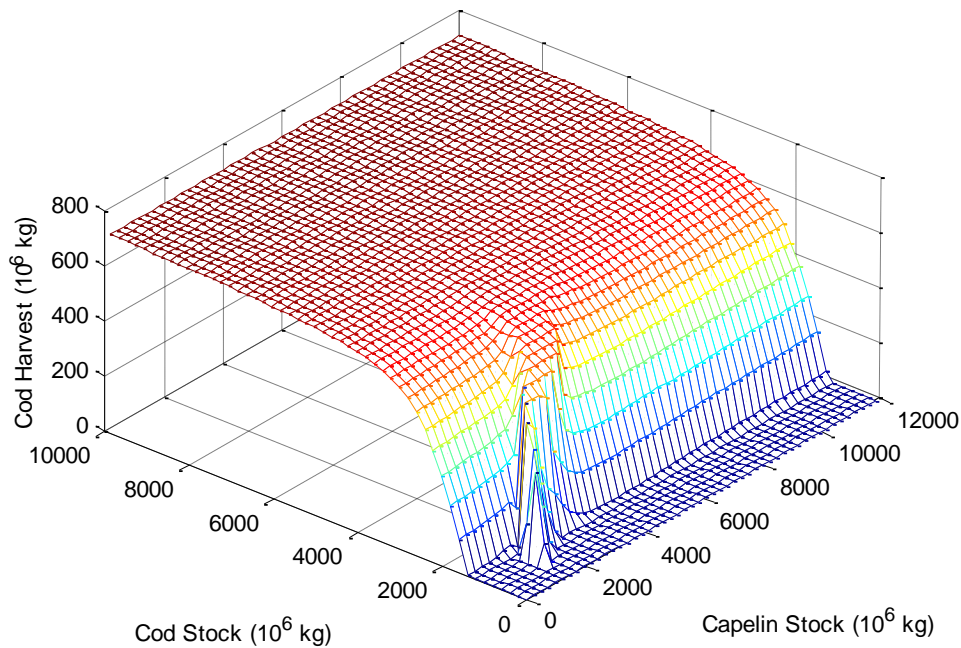


Figure 2: Optimal feedback policy for cod harvest in cod-capelin space in a deterministic model.

On the other hand, the species interaction has a minor effect on the harvest policy for cod species. In the absence of capelin or low capelin biomass (<1.6 million ton) in the ecosystem, cod moratorium is 1.414 million ton. The reason might be that the small capelin cannot be utilized by the cod which therefore is suggested to be harvested. As long as the capelin biomass increases (>1.8 million ton), the cod moratorium shifts toward lower biomass level compared to low capelin stock level. The new moratorium is 1.212 million ton for cod in the multispecies ecosystem. Furthermore, with increase in the cod biomass level, a higher exploitation is possible with more capelin in the ecosystem because of the increased food availability for cod.

***Optimal harvest in stochastic growth models***

The stochastic growth model specified in equation (2) is employed to obtain stochastic optimal policy at various level of stochasticity. Since nothing is known about the strength of stochasticity in fishery growth particularly in the selected species, capelin and cod, we employ 4 different stochasticity level and compare the stochastic optimal policy with deterministic optimal policy. The optimal solution for exploitation of capelin and cod in a two-dimensional state space with different levels of stochasticity is presented in figure 3 and 4 respectively. In the analysis, we also assume sufficiently high degree of stochasticity to compare the results with deterministic policy even though an assumption of a very high level of stochasticity may not be observed in real-world fisheries. With the precautionary principle in mind, we still found it instructive to study such high levels of stochasticity in the model. However, we put emphasis on the effect of stochasticity at a low to moderate levels of stochasticity, which is important in real-world fisheries management. To avoid many plots, we have only plotted the result for cases with  $\sigma_{11} = \sigma_{12} = \sigma_{21} = \sigma_{22} = \sigma$  to capture the general features in the result.

***Effect on the capelin policy***

For low levels of stochasticity (for example,  $\sigma \leq 0.1$ ), there is no considerable effect on the capelin harvest policy (figure 3a). However, with an increased stochasticity (for example,  $\sigma = 0.2$ ), the optimal harvest of capelin becomes more conservative compared to the deterministic optimal policy. At this stochasticity level the ‘valley region’ that appears in the deterministic growth model becomes larger suggesting no harvest of capelin in this particular region. Because by not harvesting capelin, we can harvest the high priced cod which yields higher net revenue. But in a region with a very large capelin stock, it will still be optimal to harvest capelin as ‘bliss’ along with harvest of cod (figure 3b). A further increase in stochasticity (for example,  $\sigma = 0.4$ ), makes the ‘valley region’ much wider suggesting that one should be conservative in the capelin harvest even for a very large biomass levels with almost no harvest (figure not shown). This is very intuitive because the stochastic cod requires more food than the less stochastic or deterministic cod and therefore it is optimal to be very conservative in harvest of capelin at any level except at zero cod level, so that cod can be harvested unaffected.

