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The Method of Leader's Overthrow in Networks

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THE METHOD OF LEADER'S OVERTHROW IN NETWORKS

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ABSTRACT

Methods for leader's detection and overthrow in networks are useful tools for decision-making in many real-life cases, such as criminal networks with hidden patterns or money laundering networks. In the given research, we represent the algorithms that detect and overthrow the most influential node to the weaker positions following the greedy method in terms of structural modifications. We employed the concept of Shapley value from the area of cooperative games to measure a node's leadership and used it as the core of the developed leader's overthrow algorithms. The approaches are illustrated based on the trivial network structures and tested on real-life networks. The results are represented in tabular and graphical formats.

Keywords: leadership, networks analysis, Shapley value, game theory

1. INTRODUCTION

The variety of game and graph theoretical approaches has paramount purpose in formalizing the relative importance of nodes in networks. Specifically, the analysis of centralities has a high-level significance for many real-world applications. In terms of practical use, one of the first research applications of centrality was introduced in the 1940s at the Group Networks Laboratory, M.I.T. Later, Cohn & Marriott (1958) applied the concept of network centrality to analyze the diversity of Indian social life. Pitts (1965) used the concept for examination of communication paths in the context of urban development while Czepiel (1974), applied centrality computation in the analysis of a technological innovation in the steel industry. The practical application of centrality measures has grown fast in the last fifty years. For example, Moore, Eng & Daniel (2003) used centrality scores for the estimation of aid coordination between the non-governmental organizations (NGOs) in Mozambique (i.e., NGOs involved in the flood operations). Estrada & Bodin (2008) used network centralities to manage landscape connectivity. Faris & Felmlee (2011) explored gender segregation and cross-gender aggression based on centrality measures. The diversity of centrality applications is huge and many other examples can be found in the literature.

The analysis of an agent's importance in the domain of networks is one of the core ideas in socio-economic network analysis. Different evaluation methods exist. Degree (Freeman, 1979), betweenness (Anthonisse, 1971; Freeman, 1977), and closeness (Beauchamp, 1965;

Sabidussi, 1966) are the most widely known metrics that assess the structural centralities of nodes. The algorithmic measures of nodes' authority are well represented in Kleinberg (1999) and Page, Brin, Motwani, & Winograd (1999), where the notion of authority is given based on the analysis of link structures. An interesting approach to characterize the role of nodes within networks is given by Scripps & Esfahanian (2007), where the community-based metric in the symbiosis with the degree-based measure is introduced in the context of nodes' roles classification. The analysis of the leadership position in a network is an important problem. Corresponding approaches have been presented in Balkundi & Kilduff (2006), Hoppe & Reinelt (2010), Belik & Jornsten (January, 2015) and in Belik & Jornsten (April, 2015).

In contrast, there exists yet another problem of understanding how the network's structure should be efficiently modified in order to overthrow the current network leader to the weakest position. In networks, such as criminal networks with hidden patterns, or money laundering networks, the overthrow of the detected leader may seriously damage the network or even bring about irreversible destruction. Interesting approaches and discussions regarding this topic have been presented by Bryson & Kelley (1978), Sageman (2004), and Hung, Kolitz, & Ozdaglar (2011).

In this paper, we consider the case when all nodes in the network participate in the process of the leader's overthrow. Their main goal is to create the sufficient set of links (based on greedy approach) in order to make the current leader the least powerful. To measure the level of a node's influential power we employ the concept of Shapley value (Shapley, 1952) from the area of cooperative games. Specifically, we use the Shapley value (SV) concept developed by Aadithya, Ravindran, Michalak & Jennings (2010) in order to measure the nodes' leadership positions in networks. We show the advantages of the SV-based concept compared to the traditional centralities. Based on the given game theoretic approach we developed two overthrow algorithms that establish the sets of links to overthrow the initial leader with the highest SV to the weaker positions in terms of SVs In real-life networks, the represented algorithms are not unique solutions, but they are useful methods to detect and to plan the prospective network's modifications.

Next, we test the SV-based algorithms based on the trivial network topologies and on the real-world structures retrieved from the NHH and BI co-authorship networks (Belik & Jornsten, October, 2014).

2. SHAPLEY VALUE AS THE NETWORK'S CENTRALITY

Shapley value is one of the fundamental concepts of game theory (Roth, 1988). The core idea of SV is the payoffs' distribution among players according to their personal contributions to the overall gain in a cooperative game. Since SV reflects the influential power of players based on their mutual cooperation, it is applicable in the domain of networks analysis. Specifically, SV-based centrality measures the importance of nodes within a graph (Gomez, González-Arangüena, Manuel, Owen, del Pozo & Tejada, 2003). In terms of quantitative leadership analysis, SV is an effective measure due to its well-formalized mathematical apparatus. Furthermore, its game theoretic nature helps to reflect the real-world players' interrelations because it counts mutual influence of players and their all-possible coalitions (i.e., combinations of players). For large-scale networks with high level of information lack regarding the network's nature, the network's structure becomes a very important factor in terms of quantitative leadership analysis of nodes. In many cases, structure might even be the only well-known factor. Therefore, it is important to have an efficient measure that computes the importance of nodes based on the retrieved structural details. Obviously, SV-based centrality is not the unique or only solution to estimate leadership, but it makes a high-level contribution to the multi-factor analysis of leadership formation in networks. Its game theoretic nature makes it a preferable analytical tool compared to the conventional measures.

