Discussion paper

The Multifactor Model of the Agent's Power in Social Networks

BY
Ivan Belik AND **Henry Hexmoor**

Norges
Handelshøyskole

NORWEGIAN SCHOOL OF ECONOMICS

The Multifactor Model of the Agent's Power in Social Networks

Ivan Belik[1], Henry Hexmoor[2]

[1]Norwegian School of Economics, Department of Business and Management Science,
Norway, ivan.belik@nhh.no

[2] Southern Illinois University Carbondale, Department of Computer Science,
USA, hexmoor@cs.siu.edu

ABSTRACT

The analysis of social reasoning is at the core of understanding how to manage social networks. Since interpersonal relations are composed of multiple factors with different nature (i.e., structural and social factors), we explore their influence on the strategizing processes in social networks. We formalize interpersonal relations using the methods of structural and social analysis. As a part of the research, we develop the software application for the numerical visualization of the social network functioning based on the proposed mechanism.

Key words: agent's power, social networks, structural centrality, trust

1. INTRODUCTION

The framework of the research is based on the problem of structural analysis of social networks. In general, social network is considered as a multi-agent (multi-players) system. Basically, each agent is characterized by structural metrics (i.e., centralities) and by social characteristics, such as measure of trust to other players. In fact, the research corresponds to the investigation of functional dependencies between the logical and mathematical apparatuses of two interconnected concepts described next.

1.1 Structural analysis of social networks

As was mentioned, structural analysis is a basic component of the investigation process. We use three fundamental structural measures in the given research: (a) degree-based centrality, (b) betweenness centrality and (c) closeness centrality (Cook, Emerson, Gillmore, & Yamagishi, 1983). All of these measures are the components of social power analysis. One of the goals for this research is to encapsulate structural centralities in a unified structural measure. This encapsulation is the first step in the formalization of social power.

1.2 Analysis of social networks as the networks of trust

We consider trust as a social property of interpersonal relations in networks. In fact, social networks are based on the exchange of trust between their members (i.e., agents). Trust is at the core of the decision making process of each agent in a social network (Edwards, Claire, & Temple, 2006). In trivial case, we can consider the social network with three interconnected agents: A, B and C. If agent A trusts agent B more than agent C, then the probability that agent A will prefer to interact with agent B is higher than the probability of its interaction with agent C. The given example is trivial. Agent A does not take into account the property of the structural centralities of agents B and C. However, it shows the importance of trust in the exchange of resources (i.e., material and non-material) within a social network. The conception of trust can also be used in combination with Bayesian networks. The approach is based on the method of Bayesian inference (Wang & Vassileva, 2003).

2. BACKGROUND

The analysis of social networks is basically related to their structural analysis. One of the first structural models based on the theory of directed graphs was suggested by Harary, Norman, & Cartwrigh (1965). It includes basic mathematical formalization and explanation of graph theoretic methodologies and their application in formalization of networks. Theory of directed graphs is a mathematical formalization of networks that can be applied to any types of networks represented by graphs (i.e., not only social networks). The theory of directed graphs is closely related to power networks (Emerson, 1962). According to Emerson (1962), power is an agent's ability to influence other agents and to resist an influence from other agents in the network. The computation of structural measures is considered as a basic step of the analysis of social networks. Harary's research is concentrated on the investigation of social properties of agents, such as "power", "dominance", "dependence" and "status". Power networks are based not only on the structural analysis of networks, but also on the formalization of social interrelations among agents. This approach is widely used in the analysis of social systems, such as exchange networks (Cook et al., 1983).

Exchange networks are socio-economic networks that can be characterized by five properties (Cook et al., 1983). First, an exchange network is a set of agents and interrelations between them. Second, network resources are distributed between agents. Third, each agent makes a decision regarding the exchange process according to its individual interests. Fourth, each agent has a personal history of exchange within a network. Fifth and last, all interpersonal relations are encapsulated in a unified exchange network. According to Cook et al. (1983), the formalization of exchange networks is based on two basic aspects: structural analysis and internal power of relations. Specifically, Cook et al. (1983) used three basic measures for the structural analysis: (a) degree-based centrality, (b) closeness-based centrality, and (c) betweenness-based centrality. The analysis of internal power includes two factors: power and dependence. Power is considered as an agent's potential to obtain the desired outcome from other agents in the network. Dependence implies the separability of opportunities and limitations of power distribution between different agents. It means that the relation between agent A and agent B is characterized by the dependence that is different from the dependence between agent A and agent C. According to Cook et al. (1983), structural measures and internal power of relations are interdependent and influence each other.

Social networks can be analyzed from the different angles. According to Jackson (2003), efficiency is one of the most important properties of social networks. Jackson (2003) described the efficiency of social and economic networks in three basic categories. The first is that the notion of efficiency is the Pareto efficiency. Pareto efficiency (i.e., Pareto optimality) is a specific state of social network when an improvement of an agent's condition is impossible without worsening the conditions of other agents. Pareto optimality is based on the idea that all profits from the operations of exchange within a network are exhausted. It means that if at least one agent starts to improve its condition, then it will change the state of another agent or agents in a negative way. According to Jackson (2003), an agent is a member of the Pareto efficient network if there is no other network that can guarantee a better benefit than the current network. The second definition of efficiency is related to the maximization of an agent's benefit (Jackson, 2003). It does not mean that each agent will maximize its payoff. The basic idea of such kind of efficiency is that the total amount of all payoffs should be maximized. The third conception of network efficiency is related to the availability of specific types of transactions for each agent. It means that social network is efficient if the availability to realize the specific set of transactions at any time is guaranteed to each agent. This type of network efficiency implies that agents should not be limited in the realization of the specific set of rights. For example, if any democratic society is considered as efficient, then it should guarantee the freedom of choice and freedom of action for each member. The advantage of the given research is that it includes a deep analysis of the specific models of social networks. For example, Jackson (2003) considered the Con-

nections Model (Jackson & Wolinsky, 1996) and the Co-Author Model (Jackson & Wolinsky, 1996).

