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# **Speciality oils supply chain optimization: from a decoupled to an integrated planning approach**

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# Speciality oils supply chain optimization: from a decoupled to an integrated planning approach

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## Abstract

We study a problem of tactical planning in a divergent supply chain. It involves decisions regarding production, inventory, internal transportation, sales and distribution to customers. The problem is motivated by the context of a company in the speciality oils industry. The overall objective at tactical level is to maximize contribution and, in order to achieve this, the planning has been divided into two separate problems. The first problem concerns sales where the final sales and distribution planning is decentralized to individual sellers. The second problem concerns production, transportation and inventory planning through refineries, hubs and depots and is managed centrally with the aim of minimizing costs. Due to this decoupling, the solution of the two problems needs to be coordinated in order to achieve the overall objective. In the company, this is pursued through an internal price system aiming at giving the sellers the incentives needed to align their decisions with the overall objective. We propose and discuss linear programming models for the decoupled and integrated planning problems. We present numerical examples to illustrate potential effects of integration and coordination and discuss the advantages and disadvantages of the integrated over the decoupled approach. While the total contribution is higher in the integrated approach, it has also been found that the sellers' contribution can be considerably lower. Therefore, we also suggest contribution sharing rules to

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achieve that both the company and sellers get a better outcome under the integrated planning.

*Key words:*

Supply chain management; integrated planning; decoupled planning; linear programming; contribution sharing; OR in the oil industry.

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## 1. Introduction

Integrating decisions about production with other functions in the supply chain, such as inventory and distribution, has proved to be of significant relevance in organizations. An important body of Operations Research literature has been devoted to this issue, as reviewed by Erengüç et al. [9]. The basic idea of an integrated model is to simultaneously optimize decision variables of different functions that have traditionally been optimized in a sequence where the output of one stage was used as the input to other stage (Sarmiento and Nagi [25]). Aligning decisions under the same goal can be challenging when the objectives of the different functions are in conflict. Successful implementations in practice, such as King and Love [15] in the tyre industry and Martin et al. [18] in the flat glass industry, have reported significant benefits through the use of linear programming models under an integrative perspective.

In this paper, we address a problem of tactical planning in a divergent supply chain. Our motivation comes from a project in which we are working with a company in the speciality oils industry. The logistics network is composed of refineries, hubs, depots and sales offices. Refineries and hubs act as production units. Hubs and depots serve as storage of saleable products. Sales offices are the channel for fulfilling demand from customers (but the products are never handled at the offices). Although owned by the company, the sales offices are managed independently and the decision on how to ship to customers is decentralized. According to the demand they observe, the sellers make decisions on type and amount of products to order, and from which storage location to order from. This decision is mainly driven by an internal price set by the company and the distribution cost calculated by the seller. The internal price attempts to reflect all variable costs caused by a product until it is ready to be shipped to the market. This price is set for each product and each location where it is stored. After a sale is realized, the seller receives a percentage of the contribution margin (revenue minus

the internal price and minus the cost of distribution to customers), and the rest of the revenue is received by the company itself.

The production is conditioned by fixed proportions between the output of different products, while the demand for different products does not necessarily have the same proportions as the output from production. The supply chain planning of the company thereby faces conflicts when aligning operations activities with sales requirements. Large inventories of products held in some depots during long intervals of time is one of the main consequences of performing the sales plans separated from the operation decisions.

We formulate linear programming models to represent this supply chain, considering decisions on production, inventory, internal transportation, sales and distribution to customers. In a first approach, we propose decoupled models to represent the situation where sales and distribution to customers are decided separate from the rest of the functions in the supply chain. Then, we integrate all the decisions in the same model and analyze its potential to improve the performance in comparison to the decoupled models.

Integrating planning has been one of the main topics studied by recent literature in the oil supply chain. From earlier simple representations, as in the logistic planning model by Sear [26], more complex works have been reported recently. Pinto et al. [21] work on planning and scheduling applications for refinery operations. Neuro and Pinto [19] propose a model for a petroleum supply chain in the context of the Brazilian company Petrobras, integrating sources, terminals, refineries, distribution centres and consumers. Bengtsson et al. [2] integrate production and logistics decisions under uncertainty in ship arrivals. Guyonnet et al. [13] explore the potential benefits of an integrated model involving three parts of the crude oil supply chain: unloading, oil processing, and distribution. To build the unloading model, they use the scheduling model in Reddy et al. [24] as a base, while their production planning model is based on the model by Pinto and Moro [22]. For the distribution part, they develop a third model. Then, the three models are linked assuming the unloading section, the refinery, and the distribution centre are connected by pipelines. In these works, one of the main challenges is given by the numerous non-linear constraints appearing from computing the properties of the products after being processed. When the planning horizon consists of various time periods, it becomes quite hard to solve real-world instances. In fact, a recent overview of refinery planning and scheduling by Bengtsson and Nonås [3] have identified the handling of non-linearities as one of the main issues in the agenda for future work.

A distinction of the problem we deal with is that fixed and unique recipes are used to mix each final product from semi finished products. This characteristic allows us to approach the problem by linear programming, in both the decoupled and the integrated approach. A second distinction of our problem is the sales mechanism involved in the supply chain. Normally, in the oil planning literature it has been assumed that the objectives of the sales units are aligned with the objectives of the whole company. In the decoupled version of the problem approached in this article, we give insights in the case when both parts are not aligned. This has been a research topic in other industrial contexts (see, for example, the problem of a furniture company by Ouhimmou et al. [20], and the problem of an oriented strand board manufacturing company by Feng et al. [10] and [11]).

The integrated planning approach generates higher revenue by aligning sales and operations decisions under the same objectives. However, the sellers can receive lower premiums than in the decoupled case and it would therefore be arguable whether the sellers would be motivated to implement the integrated solution. The agreement among the actors in the supply chain has been identified by Erengüç et al. [9] as a particularly important issue on the integration of production and distribution planning in supply chains, because these agreements will determine to a large extent whether each component of the chain will be motivated to achieve the cost reductions by integrating decisions across the chain. In the numerical examples of our problem, we discuss contribution sharing rules that make both the sellers and the company better off in the integrated case.

The remainder of this article is organized as follows. In Section 2, we present the production process and supply chain involved in our problem. In Section 3, we describe planning and management issues in this supply chain. In Section 4, we formulate linear programming models to represent sales and operations as decoupled problems. In Section 5, we propose a linear model that integrates sales and operations decisions. In Section 6, we provide numerical results of the models, compare their outcomes and discuss premium allocations. Our concluding remarks are presented in Section 7.

## **2. Specialty oils supply chain**

The oil industry faces a number of problems that have caught the attention of the Operations Research field. Bodington and Baker [4], Cooper [7] and Iachan [14] document that during several years the oil industry and OR

have been linked in a number of applications. The oil industry has also been identified as a typical example of divergent supply chain (Viswanadham and Raghavan [27]; Lasschuit and Thijssen [16]). This is the case of the supply chain for speciality oils that we face in our problem, which is characterized by a divergent product structure as well as a divergent physical structure. A representation of the supply chain is presented in Figure 1. Next, we describe its main parts.

