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Discussion paper

What Happened in Burlington?

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ABSTRACT

Three visualization techniques illustrate the distribution of electoral preferences over a candidate triple. Two of them, introduced here, concern an IRV tally. The conditions that may allow the "pushover strategy" and the "No-Show Paradox", are identified, and the practical consequences discussed. The controversial mayoral election of Burlington, Vermont, in 2009 is background. We see the IRV method in a legal and in a political context, presenting aspects of a judgment in the Minnesota Supreme Court 2009 and of the UK referendum over IRV in 2011. IRV is the single-seat version of STV. Both may achieve proportional voter influence with a designed disproportional distribution of seats in a legislature, e.g. as part of a potentially viable modus vivendi in ethnically divided societies.

I INTRODUCTION

The city of Burlington, Vermont, elected its mayor on Tuesday, March 3rd 2009. The election method *IRV*, i.e. *Instant Runoff Voting* (*Alternative Vote* in UK English), had been adopted at the previous election in 2006. Each voter prepared a ballot with a *ranking list* of a *freely chosen subset* from the set of candidates.

The IRV tally is in several rounds: Each round results in elimination and removal from all ballot lists of the candidate with the smallest number of first ranks. In the next round, a shortened list replaces its predecessor. Ballots that at the start ranked just a proper subset of the candidate set may become empty in the tally. The tally ends when the winner is clear, being ranked first among the remaining candidates in a majority of the ballots that are not emptied.

An arena for the IRV struggle. In Burlington 2009, there were five candidates, but three of them dominated. They were:

M (Andy Montroll, democrat); K (Bob Kiss, progressive); W (Kurt Wright, republican).

The last round but one recorded *the number of first ranks* then received by each candidate:

(1.1) 2554 for M; 2982 for K; 3297 for W.

After elimination of M, the subsidiary ranks from the 2554 M-supporters became available:

(1.2) 1332 for K; 767 for W; 455 emptied.

Then the final round was a *pairwise comparison* between K and W:

(1.3) 2982 + 1332 = 4314 votes for K; 3297 + 767 = 4064 votes for W.[1]

Thus, the incumbent K won the election for a new 3-year term.

All margins were clear. Still, within a few days there were strong reactions and attacks on the IRV-method. Already in 2010, a motion to repeal the IRV-method won with 3972 votes against 3669.

[1] Various sources give election data with small and insignificant differences. The tally report (1.1) – (1.3) is based on the data in Figure 1; these are from Rangevoting's table at http://rangevoting.org/Burlington.html

The alternative to IRV was a return to the common 2-day election, sometimes called the Two-Round System (TRS): In a *"nonpartisan primary"* on day 1 each voter supports just one candidate; in a *"general election"* on day 2 there is a runoff between the two candidates with most support on day 1, say X and Y. Thus, with 3 candidates remaining, the IRV tally simulates a 2-day election with the same voters both days, and those who voted for X or Y on day 1 vote the same way on day 2.

Actually, already in the very first IRV tally round in Burlington 2009, K and W were clearly ahead of every other candidate in number of first ranks, and they remained so. Thus, it was reason to assume that K would have won also in a 2-day election. So – why the ado?

In the public debate leading up to the repeal vote, there were groups and organizations for different electoral reforms. They attack each other's proposals, claim that wanted or unwanted properties of proposed methods are significant, and try to substantiate their claims with different means, including constructed examples, simulations and observations from real elections.

Elections also occur in private and professional organizations. Political elections attract special attention, certainly due to their importance, but perhaps also because a higher number of people have both their own experiences from political elections and a shared familiarity with elections that get an international audience, e.g. US presidential elections.

Burlington got wider attention than most arenas for the struggle over IRV. However, the turnout for the repeal referendum was only 23% of Burlington's voter roll. How many of those who supported the repeal bill had been stirred up by charges of faults in IRV, and been particularly amenable because of their frustration with the outcome in 2009? Did they assess IRV fairly, a method used for generations in e.g. Australia (single-seat constituencies for the House of Representatives) and Ireland (presidential elections)? [2] How many of the silent 77% majority found that neither side had convincingly explained its viewpoints?

Structural features in real preference distributions. There is disagreement over what properties that are most prominent in the various election methods used or proposed. Clarifying and assessing claims and counterclaims should improve the background for societies choosing between available election methods. However, such clarification should build on knowledge of the structural features that are normal in the preference distributions found in real preferential elections.

[2] The STV-method (Single Transferable Vote) is designed to elect r representatives from the same constituency, $r \geq 1$, and it specializes to IRV when $r = 1$. It is used to elect from multi-seat constituencies to the National Assembly in Australia, Ireland, and Malta; the two latter also use STV to elect representatives to the EU Parliament. To the extent that r permits, STV gives proportional representation of parties.

In the following, Burlington 2009 will be a recurring theme in an attempt to contribute towards a clarification. The preference distribution in the Burlington ballot set appears in the "*pictograms*" of Figures 1 and 2. For any candidate triple {X, Y, Z} there are six possible strict preferences:

(1.4) XYZ, XZY, ZXY, ZYX, YZX, YXZ

Let |XYZ| be the number of voters with preference order XYZ, etc. To (4) corresponds the *vote vector*:

(1.5) (|XYZ|, |XZY|, |ZXY|, |ZYX|, |YZX|, |YXZ|)

The sizes of the six components in (1.5) are proportional to six areas in Figure 1, with

(1.6) (X, Y, Z) = (M, W, K)

In Figure 2, a voter who supported only one candidate, say M, not distinguishing between second and third preference, is counted as giving one half vote for MWK and one half for MKW, etc.

A *pictogram* for (1.5) consists of a unit circle and three chords that meet pairwise inside or on the circle. If they are distinct, they form a triangle T not corresponding to any voter group. A pictogram always exists; it is unique up to reflections and rotations.

In Figures 1 and 2 T is very small and the six components of (1.5) are strictly positive. Notice the cyclic order of the six voter categories. *"Ideal points"* M, K, and W, are inserted so that the perpendicular bisectors of the *"candidate triangle"* ΔMKW almost coincide with the chords of the pictogram. *"Perfect Pie-sharing"* occurs when the chords are not distinct or T has zero area. When a candidate triangle fits, then the vote vector (1.5) also fits a simple model of *spatial voting*: The voters distribute uniformly in the unit circle, and each voter ranks the candidates according to their Euclidean distance from the voter.

A pictogram may be constructed for each of the n(n–1)(n–2)/6 candidate triples in an n-candidate election. T is usually very small for every triple. Thus very few of the possible preference distributions are realistic. A general structural feature of real data is that, for every candidate triple, the model of Perfect Pie-sharing fits much better than it does in a simulated election based on the popular probability distribution of preferences IAC (Independent Anonymous Culture).

Many voters from different parts of the political landscape have roughly the same perception of it. The 1-dimensional "Single-Peak" model assumes that voters with different subjective preferences agree on an objective ordering, $C_1, C_2 \ldots, C_n$, of the candidates along a political "left-right" axis. [3]

[3] The most familiar spatial voting model is the one of Duncan Black (1948): The candidates are represented as points on the unit line: $0 \leq C_1 < C_2 < \ldots < C_n \leq 1$. A ballot ranking is single-peaked with respect to this sequence if C_i is never ranked after both C_{i-1} and C_{i+1} $(1 < i < n)$; it is easily checked that there are 2^{n-1} single-peaked rankings. However, if the voters distribute along the line and vote according to distance, ballots only change at the n(n–1)/2 midpoints $(C_i + C_j)/2$; thus only 1+ n(n–1)/2 different ballots may occur. In Black's full Single-Peak model, we may imagine that the voters agree on the objective ordering of candidates from left to right, but not on their precise locations. Even so, it is difficult to model a useful sequence of voters on the line. To see this, consider an election with n = 4 candidates: If voter *a* ranks $C_2C_3C_4C_1$ and voter *b* ranks $C_3C_2C_1C_4$, both have a single-peaked preference. However, if we consider the pair $\{C_2, C_3\}$, it is natural modelling to put voter *a* on the left of voter *b*. Similarly, if we consider $\{C_1, C_4\}$, it is natural to put *a* on the right of *b*.

However, for every triple {C_i, C_j, C_k}, two out of six possible rankings are then supposed not to occur. Black's model is therefore not suited for adaptation to real election data. As a rough approximation that guides our thinking, it is useful, and the pictogram shows, in each case, how far the real data are from being single peaked. Thus, Figures 1 and 2 show M as intermediate between W and K, with clearly the smallest number of last ranks, but the distribution would be single-peaked only if the voter categories WKM and KWM were empty.

For candidate triples in real elections, the model of Perfect Pie-sharing usually fits quite well. A likely reason is that many independent voters with different preferences still have similar perceptions of the political landscape. This is a general rationale for spatial voting models. In most political elections, the political parties are central in the development of similarities of perception among voters with very different preference rankings. On this party-voter interaction, see e.g Caillaud and Tirole (1999, 2002).

Strategic voting. In the theory, *"strategic voting"* usually has a wide, technical meaning: *A group of voters who switch from one ranking to another, thereby obtain a better result according to the ranking they abandoned.* Usually three special types of strategic voting, S1, S2, S3, popularly called compromise, burying, and pushover, get most attention. With three candidates, they are:

(1.7) S1: Switch from XYZ to YXZ takes victory from Z to Y (*compromise*, vote change in {X, Y});

(1.8) S2: Switch from XYZ to XZY takes victory from Y to X (*burying*, vote change in {Y, Z});

(1.9) S3: Switch from XYZ to YXZ takes victory from Z to X (*pushover*, vote change in {X, Y}).

For special preference distributions, a preferential election method may give incentives to try one of the strategies S1, S2 or S3. Only in S2 and S3 may the change involve a degree of insincerity. The more efficient support of the favorite X then has a price: If the strategy attempt fails, perhaps due to wrong assumption about the distribution, then the insincere change in {Y, Z} or {X, Y} may lead to a result that is worse according to the voter's real wish.

Moreover, incentives to apply e.g. S2 have another aspect. In each candidate pair, the ballot ranking is a signal from the voter; does it matter if it is trustworthy? One of the criteria for assessment of preferential elections should be how well they let a voter combine efficient support to a favorite with true statement of preference in every candidate pair.

The typical reason for attempting strategy S1 is that the change is due to a new assessment; the voter includes *feasibility* as a ranking criterion. Y is then, in all sincerity, seen as a more feasible opponent to Z than X would be. There is no inkling of insincerity when a voter modifies the ballot because of feasibility. The ugly term *"favorite betrayal"* for strategy S1 is both untrue and unfair. With a narrow definition of *strategic voting*, as victory obtained by ranking insincerely, S1 is not included.

Of course, compared to Plurality voting (FPTP – "First Past the Post" in UK English) used in single-seat elections in very many countries, IRV goes a long way to allow voters to disregard feasibility:

The tally will still organize increasing support for compromise candidates through vote transfers after each elimination. In particular, if a minor candidate is a *spoiler* for a major candidate, as Nader perhaps was for Gore in the US presidential election in 2000, then IRV could have solved the problem with transfers in the tally. However, there are cases were one must compromise in the ballot, and not expect that the IRV-method always achieves it through the tally.

The structure of the paper is as follows:

Section II presents the main arguments adduced for or against IRV, with particular reference to Burlington 2009.

Section III, Figures 1 and 2, illustrate the Burlington election 2009. Some general structural features in 3-candidate elections appear, together with the potential for a special *nonmonotonicity effect* in IRV, a possibility that actually occurred in this particular election. However, the possibility was remote.

Section IV deals with two manifestations of *nonmonotonicity*; a preferential election method is *monotonic* if a candidate never can get a worse (better) result by getting a better (worse) rank in a ballot. It is well known, and a source of criticism, that IRV is nonmonotonic. This allows the pushover strategy S3 to work in some preference distributions, but it is hard to apply. The reverse ballot change, causing the reverse change of IRV-winner, is more significant. In order to reflect the connection between the two effects, the terms used here will be *trick effect* for pushover, i.e. Strategy 3, and *trap effect* for the reverse change.

The *trick effect* is that a suitable number of XYZ voters turn X into IRV-winner instead of Z by switching their ballot ranking from XYZ to YXZ. The point is to promote Y in order to eliminate Z; this may sometimes work if X has a sufficient advantage over Y to win in the end. The *constellation* is a 3x3-table that visualizes the IRV-tally for three candidates: It turns out that the trick effect is only possible in some of the elections in *constellation iii* in Figure 4 or its cyclic version. The *trap effect* is the opposite change; a switch from YXZ to XYZ causes Z to snatch victory from X:

The participating YXZ-voters who want to give extra support to the expected winner X in order to make sure that Z does not win, walk into a *trap*: They cause X to lose and Z to win. If 770 voters move from WKM to KWM, the trap effect will lead from Figure 2 to Figure 3b, and the *trick effect* will lead back again. The tally cannot distinguish between the preference distributions in Figures 3abc. All are trick positions; a realistic split into KMW and KWM gives a pictogram close to Figure 3c. In particular, the IRV tally (1.1) – (1.3) alone will never reveal it if a *Condorcet Paradox* has occurred, i.e. that the three candidates beat each other cyclically in pairwise comparisons (e.g. Figures 3a and 6); more ballot information is needed.

Section V concerns the *No-Show Paradox*, which occurs when a group of XYZ-voters by participating actually will cause the election result to get worse according to their own ranking. In IRV, they cause Z to win instead of Y. This is very different from the nonmonotonicity trap effect: The XYZ category is the only one that changes in size. These voters are better off if they do not show up at the polling site. However, like nonmonotonicity trick distributions, also the potential that new XYZ-voters will cause Z to win is only present in some of the elections in *constellation iii* or its cyclic version. Moreover, the new XYZ-voters actually just change the preference distribution so that X becomes a *spoiler* for Y, but of course, all XYZ-voters are equally responsible.