Another approach regarding the network power and structural measures was done by Bonacich (1987). The research is based on the abstract formalization of interdependencies between an agent's power and centrality. Bonacich (1987) did not specify which structural measures are better to be used for the structural analysis of social networks. He considered bargaining situations where agent's power is its bargaining power. According to Bonacich (1987), it is preferable for an agent to keep relations with agents who have less bargaining power. If agent A keeps relations with more powerful agents, then it will have less influence in the bargaining process. This implies the decrease of the bargaining power for agent A. Bonacich (1987) analyzed the problem of interrelations between network properties conceptually without specific computations. Mathematical formalization of interrelations between structural measures is abstracted away from the use of specific measures.

The basic mathematical formalization of social networks is based on principals and methods of graph theory and game theory. Specifically, Jackson (2008) characterized the dynamic behavior of network agents using different approaches, such as Markov chains, multiple equilibrium, local public goods model (Bramoulle & Kranton, 2006), games in normal forms, and dominant strategies. Jackson (2008) described the fundamental analysis of social and economic networks. It includes an explanation of networks representation and measuring with graph theoretic methodologies, the analysis of network models including static random-graph models and hybrid models. On each step of the analysis, Jackson (2008) gave a detailed explanation of different network properties and measures, such as socio-economic externalities, network stability, clustering, distance-based utility, and network efficiency. The analysis of social and economic network processes is combined with mathematical and logical formalization of network components. In addition to structural analysis, Jackson (2008) characterized social and economic networks in terms of game theory. It includes the description of basic game theoretic measures, properties, and methods. Defining stability and equilibrium as the basic characteristics of a game, Jackson (2008) provides the analysis of practical application of these measures in socio-economic network modeling.

Another analysis of social networks is given by Galeotti et al. (2010). They conducted a study of interconnection processes between agents in network games taking into consideration the fact that agents communicate having incomplete information about each other. The given research is based on the idea that agents have an adequate knowledge regarding their current and future structural characteristics. For example, agents know their current values of structural measures (i.e., structural centralities). Nevertheless, agents are not knowledgeable regarding the structural characteristics of their neighbors. Therefore, an agent's behavior depends on incomplete information about

2

neighbors. According to Galeotti et al. (2010), an agent's strategizing process does not depend only on the structural calculus. The computation of structural measures is combined with the analysis of social factors. Galeotti et al. (2010) considered agents' social properties as primary factors in the process of decision making. Agents are characterized by their proclivity to interact with other network agents (Galeotti et al., 2010). In fact, agents do not have complete information regarding acting, intentions, and desires of their neighbors. Therefore, Galeotti et al. (2010) analyzed social networks in terms of information incompleteness, which is considered as a combination of incomplete structural and social knowledge of agents regarding the behavior of their neighbors. The approach of Galeotti et al. (2010) is based on two aspects. First, an agent's decision-making process is based on the complete knowledge regarding its own structural degrees and incomplete knowledge regarding its neighbors' degrees. Second, agent's structural degrees are interdependent with its payoff in the game. The investigation of these two aspects is at the core of the given research.

Structural measures are not the only components of the analysis of social networks. Social factors, such as trust, are also at the core of a network's investigation. The analysis of trust in the context of social networks and semantic webs is given by Golbeck, Hendler & Parsia (2003). The research is related to the investigation of trust networks and semantic webs. The main idea is to combine the concepts of trust networks and social networks into the unified approach that is called web of trust (Golbeck et al., 2003). According to Golbeck et al. (2003), trust is a level of an agent's reliability. It has a social nature and mostly related to the confidence that agent will act in the expected way. Trust networks are considered as the networks where trust is at the core of interpersonal relations. Each agent in a trust network is represented by persons or communities. In fact, the formalization of trust networks is based on the methods of social networks' analysis. Social networks, as the second component of the given research, are considered as the networks where trust is at the core of interpersonal relations. The third component of the given research is a semantic web (Golbeck et al., 2003). Semantic web is a method of data representation for the appropriate machine processing (Daconta, Obrst, & Smith, 2003). Semantic webs are related to the advanced mechanisms of web search. For example, machines (i.e., computers) cannot analyze the data in the same way as human. The main aim of the development of semantic webs is to make the processes of data search and data classification to be similar to the human's methods. The combination of the mentioned concepts (i.e., trust, social networks, and semantic webs) is encapsulated in the formalization of the webs of trust (Golbeck et al., 2003). Each agent in the web of trust is characterized by the level of belief toward its neighbors. If this information can be collected from all agents in the network, then the interpersonal relations can be formalized and processed automatically (Golbeck et al., 2003). Golbeck et al. (2003) described nine levels of trust. The first level of trust

corresponds to the situation when agent does not trust to another agent at all. The ninth level of trust corresponds to the situation when agent absolutely trusts to another agent. According to the given research, level of trust can be measured in the specific range not only between the directly connected agents, but also between indirectly connected agents. Golbeck et al. (2003) included the description of software products that are based on the proposed methods of trust calculations. Specifically, they analyzed the application of webs of trust in email clients and in the internet relay chats (IRC). In fact, the given research represents the formalization of how the information about trust can be mined and integrated in the intelligent software applications.

