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When is it Better to Wait for a New Version? Optimal Replacement of an Emerging Technology under Uncertainty

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When is it Better to Wait for a New Version? Optimal Replacement of an Emerging Technology under Uncertainty

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Abstract

We determine the optimal timing for replacement of an emerging technology facing uncertainty in both the output price and the arrival of new versions. Via a sequential investment framework, we determine the value of the investment opportunity, the value of the project, and the optimal investment rule under three different strategies: compulsive, laggard, and leapfrog. In the first one, we assume that a firm invests sequentially in every version that becomes available, whereas in the second and third ones, it can choose an older or a newer version, respectively. We show that, under a compulsive strategy, technological uncertainty has a non–monotonic impact on the optimal investment decision. In fact, uncertainty regarding the availability of future versions may actually hasten investment in the current one. Next, by comparing the relative values of the three strategies under different rates of technological innovation, we find that, under a low output price, the compulsive strategy always dominates, whereas, at a high output price, the incentive to wait for a new version and adopt either a leapfrog or a laggard strategy increases as the rate of innovation increases, while high price uncertainty mitigates this effect.

Keywords: investment analysis, real options, emerging technologies

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1. Introduction

The implications of output price and technological uncertainty for investment and operational decisions are crucial as they are not only pertinent to various industries, e.g., renewable energy (RE), information technologies, and telecommunications, but also influence many of their participants, e.g., private investors, research and development (R&D) units, etc. For example, given a specific rate of innovation, a firm's optimal strategy for upgrading equipment may influence a manufacturer's R&D strategy and vice versa. Indeed, in the RE sector, Vestas faced unfavourable market conditions when it failed to foresee that the demand for new turbines would weaken after 2008 and continued to invest in manufacturing capacity and R&D (Financial Times, 2012). Similarly, taking into account that RE projects are capital intensive, investors in wind turbines may have foregone revenues by adopting an old technology without anticipating the likely arrival of more efficient ones. Here, we take the perspective of a firm that invests and operates a project under price and technological uncertainty in order to provide insights on how to develop an optimal strategy for technology adoption.

Although both technological decay and the random arrival of innovations reflect technological uncertainty, empirical evidence has shown that the latter presents a greater incentive for replacing a technology. For example, in the computer industry, hardware and software companies often design a new version so that the value of an earlier one is reduced. As a result, private investors replace equipment typically due to planned obsolescence and not because their lifetime has expired (The Economist, 2009). This has far–reaching consequences considering that many industries, e.g. smart phones, rely gradually more on computer technologies, and, thus, private investors are faced with the task of making timely investment and operational decisions under increasing technological uncertainty. Similarly, in the area of wind turbines, empirical research has indicated that innovation, rather than technological decay, is the primary cause of turbine replacement (Jensen *et al.*, 2002). Of course, the replacement of a RE technology may be also influenced by other factors that are beyond the scope of this paper, e.g., the limited availability of resource–rich locations. For example, the opportunity cost from delaying the replacement of wind turbines in wind–rich locations is endogenously related to the availability of land. Consequently, empirical analysis has indicated that policies that are implemented in order to encourage the scrapping of older, poorly placed turbines are inefficient as they have a larger effect on turbines that are located in areas with better wind resources (Mauritzen, 2014).

Furthermore, estimating the profit of projects based on emerging technologies is a complicated process as it typically depends on several factors. For example, in the case of a wind farm, the annual revenue may depend on electricity prices, wind speeds, and feed–in tariffs, as well as other random variables. Thus, compared to commodity–based facilities that rely on more mature technologies, RE projects are more exposed to output price and technological uncertainty as well as their endogenous relationship. Nevertheless, in order to enable mathematical tractability, investment models usually address such features separately, and, as a result, questions regarding their combined impact on investment and operational decisions remain open. In this paper, we address this disconnect by developing a real options framework in order to address the problem of optimal replacement of a technology under market and technological uncertainty. As a result, the contribution of this paper is threefold. First, we develop an analytical framework for sequential investment under price and technological uncertainty. Second, we derive analytical and numerical results on the effect of price and technological uncertainty as well as their interaction on the decisions to upgrade a technology by replacing old equipment with more efficient ones. Third, we provide managerial insights for investment and operational decisions based on analytical and numerical results. Specifically, we show that price and technological uncertainty interact to affect the optimal strategy adoption decision when the output price is high and that this decision is independent of technological uncertainty when the output price is low.

We proceed by discussing some related work in Section 2 and introduce assumptions and notation in Section 3. We address the problem of exercising a single replacement option in Section 4.1 and analyse a compulsive strategy, where a firm adopts two subsequent technologies, in Section 4.2. In Section 5, we analyse the case where a firm can adopt either a leapfrog or a laggard strategy, and in Section 6, we compare these two strategies with the compulsive one and show how the optimal strategy can be determined endogenously. Section 7 provides numerical examples for each case and illustrates the interaction between price and technological uncertainty in order to enable more informed investment and operational decisions. Section 8 concludes and offers directions for future research.

2. Related Work

Although there is significant literature in the area of sequential investment, analytical formulations of problems that combine price and technological uncertainty are limited. Early examples in the area of sequential investment include Majd and Pindyck (1987), who value a sequential investment under uncertainty and analyse the flexibility that lies within the time it takes to build an investment project, thereby showing how traditional valuation methods understate the value of a project by ignoring this flexibility. Dixit and Pindyck (1994) develop an analytical framework for sequential investment assuming that the output price follows a geometric Brownian motion (GBM), the project value depreciates exponentially, and the investor has an infinite set of options.

More recent examples include Gollier *et al.* (2005), who allow for a construction lag between subsequent stages and compare a flexible sequence of small nuclear power plants with a nuclear power plant of large capacity. By measuring the option value generated by the modularity of the first project under electricity price uncertainty, they show that modularity may even trigger investment in the initial module at a level below the now–or–never NPV threshold. Malchow–Møller and Thorsen (2005) illustrate that, due to the possibility of updating equipment when investing in an alternative energy technology, the required investment threshold is less sensitive to changes in uncertainty and resembles the investment behaviour under the simple NPV rule. By contrast, the value of waiting is reduced significantly compared to the single–option case. Heydari (2010) presents a methodology for solving a sequential decision–making problem with lags under electricity price uncertainty taking the perspective of a load–serving entity that has its representative consumer on an interruptible load contract with multiple exercise opportunities. Kort *et al.* (2010) show that, contrary to the conventional real options intuition, higher price uncertainty makes a single–stage investment more attractive relative to a more flexible stepwise investment strategy.

In the area of investment under technological uncertainty, Balcer and Lippman (1984) analyse the optimal timing of technology adoption under infinite switching options by assuming that innovations follow a discrete semi–Markov process. They find that the timing of technology adoption is influenced by expectations about future technological changes and that increasing technological uncertainty tends to delay adoption. Grenadier and Weiss (1997) develop a model for the optimal investment strategy of a firm that is confronted with a sequence of technological innovations assuming that technological progress follows a continuous–time stochastic process and that the price is normally distributed. They consider four strategies; compulsive, leapfrog, buy and hold, and laggard. In the first, a firm adopts every technology that becomes available, whereas in the second it skips an old technology in order to adopt the next one. In the third strategy, a firm purchases only an early innovation, and in the final strategy, it waits until a more efficient one becomes available before adopting the previous technology. Their results indicate that, depending on technological uncertainty, a firm may adopt an available technology even though more valuable innovations may occur in the future, while future decisions on technology adoption are path dependent.

Farzin *et al.* (1998) investigate the optimal timing of technology adoption assuming ongoing technological progress and irreversibility. Although they account for technological uncertainty by assuming that new technologies arrive according to a Poisson process, they consider a deterministic production function, thereby assuming no output price uncertainty. Doraszelski (2001) identifies an error in Farzin *et al.* (1998) and shows that, compared to the NPV approach, a firm will defer the adoption of a new technology when it takes the option value of waiting into account. Huisman and Kort (2004) analyse a duopolistic competition in which firms face price and technological uncertainty and show that a high arrival probability can turn a pre–emption game into a war of attrition and that price uncertainty induces the adoption of a new technology. Miltersen and

Schwartz (2007) develop a new real options approach for valuing R&D projects under uncertain time to completion, operational flexibility, and competition.

An implication of technological uncertainty is that a firm may have to choose between alternative projects. Dixit (1993) analyses an irreversible choice among mutually exclusive projects under output price uncertainty and finds that increasing returns and uncertainty make it optimal to wait for the largest project. Décamps *et al.* (2006) extend Dixit (1993) by providing parameter restrictions under which the optimal investment strategy is not a trigger strategy and the optimal investment region is dichotomous. Siddiqui and Fleten (2010) analyse how a firm may proceed with staged commercialisation and deployment of competing alternative energy technologies. They consider a setting where a firm can choose between a new alternative technology, which requires cost–reducing enhancement measures prior to deployment, and an existing RE technology. Although these are examples of analytical frameworks for investment in alternative projects, the availability of these projects is taken for granted as it is not subject to a probability distribution.

In this paper, we develop a framework for sequential investment in which we analyse the trade–off between continuing to run an old technology and replacing it with successively improved versions under price and technological uncertainty. The arrival of innovations is modelled via a Poisson process as it enables the analysis when firms have no information about the decisions made by R&D companies. We analyse three strategies, i.e., compulsive, leapfrog, and laggard; however, unlike Grenadier and Weiss (1997), we analyse their endogenous relation assuming a stochastic price process that facilitates the analysis of the impact of price and technological uncertainty on the optimal investment rule under each strategy. We show that, under a compulsive strategy, technological uncertainty has a non–monotonic impact on the optimal investment decision and may actually accelerate investment. Additionally, we determine the range of prices where the optimal strategy depends on technological uncertainty and find that the required rate of innovation for which a firm may consider waiting for the next technology decreases as the output price increases.

3. Assumptions and Notation

Taking the perspective of a price–taking firm, we assume that it has $n = 1, 2, 3, ..., N$ investment options available with $N < \infty$. Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we assume that technological innovations follow a Poisson process $\{M_t, t \geq 0\}$, where t is continuous and denotes time. The process M_t is defined in (1):

$$M_t = \sum_{k \geq 1} 1_{\{t \geq T_k\}} \tag{1}$$

where $T_k = \sum_{n=1}^{k} y_n$ and $\{y_n, n \geq 1\}$ is a sequence of independent and identically distributed random variables, such that $y_n \sim \exp(\lambda), \forall n \geq 1$, i.e., $f_Y(y) = \lambda e^{-\lambda y} \mathbb{1}_{\{y \geq 0\}}$. Parameter $\lambda \in \mathbb{R}^+$

denotes the intensity of the Poisson process and is independent of t. Intuitively, M_t counts the number of random times T_k that occur between 0 and t, and $y_n = T_n - T_{n-1}$ is the time interval between subsequent innovations. Hence, if no innovation has occurred for t years, then, with probability λdt, an innovation will occur within the next short interval of time dt, i.e.,

$$dM_t = \begin{cases} 1 & \text{, with probability } \lambda dt \\ 0 & \text{, with probability } 1 - \lambda dt \end{cases}$$

We assume that there is no operating cost associated with the technology, and that the revenue at time t, E_t, is independent of the Poisson process and follows a GBM that is described in (2), where μ is the annual growth rate, σ the annual volatility, dZ_t the increment of the standard Brownian motion, and ρ the subjective discount rate.

$$dE_t = \mu E_t dt + \sigma E_t dZ_t, E_0 \equiv E > 0 \tag{2}$$

The output of technology version n is D_n ($D_{n+1} \geq D_n$, $\forall n$), and the corresponding investment cost is I_n ($I_{n+1} \geq I_n$, $\forall n$). Additionally, $\tau^{(N)}_{\ell,m,n}$, where $\ell, m, n \in \mathbb{N}$, is the time at which technology m is adopted given that technology $\ell < m$ is installed while replacement n, $n \geq m > \ell$, is available, and $\epsilon^{(N)}_{\ell,m,n}$ denotes the corresponding optimal adoption threshold with N total versions available. For example, $\tau^{(N)}_{0,1,2}$ is the optimal time to invest in technology 1 when technology 2 is the latest one available and no technology is currently in operation, while $\epsilon^{(N)}_{0,1,2}$ is the corresponding optimal investment threshold. All options are perpetual and installed technologies last forever. Finally, $F^{(N)}_{\ell,m,n}(\cdot)$ is the maximised expected NPV from investment in technology m given that technology ℓ is in operation and technology n is the latest one available for adoption, while $\Phi^{(N)}_{n,n}(\cdot)$ is the expected value of an active project inclusive of embedded options when technology n is installed and is also the latest one available. Notice that, in order to have a trade off between an old and a more efficient technology, we assume that at the point of indifference, ε, where ε is such that $\Phi^{(n)}_{n-1,n-1}(\varepsilon) = \Phi^{(n)}_{n,n}(\varepsilon)$, we have $\Phi^{(n)}_{n,n}(\varepsilon) > 0$, which, in turn implies, $\frac{D_n}{\sum_{i=1}^{n} I_i} < \frac{D_{n-1}}{\sum_{i=1}^{n-1} I_i}$, $\forall n$

4. Compulsive Strategy

4.1. N = 1

We assume that a firm holds a single option to invest in a technology that will become available at a random time T_1 (Figure 1). Figure 2 indicates the different states of operation and the value function in each state. Notice that a transition due to a Poisson event is indicated by a broken arrow whereas a transition due to investment by a solid arrow. In state $(0, 0)$, the firm holds an option to invest in a technology that is not available but may arrive according to a Poisson process. When that happens, the firm moves into state $(0, 1, 1)$ where it can exercise the option by incurring a fixed cost, thus moving to state $(1, 1)$ where it continues to operate technology 1 forever.

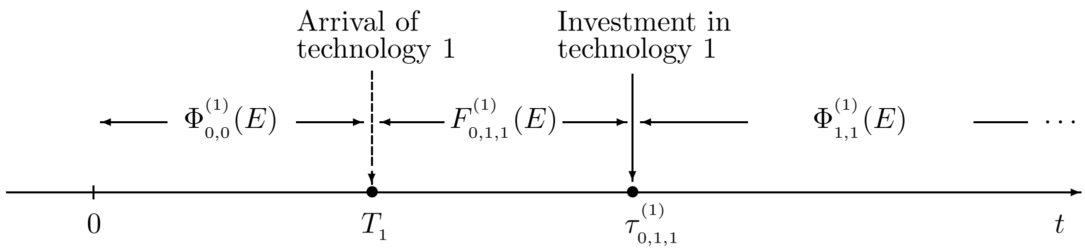

Figure 1: Single investment option

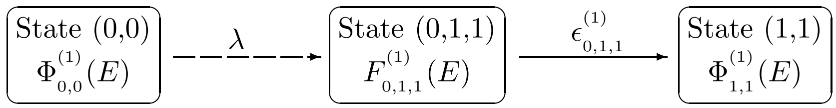

Figure 2: State transition diagram for $N = 1$

In order to determine the value function in each state, we work backwards and first consider state (1,1). When the first technology becomes available, the firm has the option to incur a fixed cost, I_1, in order to adopt it and the expected NPV from immediate investment is described in (3).

$$\Phi^{(1)}_{1,1}(E) = \frac{D_1 E}{\rho - \mu} - I_1 \tag{3}$$

Notice that at T_1, we can have either $E < \epsilon^{(1)}_{0,1,1}$ or $E \geq \epsilon^{(1)}_{0,1,1}$. Thus, the value of the investment opportunity in state $(0, 1, 1)$ is described in (4)

$$F^{(1)}_{0,1,1}(E) = \begin{cases} A^{(1)}_{0,1,1} E^{\beta_1} & , E < \epsilon^{(1)}_{0,1,1} \\ \Phi^{(1)}_{1,1}(E) & , \epsilon^{(1)}_{0,1,1} \leq E \end{cases} \tag{4}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - \rho = 0$ (all proofs can be found in the appendix). The endogenous constant, $A^{(1)}_{0,1,1}$, and the investment threshold, $\epsilon^{(1)}_{0,1,1}$, are determined via the value–matching and smooth–pasting conditions between the two branches of (4) and are indicated in (5).

$$A^{(1)}_{0,1,1} = \frac{{\epsilon^{(1)}_{0,1,1}}^{1-\beta_1}}{\beta_1} \frac{D_1}{\rho - \mu} \quad \text{and} \quad \epsilon^{(1)}_{0,1,1} = \frac{\beta_1}{\beta_1 - 1} \frac{I_1(\rho - \mu)}{D_1} \tag{5}$$

Notice that, since there are no embedded investment options to impact the initial investment decision, $\epsilon^{(1)}_{0,1,1}$ is independent of the rate of innovation, λ. Hence, a higher λ increases the likelihood of an innovation but has no effect on the optimal investment rule, which is subject to the GBM.

