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Employment, transport infrastructure and rural depopulation: a new spatial equilibrium model

BY David Philip McArthur, Inge Thorsen, AND Jan Ubøe



NORWEGIAN SCHOOL OF ECONOMICS.

Employment, transport infrastructure and rural depopulation: a new spatial equilibrium model *

David Philip McArthur,[†] Inge Thorsen[‡] and Jan Ubøe[§]

Abstract

In this paper we propose a new spatial equilibrium model, and use it to discuss issues related to rural depopulation. The discussion focuses on how investments in transport infrastructure and the spatial distribution of basic sector jobs can promote a relatively balanced growth of peripheral and central areas of a region. Through interdependencies in individual migration decisions and an economic base multiplier mechanism, negative exogenous shocks may take a peripheral zone beyond a bifurcation point, into an equilibrium of dramatically lower population and employment. We study how the location of bifurcation points depend on spatial interaction behavioural parameters and variables subject to regional policy. We also discuss the issue of the timing of interventions intended to prevent a process of rural depopulation.

1 Introduction

This paper focuses on the intraregional spatial location pattern of jobs and people. Location decisions of firms and households are typically interdependent. Profit opportunities for firms depend on their location relative to potential customers and qualified workers, while residential location choices typically reflect job opportunities within a reasonable commuting distance. A specific location pattern has its counterpart in flows of labour, capital, and goods. The current

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[†]Department of Health Management and Health Economics, University of Oslo and Stord/Haugesund University College, e-mail: d.p.mcarthur@medisin.uio.no

[‡]Stord/Haugesund University College, e-mail: inge.thorsen@hsh.no

[§]NHH Norwegian School of Economics, e-mail: jan.uboe@nhh.no

spatial distribution of employment and population reflects flows of migration, commuting, and shopping in previous periods, and spatial mobility depends on characteristics of the location pattern. Hence, a process of regional growth incorporates a complex set of interdependencies that should be accounted for in a model explaining and predicting the development of the central place system and spatial interaction in a region.

This paper uses a spatial equilibrium model to discuss the phenomenon of rural depopulation. Many regions experience centralisation. For example, centralisation has been observed in several regions of western Norway. Some towns developed many decades ago due to the presence of lowcost hydroelectricity, which attracted power-intensive industries. The comparative advantage of such locations lost significance as the transportation costs of power fell. This resulted in reduced employment in manufacturing, leading to a process of economic decline and depopulation. The issue of depopulation is explored with the introduction of a new spatial equilibrium model. This approach is used to find how the risk of depopulation depends on the local provision of basic sector jobs, and on the accessibility to jobs at alternative locations.

The literature offers some interesting operative spatial equilibrium models. One very successful set of models belong to the 'new economic geography' tradition (Krugman, 1991a,b; Venables, 1996; Fujita et al., 1999). This tradition of models normally takes a relatively macroscopic perspective. Models are specified for a multi-regional set-up, with relatively large regions. Intraregional disparities in prices of labour and housing are ignored, and they do not explicitly take account of intraregional commuting and shopping. The spatial distribution of jobs and people results from a balance between centrifugal and centripetal forces.

Large-scale models intended for urban planning also feature in the literature. Hunt et al. (2005) provide a review of six operational urban land-use-transport models. The aim of the review is to define what characterises this class of models, as well as to identify strengths and weaknesses of the different operational models. Important features identified include, inter alia, the ability to account for multi-model travel, model route choices and model land use patterns. Such factors are obviously important at the urban scale, but we argue they are less important for the applications considered in this paper. Other features of such models may be relevant for our purposes, for instance travel demand models and the presence of a basic sector (i.e. industries which export and depend on external demand) in a zone.

Analysing regional policy in a typical Norwegian region calls for a spatial dimension somewhere between new economic geography type models and urban models. Rural depopulation has been a policy challenge for the Nordic countries for many decades, where rural areas have experienced significant depopulation (Håkansson, 2000; Hjort and Malmberg, 2006). One policy option to combat depopulation would be to improve accessibility to urban labour markets. Partridge et al. (2010) find that access to urban employment can be a key source of population retention and growth. Another approach may be to increase employment into an area. Renkow (2007) found that in metropolitan areas most new jobs (60-70%) are filled by commuters rather than locals. In rural areas, an increase in the demand for labour tends to be met largely with a fall in the out-migration rate. It is therefore hard to predict what the effect of a particular policy will be.

The paper is structured as follows: Section 2 provides a non-technical outline of the basic mechanisms incorporated in the modelling framework. A technical specification of the model is provided in the Appendix. Section 3 studies how the equilibrium population and employment in a rural area depends on the generalised cost of travelling to the central business district. The existence of bifurcation points is identified in Section 4, while Section 5 focuses on the time dynamics of adjustment. Possible policy responses to depopulation are considered in Section 6, before Sections 7 and 8 address problems related to the timing of policy measures to prevent rural depopulation. Some concluding remarks are provided in Section 9.

2 A non-technical description of the modelling framework

The core of the model centres on the definition of equilibrium, involving intra-regional migration and commuting flows corresponding to a specific spatial distribution of jobs and workers between the zones of a region. To reach an operational model specification we introduce a set of reasonable hypotheses on the spatial behaviour of firms and households. Hence, we concentrate on the spatial dimension of the supply and demand for labour.

Consider the demand for labour at specific locations. Like most spatial equilibrium models, our model incorporates the core elements of economic base modelling (Lowry, 1964). The model therefore distinguishes between two types of firms. The activity of local-sector firms is determined by demand arising from within the region, while production in basic-sector firms is determined by factors unrelated to intraregional demand.

Three types of spatial mobility are considered in the model. Workers in the model may migrate between zones. In addition to migration and commuting, certain non-work travel behaviour is modelled. The spatial distribution of people, and their non-work travel behaviour, determines the distribution of local-sector activity.

The basic sector, local innovativity and competitiveness

The number of basic sector jobs at a specific location depends on local innovativity and competitiveness. This may reflect agglomeration economies, the wage level, entrepreneurial spirit, transport costs, the availability of qualified workers etc. There is no attempt to account explicitly for local variations of innovativity and competitiveness in the current version of the model. The spatial distribution of basic sector jobs is assumed exogenous.