Another investigation of trust, as the core social factor of interpersonal relations, was done by Castelfranchi & Falcone (1998). They analyzed principles of trust considering trust as a social property of multi-agent systems (MAS). The paper is based on the general analysis of trust and its importance in MAS. According to Castelfranchi & Falcone (1998), exchange, team work, and collaboration are the basic principles of the socially oriented MAS. Trust is considered as the belief that an agent will act in the most likely way. It implies the probabilistic nature of trust. Another important social factor of MAS is delegation. Delegation is considered the opportunity for agent A to use agent B in a goal achievement. This means that agent A can use the resources of agent B for its personal needs or it can force agent B to act in a desired way. According to Castelfranchi & Falcone (1998), there are two types of delegation: weak delegation and strong delegation. Weak delegation implies that agent B is not confident in the fact that it is exploited by agent A, or agent B does not know about the exploitation at all. In contrast, strong delegation means that agent B is aware of the exploitation by agent A. Castelfranchi & Falcone (1998) considered trust and delegation as interdependent notions. Generally, delegation cannot exist without trust. However, some exceptional situation can occur. For example, agent B may be dependent on agent A. It means that agent B has to work under the supervision of agent A even if it does not trust in agent A at all. There are two types of trust: non-social trust and social trust (Castelfranchi & Falcone, 1998). Non-social trust implies the lack of emotional interdependencies between agents. Trivially, non-social trust is based on the computation of belief that agent B will trust agent A, taking into consideration only practical profitability of the interaction between agents. Social trust implies the consideration of personal social characteristics of agents such as morality (Castelfranchi & Falcone, 1998). For example, agent A delegates responsibilities to agent B based on the morality or social status of agent B. The final step of the research is related to the quantification of trust. Castelfranchi & Falcone (1998) gave a mathematical and logical formalization of trust measuring. In general, the research contributed not only to the theoretical understanding of trust, but also to the understanding of its practical use in the modeling of social networks.

3

3. METHODOLOGY

A social network is a network that has a specific topology and social structure. The basic objects of social networks are agents (i.e., individuals, companies, and communities) that are represented by nodes and related by different kinds of social relations. In fact, a social network can be represented as a graph as shown in Figure 1. Every social network can be analyzed by graph theoretic methodologies. Social networks have different structural complexity, but in practice, they are considered as large-scale networks. This is due to the fact that they mimic the complexity of real-world social interdependencies.
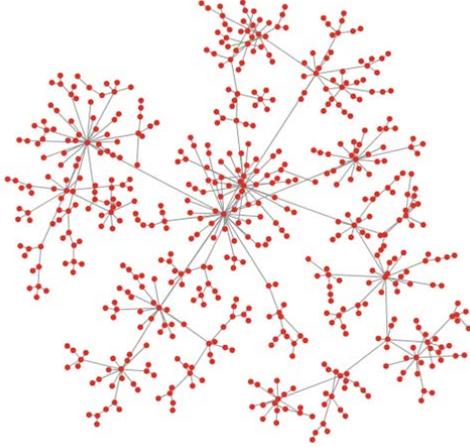

Figure 1: A Prototypical Social Network with a Mixed Topology

Quantification of social power is a multifactor analysis of the agent's role in any kind of social and economic network. It is strongly related to the level of agent's influence on each member of the network and on the integrity of the network. To put it more simply, social power captures a level of an agent's importance and an agent's opportunities within a social network.

Social power can be characterized by many measures. For example, Cook et al. (1983) used structural centrality as a primary factor for social power. They used three basic measures of (a) degree-based measure, (b) betweenness measure, and (c) closeness-based measure in order to compute the distribution of power in exchange networks. Brandes & Pich (2007) used two measures of (a) closeness and (b) betweenness for centrality estimation in large networks. Another important factor of social power is an agent's internal power, which characterizes an agent's resources (i.e., energy, knowledge, and trust).

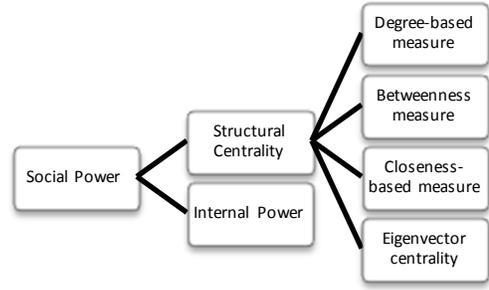Social power structure is represented in Figure 2. Next, we describe the components in detail.

Figure 2: Social Power Structure

3.1. Structural Centrality

Structural centrality is the most important concept in social power. It is based on the structural analysis of networks. Every social network can be represented as a graph. Formalization of structural centrality is closely related to the mathematical approach in graph theory. It is based on the computation of the shortest-path distances in the graphs, frequencies of nodes on the shortest paths, and connections of vertices to the low/high scoring nodes. Structural centrality is a measure of an agent's importance in terms of the structural analysis of networks.

3.1.1 Degree-based measure (degree centrality)

Degree centrality (DC) of a vertex is a number of links directly connected to it. According to Freeman (1979), DC of a vertex can be characterized as an indicator of its potentiality to interact with other vertices.

Based on the Freeman (1979) approach, DC computation for a vertex v of a graph $G(V, E)$ with n nodes can be realized by equation 1.

$$DC(v) = \frac{\deg(v)}{n-1} \tag{1}$$

where $\deg(v)$ is a number of nodes directly connected to v.

3.1.2 Betweenness measure (betweenness centrality)

Betweenness centrality (BC), as the measure of structural centrality, estimates how often the particular vertex can be visited looking through the shortest paths between all possible pairs of vertices (Freeman, 1979).