In state $(0, 0)$, an innovation has not occurred yet but is likely to occur at some random time, T_1, in the future. Thus, the value function in $(0, 0)$ is described in (6).

$$\Phi^{(1)}_{0,0}(E) = (1 - \rho dt)\lambda dt \mathbb{E}_E \left[F^{(1)}_{0,1,1}(E + dE)\right] + (1 - \rho dt)(1 - \lambda dt)\mathbb{E}_E \left[\Phi^{(1)}_{0,0}(E + dE)\right] \tag{6}$$

The first term on the right–hand side of (6) reflects the value of the option to adopt a technology if it becomes available over the time interval dt, while the second term is the value of continuing to wait if an innovation does not take place over the time interval dt. By expanding the right–hand side of (6) using Itô's lemma, we can re–write (6) as follows:

$$\frac{1}{2}\sigma^2 E^2 \Phi_{0,0}^{(1)''}(E) + \mu E \Phi_{0,0}^{(1)'}(E) - (\rho+\lambda)\Phi_{0,0}^{(1)}(E) + \lambda F_{0,1,1}^{(1)}(E) = 0 \tag{7}$$

Notice that if $E < \epsilon_{0,1,1}^{(1)}$, then, even if an innovation takes place, it cannot be adopted immediately, and $F_{0,1,1}^{(1)}(E)$ is expressed via the top part of (4). Otherwise, the expression for $F_{0,1,1}^{(1)}(E)$ is indicated in the bottom part of (4). Hence, the differential equations for $\Phi_{0,0}^{(1)}(E)$ are indicated in (8).

$$\begin{cases} \frac{1}{2}\sigma^2 E^2 \Phi_{0,0}^{(1)''}(E) + \mu E \Phi_{0,0}^{(1)'}(E) - (\rho+\lambda)\Phi_{0,0}^{(1)}(E) + \lambda A_{0,1,1}^{(1)} E^{\beta_1} = 0 & , E < \epsilon_{0,1,1}^{(1)} \\ \frac{1}{2}\sigma^2 E^2 \Phi_{0,0}^{(1)''}(E) + \mu E \Phi_{0,0}^{(1)'}(E) - (\rho+\lambda)\Phi_{0,0}^{(1)}(E) + \lambda \Phi_{1,1}^{(1)}(E) = 0 & , \epsilon_{0,1,1}^{(1)} \le E \end{cases} \tag{8}$$

The expression for $\Phi_{0,0}^{(1)}(E)$ is indicated in (9), where $A_{0,1,1}^{(1)}$ is described in (5), while $A_{0,0}^{(1)} < 0$ and $B_{0,0}^{(1)} > 0$ are determined via the value–matching and smooth–pasting conditions and are indicated in (A–11) and (A–12) respectively.

$$\Phi_{0,0}^{(1)}(E) = \begin{cases} A_{0,1,1}^{(1)} E^{\beta_1} + A_{0,0}^{(1)} E^{\delta_1} & , E < \epsilon_{0,1,1}^{(1)} \\ \frac{\lambda D_1 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + B_{0,0}^{(1)} E^{\delta_2} & , \epsilon_{0,1,1}^{(1)} \le E \end{cases} \tag{9}$$

Notice that $\delta_1 > \beta_2 > 1$ and $\delta_2 < \beta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\delta(\delta-1)+\mu\delta-(\rho+\lambda) = 0$, i.e., that $\lambda = 0 \Rightarrow \delta_1 = \beta_1$ and $\delta_2 = \beta_2$. The first term in the top part of (9) is the option to invest should an innovation arrive; however, since this option is not available yet, we need to adjust the option value via the second term. The first term in the bottom part of (9) reflects the expected present value of the revenues from the new technology, and the second term is the expected investment cost. Finally, the third term reflects the probability that the price will drop into the waiting region before the occurrence of the innovation. Notice that the relative loss in the value function $\Phi_{0,0}^{(1)}(E)$ due to technological uncertainty is described in (10).

$$\frac{F_{0,1,1}^{(1)}(E) - \Phi_{0,0}^{(1)}(E)}{F_{0,1,1}^{(1)}(E)} = -\frac{A_{0,0}^{(1)} E^{\delta_1}}{A_{0,1,1}^{(1)} E^{\beta_1}}, \quad E < \epsilon_{0,1,1}^{(1)} \tag{10}$$

Hence, if $\lambda = 0$, then no innovation will occur and $\Phi_{0,0}^{(1)}(E) = 0$. By contrast, when $\lambda \to \infty$, the loss in value due to the likelihood of an innovation converges to zero, and, thus, $\Phi_{0,0}^{(1)} \to F_{0,1,1}^{(1)}$. Consequently, $\Phi_{0,0}^{(1)} \in \left[0, F_{0,1,1}^{(1)}\right) \forall \lambda \in \mathbb{R}^+$ (see Propositions 4.1 and 4.2).

4.2. N = 2

We extend the previous framework by assuming that a firm holds two investment options and that it invests in each technology that becomes available ignoring the option to wait to choose between the two. Hence, the transition diagram of Figure 2 is extended by adding states $(1,2,2)$ and $(2,2)$.

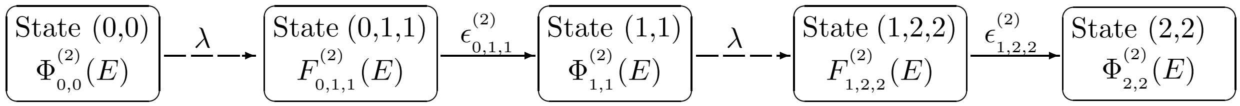

Figure 3: State transition diagram for $N = 2$

The expected NPV from immediate investment in the second technology is described in (11).

$$\Phi^{(2)}_{2,2}(E) = \frac{D_2 E}{\rho - \mu} - (I_2 + I_1) \tag{11}$$

Next, the value function in state $(1, 2, 2)$ is indicated in (12)

$$F^{(2)}_{1,2,2}(E) = \begin{cases} \Phi^{(1)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} & , E < \epsilon^{(2)}_{1,2,2} \\ \Phi^{(2)}_{2,2}(E) & , \epsilon^{(2)}_{1,2,2} \leq E \end{cases} \tag{12}$$

where the endogenous constant $A^{(2)}_{1,2,2}$ and investment threshold $\epsilon^{(2)}_{1,2,2}$ are indicated in (13).

$$A^{(2)}_{1,2,2} = \frac{{\epsilon^{(2)}_{1,2,2}}^{1-\beta_1}}{\beta_1} \frac{(D_2 - D_1)}{\rho - \mu} \quad \text{and} \quad \epsilon^{(2)}_{1,2,2} = \frac{\beta_1}{\beta_1 - 1} \frac{I_2(\rho - \mu)}{D_2 - D_1} \tag{13}$$

Thus, the value of a project with an installed first technology and a single remaining embedded investment option, given that the new technology is not available yet, is

$$\Phi^{(2)}_{1,1}(E) = \begin{cases} \Phi^{(1)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} + A^{(2)}_{1,1} E^{\delta_1} & , E < \epsilon^{(2)}_{1,2,2} \\ \frac{E\left[\lambda D_2 + (\rho-\mu) D_1\right]}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} + B^{(2)}_{1,1} E^{\delta_2} - I_1 & , \epsilon^{(2)}_{1,2,2} \leq E \end{cases} \tag{14}$$

where $A^{(2)}_{1,1} < 0$ and $B^{(2)}_{1,1} > 0$ are determined via the value–matching and smooth–pasting conditions between the two branches of (14) and are indicated in (B–6) and (B–7) respectively. The first term in the top part of (14) is the expected value from operating the first technology, while the second term reflects the option to invest in the second technology, which is not available yet, and, therefore, must be adjust via the third term. The first term in the bottom part of (14) reflects the expected present value of the revenues from the second technology and the second term is the expected investment cost. Finally, the third term is the probability that the price will drop into the waiting region before the occurrence of the innovation. As in (10), a higher λ increases the likelihood of the second innovation and reduces the relative loss in $\Phi^{(2)}_{1,1}(E)$, which, in turn, implies that $\Phi^{(2)}_{1,1}(E) \in \left[\Phi^{(1)}_{1,1}(E), F^{(2)}_{1,2,2}(E)\right) \; \forall \lambda \in \mathbb{R}^+$. These results are shown more generally in Propositions 4.1 and 4.2. Notice that, under a compulsive strategy, $m = \ell + 1$ and $n = m + 1 \; \forall \ell, m, n \in \mathbb{N}$.

Proposition 4.1. $\forall \ell, m, n \in \mathbb{N}$ *the relative loss in* $\Phi^{(n)}_{m,m}(E)$ *converges to zero as* $\lambda \to \infty$*, i.e.,*

$$\lambda \to \infty \Rightarrow \frac{F^{(n)}_{m,n,n}(E) - \Phi^{(n)}_{m,m}(E)}{F^{(n)}_{m,n,n}(E)} \to 0, \quad \forall E < \epsilon^{(n)}_{m,n,n}$$

Proposition 4.2. $\forall \ell, m, n \in \mathbb{N}$ *and* $\forall \lambda \in \mathbb{R}^+$, $\Phi^{(m)}_{\ell,\ell}(E) \in \left[\Phi^{(\ell)}_{\ell,\ell}(E), F^{(m)}_{\ell,m,m}(E)\right)$, $E \leq \epsilon^{(m)}_{\ell,m,m}$.

Next, we step back and consider the option to invest in the first technology that includes an embedded option to perform a single replacement. Notice that the value of an active project with a single embedded replacement option is described in (14) for $E < \epsilon^{(2)}_{1,2,2}$. Consequently, $F^{(2)}_{0,1,1}(E)$ is described in (15), where the top part reflects the value of the option to invest and the bottom part is the expected NPV at investment. Notice that the latter consists of the value from investment in the first technology and a single embedded option to upgrade it when an innovation occurs.

$$F^{(2)}_{0,1,1}(E) = \begin{cases} A^{(2)}_{0,1,1} E^{\beta_1} & , E < \epsilon^{(2)}_{0,1,1} \\ \Phi^{(2)}_{1,1}(E) & , \epsilon^{(2)}_{0,1,1} \leq E \end{cases} \tag{15}$$

Although the optimal investment threshold $\epsilon^{(2)}_{0,1,1}$ and the endogenous constant $A^{(2)}_{0,1,1}$ are now obtained numerically via the value–matching and smooth–pasting conditions (B–9) and (B–10), it is possible to investigate the impact of λ on $\epsilon^{(2)}_{0,1,1}$ by expressing $F^{(2)}_{0,1,1}(E)$ as in (16).

$$F^{(2)}_{0,1,1}(E) = \left(\frac{E}{\epsilon^{(2)}_{0,1,1}}\right)^{\beta_1} \left[\frac{D_1 \epsilon^{(2)}_{0,1,1}}{\rho - \mu} - I_1 + A^{(2)}_{1,2,2} \epsilon^{(2)\beta_1}_{0,1,1} + A^{(2)}_{1,1} \epsilon^{(2)\delta_1}_{0,1,1}\right], \forall E < \epsilon^{(2)}_{0,1,1} \tag{16}$$

Then, the optimal investment rule is indicated in (17) where we equate the marginal benefit (MB) of delaying investment to the marginal cost (MC).

$$\left(\frac{E}{\epsilon^{(2)}_{0,1,1}}\right)^{\beta_1} \left[\frac{D_1}{\rho - \mu} + \frac{\beta_1}{\epsilon^{(2)}_{0,1,1}} I_1 - \beta_1 A^{(2)}_{1,1} \epsilon^{(2)\delta_1 - 1}_{0,1,1}\right] = \left(\frac{E}{\epsilon^{(2)}_{0,1,1}}\right)^{\beta_1} \left[\frac{\beta_1 D_1}{\rho - \mu} - \delta_1 A^{(2)}_{1,1} \epsilon^{(2)\delta_1 - 1}_{0,1,1}\right] \tag{17}$$

The first term on the left–hand side of (17) is the incremental project value created by waiting until the price is higher, while the second term represents the reduction in the MC of waiting due to saved investment cost. Similarly, the first term on the right–hand side reflects the opportunity cost of forgone cash flows discounted appropriately. Since $A^{(2)}_{1,1} < 0$, the third and second term on the left– and right–hand side, respectively, reflect the loss in option value from not having the second technology yet. Specifically, the third term on the left–hand side is the MB from postponing the loss in value, whereas the second term on the right–hand side is the MC from a potentially greater impact of the loss from waiting for a higher threshold price. Notice that it is the impact of λ on these two terms that determines the overall behaviour of the $\epsilon^{(2)}_{0,1,1}$ with respect to λ, and, as Proposition 4.3 indicates more generally, the impact of λ on $\epsilon^{(m)}_{\ell,m,m}$ is non–monotonic.

Proposition 4.3. $\forall \ell, m, n \in \mathbb{N}$ *the impact of* λ *on* $\epsilon^{(n)}_{\ell,m,m}$ *is non–monotonic.*

Finally, we step back to state (0,0) in order to determine the value of a project with two embedded replacement options that are subject to the arrival of the corresponding technologies. Notice that, unlike (7), now the value of the first investment option, $F^{(2)}_{0,1,1}(E)$, includes a single embedded option to perform one upgrade and that, as long as $\epsilon^{(2)}_{0,1,1} < \epsilon^{(2)}_{1,2,2}$, the solution depends on whether

$E < \epsilon^{(2)}_{0,1,1}$ or $E \geq \epsilon^{(2)}_{0,1,1}$. If $E < \epsilon^{(2)}_{0,1,1}$, then, even if the first technology became available, it would still be optimal to delay investment, whereas if $E \geq \epsilon^{(2)}_{0,1,1}$, then investment should be exercised immediately. The expression of $\Phi^{(2)}_{0,0}(E)$ is indicated in (18), where $A^{(2)}_{0,0} < 0$ and $B^{(2)}_{0,0} > 0$ are obtained via the value–matching and smooth–pasting conditions between the two branches and are indicated in (B–14) and (B–15) respectively.

$$\Phi^{(2)}_{0,0}(E) = \begin{cases} A^{(2)}_{0,1,1}E^{\beta_1} + A^{(2)}_{0,0}E^{\delta_1} & , E < \epsilon^{(2)}_{0,1,1} \\ \frac{\lambda D_1 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A^{(2)}_{1,2,2}E^{\beta_1} + A^{(2)}_{1,1}E^{\delta_1} + B^{(2)}_{0,0}E^{\delta_2} & , \epsilon^{(2)}_{0,1,1} \leq E \end{cases} \tag{18}$$

Similarly, we can determine the required investment threshold and the value of the option to invest for a project with any number of replacement options under a compulsive strategy.