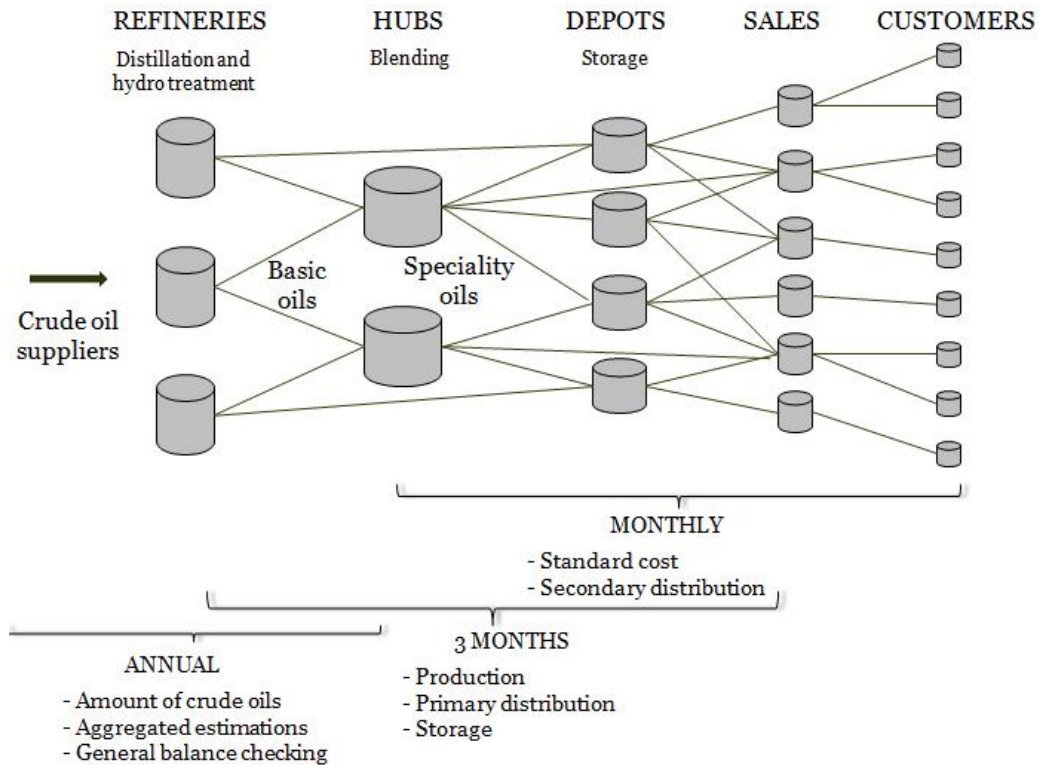


Figure 1: Supply chain for speciality oil products and planning levels.

### 2.1. Refineries and products

The refineries are supplied with crude oil from external suppliers. There are different types of crude oil, some of them containing more percentage of one or another component. This determines if a type of oil is more suitable to produce one or another final product. In the refineries, the crude oils are exposed to a series of processes, in order to generate saleable products.

There are two product segments, that we call *basic oil products* and *speciality oil products* (or simply *basic oils* and *speciality oils*). The processes in the refineries and hubs differ somewhat for different products, but they can be simplified to the following three steps: distillation, hydrotreatment and blending.

#### *2.1.1. Distillation*

During the distillation process, the crude oil is divided into several fractions. The characteristics of the fractions depend on which crude oil and *run-mode* are used. The run-mode defines the division between the fractions and the generation of different distillates. This determines the characteristics of the different fractions, for instance, in terms of the hydrocarbons that will be contained within them, viscosity and point of ignition. There are several run-mode alternatives. Given a run-mode and a type of crude oil, the proportions between the distillates obtained from the process are fixed and hence, if generation of more of a certain distillate is desired, then more of the other distillates obtained in this run-mode will also be generated.

#### *2.1.2. Hydrotreatment*

During the hydrotreatment process, the distillates obtained from the distillation receive desired properties with respect to density, volatility flash-point, pour point and colour. Impurities, such as sulphur, are also removed in this process. The products resulting from this stage correspond to the basic oil products. Some of them are already saleable products, but they can also be used for blending, in order to generate the more sophisticated speciality oil products.

#### *2.1.3. Blending*

Blending is the last part of the production process. This part does not take place at the refineries, as the distillation and hydrotreatment processes do, but in the hubs later in the supply chain. During the blending process, the basic oils are mixed with each other and sometimes with additional components to create desired properties for the speciality oil products, which are saleable products of higher value in the market.

### *2.2. Storage locations*

When the saleable products have completed processing, they are transported to depots that serve as storage locations. In addition, the hubs where

the blending process takes place, also act as storage locations of saleable products. The refineries also serve as storage locations, but only for crude oils.

From the refineries, some few products will be sent directly to the depots, while most will go through one of the hubs. Normally, the storage locations will be supplied from the closest processing unit (refinery or hub), but note that not all products are generated in each of these units.

### *2.3. Sales*

Sellers perform the product transactions with the customers, in a number of local markets. The sellers play an important role in the supply chain, since they decide from which storage location to ship a product in order to satisfy a customer requirement. We discuss this sales mechanism in more detail in Section 3.1.

### *2.4. Customers*

The customers for basic and speciality oil products include a number of firms in construction, road building, pipe coating and automotive industries.

### *2.5. Transportation*

We distinguish primary and secondary transportation or distribution. The primary distribution corresponds to the transportation of oils within the facilities (refineries, hubs and depots), while the secondary distribution corresponds to the transportation of the saleable products to the customers.

The transportation of crude oil from supply sources to refineries is carried out by ships, the same as from refineries to hubs and depots.

From hubs to depots and from these locations to the customers, the means of transport varies more, since the volumes are smaller and variable. When ships are used, the oil is usually transported in tanks or specially equipped containers that can be transported by any container ship. For the transportation of products to the customers, tank trucks are used more often. On occasion, a combination of ship and truck is used and less often, train and tank trucks are also used.

Note that transports within the company's own supply chain are managed centrally, while transports to customers are ordered by the sellers.



### 3. Planning and management of the supply chain

The current planning and management of the supply chain is performed in three main levels, as Figure 1 shows.

The strategic planning considers decision on how much crude oil will be used in a year and performs aggregated estimations in order to check that there will be a production balance between the different products.

Our research focuses on the tactical level, which includes two stages. One stage is performed by the planners at the refineries and hubs. They perform a production plan, considering a horizon of three months. Decisions involved in the plan are the amount of each product to produce in each location, the primary distribution of basic oil products between refineries and hubs or depots, and the primary distribution of speciality oils between hubs and depots. A second stage involves the planning of the secondary distribution, from hubs and depots to the customers. This planning is based on a mechanism with internal pricing. For each depot, each product is given an internal price. This internal price plus the distribution cost from storage location to the customer results in a figure that we call the *value chain cost*. The sellers' premiums depend on the margin they can achieve between the sales price and the value chain cost. We describe the computation of the value chain cost in detail below.

#### 3.1. Value chain cost description

An internal pricing mechanism considers the assignment of premiums to the sellers, depending on their sales results. One significant part of the premiums is the difference between the sales price and the value chain cost. In consequence, for each sale, a main goal of the sellers is to maximize this difference so as to maximize their own premiums.

The value chain cost is intended to reflect the variable costs of production, distribution and storage of the product within the supply chain over a five to ten-year perspective. Some of the variable costs are actual costs, while some are estimated. In addition to the variable costs, the value chain cost also includes a distributed fixed cost for all the depots where the products have been stored, based on the volumes that flowed through each depot the previous year. It is a challenge to set accurate value chain cost values per product, because of the difficulty in distributing some of the production cost among them. Some of the costs involved in the value chain cost are updated every month but others remain the same over a whole year.

The idea from the company is that this mechanism should be self regulatory and make the sellers act in such a way that, while acting in their own interests, they minimize the total long term cost of distribution for the company. In practice, however, this control mechanism is not exempt from imperfections.

The value chain cost is calculated as follows:

**Value chain cost = Cost of goods sold + Primary distribution cost + Secondary distribution cost.**

The *Cost of goods sold* (COGS) includes raw material cost, cost for externally procured products, exchange rates and processing costs in refining and blending.

The *Primary distribution cost* is related to the distribution to storage facilities, including depot freight and associated costs of running depots and hubs.

The *Secondary distribution cost* includes the transport cost to the customer, a cost for filling the product in drums and other variable costs (such as import taxes).

