Norwegian School of Economics
Bergen, Fall 2015

The Nature and Causes of the Norwegian Interbank Offered Rate

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Master thesis in Economic Analysis

NORWEGIAN SCHOOL OF ECONOMICS

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ABSTRACT

The importance of interbank rates for unsecured funding has increased vastly the last decades with the expansion of financial instruments. Today's interbank rates are arguably the most influential benchmarks in pricing of assets and an important indicator on the state an economy. In the aftermath of the financial crisis, the awareness of weaknesses of interbank rates surfaced. The awareness has led to a tightening of the regulations regarding the Norwegian Interbank Offered Rate (NIBOR). The purpose of this paper is to identify the nature of NIBOR in both a domestic and international context, and expand on NIBOR's ability to accurately reflect the lending cost between Norwegian prime banks. The first part of the paper uses the Nelson-Siegel and Vasicek models to compare offered rates against observable financing cost using unsecured corporate bonds. NIBOR has historically been quoted higher than both STIBOR and EURIBOR, and we find that Norwegian banks contributing to NIBOR and STIBOR face the same financing costs as European banks contributing to EURIBOR. This implies that the differences between interbank rates cannot be justified by higher financing costs. When comparing the interbank rates to domestic financing costs, we are unable to determine if banks contributing to NIBOR are more or less accurate in the Norwegian interbank market compared to other interbank markets where these banks are present. In the second part of the paper, we compare individual interest rate quotes to credit default swaps, and observe an inconsistent relationship between panel banks' quotes and their market priced risk over time. By applying a hidden markov model, we examine individual short term behavioral dynamics during the opening of the day, and preceding the fixing. Our results indicate that interpretation of information varies across participants, which is a possible weakness of the governance structure.

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ACKNOWLEDGEMENTS

We would like to thank our supervisor Associate Professor Aksel Mjøs for his facilitation and guidance during the writing process. The topic was originally suggested by Aksel, and we appreciate the opportunity he gave us when allowing us to use and expand on the topic he presented. Acquiring such an extensive amount of information on NIBOR had not been possible without his help.

In addition we would like to thank important contributors whom have helped us along the way with information, data and input (in alphabetical order):

Steen Alva-Jørgensen, Oslo Børs, Thore Johnsen, Trond Sannes Marthinsen, Finance Norway, Torgeir Stensaker, Per Erik Stokstad, and Thor Tellefsen.

Erlend and Anders

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I. Introduction

The aim of this paper is to examine and evaluate the validity and trustworthiness of the Norwegian interbank rate through an empirical analysis. By reviewing historical data for NIBOR itself compared to other relevant sources of data, we aim to highlight some of the consistencies and perhaps inconsistencies in the most important benchmark rate in the Norwegian money market: The Norwegian Interbank Offered Rate.

A. A brief history of interbank offered rates

It is difficult to establish a clear definition, interpretation or answer to what interest rates really represents, but their existence has been essential in the development of financial markets. During the 11th century, the first recorded deposit rate was offered by the Bank of Venice (Homer u.a., 2005). As the financial foundations in modern Europe arose during the forthcoming centuries, loans with maturity less than a year were being offered by central banks for short term funding to local and foreign merchants, and the establishment of standardized interest rate markets arose. Short term interest rates in the Norwegian money market dates back to 1818, governed solely by the central bank of Norway, Norges Bank (2015).

Our interpretation of interest rates was reformed during the 1970's and 1980's with the emergence of financial instruments such as derivatives, complex coupon structures and index-linked bonds. As financial parties actively traded derivatives over the counter in London, the British Bankers' Association established the first standardized, but decentralized interest rates. The US Dollar London Interbank Offered Rate (LIBOR) was officially established January 1. 1986 (Jordan, 2009), and has since been quoting daily interest rates for a variety of maturities. The existence of interbank rates serves several purposes. Most importantly, interest rates for unsecured lending is an accurate proxy for the state of macroeconomic and liquidity conditions in a country or market, and provides a building block for the pricing of financial assets. Since the establishment of LIBOR, nearly every developed financial ecosystem has established their own offered rate for the interbank market, such as Tokyo, Stockholm, Copenhagen, the eurozone and Norway.

During recent years, the number of exchange-traded and over-the-counter (OTC) derivatives traded between parties has increased vastly. Derivatives, which are financial instruments whose value solely rely on other factors - such as the interest rate, are popularly provided in markets for investment and risk management. The emergence of the derivatives market is illustrated with data from Bank for International Settlements (2015) in the left graph in figure 1, where the derivatives market is decomposed into interest rate, foreign exchange, equity and CDS derivatives. The derivatives market is dominated by financial instruments linked to interest rates, which is decomposed in the graph to the right in figure 1. The three main sources of derivatives are forward rate agreements, interest rate swaps, and interest rate-linked options. These derivatives with trillions

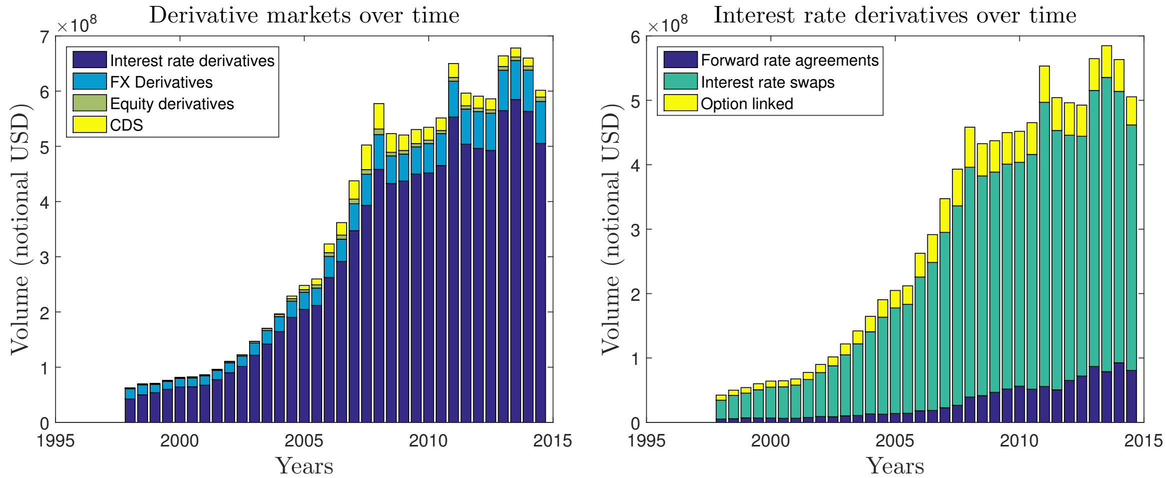

Figure 1. The evolution of OTC derivative volumes and interest rate-linked derivatives during 1998-2014 (trillions USD).

in notional value are priced contingent on the development of some underlying factor. The value of the majority of interest rate derivatives are dependent on a benchmark such as NIBOR, LIBOR and EURIBOR.

Having a well-functioning money market independent of the central bank is important to ensure that interest rates reflect the cost of unsecured loans between financial corporations. Historically, the interbank rates have proven to be a superior estimate to the market lending cost compared to government Treasury bills and bonds, and is today the benchmark for the markets interest rate. During the three last centuries, the interbank rates have been quoted daily for a variety of maturities. The interest rate for unsecured loans can be decomposed into three concepts; the risk-free interest rate, default risk premium, and currency risk.