To understand why the SV-concept is employed to estimate node's influential power, it is important to understand its advantages over the conventional centralities, such as degree, closeness, betweenness, etc. The core drawback of the conventional centralities lies in their "individualistic" nature. This means that they "fail to recognize that in many network applications, it is not sufficient to merely understand the relative importance of nodes as *stand-alone* entities. Rather, the key requirement is to understand the importance of each node in terms of its utility when combined with other nodes" (Aadithya et al., 2010). This means that conventional centralities do not consider the mutual effect of nodes' failures. They only reflect the resulting effect (i.e., after-effect) of multi-node failures in terms of a network's structure. In contrast, SV-based centrality counts the mutual effect of all nodes' combinations and the corresponding contributions to the overall network's gain. It reflects the game theoretic nature of the classical interpretation of Shapley value.

> Aadithya's (2010) study found the following:
> The SV of each agent (node) in the game is interpreted as a centrality measure because it represents the average marginal contribution made by each node to every possible combination of the other nodes. This paradigm of SV-based network centrality thus confers a high degree of flexibility (which was completely lacking in traditional centrality metrics) to model real-world network phenomena. (p. 2)

Conventional centralities have some drawbacks based on the structural factors. For example, degree centrality does not count the global networks structure, because it takes into consideration only the neighboring nodes approachable in one hop (i.e., within one-link distance). Closeness centrality is based on the calculation of the inverse sum of node's shortest distances to all other nodes, but due to its nature, it cannot be applied to the analysis of disconnected graphs. Betweenness centrality counts the frequency of a node to appear along the shortest paths between any other two nodes. It overcomes the limitations of degree and closeness centralities. However, in many real-life networks there is a great proportion of nodes that do not appear on the shortest paths between any other two nodes (Opsahl, Agneessens, & Skvoretz, 2010). This means that many nodes can get the betweenness value equal or close to zero. In contrast, SV-based centrality overcomes the structural limitations, because it counts the mutual effect of all possible nodes' combinations hesitating the game theoretic features of the classical SV concept.

It is important to note that our goal is not to discredit the applicability of conventional centralities. Their sufficiency depends on the areas of application. In many real-world cases these measures have proved their efficiency. For example, they were applied in different combinations by Cohn & Marriott (1958), Pitts (1965), Czepiel (1974), Moore, Eng & Daniel (2003), etc. The details regarding these approaches were given in the previous section. However, there are many applications, such as social networks, power transmission or communication networks, where the limitations of conventional centralities are critical.

3. LEADER'S OVERTHROW ALGORITHM

Consider graph G(V,E) and $v_i \in V$. All nodes (i.e., neighbors), which are reachable from v_i in at most one hop within G(V,E) are denoted by $N_G(v_i)$. The degree of node v_i is defined by $deg_G(v_i)$. According to Aadithya et al. (2010), the SV interpretation for node v_i in G(V,E) is the following:

$$SV(v_i) = \sum_{v_j \in \{v_i\} \cup N_G(v_i)} \frac{1}{1+deg_G(v_j)}, \quad (1)$$

Based on equation (1) Aadithya et al. (2010) introduced the algorithm to compute nodes' leadership in a network:

SV-COMPUTING (G):
for each $v \in V(G)$ do
    ShapleyValue [v]= $\frac{1}{1+deg_G(v)}$;
    For each $u \in N_G(v)$ do
        ShapleyValue [v] += $\frac{1}{1+deg_G(u)}$;
    end
end
return L=List of SV-s for all nodes;

SV-COMPUTING returns the SVs for all nodes and reflects their leadership positions within the analyzed network. The advantage of the procedure is its polynomial running time.
We represent the algorithms that calculate the sets of links to be established in order to overthrow the strongest node (i.e., leader):

I. K-OVERTHROW-COMPUTING (G, k)
The algorithm iteratively detects links to be established in order to overthrow the initial leader in terms of SV allocation within a network. Following the greedy approach, algorithm establishes k links allowed to build by the decision-maker. The practical importance of using k-parameter is a flexibility to manipulate by the SV-based leadership via adding the number of links allowed by the decision-maker.

II. MAX-OVERTHROW-COMPUTING (G)

The algorithm iteratively detects links to be established in order to overthrow the initial leader to the *weakest* position in terms of SV allocation within a network. The number of edges that needs to be created can be much bigger than the existing number of edges in the network. Creating that many links is not always feasible in the real-life scenarios, but the purpose of the algorithm is to show the ultimate set of links required to overthrow the node to its lowest leadership position. For the analytical purposes, it might be important to see the number of links that is required to be created for the ultimate overthrow of the node. For example, it is applicable in the analysis of node's resistance power. Obviously, the node with the stronger leadership position requires a bigger effort to be overthrown compared to the node with the weaker position.

The pseudocodes of the given approaches are the following:

I. K-OVERTHROW-COMPUTING (G, k):

```
1     L=SV-COMPUTING (G)
2     Target=node with MAX(L)
3     SV_intermediate=SV(Target)
4     Approved_link = no link
5     n=0
6     WHILE n < k:
7           FOR each v ∈ V(G):
8                 FOR each u ∉ N_G(v):
9                       Create trial link (v,u)
10                      L=SV-COMPUTING (G)
11                      IF SV_intermediate > SV(Target):
12                      THEN:   Approved_link = (v,u)
13                              SV_intermediate = SV(Target)
14                      ELSE: Delete trial link (v,u) from G(V,E)
15          Include Approved_link to G(V,E)
16          Update G(V,E)
17          Approved_link = no link
18          n=n+1
19    return:  Approved_link-s and the corresponding SVs of the Target-node
```

NOTATION:

Line 2:

MAX(L) detects the maximal Shapley Value (SV) in the list L. Target is the initially detected node (leader) that has to be overthrown. Its value is constant in the algorithm.

Lines 5-6:

Counter n is initially equal to zero. It is used to control the number of established links.