In practice, the company centralizes the calculation of COGS and the Primary distribution cost, resulting in what is called the *internal price*. Hence,

**Value chain cost = Internal price + Secondary distribution cost.**

For each sale opportunity the secondary distribution cost is calculated by the seller for different supply options and added to the internal price, thus completing the total value chain cost. The sale price is based on a negotiation between the seller and the customer. Of course, the sale price intends to reflect, at least, the value chain cost. When the sale is realized, the seller receives a premium, the main share of which is proportional to the gross result of the sale (revenue minus total value chain cost).

For each sale, the seller has a choice from which depot to supply the customer from (assuming availability). Both the internal price and the secondary distribution cost depend on which depot the product will be shipped from. Hence, it is not necessarily convenient for the seller to order the product from the closest depot (or the one with cheapest transportation cost), because the same product can have different internal prices in different depots. It is also not always best for the seller to order from the depot with

the lowest internal price, because the transportation cost from the depot to the customer might be too high. Normally, the seller will chose to supply from the depot with the lowest sum of these two. In practice, there might be other factors affecting the choice, such as a particular preference requested by the customer or a lead time limitation based on the terms the sale has been agreed on, but our research interest focuses on the value chain cost factor. An illustrative situation is described in Figure 2.

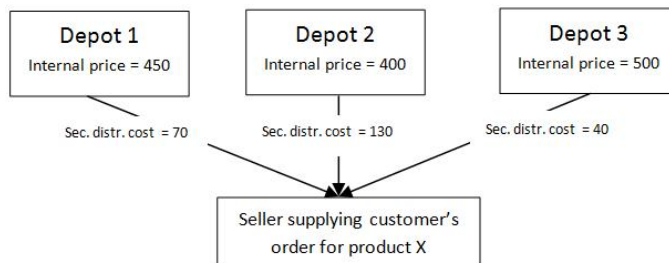


Figure 2: Example of seller choices of depots to fulfill an order from a customer.

In this example, the value chain cost from the first depot is 520; from the second depot, 530; and from the third depot, 540. Assuming a given sale price agreed with the customer, the seller will chose to ship from Depot 1, since it has the lowest value chain cost and therefore awards the highest premium. However, this is not necessarily the most cost efficient way to distribute for the company as a whole.

### 3.2. Demand

Each seller forecasts monthly sales in his area. The forecast is normally based on a one-year horizon. Each month, the sellers update the forecast information into a central system of the company. The sellers base their forecast mainly on their judgement and perceptions of last sales periods.

Depending on the type of product, different patterns of sales demand are observed; some present high seasonality in demand, with peaks during the northern hemisphere summer, while the demand for other products is more stable.

As described earlier, the refineries utilize different types of crude oil, each with different component properties. Depending on the type of crude oil and the run-mode used, the yields of basic and speciality oils are different. The high seasonality of some products impacts the production and inventories of

other products even if the seasonality of these other products is not as great. In practice, there is little flexibility to cope with the seasonal variations. However, the inflow of crude oils is more or less continuous during the year, resulting in large seasonal inventories of crude oil. High levels of inventory are the result of trying to counteract the seasonality.

At the geographical level, recall that independent of the seller's regional location, he can order products from each of the depots within the company's network. Balancing inventories in different depots, while at the same time satisfying demand, leads to a challenging problem for the planners.

#### 4. Decoupled planning models

In this section we develop models to represent the tactical planning including the refineries and echelons downstream. We formally state the problem of having sales and operations as non-coordinated units. In the operations units, we include production (at refineries and hubs), inventory (at refineries, hubs and depots) and distribution (from refineries to hubs and depots, and from hubs and depots to customers). Recalling that the procurement of crude oils is programmed at a more strategic level, in our model we thus consider the amount of crude oil incoming to the refineries as a given parameter.

We start by formulating minimum cost and maximum revenue models to motivate the different interests of sales and operations units. Then, we formulate decoupled models and the coordination constraints that the company has implemented to partly balance both perspectives. In all these cases, we formulate linear programming models.

First, we introduce the notation of sets and parameters that are used through the remainder of the article. In cost parameters and decision variables, we use superscripts to index oils and subscripts to index nodes of the supply chain network and time periods.

##### **Indexes and sets**

$a \in A$ : set of sellers.

$j \in J$ : set of geographic regions.

$a \in L_j$ : set of sellers that belong to region  $j$ .

$k \in K$ : set of customers.

$i \in I$ : set of crude oils.

$b \in B$ : set of basic oil products.

$s \in S$ : set of speciality oil products.  
 $p \in P$ : set of saleable products, the union of set  $B$  and set  $S$  (basic oils and speciality oils).  
 $r \in R$ : set of refineries.  
 $h \in H$ : set of hubs.  
 $d \in D$ : set of depots.  
 $f \in F$ : set of storage locations for basic oils, the union of sets  $R$ ,  $H$  and  $D$  (refineries, hubs and depots).  
 $g \in G$ : set of storage locations for saleable products, the union of set  $H$  and set  $D$  (hubs and depots).  
 $m \in M$ : set of run-modes in refining process.  
 $t \in T$ : set of periods.

### Parameters

$\alpha_{app\tilde{g}}$ : maximum proportion between the amount of saleable products  $p$  and  $\tilde{p}$  possible to assign to seller  $a$  from location  $g$ .  
 $\beta_a$ : fraction of the revenue that seller  $a$  receives as premium.  
 $\delta_{akpt}$ : demand of customer  $k$  to seller  $a$  for product  $p$  in period  $t$ .  
 $\eta_{irt}$ : amount of crude oil  $i$  incoming to refinery  $r$  in period  $t$ .  
 $\gamma_{bs}$ : amount of speciality oil  $s$  generated from one unit of basic oil  $b$ .  
 $\lambda_{pgjt}$ : maximum amount of product  $p$  that the company can sell from location  $g$  to region  $j$  in period  $t$ .  
 $\rho_{bim}$ : amount of basic oil  $b$  generated from one unit of crude oil  $i$  at run-mode  $m$ .  
 $\theta_{pkt}$ : sale price of one unit of product  $p$  to customer  $k$  in period  $t$ .  
 $\zeta_{pgkt}$ : value chain cost of one unit of product  $p$  if it is ordered from location  $g$  to be sold to customer  $k$  in period  $t$ .  
 $\psi_{pkt}$ : cost for unsatisfied demand of customer  $k$  for product  $p$  in period  $t$ .  
 $C_{rmt}^i$ : cost of refining one unit of crude oil  $i$  in mode  $m$  at refinery  $r$  in period  $t$ .  
 $C_{ht}^s$ : cost of producing one unit of speciality oil  $s$  at hub  $h$  in period  $t$ .  
 $C_{fgt}$ : unitary transport cost from location  $f$  to location  $g$  in period  $t$ .  
 $C_{gkt}$ : unitary transport cost from location  $g$  to customer  $k$  in period  $t$ .  
 $C_r^i$ : inventory cost from storing one unit of crude oil  $i$  in refinery  $r$ .  
 $C_f^b$ : inventory cost from storing one unit of basic oil  $b$  in location  $f$ .  
 $C_g^s$ : inventory cost from storing one unit of speciality oil  $s$  in location  $g$ .  
 $I_{r0}^i$ : initial inventory of crude oil  $i$  at refinery  $r$ .  
 $I_{f0}^b$ : initial inventory of basic oil  $b$  at location  $f$ .

$I_{g0}^s$ : initial inventory of speciality oil  $s$  at location  $g$ .  
 $\bar{Y}_{rm0}^i$ : initial amount of crude oil  $i$  refined in mode  $m$  in refinery  $r$ .  
 $\bar{Y}_{hs0}^b$ : initial amount of basic oil  $b$  used at hub  $h$  to produce speciality oil  $s$ .

#### 4.1. Min-cost and Max-revenue models

##### 4.1.1. Min-cost model

In this case, production, storage and distribution plans are decided to match estimated sales while at the same time minimizing costs. The estimated sales correspond to the forecast  $\delta$ . We propose a linear programming model for this problem and present the formulation below.