An important concept within interest rates is the risk-free interest rate. A risk-free return is defined as the financial gain from investing in an asset with no risk of financial loss. Investing in an asset with no default risk is still not necessarily risk-free, as there is interest risk if the future risk-free rate is uncertain. An investment in a risk-free asset implies that an investor chooses to receive a fixed payment equivalent to today's risk-free rate, instead of continuously investing in the future risk-free rate. The reinvestment requirement for risk-free assets states that an investor at any time should be able to reinvest in the risk-free asset. In reality, it is impossible to find an asset which is truly risk-free, but in many cases, government Treasury bills are a good proxy for the risk-free interest rate.

The interbank offered rate serves many important purposes, but there are also drawbacks to the structure and nature of the interbank offered rates. The most obvious one is having them reported at such high frequencies that they often are based on hypothetical transactions. As they are offered rates they are not binding, and serve as an estimate of the cost of lending money unsecured in the interbank market, a cost determined solely by each contributing member. Short maturities and few transactions make validating the offered rates challenging and perhaps problematic as there are derivatives worth trillions of dollars contingent on the interbank rates across the world. The awareness of the weaknesses of the interbank offered rates surfaced during the LIBOR-scandal, which uncovered comprehensive manipulation of the London Interbank Offered Rate.

To facilitate for a Norwegian interbank money market, and make the Norwegian krone less dependent upon the US Dollar, the Norwegian Interbank Offered Rate was established in 1986.

B. *What is NIBOR?*

NIBOR - Norwegian Interbank Offered Rate is a collective term for Norwegian money market rates with different maturities. The major participants in the Norwegian money and currency market, which are referred to as panel banks, determine NIBOR. Participation is voluntary, but the requirement for being a panel bank is to be active in the Norwegian money market. NIBOR is calculated as an average of the submitted quotes from each panel bank after removing the highest and lowest outlier.

Finance Norway, which is the industry organization for the financial industry in Norway, has an official mandate to administrate and govern NIBOR. According to Finance Norway, the aim of the submitted NIBOR quotes is to reflect the level of interest rate a panel bank requires to lend unsecured NOK with delivery in two days, "spot", to another leading bank that is active in the Norwegian money and currency market. NIBOR is hence defined as a lending rate, and the interest rates are based on the individual banks' lending and currency costs. The reported interest rates are the individual panel banks' best estimate on their lending cost, and are not in any form considered to be a binding offer(FinanceNorway, 2015).

Due to the dollar and euro market being the preferred liquidity source, the Norwegian Interbank Offered Rate is often approximated as a currency swap rate. Each panel bank determines their lending cost based on the current interest rates in USD/EUR money market. Using c as notation for foreign currency abroad, NIBOR can be expressed as follows

$$(1 + i_{NIBOR}) = (1 + i_{N,C})\frac{F}{S}, \tag{1}$$

where F is the forward rate and S is the spot rate in number of NOK for each unit of foreign currency, and $i_{N,C}$ is the foreign lending cost for Norwegian banks. On logarithmic form, this simplifies to

$$i_{NIBOR} = i_{N,C} + f - s. \tag{2}$$

We previously introduced the concept of risk-free interest rates, denoted rf , which is the default free compensation required to exchange a fixed rate for future floating rates. In a foreign economy with no currency risk the interest rate for unsecured loans is

$$i_{C,C} = rf_C + rp_C, \tag{3}$$

where rp_c is the premium for default risk in the foreign economy. This equation could be subject to further expansion by examining two different cases for Norwegian banks when lending in the foreign economy. If Norwegian banks can lend to domestic interbank rate, $i_{C,C} = i_{N,C}$, then NIBOR can be written as

$$i_{NIBOR} = rf_C + rp_C + f - s. \tag{4}$$

However, if lending to a Norwegian bank in a foreign economy has a different risk exposure compared to the domestic banks, $i_{C,C} \neq i_{N,C}$, Norwegian banks face a different risk premium compared to the domestic banks. This difference rp_N, can be either positive or negative depending on relative risk. This entails rewriting NIBOR as

$$i_{NIBOR} = rf_C + rp_C + f - s + rp_N. \tag{5}$$

C. History of NIBOR

The oldest official records of NIBOR from Norges Bank dates back to 1986. However, the interest rate's importance has increased vastly during the 21st century, which also has been a turbulent period for NIBOR. Preceding the financial crisis in 2008, the underlying rate was LIBOR. Due to the lack of regulatory framework, it was highly unclear what NIBOR really was expressing, and with the financial crisis came volatile times in the financial markets. As the financial instability was at its peak with the bankruptcy of Lehman Brothers, both LIBOR and the volatility of LIBOR rose significantly. The NIBOR panel banks decided to leave LIBOR as their reference rate for foreign lending cost in September 2008. The new reference rate became the dollar rate published by Carl Kliem Interbank & Security Brokers (Bernhardsen u.a., 2012).

In the aftermath of the financial crisis, the panel banks, with support from Norges Bank, expressed the need for Finance Norway to take charge of NIBOR. Finance Norway was asked to formalize NIBOR, as the importance of NIBOR in the derivatives market and as a reference rate was rapidly increasing. Finance Norway implemented the first formal NIBOR regulations in August 2011. Although the regulations only were a formalization of established practice, it now became more evident what NIBOR really was, and the panel banks' responsibilities (Stokstad, 2014). The NIBOR steering group was also implemented, and was given the role of continuously assessing both the regulations and the submitted quotes, and advice Finance Norway accordingly (FinanceNorway, 2013).

The period both preceding and following this implementation was influenced by the early suspicion and evidence of the London interbank offering rate being manipulated. During this volatile period, the NIBOR regulations were under constant assessment. Already on October 5th 2011 the *Ministry of Finance*, the ministry responsible for planning and implementing the Norwegian economic policy, requested the Norges Bank to assess Finance Norway's new rules for NIBOR. In its reply to the Ministry of Finance, Norges Bank pointed out both the lack of transparency in the interest rate setting, and insufficient guidelines and governance to address principal-agent problems that could occur within and between the banks. The Ministry of Finance formally requested *Finanstilsynet*, the Financial Supervisory Authority in Norway, to thoroughly review the NIBOR regulations (Finanstilsynet, 2012).

The conclusion of the review was forwarded to the Ministry of Finance in April 2013, pointing out several weaknesses with the current NIBOR system, and Finance Norway implemented several changes in June 2013. They introduced the NIBOR compliance committee, and increased transparency through requirements of documentation regarding the assessment of the quote. In addition, Finance Norway was now responsible for making the individual panel bank quotes available for the public (FinanceNorway, 2013). This was however only the first phase of changes to the regulations. The second phase was implemented on December 9th 2013, which in addition to previous changes included Oslo Børs as the new calculating agent, a reduction the number of maturities from ten to five(as of January 1st 2014), and guidelines for how the NIBOR quotes should be determined among the individual banks. These are the last changes done to the NIBOR regulations (FinanceNorway, 2013).

D. NIBOR today

As of 2015 there are six banks submitting NIBOR quotes each day, namely DNB Bank ASA, Danske Bank, Handelsbanken, Nordea Bank Norge ASA, SEB AB and Swedbank. These participants report their estimated interest rate for maturities 1 week, 1 month, 2 months, 3 months and 6 months to the NIBOR calculating agent, Oslo Børs. Oslo Børs continuously records submitted quotes, indicative deposit rates (IDR), during the day. The IDRs reflect what the individual banks believe the current market rate to be throughout the entire day. At 12:00 pm Oslo Børs extracts the most recent IDR from each bank, removes the highest and lowest outlier, and then calculates NIBOR based on an average of the remaining IDRs (FinanceNorway, 2013).