Local sector activity

The spatial distribution of local sector activity will depend on the residential location pattern, the transportation infrastructure and people's travel behaviour. The local sector serves the local population of a zone, and is comprised of things such as retail, schools, medical facilities etc. Willingness to travel to the locations of such activities declines with the travel costs which must be incurred. The deterrence effect of these travel costs will likely vary by activity.

One approach to modelling the ratio of local sector jobs to the population in a zone (henceforth referred to as local sector density, or LSD for short), is to assume that is is constant within a region. Assuming a uniform density of activity across spatial units may be reasonable at certain scales, but we argue that it is not in the intraregional case we consider.

We propose an alternative approach to modelling the spatial distribution of local-sector activity based on Gjestland et al. (2006). The idea is that the trip frequency to different locations will depend upon the local availability of the activity in question, and on the travel costs to each location at which the activity is available (Krizek, 2003; Handy, 1992).

A next step in deriving a spatial pattern of local-sector activities is by recognising that economies of scale, transportation costs, and agglomeration benefits allow firms in a central location to offer a wider range of goods and services, and at a lower price than firms located in more peripheral locations. Agglomeration benefits explain why some types of local sector activities will largely be concentrated in a centre. Administrative services often locate in the centre, giving rise to agglomeration benefits which in turn attract more activity. At the same time businesses in many cases choose to locate in the same area because consumers often perceive it to be beneficial if they can satisfy their demand for several goods and services with one shopping trip.

As an example, consider the number of shop-employees per resident as a proxy for the local sector density. Low prices explain a very high local sector density in the centre of the region. A significant proportion of the shopping trips emanating from a suburb will be directed towards the regional centre, since customers here can benefit from low prices in the centre at relatively low transport costs. For zones which lie far from the centre, virtually all shopping will take place within the zone. The local sector density will be high in the regional centre, low in suburbs, and it will be approaching the regional average as the generalised cost sensitivity from the centre increases. Gjestland et al. (2006) find empirical support for such an intuitively appealing pattern from observations of Norwegian regions. The Appendix explains how this reasonable hypothesis is made operational in the spatial equilibrium model.

The discussion so far means that the intraregional distribution of (local-sector) jobs reflects the residential location pattern. At the same time it makes sense to assume that residential location decisions are influenced by the job opportunities within a reasonable commuting distance. This means that the spatial distribution of jobs and people are interdependent. This is the fundamental mechanism in economic base modelling.

The decision to stay or move from a residential site

The residential location choice can be considered to result from a two-step decision process. First, a household decides whether to move from the current residential site. Second, households moving have to choose between alternative locations. Consider first the diagonal elements of a matrix of transition, migration probabilities.

One hypothesis incorporated into our model is that the probability of remaining in a zone is positively related to the labour market accessibility of the zone. This is consistent with the findings from Swedish microdata (Lundholm, 2010; Eliasson et al., 2003) and similar work in the Netherlands (Van Ham and Hooimeijer, 2009). The explanation is that labour market accessibility allows greater flexibility, and can generally be seen as a desirable attribute for a residential location.

One challenge in the model formulation is to specify an operational measure of labour market accessibility. We choose to represent accessibility by a measure of the generalised cost to reach all other zones in the region. Each zone is weighted by the number of jobs, adjusted for the competition for jobs measured by the ratio of jobs to local job seekers. In addition, the weights involve a cost deterrence function that places a relatively high weight on destinations which can be reached at a low cost from the residential location.

Finally, the measure of generalised cost is combined with information on the local labourmarket situation in the function that determines the probability that workers move from a specific zone. Assume that a zone has high unemployment. If this zone is centrally located in the region, many workers will choose to commute rather than move. If however accessibility is low, then migration will be a more frequent response to high unemployment.

Spatial equilibrium and migration flows between different zones

The migration between different zones is modelled through the introduction of a search strategy where a worker evaluates destinations successively outwards over the network. The worker will move to the first place where the conditions are 'satisfactory'. Options further out in the network will then not be evaluated. Hence, an absorption effect is introduced, analogously to the basic idea in the theory of intervening opportunities (Stouffer, 1940). This further means that the probability of moving decreases as the worker evaluates alternatives which lie progressively further out in the network.

Another central hypothesis within the regional science literature is that distance, which we convert to a generalised cost, limits spatial interaction. Accounting for the absorption effect and the cost deterrence effect forms a symmetric matrix, that is subsequently normalised into a migration probability matrix. This matrix is then used to find the equilibrium solution for the system.

The relationship between the spatial distribution of jobs and people, an economic base multiplier process

The spatial distribution of jobs are linked to the spatial distribution of people through labour market accessibility, and the simultaneity between commuting and migration flows. The economic base mechanism represents the more direct link between the location of jobs and people. At the same time workers employed in local sector firms tend to prefer a residential location close to the firm. Assume increased basic-sector activity in a zone. This causes a rise in labour demand, attracts labour to the zone, and increases the demand for goods and services produced in the local sector. This creates further demand for labour and initiates a positive growth cycle, known in the literature as an economic base multiplier process.

The equilibrium modelling approach in this paper accounts for different kinds of interdependencies in the treatment of location decisions made by firms and households. This can be argued to be preferable to introducing a specific causality on the employment-population interaction. According to Hoogstra et al. (2011), the nature of this causality differs across space and time. In a meta-analysis, they find that the empirical evidence is highly inconclusive on the jobs-people direction of causality, albeit most results point towards jobs following people.

Interdependent migration probabilities

In addition to the local labour-market situation and the position in the transportation network, the model accounts explicitly for the possibility that migration probabilities may be interdependent. The hypothesis is that the probability of migrating from a zone increases if the population falls below some critical level. If, for instance, the population falls below a certain threshold due to out-migration, institutions taking care of important community services may have to close down. The threshold may of course differ between services/businesses, such as a school, post office, bank, grocery store etc. (Henderson and Taylor, 2003; Parr, 1966; Berry and Garrison, 1958b). This is a core idea of central place theory (see Berry and Garrison (1958b) for a discussion). Berry and Garrison (1958a) provide population thresholds for a range of services. (Parr, 1966) discusses the possible adverse impacts of out-migration from a depressed region, and how this can lead to 'ghost towns' in extreme cases.