#### Decision variables

$v_{agkt}^p$ : amount of saleable product  $p$  sold from location  $g$  to customer  $k$  through seller  $a$  in period  $t$ .

$x_{fgt}^p$ : amount of saleable product  $p$  transported from location  $f$  to location  $g$  in period  $t$ .

$y_{rmt}^i$ : amount of crude oil  $i$  refined at refinery  $r$  in mode  $m$  in period  $t$ .

$y_{hst}^b$ : amount of basic oil  $b$  used at hub  $h$  to produce speciality oil  $s$  in period  $t$ .

$z_{rt}^i$ : amount of crude oil  $i$  stored in refinery  $r$  at the end of period  $t$ .

$z_{ft}^b$ : amount of basic oil  $b$  stored in location  $f$  at the end of period  $t$ .

$z_{gt}^s$ : amount of speciality oil  $s$  stored at location  $g$  at the end of period  $t$ .

#### Min-cost objective function

$$\begin{aligned}
 \min Cost = & \sum_{m \in M} \sum_{r \in R} \sum_{i \in I} \sum_{t \geq 1} C_{rmt}^i y_{rmt}^i + \sum_{h \in H} \sum_{s \in S} \sum_{t \geq 1} C_{ht}^s \left( \sum_{b \in B} \gamma_{bs} y_{hst}^b \right) \\
 & + \sum_{p \in P} \sum_{f \in F} \sum_{g \in G} \sum_{t \geq 1} C_{fgt} x_{fgt}^p + \sum_{r \in R} \sum_{i \in I} \sum_{t \geq 1} C_r^i (z_{r,t-1}^i + z_{rt}^i) / 2 \\
 & + \sum_{f \in F} \sum_{b \in B} \sum_{t \geq 1} C_f^b (z_{f,t-1}^b + z_{ft}^b) / 2 + \sum_{g \in G} \sum_{s \in S} \sum_{t \geq 1} C_g^s (z_{g,t-1}^s + z_{gt}^s) / 2 \\
 & + \sum_{a \in A} \sum_{p \in P} \sum_{k \in K} \sum_{t \geq 1} \psi_{pkt} (\delta_{akpt} - \sum_{g \in G} v_{agkt}^p)
 \end{aligned} \tag{1}$$

## Constraints

$$z_{r0}^i = I_{r0}^i \quad \forall r \in R, i \in I; \quad z_{f0}^b = I_{f0}^b \quad \forall f \in F, b \in B; \quad z_{g0}^s = I_{g0}^s \quad \forall g \in G, s \in S. \quad (2)$$

$$y_{rm0}^i = \bar{Y}_{rm0}^i \quad \forall m \in M, r \in R, i \in I. \quad (3)$$

$$y_{hs0}^b = \bar{Y}_{hs0}^b \quad \forall b \in B, h \in H, s \in S. \quad (4)$$

$$z_{r,t-1}^i + \eta_{irt} = z_{rt}^i + \sum_{m \in M} y_{rmt}^i \quad \forall r \in R, i \in I, t \geq 1. \quad (5)$$

$$z_{r,t-1}^b + \sum_{i \in I} \sum_{m \in M} \rho_{bim} y_{r,m,t-1}^i = z_{rt}^b + \sum_{h \in H} x_{rht}^b + \sum_{d \in D} x_{rdt}^b \quad \forall r \in R, b \in B, t \geq 1. \quad (6)$$

$$z_{h,t-1}^b + \sum_{r \in R} x_{rht}^b = z_{ht}^b + \sum_{d \in D} x_{hdt}^b + \sum_{a \in A} \sum_{k \in K} v_{ahkt}^b + \sum_{s \in S} y_{hst}^b \quad \forall h \in H, b \in B, t \geq 1. \quad (7)$$

$$z_{d,t-1}^b + \sum_{r \in R} x_{rdt}^b + \sum_{h \in H} x_{hdt}^b = z_{dt}^b + \sum_{a \in A} \sum_{k \in K} v_{adkt}^b \quad \forall d \in D, b \in B, t \geq 1. \quad (8)$$

$$z_{h,t-1}^s + \sum_{b \in B} \gamma_{bs} y_{h,s,t-1}^b = z_{ht}^s + \sum_{d \in D} x_{hdt}^s + \sum_{a \in A} \sum_{k \in K} v_{ahkt}^s \quad \forall h \in H, s \in S, t \geq 1. \quad (9)$$

$$z_{d,t-1}^s + \sum_{h \in H} x_{hdt}^s = z_{dt}^s + \sum_{a \in A} \sum_{k \in K} v_{adkt}^s \quad \forall d \in D, s \in S, t \geq 1. \quad (10)$$

$$\sum_{g \in G} v_{agkt}^p \leq \delta_{akpt} \quad \forall a \in A, p \in P, k \in K, t \in T. \quad (11)$$

$$\begin{aligned} & v_{agkt}^p, x_{fgt}^p, y_{rmt}^i, y_{hst}^b, z_{rt}^i, z_{ft}^b, z_{gt}^s \geq 0 \\ & \forall a \in A, b \in B, f \in F, g \in G, h \in H, i \in I, k \in K, m \in M, p \in P, r \in R, s \in S, t \in T. \end{aligned} \quad (12)$$

Objective function (1) minimizes the total cost through the whole planning horizon up to the depot level (i.e., excluding distribution cost to the customers). The first term is the cost of processing crude oils at the refineries; the second term is the cost of production at the hubs; the third term is the primary distribution transport costs; the next three terms are the total costs of the average inventory per period; the last term is the cost for unsatisfied demand.

Constraint (2) sets the initial level of inventories of crude oils, basic oils and speciality oils. Constraint (3) sets the initial values of crude oils refined in each mode and refinery. Constraint (4) sets the initial values of basic oils utilized in each hub for producing each type of speciality oil. Constraint (5) represents the flow conservation of crude oils at the refineries. Constraints (6), (7) and (8) state the conservation of flow of basic oils at the refineries, hubs and depots, respectively. Constraints (9) and (10) give the conservation of flow of speciality oils at the hubs and depots, respectively. Constraint (11) states that the company supplies a customer at most the amount he ordered, through the corresponding seller. Constraint (12) states non-negativity of the variables.

#### 4.1.2. Max-revenue model

In this case the goal is to maximize the total revenue obtained from the sales over the planning period, assuming that they will be realized as forecasted. The objective function is stated as follows:

#### Max-revenue objective function

$$\max Revenue = \sum_{a \in A} \sum_{p \in P} \sum_{g \in G} \sum_{k \in K} \sum_{t \in T} v_{agkt}^p \theta_{pkt} \quad (13)$$

The constraints from the previous model remain the same as (2)-(12).

#### 4.2. Decoupled models

We first consider a fully decoupled case, where there is no coordination between sales and operations units. While the sales units focus on their sales premiums, the operations units focus on supplying at minimum cost.



For this case, we develop a decoupled model that is composed of two sub-models: the sales sub-model and the operations sub-model.

In the sales sub-model, the sales units do their planning separate from the other echelons of the supply chain, by considering only the sales prices and the value chain costs to maximize their premiums.

In the operations sub-model, production and primary distribution are planned together and the decisions from the sales sub-model are considered as input.

#### 4.2.1. Sales sub-model

According to given demand, the sellers make decisions on which products, in what amount and from which storage location to order for maximizing their premiums.

#### Decision variables

$w_{agkt}^p$ : amount of saleable product  $p$  ordered by seller  $a$  from location  $g$  to be shipped to customer  $k$  in period  $t$ .