The governance structure of NIBOR has been strengthened significantly in recent years through establishment of an improved governance structure. Today the regulation and development of NIBOR is no longer only in the hands of Finance Norway, it is also in the hands of the NIBOR steering group, the NIBOR compliance committee, and the NIBOR monitoring body. The NIBOR steering group is a precautionary measure in the way that it is responsible of following up on the development and implementation of the regulations. The NIBOR compliance committee is on the other hand monitoring compliance with rules and the correctness of reported data. Oslo Børs has, in addition to being calculating agent, the role as NIBOR monitoring body. This role entails being a support function for the compliance committee by supplying relevant information in line with their tasks.

II. Background

The Norwegian Offered Interbank is currently administered by Finance Norway. As of 2014, they revised the calculation and control regime in order to make the fixing process more transparent, and to increase the trustworthiness of the reference rate. The Oslo Børs Market Surveillance now has a responsibility of following up on unexpected activity in the interest rate fixing. In this regard, Finance Norway has expressed the wish for a comparative analysis of cross-country governance of interbank rates in light of the LIBOR scandal. In the extension to the preceding analysis, it was desirable with suggestions for future improvement on the governance of NIBOR.

A. Research question

Norges Bank has on several occasions pointed out weaknesses in NIBOR. In their letter to Finanstilsynet (2014), they point out that the NIBOR banks' individual contribution vary without obvious logical patterns. Bernhardsen, Kloster and Syrstad (2012) pointed to the fact that NIBOR had been high relative to STIBOR when the financial situation in the eurozone was uneasy, which seems abnormal as the economic condition in Europe should affect the Norwegian and Swedish interbank rates equally. Kyrre Aamdal (2014), a representative of one of the panel banks, rationalise these differences by claiming that the banks must obey the laws of covered interest rate parity between the interest rate in Norway and interest rates in other currencies. He illustrated this by swapping NIBOR and STIBOR to euro, and thereby comparing it with EURIBOR. Syrstad u.a. (2014) responded that Aamdal does not risk adjust the alternatives, and that the NIBOR panel banks wrongly let the euro-specific risk be directly incorporated in the NIBOR rate.

Figure 2 compares the various interest rates across countries over time. Even after swapping NIBOR and STIBOR using spot and forward exchange rates, the graph to the right in figure 2 clearly shows a discrepancy between NIBOR and other interbank rates. The difference between rates seems to be persistent even after the period of financial unease referred to by both Bernhardsen et. al and Aamdal. This discrepancy is the foundation for our examination of the dynamics of NIBOR.

The paper's first research question will address NIBOR in a broader sense. We aim to determine the validity of NIBOR in an international perspective by looking at the relationship between NIBOR

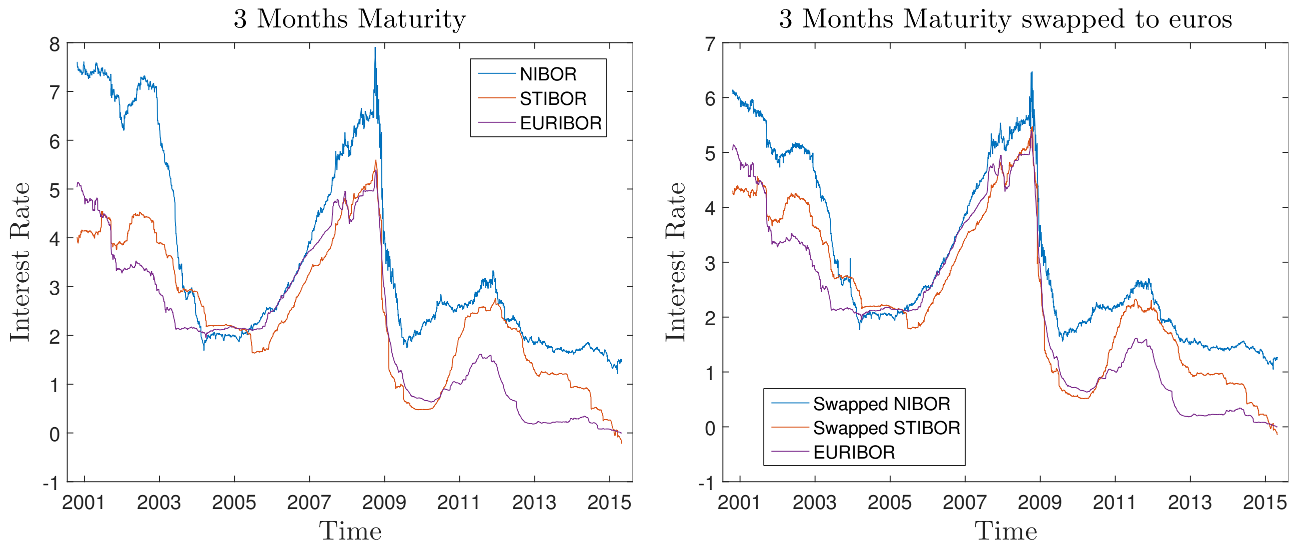

Figure 2. Cross-country interbank rates

and other sources of market information, and determine how the funding cost for the banks is reflected in the interest rates.

1. *Does NIBOR represent the true lending cost in the Norwegian interbank market?*

Although international reference rates such as NIBOR, STIBOR and EURIBOR all represent the same short interest rate for unsecured funding in their respective currencies, their governance and fixings are quite different. This give rise to our second research question, which relates to the fixing structure.

2. *How does the structure and governance of NIBOR effect the participants contribution to the interest rate?*

B. Paper structure

The paper is divided into two main chapters, one for each research question. The first perspective will examine the NIBOR fixing, while the second perspective will decompose the fixing into the individual quotes, and thereby look at the microstructure of the interest rate.

By evaluating and estimating two models for the yield curve of NIBOR, STIBOR and EURIBOR across maturities and time, we can compare the time variant dynamics. A continuous yield curve model can be compared to historical bond data for the NIBOR, STIBOR and EURIBOR panel banks. We hypothesize that corporate bonds issued by panel banks have the same risk dynamics as the interbank rates, and evaluate if NIBOR and STIBOR reflect the lending costs in the corporate bond market.

The second chapter extends the analysis by looking at the microstructure of NIBOR. This section explores the relationship between Norwegian interbank market risk, proxied by credit default swap prices, and the individual interest rate quotes to see whether the structure and governance of the interest rate makes it reflect domestic credit risk. In the extension, it explores whether there are interesting cross bank characteristics with regards to the relationship between domestic risk and the interest rates. Finally, we look at the behavior of banks on an intraday basis, to examine whether there seem to be a mutual understanding of what affects the short term development of the interest rate.

The common denominator for the analysis will be to draw inference about NIBOR by examining other sources of market information such as bond data, STIBOR, EURIBOR, credit default swaps for the panel banks and high frequency microdata for NIBOR. The paper seeks to answer the underlying research question by utilising proxies for risk present in the interbank market, and thereby draw inference about NIBOR. Based on this analysis we will make suggestions for improving NIBOR during our concluding remarks, and for future research that might expand the understanding of interest rates, and especially the Norwegian Interbank Offered Rate.

C. Out of scope

To limit the extensiveness and to enable a comprehensive evaluation of the Norwegian interbank rates, we have made some limitations to the scope of the paper. From a large sample of available interbank rates, we chose to use STIBOR and EURIBOR as relevant comparable interest rates for unsecured loans. Other closely related interbank rates are the US dollar LIBOR in London, and CIBOR for Danish krone in Copenhagen. There are two important aspects of LIBOR making it unsuitable in this paper. The somewhat turbulent and controversial fixing habits during the last decade create a bias and uncertainty regarding the validity of LIBOR itself. In addition, LIBOR by definition is a borrowing rate, whereas both NIBOR and EURIBOR are lending rates. As the Danish krone has been a part of the European Exchange Rate Mechanism since 1999, the Danish central bank ensures the exchange rate does not deviate more than ±2.25% (Nationalbank, 2009). This makes CIBOR superfluous to the superior European interbank rate.