All voters who assess both X and Y as much better than a strong opponent Z, have an incentive to consider whether it is X or Y who is most feasible, and then vote accordingly. If political commentators predict a spoiler situation, a group of voters who intend to vote XYZ, may *compromise*, vote YXZ and turn Y into IRV-winner instead of Z.

Section VI regards IRV in a wider perspective of preferential voting methods. In a *"positional"* method of preferential voting, the tally awards a candidate P(r) points for rank r in a ballot,

(1.10) $$P(1) \geq P(2) \geq \ldots \geq P(n) \geq 0$$

Three examples follow in a table:

	P(1)	P(2)	···	P(n)
Plurality/FPTP	1	0	···	0
Borda Count	n-1	n-2	···	0
Nauru Count	1	½	···	1/n

The tally ranks according to summation over all ballots. The choice between the Plurality method, still used in many countries throughout the world, and Borda's proposal of 1770, was a recurring theme in the French Academy of Sciences. Figure 8 illustrates an important difference: The Plurality method gives a very strong incentive to make feasibility a criterion and to compromise even if it hurts, e.g. switch from Nader to Gore. The Plurality method qualifies for the label *"coarseness"*. However, the Borda Count gives extremely strong incentive to bury a competitor, which means that a voter is under pressure to fake opinion on one or more candidate pairs. This deserves the label *"cruelty"*.[4] However, everybody interested in electoral reform ought to know the Borda Count well because it so clearly exhibits grave faults that society should try to avoid or at least reduce with a wise choice of preferential election method.

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[4] Obviously also the Nauru Count occasionally allows successful use of strategy S2 (burying), but this makes a difference of at most 1/2 – 1/n points in a single ballot. In all sincerity, the Nauru Count offers the country's teachers of basic math a relevant national didactic tool to create familiarity with fractions – and to make new generations aware of how one of society's basic decision mechanisms works. Teachers elsewhere, in their struggle with cohort after cohort, have a reason for envy.

Nevertheless, many of the other preferential election methods in use relate to the Plurality method or the Borda Count, the STV-methods as “Plurality light” and the Condorcet-methods as “Borda light”.

In a Plurality election, the pressure on voters to compromise can be very strong. Alleviation of this pressure is one important motivation for electoral reform. IRV reduces the pressure, and does not allow burying. In *“Conditional IRV”*, pushover cannot work either.

The constitutionality of various election methods have often been tried in US courts. The adoption of IRV/STV in Minneapolis was in 2009 adjudicated by the state Supreme Court. Parts of the ruling and reasons are also included.

Section VII deals with ballot information that cannot influence the tally for IRV. Many attacks on IRV use information not revealed in the tally. Voters whose first ranked candidate lost in the last tally round may well resent a tally method that ignores all other information in their ballots.

Subsidiary rankings by W-supporters and K-supporters are required for other pairwise comparisons or for analyzing possible effects of nonmonotonicity or No-Show. Every point in the rectangle of Figure 9 represents a preference distribution with precisely the same tally as that of Figure 2.

Figure 9 also visualizes why Condorcet cycles are rare and why Perfect Pie-sharing is a robust model.

Section VIII concerns the political problems raised by introduction of IRV. In a UK referendum of May 2010, the question was whether to elect members of the House of Commons with IRV (“Alternative Vote” in UK English). The result was an overwhelming victory for keeping the Plurality method (“First past the Post”). Why? The Opinion Polls throughout the long campaign give some clues. Not surprisingly, the poll sequence seem to confirm that the poll respondents in general had not done much homework on the technicalities of IRV. On referendum day, many voters accepted the advice from the politicians that they would lend their ears.

Often IRV replaces the 2-day election. An expectation of reduced election costs is a natural motivation. However, ranking candidates in the ballots have a *“cognitive cost”* (Gierzynski 2009, 2011). Many countries accept the costs, economic and cognitive, of a 2-day election for president, while keeping Plurality in single-seat constituencies for the legislature. A common alternative is a rough proportionality between parties of sufficient size, usually achieved with a choice between party lists. With a preferential election, one may go a step further, and let the composition of the legislature itself be a compromise between various voter groups. Especially in ethnically divided societies, this is a potential field for IRV or STV (Reilly 2002).

II VIEWPOINTS AND CLAIMED SUPPORTING FACTS

Information from the Burlington ballots that does not appear in the tally report (1.1) – (1.3) was prominent in the aftermath of the 2009 election. K was elected even though M was a clear *Condorcet winner* and W was a clear *Plurality winner*. This means that

(2.1) *M won (clearly) in pairwise comparisons against both K and W, and*

(2.2) *W had (clearly) more first ranks than both M and K.*

Fact (2.1), that M was Condorcet winner, is not in the tally report, but is clear from the complete ballots in (3.1) below. Of course, a Plurality winner may well also be Condorcet winner, and will obviously be IRV-winner too, if only three candidates remain.[5] After the Burlington result, commentators seemed surprised that the IRV-winner was neither Plurality winner nor Condorcet winner. Some attackers considered it a flaw in IRV that the rules allowed such a *"pathological"* result.

Another fact claimed by attackers to support them, concerned the *nonmonotonicity* of IRV: There exist elections with ballot sets such that a suitable *change* in some ballots in one direction will change the election result in the opposite direction. It so happens that the structure of the Burlington ballot set also illustrates this possibility: If a suitable number of voters who voted WKM or supported W without expressing a preference in the pair {M, K}, instead had voted KWM, then they would not have strengthened K. As it is often but misleadingly expressed,

(2.3) *the extra support to K would have caused M to be IRV-winner instead of K.*

Another unfortunate possibility in some IRV-elections is the *"No-Show Paradox"*;

(2.4) *a group of voters would have obtained a better result according to their own ballots by not showing up for polling day, i.e. by not participating in the election.*

None of the possibilities (2.1), (2.3), and (2.4), if they occur, will be in a tally report like (1.1) – (1.3). In a 2-day election, they are not even meaningful, since much of the relevant data are not collected. However, they are essential in the criticism of IRV.

Figure 1 visualizes all subsidiary preferences. Most supporters of K and W had expressed a subsidiary preference for one of the two other remaining candidates, but these were not tallied – for a good reason. A basic principle in IRV is that while a candidate ranked number j in a ballot, say X, is not yet eliminated, the information on who is number j+1, j+2, … in the same ballot is simply not available to the tally officers. Thus, this information cannot influence the fate of X.

Since W and K qualified for the final round, the subsidiary rankings from their supporters never got tallied, but they had to be collected to make instant runoffs possible if other eliminations had occurred. Thus, in IRV, voters cannot help their favorite e.g. by giving last rank to the candidate whom they

[5] Who is Plurality winner when four candidates remain, may become Plurality loser when three remain.

consider as the strongest opponent. Strategy S2, i.e. burying, see (1.8), is an important strategy in some methods of preferential election, but it does not work in IRV.

(2.5) ***Example*** For contrast, consider the *Borda Count*, see (1.10), which gives a particularly strong incentive to bury a strong opponent: Ranking ABCDEFGH, a voter will give them 7, 6, 5, 4, 3, 2, 1, 0 *Borda-points*, respectively. By voting ACDEFGHB instead, the voter buries B and makes up for 7 voters who rank B first, A second. A fair comparison of IRV to other methods must include the immunity of IRV to burying.

IRV often replaces the 2-day election. The voters save one trip to the polling site, but to make use of their voting power, they must put more effort into preparation of their ballots. IRV bothers voters to express preferences that will not even be considered in the tally. Understandably, that is among arguments against IRV often seen.

When comparing IRV to other methods of preferential single-seat elections, it is of course natural to consider the full distribution of voter preferences. However, in order to compare properly, it is essential to keep in mind the reasons why an IRV-election collects so much data that cannot influence the tally.

III THE BURLINGTON PREFERENCE STRUCTURE

When three candidates remained, the ballot preferences in Burlington 2009 were these:

▪ *Tallied* details of ballots from the 2554 M-supporters, see (1.1) – (1.3):

(3.1) p = 767 preferred W to K; q = 1332 preferred K to W; 455 stated no preference in {K, W}

▪ *Not tallied* details of ballots from the 2982 K-supporters:

(3.2) r = 2043 preferred M to W; s = 371 preferred W to M; 568 stated no preference in {W, M}

▪ *Not tallied* details of ballots from the 3279 W-supporters:

(3.3) t = 495 preferred K to M; u = 1513 preferred M to K; 1289 stated no preference in {M, K}

Figure 1 illustrates the structure in (3.1) – (3.3). It brings together two distinct, but related ideas. One ingredient is a *pictogram* consisting of a circle with three chords intersecting *pairwise* inside or on the circle. For any six given nonnegative numbers in a given cyclical order (p, q, r, s, t, u), a *unique* pictogram has six areas along the periphery, with areas *proportional* to p, q, r, s, t, u; the number of ballots with subsidiary preference was

(3.4) $$p + q + r + s + t + u = 6521$$

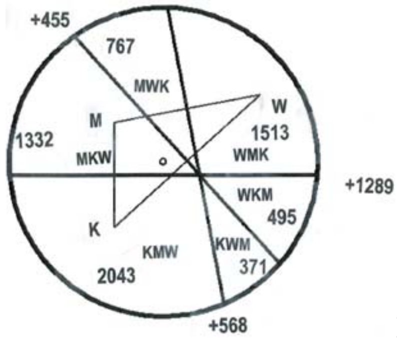
In Figure 1 the 455 + 568 + 1289 = 2312 *"don't know"*-voters in (3.1) distribute outside the pictogram according to their first preference. Except in *degenerate cases*, i.e. with two or three coinciding chords, the three chords form a triangle T. In Figure 1, T covers a fraction 0.00000019 of the circle area. An explanation of the pictogram construction is in Stensholt (1996, 2013); the latter also includes a Maple program.

Another ingredient in Figure 1 is a *model of Perfect Pie-sharing*. In this model, the areas for the six voting categories cover the whole circle; the model fits exactly in degenerate cases, but also in non-degenerate cases where T has shrunk to a point. In most pictograms from real elections with many independent voters, the perfect pie-sharing model fits visibly well, but not exactly.

In Figure 1, the model of Perfect Pie-sharing is visualized with 3 *candidate points*, for M, K, and W, that are the corners of a *candidate triangle* ΔMKW. The *perpendicular bisectors* of its sides are *concurrent*; in Figure 1 they are very close to the pictogram chords. In the model of Perfect Pie-sharing the voters are regarded as being uniformly distributed in the circle, each one ranking the candidates according to distance from the voter. Obviously, the triangle ΔMKW may change in size and still have the same perpendicular bisectors.

The size of T is a measure of how well the Perfect Pie-sharing model fits. The cyclical order in the pictogram of the six voting categories must then be compatible with the candidate triangle; the area for preference XYZ has neighbor areas for YXZ and XZY.

Like in Figures 1 and 2, real elections usually give, for every candidate triple, a T that is small enough to allow a visibly close adaptation of a candidate triangle in a model of Perfect Pie-sharing. The ballot preferences alone do not carry information that allows us to pinpoint the candidate points. One may of course consider additional questions to the voters about their perception of the political landscape, perhaps in an exit poll.



Burlington 2009

FIGURE 1

The triangle T defined by the 3 chords is usually small when the numbers are the sizes of 6 preference categories in a real election with many and independent voters. In this case, T covers a fraction 0.00000019 of the circle area.

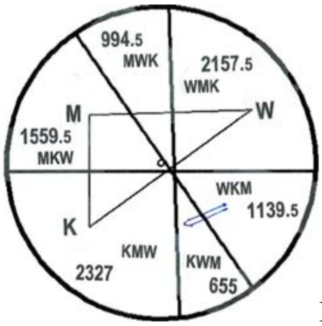
Here one cannot visually separate the perpendicular bisectors of the candidate triangle ΔMKW from the pictogram chords. If Perfect Pie-sharing fits the data exactly, the data determine the shape of ΔMKW, but the voters' views as expressed in their ballots do not determine its size.

The 455 + 568 + 1289 = 2312 voters without a second preference in {M, K, W} are outside the pictogram of Figure 1. This complicates a political analysis based on the figure. In Figure 2, we therefore count each ballot for X without subsidiary preference as half a vote for XYZ and half a vote for XZY. Thus, all 8833 voters become included in the next pictogram. The IRV tally develops the same way as before, but Figure 2, with six instead of nine preference types, is sometimes a more useful illustration.

The structure of the preference distribution, as visualized in Figure 1 and 2, fits with M and K being relatively close to each other in the political landscape, while W is somewhat further away.

It fits with the Perfect Pie-sharing that among the three candidates, M is closest to the circle center in Figure 2, and is therefore Condorcet-winner: M is preferred in pairwise contest both with K and with W by a voter at the center, and therefore, in each case, by more than one half of the voters.

Beware of the two voter categories WKM and KWM. These are not mainstream voters: They agree on ranking the Condorcet-winner M last. They prevent M from becoming IRV-winner only if they keep both K and W ahead of M in first ranks. If one of them drops behind M, then M qualifies for the final round, and will then necessarily become IRV-winner. This illustrates a general result shown below: If there is a Condorcet-winner Z, then nonmonotonicity with three candidates has to do with voters moving between XYZ and YXZ, eliminating or promoting Z.



Burlington 2009

FIGURE 2

The 455 M-voters that were not included in the pictogram of Figure 1, count as 227.5 for MWK and 227.5 for MKW. The 568 K-supporters and 1289 W-supporters outside the pictogram in Figure 1 also count the similar way. The new tally does not use the category *"emptied"* of (1.2).

The triangle T covers a fraction 0.00009772 of the circle area.

The pictogram illustrates the two final tally rounds: There is a Condorcet ranking according to distance from the center, but Condorcet winner M, the most central candidate, was squeezed between the two others, got the smallest primary support, and was eliminated. The MKW-area includes the center voter. Thus, in the final round, in pairwise comparison with W, K got support from the center voter, and thus from a majority.