Equation 2 represents BC computation (Anthonisse, 1971; Freeman, 1977):

$$BC(v) = \frac{\sum_{s \neq v \neq t} \sigma(s,t|v)}{\sigma(s,t)} \tag{2}$$

where:

$\sigma(s,t)$ is the number of the shortest paths among all paths from s to t;

$\sigma(s,t|v)$ is the number of the shortest paths starting at s, visiting v and ending in t.

3.1.3 Closeness-based measure (closeness centrality)

Closeness centrality (CC) measures how close the given vertex is to all other vertices of the graph on average. An agent with

the highest closeness can be approached from elsewhere in the network faster on average than any other agent. CC has an important practical use because it allows for determining the best position in the network from which other agents can be easily reached.

CC is inversely related to the sum of the shortest distances from vertex v to all other nodes (Beauchamp, 1965; Sabidussi, 1966). Distance is considered as a number of edges in the shortest path between two vertices.

$$CC(v) = \frac{1}{\sum_{t \in V \setminus v} d_G(v, t)} \qquad (3)$$

where $d_G(v, t)$ is the shortest distance between vertices v and t in graph G.

Equation 3 works well only with connected graphs. The modification of this formula was offered by Dangalchev (2006):

$$CC(v) = \sum_{t \in V \setminus v} 2^{-d(v, t)} \qquad (4)$$

Equation 4 is adapted to work with disconnected graphs.

3.1.4 Eigenvector centrality

Eigenvector centrality (EC) measures an agent's significance with respect to other agents in the network. It characterizes quantitative and qualitative performance capabilities of agents (Newman, 2008). In other words, more powerful agents can be more beneficial, and it is preferable to keep connections with them.

According to Newman (2008), EC of the agent i is proportional to the average total EC score of its neighbors:

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{ij} x_j \qquad (5)$$

Here:

A_{ij} is a network's adjacency matrix. If vertex i is directly connected to vertex j, then $A_{ij} = 1$; otherwise, $A_{ij} = 0$;

λ is a constant.

Some EC values for nodes are a priori known. Since equation 5 is recursive, the a priori values seed initial values used to compute values of EC for other agents.

Alternatively, equation 5 can be represented in matrix form (Newman, 2008):

$$\lambda x = A \cdot x \qquad (6)$$

Here:

x is an eigenvector of centralities;

λ is an eigenvalue of matrix A.

3.2. Internal Power (IP)

IP is the second approach for social power quantification. It characterizes the internal agent's resources. Compared to structural centralities, IP is not related to the structural features of the network, but it works with the internal characteristics of connections between agents. The specification of IP depends on the area of its application. For example, in terms of economics agent's IP can be represented by capital, money, investments, and other tangible quantities.

Current research focuses on the social foundation of agent's IP. Accordingly, we characterize IP by three internal components: energy, knowledge and trust.

3.2.1 Energy

Energy is an abstraction of social and economic resources. One of the interpretations of energy as a social category is given by Marks (1977). In the context of social analysis, energy can be represented by an agent's ambitions, willpower, and social activities. In terms of economic analysis, energy can be represented by money, time, and propensity for financial risk.

Both kinds of energy are limited. For example, an agent cannot work more than 24 hours per day or spend more money than it has. An aggregated agent's energy can be represented by any value in the range [0, 1].

3.2.2 Knowledge

Knowledge is what is known by an agent regarding its position in the network. It includes the information regarding the states of other agents, connections, and network characteristics in general. In the context of social power, knowledge can be characterized as the level of an agent's information awareness about the network. The deep analysis of knowledge as a social category is done by Berger & Luckmann (1966).

3.2.3 Trust

Trust is a basic characteristic of social networks. We consider a trust network as a directed graph, where trust can take on any value from a range [0, 1]. Therefore, a mathematical apparatus applied for directed graphs can also be used in trust networks. One of the interesting interpretations of trust is given by Edwards et al. (2006), where trust is considered as an abstract and personal category of interpersonal relations.

It is important to say that social power has already become one of the most important parameters in the analysis of social and economic networks. It is not just an abstract and uncertain philosophic term, but it is a deeply formalized concept of mathematical formalization in social and economic networks.

4. APPROACH

4.1 Formalization of Social Power

Measures that characterize social networks are often motivated independently. For example, centrality and density are heterogeneous measures of a social network and cannot be easily combined since they quantify measures of interest for different uses of social networks.

Structural network analysis attempts to understand the internode connectedness as in graph theory methodologies. The analysis of different types of structural measures in terms of social networks was done by Everett, Sinclair, & Dankelmann (2004). Graph based network methodologies cannot be applied for analysis of social factors in social network processing because social networks possess social content that cannot be reduced to measurement by structure. In contrast to structural analysis, social analysis has a different foundation and cannot be quantified by topological analysis.

4.2 Structural Centrality and Trust

Three measures of structural centrality are taken into consideration: (a) degree-based measure, (b) betweenness measure, and (c) closeness-based measure. To accomplish interdependency, these three measures are unified in one structural parameter that is called structural centrality (SC). A problem is that each measure takes its values from different numerical intervals. The process of unification is based on the idea that SC should take its value from a unified interval, say [0, 1].

The method of unification for structural parameters is based on the knowledge about the minimum and maximum values of each parameter at the particular moment (i.e., snapshot of the network).