However, at a high level of stochasticity (for example  $\sigma = 0.5$ ), the valley region in the deterministic model begin to disappear suggesting the capelin stock should be harvested myopically at small stock levels and conservatively at large stock levels (figure 3c). There is no reason to conserve highly stochastic capelin stock to another highly stochastic cod stock. At this stochasticity level both of the species be

harvested conservative at high biomass but myopic at low biomass. At low biomass, there is high risk of extinction even if not harvested. But at large biomass the extinction or collapse of stock can be avoided by being conservative. In both species the possibility of extinction of a small stock is high due to stochasticity and the capelin extinction risk is further exacerbated by cod predation.

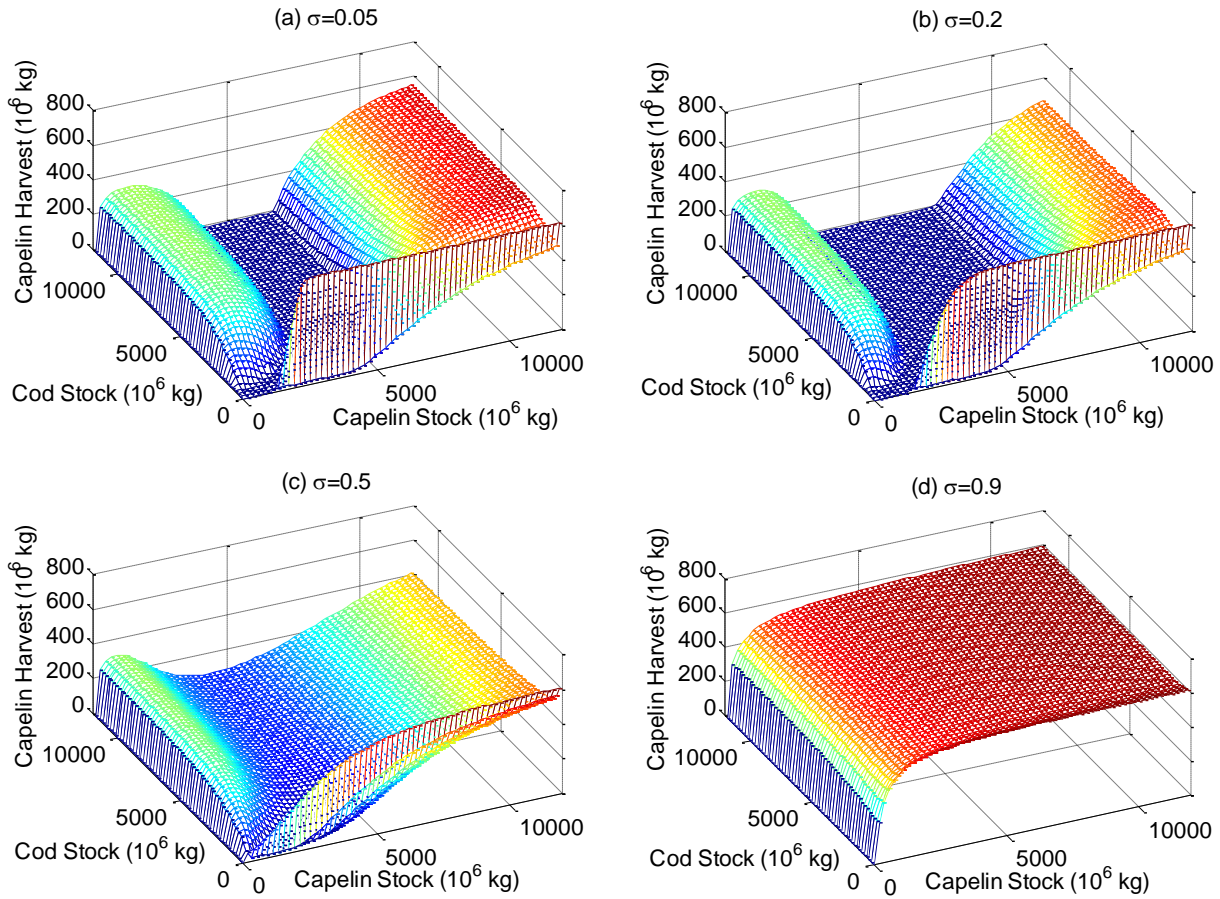


Figure 3: Effect of stochasticity in optimal harvest of capelin in multispecies model (a)  $\sigma=0.05$  (b)  $\sigma=0.2$ , (c)  $\sigma=0.5$ , and (d)  $\sigma=0.9$ .

If the stochastic volatility is substantially high (for example  $\sigma \geq 0.7$ ), there is no gain of conserving a highly stochastic capelin stock for cod food. Therefore, capelin should be managed myopically as a highly stochastic single species model, without taking into account of the future profit and the species interaction in the ecosystem (figure 3d).

*Effect on the cod harvest policy*

There is no strong influence of the stochastic volatility on the optimal harvest of cod as long as the level of stochasticity is low (for example  $\sigma \leq 0.3$ ). The main reason is that the uncertainty is managed by reducing the harvest of capelin (figure 4a,b).