3 A case where basic sector jobs are evenly spread between non-cbd zones

In this section we study how the number of jobs and workers in a zone depends on the location of the zone relative to the central business district (cbd) and the other zones of the region. This is done through simulations, where the generalised cost between a zone and the cbd is systematically varied.

3.1 An illustrative geography

Consider the simple, 5-node network in Figure 1. The central business district is located in zone B. The generalised costs from the cbd to the other zones (d_{ij}) are indicated in the figure. The suburban zone D is located only 5 units from zone B, while the zones C and E are located within a reasonable commuting distance from the cbd of the geography; 30 and 20 units, respectively. In Figure 1 zone A appears to be a peripheral rural location, where a generalised cost of 80 must be incurred to reach the cbd. The generalised cost between the zones A and B, d_{AB} , will be systematically varied to study the impact of this cost on the equilibrium employment and population in zone A. Reductions in generalised cost may occur due to investments in road infrastructure.

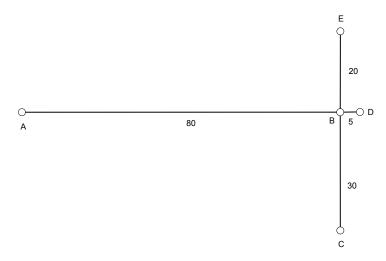


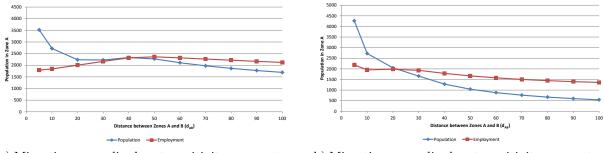
Figure 1: A 5-node network of zones.

Finding an equilibrium spatial distribution of population and workers of course calls for a parametrisation of the model. The parameters are defined in the Appendix, which provides a technical presentation of the model. The parameter values chosen for the standard case of the numerical experiments are presented in A.8.

3.2 Equilibrium population and employment is depending on the proximity to the cbd

In this section, we assume a concentration of basic sector jobs (E_i^b) in the cbd, while the basic sector jobs that are not located there are assumed to be evenly spread between the other zones: $E_B^b = 5000, E_A^b = E_C^b = E_D^b = E_E^b = 1000$. The discussion to follow focuses primarily on the location of zone A relative to the cbd. Changes in d_{AB} will affect both residential location decisions, commuting behaviour, and non-work travel behaviour. Hence, the equilibrium spatial distribution of jobs and people will also be affected.

Consider first that zone A is located very close to the cbd. As illustrated in Figure 2a, zone A takes on the role of a suburban zone, with a high population and a relatively low number of local sector jobs. The zone is attractive for commuters, while the households living here tend to do access services in the cbd rather than locally.



a) Migration generalised cost sensitivity parameter $\beta=-0.5$

b) Migration generalised cost sensitivity parameter $\beta = -1.0$

Figure 2: Equilibrium population and employment in zone A, for different values of d_{AB} .

The equilibrium employment and population is particularly sensitive to variations in the generalised cost at low levels of d_{AB} . If, for instance, d_{AB} were 10 rather than 5, zone A becomes less attractive as an origin of commuting, resulting in reduced population. On the other hand more people access services locally, which means that the zone gets more attractive for local sector activities. Hence, there are two forces pulling the equilibrium level of population in different directions. In the case shown in Figure 2a, the force explained by commuting attractive balanced for

 $30 < d_{AB} < 50$. For $d_{AB} > 50$, local sector activities are only marginally affected by variations in generalised cost. Hence, the effect on labour market accessibility dominates, explaining why both population and local sector employment are declining functions of the generalised cost from the cbd for $d_{AB} > 50$.

Figure 2a corresponds to a case where the cost deterrence effect in the migration decision is represented by a value of $\beta = -0.5$. i.e. an elasticity equal to -0.5. Plane (1984) finds a distance deterrence value of -1 for interstate migration in the US. We assume movers are less sensitive to generalised cost on an intraregional scale. Figure 2b corresponds to a case where $\beta = -1.0$. This reflects stronger preferences for living in, or close to, the current residential zone. By comparing the two parts of the figure it follows that the equilibrium population and employment of zone A is more sensitive to variations in generalised cost in the case with stronger residential site preferences.

If zone A is located close to the cbd and the other zones, workers living here can take advantage of a high labour market accessibility, without moving far from their preferred residential location. This makes zone A popular as a residential location, especially in cases with a strong cost deterrence effect for local migration. In Figure 2b this is reflected in a high equilibrium population of zone A, for low values of d_{AB} . Notice also that the equilibrium number of local sector jobs, is relatively high for low values of d_{AB} in Figure 2b. Hence, the effect of a high number of potential customers/service-users living nearby dominates the effect that people do their shopping or access local services in the cbd rather than locally.

For high values of d_{AB} , zone A is in a location of low labour-market accessibility. A high β in addition means that workers from other zones are reluctant to move to zone A, even if there is a good chance of receiving job offers. The zone offers an unfortunate combination of low labour-market accessibility and a location far from the preferred residential location for a large majority of the population. This is reflected in Figure 2b. For large generalised costs to the cbd, the equilibrium population in zone A is considerably lower than at the same cost as in part a) of the figure, where workers are assumed to be less attached to their residential location. Hence, a situation where the inhabitants of a region have strong location preferences for their current residential area is detrimental for a peripheral zone, and helps explain rural depopulation. Notice also from Figure 2b that low levels of population corresponds to low employment in zone A.

Figure 2 can be used to provide predictions about how investments in transportation infrastructure affect the equilibrium population and employment in different zones. Consider the case where the generalised cost has a low weight in the migration decision, in part a) of the figure. Assume further that the generalised cost of travelling between zone A and the cbd is 80 units, and is then reduced to 60 units as a result of road improvements. These investments are predicted to result in an increase in the working population of around 245 workers, while the number of (local sector) jobs is predicted to increase by 98. This corresponds to an increase of 13% of the working population, and 4.5% of the number of local sector jobs.

Consider next a case where zone A is initially located closer to the rest of the system, where the cost of travelling to the cbd is 30 units. Investments leading to a reduction of the generalised cost to the cbd of 20 units is then predicted to result in an equilibrium solution where 93 more workers choose to live in zone A, while the number of jobs is reduced by 134. The reduction in employment reflects the tendency that households access services in the cbd. As higher generalised costs, d_{AB} , are considered, this effect is eventually offset by the increasing population, and $d_{AB} \approx 50$ represents an optimal location if the ambition is to maximize the number of jobs in zone A.