The arrow in Figure 2 shows that a *trap effect* was possible: If sufficiently many voters had switched from WKM to KWM, perhaps after agreeing to raise their appraisal of K, they would have *walked together into a trap*, causing the elimination of W and the entry of Condorcet winner M into the final tally round. Figure 3b shows the pictogram after 770 voters hypothetically, have walked into the trap.

The break-even point is reached when 743 voters have changed from WKM to KWM, see a tally below in (4.1). The new preference distribution is shown in the *vote vector* (p, q, r, s, t, u) =

(3.5) (|MWK|, |MKW|, |KMW|, |KWM|, |WKM|, |WMK|)

= (994.5, 1559.5 , 2327 , 655+743, 1139.5–743, 2157.5)

M and W have the same number of first ranks: 994.5 + 1559.5 = 2554 = 396.5 + 2157.5. Of course, complete election rules must include a tie-break rule.

Despite being a clear Plurality winner, W is far from becoming IRV-winner. In Figure 2, Plurality winner W appears as a spoiler for Condorcet winner M. However, if just 372 voters in the WMK category had compromised and changed their ranking to MWK, there would obviously have been a final round with K and M, and M would have emerged as IRV-winner, a clear improvement for the 1513 voters in Figure 1 with ballot ranking WMK.

According to a very broad, technical definition, compromising counts as a method of *strategic voting*, S1 in (1.7). This is essential in the Gibbard-Satterthwaite impossibility theorem.

IRV and balanced incentives. With three candidates remaining in the tally, an IRV-winner can never be last in ranking by first-ranks or last in Condorcet ranking. Thus, IRV gives balanced incentives: Work for primary support from enthusiastic followers; also work for subsidiary support from political neighbors. There were three candidates all worthy of becoming mayor in Burlington 2009. However, to elect the one candidate who is not last in any of the two basic rankings, has a didactic value.

Directional or proximity ranking. Perfect Pie-sharing will usually fit quite well even if some voter groups find that the candidate triangle does not express well their perceptions of the political landscape. One may still think of the candidate triangle as a useful simplification reflecting an *average perception* of the political landscape. If T is a point, the *neutral point*, representing the *status quo*, the rays from T indicate possible *directions* from T. The pictogram is then a *directional spatial model*, where the voters rank the candidates' policies according to direction (Merrill and Grofman 1999).

Reversed ballots. Reading all ballots back to front, so that e.g. the 2327 KMW ballots in Figure 2 express *"K after M after W"*, we keep the same pictogram, but the candidate triangle ΔMKW has to be rotated 180^{o} around the neutral point, i.e. the point where the perpendicular bisectors meet. If the voter categories KWM and WKM are smaller than in Figure 2, and thus the preference distribution closer to Single Peak, then the distribution of the reversed ballots becomes quite close to a *"Single Bottom"* distribution. Then there may be insufficient space for the new candidate triangle. If we want to draw it inside the pictogram, we may have to make it very small. For this reason, it is unlikely that real elections will be close to Single-Bottom. When Duncan Black went for Single Peak rather than Single Bottom, it certainly was for good practical reasons. Both Single Peak and Single Bottom models clearly guarantee that there is no Condorcet cycle; in fact, they both are great "overkills".[6]

[6] For a Condorcet cycle with three candidates to occur, it is easy to see that the triangle T in the pictogram must cover the circle center, but usually T is very small. Moreover, the closer the preference distribution is to Single Peak, the further T will be away from the center.

IV NONMONOTONICITY IN BURLINGTON 2009

The Burlington preference distribution, shown in Figure 2, is a possible starting point for a *nonmonotonic* change: Assume that h voters move from WKM to KWM, as indicated with the arrow in Figure 2. Then the next round of the IRV tally becomes

M: 2554 first ranks ; K: 2982 + h first ranks ; W: 3297 – h first ranks

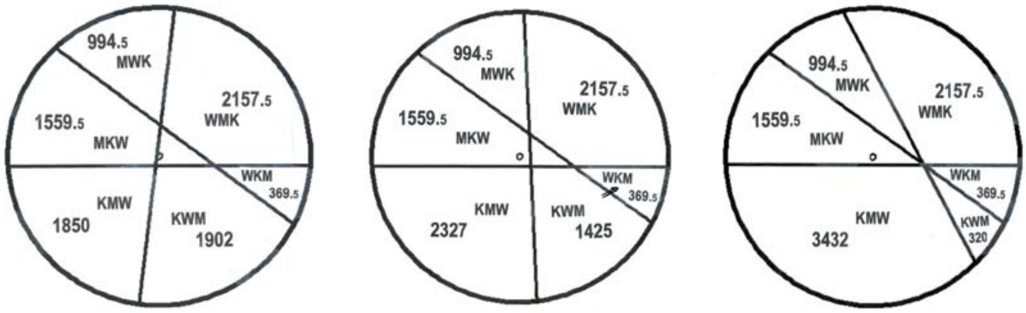
If h > 743, then W is eliminated. There are not enough voters in the original WKM-group (Figure 1), but after its extension with 644.5 *"don't know"*-supporters of W (Figure 2), the possibility is there. Then, in the last round, the tally will be:

M: 2554 + 2158 = 4712; K: (2982+h) + (1139–h) = 4121

If more than 743 anti-M voters move from WKM to KWM, supporting the expected winner K as an extra precaution against election of M, then they walk into a *nonmonotonicity trap*: They recklessly make space for M in the final, and there the Condorcet winner M snatches the IRV-victory from K.

It is common to say, as in (2.3), that such a disaster for both voter categories involved, WKM and KWM, is a consequence of the increased support for K. This is inaccurate, and even misleading. If any number of Burlington citizens who stayed home, instead had gone to the polls and supported K, they would certainly only have helped K to an even clearer IRV-victory. It is by weakening W that they let the Condorcet winner M escape from elimination.

The pictogram of Figure 3b shows the new preference distribution after h = 770 voters have switched from WKM to KWM (see Figure 2). Comparisons with Figures 3ac are made in the discussion.



3a 3b 3c

FIGURE 3

The elections in 3abc differ only in the subsidiary votes of the 3752 K-supporters; an IRV tally eliminates W, and M becomes IRV-winner. In 3b, T covers 0.00980757 of the circle area. A transfer of 770 voters from KWM to WKM leads back to the Burlington election in Figure 2. They perform a *"trick"* which works in all 3 elections, and actually 28 participants will suffice. They push W up to avoid M in the final round; even though they count for W in the final, K can afford it, and W is "pushed overboard". This trick is the pushover strategy S3; see (1.9). Election 3a has a Condorcet cycle, while 3c is close to Perfect Pie-sharing and realistic.

Tricks and traps. The critical point in (3.5), where the change takes place when voters in Figure 2 switch, one by one, from WKM to KWM, is much closer to Figure 3b than to Figure 2: When 27 voters in Figure 3b switch back from KWM to WKM, then the vote vector is as shown in (3.5).

Obviously also some of the KMW-voters in Figures 3abc may want to participate in the pushover and vote WKM, but an attempt may be overdone, and W may become IRV-winner. However, for the KWM-voters in Figure 3, even victory for W will be an improvement. We decompose a move from KMW to WKM in two shorter steps, between neighbor categories in the pictogram:

(4.1) KMW → KWM → WKM

It is the second step that helps to eliminate M. The first step is just an increase of the KWM-category. Symmetrically, WMK-voters may intervene if K is too weak: They first increase the WKM-category and then move on to KWM, but if the action is overdone, they may see that K becomes IRV-winner. Nonmonotonicity is caused by shifts between the two neighbor categories WKM and KWM, i.e. among the voters who rank the Condorcet-winner M last.

As Figures 2 and 3b together illustrate, *trick positions* and *trap positions* belong together. Shuffling of voters between the two anti-Condorcet-winner groups in Figure 2, KWM and WKM, may lead to a crossing of a border either way. Both crossings may, objectively, be seen as being in the "wrong direction", as when a short trip *due south* from downtown Detroit leads from US territory (except Alaska) to Canadian territory.

(4.2) ***The trap effect*** occurs when ranking K (or W) higher in some ballots robs K (or W) of the IRV-victory and hands it to M; this is bad for both voter groups.

(4.3) ***The trick effect*** occurs when ranking K (or W) lower in some ballots, takes the IRV-victory away from M and hands it to K (or W); this is good for both voter groups.

When there is a Condorcet winner, these two ways turn out to be the only ways that nonmonotonicity can occur in IRV with 3 candidates, and they clearly belong together.

Tricks and cycles. If we standardize all possible vote vectors, e.g. to p + q + r + s + t + u = 8833 as in Burlington, then they form a grid filling a 5-dimensional simplex:[7] Those vote vectors that allow a nonmonotonicity trick fill 6 convex subsets (polytopes), each with 16 corners. Simulation shows that 2.17% are trick situations; restriction to realistically small T brought the frequency below 1% (Stensholt 2002).

[7] The IAC probability distribution of preference distributions picks a grid point, i.e. a vote vector, with uniform density in this simplex. In the limit, when the number of voters → ∞, this results in a β-distribution for each component of the vote vector (Stensholt 1999).

As indicated in (4.1), all K-supporters are available to perform the pushover trick. Since K gets to the final tally round, the IRV tally does not reveal the numbers of KMW-voters and KWM-voters. However, as the three preference distributions in Figures 3a, 3b, 3c indicate, almost one half of the possible trick distributions are cyclic. *Equality* (here 1876 voters in KMW and in KWM) always gives a noncyclic distribution (Stensholt 2002).

Constellations. In order to investigate nonmonotonicity with 3 candidates, we consider the *constellations* of the two kinds of ranking that are invoked in the IRV-tally. The candidates are ranked according to first ranks as F_1, F_2, F_3, and ranked according to pairwise comparisons as C_1, C_2, C_3. Condorcet cycles are extremely rare in real elections; a reason is visualized Figure 9. Even if a cycle occurs, an IRV tally will not reveal it.

There are six *constellations*, i.e. ways to combine the two rankings, shown in Figure 4. Cycles are more likely to occur when relatively large voter groups agree on both first and second preference.[8] If they occur, the candidates are labelled so that, in pairwise comparisons, C_1 beats C_2 beats C_3 beats C_1, and so that C_3 also is the candidate last in first ranks: $C_3 = F_3$. Thus there are two classes of cycles, to be called *i(cyclic)* and *iii(cyclic)*, illustrated by *i* and *iii* in Figure 4, but with reversed pairwise comparison in {x, z}.

	F_1	F_2	F_3	F_1	F_2	F_3	F_1	F_2	F_3	F_1	F_2	F_3	F_1	F_2	F_3	F_1	F_2	F_3
C_1	x			x				x			x				z			z
C_2		y				z	y					z	x				x	
C_3			z		y				z	y				y		y		
		i			ii			iii			iv			v			vi	

FIGURE 4

Here the candidates are labelled x, y, z according to how they fare in the IRV-tally. Candidate z $= F_3$ is eliminated; x is the IRV-winner, winning after pairwise comparison with y. A cyclic case will look like constellation *i* or *iii*, but z defeats the IRV-winner x in pairwise comparison.

The Burlington election, Figure 2, was in constellation *vi*, with (M, K, W) = (z, x, y). This is the only constellation where Condorcet winner, Plurality winner, and IRV-winner are three different candidates. The hypothetical elections in Figures 3a, 3b, and 3c have, respectively, constellation *iii(cyclic), iii,* and *iii,* with (M, K, W) = (x, y, z).

Theorem 1 establishes that a pushover trick can only start from *iii* or *iii(cyclic)*:

[8] A natural place to search for cycles is in legislatures, when each party coordinates the voting of its members. See e.g. Stensholt (2013). However, because voting usually is done as a sequence of eliminations after pairwise comparisons, or as a sequence of "aye" or "nay" to one bill at a time until an "aye" terminates the process, complete preferences are usually not available to the public, if they are defined at all. Statements from party leaders may be more or less reliable substitutes for missing data.

THEOREM 1 The preference distributions that allow supporters of y or z to turn their favorite into an IRV-winner with any kind of strategic voting, form a subset of all preference distributions in constellation *iii* and *iii(cyclic)*. The only possibility is then that suitably many supporters of y apply the pushover strategy, yielding first rank to z.

Proof: The voters who rank z first cannot change their ballots in a way that prevents elimination of z. We must consider what may be possible for the supporters of y.

The voters who rank y first cannot make y an IRV-winner in constellations *i*, *i(cyclic)*, or *ii* because no change in their ballots can prevent that x, as Plurality winner, thus with more than 1/3 of the first ranks, qualifies for the final round. In order to win, they must ensure that y still qualifies for the final round, but no change in their ballots can prevent that x there will win over y in pairwise comparison.

Moreover, the voters who rank y first cannot make y an IRV-winner in constellations *iv*, *v* or *vi*, because no change in their ballots can change the fact that y is the Condorcet loser and, if promoted to the final round, will lose whether the opponent is z or x.

Only constellation *iii* and *iii(cyclic)* remain. The supporters of y cannot change the fact that a majority prefers x to y. The only possibility is to get rid of x: A suitable number of y-supporters yield first place to z, promote z to the final round and get x eliminated. □

This pushover trick may work for some preference distributions in constellation *iii* or *iii(cyclic)*; Figures 3abc show examples. Obviously, for this to work, x = F_2 must have less than 1/3 of the first-ranks, and y must keep a majority in the pairwise contest vs z.

According to Theorem 1, the pushover participants move from yxz or yzx to zxy or zyx.
If they succeed in eliminating x, it does not matter if they move to zxy or to zyx, but an attempt may fail, and a zxy-ballot can never be better for y than a zyx-ballot, so we only consider moves to zyx. As explained in (4.2), we then concentrate on moves between the neighbor categories zyx and yzx.

