THE HYBRID MODEL OF AGENT'S POWER IN SOCIAL NETWORKS

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ABSTRACT

The basic idea of the research is to investigate and formalize functional dependency between structural and social measures in social networks. Since interpersonal relations are composed of multiple factors with different nature (i.e., structural and social factors), the measure of social power is explored and formalized in terms of the strategizing processes in social networks.

KEYWORDS: social networks, structural centralities, social power.

1. INTRODUCTION

The framework of the research is based on the problem of structural analysis of social networks. In general, social network is considered as a multi-agent (multi-players) system. Basically, each agent is characterized by structural metrics (i.e., centralities) and by social characteristics, such as measure of trust to other players. In fact, the research corresponds to the investigation of functional dependencies between the logical and mathematical apparatuses of two interconnected concepts described next.

1. Structural analysis of social networks

As was mentioned, structural analysis is a basic component of the investigation process. We use three fundamental structural measures in the given research: (a) degree-based centrality, (b) between ness centrality and (c) closeness centrality (Cook, Emerson, Gillmore, & Yamagishi, 1983). All of these measures are the components of social power analysis. One of the goals for this research is to encapsulate structural centralities in a unified structural measure. This encapsulation is the first step in the formalization of social power.

2. Analysis of social networks as the networks of trust

We consider trust as a social property of interpersonal relations in networks. In fact, social networks are based on the exchange of trust between their members (i.e., agents). Trust is at the core of the decision making process of each agent in a social network (Edwards, Claire, & Temple, 2006). For example, we can consider the social network with three interconnected agents: A, B and C. If agent A trusts agent B more than agent C, then the probability that agent A will prefer to interact with agent B is higher than the probability of its interaction with agent C. The given example is trivial. Agent A does not take into account the property of the structural centralities of agents B and C. However, it shows the importance of trust in the exchange of resources (i.e., material and non-material) within a social network.

The conception of trust can also be used in combination with Bayesian networks. The approach is based on the method of Bayesian inference (Wang & Vassileva, 2003).

2. BACKGROUND

The analysis of social networks is basically related to their structural analysis. One of the first structural models based on the theory of directed graphs was suggested by Harary, Norman, & Cartwrigh (1965). It includes basic mathematical formalization and explanation of graph theoretic methodologies and their application in formalization of networks. Theory of directed graphs is a mathematical formalization of networks that can be applied to any types of networks represented by graphs (i.e., not only social networks). The theory of directed graphs is closely related to power networks (Emerson, 1962). According to Emerson (1962), power is an agent's ability to influence other agents and to resist an influence from other agents in the network. The computation of structural measures is considered as a basic step of the analysis of social networks. Harary's research is concentrated on the investigation of social properties of agents, such as "power", "dominance", "dependence" and "status". Power networks are based not only on the structural analysis of networks, but also on the formalization of social interrelations among agents. This approach is widely used in the analysis of social systems, such as exchange networks (Cook et al., 1983).

Exchange networks are socio-economic networks that can be characterized by five properties (Cook et al., 1983). First, an exchange network is a set of agents and interrelations between them. Second, network resources are distributed between agents. Third, each agent makes a decision regarding the exchange process according to its individual interests. Fourth, each agent has a personal history of exchange within a network. Fifth and last, all interpersonal relations are encapsulated in a unified exchange network. According to Cook et al. (1983), the formalization of exchange networks is based on two basic aspects: structural analysis and internal power of relations. Specifically, Cook et al. (1983) used three basic measures for the structural analysis of internal power includes two factors: power and dependence. Power is considered as an agent's potential to obtain the desired outcome from other agents in the network. Dependence implies the reparability of opportunities and limitations of power distribution between different agents. It means that the relation between agent A and agent B is characterized by the dependence that is different from the dependence between agent A and agent C. According to Cook et al. (1983), structural measures and internal power of relations are interdependent and influence each other.