For increased level of stochastic volatility (for example  $\sigma = 0.5$ ), the cod harvest is myopic at low stock level and conservative at high stock level (figure 4c). At low biomass, there is high risk of extinction due to volatility even if not harvested. But at large biomass the extinction or collapse of stock can be avoided by being conservative. At a substantially high stochasticity level (for example  $\sigma \geq 0.7$ ), it should be harvested myopically without considering multispecies interaction (figure 4d). Theoretically a species may collapse if it is exposed to very high stochasticity even if not harvested. As mentioned earlier, a very high level of stochasticity may not be observed in real-world fisheries, we still found it instructive to study such high levels of stochasticity in the model as a precautionary principle.

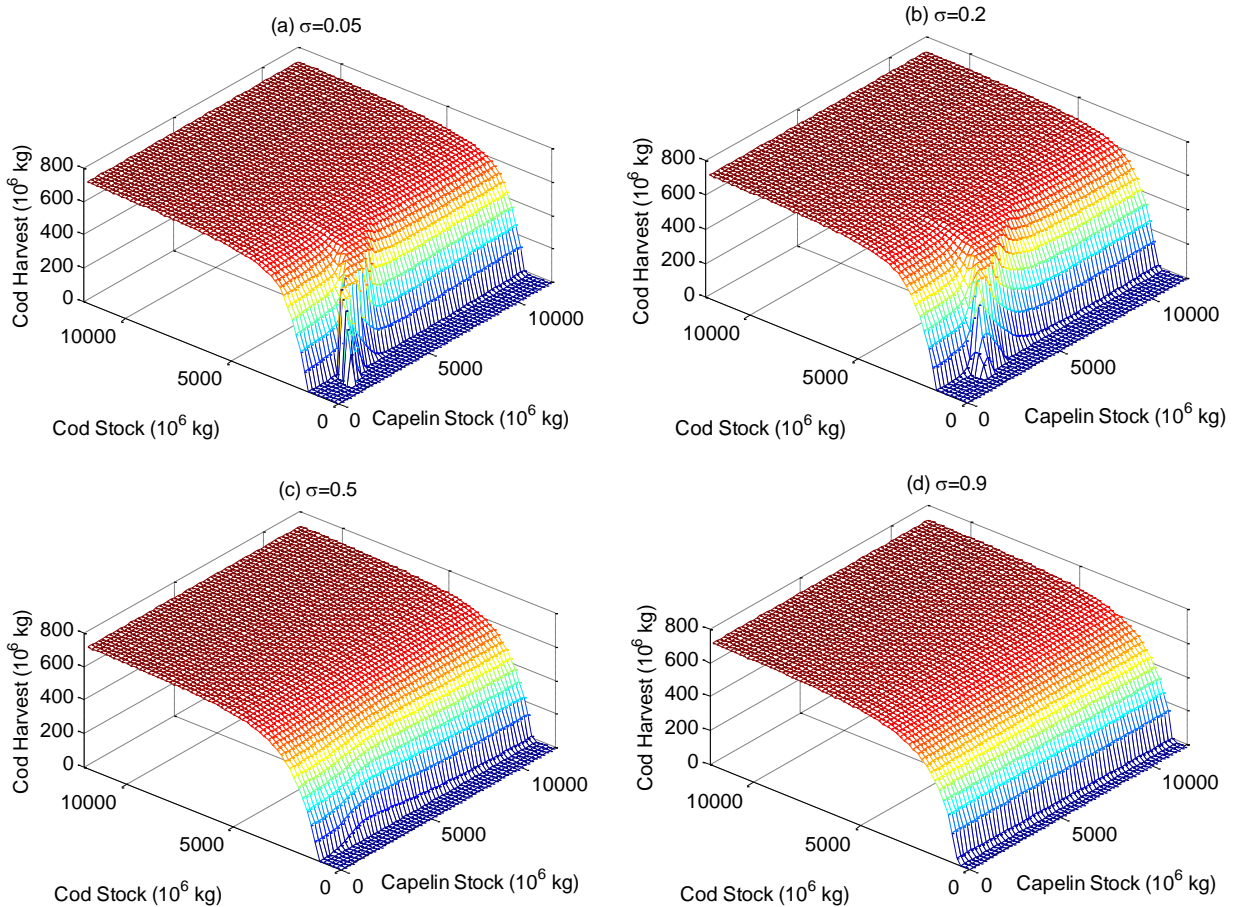


Figure 4: Effect of stochasticity in optimal harvest of cod in multispecies model (a)  $\sigma=0.05$  (b)  $\sigma=0.2$ , (c)  $\sigma=0.5$ , and (d)  $\sigma=0.09$ .

Looking at the different levels of stochasticity, we observe that there are various effects on the optimal harvests compared to the deterministic growth model. Effects naturally depend on the stock size. If the stock is small, stochasticity has a more pronounced effect. The stochastic volatility has large effect on capelin exploitation compared to cod species. There are three possible reasons. First, by conserving the low priced prey species as a food for the higher priced predator, profits can be increased. Second, due to stock independent costs of harvest, there is a lower moratorium for the prey (capelin), thus leading to a small stock, which becomes more sensitive with increasing stochasticity. Third, cod predation makes the prey more sensitive to optimal exploitation.