A realistic attempt to implement the pushover strategy on behalf of y requires reliable knowledge of the preference distribution and accurate execution. Thus, it will be a risky enterprise. However, if the ballot distribution in an election is a trick situation, a voter group may claim to be victims of a trap effect. No matter how unlikely the claim may be, it is *possibly* true.

Action space for anti-M voters in Burlington 2009. Figure 5 shows all possible switches between KWM and WKM in Burlington 2009 (Figure 2); there are 1795 preference distributions:

(4.3) (|MWK|, |MKW|, |KMW|, |KWM|, |WKM|, |WMK|)

= (994.5, 1559.5, 2327, 655 + h, 1139.5 – h, 2157.5), – 655 ≤ h ≤ 1139.5

The constellation changes when h passes – 428, – 251, 157.5, 743;

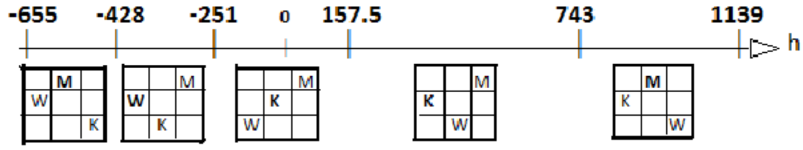


FIGURE 5

When one voter switches from WKM to KWM, h increases by one. There are 5 different scenarios, with tie-breaks required at h = – 428, – 251, 157.5, 743:

– 655 ≤ h < – 428, constellation *iii*; M is IRV-winner; it is a *trick position*: W can let a suitable number of WKM-voters switch to KWM, obtain constellation *v*, and W wins with *pushover.*

– 428 < h < – 251, constellation *v*; W is IRV-winner; it is a *trap position*: if a sufficient number of KWM-voters switch to WKM, and seem to strengthen W, they move back to constellation *iii*; they rob W of the victory and hands it to M.

– 251 < h < 157.5, constellation *vi*; K is IRV winner; h = 0 is the Burlington election; it is a *trap position*, if sufficiently many WKM-voters switch to KWM, and h passes 743, the constellation again is *iii*, but this time the roles for K and W are reversed. The pairwise comparison of K and W is switched at h = – 251; their order in first ranks is switched at h = 157.5.

157.5 < h < 743, constellation *v*; K is still IRV-winner; it is still a *trap position*, but now closer to the brink at h = 743; see (3.4).

743 < h ≤ 1139, constellation *iii*; M is IRV-winner; it is a *trick position*: K can let a suitable number of KWM-voters switch to WKM, and K wins by pushover, i.e. strategy S3, see (1.9). Depending on the number of pushover participants, one may arrive at a *trap position* in constellation *v* or *vi*. The distribution of Figure 3b is indicated by h = 770.

How bad is nonmonotonicity? The concept of nonmonotonicity is linked to a *change* in some of the ballots. What can be detected, is only if a *hypothetical change* in some ballots *could* have caused a nonmonotonic change of the result, either a *trap effect* or a *trick effect*, i.e. pushover. A trap effect creates a trick situation; a trick effect creates a trap situation.

The vote distribution in the Burlington election, shown in Figure 2 and represented by h = 0 in Figure 5, certainly was a trap. But in order to function, the trap required that a catch of at least 743 WKM voters had changed their mind, walked into the trap, i.e. voted KWM, and caused K to lose and M to win. The risk that this would happen after independent decisions seems quite insignificant.

Of the three *hypothetical* situations in Figure 3, 3b could be caused by a trap effect involving h = 770 voters in Figure 2. Only Figure 3c is realistic, close to Perfect Pie-sharing. All three elections, 3abc, are trick positions quite close to the border between M-territory and K-territory.

Natural sortition. Sortition is a planned lottery in an appointment process, e.g. used in the selection of jury members in many countries. The idea is old. Socrates criticized the practice in Athens to appoint archons entirely by lottery (*"election by beans"*).[9] Would he have accepted it if a lottery only influenced the composition of the electorate?[10] The election may then still be deterministic unless it may resort to lottery within the tally, e.g. for tie-break.

Many random events have small last day effects on the sizes p, q, r, s, t, u of the six voter categories in a 3-candidate election. Non-participation is due to accidents in the handling of advance votes, voters' mistakes in preparing their ballots, unexpected events that prevent them from showing up at the voting site, vacillation, or forgetfulness.

Together these and other random events constitute a *natural sortition* in the electorate.

Strategic voting is difficult to organize even with reliable knowledge of the preference distribution among enrolled citizens who intend to vote. Natural sortition makes it even more difficult.

With short distance to the border, natural sortition has played a rôle in landing the election shown in Figure 3c, in the territory of M. In a close race between two or more candidates, random events decide anyway, with any kind of election. Many remember the legal dispute after the presidential election in Florida 2000 (technically for the Florida seats in the Electoral College); the final and official votes became 2,912790 for Bush, 2,912253 for Gore [11] .

The Burlington election, at h = 0 (Figures 5 and 2) was more stable; we may believe that natural sortition did not decide the result. The balance between KWM and WKM may actually change in two directions; the territory of M is reached either at h = 743 or at h = – 428. (However, before crossing at h = – 428, W has taken over as IRV-winner.) Thus, the closest points in M-territory are far away from the Burlington ballot set. They also correspond to pictograms with an unusually large T, as Figure 3b.

IRV protagonists may consider that a potential for nonmonotonic events in an IRV-election is a minor nuisance; it occurs now and then, but if a trap effect realistically may have occurred, natural sortition may be an equally realistic alternative.

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[9] "But assuredly," said the accuser, "he caused those who conversed with him to despise the established laws, by saying how foolish it was to elect the magistrates of a state by beans when nobody would be willing to take a pilot elected by beans or an architect, or a flute-player, or a person in any other profession, which, if erroneously exercised, would cause far less harm than errors in the administration of the state:" and declared that "such remarks excited the young to contemn the established form of government, and disposed them to acts of violence." (Xenophon, *Memorabilia book 1*, 2.9)

[10] Until 1797, the republic of Venice had a long tradition for letting an "Electoral College" elect the Doge; the "Electoral College" was last in a sequence of elected groups. An elected group was reduced by sortition, its remaining members elected a new group etc. (Finlay, 1980; Engelstad, 1989).

[11] Figures from *uselectionatlas.org* .

However, when media report that an election landed on the trick side near the border, one must expect that some voter group gets upset, and that it is not necessarily amenable to an explanation based on natural sortition. Consider the following possibility:

(4.4) ***Example from the border.*** Assume, *counterfactually*, that Figure 3c shows the Burlington election, and that a group of 35 voters writes to the Election Board:

We are active in the political discussion forum "Publius", generally pro-republican. This year, however, due to some special local issues, we agreed that another period for the incumbent, Mr. K, was preferable. So according to our agreement, we sincerely voted KWM instead of WKM.

However, if we should believe assertions from political commentators in the days after the election, our move from WKM to KWM has caused Mr. K to lose the election. Moreover, equally mindboggling, it has caused our last ranked candidate, Mr. M, to win.

We find it very hard to believe that our move from WKM to KWM can have had these two consequences, but the assertions have reached the media and not been retracted. Our interest is politics, not election rules, but these persistent claims are too disturbing.

We are neither qualified to understand a technical explanation, nor interested in it.

We only ask for a simple clear answer to a simple clear question. If 35 of the KWM-ballots instead had been WKM-ballots, would then Mr. K have won the IRV-election?

Dear reader. You are the Chair of the Election Board. How do you answer?

V THE NO-SHOW PARADOX

(5.1) ***Definition*** Let candidate A be the winner of a preferential election. Suppose counterfactually that a group of would-be voters who actually stayed home, instead had shown up at the polls and delivered ballots with candidate A ranked before candidate B, and that their participation then caused B to win instead of A. According to their own planned voting, they were all better off by not showing up at the polls. This is the *No-Show Paradox*.[12]

(5.2) ***Example*** All three elections of Figure 3 allow the No-Show Paradox. The constellation is *iii* in Figure 4 or *iii(cyclic)*, with (M, K, W) = (x, y, z). Assume that k potential voters planned to vote WMK, thus with candidate K ranked after the IRV-winner M, but that they actually stayed home. If they had participated, the first ranks would have become

K: 3752; M: 2554; W: 2527+k

Figure 6 shows the situation after k = 1200 new WMK voters join the election of Figure 3b. Now assume 27 < k < 1790. Then there is a No-Show Paradox: Since k > 27, participation will cause elimination of M. In the final round, the pairwise comparison will be:

K: 5311.5; W: 3521.5 + k

Since k < 1790, participation of k new voters with WMK-ballots would have caused K to win instead of the original IRV-winner M; by not showing up, they kept M as a winner, which is a better result according to their own WMK-ranking.

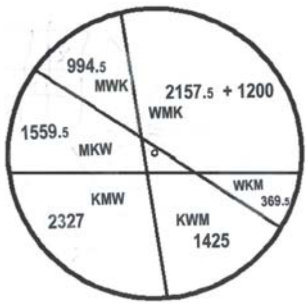

FIGURE 6

T covers a fraction 0.01061401 of the circle area. The pictogram shows the election after addition of k = 1200 WMK-ballots to the imagined election in Figure 3b. There is a cycle: With "rotating majorities", M beats K beats W beats M. After elimination of M, K is the new IRV-winner. Many WMK-voters may have a strong preference for M in pairwise comparison with K. However, a more efficient strategy than not participating is to compromise and support M by voting MWK, i.e. to apply strategy S1, see (1.7).

It is practical to extend the range of k and include all sizes of the WMK-category:

[12] If the unfortunate new voters gave A their first rank, the No-Show Paradox is of the *strong* kind; otherwise it is of the *weak* kind. It follows from Theorem 2 that only the weak kind is possible in IRV.

$$-2157.5 \leq k < \infty$$

Thus, k + 2157.5 is the number of WMK-voters. Figure 7 is similar to Figure 5, and shows where the constellation changes. The No-Show effect starts at k = 27:

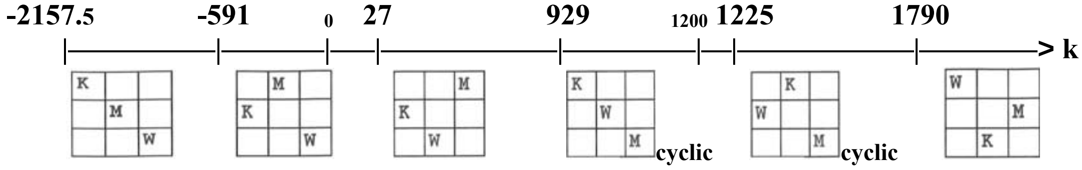

FIGURE 7

Figures 3b and 6 correspond to k = 0 and k = 1200. The constellation changes 5 times as new voters join the election in Figure 3b, vote WMK and increase k. The No-Show Paradox starts at k = 27 and ends at k = 1790. Depending on how many new "reckless" WMK-voters that enter in Figure 3b (constellation *iii,* k = 0), and cause the paradox, the result may, in this case, be in constellation *v*, *i(cyclic)* or *iii(cyclic)*. Obviously, W will eventually win (k > 1790) but the progress is slow:

k = – 591, M passes K in pairwise comparison;
k = 27, W passes M in first ranks (W qualifies for the final and spoils for M);
k = 929, W passes M in pairwise comparison (a cycle is formed);
k = 1225, W passes K in first ranks (the cycle persists);
k = 1790, W passes K in pairwise comparison (the cycle ends).

(5.3) ***Definition***. A method for preferential election satisfies the *participation criterion* if it does not allow the No-Show Paradox:

A voter can never get a worse result, according to the voter's ballot, by participating than by not participating.

Example (5.2) shows that IRV does not satisfy the participation criterion.

What are the preference distributions that may allow violation of the participation criterion in IRV? Let x, y, z be defined by the IRV tally before the new voters are added, i.e. z is eliminated and x is IRV winner. For a 3-candidate IRV-election, the possibilities are as in the next result.

THEOREM 2 The preference distributions that allow new voters to be added to one of the six voter categories and cause a candidate whom they the rank after the IRV-winner x to become new IRV-winner, form a subset of all preference distributions in constellation *iii* and *iii(cyclic)* in Figure 4. The only possibility is then that the new voters have preference zxy.

Proof: We first establish that the new voters must have preference zxy. The new voters cannot give first rank to IRV-winner x, because with higher margin over z than before, x would qualify for the final round, and then win against y with higher margin than before.

They cannot rank x last either, because then there cannot be a new winner whom they rank *after* x. Thus the extra voters must give x second rank and vote either yxz or zxy.

If they vote yxz, then z still is last ($z = F_3$), and the new winner is not the one they rank after x. Therefore, the only possibility is that they vote zxy, eliminate x, and that y becomes new IRV-winner.

In what constellations from Figure 4 may additional zxy-ballots change the IRV-winner from x to y? In constellations *i*, *i(cyclic)*, *ii*, and *v*, x is ahead of y in first ranks, and cannot possibly be eliminated.

In constellations *iv* and *vi* y is Condorcet loser ($y = C_3$) and cannot possibly win in the final tally round. Thus only *iii* and *iii(cyclic)* remain. □

In both constellations, *iii* and *iii(cyclic)*, this may actually happen; see example (5.2).

How bad is the No-Show Paradox? If the k potential voters in example (5.2), $27 < k < 1790$, had shown up and voted WMK in any of the 3-candidate IRV-elections in Figure 3, they would have been unfortunate and caused K to snatch victory from M.

This popular dramatization of the No-Show Paradox may have a didactic point, but with an unfortunate side effect: it suggests that the natural remedy would be a special kind of strategy, i.e. that more than k–27 out of k potential WMK-voters stay home.

