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Coordination between production and sales planning in an oil company based on Lagrangean Decomposition

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Abstract

In our work, we study the issues between production and sales planning processes in an oil company. The planning problems involve decisions regarding procurement of crude oil, generation of components, blending of products, internal transportation of components and products, operation of depots, and sales and distribution of products to markets. We formulate two separated planning problems in a decoupled setting i.e. production model solved by the production department (PD) and sales model solved by the sales department (SD). Sales planning problem is formulated in several ways, considering different scenarios for allocation of depot operation decision and calculation of departmental premium. In addition, we consider two different formulations of revenue functions in each of the sales problems. The first way assumes quadratic programming model with linear demand functions, whereas the second one assumes a piecewise linear approximation of the revenue function and is a mixed integer programming model. The sales model maximizes the premium received by SD, whereas the production model minimizes the costs based on the demand from SD. We also present integrated models that assume centralized planning and maximize the company's total profit. Because in many cases integrated planning is not possible in practice, these models serve only as a theoretical benchmark.

We assume that coordination between the departments is achieved through internal prices. We propose two mechanisms for setting internal prices. The first mechanism includes two cost based-methods, whereas the second mechanism is based on Lagrangean Decomposition (LD). Then we present numerical example to illustrate the methodologies. We study the performance of each of the mechanisms and compare the results achieved under different scenarios. We illustrate the potential advantages and possible disadvantages of LD over the cost-based methods and discuss the allocation of decision-making and sharing rule, in which the company attains a better outcome under the decoupled planning.

Preface

This thesis is written as a part of the master profile Economic Analysis at the Norwegian School of Economics.

Working with this thesis has been a very rewarding and useful experience. We both have a keen interest in optimization, mathematical modeling, and petroleum industry, and have enjoyed exploring these fields further.

Work on the thesis has been a learning and challenging process. In addition to use of knowledge we previously have acquired, we also have got an excellent opportunity to expand our expertise in the petroleum industry, optimization modelling techniques, and AMPL modeling language. In addition, we have become more familiar with the various aspects that affect the petroleum industry, and realized the complexities and challenges prevailing in oil companies.

We would like to thank our supervisor, Kurt Jörnsten, for his academic guidance and his feedback throughout the entire process. Further, we would like to thank, Mario Guajardo, for the great ideas on the topic for our thesis. Finally, we would like to express our appreciation to Julia's mom for the support she has given us.

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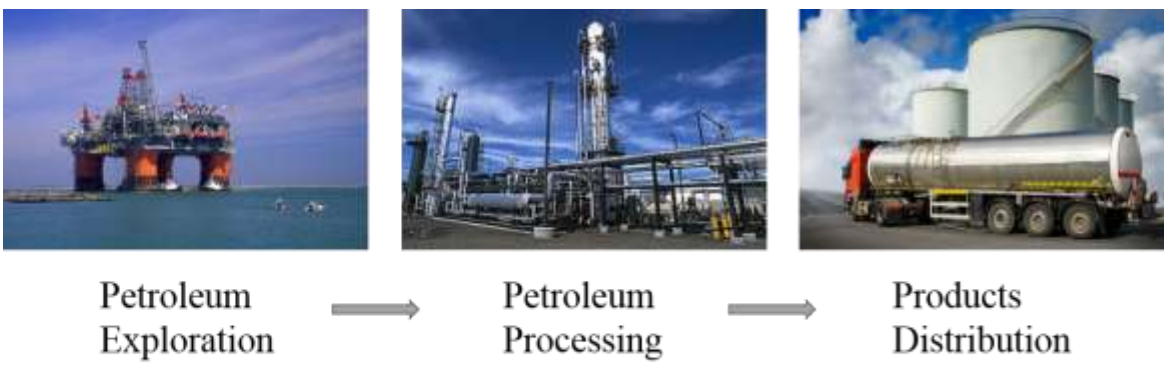
Introduction

1 Motivation

Petroleum is a huge and global industry. Major companies operating in this industry have complicating supply chains and have their facilities spread all over the world. Supply chain in petroleum companies includes many different processes. At the highest level of the chain is exploration of potential petroleum fields, where decisions regarding design and planning of oil field infrastructure must be taken. Next processes are drilling and operating of wells together with extraction of oil and gas. These processes are followed by transportation of raw materials with tankers to terminals, which are connected to refineries through a pipeline network. Some of the decisions at these levels are transportation nodes and supply scheduling. Next processes are: refinery operations, transportation of products to distribution centers, and marketing of petroleum products. Planning and controlling of all these processes create many challenges. Some of these challenges are: uncertainty in wells productivity, finding the optimal schedule for company's rigs, dealing with complicated equipment, uncertainty in demand and oil/gas prices, government regulations, and many others. Because of global competition and high turnover of products, petroleum companies have to find an effective way to deal with these challenges and be able to provide a rapid response to change in environments. To address the challenges quantitative models and mathematical programming techniques have been developed for several decades. Their use have significantly increased company's ability to plan and control their activities (Bengtsson and Nonås, 2009). Van den Heever and Grossmann (2001) have proposed multi-period MINL problem for the long-term design and planning of offshore hydrocarbon field infrastructures with complex economic objectives. As a solution method the authors have used a specialized heuristic algorithm which relies on the concept of Lagrangean decomposition. Neiro and Pinto (2006) have presented a stochastic multiperiod model for representing a petroleum refinery. Uncertainty has been taken into account in parameters such as demands, product sale prices and crude oil prices. Lagrangean Decomposition has been applied in order to reduce computational effort.

Oil refinery system is a part of petroleum supply chain, it stretches from the purchase of crude oil to the sale of petrochemical products. To build a modern refinery is a huge investment and requires covering of fixed cost in the future. In addition to determining efficient processes within the refineries itself, an important task that major refining companies have been focusing in the past few years is to integrate those processes with other functions in the supply chain, such as distribution and sale to markets (Bengtsson and Nonås, 2009). Integrated planning has proved to be of significant relevance, where the basic idea is to optimize simultaneously decisions of different functions, which traditionally have been optimized independently of each other (Erengüc et al. 1999). Guyonnet et al. (2009) have explored the potential benefits of an integrated model involving three parts of the crude oil supply chain: unloading, oil processing, and distribution. The authors have argued, that integrated model would achieve better functional cooperation between different planning problems and avoid suboptimal solutions.



2 Background

Refinery production system is a part of Supply Chain in Petroleum Industry. Refinery process is linked *up-stream* with *oil platforms,* which produce crude oil of different qualities (Bredström and Rönnqvist, 2008). Refinery converts crude oil into components, which are blended into products in hubs. *Downstream* the refinery system is linked with *sales and distribution processes*. The part of logistic network that we consider in our work is composed of refineries, hubs, depots and sale offices, owned by the same company. Refineries and hubs are production units where the inflow of raw material is transformed through several processes into multiple products. Finished products are stored in depots close to customer regions. Sale offices are the channel responsible for distribution of products from depots to customers.

Supply chain management of this logistic network involves many decisions, both short term and long term. These decisions are usually taken on different levels in the company. Supply chain literature distinguishes between three main decision levels: *strategic, tactical* and *operational* (Simchi-Levi et al. 2003).

The strategic level deals with decisions that have a long-lasting effect on the firm (Simchi-Levi et al. 2003). This includes decisions regarding manufacturing and logistics investments, such as utilization of production capacities and nodes of transport, location and size of new depots, product development, and entrance to new markets. Not only existing capabilities have to be considered, but also new opportunities in all areas have to be evaluated. Fernandes et al. (2013) and Oliveira et al. (2012) have raised some of the problems related to this decision level. Fernandes et al. (2013) have developed a deterministic MILP for strategic design and planning of downstream petroleum supply chain network to decide optimal depot locations, transport modes, resource capacities, routes and network affectations for long term planning. Oliveira et al. (2012) have addressed the strategic multi-product, multi-period oil supply chain investment planning problem of network design and discrete capacity expansion under demand uncertainty.

The tactical level includes purchasing and production decisions, inventory policies and transportation strategies based on forecasts of future demand. Examples of decisions at this level are: amount of each product to produce, distribution and storage of products and other materials, and pricing of products. Normally time horizon for such decisions in an oil company is 3 month (Guajardo et al. 2013a). Neiro and Pinto (2004) have developed a multi-period MINL model for petroleum supply chain in context of the Brazilian company, Petrobas. This model considered several refineries connected with pipelines and storage tanks, and included decisions regarding oil type selection, production levels, operating of processing units at refineries, product distribution plan and inventory management.

The operational level refers to day–to-day decisions such as scheduling, lead time quotations, routing, and truck loading (Simchi-Levi et al. 2003). Due to complexity of the supply chain in an oil company and huge amount of data that needs to be manipulated at this level, operational planning is often separated into different subproblems. As Alabi and Castro (2009) have pointed out, in most cases the refinery-planning problem is decomposed into three subproblems: crude oil supply, refining and blending, and product distribution. Jia and Ierapetritou (2004) have developed a continuous time MILP model for the efficient scheduling of oil refinery operations. In their approach the authors decomposed the overall problem into three subproblems: the crude-oil unloading and blending, the production unit operations, and the product blending and delivery. Each of these subproblems has been modeled and solved in a most efficient way. Alabi and Castro (2009) have modeled and implemented an integrated refinery-planning problem, in which the authors have considered decisions from crude oil purchase through to products distribution. The problem has been approached by interior-point algorithms and two decomposition techniques, Dantzig–Wolfe and block coordinate-descent.

The tactical level connects long-term strategic level to day-to-day operational level: it ensures that operative planning follows the direction that has been set out at strategic level (Bredström and Rönnqvist, 2008). This issue has been raised by Mouret et al. (2011). The authors of the paper have used Lagrangean decomposition approach to integrate and solve two main optimization problems appearing in the oil refining industry: refinery planning and crude-oil operational scheduling.

Guajardo et al. (2013a) have studied another coordination issue at tactical level in an oil company. The authors have considered a decoupled setting in which decisions about production and distribution of products down to depots were taken by operational unit, while decisions about distribution of products from depots to customers were decentralized to individual sellers. In a numerical example the authors have showed that an integrated modeling approach, where decisions about production and sale were made simultaneously, outperform the decoupled planning.

In the real world due to large size and complexity of organizations, an integrated optimization model would be significantly challenging. In such cases coordination between divisions in a decentralized company can be achieved through the use of transfer pricing system, also called internal prices. Dean (1955), referred in (Abdel-Khalik and Lusk, 1974; p.8), has argued that a rationally conceived and systematically applied transfer pricing system would allow divisions to maintain their autonomy while making decisions that benefit the entire organization.

3 Aim of the Dissertation

The focus of our work is coordination between production and sales divisions at the tactical level, in an integrated oil company. We use internal prices as the main coordination mechanism between these two divisions. In our thesis we study two planning mechanisms for setting internal prices. The first one, is a pure cost based mechanism. However, our main focus is on the second mechanism in which we employ Lagrangean decomposition (LD). We also compare these two mechanisms.

Each of the mechanism includes several methods. The methods we consider, are possible to realize in a decoupled setting, without knowledge of the optimal solution. In order to measure results from the proposed methods, we develop integrated models in which decisions about production and sales are made simultaneously, and use these as the theoretical benchmark for performance.

In our work we use relative simple models to represent the tactical planning, without going into too much details about production specifics. As the base for our models, we adapt models from previous studies of coordination between production and sales divisions in oil companies (Guajardo et al., 2013a, 2013b; Bredström and Rönnqvist, 2008). We also make an extension of those models, by introducing fixed costs associated with operations of distribution centers, called depots and include possibility of closing them down. Based on this extension we consider various scenarios assuming different allocation of decisions and premium rules. We study the performance of the proposed methods according to these scenarios.

The aim of our master thesis is to ***investigate how Lagrangean decomposition mechanism can be used to find internal prices and how does the efficiency of this mechanism changes with different model formulations.*** In addition we also ***compare LD with the cost based mechanism, and try to determine which allocation of decisions and premium rule are best.***

4 Disposition

The remaining of our work is organized as follows.

Part 1 – Problem Description. We start by general description of parts of petroleum supply chain which will be analyzed, in chapter 1. In chapter 2, we introduce coordination problem and describe what is meant by internal price. In chapter 3, some of the constraints which are used in our models are formulate and described. We start with constraints associated with production process and depots use in 3.1. Next in 3.2 we give an overview over how our models pick up competition in markets, before we formulate related constraints. In 3.3 we introduce a piecewise linear revenue function and associated constraints. In chapter 4, the production model, together with sales models are formulated. Integrated models which serve as our benchmarks are formulated in chapter 5.

Part 2 – Internal Price mechanisms. We propose our cost based methods for setting of internal prices in chapter 6. In chapter 7, we give theoretical explanation of LD, before we apply this mechanism to our problem.

Part 3 – Computational study. In chapter 8, data used in our models are described. In Chapter 9, we present results from cost-based methods that have been described in chapter 6. In chapter 10, we make computational experiments with LD methods.

Part 4 – Conclusion. Concluding remarks are presented in chapter 11.

PART 1: PROBLEM DESCRIPTION

Chapter 1 - Oils Supply Chain

In this chapter we give a brief description of the part in an oil company that represents the application to be analyzed. We start by explaining what we mean by *crude oil*, *components* and *products,* terms which we use through the rest of our work.

1.1 Crude oil, components, and products

The basic raw material for refineries is ***crude oil***. The price of each crude oil is a function of its quality. Crude oils which are easier to refine have a higher price in the market relative to crudes which require extra treating (Kutz et al. 2014). Acquisition of crude oils account for a large portion of refineries costs (Bengsson and Nonås, 2009). Two properties that have greatest influence on the value of crude oil are sulfur content and density (expressed in terms of API gravity). Sulfur content is expressed as percent sulfur weight and varies from less than 0.1% to greater than 5%. Crudes with greater than 0.5% sulfur generally require more expensive processing than those with lower sulfur content (Gary and Handwerk, 2001).

Components refer to semi-finished products. In a refinery, components can be obtained from crude oils or they can be purchased from outside. Qualities of components depend upon the crude oil they are obtained from. Components can either be used as input to processing unit or as blending components. Examples of components, which are obtained from crude distillation unit, are *light* and *heavy naphtha*. Both are used in gasoline blending.

Products refer to finished goods which are saleable in the markets. The basic refinery processes are based on large-quantity products such as gasoline, diesel, jet fuel etc. The value of products depends on location, demand in the markets, products characteristics, and other things. Gasoline is one of the most high-valued due to large margins and high volumes (Bredström et al., 2008). The main part of *gasoline* made by refineries is used as fuel in automobiles. Most refineries produce gasoline in three grades: regular, premium, and super-premium. *Jet fuel* is used by both commercial aviation and military aircraft. *Automotive Diesel Fuels* is used in high-speed engines in automobiles, trucks, and buses. (Gary and Handwerk, 2001)

In the rest of this chapter we describe the main parts of the supply chain (Figure 1.1) of an oil company which we later include in our model. We start by giving a general overview of these parts, before we explain in chapter 3 how these are modeled.

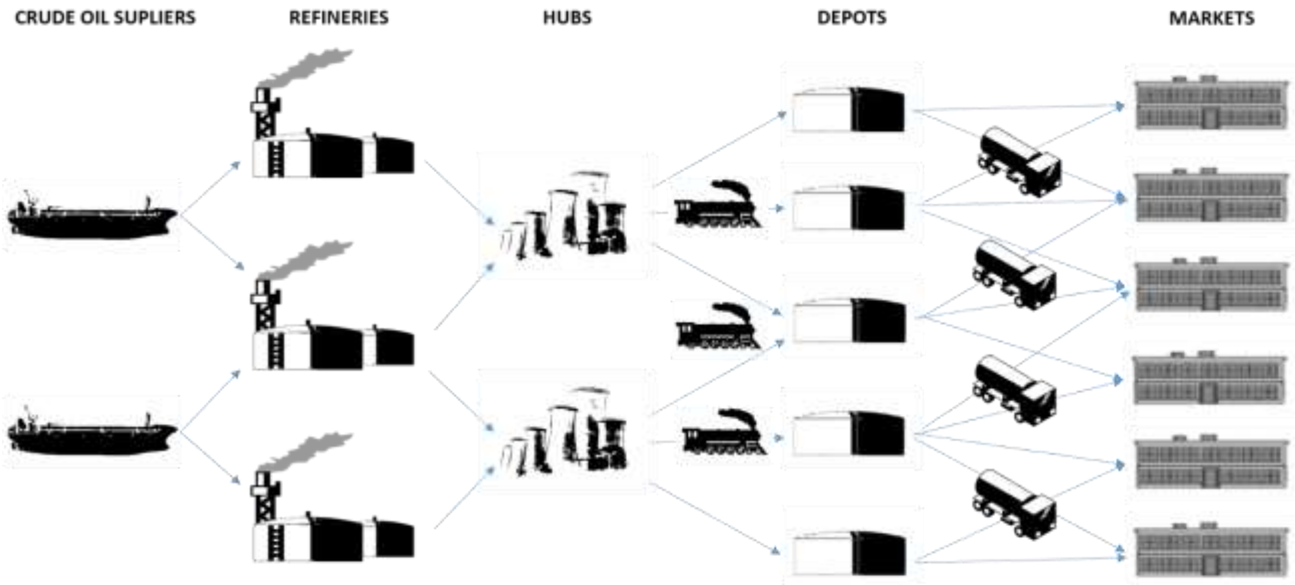


Figure 1.1- Supply Chain

1.2 Procurement of crude oil

Because of scheduling and transportation time, an oil company orders crude oils two-four months before processing (Kutz et al. 2014). Amount and type of crude that should be purchased is a huge decision for an oil company, because crude oil costs typically represent 70-80 % of company's total costs (Kutz et al. 2014). It's important to choose crude oils that make the production cheaper: if the company manages to get small reduction in production costs, it will lead to huge increase of profit because of large scale of production. To order the "right" amount of crude oils, company must have some strategic forecasts of future demand. Some parts of future demand can be known, while other parts may be unknown. For example, company may already have some ordered amount of products which must be delivered, on the other hand company may have customers who buys different amount of products each time. Too little amount of purchased crude oil leads to unsatisfied demand, unsatisfied demand leads to loose in profit and in addition possible loose of reputation and customers. On the other hand, too high purchase volume leads to additional storage costs.

1.3 Processing of crude oil

When crude oils have been delivered at refineries, they are exposed to a series of processes in order to generate salable products. The first major step in refinery is to separate crude oils by distillation into fraction according to their boiling points. Gary and Handwerk, (2001) have described this step as follows. Volumes of crudes are processed through Crude Oil Distillation Unit (CDU) where different components are produced. During this process many compounds that are present in crude oil are separated. The longer the carbon chain is, the higher is the temperature at which the compounds will boil. The crude oil is heated and changed into a gas. The gasses are passed through a distillation column which becomes cooler as the height increase. When a compound in the gaseous state cools below its boiling point, it condenses into a liquid. The liquids may be drawn off the distilling column at various heights.

These liquids are the components. The characteristics of the components depend on which crude oil has been used. Some of these components can be directly used in blending, however most are used further in processes where properties of components changes (Bredström et al., 2008). One typical example of further processes is *cracking process* (Kutz et al. 2014) where heavy molecules are cracked into lower molecular weight. From cracking unit components are improved in qualities by *hydro treatment process*, where components receive desired properties such as density and some of sulfur content is removed, and *reforming process* where components are reformed to meet octane specification (Kutz et al. 2014). In reality refinery operations are very complex and no one of the refineries are identical in their operations (Kutz et al. 2014). A refinery has available crude oils and information about products that must be produced, based on that the refinery must find an economic practical way to process crude oils and generate components, which will further be used in blending of products.

1.4 Blending

From refineries, components are sent to hubs, where blending takes place. Relevant decision in the blending process is how to blend components in order to meet all critical specifications and demand requirements most economically. Each product has specific quality restriction on e.g. density, octane, and sulfur content, while each component has some values of these qualities (Gary and Handwerk, 2001). However, in reality some qualities may be unknown and may not show linear relationships, what makes the problem nonlinear. For example octane limits are typically specified with fourth-order expressions, while volatility quality measure perform logarithmically (DeWitt et al. 1989) In addition one product may be blended in many different ways, what makes blending a complicated process.

1.5 Storage

Storage locations are important in order to achieve flexibility in manufacturing and in case of shutdowns. Both crude oils, components, and finished products are possible to store. As Hu et al. (2011) have pointed out, in a firm a potential conflict exists between manufacturing and sales departments: salespersons prefer to order from manufacturing departments in advance so that they can secure products in the amount they need to satisfy customers in time. While this strategy is good for the sales department to guarantee the right quantity at the right time for customers, it adds additional costs and pressure to the manufacturing department.
Therefore it should be a balance between benefit from storage availability against cost of holding extra stock.

While crude oils and semi-finished products are usually stored in tanks at refineries, salable products are sent to depots for storage. Depots are warehouses which serve as "distribution centers and storage locations for final products" (Guajardo et al. 2013b; p.892). Depots are located closer to the markets than refineries and hubs.

1.6 Transportation

In general, crude oils are supplied to refineries with two type of ships (Bengtsson and Nonås, 2009): very large crude carriers and small vessels carries. Large crude carriers may carry different crudes, while small vessels carries only single crude (Bengtsson and Nonås, 2009). Typically oil refineries receive crude oils from ships through a pipeline. From refineries products are sent to hubs, and thence to depots. Because of large volumes that needs to be distributed, transportation is generally undertaken using pipeline, maritime ships or railway (Fernandes et al., 2013). From depots products are sent to customers in different markets. Volumes are typically smaller and transportation is normally undertaken by road using tank trucks (Fernandes et al., 2013). However, when large volumes are transported in the case of Jet Fuel to airports, pipeline or railroad may be used (Fernandes et al., 2013).

In our work transportation from refineries to depots is called ***primary transportation***.
Transportation from depots to markets is called ***secondary transportation***.

Chapter 2 - Coordination between departments

2.1 The planning problem

In large organizations, it is usual that different functions are divided between organizational subunits called departments or divisions. In petroleum firms, such divisions may include one or several echelons of the supply chain. A firm may consist of a headquarters group and several departments. Each department controls a set of activities. In the case of petroleum firm these activities may relate to purchase of crude oil, production of petroleum products, transportation planning, and sales to outside customers. Also each department usually has limited local resources. Such restrictions can be storage capacity, customers demand etc. In addition to local restrictions, it may exist corporation restrictions which affect all departments. Joint resources, for example available crude oil, may restrict amount of products it is possible to produce and hence sale to customers. Another example is coupling constraints, which affect resource exchange between departments. We can formulate a general planning problem (M) in the following abstract way:

$$\begin{array}{lll} \mathit{maximize}: & z = x_1 \times c_1 + x_2 \times c_2 & (M) \\ \mathit{subject\ to}: & f(x_1, x_2) \leq a & (M1) \\ & g_1(x_1) \leq b_1 & (M2) \\ & g_2(x_2) \leq b_2 & (M3) \\ & x_1, x_2 \geq 0 & (M4) \end{array}$$

This problem formulation has the following interpretation. The company consists of two departments. Each of these departments has some activity levels x_j which it has control over. The objective of this problem is to maximize the total contribution of the company z, where the c_j vector expresses the contribution from the activities. The constraint (M1) is a corporation constraint which affect all departments, while constraints (M2) and (M3) are departmental constraints, where b_j represents the available resources at department j. Dirickx and Jennergren (1979) have pointed out that this type of problem formulation does not contain a detailed scheduling of individual jobs.

One problem that arises in this type of model formulation is that it is not possible to solve the model in one place. This may be because information is dispersed between different subunits in organization. For example a production process of products may be known only in production department, while the demand forecast is information available only for sales division. In addition some departments may not be willing to share some of the information with other units in organization.

When it is not possible to solve the overall problem in one place, we can divide it into several smaller subproblems. Dirickx and Jennergren (1979) have distinguished between three different situations which can appear when the overall problem is divided: (1) the subproblems do not correspond to organizational subunits in the real world and the subproblems has no meaningful institutional interpretation; (2) the subproblems do correspond to organizational subunits, but this correspondence is not used in the actual solution process; (3) there is correspondence between organizational subunits and the subproblems, and this correspondence is utilized in the solution process.

2.2 Decomposition methods

In large scale optimizations, one of the fundamental techniques are decompositions. Decomposition methods use different relaxations and decompose the original problem into smaller subproblems. Then, the subproblems are solved repeatedly in a systematic way until an optimal solution is found (Lundgren et al. 2009). Coordinating divisions in a multi-divisional firm using mathematical decomposition has been a subject for OR research (Karabuk and Wu, 2000). Dirickx and Jennergren (1979) have pointed out that it is customary to divide planning procedures for solving problems like (M) founded on decomposition methods into two groups: price-directive and resource-directive. The main difference between these two groups is in information exchange between the headquarters and the departments. Dirickx and Jennergren, (1979) have described that in a price-directive approach headquarter sends price information to divisions. Based on the prices announces by headquarter, the divisions decide on quantities and send this information back to headquarter. The Dantzig-Wolfe and Lagrangean decomposition methods are used as price-directive approaches (Dirickx and Jennergren, 1979), we refer to these in the next subchapter. The second resource-directive approach is based on budget, and involves headquarter distribute the common resources directly among the subdivisions and requires from them fixed contributions to the common aims. Then subdivisions calculate their respective optimal programs and report the prices they can pay for the common constraints to headquarter. This decomposition technique has been presented by ten Kate (1972).

Dirickx and Jennergren (1979) have pointed out that one important property of the solution method to the planning problem is that a relative “good” solution should be obtained with only small number of iterations of information exchange. The authors have claimed that not many iterations of information exchange between different organizational subunits will be undertaken in a real company. Another aspect for a solution methods in real companies is that each department should have a clear information about what it supposed to do in each planning stage and what information it must exchange with other units. An assumption that is implied here is that each department send the true information, and doesn’t act out of self-interests.

2.3 Internal Price Mechanisms

In many of multidivisional firms, there are two profit centers: manufacturing cost center that seeks lower costs and operational efficiency, and marketing revenue center that controls pricing and other marketing elements (Balasubramanian and Bhardwaj, 2004). The benefits of decentralization are for instance: (1) greater responsiveness to local needs; (2) quicker decision making; and (3) sharpened focus of business unit managers (Pfeiffer, 1999). However, decentralization can also lead to suboptimal solutions, which are not necessarily in line with firm’s goals. Transferring (internal) pricing mechanism is supposed to deal with this coordination issue. Transfer pricing mechanism attempt to “generate prices for internally produced and consumed commodities” (Abdel-Khalik and Lusk, 1974; p.8). Also the mechanism should motivate, coordinate, and control the allocation of economic resources and factors of production so that the overall organizational goals can be achieved (Abdel-Khalik and Lusk, 1974).

Different approaches of transfer pricing models have been developed. Kouvelis and Lariviere (2000) have proposed a mechanism based on linear transfer prices for the intermediate output. This mechanism has been implemented through an internal market. In this internal market the authors have assumed a principal who acted as market maker, buying all output from upstream managers and reselling it to downstream managers. The principal was not obliged to buy and sell at the same price. Dorestani (2004) has considered two divisions: one of the divisions produced, while the other used an intermediate good. Each of the divisions had some information which was not available to the center. Dorestani has showed how the center of the firm can 'control' division's actions with transfer price and a penalty factor, assuring that divisions are sensitive both to profit opportunities of seeking outside trades and to benefits of internal trade.

In some of the papers, game theory has been applied in order to deal with the coordination issue. Erickson (2012) has proposed a transfer price mechanism to coordinate the strategies of marketing and operations functional areas, recognizing differing and often conflicting objectives of these areas in a decentralized firm. The transfer price was included in the differential game model, which allowed the coordination of equilibrium marketing and production strategies to achieve a maximum profit for the firm. Hu et al. (2011) have considered potential issues between manufacturing and sales departments as a result of ''lead-time hedging" strategy which has been used by sales department. The authors have introduced internal price in two different coordination models for different structure of the firm. In the Nash game model, the manufacturing and the sales departments decided the internal price and the lead-time hedging simultaneously. In the Stackelberg game model, the manufacturing department served as the leader and the sales department acted as the follower. It has been showed that the suggested approaches, compared to the traditional model, are effective to reduce the lead-time hedging and improve the entire firm's profit. Pekgün et al. (2008) have studied a decentralized system where price and lead time decisions have been made by the marketing and the production departments in an MTO firm. The authors have focused on evaluating marketing as a revenue center and production as a cost center, and have formulated the problem as a Stackelberg game with alternative decision making sequences. The authors have showed that coordination can be achieved using a transfer price contract with bonus payments, as long as production receives a satisfactory incentive as a fraction of total revenues.

Guajardo et al. (2013b) have presented models for joint optimization of internal pricing and planning decisions in an oil company. In the described approach, producer incorporated sellers' behavior by expressing demand as a function of the internal price. The authors have showed that this joint optimization model outperform traditional cost-based methods.

Also decomposition methods have been used to determine internal/transfer prices. Baumol and Fabian (1964) have suggest utilization of the Dantzig-Wolfe decomposition procedure to provide internal prices for decentralized decision making in a multidivisional firm. Karabuk and Wu (2002) have studied the coordination issues between local marketing and manufacturing decision problems as separate stochastic programs. In their models the authors have considered uncertainty of demand and capacity in a semiconductor industry. Two coordination mechanisms have been presented, in which transfer prices have been used in order to achieve coordination. Mechanisms were motivated by mathematical decomposition via Augmented Lagrangean (nonlinear penalty methods). Bredström and Rönnqvist (2008) have studied coordination issue between refinery production and sales planning. The authors have showed that Lagrangean decomposition can provide a more stable methodology than standard approaches used in many industries. Kong and Rönnqvist (2012) have studied coordination between sales and production planning at a refinery, in a working paper. The authors have proposed two mechanisms for setting internal prices. The first mechanism used marginal values as internal prices whereas the second employed Lagrangean decomposition.

2.4 Coordination between Sales and Production Departments in an Oil Company

In our work we consider an oil company in which sales and production departments make their decisions separately and each of the departments have their own objectives. Owned by the company, the sales department (SD) is managed independently and is seeking to maximize its profit from sales. Based on the estimated demand in markets, costs associated to purchase of products, and secondary transportation costs, the sales department makes orders from the production department (PD). These orders include type and amount of each product, in addition to locations products must be available at. According to the orders from SD, PD decides how to produce and distribute the products such that costs associated to production and primary transportation are minimized.

We assume that coordination between the departments is achieved through the use of internal prices of products. In our models, the internal prices are the costs that SD has to pay in order to "buy" products from PD. These internal prices must be decided for each product at each storage location, and stimulate the sales department to make decisions that will maximize the profit for the whole company.

We assume that the production department is integrated with headquarter (the company). In addition to satisfy orders from SD, the department decides internal prices.

In the same way as Guajardo et al. (2013a, 2013b) we assume that SDs premium to a great deal depends on the margin between the sale price and the value chain cost of the products. The value chain cost includes the price SD pays to PD for the product (internal price) and secondary distribution costs.

Value chain cost = internal price + secondary distribution cost

The sales department receives a percentage premium *Δ* from the “profit” it achieves.

$$Premium = \Delta(sale\ price - value\ chain\ cost) * sold\ amount \qquad (E2.1)$$

In our model we also consider possibility in which SD takes into account fixed costs associated with operation of depots.

$$Premium = \Delta\{(sale\ price - value\ chain\ cost) * sold\ amount - fixed\ cost\} \qquad (E2.2)$$

We can argue that the premium proportion Δ, doesn’t affect decisions made by SD, because the department will always maximize $(sale\ price - value\ chain\ cost) * sold\ amount - fixed\ cost$ independently of Δ (as long as Δ is a positive number), therefore we can ignore it.

In order to increase the difference between price and cost, SD will tend to choose the lowest possible combination of internal price + transport cost, for each product that it sells. As Guajardo et al. (2013b) have pointed out, this way may not be the most cost efficient way to distribute for the company as whole. If the sales department takes fixed costs into account then the problem is no longer be straight forward. However, the conclusion doesn't change: if internal prices are "wrong" it may exist conflicts between the objective of SD and the company's goals, because SD will not fully take into account the costs imposed by its activities on the production. Therefore, it is important to find an appropriate mechanism for setting of internal prices.

Often economic literature that has studied transferring pricing, has made following assumptions (Abdel-Khalik and Lusk, 1974): selling division produce only one type of product, it is possible to determine unit variable cost of the product, and the product of one (or both) divisions has external markets. In our problem there are multiple products and there are dependencies between them, unit variable cost varies and it is difficult to determine. In addition, we assume that there are no external market company can sell or buy intermediate product to/from.

As it was pointed out by Erickson (2012, p.226): "If there is no market outside the firm for the selling profit center's product, the transfer price needs to be determined internally. In such a case, an appropriate transfer price is one that maximizes the firm's profit".

In practice in oil companies, it is not uncommon that internal prices are decided manually or through a simple cost based method (Guajardo et al., 2013b). These prices are intended to reflect costs caused by products up to the depot locations. However, because of divergent supply chain which is characterized by multiple components and products, and dependencies between products, it is difficult to distribute costs among these products.

If we assume that PD knows the mechanism used by SD when it makes decisions, then PD has indirect control over SDs decisions, because it can change input factors to sales optimization model. The input factors that PD has control over are internal prices. By changing internal prices PD can force SD to act in company's best interests, while at the same time SD will be able to make decisions which will maximize its own premium. These assumptions were made by Guajardo et al. (2013b) in their model formulation, in which the authors suggested method of setting internal prices. They assumed that demand functions in markets are known by PD, together with secondary distribution costs.

In our models we do not make the same assumptions, and instead assume that some information, like demand in the markets, is available only for the sales department. Therefore, company will not be able to predict the response from the sales department, unless information exchange between departments would find place in some step of the planning process.

2.5 The Planning Approach

In the rest of our work we consider a two level approach for planning in sales and production departments. At the first level, the internal prices for products are decided by production department. We consider two possible mechanisms of how these prices can be decided. The first mechanism is based on costs incurred at production department and doesn't require information exchange between departments. The second mechanism is based on Lagrangean decomposition and requires information exchange between production and sales departments. This mechanism can involve solution of real divisional local problems corresponding to category (3) defined in 2.1, as well as other subproblems which may correspond to category (2), implying that representation of such problems is not used in the actual solution process. This level is a pure planning level, later denoted as ***planning level***, and no concrete actions are taken at this level.

At the second level of the approach, when internal prices are decided, the production department announces these prices for the sales department. Based on it, the sales department solves its "naturally" local subproblem and makes decisions on type and amount of products to order, and which locations to order from. Then, the production department solves its subproblem in which it should match the orders from SD at the lowest possible costs. This level will be denoted as ***decision level***.

Whenever internal or external market conditions change, the internal prices or/and production and sales plans can be decided again. Depending on the changes that have occurred, parts or the whole two level approach is recalculated. Also we assume that the company repeats the whole procedure after a certain amount of time has passed.

Local subproblems, which are solved by departments at the decision level are formulated in chapter 4. In chapter 6 and chapter 7 we suggest planning processes which correspond to the planning level, in which internal prices are determined. But first, we formulate departmental constraints and explain the intuition behind these in chapter 3.

Chapter 3 - Problem Formulation

The processes that we have described in chapter 1 are very complex, and there are some aspects that are difficult to include in an optimization model. Therefore, we have identified aspects that are relevant for our problem and at the same time decided which aspects are less important. Increased level of details leads to better realism in the model, but at the same time it leads to a larger model with decreased solvability. Therefore it is important to identify the formulation of real problem with reasonable level of details and complexity. In this chapter we formulate the basic constraints that must be taken into account by the departments. These constraints are used as fundament for the models formulated throughout our work.

Because, the aim of our thesis is to study coordination between two departments, we use formulation in which it is more clear which parts affect the coordination issue. Also, because later in our work we formulate integrated models, which include all decisions, we need to specify the problem such that the optimization models are possible to solve.

In what follows, we introduce the notation of sets and parameters that are used through the remainder of our work.

3.1 Indexes

Sets

$r \in R$:	*Set of refineries*
$h \in H$:	*Set of hubs*
$i \in I$:	*Set of crude oils*
$d \in D$:	*Set of depots*
$k \in K$:	*Set of markets*
$a \in A$:	*Set of components which cannot be processed*
$b \in B$:	*Set of components which can either be processed or directly used in blending*
$c \in C$:	*Set of components which are generated from components from set B*
$p \in P$:	*Set of products*

$e \in E$:	*Union of sets A and B*
$qmin \in QMIN$:	*Set of minimum qualities*
$qmax \in QMAX$:	*Set of maximum qualities*
$n \in M$:	*Number of breakpoints*

Parameters

$sup_{i,r}$:	*Available volume of crude oil i at refinery r*
$\rho_{i,e}$:	*Amount of component e generated from one unit of crude oil i*
$\rho^2{}_{c,b}$:	*Amount of component c generated from one unit of component b*
$spm_{p,qmin}$:	*Value of required quality qmin in product p*
$sbm_{e,qmin}$:	*Value of quality qmin in component e*
$sam_{c,qmin}$:	*Value of quality qmin in component c*
$spma_{p,qmax}$:	*Value of required quality qmax in product p*
$sbma_{i,e,qmax}$:	*Value of quality qmax in component e obtained from crude oil i*
$sama_{i,c,b,qmax}$:	*Value of quality qmax in component c obtained from component b which is again obtained from crude oil i*
$C_{i,r}^{Buy}$:	*Cost of purchasing one unit of crude oil i at refinery r*
$\mathrm{C}_{\mathrm{i,r}}^{\mathrm{PRO}}$:	*Cost of processing one unit of crude oil i at refinery r*
$\mathrm{C}_{\mathrm{b,r}}^{\mathrm{PRO2}}$:	*Cost of processing one unit of component b at refinery r*
$\mathrm{C}_{\mathrm{p,h}}^{\mathrm{BLEND}}$:	*Cost of producing one unit of product p at hub h*
$\mathrm{C}_{\mathrm{r,h}}^{\mathrm{TRAN1}}$:	*Cost of transporting one unit of any product from refinery r to hub h*
$\mathrm{C}_{\mathrm{h,d}}^{\mathrm{TRAN2}}$:	*Cost of transporting one unit of any product from hub h depot d*
$\mathrm{C}_{\mathrm{d,k}}^{\mathrm{TRAN3}}$:	*Cost of transporting one unit of any product from depot d to market k*
C_d^{FIX}:	*Fixed cost to operate depot d*
m_d:	*Maximum capacity at depot d*

A_p *and* b_p: *Coefficients of price function*

$\underline{dem}_{p,k}, \overline{dem}_{p,k}$: *Minimum and maximum demand of product p at market k*

$amount_{m.p}$: *Sold amount of product p corresponding to breakpoint m*

$rev_{m.p}$: *Revenue from product p corresponding to breakpoint m*

Decision variables

$x_{i,r}$: *Amount of crude oil i purchased and processed at refinery r*

$y_{e,i,r}$: *Amount of component e generated from crude oil i at refinery r*

$v_{b,i,r}$: *Amount of component b generated from crude oil i, used for further processing at refinery r*

$\tilde{v}_{c,b,i,r}$: *Amount of component c generated from component b which is again produced form crude oil i at refinery r*

$\tilde{y}_{e,i,r}$: *Amount of component e obtained from crude oil I at refinery r, which is sent directly to blending*

$\overline{y}_{p,e,i,r,h}$: *Amount of component e obtained from crude oil i used in product p sent from refinery r to hub h*

$\overline{v}_{p,c,b,i,h}$: *Amount of component c obtained from component b which is again obtained from crude oil i, used in product p which is sent from refinery r to hub h*

$\tilde{q}_{p,d,h}$: *Amount of product p at depot d which is sent from hub h*

$q_{p,d}$: *Amount of product p at depot d*

$z_{p,d,k}$: *Amount of product p transported from depot d to market k*

h_d: *Binary variable which have value 1 when depot d is used, and 0 otherwise*

$revenue_{p,k}$: *Revenue from product p at market k*

$w_{m,p,k}$: *Weight for product p, breakpoint m*

$l_{m,p,k}$: *Binary variable, takes value 1 if segment m for product p is used, and 0 otherwise*

$\theta_{p,k}$: *Price of product p at market k*

3.2 The supply chain

This section presents the mathematical formulation of each element in the supply chain and highlights its particularities. Constraints for purchase and processing of crude oils, together with blending constraints correspond to the production problem. While correspondence of depot operation constraint will be discussed.

3.2.1 Procurement of crude oil

In our model there is a limitation on supply of each type of crude oil at each refinery. We assume that the decision about supply of crude oil is made on strategic decision level and is based on historical data or some forecasts. We also assume that quality of crude oils are well known. The following constraint is formulated:

$$\mathbf{x}_{\mathbf{i},\mathbf{r}} \leq \boldsymbol{sup}_{\boldsymbol{i},\boldsymbol{r}} \qquad \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{P1})$$

(P1) states that amount of each crude oil that is used in each refinery, must be less than purchased amount for that crude oil. Quality of crude oils are not given directly in our model, but instead considered indirectly through qualities of components generated from the crude oils. We assume that it is not possible to exchange crude oils, and other commodities between refineries.

In order to simplify our model we assume that (P1) only concerns PD in such a way that it only sets a restriction on purchase of crude oil type. But in general, demand constraints (described in section 3.3) are given in a way that there is always enough crude oil to satisfy the demand from SD. Thus constraint (P1) will only affect the production department.

3.2.2 Processing of crude oil

Depending on properties of crude oil, fixed fractions of components are generated from it. Use of such conversion factor is common in optimization models for supply chain planning involving natural resources, and is used by e.g. Guajardo et al. (2013a, 2013b) and Bredström and Rönnqvist, (2008). In our model we assume that there is only one possible way to separate each type of crude oil into components. In reality there can be several ways to divide the fractions contained within a crude oil, depending for example on refinery and which CDU crude oil goes through.

We assume that qualities of components which are generated from crudes are known and that these qualities vary according to characteristics of crude oils.

Some of the components obtained from crude oils can be processed further, in order to improve qualities. In our model we divide the components into three groups:

- A components - components that can be used in blending only
- B components - components those qualities can be improved, or alternatively components can be used directly in blending
- C components – “new” components with improved qualities generated from A components

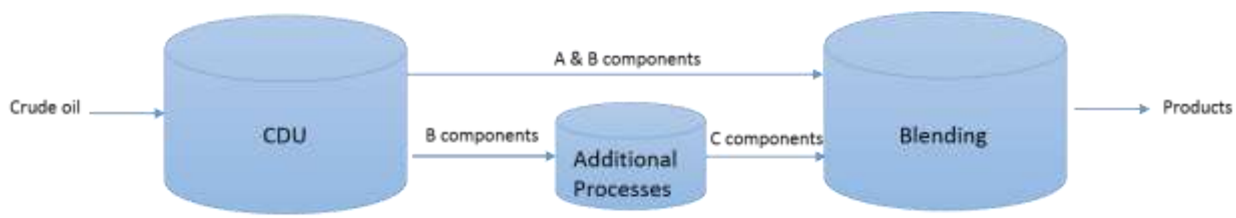


Figure 3.1 – Processing of components

We assume that each B component can generate one or several C components. Some of C components can be generated from different B components. Further we assume that there is only one way to break B component into C components. Characteristics of C component depend both on characteristics of B component and crude oil B component is generated from. These characteristics are known. In order to simplify our model we do not distinguish between cracking process, hydro treatment process, and reforming process, but instead combine them into one additional process.

$$\boldsymbol{\rho}_{\mathbf{i,e}} \times \mathbf{x}_{\mathbf{i,r}} = \mathbf{y}_{\mathbf{e,i,r}} \qquad \forall \mathbf{e} \in \mathbf{E}, \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{P2})$$

$$\tilde{\boldsymbol{y}}_{\mathbf{a,i,r}} \leq \mathbf{y}_{\mathbf{a,i,r}} \qquad \forall \mathbf{a} \in \mathbf{A} \subseteq \mathbf{E}, \ \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{P3})$$

$$\tilde{\boldsymbol{y}}_{\mathbf{b,i,r}} + \mathbf{v}_{\mathbf{b,i,r}} \leq \mathbf{y}_{\mathbf{b,i,r}} \qquad \forall \mathbf{b} \in \mathbf{B} \subseteq \mathbf{E}, \ \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{P4})$$

$$\boldsymbol{\rho}^{2}_{\mathbf{c,b}} \times \mathbf{v}_{\mathbf{b,i,r}} = \tilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}} \qquad \forall \mathbf{b} \in \mathbf{B} \subseteq \mathbf{E},\ \forall \mathbf{i} \in \mathbf{I},\forall \mathbf{c} \in \mathbf{C}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{P5})$$

Constraint (P2) sets the amount of each component which is produced from each crude oil. Constraint (P3) ensures that amount of each A component obtained from a given crude oil and used in blending, cannot be more than the actual amount of this A component obtained from this crude oil. Constraint (P4) ensures that amount of each B component which is used in blending plus amount of the same B component used in processing, cannot be more than the actual amount of this B component which is generated in the refinery. Constraint (P5) sets the amount of each C component which is obtained from each B component which is used in processing.