#### Max-premium objective function

$$\max Premium = \sum_{a \in A} \sum_{p \in P} \sum_{g \in G} \sum_{k \in K} \sum_{t \in T} \beta_a w_{agkt}^p (\theta_{pkt} - \zeta_{pgkt}) \quad (14)$$

#### Constraints

$$\sum_{g \in G} w_{agkt}^p \leq \delta_{akpt} \quad \forall a \in A, p \in P, k \in K, t \in T. \quad (15)$$

$$w_{agkt}^p \geq 0 \quad \forall a \in A, p \in P, g \in G, k \in K, t \in T. \quad (16)$$

The objective function (14) maximizes the total premium obtained by all the sellers, through the whole planning horizon. Constraint (15) states that each seller will order for each customer at most the amount that this customer demanded, considering that it is possible to serve the same customer from different depots. Constraint (16) corresponds to the non-negativity of

the variables.

#### 4.2.2. Operations sub-model

The quantities  $w_{agkt}^p$  ordered by the sellers play the role of demand parameters in the operations sub-model (from the solution to the sales sub-model, the sellers have already decided on the location from which to order). The production, storage and primary distribution plans of the company are decided to match such a demand, while at the same time minimizing costs. This operations sub-model corresponds to the same formulation as the min-cost model in Section 4.1.1, but the demand fulfilment constraint (11) is replaced by constraint (17) as follows:

$$v_{agkt}^p \leq w_{agkt}^p \quad \forall a \in A, p \in P, g \in G, k \in K, t \in T. \quad (17)$$

#### 4.2.3. Coordination constraints

In practice, the company attempts to set conditions in order to achieve certain balance between production and sales of different products from different depots. These conditions work as recommendations and encouragement to the sellers, that we incorporate as two coordination constraints in our decoupled approach. The first coordination constraint is given by an upper bound  $\alpha$  on the proportion between two different products that the same seller can order from the same depot. The second coordination constraint imposes a maximum quantity  $\lambda$  for each product that can be ordered in total from sellers in the same region. We introduce these conditions into the sales sub-model, by the formulation of constraints (18) and (19) below.

$$\sum_{k \in K} w_{agkt}^p \leq \alpha_{ap\tilde{p}g} \sum_{k \in K} w_{agkt}^{\tilde{p}} \quad \forall a \in A, p \in P, \tilde{p} \in P, g \in G, t \in T. \quad (18)$$

$$\sum_{k \in K} \sum_{a \in L_j} w_{agkt}^p \leq \lambda_{pgjt} \quad \forall j \in J, p \in P, g \in G, t \in T. \quad (19)$$

#### 4.3. Discussion

The fulfilment of demand in the min-cost model of Section 4.1.1 will be driven by the penalization  $\psi$  incorporated in the objective function on the deviation of production with respect to the estimated sales. In fact,

provided that the initial inventories, the crude oils supplies and  $\psi$  are large enough, the optimal solution to this model will be such that the production is as little as possible to fulfil the estimated forecast in the cheapest feasible way (down-to-depot costs). However, since the objective function does not consider secondary distribution costs, in reality, the implementation might be inconvenient for the company.

When using the max-revenue model of Section 4.1.2, assuming that the initial inventories and crude oils supplies are large enough, the production will reach the level of demand, without concern for any of the costs. In practice this is, of course, inconvenient for the company.

The max-revenue and min-cost problems illustrate a classical conflict between operation and commercial units. The decoupled models of Section 4.2 illustrate the conflict appearing in the company at the tactical level.

Note that the sales sub-model problem in the fully decoupled approach is separable in the sellers, thus solving the problem for each single seller  $a$  will lead to the same solution as when solving for all of them together. The trivial solution is that the sellers place all the orders such that the price  $\theta$  is higher than the value chain cost  $\zeta$ , and they do it from the location such that this difference is the highest. With these orders as input in the operations sub-model, the secondary distribution has been conditioned by the decisions of the sellers.

When there is not enough supply of products to fulfil all the demand, the company may assign priority to some of the customers. In the model, this can be managed by the penalization  $\psi$ .

The decoupled models we have formulated illustrate the issues created by the non-coordinated planning in the company. In order to come up with better coordination mechanisms, the coordination constraints have been implemented gradually for all those cases in which the sales orders for some products have undesirably unbalanced the inventories in depots and hubs. Although they mitigate the undesired effects, limiting sales leads to a sub-optimal solution from an integrated perspective. Currently, there is no integrated planning model implemented at the company. We tackle this problem in the next section.

## 5. Integrated planning model

In this section, we propose a model that integrates sales and operations decisions, under the same objective of maximizing the resulting contribution

of sales minus variable costs over the planning horizon. Sales and operation units are modelled together, in the same model. The decision on how to fulfil demand is made centrally, as well as the decisions on production, inventory and primary and secondary distribution. The integrated model will be used to identify the potential of an integrated planning over decoupled planning, and used as a benchmark for evaluating coordination constraints.

We maintain the notation and definitions from the previous section for all parameters, sets and variables, but for explicit differentiation in the decision variable on demand fulfilment between this integrated model and the previous cases, instead of using  $v_{agkt}^p$  we use the notation  $\bar{v}_{agkt}^p$  as decision variable for the amount of saleable product  $p$  sold from location  $g$  to customer  $k$  through seller  $a$  in period  $t$ .

### Objective Function

$$\begin{aligned}
\max \text{Contribution} = & \sum_{a \in A} \sum_{p \in P} \sum_{g \in G} \sum_{k \in K} \sum_{t \geq 1} \bar{v}_{agkt}^p \theta_{pkt} - \sum_{m \in M} \sum_{r \in R} \sum_{i \in I} \sum_{t \geq 1} C_{rmt}^i y_{rmt}^i \\
& - \sum_{h \in H} \sum_{s \in S} \sum_{t \geq 1} C_{ht}^s \left( \sum_{b \in B} \gamma_{bs} y_{hpt}^b \right) - \sum_{p \in P} \sum_{f \in F} \sum_{g \in G} \sum_{t \geq 1} C_{fgt} x_{fgt}^p \\
& - \sum_{a \in A} \sum_{g \in G} \sum_{k \in K} \sum_{p \in P} \sum_{t \geq 1} C_{gkt} \bar{v}_{agkt}^p - \sum_{r \in R} \sum_{i \in I} \sum_{t \geq 1} C_r^i (z_{r,t-1}^i + z_{rt}^i) / 2 \\
& - \sum_{f \in F} \sum_{b \in B} \sum_{t \geq 1} C_f^b (z_{f,t-1}^b + z_{ft}^b) / 2 - \sum_{g \in G} \sum_{s \in S} \sum_{t \geq 1} C_g^s (z_{g,t-1}^s + z_{gt}^s) / 2
\end{aligned} \tag{20}$$

### Constraints

$$z_{r0}^i = I_{r0}^i \quad \forall r \in R, i \in I; \quad z_{f0}^b = I_{f0}^b \quad \forall f \in F, b \in B; \quad z_{g0}^s = I_{g0}^s \quad \forall g \in G, s \in S. \tag{21}$$

$$y_{rm0}^i = \bar{Y}_{rm0}^i \quad \forall m \in M, r \in R, i \in I. \tag{22}$$

$$y_{hs0}^b = \bar{Y}_{hs0}^b \quad \forall b \in B, h \in H, s \in S. \tag{23}$$

$$z_{r,t-1}^i + \eta_{irt} = z_{rt}^i + \sum_{m \in M} y_{rmt}^i \quad \forall r \in R, i \in I, t \geq 1. \quad (24)$$

$$z_{r,t-1}^b + \sum_{i \in I} \sum_{m \in M} \rho_{bim} y_{r,m,t-1}^i = z_{rt}^b + \sum_{h \in H} x_{rht}^b + \sum_{d \in D} x_{rdt}^b \quad \forall r \in R, b \in B, t \geq 1. \quad (25)$$

$$z_{h,t-1}^b + \sum_{r \in R} x_{rht}^b = z_{ht}^b + \sum_{d \in D} x_{hdt}^b + \sum_{a \in A} \sum_{k \in K} \bar{v}_{ahkt}^b + \sum_{s \in S} y_{hst}^b \quad \forall h \in H, b \in B, t \geq 1. \quad (26)$$