(5.4) ***Example*** The arrival order at the polling site is of course immaterial; and it is artificial to analyze the election of Figure 6 with a focus on 1200 WMK voters. One may instead remove 1200 KWM-voters in Figure 6, and then add them in, one by one, running through the following preference distributions, $0 \leq g \leq 1200$:

$$(|MWK|, |MKW|, |KMW|, |KWM|, |WKM|, |WMK|) =$$
$$(994.5,\ 1559.5,\ 2327,\ 225+g,\ 369.5,\ 3357.5)$$

The starting point, g = 0, is a realistic preference distribution. In its pictogram, T covers 0.00003355 of the circle area. At g = 0, K gets eliminated, and the Condorcet-winner M also becomes IRV-winner. The new KWM voters cause a normal change: K wins instead of M. Nothing funny happens on the way from M-territory to Figure 6.

What the preference distributions for $27 < k < 1791$, Figure 7, have in common, is that the WMK-category has an unfortunate size for its own members: It is too small to make W an IRV-winner, but large enough to spoil for M. They may win with the perfectly normal strategy of *compromising*: If $k > 27$ new WMK-voters show up in addition to the preference distribution of Figure 3, it is enough that more than (k-27)/2 voters switch from WMK to MWK; then they obtain that M becomes IRV-winner.

Compromising is a much more efficient strategy than non-participation. Arguably, participation and compromising serve democracy better than non-participation does. For these reasons, the strategy of staying home does not have independent interest in 3-candidate IRV.
Moreover, there are preference distributions which give a voter group incentive to compromise, but which could not possibly be the result of a No-Show drama:

(5.5) ***Example*** The real Burlington election in Figure 2 is an example: W spoils for M, but to turn M into an IRV-winner, it suffices that 372 voters compromise and switch from WMK to MWK. However, if we remove WMK-voters one by one, and illustrate the same way as in Figure 7, we never reach constellation *iii*. Thus, by Theorem 2, reckless new WMK-voters performing the No-Show Paradox cannot have caused the preference distribution in Figure 2.[13]

Preference distributions where one voter group may gain through compromising, under IRV, may come about in different ways. The Burlington election, Example (5.4), could not be due to the No-Show Paradox. In theory, the hypothetical situation of Figure 6 can be due to the No-Show Paradox. However, there is a different explanation in Example (5.4). The drama version is entertaining, but to focus on the arrival order of the voters is artificial.

The problem in the Burlington preference distribution of Figure 2 is how to make WMK-voters aware that they have a problem: Should they submit an *"expressive"* ballot WMK or compromise with an *"instrumental"* ballot MKW? Perhaps pre-election polls in Burlington 2009 might have established how far W was from election, and, in fact, was a spoiler for M. Then voters with expressive ranking WMK and a strong wish to avoid K might have compromised and got M elected.

[13] Gierzynski (2011), states on page 165-166: *"The election* [Burlington 2009] *also featured a* no-show paradox *(supporters of the Republican candidate would have served their own interests better by not voting) …"*

That this is false is visible in Figure 2. Reckless WMK voters could not possibly have caused K to snatch victory from M. They could not have arrived at a stage where M was IRV winner: Unless removal of W-supporters give space for M in the final, they will not get a better result. Thus, we must remove at least 743 W-supporters. Among them, only the WMK-voters will think that a final round without W can give a better winner than K. However, if the category WMK is reduced by >743, so that M qualifies for the final round, then M is no longer Condorcet winner, and K wins over M after the removals.

VI COARSENESS VERSUS CRUELTY IN PREFERENTIAL ELECTIONS

Nobody minds coarseness but one must draw the line at cruelty, said Lord Peter Wimsey, the amateur detective created by Dorothy L Sayers. A simple procedure may well lead to coarseness only. Mindless sophistication may result in cruelty – in election methods as in the world of Lord Peter.[14]

Consider a race between center-left candidate L and center-right candidate R, where R is about to win with 51.6% to 48.4%. Example (6.1) shows what may happen, under the Plurality rule and under the Borda Count, when a third candidate enters far from the political center, outflanking R or L. We assume that most voters have the same perception of the left-right structure, so that the Perfect Pie-sharing model fits reasonably well.

(6.1) ***Example*** Locate two "pragmatics", L and R, respectively at (– 0.2, 0.25) and (– 0.2, – 0.2) in the unit circle. In pairwise comparison R gets 516 of 1000 votes, L gets 484. With just two candidates, the Plurality method and the Borda Count agree: R is the winner.

In a Plurality election, R may lose to L if a third candidate R' (far right) joins the election; in the pictogram of Figure 8a, R' is placed at (0.3, – 0.8). The vote vector becomes

(|LR'R|, |LRR'|, |RLR'|, |RR'L|, |R'RL|, |R'LR|) =

(5 , 472 , 193 , 92 , 231 , 7)

The presence of R' takes 231 first ranks away from R and only 7 from L; L becomes clear Plurality winner. However, the right side itself caused it. The Plurality method gives a coarse message: Stick together or lose!

But in a Borda Count, R may lose to L if a third candidate L' (far left) joins the election; in the pictogram of Figure 8b, L' is placed at (0.3, 0.8). The vote vector becomes

(|LL'R|, |LRL'|, |RLL'|, |RL'L|, |L'RL|, |L'LR|) =

(87 , 179 , 494 , 9 , 13, 218)

The presence of L' causes 87 L-supporters to vote LL'R. Whatever reason there was for nominating L', in effect they get a *double vote* in the pair {R, L}. The Borda Count gives a cruel message to the right: Either bury L, vote RL'L and report wrongly your opinion in {L, L'} or enter a fourth candidate R' to match L' in a rôle similar to that of a *domestique* in a bicycle road race. Both of these actions mean to follow up in a *"chicken game"* initiated with the entry of L'.

[14] Sayers: "The Abominable History of the Man with Copper Fingers", in *Lord Peter views the Body* (1928).

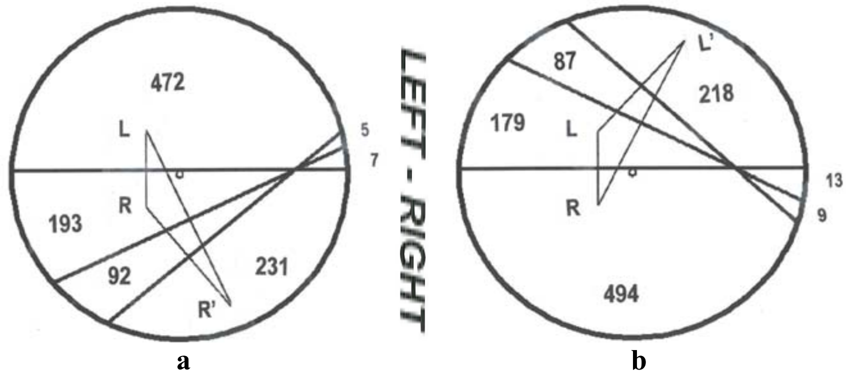

FIGURE 8

L at (– 0.2, 0.25); R at (– 0.2, – 0.2); L' at (0.3, 0.8); R' at (0.3, – 0.8)

R wins against L in pairwise comparison, 516 to 484.

In election 8a, L, R, R' have, respectively, 477, 285, 238 first ranks; R' outflanks R, and then L *becomes Plurality winner*.

In election 8b, L' outflanks L and the Borda Count is:

R: (494 + 9)·2 + (179 + 13) = 1198 Borda points;

L: (179 + 87) ·2 + (218 + 494) = 1244 Borda points;

L': (13 + 218) ·2 + (9 + 87) = 558 Borda points.

The Borda Count gives the 87 LL'R-voters a double vote for L over R; only 9 RL'L-voters get a double vote for R over L, and *L becomes Borda-winner*.

Moreover, Example (6.2) shows how a third candidate may also cause the change of Borda-winner through a *"clone effect"*. In general, k candidates form a set of *clones* if they occupy k consecutive ranks in every ballot (Tideman 1987).

(6.2) ***Example*** Sports journalists elect their country's "athlete of the year". Comparison of candidates in the same sport may be objective, based on results. Comparison of candidates from different sports is much more demanding and cannot be equally fine-tuned. Candidate A in sport I has 60% of the first-ranks, and B in sport II has 40%. However, in everybody's mind, C in sport II is very close to B, just slightly behind B according to objective results. Thus, every ballot is either ABC… or BCA…; the Borda Count gives every B-supporter a double vote in the pair {A, B} and B becomes Borda-winner.

There is no strategic voting in Example (6.2) and no agenda manipulation (strategic nomination) either. However, the Borda protagonist Michael Dummett (1998) was still concerned about the *(dis)similarity effect*, which may also come through agenda manipulation: A loses to B because of dissimilarity between a small group of candidates similar to A and a larger group of candidates similar to B. Clones form a special case suited for theoretical study. Moreover, the pair of clones {B, C} in Example (6.2) is extreme in the sense that B gets the maximal advantage of the clone effect. Example

(6.2) also emphasizes that the dissimilarity effect may very well occur naturally, albeit usually not in the pure and easily recognizable form of cloning.

Borda's result in Example (6.2), BAC…, should be compared to Condorcet's result, ABC… . At the time of Borda and Condorcet, the discussion seems to have been over the kind of strategic voting that we now call burying, not over strategic nomination or the natural occurrence of outflanking or (dis)similarity. Of course, according to Arrow's IIA-axiom, C should be irrelevant to A v B.

Because of the effects in Example (6.1) and (6.2), the Borda Count is a prime example of cruelty in preferential elections. Therefore, it is important to understand it well. The Borda Count lines up the voters for an unavoidable "chicken game", which may already have begun with the nomination, and which will continue if they make use of the multiple votes that they will obtain with burying. Voters who were not aware of the game – or recognized it but chickened out – must accept a loss due to the winner's advantages in outflanking, in (dis)similarity, or in combative supporters.

On Borda's suggestion, the French Academy of Sciences for a while used the controversial Borda Count for its election of new members. The combined efforts of two other prominent members caused its repeal: Pierre-Simon de Laplace and his younger associate Napoleon Bonaparte (Szpiro 2010).

The immediate appeal of the method that Borda proposed in 1770, should not be underestimated, and neither should the enhanced appeal that follows from conspicuous claims of the method's use in our time. According to the Eurovision, the Eurovision Song Contest 2014 reached 195 million viewers. In particular, statements like the following flourish: *"The Eurovision Song Contest also uses a positional voting method similar to the Borda Count, with a different distribution of points."* Anybody can still (2015) check the web and make an anthology of such statements.

However, these statements are false: They dress up the Borda Count in borrowed feathers. The difference noticed in the quotation is just the one between the Borda *scale* (n-1, n-2, …, 1, 0) and the Eurovision *scale* (12, 10, 8, 7, …, 1, 0, 0, …, 0).

The Eurovision *does not* use a positional method at all. The 195,000,000 viewers watched the results from the national, intermediate tallies and the attachment of Eurovision points to each national *tally ranking*. In a positional method, like Borda's, the points are attached to the *ballot ranking* from each single voter. A Eurovision member nation lets its "tele-voters" support one single candidate song, as in the Plurality election. Obviously, in the Eurovision Song Contest, the possibilities for strategic voting are *not* as in Borda-like methods.

The derived methods: STV and Condorcet. Despite the coarseness of the Plurality Method and the cruelty of the Borda Count, each of them is a natural basis for one important family of preferential election methods. Many of these methods are in practical use.

IRV is a "transformation" of the Plurality Method. It eliminates repeatedly the candidate last in first ranks. A faster variation eliminates immediately all but the two best candidates, and it actually simulates a 2-day election for an arbitrary number of candidates.[15] The same transformation idea is behind all STV-methods (Single Transferable Vote). These are methods are to elect candidates for s seats, s > 1; IRV is the single-seat version of STV. Like Plurality, STV and IRV emphasize the importance of high ranks, but does so in a gentler way.

A similar transformation of the Borda Count, eliminating repeatedly the candidate last in Borda-points, is the *Baldwin* method. The *Nanson* method is a faster variation that eliminates repeatedly all candidates not above average Borda-score. The well-known central fact is that a Condorcet winner always gets above average Borda-score and avoids elimination.[16] Thus, the methods of Nanson and Baldwin belong to the family of Condorcet methods; i.e. they elect the Condorcet winner if one exists. Like Borda, also the Condorcet methods emphasize the importance of the voters' ranking in every candidate pair {X, Y}. However, Condorcet is much less burdensome for the voters: Notice e.g. that neither the outflanking L' in Example (6.1) nor the clone C in Example (6.2) will give a new Condorcet-winner.

In real elections with many independent voters, it is extremely rare not to have a Condorcet winner.[17] Generally, voters regard a Condorcet winner as the candidate closest to the center; see Figures 1 and 2 for a candidate triple. Condorcet methods give every candidate an obvious incentive to appear as the one closest to the political center, and competition may result in a Condorcet winner with very low primary support. It is easy to concoct preference distributions (with four and more candidates) where the Condorcet winner does not get any first rank at all.

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[15] This is *the Contingent Vote*, which simulates a 2-day election with n candidates: Only the two candidates with most first ranks advance to the second and last tally round. In comparison, IRV tallies much more ballot information, and still saves time and expense.

[16] Reason: v voters give vn(n-1)/2 Borda-points to n candidates, one point from voter i to candidate A for every pair {A, X} where i prefers A. Thus, on average, candidates receive (n-1)v/2 points, but in each of n-1 pairs, a Condorcet winner is preferred by more than v/2 voters.