5. Leapfrog versus Laggard Strategy

It is possible that a better technology becomes available while a firm waits in order to invest in an existing one, thus replacing the initial investment option with the option to choose between two alternative technologies. Here, we assume that a firm would not want to adopt an existing technology before comparing it to the next one. Consequently, the transition from (0,1,1) to (1,1) is not possible, and the only state prior to (1,2,2) is (0,1∨2), from which the firm may either adopt a laggard strategy and invest in the first technology with the embedded option to upgrade to the second or adopt a leapfrog strategy and invest directly in the second technology (Figure 4). Since the analysis related to states (2,2) and (1,2,2) is the same as in Section 4.2, we proceed directly to state $\overline{(2,2)}$. Notice that if the firm adopts the second technology directly from (0,1∨2), then it does not incur the cost I_1, and the expected NPV from immediate investment is indicated in (19).

$$\Phi^{(2)}_{\overline{2,2}}(E) = \frac{D_2 E}{\rho - \mu} - I_2 \tag{19}$$

Next, we consider state (0,1∨2) where the firm has the option to choose either the first technology with the option to switch to the second or the second technology directly. Due to the presence of the second technology, there exist two waiting regions, i.e., $\left(0, \underline{\epsilon}^{(2)}_{0,1,2}\right]$ and $\left[\overline{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{0,2,2}\right]$. If $E < \underline{\epsilon}^{(2)}_{0,1,2}$, then the firm will adopt a laggard strategy, i.e., wait until $E = \underline{\epsilon}^{(2)}_{0,1,2}$ and then invest in the first technology. If $E \in \left[\overline{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{0,2,2}\right]$, then the firm can either adopt a laggard or a leapfrog strategy. Specifically, if the output price increases to $\epsilon^{(2)}_{0,2,2}$, then the firm will invest in the second technology, but if it drops to $\overline{\epsilon}^{(2)}_{0,1,2}$, then it will invest in the first one. Consequently, the laggard strategy is adopted either when the output price is low, i.e., $E < \underline{\epsilon}^{(2)}_{0,1,2}$, and increases to $\underline{\epsilon}^{(2)}_{0,1,2}$ or when the output price is high, i.e., $E \in \left[\overline{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{0,2,2}\right]$, and decreases to $\overline{\epsilon}^{(2)}_{0,1,2}$.

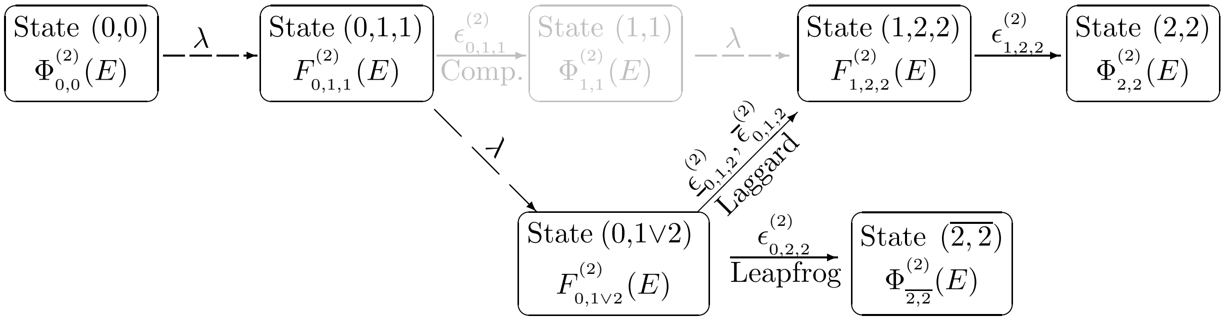

Figure 4: State transition diagram for $N = 2$ under leapfrog and laggard strategy

Hence, assuming that $\underline{\epsilon}^{(2)}_{0,1,2} < \epsilon^{(2)}_{1,2,2}$, then according to Décamps *et al.* (2006), the value function in state (0,1∨2) is indicated in (20), where $A^{(2)}_{0,1\vee 2}$ and $\underline{\epsilon}^{(2)}_{0,1,2}$ are determined via the value–matching and smooth–pasting conditions between the first two branches, while $\overline{\epsilon}^{(2)}_{0,1,2}$, $\epsilon^{(2)}_{0,2,2}$, $G^{(2)}_{0,1\vee 2}$, and $H^{(2)}_{0,1\vee 2}$ via the second, third, and fourth branch.

$$F^{(2)}_{0,1\vee 2}(E) = \begin{cases} A^{(2)}_{0,1\vee 2} E^{\beta_1} & , E < \underline{\epsilon}^{(2)}_{0,1,2} \\ F^{(2)}_{1,2,2}(E) & , \underline{\epsilon}^{(2)}_{0,1,2} \leq E \leq \overline{\epsilon}^{(2)}_{0,1,2} \\ G^{(2)}_{0,1\vee 2} E^{\beta_2} + H^{(2)}_{0,1\vee 2} E^{\beta_1} & , \overline{\epsilon}^{(2)}_{0,1,2} < E < \epsilon^{(2)}_{0,2,2} \\ \Phi^{(2)}_{\overline{2,2}}(E) & , \epsilon^{(2)}_{0,2,2} \leq E \end{cases} \tag{20}$$

Interestingly, although now both technologies are available, and, as a result, there is no loss in the value of the option to invest in the first one, the corresponding investment threshold under a laggard strategy when $E < \underline{\epsilon}^{(2)}_{0,1,2}$ is greater than that under a compulsive strategy when the arrival of the second innovation is uncertain, i.e., $\underline{\epsilon}^{(2)}_{0,1,2} > \epsilon^{(2)}_{0,1,1}$, whereas $\underline{\epsilon}^{(2)}_{0,1,2} = \epsilon^{(2)}_{0,1,1}$ when $\lambda = 0$ or $\lambda \to \infty$. More generally, Proposition 5.1 shows that the absence ($\lambda = 0$) or presence ($\lambda \to \infty$) of the second technology does not affect the decision to invest in the first one and indicates that a firm is more willing to adopt the current technology when the arrival of a subsequent one is uncertain.

Proposition 5.1. *$\forall \ell, m, n \in \mathbb{N}$ we have $\epsilon^{(n)}_{\ell,m,m} < \underline{\epsilon}^{(n)}_{\ell,m,n}$ $\forall \lambda \in (0, +\infty)$, whereas $\lambda = 0 \Rightarrow \epsilon^{(n)}_{\ell,m,m} = \underline{\epsilon}^{(n)}_{\ell,m,n}$ and $\lambda \to \infty \Rightarrow \epsilon^{(n)}_{\ell,m,m} \to \underline{\epsilon}^{(n)}_{\ell,m,n}$.*

The value function in state (0,1,1) is described in (21). The first term on the right–hand side is the option to invest in the first technology with an embedded option to upgrade to the second one if no innovation takes place within the time interval dt. However, as the second term indicates, if during dt an innovation occurs, then the firm obtains the option to choose between two technologies.

$$F^{(2)}_{0,1,1}(E) = (1 - \rho dt)(1 - \lambda dt)\mathbb{E}_E \left[F^{(2)}_{0,1,1}(E + dE) \right] + (1 - \rho dt)\lambda dt \mathbb{E}_E \left[F^{(2)}_{0,1\vee 2}(E + dE) \right] \tag{21}$$

Notice that (21) has to be solved separately for each of the four regions of E that are indicated in

(20). By substituting for $F^{(2)}_{0,1\vee 2}(E)$ in (21), we obtain the four differential equations for $F^{(2)}_{0,1,1}(E)$.

$$\begin{cases} \frac{1}{2}\sigma^2 E^2 F^{(2)''}_{0,1,1}(E) + \mu E F^{(2)'}_{0,1,1}(E) - (\rho+\lambda) F^{(2)}_{0,1,1}(E) + \lambda A^{(2)}_{0,1\vee 2,2} E^{\beta_1} = 0 & , E < \underline{\epsilon}^{(2)}_{0,1,2} \\ \frac{1}{2}\sigma^2 E^2 F^{(2)''}_{0,1,1}(E) + \mu E F^{(2)'}_{0,1,1}(E) - (\rho+\lambda) F^{(2)}_{0,1,1}(E) + \lambda F^{(2)}_{0,2,2}(E) = 0 & , \underline{\epsilon}^{(2)}_{0,1,2} \leq E \leq \overline{\epsilon}^{(2)}_{0,1,2} \\ \frac{1}{2}\sigma^2 E^2 F^{(2)''}_{0,1,1}(E) + \mu E F^{(2)'}_{0,1,1}(E) - (\rho+\lambda) F^{(2)}_{0,1,1}(E) & \\ + \lambda \left[G^{(2)}_{0,1\vee 2} E^{\beta_2} + H^{(2)}_{0,1\vee 2} E^{\beta_1} \right] = 0 & , \overline{\epsilon}^{(2)}_{0,1,2} < E < \epsilon^{(2)}_{0,2,2} \\ \frac{1}{2}\sigma^2 E^2 F^{(2)''}_{0,1,1}(E) + \mu E F^{(2)'}_{0,1,1}(E) - (\rho+\lambda) F^{(2)}_{0,1,1}(E) + \lambda F^{(2)}_{\overline{2,2}}(E) = 0 & , \epsilon^{(2)}_{0,2,2} \leq E \end{cases} \tag{22}$$

By solving for $F^{(2)}_{0,1,1}(E)$, we obtain the solution indicated in (23), where $A^{(2)}_{1,2,2}$ is described in (13) and the endogenous constants $A^{(2)}_{0,1,1}$, $L^{(2)}_{0,1,1}$, $P^{(2)}_{0,1,1}$, $Q^{(2)}_{0,1,1}$, $R^{(2)}_{0,1,1}$, and $J^{(2)}_{0,1,1}$ are determined numerically via the value–matching and smooth–pasting conditions between the branches of (23).

$$F^{(2)}_{0,1,1}(E) = \begin{cases} A^{(2)}_{0,1\vee 2} E^{\beta_1} + P^{(2)}_{0,1,1} E^{\delta_1} & , E < \underline{\epsilon}^{(2)}_{0,1,2} \\ \frac{\lambda D_1 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A^{(2)}_{1,2,2} E^{\beta_1} + L^{(2)}_{0,1,1} E^{\delta_1} + B^{(2)}_{0,1,1} E^{\delta_2} & , \underline{\epsilon}^{(2)}_{0,1,2} \leq E \leq \overline{\epsilon}^{(2)}_{0,1,2} \\ G^{(2)}_{0,1\vee 2} E^{\beta_2} + H^{(2)}_{0,1\vee 2} E^{\beta_1} + Q^{(2)}_{0,1,1} E^{\delta_1} + R^{(2)}_{0,1,1} E^{\delta_2} & , \overline{\epsilon}^{(2)}_{0,1,2} < E < \epsilon^{(2)}_{0,2,2} \\ \frac{\lambda D_2 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} + J^{(2)}_{0,1,1} E^{\delta_2} & , \epsilon^{(2)}_{0,2,2} \leq E \end{cases} \tag{23}$$

Finally, following the same steps as in the case of $F^{(2)}_{0,1,1}(E)$, we can determine the value function in state $(0,0)$. Notice that, without loss of generality, the comparison of $F^{(2)}_{0,1,1}(E)$ under the two strategies can be done in state $(0,1,1)$, and, therefore, the analysis of state $(0,0)$ is omitted. In fact, since we know from Proposition 5.1 that $\epsilon^{(2)}_{0,1,1} \leq \underline{\epsilon}^{(2)}_{0,1,2}$, the comparison of the strategies at $(0,1,1)$ can be made separately for each of the regions of E that are indicated in (15) and (23).

6. Comparison of the Strategies

Despite the incentive to delay investment in an old technology in order to compare it with a newer one, it is possible that by the time that the latter becomes available it is already optimal to invest in the former. Here, we extend Section 5 by assuming that the choice of strategy depends on E and λ, and, thus, it is not determined exogenously. Hence, both $(0,1,1) \rightarrow (1,1)$ and (0,1,1)$\rightarrow$ (0,1$\vee$2) are possible transitions (Figure 5), and the final choice of strategy is endogenous.

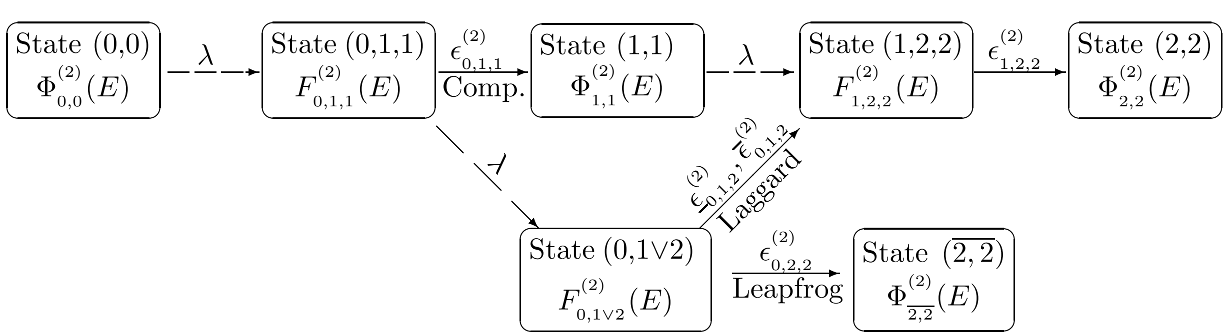

Figure 5: State transition diagram for $N = 2$ under leapfrog and laggard strategy

Figure 6 summarises the possible strategies for different values of E and λ. As shown in Proposition 6.1, the compulsive strategy is optimal $\forall \lambda \in \mathbb{R}^+$ within the first two price regions, i.e., $E \leq \overline{\epsilon}^{(2)}_{0,1,2}$. Intuitively, even if a second technology were available, then a firm would have to wait long before the output price reaches the corresponding investment threshold and the expected payoff from investment in the second technology does not offset the forgone revenues from skipping the first one. By contrast, in the third and fourth region, i.e., $E > \overline{\epsilon}^{(2)}_{0,1,2}$, the optimal strategy depends on λ. Indeed, the required λ for which a firm may consider waiting for the next technology decreases as the output price increases, and, as shown in Proposition 6.2, it is possible to determine the required value of λ $\forall E > \overline{\epsilon}^{(2)}_{0,1,2}$. Additionally, high price uncertainty delays investment in both the first and the second technology and facilitates a laggard strategy in state $(0,1 \vee 2)$, thereby increasing the range of prices where a leapfrog or a laggard strategy may be considered.

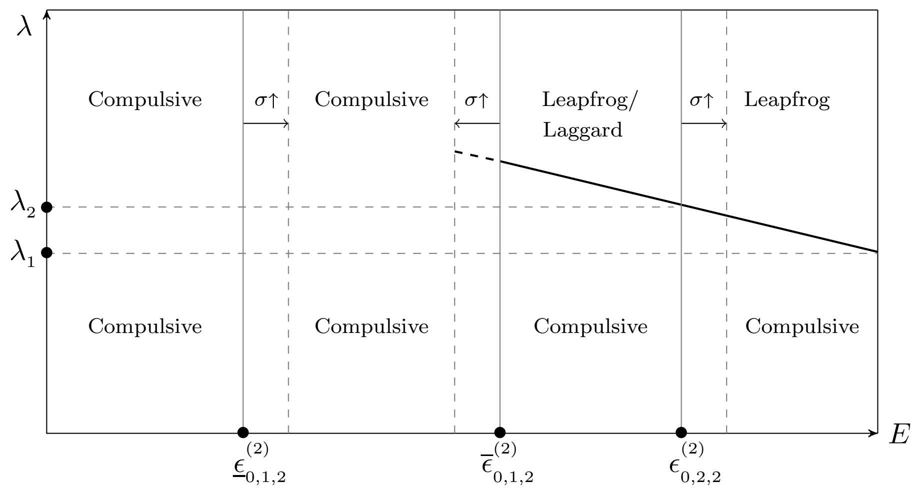

Figure 6: Comparison of the strategies

More specifically, notice that in the first two regions, i.e., $E \in \left(0, \overline{\epsilon}^{(2)}_{0,1,2}\right]$, the compulsive strategy always dominates the leapfrog/laggard strategy (Proposition 6.1). Indeed, under a leapfrog/laggard strategy, $F^{(2)}_{0,1,1}(E)$ consists of the expected payoff from investment in the first technology with a single embedded option to replace it conditional on the arrival of the second innovation. Hence, $F^{(2)}_{0,1,1}(E)$ is greater under a compulsive strategy $\forall \lambda \in \mathbb{R}^+$, since the firm receives the same payoff without having to wait for the second innovation. Consequently, if $E \in \left(0, \overline{\epsilon}^{(2)}_{0,1,2}\right]$ the firm does not need to wait until $\overline{\epsilon}^{(2)}_{0,1,2}$ in order to invest in the first technology (Décamps *et al.,* 2006), since, under a compulsive strategy, the investment option should be exercised at $\epsilon^{(2)}_{0,1,1} \leq \underline{\epsilon}^{(2)}_{0,1,2}$.