In our models it is not possible to buy components or other commodities except the crude oils. In reality refineries can have possibility to buy some components which are ready for blending externally. Also refineries can have possibility to exchange components between refineries internally.

3.2.3 Blending

From the refineries components are sent to hubs, where blending takes place. In contrast to the models used by Guajardo et al. (2013a, 2013b), we do not use fixed recipes for how products should be mixed. In our models, we specify quality requirements for final products, for example maximum percent of sulfur content and minimum amount of octane. According to these requirements products can be mixed in any suitable way. Because characteristics of the components are fixed, the blending problem doesn’t create non-linearity. This method of blending problem formulation in optimization models has been used by Bredström and Rönnqvist (2008) and Bredström et al. (2008).

$$\sum_{p \in P} \sum_{h \in H} \bar{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}} \leq \tilde{\boldsymbol{y}}_{\mathbf{e,i,r}} \qquad \forall \mathbf{e} \in \mathbf{E}, \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{P6})$$

$$\sum_{p \in P} \sum_{h \in H} \bar{\boldsymbol{v}}_{\mathbf{p,c,b,i.r}} \leq \tilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}} \qquad \forall \mathbf{b} \in \mathbf{B} \subseteq \mathbf{E}, \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{c} \in \mathbf{C}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{P7})$$

$$\sum_{\mathbf{d}\in\mathbf{D}} \tilde{q}_{p,d,h}\, spm_{p,qmin} \leq \sum_{\mathbf{e}\in\mathbf{E}} \sum_{\mathbf{i}\in\mathbf{I}} \sum_{r\in R} \bar{y}_{\mathbf{p,e,i,r,h}}\ sbm_{e,qmin} + \sum_{i\in I} \sum_{b\in B} \sum_{c\in C} \sum_{r\in R} \bar{v}_{\mathbf{p,c,b,i,r,h}}\ sam_{c,qmin}$$

$$\forall p \in P,\ \forall qmin \in QMIN,\ \forall \mathbf{h} \in \mathbf{H} \qquad (\mathbf{P8})$$

$$\sum_{\mathbf{d}\in\mathbf{D}} \tilde{q}_{p,d,h}\, spma_{p,qmax} \geq \sum_{\mathbf{e}\in\mathbf{E}} \sum_{\mathbf{i}\in\mathbf{I}} \sum_{r\in R} \bar{y}_{\mathbf{p,e,i,r,h}}\ sbma_{i,e,qmax} + \sum_{i\in I} \sum_{b\in B} \sum_{c\in C} \sum_{r\in R} \bar{v}_{\mathbf{p,c,b,i,r,h}}\ sama_{i,c,b,qmax}$$

$$\forall p \in P,\ qmax \in QMAX,\ \forall \mathbf{h} \in \mathbf{H} \qquad (\mathbf{P9})$$

$$\sum_{\mathbf{d}\in\mathbf{D}} \tilde{q}_{p,d,h} \leq \sum_{\mathbf{e}\in\mathbf{E}} \sum_{\mathbf{i}\in\mathbf{I}} \sum_{r\in R} \bar{y}_{\mathbf{p,e,i,r,h}} + \sum_{i\in I} \sum_{b\in B} \sum_{c\in C} \sum_{r\in R} \bar{v}_{\mathbf{p,c,b,i,r,h}} \qquad \forall \mathbf{p} \in \mathbf{P}, \forall \mathbf{h} \in \mathbf{H} \qquad (\mathbf{P10})$$

Constraints (P6) and (P7) ensure that the amount of each component sent to hubs and used for product blending cannot be more than produced amount of this component. Constrains (P8) and (P9) make sure that product quality is reached in the blending process. Constraint (P10) sets mass balance.

3.2.4 Depots

From hubs, products are sent to depots. Transportation costs depend both on hubs and depots locations. In our model we use already existing depots: location and capacity of depots are decided at strategic level. Each depot has fixed costs associated to its operation. If depot is used, these costs are higher than when depot is not used. We focus only on the extra charge of using depots. Because an oil company has a large number of depots, it can be reasonable to assume that the company doesn’t need to have all depots open at all times. Hence, for a given planning period company can decide to close down some of available depots.

In our work we investigate the behavior of solution under alternative allocations of the decision regarding operation of depots. We consider that this decision can be either taken by PD or SD.

$$\sum_{\mathbf{h}\in\mathbf{H}} \tilde{\boldsymbol{q}}_{\boldsymbol{p},\boldsymbol{d},\boldsymbol{h}} = \boldsymbol{q}_{\boldsymbol{p},\boldsymbol{d}} \qquad \forall \mathbf{p} \in \mathbf{P}, \forall \mathbf{d} \in \mathbf{D} \qquad (\mathbf{P11})$$

Constraint (P11) sets a mass balance between products produced at hubs and products available at depots.

In order to deal with fixed costs it is necessary to include binary variables in the model.

$$\sum_{\mathbf{d}\in\mathbf{D}} \boldsymbol{q}_{\boldsymbol{p},\boldsymbol{d}} \leq \boldsymbol{m}_{\boldsymbol{d}} \cdot \boldsymbol{h}_{\boldsymbol{d}} \qquad \forall \mathbf{d} \in \mathbf{D} \qquad (\mathbf{G1})$$

Constrain (G1) ensures that company only uses depots which are open. When the depot is used the binary variable h_d is 1, and 0 otherwise. The constraint also ensures that depots capacity is not exceeded.

If SD doesn't include fixed costs into its model the constraint (G1) will be replaced with the following constraint:

$$\sum_{\mathbf{d}\in\mathbf{D}} \boldsymbol{q}_{\boldsymbol{p},\boldsymbol{d}} \leq \boldsymbol{m}_{\boldsymbol{d}} \qquad \forall \mathbf{d} \in \mathbf{D} \qquad (\mathbf{G2})$$

Constraint (G2) sets a restriction on amount of products that can be stored at depots.

3.3 Product demand

In our models we consider deterministic demand in markets. This assumes that SD have a perfect information about the demand process in the markets, which is a strong assumption. However, this assumption makes our model easier to analyze, also deterministic models are commonly used in practice (Bitran and Caldentey, 2003). Further we assume that demand is given exogenously and customers are price takers, meaning that they observe price set by seller and react by buying or not buying the product. Another assumption that we make, is that demand between products and markets is not correlated. In other words, markets are isolated from each other, thus customers from one market cannot buy products from other markets. This is a reasonable assumption because markets are geographically spread. Similarly, price of one type of product doesn't affect demand for other products. This assumption is reasonable for a short run, because if for example diesel becomes relatively cheaper than gasoline, it should not affect demand for gasoline in a short run. In a long run, this assumption may not be reasonable because demand for diesel transport can go up and demand for gasoline will decline. However, we do not take this possibility into account because we consider only short-run models.

Petroleum industry has high barrier to entry as a result of large investment costs and government regulations. We assume therefore, that firms are strategically adapted in markets, and markets are typically characterized by competition between a small numbers of firms. Each type of product offered by oil companies has the same qualities and covers the same customer needs. Because petroleum's products can be considered as substitutes, it is reasonable to assume that all companies are facing the same price. Therefore, we assume that competition occurs only through quantum.

If we assume that each firm operating in a given market maximizes its profit and takes into account the quantities chosen by its competitors in the same market, and at the same time the company knows that other companies will also take into account its actions, then we can use Cournot model (Tirol, 1988). We will now analyze one-stage game in which firms choose their quantities simultaneously. In order to describe demand in a market for a given product we assume that demand function looks as follows:

$$\theta = a - b \sum_{i=1}^{n} z_i \qquad (E3.1)$$

Where θ is the price of the product, z_i is quantity produced by company *i*, *n* is total number of companies operating in market and *a* and *b* are demand parameters.

As we can observe from (E3.1) price decreases when quantity sold in the market increases. Hence one firm cannot decide price alone. As long as companies are rational, each of them will optimize their own profit. Profit for company *i* is given by (E3.2):

$$\pi_i = \theta \times z_i - \mathrm{C}(z_i) \qquad \text{(E3.2)}$$

Where C (x_i) is the cost function of company *i*.

Assuming that the profit function is strictly concave in x_i and twice differentiable, from the production theory we know that profit is maximized when $\frac{\partial \pi_i}{\partial z_i} = 0$.

$$\frac{\partial \pi_i}{\partial z_i} = a - b \sum_{j=1}^{n-1} z_j - 2bz_i - \frac{\partial C_i}{\partial z_i} = 0 \qquad (E3.3)$$

From (E3.3) follows:

$$z^*{}_i = \frac{\mathrm{a} - \mathrm{b} \sum_{j=1}^{n-1} z_j - \frac{\partial C_i}{\partial z_i}}{2\mathrm{b}} \qquad (E3.4)$$

As we can observe from (E3.4) optimal quantity produced by company i ,z_i, is a function of production in other companies$\sum_{j=1}^{n-1} z_j$. (E3.4) indicates how much company i should produce given the quantity produce by other companies.

To illustrate our point, we use example of two firms and the Cournot model. However, the conclusion will not change if there are more than two firms. One important assumption that we make here is that companies know marginal cost functions of other companies. This is a strong assumption, which may not correspond to the real world.

We start by claiming that company 1 knows that company 2 has the following response function (based on the assumption that company 2 is rational and maximizes its profit):

$$z_2 = \frac{\mathrm{a} - \mathrm{b}\, z_1 - \frac{\partial C_2}{\partial z_2}}{2\mathrm{b}} \qquad (E3.5)$$

Further we assume that company 2 has the following cost function $C(z_2) = D_2 + c_2 z_2$. We assume linear form, and hence constant return to scale, in order to simplify calculation.

Then $\frac{\partial C_2}{\partial x_2} = c_2$. Now we can rewrite (E3.5).

$$z_2 = \frac{\mathrm{a} - \mathrm{b}z_1 - c_2}{2\mathrm{b}} \qquad (E3.6)$$

Assume that cost function of company 1 is of the same form: $C(z_1) = D_1 + c_1 z_1$

Then company 1 has following response function:

$$z_1 = \frac{\mathrm{a} - \mathrm{b}z_2 \ - c_1}{2\mathrm{b}} \qquad (E3.7)$$

Because we have made assumption that company 1 knows the response function for company 2, we can insert (E3.6) instead of z_2 in (E3.7).

$$z_1 = \frac{\mathrm{a} - \mathrm{b}\ \frac{\mathrm{a} - \mathrm{b}z_1 \ - \ c_2}{2\mathrm{b}} \ - c_1}{2\mathrm{b}} \qquad (E3.8)$$

We simplify (E3.8):

$$z_1 = \frac{\mathrm{a} + c_2 \ - 2c_1}{3\mathrm{b}} \qquad (E3.9)$$

Then we do exactly the same for company 2 and get:

$$z_2 = \frac{\mathrm{a} + c_1 \ - 2c_2}{3\mathrm{b}} \qquad (E3.10)$$

We have found optimal quantities for both firms.

On the other hand, if we assume that company 1 knows that company 2 will produce quantum calculated in (E3.10) then company 1 can calculate optimum quantum in the following way.

Insert (E3.10) into (E3.2) and derivate with respect to z_1.

$$\frac{\partial \pi_1}{\partial z_1} = \frac{2\mathrm{a} - c_1 + 2c_2}{3} - 2bz_1 - c_1 = 0 \qquad (E3.11)$$

Derive z_1 from (E3.11):

$$z_1 = \frac{\mathrm{a} - 2c_1 + c_2}{3} \qquad (E3.12)$$

As we can observe (E3.12) is identical to (E3.9).

In analogues manner, equilibrium price and quantities can be derived for a larger number of firms and costs functions with decreasing return to scale.

In our example we have used Cournot Analysis and showed how optimal quantity can be derived. This implies that a company makes estimates of quantum to other companies and chooses its quantum according to these estimates. In our models we assume that company has done some forecasts on produced quantity of other companies, without going in details of how this is done.

So, back to our problem. Price for a given type of product at a given market is given as:

$$\theta = a - b\left(z_1 + \sum_{i=2}^{n} z_i\right) \qquad (E3.13)$$

Assume that company 1 makes forecast on aggregated production from other companies $\hat{z} = \sum_{i=2}^{n} \widehat{z_i}$

Then estimated revenue from sales for company 1 is:

$$Revenue_1 = ax_1 - bz_1{}^2 - b\hat{z}z_1 = (a - b\hat{z})z_1 - bz_1{}^2 \qquad \text{(E3.14)}$$

Let $A = a - b\hat{z}$

Then we can rewrite revenue function:

$$Revenue_1 = Az_1 - bz_1{}^2 \qquad (E3.15)$$

In our work we use A and b as demands parameters, where we assume that A picks up competitors action. We assume that A and b parameters are given in our models and we do not focus on their calculations.

So, we are ready to formulate constraint which will be used in our models:

$$\boldsymbol{\theta_{p,k} = A_p - b_p \sum_{d \in D} \mathbf{z_{p,d,k}}} \qquad \forall \boldsymbol{p} \in \boldsymbol{P}, \forall \boldsymbol{k} \in \boldsymbol{K} \qquad (\boldsymbol{S1})$$

Constraint (S1) gives the market price for products.

Revenue will be:

$$\sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\theta_{p,k}\mathrm{z}_{\mathrm{p,d,k}} \qquad (E3.16)$$

$$\sum_{p\in P}\sum_{k\in K}\left\{\left(A_p - b_p\sum_{d\in D}\mathrm{z}_{\mathrm{p,d,k}}\right)\sum_{d\in D}\mathrm{z}_{\mathrm{p,d,k}}\right\} = \sum_{p\in P}\sum_{k\in K}\left(A_p\sum_{d\in D}\mathrm{z}_{\mathrm{p,d,k}} - b_p\sum_{d\in D}\mathrm{z}_{\mathrm{p,d,k}}{}^2\right) \qquad (E3.17)$$

This is a quadratic problem. In order to avoid non-convexity we use equation (E3.17) in our objective function instead of (E3.16).

In our model we assume that for each product and at each market is exists some minimum and maximum demand quantities which company must adhere to. Minimum demand may arise naturally in a short-run planning situation where company may have contracted to deliver certain minimum quantities of each products to the markets. We formulate the following constraint:

$$\underline{\boldsymbol{dem}}_{\boldsymbol{p,k}} \le \sum_{\mathbf{d}\in\mathbf{D}}\mathbf{z}_{\mathbf{p,d,k}} \le \overline{\boldsymbol{dem}}_{\boldsymbol{p,k}} \qquad \forall\mathbf{p}\in\mathbf{P} \quad \forall\mathbf{k}\in\mathbf{K} \qquad (\mathbf{S2})$$

(S2) Ensures that distribution from depots to market is within upper and lower bound for each product at each market.

3.4 Piecewise linear Revenue Function

In this section we formulate an alternative way of describe revenue, by use of piecewise linear approximation of revenue function. This will allow our models to remain linear. However, this approach will require inclusion of binary variables.

We can avoid non-linearity in (E3.17) by dividing the revenue function in a number of segments. The new revenue function has the following interpretation. If sold amount of a product lies within a given segment then the revenue from this amount will be within corresponding segment. In this segment the relationship between sold amount and revenue is linear. We find values for revenue by weighting the breakpoints. The following numerical example illustrate the method:

Suppose that for a given product at a given market the demand lies within following interval $[0,300]$. Further this interval is divided into three equal segments, illustrated by l_m in Figure 3.2.

For each breakpoints we define a variable w_j

Suppose sold amount is 150, which is within segment $l_2 = [100,200]$. Corresponding revenue segment is $[1000,1500]$. The weights between breakpoints for sold amount are $w_2 = w_3 = 0.5$ $100 * 0,5 + 200 * 0,5 = 150$. In order to find revenue we use the same weights: $1000 * 0,5 + 1500 * 0,5 = 1250$

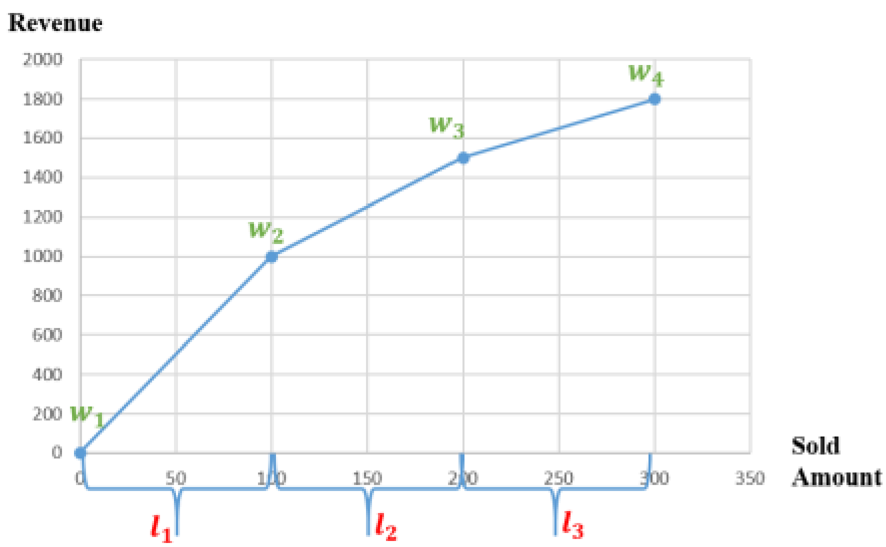


Figure 3.2 – Piecewise linear Revenue function

The more breakpoints we include, the more precise linear approximation of revenue will be. However, this will be achieved at the expense of increased number of integer variables.

How will approximation of revenue influence the solution of the problem? A linear approximation will not give precise values for revenue, and hence the sold amount and explicitly calculated price can deviate from values found with quadratic revenue function. However, as we have pointed out earlier, the demand functions are forecasts made by sales department, hence the values calculated in these functions do not fully represent the reality. Also, in the real world the price can be fixed for a given quantity segment, and not change with each sold amount.

We are now ready to formulate constraints which will be used in linear-revenue sales model.

$$\sum_{d \in D} \mathbf{z_{p,d,k}} = \sum_{m \in M} \boldsymbol{amount_{m.p} w_{m,p,k}} \qquad \forall \mathbf{p} \in \mathbf{P}, \ \forall \mathbf{k} \in \mathbf{K} \qquad (\mathbf{SL1})$$

$$\boldsymbol{revenue_{p,k}} = \sum_{m \in M} \boldsymbol{rev_{m.p} w_{m,p,k}} \qquad \forall \mathbf{p} \in \mathbf{P}, \ \forall \mathbf{k} \in \mathbf{K} \qquad (\mathbf{SL2})$$

$$\sum_{m \in M} \boldsymbol{w_{m,p,k}} = \mathbf{1} \qquad \forall \mathbf{p} \in \mathbf{P}, \forall \mathbf{k} \in \mathbf{K} \qquad (\mathbf{SL3})$$

$$\sum_{m \in iM} \boldsymbol{l_{m,p,k}} = \mathbf{1} \qquad \forall \mathbf{p} \in \mathbf{P}, \ \forall \mathbf{k} \in \mathbf{K} \qquad (\mathbf{SL4})$$

$$\boldsymbol{w_{m,p,k}} \leq \boldsymbol{l_{m-1,p,k}} + \boldsymbol{l_{m,p,k}} \qquad \forall \mathbf{p} \in \mathbf{P}, \forall \boldsymbol{m} \in \boldsymbol{M}, \ \forall \mathbf{k} \in \mathbf{K} \qquad (\mathbf{SL5})$$

Sold amount lies between two breakpoints (can also be only at one point), constrain (SL1) finds how these breakpoints are weighted for each product. Each sold amount corresponds to a certain revenue value, constrain (SL2) calculates this revenue value. Constraint (SL3) ensures that the sum of weights sums to one. (SL4) ensures that sold amount can correspond to only one segment. (SL5) ensures that the weights which are used to create a point can lie only between two breaking points which correspond to one segment

Revenue term in the objective function will look as follows:

$$\sum_{p \in P} \sum_{k \in K} revenue_{p,k} \qquad (E3.18)$$

Because revenue is a variable which is calculated in constraint (SL2) for each product at each market, in the objective function we sum over all products and all markets in order to calculate the total revenue.

Chapter 4– Production and Sales Models

In this chapter, we develop models to represent two divisional subproblems. Production model (PM) includes production at refineries, blending at hubs, and primary distribution. Sales models (SM) include sales planning and secondary distribution. Under such decoupled setting, divisional planning is performed separately. Because the sales department is assumed to be managed independent from the rest of the company, its objective is seeking for local optimality.

This way of problem formulation has a lot in common with decoupled models used in Guajardo et al. (2013a, 2013b). The main differences are in a way we formulate blending constraints, further we consider only one time period, and we include possibility of closing depots. We consider two possible ways the sales department's premium can be calculated: in the first alternative sales department takes fixed costs associated with depots operation into account, while in the second alternative premium doesn't depend on fixed costs and hence department doesn't include these costs in its objective function. Therefore we formulate two alternative sales models: SM1 and SM2.

We assume that there is infinite capacity for processing crude oils at refineries and producing products at hubs. Further we assume that feedstock and commodities that have not been used in processing or blending, and products that have not been sold, have zero value.

4.1 Production model (PM)

In the production model the main goal is to match estimated sales from SD while at the same time minimizing costs. The estimated sales are given by the parameter $\overline{q}^{S}_{p,d}$.

New parameters:

$\overline{q}^{S}_{p,d}$: *Amount of product p which must be available at depot d*

<u>Production model (PM)</u>

Objective function

$$\textit{Minimize cost}^{PM}: \quad \sum_{i\in I}\sum_{r\in R} C_{i,r}^{Buy}\,\mathbf{x}_{\mathbf{i,r}} + \sum_{i\in I}\sum_{r\in R} C_{i,r}^{PRO}\,\mathbf{x}_{\mathbf{i,r}} + \sum_{e\in E}\sum_{i\in I}\sum_{r\in R} \mathbf{C}_{\mathbf{e,r}}^{\mathbf{PRO2}}\,\mathbf{v}_{\mathbf{e,i,r}}$$

$$+ \sum_{p\in P}\sum_{d\in D}\sum_{h\in H} \mathbf{C}_{\mathbf{p,h}}^{\mathbf{BLEND}}\,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}}$$

$$+ \sum_{r\in R}\sum_{h\in H} \mathbf{C}_{\mathbf{r,h}}^{\mathbf{TRAN1}} \left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P} \overline{y}_{\mathbf{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B} \overline{v}_{\mathbf{p,c,b,i.r}} \right)$$

$$+ \sum_{h\in H}\sum_{d\in D}\sum_{p\in P} \mathbf{C}_{\mathbf{h,d}}^{\mathbf{TRAN2}}\,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}} \qquad (\boldsymbol{PM})$$

Subject to (P1) to (P11)

$$\overline{\mathbf{q}}^{S}{}_{\mathbf{p,d}} \leq q_{\mathbf{p,d}} \qquad \forall \mathbf{p} \in \mathbf{P}, \forall \mathbf{d} \in \mathbf{D} \qquad (\mathbf{P12})$$

$$\mathbf{x}_{\mathbf{i,r}}, \mathbf{y}_{\mathbf{e,i,r}}, \mathbf{v}_{\mathbf{b,i,r}}, \widetilde{y}_{\mathbf{e,i,r}}, \widetilde{v}_{\mathbf{c,b,i,r}}, \overline{y}_{\mathbf{p,e,i,r,h}}, \overline{v}_{\mathbf{p,c,b,i,r,h}}, \widetilde{\mathbf{q}}_{\mathbf{p,d,h}}, q_{\mathbf{p,d}} \geq \mathbf{0}$$

The first term of the objective function represents costs related to purchasing of crude oil. Second term represents costs of processing crude oils. The next term is processing cost for components. Fourth term represents blending costs. The last two terms represent transportation costs.

We have included costs of purchasing crude oils in the model, because this ensure a more efficient use of crude oils. If these costs have not been included, the crude oil of “good” quality would be “overused”, because it's cheaper to process and easier to satisfy quality requirements of final products with.

Constraints (P1) to (P11) are explained in details in chapter 3.2. Constraint (P12) ensures that producer generates at least what is ordered from SD.

PM is a linear problem. That makes it “easy” to solve and we can be sure that the solution of this model is optimal (but may not be unique).

4.2 Sales Models (SM)

The goal of SD is to maximize its premium, assuming that forecasts about demand will be realized. As we have argued in chapter 2, the premium proportion doesn't affect departmental decisions, and therefore is not included in the model. One of the decisions that sales department makes is type and amount of products, and which depots to order from. It is given by variable $q^{S}_{p,d}$.

<u>*New parameters:*</u>

$\pi_{p,d}$: *Internal price sales department has to pay for product p at depot d*

<u>*New variables:*</u>

$q^{S}_{p,d}$: *Amount of product p ordered by sales department at depot d*

<u>**Sales Model 1 (SM1)**</u>

Objective function

$$\boldsymbol{Maximize\ Profit^{SM1}}: \quad \sum_{p\in P}\sum_{k\in K}\left(A_p \sum_{d\in D} \mathbf{z}_{\mathbf{p,d,k}} - b_p \sum_{d\in D} \mathbf{z}_{\mathbf{p,d,k}}{}^{2}\right) - \sum_{p\in P}\sum_{d\in D} \pi_{p,d} \times \mathbf{q}^{S}{}_{\mathbf{p,d}}$$

$$-\sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathbf{C}^{\mathbf{TRAN3}}_{\mathbf{d,k}} \times \mathbf{z}_{\mathbf{p,d,k}} - \sum_{d\in D} C^{FIX}_{d} h_d \qquad (SM1)$$

Subject to (S1) to (S2) and (G1)

$$\sum_{k\in K} \mathbf{z}_{\mathbf{p,d,k}} \le \mathbf{q}^{S}{}_{\mathbf{p,d}} \qquad \forall \mathbf{p} \in \mathbf{P},\ \ \forall \mathbf{d} \in \mathbf{D} \qquad (\mathbf{S3})$$

$$\mathbf{z}_{\mathbf{p,d,k}} \ge \mathbf{0},\ \mathbf{q}^{S}{}_{\mathbf{p,d}} \ge \mathbf{0},\ \theta_{p,k} \ge \mathbf{0} \qquad h_d \in \{\mathbf{0},\mathbf{1}\}$$

Objective function (SM1) expresses SDs maximization function. The first term includes revenue from sales which is explained in more details in chapter 3.3. The second term represents products purchase costs. The third term represents fixed costs from depots, and the last term represents transportation costs from depots to markets. Constraint (G1) is explained in chapter 3.2, while (S1) and (S2) in chapter 3.3. Constraint (S3) ensures that the amount of ordered products is bigger or equal than the amount of sold products.

__Sales Model 2 (SM2)__

Objective function

$$\boldsymbol{Maximize\ Profit^{SM2}}: \quad \sum_{p\in P}\sum_{k\in K}\left(A_p \sum_{d\in D} \mathbf{z_{p,d,k}} - b_p \sum_{d\in D} \mathbf{z_{p,d,k}}^2\right) - \sum_{p\in P}\sum_{d\in D} \boldsymbol{\pi_{p,d}} \times \mathbf{q^{S}}_{\mathbf{p,d}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathbf{C^{TRAN3}_{d,k}} \times \mathbf{z_{p,d,k}} \qquad (\boldsymbol{SM2})$$

Subject to (S1) to (S3) and (G2).

$$\mathbf{z_{p,d,k}} \geq \mathbf{0},\ \mathbf{q^{S}}_{\mathbf{p,d}} \geq \mathbf{0} \quad \boldsymbol{\theta_{p,k}} \geq \mathbf{0} \qquad \textbf{(S5)}$$

SM2 differs from SM1 in the objective function: the part that is associated with fixed costs is removed in SM2. Also constraint (G2) is used instead of (G1).

Discussion about the sales subproblem

SM1 is used when SD chooses depots for operation and at the same its premium depends upon fixed costs. SM2 is used when premium to SD doesn't depend on fixed costs. In this case we consider two possibilities. The first one is that production department decides in advance which depots will be in operation. In this case, the available depots in SM2 are determined by PD. The second possibility is that sales department makes decision about the operation of depots. In this case there are no binary variables because fixed costs are not taken into account in SDs objective function. Depots that sales department doesn't choose for operation will be indicated by zero inventory.

If we consider SM2, where fixed costs from depots are ignored, and assume that $Profit^{SM2}(\mathrm{z_{p,d,k}})$ is concave for $\mathrm{z_{p,d,k}} \geq 0$, we know that function has its maximum when $\frac{\partial Profit^{SM2}}{\partial \mathrm{z_{p,d,k}}} = 0$. If the feasible region to this maximization problem defined by the constraints is a convex set then we have a convex problem, and a local maximum is also a global maximum (Lundgren et al. 2010). Because constraints (S1) and (S3) are linear we know that we have a convex set. Hence when we are ignoring fixed costs from depots, we can easily find the global optimal solution to this problem. When we include binary variables to our problem, in SM1, we make the problem mixed integer nonlinear. However under the assumptions that we have just made the problem SM1 is convex MINL. For problems of this type there exist some solution methods which guarantee to find an optimal solution. One such method is branch and bound for nonlinear problems (Gupta and Ravindran, 1985). By relaxing the integer constraints, subproblems which are solved are convex-problems.

Linear Sales Problem

Below we formulate sales model based on piecewise linear revenue forecasts.

<u>Linear Sales Model 1 (SM1-Linear)</u>

Objective function

$$\boldsymbol{Maximize\ Sale\ Profit^{SM1-L}}: \sum_{p\in P}\sum_{k\in K} \boldsymbol{revenue_{p,k}} - \sum_{d} \boldsymbol{C_d^{FIX} h_d}$$

$$- \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathbf{C_{d,k}^{TRAN3}} \times \mathbf{z_{p,d,k}}$$

$$- \sum_{p\in P}\sum_{d\in D} \boldsymbol{\pi_{p,d}} \times \mathbf{q^{S}}_{\mathbf{p,d}} \qquad (\boldsymbol{SM1-L})$$

Subject to (SML1) to (SML5), (S3) and (G1)

$$\mathbf{z_{p,d,k}}, \mathbf{q^{S}}_{\mathbf{p,d}}, \boldsymbol{w_{m,p,k}},\ \boldsymbol{revenue_{p,k}} \geq \mathbf{0} \qquad \boldsymbol{h_d}, \boldsymbol{l_{m,p,k}} \in \{\mathbf{0},\mathbf{1}\}$$

The objective function to SM1-L differs from SM1 only in the first term, because revenue is defined as the variable. Constraints (SML1) to (SML5) are explained in chapter 3.4. (S3) and (G1) are also used in SM1. The linear sales model SM2-L, which is the alternative to SM2, is formulated in Appendix A.

4.3 Coordination between PM and SM/SM-L

In SM/SML internal prices for products are given as a parameters $\pi_{p,d}$. How these prices are calculated, is discussed in part 2, which concerns the planning level. The models that we have defined in this chapter, are solved at the decision level. At this level internal prices are already decided.

Solution of SM/SML gives quantities $q^{S}{}_{p,d}$ ordered by the sales department. These are used as demand in PM. In other words $q^{S}{}_{p,d} = \bar{q}^{S}{}_{p,d}$ and constraint (P12) ensures that produced amount satisfies this demand.

Because at the decision level, the production department has to satisfy a concrete demand from the sales department, PM doesn't include the depot operation constraint.

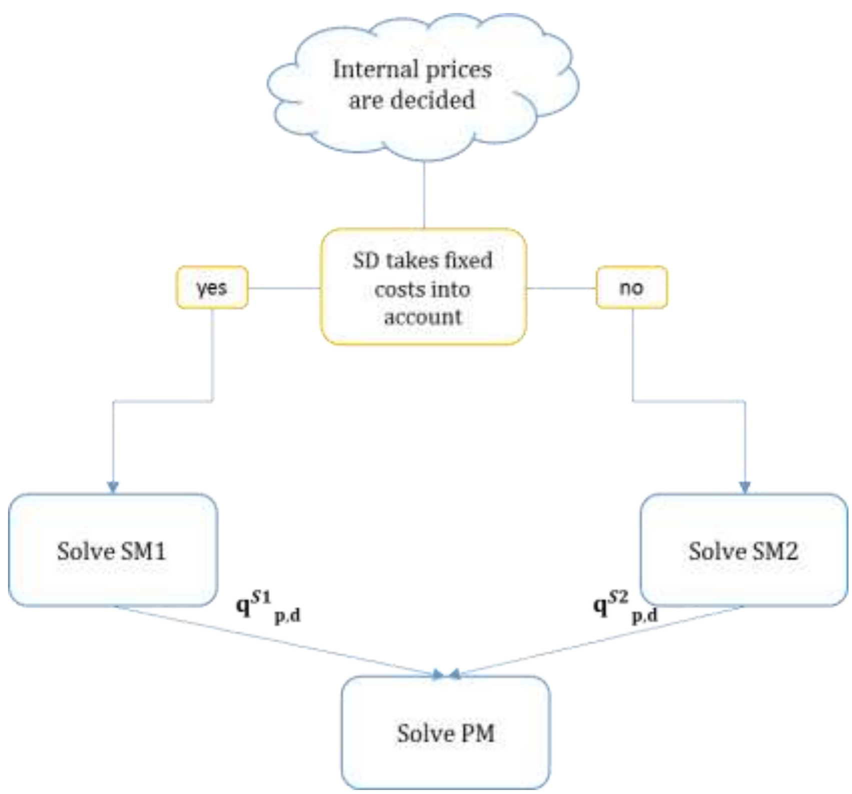


Figure 4.1 – Decision level

Chapter 5 – Integrated Models (IM)

In this chapter, we propose an Integrated model (IM1). The optimal solution to IM1 is the best achievable result for the whole company. When sales and production planning are integrated into one model, the company as whole decides on production, distribution and sales, and maximizes the total profit of the organization. The advantage with integrated planning is that decisions about distribution to markets are made together with production decisions, and hence provide a better match between sale and production. However, as we have pointed out earlier, because such planning is not possible in practice we can only use solution from this model as theoretic benchmark for performance of other approaches.

We maintain the notation and definition from previous chapters for all parameters, sets and variables.

Integrated Model 1 (IM1):

Objective function

$$
\begin{aligned}
Maximize\ Contribution^{IM1}: & \sum_{p\in P}\sum_{k\in K}\left(\boldsymbol{A_p}\sum_{d\in D}\mathbf{z_{p,d,k}} - \boldsymbol{b_p}\sum_{d\in D}\mathbf{z_{p,d,k}}^{2}\right) - \sum_{d\in D}\boldsymbol{C_d^{FIX}}\boldsymbol{h_d} \\
& - \sum_{i\in I}\sum_{r\in R}\boldsymbol{C_{i,r}^{Buy}}\,\mathbf{x_{i,r}} - \sum_{i\in I}\sum_{r\in R}\boldsymbol{C_{i,r}^{PRO}}\,\mathbf{x_{i,r}} \\
& - \sum_{e\in E}\sum_{i\in I}\sum_{r\in R}\mathbf{C_{e,r}^{PRO2}}\,\mathbf{v_{e,i,r}} - \sum_{p\in P}\sum_{d\in D}\sum_{h\in H}\mathbf{C_{p,h}^{BLEND}}\,\mathbf{\tilde{q}_{p,d,h}} \\
& - \sum_{r\in R}\sum_{h\in H}\mathbf{C_{r,h}^{TRAN1}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P}\boldsymbol{\bar{y}}_{\mathbf{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B}\boldsymbol{\bar{v}}_{\mathbf{p,c,b,i.r}}\right) \\
& - \sum_{h\in H}\sum_{d\in D}\sum_{p\in P}\mathbf{C_{h,d}^{TRAN2}}\,\mathbf{\tilde{q}_{p,d,h}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\mathbf{C_{d,k}^{TRAN3}}\times\mathbf{z_{p,d,k}} \qquad (\boldsymbol{IM1})
\end{aligned}
$$

Subject to (P1) to (P11), (S1) to (S2), (G1)

$$\boldsymbol{q_{p,d}} = \sum_{k \in K} \mathbf{z_{p,d,k}} \qquad \forall \mathbf{p} \in \mathbf{P} \quad \forall \mathbf{d} \in \mathbf{D} \qquad (\mathbf{I1})$$

$$\mathbf{x_{i,r}}, \mathbf{y_{e,i,r}}, \mathbf{v_{b,i,r}}, \tilde{\boldsymbol{y}}_{\mathbf{e,i,r}}, \tilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}}, \bar{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}}, \bar{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}}, \mathbf{z_{p,d,k}}, \tilde{\mathbf{q}}_{\mathbf{p,d,h}}, \boldsymbol{q_{p,d}}, \boldsymbol{\theta_{p,k}} \geq \mathbf{0} \quad \textbf{and } \boldsymbol{h_d} \in \{\mathbf{0}, \mathbf{1}\}$$

In IM1 the objective function represents the goal of the company: maximize contribution, which is the revenue from sales minus the costs. No internal price for products are present, as there are no need for them in integrated planning.

The IM1 has almost the same constraints as SM1 and PM. The only difference is that constraints (S3) and (P12) are replaced with the new constraint (I1).

Similar as SM1, IM1 is a quadratic programming (QP) problem with linear constraints.

In the same way as we did in SM1-L, we can formulate Linear Integrated Model (IM1-L), assuming piecewise revenue function. This model is formulated in Appendix A.

Also we analyze a case in which decision about depots operation is taken by PD in advance. The best achievable solution, in this case, may be different from the solution of IM1. Therefor in order to know what the optimal result is in this case we need to formulate another model, which we call IM2. The objective function of IM2 is the same as in IM1 except the fixed cost term which is removed. Also the constraints are almost the same, the only difference is that instead of (G1), (G2) is used. We note, that because which depots are used in operation is decided in advance, the set of available depots *D*, may be different from the one which is used in IM1. IM2 together with L-IM2 are formulated in Appendix A.

PART 2: INTERNAL PRICES MECHANISMS

In chapter 4 we have formulated two different sales models: SM1 and SM2, each corresponding to different ways how premium received by the sales department is calculated. Depending on which of the models is used and how decisions are allocated, internal prices can be calculated in different ways at the planning level. When we formulate methods for calculation of internal prices, we consider four different scenarios for how information is taken into account by the departments.

Scenario 1 – Both production and sales departments take into account operating costs of depots

Scenario 2 – Only production departments takes these costs into account.

Scenario 3 - Only sales department takes these costs into account.

Scenario 4 – The decision about which depot will operate, is taken by production department in advance and none of the departments take these costs into account when internal prices are decided.

If SM1, where sales department takes fixed costs into account, is used then the scenarios 1 or 3 can be used in the planning process. If instead of SM1, SM2 is used and PD decides which depots should operate, then scenario 4 is possible to use in the planning process. If SM2 is used, but all depots are available for use, then scenario 2 should be used to decide internal prices. The intuition behind scenario 2 is as follows: the production department should set internal prices in such a way that depots operating costs should be reflected in these prices. Internal prices should stimulate the sales department to choose "right" depots, hence it should not be necessary for SD to take depot costs into account directly. Figure 6.1 summarizes the choice of scenarios.

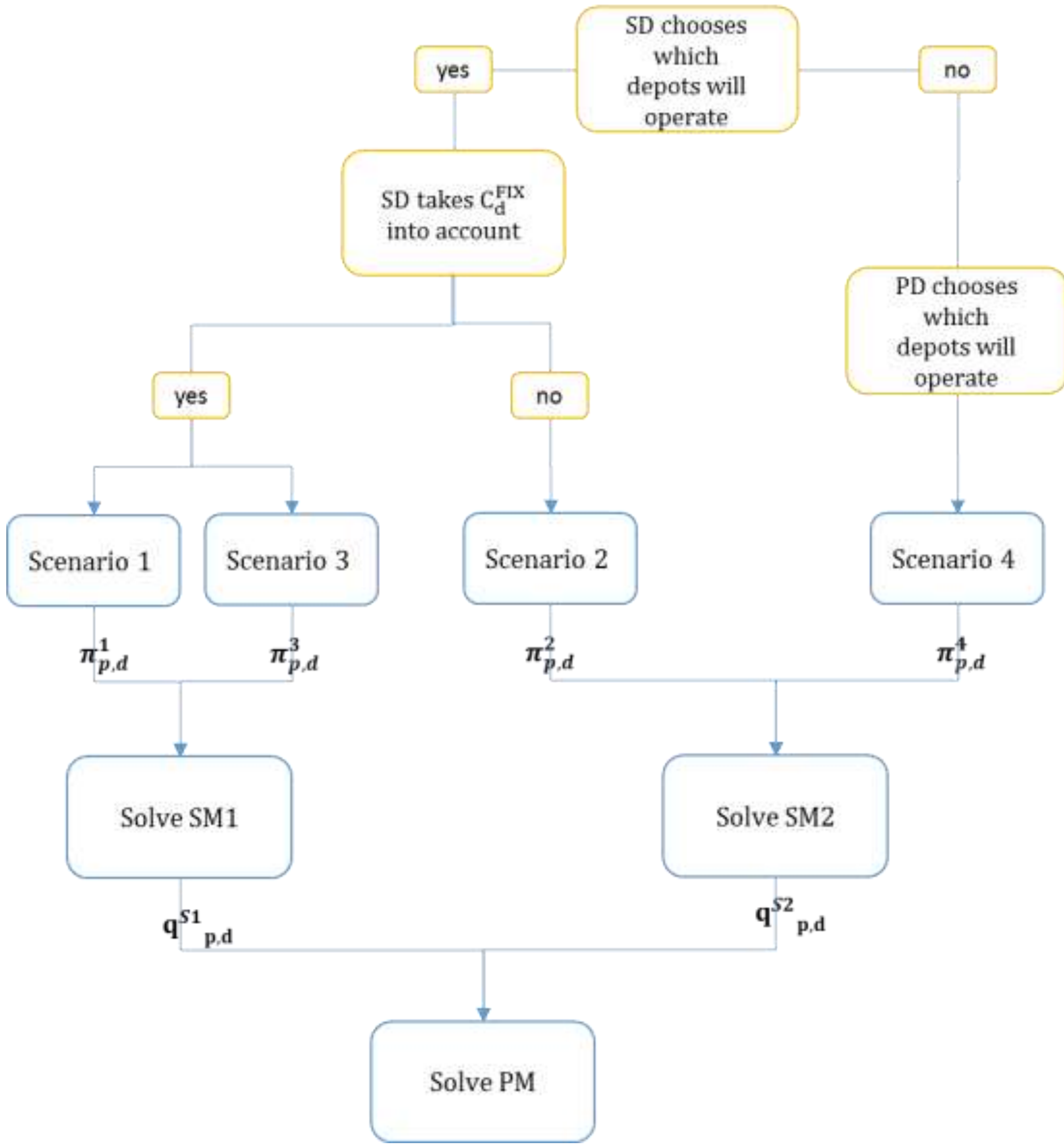


Figure 6.1- Choice Scenarios

Chapter 6 – The Cost-Based Mechanism

We starting with proposition of cost-based pricing mechanism, which assigns internal price for each product *p* at each depot *d*, based on the cost that the product has incurred in the supply chain until the depot stage.

Bitran and Caldentey (2003) have pointed out that there is a tendency in practice to set prices based on costs. The authors have asserted that the reason for that is probably "a mixture of managers' incentives based on margins and the classical advice from economic theory where marginal cost plays a central role in pricing decisions" (Bitran and Caldentey, 2003; p.211). Bredström and Rönnqvist (2008) have claimed that a common approach to find internal prices is to use shadow prices or marginal values from models used in production planning. However, as Guajardo et al. (2013b) have pointed out, calculating a true marginal cost becomes a difficult task in a situation with multiple products, echelons and locations.

Guajardo et al. (2013b) have presented three methods of setting the internal prices based on cost calculations. In these methods the authors took into account dependencies between products due to one raw material yielding several products. The dependencies between products are also present in our problem. But since in those three methods the final products were blended according to well specified recipes, we cannot use them directly, because our models don't include such recipes. Therefore we will introduce two methods that are more useful in our case.

These relative simple methods will be used as a comparison to Lagrangean decomposition method which is described in the next chapter. Cost based methods do not require any exchange of information between departments at the planning level. And PD doesn't take into account possible orders from SD when it decides internal prices.

6.1 Method 1

In our first cost based method, we start by calculating how much it costs to produce one unit of product *p*, when company produces *only* this type of product. In other words, we assume that the company produces only one type of product at a time. This calculation is relative easy, because when company produces just one product there are no dependencies between products. Therefore we can easily calculate variable costs related to the product. In our problem, the production model (PM) is linear. It implies that producing 10 units of one product, costs 10 times more than producing one unit of the same product, under the assumption that company produces only one type of product. The linear model also guarantees that the company is able to find the lowest possible production cost.