Although we can harvest capelin at higher level by ignoring the interaction compared to multispecies solution, it is important to note that it is profitable to increase the harvest of cod by one kg while sacrificing up to 12 kg of capelin because of the price difference. A similar result found by [Kellner, Sanchirico, Hastings, and Mumby \(2010\)](#) in multispecies fishery where the predator and one of its prey increases the predator's net present value although net present value of prey is decreases. The general intuition is that ecosystem management is more profitable than managing individual stocks separately when there is limited uncertainty, because the single species model imposes constraint on the ecosystem and the profits will be less than or equal to multispecies management that does not impose that constraint.

The main findings of the stochastic analysis of the article is that at low level of stochasticity there is little or no effect on the exploitation policy, particularly in the multispecies ecosystem unlike in single species management. While an assumption of very high stochasticity and its effect on the policy is suggested for precautionary principle. An assumption of a moderate level of stochasticity is highly relevant for fishery management. We find that at moderate level of stochasticity both non conservative and conservative policy are optimal. For example at small stock one should be non-conservative but at higher stock biomass one should be conservative in its exploitation. In real world fisheries, particularly the Barents Sea fishery is seen as one of the best-managed international fisheries, with large fish stocks providing sustainable and high annual yields ([Hønneland, 2012](#)). Therefore it requires a conservative policy to avoid collapse due to stochasticity, unless there is unreasonably very high stochasticity.

### **Long-term sustainable optimal states (LSOS)**

In this section, we discuss long-term sustainable state of stocks and harvest. To sustain resources over a long time horizon, it is important to manage them optimally. Therefore, it is important to determine sustainable levels for the resource and their harvest. To determine the optimal sustainable levels, we simulated the system forward in time with optimal solutions implemented. We conducted Monte Carlo

simulation for a range of initial stock levels. The LSOS for the deterministic and stochastic models are obtained as follows.

### *Deterministic LSOS*

In the deterministic setting, the simulated paths approach a certain level over time which is defined as LSOS. In other words, LSOS is the steady state<sup>2</sup> or the equilibrium level. The LSOS in the deterministic setting are presented in table 1.

Table 1: Non-trivial LSOS for capelin and cod stock and their harvest in a deterministic model

Species and Harvest	Long-term sustainable Optimal States(LSOS)	
	LSOS I	LSOS II
Capelin ( $10^6$ kg)	0	9165.5
Cod ( $10^6$ kg)	2507.1	2838.9
Capelin harvest ( $10^6$ kg/year)	0	495.1
Cod harvest ( $10^6$ kg/year)	484.5	592.8

Table 1 reveals that there are two non-trivial LSOS for capelin and cod species. The LSOS I is the state where capelin become extinct or collapsed in the long-term due to predation of cod, which means that there is no long term steady state in the capelin species for some initial stock levels. The risk of reaching to LSOS I is further increased if exploited when the stock is small. The initial stock levels that go to zero in the long-term are shown in figure 5 and discussed later. While, the LSOS II reveals that the long term steady state for capelin species is  $9165.5 \times 10^6$  kg and for cod species is  $2838.3 \times 10^6$  kg. The cod LSOS II is higher than the LSOS I. The reason why the LSOS I is lower is that in the long-term there is no capelin for its consumption, thus decreasing the growth of Cod. Similarly, the LSOS for harvest level is higher in LSOS II. Therefore LSOS II is the important for the management point of view. By managing both species at higher stock level, both species can be harvested in the long-term while cod can be harvested at higher level increasing value of the fishery.

<sup>2</sup> Steady state is defined as (single or multiple) stable equilibrium solution(s), where the natural population growth is exactly offset by the rate of harvest, so as to maintain the biomass constant.



*Stochastic LSOS level:*

In the stochastic setting, there is no equilibrium but most paths become confined to the same level after some period. This is a region around a zero drift level. This region is characterized as a long-term sustainable optimal state (LSOS) ([Poudel, Sandal, Kvamsdal, & Steinsham, 2013](#)), which is a mean of thousands of realizations. In other words, it can be defined as the optimal stochastic stationary state ([Smith, 1986](#)). The non-trivial stochastic LSOS of stocks and exploitations are presented in table 2.