The loop continues while the number of established links (i.e., n) is not equal to the allowed number of links (i.e., k). In each iteration of the WHILE loop, algorithm approves the link that gives the maximal decrease of SV(Target)-value. We consider k as a constraint for the number of links to be established. To reflect the real-life cases, the value of k cannot be greater than the existing number of edges in the initial network G: $1 \leq k \leq |G.E|$

II. MAX-OVERTHROW-COMPUTING (G):

```
1       L=SV-COMPUTING (G)
2       Target=node with MAX(L)
3       SV_intermediate=SV(Target)
4       Approved_link = no link
5       WHILE  SV(Target) ≠ MIN(L) OR
6               [SV(Target)= MIN(L) AND [SV(Target)=SV(j) AND j ≠ Target]
7               AND G is NOT complete]:
8               FOR each v ∈ V(G):
9                       FOR each u ∉ N_G(v):
10                              Create trial link (v,u)
11                              L=SV-COMPUTING (G)
12                              IF SV_intermediate > SV(Target):
13                              THEN:  Approved_link = (v,u)
14                                     SV_intermediate = SV(Target)
15                              ELSE: Delete trial link (v,u) from G(V,E)
16              Include Approved_link to G(V,E)
17              Update G(V,E)
18              Approved_link = no link
19      return:  Approved_link-s and the corresponding SVs of the Target-node
```

NOTATION:

Line 2:

MAX(L) detects the maximal Shapley Value (SV) in the list L. Target is the initially detected node (leader) that has to be overthrown. Its value is constant in the algorithm.

Line 5-7:

WHILE SV(Target) ≠ **MIN**(L) **OR**
[SV(Target)= **MIN**(L) **AND** [SV(Target)=SV(j) **AND** j ≠ Target] **AND** G is **NOT** complete]

In each iteration of the WHILE loop, algorithm approves the link that gives the maximal decrease of SV(Target)-value. The compound WHILE loop checks two main conditions:

1. SV(Target) ≠ **MIN**(L):

MIN(L) detects the minimal Shapley Value (SV) in the list L.

The loop continues while the Shapley value of the initially detected leader is not the minimum Shapley value.

2. [SV(Target)= **MIN**(L) **AND** [SV(Target)=SV(j) **AND** j ≠ Target] **AND** G is **NOT** complete]:

This condition is required to control cases, when the Target-node approached the lowest SV, but there exist other node(s) with the same SV: SV(Target)=SV(j) **AND** j ≠ Target. In other words, it is required to check if Target has a potential to get a lower Shapley Value. It is possible only if the updated G-graph is not complete (G is **NOT** complete).

The computational mechanism of the leadership measurement in K-OVERTHROW-COMPUTING (G,k) and MAX-OVERTHROW-COMPUTING (G) are based on the SV-COMPUTING procedure. SV-COMPUTING (G) procedure allows computing the exact SVs for the nodes in the large-scale networks in the polynomial running time (Aadithya et al., 2010).
In addition, it is important to notice that the given "overthrow" algorithms are applicable to connected graphs. We show how the given algorithms work on the trivial networks. Next, we test them on the real-life networks.

4. LEADER'S OVERTHROW IN DIFFERENT NETWORK TOPOLOGIES

Any large-scale network consists of the trivial topologies with different characteristics (Haddadi, Rio, Iannaccone, Moore, & Mortier, 2008):

- "point-to-point", or "line";
- "star";
- "ring";
- mesh, i.e., topologies that are based on the previous three types.

Since the number of links in the tested trivial topologies is small (i.e., in the range between two and eight), we show how the SV-based leader's overthrow procedure works running MAX-OVERTHROW-COMPUTING (G) algorithm. Our main goal in this section is to explain the computational SV-based mechanism step-by-step.
In section 5 we give the detailed results running both algorithms (i.e., K-OVERTHROW-COMPUTING (G,k) and MAX-OVERTHROW-COMPUTING (G)) on the real-life networks.

4.1 "Point-to-point" topology

The "Point-to-point" topology is represented in Figure 1.

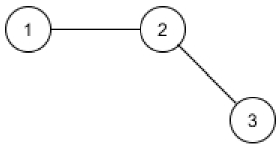

Figure 1. "Point-to-point" network topology in the initial state

Initially, the MAX-OVERTHROW-COMPUTING (G) algorithm calculates the SVs for the given topology.
It is detected that node 2 is the most powerful (i.e., it has the highest SV). Next, the link (1,3) is established in order to decrease the power of node 3. Since we get the complete graph, the algorithm stops, and we get SV(1)=SV(2)=SV(3)=1. The results for all algorithm's steps are represented in Table 1 and in Figure 2.

Table 1. Results for the "point-to-point" topology

INITIAL		OVERTHROW			FINAL	
Node	**Shapley Value**	**Link**	**SV(Target)**	**Decrease**	**Node**	**Shapley Value**
1	0.83	(1,3)	1.00	0.33	1	1.00
2	1.33				2	1.00
3	0.83				3	1.00

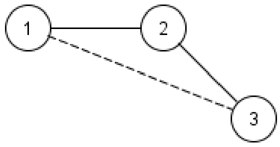

Figure 2. Modified “Point-to-point” network topology

4.2 “Star” topology

The “Star” topology is characterized by the existence of central hub that is represented by node 1 in Figure 3.

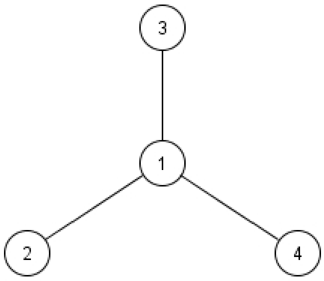

Figure 3. “Star” network topology in the initial state

Following the MAX-OVERTHROW-COMPUTING (G) algorithm, we get the results represented in Table 2.