Population is more sensitive to changes in generalised cost the higher the weight it received in the migration decision, see Figure 2b. This also means that the potential benefits are higher, in terms of population growth in the zone that is better connected to the cbd, of investments in the transportation network. The number of jobs in zone A is, on the other hand, less sensitive to changes in d_{AB} in this case. Notice in particular that the number of jobs is no longer predicted to be reduced as a result of road improvements, in a situation where zone A is initially located within 50 units from the cbd.

4 The existence of a bifurcation point

Assume that zone A has 150 rather than 1000 basic-sector jobs, and that the difference of 850 jobs is proportionally distributed among the other zones. As equilibrium solutions are calculated for increasing values of d_{AB} , a situation emerges where a process of interdependent migration decisions comes into action. Section A.4 explains how these interdependent migration decisions are specified. This bifurcation leads the zone into a very different kind of equilibrium, with a

dramatically lower level of population and local-sector jobs.

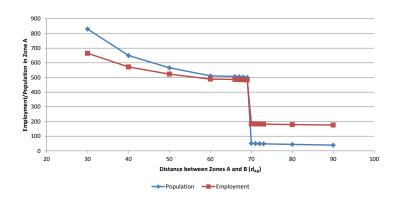
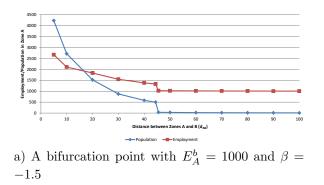
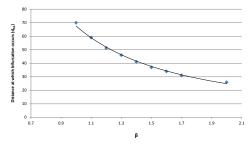


Figure 3: A bifurcation point with $E_A^b = 150$

In Figure 3, the bifurcation point is located at $d_{AB} \approx 69$. The figure illustrates how the character of the equilibrium solution differs dramatically if the generalised cost between A and the cbd is 70 rather than 68. We will return to the time dynamics, and potential transition problems, if changes in the road network bring the relevant generalised cost beyond the bifurcation point. The position of this bifurcation point was found numerically, rather than attempting to derive it analytically. The non-linear nature of the model would make such a derivation prohibitively complex.

Consider the impact of variations generalised cost sensitivity parameter in the migration function, β , on the likelihood of a dramatic rural depopulation. Figure 4a illustrates the results based on the same experiment that was illustrated in Figure 3, with the exception that the cost deterrence parameter is now represented by $\beta = -1.5$, rather than $\beta = -0.5$. By comparing the two figures, it becomes clear that a stronger deterrence effect brings the bifurcation point closer to the cbd. In other words, a strong deterrence parameter extends the range of geographies where rural areas will be depopulated. This is made more clear in Figure 4b, where the location of the bifurcation point is plotted as a function of the cost deterrence parameter. The figure suggests that this relationship follows a power law, where the elasticity of the generalised cost to the cbd with respect to β is about 1.4.





b) The cost determence effect in the migration decision affects the location of the bifurcation point; $E_A^b = 1000$

Figure 4: The bifurcation point and the cost deterrence effect in the migration decision.

5 Time dynamics and hysteresis related to shocks in the road transportation network

The results of the experiments carried out above were presented in a comparative static framework, comparing equilibrium solutions for different values of an exogenous parameter. The underlying model accounts for time dynamics, for instance in terms of the probability that a household will stay in a zone within the given period. We interpret a time period in our model as one year.

Figure 5 explicitly takes into account the time dimension of the transition from one equilibrium to another after an exogenous shock. The time dimension is represented on the horizontal axis, while the vertical axis measures the population in zone A. The first shock occurs in year 10. The generalised cost between zone A and the cbd, d_{AB} , is increased from 60 to 75.

According to Figure 5, it takes about 15 years from when the shock is introduced until a new equilibrium reached. The process is driven by the interdependent migration decisions and the economic base multiplier mechanism. One interesting question is whether this process is reversed if the initial generalised cost (d_{AB}) of 60 is restored, through investments in the road network for example. These investments are made in year 50. It follows from Figure 5, however, that the investment has a very marginal impact on the equilibrium population in zone A; an improvement of the road network back to the previous standard does not initiate a process taking zone A back to the "high-level" equilibrium with a population of more than 500. A new shock is introduced in year 84, once again causing an increase in the generalised cost from $d_{AB} = 60$ to $d_{AB} = 75$. This restores the previous low-level equilibrium.

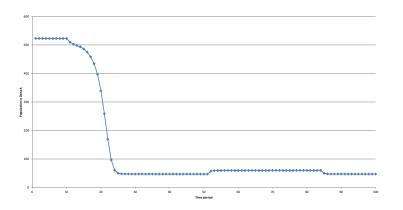


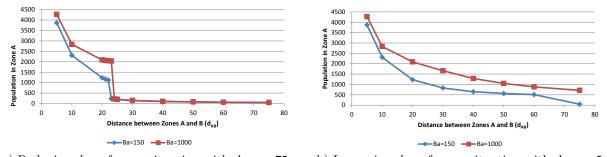
Figure 5: Consequences on equilibrium population in zone A of shocks in the road network; $d_{AB} = 60$ for t < 10 and t > 84, $d_{AB} = 75$ for 10 < t < 84. $E_A^b = 150$.

The numerical experiments illustrated in Figure 5 demonstrate that negative and positive shocks are not necessarily symmetric in their effect on the equilibrium population of a rural zone. Once the low-level equilibrium is reached as a consequence of a negative shock in the road standard, a corresponding positive shock does not bring the equilibrium solution back to the high-level equilibrium. This reflects a case of hysteresis, where a transitory shock results in a permanently different equilibrium solution. Such phenomena can be captured in non-linear mathematical models with a multitude of equilibria.

The phenomenon demonstrated in Figure 5 also means that it is not straightforward to explain the emergence of a new central place from the mechanisms which are built into the model. It would probably help if possible spatial disparities in wages and housing prices were taken into account, but this is left for future research. In the next section we will discuss whether combinations of new basic sector jobs and improvements in the road network can bring zone A into a high-level equilibrium, from a state where the zone is depopulated.