$$z_{d,t-1}^b + \sum_{r \in R} x_{rdt}^b + \sum_{h \in H} x_{hdt}^b = z_{dt}^b + \sum_{a \in A} \sum_{k \in K} \bar{v}_{adkt}^b \quad \forall d \in D, b \in B, t \geq 1. \quad (27)$$

$$z_{h,t-1}^s + \sum_{b \in B} \gamma_{bs} y_{h,s,t-1}^b = z_{ht}^s + \sum_{d \in D} x_{hdt}^s + \sum_{a \in A} \sum_{k \in K} \bar{v}_{ahkt}^s \quad \forall h \in H, s \in S, t \geq 1. \quad (28)$$

$$z_{d,t-1}^s + \sum_{h \in H} x_{hdt}^s = z_{dt}^s + \sum_{a \in A} \sum_{k \in K} \bar{v}_{adkt}^s \quad \forall d \in D, s \in S, t \geq 1. \quad (29)$$

$$\sum_{g \in G} \bar{v}_{agkt}^p \leq \delta_{akpt} \quad \forall a \in A, p \in P, k \in K, t \in T. \quad (30)$$

$$\bar{v}_{agkt}^p, x_{fgt}^p, y_{rmt}^i, y_{hst}^b, z_{rt}^i, z_{ft}^b, z_{gt}^s \geq 0 \quad \forall a \in A, b \in B, f \in F, g \in G, h \in H, i \in I, k \in K, m \in M, p \in P, r \in R, s \in S, t \in T. \quad (31)$$

In the integrated model, the objective function (20) maximizes the total contribution of the company over the planning horizon. The first term is the revenue obtained from the sales of all the products through all the sellers. The second term is the variable cost of processing crude oils at the refineries; the third term is the variable cost of production at the hubs; the fourth term is the primary distribution costs; the fifth term is the secondary distribution cost; the last three terms are the total costs of the average inventory per period. Constraints (21) - (25) are the same as constraints (2) - (6) of the min-cost model. Constraints (7) - (12) are also considered, but now the

variables  $v$  in the formulations are changed to  $\bar{v}$ , as stated in constraints (26) - (31).

Note that constraint (30) will drive decisions on distribution from storage locations to the customers, from an integrated perspective, involving operations and sales decisions under the max-contribution objective function. This was not the case in the decoupled model under max-premium and min-cost separated objective functions, where constraint (17) limited the distribution from a given location to a given customer by the corresponding amount  $w$  ordered by the seller. Moreover, the optimal solution to the decoupled model is feasible in the integrated model. In fact, if  $v$  values respect constraint (17) in the operations sub-model, it holds that  $\sum_{g \in G} v_{agkt}^p \leq \sum_{g \in G} w_{agkt}^p$ . Then, because the  $w$  values respect constraint (15) in the sales sub-model, it follows that the optimal  $v$  values of the decoupled solution satisfy  $\sum_{g \in G} v_{agkt}^p \leq \delta_{akpt}$ , which matches with constraint (30) of the integrated model. Thus, the integrated solution can not be worse than the decoupled solution.

The integrated planning approach leads to an integrated inventory and network flow problem and it describes a standard large scale tactical planning problem in the supply chain management literature. The network model describes a divergent supply chain and similar models have been solved efficiently earlier (see, e.g., the tactical planning problems in Bredström et al. [5] and Broman et al. [6]).

## 6. Numerical results

We provide numerical results for the implementation of the models of sections 4 and 5 in two instances. A description on the numerical dimension of these instances is given in Table 1. We consider a time horizon of three months split in 12-week periods and weekly demand forecasts as given. Instance I1 will serve as an illustrative example to show the advantages of the integrated over the decoupled planning. Instance I2 enlarges the dimension of the network from instance I1, with higher number of depots, regions, sellers and customers. We use instance I1 as the base of our discussion and instance I2 to strengthen the results.

AMPL has been used to code the models and CPLEX 10.0 to solve them on an Intel Core2 Duo 2.27GHz processor with 2GB of RAM. It took less than a second to find the optimal solution to each of the instances.

Table 1: Instances description.

Instance	I1	I2
Refineries	2	2
Hubs	2	2
Depots	3	15
Crude oils	2	2
Basic oils	2	2
Specialty oils	4	4
Regions	3	5
Sellers	3	5
Customers	9	40
Periods	12	12

### 6.1. Instance I1

The results for the illustrative instance I1 are shown in Table 2.

Table 2: Costs, revenue and contribution of the optimal solution to instance I1.

	DM	DC $\alpha$	DC $\lambda$	DC $\alpha\lambda$	Integrated
Down-to-depot costs	30,469	30,007 -1.52%	30,375 -0.31%	29,947 -1.71%	30,375 -0.31%
2ry distribution costs	4,079	3,930 -3.67%	3,997 -2.02%	3,856 -5.48%	925 -77.33%
Total costs	34,549	33,937 -1.77%	34,372 -0.51%	33,803 -2.16%	31,300 -9.40%
Revenue	68,352	68,352 0.00%	68,352 0.00%	68,352 0.00%	68,352 0.00%
Contribution	33,803	34,415 1.81%	33,980 0.52%	34,549 2.21%	37,052 9.61%

The second column of the table shows the result obtained for the fully decoupled model (DM), expressed in monetary units. The next column corresponds to the solution of the decoupled model with the coordination constraint imposing the bound  $\alpha$  as in (18). The percentage figure corresponds to the difference between this solution and the DM solution. Note a reduction of 1.77% in total costs is achieved when introducing this coordination constraint to the sales sub-model.

The column labelled as DC $\lambda$  shows the results when the coordination constraint (19) introducing the regional limit is considered. In this case, a reduction of 0.51% in total costs is obtained compared to the DM case.

The column labelled DC $\alpha\lambda$  corresponds to the solution when both coordination constraints (18) and (19) are considered simultaneously, leading

to a drop of 2.16% in total costs and an increase of 2.21% in contribution, compared to the DM case.

The last column shows the results obtained from the integrated model. This case outperforms all previous ones. A reduction of 9.40% in total costs and an increase of 9.61% in contribution are achieved by the integrated model compared to the DM case. Note the secondary distribution cost from the integrated solution is dramatically lower than in the DM case, with a 77.3% reduction. This is the result of planning the distribution to the customers taking into account not only the particular revenues of each sale, but also the involved costs of the company as a whole.

Still, when compared to the  $DC\alpha\lambda$  case, the integrated model leads to a reduction of 7.40% in total costs and an increment of 7.24% in contribution, thus outperforming the decoupled approach with coordination constraints.

Note that for a fair comparison, we have considered a penalization on unfulfilled demand  $\psi$  high enough to satisfy all demand, thus the revenue result remains unaffected in all cases. Also, the prices considered are such that it is convenient for the sellers to accept all demand from their customers. An observation must be made on setting the value chain costs for the sales sub-model. Its estimation will not necessarily match the costs in the operating sub-model and the integrated model. Though this may cause some distortion for comparison purposes, the large differences obtained suggest that the current situation could be significantly improved by integrating commercial decisions with operations decisions. Moreover, we would expect that in practice, it is harder to get accurate estimation for the value chain costs than for each of the other costs (processing, production, transport, inventory) used by the integrated model.

Another observation concerns the premium amounts obtained by the sellers in each model. Table 3 shows and compares these amounts.

Table 3: Premium amounts obtained by the sellers in each model.