Proposition 6.1. $\forall E \in \left(0, \overline{\epsilon}^{(2)}_{0,1,2}\right]$, *if the investment region in state* $(0, 1 \vee 2)$ *is dichotomous, then the compulsive strategy dominates the leapfrog/laggard strategy* $\forall \lambda \in \mathbb{R}^+$.

By contrast, when the output price is high, i.e., $E > \overline{\epsilon}^{(2)}_{0,1,2}$, it is possible that a laggard or a leapfrog strategy dominates. Intuitively, a high output price compensates for a low λ and increases the incentive to wait for the second technology, while a low output price increases the rate λ for which a firm may consider waiting for the second technology. More specifically, if $E \in \left(\bar{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{0,2,2}\right]$, then the payoff from a compulsive strategy must be compared to the value of the option to choose either one of the two technologies. In effect, the value of the option to choose between two technologies may be higher than the expected NPV from adopting the older one if the rate of innovation in high. Similarly, if $E \in \left(\epsilon^{(2)}_{0,2,2}, \infty\right)$, then the value from immediate investment in the second technology, which, under a leapfrog strategy, is contingent upon its arrival, must be compared to the value from immediate investment in the first technology with an embedded option to upgrade it. If λ is high, then it may be preferable to wait for the second innovation rather than invest in the first one. As Proposition 6.2 indicates, $\forall E \in \left(\bar{\epsilon}^{(2)}_{0,1,2}, \infty\right]$ it is possible to determine the optimal strategy for each pair (E, λ) as well as the minimum rates λ_1 and λ_2 for which a firm would adopt a leapfrog or a leapfrog/laggard strategy respectively.

Proposition 6.2. $\exists\ \lambda_1, \lambda_2 \in \mathbb{R}^+$ *with* $\lambda_1 \leq \lambda_2$:

i : $\forall \lambda > \lambda_1$, $\exists \mathcal{B} \subseteq \left(\epsilon^{(2)}_{0,2,2}, \infty\right)$: $\forall E \in \mathcal{B}$, *the leapfrog strategy dominates*

ii : $\forall \lambda > \lambda_2$, $\exists \mathcal{A} \subset \left(\bar{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{0,2,2}\right]$: $\forall E \in \mathcal{A}$, *the leapfrog/laggard strategy dominates*

Hence, by taking into account the arrival of innovations, we obtain the maximised value in state $(0,1,1)$, which is described in (24). The first two branches of (24) indicate that the compulsive strategy is always better for $E < \bar{\epsilon}^{(2)}_{0,1,2}$ regardless of λ. However, as the bottom two branches indicate, for $E > \bar{\epsilon}^{(2)}_{0,1,2}$ the optimal strategy depends on E and λ.

$$F^{(2)}_{0,1,1}(E) = \begin{cases} A^{(2)}_{0,1,1} E^{\beta_1} & , E < \underline{\epsilon}^{(2)}_{0,1,2} \\ \Phi^{(2)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} + A^{(2)}_{1,1} E^{\delta_1} & , \underline{\epsilon}^{(2)}_{0,1,2} < E < \bar{\epsilon}^{(2)}_{0,1,2} \\ \max_{E,\lambda} \Big\{ \Phi^{(2)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} + A^{(2)}_{1,1} E^{\delta_1}, & \\ \qquad G^{(2)}_{0,2} E^{\beta_2} + H^{(2)}_{0,2} E^{\beta_1} + Q^{(2)}_{0,1,1} E^{\delta_1} + R^{(2)}_{0,1,1} E^{\delta_2} \Big\} & , \bar{\epsilon}^{(2)}_{0,1,2} < E < \epsilon^{(2)}_{0,2,2} \\ \max_{E,\lambda} \Big\{ \Phi^{(2)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} + A^{(2)}_{1,1} E^{\delta_1}, & \\ \qquad \frac{\lambda D_2 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} + J^{(2)}_{0,1,1} E^{\delta_2} \Big\} & , \epsilon^{(2)}_{0,2,2} \leq E \end{cases} \quad (24)$$

7. Numerical Results

7.1. Compulsive Strategy with N = 1

For the numerical results, the values of the different parameters are $\rho = 0.1, \mu = 0.01$, and $\sigma \in [0, 0.3]$. Also, for the purposes of the analysis we assume that $\lambda \in [0,1]$. We begin with the case of $N = 1$ and set $I_1 = 1500$ and $D_1 = 16$. Figure 7 illustrates the value of a single investment option, $F^{(1)}_{0,1,1}(\cdot)$, the expected NPV from exercising it, $\Phi^{(1)}_{1,1}(\cdot)$, and the value of an option to perform a single upgrade which is not available yet, $\Phi^{(1)}_{0,0}(\cdot)$. Notice that the value function in state $(0,0)$

increases with higher λ. Indeed, for $\lambda = 0$, no innovation will take place, and, as a result, the firm will continue to wait forever. By allowing for $\lambda > 0$, the likelihood of an innovation increases, thereby raising the value function $\Phi_{0,0}^{(1)}(\cdot)$. Once a new technology arrives, the firm has the option to adopt it, and the required investment threshold is $\epsilon_{0,1,1}^{(1)} = 14.06$. Notice that there are no additional investment options available to influence the value of the initial investment decision. As a result, conditional on the arrival of an innovation, the likelihood of exercising the investment option in state (0,1,1) is subject only to the underlying stochastic process. Consequently, as indicated in (5), λ impacts the likelihood of an innovation but not the required investment threshold.

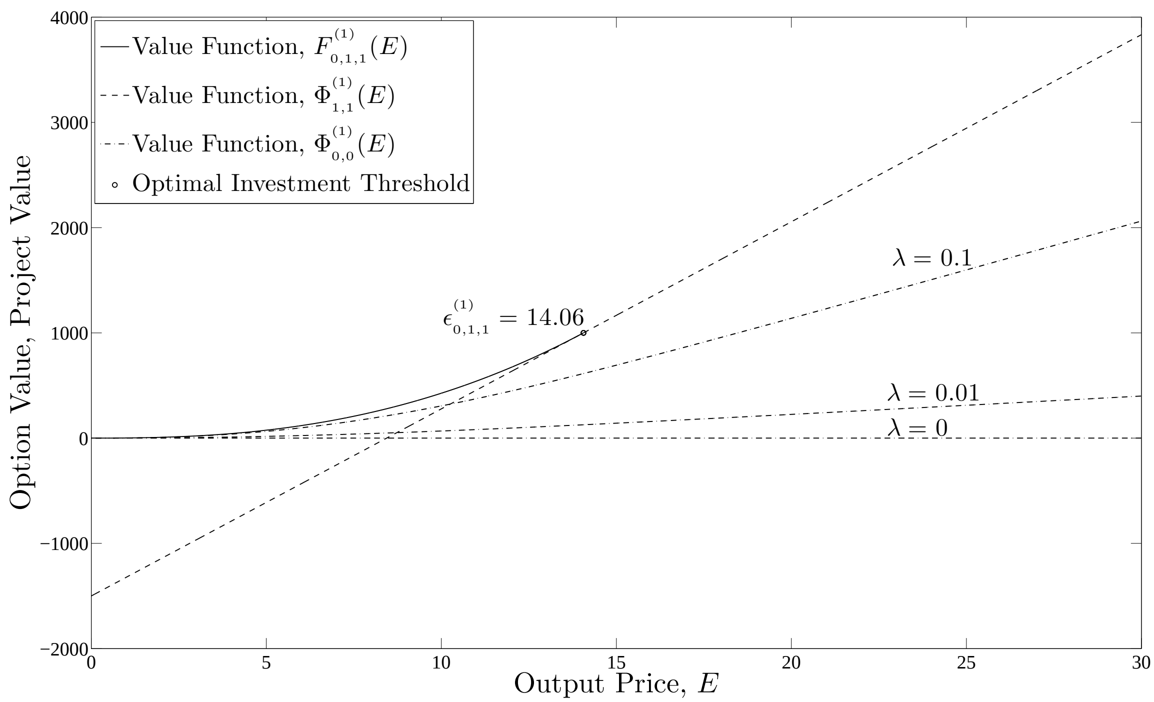

Figure 7: Option and project value under a compulsive strategy with $N = 1$, $\sigma = 0.2$, and $\lambda = 0, 0.01, 0.1$

7.2. Compulsive Strategy with N = 2

Next, we extend the case of compulsive strategy by assuming that a firm holds two investment options. Figure 8 illustrates the value functions in states $(2,2), (1,2,2), (1,1), (0,1,1)$, and $(0,0)$. The numerical assumptions $I_1 = 500, D_1 = 8, I_2 = 1500$, and $D_2 = 16$, imply that the second technology is three times as expensive as the first one and twice as efficient, i.e., they satisfy the assumption $\frac{D_2}{I_1+I_2} < \frac{D_1}{I_1}$. For $\lambda = 0.01$, Figure 8 illustrates the value of the investment options $F_{0,1,1}^{(2)}(\cdot)$ and $F_{1,2,2}^{(2)}(\cdot)$, the corresponding expected NPVs from exercising them, i.e., $\Phi_{1,1}^{(2)}(\cdot)$ and $\Phi_{2,2}^{(2)}(\cdot)$ respectively, and, finally, the value of a project with two embedded investment options, $\Phi_{0,0}^{(2)}(\cdot)$, that are subject to the arrival of the corresponding technologies. Like in the case $N = 1$, the value function $F_{1,2,2}^{(2)}(\cdot)$ and the investment threshold $\epsilon_{1,2,2}^{(2)}$ are independent of λ. For $\lambda = 0.01$, the value function in state (0,0) increases with E since now both of the embedded investment options have a positive value. Additionally, in state $(0,1,1)$ the value of the embedded investment option is positive but, since the arrival of the second technology remains uncertain, $F^{(2)}_{0,1,1}(\cdot)$ lies between $\Phi^{(2)}_{0,0}(\cdot)$ and $F^{(2)}_{1,2,2}(\cdot)$, as shown in Proposition 4.2. In fact, $F^{(2)}_{0,1,1}(\cdot)$ is greater than $\Phi^{(2)}_{0,0}(\cdot)$ since in state $(0,0)$ the availability of the two technologies is uncertain, while, at the same time, it is lower than $F^{(2)}_{1,2,2}(\cdot)$ where both technologies are available.

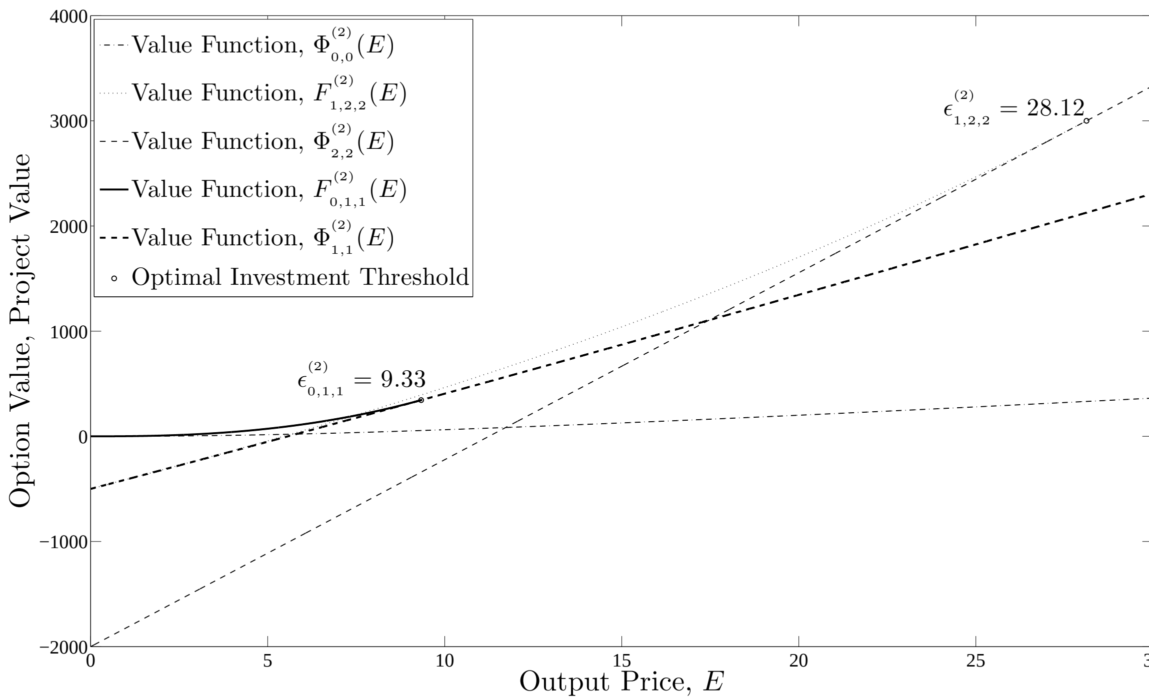

Figure 8: Option and project value under a compulsive strategy with $N = 2$, $\sigma = 0.2$, and $\lambda = 0.01$

The left panel in Figure 9 illustrates the relative loss in the value function, $\Phi^{(2)}_{1,1}(\cdot)$, due to the likelihood associated with the arrival of the second innovation. This is expressed in (25), where $F^{(2)}_{1,2,2}(E)$ is the value of the option to invest when the first technology is in operation and the second technology is available for adoption.

$$\frac{F^{(2)}_{1,2,2}(E) - \Phi^{(2)}_{1,1}(E)}{F^{(2)}_{1,2,2}(E)}, \quad E \leq \epsilon^{(2)}_{1,2,2} \tag{25}$$

Notice that for $\lambda = 0$, we have $A^{(2)}_{1,1} = -A^{(2)}_{1,2,2}$, and, thus, the relative loss in $\Phi^{(2)}_{1,1}(\cdot)$ is maximised since the value of the embedded option to invest in the second technology is zero. Then, for low values of λ, the relative loss in $\Phi^{(2)}_{1,1}(\cdot)$ decreases quickly because a higher λ increases the likelihood of the arrival of the second technology. Above a certain value of λ, the decrease is less pronounced since the likelihood of at least one innovation occurring converges to one, and, as a result, the relative loss in option value converges to zero, as shown in Proposition 4.1. As the right panel illustrates, for low values of λ the rapid decrease in the relative loss in $\Phi_{1,1}^{(2)}(\cdot)$ increases the incentive to invest and lowers the required investment threshold, $\epsilon_{0,1,1}^{(2)}$. Surprisingly, however, $\epsilon_{0,1,1}^{(2)}$ does not always decrease with higher λ. Indeed, when λ is low, $\epsilon_{0,1,1}^{(2)}$ decreases as the rate of innovation increases; however, as λ increases further, $\epsilon_{0,1,1}^{(2)}$ increases and converges to its value for $\lambda = 0$, as shown in Proposition 5.1. Intuitively, when the rate of innovation is low, the extra benefit due to the decrease in the relative loss in $\Phi_{1,1}^{(2)}(\cdot)$ is more pronounced than the incentive to delay investment. By contrast, when innovations arrive more frequently, the decrease in the relative loss in $\Phi_{1,1}^{(2)}(\cdot)$ is less pronounced than the value of waiting, which increases with higher uncertainty. Finally, uncertainty raises the value of the investment opportunity, thereby increasing the incentive to delay investment, and, in turn, the required investment threshold, while, at the same time, it makes the non–monotonic impact of λ on $\epsilon_{0,1,1}^{(2)}$ more pronounced.