D. Data sources

The primary data sources for the paper are Oslo Børs ASA, Thomson Reuters and Bloomberg. In addition to the primary data sources, additional information and data has been provided from panel banks regarding guarantees for bonds and insight to the fixing process, and Finance Norway regarding NIBOR regulations.

High frequency data (indicative deposit rates) for NIBOR with maturities 1 week, 1 month, 2 months, 3 months and 6 months has been supplied by Oslo Børs ASA, and was acquired 18.09.2015 with financial support from Finance Norway.

Historical fixing data for NIBOR, STIBOR and EURIBOR and currency exchange rates has been acquired from Thomson Reuters Datastream between 21.10.2015 and 10.12.2015. The underlying official sources for the interbank rates are Norges Bank for NIBOR, Stockholm Chamber of Commerce for STIBOR, and EBF - European Banking Federation/ACI - The Financial Markets Association for EURIBOR. The source for NOK/EUR and SEK/EUR exchange rates is WM/Reuters. Bond data has been gathered from Thomson Reuters Datastream between 20.10.2015 and 15.12.2015, and there is no underlying source for bond data beyond Reuters. Prices are obtained using Thomson Reuters Pricing Service for daily pricing, and the extracted prices are Thomson Reuters Price Service Evaluated Bid (TRPB). TRPB are considered to be end of day bid prices, and a more detailed explanation of TRPB is available in the Datastream Fixed Income Content information guide (2011). Treasury bill data for the currencies NOK, EUR and SEK has been acquired from Thomson Reuters Datastream between 20.10.2015 and 10.12.2015, and the source is Thomson Reuters. Prices are denoted as government bond indices calculated by Thomson Reuters. Detailed explanation of how the indices are calculated is available in the Thomson Reuters Government Bond Indices user guide (2013), and quoted prices are considered as annualized interest rates.

Credit default swap prices were supplied with the help from one of the panel banks on 26.11.2015, where the original source is Bloomberg. Due to strict limitations on credit default swap data, the quotes were supplied in graphs. The values were extracted from the graphs using measuring tools in Adobe Illustrator®. This ensured that the data extracted was as exact as possible. The extraction resulted in potentially small deviations, but the magnitude of the deviation is small compared to the size of the prices measured.

Before transforming and altering the data for more useful purposes, descriptive statistics for the original unedited data will be presented.

III. Descriptive statistics

To introduce the data material used in the paper, some descriptive statistics and highlights of key characteristics are presented for the interbank rates, fixing quotes, indicative deposit rates, bond prices and credit default swaps. Cross-country data will be presented first, containing information about NIBOR, STIBOR, EURIBOR, exchange rates and bonds in a broader perspective. After presenting the data for interbank rates in a global context, we will dig deeper into the data regarding individual NIBOR panel banks and the fixing process of NIBOR.

A. *Cross-country data*

In the paper, we have used historical data for the current maturities and previously discontinued maturities for NIBOR, STIBOR and EURIBOR. This includes tomorrow-next, 1 week, 2 weeks, 3 weeks, and 1-12 months, depending on the availability. The complete sample period is exactly 15 years, from 20.10.2000 to 20.10.2015. The sample is divided into two components: in-sample estimation and out-of-sample forecast for evaluation. The in-sample period is 20.10.2000 to 28.04.2015, which leaves 125 observations for the out-of-sample forecast.

Table I Descriptive statistics for structure of the interbank rates

	NIBOR	STIBOR	EURIBOR
Loan type	Offered unsecured interbank loan	Offered unsecured interbank loan	Offered unsecured interbank loan
Current maturities	1W 1M 2M 3M 6M	T/N 1W 1M 2M 3M 6M	1W 2W 1M 2M 3M 6M 9M 1Y
Discontinued maturities	T/N 2W 9M 1Y	9M 1Y	3W 4M 5M 7M 8M 10M 11M
Number of panel banks	6	6	24

Note: Some interbank rates have changed panel banks during the period.

In table I we have summarized relevant information regarding the availability of maturities for the various interbank rates. All interbank rates have decreased the number of quoted maturities during the period 20.10.2000 to 28.04.2015. Unconventional maturities have been dropped over the last years, and the most common maturities are 1 week, 1 month, 2 months, 3 months and 6 months. There is a considerable difference between EURIBOR and NIBOR/STIBOR regarding number of contributing banks, as EURIBOR reflects the interbank rate across the whole Economic and Monetary Union.

Table II displays NIBOR decomposed for the individual maturities, and equivalent tables for STIBOR and EURIBOR are presented in table XXIX and XXX, appendix A.A. Interestingly, the average interest rate for maturity tomorrow-next is higher than almost all other maturities. As there have been changes in the number of maturities during the time period, this representation of the descriptive statistics for NIBOR do not tell the whole story. To get a feeling of the behavior of the average yield curve for NIBOR we have divided the sample into different periods dependent on when maturities were excluded from NIBOR, and the average interest rates for the periods are shown in table III. We observe that for the two most recent periods the yield curve is strictly increasing and concave.

Table II Descriptive statistics for NIBOR

Maturity(months)	Time Period (obs)	Mean	Std. dev.	Minimum	Maximum
T/N	20.10.2000 - 18.11.2011 (2891)	4.04	2.05	0.57	9.37
1 Week	20.10.2000 - 28.04.2015 (3788)	3.38	2.03	1.28	9.1
2 Week	20.10.2000 - 30.12.2013 (3442)	3.66	2.02	1.49	8.25
1 Month	20.10.2000 - 28.04.2015 (3788)	3.42	2.02	1.25	9.13
2 Month	20.10.2000 - 28.04.2015 (3788)	3.46	2.01	1.25	8.46
3 Month	20.10.2000 - 28.04.2015 (3788)	3.51	1.99	1.21	7.91
6 Month	20.10.2000 - 28.04.2015 (3788)	3.60	1.95	1.15	7.95
9 Month	20.10.2000 - 30.12.2013 (3442)	3.96	1.86	1.66	7.79
1 Year	20.10.2000 - 30.12.2013 (3442)	4.05	1.81	1.72	7.9

Note: Number of observations in parentheses.

Table III Descriptive statistics for each sub period, NIBOR

Period	T/N	1W	2W	1M	2M	3M	6M	9M	1Y
20.10.2000 - 18.11.2011	4.04	4.00	4.03	4.04	4.06	4.12	4.20	4.27	4.35
21.11.2011 - 30.12.2013		1.76	1.80	1.88	1.95	2.05	2.24	2.36	2.48
31.12.2013 - 28.04.2015		1.56		1.57	1.60	1.63	1.65		

For STIBOR and EURIBOR the average interest rate curve is similar to NIBOR, and the average interest rates are lower than NIBOR for the same maturities. The trend for all interest rates is decreasing variation as time to maturity increases, which is expected as we know long term yields are more persistent.

In total, the panel banks for NIBOR, STIBOR and EURIBOR issued several thousand bonds during the period 20.10.2000 to 20.10.2015. This sample was substantially reduced after removing index linked, floating rate, graduate rate, and complex coupon bonds. The sample was further decreased, as some pricing data for earlier periods were unreliable due to lack of accurate prices or lack of liquidity in the bond market. All bonds are confirmed to be unsecured senior loans by either Thomson Reuters, Bloomberg or the panel banks. As the underlying risk is very important when comparing the unsecured interbank rate against other sources of data, we have excluded any bonds where we have been unable to confirm if the bonds are senior uncovered.