	DM	$DC\alpha$		$DC\lambda$		$DC\alpha\lambda$		Integrated	
Premium seller 1	607	607	0.00%	600	-1.09%	595	-1.96%	227	-62.62%
Premium seller 2	647	626	-3.17%	641	-0.92%	626	-3.17%	427	-33.91%
Premium seller 3	596	596	0.00%	596	0.00%	596	0.00%	105	-82.42%
Total premium	1,850	1,829	-1.11%	1,837	-0.68%	1,817	-1.75%	759	-58.97%

The premiums obtained by the sellers in the decoupled models with coordination constraints differ slightly from what they get in the fully decoupled



model, keeping at least one of the sellers with the same premium, and with a variation of the total premium obtained by the sellers not greater than 1.75%. On the other hand, the premiums obtained by the sellers in the integrated model exhibit high differences in comparison to all the other cases. In particular, when comparing with the fully decoupled case, in the integrated case the sellers receive between 33.91% and 82.42% lower premium. It is therefore arguable whether, under the premiums obtained in the integrated model, the sellers would still be encouraged to sell high volumes or not. However, given that the integrated model leads to higher total contribution, finding another mechanism to share the contribution among sellers and the rest of the company could keep the incentives for the sellers to achieve high sales volumes. Then, the question arises of how to find an allocation such that all stakeholders are motivated to use the integrated model.

Table 4: Premium of the sellers as percentage of the total contribution.

	DM	DC $\alpha$	DC $\lambda$	DC $\alpha\lambda$	Integrated
%P/C Seller 1	1.79%	1.76%	1.77%	1.72%	0.61%
%P/C Seller 2	1.91%	1.82%	1.89%	1.81%	1.15%
%P/C Seller 3	1.76%	1.73%	1.75%	1.73%	0.28%
%P/C Total	5.47%	5.31%	5.41%	5.26%	2.05%

Table 4 shows the percentage of the contribution that the premium of the sellers represents in each problem (labelled as “%P/C”). It can be observed that the share that the sellers get in the integrated solution is considerably lower than in the other cases. In total, the drop goes from figures in the order of 5.26% or higher to only 2.05%.

Table 5: Equivalent premium amounts.

	DM	DC $\alpha$	DC $\lambda$	DC $\alpha\lambda$
Equivalent premium seller 1	665	653	654	638
Equivalent premium seller 2	709	674	699	672
Equivalent premium seller 3	654	642	650	639
Total equivalent premium	2,027	1,969	2,003	1,949

Table 5 shows the *equivalent premium*, that we define as the premium amount obtained by the sellers considering the same percentage they received in the original case of the corresponding problem (DM, DC $\alpha$ , DC $\lambda$ ,

DC $\alpha$  $\lambda$ ) but applied to the contribution from the integrated solution. For example, seller 1 in the DM case received 1.79% of the contribution as premium (Table 4). The contribution in the integrated solution was 37,052 (Table 2). Then the equivalent premium of seller 1 from the DM case in the integrated solution would be 1.79% of 37,052, thus equal to 665. Note that all the equivalent premiums so obtained are greater than the premiums received by the sellers in the original case that were presented in Table 3.

Table 6 shows the contribution after total premium for each of the other problems according to different ways of calculating the premiums. The contribution after premiums in the integrated solution corresponds to  $37,052 - 759 = 36,293$ .

Table 6: Contribution after premium allocations.

	DM		DC $\alpha$		DCA		DC $\alpha$ $\lambda$	
Original contribution after premium	31,954		32,586		32,143		32,732	
Integrated's contrib. after equivalent premium	35,025	9.61%	35,083	7.66%	35,049	9.04%	35,103	7.24%
Integrated's contrib. after identical premium	35,202	10.17%	35,223	8.09%	35,215	9.56%	35,235	7.65%

The first row shows the contribution after premium in the original case, derived from subtracting the total premium values of Table 3 from the contribution values of Table 2.

The second row of Table 6 shows the result of the contribution from the integrated case (37,052) minus the total equivalent premiums of Table 5. We have also specified the percentage of improvement achieved by using the equivalent premium rule with respect to the original case. The resulting contribution after premium computed by this rule outperforms the original cases with an increase ranging from 7.24% to 9.61%.

The last row of Table 6 shows the result of the contribution from the integrated case (37,052) minus the *identical premium*, that we define as the same absolute premium amount obtained by the sellers in the original case (the total values in Table 3). The resulting contribution after premium in this case is between 7.65% and 10.17% better than in the corresponding original cases.

Reallocation rules based on equivalent premiums and identical premiums are two examples of simple ways of reallocating the additional contribution among the sellers and the company, such that all the sellers in this instance are better off than in the decoupled case while the company also obtains a better result.

## 6.2. Instance I2

The results of the extended instance I2 are shown in Table 7. As in the first instance, in this second instance we also considered a penalization on unfulfilled demand  $\psi$  large enough to satisfy all demand, and prices such that it is profitable for the sellers to accept all demand from their customers.

Table 7: Costs, revenue and contribution of the optimal solution to instance I2.

	DM	DC $\alpha$		DC $\lambda$		DC $\alpha\lambda$		Integrated	
Down-to-depot costs	80,215	78,032	-2.72%	80,084	-0.16%	78,024	-2.73%	73,179	-8.77%
2ry distribution costs	9,992	9,373	-6.20%	10,056	0.64%	9,378	-6.15%	4,113	-58.84%
Total costs	90,207	87,405	-3.11%	90,140	-0.07%	87,401	-3.11%	77,292	-14.32%
Revenue	246,921	246,921	0.00%	246,921	0.00%	246,921	0.00%	246,921	0.00%
Contribution	156,714	159,517	1.79%	156,781	0.04%	159,520	1.79%	169,629	8.24%

The solution of the decoupled model with the coordination constraint (18) imposing the bound  $\alpha$  implies a reduction of 3.11% in total costs, compared to the solution of the fully decoupled model. The effect of the coordination constraint (19) on regional limits is more moderate, with savings of only 0.07% in total costs with respect to the solution of the fully decoupled model. When both coordination constraints (18) and (19) are considered together, the savings in total costs are 3.11% and the contribution increases by 1.79%.

The solution to the integrated model in instance I2 outperforms all the decoupled cases, as also was the case in instance I1. Compared to the DM case, a reduction of 14.32% in total costs and an increase of 8.24% in contribution is achieved by the integrated model.

When comparing the integrated solution to the decoupled case with both coordination constraints (DC $\alpha\lambda$ ) the total cost of the integrated solution is 11.57% lower and its contribution is 6.34% higher.

Note the first two values of the last column in Table 7 indicate savings of 8.77% in down-to-depot-costs and 58.84% in secondary distribution costs, when comparing the integrated to the fully decoupled solution. These results reinforce the advantages of planning the distribution to the customers taking into account the costs of the company as a whole instead of the particular contribution of each sale.

An analysis of the premium of the sellers for instance I2 would lead to similar observations as for instance I1, with worse premium allocations in the integrated case. The five sellers in instance I2 get, in total, 2.95% of

the contribution in the integrated case, while in the fully decoupled case they get 4.96%. However, by reallocating the contribution of the integrated solution using the equivalent premium or the identical premium rules, all the sellers are better off than in the decoupled cases. Also, by using these rules, the contributions after premiums for the company in the integrated case are approximately between 6% and 9% larger than in the decoupled cases.

## 7. Concluding remarks

By using linear programming, we have formulated decoupled and integrated planning models for a divergent supply chain of speciality oil products. In the decoupled approach, we separated sales from the rest of the operations, thus formulating two sub-models: the sales sub-model and the operations sub-model. The sales sub-model decides, given the demand from the customers, the amount of products ordered for each seller from each storage facility in each period of time in the horizon to maximize their total premium. The optimal solution to the sales sub-model problem is used as a given parameter in the operations sub-model, which decides production, primary distribution, inventory and the amount to be assigned to each of the sellers, while at the same time minimizing costs. We also incorporated coordination constraints, by an upper bound on the proportion of two different products assigned to a certain seller from the same depot and an upper bound on the amount of each product that can be sold in the same region.