[17] In an election with fifteen candidates to the Wikimedia Foundation Board of Trustees 2008, four candidates formed a cycle, i.e. two overlapping cyclic triples. Thus 2 of the 15▪14▪13/6 = 455 triples were cyclic. The closest pairwise win in the quartet, 745 – 737, was common to both cyclic triples; a reversal in this pair would give a transitive tally preference. All four lost to the five first candidates, and won against the last six. Similar observations suggest the hypothesis that voters reduce their effort in ranking the middle range of candidates. See https://meta.wikimedia.org/wiki/Wikimedia_Foundation_elections/Board_elections/2008/Results/en

Burlington 2009 shows a real IRV-tally eliminating a Condorcet winner with 29% of the first ranks. However, this reminds of another crucial fact: The center voters got a correspondingly strong impact through their subsidiary votes. In IRV, wing candidates cannot afford to disregard center voters.

Some properties to consider. A first step in the choice of a method for preferential election may be between the STV-family and the Condorcet-family; what kind of winner does best serve the purpose of the election? How should a committee rank candidates for a job in a university department? Should all, some, or none of the members of a legislature be Condorcet winners or, perhaps, IRV-winners, in single-seat constituencies?

A second step may be to consider what properties various methods in the family have. What incentives does a method give to the candidates and their parties? It is convenient to focus on the conditions for strategic voting, e.g. as in Theorem 1; a missed opportunity for strategic voting (i.e. a trick situation) may *possibly* be due to a trap effect. Although events like example (4.4) may be extremely rare, one should assess their effect on the voters' trust in IRV.

Burying does not work in the usual STV-methods. With Condorcet, the theoretical possibility of successful burying depends very much of the particular method chosen (Stensholt 2013). In a situation with a Condorcet-winner, burying involves the creation of a cycle, but an attempt to organize burying will probably be hazardous.

Nonmonotonicity is a central theme in the struggle over IRV. This fact enhances its importance, particularly in the political arenas, as shown in the repeal vote in Burlington 2010.

"Conditional IRV" may be a worthwhile modification (Stensholt 2010): Tally round 1 ranks the candidates according to their first-ranks. Let $F_1, F_2, F_3, \dots F_n$ receive $v_1 > v_2 > v_3 \dots > v_n$ first-ranks. The rule for conditional IRV is:

(6.3) *If* $(v_1 - v_2) < (v_2 - v_3)$, *then have a final round with* $\{F_1, F_2\}$, *otherwise declare* F_1 *as winner*.

The idea is that F_1 cannot afford the nonmonotonicity trick if condition (6.3) is satisfied. By yielding first ranks to someone in order to eliminate F_2, F_1 would drop to third place or worse. Without trick situations, there cannot be trap situations. Neither burying nor pushover works.

In Burlington 2009, K would have won also with conditional IRV; the first round votes were 2951, 2585, and 2063 for respectively, W, K, and M. Clearly, Conditional IRV simulates a 2-day election if (6.3) is satisfied, and otherwise becomes a Plurality election.

With Conditional IRV, there is of course a risk that the voters' subsidiary rankings are not tallied at all. Voters, who have spent time pondering their ballots, may resent this. Perhaps, like Plurality, Conditional IRV will give a stronger incentive for compromise and a development towards two major parties, so that a normal tally will include a final between F_1 and F_2. Anyway, it seems likely that on average, F_2 will be a tougher challenger for F_1 when condition (6.3) is satisfied than when it is not.

However, voters with an ideological attachment do not easily switch from *"expressive"* to *"instrumental"* modus. In a 2-day election, the primary on day 1 is much coarser than the gentle sequence of transfers in IRV after single eliminations. A subset of candidates, that are politically close, may all be eliminated on day 1, while ordinary IRV may let their supporters gather around one candidate who reaches the final tally round.

(6.4) ***Example*** In the French presidential election 2002, the top three in the primary on day 1 were:

Chirac 19.88 %; LePen 16.86 %; Jospin 16.18 %.

The runoff on day 2 gave

Chirac (center-right) 82.21 %; LePen (far right) 17.79 %.

Despite clear warnings from pre-election opinion polls, supporters of the many parties of the left voted overwhelmingly according to their expressive rankings and self-destructed on day 1, failing to challenge the incumbent Chirac with a feasible candidate. Jospin was prime minister, and actually had opposed Chirac in the runoff of the previous presidential election, in 1997.

Practical modifications of the modification (6.3) may be either to follow ordinary IRV until 3 candidates remain or run an STV-election for 3 seats, and then switch to Conditional IRV. There will certainly exist preference distributions that allow pushover. Perhaps simulations can indicate if there is any reason to be concerned.[18]

Because of the elimination rule, nonmonotonic changes may occur also in Baldwin's method (with 3 or more candidates) and in Nanson's method (with 4 or more candidates), but many Condorcet methods are monotonic.

A third step might concern the No-Show, but this is mainly of theoretical interest.[19]

Sincerity in voting. Too often, the expression *"insincerity"* is a synonym for strategic voting. However – in what matters should a "sincere voter" be sincere?

[18] A possible analogue to Condorcet methods, with pairwise comparisons as guiding principle, may be first to tally all triples with IRV or Conditional IRV, and investigate various possibilities for aggregating the outcomes.

[19] As argued above, the No-Show paradox should not be seen as a practical problem, at least not in ordinary 3-candidate IRV. However, Moulin (1988) has shown that all Condorcet methods, with at least 4 candidates, also allow the No-Show paradox in some situations. In Condorcet methods, No-Show has to do with the existence or creation of cycles, and the essential part of Moulin's proof is a delightful combinatorial artwork, a very special preference distribution with two overlapping cyclic triples.

In a Plurality election for a public office with just one candidate pair {N, G}, a voter, right or wrong, may vote for candidate N, *sincerely* thinking than N is better qualified than G for the office. However, if the set of candidates is extended to a triple {N, G, B}, then the same voter may *sincerely* switch ranking from NG to GNB and vote accordingly.

The reason is that the presence of B creates a *feasibility criterion* for the ranking: Our voter considers that both N and G are significantly better qualified than B for the office, but also that G and B are the only feasible candidates, i.e. who have a realistic chance to win. A switch of preference within {N, G} *is sincerely meant*; the voter assesses that G is better qualified than N to gather enough support to win the election.

Our voter would rank N before G if {N, G} were the full candidate set, G before B if {G, B} were the full candidate set, and N before B if {N, B} were the full candidate set: The voter then has an *expressive ranking* NGB. However, because of the feasibility criterion, the same voter has an *instrumental ranking* GNB in the candidate triple {N, G, B}. The decision, instrumental or expressive *voting*, may cause anguish, but anguish does not diminish sincerity. Here it is a sign of sincerity.

Voting according to instrumental ranking, i.e. compromising, is the mechanism behind Duverger's law: *Plurality elections tend to favor a two-party system*. The voters' behavior in the US presidential election in 2000, with {N, G, B} = {Nader, Gore, Bush}, has gotten an almost archetypal status in the field of Social Choice. Of course, there were also voters with expressive ranking NBG and instrumental ranking BNG. Voting according to instrumental ranking helped Gore much more than Bush, but not quite enough to make a difference.

When a Plurality election is a close race between two "front runners", the difference between instrumental and expressive ranking may become relevant already when a third candidate has quite small primary support. Small emerging parties may easily be "nipped in the bud" because many voters *sincerely* change their ranking from expressive to instrumental, and vote accordingly.

With IRV in the US presidential election of 2000, the N-supporters could very well have voted according to their expressive rankings NGB or NBG, and after elimination of N they would have been tallied for their preferred feasible candidate. With IRV, a candidate from a minor party will get eliminated, but larger parties become rivals for subsidiary support.

Thus, with IRV small parties may grow. But when a third party becomes strong enough to challenge a front runner for promotion to the final pair, then changes from expressive to instrumental ranking become relevant. This happened in Burlington 2009: If 25% of the WMK-voters in Figure 1 had recognized how far the Plurality winner W was from becoming IRV-winner, they might have chosen the instrumental ranking MWK, compromised and let M become IRV-winner. Of course, it is likely that some of the 767 MWK-voters in Figure 1 actually had switched from expressive WMK to instrumental MWK.

There is a practical difference between compromising under Plurality and under IRV. In a Plurality election, the relevance of compromising is obvious at an early stage: Those who preferred N and considered whether to compromise could be certain that N had no realistic chance to win. When compromising becomes relevant in IRV, the picture is less clear, but Example (5.5) shows, in hindsight, that a relatively small fraction of the WMK-voters could have compromised and gotten M as mayor instead of K. It seems likely that the category WMK contained enough voters not firmly attached to W, i.e. voters who might have compromised without much anguish. However, W in 2009 did not give the same immediate appearance of infeasibility as N did in 2000.

Strategy – by sincere voters. Strategic voting is more likely due to fear than to sinister scheming. Victims of a missed opportunity for burying or pushover may perceive it as a punishment for being naïve. However, in the Gibbard-Satterthwaite impossibility theorem, the question of sincerity is irrelevant. Its definition of strategic voting is very wide and entirely technical:

(6.5) ***Definition****. A voter's change from one ranking, where A ranks before B, to a new ranking, is* ***strategic*** *if it changes the winner from B to A, an improvement in terms of the original ranking.* [20]

Too often, a wrong interpretation of the Gibbard-Satterthwaite theorem claims it says that, for any method of preferential election, conditions exist under which a voter *would have an incentive to vote in a manner that does not reflect his or her preferences,* unless there is a dictator or some other unpalatability.

Obviously, the definition (6.5) includes burying and pushover, possibilities that are generally unwanted. However, in this wide definition, also compromising gets the status of "strategic voting",

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[20] In the main types of strategic voting, (1.7) – (1.9), no voter changes the internal ranking in {A, B}. This shows that the election method does not satisfy Arrow's IIA-axiom. The Condorcet method BPW allows very few opportunities for burying a Condorcet winner. However, in a cycle, it may allow a type of strategic voting which exploits nonmonotonicity, but still does not violate IIA. Example 8.3 in Stensholt (2013) is from a voting in the Norwegian legislature 1992, with (estimated) vote vector

$$(|FGH|, |FHG|, |HFG|, |HGF|, |GHF|, |GFH|) = (0, 42, 22, 37, 1, 63).$$

Here is a cycle HGFH with large pairwise wins. In a 3-cycle, the BPW-winner is the alternative that beats the Plurality winner in pairwise comparison: here H beats Plurality winner G and becomes BPW-winner. A transfer of t ballots from FHG to HFG, $5 < t < 23$, lets the cycle persist. It makes H Plurality winner. Thus, it makes F BPW-winner. Moreover, if H "returns the favor", changes t ballots from HFG to FHG, and thus restores the 3-candidate distribution above, then the BPW-winner changes back from F to H. The point of IIA is that only F and H change positions in some ballots and only F and H change positions as BPW-winner. Of course, IIA is not satisfied (Arrow's theorem). However, this particular strategy does not demonstrate it. Actually G won in 1992 after a sequence of "ayes" or "nays" in a field starting with five alternatives.

but then in a purely technical sense. Definition (6.5) is important as a suitable basis for a chain of reasoning that leads to the Gibbard-Satterthwaite impossibility theorem.

In particular, definition (6.5) includes a sincere switch from expressive to instrumental ranking: Then the voter does not change any underlying "true" ranking. It seems unlikely that it would serve any purpose to restrict the definition (6.5) to cases where the original ranking was "sincere", while the new one is not. As emphasized by Dowding and Van Hees (2008) there is a difference between *"sincere manipulation"* and other manipulation.

In preferential elections, a voter is sincere *unless* the ranking in at least one candidate pair in the ballot is not the true ranking. The feasibility criterion then puts compromise in a different category than burying or pushover. When voters move from ABC to BAC in order to have B elected instead of C, their change is in {A, B}. The participants switch from their expressive to their instrumental ranking. They find that B has a relevant advantage over A, and they sincerely prefer B for first rank.

For contrast, consider burying and pushover:

When voters move from ABC to ACB in order to help A against the main competitor B by burying B, then they also change their ballot in one pair only: {B, C}. But they do not sincerely prefer C to B. It only appears that way; this appearance is a price they must pay for giving efficient support to A. It is a major advantage of IRV that burying never works; IRV never burdens a voter with this cruel dilemma.

When voters move from ABC to BAC in order to help A against the main competitor C by pushover of B, then they change their ballot in one pair only: {A, B}. But they do not sincerely prefer B to A; the pushover trick may be seen as a price for giving efficient support to A. However, if they overdo it and cause B to win instead of C, they have in effect had success with compromising. Whether it is A or B who takes over for C, both ABC-voters and BAC-voters have gained. However, the trap effect is unfortunate. Anyway, nonmonotonicity will disappear from IRV with the modification (6.3), i.e. Conditional IRV.

Voter's burden; a legal problem. Is IRV unconstitutional? Minnesota's Supreme Court treated this question in 2009, in the case

Minnesota Voters Alliance, et al., (appellants) v
The City of Minneapolis, et al., (respondents)

In a referendum, Minneapolis had adopted IRV for its municipal elections. *FairVote Minnesota, Inc.* was intervenor-defendant. FairVote works for the introduction of STV-methods in American elections. Below are some aspects of the ruling that concern IRV and some arguments raised against it.

The challenge that the adoption of IRV/STV was unconstitutional was a *facial* challenge, not just an *as-applied* challenge; if successful, the appellants would have struck out all intended use of the method. This may look like an uphill fight, but the appellants claimed there was a *precedent* in the same court from 1915, in the case *Brown v Smallwood.*

In that case, the state's Supreme Court declared another voting method, adopted in Duluth, to be unconstitutional. In 2009, the sides claimed similarities or differences between the two election methods to be relevant facts. The Duluth method belongs to the family of Bucklin methods; it is neither an STV-method nor a Condorcet method. The Duluth rules appear from the tally reported in the Supreme Court decision from 1915:

Duluth 1915	First Choice	Second Choice	1st&2nd Choice	Add'l Choice	1st, 2nd & Add'l Choice
Louisell	992	734	1,726	402	2,128
Norton	3,417	1,501	4,918	167	5,085
Smallwood	3,496	2,845	6,341	240	6,581
Windom	4,408	604	5,012	54	5,066
	12,313	5,684	17,997	863	18,860

A voter made one first choice, had the option to name another candidate for second choice, and also to add more names for additional choice. The tally was in 3 rounds; no candidate got ½ of the 1st choices; no candidate got ½ of the 1st & 2nd choices, and finally Smallwood won with a plurality of the 1st, 2nd & additional choices. The table does not show, e.g., how many of the 2,845 2nd choices for Smallwood that were combined with 1st choice for Louisell, Norton, or Windom, respectively.