Because of many variables and constraints, which are involved in the production process, to ensure that the lowest cost is achieved we use PM to calculate product costs. The PM requires an input matrix $\bar{q}^{S}{}_{p,d}$, which specifies how many units of product *p* should be delivered to depot *d*. As long as we are interesting in cost of one product at a time, the input is one unit of product *p* to depot *d* and zero to other depots and other products.

The objective function value of PM indicates how much it costs to produce and deliver ordered products to the depots. When only one product is ordered at one depot, the objective of PM will show exactly what we are looking for. Because we are interesting to price product at all depots, we solve PM *n* times, where *n* indicates how many depots company has.

Then we repeat the same procedure for other products, and at the end we have a matrix with internal prices for each product at each depot.

6.2 Method 2

Method 2 is a modification of method 1. It considers possibility that several products can be ordered simultaneously.

The idea behind this modification comes from the cost allocation methods. Due to dependencies between products it can be cheaper to produce two type of products at the same time than each product separately. This is the case because, when one type of product is produced there can be some components that are left. These components can be used in production of other type of products.

The cost allocation problems arise when individuals, all with their own propose, decide to work together. One problem that arise in this situation, is how to divide the joint costs and the costs savings which results from the cooperation, among the participants (Tijs and Driessen, 1986). We have a similar problem: some costs are related to several products and the question is how to allocate these costs between these products.

Above we have described how to calculate costs of producing and transport product *p* to depot *d*, denoted later as $C_{p,d}$. Supposing that the company produces four types of products we can find $C_{1,d}$, $C_{2,d}$, $C_{3,d}$ and $C_{4,d}$ for all depots *d*. In what follows, we describe how we can calculate how much it costs to produce two different products at the same time.

In the same way as we have done in method 1, we use PM. But now we require that two products must be available at one depot *d*. The cost of producing and transport product 1 and product 2 to depot *d*, is denoted as $C_{1,2,d}$. As we have mentioned we expect that $C_{1,2,d} \leq C_{1,d} + C_{2,d}$.

When we have calculated $C_{1,2,d}$, we calculate a new term: $D_{1,2,d} = (C_{1,d} + C_{2,d}) - C_{1,2,d}$.

This term can be interpreted as a *discount* which company gets when product 1 and product 2 are produced at the same time. The main idea behind the modification is to find discounts for each product type when we consider production of two products at the same time, and hence more correct cost than we have found in method 1.

In order to calculate internal prices with this method, we need to find discount factors for all possible combination of two products at each depot. Therefore we repeat the same procedure for $C_{1,3,d}$ and $C_{1,4,d}$. When we have calculated $D_{1,2,d}$, $D_{1,3,d}$ and $D_{1,4,d}$ we can find "the final discount" for product 1 at depot *d*. Because $D_{1,2,d}$ includes discount for both product 1 and product 2, we divide it by two to find discount for product 1 and 2 separately.

Then the total discount for product 1 at depot *d* will equal to the average of $\frac{D_{1,2,d}}{2}$, $\frac{D_{1,3,d}}{2}$, $\frac{D_{1,4,d}}{2}$.

$$D_{1,d} = \frac{\sum_{j=2}^{4} \frac{D_{1,j,d}}{2}}{3}$$

Now we are ready to calculate internal price of product 1 at depot *d*.

$$\pi_{1,d}{}^{*} = C_{1,d} - D_{1,d}$$

We use this procedure to find internal prices for other products at each depot. For each next product we have to do less calculations, because the production of product 1 + product 2 gives exactly the same costs as the production of product 2 + product 1. Therefore, we don’t need to calculate discount $D_{2,1,d}$, because it's exactly the same as $D_{1,2,d}$.

6.3 Comments on the cost-based methods

The two methods that we have described, don’t fully represent costs related to production of products, and therefore give just some indication of costs created by each product. Methods don’t take into account restriction on purchase of crude oils at refineries. As long as the methods calculate how much it will cost to produce only one unit of each product, the solution will always be based on the most profitable crude oil type and the "cheapest" refinery for this type of product. In reality, amount of each product ordered by SD is big, and it cannot be possible to produce all products in the cheapest way. Also, the second method (and the first method) doesn’t take into account that when more than two types of products are ordered at the same time the costs may decrease even more. We don’t expect that our methods of internal price calculation will give an optimal solution to the overall company's problem. However, these methods are relative simple and don't require information exchange between SD and PD at the planning level. These methods can also serve as a starting point for more advanced methods. For example, internal prices calculated in these methods can be used as initial values of Lagrangean multipliers in the Lagrangean decomposition method which we describe in the next chapter.

Chapter 7 - Lagrangean Decomposition

7.1 – The Theory behind LD

Lagrangean relaxation is a solution strategy used for solving large structured problems. The idea behind this method is to relax some constraints in the original problem formulation and consider them implicit through the objective in the *Lagrangean function.* The use of *Lagrangean multipliers* lead to penalty of the objective function if the relaxed constrains are violated (Lundgren et al. 2009).

Guignard and Kim (1987b) have proposed Lagrangean decomposition as a generalization of conventional Lagrangean relaxation. The authors have introduced copies of the original variables for a subset of constraints and dualized the equivalence conditions between the original variables and the copies.

In the Lagrangean decomposition method, Lagrangean subproblems keep all the original constraints. The method is applicable when the original model consists of two (or more) subproblems with common variables. The method involves reformulation of the original problem using variable splitting. The original variables that occur in both subproblems are replaced with copies, and at the same time a coupling constraints are added. These new constraints then are relaxed and the original model is decomposed into separate subproblems. This implies that we get subproblems that have identical constraints with the original model, but can be solved individually now. When variables are duplicated the problem becomes larger, but on the other hand duplication enables decomposition of the model into parts which will be easier to solve.

Guignard and Kim (1987a) have showed that applying the Lagrangean decomposition method to integer programming problems may yield a stronger bounds than the conventional Lagrangean relaxation method.

The Lagrangean decomposition technique has been applied in many different applications. For instance, Lidestam and Rönnqvist (2011) have applied Lagrangean heuristic method based on Lagrangean decomposition to an integrated planning problem in a supply chain for a large pulp company. As a result of applying the proposed approach, feasible solution of high quality was generated in a short time. As we have mentioned in the introduction, Mouret et al. (2011), Neiro and Pinto (2006), and Oliveira et al. (2012), have used different versions of LD method to solve various problems that arise in oil companies. Also, as it has been pointed out in chapter 2, the method has been used as price-directive approach for coordination between departments. LD has been applied as a coordination approach between refinery production planning and sales planning in a SNF report written by Bredström and Rönnqvist, (2008), and in a working paper written by Kong and Rönnqvist (2012).

We use an example to illustrate Lagrangean decomposition method. Consider the following optimization problem [P]

$$\max_{x}: \; z = c^T x \qquad [P]$$

$$s.t: \quad Ax = a, \, Bx = b, \qquad x \in X$$

Where *c*, *a*, *b*, *A* and *B* are vectors and matrices, and *X* is integer requirements on variables.

We start with coping of variables $x \rightarrow \; x = y$

And express the objective function coefficients in the following way: $c = c_1 + c_2$.

The reformulated problem [P] looks now as following:

$$\max_{x}: z = {c_1}^T x + {c_2}^T y \qquad [PR]$$

$$s.t: \quad Ax = a, \, By = b, \qquad x = y, \, x \in X, y \in Y$$

The problem [PR] is equivalent to [P] for any set Y containing X (Guignard and Kim, 1987a).

The next step is to relax the new constraint with Lagrangean multiplier λ:

$$\max_{x,y}: L(x, y, \lambda) = {c_1}^T x + {c_2}^T y \; + \; \lambda(x - y) \qquad [PL]$$

$$s.t: \quad Ax = a, \, By = b, \qquad x \in X, y \in Y$$

We can now easily split the problem into two subproblems

$$\max_{x}: L_1 = \; (c_1 + \lambda)^T x \qquad s.t: \quad Ax = a, \qquad x \in X$$

$$\max_{y}: L_2 = \; (c_2 + \lambda)^T y \qquad s.t: \quad By = b, \qquad y \in Y$$

From the optimization theory we know that in order to find an optimal solution to problem [P], which we call for the primal problem, we need to maximize the Lagrangean function, $L(x, y, \lambda)$ with respect to x, y and minimize it with respect to λ (Lundgren et al. 2009). Hence we are interesting to find solution to the following problem:

$$\min_{\lambda} \quad \max_{x,y} L(x, y, \lambda)$$

The Lagrangean decomposition dual function $[D]$ to the problem $[P]$, which is the problem of finding the tightest upper bound on problem $[P]$ is defined as:

$$\min_{\lambda} h(\lambda) \qquad [D]$$

Where $h(\lambda) = \max_{x,y} L(x, y, \lambda)$.

For each dual solution λ, the Lagrangean dual problem provides an optimistic bound on the optimal objective function value $z(x^*, y^*)$ (Lundgren et al. 2009).

If the original problem $[P]$ is convex, the optimal objective function value of dual and primal problems are equal: $h(\lambda^*) = z(x^*, y^*)$. However, if the problem is non-convex, then we may get a duality gap:

$$h(\lambda^*) > z(x^*, y^*)$$

Suppose, we have found optimal solution of $[PL]$: $(x(\hat{\lambda}), y(\hat{\lambda}))$. If $x(\hat{\lambda})$ and $y(\hat{\lambda})$ are identical, then $x(\hat{\lambda})$ is an optimal solution of $[P]$, $\hat{\lambda}$ is an optimal solution of $[D]$ and there is no duality gap (Guignard and Kim, 1987a; 1987b).

When we use Lagrangean decomposition, one important question that we need to answer is which variables and constraints should be duplicated. One criteria is to achieve subproblems which are relative easy to solve. If we relax too many constraints, problems will be easy to solve, however the quality of optimistic bound will be worse.

When we have decided which variables and constraints should be duplicated and formulated the dual function, we need to choose some initial Lagrangean multipliers. Often these are set to zero as a starting point, meaning that relaxed constraints are ignored (Lundgren et al. 2009). Once we solve the dual problem with some chosen multipliers we obtain an optimistic bound, which is given by the objective function value of the Lagrangean problem. If the solution is feasible we can also calculate a pessimistic bound. However, if the solution is infeasible another question arise: How to deduce a good feasible solution to the original problem given the solution to the relaxed problem? Often, some heuristic methods are used to make the solution feasible. As Guignard (2003) has pointed out, Lagrangean heuristics are essentially problem dependent.

A standard convergence criterion is to stop when difference between lower bound (LBD) and upper bound (UBD) is small enough. A difficulty in the solution process is how to update and compute good Lagrangean multipliers. Because we are interesting in that the dual objective function moves in the direction of the optimal objective function of the primal problem, we must find a way in which we choose the right search direction and step length for the Lagrangean multipliers. In other words we are interesting to obtain the tightest bound by adjusting λ such that:

$$h(\lambda) = \min_{\lambda} L(\lambda, x, y)$$

Some of Lagrangean dual problems may be non-differentiable. This can be the case if we solve an integer programming problem, where dual problem may give multiple solutions for some multipliers and hence not be differentiable at these points (Lundgren et al. 2009). This implies that gradient method cannot be used to find a search direction. A typically iterative technique which is employed in such cases is a subgradient method, which can be thought as a gradient method with some adaptation at the points where Lagrangean function is non-differentiable (Fisher, 1985; Guignard 2003; Lundgren et al., 2009). Subgradient optimization has a wide acceptability among researches and is one of the most elective and useful technique for large problems with complex structure (Fumero, 2001).
The subgradient is calculated in the same way as the gradient:

$$\gamma = \frac{\partial L(x, y, \lambda)}{\partial \lambda}$$

However at points where $L(x, y, \lambda)$ is non-differentiable, the subgradient method chooses arbitrarily from the set of alternative optimal solutions. In our example $\gamma^{(k)} = x^{(k)} - y^{(k)}$.

In the subgradient method the sequence of multipliers is generated as follows:

$$\lambda^{(k+1)} = \lambda^{(k)} + t^{(k)}\gamma^{(k)}$$

Where $t^{(k)}$ is the step length at iteration *k*.

One of the problems with subgradients is that they cannot guarantee that found search direction is ascent and hence that the objective function will be improved in each iteration (Lundgren et al. 2009). It is therefore important to choose step length carefully. In order to guarantee convergence, theoretical requirement is that the step length $t^{(k)}$should be selected so that it converge to 0, but not too quickly. The results has shown that if $t^{(k)} \to 0$ and $\sum_{i=1}^{k} t^{(k)} \to \infty$ when $k \to \infty$ then $L(x, y, \lambda^{(k)})$ will converge to its optimal value (Fisher, 1985). The suitable step size may be determined by the following formula (Fisher, 1985):

$$t^{(k)} = \frac{\sigma^{(k)}\left(h\left(\lambda^{(k)}\right) - z^{*}\right)}{\parallel \gamma^{(k)} \parallel^{2}}$$

Where $\varepsilon_1 \leq \sigma^{(k)} \leq 2 - \varepsilon_2$ $(\varepsilon_1, \varepsilon_2 > 0)$ is a scalar and z^{*} is the optimal solution of (P). If we know the optimal solution, it will be possible to assure geometric convergence to the optimal point (Fumero, 2001).

Obviously, the optimal solution in most cases is unknown. In these cases, the formula for step length that works well in practice (Fisher, 1985) is:

$$t^{(k)} = \frac{\sigma^{(k)}\left(h\left(\lambda^{(k)}\right) - z^{LBD}\right)}{\parallel \gamma^{(k)} \parallel^{2}}$$

Where z^{LBD} is a best known feasible solution. Frequently, one start with $\sigma^{(k)} = 2$, and then reduce $\sigma^{(k)}$ by some factor whenever $L(x, y, \lambda^{(k)})$ has failed to decrease in a specified number of iterations (Fisher, 1985).

Guignard (2003) has mentioned that many authors have studied subgradient method and improved its algorithmic behavior. Fumero (2001) has suggested a modified subgradient algorithm, in which the author presented more accurate step length calculation and search direction. Also other methods has been proposed to solve Lagrangean duals. Some of these, combined subgradient method with other methods. Oliveira et al. (2012) suggested a novel hybrid algorithmic framework for updating the Lagrangean multiplier based on the combination of cutting-plane, subgradient and trust-region strategies. The authors have showed in a numerical example that this framework may lead to significant savings in computational times compared with the traditional subgradient algorithm. Mouret et al. (2011) have introduced a new hybrid dual problem to update the Lagrangean multipliers, in which the authors used the classical concepts of cutting planes, subgradient, and boxstep. The results obtained in a case study showed that the new Lagrangean decomposition algorithm was more robust than the other approaches and produces better solutions in reasonable times.

In our work we consider only traditional subgradient method.

7.2 - Implementation of Lagrangean Decomposition (LD)

The main idea of using LD in our problem, is to find values for internal prices of products, which will be used as input to optimization model used by sales department. These prices should stimulate SD to make optimal orders.

Because the solution of IM1/IM2 is the best achievable solution, we will use these models as the base for the decomposition. The integrated models consist of two interesting subproblems: sales and production subproblems. As we have pointed out in the previous section, one should carefully consider which variable and constraints should be duplicated and which subproblems will appear as the result duplication. When we apply LD method to planning problems, it can be a good idea to create subproblems that will correspond to the local problems of the departments, because it will give a realistic interpretation of these subproblems. It's desirable that constraints associated to each of the departments remain in the departmental subproblems. We are therefore interesting to decouple IM1/IM2 in such a way that obtained subproblems will be as close as possible to each of the departmental local problems, while at the same time we are still interesting to obtain a solution which will be optimal to IM1/IM2.

7.2.1 Decomposition of IM1/IM2

As we have discussed in Chapter 4, there are several possible ways to allocate some of the decisions between departments. Therefore, we will consider four alternative scenarios (described in at the beginning of part 2) of decompose IM1/IM2.

From IM1/IM2 we can observe that activities in PD and SD are linked together through variable $q_{p,d}$, which shows how many units of product *p* are delivered to depot *d*. We choose to split this common variable and add a coupling constraint $q^{(P)}{}_{p,d} = q^{(S)}{}_{p,d}$. Then, this copy constraint is dualized (relaxed) with Lagrangean multiplier $\lambda_{p,d}$. We find it reasonable to do in all four scenarios. The base for scenarios 1, 2 and 3 is IM1, while IM2 is the base for scenario 4.

In what follows, we start by formulating subproblems for each of the four scenarios before we describe the solution algorithm. The linear sales models, which are decomposed from IM1-L and IM2-L, are formulated in Appendix A (the production models are the same in both nonlinear and linear cases).

Scenario 1

If both departments take into account fixed costs associated with operation of depots, in addition to $q_{p,d}$ variable, we duplicate the decision variable h_d which shows whether depot *d* is operating or not. We define copies of variable and add a coupling constraint: $h_d{}^{(P)} = h_d{}^{(S)}$, then we relax this new constraint with Lagrangean multiplier μ_d . Because we include these variables in both production and sales subproblems, we replicate the associated objective function coefficient, C_d^{FIX}. As it was pointed out by Guignard and Kim (1987a), when some of the constraints are kept in both subproblems a stronger bound is usually obtained at the expense of having possibly more difficult problems to solve.

New parameters

$\underline{prod}_p, \overline{prod}_p$; *Minimum and maximum production of product p*

Lagrangean 1 IM

Objective function

$$Maximize\ Contribution^{L1IM}: \sum_{p\in P}\sum_{k\in K}\left(A_p \sum_{d\in D} \mathbf{z}_{\mathbf{p,d,k}} - b_p \sum_{d\in D} \mathbf{z}_{\mathbf{p,d,k}}{}^2\right) - \sum_{i\in I}\sum_{r\in R} C_{i,r}^{Buy}\, \mathbf{x}_{\mathbf{i,r}}$$

$$- \sum_{i\in I}\sum_{r\in R} C_{i,r}^{PRO}\ \mathbf{x}_{\mathbf{i,r}} - \sum_{e\in E}\sum_{i\in I}\sum_{r\in R} \mathbf{C}_{\mathbf{e,r}}^{\mathbf{PRO2}}\, \mathbf{v}_{\mathbf{e,i,r}} - \sum_{p\in P}\sum_{d\in D}\sum_{h\in H} \mathbf{C}_{\mathbf{p,h}}^{\mathbf{BLEND}}\, \widetilde{\mathbf{q}}_{\mathbf{p,d,h}}$$

$$- \sum_{r\in R}\sum_{h\in H} \mathbf{C}_{\mathbf{r,h}}^{\mathbf{TRAN1}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P} \overline{y}_{\mathbf{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B} \overline{v}_{\mathbf{p,c,b,i.r}}\right)$$

$$- \sum_{h\in H}\sum_{d\in D}\sum_{p\in P} \mathbf{C}_{\mathbf{h,d}}^{\mathbf{TRAN2}}\, \widetilde{\mathbf{q}}_{\mathbf{p,d,h}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathbf{C}_{\mathbf{d,k}}^{\mathbf{TRAN3}} \times \mathbf{z}_{\mathbf{p,d,k}} - \sum_{d} \frac{C_d^{FIX}}{2} h^{(P)}{}_d$$

$$- \sum_{d} \frac{C_d^{FIX}}{2} h^{(S)}{}_d - \sum_{p\in P}\sum_{d\in D} \lambda_{p,d}\left(\mathbf{q}^{(S)}{}_{\mathbf{p,d}} - \mathbf{q}^{(P)}{}_{\mathbf{p,d}}\right) - \sum_{d\in D} \mu_d (h_d{}^{(S)} - h_d{}^{(P)})$$

Subject to:

$$\mathbf{x}_{\mathbf{i,r}} \le sup_{i,r} \qquad \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{L1})$$

$$\boldsymbol{\rho}_{\mathbf{i,e}} \times \mathbf{x}_{\mathbf{i,r}} = \mathbf{y}_{\mathbf{e,i,r}} \qquad \forall \mathbf{e} \in \mathbf{E}, \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{L2})$$

$$\widetilde{y}_{\mathbf{a,i,r}} \le \mathbf{y}_{\mathbf{a,i,r}} \qquad \forall \mathbf{a} \in \mathbf{A} \subseteq \mathbf{E},\ \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{L3})$$

$$\widetilde{y}_{\mathbf{b,i,r}} + \mathbf{v}_{\mathbf{b,i,r}} \le \mathbf{y}_{\mathbf{b,i,r}} \qquad \forall \mathbf{b} \in \mathbf{B} \subseteq \mathbf{E},\ \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{L4})$$

$$\boldsymbol{\rho}^{\mathbf{2}}{}_{\mathbf{c,b}} \times \mathbf{v}_{\mathbf{b,i,r}} = \widetilde{v}_{\mathbf{c,b,i,r}} \qquad \forall \mathbf{b} \in \mathbf{B} \subseteq \mathbf{E},\ \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{c} \in \mathbf{C}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{L5})$$

$$\sum_{p\in P}\sum_{h\in H} \overline{y}_{\mathbf{p,e,i,r,h}} \le \widetilde{y}_{\mathbf{e,i,r}} \qquad \forall \mathbf{e} \in \mathbf{E}, \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{L6})$$

$$\sum_{p\in P}\sum_{h\in H} \overline{v}_{\mathbf{p,c,b,i.r}} \le \widetilde{v}_{\mathbf{c,b,i,r}} \qquad \forall \mathbf{b} \in \mathbf{B} \subseteq \mathbf{E}, \forall \mathbf{i} \in \mathbf{I}, \forall \mathbf{c} \in \mathbf{C}, \forall \mathbf{r} \in \mathbf{R} \qquad (\mathbf{L7})$$

$$\sum_{\mathbf{d\in D}} \widetilde{q}_{p,d,h}\, spm_{p,qmin} \le \sum_{\mathbf{e\in E}}\sum_{\mathbf{i\in I}}\sum_{r\in R} \overline{y}_{\mathbf{p,e,i,r,h}}\ sbm_{e,qmin} + \sum_{i\in I}\sum_{b\in B}\sum_{c\in C}\sum_{r\in R} \overline{v}_{\mathbf{p,c,b,i,r,h}}\ sam_{c,qmin}$$

$$\forall \boldsymbol{p} \in \boldsymbol{P},\ \ \forall \boldsymbol{qmin} \in \boldsymbol{QMIN}\ ,,\forall \mathbf{h} \in \mathbf{H} \qquad \mathbf{(L8)}$$

$$\sum_{\mathbf{d}\in\mathbf{D}} \tilde{\boldsymbol{q}}_{\boldsymbol{p,d,h}}\, \boldsymbol{spma}_{\boldsymbol{p,qmax}} \geq \sum_{\mathbf{e}\in\mathbf{E}} \sum_{\mathbf{i}\in\mathbf{I}} \sum_{\boldsymbol{r}\in\boldsymbol{R}} \overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}}\ \ \boldsymbol{sbma}_{\boldsymbol{i,e,qmax}} + \sum_{\boldsymbol{i}\in\boldsymbol{I}} \sum_{\boldsymbol{b}\in\boldsymbol{B}} \sum_{\boldsymbol{c}\in\boldsymbol{C}} \sum_{\boldsymbol{r}\in\boldsymbol{R}} \overline{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}}\ \ \boldsymbol{sama}_{\boldsymbol{i,c,b,qmax}}$$

$$\forall \boldsymbol{p} \in \boldsymbol{P},\ \ \forall \boldsymbol{qmax} \in \boldsymbol{QMAX},\ \forall \mathbf{h} \in \mathbf{H} \qquad \mathbf{(L9)}$$

$$\sum_{\mathbf{d}\in\mathbf{D}} \tilde{\boldsymbol{q}}_{\boldsymbol{p,d,h}} \leq \sum_{\mathbf{e}\in\mathbf{E}} \sum_{\mathbf{i}\in\mathbf{I}} \sum_{\boldsymbol{r}\in\boldsymbol{R}} \overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}} + \sum_{\boldsymbol{i}\in\boldsymbol{I}} \sum_{\boldsymbol{b}\in\boldsymbol{B}} \sum_{\boldsymbol{c}\in\boldsymbol{C}} \sum_{\boldsymbol{r}\in\boldsymbol{R}} \overline{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}} \qquad \forall \mathbf{p} \in \mathbf{P}, \forall \mathbf{h} \in \mathbf{H} \qquad \mathbf{(L10}$$

$$\sum_{\mathbf{h}\in\mathbf{H}} \tilde{\boldsymbol{q}}_{\boldsymbol{p,d,h}} = \boldsymbol{q}^{(\boldsymbol{P})}{}_{\boldsymbol{p,d}} \qquad \forall \mathbf{p} \in \mathbf{P}, \forall \mathbf{d} \in \mathbf{D} \qquad \mathbf{(L11)}$$

$$\sum_{\mathbf{p}\in\mathbf{P}} \boldsymbol{q}^{(\boldsymbol{P})}{}_{\boldsymbol{p,d}} \leq \boldsymbol{m}_{\boldsymbol{d}} \cdot \boldsymbol{h}^{(\boldsymbol{P})}{}_{\boldsymbol{d}} \qquad \forall \mathbf{d} \in \mathbf{D} \qquad \mathbf{(L12)}$$

$$\sum_{\mathbf{p}\in\mathbf{P}} \boldsymbol{q}^{(\boldsymbol{S})}{}_{\boldsymbol{p,d}} \leq \boldsymbol{m}_{\boldsymbol{d}} \cdot \boldsymbol{h}^{(\boldsymbol{S})}{}_{\boldsymbol{d}} \qquad \forall \mathbf{d} \in \mathbf{D} \qquad \mathbf{(L13)}$$

$$\boldsymbol{q}^{(\boldsymbol{S})}{}_{\boldsymbol{p,d}} = \sum_{\boldsymbol{k}\in\boldsymbol{K}} \mathbf{z}_{\mathbf{p,d,k}} \qquad \forall \boldsymbol{p} \in \boldsymbol{P}, \forall \boldsymbol{k} \in \boldsymbol{K} \qquad (\boldsymbol{L14})$$

$$\boldsymbol{\theta}_{\boldsymbol{p}} = \boldsymbol{A}_{\boldsymbol{p}} - \boldsymbol{b}_{\boldsymbol{p}} \sum_{\boldsymbol{d}\in\boldsymbol{D}} \mathbf{z}_{\mathbf{p,d,k}} \qquad \forall \boldsymbol{p} \in \boldsymbol{P}, \forall \boldsymbol{k} \in \boldsymbol{K} \qquad (\boldsymbol{L15})$$

$$\underline{\boldsymbol{dem}}_{\boldsymbol{p,k}} \leq \sum_{\mathbf{d}\in\mathbf{D}} \mathbf{z}_{\mathbf{p,d,k}} \leq \overline{\boldsymbol{dem}}_{\boldsymbol{p,k}} \qquad \forall \mathbf{p} \in \mathbf{P}\ \ \forall \mathbf{k} \in \mathbf{K} \qquad \mathbf{(L16)}$$

$$\underline{\boldsymbol{prod}}_{\boldsymbol{p}} \leq \sum_{\mathbf{d}\in\mathbf{D}} \mathbf{q}^{(\boldsymbol{P})}{}_{\mathbf{p,d}} \leq \overline{\boldsymbol{prod}}_{\boldsymbol{p}} \qquad \forall \mathbf{p} \in \mathbf{P} \qquad \mathbf{(L17)}$$

$$\mathbf{x}_{\mathbf{i,r}}, \mathbf{y}_{\mathbf{e,i,r}}, \mathbf{v}_{\mathbf{b,i,r}}, \tilde{\boldsymbol{y}}_{\mathbf{e,i,r}}, \tilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}}, \overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}}, \overline{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}}, \tilde{\boldsymbol{q}}_{\boldsymbol{p,d,h}}, \mathbf{q}^{(\boldsymbol{S})}{}_{\mathbf{p,d}}, \mathbf{q}^{(\boldsymbol{P})}{}_{\mathbf{p,d}}, \boldsymbol{\theta}_{\boldsymbol{p,k}} \geq \mathbf{0}$$

$$\boldsymbol{h}^{(\boldsymbol{S})}{}_{\boldsymbol{d}}, \boldsymbol{h}^{(\boldsymbol{P})}{}_{\boldsymbol{d}} \in \{\mathbf{0}, \mathbf{1}\}$$

Constraints (L1)-(L11) and (L12)-(L13) are the same/ have the same interpretation as constraints (P1)-(P11) and (G1) described in chapter 3.2. Constraints (L14)-(L16) have the same interpretation as constraints (S1)-(S3) described in chapter 3.3 and chapter 4. We have also added a new constraint (L17). This constraint tells what should be the minimum and maximum production of each product and is aligned with constraint about minimum and maximum demand in the markets (L16) from the sales model.

$$\sum_{k\in K} \underline{dem}_{p,k} = \underline{prod}_{p} \forall \mathrm{p} \in \mathrm{P}$$

$$\sum_{k\in K} \overline{dem}_{p,k} = \overline{prod}_{p} \forall \mathrm{p} \in \mathrm{P}$$

This new constraint should ensure a better convergence. Without it, the solution could be that ${q^{(P)}}_{p,d}$ doesn't meet the minimum or maximum demand requirements. We note, that when we add this constraint we don't restrict the original feasible set of IM1.

The solution of L1-IM will be an upper bound of the solution of IM1. It is obviously that model L-IM1 can be decomposed into two subproblems. One for the production department and one for the sales department, each with corresponding constraint sets.

<u>Lagrangean 1 Production Model (L1-PM)</u>

Objective function

$$L1PM\ Profit: \quad \sum_{p\in P}\sum_{d\in D} \lambda_{p,d} {\mathrm{q}^{(P)}}_{\mathrm{p,d}} - \sum_{i\in I}\sum_{r\in R} C_{i,r}^{Buy}\ \mathrm{x_{i,r}}$$

$$- \sum_{i\in I}\sum_{r\in R} C_{i,r}^{PRO}\ \mathrm{x_{i,r}} - \sum_{e\in E}\sum_{i\in I}\sum_{r\in R} \mathrm{C_{e,r}^{PRO2}}\ \mathrm{v_{e,i,r}} - \sum_{p\in P}\sum_{d\in D}\sum_{h\in H} \mathrm{C_{p,h}^{BLEND}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}}$$

$$- \sum_{r\in R}\sum_{h\in H} \mathrm{C_{r,h}^{TRAN1}} \left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P} \bar{y}_{\mathrm{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B} \bar{v}_{\mathrm{p,c,b,i.r}} \right)$$

$$- \sum_{h\in H}\sum_{d\in D}\sum_{p\in P} \mathrm{C_{h,d}^{TRAN2}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}} - \sum_{d} \frac{C_d^{FIX}}{2} {h^{(P)}}_d + \sum_{d\in D} \mu_d {h_d}^{(P)}$$

Subject to (L1) to (L12) and (L17)

$\mathbf{x_{i,r}}, \mathbf{y_{e,i,r}}, \mathbf{v_{b,i,r}}, \widetilde{\boldsymbol{y}}_{\mathbf{e,i,r}}, \widetilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}}, \overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}}, \overline{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}}, \ , \widetilde{\mathbf{q}}_{\mathbf{p,d,h}}, \mathbf{q}^{(\boldsymbol{P})}{}_{\mathbf{p,d}} \ \geq \mathbf{0} \quad$ **and** $\boldsymbol{h_d}^{(\boldsymbol{P})} \in \{\mathbf{0}, \mathbf{1}\}$

<u>Lagrangean 1 Sales Model (L1-SM)</u>

Objective function

$$L1SM\ Profit: \qquad \sum_{p\in P}\sum_{k\in K}\left(\boldsymbol{A_p}\sum_{d\in D}\mathbf{z_{p,d,k}} - \boldsymbol{b_p}\sum_{d\in D}\mathbf{z_{p,d,k}}^{\mathbf{2}}\right) - \sum_{p\in P}\sum_{d\in D}\lambda_{p,d}\mathrm{q}^{(S)}{}_{\mathrm{p,d}}$$

$$-\sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\boldsymbol{C^{TRAN2}_{d,k}}\times\boldsymbol{z_{p,d,k}} - \sum_{d}\frac{\boldsymbol{C^{FIX}_d}}{\mathbf{2}}\boldsymbol{h}^{(\boldsymbol{S})}{}_{\boldsymbol{d}} - \sum_{d\in D}\boldsymbol{\mu_d h_d}^{(\boldsymbol{S})}$$

Subject to (L13) to (L16)

$\mathbf{z_{p,d,k}}, \mathbf{q}^{(\boldsymbol{S})}{}_{\mathbf{p,d}}, \boldsymbol{\theta_{p,k}} \geq \mathbf{0} \quad$ **and** $\boldsymbol{h_d}^{(\boldsymbol{S})} \in \{\mathbf{0}, \mathbf{1}\}$

We make assumption that information about minimum and maximum demand of each product is available at production department.

Scenario 2

Assuming that sales department doesn't include information about operating costs of depots in their model, subproblems are formulated in the following way:

<u>Lagrangean 2 Production Model (L2-PM)</u>

Objective function

$$Maximize\ L2PM\ Profit: \quad \sum_{p\in P}\sum_{d\in D}\lambda_{p,d}\mathrm{q}^{(P)}{}_{\mathrm{p,d}} - \sum_{i\in I}\sum_{r\in R}\boldsymbol{C^{Buy}_{i,r}}\,\mathbf{x_{i,r}} - \sum_{i\in I}\sum_{r\in R}\boldsymbol{C^{PRO}_{i,r}}\ \mathbf{x_{i,r}}$$

$$-\sum_{e\in E}\sum_{i\in I}\sum_{r\in R}\mathbf{C^{PRO2}_{e,r}}\ \mathbf{v_{e,i,r}} - \sum_{p\in P}\sum_{d\in D}\sum_{h\in H}\mathbf{C^{BLEND}_{p,h}}\ \widetilde{\mathbf{q}}_{\mathbf{p,d,h}}$$

$$-\sum_{r\in R}\sum_{h\in H}\mathbf{C^{TRAN1}_{r,h}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P}\overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B}\overline{\boldsymbol{v}}_{\mathbf{p,c,b,i.r}}\right)$$

$$-\sum_{h\in H}\sum_{d\in D}\sum_{p\in P}\mathbf{C}^{\mathbf{TRAN2}}_{\mathbf{h,d}}\,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}}-\sum_{d}C^{FIX}_{d}h^{(P)}{}_{d}$$

Subject to (L1) to (L12) and (L17)

$\mathbf{x_{i,r}}, \mathbf{y_{e,i,r}}, \mathbf{v_{b,i,r}}, \widetilde{\boldsymbol{y}}_{\mathbf{e,i,r}}, \widetilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}}, \overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}}, \overline{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}}, , \widetilde{\mathbf{q}}_{\mathbf{p,d,h}}, \mathbf{q}^{(\boldsymbol{P})}{}_{\mathbf{p,d}} \geq \mathbf{0}$ **and** $\boldsymbol{h_d}^{(\boldsymbol{P})} \in \{\mathbf{0},\mathbf{1}\}$

__Lagrangean 2 Sales Model (L2-SM)__

Objective function

$$Maximize\ L2SM\ Profit:\quad \sum_{p\in P}\sum_{k\in K}\left(\boldsymbol{A_p}\sum_{d\in D}\mathbf{z_{p,d,k}}-\boldsymbol{b_p}\sum_{d\in D}\mathbf{z_{p,d,k}}^{2}\right)$$

$$-\sum_{p\in P}\sum_{d\in D}\boldsymbol{\lambda_{p,d}}\mathbf{q}^{(\boldsymbol{S})}{}_{\mathbf{p,d}}-\sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\mathbf{C}^{\mathbf{TRAN2}}_{\mathbf{d,k}}\times\mathbf{z_{p,d,k}}$$

Subject to (L14) to (L16)

$$\sum_{\mathbf{p}\in\mathbf{P}}\boldsymbol{q}^{(\boldsymbol{S})}{}_{\boldsymbol{p,d}}\leq\boldsymbol{m_d}\qquad\qquad\forall\mathbf{d}\in\mathbf{D}\qquad\qquad(\mathbf{L18})$$

$\mathbf{z_{p,d,k}}, \mathbf{q}^{(\boldsymbol{S})}{}_{\mathbf{p,d}}, \boldsymbol{\theta_{p,k}} \geq \mathbf{0}$

When only the production department takes fixed costs into consideration there is no need for duplicating of $h_{p,d}$ variable, and hence corresponding Lagrangean multiplier. Production model L2-PM is almost identical to L1-PM, the only difference is in the objective function: the term with Lagrangean multiplier $\mu_d$is removed and the fixed costs coefficient of depots is not divided by 2. All constraints remain the same.

In the sales model L2-SM, the expressions connected to depot operation do not exist anymore. However, because maximum capacity of depots should still be taken into account we add another constraint (L18), which sets a restriction on the amount of products that can be stored at depots. This constraint has the same interpretation as (G2) described in chapter 3.2. Again, we note that the new constraint doesn't restrict the original problem.

In this scenario, sales subproblem L2-SM doesn't include any binary variables and hence is now a simple QP problem. This of course simplify the solution of the model. On the other hand because now the sales department doesn't consider information about how costly it is to operate depots, the subproblems will be less coordinated and we expect that it will take longer time to achieve "a good solution". In this situation coordination between departments takes place only through Lagrangean multipliers $\lambda_{p,d}$.

Scenario 3

Assuming that production department doesn't take into account information about fixed costs of depots in their model, IM1 is decomposed as follows:

<u>Lagrangean 3 Production Model (L3-PM)</u>

Objective function

$$Maximize\ L2PM\ Profit\!: \sum_{p\in P}\sum_{d\in D}\boldsymbol{\lambda_{p,d}}\mathbf{q}^{(P)}{}_{\mathbf{p,d}} - \sum_{i\in I}\sum_{r\in R}\boldsymbol{C^{Buy}_{i,r}}\,\mathbf{x_{i,r}}$$

$$-\sum_{i\in I}\sum_{r\in R}\boldsymbol{C^{PRO}_{i,r}}\ \mathbf{x_{i,r}} - \sum_{e\in E}\sum_{i\in I}\sum_{r\in R}\mathbf{C^{PRO2}_{e,r}}\,\mathbf{v_{e,i,r}} - \sum_{p\in P}\sum_{d\in D}\sum_{h\in H}\mathbf{C^{BLEND}_{p,h}}\,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}}$$

$$-\sum_{r\in R}\sum_{h\in H}\mathbf{C^{TRAN1}_{r,h}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P}\overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B}\overline{\boldsymbol{v}}_{\mathbf{p,c,b,i.r}}\right)$$

$$-\sum_{h\in H}\sum_{d\in D}\sum_{p\in P}\mathbf{C^{TRAN2}_{h,d}}\,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}}$$

Subject to (L1) to (L11), (L17)

$$\sum_{\mathbf{p}\in\mathbf{P}}\boldsymbol{q}^{(\boldsymbol{P})}{}_{\boldsymbol{p,d}} \le \boldsymbol{m_d} \qquad \forall\mathbf{d}\in\mathbf{D} \qquad (\mathbf{L19})$$

$$\mathbf{x_{i,r}},\ \mathbf{y_{e,i,r}},\ \mathbf{v_{b,i,r}},\ \widetilde{\boldsymbol{y}}_{\mathbf{e,i,r}},\ \widetilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}},\ \overline{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}},\ \overline{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}},\ ,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}},\ \mathbf{q}^{(\boldsymbol{P})}{}_{\mathbf{p,d}}\ \ge \mathbf{0}$$

Lagrangean 3 Sales Model (L3-SM)

Objective function

$$Maximize\ L2SM\ Profit\colon \sum_{p\in P}\sum_{k\in K}\left(A_p\sum_{d\in D}\mathbf{z_{p,d,k}} - b_p\sum_{d\in D}\mathbf{z_{p,d,k}}^2\right)$$

$$-\sum_{p\in P}\sum_{d\in D}\lambda_{p,d}\mathbf{q^{(S)}}_{\mathbf{p,d}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\mathbf{C^{TRAN2}_{d,k}}\times\mathbf{z_{p,d,k}} - \sum_{d}C^{FIX}_{d}h^{(S)}_{d}$$

Subject to (L13) to (L16)

$\mathbf{z_{p,d,k}}, \mathbf{q^{(S)}}_{\mathbf{p,d}}, \theta_{p,k} \geq 0$ **and** $h_d{}^{(S)} \in \{0,1\}$

In the same way as in scenario 2, when only one department takes fixed costs into consideration, there is no need for duplicating of $h_{p,d}$ variable, and hence corresponding Lagrangean multiplier. Production model L3-PM doesn't include any binary variables, while L3-SM is now taking into account fixed costs from depots operation. We add a new constraint (L19) to the production subproblem, which is of the same type as (L18).

Scenario 4

Lagrangean 4 Production Model (L4-PM)

Objective function

$$Maximize\ L4PM\ Profit\colon \sum_{p\in P}\sum_{d\in D}\lambda_{p,d}\mathbf{q^{(P)}}_{\mathbf{p,d}} - \sum_{i\in I}\sum_{r\in R}C^{Buy}_{i,r}\,\mathbf{x_{i,r}}$$

$$-\sum_{i\in I}\sum_{r\in R}C^{PRO}_{i,r}\ \mathbf{x_{i,r}} - \sum_{e\in E}\sum_{i\in I}\sum_{r\in R}\mathbf{C^{PRO2}_{e,r}}\,\mathbf{v_{e,i,r}} - \sum_{p\in P}\sum_{d\in D}\sum_{h\in H}\mathbf{C^{BLEND}_{p,h}}\,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}}$$

$$-\sum_{r\in R}\sum_{h\in H}\mathbf{C^{TRAN1}_{r,h}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P}\overline{y}_{\mathbf{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B}\overline{v}_{\mathbf{p,c,b,i.r}}\right)$$

$$-\sum_{h\in H}\sum_{d\in D}\sum_{p\in P}\mathbf{C^{TRAN2}_{h,d}}\,\widetilde{\mathbf{q}}_{\mathbf{p,d,h}}$$

Subject to (L1) to (L11), (L17) and (L19)

$$\mathbf{x}_{\mathbf{i,r}}, \mathbf{y}_{\mathbf{e,i,r}}, \mathbf{v}_{\mathbf{b,i,r}}, \tilde{\boldsymbol{y}}_{\mathbf{e,i,r}}, \tilde{\boldsymbol{v}}_{\mathbf{c,b,i,r}}, \bar{\boldsymbol{y}}_{\mathbf{p,e,i,r,h}}, \bar{\boldsymbol{v}}_{\mathbf{p,c,b,i,r,h}}, \tilde{\mathbf{q}}_{\mathbf{p,d,h}}, \mathbf{q}^{(\boldsymbol{P})}{}_{\mathbf{p,d}} \geq \mathbf{0}$$

__Lagrangean 4 Sales Model (L4-SM)__

Objective function

$$Maximize\ L4SM\ Profit: \quad \sum_{p\in P}\sum_{k\in K}\left(\boldsymbol{A}_{\boldsymbol{p}}\sum_{d\in D}\mathbf{z}_{\mathbf{p,d,k}} - \boldsymbol{b}_{\boldsymbol{p}}\sum_{d\in D}\mathbf{z}_{\mathbf{p,d,k}}{}^{\mathbf{2}}\right)$$
$$-\sum_{p\in P}\sum_{d\in D}\boldsymbol{\lambda}_{\boldsymbol{p,d}}\mathbf{q}^{(\boldsymbol{S})}{}_{\mathbf{p,d}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\mathbf{C}_{\mathbf{d,k}}^{\mathbf{TRAN2}} \times \mathbf{z}_{\mathbf{p,d,k}}$$

Subject to (L14) to (L16) and (L18)

$$\mathbf{z}_{\mathbf{p,d,k}}, \mathbf{q}^{(\boldsymbol{S})}{}_{\mathbf{p,d}}, \boldsymbol{\theta}_{\boldsymbol{p,k}} \geq \mathbf{0}$$

In this scenario, the base for decomposition is IM2. If none of the departments take into account costs connected to operation and possibility to close and open depots at this planning stage, decision variable h_d should not appear in any of the subproblems. The sales subproblem is the same as under scenario 2 and the production subproblem is the same as under scenario 3. However, because depot usage is predetermined, the set of available depots in the subproblems under this scenario may be smaller than in other scenarios. This new set should be within the set used in other scenarios. In our work we are not going into explanations of how PD can choose which depots should be in operation. In our examples, we will assume that depots with highest fixed costs are closed.