Each agent is characterized by values of three structural measures mentioned, and each structural measure may have any value greater than or equal to zero at a particular moment (i.e., at a snapshot) of the network state. The agent with the minimum value of the particular structural measure will set the lowest value of this structural measure corresponding to "0" value equivalent in the range [0, 1]. Accordingly, the agent with the maximum value of the considered structural measure will set the highest value "1" in the range [0, 1]. For example, let's consider the betweenness centrality (i.e., measure) in the trivial network consisting of three agents shown in Figure 3. Numbers inside nodes represent centrality values.
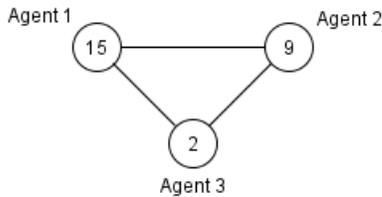

Figure 3: A Trivial Network Example with Betweenness Centrality Values

According to Figure 3, agent 1 has a maximum betweenness centrality value of 15. This means that value of 15 will be mapped to 1 in the range [0, 1]. Agent 3 has a minimum betweenness centrality value of 2. Value of 2 will be mapped to 0 in the range [0, 1].

Having upper- and lower- bounds of the betweenness centrality, all other intermediate values can be computed in the interval [0, 1]. Particularly, agent 2 will have betweenness centrality value interpolated to 0.69.

The methodology we described above is applied for all three measures taken into consideration. The unified value of social centrality (SC) is determined by equation 7.

$$SC = \frac{DC + BC + CC}{3} \qquad (7)$$

where $SC \in [0, 1]$.

Equation 7 is founded on the idea that all structural measures contribute equally to the general SC. Our formulation specifies a linear composition between them. A linear composition is stipulated by structurally equal importance of degree-based, betweenness and closeness-based measures for an agent's structural centrality. Structural analysis is at the core of each centrality measure, but the difference is that each measure is based on the consideration of network structure from a specific angle. Each of these measures is a quantitative characteristic of an agent's structural centrality. An arithmetic mean computation (i.e., equation 7) is a method to avoid the prioritizing of their contributions to a general agent's structural centrality. In fact, the consideration of non-linear composition implies different levels of structural measures' importance. In this case, each structural measure should have some specific characteristics (excepting structural) to be considered as a more or less important measure. A good example is an eigenvector centrality (equation 5) that is not only quantitative, but also qualitative structural measure. Equation 7 cannot have a linear composition if it includes an eigenvector centrality. Nevertheless, an eigenvector centrality is not used in equation 7.

Once we consider a network that represents social nature of interactions, we can interpret such a network to be a network of trust (Gambetta, 1988; Sato, 2002; Wang & Vassileva, 2003; Golbeck et al., 2003). Basically, agents can measure trust and represent values in the range [0, 1]. An agent lacks trust at all (i.e., the fewest trust) or has an abundant trust (i.e., the most trust) to another agent if the values of trust are equal to 0 and 1 respectively.

4.3. Formalization of Social Power

Having unified values of structural measures and trust, it is necessary to amalgamate them into a single function:

$$Y = f(SC, T) \qquad (8)$$

One of the basic analyses of interdependencies between structural centrality and trust was done by Buskens (1998). Buskens (1998) investigated the interdependencies between two components of social networks: structural measures and trust. The

functional dependencies were formalized for the relations between buyers and sellers. The conception of equation 8 is another point of view for the interpretation among social network relations. It is not limited by the consideration of specific socio-economic interactions, because it is based on the conceptual analysis of social relations.

The proposed idea in this research is to consider equation 8 as the combined social power of agent A (see Figure 4). According to Figure 4, social power of agent A (i.e., computed using equation 8) depends not only on the current structural centrality of agent A and its trust (T) with respect to other agents (namely B and C in Figure 4), but also on the current structural centralities of the other agents and their trust on agent A.
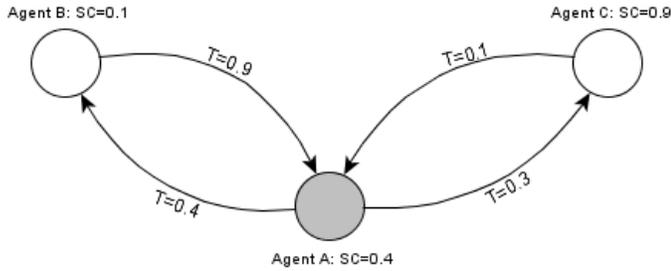

Figure 4: A Trivial Example of Network with Trust and Social Centrality Relations

In fact, the combination of T and SC can be termed as an agent's social centrality or social power (SP). Equation 9 elaborates equation 8.

$$SP_A = \frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1} \times SC_A + \frac{\sum_{i=1}^{N-1}(T_{A,i} \times SC_i - T_{i,A} \times SC_A)}{N-1} \quad (9)$$

Here,
N is a number of agents;
$T_{i,A}$ is a trust from agent i to agent A;
$T_{A,i}$ is a trust from agent A to agent i;
SC_A is a structural centrality of agent A;
SC_i is a structural centrality of agent i.

Equation 9 consists of two main components.

1. $\left(\frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1} \times SC_A\right)$. This encapsulates the basic interdependency between SC for agent A and T to agent A from all other agents. Agent A may have the highest SC in the network. However, if no one trusts it, A will not experience any social power. $\left(\frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1}\right)$ computes an average T from all agents at the network toward agent A.

2. $\left(\frac{\sum_{i=1}^{N-1}(T_{A,i} \times SC_i - T_{i,A} \times SC_A)}{N-1}\right)$. Social power of agent A can be consistent with the influences from all other agents. I.e., current structural centrality of the other agents and their levels of trust to agent A. This influence makes social power more sen-

sitive to feedback from other agents and their current conditions compared with the current individual outcomes from agent A to each agent. This component can take on a positive or negative value.