Table 2: Non-trivial LSOS for capelin, cod and their harvest level under different degree of stochasticity

Species and Harvest	At 5% stochasticity level		At 20% stochasticity level	
	LSOS I	LSOS II	LSOS I	LSOS II
Capelin ( $10^6$ kg)	0	9125.3 (470.91)	0	8552.1 (1787.6)
Cod ( $10^6$ kg)	2497.6 (117.4)	2829.8 (120.5)	2340.2 (464.1)	2615.4 (502)
Cap harvest ( $10^6$ kg)	0	459.9 (33.1)	0	281.01 (142.8)
Cod harvest ( $10^6$ kg)	461.5 (37.2)	577.2 (18.5)	365.06 (152.4)	509.09 (104.6)

† Figures in the parentheses represent standard deviation

Table 2 indicates stochastic long-term sustainable optimal states for cod and capelin at different stochasticity level. As in the deterministic model, there are two non-trivial stochastic LSOS in the model. LSOS I is zero for capelin species but it is lower in the cod stock. It can also be seen that the LSOS I in stochastic setting is lower compared to the deterministic model. Furthermore, with increasing stochasticity the stock and harvest are both decreasing. The stock decreases as a result of the stochastic downward drag on the stock ([Poudel, Sandal, & Kvamsdal, 2012](#)), therefore a decreased harvest is the optimal for the management of the stochastic stock at moderate degree of stochasticity, which has also been reported in the literature ([for example, see Reed, 1978](#)).

Particularly, the harvest in the capelin species is decreased from  $459.9 \times 10^6$  kg to  $281 \times 10^6$  kg as the stochasticity increases from 5 -20 percent. This indicates that there is stronger effect of the stochastic in capelin management. We should reduce the 39 percent of the catch at 20 percent stochasticity. While the catch decline in the cod species is relatively low (7.5 percent) suggesting that cod is less prone to stochasticity compared to capelin.

## Evolution of capelin and cod over time in multispecies ecosystem

We have illustrated how different stocks evolve over time as they approach the LSOS in the prey-predator ecosystem. Several different optimal paths for various combinations of initial biomass levels are shown in figures 5 and 6 for deterministic and stochastic model overtime for different initial stock levels. The paths in the stochastic setting were obtained by taking an average of thousand realizations from the same point.

### *Capelin evolution over time:*

It can be clearly observed that for certain level of the initial stock of predators, the capelin goes extinct over time. Either if the initial cod is higher than 6 million ton or the initial capelin is lower than the 3 million ton, in both cases capelin goes extinct in the long run. Similarly a very high level of cod and medium level capelin, there is still risk of extinction in the long-run. However, if capelin biomass is above 5 million ton with low cod in the ecosystem, it evolves to LSOS II and any capelin biomass above 7.5 million ton reaches to LSOS II in the long term even for any level of cod biomass in the ecosystem.

Furthermore, the persistence of high cod stock leads to slow growth of the capelin towards the LSOS II. For example, 7 million tons of capelin takes over 20 years to reach LSO when there is high cod compared to 5 million ton of capelin that reaches LSOS in 12 years when there is very low cod in the ecosystem. This is trivial. If the cod stock is high, cod predation affects capelin to reach to LSOS.

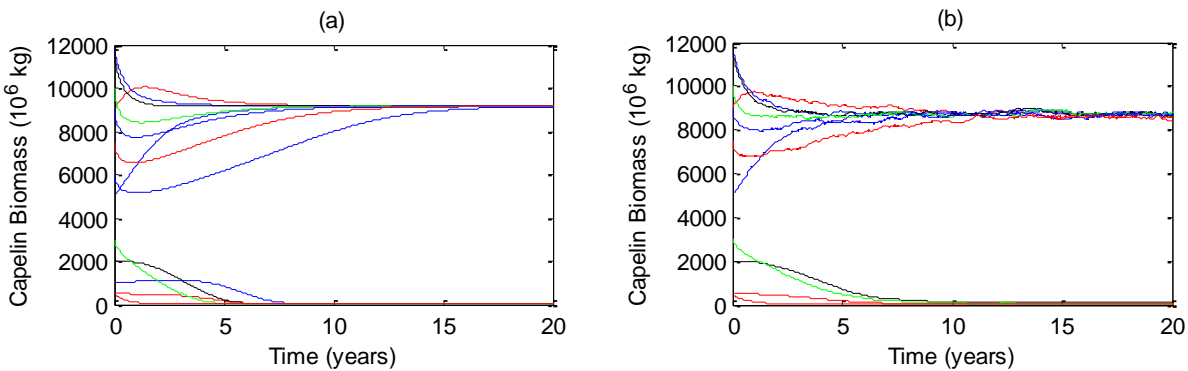


Figure 5: Evolution of capelin stock over time towards LSOS (a) Deterministic model (b) Stochastic model

The trend of stock evolution in the stochastic and deterministic models is similar but the stochastic stock evolution seems to take longer time to reach the LSOS compared to the deterministic evolution and has lower LSOS, thus suggesting lower exploitation that allows for the stock to rebuild to LSOS.