Table 2. Results for the “Star” topology

INITIAL		OVERTHROW			FINAL	
Node	**Shapley Value**	**Link**	**SV(Target)**	**Decrease**	**Node**	**Shapley Value**
1	1.75	(2,3)	1.42	0.33	1	1.00
2	0.75	(2,4)	1.17	0.25	2	1.00
3	0.75	(3,4)	1.00	0.17	3	1.00

Node 1 was detected by the algorithm as the most powerful one. The algorithm created three links in order to overthrow node 1 to the weakest position (i.e., SV(1)=1). It stopped on the iteration when the graph became complete and no more links could be established. The resulting modified “Star” topology is represented in Figure 4.

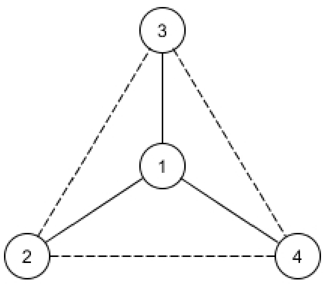

Figure 4. Modified “Star” network topology

4.3 “Ring” topology

The “Ring” topology is characterized by sequential connections of odd or even numbers of nodes forming the cycle. First, we consider the structure with an even number of nodes that is represented in Figure 5.

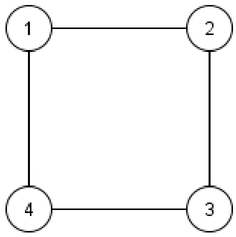

Figure 5. The initial state of the “Ring” network topology with even number of nodes

Running the MAX-OVERTHROW-COMPUTING (G) algorithm, we get the following results (see Table 3).

Table 3. Results for the “Ring” topology with even number of nodes

INITIAL		OVERTHROW			FINAL	
Node	**Shapley Value**	**Link**	**SV(Target)**	**Decrease**	**Node**	**Shapley Value**
1	1.00	(2,4)	0.83	0.17	1	0.83
2	1.00				2	1.17
3	1.00				3	0.83
4	1.00				4	1.17

According to Table 3, initially all nodes have equal SVs. The algorithm chooses node 1 as the Target from the list. By establishing link (2,4) SV(1) decreased by 0.17, and the resulting SV(1) became equal to 0.83. Link (1,3) is not created by the MAX-OVERTHROW-COMPUTING (G) algorithm, because it increases SV(1) back to the initial value that is equal to 1.00. For the given “Ring” network topology with an even number of nodes the SV(Target) is decreased to its minimum value of 0.83. The resulting network is represented in Figure 6.

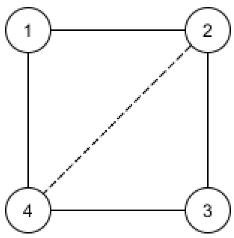

Figure 6. Modified “Ring” network topology with even number of nodes

Next, we test the “Ring” structure with an odd number of nodes (see Figure 7).

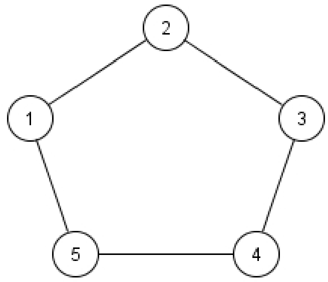

Figure 7. The initial state of the "Ring" network topology with odd number of nodes

Applying the MAX-OVERTHROW-COMPUTING (G) algorithm to the graph represented in Figure 7, we get the following results (see Table 4).

Table 4. Results for the "Ring" topology with odd number of nodes

INITIAL		OVERTHROW			FINAL	
Node	**Shapley Value**	**Link**	**SV(Target)**	**Decrease**	**Node**	**Shapley Value**
1	1.00	(2,5)	0.83	0.17	1	0.83
2	1.00				2	1.17
3	1.00				3	0.92
4	1.00				4	0.92
5	1.00				5	1.17

According to Table 4, link (2,5) was sufficient to overthrow node 1 to the weakest position in the network. Specifically, ΔSV(1) = -0,17. The resulting graph is represented in Figure 8.

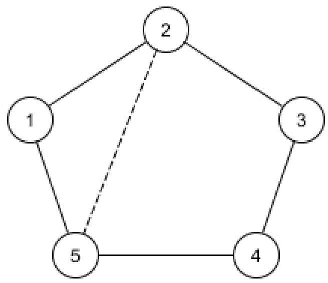

Figure 8. Modified "Ring" network topology with odd number of nodes

4.4 Mixed topology

We analyze the symmetric mixed topology that includes "Point-to-point", "Star" and "Ring" based sub-graphs. The given network is represented in Figure 9.

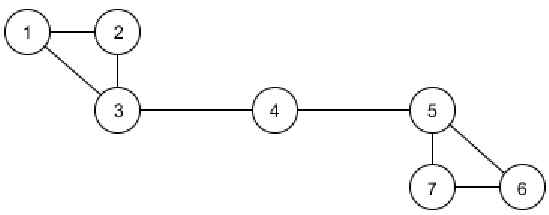

Figure 9. Mixed network topology in the initial state

The results of the algorithm's application are represented in Table 5.

Table 5. Results for the mixed topology

INITIAL		OVERTHROW			FINAL	
Node	**Shapley Value**	**Link**	**SV(Target)**	**Decrease**	**Node**	**Shapley Value**
1	0.92	(1,4)	1.08	0.17	1	1.24
2	0.92	(2,4)	0.95	0.13	2	1.04
3	1.25	(1,5)	0.90	0.05	3	0.7
4	0.83	(2,5)	0.85	0.05	4	1.24
5	1.25	(1,6)	0.82	0.03	5	0.99
6	0.92	(2,6)	0.78	0.03	6	0.99
7	0.92	(4,6)	0.75	0.03	7	0.82
		(4,7)	0.73	0.02		
		(1,7)	0.70	0.02		

According to Table 5, node 3 was detected as the most influential (the initial SV(3) = 1.25). Following the algorithm, nine links were created to overthrow node 3 to the weakest position with SV=0.7. The resulting network is represented in Figure 10.