Interpretation of Lagrangean Multipliers

The Lagrangean multipliers penalize the objective functions when the relaxed constraints are violated. In the production models the Lagrangean multiplier $\lambda_{p,d}$ can be interpreted as the price PD receives for selling products from depots to SD. In the sales models, $\lambda_{p,d}$ can be compared with costs of buying products from PD. Hence, this multiplier can be referred to as internal price for specific product at specific depot. When we solve the sales and production subproblems independently to each other, it is high possibility that PD wants to produce and deliver products to other depots than SD wants to purchase these products at. This difference should be regulated by setting the right Lagrangean multipliers. If PD produces and delivers more product to depot than SD wants to buy of this product at this depot, then the Lagrangean multiplier (for this specific product and depot) should be reduced. If the Lagrangean multiplier will be reduced, it will be less attractive for PD to produce this product and deliver to this depot, while on the other hand for SD it will be cheaper and hence more attractive to buy this product at this depot. If the opposite is the case: SD wants to buy more than PD wants to produce, then the corresponding multiplier should increase.

In scenario 1, we have also used another Lagrangean multiplier μ_d. It can be interpreted as extra value or extra cost for using depots. The updating mechanism for this multiplier is the same as for the first one: if production and sales departments have decided to use different depots, Lagrangean multiplier should be updated in such a way that depots which have been attractive for one part and not used by the other part, should become more attractive for the last mentioned part and less attractive for the first part. In other words, if production department decides to use one depot which has very high Lagrangean multiplier (high value has positive effect on the objective function in L1-PM) then sales department will not use this depot because high Lagrangean multiplier represents costs. If depot is chosen only by production department, then Lagrangean multiplier will be reduced. This will make depot less attractive for PD but more attractive for SD. Opposite happens if depot is used just by SD. When we include the depot operation constraint in both subproblems we get a double regulation for depots use, because we use two Lagrangean multipliers which should stimulate departments to choose the same depots.

The regulation of Lagrangean multipliers will be done through subgradient method which will be described below.

7.2.2 The Solution Algorithm

We start by initialize Lagrangean multipliers $\lambda_{p,d}$ (and μ_d in scenario 1). Then sales and production subproblems are solved separately with given Lagrangean multipliers.

When these two subproblems are solved, the sum of the objective function values, that is the objective value of Lagrangean function, will give an optimistic bound for the objective function of IM1 in scenarios 1,2 and 3 and IM2 in scenario 4.

$$Contribution^{Lagrange} = UB = Profit\ Sale + Profit\ Production$$

Because we have a maximization problem, the optimistic bound is an upper bound. We update upper bound in each iteration by choosing the lowest of the new calculated UB and the previous UBD.

$$UBD = \min(UB, UBD)$$

As we have mentioned above, the solution that we obtain may be infeasible: ${q^{(P)}}_{p,d} < {q^{(S)}}_{p,d}$ or/and ${h_d}^{(P)} \neq {h_d}^{(S)}$ for any p and d. If this is the case, we will use a heuristic method to modify the solution into a feasible solution. When this is done, we obtain a pessimistic bound which in our case is a lower bound of the original problem.

Heuristic used to obtain feasible solutions

After both subproblems are solved, we obtain amount of supplied products ${q^{(P)}}_{p,d}$ from the production model and amount ordered products ${q^{(S)}}_{p,d}$ from the sales model. As we have mention, if ${q^{(P)}}_{p,d} \geq {q^{(S)}}_{p,d}$ then the solution is feasible. If this is not the case, the easiest way to obtain a feasible solution is to use one of these outputs (either from PM or SM) and define it as amount that have to be supplied/sold.

In our heuristic method we use local optimal values $q^{(S)}$ from the sales subproblem. We use these values as input to PM, described in chapter 4 by requiring that PD has to satisfy this demand from the sales department. In particular we set parameter ${\overline{\mathrm{q}}^{S}}_{\mathrm{p,d}}$ in PM equal to $q^{(S)}$ from the Lagrangean sale subproblem. The objective function value of PM will represent production costs, $Costs\ PM$. Below, we formulate expressions for calculation of feasible solutions for each of the scenarios:

Scenario 1:

$$LB = Profit\ L1SM - Costs\ PM + \sum_{p \in P} \sum_{d \in D} \lambda_{p,d} \mathrm{q}^{(S)}{}_{\mathrm{p,d}} + \sum_{d \in D} \mu_d h_d{}^{(S)} - \sum_{d} \frac{C_d^{FIX}}{2} h^{(S)}{}_d \qquad (\boldsymbol{D1-1})$$

From the profit of L1-SM we subtract costs of producing ordered products. However, because we are interesting to find the solution which concerns the whole company, we should add back costs sales department paid for products to PD and extra costs for using depots in scenario 1. This is because they are internal transactions, which do not affect the profit of the company. Because in the sales objective function we have only considered half of the fixed costs, we also need to subtract the other part of these costs.

Scenario 2:

$$LB = Profit\ L1SM - Costs\ PM + \sum_{p \in P} \sum_{d \in D} \lambda_{p,d} \mathrm{q}^{(S)}{}_{\mathrm{p,d}} - \sum_{d} C_d^{FIX} h^{(S)}{}_d \qquad (\boldsymbol{D1-2})$$

In this scenario fixed costs are not included in the sales subproblem, and we need to subtract them. In the same way as in (D1-1) we add back internal price costs.

Scenario 3:

$$LB = Profit\ L3SM - Costs\ PM + \sum_{p \in P} \sum_{d \in D} \lambda_{p,d} \mathrm{q}^{(S)}{}_{\mathrm{p,d}} \qquad (\boldsymbol{D1-3})$$

In this scenario fixed costs are included in the sales subproblem and we need only to add back internal price costs.

Scenario 4

$$LB = Profit\ L4SM - Costs\ PM + \sum_{p \in P} \sum_{d \in D} \lambda_{p,d} \mathrm{q}^{(S)}{}_{\mathrm{p,d}} - \ fixed\ costs \qquad (\boldsymbol{D1-4})$$

In this scenario, PD has decided in advance which depots will be in operation, hence fixed costs is a constant.

Now we have found the feasible solution. In each scenario, in each iteration the lower bound (LBD) is calculated as the highest of LB value (fount in this iteration) and the previous lower bound. We update the lower bound as follows:

$$LBD = \max(LB, LBD)$$

Instead of using output from the sales subproblem we could use output from the production subproblem $q^{(P)}{}_{p,d}$ as a starting point for modification, and obtain a feasible solution based on production department's result. The feasible solution could be obtained by requiring that sales department must distribute products that production department has found optimal to produce. But as long as we are interesting in method for setting internal prices, this heuristic will not provide results which we are looking for. Lower bound should represent a feasible solution, however in our problem it should be possible to obtain this feasible solution by use of internal prices which we obtain at each iteration. If we insert these internal prices in SM the results may differ from the lower bound found by the heuristic, and the total profit may not be the same. Hence this way of calculating of lower bound doesn't make any sense, because the company will not be able to obtain the same result in reality.

Step length calculation

After each iteration we compute new values for Lagrangean multipliers, which will be used as input to subproblems in the next iteration. We do it according to the subgradient method described in the previous section. In our decomposition we have added an equality constraint $q^{(P)}{}_{p,d} = q^{(S)}{}_{p,d}$. However, because the solution can still be feasible if $q^{(P)}{}_{p,d} \geq q^{(S)}{}_{p,d}$, the corresponding Lagrangean multiplier should be restricted in sign $\lambda^{(n)}{}_{p,d} \geq 0$. Hence:

$$\lambda^{(n+1)}{}_{p,d} = max\left\{0, \lambda^{(n)}{}_{p,d} + t^{(n)}\gamma_{p,d}{}^{(n)}\right\} \qquad (\boldsymbol{D2})$$

where $\gamma_{p,d}$ is the subgradient:

$$\gamma_{p,d}{}^{(n)} = q^{(S)}{}_{p,d} - q^{(P)}{}_{p,d} \qquad (\boldsymbol{D3})$$

and t is the step length:

$$t^{(n)} = \frac{\sigma^{(n)}\left(UB^{(n)} - LBD\right)}{\sum_{p\in P}\sum_{d\in D}\left(\lambda_{p,d}{}^{(n)}\right)^2} \qquad (\boldsymbol{D4})$$

In scenario 1 we also need to update another multiplier:

$$\mu^{(n+1)}{}_d = \mu^{(n)}{}_d + t_2{}^{(n)}\eta_d{}^{(n)} \qquad (\boldsymbol{D5})$$

where $\eta_d$is the subgradient:

$$\eta_d{}^{(n)} = h^{(S)}{}_d - h^{(P)}{}_d \qquad (\boldsymbol{D6})$$

Because the subgradient $\eta_d{}^{(n)}$ will always be 1 or 0, the use of standard step length formula may lead to very large step length. A numerator will be very big when there is a big gap between upper and lower bounds, while denominator will be relative small. Therefore we use another formula that has showed a better convergence. This step length calculation is a version of formula used by (Jörnsten and Nasberg, 1986), where it has been applied for calculation of step length for binary variables in LD of generalized assignment problem.

$$t_2{}^{(n)} = \frac{\frac{1}{k}\sum_{d\in D} C_d^{FIX}}{1+n} \qquad (\boldsymbol{D7})$$

Where *k* is the total number of depots. The numerator of (D7) calculates the average fixed costs of depots. Denominator is the number of current iteration plus one.

In addition, we have tried an alternative way of calculating the step length $t_1{}^{(n)}$ for Lagrangean multiplier $\lambda_{p,d}{}^{(n)}$, which has been used in the working paper written by Kong and Rönnqvist (2012). In this alternative way of calculating the step length, instead of using upper bound obtained in the current iteration $UB^{(n)}$, we use the best achievable upper bound so far (the lowest upper bound) UBD.

$$\boldsymbol{t^{(n)}} = \frac{\boldsymbol{\sigma^{(n)}(UBD - LBD)}}{\boldsymbol{\sum_{p\in P}\sum_{d\in D}\left(\lambda_{p,d}{}^{(n)}\right)^2}} \qquad (\boldsymbol{D8})$$

We use the following convergence criteria: $\frac{UBD-LBD}{LBD} \leq \varepsilon$ or $n = n_{max}$

If UBD has not been improved during the last k iterations, $\sigma^{(n)}$ should be updated as follows:

$$\sigma^{(n+1)} := \alpha\sigma^{(n)} \quad (\boldsymbol{D9}) \text{ where } 0 < \alpha \leq 1$$

The Algorithm

The algorithm is summarized below.

Step 0 Choose initial multipliers $\lambda^{(0)}$ (and $\mu^{(0)}{}_{d}$ in scenario 1). Set $\mathrm{n} = 0, LBD = -\infty$ $and\ UBD = +\infty$. Choose initial values for $\sigma^{(0)} \in (0,2]$, and decide ε, n_{max} $and\ k$

Step 1 Solve the subproblems for a given $\lambda^{(n)}{}_{p,d}$ and (and $\mu^{(n)}{}_{d}$ in scenario 1). Let $UB^{(n)} = Profit$ Production + Profit Sale. Update UBD = min(UBD, $UB^{(n)}$)

Step 2 Let parameter $\overline{\mathrm{q}}^{S}{}_{\mathrm{p,d}}$ in PM be equal to $\mathrm{q}^{(S)}{}_{\mathrm{p,d}}$ from the sales subproblem. Solve PM. Calculate LB according to (D1-i)

Step 3 Update $LBD = \max(LB^{(n)}, LBD)$

Step 4 Check the convergence criteria. If $\frac{UBD-LBD}{LBD} \leq \varepsilon$ or $n = n_{max} \Longrightarrow$ Stop. Let $\lambda^{(n)}{}_{p,d}$ be the internal prices for products

Step 5 Update $\sigma^{(n)}$ according to (D9) if UBD has not been improved during the last k iterations

Step 6 Compute the subgradient according to (D3) (and (D6) in scenario 1)

Step 7 Determine the step length according to (D4)/(D8) and (and (D7) in scenario 1)

Step 8 Update the Lagrangean multipliers according to (D2) and (and (D5) in scenario 1). Set n = n+1 and go to Step 1.

7.2.3 Comments on the solution of LD

Information exchange between departments

As it was pointed out in chapter 2, each department should have a clear information about what it supposed to do at each planning stage and what information it should exchange with other departments. In our approach the planning stage should start with definition of the departmental subproblems, which departments will solve at the planning level.

In the production subproblems we have added one extra constraint, which define the minimum and maximum production of each product. Except for this constraint, the constraints that appear in the subproblems under LD, are the same as in PM and SM. This implies that the departmental constraints have to be known only by the departments themselves.

The process of information exchange between the departments is as follows:

1. Production department initializes some values of internal prices. These could be for example some estimates or historical prices. These values are sent to sales department.
2. Each of the departments solves its own subproblems, based on the internal price values decided by PD. After that, SD submits information about its objective function value and the quantities it will order to PD.
3. According to the information received from SD and its own calculations, PD calculates upper bound, lower bound, and new internal prices.
4. New internal prices are sent back to SD. And departments resolve their subproblems, based on new internal prices. SD submits new plans to PD.
5. The procedure is repeated (with recalculation of subproblems, internal prices, and upper and lower bounds) until the company arrives at the optimal solution or maximum number of iterations is reached.

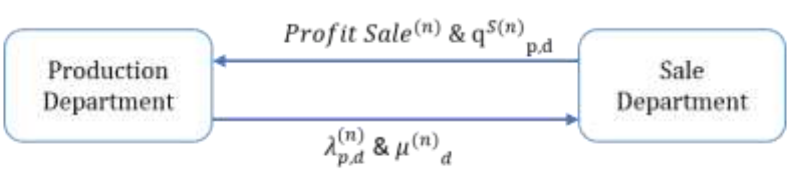


Figure 7.1 – Information Exchange

In our work we have assumed that the production department is connected to the rest of the company, and therefore is interested to obtain the best possible result for the whole company. While the sales department is more independent unit that maximizes only its premium. We could also assume that both departments act separately and optimize their own goals. In this case all information should go through headquarter, which would be responsible for calculation of Lagrangean multipliers, and upper and lower bounds.

Comments to scenarios

Sales subproblems in scenarios 2, 3 and 4 correspond to the sales models formulated in chapter 4. Sales model in scenario 1 includes two Lagrangean multipliers at the planning level. These require more information exchange between the departments at the planning level, at the same time SM solved in the decision phase doesn't include second multiplier associated with depots. Therefore, the second Lagrangean multipliers should be incorporated into fixed cost parameters, which are used as input to SM in the decision phase. Otherwise, the solution from SM may deviate from the lower bound obtained in LD. So, if the company uses scenario 1 at the planning level, fixed cost parameters used by SD at the decision level should be adjusted according to the optimal Lagrangean multiplier values μ_d. When this is done, the subproblem solved by SD at the planning level in scenario 1, will also correspond to the sales model solved at the decision level.

The production subproblems in LD at the planning level deviate from PM at the decision level, in all scenarios. The main difference is that at the planning stage, the department decides how many units of products it will supply, while in the decision stage the supply is predetermined by orders from the sales department. It means that under the decision stage, the production department has no influence on how much of each product will be produced and which depots these will be delivered to. However, during the planning phase the department has a clear influence on the determination of internal prices. This way of formulating subproblems, corresponds to the second situation, described in chapter 2: representation of the subproblems corresponds to organizational subunits but this correspondence is not used in the actual solution process.

In scenario 4 it may not be possible to obtain the same solution as in other scenarios, if "wrong" depots have been closed. However, the problem excludes binary variable (in nonlinear case) and we expect that after a small number of iterations the solution will be better than in other scenarios. Also we expect a faster convergence.

Depot usage in different scenarios

When the relaxed constraints are violated the objective function is penalized through Lagrangean multipliers. In our problem the relaxed constraints are violated when the demand from the sales department is bigger than the supply from the production department for any of the products at any of the depots. This implies that the departments must supply products at and order products from the same depots. Below we discuss which conditions must be satisfied in order this should hold.

The sales model in scenarios 2 and 4 doesn't include any costs associated with depots. If internal prices of products are similar for all depots, SD will use all depots in order to reduce transportation costs (we assume that for each depot there is at least one market, for which this depot is the most profitable starting point). In these scenarios SD will not use depot, if the internal prices of all products at this depot are significantly higher than internal prices for these products at other depots. With significantly higher, we mean that savings from the transportation from this depot are less than the difference between internal prices

The PDs subproblem in scenarios 3 and 4, is formulated in a similar way. The department will choose to deliver products to depots in which it can obtain the highest difference between internal prices and transportation costs. At the same time PD will close depots where the difference is small.

Fixed costs add more complexity to these problems. In PDs subproblems (in scenarios 1 and 2), low difference between internal prices and transportation costs in a depot, and at the same time low fixed costs of this depot can be more profitable than high difference and high fixed costs. In the same way, in SDs subproblems (in scenarios 1 and 3) depots that have a good combination of internal prices and transportation costs can be closed because of high fixed costs.

In order that both departments choose the same depots, the internal prices in these depots must be high enough to ensure that PD will use them, but at the same time these prices must be low enough to ensure that SD will use them as well. If this is the case, internal prices are balanced and it is possible to obtain a more realistic upper bound, because none of the departments make a super profit.

We illustrate with a small example, that it may exist a case in scenario 2 in which it is not possible for both departments to choose the same depots.

Suppose that we have: one hub, two depots, two markets, and one product. Let's assume that demand at both markets is fixed at 50 units, so that the production department always produces 100 units, and sales department always sales 50 units to each market. Fixed costs of depots are 1000 and 1500.

Transportation costs to depots:

Depot 1	10,00
Depot 2	4,00

Transportation costs to markets:

	Market 1	**Market 2**
Depot 1	4,00	5,00
Depot 2	6,00	4,00

Let *X* be the internal price at depot 1 and *Y* the internal price at depot 2. We can easily calculate that costs to the production department will be 2000 if it decides to use depot 1 and 1900 if depot 2 is used. If both depots are used, the costs will be 2500+. We can exclude the last possibility because it isn't optimal.

The production department has the following simple algorithm:

Condition	Choice
$X-Y > 1$	Depot 1
$X-Y \leq 1$	Depot 2

Table 7.1 – Example 1

Based on the transportation costs to the markets, the sales department has the following algorithm:

Condition	Choice
2 < X-Y	Depot 2
-1< X-Y ≤2	Depot 1 & 2
X-Y ≤ -1	Depot 1

Table 7.2 – Example 2

If we combine these to algorithms we obtain the following:

Condition	Choice S	Choice P
2< X-Y	Depot 2	Depot 1
1< X-Y ≤ 2	Depot 1 & 2	Depot 1
-1< X-Y ≤ 1	Depot 1 & 2	Depot 2
X-Y ≤ -1	Depot 1	Depot 2

Table 7.3 – Example 3

From the Table 7.3 we observe that there are no internal prices for which it would be optimal for both departments to choose the same depots.

When instead sales department takes fixed costs into account, it is easier to find internal prices that will lead to the same depot use. Also, in scenarios where fixed costs are included in both models or there are no fixed costs, it is more likely that such internal prices will exist. In scenario 4 there are no extra costs for using depots and the set of depots is smaller than in other scenarios, while in scenario 1 in addition to fixed costs the choice of depots is also regulated by additional multiplier.

Upper bound

In addition to the criteria of the same depot usage, we are also interesting that the departments supply and order the same amount of products from the same depots (or the supply is bigger than the demand). This is more strong criteria, and it can be the case that it doesn’t exist internal prices (Lagrangean multipliers) that would ensure that it holds, in any of the scenarios. If we discover price values for which the supply and the demand are equal or very close to each other, then these values will also give a tighter bound. Instead, upper bound becomes high and unrealistic when internal prices are low at some depots and SD orders too much products from these depots, while at other depots the prices are high and PD supplies too much to these depots. In this case both departments are making good profits which are impossible to realize in reality.

Lower bound

In our models it is important to find internal prices that will ensure that SD makes a good choice. Because the sales subproblems correspond to the real problems SD solves at the decision level, it is also possible to calculate the real profit the company would obtain at each iteration. And because we know that upper bound calculated during each iteration is greater or equal to the optimal profit that can be achieved by the company, we also know when internal prices lead to "a near" optimal solution. That is when feasible solution is very close to the upper bound. When this is the case, we can stop and let Lagrangean multipliers be internal prices.

Alternative decompositions of the original problem

In addition to the alternatives that we have considered, there are of course other possible ways of decompose IM1/IM2. More variables could be duplicated and subproblems could have more constraints in common. For example, Bredström and Rönnqvist, (2008) and Kong and Rönnqvist (2012) have included blending decisions in both sales and production subproblems, in order to achieve a better convergence. We have not considered such alternatives because it would lead to unrealistic subproblems. Information about blending (or other production activities) may be complex, and it may require a lot of effort to incorporate such information in sales subproblem. And vice versa. Therefore, in our decomposition we have not considered alternatives in which departments would be dealing with the constraints of other departments.

PART 3: COMPUTATIONAL STUDY

We divide the computational study into three parts. First, we give a description of the refinery system and data used in our models. Next, we employ the cost-based methods and study the results. Then, we employ Lagrangean decomposition methods on our numerical example and study the results, as well as compare with the previous results.

The mathematical models are programmed by AMPL modeling language (version 20140224). The Linear models and Mixed-Integer Linear models are solved by CPLEX 12.6 and the Quadratic models by MINOS 5.51. Models which are Non-Linear Mixed Integer are solved by KNITRO 9. KNITRO MINL code is designed for convex mixed integer programming. As we have discussed earlier our MINL models are only of this type and hence can be solved by KNITRO to find global optimum. Also KNITRO has been used when several functional forms are used in the same “run file”. We have used the default parameters of solvers.

Chapter 8 – Numerical Example

To describe the refinery process we have used case study in Bredström et al. (2008). The explanation of abbreviations which are used further, is given in Table 8.1

Abbreviation	Explanation	Abbreviation	Explanation
Raw material		Additional components out from reformer and cracker	
CR1	Crude oil 1	C1	Reformulated FG
CR2	Crude oil 2	C2	Reformulated GA
		C3	Cracker FG
Components out from CDU		C4	Cracker GA
A1	Fuel Gas fraction	C5	Cracker GO
A2	Gasoline fraction		
A3	Residuals bottoms	Blended products	
B1	Naphtha fraction	P1	Premium Gasoline
B2	Light distillates fraction	P2	Regular Gasoline
B3	Heavy distillates fraction	P3	Distillates
		P4	Fuel Oil

Table 8.1 - Explanation of abbreviations

We consider a company with two refineries: r1 and r2. Each refinery purchases two types of crude oil: CR1 and CR1. CR1 costs 108$ and 107.5$ per barrel at r1 and r2, and has lower sulfur content than CR2 which costs 69.5 $ and 70.1$ per barrel at r1 and r2 respectively. Maximum supply of CR1 per period is 12 000 barrels, while it is 25 000 barrels for CR2 at each of the refineries. Crude oils are used as input to CDU, where they are broken into six components. It costs 26$ and 27$ per barrel to process CR1 at CDU at r1 and r2, while it costs 29$ and 28$ per barrel to process CR2 at r1 and r2. The fraction and the quality of generated components depend on the type of crude oil.

Table 8.2 shows the fraction coefficients of output for each crude oil type. Table 8.3 and Table 8.4 yield values of sulfur content and density of each component generated from CR1 and CR2, while Table 8.5 yields values of octane concentration. We assume that processes are identical in both refineries.

	A1	A2	A3	B1	B2	B3
CR1	0.029	0.236	0.314	0.223	0.087	0.111
CR2	0.017	0.180	0.443	0.196	0.073	0.091

Table 8.2 – Yields from CDU for one unit of CR1 and CR2

	Sulfur	Density
A1	0	0
A2	0	0
A3	4,7	343
B1	0,283	272
B2	0,526	292
B3	0,98	295

Table 8.3 – Sulfur content and density of components generated from CR1

	Sulfur	Density
A1	0	0
A2	0	0
A3	1,48	272
B1	2,83	297,6
B2	5,05	303,3
B3	11	365

Table 8.4 – Sulfur content and density of components generated from CR2

	Octane
A1	0
A2	78,5
A3	0
B1	65
B2	0
B3	0

Table 8.5 – Octane concentration in components

As shown in Table 8.5, octane concentration in components is independent on the crude oil these components are generated from.

Generated A components, are used directly in blending. B components can be processed further at the refineries. There are three type of each A and B components. The cost of processing one barrel of B component is 25$, 25$ and 24$ for B1, B2 and B3 at r1, and 24$, 25$ and 26$ at r2 respectively. Table 8.6 shows the fraction coefficients of output for each B component.

	C1	**C2**	**C3**	**C4**	**C5**
B1	0,129	0,807	0	0	0
B2	0	0	0,3	0,59	0,21
B3	0	0	0,31	0,59	0,22

Table 8.6 – Amount of C component generated from one unit of B component

We assume that there are five C components, which can be generated from B components. Table 8.7, Table 8.8 and Table 8.9 show density, sulfur and octane values of C components. Density and sulfur content in C components depend both on B components and crude oil B components are generated from. While octane concentration in C components is independent of which components these are obtained from.

	CR1-B1	**CR1-B2**	**CR1-B3**	**CR2-B1**	**CR2-B2**	**CR2-B3**
C1	272	0	0	272	0	0
C2	272	0	0	272	0	0
C3	0	292	295	0	297,6	303,3
C4	0	292	295	0	297,6	303,3
C5	0	294,9	292,1	0	300,6	300,3

Table 8.7 – Density of C components

	CR1-B1	CR1-B2	CR1-B3	CR2-B1	CR2-B2	CR2-B3
C1	0,283	0	0	1,48	0	0
C2	0,283	0	0	1,48	0	0
C3	0	0,526	0,98	0	2,83	5,05
C4	0	0,526	0,98	0	2,83	5,05
C5	0	0,3	0,304	0	1,61	1,57

Table 8.8 – Sulfur content of C components

	Octane
C1	65
C2	104
C3	0
C4	93,7
C5	0

Table 8.9 – Octane concentration in C components

From the refineries components are sent to hubs. In our example we consider two hubs: h1 and h2. We assume that company produces four types of products. Table 8.10 shows maximum quality specifications related to density and sulfur, and minimum specifications of octane.

	Octane	Sulfur	Density
P1	90	3	500
P2	86	3	500
P3	0	0,5	306
P4	0	3,5	352

Table 8.10 – Quality requirements

Our example consists of five depots and five markets. We assume that maximum capacity for each of the depot is 10 000 units of products. Cost data that is used in our example is generated. Cost values are generated such that there are reasonable proportions between different costs. Costs related to blending, transportation, and operation of depots are shown in Appendix B.

Demand parameters which are used in our models, have been generated. These are chosen such that they have moderate price elasticity. In Appendix B we have presented parameters used in price functions together with price elasticity, and minimum and maximum demand for each of these four products.

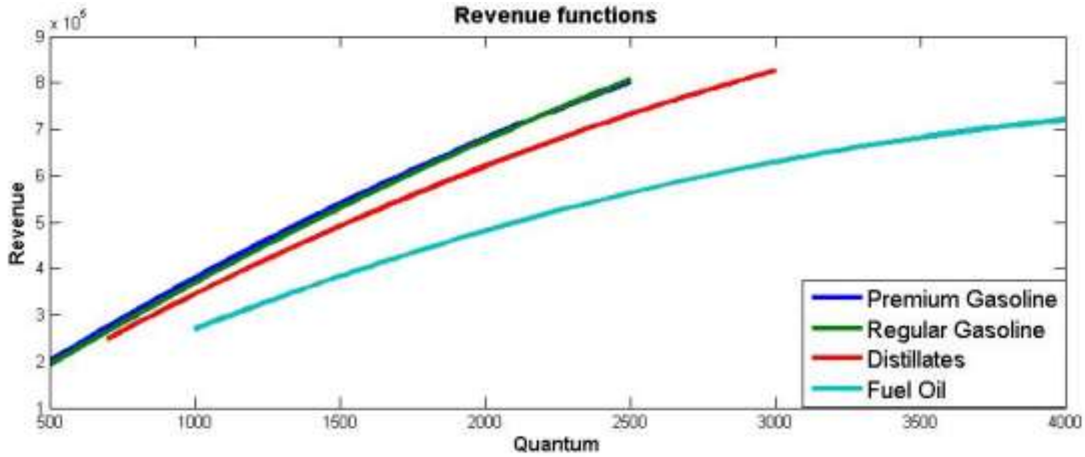


Figure 8.1- Revenue functions

Figure 8.1 shows nonlinear revenue functions for each of the four products used in our example. The piecewise linear functions are approximations of these four revenue functions, where each of the functions is divided into 10 equal segments. These parameters are also presented in Appendix B.

Chapter 9 - Numerical Tests on Cost-Based Methods and Benchmark Models

In this chapter in addition to consider nonlinear and linear revenue functions, we consider two alternative cases: in the first case SD chooses which depots should operate and at the same time its premium depends on fixed costs, while in the second case this decision is not included in this planning process and is made by PD in advance. Because in the cost-based methods it is not possible to include fixed costs from depots in an appropriate way, we exclude the alternatives when production department takes these costs into account at the planning level. Hence, we consider only scenarios 3 and 4. Depending on the case, different benchmark values should be used. In the first case we use the solution from IM1, while in the second case we use the solution from IM2.

In our example we consider the alternative when PD decides to close two depots with highest operational costs: depot 2 and depot 4. Because feasible set in IM2 is smaller than in IM1, we expect that the solution of IM2 will be worse or equal to the solution of IM1.

We start by calculating internal prices for each of these four products, with the procedures described in section 6. Then, we use these price values as input to SM1 and SM2. The solution of SM1/2 provides the information about the amount of each product at each depot, which the sales departments wants to order. This output is used as input to PM.

The calculated internal prices are presented in Appendix C.

We also solve IM1 and IM2.

We start by presenting the results for nonlinear revenue models. Table 9.1 presents the results from benchmark models IM1 and IM2.

	Benchmark	
	IM1	IM2
Sale profit	8 622 383	8 618 889.576
Fixed costs	-730 000	-700 000
Production costs	-6 629 268,71	- 6 694 136.372
Profit BM	**1 263 114,09**	**1 224 753.204**

Table 9.1 – Results: Nonlinear Benchmark

In the optimal solution of IM1, depots 3 and 4 are not in operation. Because we have assumed that PD has closed another depots, the optimal solution of IM2 is lower.

The next table presents the results from our cost-based methods:

	Case 1 (scenario 3)		**Case 2 (scenario 4)**	
	Method 1	Method 2	Method 1	Method 2
Profit SM	481 065,0442	922 452,418	989 092,7052	1 570 561,044
INT.PRICE	5 195 803,513	4 673 101,981	6 347 221,703	6 893 040,998
Fixed cost	Included in SM	Included in SM	700 000	700 000
Cost PM	4,542,508,711	4,435,531,738	5 563 954,795	6 567 031,947
Profit	**1 134 359.846**	**1 160 022.661**	**1 072 359.613**	**1 196 570.095**
%deviation from BM	-10.19 %	-8.16 %	-12.44 %	-2.30 %

Table 9.2 – Results: Nonlinear Cost-Based Methods

Profit SM represents the objective function value of SD. Because this *Profit* includes internal costs (in term of internal prices SD paid to PD), we must add them back in order to calculate company's profit. *Cost PM*, represents the cost to PD associated with production and primary transportation of products ordered by SD. As we can observe from table 9.2, method 2 gives better results in term of profit. In the solution of SM1 sales department found it optimally that only two depots should operate: depot 2 and depot 5. An interesting observation is that under method 2, the profit in case 2 is higher than in case 1. This happens because SD failed to choose the "right" depots, also the sold amount of products is lower than in case 2. We note, that if SD closed depot 2 and depot 4, then *Profit SM* would be lower, while the company's profit would be higher. This example emphasizes the importance of finding the correct values of internal prices. We can observe that in the benchmark solutions sold amount is bigger than under the cost methods. The data indicates that some internal prices are set too high, and therefore SD has chosen to decrease the sale.

We repeat calculations with piecewise linear revenue function, and use SM1-L and SM2-L instead of SM1 and SM2. We also use linear benchmark models: IM1-L and IM2-L. The results are presented in Table 9.3 and Table 9.4.

	Benchmark	
	IM1L	IM2L
Sale profit	8 616 537.415	8 614 697
Fixed costs	730 000	700 000
Production costs	6 627 785.873	6 693 821
Profit BM-L	**1 258 751.542**	**1 220 876**

Table 9.3 – Results: Linear Benchmark

The solutions of IM1 and IM1-L are very similar, and it is optimal to close the same depots. As expected the profit is lower than in nonlinear case because linear approximation of concave function leads to under approximation.

	Case 1 (scenario 3)		**Case 2 (scenario 4)**	
	Method 1	Method 2	Method 1	Method 2
Profit SM	477 690,1013	919 274,8195	9,844,860,001	1,566,950,014
INT.PRICE	5 184 328,399	4 681 644,181	6275875,5	6,836,507,486
Fixed cost	Included in SM	Included in SM	700000	700000
Cost PM	4 528 494,118	4 447 077,819	5,519,833,328	6,515,322,669
Profit	**1133524.382**	**1153841.182**	**1040528.172**	**1188134.831**
%deviation from BM	-9.95 %	-8.33 %	-14.77 %	-2.68 %

Table 9.4 –Results: Linear Cost-Based Methods

We observe the same pattern in linear sales problems. Method 1 gives lower profit than method 2. As in nonlinear case, SD finds it optimal to use depot 2 and depot 5 for operation, under both methods. Again, under method 2, the company's profit is higher in case 2 than in case 1.

Chapter 10 - Numerical Tests on Lagrangean Decomposition

In our computational study of LD we try two alternative ways of selecting step length, for calculation of Lagrangean multipliers $\lambda_{p,d}$ in each iteration.

$$(\boldsymbol{t1}):\ \boldsymbol{t^{(n)}} = \frac{\boldsymbol{\sigma^{(n)}\left(UB^{(n)} - LBD\right)}}{\boldsymbol{\sum_{p\in P}\sum_{d\in D}\left(\lambda_{p,d}{}^{(n)}\right)^2}}$$

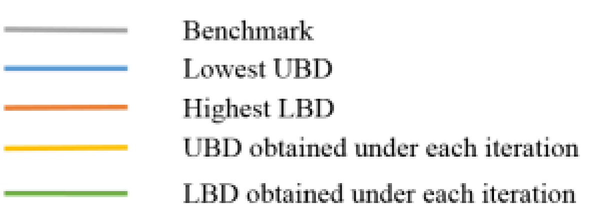
$$(\boldsymbol{t2}):\ \boldsymbol{t^{(n)}} = \frac{\boldsymbol{\sigma^{(n)}(UBD - LBD)}}{\boldsymbol{\sum_{p\in P}\sum_{d\in D}\left(\lambda_{p,d}{}^{(n)}\right)^2}}$$

Initial values of Lagrangean multipliers μ_d are set to zero. We try three different kind of initial values for $\lambda_{p,d}$. The first alternative is to set initial values of the multipliers to zero, later denoted $(\lambda 0)$. The second alternative is to start with multiplier values set to the shadow price values for constraints from the integrated model, later denoted as (λS). And the third alternative is to start with internal prices found in the cost-based method in chapter 9, later denoted (λC)**.** Initial start value for σ is 2. We update $\sigma \leftarrow 0{,}9\sigma$ if during the last five iterations change in UBD is less than 10. We set maximum number of iterations equal to 200, in order to compare how fast different models converge.

As in the previous chapter, depot 2 and depot 4 are closed in scenario 4.

10.1 – Initial Lagrangean multiplier values $(\boldsymbol{\lambda 0})$

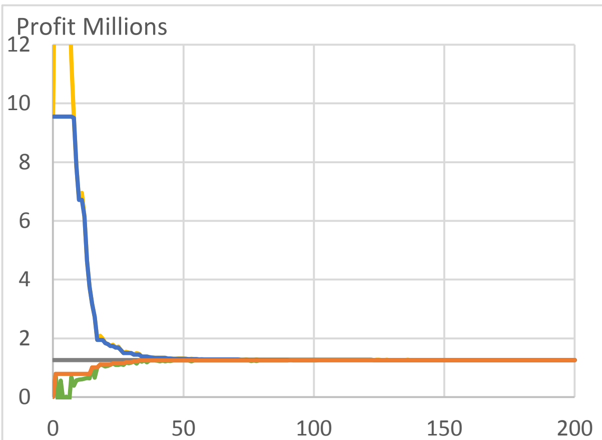
In all convergence plots we use the following designations:



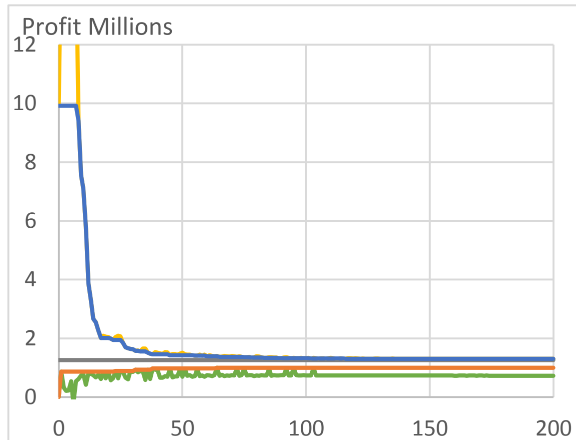
Nonlinear Sales Problems

Next graphs show convergence of lower and upper bounds according to the four scenarios and two alternative calculations of step length. Initial values of all Lagrangean multipliers are set to zero and sales models are quadratic programming (QP) problems.

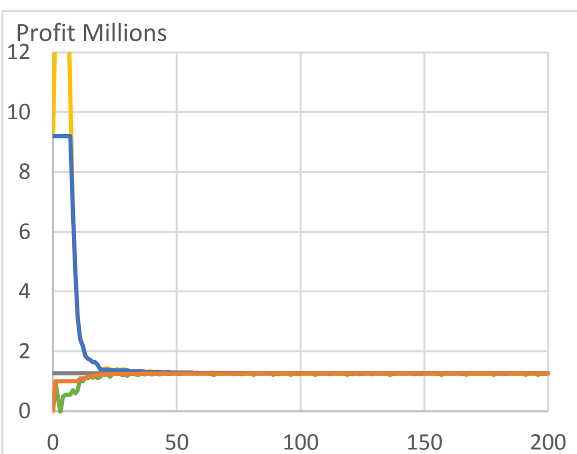
Convergence plot - Scenario 1 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 2$(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 3$(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 4 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$

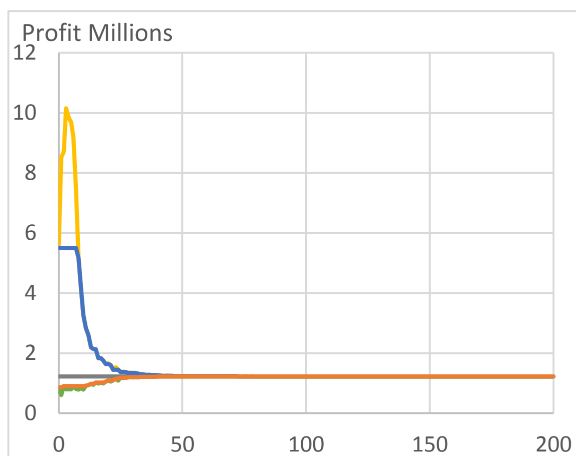
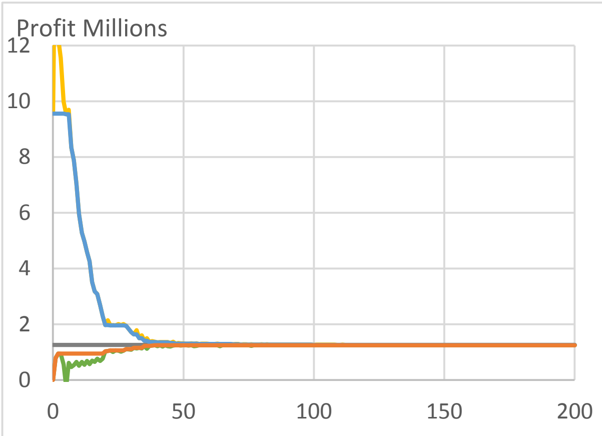
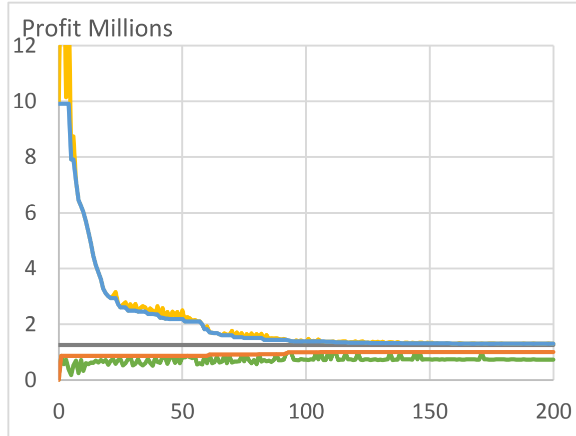


Figure 10.1 – Convergence plots $(t1), (\lambda 0)$

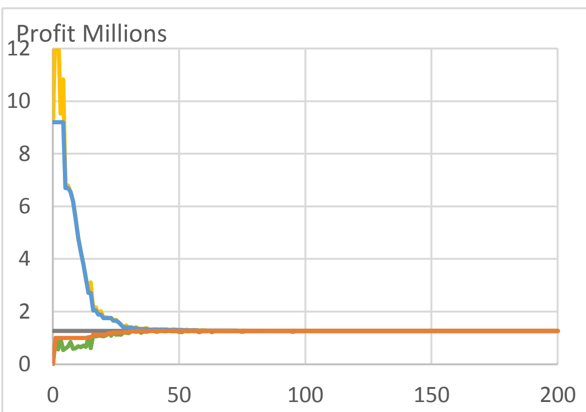
Convergence plot - Scenario 1 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 2 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 3 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 4 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$

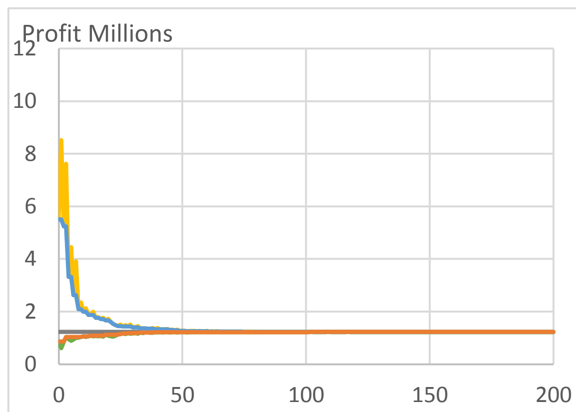


Figure 10.2 - Convergence plots $(t2), (\lambda 0)$

From the graphs we can observe that different choices of decomposition of the original problem lead to different results. It emphasizes the importance of carefully selection of variables and constraints that are duplicated. Also different choices of step length lead to different convergence in different models.

Table 10.1 shows the highest lower bound (feasible solution), the percent deviation of highest LBD and lowest UBD, and the percent deviation of the highest LBD from the optimal solution for each of scenarios after 5, 50 and 200 iterations, for each of the alternatives of step length calculations. In the last two rows we have showed iteration number when the gap between UBD and LBD is within the tolerance of 1%. Note that for scenarios 1, 2 and 3 the optimal solution is found by IM1, while for scenario 4 IM2 is used.