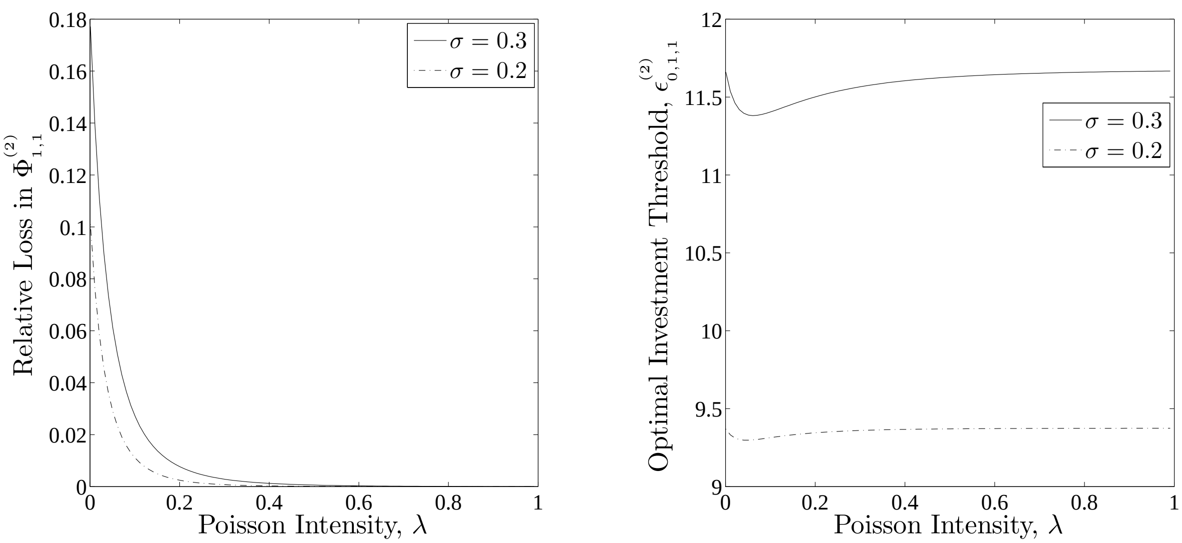

Figure 9: Relative loss in option value (left) and optimal investment threshold (right) versus σ, λ

In order to obtain additional insights on the impact of λ on $\epsilon_{0,1,1}^{(2)}$, Figure 10 illustrates the MB and MC of delaying investment for $\sigma = 0.2$ and $\lambda = 0, 0.1, 0.3$. Notice that, as λ increases, both the MB and the MC of delaying investment decrease because the frequent arrival of new technologies erodes the value of waiting to invest in the currently available technology and lowers the forgone revenues from waiting. Intuitively, for low λ, technologies are not arriving frequently, and, as a result, a firm would be more willing to adopt the current technology sooner in order to have a shot at the yet unreleased version. By contrast, as λ increases, innovations take place more frequently, thereby increasing the incentive to delay investment in order to avoid making a mistake.

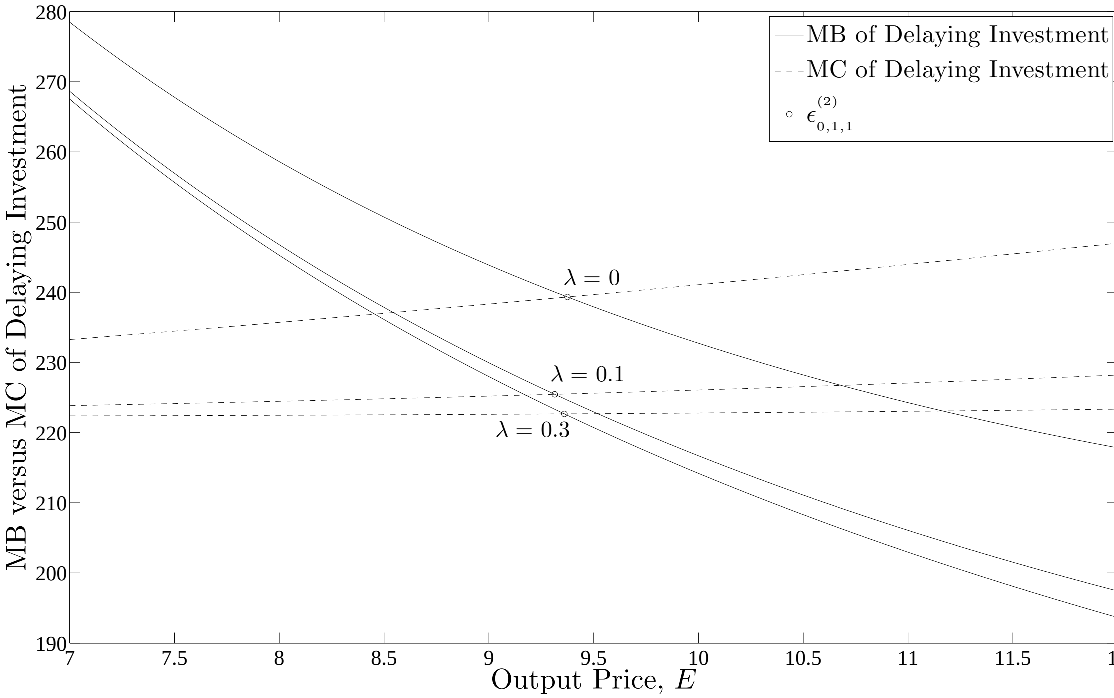

Figure 10: MB and MC of delaying investment for, $\sigma = 0.2$ and $\lambda = 0, 0.1, 0.3$

7.3. Leapfrog versus Laggard Strategy

Figure 11 illustrates the value functions in the states $(0, 1 \vee 2)$ and $(0, 1, 1)$, when the transition from (0,1,1) to (1,1) is not considered. If $E < 9.37$, then the firm must wait until $E = 9.37$ and adopt the first technology. Also, if $E \in [10.96, 14.51]$, then, due to the presence of the second technology, the firm has to wait again, i.e., refrain from adopting any technology. Specifically, if E increases to 14.51, then the firm will invest directly in the second technology, but if E drops to 10.96, then it will invest in the first technology while holding the option to switch to the second. Notice that the value function $F^{(2)}_{0,1,1}(\cdot)$ is lower than $F^{(2)}_{0,1\vee 2}(\cdot)$ because $F^{(2)}_{0,1\vee 2}(\cdot)$ reflects the value of the option to choose between two technologies that are available, whereas $F^{(2)}_{0,1,1}(\cdot)$ reflects the same value function in the absence of the second technology. However, as λ increases, the loss in value due to the likelihood associated with the arrival of the second technology decreases and $F^{(2)}_{0,1,1}(\cdot)$ converges to $F^{(2)}_{0,1\vee 2}(\cdot)$.

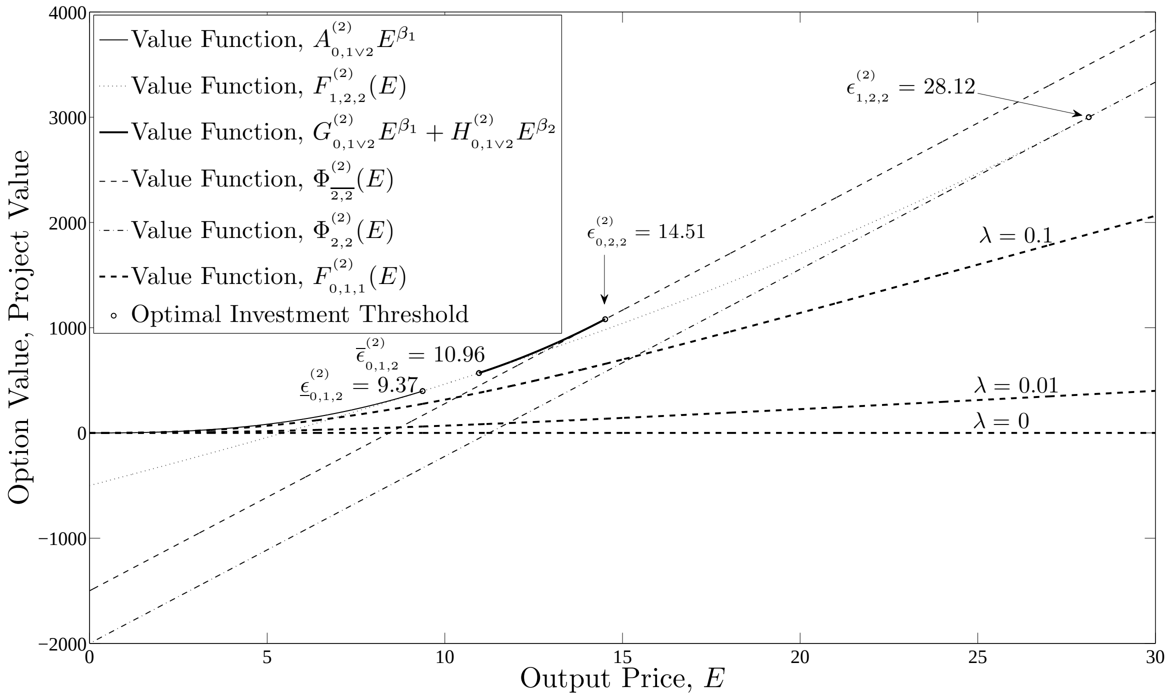

Figure 11: Option and project value for $N = 2$, $\sigma = 0.2$ under leapfrog and laggard strategy

The impact of volatility on the optimal investment thresholds $\underline{\epsilon}^{(2)}_{0,1,2}, \overline{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{1,2,2}$, and $\epsilon^{(2)}_{0,2,2}$ is illustrated in Figure 12. Notice that, since the investment thresholds $\underline{\epsilon}^{(2)}_{0,1,2}$ and $\overline{\epsilon}^{(2)}_{0,1,2}$ move in opposite directions with higher volatility, it is possible that $\underline{\epsilon}^{(2)}_{0,1,2} > \overline{\epsilon}^{(2)}_{0,1,2}$. In that case, the region of direct investment in the first technology disappears, and, therefore, it is optimal to adopt a leapfrog strategy, i.e., wait until $E = \epsilon^{(2)}_{0,2,2}$ and then invest in the second technology.

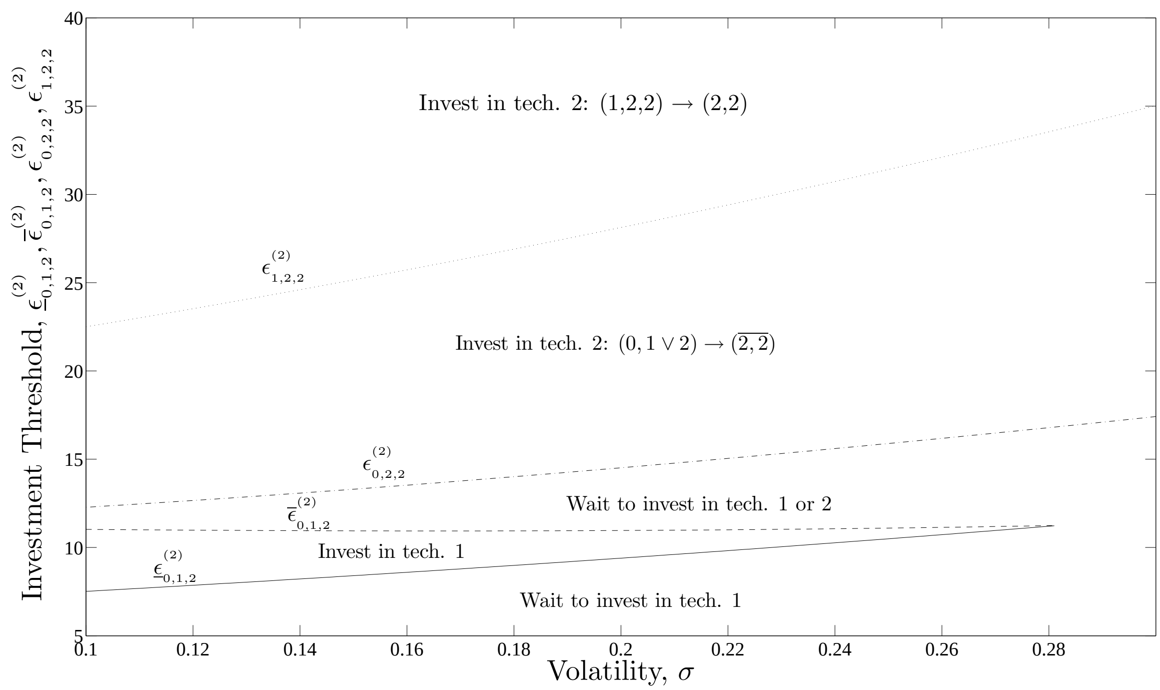

Figure 12: Impact of σ on $\underline{\epsilon}^{(2)}_{0,1,2}, \overline{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{0,2,2}$, and $\epsilon^{(2)}_{1,2,2}$ for $\lambda = 0.1$

7.4. Comparison of the Strategies

Figure 13 illustrates the value function $F^{(2)}_{0,1,1}(\cdot)$ under a compulsive and a leapfrog/laggard strategy for $\lambda = 0.1$ and 0.6. Notice that an increase in λ, which is indicated by the direction of the arrows, increases the value of the embedded options and causes both value functions to shift upward, yet the value function under a compulsive strategy is less responsive. Interestingly, when $\lambda = 0.1$, the value function in state (0,1,1) is always greater under a compulsive strategy than under a leapfrog/laggard strategy, while, for $\lambda = 0.6$, the leapfrog/laggard strategy dominates when the output prices is high. Notice that, as shown in Proposition 6.2, the leapfrog/laggard strategy can dominate only for $E > \overline{\epsilon}^{(2)}_{0,1,2}$, whereas the compulsive strategy dominates always when $E < \overline{\epsilon}^{(2)}_{0,1,2}$ as shown in Proposition 6.1.