Social networks can be analyzed from the different angles. According to Jackson (2003), efficiency is one of the most important properties of social networks. Jackson (2003) described the efficiency of social and economic networks in three basic categories. The first is that the notion of efficiency is the Pareto efficiency. Pareto efficiency (i.e., Pareto optimality) is a specific state of social network when an improvement of an agent's condition is impossible without worsening the conditions of other agents. Pareto optimality is based on the idea that all profits from the operations of exchange within a network are exhausted. It means that if at least one agent starts to improve its condition, then it will change the state of another agent or agents in a negative way. According to Jackson (2003), an agent is a member of the Pareto efficient network if there is

no other network that can guarantee a better benefit than the current network. The second definition of efficiency is related to the maximization of an agent's benefit (Jackson, 2003). It does not mean that each agent will maximize its payoff. The basic idea of such kind of efficiency is that the total amount of all payoffs should be maximized. The third conception of network efficiency is related to the availability of specific types of transactions for each agent. It means that social network is efficient if the availability to realize the specific set of transactions at any time is guaranteed to each agent. This type of network efficiency implies that agents should not be limited in the realization of the specific set of rights. For example, if any democratic society is considered as efficient, then it should guarantee the freedom of choice and freedom of action for each member. The advantage of the given research is that it includes a deep analysis of the specific models of social networks. For example, Jackson (2003) considered the Connections Model (Jackson & Wolinsky, 1996) and the Co-Author Model (Jackson & Wolinsky, 1996).

Another approach regarding the network power and structural measures was done by Bonacich (1987). The research is based on the abstract formalization of interdependencies between an agent's power and centrality. Bonacich (1987) did not specify which structural measures are better to be used for the structural analysis of social networks. He considered bargaining situations where agent's power is its bargaining power. According to Bonacich (1987), it is preferable for an agent to keep relations with agents who have less bargaining power. If agent A keeps relations with more powerful agents, then it will have less influence in the bargaining process. This implies the decrease of the bargaining power for agent A. Bonacich (1987) analyzed the problem of interrelations between network properties conceptually without specific computations. Mathematical formalization of interrelations between structural measures is abstracted away from the use of specific measures.

3. METHODOLOGY

A social network is a network that has a specific topology and social structure. The basic objects of social networks are agents (i.e., individuals, companies, and communities) that are represented by nodes and related by different kinds of social relations (i.e., friendship, love, trust, business, and knowledge). In fact, a social network can be represented as a graph as shown in Figure 1. Every social network can be analyzed by graph theoretic methodologies. Social networks have different structural complexity, but in practice, they are considered as large-scale networks. This is due to the fact that they mimic the complexity of real-world social interdependencies.

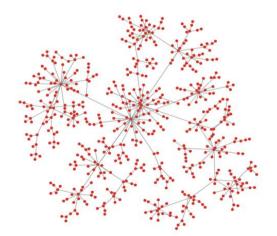


Figure 1. A Prototypical Social Network with a Mixed Topology

Quantification of social power is a multifactor analysis of the agent's role in any kind of social and economic network. It is strongly related to the level of agent's influence on each member of the network and on the integrity of the network. To put it more simply, social power captures a level of an agent's

Importance and an agent's opportunities within a social network. Social power can be characterized by many measures. For example, Cook et al. (1983) used structural centrality as a primary factor for social power. They used three basic measures of (a) degree-based measure, (b) between ness measure, and (c) closeness-based measure in order to compute the distribution of power in exchange networks. Brandes & Pich (2007) used two measures of (a) closeness and (b) between ness for centrality estimation in large networks. Another important factor of social power is an agent's internal power, which characterizes an agent's resources (i.e., energy, knowledge, and trust). Social power structure is represented in Figure 2. Next, we describe the components in detail.

3.1. STRUCTURAL CENTRALITY

Structural centrality is the most important concept in social power. It is based on the structural analysis of networks. Every social network can be represented as a graph. Formalization of structural centrality is closely related to the mathematical approach in graph theory. It is based on the computation of the shortest-path distances in the graphs, frequencies of nodes on the shortest paths, and connections of vertices to the low/high scoring nodes. Structural centrality is a measure of an agent's importance in terms of the structural analysis of networks.

Degree-based measure (degree centrality)

Degree centrality (DC) of a vertex is a number of links directly connected to it. According to Freeman (1979), DC of a vertex can be characterized as an indicator of its potentiality to interact with other vertices. Based on the Freeman (1979) approach, DC computation for a vertex v of a graph G (V, E) with n nodes can be realized by equation 1.

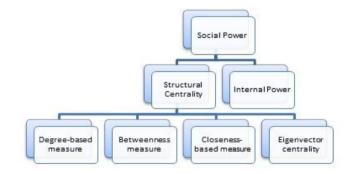


Figure 2. Social Power Structure

$$DC(v) = \frac{\deg(v)}{n-1} \tag{1}$$

Where deg(v) are a number of nodes directly connected to? v.