Central to the court's judgment in 1915 was its understanding of the state's Constitution from 1859. The Constitution was not explicit on voting methods, but the majority of the court took judicial notice of the meaning of the word "vote" fifty-six years earlier, before the "Progressive era" with its many electoral reforms:

> *When the Constitution was framed, and as used in it, the word "vote" meant a choice for a candidate by one constitutionally qualified to exercise a choice. Since then it has meant nothing else. It was never meant that the ballot of one elector, cast for one candidate, could be of greater or less effect than the ballot of another elector cast for another candidate. It was to be of the same effect.*

The court went on to point out that, with the method under consideration, a voter may harm the first-ranked candidate with a second rank to another candidate, etc. From the court's reasoning, the continuation of the quotation above is relevant in our context, and in particular the last sentence:

> *It was never thought that with four candidates, one elector could vote for the candidate of his choice, and another elector could vote for three candidates against him. The preferential system*

[the Duluth method] *directly diminishes the right of an elector to give an effective vote for the candidate of his choice. If he votes for him once, his power to help him is exhausted. If he votes for other candidates he may harm his choice, but cannot help him.*

IRV, which came under consideration 94 years later, is designed so that a ballot's second choice cannot harm the ballot's first choice. However, in the pair {Windom, Smallwood}, a first choice for Windom in a ballot would, with the Duluth system, be annulled in tally round 2 by a second choice for Smallwood in the same ballot. Like the Borda Count, Bucklin methods are designed not to consistently respect a ballot ranking of the candidates; the result depends only on the *number* of first ranks, the *number* of second ranks, etc., that each candidate receives.

In 2009, the same court summarized its finding in a syllabus:

Instant Runoff Voting as adopted in Minneapolis is not facially invalid under the United States or Minnesota Constitution, and does not contravene any principles established by this court in Brown v. Smallwood, 130 Minn. 492, 153 N.W. 953 (1915)

The court considered "*the burdens that appellants contend IRV imposes on the right to vote"*. In particular, the eliminations in IRV meant that no ballot counted as supporting several candidates at the same time. The court in 2009 also compared IRV to the 2-day election, which had been used in Minneapolis before IRV was adopted, and still was the alternative. Obviously, the 2-day election has many features in common with IRV, but it was not under attack.

However, nonmonotonicity was a new legal problem, not considered in 1915. The last of the claimed burdens on the voter concerned nonmonotonicity, i.e.

▪ *by creating the possibility that casting a vote for a preferred candidate may harm the chances for that candidate to win office.*[21]

Moreover, a 2-day election may allow a pushover trick, based on an *expectation* of how the votes will be on day 2. Thus, in Example (6.4), LePen passed Jospin on day 1. Possibly, some supporters of Chirac contributed to this. However, pre-election polls showed a lead for LePen anyway, so there is no reason to think that a 2-day variety of pushover decided the election, but the figures illustrate the possibility. The court remarks:

Appellants' response fails to address the candidate-elimination function of the nonpartisan primary. It is at that stage that the primary/general election system is non-monotonic. This is illustrated by the fact that in some circumstances, a voter can increase her preferred candidate's chance to win office by voting in the primary for a non-preferred candidate who

[21] The context indicates that the court had the nonmontonicity trap effect in mind, but the trap effect requires more than *"casting a vote for the preferred candidate"*; the essential change is that number two in first ranks loses support and the Condorcet winner qualifies for the final tally round. The quoted formulation actually fits with the strong version of No-Show Paradox, which, however, does not occur in IRV.

would be a weaker opponent for her preferred candidate. By helping the non-preferred, but weaker, candidate succeed in the primary, the voter can help her preferred candidate win the general election.

The court might also have pointed out that a 2-day election actually gives *better conditions* for a trick similar to pushover than IRV does.[22] If some Chirac supporters had voted for LePen on day 1, they would have been able to vote for Chirac on day 2, while in IRV their ballots would have counted for LePen also in the final round.

In most civilian cases, courts are limited to treat claims and evidence brought in by the parts. , The Minnesota Supreme Court found that the appellants had not submitted sufficient evidence:

Although it is apparently undisputed that the IRV methodology has potential for a non-monotonic effect, there is no indication, much less proof, of the extent to which it might occur, and so there is no way to know whether the alleged burden will affect any significant number of voters. Accordingly, appellants have not established that non-monotonicity imposes a severe burden on the right to vote.

From the ballots one can only see if the preference distribution was a possible start or end point for a nonmonotonic change: Theoretically, the Burlington election of March 3rd could be due to a pushover trick where 770 voters (say) out of 1425 had switched from KWM to WKM, changing from Figure 3b to Figure 2. Also theoretically, this could have been submitted as real evidence before the Minnesota verdict of June 11th. It is far-fetched to think that would have supported the *"facial challenge"* against the constitutionality of IRV.

A very close tally, like Bush v Gore in Florida 2000, is a reminder: The preference distributions of Figure 3abc are close to the border, and a scenario like Example (4.4) will occur on rare occasions. Will a court then consider a rare but likely occurrence of such an event as a violation of voters' trust in the election method, and a basis for an *"as-applied challenge"*?

[22] The court generalizes the concept of nonmonotonicity from IRV so that it covers the 2-day election and even its primary part. At this stage, voters have not revealed any subsidiary preferences. Not even new information from day 2 will reveal the possibility of a nonmonotonic effect, but opinion polls show that Chirac *could* have won by "generalized" pushover. It is natural to generalize the concept this way, building on *estimates* of the preferences. The court declares the primary part of the 2-day election "guilty" of nonmonotonicity, although a proof would require more information about preferences than the voters reveal. For this use, *"potentially nonmonotonic"* may be a more accurate term, keeping a "presumption of innocence".

Not even the Burlington tally in (1.1) – (1.3) is enough to show that a nonmonotonic effect is possible. Only with information from Figure 2 about the two anti-M categories WKM and KWM, can we prove that there was a trap situation in Burlington 2009.

The IRV tally is consistent with entirely different election properties. Thus, with (x, y) = (1900, 1800), the vote vector (7.1) will be in constellation *iv,* and the election will be in the bottom right sub-rectangle of Figure 9:

$$(|MWK|, |MKW|, |KMW|, |KWM|, |WKM|, |WMK|) = (994.5, 1559.5, 1900, 1082, 1497, 1800)$$

(|Here T covers 0.00001839 of the circle area). K is Condorcet winner and has more than 1/3 of the first ranks. Thus K is also IRV winner. No nonmonotonicity effect can change this result.

VII WHAT TALLIES CANNOT TELL

With 3 candidates, A, B, and C, it is convenient to think of a ballot supporting A without subsidiary preference as two ballots, ABC and ACB, both of weight ½: The number of voter categories is reduced from 9 in Figure 1 to 6 in Figure 2, without changing the result.

However, a tally like (1.1) – (1.3) in Burlington 2009 does not reveal the preference distribution; the number x of KMW-votes and the number y of WMK-votes remain unknown. All the following vote vectors are compatible with the tally:

(7.1) $$(|MWK|, |MKW|, |KMW|, |KWM|, |WKM|, |WMK|) = (994.5, 1559.5, x, 2982-x, 3297-y, y), \quad 0 \le x \le 2982, \; 0 \le y \le 3297.$$

The two pairwise comparisons, in {W, M} and in {M, K}, change at x = 1862.5 and at y = 1862.5, respectively. These lines partition the rectangle in the xy-plane (Figure 9) into four sub-rectangles:

Top-left: Condorcet cycles MKWM	Top-right: Condorcet ranking MKW
Bottom-left: Condorcet ranking KWM	Bottom-right: Condorcet ranking KMW

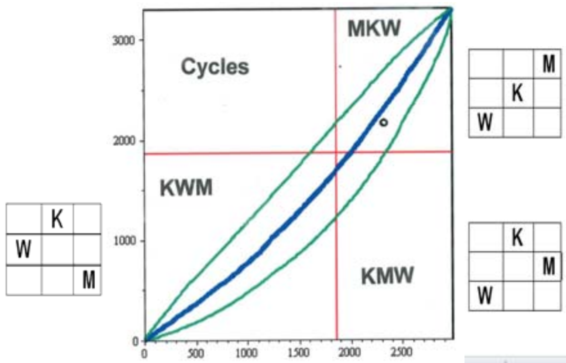

FIGURE 9

For all (x, y) the IRV-tally develops exactly the same way as in Figure 2. According to the constellation diagrams (Figure 4), M is eliminated, and K defeats W in the final tally round.

In the bottom-left corner, (x, y) = (0, 0), nobody ranks M second; the pictogram is *degenerate*, and there is *Perfect Pie-sharing*. In the top-right corner, (x, y) = (2982, 3297), nobody ranks M last: the preference distribution is *single-peaked,* and there is *Perfect Pie-sharing*.

Imagine that along the middle curve, from bottom-left to top right, the supporters of K and W coordinate their increase in x and y, i.e. their subsidiary support for M, so that Perfect Pie-sharing is maintained. The two other curves mark elections where the triangle T in the pictogram covers 0.001 of the circle area. The Burlington election 2009, of Figure 2, is marked at the point (x, y) = (2327, 2157.5), close to the middle curve.

In every 3-candidate election where no candidate gets 50% of the first-ranks, a similar figure may be drawn. Except for the Burlington point, the tally information (1.1) – (1.3) determines the figure. Usually, the real (x, y) is well within the "0.001-zone" defined by the upper and lower curve.

Because of changed first-ranks, one cannot mark the imagined elections of Figure 3 in this graph. In a similar graph, elections 3a and 3b would be far outside the 0.001-zone, and election 3c would be close to the middle curve.

(7.2) ***Why Condorcet cycles are rare.*** When (x, y) is on the curve for Perfect Pie-sharing in Figure 9, a candidate triangle may be fitted exactly to the pictogram, and the candidate closest to the center becomes Condorcet-winner. Therefore, the curve for Perfect Pie-sharing does not pass through the top-left sub-rectangle. If the 0.001-zone has a positive area in common with this sub-rectangle, it must be very small. It is normal that the Perfect Pie-sharing model gives a good approximation, and we see why Condorcet cycles are very rare.

(7.3) ***Robustness of the Perfect Pie-sharing model.*** In Figure 9 we will cross the 0.001-zone from SE to NW through the Burlington point, and calculate the size of T at 9 points. The vote vectors are:

(7.4) $$(|MWK|, |MKW|, |KMW|, |KWM|, |WKM|, |WMK|) = (994.5,\ 1559.5,\ 2327-z,\ 655+z,\ 1139.5-z,\ 2157.5+z)$$

The Burlington election of Figure 2 is at z = 0. The middle curve in Figure 9 is crossed at z = 59.
We cross a U-valley with a roughly parabolic cross section. [23] Let |T| be the area of T:

Z	–141	–91	–41	9	59	109	159	209	259
$\lvert T\rvert/\pi$	0.00123	0.00067	0.00029	0.00007	0	0.00007	0.00028	0.00065	0.00116

For a preference distribution on the middle curve, i.e. with Perfect Pie-sharing, the chords adjust themselves after a normal perturbation, and the new pictogram has a very small T.

[23] This is not as obvious as one may think. Along the middle curve of Figure 9, where |T| = 0, the differentiability properties of |T| as a function of (x, y) are not firmly established.

The calculation of the pictogram for a given vote vector (1.5) is essentially an iterative determination of |T| to a chosen accuracy. An iteration starts with calculation of the three circle segments. SE of the middle curve in Figure 9, e.g in Burlington 2009, these segments are

$$MK = WMK \cup MWK \cup MKW, \quad KW = MKW \cup KMW \cup KWM, \quad WM = KWM \cup WKM \cup WMK$$

Thus, the triangle T points downwards in Figure 2. NW of the middle curve, they are

$$KM = KMW \cup KWM \cup WKM, \quad WK = WKM \cup WMK \cup MWK, \quad MW = MWK \cup MKW \cup KMW$$

Thus, in the NW, T points upwards. (T points upwards also in Figure 1, 3abc, and 6, but these pictograms do not correspond to any point in Figure 9, because they have different numbers of first ranks.)

The possible vote vectors (1.5), with real components, and scaled to component sum = 1, form a standard 5D-simplex Σ. Σ is cut in two halves by a 4D-manifold, which is the locus of vote vectors with |T| = 0, i.e. Perfect Pie-sharing. On this 4D-manifold, both segment definitions may be used, and result in the same pictogram. Thus, |T| is a continuous function of the vote vector. Difference schemes suggest that gluing together the two functions thus defined, each in its own half of Σ, actually makes |T| twice continuously differentiable as a function of a single variable, along straight lines.

VIII THE WORLD OUTSIDE THE WALLS OF VERMONT

Indeed, one year after the Burlington repeal, IRV met an even tougher environment in the world outside. After years of debate over an electoral reform in UK, there was a referendum on May 5th 2011. The question posed was

(8.1) *"At present, the UK uses the 'first past the post' system to elect MPs to the House of Commons. Should the 'alternative vote' system be used instead?"*

There were 6,152,607 votes for IRV (Alternative Vote), 13,013,123 for Plurality (First Past the Post). Thus, Plurality remains the method for elections to the House of Commons.

The referendum was part of the agreement between the new coalition partners, who in 2010 had won a combined majority of the 650 seats (Conservatives 306, Liberal Democrats 57). The Liberal Democrats wanted IRV, the Conservatives wanted to keep Plurality. Labour, the major opposition party, had split, and gave no recommendation to its supporters.