Social power is the formalization of functional interdependencies between attributes that have different nature (i.e., structural vs. social). For example, if agent A is in the same structural condition (i.e., SP=1) as all other agents and its trust to other agents is at maximum level, then agent A possess the biggest social power in the network even if all other agents do not trust agent A at all (i.e., $\sum_{i=1}^{N-1} T_{i,A} = 0$).

It is important to notice that the given model of social power can be augmented by the extended considerations of social factors. If any other social relations can be measured numerically, unified to the range [0, 1] and represented by functional interdependency, defined by Z=(social factor 1,..., social factor N), then T in equation 11 can be replaced by Z. It means that T in equation 9 can be replaced by a multi-factor model of encapsulated social factors like it is done by the implementation of SC multi-factor model (equation 7) for structural factors.

The main limitation here is that many social factors cannot be easily measured numerically. This replacement possibility shows that the proposed SP-function is flexible for multi-factor analysis of social networks and can be operated with different social and structural parameters without radical change.

5. SOFTWARE APPLICATION FOR THE NUMERICAL VISUALISATION

5.1 Concept Description

The computer-numerical visualization corresponds to the model of a large-scale social network with a dynamic monitoring of social power, trust and structural centrality variations for a specific agent. We implemented the software application for the purpose of the numerical visualization of our approach using the integrated modeling environment NetLogo. The interface with initial settings and initial agents' positions is represented in Figure 4.

Before the beginning of the iterations' run the user has the option to manipulate settings using the control panel (see Figure 5). The control panel consists of a "setup" button, "go" button, "number_of_agents" slider and "show_social_power?" switch.

Figure 4. The Software Application Interface with Initial Settings

Figure 5: Control Panel

First, the user chooses the desired number of agents using the "number-of-agents" slider. The minimum number of agents that can be set up is 2 and the maximum number is 25.

Second, there is an option to show or not show the label of the current value of social power for each agent using "show_social_power?" switch. If the user chooses to show social power, then the label corresponding to the current SP will be shown for each agent. Social power labels show the changing values dynamically after each iteration.

Third, the user should press the "setup" button to initialize the numerical visualization process according to set up settings. Also, this step includes the selection of the monitored agent i. Since all agents are characterized by the same set of parameters, we monitor only one agent i. This helps to avoid the complexity of visual monitoring. It is especially reasonable when we have more than ten agents. Since the nature of agents' characteristics and relations are homogeneous, selection of one agent for monitoring simplifies a controlling process. The monitored agent i is chosen randomly after pressing "setup" button.

Fourth, the user presses the "go" button to start process. The numerical visualization is represented by the set of iterations. The number of iterations is unlimited.

The process can be stopped by the user at any moment by pressing the "go" button again.

5.2 Process Description

Initially, each agent randomly chooses the level of its personal trust to all other agents in the beginning of the iteration. Since trust selection is an agent's strategy, the application run procedure is based on the interchange of trust between agents. The next step is SP computation that is based on the values of chosen T (i.e. trust) and current SC for each agent. SP is a result of interdependency computations (i.e., equation 9) between T and SC of agent i and all agents connected to the current agent i (i.e., SP=f(T,SC)).

After T selection and SP computation for each agent the system updates SP labels for each agent (see Figure 6). Also, the system updates the shapes of edges between the monitored agent i that is depicted by a circular shape and all other agents that have a person-shape.

The visualization of interrelations between the monitored agent i and all other agents is represented by four types of edges:

· Solid straight line shows the potential relations between agents.

· Solid curved line shows that the SP of agent i in the current iteration is greater-than the SP of another agent.

· Dashed curved line shows that the SP of agent i in the current iteration is less-than the SP of another agent.

8

· Dashed straight triple-line shows that the SP of agent *i* in the current iteration is equal to the SP of another agent.
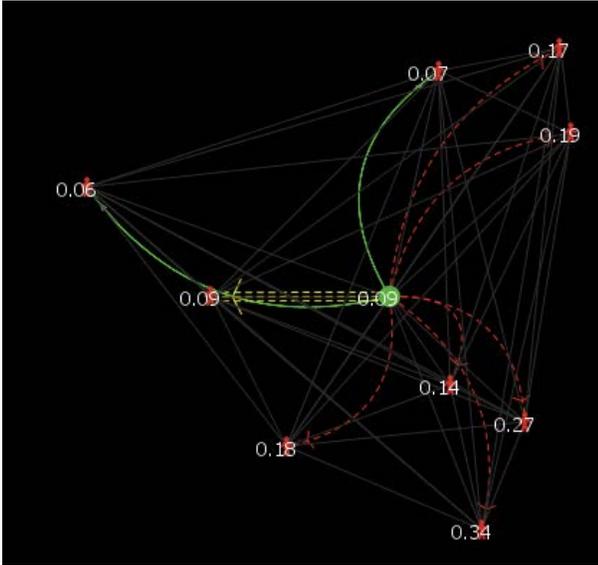

Figure 6: The Screenshot of the n-th Iteration

The process of SP and edges updating is iterative. The next iteration starts from the same procedure for T selection and SP computation.

5.3 The Monitoring of the Results

The monitoring of the results consists of two parts.

First, the application shows social power, structural centrality, and trust monitoring of agent i during the current iteration (see Figure 7). The user can supervise the number of the current iteration and the detailed information regarding the current iteration. There are three types of foreground data represented by three histograms: "Social Power", "Structural Centrality" and "Trust". Each histogram corresponds to the time history of SP, SC and T parameters computations. The results of computations for the current iteration are represented in the "Current" fields in the upper-right corners of each histogram. Each histogram bar corresponds to the computed value of SP, SC or T in the separate iteration. The rightmost histogram bars show the current parameters values of the current iteration.