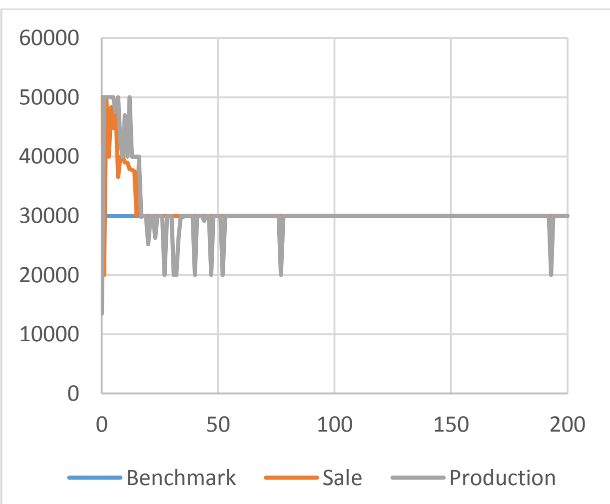
	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Alt.1 After 5 iterations				
LBD	790456,4043	868557,008	998068,67	901906,07
(UBD-LBD)/LBD	1108,73 %	1041,78 %	821,68 %	510,13 %
LBD/BM-1	-37,42 %	-31,24 %	-20,98 %	-26,36 %
Alt.2 After 5 iterations				
LBD	952973,65	868557,008	998068,67	968112,7
(UBD-LBD)/LBD	902,60 %	1041,78 %	821,68 %	223,16 %
LBD/BM-1	-24,55 %	-31,24 %	-20,98 %	-16,01 %
Alt.1 After 50 iterations				
LBD	1252310,947	976926,328	1257275,6	1221800,5
(UBD-LBD)/LBD	4,24 %	46,31 %	2,62 %	1,09 %
LBD/BM-1	-0,86 %	-22,66 %	-0,46 %	-0,24 %
Alt.2 After 50 iterations				
LBD	1251114,418	868557,008	1259593,7	1219485,5
(UBD-LBD)/LBD	4,61 %	153,22 %	2,79 %	3,53 %
LBD/BM-1	-0,95 %	-31,24 %	-0,28 %	-0,43 %
Alt.1 After 200 iterations				
LBD	1255414,896	1002181,81	1260566,2	1222365
(UBD-LBD)/LBD	0,74 %	29,64 %	0,57 %	0,21 %
LBD/BM-1	-0,61 %	-20,66 %	-0,20 %	-0,19 %
Alt.2 After 200 iterations				
LBD	1257420,721	1005001,43	1259628,7	1222097,1
(UBD-LBD)/LBD	0,53 %	29,77 %	0,29 %	0,27 %
LBD/BM-1	-0,45 %	-20,43 %	-0,28 %	-0,22 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	106	-	83	56
Alt 2	93	-	70	76

Table 10.1 – Summary: Nonlinear SM ($\lambda 0$)

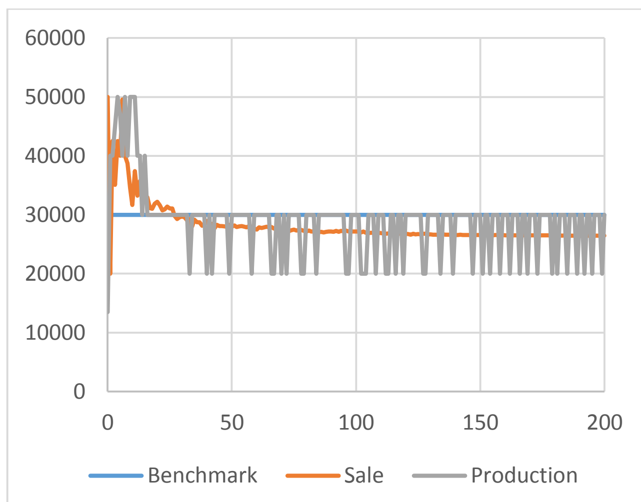
The best convergence, as expected, is obtained in scenario 4. In this scenario it is easier for departments to come to a common solution because fixed costs are excluded. However, as we have already pointed out in the previous chapter, the optimal solution under this scenario may be lower, hence one can obtain a better results in other scenarios. Scenario 3 has showed the next best convergence. Unlike scenario 4, the best convergence is obtained under the second alternative of step length calculation *(t2).* Also, Scenario 3 has provided the highest feasible solutions in many of iterations. Scenario 1 has showed a slower convergence than scenario 3. Scenario 2 has showed the worse convergence, and after 200 iterations the gap is still around 30%. Under the best three scenarios deviation of feasible solution from the optimal one is at most 0.86% after 50 iterations and 0.45% after 200 iterations. We notice that upper bounds in all four scenarios are relative similar, also in scenario 2. Therefore, the main reason for poor results under scenario 2 are low feasible solutions.

To get more insights about the solutions in different scenarios, we have included several graphs which show supply/ demand of the departments. The first four graphs illustrate the total volume of products supplied by PD and ordered by SD in each of iteration. We have also included the total volume of products from the solutions of the benchmark models. The next four graphs illustrate demand and supply only for one type of product. Because the graphs for all types of products are very similar we choose to include graphs for only one product, P1. All graphs are taken from the solution in which (t1) is used for step length calculation.

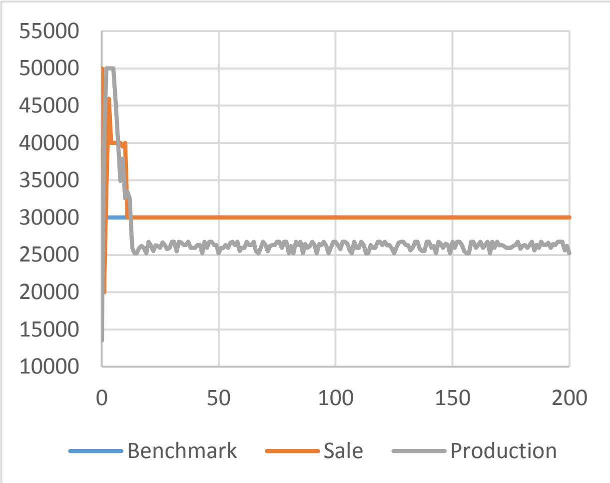
Total volume Scenario 1 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Total volume Scenario 2 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Total volume scenario 3 $(\boldsymbol{t1}), (\boldsymbol{\lambda0})$



Total volume scenario 4 $(\boldsymbol{t1}), (\boldsymbol{\lambda0})$

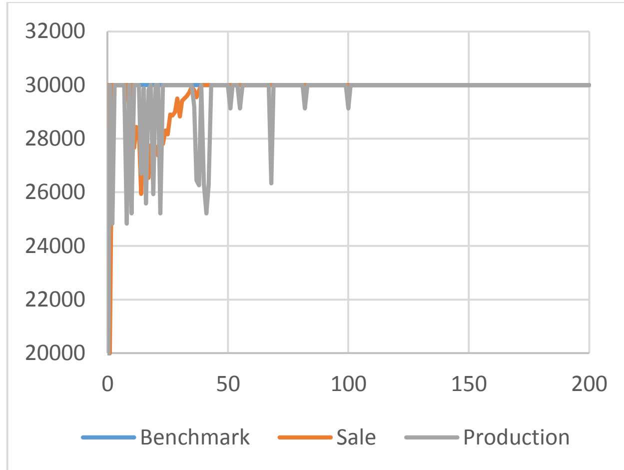
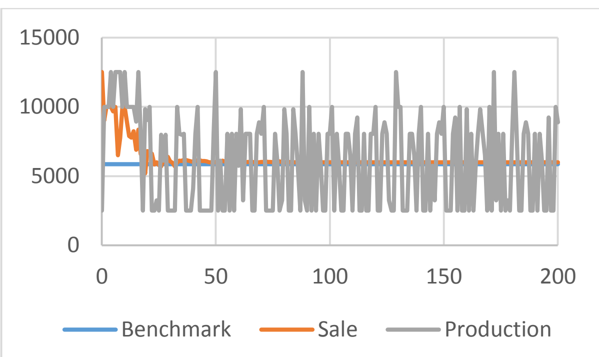
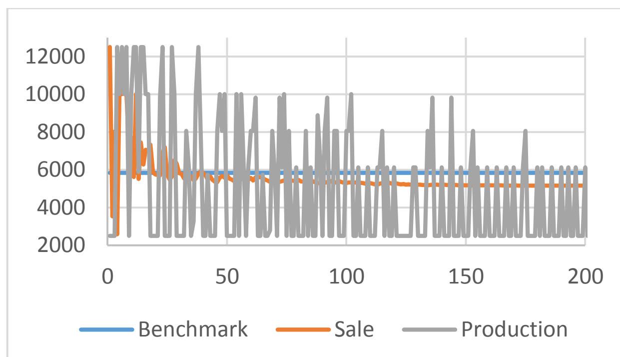


Figure 10.3- Total Volume $(t1), (\lambda0)$

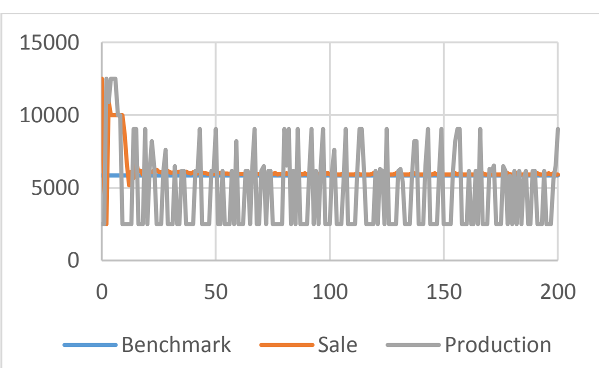
P1 Scenario 1 $(\boldsymbol{t1}), (\boldsymbol{\lambda0})$



P1 Scenario 2 $(\boldsymbol{t1}), (\boldsymbol{\lambda0})$



P1 Scenario 3 $(\boldsymbol{t1}), (\boldsymbol{\lambda0})$



P1 Scenario 4 $(\boldsymbol{t1}), (\boldsymbol{\lambda0})$

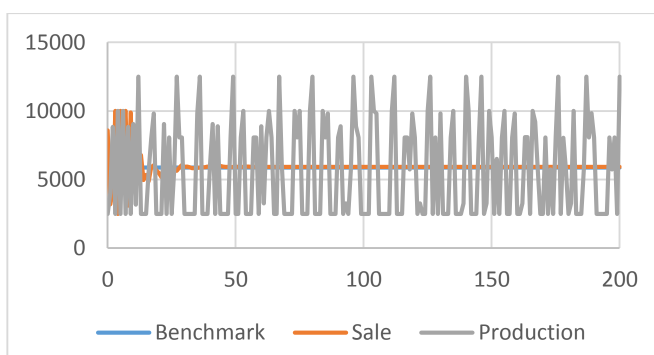


Figure 10.4- Volume P1 $(t1), (\lambda0)$

From the graphs, we can observe that the volume ordered by SD varieties less than the volume supplied by PD. Possible explanation, is that SD can control prices in the markets, while PD has to take product prices as given. Therefore when internal prices change, SD can also change the prices in the markets by increase/decrease the volume ordered (and sold), while PD doesn't has this flexibility.

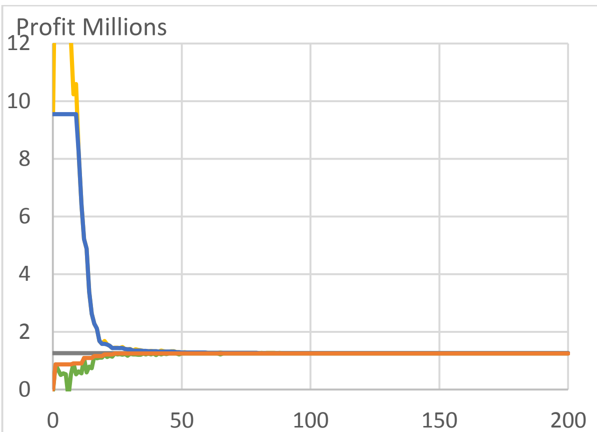
If we study the graphs for the total volume in scenarios 2 and 3, which are the scenarios in which only one department is taking fixed costs into account, we observe the following: the department that doesn't take fixed costs into account supply/order less than it is optimal. In scenario 2 the amount ordered by SD, when number of iteration is high, is always lower than the optimal amount from IM1. The same is true for the supply from PD in scenario 3. In scenario 1, in which both departments take fixed costs into account and at the same time it is one more Lagrangean multiplier that provides a better coordination of the depot use, departments succeed to supply and demand equal total amount of products. The same is true in scenario 4.

As we can notice from the graphs the main difference between scenario 2 and other scenarios is that the demand from SD is lower than the optimal. As we know a good feasible solution depends on good orders from SD, hence this explain the poor performance of scenario 2.

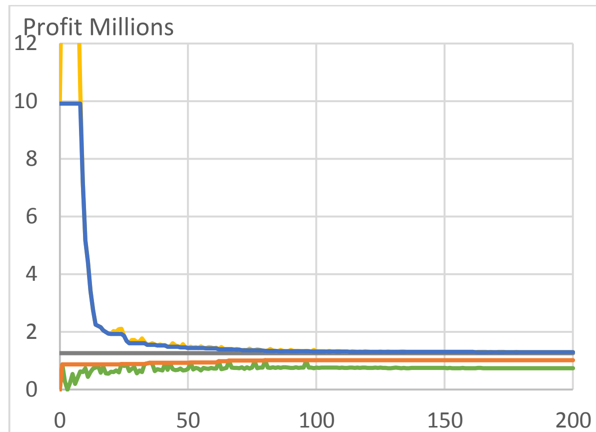
Linear Sales Problems

The next graphs show the convergence plots under the linear sales models. In the same way as above two alternative calculations of step length are considered and all initial values of Lagrangean multipliers are set to zero.

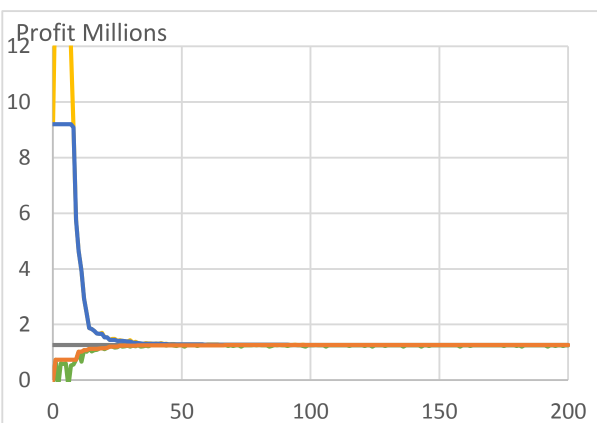
Convergence plot - Scenario 1 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 2 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 3 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 4 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$

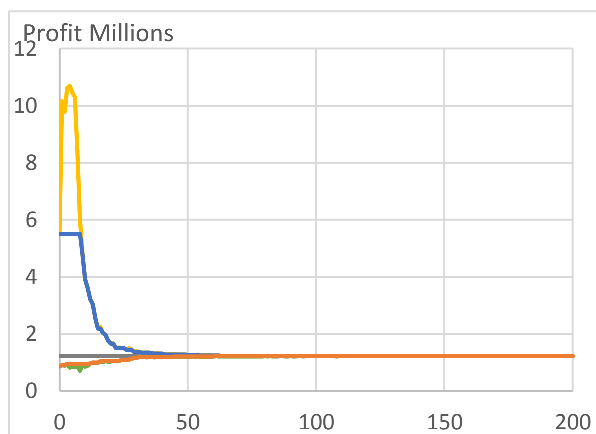
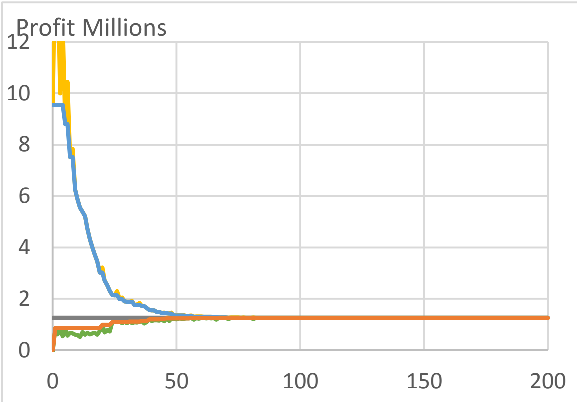
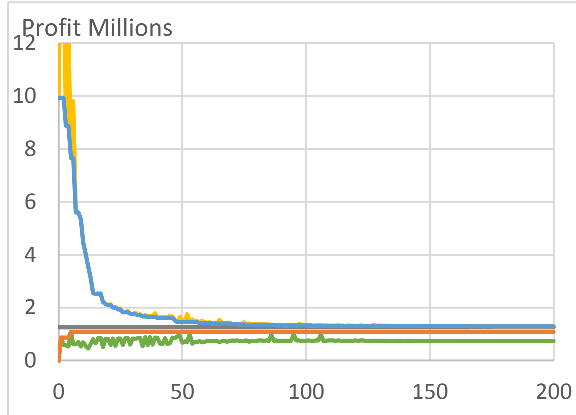


Figure 10.5- Convergence plots: Linear $(t1), (\lambda 0)$

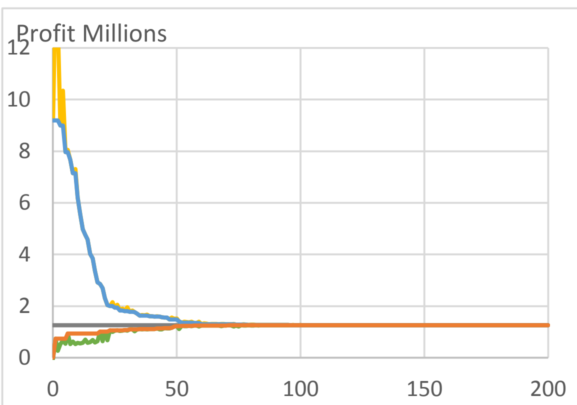
Convergence plot - Scenario 1 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 2 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 3 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Convergence plot - Scenario 4 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$

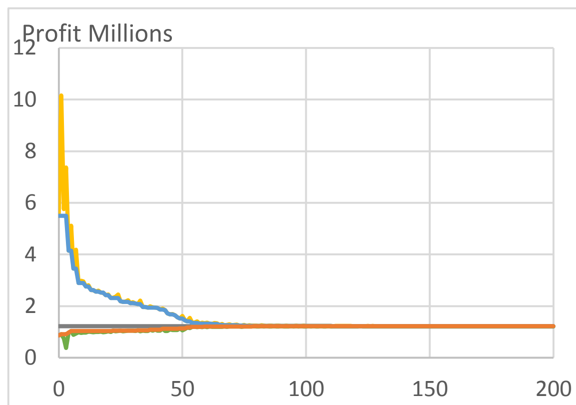


Figure 10.6 – Convergence plots: Linear $(t2), (\lambda 0)$

Table 10.2 shows the results from linear sales subproblems and has the same structure as Table 10.1.

Under linear sales problems the convergence in scenarios 1, 3 and 4 seems to be very similar. Under QP sales problems the tolerance of 1% is reached more quickly in scenario 4. While the convergences in scenario 1 and 3 are almost the same under both sales problems. Again scenario 2 has showed a bad convergence with a significant gap after 200 iterations.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Alt.1 After 5 iterations				
LBD	867519,7946	866979,795	736675,7	948397,775
(UBD-LBD)/LBD	1000,79 %	1043,29 %	1148,05 %	479,95 %
LBD/BM-1	-31,08 %	-31,12 %	-41,48 %	-22,32 %
Alt.2 After 5 iterations				
LBD	867519,7946	866979,795	736675,7	968112,696
(UBD-LBD)/LBD	1000,79 %	923,93 %	1121,30 %	223,16 %
LBD/BM-1	-31,08 %	-31,12 %	-41,48 %	-20,70 %
Alt.1 After 50 iterations				
LBD	1249350,661	921053,904	1257046,69	1204487,26
(UBD-LBD)/LBD	3,17 %	58,52 %	2,01 %	5,56 %
LBD/BM-1	-0,75 %	-26,83 %	-0,14 %	-1,34 %
Alt.2 After 50 iterations				
LBD	1239934,056	1080868,13	1203570,06	1128366,81
(UBD-LBD)/LBD	8,56 %	34,49 %	23,20 %	34,98 %
LBD/BM-1	-1,49 %	-14,13 %	-4,38 %	-7,58 %
Alt.1 After 200 iterations				
LBD	1251499,661	1012103,11	1257923,21	1217586,38
(UBD-LBD)/LBD	0,62 %	27,92 %	0,61 %	0,36 %
LBD/BM-1	-0,58 %	-19,59 %	-0,07 %	-0,27 %
Alt.2 After 200 iterations				
LBD	1256949,138	1080868,13	1257099,52	1217586,38
(UBD-LBD)/LBD	0,17 %	19,83 %	0,14 %	0,36 %
LBD/BM-1	-0,14 %	-14,13 %	-0,13 %	-0,27 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	106	-	84	81
Alt 2	93	-	85	105

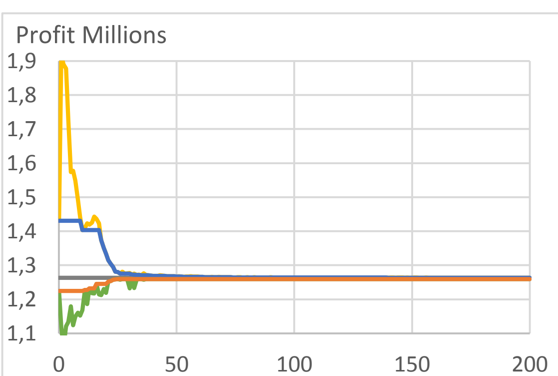
Table 10.2 – Summary: Linear SM ($\lambda 0$)

10.2 Initial Lagrangean multiplier values different from zero

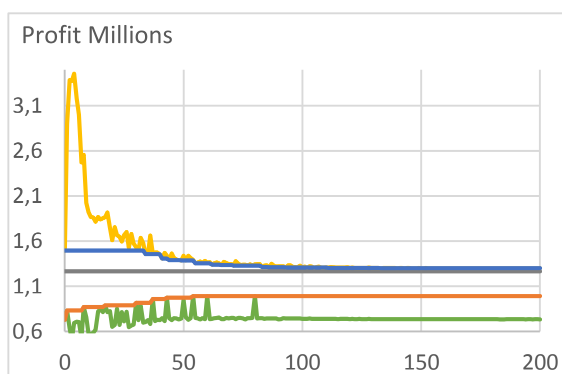
When initial Lagrangean multiplier values have been set to zero $(\lambda 0)$, LD method has started by producing bad optimistic bounds and bad feasible solutions (pessimistic bounds) with big gaps between these two bounds. Starting with values of initial multipliers set to zero can be interpreted as assuming that production managers know nothing about a good solution. This may not be reasonable assumption, and normally managers should have some ideas about the values of internal prices. The ideas can be based on historical information or some other estimates. These estimates could be used as initial values for Lagrangean multipliers. In our work we use two different initial values in addition to zero. The first alternative is to use shadow price values of constrain I1 from IM2 as starting values (when all depots are open), (λS). The shadow price for a constraint, also cold dual value, can be interpreted as the change of the optimal objective function value when the right-hand-side is increased by one unit (Lundgren et al. 2010). We note that these values are unknown in practice, because the integrated model doesn't exist and optimal solution is not known *a priori*. Therefore, the obtained results in our example may be better than in practice. The second alternative is to use internal prices which we have found in method 2 (in chapter 9), (λC). The initial values for the second Lagrangean multiplier μ_d used in scenario 1, are set to zero as previously. Hence, we have assumed that managers don't have any additional information about depots other than fixed costs.

Below we have presented graphs, in which we have used $(t1)$ for step length calculation and QP sales problems. Other graphs together with tables are presented in Appendix D.

Convergence plot - Scenario 1 $(\boldsymbol{\lambda S}), (\boldsymbol{t1})$



Convergence plot - Scenario 2 $(\boldsymbol{\lambda S}), (\boldsymbol{t1})$



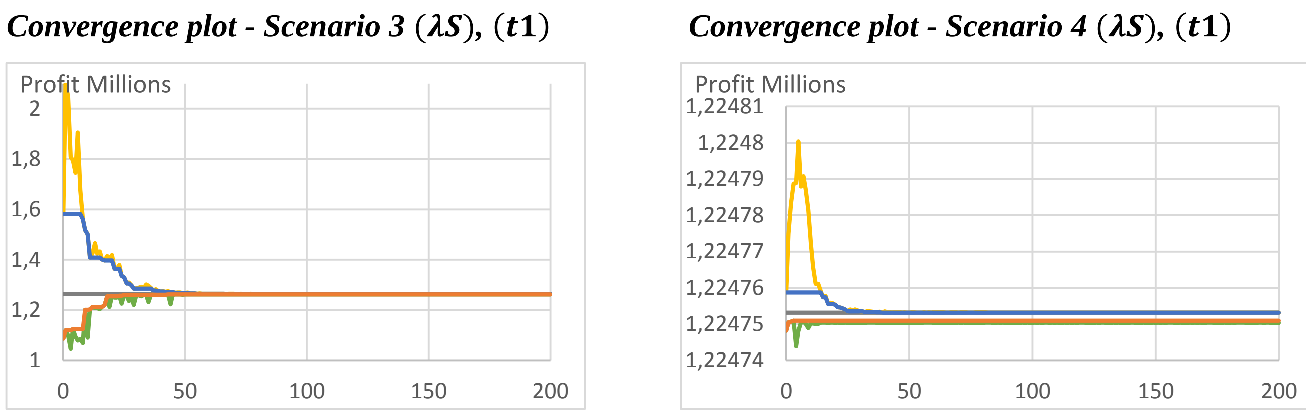


Figure 10.7 – Convergence plots $(t1), (\lambda S)$

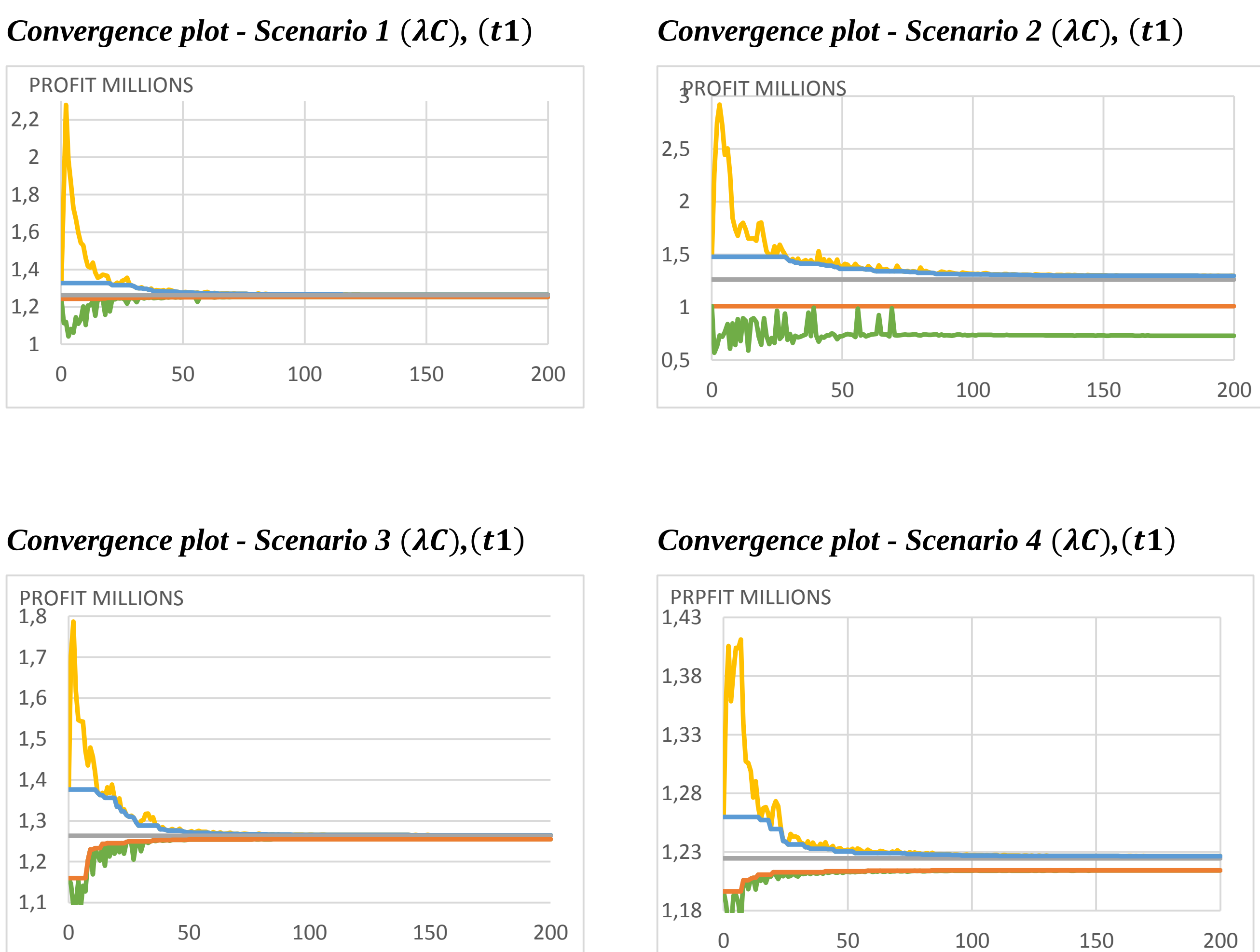


Figure 10.8 – Convergence plots $(t1), (\lambda C)$

In all scenarios, the fastest convergence is obtained when λS are used as initial values for Lagrangean multipliers. In scenario 4 the difference between lower and upper bonds is very small already after the first iteration, while the obtained feasible solution is optimal. This can be explained by the fact that the initial internal prices have been taken from the optimal solution of benchmark model for this scenario.

As expected when λS or λC are used as initial values, after a small number of iterations the gap between lower and upper bounds is significantly smaller than when $\lambda 0$ is used. Also after a few iterations, in all scenarios except scenario 4, λC gives better results than λS. We can also observe that after "a big" number of iterations, $\lambda 0$ outperforms λC. In some of the cases, method with λC doesn't manage to achieve 1% tolerance between lower and upper bounds, whereas in the same cases the tolerance is achieved when $\lambda 0$ is used. In these cases with (λC), we can observe that the lower bound gets stuck at some values while the upper bound continues to decrease. We have not found any significant differences between the results from linear and QP sales problems.

Burton and Obel (1980) have investigated the behavior of different decomposition models (applied to different planning approaches) in the first few iterations under varying types of *a priori* information. The authors have showed that *a priori* information used by companies has effect on the performance of companies' plans. Burton and Obel have claimed that relevant information leads to better results. From our observations we obtain a similar results after a small number of iteration. If company uses up to 50-60 iterations, then the best results are obtained when initial values of Lagrangean multipliers are based on some estimates or other available information. However, if company uses a large number of iterations or it uses convergence criteria for stopping, then starting with the estimates made by the company doesn't necessarily lead to a better results.

10.3 Comments on the Calculation

Step length calculation

As we have pointed out in chapter 7, the selection of step length is important to guarantee convergence. From our example we can observe that under different step length selections the convergence of lower and upper bounds have showed different behaviors. The most obvious difference between $(t1)$ and $(t2)$ alternatives is under method (λC), where the 1% tolerance under QP sales problem is obtained only with $(t1)$. In linear sales problems, this tolerance has not been met, however in all scenarios, $(t1)$ gives tighter bounds than $(t2)$. Also under method (λS) with linear sales problems, $(t1)$ seems to outperform $(t2)$. While in QP sales problems we cannot observe the same pattern. Also in method $(\lambda 0)$, we cannot conclude that one of the alternatives has showed better results than the other. In some cases $(t1)$ gives better results, in other cases $(t2)$ does it.

The calculation of step length for the second Lagrangean multiplier μ_d, associated with duplication of binary variables, is not carried out with the formulas $(t1)$ or $(t2)$. The results obtained with these formulas are very poor. In Appendix D we have showed these results under method $(\lambda 0)$ for QP sales problem.

Also we have tried another updating of σ multiplier from the step length formula: $\sigma \leftarrow 0{,}5\sigma$. The results obtained under LD$(t2)$ are similar to the results under 0.9 updating rule. However, a much worse convergence is obtained under $(t1)$ alternative of step length calculation, which is a more traditional choice (Fisher, 1985). The results with 0.5 updating rule are presented in Appendix D for LD$(\lambda 0)$. Possible explanation for this outcome is that under this approach step length convergences to zero too quickly, and therefore we obtain a convergence to a point different from the optimal solution.

As we have seen, the calculation of step length affects the solution process. Different problems formulation require different selection of step length in order to obtain fast convergence. The standard methods for selection of step length may not give the best convergence. LD will therefore require some computational effort in order to find a good step length selection

Linear vs nonlinear sales problems

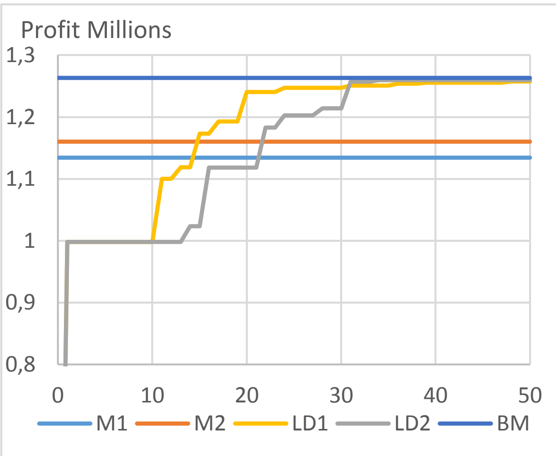
In our example, the convergence of the lower and upper bounds occurs more slowly in cases with linear sales problems. Because in our nonlinear models we have used linear demand functions, the revenue function is quadratic and the sales subproblems are relative easy to solve. In reality demand functions can be more complicated and in order to model them SD would have to use a complicated function which could lead to non-convex sales subproblem, and hence optimal solution could not be guaranteed. In these cases it could be easier to model revenue with piecewise linear function, and avoid complex demand functions and non-convex problems.

10.4 Comparison of the results between the mechanisms

The main idea of the cost-based methods is to reflect all variable costs associated with products down to depot stage. Internal prices which have been calculated by these methods, take into account the costs related to production and primary transportation. In contrast to these methods, LD also takes into account secondary distribution costs and revenue created by products in markets.

In this section we compare the results from cost-based methods and LD. In chapter 9 we have assumed scenarios 3 and 4, when we have applied these methods. Hence we compare corresponding results from LD. We start with $\lambda 0$ as initial values for Lagrangean multipliers in LD method.

Non-linear - Scenario 3 $(\boldsymbol{\lambda 0})$



Non-linear - Scenario 4 $(\boldsymbol{\lambda 0})$

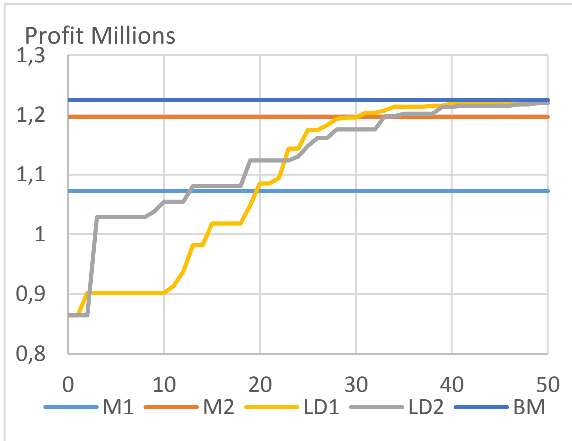
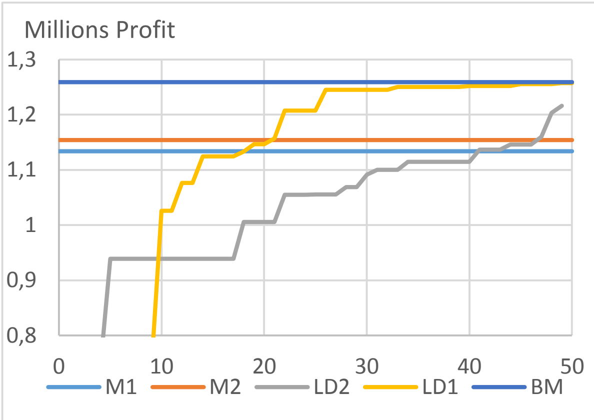


Figure 10.9 – Nonlinear: Cost-Based vs LD $(\lambda 0)$

As we have showed in chapter 9, cost-based method 2 has provided a relative good solution in scenario 4 relative to scenario 3. This explains that, the performance of method 2 compared to LD, is significantly better in scenario 4 than in scenario 3. In scenario 3 it takes approx. 15 iterations before LD outperforms both cost-based methods. Whereas in scenario 4, LD outperforms method 2 only after approx. 40 iterations. .

Linear - Scenario 3 $(\boldsymbol{\lambda 0})$



Linear – Scenario 4 $(\boldsymbol{\lambda 0})$

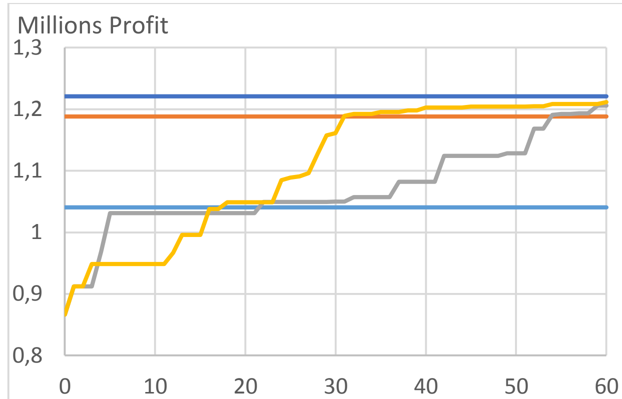


Figure 10.10 – Linear: Cost-Based vs LD $(\lambda 0)$

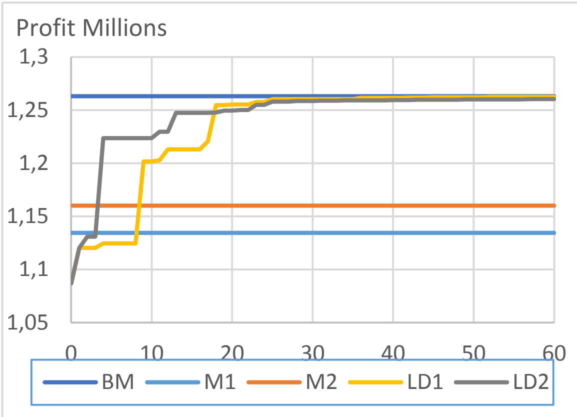
In the linear sales problems in scenario 3, LD has outperformed both cost-based methods after approx. 20 iterations, when step length is calculated according to $(t1)$. If step length is calculated according to *(t2)*, it takes a significantly higher number of iterations to outperform both cost-based methods, approx. 40. In scenario 4 it takes approx. 20 iterations before LD outperforms method 1. Because method 2 has performed well in this case, it takes more iterations to outperform this method, especially under second alternative of step length calculation.

In general, cost-based methods outperform LD $(\lambda 0)$ when the number of iterations is small, and opposite when the number of iterations is relative big.

It doesn’t make sense to compare results from cost-based methods with LD(λC), because in this case LD will obtain the same result as in method 2 in the first iteration. Therefore the best feasible solutions of LD(λC) will never be worse than in the cost-based methods.

Below, we compare results from LD(λS) with the cost-based methods. As we have showed in the previous section, LD(λS) gives optimal results from the first iteration in scenario 4. Therefore we compare results only from scenario 3.

Scenario 3 - Non-linear ($\boldsymbol{\lambda S}$)



Scenario 3 – Linear ($\boldsymbol{\lambda S}$)

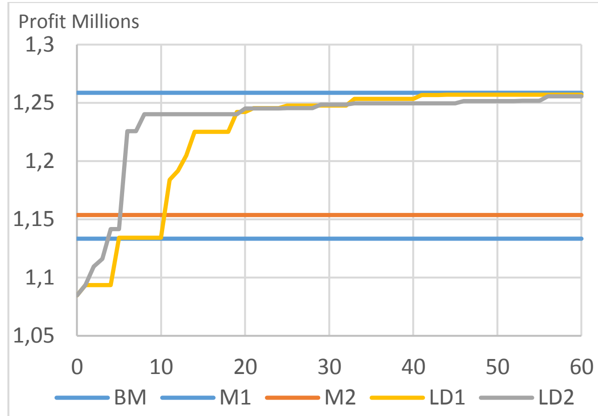


Figure 10.11- Scenario 3: Cost-Based vs LD (λS)

As we can observe from figure 10.11, in both linear and QP sales problems, LD ($t1$) outperforms both cost-based methods after 10 iterations. LD ($t2$) has the fastest convergence at the beginning and outperforms cost-based methods already after approx.5 iterations.

From these results we have observed that LD(λS) outperforms cost-based methods after a few iterations. As expected the number of iterations required to outperform cost-based methods is significantly smaller in LD(λS) than in LD ($\lambda 0$). We have already pointed out that (λS) values would not be known in the absence of the optimal solution. However, we expect that the company is able to set realistic estimates of internal prices, and based on these estimates it would take less iterations to outperform cost-based methods, than when ($\lambda 0$) is used.

PART 4: CONCLUSION

Chapter 11 – Summary of Findings

In this chapter we summarize our findings. We start with discussion about allocation of decisions and premium sharing rules, which are assumed in different scenarios. Then we discuss the main advantages and disadvantages with mechanisms proposed in our thesis. We conclude by summing up the work we have done.

11.1 Discussion about Scenarios

Allocation of decisions

In scenario 4 we consider a case in which PD/company does not want to involve SD in the decision process about operation of depots. The reason behind this allocation of decision can be that the decision about drift of depots is of operational type, and therefore should be taken by operational units. Also such allocation of decision may lead to an easier planning process ensuring faster decisions making, and can therefore be preferred by the company. From the solution under the cost-based mechanism, we observe that in scenario 4 the profit obtained by the company is higher than in scenario 3, when Method 2 is applied. It can partly be explained by a relative good choice made by PD about operation of depots. This illustrates that the performance of the company in scenario 4 depends on the choice made by PD, and the solution can be both very good and very poor depending on which depots are closed. Therefore, when such distribution of decision is used, PD should have an idea about depots that should operate. Simple methods, like close down depots with highest fixed costs may lead to wrong decisions, which can be crucial for the company's performance.

Because SD chooses from which locations products will be transported to the markets, we can also argue that SD should be responsible for the decision about operation of depots. This is assumed in scenarios 1, 2 and 3. As we have showed in our numerical example of LD mechanism, after some number of iterations scenarios 1 and 3 outperform scenario 4 in term of company's profit, in all cases. However, under scenario 4 we obtain the fastest convergence.

Premium sharing Rule

Because in scenario 4 the decision about operation of depots is taken by PD, premium to SD doesn't depend on fixed costs. In scenarios 1, 2 and 3 SD is involved in the decision about depots, while fixed costs are taken into account only in scenarios 1 and 3. In scenario 2 sales department makes the decision about depots but doesn't take into account any costs associated to them (except those that are incorporated in internal prices). As we have showed, in this scenario the solution is very poor relative to other scenarios. The main reason for that is that SD chooses too many depots for operation. If SD is responsible for operation of depots, it can also be reasonable that the department should take into account fixed costs. At the same time, it may be reasonable to assume that in order to achieve coordination and avoid suboptimal solutions PD should also take these costs into account. Therefore we have expected that in scenario 1 we will obtain a faster convergence than in scenario 3. But as we can observe from the results, scenario 3 seems to outperform scenario 1 in nonlinear case. When linear sales problems are used the results between these two scenarios are very similar. Because scenario 3 provides the best results, it is very attractive to conclude that this is the best decomposition choice in this situation. However, we should be carefully to make such conclusion as the results obtained in our example may change with input data, and because it is clear that scenario 1 provides a better coordination between departments we should not exclude possibility that this scenario may be a better choice in some situations. On the other hand, scenario 1 may have a disadvantage in terms of premium calculation. Because in this scenario at the decision level, fixed costs are adjusted with Lagrangean multipliers, the premium to SD may be unreasonably low or high if the values of these multipliers are very small/large.

In the cost-based methods, in scenario 3, the calculated internal prices have led to relative small sales in the markets and low profit. This is partly caused by fixed costs to depots. When premium to SD depends on fixed costs there is a risk that the department will decrease the sale in order to get lower fixed costs, and at the same time higher prices in the markets. By doing this SD will increase its premium, while profit to the company will not necessarily be improved.

Summary

We sum up our findings. The choice of allocation of the depot decision should be seen in context with mechanism used to decide internal prices. As we have seen, the results in different scenarios depend on how internal prices are determined. When the cost-based methods are used, there is a relative high probability to obtain a solution which is far away from the optimum. Therefore, to entrust SD the decision about depots may be not the best choice. On the other hand, when LD is used, choices made by departments are corrected through the updating of internal prices (Lagrangean multipliers). In this situation it may be better to involve SD in the decision process about operation of depots. Fast convergence and easy planning process in scenario 4 must be seen in context with PDs possibility to make good choices about which depots should be operating without involving SD.

If the company decides to delegate the depot decision to the sales department, then the costs associated with depots drift should be reflected in the premium of the department. In our numerical example, we have not found any evidence for why it would be better to assign the depot decision to both departments at the planning level, and at the same time we have argued that this could lead to an unrealistic SDs premium.

11.2 Discussion about LD and Cost-based methods

Ability to know when the optimal solution is found

When the optimal solution is unknown, it's difficult to make any conclusions about how good are the solutions provided by methods in a decoupled setting. One of the advantages with LD is that we can observe when the solution is the optimal one (or is close to optimal). LD provides an optimistic bound which is an upper bound in our problem (lower bound in minimization problem). If the value of the optimistic bound equals to the value of feasible solution then we can conclude that the optimal solution has been found. However, because the approach that we have used in order to find feasible solutions is a heuristic, an optimal solution cannot be guaranteed. We also expect that many of the real world problems in oil companies have non-convex constraints (for example blending constraints). In non-convex problems LD is still applicable, but as we have pointed out in chapter 7.1, upper bounding will not be guaranteed with the subgradient method. This implies that optimistic bound may not converge to the optimal solution and a *duality gap* will occur (Lundgren et al, 2009). As Guignard (2003) has pointed out, clever implementations of solving Lagrangean dual, with powerful heuristics imbedded at every iteration, is important to achieve good solutions. In the cost-based methods proposed in our work, or some other cost-based approaches it is difficult to say whether the solution is near or far from optimum.