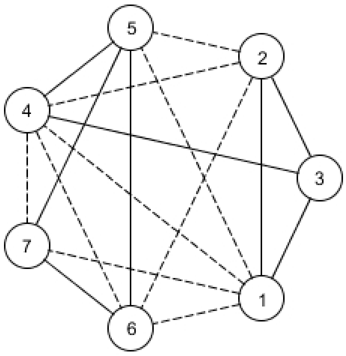

Figure 10. Modified mixed topology

5. TESTING ON THE REAL-LIFE NETWORKS

We illustrate K-OVERTHROW-COMPUTING (G, k) and MAX-OVERTHROW-COMPUTING (G) algorithms based on two real-life networks. The first network is the largest connected component of the NHH interdepartmental co-authorship network and the second one is the largest component of the BI interdepartmental co-authorship network. The detailed analysis of the NHH and BI networks is represented in Belik & Jornsten (October, 2014).

5.1 NHH network

The network structure of the NHH largest component is represented in Figure 11.

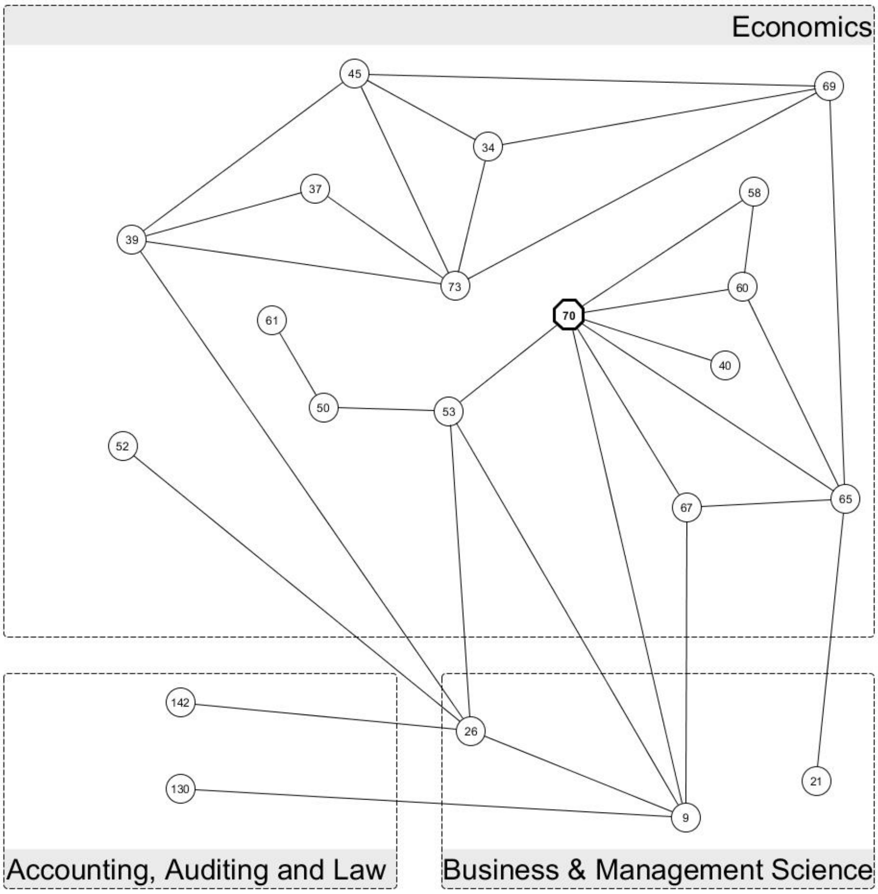

Figure 11. The NHH largest component

First, we test K-OVERTHROW-COMPUTING (G, k) in order to detect and overthrow the most powerful node applying different k-values. Since the NHH largest component has 32 links connecting 21 nodes, we run the algorithm for all k-s in the range [1, 32].
First, the algorithm calculates the initial SVs (see Table 6).

Table 6. Initial results for the NHH largest component

Node	Shapley Value	Node	Shapley Value	Node	Shapley Value	Node	Shapley Value
9	1.41	40	0.62	58	0.71	69	0.98
21	0.67	45	1.02	60	0.87	70	1.99
26	1.73	50	1.03	61	0.83	73	1.35
34	0.82	52	0.67	65	1.49	130	0.67
37	0.7	53	0.99	67	0.71	142	0.67
39	1.07						

Node 70 is detected as the Target-node with SV=1.99. Next, the algorithm establishes k-links allowed to build in order to overthrow node 70. Table 7 shows the list of consequently established links. For each link, we provide the following details:

1. Current SV of the Target-node for the latest established link;
2. The difference between SVs of the Target-node before and after the link was established (i.e., "Decrease")
3. The current position (i.e., SV-based rank) of the Target-node within the network. For example, "Position=3" means that the node is the third-most influential (out of 21 nodes) in terms of SV-based analysis.