*Cod evolution of over time:*

The cod evolution is affected by prey or capelin in the ecosystem. There are two different LSOS in the cod for different level of capelin in the ecosystem. The LSOS II is only achieved if the capelin stock or the prey in the ecosystem is very high for example above 6 million ton. But if the initial capelin biomass in the ecosystem is below 6 million ton, there is Lower LSOS in the cod species (figure and Table). The lower LSOS could be due to the lack of capelin in the ecosystem, causing food competition or cannibalism that drags to cod LSOS to a low level because the decreased capelin biomass affects the cod recruitment negatively through cannibalism ([Hjermann et al., 2007](#)). For higher presence of capelin biomass, cod has higher LSOS because the juvenile cod needs capelin for better growth ([Dalpadado & Bogstad, 2004](#); [Hamre, 2003](#)). Furthermore, with high capelin stock in the ecosystem, cod reaches to LSOS fast compared to low capelin. However, its own initial stock level influences the evolution directly. A small initial stock takes longer compared to the higher initial stock level. Any stock level above the LSOS reaches to LSOS quickly. The reasons are food and space competition and cannibalism along with the suggested optimal harvest level. At higher stock a higher harvest is suggested in the model, which is taken into account investigating the LSOS.

In the stochastic stock, the LSOS is lower but having other features similar to the deterministic growth model. In stochastic model, the LSOS is lower and also takes further longer duration to reach to LSOS for given initial stock level compared to the deterministic stock level (figure 6). However compared to the capelin stochastic evolution to LSOS, the cod stochastic evolution takes shorter period. This could be because capelin growth is negatively affected by the cod stock as the stochastic cod needs more food.

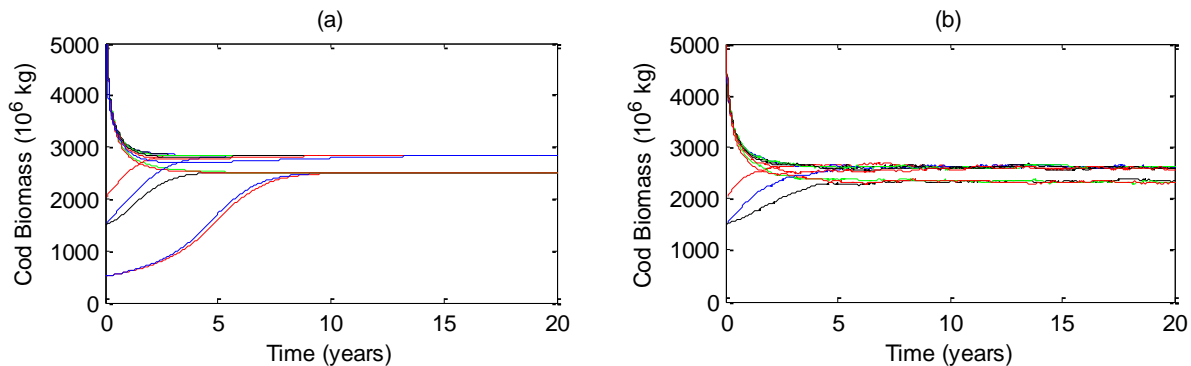


Figure 6: Evolution of cod stock over time towards LSOS (a) Deterministic model (b) Stochastic model.

## **Concluding Remarks**

We have demonstrated the applicability and usefulness of dynamic programming to multispecies management under stochasticity. While biological predator-prey interactions may have been well understood, much remains to be done for a comprehensive understanding of their economic consequences. Our approach sheds light on the interaction of economics with a complex biology.

Despite that the eco-biological system is relatively complex; the nature of our findings is intuitive. It is relatively more profitable to manage the ecosystem in stochastic multispecies fishery than individual stocks separately. Our results show fundamentally different conclusions with and without stochasticity. We conclude that ignoring biological interactions never makes sense and that the level of stochasticity will have an impact on the optimal strategy for management. We believe that policy makers would benefit from an increased appreciation of the effects of stochasticity and the consequences of ignorance. Although the general result might not reflect the best management due to the exogenous parameters adopted in the work, our paper is a breakthrough in complicated, stochastic multispecies modeling which combines both biology and economics and provides a basis to answer what the optimal response is for a bioeconomic manager of renewable resources.

The estimation of stochastic parameters in a multispecies model is a very difficult and complex task. But at the same time its accurate estimation is crucial for further improvement of the model. The stochastic parameter estimation, therefore, remains to be a future scope for the identification of optimal policy for stochastic multispecies fisheries.

Finally, we submit that real world fisheries management is concerned with a limited number of fish stocks that are only a part of a larger ecosystem. To analyze the effect of stochasticity and to assess risk of stock collapse in an ecosystem framework is beyond our scope here. We are only on one of the first steps on a long ladder towards the ultimate goal of ecosystem management under uncertainty. But, it is an important step in the right direction.

## **Acknowledgement**

The author(s) are also thankful for support from the NFR Project No. 196433 and 216571/E40.