"Trust" histogram is a visualization of agent's strategies. It shows the average values of T from agent *i* to all other agents in each iteration. In fact, it shows the history of T computations. "Structural centrality" histogram corresponds to the computation of SC of agent *i*. As was mentioned, social power computation is based on trust and structural centrality interdependencies. The history of SP computations is represented in "Social Power" diagram.
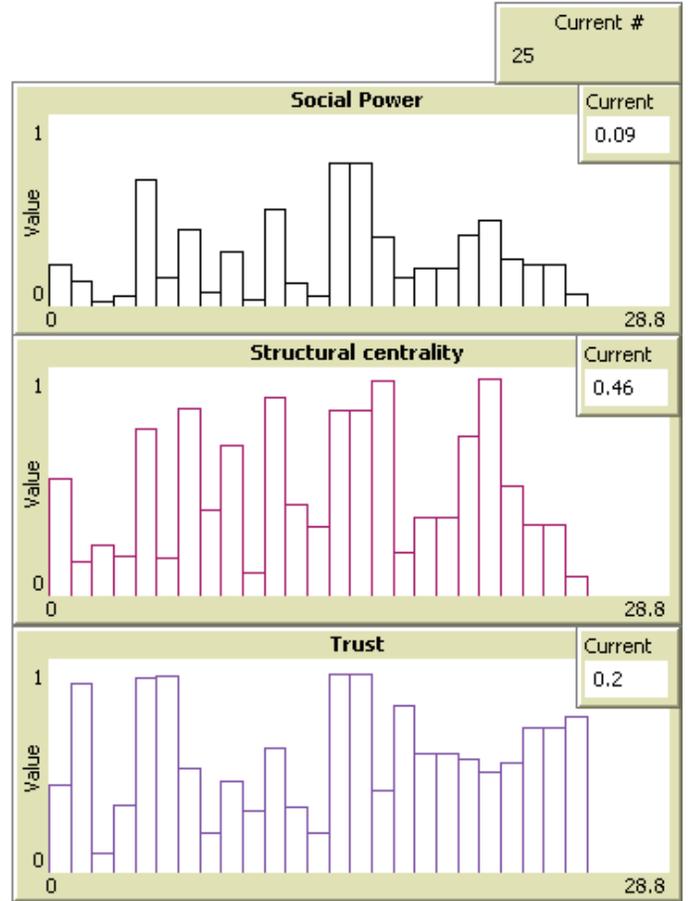

Figure 7: The State of Agent *i*
during the Current Iteration

The second part of the monitoring process includes statistics about the average values of SP, SC and T of agent *i* after n iterations.

The monitoring process is represented in Figure 8. The time histories of the average values of SP, SC and T after all finished iterations are represented in "Social power: average", "Structural centrality: average" and "Trust: average" diagrams respectively.

For example, if three iteration are finished then "Social power: average" diagram will show the average time history of SP after three iterations. The right-most point of SP average value after three iterations will be computed according to equation 10.

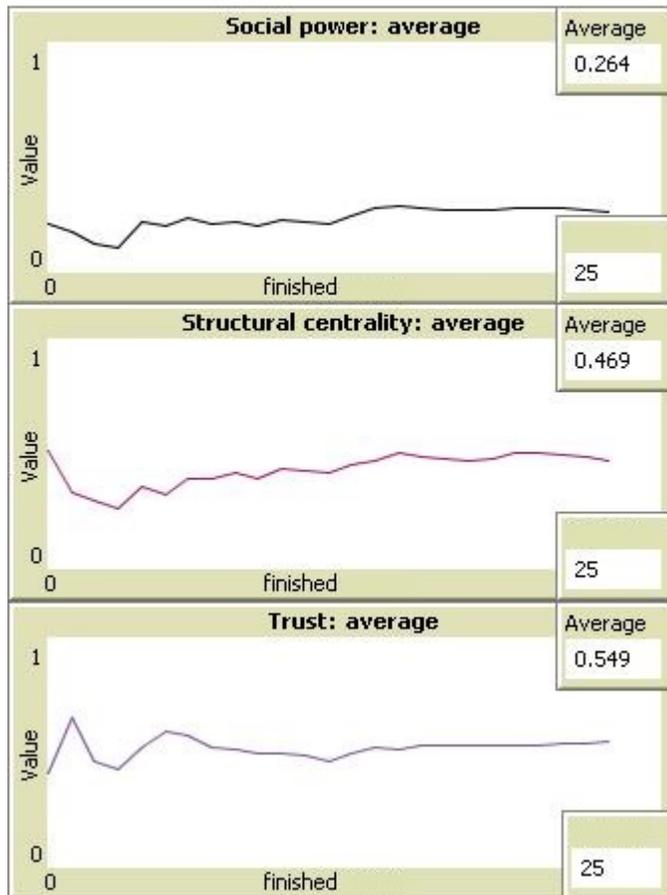$$SP_{average} = \frac{SP_1 + SP_2 + SP_3}{3} \qquad (10)$$

9

Figure 8: The Average Statistics
of Agent *i* After n-iterations

As it is shown in Figure 8, each diagram shows the current average value of the corresponding parameter in the "Average" field.

6. CONCLUSION

The analysis of social systems is based on the interdisciplinary approach. It includes not only the social analysis of interpersonal relations, but also graph theoretic methods and concepts. The basic idea of the research was to combine structural and social properties of agents in a single parameter (i.e., social power). It was approached by the unification of trust as the basic measure of interpersonal exchange and three basic structural measures of social networks, such as degree-based measure, betweenness measure, and closeness-based measure.