Table 7. Established links in the NHH network following K-OVERTHROW-COMPUTING (G, k)

k	Link	Target			k	Link	Target		
		SV	Decrease	Position			SV	Decrease	Position
1	(40,58)	1.742	0.248	1	17	(21,67)	0.911	0.014	11
2	(40,60)	1.608	0.133	2	18	(21,40)	0.897	0.014	12
3	(40,67)	1.508	0.1	2	19	(21,58)	0.883	0.014	13
4	(53,58)	1.425	0.083	2	20	(21,60)	0.869	0.014	14
5	(58,67)	1.358	0.067	2	21	(26,58)	0.858	0.011	14
6	(40,65)	1.301	0.057	4	22	(26,40)	0.847	0.011	14
7	(53,60)	1.244	0.057	6	23	(26,60)	0.836	0.011	14
8	(60,67)	1.196	0.048	6	24	(26,67)	0.825	0.011	15
9	(9,58)	1.149	0.048	6	25	(9,21)	0.816	0.009	16
10	(9,40)	1.107	0.042	6	26	(21,53)	0.807	0.009	17
11	(40,53)	1.071	0.036	7	27	(34,40)	0.798	0.009	17
12	(58,65)	1.036	0.036	7	28	(26,65)	0.789	0.009	17
13	(9,60)	1.004	0.032	8	29	(34,58)	0.78	0.009	17
14	(53,67)	0.972	0.032	8	30	(34,60)	0.77	0.009	17
15	(9,65)	0.947	0.025	9	31	(34,67)	0.761	0.009	17
16	(53,65)	0.925	0.022	9	32	(37,58)	0.754	0.008	17

Each value in the "Link"-column shows the latest link established for the current k. For example, for k=3 three links were established. First two links (i.e., (40,58) and (40,60)) are reflected in the previous rows, and the latest link (i.e., (40,67)) is represented in the row k=3.

It is important to notice that each approved link guarantees the SV-decrease of the initial leader (i.e., Target-node), but it is not necessary that each approved link gives a decrease in terms of its "Position"-value. In fact, each approved link makes the Target-node weaker, but it also affects the rearrangement of SVs for all other nodes in the network. Therefore, it is a very common situation when more than one link has to be established in order to decrease the "Position"-value of the Target-node.

Next, we apply the MAX-OVERTHROW-COMPUTING (G) algorithm to the NHH largest component in order to detect and overthrow the most powerful node to its weakest position.

First, the algorithm calculates the initial SVs. The results were represented in Table 6.
Node 70 is detected as the Target-node with SV=1.99. Next, the algorithm establishes the set of links in order to overthrow node 70 to the weakest position. The list of consequently established links is represented in Appendix A. For each link we provide the details about the current SV(Target) and the difference between SVs of the Target-node before and after the link was established.

According to Appendix A, sixty seven links were created to overthrow node 70 from the position of the most powerful node to the weakest position in the network. The resulting SVs for all nodes in the network are represented in Table 8.

Table 8. Resulting SVs for the NHH largest component based on the MAX-OVERTHROW-COMPUTING (G)

Node	Shapley Value	Node	Shapley Value	Node	Shapley Value	Node	Shapley Value
9	1.87	40	1.37	58	1.37	69	0.6
21	0.56	45	1.06	60	1.37	70	0.56
26	1.23	50	1.04	61	0.61	73	0.72
34	0.97	52	0.59	65	1.44	130	0.56
37	0.78	53	1.37	67	1.37	142	0.59
39	0.96						

5.2 BI network

The network structure of the BI largest component is represented in Figure 12.

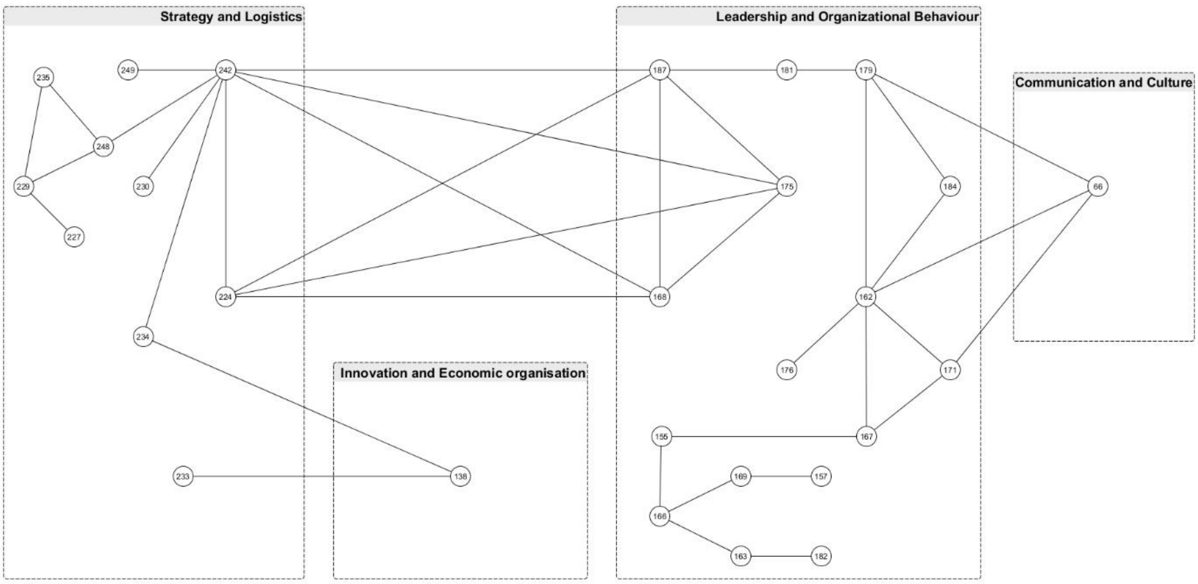

Figure 12. The BI largest component

Applying the K-OVERTHROW-COMPUTING (G, k) algorithm we get the initial SVs on the first step (see Table 9). Since the BI largest component has 38 links connecting 28 nodes, we run the algorithm for all k -s in the range [1, 38].

Table 9. Initial results for the BI largest component

Node	Shapley Value	Node	Shapley Value	Node	Shapley Value	Node	Shapley Value
66	0.84	167	0.98	181	0.7	230	0.61
138	1.17	168	0.88	182	0.83	233	0.83
155	0.83	169	1.08	184	0.68	234	0.78
157	0.83	171	0.89	187	1.21	235	0.83
162	1.93	175	0.88	224	0.88	242	2.46
163	1.08	176	0.64	227	0.75	248	0.94
166	1.25	179	1.26	229	1.33	249	0.61

According to Table 9, node 242 is detected as the most influential (SV(242) = 2.46). Next, the algorithm establishes k-links allowed to build in order to overthrow node 242. Table 10 shows the list of consequently established links.