Between ness measure (between ness centrality)

Between ness centrality (BC), as the measure of structural centrality, estimates how often the particular vertex can be visited looking through the shortest paths between all possible pairs of vertices (Freeman, 1979).

Equation 2 represents BC computation (Anthony's, 1971; Freeman, 1977):

$$BC(v) = \frac{\sum_{s \neq v \neq t} \sigma(s, t|v)}{\sigma(s, t)}$$
(2)

Where:

 $\sigma(s,t)$ is the number of the shortest paths among all paths from s to t;

 $\sigma(s,t|v)$ is the number of the shortest paths starting at s, visiting v and ending in t.

Closeness-based measure (closeness centrality)

Closeness centrality (CC) measures how close the given vertex is to all other vertices of the graph on average. An agent with the highest closeness can be approached from elsewhere in the network faster on average than any other agent. CC has an important practical use because it allows for determining the best position in the network from which other agents can be easily reached.

CC is inversely related to the sum of the shortest distances from vertex v to all other nodes (Beauchamp, 1965; Sabidussi, 1966). Distance is considered as a number of edges in the shortest path between two vertices.

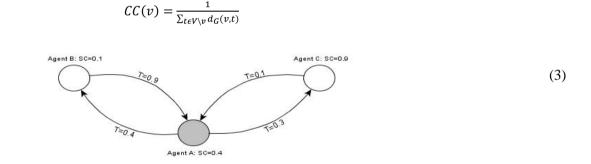


Figure 4. A Trivial Example of Network with Trust and Social Centrality Relations

Where

 $d_G(v, t)$ Is the shortest distance between vertices 'v' and t' in graph G.

Equation 3 works well only with connected graphs. The modification of this formula was offered by Dangalchev (2006):

$$\mathcal{CC}(v) = \sum_{t \in V \setminus v} 2^{-d(v,t)}$$
(4)

Equation 4 is adapted to work with disconnected graphs.

Eigenvector centrality

Eigenvector centrality (EC) measures an agent's significance with respect to other agents in the network. It characterizes quantitative and qualitative performance capabilities of agents (Newman, 2008). In other words, more powerful agents can be more beneficial, and it is preferable to keep connections with them. According to Newman (2008), EC of the agent i is proportional to the average total EC score of its neighbors:

$$x_i = \frac{1}{\lambda} \sum_{j=1}^n A_{ij} x_j \tag{5}$$

Here:

 A_{ij} is a network's adjacency matrix. If vertex *i* is directly connected to vertex *j*, then $A_{ij} = 1$; otherwise, $A_{ij} = 0$; λ is a constant.

Some EC values for nodes are a priori known. Since equation 5 is recursive, the a priori values seed initial values used to compute values of EC for other agents.

Alternatively, equation 5 can be represented in matrix form (Newman, 2008):

 $\lambda x = A \cdot x$ Here: (6)

x is an eigenvector of centralities;

 λ is an eigenvalue of matrix A.

3.2. INTERNAL POWER (IP)

IP is the second approach for social power quantification. It characterizes the internal agent's resources. Compared to structural centralities, IP is not related to the structural features of the network, but it works with the internal characteristics of connections between agents. The specification of IP depends on the area of its application. For example, in terms of economics agent's IP can be represented by capital, money, investments, and other tangible quantities.

Current research focuses on the social foundation of agent's IP. Accordingly, we characterize IP by three internal components: energy, knowledge and trust.

Energy

Energy is an abstraction of social and economic resources. One of the interpretations of energy as a social category is given by Marks (1977). In the context of social analysis, energy can be represented by an agent's ambitions, willpower, and social activities. In terms of economic analysis, energy can be represented by money, time, and propensity for financial risk.

Both kinds of energy are limited. For example, an agent cannot work more than 24 hours per day or spend more money than it has. An aggregated agent's energy can be represented by any value in the range [0, 1].

Knowledge

Knowledge is what is known by an agent regarding its position in the network. It includes the information regarding the states of other agents, connections, and network characteristics in general. In the context of social power, knowledge can be characterized as the level of an agent's information awareness about the network. The deep analysis of knowledge as a social category is done by Berger & Luckmann (1966).