Table IV summarizes the total number of bonds from each bank, and the distribution of bonds used over banks and across years. The number of bonds are aggregated across the currencies NOK, SEK and EUR. The distribution across the years from 2010 to 2015 is sufficient, however there are large variations between banks, which is a weakness of the data set. Three European banks have been included in our data set, Barclays, Deutsche Bank and ING Bank, which will be discussed in greater detail during the evaluation of the bond data. The European banks provide a substantially larger sample of bonds than the Nordic banks.

Table IV Distribution of bonds across banks and years

Bank/Year	2010	2011	2012	2013	2014	2015	Total	Out-of-sample
DNB	0	7	1	0	2	2	12	
Nordea	0	4	7	0	1	1	13	6
Handelsbanken	0	1	0	0	0	0	1	
SEB	0	2	1	0	1	0	4	3
Swedbank	0	0	0	1	1	0	2	
Danske Bank	0	0	0	1	0	0	1	4
Barclays	0	1	6	5	7	4	23	
Deutsche Bank	8	6	1	2	3	1	21	
ING Bank	0	3	3	2	1	1	10	
Total	8	24	19	11	16	9	87	13

Note: 2015 matures before 28.04.2015. Out-of-sample matures after 20.10.2015.

In table V the bond data has been sorted based on currency, and we observe that the NIBOR banks have issued far more unsecured bonds in euros than NOK and SEK from 2010 to 2015. This is not a representative currency distribution for corporate bonds in general, as covered corporate bonds are not included. We use bonds with zero coupon when maturity is after the in-sample period, shown in the right column in table V. The out-of-sample bonds will be used separately when applying a secondary framework. Bonds will be referred to by the Reuters ID number throughout the paper. The ID can be used to identify the issuer, ISIN-identification number, coupon, currency and maturity date. A complete list of bonds is available in table XXXI, XXXII, XXXIII and XXXIV in appendix A.A.

Table V Distribution of bonds across banks and years

Currency/Characteristics	Fixed coupon	Zero coupon	Average coupon	Total no. bonds
NOK	6	1	5,4%	7
SEK	3	2	3,4%	5
EUR NIBOR banks	15	6	3,6%	21
EUR EURIBOR banks	41	13	3,1%	54
EUR out-of-sample	0	13	0%	13

Note: Aggregate across years.

B. NIBOR quotes

Our statistics on the individual quotes underlying the NIBOR fixing are daily observations dating back to 01.10.2012 across all maturities, except for the one week maturity which date back to 09.12.2013. Table VI presents a brief summary of the NIBOR fixings across maturities. We will when appropriate use the abbreviation DDB for Danske Bank, DNM for DNB, NDA for Nordea, SEB for SEB, SHB for Handelsbanken and SWD for Swedbank, as these are the formal abbreviations used by Oslo Børs.

Table VI Summary NIBOR fixing quotes, averages

interest rate	Time period(obs)	Fixing	Max	Min	Max/Min spread
1 week	09.12.2013 - 21.11.2015	1.44	1.51	1.38	0.13
	(490)	(0.224)			(0.0566)
1 month	01.10.2012 - 21.11.2015	1.55	1.60	1.51	0.09
	(789)	(0.219)			(0.0304)
2 month	01.10.2012 - 21.11.2015	1.58	1.62	1.55	0.07
	(789)	(0.2203)			(0.0278)
3 month	01.10.2012 - 21.11.2015	1.61	1.66	1.59	0.07
	(789)	(0.228)			(0.0262)
6 month	01.10.2012 - 21.11.2015	1.68	1.72	1.63	0.08
	(789)	(0.286)			(0.030)

Note: Observations and standard errors in parentheses.

Observe from VI that there does not seem to be any differences in the volatility across maturities. Individual quotes suggest that the spread between the maximum and minimum quotes is higher for short maturities than long maturities. Within our panel, this has been especially prevailing for the one week rate, but also to some extent for the one week rate. The spread has however been reduced in 2015 for the short maturities, and plots of spreads across maturities are available in figure 24 in appendix A.B. Note that this spread is not directly reflected in the interest rate in table VI as the highest and lowest values are discarded when fixing NIBOR.

Looking closer at the highest and lowest quotes, there seem to be some consistency in the relative level of the interest rate across the participants' contributions. Figure 3 shows the aggregate distribution across maturities of banks having the highest and lowest quote. The high end of the scale is mostly dominated by DDB which accounts for the highest quote roughly 60% of the time. The only exception is the one week rate, where DNM is most frequently the highest bidder. On the opposite side of the scale, the most dominant low bidder is SHB with an average of 40%. SHB is hence not as dominant on the low side as DDB is on the high side. Both SWD and SEB take turns being the most frequent low bidder dependent on maturity. The distribution divided by maturities may be found in figure 25 and 26 in appendix A.B.

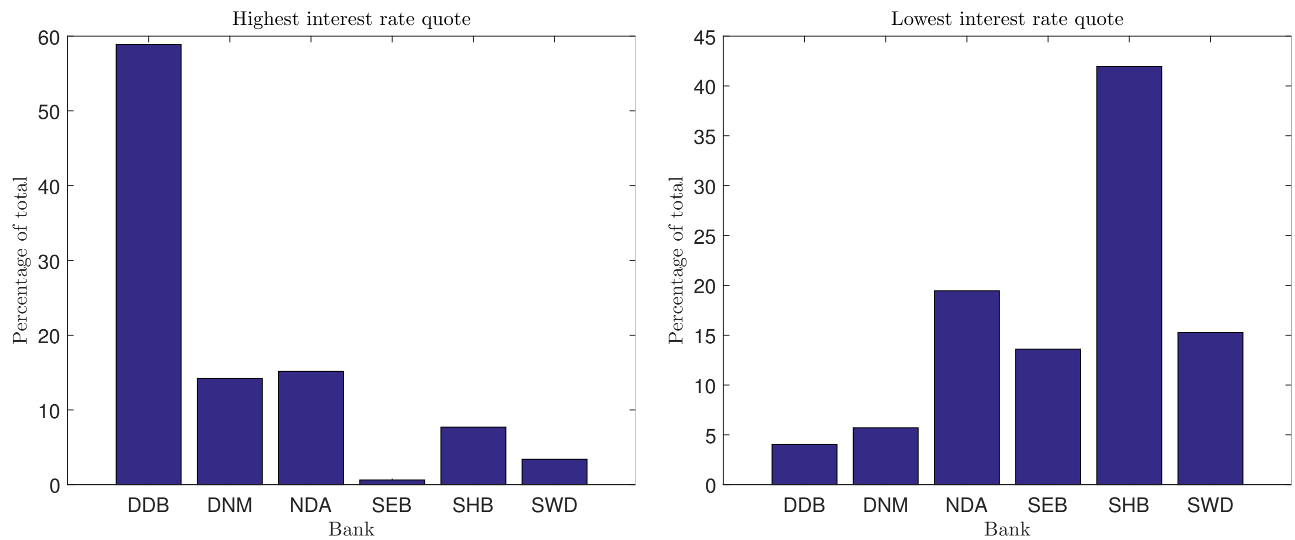

Figure 3. Note: Aggregate distribution of highest/lowest quotes divided by banks.