## References

- Aanestad, S., Sandal, L. K., & Eide, A. (2007). Optimal Fishing Policy for Two Species in a Three-Species Predator-Prey Model The case of Capelin, Cod and Juvenile Herring in the Barents Sea. *Discussion Papers -no 29, NHH, Bergen Norway*.
- Agnarsson, S., Arnason, R., Johannsdottir, K., Ravn-Jonsen, L., Sandal, L. K., Steinshamn, S. I., & Vestergaard, N. (2008). Comparative evaluation of the fisheries policies in Denmark, Iceland and Norway: multispecies and stochastic issues: SNF, Bergen, Norway.
- Bogstad, B., Hauge, K. H., & Ulltang, Ø. (1997). MULTSPEC—a multispecies model for fish and marine mammals in the Barents Sea. *Journal of Northwest Atlantic Fishery Science*, 22, 317-341.
- Charles, A. T. (1983). Optimal fisheries investment under uncertainty. *Canadian Journal of Fisheries and Aquatic Sciences*, 40(12), 2080-2091.
- Charles, A. T., & Munro, G. R. (1985). Irreversible investment and optimal fisheries management: a stochastic analysis. *Marine Resource Economics*, 1(3), 247-264.
- Clark, C. W. (1990). *Mathematical bioeconomics: the optimal management of renewable resources*: John Willey and Sons, Inc.
- Clark, C. W., & Kirkwood, G. P. (1986). On uncertain renewable resource stocks: Optimal harvest policies and the value of stock surveys. *Journal of Environmental Economics and Management*, 13(3), 235-244.
- Dalpadado, P., & Bogstad, B. (2004). Diet of juvenile cod (age 0–2) in the Barents Sea in relation to food availability and cod growth. *Polar biology*, 27(3), 140-154.
- FAO. (2008). The State of the World Fisheries and Aquaculture: FAO Fisheries and Aquaculture Dept, Food and Agriculture Organization of the United Nations.
- Fleming, C. M., & Alexander, R. R. (2003). Single-species versus multiple-species models: the economic implications. *Ecological modelling*, 170(2-3), 203-211.

- Gjørseter, H. (1998). The population biology and exploitation of capelin (*Mallotus villosus*) in the Barents Sea. *Sarsia: a Norwegian journal of marine biology*, 453-496.
- Gjørseter, H., & Bogstad, B. (1998). Effects of the presence of herring (*Clupea harengus*) on the stock-recruitment relationship of Barents Sea capelin (*Mallotus villosus*). *Fisheries Research*, 38(1), 57-71.
- Hamre, J. (2003). Capelin and herring as key species for the yield of north-east Arctic cod. Results from multispecies model runs. *Scientia Marina*, 67(S1), 315-323.
- Hannesson, R. (1987). Optimal catch capacity and fishing effort in deterministic and stochastic fishery models. *Fisheries Research*, 5(1), 1-21.
- Hjermann, D. Ø., Bogstad, B., Eikeset, A. M., Ottersen, G., Gjørseter, H., & Stenseth, N. C. (2007). Food web dynamics affect Northeast Arctic cod recruitment. *Proceedings of the Royal Society B: Biological Sciences*, 274(1610), 661-669.
- Hoff, A., Frost, H., Ulrich, C., Damalas, D., Maravelias, C. D., Goti, L., & Santurtún, M. (2010). Economic effort management in multispecies fisheries: the FcubEcon model. *ICES Journal of Marine Science: Journal du Conseil*, 67(8), 1802.
- Hollowed, A. B., Bax, N., Beamish, R., Collie, J., Fogarty, M., Livingston, P., . . . Rice, J. C. (2000). Are multispecies models an improvement on single-species models for measuring fishing impacts on marine ecosystems? *ICES Journal of Marine Science: Journal du Conseil*, 57(3), 707.
- Huag, T., & Nilssen, K. T. (1995). Ecological implications of harp seal *Phoca groenlandica* invasions in northern Norway. *Developments in Marine Biology*, 4, 545-556.
- Hønneland, G. (2012). *Making Fishery Agreements Work: Post-Agreement Bargaining in the Barents Sea*. Massachusetts Edward Elger Publishing Inc.
- ICES. (2004). Report of the Arctic Fisheries Working Group. *ICES CM 2004/ACFM:28*. (pp. 418-424).



- Iversen, R. S. (2006). Some implications of predation to optimal management of marine resources.: SNF Report 28/06 , Bergen.
- Judd, K. L. (1998). *Numerical methods in economics*: The MIT press, Cambridge, Mass.
- Kar, T., & Chaudhuri, K. (2004). Harvesting in a two-prey one-predator fishery: a bioeconomic model. *The ANZIAM Journal*, 45(03), 443-456.
- Kasperski, S. (2010). *Optimal Multispecies Harvesting in the Presence of a Nuisance Species*. Paper presented at the Agricultural & Applied Economics Association, Denver, Colorado July 25-27, 2010.
- Kellner, J. B., Sanchirico, J. N., Hastings, A., & Mumby, P. J. (2010). Optimizing for multiple species and multiple values: tradeoffs inherent in ecosystem-based fisheries management. *Conservation Letters*, 4(1), 21-30.
- Kugarajh, K., Sandal, L. K., & Berge, G. (2006). Implementing a stochastic bioeconomic model for the North-East Arctic cod fishery. *Journal of Bioeconomics*, 8(1), 35-53.
- Kushner, H. J., & Dupuis, P. (2001). *Numerical methods for stochastic control problems in continuous time* (Vol. 24). Berlin: Springer Verlag.
- May, R. M., Beddington, J. R., Clark, C. W., Holt, S. J., & Laws, R. M. (1979). Management of multispecies fisheries. *Science*, 205(4403), 267.
- McGough, B., Plantinga, A. J., & Costello, C. (2009). Optimally managing a stochastic renewable resource under general economic conditions. *The BE Journal of Economic Analysis & Policy*, 9(1), 56.
- O'Brien, K., Tompkins, H., Eriksen, S., & Prestrud, P. (2004). Climate Vulnerability in the Barents Sea Ecoregion: A Multi-Stressor Approach. *Report Center for International Climate and Environmental Research-Oslo (CICERO), Oslo*, 35.