<u>Trust</u>

Trust is a basic characteristic of social networks. If agent A does not trust agent B, then agent B will not get any benefit from agent A, which includes energy and knowledge. We consider a trust network as a directed graph, where trust can take on any value from a range [0, 1]. Therefore, a mathematical apparatus applied for directed graphs can also be used in trust networks. One of the interesting interpretations of trust is given by Edwards et al. (2006), where trust is considered as an abstract and personal category of interpersonal relations. It is important to say that social power has already become one of the most important parameters in the analysis of social and economic networks. It is not just an abstract and uncertain philosophic term, but it is a deeply formalized concept of mathematical formalization in social and economic networks.

4. APPROACH

4.1 FORMALIZATION OF SOCIAL POWER

Measures that characterize social networks are often motivated independently. For example, centrality and density are heterogeneous measures of a social network and cannot be easily combined since they quantify measures of interest for different uses of social networks. Structural network analysis attempts to understand the internodes connectedness as in graph theory methodologies. The analysis of different types of structural measures in terms of social networks was done by Everett, Sinclair, & Dankelmann (2004). Graph based network methodologies cannot be applied for analysis of social factors in social network processing because social networks possess social content that cannot be reduced to measurement by structure. In contrast to structural analysis, social analysis has a different foundation and cannot be quantified by topological analysis.

4.2 STRUCTURAL CENTRALITY AND TRUST

Three measures of structural centrality are taken into consideration: (a) degree-based measure, (b) between's measure, and (c) closeness-based measure. To accomplish interdependency, these three measures are unified in one structural parameter that is called *structural centrality (SC)*. A problem is that each measure takes its values from different numerical intervals. The process of unification is based on the idea that SC should take its value from a unified interval, say [0, 1]. The method of unification for structural parameters is based on the knowledge about the minimum and maximum values of each parameter at the particular moment (i.e., snapshot of the network). Each agent is characterized by values of three structural measures mentioned, and each structural measure may have any value greater than or equal to zero at a particular moment (i.e., at a snapshot) of the network game. The agent with the minimum value of the particular structural measure will set the lowest value of this structural measure corresponding to "0" value equivalent in the range [0, 1]. Accordingly, the agent with the maximum value of the considered structural measure will set the highest value "1" in the range [0, 1]. For example, let's consider the between ness centrality (i.e., measure) in the trivial network consisting of three agents shown in Figure 3. Numbers inside nodes represent centrality values.

According to Figure 3, agent 1 has a maximum between ness centrality values of 15. This means that value of 15 will be mapped to 1 in the range [0, 1]. Agent 3 has a minimum between ness centrality values of 2. Value of 2 will be mapped to 0 in the range [0, 1]. Having upper- and lower- bounds of the between ness centrality, all other intermediate values can be computed in the interval [0, 1]. Particularly, agent 2 will have between ness centrality values interpolated to 0.69. The methodology we described above is applied for all three measures taken into consideration. The unified value of social centrality (SC) is determined by equation 7.

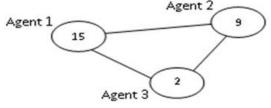


Figure 3. A Trivial Network Example with betweenness Centrality Values

Where $SC \in [0, 1]$.

Equation 7 is founded on the idea that all structural measures contribute equally to the general SC. Our formulation specifies a linear composition between them. A linear composition is stipulated by structurally equal importance of degree-based, between ness and closeness-based measures for an agent's structural centrality. Structural analysis is at the core of each centrality measure, but the difference is that each measure is based on the consideration of network structure from a specific angle. Each of these measures is a quantitative characteristic of an agent's structural centrality. An arithmetic mean computation (i.e., equation 7) is a method to avoid the prioritizing of their contributions to a general agent's structural centrality. In fact, the consideration of non-linear composition implies different levels of structural measures' importance. In this case, each structural measure should have some specific characteristics (excepting structural) to be considered as a more or less important measure. A good example is an eigenvector centrality (equation 5) that is not only quantitative, but also qualitative structural measure. Equation 7 cannot have a linear composition if it includes an eigenvector centrality. Nevertheless, an eigenvector centrality is not used in equation 7.

 $SC = \frac{DC + BC + CC}{3}$

Once we consider a network that represents social nature of interactions, we can interpret such a network to be a network of trust (Gambetta, 1988; Sato, 2002; Wang & Vassileva, 2003; Golbeck et al., 2003). Basically, agents can measure trust and represent values in the range [0, 1]. An agent lacks trust at all (i.e., the fewest trust) or has an abundant trust (i.e., the most trust) to another agent if the values of trust are equal to 0 and 1 respectively.