C. Indicative Deposit Rates

The indicative deposit rates were implemented 09.12.2013, and the data ranges from its introduction until 22.09.2015. Adjusting for reporting errors, we are left with roughly twenty thousand observations for each maturity. Observe from table VII that 6 month NIBOR is the only maturity with significantly more observations than other maturities. It has roughly thirty percent more changes. Observe also that the size of changes are relatively stable around one basis point, with quite heavy outliers connected with big economic events like key interest rate meetings. Within each of the maturities, the frequency distribution among the panel banks seem fairly constant. The common trend across all maturities is that NDA most frequently change their interest rates with roughly 30% of the total, and SWD being the least active with roughly 6%. Other panel banks fluctuate in the range of 10% to 20% of the total. Summary statistics on the relative frequency among individual banks may be found in table XXXV appendix A.C.

Table VII Summary Indicative Deposit Rates

interest rate	Time period(obs)	avg.obs/day	Mean change	Max increase	Max decrease
1 week	09.12.2013 - 22.09.2015	43.29	±0.01239	0.27	-0.27
	(21213)		(0.0179)		
1 month	09.12.2013 - 22.09.2015	44.83	±0.011	0.35	-0.3
	(21967)		(0.0148)		
2 month	09.12.2013 - 22.09.2015	41.67	±0.010	0.31	-0.28
	(20420)		(0.014)		
3 month	09.12.2013 - 22.09.2015	42.14	±0.010	0.3	-0.26
	(20649)		(0.0163)		
6 month	09.12.2013 - 22.09.2015	56.60	±0.0105	0.28	-0.28
	(27737)		(0.0138)		

Note: Number of observations and standard errors in parentheses.

From the frequency distribution for changes to the left in figure 4 the common trend is a peak during the two opening hours followed by a steady decline throughout the day. The activity in the last hour before the fixing seem to be reduced, while the activity during the last hour of the day is significantly lower than for the rest of the day. A summary of the aggregate frequency distribution across maturities may be found in figure 27 in appendix A.C. Knowing what the distribution of changes look like, another interesting property to examine is the size of the changes. As the index is operating at a two decimal accuracy level, any change must be least one basis point. Hence, a vast majority of the changes is one basis point in either direction as illustrated in the right figure 4. The distribution of change sizes across maturities may be found figure 28 appendix A.C.

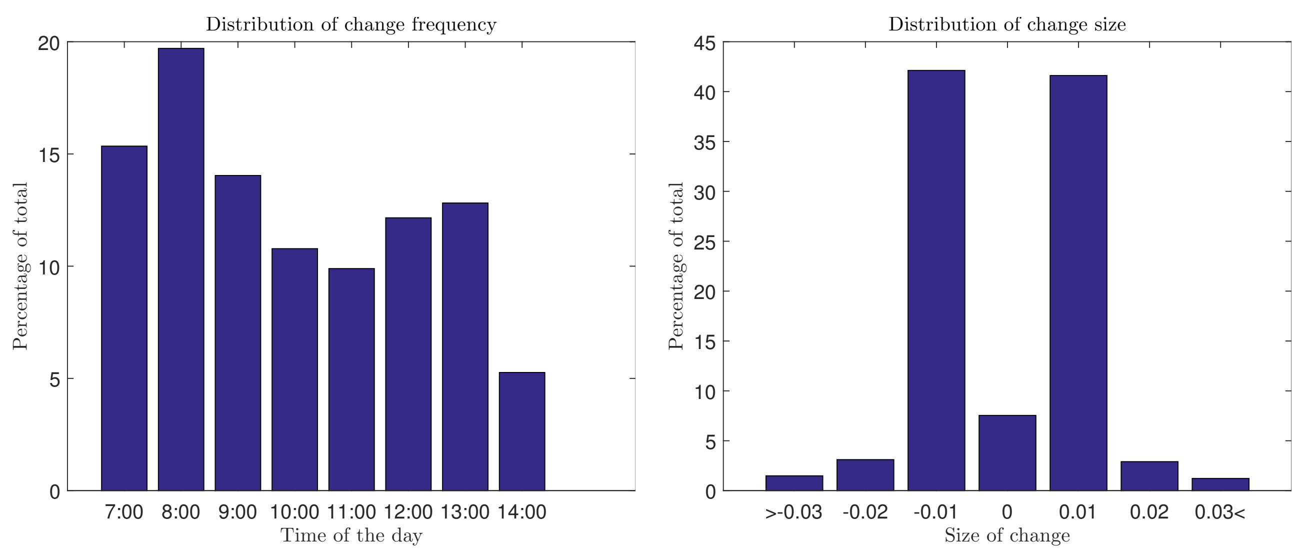

Figure 4. Empirical distribution properties of IDR quotes

D. Credit default swap spreads

The credit default swap (CDS) spreads are weekly observations dating back to 01.10.2012. These derivatives express the price of insuring a senior corporate bond from defaulting within one year. In other words, to insure 100 $ from defaulting within one year for any of the panel banks, you will need to pay 100 $ multiplied with the spread for the given bank. We have credit default swaps across all banks except Nordea, which was not available until late 2015. Nordea is thereby left out of the part of the analysis where the CDS spreads are utilised.

From the CDS spread development over time in figure 5 we observe that the spreads were significantly higher during the first part of our sample. Especially in the period preceding February 2013. While this could probably be attributed to macroeconomic factors, properties within banks, and a persistent aftershock of the credit crisis, bear in mind that there was not nearly such a difference in the spread across reported interest rates with respect to neither maturities nor time. A summary of the CDS spreads across time and banks may be found in appendix A.D. Regarding the individual characteristics, observe from figure 5 that SHB dominates having the lowest CDS spread, similarly they also dominated having the lowest interest rate bid. On the top side however, there is seemingly no consistency in having the highest spread, and Danske Bank is not nearly as dominant as with their reported rates.

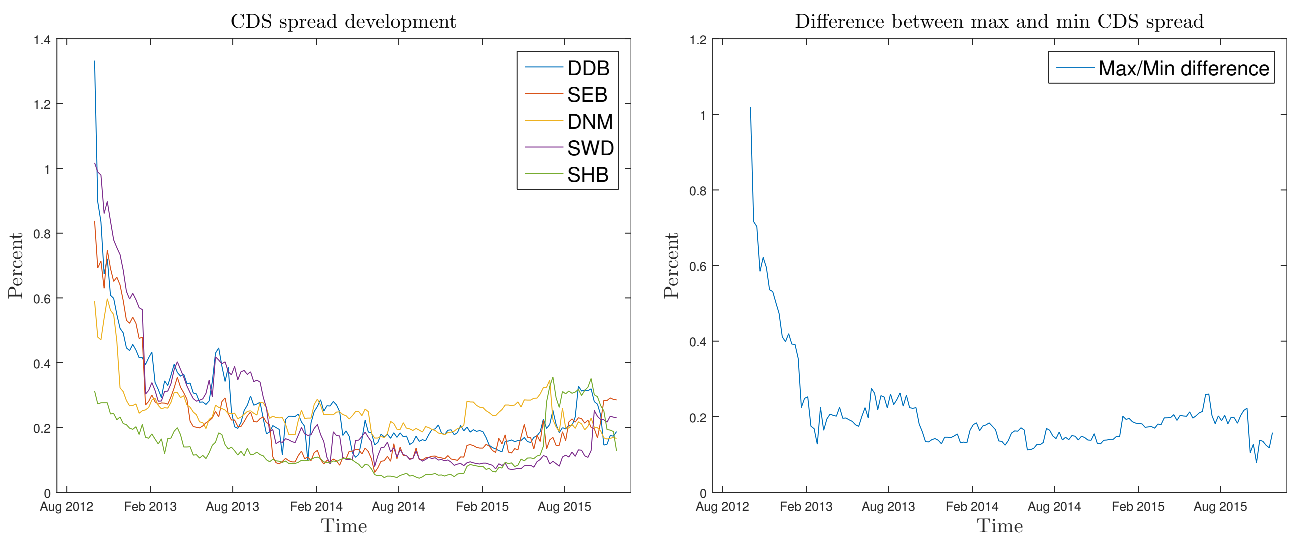

Figure 5. CDS spread development over time

IV. Chapter 1: Dynamics of interest rates

In this chapter, we wish to investigate the dynamics of NIBOR by exploring how the interest rate behaves across maturities and time, and how interest rates vary between currencies. In section II the interest rates across currencies were compared, and we observe differences between the swapped interest rate that we could not explain. This raises two important questions: Does NIBOR accurately reflect the lending cost in the Norwegian money market? The NIBOR panel banks state that foreign currency is the primary liquidity source. Does the swap rate reflect the financing cost in a foreign currency money market?