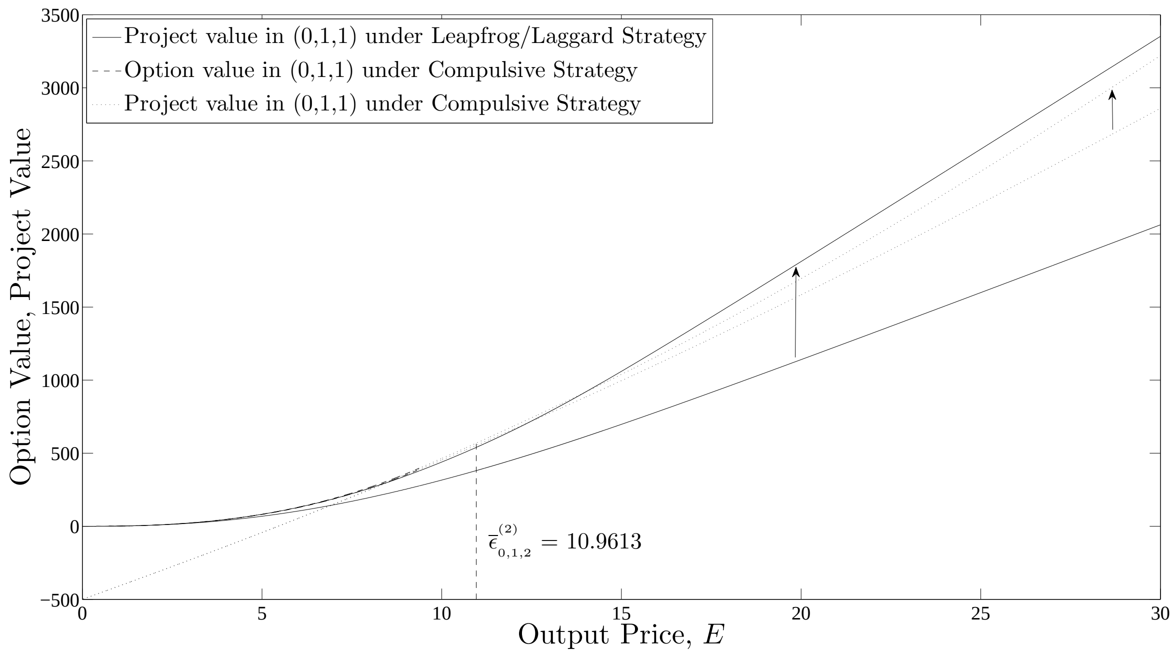

Figure 13: Comparison of the value function $F^{(2)}_{0,1,1}(E)$ under the two strategies for $\sigma = 0.2$ and $\lambda = 0.1, 0.6$

The left panel in Figure 14 illustrates the relative value of the two strategies for $E < \underline{\epsilon}^{(2)}_{0,1,2}$. More specifically, (26) compares the first branch of (15) with the first branch of (23).

$$\frac{A^{(2)}_{0,1,1}E^{\beta_1}}{A^{(2)}_{0,1\vee 2}E^{\beta_1} + P^{(2)}_{0,1,1}E^{\delta_1}} \tag{26}$$

Similarly, the right panel compares the two strategies for $E > \epsilon^{(2)}_{0,2,2} > \epsilon^{(2)}_{0,1,1}$, i.e., the second branch of (15) with the fourth branch of (23).

$$\frac{\Phi^{(1)}_{1,1}(E) + A^{(2)}_{1,2,2}E^{\beta_1} + A^{(2)}_{1,1}E^{\delta_1}}{\frac{\lambda D_2 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} + J^{(2)}_{0,1,1}E^{\delta_2}} \tag{27}$$

According to the left panel, it is always better to adopt a compulsive strategy when the output price is low because, even if a second technology were available, it would still be optimal to delay investment until $E = \underline{\epsilon}^{(2)}_{0,1,2}$ and then invest in the first technology. Intuitively, the firm would have to wait too long before adopting the second technology and the corresponding revenues do not offset the foregone cash flows from ignoring the first one. Notice also that the relative value of the compulsive strategy increases as output price uncertainty decreases. This happens because a more stable economic environment reduces the likelihood of an unexpected increase in the output price, and, in turn, the opportunity cost from adopting the strategy. By contrast, as the right panel illustrates, if the output price is high, then the relative value of the two strategies can drop below one when λ is high, thereby indicating that the expected value from adopting a leapfrog or a laggard strategy exceeds that of the compulsive strategy. This happens because a higher output price reduces the expected time until investment in the second technology is justified, while, at the same time, a high λ decreases the feasibility of the compulsive strategy by reducing the expected time between subsequent innovations. Again, this result is more pronounced under lower price uncertainty since this decreases the likelihood of an unexpected downturn.

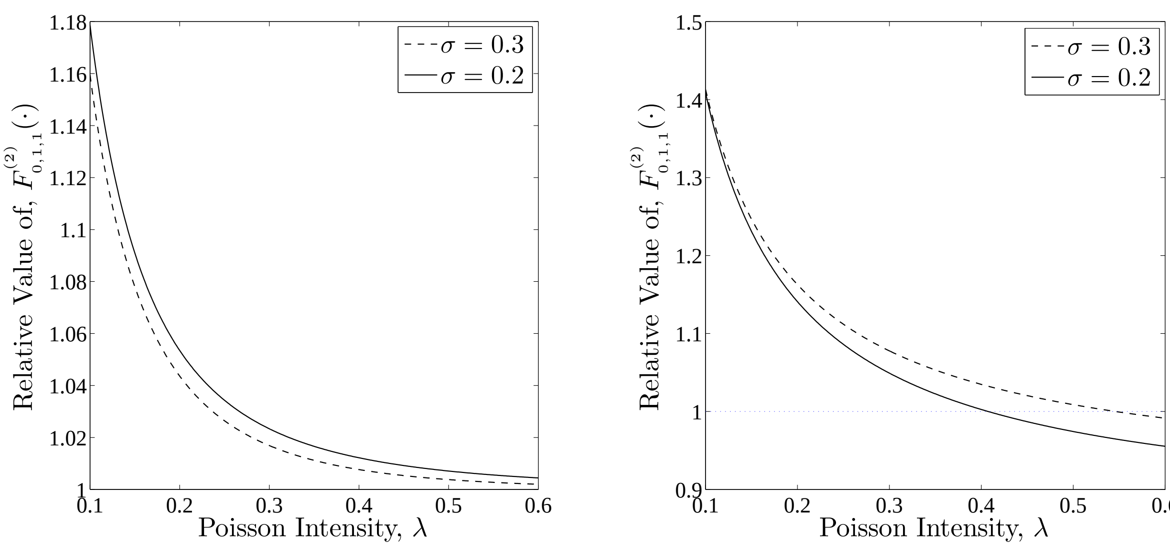

Figure 14: Relative value of $F^{(2)}_{0,1,1}(\cdot)$ under the two strategies under low output price, i.e., $E < \underline{\epsilon}^{(2)}_{0,1,2}$, (left) and high output price, i.e., $E > \epsilon^{(2)}_{0,2,2}$, (right)

Notice that these results are in line with Grenadier and Weiss (1997) who find that, as the expected time of arrival of an innovation increases, the probability of adopting a leapfrog or a laggard strategy drops to zero while the probability of adopting a compulsive strategy increases. Here, we extend Grenadier and Weiss (1997) by allowing for price uncertainty via a continuous time stochastic process, thereby relaxing the assumption of the static representation of price uncertainty. This not only enables the derivation of price thresholds corresponding to each strategy but also facilitates the analysis of the endogenous relation between price and technological uncertainty and its impact on the optimal strategy selection decision.

8. Conclusions

We develop a real options framework for sequential investment in order to address the problem of optimal replacement of an emerging technology under price and technological uncertainty. Although such features are crucial for investment in many industries, analytical formulations of sequential investment that include both of them are limited. Consequently, we extend the real options approach by incorporating these features into an analytical framework for sequential investment, in order to obtain insights on their combined impact on the investors' propensity to upgrade equipment. We analyse three investment strategies; compulsive, laggard, and leapfrog. In the first one, a firm invests in every technology that becomes available, whereas in the second and third ones it can choose between an older and a newer technology respectively.

We find that, under a compulsive strategy, an increase in the rate of innovation does not affect the optimal investment threshold when a firm holds a single investment option, even though it increases the likelihood of an innovation. Interestingly, under multiple investment opportunities, the impact of the rate of innovation on the initial investment decision is ambiguous. Indeed, although a higher rate increases the likelihood of an innovation, and, in turn, the value of an investment opportunity, the corresponding investment threshold may either increase or decrease. In fact, uncertainty regarding the arrival of innovations may actually accelerate investment in a technology. Moreover, a comparison of the strategies indicates that, when the output price is low, the compulsive strategy is always better as long as, under a leapfrog/laggard strategy, the investment region is dichotomous. However, when the rate of innovation is high, it is possible that the option to choose between two technologies is more valuable than the immediate payoff from investment in the first technology with an embedded option to switch to the second. The implications of the results are crucial for the participants of many industries. For example, manufacturers of RE equipment can develop more informed R&D strategies by anticipating the investors' response to changes in the rate that technologies become available, while policy measures for supporting RE via R&D funding can become more efficient by taking into account the investors' propensity to adopt new technologies.

Apart from market and technological uncertainty, several other uncertainties related to RE projects are amenable to real options, which, in turn, offers directions for further research. For example, it is possible to analyse policy uncertainty with respect to any change of a support scheme via a regime–switching model based on a Markov–modulated Brownian motion. Hence, the current framework can be extended to analyse not only the impact of policy uncertainty on investment and operational decisions but also the endogenous relation between technological and policy uncertainty as well as strategic interactions between competing investors and developers. Finally, it would be interesting to allow for a different stochastic process, e.g., mean reverting process, in order to relax the limitations inherent in the geometric Brownian motion.

APPENDIX

Compulsive Strategy with N = 1

The value of the investment option in state $(0,1,1)$ is indicated in (A–1).

$$F^{(1)}_{0,1,1}(E) = \begin{cases} (1-\rho dt)\mathbb{E}_E\left[F^{(1)}_{0,1,1}(E+dE)\right] & , E < \epsilon^{(1)}_{0,1,1} \\ \Phi^{(1)}_{1,1}(E) & , E \geq \epsilon^{(1)}_{0,1,1} \end{cases} \tag{A–1}$$

By expanding the first branch on the right–hand side of (A–1) using Itô's lemma, we obtain the differential equation (A–2)

$$\frac{1}{2}\sigma^2 E^2 F^{(1)''}_{0,1,1}(E) + \mu E F^{(1)'}_{0,1,1}(E) - \rho F^{(1)}_{0,1,1}(E) = 0 \tag{A–2}$$

which, for $E < \epsilon^{(1)}_{0,1,1}$, has the general solution that is indicated in (A–3).

$$F^{(1)}_{0,1,1}(E) = A^{(1)}_{0,1,1} E^{\beta_1} + C^{(1)}_{0,1,1} E^{\beta_2} \tag{A–3}$$

The second term on the right–hand side of (A–3) can be ruled out by noticing that as $E \to 0$ the value of the project becomes very small. However, since $\beta_2 < 0$, we have $C^{(1)}_{0,1,1} E^{\beta_2} \to \infty$ as $E \to 0$. Consequently, we must have $C^{(1)}_{0,1,1} = 0$, and, thus,

$$F^{(1)}_{0,1,1}(E) = \begin{cases} A^{(1)}_{0,1,1} E^{\beta_1} & , E < \epsilon^{(1)}_{0,1,1} \\ \Phi^{(1)}_{1,1}(E) & , E \geq \epsilon^{(1)}_{0,1,1} \end{cases} \tag{A–4}$$

where $A^{(1)}_{0,1}$ and $\epsilon^{(1)}_{0,1,1}$ are determined via the value–matching and smooth–pasting conditions between the two branches of (A–1) and are indicated in (A–5).

$$\left.\begin{aligned} A^{(1)}_{0,1,1} \epsilon^{(1)^{\beta_1}}_{0,1,1} &= \Phi^{(1)}_{1}\left(\epsilon^{(1)}_{0,1,1}\right) \\ \beta_1 A^{(1)}_{0,1,1} \epsilon^{(1)^{\beta_1 - 1}}_{0,1,1} &= \Phi^{(1)'}_{1}\left(\epsilon^{(1)}_{0,1,1}\right) \end{aligned}\right\} \Rightarrow \begin{aligned} \epsilon^{(1)}_{0,1,1} &= \frac{\beta_1}{\beta_1 - 1}\frac{I_1(\rho-\mu)}{D_1} \\ A^{(1)}_{0,1,1} &= \frac{\epsilon^{(1)^{1-\beta_1}}_{0,1,1}}{\beta_1}\frac{D_1}{\rho-\mu} \end{aligned} \tag{A–5}$$

Next, we consider the value function in state $(0,0)$ where the first technology has yet to become available. The value function in state $(0,0)$ is described in (A–6)

$$\Phi^{(1)}_{0,0}(E) = (1-\rho dt)(1-\lambda dt)\mathbb{E}_E\left[\Phi^{(1)}_{0,0}(E+dE)\right] + (1-\rho dt)\lambda dt \mathbb{E}_E\left[F^{(1)}_{0,1,1}(E+dE)\right] \tag{A–6}$$

and by expanding the right–hand side using Itô's lemma we obtain (A–7).

$$\begin{aligned} \Phi^{(1)}_{0,0}(E) &= (1-(\rho+\lambda)dt)\left[\Phi^{(1)}_{0,0}(E) + \frac{1}{2}\sigma^2 E^2 \Phi^{(1)''}_{0,0}(E)\,dt + \mu E \Phi^{(1)'}_{0,0}(E)\,dt\right] \\ &+ (1-\rho dt)\lambda dt\left[F^{(1)}_{0,1,1}(E) + \frac{1}{2}\sigma^2 E^2 F^{(1)''}_{0,1,1}(E)\,dt + \mu E F^{(1)'}_{0,1,1}(E)\,dt\right] \end{aligned} \tag{A–7}$$

After simplifying (A–7), we obtain the differential equation that describes $\Phi^{(1)}_{0,0}(E)$.

$$\frac{1}{2}\sigma^2 E^2 \Phi^{(1)''}_{0,0}(E) + \mu E \Phi^{(1)'}_{0,0}(E) - (\rho+\lambda)\Phi^{(1)}_{0,0}(E) + \lambda F^{(1)}_{0,1,1}(E) = 0 \qquad \text{(A–8)}$$

Notice that the solution depends on whether $E < \epsilon^{(1)}_{0,1,1}$ or $E \geq \epsilon^{(1)}_{0,1,1}$, i.e.,

$$\begin{cases} \frac{1}{2}\sigma^2 E^2 \Phi^{(1)''}_{0,0}(E) + \mu E \Phi^{(1)'}_{0,0}(E) - (\rho+\lambda)\Phi^{(1)}_{0,0}(E) + \lambda A^{(1)}_{0,1,1} E^{\beta_1} = 0 & , E < \epsilon^{(1)}_{0,1,1} \\ \frac{1}{2}\sigma^2 E^2 \Phi^{(1)''}_{0,0}(E) + \mu E \Phi^{(1)'}_{0,0}(E) - (\rho+\lambda)\Phi^{(1)}_{0,0}(E) + \frac{\lambda D_1 E}{\rho-\mu} - \lambda I_1 = 0 & , \epsilon^{(1)}_{0,1,1} \leq E \end{cases} \qquad \text{(A–9)}$$

The solution for $\Phi^{(1)}_{0,0}(E)$ is indicated in (A–10), where, following the same reasoning as in (A–3), we can rule out the terms containing the negative exponents, β_2 and δ_2, in the top part of (A–10) and the term containing the positive exponent δ_1 in the bottom part of (A–10).

$$\Phi^{(1)}_{0,0}(E) = \begin{cases} A^{(1)}_{0,1,1} E^{\beta_1} + A^{(1)}_{0,0} E^{\delta_1} & , E < \epsilon^{(1)}_{0,1,1} \\ \frac{\lambda D_1 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + B^{(1)}_{0,0} E^{\delta_2} & , \epsilon^{(1)}_{0,1,1} \leq E \end{cases} \qquad \text{(A–10)}$$

The endogenous constants $A^{(1)}_{0,0} < 0$ and $B^{(1)}_{0,0} > 0$ are determined via the value–matching and smooth–pasting conditions between the two branches of (A–10) at $\epsilon^{(1)}_{0,1,1}$ and are indicated in (A–11) and (A–12) respectively.

$$A^{(1)}_{0,0} = \frac{\epsilon^{(1)^{-\delta_1}}_{0,1,1}}{\delta_2 - \delta_1} \left[\frac{\lambda(\delta_2 - 1) D_1 \epsilon^{(1)}_{0,1,1}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\beta_1 - \delta_2) A^{(1)}_{0,1,1} \epsilon^{(1)^{\beta_1}}_{0,1,1} - \frac{\delta_2 \lambda I_1}{\rho+\lambda} \right] \qquad \text{(A–11)}$$

$$B^{(1)}_{0,0} = \frac{\epsilon^{(1)^{-\delta_2}}_{0,1,1}}{\delta_1 - \delta_2} \left[\frac{\lambda(1 - \delta_1) D_1 \epsilon^{(1)}_{0,1,1}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\delta_1 - \beta_1) A^{(1)}_{0,1,1} \epsilon^{(1)^{\beta_1}}_{0,1,1} + \frac{\delta_1 \lambda I_1}{\rho+\lambda} \right] \qquad \text{(A–12)}$$