(8.2) ***Opinion polls before UK's IRV referendum.*** Media and various organizations ordered opinion polls. The 40 polls shown below are those from the market research company YouGov.[24] Two reasons for this selection are:

▪ It avoids any systematic difference between companies in the unavoidable bias.

▪ The YouGov polls cover the whole campaign period.

Date	14.06	28.06	05.07	19.07	02.08	16.08	31.08	01.09	06.09	20.09
Day	**0**	**14**	**21**	**35**	**49**	**63**	**78**	**79**	**84**	**98**
IRV	**44**	**42**	**45**	**39**	**41**	**37**	**37**	**32**	**35**	**36**
Plur	**34**	**34**	**32**	**38**	**36**	**38**	**39**	**33**	**39**	**39**
Ratio	**1.29**	**1.24**	**1.41**	**1.03**	**1.14**	**0.97**	**0.95**	**0.97**	**0.90**	**0.92**

Date	04.10	18.10	01.11	15.11	29.11	13.12	14.12	10.01	24.01	07.02
Day	**112**	**126**	**140**	**154**	**168**	**182**	**183**	**210**	**224**	**238**
IRV	**35**	**33**	**32**	**33**	**35**	**33**	**32**	**32**	**32**	**38**
Plur	**40**	**41**	**43**	**40**	**41**	**39**	**40**	**41**	**41**	**39**
Ratio	**0.88**	**0.80**	**0.74**	**0.83**	**0.85**	**0.85**	**0.80**	**0.78**	**0.78**	**0.97**

Date	08.02	21.02	28.02	01.03	07.03	10.03	18.03	21.03	28.03	30.03
Day	**234**	**252**	**259**	**260**	**266**	**269**	**277**	**280**	**287**	**289**
IRV	**37**	**34**	**32**	**33**	**30**	**37**	**39**	**32**	**32**	**31**
Plur	**38**	**41**	**43**	**30**	**47**	**32**	**37**	**44**	**44**	**44**
Ratio	**0.97**	**0.83**	**0.74**	**1.10**	**0.64**	**1.16**	**1.05**	**0.73**	**0.73**	**0.70**

Date	01.04	04.04	08.04	11.04	12.04	15.04	19.04	26.04	29.04	04.05
Day	**291**	**294**	**298**	**301**	**302**	**305**	**309**	**316**	**319**	**324**
IRV	**40**	**34**	**39**	**33**	**37**	**40**	**42**	**41**	**45**	**40**
Plur	**37**	**42**	**38**	**45**	**44**	**41**	**58**	**59**	**55**	**60**
Ratio	**1.08**	**0.81**	**1.03**	**0.73**	**0.84**	**0.98**	**0.72**	**0.69**	**0.86**	**0.67**

The categories *"Would not vote"* and *"Don't know"* vary in size. Thus, the sum of percentages for IRV and Plurality varies from 65 to 100. For better comparisons, their ratio is included.

[24] Wikipedia article «United Kingdom Alternative Vote referendum, 2011.

Marked with underline are the polls that posed the actual referendum question (8.1). YouGov has also published other formulations presented to their opinion poll respondents.[25] The cases of two polls in a row with different questions posed are:

(8.3) **{0.74, 1.10}; {1.10, 0.64}; {0.64, 1.16}; {1.05, 0.73};**
{0.70, 1.08}; {1.08, 0.81}; {0.81, 1.03}; {1.03, 0.73}; {0.73, 0.84}

In each pair, IRV had its clearly best result in the poll that used the referendum formulation (8.1).

Voter's burden; a political problem. What can a political system ask of its citizens? Looking for a significant difference in the questions used, one may notice that the alternative to (8.1) was to give the respondent a brief but technical introduction to IRV:

> *"Voters would RANK a number of candidates from a list. If a candidates wins more than half of the '1st' votes, a winner is declared. If not, the least popular candidates are eliminated from the contest, and their supporters' subsequent preferences counted and shared accordingly between the remaining candidates. This process continues until an outright winner is declared."*

It is unlikely that a respondent had time and willingness to digest what might be new information in this. More likely, it stimulated confusion and awareness of being unprepared. Without a personal opinion based on understanding, it seemed safest to follow the advice of trusted politicians. For most voters that meant to keep the status quo.

How to explain the difference between opinion poll ratio 0.67 on the referendum eve (day 324) and the result next day, 6152607/13013123 = 0.47? In the referendum formulation (8.1), without any accompanying technicality, the key word *"alternative"* is likely to be received as a message that the new system will provide voters with a new and democratic freedom of expression. Respondents who planned to reject IRV might well have done so based on impressions from media, without even knowing (8.1). A change of modus favored IRV when a YouGov interviewer pulled their attention to the ballot question, and may explain the difference.

What caused the decline of support for IRV, which is obvious despite a more favorably formulated question towards the end? Many voters were certainly comfortable with a simple choice between two familiar parties, both capable of forming a government. With time passing, more of them understood that a new system might force them to spend more time preparing their ballots.

Gierzynski (2009) is also concerned about these *cognitive costs of voting*:

[25] See "UK Polling Report – AV Referendum polls". (Ukpollingreport.co.uk.)

> *"If states in the US were to adopt IRV for all (or even some) of their elections, the situation would only be made worse. Instead of simply choosing the preferred candidate for president, senator, representative, governor, lieutenant governor, secretary of state, treasurer, and so on, the public would be asked to rank each candidate. Ranking each candidate in all these races means that the cognitive costs of voting would double, triple, or even quadruple."*

Gierzynski emphasized the importance of *"recognizing the limits to what a political system can ask of its citizens and recognizing that adding complexity to an already complex ballot will disproportionately harm some groups of people more than others."*

Moreover, the UK referendum campaign reflected the strained relations between the coalition partners. Only the Liberal Democrats favored a change. Would a change favor them? We have to think so. To win a Plurality election, they must become the largest party. To win IRV in a 3-party setting, it might well suffice for them to become second in first ranks. The reason is that the Liberal Democrat candidate is a natural second choice both for Conservative and for Labour voters, and is often Condorcet winner; then it suffices to qualify for the final tally round. Many voters must have thought that, in reality, the referendum was about introducing a lower threshold for MP status for Liberal Democrats than for the two main parties.

Plurality, majority, or proportionality? These words indicate basic criteria often invoked in judgments of an election method. The public's judgment reflects basic values, and is a main component of the method's legitimacy. However, the political parties that will lose seats with a proposed new method have an obvious self-interest in preserving the old method.

Plurality is the mainstay of some large democracies, and its coarseness favors a two party system. The government party is under scrutiny from the opposition. The major opposition party may even form a "shadow cabinet". Already before the election, those voters who accept some amount of cognitive costs may get relatively reliable knowledge of what kind of government policy that will follow from each of the two possible outcomes. See e.g. Gierzynski (2011, 200–214).

Small "third" parties face an uphill fight. Their supporters get harassed with claims that they either "waste their votes" or "betray their favorite". To what extent does the Plurality method also provide dynamics of a disruptive political polarization? Moreover, in a close race, it may very well happen that one party is national Plurality winner while another wins more than half of the single-seat constituencies. Similarly, in the US presidential election of 2000, the winner Bush had 47.9%, defeating Gore who was Plurality winner with 48.4%.

On the other hand, an IRV-winner is preferred to the closest competitor by a majority. There is a wide acceptance for 50% as a decisive criterion, clearly preferable to a plurality. A table in the *IDEA Handbook* (editors Reynolds, Reilly, and Ellis, 2005) shows election methods in 213 independent countries and related territories. There are 78 cases of (some variant of) a 2-day presidential election, and 22 with (some variant of) Plurality.

Elections to national legislatures is the main concern in the *Handbook*. The table records 90 cases in the "Plurality/Majority family". The constituencies are single-seat. Plurality elections are most common (47). However, 2-day elections are also used (20), with final support > 50% for the winner. The figures may indicate a widespread preference for politicians with more than 50% support, but less motivation to pay for a 2-day election to a large number of parliament seats than to the president's office. One should also recognize the voters' time and cognitive costs.

Moreover, there are 73 cases in the Proportionality family. The constituencies have more than one seat. Usually each voter supports one party list, but there are a few cases of STV. The number of seats in a constituency may bar small parties from representation, unless special seats are a compensation, e.g. based on a party's result nationwide.

The Condorcet methods. These methods consistently follow the "50% philosophy". However, they are not included in the *Handbook.* It is hard to find examples from political elections. The reason may be that Condorcet methods do not necessarily encourage politicians to campaign for development in any clear direction. They may actually urge a candidate to shy away from controversial questions in order to appear to the electorate as being the one closest to the center voter.

IRV and STV urge candidates to conduct a balanced campaign, in order to obtain both primary and subsidiary support. In political elections, one may consider narrowing down the field with an STV-tally and then get a Condorcet winner with a clear but moderate political profile. To calibrate, one may consider the following procedure:

(8.4) ***STV/Condorcet.*** Choose $r \in \{2, 3, \dots\}$. Use an r-seat STV-method to fill r spots in a final tally round; then pick the Condorcet winner. With $r = 3$, M would have won in Figure 2. [26]

Many election scholars see proportional representation of various groups as the best way for societies divided according to ethnicity, language, or other criteria. An alternative is to give incentives for parties to make deals and for voters to vote across division lines. (Reilly 2002).These societies may assess how well various preferential election methods encourage such cooperation. IRV-variations, Condorcet methods, and hybrids like (8.4) may be considered. Secondary preferences from a 40%

[26] To qualify for the final round, a candidate needs a quota of first ranks. A quota is between $1/r$ and $1/(r+1)$ of the total. In Figure 2, K and W both have above 1/3. With $r = 2$, only M can be eliminated, and K still wins.

minority may well decide between competing candidates from a 60% majority.

Nonpolitical elections. Some elections apply a Condorcet method in order to rank candidates according to generally accepted criteria for competence. There may be many relevant criteria. Thus, for applicants to an academic position, scientific production, teaching ability, and administrative experience may all be relevant.

An accepted criterion for competence is very different from a candidate's location in a political landscape: In a political election, it is legitimate that voters agree on candidate locations in the landscape and still express different preferences. Figures 1 and 2 are models where different voters view a candidate triangle from different angles and vote accordingly. This is different from ranking job candidates. Together with the robustness (7.3), it explains the small T in Figures 1 and 2.

Thus, one should not expect the model of Perfect Pie-sharing to fit equally well in each triple when candidates are ranked according to criteria of competence. On the other hand, with such ranking, we should expect that a Condorcet winner is also Plurality winner. The squeeze of Condorcet winner M between W and K in Figure 2 is caused by legitimate *political* differences.

Thus, in elections according to ability criteria, the structure of the preference distribution is likely to be very different from what we see in real political elections, but a Condorcet winner is still likely to exist, and to win *clearly* in every pairwise comparison. [27] However, Example (6.2) is rather extreme.[28]

Proportional influence. According to *Handbook*, proportional representation, obtained with party lists, is common in European countries. The composition of the legislature becomes, more or less, a scaled down copy of the electorate. However, politics is also an art of compromise. Compromise may start with a suitable preferential election. The resulting composition then reflects a political compromise obtained with the ballots as tools. Voter influence may be proportional with designed disproportionality in representation according to first ranks only.

With IRV/STV, one may follow each single ballot through the tally rounds. Every time a ballot supports the election of a candidate, it costs some of its voting power. In (multi-seat) STV, just a

[27] For these reasons, the use of Condorcet methods, often the one of Markus Schulze (2003), in many professional organizations seem natural, while they hardly occur in political elections. The only known example of political use was Nanson's method for city elections in Marquette, Michigan, in the 1920s (McLean 2002). It very rarely happens that there is no Condorcet winner, so the choice of a particular method does not seem important. If it happens, there will most likely be one cyclic triple on top, and it is reasonable to overrule the smallest pairwise defeat, which is what many common methods do. Another possible criterion is to minimize the number of cases where a Condorcet winner may be defeated with Strategy 2, i.e. burying (Stensholt 2013).

[28] The vote vector (|ABC|, |ACB|, |CAB|, |CBA|, |BCA|, |BAC|) = (60, 0, 0, 0, 40, 0) from Example (6.2) has a pictogram where two chords coincide and the third is a tangent. To adapt a candidate triangle makes no sense. In this kind of election there is no "central candidate" that may be seen as a compromise. In a more normal situation, B is more clearly ahead of C, and one should expect that some voters rank BAC. A vote vector close to (|ABC|, |ACB|, |CAB|, |CBA|, |BCA|, |BAC|) = (58, 1, 1, 1, 29, 10), with $|T|/\pi = 0.0165$, should not be surprising.

small fraction of the total voting power of the electorate remains with the voters, not contributing to the election of a candidate. Thus, the average voter has significantly influenced the result, and each voter may check the details if the relevant election parameters are published. The mathematical fact that Strategy 2, i.e. burying, is impossible, is a guarantee to a voter of "later-no-harm": A ballot ranking counts only after all candidates originally with better ranking, are elected or eliminated.

The risk that an election has no Condorcet winner is small; see (7.2). Moreover, with a Condorcet method each ballot counts in every candidate pair. A Condorcet candidate wins against different competitors with support from different majorities, and usually a large number of voters will see that their ballot has had some influence on the result. For every Condorcet method there will be preference distributions where burying (Strategy 2) is possible, but it is not easy to apply it. One may also choose a method that allows this for relatively few preference distributions (Stensholt 2013).

In ethnically divided societies, one may hope to achieve a composition of the legislature that facilitates a modus vivendi. However, the cognitive costs of preferential voting may be a problem. Adaptation of the Australian practice with "how-to-vote" cards received from the party whose candidate(s) a voter intends to rank first, should be a relief.

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