Ability to obtain the optimal solution in a decoupled setting

Often cost-based methods are used because they are easy to apply in a decoupled setting. However, as has been showed by Guajardo et al (2013b) traditional approaches for cost based pricing lack clarity in divergent supply chain problems and do not provide optimal results. Also in our work the proposed cost-based methods don't achieve the optimal solution. The optimal solution (may in some cases be local optimal if the overall problem is non-convex) can be achieved if it is possible to integrate sales and production decisions into one problem. Also, as it was showed by Guajardo et al (2013b) if producer could incorporate information about sellers' behavior when deciding the internal prices the resulting model could outperform traditional cost methods. In our work we have showed that by implementing the LD mechanism in a decoupled setting it is possible to achieve the same solution as in integrated planning and at the same time allow departments to maintain their autonomy.

Solution after a few iterations

In chapter 2 we have mention that one important property of the decomposition method is that a relative "good" solution should be obtained with a small number of information exchanges between departments. In the LD mechanism, difference between UBD and LBD indicates how good the solution is. Usually a good solution is characterized by a "small" gap between UBD and LBD. As we can observe from our numerical example, in most of the cases 50-100 iterations are required to obtain 1% tolerance ($\frac{\mathrm{UBD}-\mathrm{LBD}}{\mathrm{LBD}} \leq 0.01$), while in some cases this tolerance is not reached. How many iterations are required to obtain a "good" solution depends on the percentage tolerance required. For instance, in our example 2% tolerance would require a significantly lower number of iterations, and would be reached in all cases except scenario 2. We also expect that the number of iterations required to obtain a given level of tolerance will increase with the size of problem. If company doesn't has possibility to carry out many information exchanges, the company can restrict the number of iterations to some suitable number. In the situations with relative small number of iterations, as we have showed the bests results are achieved when production managers use some estimates to find initial internal prices. The drawback of the restriction on information exchange, is that it may not be possible to say how close the obtained solution is to the optimal one, because even if the gap between optimistic and pessimistic bounds is big, the pessimistic bound may still be close to the optimal solution. Use of traditional cost-based methods do not require exchange of information at the planning level, and therefore makes computation at this level much easier. Also if *a priori* information is not used in the LD mechanism, cost-based methods will most likely outperform LD if the number of iterations is small.

Implementation of mechanisms

Optimistic bounds obtained during the solution procedure are rarely (or never) feasible. Because of this, heuristics are implemented. In our problem we are interesting to find internal prices, which will ensure that SD makes profitable decisions. Therefore we choose the heuristic which is quite specific. In our model we have made assumption that amount of available crude oil doesn't restrict SD, in other words there are always enough resources to satisfy demand faced by the company. When this assumption is in place all solutions generated by the heuristic are feasible. If we had not made this assumption, then in many of iterations, the heuristic would have difficulties to find feasible solutions. This situation would arise, including the case in which $\lambda 0$ are used as initial values for the Lagrangean multipliers, because low internal prices lead to huge orders from SD, which cannot be satisfied by the company with limited resources. Also as we have showed the step length calculation is important to guarantee convergence, and should be selected carefully according to specific of the problem. Because the solution process of LD demands a great amount of calculations and may be difficult to understand for practitioners, the cost-based methods may be preferred by the companies.

11.3 Summary and Further Research

In our work, we have studied the issues between production and sales planning processes in an oil company. In addition to decisions about production, distribution and sale we have considered possibility to include fixed costs associated to operation of depots (warehouses). We have considered several possible ways the decision about operation of depots can be allocated between production and sales departments. Depending on this allocation, we have also considered different ways of calculation of sales department's premium. According to the distribution of decisions and premium calculation we have formulated sales and production subproblems in a decoupled setting. In order to achieve coordination between departments we have used internal prices. We have proposed two mechanism for setting internal prices.

To measure the results achieved in these mechanisms, we have also developed integrated models. These models have assumed centralized planning and have been used as theoretical benchmarks, because such planning may not be possible in reality.

The idea of the first mechanism has been to find internal prices based on costs faced by PD, and we have suggested two such cost-based methods. The first costs-based method has not taken into account the cost interrelation between products at all, while the second method has incorporated this interrelation to some extent. In the numerical example, the second method has showed a better results than the first one in term of companies' profit. However, the results have been below optimum.

In the second mechanism, Lagrangean decomposition has been used to determine internal prices. According to the allocation of decisions and premium calculations several decompositions of the original problems have been considered. In our numerical example, we have showed that use of LD makes it possible to achieve equally good results in the decoupled planning as in the centralized planning, if company allows for some information exchange between departments at the planning level. We have also showed that formulation of subproblems affects the convergence behavior. Also fixed costs associated to operation of depots lead to slower convergence. Several alternatives of updating convergence parameters in LD and different initial values of Lagrangean multipliers, have been considered. We have showed that the most common choice of step length calculation in subgradient method doesn't necessarily lead to the best results. As we have expected, when values calculated on the base of some available information have been used as initial Lagrangean Multipliers, better results have been achieved after a few iterations. However, starting with multiplier values based on some *a priori* information doesn't necessarily lead to a tighter bound.

In the numerical example we have also showed, if company allows a moderate number of information exchange between departments, LD outperforms the cost-based methods.

In all models we have considered two different ways of revenue functions formulation. The first way has assumed quadratic model with linear demand functions, while the second one has assumed a piecewise linear approximation of the revenue functions, and has been modeled with binary variables. The linear models have showed somewhat slower convergence in LD in some of the cases. However, in most of the cases models have showed a similar results.

All models presented in our work are single period models and don't considered inventory possibility. In reality companies are dealing with multi-period models, which include inventory planning. Also a full size problems include much more variables and parameters. As we have seen in our work, in some cases, small changes in problem formulation and input have led to a significant change in the results. Hence, the question for further research is how to apply the suggested methodology to a more sophisticated problems, and how this sophistication will affect the solution processes and results. Another direction for further research is to consider other methodologies than traditional subgradient method to solve Lagrangean duals.

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Appendices

Appendix A

Appendix B

Appendix C

Appendix D

Appendix E

Appendix A

SM2 Linear

Objective function

$$Maximize\ Sale\ Profit^{SM2-Linear}: \sum_{p\in P}\sum_{k\in K} revenue_{p,k} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathrm{C}^{\mathrm{TRAN3}}_{\mathrm{d,k}} \times \mathrm{z}_{\mathrm{p,d,k}}$$
$$-\sum_{p\in P}\sum_{d\in D} \pi_{p,d} \times \mathrm{q}^{S}{}_{\mathrm{p,d}}$$

Subject to (SML1) to (SML5), (S3) and (G2) and

$\mathrm{q}^{S}{}_{\mathrm{p,d}}, \mathrm{z}_{\mathrm{p,d,k}}, revenue_{p,k}, w_{m,p,k} \geq 0 \qquad l_{m,p,k} \in \{0,1\}$

***IM2*:**

Objective function

$$Maximize\ Contribution^{IM2}: \sum_{p\in P}\sum_{k\in K}\left(A_p \sum_{d\in D} \mathrm{z}_{\mathrm{p,d,k}} - b_p \sum_{d\in D} \mathrm{z}_{\mathrm{p,d,k}}{}^{2}\right) - \sum_{i\in I}\sum_{r\in R} C^{Buy}_{i,r}\ \mathrm{x}_{\mathrm{i,r}}$$
$$-\sum_{i\in I}\sum_{r\in R} C^{PRO}_{i,r}\ \mathrm{x}_{\mathrm{i,r}} - \sum_{e\in E}\sum_{i\in I}\sum_{r\in R} \mathrm{C}^{\mathrm{PRO2}}_{\mathrm{e,r}}\ \mathrm{v}_{\mathrm{e,i,r}}$$
$$-\sum_{p\in P}\sum_{d\in D}\sum_{h\in H} \mathrm{C}^{\mathrm{BLEND}}_{\mathrm{p,h}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}}$$
$$-\sum_{r\in R}\sum_{h\in H} \mathrm{C}^{\mathrm{TRAN1}}_{\mathrm{r,h}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P} \bar{y}_{\mathrm{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B} \bar{v}_{\mathrm{p,c,b,i.r}}\right)$$
$$-\sum_{h\in H}\sum_{d\in D}\sum_{p\in P} \mathrm{C}^{\mathrm{TRAN2}}_{\mathrm{h,d}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathrm{C}^{\mathrm{TRAN3}}_{\mathrm{d,k}} \times \mathrm{z}_{\mathrm{p,d,k}} \qquad (IM2)$$

Subject to (P1) to (P11), (S1) to (S2), (I1) and (G2) and

$\mathrm{x}_{\mathrm{i,r}}, \mathrm{y}_{\mathrm{e,i,r}}, \mathrm{v}_{\mathrm{b,i,r}}, \widetilde{y}_{\mathrm{e,i,r}}, \widetilde{v}_{\mathrm{c,b,i,r}}, \overline{y}_{\mathrm{p,e,i,r,h}}, \overline{v}_{\mathrm{p,c,b,i,r,h}}, \mathrm{z}_{\mathrm{p,d,k}}, \tilde{\mathrm{q}}_{\mathrm{p,d,h}}, q_{p,d}, \theta_{p,k} \geq 0$

IM Linear

Objective function

$$Maximize\ Contribution^{IML}: \sum_{p\in P}\sum_{k\in K} revenue_{p,k} - \sum_{i\in I}\sum_{r\in R} C_{i,r}^{Buy}\ \mathrm{x}_{\mathrm{i,r}} - \sum_{i\in I}\sum_{r\in R} C_{i,r}^{PRO}\ \mathrm{x}_{\mathrm{i,r}}$$
$$- \sum_{e\in E}\sum_{i\in I}\sum_{r\in R} \mathrm{C}_{\mathrm{e,r}}^{\mathrm{PRO2}}\ \mathrm{v}_{\mathrm{e,i,r}}$$
$$- \sum_{p\in P}\sum_{d\in D}\sum_{h\in H} \mathrm{C}_{\mathrm{p,h}}^{\mathrm{BLEND}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}}$$
$$- \sum_{r\in R}\sum_{h\in H} \mathrm{C}_{\mathrm{r,h}}^{\mathrm{TRAN1}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P} \bar{y}_{\mathrm{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B} \bar{v}_{\mathrm{p,c,b,i.r}}\right)$$
$$- \sum_{h\in H}\sum_{d\in D}\sum_{p\in P} \mathrm{C}_{\mathrm{h,d}}^{\mathrm{TRAN2}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathrm{C}_{\mathrm{d,k}}^{\mathrm{TRAN3}} \times \mathrm{z}_{\mathrm{p,d,k}} - \sum_{d\in D} C_d^{FIX} h_d \quad (IML)$$

Subject to (P1) to (P11), (SML1) to (SML5), (I1) and (G1) and

$\mathrm{x}_{\mathrm{i,r}}, \mathrm{y}_{\mathrm{e,i,r}}, \mathrm{v}_{\mathrm{b,i,r}}, \tilde{y}_{\mathrm{e,i,r}}, \tilde{v}_{\mathrm{c,b,i,r}}, \bar{y}_{\mathrm{p,e,i,r,h}}, \bar{v}_{\mathrm{p,c,b,i,r,h}}, \tilde{\mathrm{q}}_{\mathrm{p,d,h}}, q_{p,d}, \mathrm{z}_{\mathrm{p,d,k}}, revenue_{p,k}, w_{m,p,k} \geq 0$

$h_d, l_{m,p,k} \in \{0,1\}$

IM2 - Linear:

Objective function

$$Maximize\ Contribution^{IM2}: \sum_{p\in P}\sum_{k\in K} revenue_{p,k} - \sum_{i\in I}\sum_{r\in R} C_{i,r}^{Buy}\ \mathrm{x}_{\mathrm{i,r}} - \sum_{i\in I}\sum_{r\in R} C_{i,r}^{PRO}\ \mathrm{x}_{\mathrm{i,r}}$$
$$- \sum_{e\in E}\sum_{i\in I}\sum_{r\in R} \mathrm{C}_{\mathrm{e,r}}^{\mathrm{PRO2}}\ \mathrm{v}_{\mathrm{e,i,r}}$$
$$- \sum_{p\in P}\sum_{d\in D}\sum_{h\in H} \mathrm{C}_{\mathrm{p,h}}^{\mathrm{BLEND}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}}$$
$$- \sum_{r\in R}\sum_{h\in H} \mathrm{C}_{\mathrm{r,h}}^{\mathrm{TRAN1}}\left(\sum_{e\in E}\sum_{i\in I}\sum_{p\in P} \bar{y}_{\mathrm{p,e,i,r,h}} + \sum_{c\in C}\sum_{i\in I}\sum_{p\in P}\sum_{b\in B} \bar{v}_{\mathrm{p,c,b,i.r}}\right)$$
$$- \sum_{h\in H}\sum_{d\in D}\sum_{p\in P} \mathrm{C}_{\mathrm{h,d}}^{\mathrm{TRAN2}}\ \tilde{\mathrm{q}}_{\mathrm{p,d,h}} - \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathrm{C}_{\mathrm{d,k}}^{\mathrm{TRAN3}} \times \mathrm{z}_{\mathrm{p,d,k}} \qquad (IM2)$$

Subject to (P1) to (P11), (SML1) to (SML5), (I1) and (G2) and

$\mathrm{x}_{\mathrm{i,r}}, \mathrm{y}_{\mathrm{e,i,r}}, \mathrm{v}_{\mathrm{b,i,r}}, \tilde{y}_{\mathrm{e,i,r}}, \tilde{v}_{\mathrm{c,b,i,r}}, \bar{y}_{\mathrm{p,e,i,r,h}}, \bar{v}_{\mathrm{p,c,b,i,r,h}}, \tilde{\mathrm{q}}_{\mathrm{p,d,h}}, q_{p,d}, \mathrm{z}_{\mathrm{p,d,k}}, revenue_{p,k}, w_{m,p,k} \geq 0$ and $l_{m,p,k} \in \{0,1\}$

Scenario 1 – L1SM-Linear

Objective function

$$L1SML\ Profit: \quad \sum_{p\in P}\sum_{k\in K} revenue_{p,k} - \sum_{p\in P}\sum_{d\in D}\lambda_{p,d}\mathrm{q}^{(S)}{}_{\mathrm{p,d}}$$

$$-\sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\mathrm{C}_{\mathrm{d,k}}^{\mathrm{TRAN2}}\times \mathrm{z}_{\mathrm{p,d,k}} - \sum_{d}\frac{C_d^{FIX}}{2}h^{(S)}{}_d - \sum_{d\in D}\mu_d h_d{}^{(S)}$$

Subject to (L13) to (L14), (SL1)-(SL5) and

$\mathrm{q}^{S}{}_{\mathrm{p,d}}, \mathrm{z}_{\mathrm{p,d,k}}, revenue_{p,k}, w_{m,p,k} \geq 0$ and $l_{m,p,k}, h_d{}^{(S)} \in \{0,1\}$

Scenario 2 – L2SM-Linear

Objective function

$$Maximize\ L2SML\ Profit: \quad \sum_{p\in P}\sum_{k\in K} revenue_{p,k} - \sum_{p\in P}\sum_{d\in D}\lambda_{p,d}\mathrm{q}^{(S)}{}_{\mathrm{p,d}}$$

$$-\sum_{\boldsymbol{p\in P}}\sum_{\boldsymbol{k\in K}}\sum_{\boldsymbol{d\in D}}\mathbf{C}_{\mathbf{d,k}}^{\mathbf{TRAN2}}\times \mathbf{z}_{\mathbf{p,d,k}}$$

Subject to (L14), (L18), (SL1)-(SL5) and

$\mathrm{q}^{S}{}_{\mathrm{p,d}}, \mathrm{z}_{\mathrm{p,d,k}}, revenue_{p,k}, w_{m,p,k} \geq 0$ and $l_{m,p,k} \in \{0,1\}$

Scenario 3 – L3SM-Linear

Objective function

$$L3SML\ Profit: \quad \sum_{p\in P}\sum_{k\in K} revenue_{p,k} - \sum_{p\in P}\sum_{d\in D}\lambda_{p,d}\mathrm{q}^{(S)}{}_{\mathrm{p,d}}$$

$$-\sum_{p\in P}\sum_{k\in K}\sum_{d\in D}\mathrm{C}_{\mathrm{d,k}}^{\mathrm{TRAN2}}\times \mathrm{z}_{\mathrm{p,d,k}} - \sum_{d}\frac{C_d^{FIX}}{2}h^{(S)}{}_d$$

Subject to (L13) to (L14), (SL1)-(SL5) and

$\mathrm{q}^{S}{}_{\mathrm{p,d}}, \mathrm{z}_{\mathrm{p,d,k}}, revenue_{p,k}, w_{m,p,k} \geq 0$ and $l_{m,p,k}, h_d{}^{(S)} \in \{0,1\}$

Scenario 4 – L3SM-Linear

Objective function

$$Maximize\ L2SML\ Profit\!: \quad \sum_{p\in P}\sum_{k\in K} revenue_{p,k} - \sum_{p\in P}\sum_{d\in D} \lambda_{p,d} \mathrm{q}^{(S)}{}_{\mathrm{p,d}}$$

$$- \sum_{p\in P}\sum_{k\in K}\sum_{d\in D} \mathrm{C}_{\mathrm{d,k}}^{\mathrm{TRAN2}} \times \mathrm{z}_{\mathrm{p,d,k}}$$

Subject to (L14), (L18), (SL1)-(SL5) and

$\mathrm{q}^{S}{}_{\mathrm{p,d}}, \mathrm{z}_{\mathrm{p,d,k}}, revenue_{p,k}, w_{m,p,k} \geq 0$ and $l_{m,p,k} \in \{0,1\}$

Appendix B

	p1	p2	p3	p4
h1	30	29	29	21
h2	32	30	28	29

Table 1.B – Blending costs of products

	r1	r2
h1	25	25.5
h2	24.5	25

Table 2.B – Transportation costs from refineries to hubs

	d1	d2	d3	d4	d5
h1	25	22	28	24	25
h2	22	21	30	28	25.5

Table 3.B – Transportation costs from hubs to depots

	k1	k2	k3	k4	k5
d1	24.9	29.5	37	39.1	36.9
d2	35.5	26	27.1	39	29
d3	39.3	27.2	25.9	35.3	38.5
d4	38.7	39.4	35.7	25.5	26.4
d5	37.6	38.9	36	25	24.9

Table 4.B – Transportation costs from depots to markets

d1	250000
d2	255000
d3	225000
d4	250000
d5	225000

Table 5.B – Fixed costs from depot operation

Price functions:

P1: $\theta_1 = 420 - 0.04z_1$

P2: $\theta_2 = 400 - 0.031z_2$

P3: $\theta_3 = 380 - 0.035z_3$

P4: $\theta_4 = 300 - 0.03z_4$

$\boldsymbol{Price\ elasticity\ \ E_i = \frac{\partial z_i}{\partial \theta_i}\frac{\theta_i}{z_i}}$:

P1: $E_1 = -7.5\%$

P2: $E_2 = -7.9\%$

P3: $E_3 = -6.2\%$

P4: $E_4 = -3.78\%$

Elasticity was calculated according to the optimal quantum and price in IM for market 1.

	min	max
P1	500	2500
P2	500	2500
P3	700	3000
P4	1000	4000

Table 6.B – Minimum and maximum demand for each product at each market

Premium Gasoline											
Amount	500	700	900	1100	1300	1500	1700	1900	2100	2300	2500
Revenue	200000	274400	345600	413600	478400	540000	598400	653600	705600	754400	800000
Regular Gasoline											
Amount	500	700	900	1100	1300	1500	1700	1900	2100	2300	2500
Revenue	192250	264810	334890	402490	467610	530250	590410	648090	703290	756010	806250
Distillates											
Amount	700	930	1160	1390	1620	1850	2080	2310	2540	2770	3000
Revenue	248850	323128.5	393704	460576.5	523746	583212.5	638976	691036.5	739394	784048.5	825000
Fuel Oil											
Amount	1000	1300	1600	1900	2200	2500	2800	3100	3400	3700	4000
Revenue	270000	339300	403200	461700	514800	562500	604800	641700	673200	699300	720000

Table 7.B – Amount and revenue break point in piecewise linear revenue function

Appendix C

	p1	p2	p3	p4
d1	312.2973599	299.84681	261.3083814	190.4932129
d2	310.7989303	298.84681	260.3083814	186.4932129
d3	316.7989303	305.34681	268.8083814	192.4932129
d4	312.7989303	301.34681	264.8083814	188.4932129
d5	313.7989303	302.34681	264.8083814	189.4932129

Table 1.C – Internal prices calculated with Method 1 (without discount)

	p1	p2	p3	p4
d1	294.0051203	282.6039649	241.8003314	160.304109
d2	292.5059055	281.6037031	240.8000697	156.3038473
d3	298.5059055	288.1037031	249.3000697	162.3038473
d4	294.5059055	284.1037031	245.3000697	158.3038473

Table 2.C – Internal prices calculated with Method 2 (with discount)

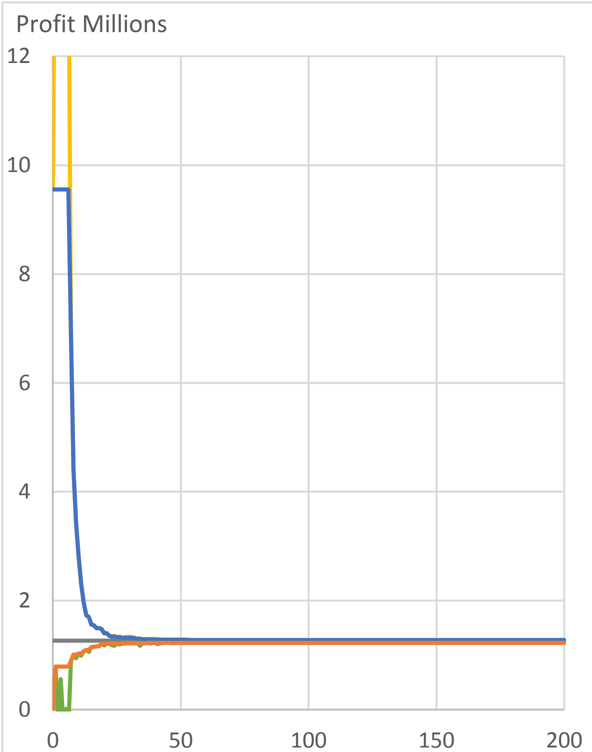
Appendix D

Step length: $\boldsymbol{\sigma \leftarrow 0,5\sigma}$

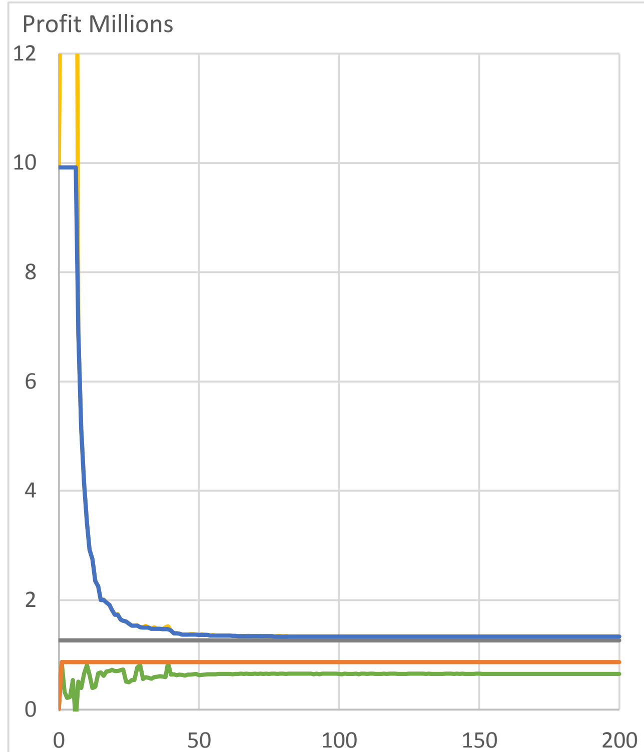
	Scenario 1	***Scenario 2***	***Scenario 3***	***Scenario 4***
Alt.1 After 5 iterations				
LBD	790456.4043	868557.008	998068.67	901906.07
(UBD-LBD)/LBD	1108.73 %	1041.78 %	821.68 %	510.13 %
LBD/BM-1	-37.42 %	-31.24 %	-20.98 %	-26.36 %
Alt.2 After 5 iterations				
LBD	952973.65	868557.008	998068.67	968112.7
(UBD-LBD)/LBD	902.60 %	1041.78 %	821.68 %	223.16 %
LBD/BM-1	-24.55 %	-31.24 %	-20.98 %	-16.01 %
Alt.1 After 50 iterations				
LBD	1224383.403	868557.008	1247419	1175944.9
(UBD-LBD)/LBD	4.55 %	57.74 %	6.53 %	7.02 %
LBD/BM-1	-3.07 %	-31.24 %	-1.24 %	-3.99 %
Alt.2 After 50 iterations				
LBD	1250681.008	868557.008	1259593.7	1219485.5
(UBD-LBD)/LBD	3.50 %	153.22 %	2.79 %	3.53 %
LBD/BM-1	-0.98 %	-31.24 %	-0.28 %	-0.43 %
Alt.1 After 200 iterations				
LBD	1224727.171	868557.008	1248802.5	1185749.7
(UBD-LBD)/LBD	4.03 %	53.64 %	4.86 %	4.52 %
LBD/BM-1	-3.04 %	-31.24 %	-1.13 %	-3.18 %
Alt.2 After 200 iterations				
LBD	1254449.255	995029.247	1259593.7	1220798.5
(UBD-LBD)/LBD	0.83 %	30.69 %	0.42 %	0.39 %
LBD/BM-1	-0.69 %	-21.22 %	-0.28 %	-0.32 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	-	-	-	-
Alt 2	87	-	66	66

Table 1.D – Summary: Nonlinear SM $(\lambda 0)$ *step length 0.5*

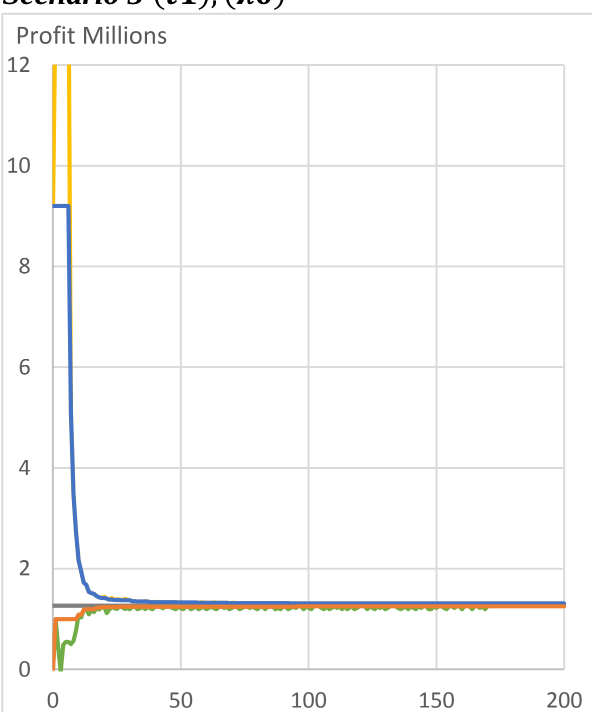
Scenario 1 $(t1), (\lambda 0)$



Scenario 2 $(t1), (\lambda 0)$



Scenario 3 $(t1), (\lambda 0)$



Scenario 4 $(t1), (\lambda 0)$

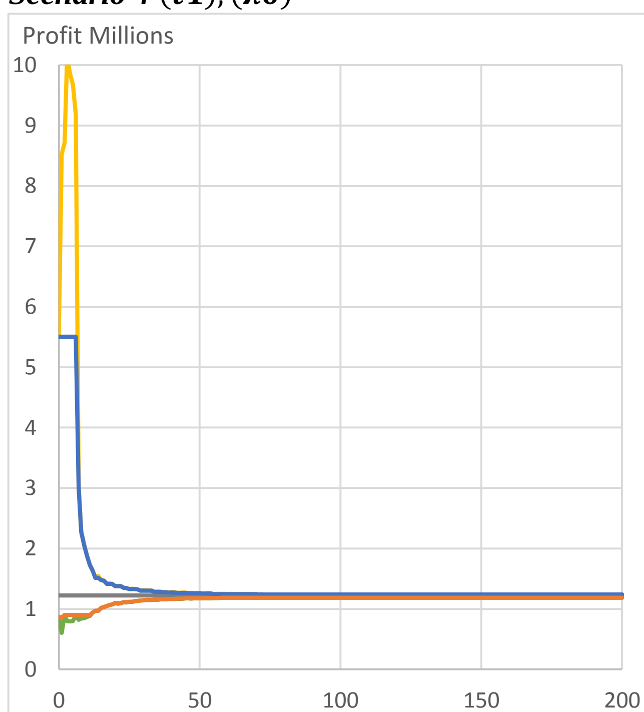
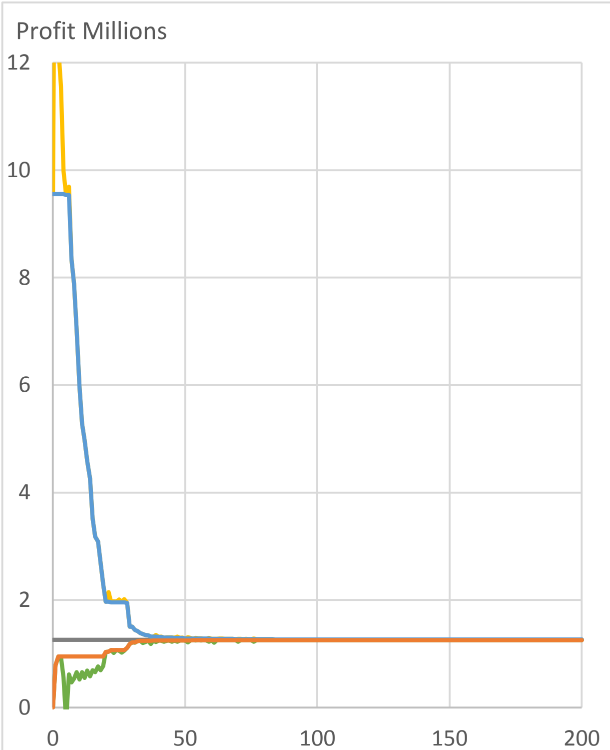
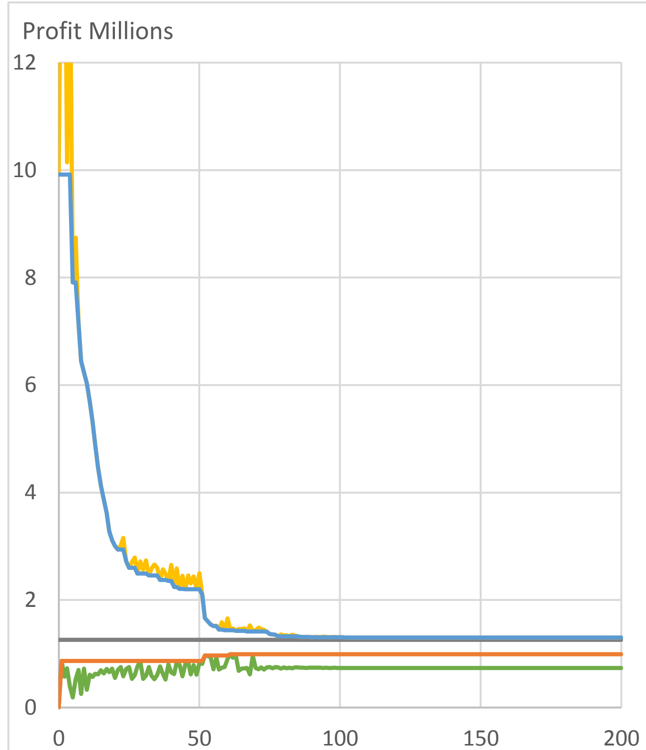


Figure 1.D – Convergence plots $(t1), (\lambda 0)$ *step length 0.5*

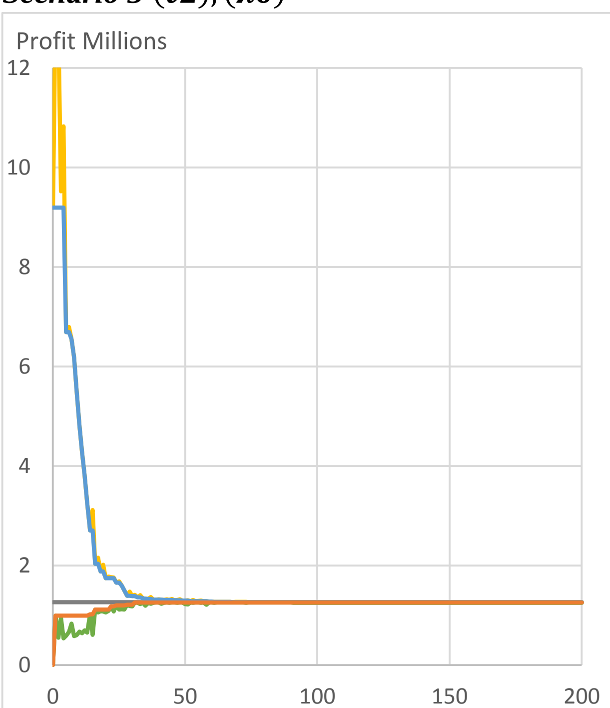
Scenario 1 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Scenario 2 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Scenario 3 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$



Scenario 4 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$

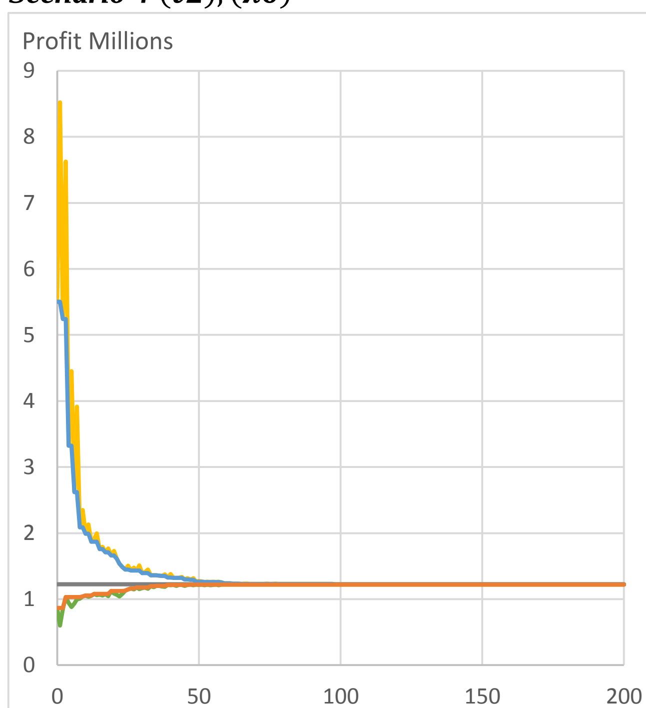
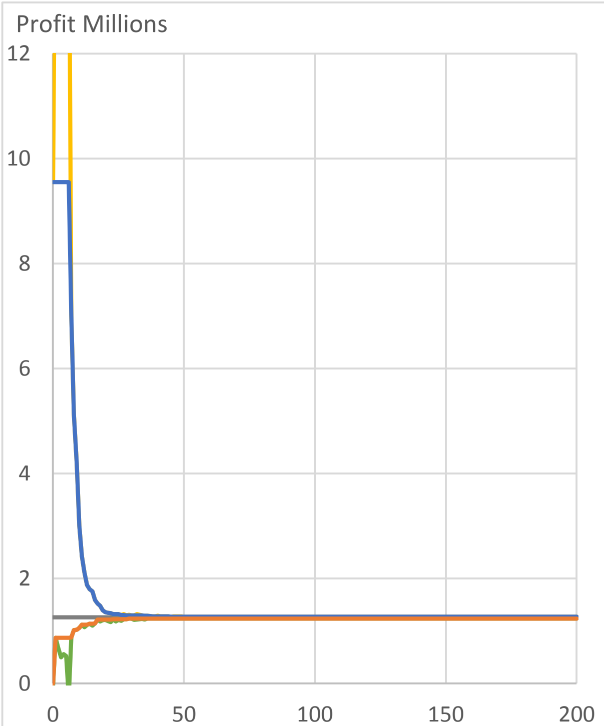


Figure 2.D – Convergence plots $(t2), (\lambda 0)$ *step length 0.5*

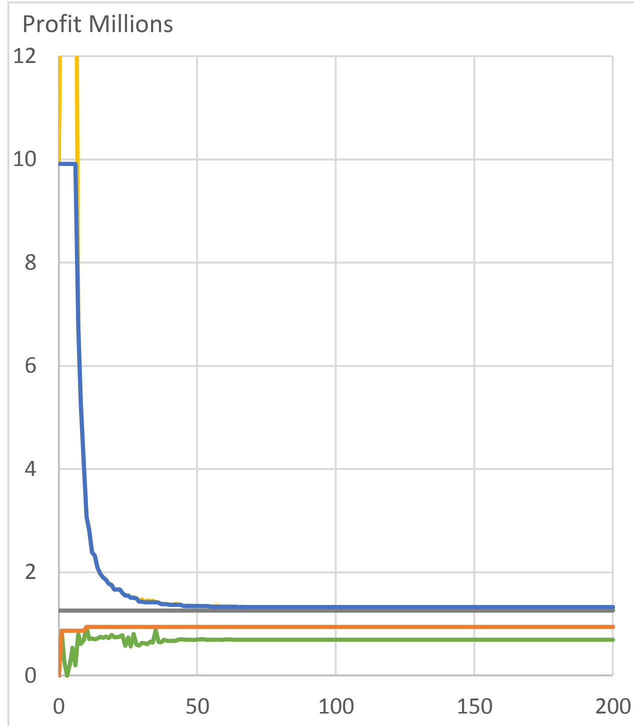
	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Alt.1 After 5 iterations				
LBD	867519.7946	866979.795	736675.7	948397.775
(UBD-LBD)/LBD	1000.79 %	1043.29 %	1148.05 %	479.95 %
LBD/BM-1	-31.08 %	-31.12 %	-41.48 %	-22.32 %
Alt.2 After 5 iterations				
LBD	867519.7946	866979.795	736675.7	968112.696
(UBD-LBD)/LBD	1000.79 %	923.93 %	1121.30 %	223.16 %
LBD/BM-1	-31.08 %	-31.12 %	-41.48 %	-20.70 %
Alt.1 After 50 iterations				
LBD	1237940.112	939088.215	1190967.53	1127849.64
(UBD-LBD)/LBD	2.95 %	43.34 %	10.01 %	12.21 %
LBD/BM-1	-1.65 %	-25.40 %	-5.39 %	-7.62 %
Alt.2 After 50 iterations				
LBD	1239934.056	1080868.13	1203570.06	1128366.81
(UBD-LBD)/LBD	8.56 %	26.97 %	23.20 %	34.98 %
LBD/BM-1	-1.49 %	-14.13 %	-4.38 %	-7.58 %
Alt.1 After 200 iterations				
LBD	1237940.112	939088.215	1199420.65	1144460.34
(UBD-LBD)/LBD	2.66 %	40.78 %	7.48 %	9.98 %
LBD/BM-1	-1.65 %	-25.40 %	-4.71 %	-6.26 %
Alt.2 After 200 iterations				
LBD	1256949.138	1080868.13	1257099.52	1217586.38
(UBD-LBD)/LBD	0.17 %	19.83 %	0.14 %	0.36 %
LBD/BM-1	-0.14 %	-14.13 %	-0.13 %	-0.27 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	-	-	-	-
Alt 2	79	-	85	98

Table 2.D – Summary: Linear SM ($\lambda 0$) step length 0.5

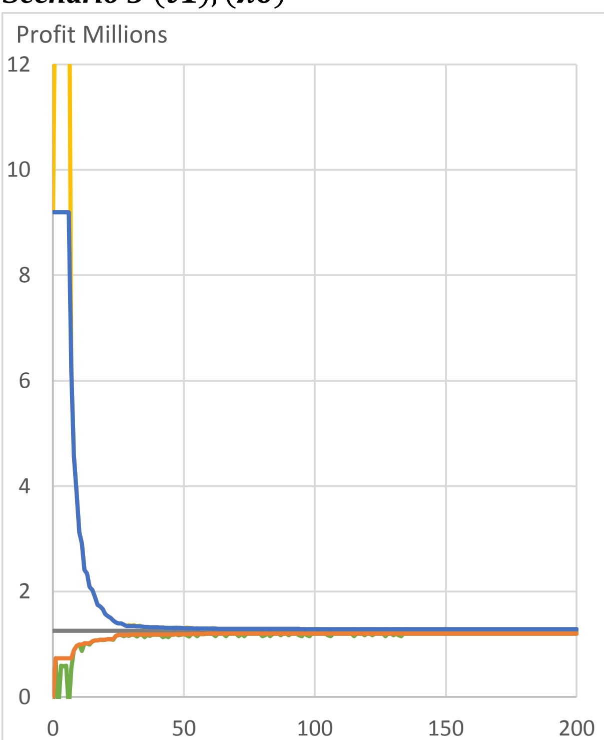
Scenario 1 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Scenario 2 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Scenario 3 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Scenario 4 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$

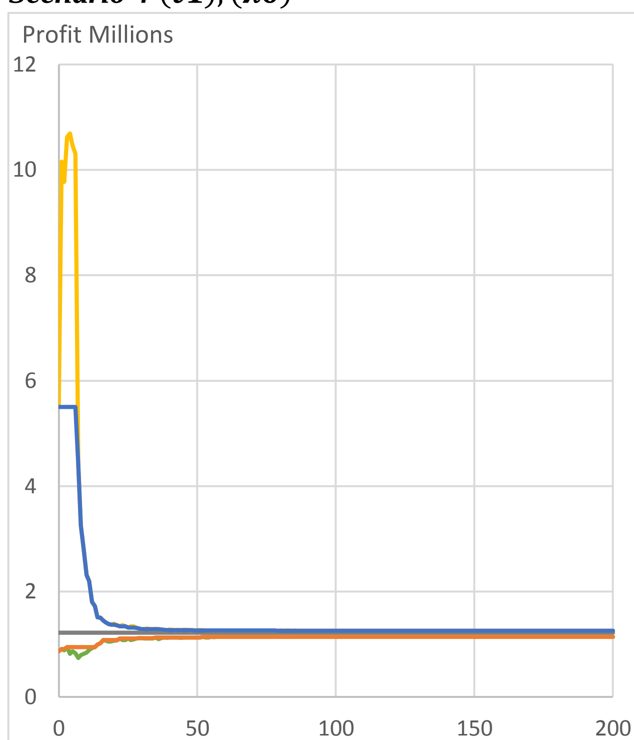
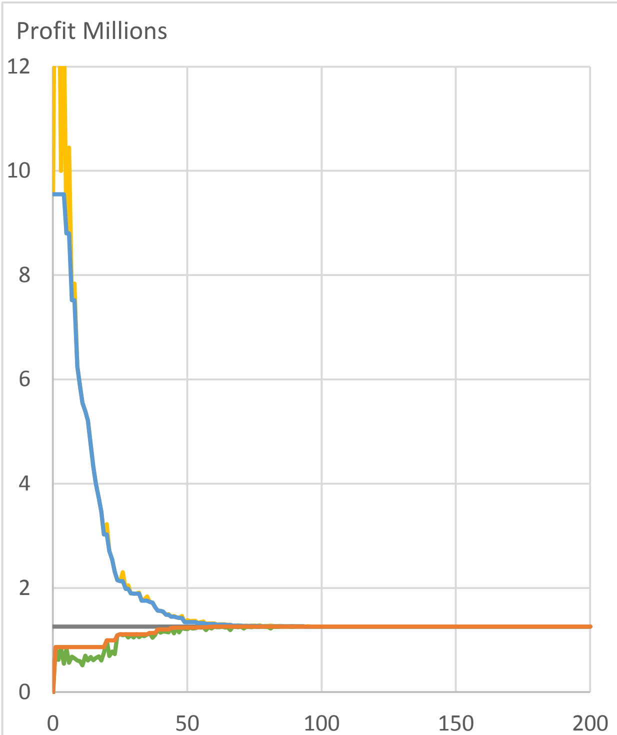
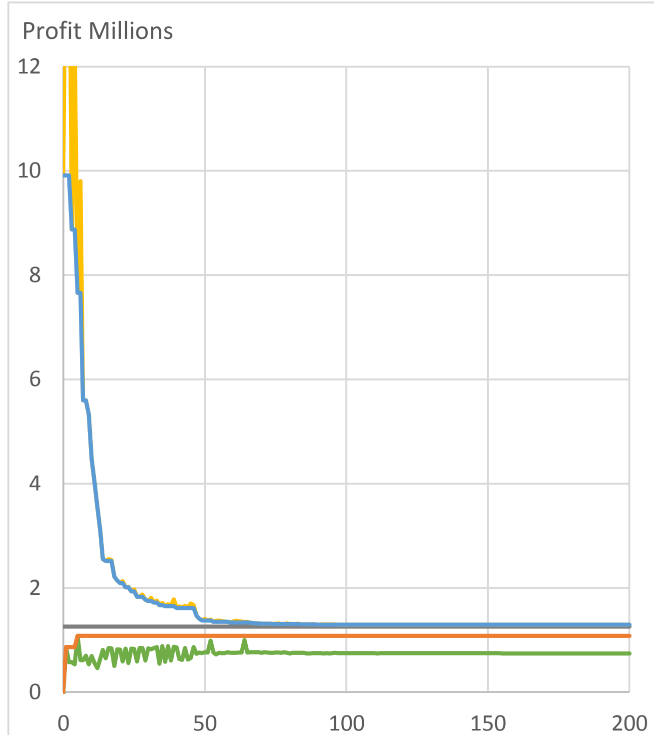