□

Compulsive Strategy with N = 2

The value function in state $(1,2,2)$ is indicated in (B–1)

$$F^{(2)}_{1,2,2}(E) = \begin{cases} \Phi^{(1)}_{1,1}(E) + (1-\rho dt)\mathbb{E}_E \left[F^{(2)}_{1,2,2}(E + dE) \right] \\ \Phi^{(2)}_{2,2}(E) \end{cases} = \begin{cases} \Phi^{(1)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} & , E < \epsilon^{(2)}_{1,2,2} \\ \Phi^{(2)}_{2,2}(E) & , E \geq \epsilon^{(2)}_{1,2,2} \end{cases} \qquad \text{(B–1)}$$

where the endogenous constant $A^{(2)}_{1,2,2}$ and investment threshold $\epsilon^{(2)}_{1,2,2}$ are determined via value–matching and smooth–pasting conditions between the two branches of (B–1) and are indicated in (B–2).

$$A^{(2)}_{1,2,2} = \frac{\epsilon^{(2)^{1-\beta_1}}_{1,2,2}}{\beta_1} \frac{D_2 - D_1}{\rho - \mu} \quad \text{and} \quad \epsilon^{(2)}_{1,2,2} = \frac{\beta_1}{\beta_1 - 1} \frac{I_2(\rho-\mu)}{D_2 - D_1} \qquad \text{(B–2)}$$

Next, we step back and consider the value function in state $(1,1)$, where the first technology has already been adopted but the second one is not available yet.

$$\begin{aligned} \Phi^{(2)}_{1,1}(E) = D_1 E dt - \rho I_1 dt &+ (1-\rho dt)\lambda dt \mathbb{E}_E \left[F^{(2)}_{1,2,2}(E+dE) \right] \\ &+ (1-\rho dt)(1-\lambda dt)\mathbb{E}_E \left[\Phi^{(2)}_{1,1}(E+dE) \right] \end{aligned} \quad \text{(B–3)}$$

Expanding the right–hand side of (B–3) using Itô's lemma we obtain (B–4)

$$\frac{1}{2}\sigma^2 E^2 \Phi^{(2)''}_{1,1}(E) + \mu E \Phi^{(2)'}_{1,1}(E) - (\rho+\lambda)\Phi^{(2)}_{1,1}(E) + D_1 E - \rho I_1 + \lambda F^{(2)}_{1,2,2}(E) = 0 \quad \text{(B–4)}$$

and by solving (B–4) separately for the two regions of E that are indicated in (B–1) we have

$$\Phi^{(2)}_{1,1}(E) = \begin{cases} \Phi^{(1)}_{1,1}(E) + A^{(2)}_{1,2,2}E^{\beta_1} + A^{(2)}_{1,1}E^{\delta_1} & , E < \epsilon^{(2)}_{1,2,2} \\ \frac{E[\lambda D_2 + (\rho-\mu)D_1]}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} + B^{(2)}_{1,1}E^{\delta_2} - I_1 & , \epsilon^{(2)}_{1,2,2} \leq E \end{cases} \quad \text{(B–5)}$$

where the endogenous constants $A^{(2)}_{1,1} < 0$ and $B^{(2)}_{1,1} > 0$ are obtain via the value–matching and smooth–pasting conditions between the two branched of (B–5) and are indicated in (B–6) and (B–7).

$$\begin{aligned} A^{(2)}_{1,1} &= \frac{\epsilon^{(2)^{-\delta_1}}_{1,2,2}}{\delta_2 - \delta_1}\left[\frac{\lambda(\delta_2 - 1)(D_2 - D_1)\epsilon^{(2)}_{1,2,2}}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\delta_2 \lambda I_2}{\rho+\lambda} - (\delta_2 - \beta_1) A^{(2)}_{1,2,2}\epsilon^{(2)^{\beta_1}}_{1,2,2} \right] && \text{(B–6)} \\ B^{(2)}_{1,1} &= \frac{\epsilon^{(2)^{-\delta_2}}_{1,2,2}}{\delta_1 - \delta_2}\left[\frac{\lambda(1-\delta_1)(D_2 - D_1)\epsilon^{(2)}_{1,2,2}}{(\rho+\lambda-\mu)(\rho-\mu)} + \frac{\delta_1 \lambda I_2}{\rho+\lambda} + (\delta_1 - \beta_1) A^{(2)}_{1,2,2}\epsilon^{(2)^{\beta_1}}_{1,2,2} \right] && \text{(B–7)} \end{aligned}$$

Next, the value of the option to invest in the first technology with an embedded option to invest in the second, $F^{(2)}_{0,1,1}(E)$, is indicated in (B–8)

$$F^{(2)}_{0,1,1}(E) = \begin{cases} A^{(2)}_{0,1,1}E^{\beta_1} & , E < \epsilon^{(2)}_{0,1,1} \\ \Phi^{(2)}_{1,1}(E) & , \epsilon^{(2)}_{0,1,1} \leq E \end{cases} \quad \text{(B–8)}$$

where $\Phi^{(2)}_{1,1}(E)$ is described in (B–5), while $A^{(2)}_{0,1,1}$ and $\epsilon^{(2)}_{0,1,1}$ are determined numerically via the value–matching and smooth–pasting conditions (B–9) and (B–10) respectively.

$$\begin{aligned} A^{(2)}_{0,1,1}\epsilon^{(2)^{\beta_1}}_{0,1,1} &= \Phi^{(2)}_{1,1}\left(\epsilon^{(2)}_{0,1,1}\right) + A^{(2)}_{1,2,2}\epsilon^{(2)^{\beta_1}}_{0,1,1} + A^{(2)}_{1,1}\epsilon^{(2)^{\delta_1}}_{0,1,1} && \text{(B–9)} \\ \beta_1 A^{(2)}_{0,1,1}\epsilon^{(2)^{\beta_1}}_{0,1,1} &= \frac{D_1 \epsilon^{(2)}_{0,1,1}}{\rho-\mu} + \beta_1 A^{(2)}_{1,2,2}\epsilon^{(2)^{\beta_1}}_{0,1,1} + \delta_1 A_{1,1}\epsilon^{(2)^{\delta_1}}_{0,1,1} && \text{(B–10)} \end{aligned}$$

Finally, the value function in state $(0,0)$ is described in (B–11)

$$\Phi^{(2)}_{0,0}(E) = (1-\rho dt)\lambda dt \mathbb{E}_E\left[F^{(2)}_{0,1,1}(E+dE) \right] + (1-\rho dt)(1-\lambda dt)\mathbb{E}_E\left[\Phi^{(2)}_{0,0}(E+dE) \right] \quad \text{(B–11)}$$

and the differential equation for $\Phi^{(2)}_{0,0}(E)$ is indicated in (B–12).

$$\frac{1}{2}\sigma^2 E^2 \Phi^{(2)''}_{0,0}(E) + \mu E \Phi^{(2)'}_{0,0}(E) - (\rho+\lambda)\Phi^{(2)}_{0,0}(E) + \lambda F^{(2)}_{0,1,1}(E) = 0 \quad \text{(B–12)}$$

The expression of $\Phi^{(2)}_{0,0}(E)$ is indicated in (B–13)

$$\Phi^{(2)}_{0,0}(E) = \begin{cases} A^{(2)}_{0,1,1} E^{\beta_1} + A^{(2)}_{0,0} E^{\delta_1} & , E < \epsilon^{(2)}_{0,1,1} \\ \frac{\lambda D_1 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_1}{\rho+\lambda} + A^{(2)}_{1,2,2} E^{\beta_1} + A^{(2)}_{1,1} E^{\delta_1} + B^{(2)}_{0,0} E^{\delta_2} & , \epsilon^{(2)}_{0,1,1} \le E \end{cases} \tag{B–13}$$

where the endogenous constants $A^{(2)}_{0,0} < 0$ and $B^{(2)}_{0,0} > 0$ are determined via the value–matching and smooth–pasting conditions between the two branches of (B–13) and are indicated in (B–14) and (B–15) respectively.

$$\begin{aligned} A^{(2)}_{0,0} &= \frac{\epsilon^{(2)^{-\delta_1}}_{0,1,1}}{\delta_2 - \delta_1} \left[\frac{\lambda(\delta_2 - 1) D_1 \epsilon^{(2)}_{0,1,1}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\beta_1 - \delta_2) A^{(2)}_{0,1,1} \epsilon^{(2)^{\beta_1}}_{0,1,1} - \frac{\delta_2 \lambda I_1}{\rho+\lambda} + (\delta_2 - \beta_1) A^{(2)}_{1,2,2} \epsilon^{(2)^{\beta_1}}_{0,1,2} \right. \\ & \qquad \left. + (\delta_2 - \delta_1) A^{(2)}_{1,1} \epsilon^{\delta_1}_{0,1,1} \right] \qquad \text{(B–14)} \\ B^{(2)}_{0,0} &= \frac{\epsilon^{(2)^{-\delta_2}}_{0,1,2}}{\delta_1 - \delta_2} \left[\frac{\lambda(1 - \delta_1) D_1 \epsilon^{(2)}_{0,1,1}}{(\rho+\lambda-\mu)(\rho-\mu)} + (\delta_1 - \beta_1) A^{(2)}_{0,1,1} \epsilon^{(2)^{\beta_1}}_{0,1,1} + \frac{\delta_1 \lambda I_1}{\rho+\lambda} - (\delta_1 - \beta_1) A^{(2)}_{1,2,2} \epsilon^{(2)^{\beta_1}}_{0,1,1} \right] \quad \text{(B–15)} \end{aligned}$$

□

Proposition 4.1 $\forall \ell, m, n \in \mathbb{N}$ *the relative loss in* $\Phi^{(n)}_{m,m}(E)$ *converges to zero as* $\lambda \to \infty$*, i.e.,*

$$\lambda \to \infty \Rightarrow \frac{F^{(n)}_{m,n,n}(E) - \Phi^{(n)}_{m,m}(E)}{F^{(n)}_{m,n,n}(E)} \to 0, \quad \forall E < \epsilon^{(n)}_{m,n,n}$$

Proof: The value of the option to invest in state (m, n, n) is described in (B–16).

$$F^{(n)}_{m,n,n}(E) = \Phi_m(E) \mathbb{1}_{m>0} + A_{m,n} E^{\beta_1}, \quad \forall E < \epsilon^{(n)}_{m,n,n} \tag{B–16}$$

Following from (9), (14), and (18), the value function in state (m, m) under a compulsive strategy is described in (B–17)

$$\Phi^{(n)}_{m,m}(E) = \Phi^{(m)}_{m,m}(E) \mathbb{1}_{m>0} + A^{(n)}_{m,n,n} E^{\beta_1} + A^{(n)}_{m,m} E^{\delta_1}, \quad \forall E < \epsilon^{(n)}_{m,n,n} \tag{B–17}$$

where the first term on the right–hand side is the expected value of technology m and the second term is the value of the option to invest in technology n, which is not available yet and thus the option must be adjusted via the third term. Consequently, the relative loss in the value function $\Phi^{(n)}_{m,m}(E)$ due to technological uncertainty is indicated in (B–18).

$$\frac{F^{(n)}_{m,n,n} - \Phi^{(n)}_{m,m}}{F^{(n)}_{m,n,n}} = \frac{-A^{(n)}_{m,m} E^{\delta_1}}{\Phi^{(m)}_{m,m}(E) \mathbb{1}_{\ell>0} + A^{(n)}_{m,n,n} E^{\beta_1}} \tag{B–18}$$

Notice that $\lambda \to \infty \Rightarrow A^{(n)}_{m,m} \to 0$. This happens because $\lambda \to \infty \Rightarrow E^{\delta_1} \to \infty$. Hence, if $\lambda \to \infty$ and $A^{(n)}_{m,m} < 0$, then $A^{(n)}_{m,m} E^{\delta_1} \to -\infty$. Hence, we conclude that $\lambda \to \infty \Rightarrow A^{(n)}_{m,m} \to 0$. Consequently, $\lambda \to \infty \Rightarrow \frac{F^{(n)}_{m,n,n} - \Phi^{(n)}_{m,m}}{F^{(n)}_{m,n,n}} \to 0$. □

Proposition 4.2: $\forall \ell, m, n \in \mathbb{N}$ *and* $\forall \lambda \in \mathbb{R}^+$, $\Phi^{(m)}_{\ell,\ell}(E) \in \left[\Phi^{(\ell)}_{\ell,\ell}(E), F^{(m)}_{\ell,m,m}(E)\right)$, $E \le \epsilon^{(m)}_{\ell,m,m}$.

Proof: From (12) and (14) we have that the general expression for $F^{(m)}_{\ell,m,m}(E)$ and $\Phi^{(m)}_{\ell,\ell}(E)$ are:

$$F^{(m)}_{\ell,m,m}(E) = \Phi^{(\ell)}_{\ell,\ell}(E) + A^{(m)}_{\ell,m,m}E^{\beta_1} \tag{B–19}$$

$$\Phi^{(m)}_{\ell,\ell}(E) = \Phi^{(\ell)}_{\ell,\ell}(E) + A^{(m)}_{\ell,m,m}E^{\beta_1} + A^{(m)}_{\ell,\ell}E^{\delta_1} \tag{B–20}$$

(i) Notice that $\lambda = 0 \Rightarrow \delta_1 = \beta_1, \delta_2 = \beta_2, A^{(m)}_{\ell,\ell} = -A^{(m)}_{\ell,m,m}$ and $B^{(n)}_{m,m} = 0$. Thus, $\Phi^{(m)}_{\ell,\ell}(E) = \Phi^{(\ell)}_{\ell,\ell}(E)$, $\forall E \le \epsilon^{(m)}_{\ell,m,m}$, i.e., the value function at each state consists only of the value of the active project since no embedded options are available.