6 Stimulating rural growth through road infrastructure investments and/or a redistribution of basic sector jobs

Consider a situation where $d_{AB} = 75$, and zone A is depopulated, as in the low-level equilibrium in Figure 5. The experiments in the previous section demonstrated that a restoration of the road network, back to the situation where $d_{AB} = 60$, was not enough to trigger a process of population and employment growth in zone A. As an alternative to heavy investments in transportation infrastructure, it is worth considering the possibility of rural growth through a redistribution of basic sector jobs in favour of zone A. For example, decentralising employment was a strategy used by the Norwegian government when it moved several government departments out of the capital. The impact of these two regional policy instruments is illustrated in Figure 6a. Consider first an increase in the number of basic sector jobs in zone A, from $E_A^b = 150$ to $E_A^b = 1000$. This increase is matched by a proportional reduction of a total of 850 jobs in the other zones of the region. According to Figure 6a, the increased number of basic sector jobs has no impact on the equilibrium population in zone A, unless this policy is accompanied by heavy investments in the transportation infrastructure. For $d_{AB} > 24$, zone A is a basic sector employment centre, where workers are recruited through in-commuting.



a) Reducing d_{AB} , from a situation with $d_{AB} = 75$ b) Increasing d_{AB} , from a situation with $d_{AB} = 5$ Figure 6: The impact of hypothetical variations in d_{AB} on the equilibrium population in zone A

Figure 6a further illustrates that investments in transportation infrastructure have no impact on the equilibrium population in zone A, unless $d_{AB} < 22$. The bifurcation point, where a further reduction in generalised cost takes the zone to a high-level equilibrium, is remarkably insensitive to variations in the number of basic sector jobs. A situation with 1000 basic sector jobs gives approximately the same bifurcation point as in the situation with only 150 basic sector jobs. Once the bifurcation point has been passed, however, zone A benefits from the higher number of basic sector jobs.

The results of the reversed experiment are illustrated in Figure 6b. Here, $d_{AB} = 5$ initially, and the generalised cost has been increased towards $d_{AB} = 75$. Comparing both parts of Figure 6, it is once again demonstrated that positive and negative shocks do not, in general, have symmetric effects on the equilibrium population in the highly non-linear model that is being used. Increased generalised cost provides a more gradual trend towards a lower equilibrium level, and the low-level equilibrium emerges for rather high values of d_{AB} .

7 The timing of investments in road infrastructure

The experiments in the two previous sections demonstrate that in practice, it may be difficult to reverse a process where a rural area has been totally depopulated. What if action is taken before the area becomes depopulated? The situation that is illustrated in Figure 7 shows that the timing of intervention can play a pivotal role in determining the outcome.

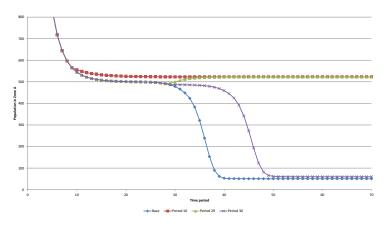


Figure 7: The population in zone A, and the timing of investments in road infrastructure.

Assume a situation where zone A is moving towards an unfavourable equilibrium, more or less totally depopulated. This may be due to deteriorated road standards, and/or a reduction in the number of basic sector jobs. The development without intervention is represented by the "base" curve in Figure 7.

Assume further that the authorities are considering intervening to prevent a process of depopulation. In Figure 7, the intervention involves investing in transport infrastructure to improve the accessibility, and hence attractiveness, of zone A. There are lags associated with such investments, however. It takes time to observe and identify a process leading to a total rural depopulation, and it takes time to make decisions, to plan and finance the investments, in addition to the time involved in the construction of new infrastructure.

According to Figure 7, new infrastructure introduced in period 10 prevents zone A from being totally depopulated, and helps preserve a high-level equilibrium. The same conclusion is valid as long as the new transport network is completed within period 29. If it takes 30 or more periods to complete the project, the investments cannot prevent total depopulation of zone A. In such a case, the process towards a depopulation may be delayed, but the point of no return bas been passed, and the process can only be reversed through massive investments in road infrastructure and/or basic sector jobs.

8 Firm closure and the timing of intervention

Assume once again that zone A is initially close to a point of bifurcation, corresponding to the situation in the first 10 periods in Figure 8. In this situation a firm with 50 closes down. If no new jobs are created, this takes the zone to a low-level equilibrium, more or less totally depopulated.

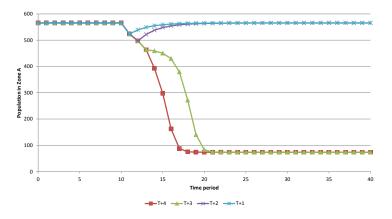


Figure 8: A firm with 50 employees is closed down in period 10; the figure illustrates how the effects of a new firm depends on the timing of the establishment.

What if the closed firm is replaced by a new firm, with the same number of jobs? According to the numerical experiments, illustrated in Figure 8, this depends on the timing of the establishment of the new firm. If the firm is established 4 periods after the initial shock, the process towards a total depopulation is not reversed. This also applies if the firm is established 3 periods after the initial shock, but the process towards a total depopulation is then delayed compared to the case represented by the curve T + 4 in Figure 8. The original, pre-shock equilibrium is re-established only if the investments are made no later than two periods after the closure of the firm.

The numerical experiments which underlie Figure 8 demonstrate that the timing of inter-

vention is crucial if the policy is to be successful. If intervention is slow, the point of no return might be passed, and the result will then be rural depopulation. Hence, there is a need for quick intervention. Recall, however, that prices, and wages, are assumed to be fixed. An alternative to public investments in private entities is to create flexible markets, so prices and wages are adjusted quickly to the new situation. New jobs will then result from private investments, as a response to the negative shock. Adjustments through the market may, however, be too slow to prevent that a low-level equilibrium follows from the negative externalities between migration decisions of individual workers. Hence, it can be argued that authorities should monitor the market closely, and encourage a rapid response to situations with closures of firms. This may prevent an unfortunate local development where a transitory shock has a permanent effect.