Figure 3.D – Convergence plots: Linear $(t1), (\lambda 0)$ *step length 0.5*

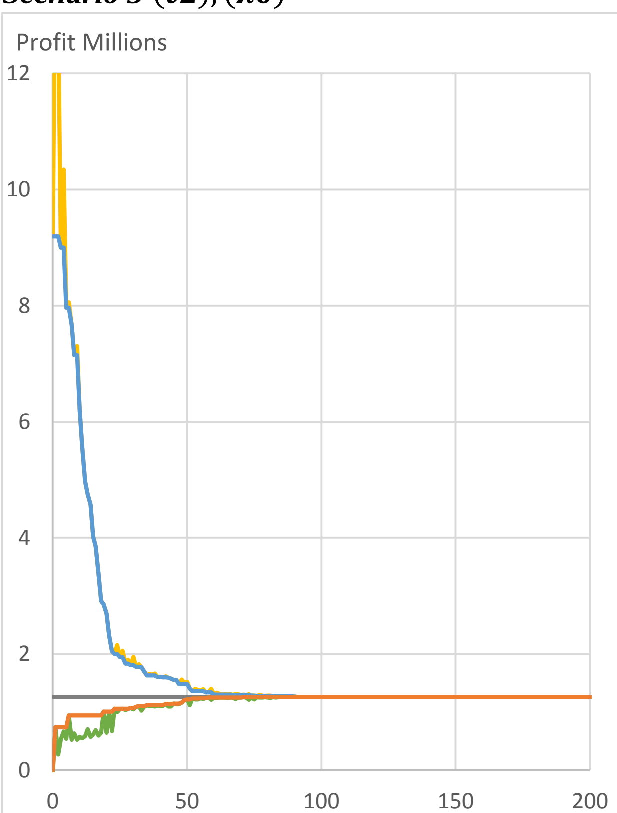
Scenario 1 $(t2), (\lambda 0)$



Scenario 2 $(t2), (\lambda 0)$



Scenario 3 $(t2), (\lambda 0)$



Scenario 4 $(t2), (\lambda 0)$

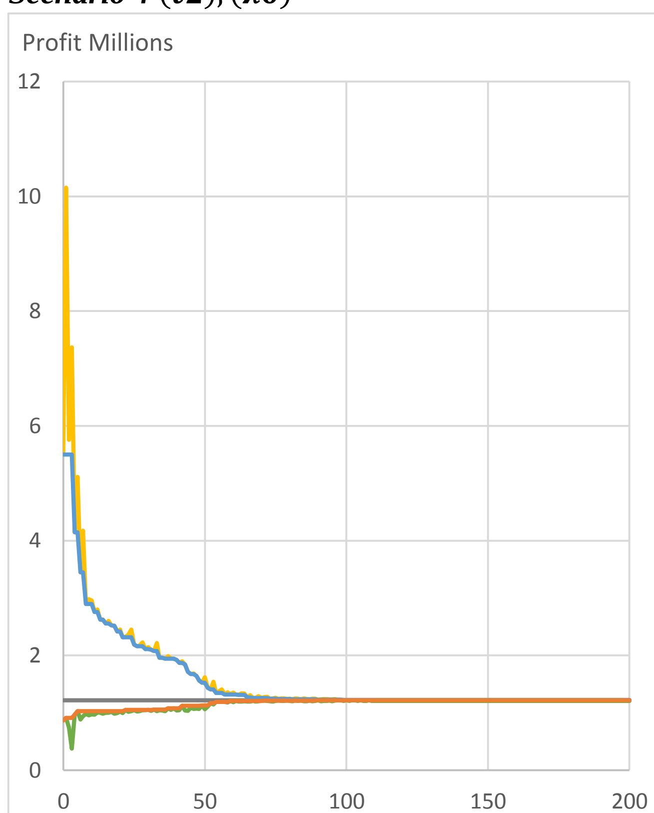


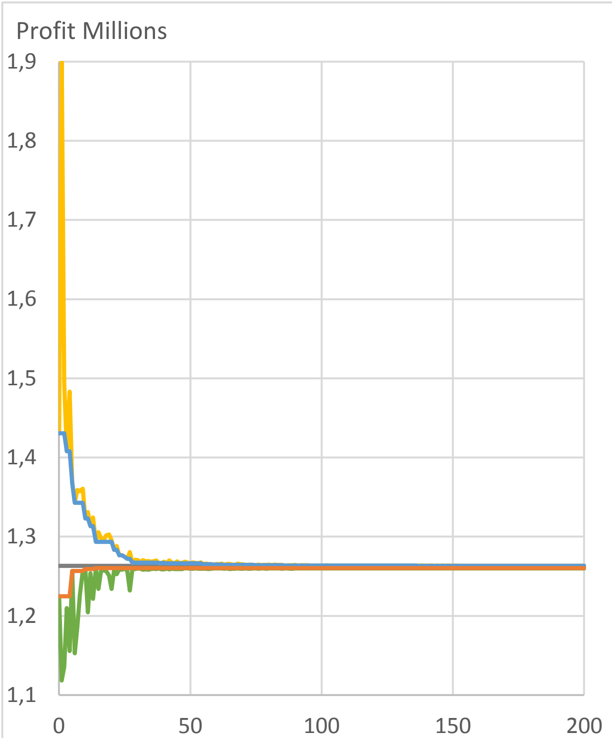
Figure 4.D – Convergence plots: Linear $(t2), (\lambda 0)$ *step length 0.5*

Initial Lagrange multipliers – Shadow prices

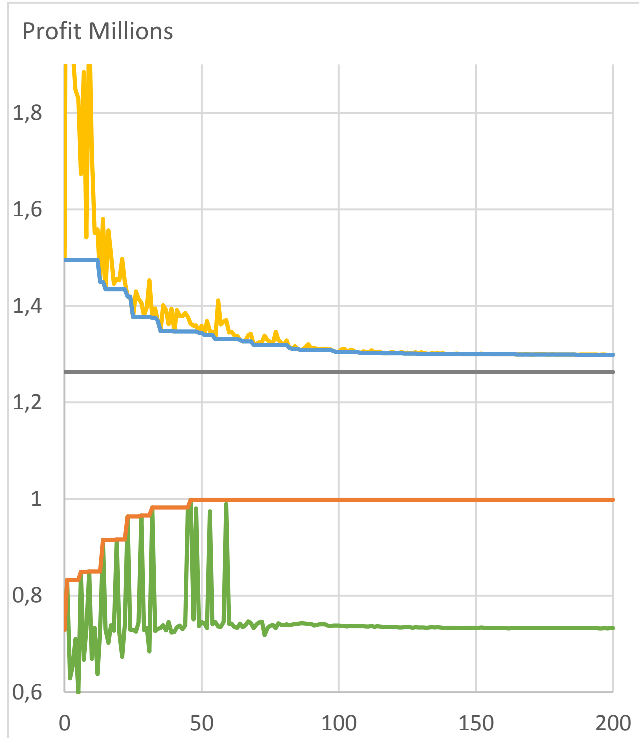
	Scenario 1	***Scenario 2***	***Scenario 3***	***Scenario 4***
Alt.1 After 5 iterations				
LBD	1224750.914	832364.3	1124589.7	1224751
(UBD-LBD)/LBD	16.82 %	79.58 %	40.68 %	0.00 %
LBD/BM-1	-3.04 %	-34.10 %	-10.97 %	0.00 %
Alt.2 After 5 iterations				
LBD	1224750.914	832364.3	1223488	1218999.3
(UBD-LBD)/LBD	14.97 %	79.58 %	28.14 %	0.00 %
LBD/BM-1	-3.04 %	-34.10 %	-3.14 %	0.00 %
Alt.1 After 50 iterations				
LBD	1259519.299	973849.392	1262131.5	1224751
(UBD-LBD)/LBD	0.62 %	42.05 %	0.43 %	0.00 %
LBD/BM-1	-0.28 %	-22.90 %	-0.08 %	0.00 %
Alt.2 After 50 iterations				
LBD	1260244.685	998227.373	1259939	1224750.7
(UBD-LBD)/LBD	0.48 %	34.69 %	0.82 %	0.00 %
LBD/BM-1	-0.23 %	-20.97 %	-0.25 %	0.00 %
Alt.1 After 200 iterations				
LBD	1259581.735	991457.894	1262385.2	1224751
(UBD-LBD)/LBD	0.31 %	31.01 %	0.06 %	0.00 %
LBD/BM-1	-0.28 %	-21.51 %	-0.06 %	0.00 %
Alt.2 After 200 iterations				
LBD	1260244.685	998227.373	1260362.3	1224750.7
(UBD-LBD)/LBD	0.24 %	30.11 %	0.24 %	0.00 %
LBD/BM-1	-0.23 %	-20.97 %	-0.22 %	0.00 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	34	-	41	1
Alt 2	27	-	49	1

Table 3.D – Summary: Nonlinear SM (λS)

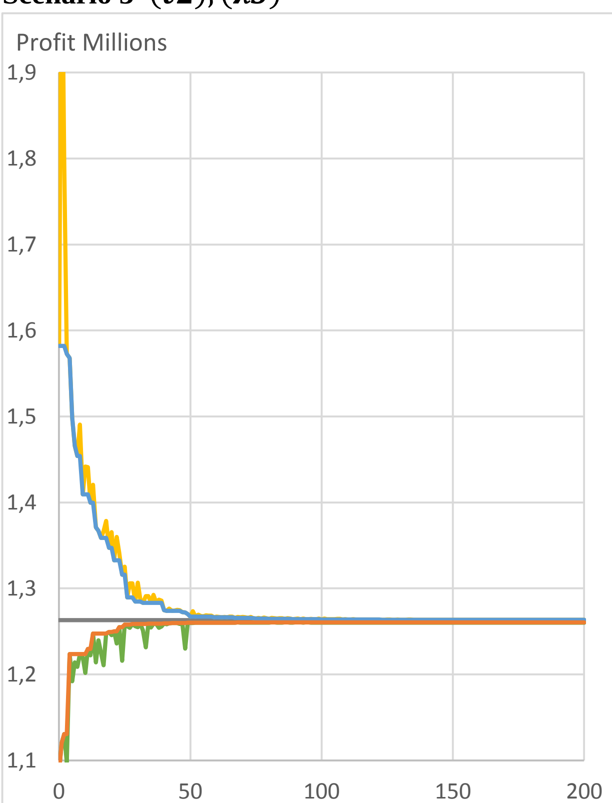
Scenario 1 $(t2), (\lambda S)$



Scenario 2 $(t2), (\lambda S)$



Scenario 3 $(t2), (\lambda S)$



Scenario 4 $(t2), (\lambda S)$

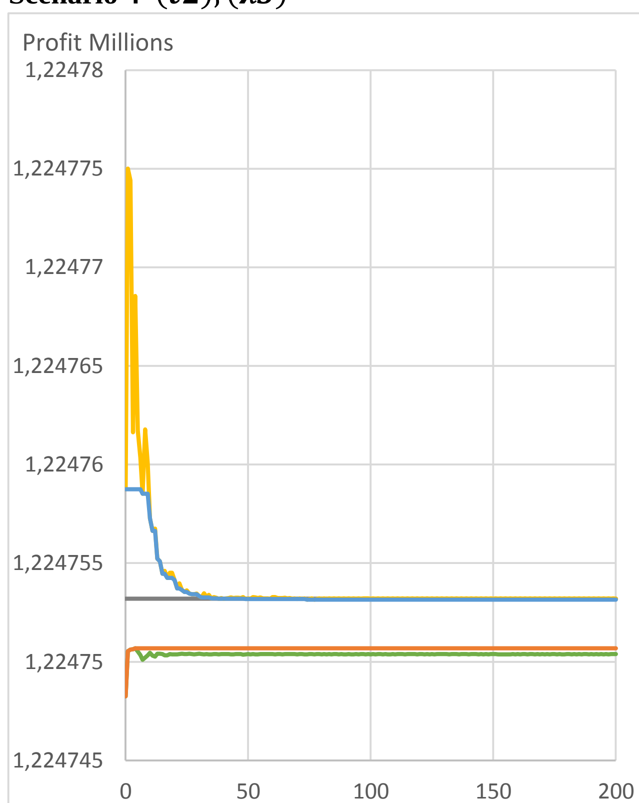
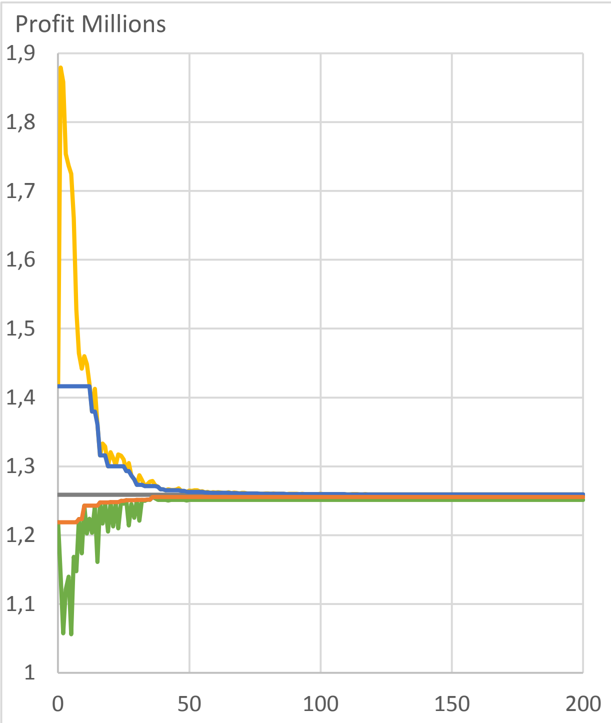


Figure 5.D – Convergence plots $(t2), (\lambda S)$

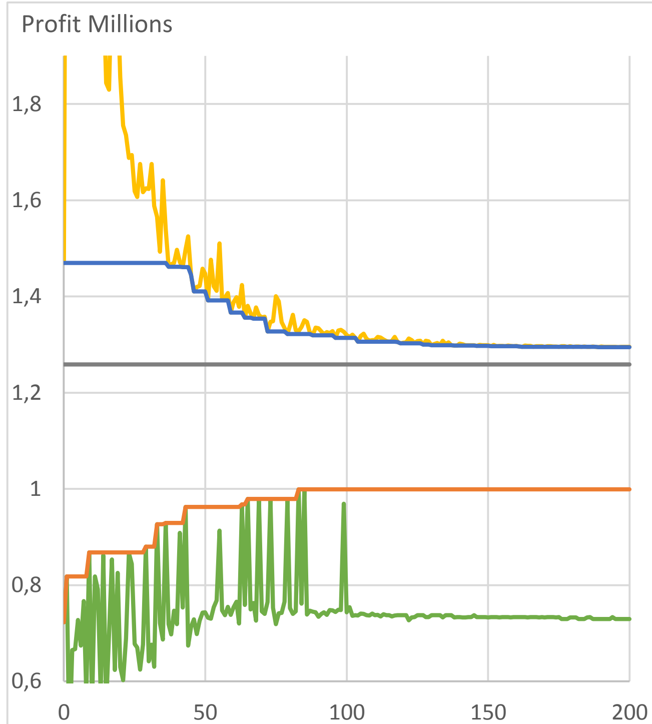
	Scenario 1	*Scenario 2*	*Scenario 3*	*Scenario 4*
Alt.1 After 5 iterations				
LBD	1218530.3	818416.223	1093601.95	1220099.4
(UBD-LBD)/LBD	16.24 %	79.59 %	45.30 %	0.06 %
LBD/BM-1	-3.20 %	-34.98 %	-13.12 %	-0.06 %
Alt.2 After 5 iterations				
LBD	1218530.3	866181.427	1141628.35	1218999.3
(UBD-LBD)/LBD	15.08 %	69.69 %	35.42 %	0.00 %
LBD/BM-1	-3.20 %	-31.19 %	-9.30 %	-0.15 %
Alt.1 After 50 iterations				
LBD	1255275.789	962358.098	1257046.69	1220289.76
(UBD-LBD)/LBD	0.62 %	46.54 %	1.02 %	0.05 %
LBD/BM-1	-0.28 %	-23.55 %	-0.14 %	-0.05 %
Alt.2 After 50 iterations				
LBD	1251440.963	1211424.84	1251470.46	1220289.76
(UBD-LBD)/LBD	1.44 %	12.09 %	1.93 %	0.05 %
LBD/BM-1	-0.58 %	-3.76 %	-0.58 %	-0.05 %
Alt.1 After 200 iterations				
LBD	1255275.789	998843.932	1257099.52	1220289.76
(UBD-LBD)/LBD	0.30 %	29.64 %	0.14 %	0.05 %
LBD/BM-1	-0.28 %	-20.65 %	-0.13 %	-0.05 %
Alt.2 After 200 iterations				
LBD	1251440.963	1211424.84	1255680.79	1220289.76
(UBD-LBD)/LBD	0.61 %	6.86 %	0.26 %	0.05 %
LBD/BM-1	-0.58 %	-3.76 %	-0.24 %	-0.05 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	40	-	51	1
Alt 2	74	-	60	1

Table 4.D - Summary: Linear SM (λS)

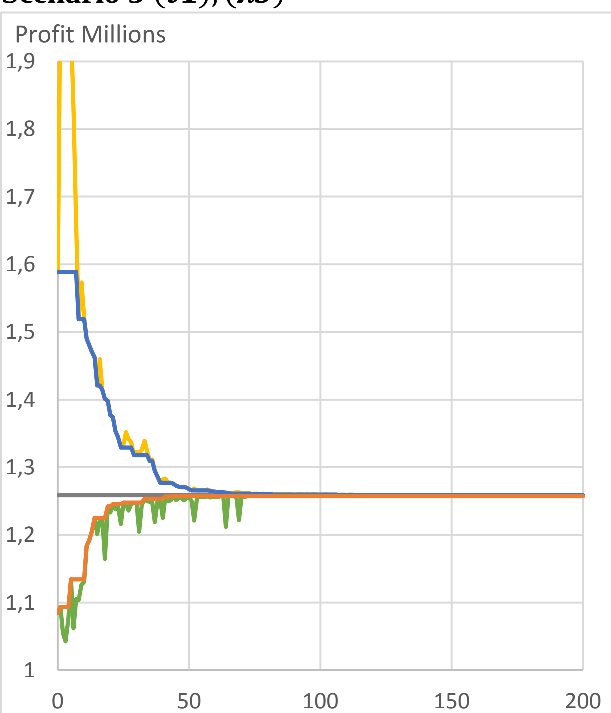
Scenario 1 $(t1), (\lambda S)$



Scenario 2 $(t1), (\lambda S)$



Scenario 3 $(t1), (\lambda S)$



Scenario 4 $(t1), (\lambda S)$

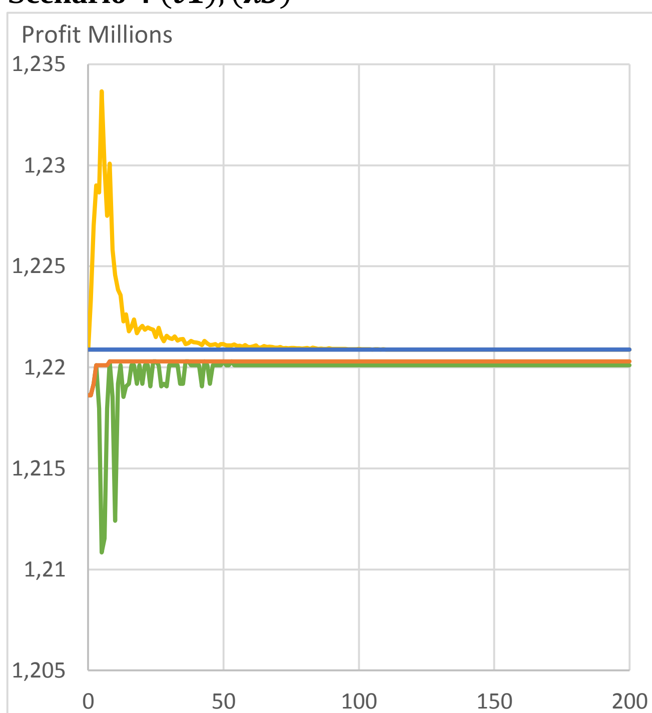
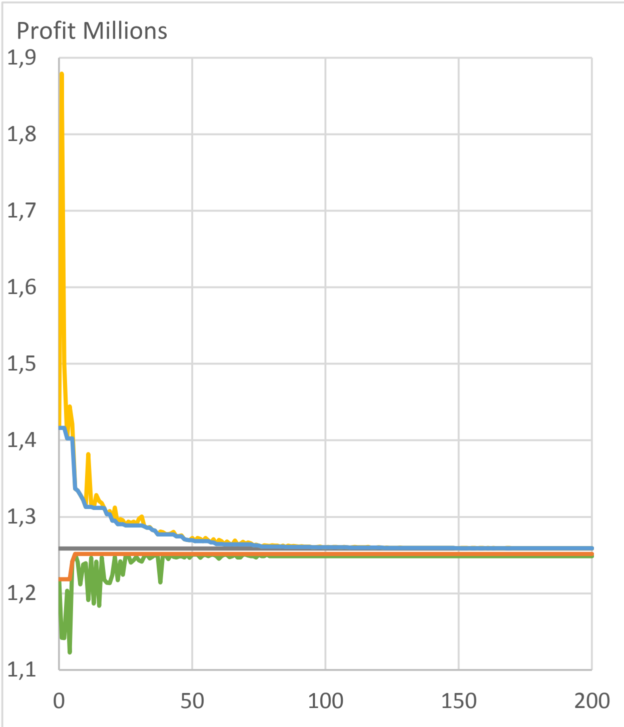
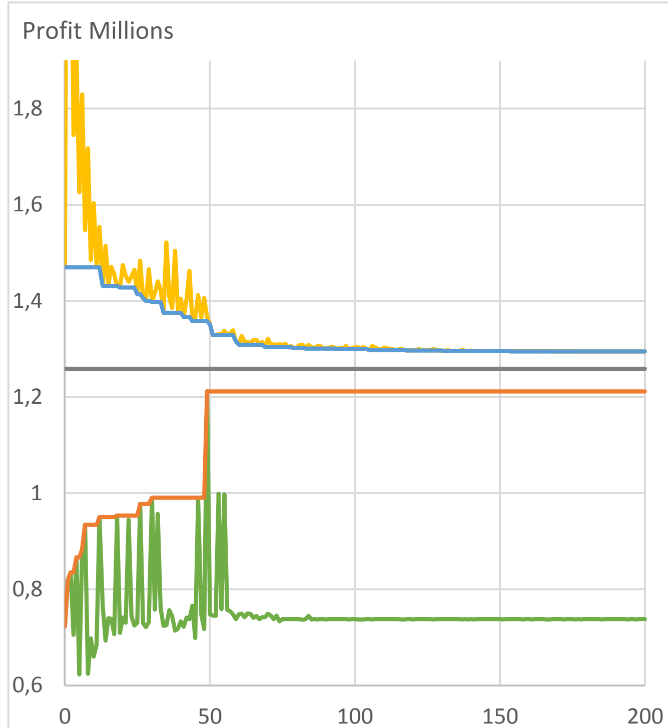


Figure 6.D - Convergence plots: Linear $(t1), (\lambda S)$

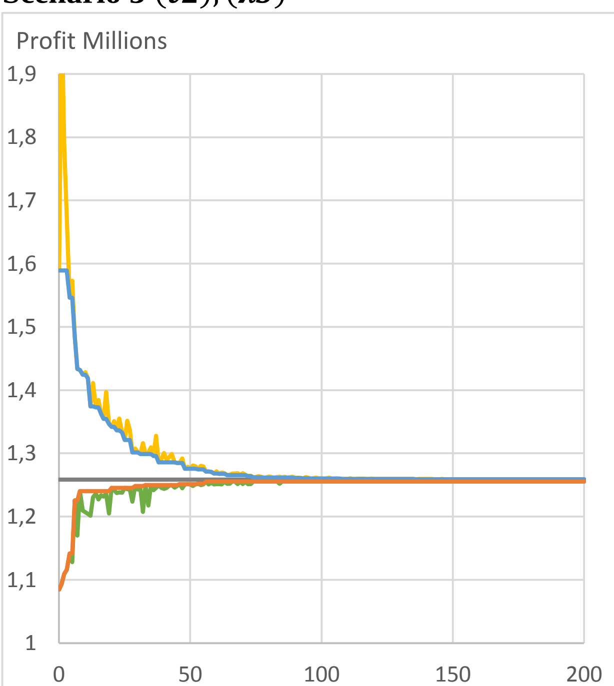
Scenario 1 $(t2), (\lambda S)$



Scenario 2 $(t2), (\lambda S)$



Scenario 3 $(t2), (\lambda S)$



Scenario 4 $(t2), (\lambda S)$

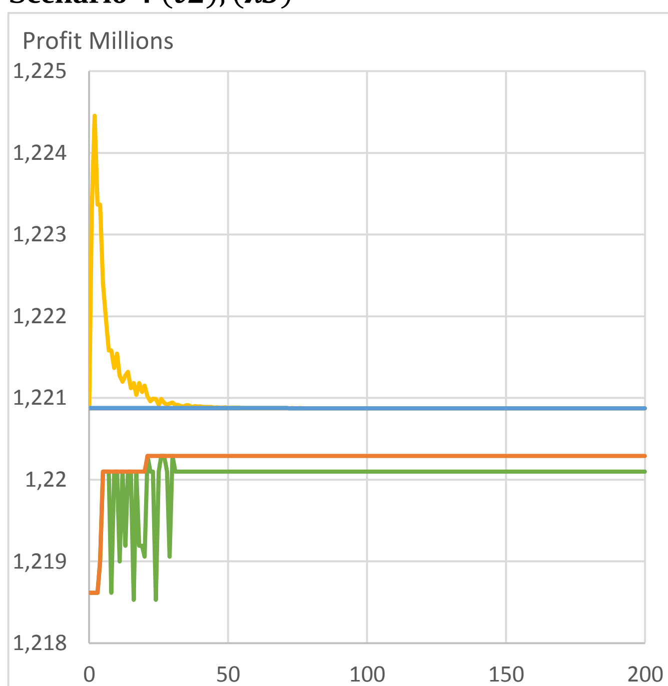


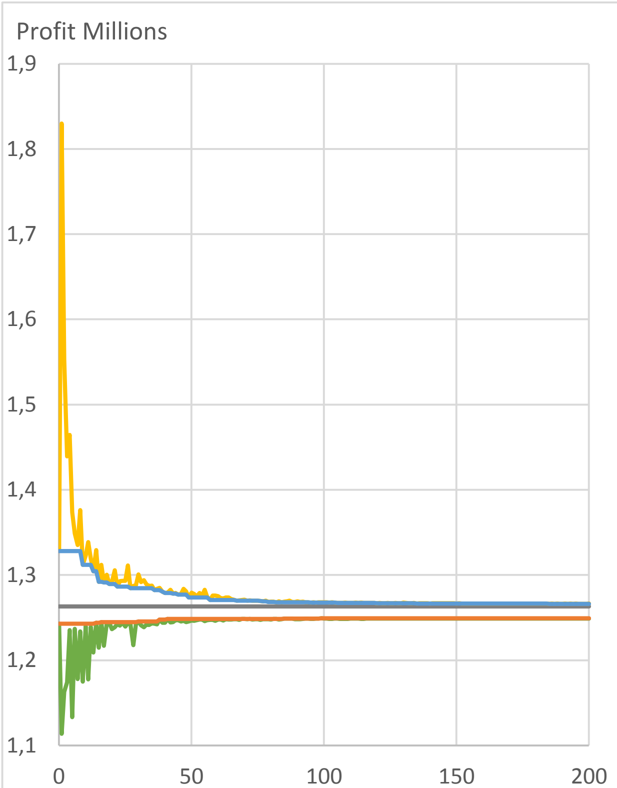
Figure 7.D - Convergence plots: Linear $(t2), (\lambda S)$

Initial Lagrange multipliers – Cost based internal prices

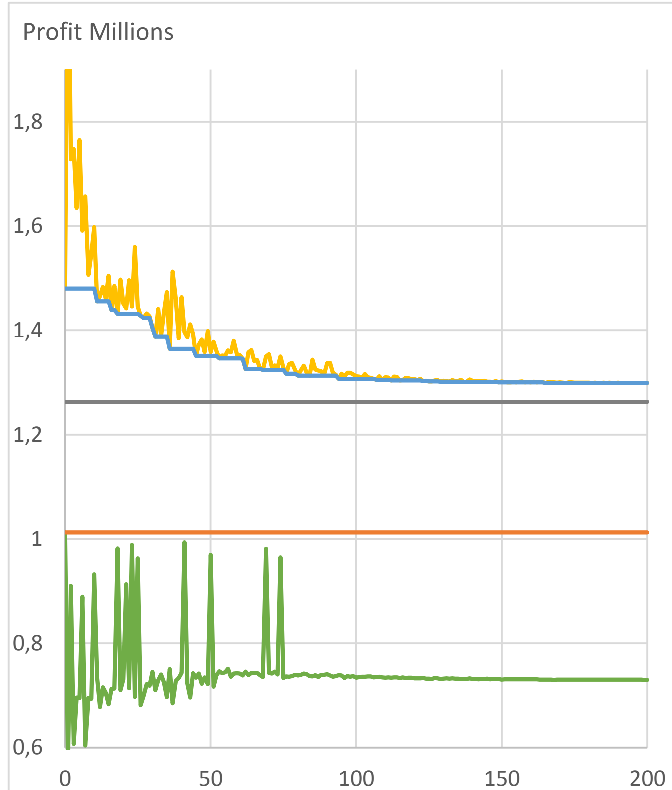
	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Alt.1 After 5 iterations				
LBD	1243001,306	1012819,32	1160022,7	1196570,3
(UBD-LBD)/LBD	6,86 %	46,13 %	18,64 %	5,29 %
LBD/BM-1	-1,59 %	-19,82 %	-8,16 %	-2,30 %
Alt.2 After 5 iterations				
LBD	1243001,306	1012819,32	1217759,1	1203197
(UBD-LBD)/LBD	6,86 %	46,13 %	13,02 %	4,84 %
LBD/BM-1	-1,59 %	-19,82 %	-3,59 %	-1,88 %
Alt.1 After 50 iterations				
LBD	1251983,794	1012819,32	1253639,4	1213649
(UBD-LBD)/LBD	2,00 %	34,87 %	1,44 %	1,39 %
LBD/BM-1	-0,88 %	-19,82 %	-0,75 %	-0,91 %
Alt.2 After 50 iterations				
LBD	1248489,593	1012819,32	1250293,9	1210296,2
(UBD-LBD)/LBD	2,03 %	33,44 %	2,01 %	1,86 %
LBD/BM-1	-1,16 %	-19,82 %	-1,01 %	-1,18 %
Alt.1 After 200 iterations				
LBD	1253747,7	1012819,32	1254706,3	1214374,8
(UBD-LBD)/LBD	0,91 %	28,27 %	0,81 %	1,00 %
LBD/BM-1	-0,74 %	-19,82 %	-0,67 %	-0,85 %
Alt.2 After 200 iterations				
LBD	1249166,879	1012819,32	1251548,6	1211801,3
(UBD-LBD)/LBD	1,37 %	28,26 %	1,15 %	1,29 %
LBD/BM-1	-1,10 %	-19,82 %	-0,92 %	-1,06 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	106	-	83	56
Alt 2	-	-	-	-

Table 5.D – Summary: Nonlinear SM (λC)

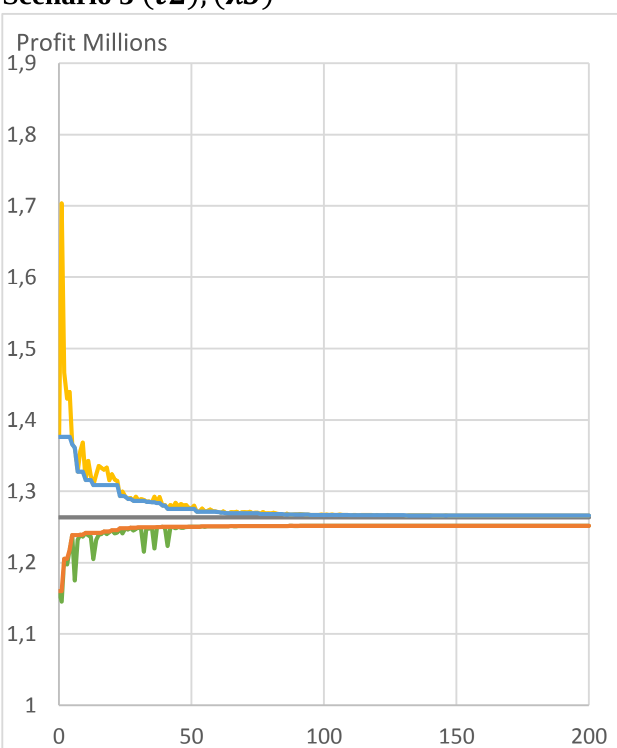
Scenario 1 $(t2), (\lambda S)$



Scenario 2 $(t2), (\lambda S)$



Scenario 3 $(t2), (\lambda S)$



Scenario 4 $(t2), (\lambda S)$

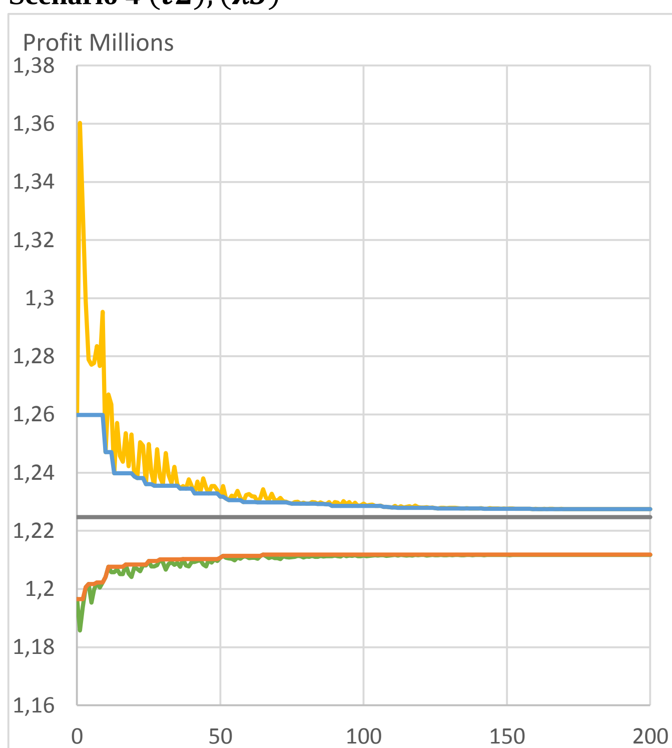
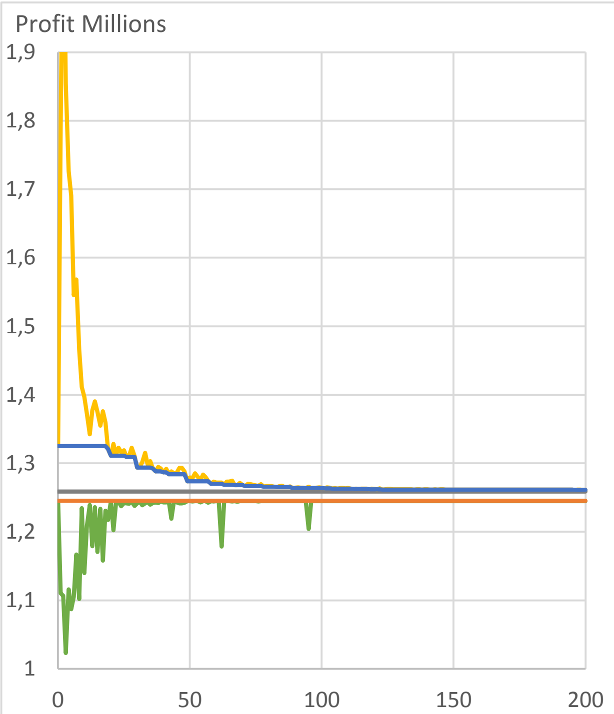


Figure 8.D – Convergence plots $(t2), (\lambda S)$

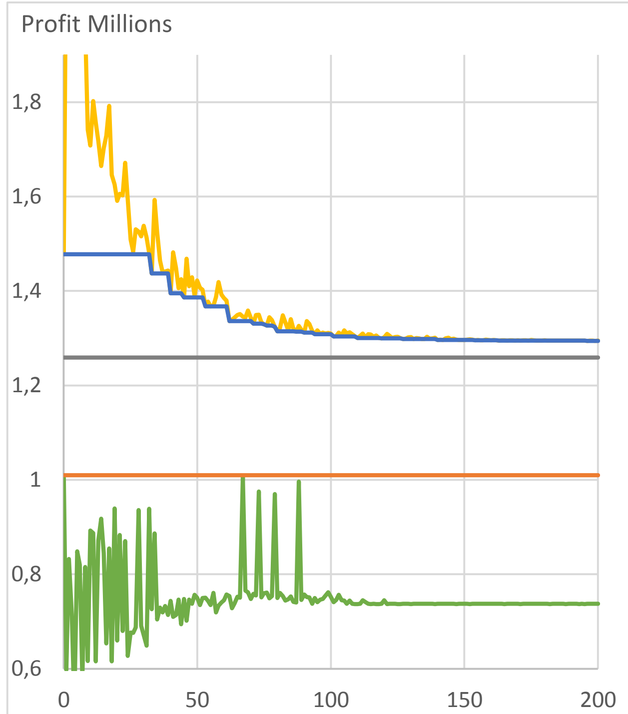
	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Alt.1 After 5 iterations				
LBD	1245122,635	1009837,21	1153841,18	1189860,14
(UBD-LBD)/LBD	6,40 %	46,34 %	19,00 %	5,58 %
LBD/BM-1	-1,08 %	-19,77 %	-8,33 %	-2,54 %
Alt.2 After 5 iterations				
LBD	1245122,635	1009837,21	1236671,67	1203196,99
(UBD-LBD)/LBD	6,40 %	46,34 %	10,03 %	4,84 %
LBD/BM-1	-1,08 %	-19,77 %	-1,75 %	-1,45 %
Alt.1 After 50 iterations				
LBD	1245122,635	1009837,21	1245135,68	1207443,58
(UBD-LBD)/LBD	2,31 %	37,28 %	2,66 %	1,73 %
LBD/BM-1	-1,08 %	-19,77 %	-1,08 %	-1,10 %
Alt.2 After 50 iterations				
LBD	1245122,635	1009837,21	1244805,2	1207443,58
(UBD-LBD)/LBD	1,99 %	36,64 %	2,12 %	1,85 %
LBD/BM-1	-1,08 %	-19,77 %	-1,11 %	-1,10 %
Alt.1 After 200 iterations				
LBD	1245122,635	1009837,21	1245135,68	1207443,58
(UBD-LBD)/LBD	1,30 %	28,19 %	1,30 %	1,20 %
LBD/BM-1	-1,08 %	-19,77 %	-1,08 %	-1,10 %
Alt.2 After 200 iterations				
LBD	1245122,635	1009837,21	1244805,2	1207443,58
(UBD-LBD)/LBD	1,48 %	28,19 %	1,55 %	1,47 %
LBD/BM-1	-1,08 %	-19,77 %	-1,11 %	-1,10 %
Iteration number when UBD-LBD/LBD<=1%				
Alt 1	-	-	-	-
Alt 2	-	-	-	-

Table 6.D - Summary: Linear SM (λC)

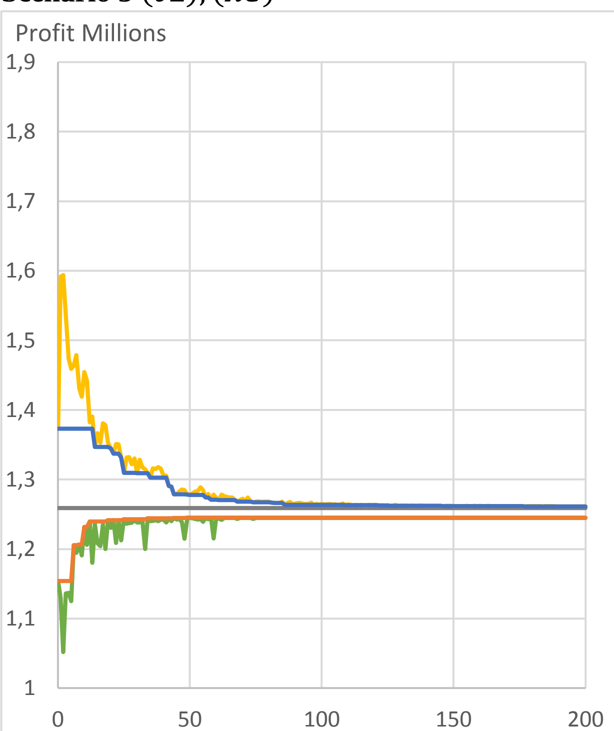
Scenario 1 $(t1), (\lambda C)$



Scenario 2 $(t1), (\lambda C)$



Scenario 3 $(t1), (\lambda C)$



Scenario 4 $(t1), (\lambda C)$

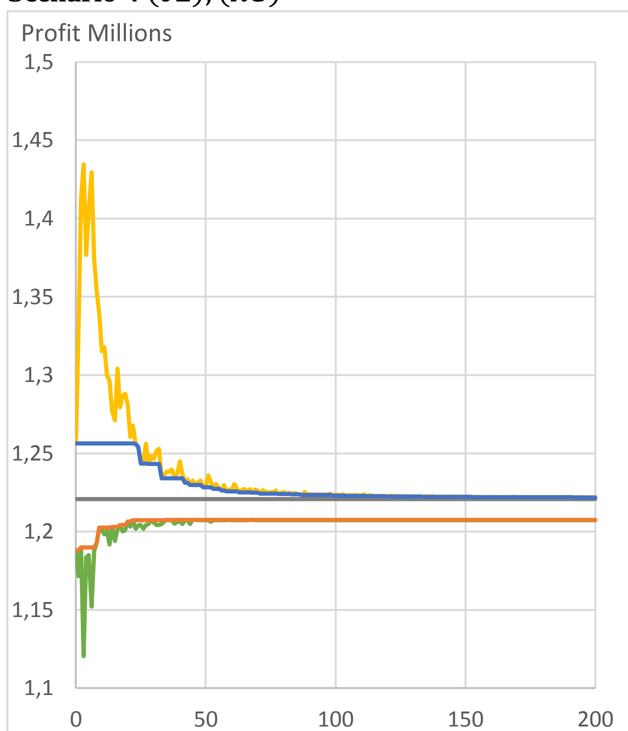
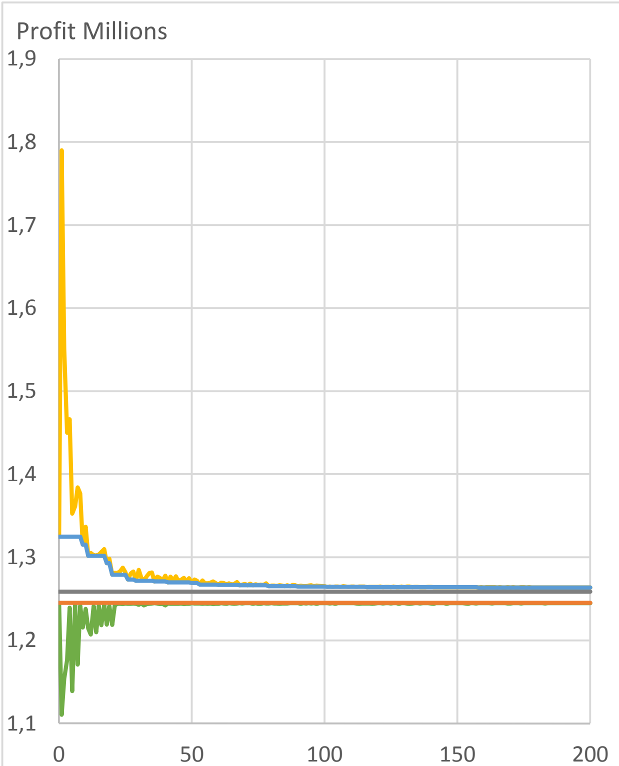
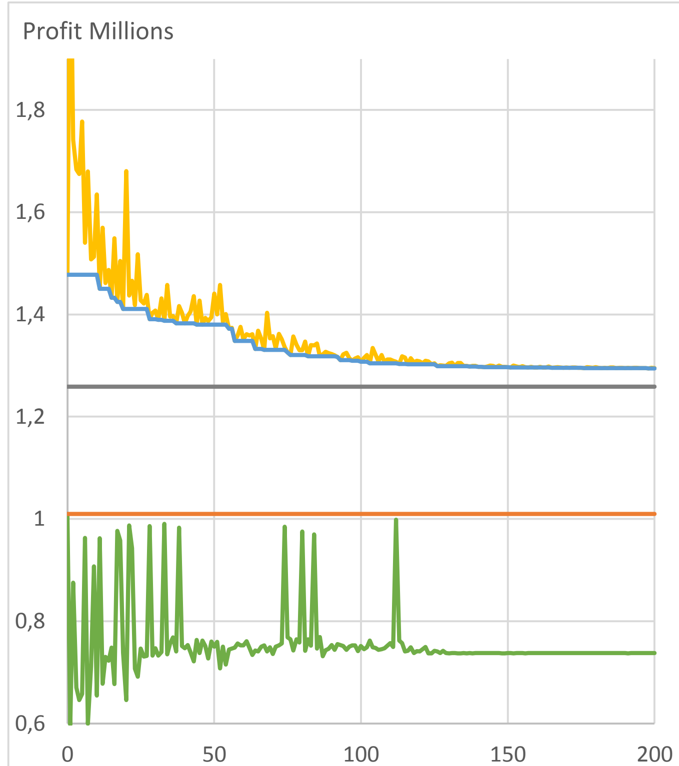