We will try to answer the first question by identifying how the Norwegian and foreign interbank rates behave across maturities and time, and continue by finding common factors that influence the interbank rates. Identifying the behavior of NIBOR and the country specific factors will help us determine the accuracy of NIBOR as a Norwegian money market lending rate. By comparing the identified factors against observable financing cost, we can evaluate if the interbank rates are consistent with market data. To answer the second question, we will compare interbank rates across countries under a common currency to identify variations not caused by currency differences. To elaborate on these two important questions, we will need to take a step back to our interpretation of what interest rates really are. Why do interest rates vary across countries, and how can we capture these dynamics in a financial framework?

A. A brief history of interest rate theory

Since the modern interpretation of banking and the bond market arose during the 11th century in today's Italy, the importance of interest rate has become imminent. During the last century, there has been extensive research and contributions to our understanding of interest rates. Since the rise of modern derivative pricing theory by Black, Scholes (1973) and Merton (1973), interest rate dynamics have become more meaningful due to the importance of the existence of a risk-free asset, and for modern bond portfolio management. During this time economists understood that an asset with no default risk is not necessarily risk-free, it is still exposed to interest rate risk tied to the assets' underlying interest rate dynamics. There exists two important frameworks in financial economics, which are applicable to understanding interest rate dynamics: the *no-arbitrage* approach and *equilibrium* approach. The equilibrium approach has its roots to the Arrow-Debrew equilibrium model (1954), and notable contributors are Vasicek (1977) and Cox, Ingersoll and Ross (CIR) (1985). The first no-arbitrage framework for interest rates was introduced by Ho and Lee (1986), and other notable contributors have been Hull and White (1990) who were the minds behind the extended Vasicek model consistent with no arbitrage. Heath, Jarrow and Morton (1992) generalized this through defining an explicit relationship between the drift and volatility for the short-rate dynamics consistent with no-arbitrage. For the most common models, the innovation of the short-rate can all be generalized with

$$dr(t) = k(\theta(t, r(t)) - r(t))dt + r(t)\sigma(t, r(t))dW_t, \tag{6}$$

where $r(t)$ is the short-rate at time t, θ is the long term trend, k is the mean reversion speed, σ is the short-rate volatility, and W_t is geometric Brownian Motion. By imposing different assumptions for the dynamics of each component, we can derive all the one-factor Gaussian models mentioned above.

Both the no-arbitrage and equilibrium framework build on fundamental assumptions about the short-rate consistent with some variation of the stochastic differential equation above. In 1987 an alternative approach was introduced by Nelson and Siegel (1987). They recognized the demand for identifying a simple model to represent a wide variety of yield curve shapes using different components for long, medium and short term yields. Nelson and Siegel presented a model for approximating the yield curve, with characteristics dynamic enough to incorporate increasing, decreasing and S-formed yield curves. Nelson and Siegel started in many ways in the opposite end of Vasicek and Cox by addressing what parametric model fits the yield data we actually have. The model has its roots from beliefs about the short-rate, but the underlying framework is not as complex as those presented by scholars before them are. After specifying the model, they asked themselves how the model fit with financial theory of the yield curve. The parametric model was empirically tested by Litterman, Scheinkman and Knez (1991; 1994) who identified that the three common factors could explain a substantial amount of variation in bond yields. The model was later refined for better economic interpretation and popularized by Diebold and Li (2006).

B. The yield curve

To make the models and results more applicable across securities, we will operate with yields instead of interest rates. Our definition of yield is the annualized total rate of return on a security. For interbank rates this corresponds to the interest rate offered to the interbank market, and for bonds it is the rate of return, which is given by the market price.

As financial interest bearing instruments both vary in structure (cash flow payments, coupon characteristics) and in time (maturity), they must be standardized by introducing a yield curve. As the offer rates for various currencies such as NIBOR, STIBOR and EURIBOR are all observable in the market, they can be converted to represent yields. By representing our interest rates using the yield curve, we can apply all the mentioned frameworks. The no-arbitrage framework by perfectly fitting the yield curve to observed data to prevent arbitrage, and the equilibrium framework to model the dynamics of the short-rate and deriving yields for other maturities by introducing some assumption about the markets risk premium. Lastly, we can use the yield observations to calibrate our parametric model to derive the time-varying factors, and later apply an autoregressive framework for the factors themselves.

C. Choice of framework

As mentioned, there are numerous models to choose from. In this paper, we have chosen to apply the model introduced by Nelson and Siegel, and the quilibrium model introduced by Vasicek. There are three key reasons that we chose not to apply any of the no-arbitrage models, which are widely used and certainly would be applicable in this case. The financial interbank market is definitely efficient enough to impose no arbitrage restrictions, however, our first reason for discarding this approach is the poor forecasting and empirical characteristics (especially out-of-sample) of the models, which has been pointed out in many previous papers (when applied to bond yields) (Duffee, 2002; Christensen u.a., 2011). Our second reason for not using a no-arbitrage model is that no-arbitrage restrictions can be incorporated into the Nelson-Siegel model, as shown by Christensen, Diebold and Rudebusch (2011) who introduce an affine arbitrage-free Nelson-Siegel model. Lastly, many of the no-arbitrage frameworks contain parameters that change over time, such as the Hull & White model with time dependent long term equilibrium. In many cases, this introduces a new stochastic differential equation and additional parameters, which increases the complexity of the model. Our second choice is the Vasicek model introduced in 1977. This model is commonly used in interest rate theory, and is a one-factor model with a few assumptions regarding the dynamics of the short-rate. There are several reasons for choosing the Vasicek framework; it is a one-factor model with few parameters, which makes it viable for few maturities. It portraits very similar characteristics as the Nelson-Siegel model which is convenient when comparing performance and results, and allows for negative interest rates.

Despite the differences between our two frameworks, there are fundamental building blocks, which are common for all interest rate models - modern interest rate theory.