9 Conclusions

In the paper we propose a new spatial equilibrium model and use it to explore how changes in the transportation network and the distribution of jobs may cause a rural area to become depopulated. The analysis demonstrated how complex non-linearities being captured by the model can generate seemingly counter-intuitive results. Understanding the reasons for these results is crucial for policy makers.

In a case where a peripheral zone has a low number of basic sector jobs, a situation may emerge where a process of interdependent migration decisions and an economic base multiplier mechanism comes into action, taking the zone into an equilibrium solution with a dramatically lower level of population and employment. The approach with numerical experiments gives an opportunity to study how the existence and location of such a bifurcation point depends on aspects of migration, travel, and labour-market behaviour. We find, for instance, that a process where a rural zone is being more or less totally depopulated gets more likely as the willingness to migrate over distance falls.

A negative shock which increases the generalised cost of travelling between two zones may lead to a process of rural depopulation. Negative and positive shocks are not necessarily symmetric in their effect on equilibrium population of a rural zone. This reflects a case of hysteresis, and typically occurs in non-linear mathematical models with multiple equilibria.

The timing of intervention to prevent depopulation was found to be very important. If

interventions are introduced too late, the process can be delayed, but not stopped or reversed without massive investments in road infrastructure and/or basic sector jobs. The process of rural depopulation can be initiated both by a negative shock in generalised travelling costs, or by the closure of a basic sector firm. In the latter case, one type of intervention is public investments in private entities. Alternatively, it may help to create flexible markets, so that prices and wages adjust, and the zone attracts private investment.

These conclusions are based on a set of simplifying assumptions. One obvious model extension would be to take into account that the effects of local exogenous shocks can be attenuated through price changes in labour and housing markets. Such extensions would render the proposed model a spatial *general* equilibrium model. In some economies, like the Norwegian, wages are resulting from centralised wage-setting, however, and intraregional wage disparities have no major impact on the spatial allocation of labour. In such economies, housing market effects may have a more substantial influence on the intraregional distribution of jobs and people. The discussion of potential effects through wages and housing prices is, however, left for future research.

A A technical presentation of the model

This appendix provides a technical presentation of the model. The spatial distribution of basic sector firms is treated as exogenous, which means that aspects of local innovativity and competitiveness are not explicitly accounted for. In this version of the model we further ignore the possibility that migration decisions are affected by job diversity and local amenities. House prices and wages are assumed to be fixed and exogenous.

While the technical presentation below might appear somewhat elaborate, it is not. We coded this in the programme Mathematica, with around one page of code. Mathematica was chosen for convenience, certainly other programs would be more efficient. Speed could be increased significantly if the code was translated into, e.g. C, but the Mathematica code was sufficient for our purposes.

A.1 Basic- and local-sector firms

Total employment in zone i (E_i) is defined as the sum of basic-sector employment (E_i^b) and local-sector employment (E_i^l) in the zone:

$$E_i \equiv E_i^b + E_i^l \tag{1}$$

Let $\mathbf{L} = (L_1, \dots, L_n)$ be a vector representing a given residential location pattern of workers, while T_{ij} is the probability that a worker lives in zone *i* and works in zone *j*. Hence, $\mathbf{T} = [T_{ij}]$ represents the commuting matrix, and by definition:

$$L_i = \sum_{j=1}^{N} T_{ij} E_j \qquad \text{i.e.} \qquad \mathbf{T} \mathbf{E} = \mathbf{L}$$
(2)

The spatial distribution of local-sector activities reflects both the spatial residential pattern and non-work travel behaviour. Assume that the number of workers living in a zone is proportional to the number of residents/consumers in the zone, and let C_{ij} be the number of local sector jobs in zone *i* which are supported by consumption from workers living in zone *j*. Hence, $C = [C_{ij}]$ is a non-work travel matrix, and the spatial distribution of employment in local-sector activities is given by:

$$E_j^l = \sum_{i=1}^N C_{ij} L_i \qquad \text{i.e.} \qquad \boldsymbol{E}^l = \boldsymbol{C} \boldsymbol{L}$$
(3)

A.2 Interzonal migration flows and spatial equilibrium

In modelling migration probabilities Nævdal et al. (1996) introduced a nice trick to facilitate construction of Markov chains. The construction uses a symmetric matrix $\boldsymbol{Q} = \{Q\}_{i,j=1}^{N}$, where all the elements are dependent on the characteristics of the geography. Nævdal et al. (1996) showed that any assumption about the coefficients Q_{ij} can be interpreted as an assumption about migration flows in the equilibrium state.

As a next step Nævdal et al. (1996) introduced some network characteristics which are symmetric between zones, and which are relevant in explaining the relevant kind of spatial interaction. For a connected network with fixed Q_{ij} -s, the construction produces regular Markov chains, and regular Markov chains always have unique equilibria. Here is how we implemented this construction in our model:

The connectivity matrix of the network is defined as follows:

$$\operatorname{con}_{ij} = \begin{cases}
1 & \text{if } i \text{ and } j \text{ are connected by a direct link} \\
0 & \text{otherwise}
\end{cases} \tag{4}$$

The construction needs the neighbour levels m_{ij} . These can be found efficiently by comparing powers of the connectivity matrix, i.e., the neighbour level is the lowest power of the connectivity matrix producing a non-zero entry. The symmetric matrix \mathbf{Q} is then defined by

$$Q_{ij} = \begin{cases} s^{m_{ij}} d_{ij}^{-\beta} & \text{if } i \neq j \text{ and } m_{ij} \leq n \\ 0 & \text{otherwise} \end{cases}$$
(5)

Here s is an absorption parameter, n is the maximum transition length, d_{ij} is the generalized cost of travelling from i to j, and β is a distance deterrence parameter.