The proposed formalization of the social power is a multifactor model that is based on the combination of the social network characteristics that have the different natures. The given multidisciplinary approach is an attempt to formalize a social power as a numerical measure that can be used in the analysis of social networks in terms of mathematical and graph theoretical apparatuses.

Furthermore, we developed the software application for the numerical visualization of our approach applied in social networks.

The future work is related to the improvement of the equation of social power. Since social power is a multi-factor model of an agent's capabilities within a network, its current components can be modified and new components can be added. Specifically, we considered trust as a basic social factor of interpersonal relations. However, the other factors can be added to the model if they are measured.

REFERENCES

Anthonisse, J. M. (1971). *The Rush in a Directed Graph*. Tech. Rep. BN 9/71, Stichting Mathematisch Centrum, 2e Boerhaavestraat 49 Amsterdam.

Beauchamp, M. A. (1965). An Improved Index of Centrality. *Behavioral Science, 10*, 161–163.

Berger, P., & Luckmann, T. (1966). *The Social Construction of Reality: A Treatise in the Sociology of Knowledge*. New York: Doubleday.

Bonacich, P. (1987). Power and Centrality: a Family of Measures. *The American Journal of Sociology, 92 (5)*, 1170–82.

Bramoullé, Y., & Kranton, R. (2007). Public Goods in Networks. *Journal of Economic Theory, 135*, 478–494.

Brandes, U., & Pich, C. (2007). Centrality Estimation in Large Networks. *Intl. Journal of Bifurcation and Chaos, Special Issue on Complex Networks' Structure and Dynamics, 17 (7)*, 2303–2318.

Buskens, V. (1998). The Social Structure of Trust. *Social Networks, 20*, 265–289.

Castelfranchi, C., & Falcone, R. (1998). Principles of Trust for MAS: Cognitive Anatomy, Social Importance, and Quantification. *Multi Agent Systems, Proceedings. International Conference on MAS – ICMAS-98, Paris.* AAAI/MIT Press, Cambridge, MA.

Cook, K. S., Emerson, R. M., Gillmore, M. R., & Yamagishi, T. (1983). The Distribution of Power in Exchange Networks: Theory and Experimental Results. *The American Journal of Sociology, 89 (2)*, 275–305.

Daconta, M. C., Obrst, L. J., & Smith, K. T. (2003). *The Semantic Web: A Guide to the Future of XML, Web Services, and Knowledge Management*. Wiley Publishing.

Dangalchev, C. (2006). Residual Closeness in Networks. *Physica A: Statistical* Mechanics and its Applications, 365 (2), 556–564.

Edwards, R., Claire, A., & Temple, B. (2006). Interpreting Trust: Abstract and Personal Trust for People who Need Interpreters to Access Services. *Sociological Research Online*, *11 (1)*. Retrieved May 2, 2013, from http://www.socresonline.org.uk/11/1/edwards.html

10

Emerson, R. M. (1962). Power-Dependence Relations. *American Sociological Review, 27 (1)*, 31–41.

Everett, M. G., Sinclair, P. & Dankelmann, P. (2004). Some Centrality Results New and Old. *The Journal of Mathematical Sociology, 28 (4)*, 215–227.

Freeman, L. C. (1979). Centrality in Social Networks: Conceptual Clarification. *Social Networks, 1*, 241–256

Galeotti, A., Goyal, S., Jackson, M. O., Vega-Redondo, F., & Yariv, L. (2010). Network Games. *Review of Economic Studies, Blackwell Publishing, 77 (1)*, 218–244.

Gambetta, D. (1988). Can We Trust Trust? in D. Gambetta (ed.) *Trust: Making and Breaking Co-operative Relations*, Oxford: Blackwell, 213–237.

Golbeck, J., Parsia, B., & Hendler, J. (2003). *Trust networks on the semantic web* (pp. 238-249). Springer Berlin Heidelberg.

Harary, F., Norman, R. Z., & Cartwrigh, D. (1965). *Structural Models: An Introduction to the Theory of Directed Graphs.* New York: Wiley.

Jackson, M. O., & Wolinsky, A. (1996). A Strategic Model of Social and Economic Networks. *Journal of Economic Theory, 71*, 44–74.

Jackson, M. O. (2003). The Stability and Efficiency of Economic and Social Networks, in: M. Sertel, S. Koray (Eds.), *Advances in Economic Design*, Springer-Verlag, Heidelberg.

Jackson, M. O. (2008). *Social and Economic Networks.* Princeton University Press: Princeton, NJ.

Marks, S. R. (1977). Multiple Roles and Role Strain: Some Notes on Human Energy, Time and Commitment. *American Sociological Review, 42 (6)*, 921–936.

Newman, M. E. J. (2008). Mathematics of Networks, in *The New Palgrave Encyclopedia of Economics, 2nd edition, L. E. Blume and S. N. Durlauf (eds.), Palgrave Macmillan, Basingstoke.* Retrieved May 2, 2013, from http://www-personal.umich.edu/~mejn/papers/palgrave.pdf

Sabidussi, G. (1966). The Centrality Index of a Graph. *Psychometrika, 31*, 581–603.

Sato, Y. (2002). Trust, Assurance and Inequality: A Rational Choice Model of Mutual Trust. *Journal of Mathematical Sociology, 26 (1)*, 1–16.

Wang, Y., & Vassileva, J. (2003). Bayesian Network-Based Trust Model. *Proceedings form IEEE/WIC International Conference Web Intelligence.*