Then, we proposed a linear model that integrates the sales with the operations planning. In this model, the company as a whole decides on production, inventory, primary distribution and secondary distribution, to maximize the total contribution of the organization over the planning horizon. The advantages of this integrated approach over the decoupled planning is that the decisions on secondary distribution are made together with previous echelons in the supply chain, thus providing a better match with production and storage units. These advantages were reflected in illustrative examples, where the integrated model achieved important decreases in total costs and increases in contribution in comparison to the decoupled models.

We also discussed the premiums obtained by the sellers. In the numerical examples, we observed that, under the current premium allocations based on value chain cost, the sellers were left with much lower premiums in the integrated solution; thus the practical situation may not allow an integrated model to be implemented. In order to motivate the sellers, the develop-

ment of a revenue/contribution sharing principle between the sellers and the company might be required. This has successfully been developed in other industries; see for example Frisk et al. [12]. In Section 6 we explored two simple rules, which served as reallocation of the contribution such that all the sellers and the company were better off in the integrated planning than in the decoupled planning. Developing pricing mechanisms with allocation of premiums is part of our future research agenda. The integrated model can be a basis for developing methods to calculate internal prices that coordinate the two sub-models to the integrated solution. Ideally, such internal prices should generate solutions where the decoupled and integrated models provide the same solutions. In the literature, there are several decomposition schemes that can be useful for this purpose, for example, based on Lagrangian relaxation (Lidestam and Rönnqvist [17], Pirkul and Jayaraman [23]). This coordination could also be integrated with the work on finding a revenue/contribution sharing principle.

We are currently collaborating with a main company in the speciality oils industry. We believe the proposed integrated model has the potential to improve the understanding of the planning process and also to improve the current supply chain planning in the company. The improvements go in the direction of, for example, how to achieve a better mix of products, the timing at which they are produced, product focus in geographical areas, and how to distribute basic oils and speciality oils among the facilities and the final customers. A further research issue is the possibility to delay mixing of some oils to the depots.

In future work we also consider incorporating the uncertainty of demand. This issue has been considered in the oil planning literature, such as the stochastic programming approaches by Al-Othman et al. [1] and Dempster et al. [8]. Research has also been done on integrating planning, as we referred to in the introduction (Pinto and Moro [22]; Neiro and Pinto [19]; Guyonnet et al. [13]) but, as far as we know, none of these and related references in the oil industry have studied the integration of operation and sales decisions under an internal pricing mechanism as described in our paper.

Other possible extension is the integration of the procurement decision to the downstream planning, which in our model would be possible to achieve by defining  $\eta$  as a decision variable instead of a parameter, and adding the corresponding cost (purchase and transport) in the objective function. Finally, a further interesting network design question is the possibility of closing down or opening depots. This requires the model to include binary decision

variables and to assign fixed cost for the use of depots.

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## References

- [1] Al-Othman, W. B. E., H. M. S. Lababidi, I. M. Alatiqi, and K. Al-Shayji (2008). Supply chain optimization of petroleum organization under uncertainty in market demands and prices. *European Journal of Operational Research* 189, 822–840.
- [2] Bengtsson, J., P. Flisberg, and M. Rönnqvist (2011). Robust planning of blending activities at refineries. *Working paper*.
- [3] Bengtsson, J. and S. Nonås (2010). Refinery planning and scheduling: An overview. In E. Bjørndal and M. Bjørndal and P. M. Pardalos and M. Rönnqvist (Eds.) *Energy, Natural Resources and Environmental Economics Springer-Verlag Berlin Heidelberg*, 115–130.
- [4] Bodington, C. E. and T. E. Baker (1990). A history of mathematical programming in the petroleum industry. *Interfaces* 20, 117–127.
- [5] Bredström, D., J. T. Lundgren, M. Rönnqvist, D. Carlsson, and A. Mason (2004). Supply chain optimization in the pulp mill industry – ip models, column generation and novel constraint branches. *European Journal of Operational Research* 156, 2–22.
- [6] Broman, H., M. Frisk, and M. Rönnqvist (2009). Supply chain planning of harvest operations and transportation after the storm gudrun. *INFOR* 47, 235–245.
- [7] Cooper, W. W. (2002). Abraham charnes and w. w. cooper (et al.): A brief history of a long collaboration in developing industrial uses of linear programming. *Operations Research* 50, 35–41.

- [8] Dempster, M., N. Pedron, E. Medova, J. Scott, and A. Sembos (2000). Planning logistics operations in the oil industry. *Journal of the Operational Research Society* 51, 1271–1288.
- [9] Erengüç, S. S., N. C. Simpson, and A. J. Vakharia (1999). Integrated production/distribution planning in supply chains: An invited review. *European Journal of Operational Research* 115, 219–236.
- [10] Feng, Y., S. D’Amours, and R. Beauregard (2008). The value of sales and operations planning oriented strand board industry with make-to-order manufacturing system: Cross functional integration under deterministic demand and spot market recourse. *International Journal of Production Economics* 115, 189–209.
- [11] Feng, Y., S. D’Amours, and S. Beauregard (2010). Simulation and performance evaluation of partially and fully integrated sales and operations planning. *International Journal of Production Research* 48, 5859–5883.
- [12] Frisk, M., M. Göthe-Lundgren, K. Jörnsten, and M. Rönnqvist (2010). Cost allocation in collaborative forest transportation. *European Journal of Operational Research* 205, 448–458.
- [13] Guyonnet, P., G. Hank, and M. Bagajewicz (2009). Integrated model for refinery planning, oil procuring, and product distribution. *Industrial and Engineering Chemistry Research* 48, 463–482.
- [14] Iachan, R. (2009). A brazilian experience: 40 years using operations research at petrobras. *International Transactions in Operational Research* 16, 585–593.
- [15] King, R. H. and R. R. J. Love (1980). Coordinating decisions for increased profits. *Interfaces* 10, 4–19.
- [16] Lasschuit, W. and N. Thijssen (2004). Supporting supply chain planning and scheduling decisions in the oil and chemical industry. *Computers and Chemical Engineering* 28, 863–870.
- [17] Lidestam, H. and M. Rönnqvist (2011). Use of lagrangian decomposition in supply chain planning. *Mathematical and Computer Modelling* 54, 2428–2442.

- [18] Martin, C. H., D. C. Dent, and J. C. Eckhart (1993). Integrated production, distribution, and inventory planning at libbey-owens-ford. *Interfaces* 23, 68–78.
- [19] Neiro, M. and J. Pinto (2004). A general framework for the operational planning of the petroleum supply chains. *Computers and Chemical Engineering* 28, 871–896.
- [20] Ouhimmou, M., S. D’Amours, R. Beauregard, D. Ait-Kadi, and S. Chauhan (2008). Furniture supply chain tactical planning optimization using a time decomposition approach. *European Journal of Operational Research* 189, 952–970.
- [21] Pinto, J., M. Joly, and L. Moro (2000). Planning and scheduling models for refinery operations. *Computers and Chemical Engineering* 24, 2259–2276.
- [22] Pinto, J. and L. Moro (2000). A planning model for petroleum refineries. *Brazilian Journal of Chemical Engineering* 17, 575–586.
- [23] Pirkul, H. and V. Jayaraman (1998). A multi-commodity, multi-plant, capacitated facility location problem: formulation and efficient heuristic solution. *Computers and Operations Research* 25, 869–878.
- [24] Reddy, P., I. Karimi, and R. Srinivasan (2004). Novel solution approach for optimizing crude oil operations. *AIChE Journal* 50, 1177–1197.
- [25] Sarmiento, A. M. and R. Nagi (1999). A review of integrated analysis of production-distribution systems. *IIE Transactions* 31, 1061–1074.
- [26] Sear, T. (1993). Logistics planning in the downstream oil industry. *Journal of the Operational Research Society* 44, 9–17.
- [27] Viswanadham, N. and N. S. Raghavan (2000). Performance analysis and design of supply chains: A petri net approach. *Journal of the Operational Research Society* 51, 1158–1169.