Table 10. Established links in the BI network following K-OVERTHROW-COMPUTING (G, k)

k	Link	Target			k	Link	Target		
		SV	Decrease	Position			SV	Decrease	Position
1	(230,249)	2.128	0.332	1	20	(175,248)	1.003	0.025	10
2	(230,234)	1.961	0.167	1	21	(248,249)	0.980	0.023	10
3	(234,249)	1.828	0.133	2	22	(187,234)	0.958	0.022	11
4	(230,248)	1.728	0.100	2	23	(66,224)	0.946	0.011	12
5	(168,249)	1.644	0.083	2	24	(66,168)	0.935	0.011	14
6	(224,230)	1.578	0.067	2	25	(66,175)	0.924	0.011	16
7	(234,248)	1.511	0.067	2	26	(66,230)	0.913	0.011	17
8	(175,249)	1.444	0.067	2	27	(66,249)	0.902	0.011	18
9	(168,234)	1.397	0.048	2	28	(66,187)	0.893	0.009	18
10	(175,230)	1.349	0.048	2	29	(66,234)	0.884	0.009	18
11	(187,248)	1.302	0.048	2	30	(138,224)	0.875	0.009	18
12	(224,249)	1.254	0.048	4	31	(138,168)	0.866	0.009	18
13	(168,230)	1.218	0.036	6	32	(138,175)	0.857	0.009	18
14	(175,234)	1.183	0.036	7	33	(138,230)	0.847	0.009	18
15	(187,249)	1.147	0.036	8	34	(138,249)	0.838	0.009	18
16	(224,248)	1.111	0.036	8	35	(155,234)	0.831	0.008	21
17	(187,230)	1.083	0.028	8	36	(66,248)	0.823	0.008	21
18	(168,248)	1.056	0.028	10	37	(138,187)	0.816	0.008	21
19	(224,234)	1.028	0.028	10	38	(155,168)	0.808	0.008	21

Next, we apply the MAX-OVERTHROW-COMPUTING (G) algorithm to the BI largest component in order to detect and overthrow the most powerful node to its weakest position. First, the algorithm calculates the initial SVs. The results were represented in Table 9.

Node 242 is detected as the Target-node with SV=2.46. Next, the algorithm establishes the set of links in order to overthrow node 242 to the weakest position. The list of consequently established links is represented in Appendix B. Ninety six links were created to overthrow node 242 from the position of the most powerful node to the weakest position in the network. The resulting SVs for all nodes in the network are represented in Table 11.

Table 11. Resulting SVs for the BI largest component based on the MAX-OVERTHROW-COMPUTING (G)

Node	Shapley Value	Node	Shapley Value	Node	Shapley Value	Node	Shapley Value
66	0.9	167	0.79	181	0.59	230	1.46
138	1.03	168	1.46	182	0.59	233	0.6
155	0.68	169	0.66	184	0.6	234	1.34
157	0.63	171	0.57	187	1.67	235	0.64
162	1.82	175	1.46	224	1.46	242	0.54
163	1.1	176	0.57	227	0.75	248	1.82
166	0.79	179	1.02	229	1.14	249	1.34

6. CONCLUSION

An important factor in the analysis of leadership formation is to use a suitable measure. For this purpose, we employed the concept of Shapley value in the interpretation of Aadithya et al. (2010). Specifically, based on the SV concept we developed the algorithms that detect the network's most influential nodes and overthrow them to the weaker positions. Specifically, the K-OVERTHROW-COMPUTING (G, k) establishes k-number of links allowed to build by the decision-maker and MAX-OVERTHROW-COMPUTING (G) algorithm establishes the set of links to get the leader's SV to its minimally possible value. Both algorithms are based on the greedy approach.

Initially, we showed how our approaches work based on the trivial network topologies. Next, we tested them based on two real-life networks. Specifically, we applied the algorithms to the NHH and BI largest connected components.

The represented algorithms are applicable in the analysis of real-life cases, such as criminal networks with hidden patterns or money laundering networks. In these kind of networks, the overthrow of the detected leader may cause serious damage. In the real-life networks, the represented algorithms are not the unique solutions, but they are useful methods to detect and to plan the prospective network's modifications.

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APPENDIX A. *MAX-OVERTHROW-COMPUTING (G) applied to the NHH largest component*