(ii) Next, we consider the case $\lambda \to \infty$. If $\lambda > 0$, then $A^{(m)}_{\ell,\ell} < 0$ and, since $A^{(m)}_{\ell,m,m} > 0$, we have $\Phi^{(m)}_{\ell,\ell}(E) = \Phi^{(\ell)}_{\ell,\ell}(E) + A^{(m)}_{\ell,m,m}E^{\beta_1} + A^{(m)}_{\ell,\ell}E^{\delta_1} < \Phi^{(m)}_{\ell,\ell}(E) + A^{(m)}_{\ell,m,m}E^{\beta_1} = F^{(m)}_{\ell,m,m}(E), \forall E < \epsilon^{(n)}_{\ell,m,m}$. According to the probability of no event occurring in the unit of time, we have $\lambda \to \infty \Rightarrow \mathbb{P}[N=0] = e^{-\lambda} \to 0$. Hence, $\lambda \to \infty \Rightarrow A^{(m)}_{\ell,\ell}E^{\delta_1} \to 0$ and, thus, $\lim_{\lambda\to\infty} \Phi^{(m)}_{\ell,\ell}(E) = F^{(m)}_{\ell,m,m}(E)$, $\forall E < \epsilon^{(m)}_{\ell,m,m}$. □

Proposition 4.3 $\forall \ell, m, n \in \mathbb{N}$ *the impact of* λ *on* $\epsilon^{(n)}_{\ell,m,m}$ *is non–monotonic.*

Proof: From (B–17), the value of the option to invest in state (ℓ, m, m) can be written as in (B–21).

$$\begin{aligned} F^{(n)}_{\ell,m,m}(E) &= \Phi^{(\ell)}_{\ell,\ell}(E)\mathbb{1}_{\ell>0} + A^{(n)}_{\ell,m,m}E^{\beta_1} \\ &= \Phi^{(\ell)}_{\ell,\ell}(E)\mathbb{1}_{\ell>0} + \left(\frac{E}{\epsilon^{(n)}_{\ell,m,m}}\right)^{\beta_1}\left[\Phi^{(m)}_{m,m}\left(\epsilon^{(n)}_{\ell,m,m}\right) + A^{(n)}_{m,n,n}\epsilon^{\beta_1}_{\ell,m,m} + A^{(n)}_{m,m}{\epsilon^{(n)}_{\ell,m,m}}^{\delta_1}\right] \end{aligned} \tag{B–21}$$

In order to investigate the impact of λ on the optimal investment threshold, we rewrite the MB and MC of delaying investment. Notice that λ impacts only the last term of the left– and right–hand side of (B–22).

$$\left(\frac{E}{\epsilon^{(n)}_{\ell,m,m}}\right)^{\beta_1}\left[\frac{D_m}{\rho-\mu} + \frac{\beta_1}{\epsilon^{(n)}_{\ell,m,m}}I_m - \beta_1 A^{(n)}_{m,m}{\epsilon^{(n)}_{\ell,m,m}}^{\delta_1 - 1}\right] = \left(\frac{E}{\epsilon^{(n)}_{\ell,m,m}}\right)^{\beta_1}\left[\frac{\beta_1 D_m}{\rho-\mu} - \delta_1 A^{(n)}_{m,m}{\epsilon^{(n)}_{\ell,m,m}}^{\delta_1 - 1}\right] \tag{B–22}$$

Consequently, the overall impact of λ on $\epsilon^{(n)}_{\ell,m,m}$ can be determined by its impact on these two terms. Therefore, we define the function $h(\cdot)$ as in (B–23).

$$h(\lambda) = \delta_1 A^{(n)}_{m,m} - \beta_1 A^{(n)}_{m,m} \Rightarrow \frac{\partial h(\lambda)}{\partial \lambda} = \frac{\partial \delta_1}{\partial \lambda}A^{(n)}_{m,m} + \delta_1\frac{\partial A^{(n)}_{m,m}}{\partial \lambda} - \beta_1\frac{\partial A^{(n)}_{m,m}}{\partial \lambda} \tag{B–23}$$

As (B–24) indicates, when λ is small, $\beta_1 A^{(n)}_{m,m}$ decreases by more than $\delta_1 A^{(n)}_{m,m}$, which implies that the MB decreases by more than the MC, and, as a result, the marginal value of delaying investment decreases, thereby lowering the required investment threshold.

$$\begin{aligned} \lim_{\lambda\to 0}\frac{\partial h(\lambda)}{\partial \lambda} &= \lim_{\lambda\to 0}\left\{\frac{\partial \delta_1}{\partial \lambda}A^{(n)}_{m,m} + \delta_1\frac{\partial A^{(n)}_{m,m}}{\partial \lambda} - \beta_1\frac{\partial A^{(n)}_{m,m}}{\partial \lambda}\right\} \\ &= \lim_{\lambda\to 0}\frac{\partial \delta_1}{\partial \lambda}A^{(n)}_{m,m} + \beta_1\lim_{\lambda\to 0}\frac{\partial A^{(n)}_{m,m}}{\partial \lambda} - \beta_1\lim_{\lambda\to 0}\frac{\partial A^{(n)}_{m,m}}{\partial \lambda} \\ &= \beta_1 \times \left(-A^{(n)}_{m,n,n}\right) < 0 \end{aligned} \tag{B–24}$$

Similarly, at high values of λ, (B–25) indicates that the MC decreases by more than the MB and, thus, the marginal value of delaying investment decreases.

$$\begin{aligned}
\lim_{\lambda\to\infty}\frac{\partial h(\lambda)}{\partial\lambda} &= \lim_{\lambda\to\infty}\left\{\frac{\partial\delta_1}{\partial\lambda}A^{(n)}_{m,m}+\delta_1\frac{\partial A^{(n)}_{m,m}}{\partial\lambda}-\beta_1\frac{\partial A^{(n)}_{m,m}}{\partial\lambda}\right\}\\
&= \lim_{\lambda\to\infty}\frac{\partial\delta_1}{\partial\lambda}A^{(n)}_{m,m}+\lim_{\lambda\to\infty}\delta_1\times\lim_{\lambda\to\infty}\frac{\partial A^{(n)}_{m,m}}{\partial\lambda}-\beta_1\lim_{\lambda\to\infty}\frac{\partial A^{(n)}_{m,m}}{\partial\lambda}\\
&= \lim_{\lambda\to\infty}\underbrace{\left(\frac{\partial\delta_1}{\partial\lambda}+\delta_1-\beta_1\right)}_{>0}\times\lim_{\lambda\to\infty}\underbrace{\frac{\partial A^{(n)}_{m,m}}{\partial\lambda}}_{>0}>0
\end{aligned}\tag{B–25}$$

□

Proposition 5.1: *$\forall \ell, m, n \in \mathbb{N}$ we have $\epsilon^{(n)}_{\ell,m,m} < \underline{\epsilon}^{(n)}_{\ell,m,n}$ $\forall\lambda\in(0,+\infty)$, whereas $\lambda = 0 \Rightarrow \epsilon^{(n)}_{\ell,m,m} = \underline{\epsilon}^{(n)}_{\ell,m,n}$ and $\lambda\to\infty \Rightarrow \epsilon^{(n)}_{\ell,m,m}\to\underline{\epsilon}^{(n)}_{\ell,m,n}$.*

Proof: First, $\epsilon^{(n)}_{\ell,m,m} < \underline{\epsilon}^{(n)}_{\ell,m,n}$ follows from Proposition 4.3. In order to determine $\underline{\epsilon}^{(n)}_{\ell,m,n}$, we rewrite the value of the option to invest in state (ℓ, m, n) as in (B–26).

$$F^{(n)}_{\ell,m,n}(E) = \max_{\underline{\epsilon}^{(n)}_{\ell,m,n}>E}\left(\frac{E}{\underline{\epsilon}^{(n)}_{\ell,m,n}}\right)^{\beta_1}\left[\frac{\underline{\epsilon}^{(n)}_{\ell,m,n}D_m}{\rho-\mu}-I_m+A^{(n)}_{m,n,n}\underline{\epsilon}^{(n)\beta_1}_{\ell,m,n}\right]\tag{B–26}$$

We can express the FONC by equating the MB of delaying investment to the MC as in (B–27). Notice that the extra benefit from the embedded option to invest in the second technology gets cancelled with the extra cost, which implies that when the second technology is available it does not affect the decision to invest in the first one.

$$\left(\frac{E}{\underline{\epsilon}^{(n)}_{\ell,m,n}}\right)^{\beta_1}\left[\frac{\beta_1 I_m}{\underline{\epsilon}^{(n)}_{\ell,m,n}}+\frac{D_m}{\rho-\mu}+\beta_1 A^{(n)}_{m,n,n}\underline{\epsilon}^{(n)\beta_1-1}_{\ell,m,n}\right]=\left(\frac{E}{\underline{\epsilon}^{(n)\beta_1}_{\ell,m,n}}\right)\left[\frac{\beta_1 D_m}{\rho-\mu}+\beta_1 A^{(n)}_{m,n,n}\underline{\epsilon}^{(n)\beta_1-1}_{\ell,m,n}\right]\tag{B–27}$$

Consequently, solving with respect to $\underline{\epsilon}^{(n)}_{\ell,m,n}$ we have:

$$\underline{\epsilon}^{(n)}_{\ell,m,n}=\frac{\beta_1}{\beta_1-1}\frac{I_m(\rho-\mu)}{D_m}\tag{B–28}$$

Notice that $\epsilon^{(n)}_{\ell,m,m} = \underline{\epsilon}^{(n)}_{\ell,m,n}$ for $\lambda = 0$. Similarly, as $\lambda\to\infty$ the likelihood of at least one innovation occurring converges to one, and, therefore, we have $\epsilon^{(n)}_{\ell,m,m}\to\underline{\epsilon}^{(n)}_{\ell,m,n}$. □

Proposition 6.1: *$\forall E \in \left(0, \overline{\epsilon}^{(2)}_{0,1,2}\right]$, if the investment region in state $(0, 1 \vee 2)$ is dichotomous, then the compulsive strategy dominates the leapfrog/laggard strategy $\forall\lambda\in\mathbb{R}^+$.*

Proof: Notice that for the compulsive strategy to dominate the leapfrog/laggard strategy, the investment region in state (0,1∨2) must be dichotomous (Décamps *et al.*, 2006). Otherwise, the second waiting region does not exist, and, then, it is optimal to wait until $E = \epsilon^{(2)}_{0,2,2}$ and then invest in the second technology (Dixit, 1993). Provided that this condition is satisfied, we have:

(i) $E \in \left(\underline{\epsilon}^{(2)}_{0,1,2}, \overline{\epsilon}^{(2)}_{0,1,2}\right]$: Notice that in state (0,1,1) the expected NPV from investment in the first technology under a compulsive strategy is $F^{(2)}_{0,1,1}(E) = \Phi^{(2)}_{1,1}(E)$, where $\Phi^{(2)}_{1,1}(E)$ indicated in (B–29).

$$\begin{aligned}\Phi^{(2)}_{1,1}(E) &= D_1 E dt - \rho I_1 dt + (1-\rho dt)\lambda dt \mathbb{E}_E\left[F^{(2)}_{1,2,2}(E+dE)\right] \\ &+ (1-\rho dt)(1-\lambda dt)\mathbb{E}_E\left[\Phi^{(2)}_{1,1}(E+dE)\right], \quad E > \underline{\epsilon}^{(2)}_{0,1,2} \geq \epsilon^{(2)}_{0,1,1} \end{aligned} \tag{B–29}$$

By contrast, under a laggard strategy, the expected value function in state (0,1,1) is the same as (B–29) but is conditional on the arrival of the second technology, as indicated in (B–30).

$$\begin{aligned}F^{(2)}_{0,1,1}(E) &= (1-\rho dt)\lambda dt \mathbb{E}_E\left[F^{(2)}_{1,2,2}(E+dE)\right] \\ &+ (1-\rho dt)(1-\lambda dt)\mathbb{E}_E\left[F^{(2)}_{0,1,1}(E+dE)\right], \quad \underline{\epsilon}^{(2)}_{0,1,2} \leq E \leq \overline{\epsilon}^{(2)}_{0,1,2}\end{aligned} \tag{B–30}$$

From (B–29) and (B–30) it follows that $\Phi^{(2)}_{1,1}(E) \geq F^{(2)}_{0,1,1}(E), \forall E > \underline{\epsilon}^{(2)}_{0,1,2}$.

(ii) $E \in \left(0, \underline{\epsilon}^{(2)}_{0,1,2}\right]$: The derivation is similar to (i). Intuitively, the compulsive strategy dominates $\forall E < \epsilon^{(2)}_{0,1,1}$, since, unlike in the leapfrog/laggard strategy, the option to invest in the first technology is not contingent upon the arrival of the second one. □

Proposition 6.2: $\exists\ \lambda_1, \lambda_2 \in \mathbb{R}^+$ *with* $\lambda_1 \leq \lambda_2$:

i : $\forall \lambda > \lambda_1$, $\exists \mathcal{B} \subseteq \left(\epsilon^{(2)}_{0,2,2}, \infty\right)$: $\forall E \in \mathcal{B}$, *the leapfrog strategy dominates*

ii : $\forall \lambda > \lambda_2$, $\exists \mathcal{A} \subset \left[\overline{\epsilon}^{(2)}_{0,1,2}, \epsilon^{(2)}_{0,2,2}\right)$: $\forall E \in \mathcal{A}$, *the leapfrog/laggard strategy dominates*

Proof: (i) Notice that under a compulsive strategy and $E \geq \epsilon^{(2)}_{0,2,2}$

$$F^{(2)}_{0,1,1}(E) = \Phi^{(1)}_{1,1}(E) + A^{(2)}_{1,2,2}E^{\beta_1} + A^{(2)}_{1,1}E^{\delta_1} \Rightarrow \begin{cases} \lambda = 0 \Rightarrow F^{(2)}_{0,1,1}(E) = \Phi^{(1)}_{1,1}(E) \\ \lim_{\lambda\to\infty} F^{(2)}_{0,1,1}(E) = \Phi^{(1)}_{1,1}(E) + A^{(2)}_{1,2,2}E^{\beta_1} \end{cases} \tag{B–31}$$

and, according to Proposition 4.3, $\lambda \nearrow \Rightarrow \left|A^{(2)}_{1,1}\right| \searrow$ which implies that $\frac{\partial F^{(2)}_{0,1,1}(E)}{\partial \lambda} > 0\ \forall \lambda \in \mathbb{R}^+$. Additionally, under a leapfrog strategy and $E \geq \epsilon^{(2)}_{0,2,2}$

$$F^{(2)}_{0,1,1}(E) = \frac{\lambda D_2 E}{(\rho+\lambda-\mu)(\rho-\mu)} - \frac{\lambda I_2}{\rho+\lambda} + J^{(2)}_{0,1,1}E^{\delta_2} \Rightarrow \begin{cases} \lambda = 0 \Rightarrow F^{(2)}_{0,1,1}(E) = 0 \\ \lim_{\lambda\to\infty} F^{(2)}_{0,1,1}(E) = \frac{D_2 E}{\rho-\mu} - I_2 \end{cases} \tag{B–32}$$

Consequently,

1. If λ is low, e.g., $\lambda = 0$, then the compulsive strategy dominates since $\Phi^{(2)}_{1,1}(E) \geq 0,\ \forall E \geq 0$
2. As $\lambda \to \infty$, the leapfrog strategy dominates $\forall E \in \left(\epsilon^{(2)}_{0,2,2}, \infty\right)$. Indeed, if we denote by ε the output price that satisfies (B–33), i.e.,

$$\frac{D_2 \varepsilon}{\rho-\mu} - I_2 = \frac{D_1 \varepsilon}{\rho-\mu} - I_1 + A^{(2)}_{1,2,2}\varepsilon^{\beta_1} \Leftrightarrow \Phi^{(2)}_{\overline{2,2}}(\varepsilon) = F^{(2)}_{1,2,2}(\varepsilon) \tag{B–33}$$

then ε is the point of indifference between the NPV in state $\overline{(2,2)}$ and (1,2,2), and, as such, $\varepsilon < \epsilon^{(2)}_{0,2,2}$

Next, for positive and finite values of λ, we set $\mathbb{L} = \left\{\lambda | \; \exists \mathcal{B} \subseteq \left(\epsilon^{(2)}_{0,2,2}, \infty\right) : \text{leapfrog dominates } \forall E \in \mathcal{B}\right\}$. Then, from (1) and (2), $\exists \lambda_1 \in \mathbb{R}^+ : \lambda_1 = \min \lambda \in \mathbb{L}$ and

$$\frac{\lambda_1 D_2 E}{(\rho + \lambda_1 - \mu)(\rho - \mu)} - \frac{\lambda_1 I_2}{\rho + \lambda_1} + J^{(2)}_{0,1,1} E^{\delta_2} = \Phi^{(2)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} + A^{(2)}_{1,1} E^{\delta_1} \tag{B–34}$$

while $\forall \lambda \in \mathbb{L} > \lambda_1 \; \exists \mathcal{B} \subseteq \left(\epsilon^{(2)}_{0,2,2}, \infty\right)$ such that the leapfrog strategy dominates $\forall E \in \mathcal{B}$, i.e.,

$$\frac{\lambda D_2 E}{(\rho + \lambda - \mu)(\rho - \mu)} - \frac{\lambda I_2}{\rho + \lambda} + J^{(2)}_{0,1,1} E^{\delta_2} \geq \Phi^{(2)}_{1,1}(E) + A^{(2)}_{1,2,2} E^{\beta_1} + A^{(2)}_{1,1} E^{\delta_1}, \; \forall \lambda \in \mathcal{B} \tag{B–35}$$

(ii) The derivation is similar to (i) and follows from the convexity of $F^{(2)}_{0,1,1}$ and the value–matching and smooth–pasting conditions ensure that $F^{(2)}_{0,1,1}$ is $\mathcal{C}^1 \; \forall E > 0$. □

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