- Posch, O., & Trimborn, T. (2010). Numerical solution of continuous-time DSGE models under Poisson uncertainty *Diskussionspapiere der Wirtschaftswissenschaftlichen Fakultät der Universität Hannover*.
- Poudel, D., Sandal, L. K., & Kvamsdal, S. F. (2012). Analyzing Risk of Stock Collapse in a Fishery Under Stochastic Profit Maximization *Discussion Series 4/ 2012*. Bergen: NHH Department of Finance and Management Science.
- Poudel, D., Sandal, L. K., Kvamsdal, S. F., & Steinsham, S. I. (2013). Fisheries Management under Irreversible Investment: Does Stochasticity Matter? *Marine Resource Economics*, 28, 83-103.
- Reed, W. J. (1978). The steady state of a stochastic harvesting model. *Mathematical Biosciences*, 41(3), 273-307.
- Reed, W. J. (1979). Optimal escapement levels in stochastic and deterministic harvesting models. *Journal of Environmental Economics and Management*, 6(4), 350-363.
- Sanchirico, J. N., & Springborn, M. (2011). How to get there from here: Ecological and economic dynamics of ecosystem service provision. *Environmental and Resource Economics*, 1-25.
- Sandal, L. K., & Steinshamn, S. I. (1997a). A feedback model for the optimal management of renewable natural capital stocks. *Canadian Journal of Fisheries and Aquatic Sciences*, 54(11), 2475-2482.
- Sandal, L. K., & Steinshamn, S. I. (1997b). A stochastic feedback model for optimal management of renewable resources. *Natural Resource Modeling*, 10(1), 31-52.
- Sandal, L. K., & Steinshamn, S. I. (2001). A simplified feedback approach to optimal resource management. *Natural Resource Modeling*, 14(3), 419-432.
- Sandal, L. K., & Steinshamn, S. I. (2010). Rescuing the Prey by Harvesting the Predator: Is It Possible? In E. Bjørndal, M. Bjørndal, P. M. Pardalos & M. Ronnqvist (Eds.), *Energy, Natural Resources and Environmental Economics* (pp. 359-378). Berlin: Springer

- Sethi, G., Costello, C., Fisher, A., Hanemann, M., & Karp, L. (2005). Fishery management under multiple uncertainty. *Journal of environmental economics and management*, 50(2), 300-318.
- Singh, R., Weninger, Q., & Doyle, M. (2006). Fisheries management with stock growth uncertainty and costly capital adjustment. *Journal of Environmental Economics and Management*, 52(2), 582-599.
- Smith, J. B. (1986). Stochastic steady-state replenishable resource management policies. *Marine Resource Economics*, 3(2), 155-168.
- Song, Q. S. (2008). Convergence of Markov chain approximation on generalized HJB equation and its applications. *Automatica*, 44(3), 761-766.
- Speer, L. (1995). Marine Fisheries, Population and Consumption: Science and Policy Issues, . Retrieved August 2011, 2010, from <http://www.aaas.org/international/ehn/fisheries/speer.htm>
- Vilhjálmsson, H. (2002). Capelin (*Mallotus villosus*) in the Iceland–East Greenland–Jan Mayen ecosystem. *ICES Journal of Marine Science: Journal du Conseil*, 59(5), 870-883. doi: 10.1006/jmsc.2002.1233
- Woodward, R. T., & Bishop, R. C. (1999). Optimal sustainable management of multispecies fisheries: lessons from a predator prey model. *Natural Resource Modeling*, 12(3), 355-377.
- Yodzis, P. (1994). Predator-prey theory and management of multispecies fisheries. *Ecological Applications*, 51-58.



Multispecies fisheries management is the first step towards ecosystem-based fisheries management. Multispecies management accounts for a number of species and their physical, biological, and economic interactions. These interactions increase complexity in understanding stock dynamics and optimal catch. To address the issue of identifying optimal catch of stochastically growing multi stocks, we have formulated and applied a time-continuous stochastic model. The model, applied in prey-predator ecosystem, contributes to sustainable yet optimal management of multispecies marine fisheries. The findings suggest that the optimal catch for stochastically growing stocks in a multispecies interaction ecosystem is different from the deterministic catch.

# SNF



**Samfunns- og næringslivsforskning AS**

Centre for Applied Research at NHH

Helleveien 30  
NO-5045 Bergen  
Norway

P +47 55 95 95 00  
E [snf@snf.no](mailto:snf@snf.no)  
W [snf.no](http://snf.no)

Trykk: Allkopi Bergen