Figure 9.D - Convergence plots: Linear $(t1), (\lambda C)$

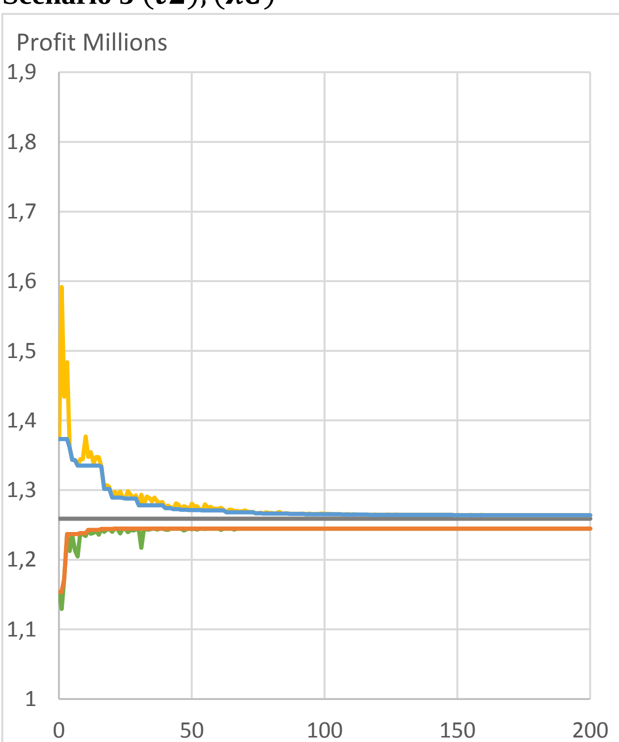
Scenario 1 $(t2), (\lambda C)$



Scenario 2 $(t2), (\lambda C)$



Scenario 3 $(t2), (\lambda C)$



Scenario 4 $(t2), (\lambda C)$

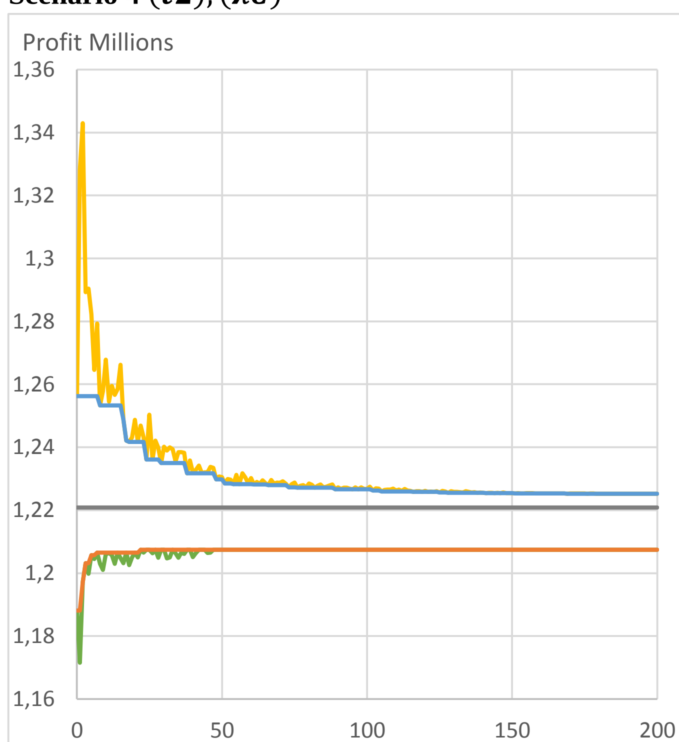


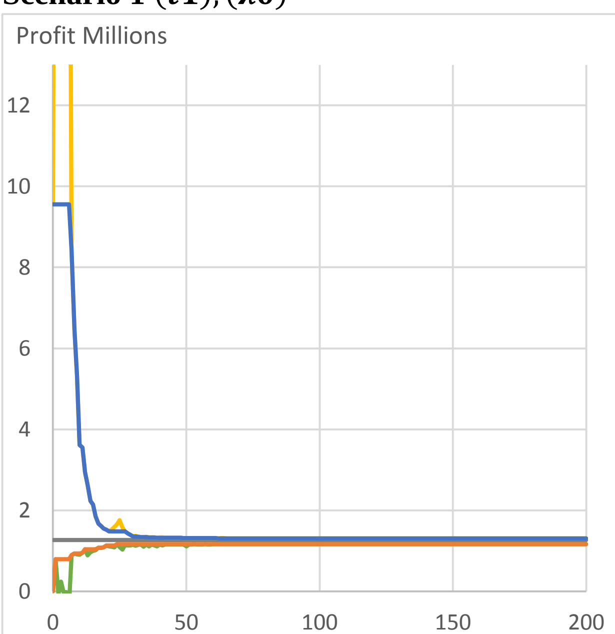
Figure 10.D - Convergence plots: Linear $(t2), (\lambda C)$

Alternative step length calculation for μ

	Alt.1	*Alt.2*
After 5 iterations		
LBD	790456,4	952973,7
(UBD-LBD)/LBD	11,08731	9,025974
LBD/BM-1	-0,3742	-0,24554
After 50 iterations		
LBD	1167496	1235546
(UBD-LBD)/LBD	0,127424	0,053859
LBD/BM-1	-0,0757	-0,02183
After 200 iterations		
LBD	1167496	1245443
(UBD-LBD)/LBD	0,121471	0,016964
LBD/BM-1	-0,0757	-0,01399

Table 7.D - Summary: Alternative step length calculation for μ

Scenario 1 $(\boldsymbol{t1}), (\boldsymbol{\lambda 0})$



Scenario 1 $(\boldsymbol{t2}), (\boldsymbol{\lambda 0})$

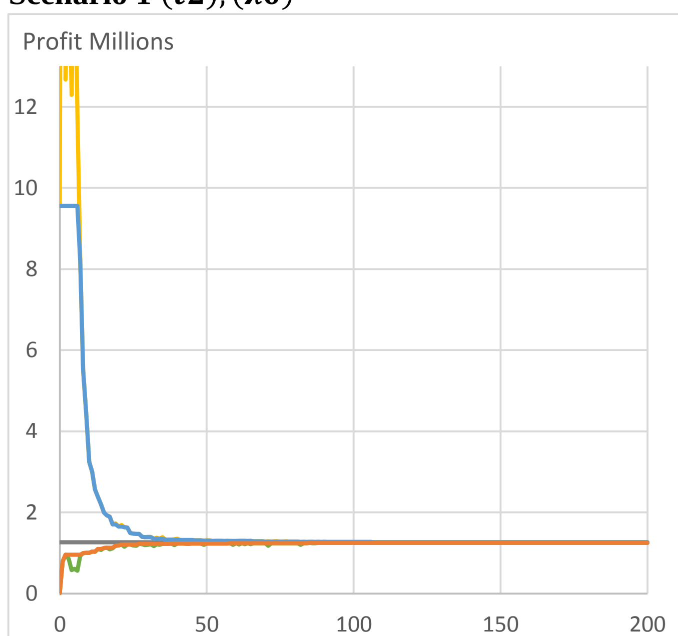


Figure 11.D - Alternative step length calculation for μ

Appendix E

AMPL code – Integrated Model 1 (IM1):

Mod file

```
# Sets

set I;               # Set of crude oils
set R;               # Set of refineries
set H;               # Set of hubs
set D;               # Set of depots
set K;               # Set of markets
set A;               # Set of components that cannot be processed
set C;               # Set of components that was generated from components from set B
set B;               # Set of components that can either be processed or directly used  in blending
set E:=A union B;      # Union of sets A and B
set P;               # Set of  products
set QMIN;            # Set of minimum qualities
set QMAX;            # Set of maximum qualities

# Parameters

param sup{I,R};              # Available volume of crude oil i at refinery r
param rho{I,E};              # Amount of component e generated from one unit of crude oil i
param rho2{B,C};             # Amount of component c generated from one unit of component b
param s_p_m{P,QMIN};          # Value of required quality qmin in product p
param s_b_m{E,QMIN};          # Value of quality qmin in component e
param s_b2_m{C,QMIN};         # Value of quality qmin in component c
param s_p_ma{P,QMAX};         # Value of required quality qmax in product p
param s_b_ma{I,E,QMAX};       # Value of quality qmax in component e obtained from crude oil i
param s_b2_ma{I,QMAX,C,B};    # Value of quality qmax in component c obtained from component b which was
again obtained from crude oil i
param C_BUY{I,R};            # Cost of purchasing one unit of crude oil i at refinery r
param C_PRO{I,R};            # Cost of processing one unit of crude oil i at refinery r
param C_PRO2{B,R};           # Cost of processing one unit of component b at refinery r
param C_BLEND{P,H};          # Cost of producing one unit of product p at hub h
param C_TRAN1{R,H};          # Cost of transporting one unit of any product from refinery r to hub h
param C_TRAN2{H,D};          # Cost of transporting one unit of any product from hub h depot d
param C_TRAN3{D,K};          # Cost of transporting one unit of any product from depot d to market k
param a_demand {P};          # Coefficients of price function, where a picks up competitors action
param b_demand {P};          # Coefficients of price function, where a picks up competitors action
param max_dep{D};            # Maximum capacity at depot d
param min_dem{P};            # Minimum demand of product p at market k
param max_dem2{P};           # Maximum demand of product p at market k
param C_FIX{D};              # Fixed cost to operate depot d

# Variables

var x{I,R} >= 1;             # Amount of crude oil i purchased and processed at refinery r
var y{E,I,R} >= 0;           # Amount of component e generated from crude oil i at refinery r
var yp{B,I,R} >= 0;          # Amount of component b generated from crude oil i, used for further processing at
refinery r
var yp1{C, B, I,R} >= 0;     # Amount of component c generated from component b which was again produced
form crude oil i at refinery r
var y2{E,I,R} >= 0;          # Amount of component e obtained from crude oil i sent directly to blending at refinery
```

```
var y3{P,E,I,R,H} >= 0;        # Amount of component e obtained from crude oil i used in product p sent from refinery r to hub h
var yp3{P,C,B,I,R,H} >= 0;     # Amount of component c obtained from component b which was again obtained from crude oil i, used in product p which was sent from refinery r to hub h
var q{P,H,D}>= 0;              # Amount of product p at depot d which was sent from hub h
var q1{P,D}>= 0;               # Amount of product p at depot d
var z{P,D,K} >= 0;             # Amount of product p transported from depot d to marked k
var h{D} binary;               # Binary variable which have value 1 when depot d is used, and 0 otherwise
var sold_k{P,K};               # Amount of product p that was sold at market k
var price{P,K};                # Price of product p at market k

# Objective function IM

maximize Contribution:
            sum{p in P, k in K} (sold_k[p,k]*(a_demand[p])- (sold_k[p,k]^2)*(b_demand[p]))
            - sum{i in I,r in R} C_BUY[i,r]*x[i,r]
            - sum{i in I, r in R} C_PRO[i,r]*x[i,r]
            - sum{b in B,i in I,r in R} C_PRO2[b,r]*yp[b,i,r]
            - sum{p in P, d in D, w in H} C_BLEND[p,w]*q[p,w,d]
            - sum{p in P, r in R, w in H, e in E, i in I} C_TRAN1[r,w]*y3[p,e,i,r,w]
            - sum{p in P, r in R, w in H, c in C, b in B, i in I} C_TRAN1[r,w]*yp3[p,c,b,i,r,w]
            - sum{p in P, d in D, w in H} C_TRAN2[w,d]*q[p,w,d]
            - sum{p in P, k in K, d in D} C_TRAN3[d,k]*z[p,d,k]
            - sum{d in D}C_FIX[d]*h[d];

# Constrains

subject to supply{i in I, r in R}:                          # (P1)
    x[i,r] <= sup[i,r];
subject to run_modes{e in E,  i in I, r in R}:              # (P2)
    rho[i,e]*x[i,r] = y[e,i,r];
subject to component1{a in A, i in I, r in R}:              # (P3)
    y[a,i,r] >=  y2[a,i,r];
subject to component2{b in B, i in I, r in R}:              # (P4)
    y[b,i,r] >=  y2[b,i,r] + yp[b,i,r];
subject to processing{c in C, b in B, i in I, r in R}:      # (P5)
    rho2[b,c]* yp[b,i,r] = yp1[c,b,i,r];
subject to products{e in E, i in I,r in R}:                 # (P6)
    y2[e,i,r] >= sum{p in P, w in H} y3[p,e,i,r,w];
subject to products2{c in C, b in B, i in I, r in R}:       # (P7)
    yp1[c,b,i,r] >= sum{p in P,w in H} yp3[p,c,b,i,r,w];
subject to blending_min {p in P, qm in QMIN, w in H}:       # (P8)
    sum{d in D}q[p,w,d]*s_p_m[p,qm] <= sum{e in E, i in I, r in R}(y3[p,e,i,r,w]*s_b_m[e,qm])+sum{c in C, b in B, i in I, r in R}(yp3[p,c,b,i,r,w] * s_b2_m[c,qm]);
subject to blending_max{p in P, qm in QMAX, w in H}:        # (P9)
    sum{d in D}q[p,w,d]*s_p_ma[p,qm] >= sum{e in E, i in I, r in R}(y3[p,e,i,r,w]*s_b_ma[i,e,qm])+sum{c in C, b in B, i in I, r in R}(yp3[p,c,b,i,r,w]*s_b2_ma[i,qm,c,b]);
subject to mass_balance{p in P, w in H}:                    # (P10)
    sum{d in D}q[p,w,d] <= sum{e in E, i in I, r in R}y3[p,e,i,r,w]+sum{c in C, b in B, i in I, r in R}yp3[p,c,b,i,r,w];
subject to mass_balance2{p in P, d in D}:                   # (P11)
    sum{w in H}q[p,w,d] = q1[p,d];
subject to fixed_cost{d in D}:                              # (G1)
    sum{p in P}q1[p,d] <= max_dep[d]*h[d];
subject to mass_balance3 {p in P, d in D}:                  # (I1)
    sum{k in K}z[p,d,k] = q1[p,d];
```

```
subject to price_calc{p in P, k in K}:                    # (S1)
	price[p,k] = a_demand[p]-b_demand[p]*sum{d in D}z[p,d,k];
subject to maximum_dem {p in P, k in K}:                   # (S2)
	sum{d in D}z[p,d,k] <= max_dem2[p];
subject to maximum_dem23 {p in P, k in K}:                 # (S2)
	min_dem[p]<= sum{d in D}z[p,d,k] ;
subject to demand_amount {p in P, k in K}:                 # Extra constrain which are used just to calculete amount of product p which was sold at market k
	sum{d in D}z[p,d,k] = sold_k[p,k];

# Parameters which will be used to extra calculations
param total{P};                                 # Amount of sold product p that was sold at all markets
```

AMPL code – Linear Integrated Model 1 (IM1-L):

Mod file

```
# Sets

set I;              # Set of crude oils
set R;              # Set of refineries
set H;              # Set of hubs
set D;              # Set of depots
set K;              # Set of markets
set A;              # Set of components that cannot be processed
set C;              # Set of components that was generated from components from set B
set B;              # Set of components that can either be processed or directly used  in blending
set E:=A union B;      # Union of sets A and B
set P;              # Set of  products
set QMIN;           # Set of minimum qualities
set QMAX;           # Set of maximum qualities
set N := 1..11;     # Number of breakpoints
set M := 1..10;     # Set within M  –  number of segments (there are one less segment then there are breakpoints)

# Paramenters

param sup{I,R};              # Available volume of crude oil i at refinery r
param rho{I,E};              # Amount of component e generated from one unit of crude oil i
param rho2{B,C};             # Amount of component c generated from one unit of component b
param s_p_m{P,QMIN};         # Value of required quality qmin in product p
param s_b_m{E,QMIN};         # Value of quality qmin in component e
param s_b2_m{C,QMIN};        # Value of quality qmin in component c
param s_p_ma{P,QMAX};        # Value of required quality qmax in product p
param s_b_ma{I,E,QMAX};      # Value of quality qmax in component e obtained from crude oil i
param s_b2_ma{I,QMAX,C,B};   # Value of quality qmax in component c obtained from component b which was
again obtained from crude oil i
param C_BUY{I,R};            # Cost of purchasing one unit of crude oil i at refinery r
param C_PRO{I,R};            # Cost of processing one unit of crude oil i at refinery r
param C_PRO2{B,R};           # Cost of processing one unit of component b at refinery r
param C_BLEND{P,H};          # Cost of producing one unit of product p at hub h
param C_TRAN1{R,H};          # Cost of transporting one unit of any product from refinery r to hub h
param C_TRAN2{H,D};          # Cost of transporting one unit of any product from hub h depot d
param C_TRAN3{D,K};          # Cost of transporting one unit of any product from depot d to market k
param amount{N,P};           # Sold amount of product p corresponding to breakpoint m
param rev {N,P};             # Revenue from product p corresponding to breakpoint m
param max_dep{D};            # Maximum capacity at depot d
param min_dem{P};            # Minimum demand of product p at market k
param max_dem2{P};           # Maximum demand of product p at market k
param C_FIX{D};              # Fixed cost to operate depot d

# Variables

var x{I,R} >= 1;             # Amount of crude oil i purchased and processed at refinery r
var y{E,I,R} >= 0;           # Amount of component e generated from crude oil i at refinery r
var yp{B,I,R} >= 0;          # Amount of component b generated from crude oil i, used for further processing at
refinery r
var yp1{C, B, I,R} >= 0;     # Amount of component c generated from component b which was again produced
form crude oil i at refinery r
```

```
var y2{E,I,R} >= 0;          # Amount of component e obtained from crude oil i sent directly to blending at refinery
var y3{P,E,I,R,H} >= 0;      # Amount of component e obtained from crude oil i used in product p sent from
refinery r to hub h
var yp3{P,C,B,I,R,H} >= 0;    # Amount of component c obtained from component b which was again obtained
from crude oil i, used in product p which was sent from refinery r to hub h
var q{P,H,D}>= 0;            # Amount of product p at depot d which was sent from hub h
var q1{P,D}>= 0;             # Amount of product p at depot d
var z{P,D,K} >= 0;           # Amount of product p transported from depot d to marked k
var h{D} binary;             # Binary variable which have value 1 when depot d is used, and 0 otherwise
var w {N,P,K} >=0;           # Weight for product p, breakpoint m
var revenue {P,K} >= 0;      # Revenue from product p at market k
var y10 {M,P,K} binary;      # Binary variable, takes value 1 if segment m for product p is used, and 0 otherwise
var price{P,K};              # Price of product p at market k


# Objective function (SM1-Linear)

maximize Contribution:
                    sum{p in P, k in K} revenue[p,k]
                    - sum{i in I,r in R} C_BUY[i,r]*x[i,r]
                    - sum{i in I, r in R} C_PRO[i,r]*x[i,r]
                    - sum{b in B,i in I,r in R} C_PRO2[b,r]*yp[b,i,r]
                    - sum{p in P, d in D, w2 in H} C_BLEND[p,w2]*q[p,w2,d]
                    - sum{p in P, r in R, w2 in H, e in E, i in I} C_TRAN1[r,w2]*y3[p,e,i,r,w2]
                    - sum{p in P, r in R, w2 in H, c in C, b in B, i in I} C_TRAN1[r,w2]*yp3[p,c,b,i,r,w2]
                    - sum{p in P, d in D, w2 in H} C_TRAN2[w2,d]*q[p,w2,d]
                    - sum{p in P, k in K, d in D} C_TRAN3[d,k]*z[p,d,k]
                    - sum{d in D}C_FIX[d]*h[d];

# Constrains

subject to supply{i in I, r in R}:                              # (P1)
       x[i,r] <= sup[i,r];
subject to run_modes{e in E,  i in I, r in R}:                  # (P2)
       rho[i,e]*x[i,r] = y[e,i,r];
subject to component1{a in A, i in I, r in R}:                  # (P3)
       y[a,i,r] >=  y2[a,i,r];
subject to component2{b in B, i in I, r in R}:                  # (P4)
       y[b,i,r] >=  y2[b,i,r] + yp[b,i,r];
subject to processing{c in C, b in B, i in I, r in R}:          # (P5)
       rho2[b,c]* yp[b,i,r] = yp1[c,b,i,r];
subject to products{e in E, i in I,r in R}:                     # (P6)
       y2[e,i,r] >= sum{p in P, w2 in H} y3[p,e,i,r,w2];
subject to products2{c in C, b in B, i in I, r in R}:           # (P7)
       yp1[c,b,i,r] >= sum{p in P,w2 in H} yp3[p,c,b,i,r,w2];
subject to blending_min{p in P, qm in QMIN, w2 in H}:                     # (P8)
       sum{d in D}q[p,w2,d]* s_p_m[p,qm] <= sum{e in E, i in I, r in R}(y3[p,e,i,r,w2]*s_b_m[e,qm])+sum{c in C, b
in B, i in I, r in R}(yp3[p,c,b,i,r,w2]*s_b2_m[c,qm]);
subject to blending_max{p in P, qm in QMAX, w2 in H}:                     # (P9)
       sum{d in D}q[p,w2,d]* s_p_ma[p,qm] >= sum{e in E, i in I, r in R}(y3[p,e,i,r,w2]*s_b_ma[i,e,qm])+sum{c in
C, b in B, i in I, r in R}(yp3[p,c,b,i,r,w2]*s_b2_ma[i,qm,c,b]);
subject to mass_balance{p in P, w2 in H}:                       # (P10)
       sum{d in D}q[p,w2,d] <= sum{e in E, i in I, r in R} y3[p,e,i,r,w2] + sum{ c in C, b in B, i in I, r in
R}yp3[p,c,b,i,r,w2];
subject to mass_balance2 {p in P, d in D}:                      # (P11)
       sum{w2 in H}q[p,w2,d] >= q1[p,d];
```

```
subject to fixed_cost {d in D}:                                  # (G1)
    sum{p in P}q1[p,d] <= max_dep[d]*h[d];
subject to mass_balance3 {p in P, d in D}:                       # (I1)
    sum{k in K}z[p,d,k] = q1[p,d];
subject to demand_amount {p in P, k in K}:                       # (SL1)
    sum{d in D}z[p,d,k] = sum{n in N} amount[n,p]*w[n,p,k];
subject to demand_revenue {p in P, k in K}:                      # (SL2)
    revenue[p,k] = sum{n in N} rev[n,p]*w[n,p,k];
subject to weights {p in P, k in K}:                             # (SL3)
    sum{n in N}w[n,p,k] = 1;
subject to segment {p in P, k in K}:                             # (SL4)
    sum{m in M}y10[m,p,k] = 1;
subject to logic1{p in P, n in N, m in M, k in K: n = 1 and m=1}:          # (SL5)
    w[n,p,k] <= y10[m,p,k];
subject to logic2{p in P, n in N, m in M, k in K: n = m and n>=2 and n<=10}:    # (SL5)
    w[n,p,k] <= y10[m,p,k]+y10[m-1,p,k];
subject to logic3{p in P, n in N, m in M, k in K: n =11 and m=10}:         # (SL5)
    w[n,p,k] <= y10[m,p,k];

# Parametrs withc are used for extra calculation
param sale_prof;
param fixed;
param prod_cost;
```

AMPL code – Mod File: Scenario 1 (Nonlinear):

The following mod file includes three models, which are used in LD calculations in scenario 1: L1-SM, L1-PM, and PM.

```
# Extra Set

param rep >=0 integer;  # Number of iteration
set REP:= 0..rep;       # Iteration number

# Sets which are used in LSM

set D;              # Set of depots
set K;              # Set of markets
set P;              # Set of  products

# Sets which are used in LPM

set I;              # Set of crude oils
set A;              # Set of components that cannot be processed
set C;              # Set of components that was generated from components from set B
set B;              # Set of components that can either be processed or directly used  in blending
set E:=A union B;       # Union of sets A and B
set QMIN;           # Set of minimum qualities
set QMAX;           # Set of maximum qualities
set R;              # Set of refineries
set H;              # Set of hubs

# Parameters which are used in LSM

param lamda{P,D,REP};   # Value for Lagrangian multiplier lamda
param lamda2{D,REP};    # Value for Lagrangian multiplier mu
param C_TRAN3{D,K};     # Cost of transporting one unit of any product from depot d to market k
param a_demand {P};     # Coefficients of price function, where a picks up competitors action
param b_demand {P};     # Coefficients of price function, where a picks up competitors action
param max_dep{D};       # Cost of transporting one unit of any product from depot d to market k
param C_FIX{D};         # Fixed cost to operate depot d
param min_dem2{P};      # Minimum demand of product p at market k
param max_dem2{P};      # Maximum demand of product p at market k

# Parameters which are used in LPM

param sup{I,R};               # Available volume of crude oil i at refinery r
param rho{I,E};               # Amount of component e generated from one unit of crude oil i
param rho2{B,C};              # Amount of component c generated from one unit of component b
param s_p_m{P,QMIN};          # Value of required quality qmin in product p
param s_b_m{E,QMIN};          # Value of quality qmin in component e
param s_b2_m{C,QMIN};         # Value of quality qmin in component c
param s_p_ma{P,QMAX};         # Value of required quality qmax in product p
param s_b_ma{I,E,QMAX};       # Value of quality qmax in component e obtained from crude oil i
param s_b2_ma{I,QMAX,C,B};    # Value of quality qmax in component c obtained from component b w
param C_BUY{I,R};             # Cost of purchasing one unit of crude oil i at refinery r
param C_PRO{I,R};             # Cost of processing one unit of crude oil i at refinery r
param C_PRO2{B,R};            # Cost of processing one unit of component b at refinery r
param C_BLEND{P,H};           # Cost of producing one unit of product p at hub h
param C_TRAN1{R,H};           # Cost of transporting one unit of any product from refinery r to
```

```
param C_TRAN2{H,D};           # Cost of transporting one unit of any product from hub h depot d
param min_dem{P};             # Minimum production of product p
param max_dem{P};             # Maximum production of product p

# Parameters which are used in PM

param lev{P,D};               # Amount of product p that should be available at depot d

# Extra parametrs

param UB{REP};                # Parameters with are used in calculations
param UB1{REP};               # Parameters with are used in calculations
param Min_UB{REP};            # Parameters with are used in calculations
param Revenue_sale;           # Parameters with are used in calculations
param LB{REP};                # Parameters with are used in calculations
param Max_LB{REP};            # Parameters with are used in calculations
param LB1{REP};               # Parameters with are used in calculations
param mult{REP};              # Parameters with are used in calculations
param dif{P,D,REP};           # Parameters with are used in calculations
param dif_2;                  # Parameters with are used in calculations
param step{REP};              # Parameters with are used in calculations
param dif_lamda2{D};          # Parameters with are used in calculations
param dif_2_lamda2;           # Parameters with are used in calculations
param step_lamda2 {REP};      # Parameters with are used in calculations
param cost_LB;                # Parameters with are used in calculations
param start_lamda{P,D};       # Initial value for Lagrangian multiplier lamda
param start_lamda2{D};        # Initial value for Lagrangian multiplier mu

# Variables which are used in LSM

var buy{P,D} >= 0;            # Amount of product p orders by sale department at depot d
var z{P,D,K} >= 0;            # Amount of product p transported from depot d to marked k
var h{D} binary;              # Binary variable which have value 1 when depot d is used by sale department , and 0
otherwise
var sold_k {P,K};             # Amount of product p that was sold at market k

# Variables which are used in LPM

var x{I,R} >= 1;              # Amount of crude oil i purchased and processed at refinery r
var y{E,I,R} >= 0;            # Amount of component e generated from crude oil i at refinery r
var yp{B,I,R} >= 0;           # Amount of component b generated from crude oil i, used for further processing at
refinery r
var yp1{C, B, I,R} >= 0;      # Amount of component c generated from component b which was again produced
form crude oil i at refinery r
var y2{E,I,R} >= 0;           # Amount of component e obtained from crude oil i sent directly to blending at refinery
var y3{P,E,I,R,H} >= 0;       # Amount of component e obtained from crude oil i used in product p sent from
refinery r to hub h
var yp3{P,C,B,I,R,H} >= 0;    # Amount of component c obtained from component b which was again obtained
from crude oil i, used in product p which was sent from refinery r to hub h
var q{P,H,D}>= 0;             # Amount of product p at depot d which was sent from hub h
var q1{P,D}>= 0;              # Amount of product p at depot d
var h_pl{D} binary;           # Binary variable which have value 1 when depot d is used by production department ,
and 0 otherwise

# Variables which are used in PM
```

```
var x_p{I,R} >= 0;             # Amount of crude oil i purchased and processed at refinery r
var y_p{E,I,R} >= 0;            # Amount of component e generated from crude oil i at refinery r
var yp_p{B,I,R} >= 0;           # Amount of component b generated from crude oil i, used for further processing at refinery r
var yp1_p{C, B, I,R} >= 0;      # Amount of component c generated from component b which was again produced form crude oil i at refinery r
var y2_p{E,I,R} >= 0;           # Amount of component e obtained from crude oil i sent directly to blending at refinery
var y3_p{P,E,I,R,H} >= 0;       # Amount of component e obtained from crude oil i used in product p sent from refinery r to hub h
var yp3_p{P,C,B,I,R,H} >= 0;    # Amount of component c obtained from component b which was again obtained from crude oil i, used in product p which was sent from refinery r to hub h
var q_p{P,H,D}>= 0;             # Amount of product p at depot d which was sent from hub h
var q1_p{P,D}>= 0;              # Amount of product p at depot d


# Objective function L1-SM

maximize Contribution:
                 sum{p in P, k in K} (sold_k[p,k]*a_demand[p]- sold_k[p,k]^2*b_demand[p])
                 - sum{p in P, d in D, per in REP: per=rep} lamda[p,d,per]*buy[p,d]
                 - sum{p in P, k in K, d in D} C_TRAN3[d,k]*z[p,d,k]
                 - sum{d in D}0.5*C_FIX[d]*h[d]
                 - sum{d in D, per in REP: per=rep}lamda2[d,per]*h[d];
# Constrains L1-SM

subject to inn{p in P, d in D}:                    # (L14)
      sum{k in K} z[p,d,k] = buy[p,d];
subject to demand_amount{p in P, k in K}:            # Extra constrain which are used just to calculete amount of product p which was sold at market k
      sum{d in D}z[p,d,k] = sold_k[p,k];
subject to fixed_cost {d in D}:                     # (L13)
      sum{p in P, k in K}z[p,d,k] <= max_dep[d]*h[d];
subject to maximum_dem {p in P, k in K}:             # (L16)
      sold_k[p,k] <= max_dem2[p];
subject to maximum_dem23 {p in P, k in K}:            # (L16)
      min_dem2[p] <= sold_k[p,k] ;

# Objective function L1-PM

maximize Prod_lag:      sum{p in P, d in D, per in REP: per=rep} lamda[p,d,per]*q1[p,d]
                 - sum{i in I,r in R} C_BUY[i,r]*x[i,r]
                 - sum{i in I, r in R} C_PRO[i,r]*x[i,r]
                 - sum{b in B,i in I,r in R} C_PRO2[b,r]*yp[b,i,r]
                 - sum{p in P, d in D, w in H} C_BLEND[p,w]*q[p,w,d]
                 - sum{p in P, r in R, w in H, e in E, i in I} C_TRAN1[r,w]*y3[p,e,i,r,w]
                 - sum{p in P, r in R, w in H, c in C, b in B, i in I} C_TRAN1[r,w]*yp3[p,c,b,i,r,w]
                 - sum{p in P, d in D, w in H} C_TRAN2[w,d]*q[p,w,d]
                 - sum{d in D}0.5*C_FIX[d]*h_pl[d]
                 + sum{d in D, per in REP: per=rep}lamda2[d,per]*h_pl[d];


# Constrains L1-PM

subject to supply{i in I, r in R}:                        # (L1)
      x[i,r] <= sup[i,r];
subject to run_modes {e in E,  i in I, r in R}:               # (L2)
```

```
	rho[i,e]*x[i,r] = y[e,i,r];
subject to component1{a in A, i in I, r in R}:                         # (L3)
	y[a,i,r] >=  y2[a,i,r];
subject to component2{b in B, i in I, r in R}:                         # (L4)
	y[b,i,r] >=  y2[b,i,r] + yp[b,i,r];
subject to processing {c in C, b in B, i in I, r in R}:                # (L5)
	rho2[b,c]* yp[b,i,r] = yp1[c,b,i,r];
subject to products{e in E, i in I,r in R}:                            # (L6)
	y2[e,i,r] >= sum{p in P, w in H} y3[p,e,i,r,w];
subject to products2{c in C, b in B, i in I, r in R}:                  # (L7)
	yp1[c,b,i,r] >= sum{p in P,w in H} yp3[p,c,b,i,r,w];
subject to blending_min{p in P, qm in QMIN, w in H}:                   # (L8)
	sum{d in D}q[p,w,d]* s_p_m[p,qm] <= sum{e in E, i in I, r in R} (y3[p,e,i,r,w] * s_b_m[e,qm]) + sum{c in C,
b in B, i in I, r in R} (yp3[p,c,b,i,r,w] * s_b2_m[c,qm]) ;
subject to blending_max {p in P, qm in QMAX, w in H}:                  # (L9)
	sum{d in D}q[p,w,d]* s_p_ma[p,qm] >= sum{e in E, i in I, r in R} (y3[p,e,i,r,w]* s_b_ma[i,e,qm]) + sum{c in
C, b in B, i in I, r in R} (yp3[p,c,b,i,r,w] * s_b2_ma[i,qm,c,b]) ;
subject to mass_balance {p in P, w in H}:                              # (L10)
	sum{d in D}q[p,w,d] <= sum{e in E, i in I, r in R} y3[p,e,i,r,w] + sum{ c in C, b in B, i in I, r in
R}yp3[p,c,b,i,r,w];
subject to mass_balance2 {p in P, d in D}:                             # (L11)
	sum{w in H}q[p,w,d] = q1[p,d];
subject to demand {p in P}:                                            # (L17)
	min_dem[p] <= sum{d in D} q1[p,d] <= max_dem[p];
subject to fixed_cost2 {d in D}:                                       # (L12)
	sum{p in P}q1[p,d] <= max_dep[d]*h_pl[d];

# Objective function PM

minimize Cost:
		sum{i in I,r in R} C_BUY[i,r]*x_p[i,r]
		+ sum{i in I, r in R} C_PRO[i,r]*x_p[i,r]
		+ sum{b in B,i in I,r in R} C_PRO2[b,r]*yp_p[b,i,r]
		+ sum{p in P, d in D, w in H} C_BLEND[p,w]*q_p[p,w,d]
		+ sum{p in P, r in R, w in H, e in E, i in I} C_TRAN1[r,w]*y3_p[p,e,i,r,w]
		+sum{p in P, r in R, w in H, c in C, b in B, i in I} C_TRAN1[r,w]*yp3_p[p,c,b,i,r,w]
		+ sum{p in P, d in D, w in H} C_TRAN2[w,d]*q_p[p,w,d];

# Constrains PM
subject to supply_p{i in I, r in R}:                                   # (P1)
	x_p[i,r] <= sup[i,r];
subject to run_modes_p {e in E,  i in I, r in R}:                      # (P2)
	rho[i,e]*x_p[i,r] = y_p[e,i,r];
subject to component1_p{a in A, i in I, r in R}:                       # (P3)
	y_p[a,i,r] >=  y2_p[a,i,r];
subject to component2_p {b in B, i in I, r in R}:                      # (P4)
	y_p[b,i,r] >=  y2_p[b,i,r] + yp_p[b,i,r];
subject to processing_p {c in C, b in B, i in I, r in R}:              # (P5)
	rho2[b,c]* yp_p[b,i,r] = yp1_p[c,b,i,r];
subject to products_p{e in E, i in I,r in R}:                          # (P6)
	y2_p[e,i,r] >= sum{p in P, w in H} y3_p[p,e,i,r,w];
subject to products2_p{c in C, b in B, i in I, r in R}:                # (P7)
	yp1_p[c,b,i,r] >= sum{p in P,w in H} yp3_p[p,c,b,i,r,w];
subject to blending_min_p {p in P, qm in QMIN, w in H}:                # (P8)
	sum{d in D}q_p[p,w,d]* s_p_m[p,qm] <= sum{e in E, i in I, r in R}(y3_p[p,e,i,r,w]*s_b_m[e,qm])+sum{c in
C, b in B, i in I, r in R}(yp3_p[p,c,b,i,r,w]*s_b2_m[c,qm]);
```

```
subject to blending_max_p {p in P, qm in QMAX, w in H}:              # (P9)
      sum{d in D}q_p[p,w,d]* s_p_ma[p,qm] >= sum{e in E, i in I, r in R}(y3_p[p,e,i,r,w]* s_b_ma[i,e,qm])+sum{c
in C, b in B, i in I, r in R}(yp3_p[p,c,b,i,r,w]*s_b2_ma[i,qm,c,b]);
subject to mass_balance_p {p in P, w in H}:                  # (P10)
      sum{d in D}q_p[p,w,d] <= sum{e in E, i in I, r in R} y3_p[p,e,i,r,w]+sum{ c in C, b in B, i in I, r in
R}yp3_p[p,c,b,i,r,w];
subject to mass_balance2_p {p in P, d in D}:                 # (P11)
      sum{w in H}q_p[p,w,d] >= q1_p[p,d];
subject to demand_p{p in P, d in D}:                      # (P12)
      q1_p[p,d]  >= lev[p,d];

# Parameters which will be used to extra calculations

param total_s{P,REP};
param total_p{P,REP};
```

AMPL code – Run File: Scenario 1 (Nonlinear):

The following run file includes the algorithms, which is used in scenario 1. The run file is connected with the mod file presented above.

```
# Ampl syntax commands
reset;
model C:\Users\Julia\Desktop\Masteroppgave\ampl\Ref\lag\allknitro.mod;
data C:\Users\Julia\Desktop\Masteroppgave\ampl\Ref\lag\allknitro.dat;
option solver knitro;
option display_precision 0;
option display_round 5;
problem Prodproblem: x, y, yp, yp1,  y2,  y3,  yp3,  q, q1, h_pl, Prod_lag ,
supply, run_modes, component1, component2, processing , products, products2,
blending_min, blending_max, mass_balance,mass_balance2, fixed_cost2, demand ;
problem Saleproblem: buy, z, h,sold_k, Contribution, inn, demand_amount,
fixed_cost,maximum_dem,maximum_dem23;
problem Costprod: x_p, y_p, yp_p, yp1_p,  y2_p,  y3_p,  yp3_p, q_p, q1_p,
Cost, supply_p, run_modes_p, component1_p, component2_p, processing_p,
products_p, products2_p, blending_min_p, blending_max_p, mass_balance_p,
demand_p, mass_balance2_p ;

# Script
let rep := 0;
# Iteration 0
let {p in P, d in D, per in REP: per=rep} lamda[p,d,per] := start_lamda[p,d];
# Values for lamda in 1 iteration
let {d in D, per in REP: per=rep} lamda2[d,per] := start_lamda2[d];
# Values for mu in 1 iteration
let Min_UB[rep] := 100000000000000  ;
# Start value for UBD
let Max_LB[rep] := 0 ;
# Start value for LBD
let mult[rep] := 2;
# Initial value for siqma

repeat {
# Start loop
solve Saleproblem;
# Solve LSM
```

```
solve Prodproblem;
# Solve LPM

for {p in P, d in D}{
# Calculation of input for PM from output of LSM
if  buy[p,d] <= 0 then
        let lev[p,d] := 0;
        else { let lev[p,d] := buy[p,d]};};

solve Costprod;
# Solve PM

let UB1[rep]  := Contribution + Prod_lag;
# Calculation of UBD responding to this iteration
#let UB[rep]  := min(Contribution + Prod_lag,Min_UB[rep]);
# Calculation of LBD which will used in step calculation alternative 2
let UB[rep]  := Contribution + Prod_lag;
# Calculation of LBD which will used in step calculation alternative 1
let{p in P}total_s[p,rep] := sum{d in D}buy[p,d];
# Calculation of all sold products by sale department
let{p in P}total_p[p,rep] := sum{d in D}q1[p,d];
# Calculation of all produced products by production department
let Revenue_sale := sum{p in P, k in K} (sold_k[p,k]*(a_demand[p])-
sold_k[p,k]^2*(b_demand[p]))
        -sum{p in P, k in K, d in D} C_TRAN3[d,k]*z[p,d,k]- sum{d in
D}C_FIX[d]*h[d];   # Calculation of profit which sale department  have
generated under this iteration
let LB1[rep] := Revenue_sale - Cost;
# Calculation of LBD under this iteration
# Calculation of gradient
let {p in P,d in D} dif[p,d,rep] := buy[p,d]-q1[p,d];
let dif_2 := sum{p in P,d in D}(dif[p,d,rep]^2);
let {d in D} dif_lamda2[d] := h[d]-h_pl[d];

let LB [rep] := max(LB1[rep],Max_LB[rep]);
# Calculation of highest LBD
let step[rep] := (UB[rep]-LB [rep])*mult[rep]/dif_2;
# Step size for lamda
let step_lamda2[rep] := 482000/(2*(rep +1 ));
# Step size for mu


if rep = 200 then break;
# Stop loop if rep equals 200(can be any number)
else {
        let rep := rep + 1;
# Next iteration
        let Min_UB[rep] := UB[rep-1];
# Update lowest UBD
        let Max_LB[rep] := LB[rep-1];
# Update highest LBD
        # Calculation of lamda for next iteration
        for {p in P, d in D, per in REP: per=rep}{
        if  lamda[p,d,per-1]+step[per-1]*dif[p,d,per-1] >= 0 then
                let lamda[p,d,per] := lamda[p,d,per-1]+step[per-
1]*dif[p,d,per-1];
        else { let lamda[p,d,per] := 0.01};}
```

```
        # Calculation of mu for next iteration
        for {d in D, per in REP: per=rep}{
                if  lamda2[d,per-1]+step_lamda2[per-1]*dif_lamda2[d] > 0 then
                        let lamda2[d,per] := lamda2[d,per-1]+step_lamda2[per-
1]*dif_lamda2[d];
        else { let lamda2[d,per] := 0.01};}
        # Update sigma if changes in UBD was small under last 5 iteration
        if rep >= 6 then
        if sum{per in REP:per<= rep-1 and per>= rep-5}(UB[per-1]-UB[per])< 10
then let mult[rep] := mult[rep-1]*0.9 ;
        else {let mult[rep] := mult[rep-1]      ;}
        else {
        let mult[rep] := mult[rep-1]; };};};

# Display comands
display UB,LB,UB1,LB1;
#display total_s,total_p;
```