Let $\alpha_i > 0$ denote the probability that a person will move from zone *i* within the given time-frame. Our Markov transition **M** is defined as follows:

$$M_{ij} = \begin{cases} 1 - \alpha_i & \text{if } i = j \\ \frac{Q_{ij}}{\sum_k, k \neq j Q_{kj}} \cdot \alpha_i & \text{otherwise} \end{cases}$$
(6)

As an illustration, the transition matrix for a simple linear three-node system is given by:

$$\begin{bmatrix} 1 - \alpha_1 & \frac{s\alpha_2}{d_{21}^{\beta}} \left(\frac{s}{d_{12}^{\beta}} + \frac{s}{d_{32}^{\beta}} \right) & \frac{s^2\alpha_3}{(d_{12}+d_{23})^{\beta}} \left(\frac{s}{d_{23}^{\beta}} + \frac{s^2}{(d_{12}+d_{23})^{\beta}} \right) \\ \frac{s\alpha_1}{d_{21}^{\beta}} \left(\frac{s}{d_{21}^{\beta}} + \frac{s^2}{(d_{21}+d_{32})^{\beta}} \right) & 1 - \alpha_2 & \frac{s\alpha_3}{d_{23}} \left(\frac{s}{d_{23}^{\beta}} + \frac{s^2}{(d_{12}+d_{23})^{\beta}} \right) \\ \frac{s^2\alpha_1}{(d_{21}+d_{32})^{\beta}} \left(\frac{s}{d_{21}^{\beta}} + \frac{s^2}{(d_{21}+d_{32})^{\beta}} \right) & \frac{s\alpha_2}{\left(\frac{s}{d_{21}^{\beta}} + \frac{s}{d_{32}^{\beta}} \right) d_{32}^{\beta}} & 1 - \alpha_3 \\ \end{bmatrix}$$

As noted above, this may appear quite elaborate, but our program only uses about 5 lines of code to construct \mathbf{M} for networks of arbitrary dimension.

In our paper the coefficients will be state dependent, i.e., the transition probabilities are functions of E and L. In that case the equilibria are no longer unique, but the interpretation

in terms of the strength of migration flows in the equilibrium state remains valid, see Nævdal et al. (1996).

A.3 The decision to stay or move from a zone

A critical part of the construction is to specify the probabilities for movement, α_i , and these will need special attention. It is a central hypothesis in the model that the decision to stay or move from a zone depends on the labour-market accessibility of the zone. Labour-market accessibility is introduced by a measure of average generalised cost, rather than for example by a gravity-based accessibility measure (e.g. Hansen, 1959). The average generalised cost from zone *i* is given by:

$$d_i = \sum_{j \neq i} \frac{W_j}{\sum_{k \neq i} W_k} d_{ij} \tag{7}$$

Labour-market accessibility is of course not just a matter of generalised costs, the weights W_i represents the size of alternative job destinations. The size, and thickness, of a potential destination is assumed to be represented by the number of jobs; $W_j = E_j, j = 1, 2, ..., N$, defining d_i as the generalised cost to potential employment opportunities. In a spatial labour-market context, however, it can be argued that potential destinations with a low generalised cost should be put more weight on than more costly alternatives. This is done through the introduction of a generalised cost deterrence function $D(d_{ij})$, that places a relatively high weight on destinations which can be reached at a low cost from the residential location:

$$W_j = E_j(1 - D(d_{ij})) \tag{8}$$

The sensitivity to generalised cost and the weights are parameterised by d_{∞} , d_0 and μ in the following logistic expression:

$$D(x) = \frac{1}{1 + e^{-k(x - x_0)}} \quad x_0 = \frac{1}{2}(d_0 + d_\infty), k = \frac{2\log(\frac{1}{\mu} - 1)}{d_\infty - d_0}$$
(9)

 d_{∞} is the upper limit for how much cost workers, as a rule, are willing to incur to commute on a daily basis, d_0 is the lower limit (internal generalised cost) where people are insensitive to further decreases in cost, while μ captures friction effects in the system. The values of x_0 and k are given to satisfy the conditions $D(d_0) = \mu$ and $(1 - D(d_\infty)) = \mu$. If , e.g., $\mu = 0.05$, this means that the function will fall to 5% of its value outside the range where $d_0 \le x \le d_\infty$. Glenn et al. (2004) give a microeconomic and geometric justification for the use of such a function.

Finally, the definition of average generalised cost also accounts for the competition for jobs at alternative locations (Liu and Zhu, 2004; Shen, 2011), represented in the model by the proportion of the total number of job seekers in each potential destination, $\frac{E_j}{L_j}$:

$$W_{j} = E_{j}(1 - D(d_{ij}))\frac{E_{j}}{L_{j}}$$
(10)

The definition of average generalised cost is included in the diagonal elements of the migration matrix, reflecting workers' spatial interaction responses to an unfortunate local labour-market situation $(L_i > E_i)$. A high value of d_i (and $D(d_i)$) means that the migration decisions are very sensitive to the local labour market situation. On the other hand, a high local unemployment does not in itself bring about a significant out-migration from zones in highly accessible labourmarket locations (low d_i), with an excellent commuting potential. This is captured by the following specification of α_i :

$$\alpha_i = \alpha_i(L_i) + D(d_i) \max\{\rho\left(\frac{L_i - E_i}{L_i}\right), 0\}$$
(11)

Here, the parameter ρ reflects the speed of adjustment to an unfortunate labour market situation, towards a situation with a balance in the local labour market, $L_i = E_i$.

A.4 Interdependent migration probabilities

In this version of the model we assume that workers are homogeneous, and that wages and house prices are fixed and uniform across space. For $E_i > L_i$ there is then no reason to suppose that out-migration from a zone will be larger than that which is given by the first term of Equation (11); labour-market accessibility is not relevant in such a case. It can be argued, however, that migration probabilities are interdependent, state-dependent and that out-migration may accelerate when services are closed down in a zone. This effect could be taken care of through a complex, stepwise non-negative function of the local population, L. To simplify the analysis, however, the relevant effect is represented by the following function:

$$\alpha_i(L_i) = \begin{cases} \alpha^{\min(m)} & \text{if } L > L^{\text{critical}} \\ 1 + \frac{\alpha^{\min(m)} - 1}{L^{\text{critical}}} \cdot L & \text{if } L < L^{\text{critical}} \end{cases}$$

Hence, in a case with $E_i > L_i$ out-migration is assumed to be constant equal to α^{minimum} when L is above the threshold L^{critical} . If $L < L^{\text{critical}}$, however, out-migration increases linearly with a falling population. This can lead to a bifurcation if the population of a zone falls below this threshold. In this paper, we set $L^{\text{critical}} = 500$. The function could be made more complex to reflect the numerous thresholds presented in Berry and Garrison (1958a). However, we use a single threshold for simplicity. This still allows us to explore the general principles behind the idea. It is important to note that the reverse is also true, so while a service may close when the population falls below the threshold, services may open up when the population passes the threshold.