#	Link	SV(Target)	Decrease	#	Link	SV(Target)	Decrease
1	(40,58)	1.742	0.248	35	(34,65)	0.731	0.008
2	(40,60)	1.608	0.133	36	(37,40)	0.723	0.008
3	(40,67)	1.508	0.100	37	(37,60)	0.716	0.008
4	(53,58)	1.425	0.083	38	(37,67)	0.708	0.008
5	(58,67)	1.358	0.067	39	(39,60)	0.702	0.006
6	(40,65)	1.301	0.057	40	(37,65)	0.696	0.006
7	(53,60)	1.244	0.057	41	(39,67)	0.689	0.006
8	(60,67)	1.196	0.048	42	(9,37)	0.683	0.006
9	(9,58)	1.149	0.048	43	(37,53)	0.676	0.006
10	(9,40)	1.107	0.042	44	(39,40)	0.670	0.006
11	(40,53)	1.071	0.036	45	(39,58)	0.663	0.006
12	(58,65)	1.036	0.036	46	(9,39)	0.658	0.005
13	(9,60)	1.004	0.032	47	(39,53)	0.652	0.005
14	(53,67)	0.972	0.032	48	(39,65)	0.647	0.005
15	(9,65)	0.947	0.025	49	(40,45)	0.641	0.005
16	(53,65)	0.925	0.022	50	(45,58)	0.636	0.005
17	(21,67)	0.911	0.014	51	(45,60)	0.630	0.005
18	(21,40)	0.897	0.014	52	(45,67)	0.625	0.005
19	(21,58)	0.883	0.014	53	(9,45)	0.620	0.005
20	(21,60)	0.869	0.014	54	(40,50)	0.615	0.005
21	(26,58)	0.858	0.011	55	(45,53)	0.611	0.005
22	(26,40)	0.847	0.011	56	(45,65)	0.606	0.005
23	(26,60)	0.836	0.011	57	(50,58)	0.601	0.005
24	(26,67)	0.825	0.011	58	(50,60)	0.596	0.005
25	(9,21)	0.816	0.009	59	(50,67)	0.592	0.005
26	(21,53)	0.807	0.009	60	(9,50)	0.587	0.004
27	(34,40)	0.798	0.009	61	(40,52)	0.583	0.004
28	(26,65)	0.789	0.009	62	(50,65)	0.579	0.004
29	(34,58)	0.780	0.009	63	(52,53)	0.575	0.004
30	(34,60)	0.770	0.009	64	(52,58)	0.571	0.004
31	(34,67)	0.761	0.009	65	(52,60)	0.567	0.004
32	(37,58)	0.754	0.008	66	(52,67)	0.563	0.004
33	(9,34)	0.746	0.008	67	(9,52)	0.559	0.004
34	(34,53)	0.739	0.008				

APPENDIX B. *MAX-OVERTHROW-COMPUTING (G) applied to the BI largest component*

#	Link	SV(Target)	Decrease	#	Link	SV(Target)	Decrease
1	(230,249)	2.128	0.332	42	(155,249)	0.778	0.008
2	(230,234)	1.961	0.167	43	(157,168)	0.771	0.006
3	(234,249)	1.828	0.133	44	(155,187)	0.765	0.006
4	(230,248)	1.728	0.100	45	(157,175)	0.759	0.006
5	(168,249)	1.644	0.083	46	(138,248)	0.752	0.006
6	(224,230)	1.578	0.067	47	(157,224)	0.746	0.006
7	(234,248)	1.511	0.067	48	(157,230)	0.739	0.006
8	(175,249)	1.444	0.067	49	(157,234)	0.733	0.006
9	(168,234)	1.397	0.048	50	(157,249)	0.726	0.006
10	(175,230)	1.349	0.048	51	(155,248)	0.721	0.005
11	(187,248)	1.302	0.048	52	(157,187)	0.716	0.005
12	(224,249)	1.254	0.048	53	(162,168)	0.710	0.005
13	(168,230)	1.218	0.036	54	(162,175)	0.705	0.005
14	(175,234)	1.183	0.036	55	(162,224)	0.699	0.005
15	(187,249)	1.147	0.036	56	(162,230)	0.694	0.005
16	(224,248)	1.111	0.036	57	(162,234)	0.688	0.005
17	(187,230)	1.083	0.028	58	(162,249)	0.683	0.005
18	(168,248)	1.056	0.028	59	(157,248)	0.678	0.005
19	(224,234)	1.028	0.028	60	(162,187)	0.673	0.005
20	(175,248)	1.003	0.025	61	(163,168)	0.668	0.005
21	(248,249)	0.980	0.023	62	(163,175)	0.663	0.005
22	(187,234)	0.958	0.022	63	(163,224)	0.659	0.005
23	(66,224)	0.946	0.011	64	(163,230)	0.654	0.005
24	(66,168)	0.935	0.011	65	(163,234)	0.649	0.005
25	(66,175)	0.924	0.011	66	(163,249)	0.644	0.005
26	(66,230)	0.913	0.011	67	(162,248)	0.640	0.004
27	(66,249)	0.902	0.011	68	(163,187)	0.636	0.004
28	(66,187)	0.893	0.009	69	(166,168)	0.632	0.004
29	(66,234)	0.884	0.009	70	(166,175)	0.628	0.004
30	(138,224)	0.875	0.009	71	(166,224)	0.624	0.004
31	(138,168)	0.866	0.009	72	(166,230)	0.619	0.004
32	(138,175)	0.857	0.009	73	(166,234)	0.615	0.004
33	(138,230)	0.847	0.009	74	(166,249)	0.611	0.004
34	(138,249)	0.838	0.009	75	(166,187)	0.607	0.004
35	(155,234)	0.831	0.008	76	(163,248)	0.604	0.004
36	(66,248)	0.823	0.008	77	(167,168)	0.600	0.004
37	(138,187)	0.816	0.008	78	(167,175)	0.596	0.004
38	(155,168)	0.808	0.008	79	(167,224)	0.593	0.004
39	(155,175)	0.801	0.008	80	(167,230)	0.589	0.004
40	(155,224)	0.793	0.008	81	(167,234)	0.585	0.004
41	(155,230)	0.785	0.008	82	(167,249)	0.582	0.004

APPENDIX B. ***Continued***

#	Link	SV(Target)	Decrease	#	Link	SV(Target)	Decrease
83	(166,248)	0.578	0.003	90	(169,249)	0.556	0.003
84	(167,187)	0.575	0.003	91	(167,248)	0.553	0.003
85	(168,169)	0.572	0.003	92	(169,187)	0.550	0.003
86	(169,175)	0.569	0.003	93	(171,224)	0.547	0.003
87	(169,224)	0.565	0.003	94	(168,171)	0.544	0.003
88	(169,230)	0.562	0.003	95	(171,175)	0.541	0.003
89	(169,234)	0.559	0.003	96	(171,230)	0.538	0.003