4.3. FORMALIZATION OF SOCIAL POWER

Having unified values of structural measures and trust, it is necessary to amalgamate them into a single function:

$$Y=f(SC,T)$$
(8)

One of the basic analyses of interdependencies between structural centrality and trust was done by Buskens (1998). Buskens (1998) investigated the interdependencies between two components of social networks: structural measures and trust. The functional dependencies were formalized for the relations between buyers and sellers. The conception of equation 8 is another point of view for the interpretation among social network relations. It is not limited by the consideration of specific socio-economic interactions, because it is based on the conceptual analysis of social relations. The proposed idea in this research is to consider equation 8 as the combined social power of agent A (see Figure 4). According to Figure 4, social power of agent A (i.e., computed using equation 8) depends not only on the current structural centrality of agent A and its trust (T) with respect to other agents (namely B and C in Figure 4), but also on the current structural centralities of the other agents and their trust on agent A.

In fact, the combination of T and SC can be termed as an agent's social centrality or social power (SP). Equation 9 elaborates equation 8.

$$SP_{A} = \frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1} \times SC_{A} + \frac{\sum_{i=1}^{N-1} (T_{A,i} \times SC_{i} - T_{i,A} \times SC_{A})}{N-1}$$
(9)

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Here,

N is a number of agents;

 $T_{i,A}$ is a trust from agent *i* to agent A;

 $T_{A,i}$ is a trust from agent A to agent *i*;

SC_A is a structural centrality of agent A;

 SC_i is a structural centrality of agent *i*.

Equation 9 consists of two main components.

1. $\left(\frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1} \times SC_A\right)$. This encapsulates the basic interdependency between SC for agent A and T to agent A from all other agents. Agent A may have the highest SC in the network. However, if no one trusts it, A will not experience any social power. $\left(\frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1}\right)$ Computes an average T from all agents at the network toward agent A.

 $2.\left(\frac{\sum_{i=1}^{N-1}(T_{A,i}\times SC_i-T_{i,A}\times SC_A)}{N-1}\right).$ Social power of agent A can be consistent with the influences from all other agents. I.e., current structural centrality of the other agents and their levels of trust to agent A. This influence makes social power more sensitive to feedback from other agents and their current conditions compared with the current individual outcomes from agent A to each agent. This component can take on a positive or negative value.

Social power is the formalization of functional interdependencies between attributes that have different nature (i.e., structural vs. social). For example, if agent A is in the same structural condition (i.e., SP=1) as all other agents and its trust to other agents is at maximum level, then agent A possess the biggest social power in the network even if all other agents do not trust agent A at all (i.e., $\sum_{i=1}^{N-1} T_{i,A} = 0$).

It is important to notice that the given model of social power can be augmented by the extended considerations of social factors. If any other social relations (e.g., knowledge, friendship, and love) can be measured numerically, unified to the range [0, 1] and represented by functional interdependency, defined by Z=(social factor 1,..., social factor N), then T in equation 11 can be replaced by Z. It means that T in equation 9 can be replaced by a multi-factor model of encapsulated social factors like it is done by the implementation of SC multi-factor model (equation 7) for structural factors.

The main limitation here is that many social factors cannot be easily measured numerically. This replacement possibility shows that the proposed SP-function is flexible for multi-factor analysis of social networks and can be operated with different social and structural parameters without radical change.

5. CONCLUSION

The analysis of social reasoning is at the core of understanding how to manage social networks. The research is based on the analysis of the related works required for the understanding of the approaches that contributed expressly or by implication to the modeling of the mechanisms of social reasoning. The basic idea of the research was to combine structural and social properties of agents in a single parameter (i.e., social power). It was approached by the unification of trust as the basic measure of interpersonal exchange and three basic structural measures of social networks:

- Betweenness measure;
- Closeness-based measure;
- Eigenvector centrality.

The proposed formalization of the social power is a multifactor model that is based on the combination of the social network characteristics that have a different nature. The given multidisciplinary approach is an attempt to formalize a social power as a numerical measure that can be used in the analysis of social networks in terms of mathematical and graph theoretical apparatuses.

The future work is related to the improvement of the equation of social power. Since social power is a multifactor model of an agent's capabilities within a network, its current components can be modified and new components can be added. Specifically, we considered trust as a basic social factor of interpersonal relations. However, the other factors can be added to the model if they can be measured.

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