A.5 The spatial distribution of local sector employment

It is reasonable to assume that local sector activities in a whole region (E_r^l) are proportional to population in the region (L_r) :

$$E_{r}^{l} = \sum_{i}^{n} E_{i}^{l} = b \sum_{i}^{n} L_{i} = bL_{r} \qquad b > 0$$
(12)

where b is the proportion parameter. Let the spatial distribution of local-sector employment be represented by $\frac{E_i^l}{L_i}$, that is the number of local-sector employees per resident at location *i*. Assume, as a simplification, a monocentric region, offering agglomeration benefits for local sector firms and price savings for households in shopping. Shopping/consumption decisions then results from a trade-off between price savings and transport costs.

Transportation costs provide an incentive for local-sector firms to decentralise in order to cater for local demand. The trade-off between transport costs and potential price savings plays a central role in Gjestland et al. (2006), providing a theoretical base in favour of the hypothesis that the frequency of shopping/consuming locally is a smooth, concave, function of the generalised cost to the cbd.

In our paper we assume that there is only one cbd, and define the local sector density by:

Local sector density
$$= \frac{E^l}{L}$$
 (generalised cost to cbd) (13)

$$= R_{\infty}(1 - \exp[-\beta_{\rm cbd} \cdot \text{generalised cost to cbd}])$$
(14)

$$+ C \cdot \exp[-(\gamma \cdot \text{generalised cost to } \operatorname{cbd}/d_{\operatorname{dispersion}})^2]$$
 (15)

The only free parameter is β_{cbd} which controls the decay in the local sector density curve. The other parameters are defined as follows:

$$R_{\infty} = \frac{\sum_{i=1}^{N} E_{i}^{l}}{\sum_{i=1}^{N} L_{i}} \qquad (\text{average local sector density in the system as a whole}) \qquad (16)$$

 $d_{\text{dispersion}}$ is the spatial extension of the cbd, $\gamma = \sqrt{-\ln[\kappa]}$ forces the effect of the second term (15) down to $\kappa\%$ of its peak value at the boundary of the cbd. Given values for $\beta_{\text{cbd}}, R_{\infty}$ and $d_{\text{dispersion}}, C$ is chosen such that the integral

$$\int_{0}^{d_{\text{dispersion}}} \text{Local sector density}(r) \cdot \frac{2r}{d_{\text{dispersion}}^2} \cdot L_{\text{cbd}} dr = E_{\text{cbd}}^l$$
(17)

The spatial distribution of local-sector activities reflect the net effect of the price savings resulting from agglomeration forces and the transport costs of procuring goods/services in the cbd rather than locally. Harris and Wilson (1978) model the demand for local sector jobs in zone j using a gravity model. Their basic assumption is that demand is deterred by travelling costs to j and enhanced by the capacity for retail in j. Our approach is different in that we focus a cbd with a spatial extension. The balancing mechanisms are largely the same, however, and the Harris/Wilson approach could replace our LSD construction without much difficulty.

A.6 Commuting flows

In our model, the location of workers and jobs are assumed to be fixed when calculating commuting flows, calling for a doubly-constrained version of the gravity model. The following model specification ensures that the column sums of the predicted commuting flow matrix equal the total number of jobs at the corresponding destinations, and that each row sum equals the number of workers residing in the corresponding zone:

$$T_{ij} = A_i O_i B_j D_j e^{(-\beta_{\text{gravity}} d_{ij})}$$
(18)

$$A_i = \left[\sum_j B_j D_j e^{(-\beta_{\text{gravity}} d_{ij})}\right]^{-1}$$
(19)

$$B_j = \left[\sum_i A_i O_i e^{(-\beta_{\text{gravity}} d_{ij})}\right]^{-1}$$
(20)

Here:

 $T_{ij}\,$ is the number of commuters from origin i to destination j

 ${\cal O}_i$ is the observed number of commuting trips originating from zone i

 D_j is the observed number of commuting trips terminating in zone j

 d_{ij} is the travel time from origin *i* to destination *j*

 A_i and B_j are the balancing factors which ensure the fulfilment of the marginal total constraints; $\sum_j T_{ij} = O_i$ and $\sum_i T_{ij} = D_j$.

A.7 An iterative process towards spatial equilibrium

To initiate the iterative process, we specify some initial values for employment and population $(E^{\text{initial}} = E^{\text{initial,l}} + E^{\text{initial,b}}$ and $L^{\text{initial,b}}$). The matrix M depends on these quantities, but once these are given, numerical values for the entries in M are computed by the formula specified in A.2. We can then compute new values for the population in the next period by

$$L_i^{\text{new}} = \sum_{j=1}^N M_{ij} L_j^{\text{old}}$$
(21)

To update the employment entries, we first find parameters such that the LSD function specified in A.5 provides the best possible fit to the old employment entries using the new population values. Once these parameters are found, we put

$$E_j^{l(\text{new})} = \text{LSD}^{\text{new}}(d_{\text{CBD } j}) \cdot L_i^{\text{new}} \qquad j \neq \text{CBD}$$
(22)

and

$$E_j^{(\text{new})} = E_j^{l(\text{new})} + E_j^b \tag{23}$$

Note that the local-sector employment in the cbd is assumed to be fixed throughout. Once new values for population and employment have been found, we can compute new values for the entries in M, and repeat the steps above until the system no longer changes. To speed up calculations we repeated the transitions in (21) 100 times for each step in the algorithm.

A.8 Parameter values chosen for the numerical experiments in this paper

Absorption effects are ignored in the very simple transportation network (Figure 1), s = 1. The aversion to generalised cost in the internal migration flows is represented by an elasticity of $\beta = -1.0$, unless else is stated. The logistic generalised cost deterrence function involved in determining the decisions to stay or move from a zone is specified by $d_{\infty} = 80, d_0 = 5$, and $\mu = 0.05$, while the speed of adjustment to an unfortunate labour-market situation is given by $\rho = 1$. The form of the local sector density function is given by $\kappa = 0.05$ and a spatial extent of the cbd of $d_{\text{dispersion}} = 4$. Estimated commuting flows reflect a generalised cost deterrence parameter of $\beta_{\text{gravity}} = 0.07$.

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