D. Modern interest rate theory

An important concept when addressing interest rates is the *short-rate*, or *instantaneous* interest rate $r(t)$. The literature often uses the terminology instantaneous (nominal) forward rate, which is equivalent to short-rate. We define $r(t)$ as the interest rate on a risk-free asset between t and $t + dt$. Thus we can define the innovation of a bank account $B(t)$ equal to

$$\mathrm{d}B(t) = r(t)B(t)\mathrm{d}t, \tag{7}$$

and with the boundary condition $B(0) = 0$, the value of a bank account $B(t)$ can be expressed as

$$B(t) = e^{-\int_t^T r(s)\,\mathrm{d}s}. \tag{8}$$

Let $P(t)$ be the price of a zero coupon bond maturing at time T. Our formal definition for bond yields y_{tT}, by simply discounting gains, is

$$P(t) = P(T)e^{-y_{tT}(T-t)} \tag{9}$$

Thus we can elaborate on the relationship between the zero coupon bonds yield and the expansion of the short-rate by pricing 1 \$ at time t to be paid at time T using (8) and (9)

$$e^{-y_{tT}(T-t)} = e^{-\int_t^T r(s)\,\mathrm{d}s},$$

$$y_{tT} = \frac{1}{T-t}\int_t^T r(s)\mathrm{d}s, \tag{10}$$

and establish a relationship between the yield y_{tT} and the future short-rate $r(t)$. To further elaborate on the expansion of the yield curve we must impose some assumptions regarding the dynamics of the short-rate. The Nelson-Siegel model expresses the yield curve directly as a function of a set of parameters and time to maturity. Using this yield curve model we can trace the assumptions regarding the short rate. The Vasicek model denote the change in short-rate as a stochastic differential equation, and we can derive the short-rate and yield curve from this.

For valuation of bonds, we will use a simple framework for cash flow valuation to express bond values as yields. As bonds are not continuously compounded, we will use the discrete version of the yield model

$$P_t = \frac{100}{(1+y_t)^t}. \tag{11}$$

This allows us to calculate yield curve for bonds after adjusting for accrued interest (European 30/360 which is the most common day count conversion for bonds in euros).

E. *Function based yield curve model*

The function based yield curve model introduced by Nelson and Siegel (1987) and adjusted by Diebold and Li (2006; 2006) aims to approximate the yield curve using three factors which are financially sensible. The three-factor model aims to express the future short-rate using a parametric yield curve for modeling and forecasting. Diebold and Li expresses the yield curve using a Laguerre function with a constant:

$$y_t(T) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t T}}{\lambda_t T}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t T}}{\lambda_t T} - e^{-\lambda_t T}\right) \tag{12}$$

The specification above is popular for mathematical approximation (Francis X. Diebold u.a., 2006), and allows for non-linearities in the yield curve. In the model we have three factors, β_{1t}, β_{2t}, β_{3t}, and a decay parameter λ_t which determines the magnitude of the exponential decay rate. The three factors have been investigated by Litterman (1991) and by Diebold, and is commonly used by institutional financial organizations such as the IMF (Gasha u.a., 2010), ECB (Modugno u.a., 2009), Federal Reserve Bank of San Francisco (Diebold u.a., 2006). We can interpret the factors β_1, β_2 and β_3 as level, steepness and curvature.

Litterman and Diebold's approaches to evaluating the accuracy of the model are different, but their conclusions are the same. Litterman applies modern portfolio theory to create hedged bond portfolios that eliminate the exposure to the different factors and uses the portfolio out-of-sample to evaluate the performance compared to a normal duration convexity-hedged portfolio. Diebold uses ordinary least squares to estimate the factors based yield data, and an autoregressive framework for out-of-sample forecasting. We can investigate if the model inhabits the characteristics we know portraits yield curves by starting to identify key features for the yield curve:

1. The average yield curve is increasing and concave.
2. The yield curve can have a variety of shapes with increasing maturity, including upward and downward sloping, humped and inverted humped.
3. The yield curve must reflect that negative interest rates may occur with probability greater than 0.
4. We know that spread dynamics (short term) are less persistent while yield dynamics are more persistent (long term).

The model does satisfy the shapes mentioned in 1 and 2 as $\frac{\partial y_t(T)^2}{\partial^2 T} > 0$, $y_t(T)$ can exhibit concavity and the property $\frac{\partial y_t(T)}{\partial T} > 0$ depending on the specification of β_{t2} and β_{t3}. Unlike interest models such as the CIR-model, this approach allows for negative interest rates, which is important due to the current interest level in the eurozone. To understand more of the dynamics in the model, and the economic interpretation of β_1, β_2, and β_3 it is useful to examine the behavior of the three factors for different conditions of $y_t(T)$. When we approach maturity 0, we can see that $y_t(0) = \beta_{1t} + \beta_{2t}$, and is the initial level for all yields regardless of maturity. This can be interpreted as the initial market premium required for lending money in the interbank market, before addressing any time contingent premium. When $T \to \infty$, $y_t(\infty) = \beta_{1t}$ we have the long term yield. β_1 effects all yields equally across maturities. It is the only factor for long maturities, and hence the long term level factor. Thus the three factors influence the yield curves differently; β_1, the level factor effects the yield on long term, β_2, the slope factor effects the yield on medium term, while β_3, the curvature factor effects the yield on short term.

To expand on the assumptions on the short-rate r_t in the model, we can derive the relationship between the short-rate and the yield curve. Let $t = 0$, and thus

$$y_t(T) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda_t T}}{\lambda_t T}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda_t T}}{\lambda_t T} - e^{-\lambda_t T}\right) = \frac{1}{T}\int_0^T r(s)ds \tag{13}$$

and we can derive the process for the short-rate:

$$r(T) = \frac{d}{dT} T y_t(T) = \beta_{1t} + \beta_{2t} e^{-\lambda_t T} + \beta_{3t}\lambda_t T e^{-\lambda_t T} \tag{14}$$

We can see that $\lim_{T\to 0} r(t) = \beta_{1t} + \beta_{2t}$ and $\lim_{T\to\infty} r(t) = \beta_{1t}$, similarly as the yield curve. Let

θ_{NS} be the long term trend for $r(t)$, and $r(0) = r_0$. Then $\theta_{NS} = \beta_{1t}$ and $r_0 = \theta_{NS} + \beta_{2t}$. Then our short-rate model is

$$r(t) = \theta_{NS}(1 - e^{-\lambda_t t}) + r_0 e^{-\lambda t} + \beta_{3t} T \lambda e^{-\lambda t} \tag{15}$$

This is an interesting result, as this is closely related to the Vasicek model shown later, despite the two models being derived in different ways. The two first components containing θ_{NS} and r_0 are similar, but the third element will deviate.

The Nelson-Siegel (and the modified Diebold-Li) model is fairly simple to estimate using non-linear optimization, however a complication using this approach is over-fitting. We aim to estimate four parameters using from 5 to 9 data points for NIBOR, which for some time periods creates suspiciously high variance in the estimated coefficients (the other interbank rates provide us with with more maturities on average, but do at times also lack the data to allow estimation of all four parameters). To address this issue there are two possibilities; increase the number of maturities or decrease the number of parameters. If we chose to increase maturities, interpolation will not create any additional information, so the only way would be to incorporate other sources of information such as yield data for Norwegian Treasury bills. As the underlying risks are quite different in the interbank market and government bonds - we first try to reduce the number of parameters before integrating additional data. It is popular in the literature to fix the decay parameter λ_t (Nelson u.a., 1987; Francis X. Diebold u.a., 2006), which determines at what maturity the loading on the curvature is at its maximum. When we ran the estimation for all four parameter, we can get a ballpark feeling of what value is fitting for the decay parameter. Diebold argues that $\lambda_t = 0.0609$, however we expect a higher decay parameter as we are working with far shorter maturities than for government bonds. Using a sample of 200 observations, after removing extreme results, we estimate λ to be 0.1157. A second argument for using a deterministic decay parameter is how this simplifies and increases the robustness of our solution. This is shown by looking a bit closer in the estimation technique we have to use if all four parameters are estimated - non-linear least squares. In appendix B.A we have derived the theoretical solution to our optimization problem, and from the Hessian matrix the problem is vastly simplified by fixing λ_t. This does influence the dynamics of the model, but as we do not know the behaviour of our yield curve outside the neighbourhood of the optimal solution, it is hard to evaluate if this is the global minimum.

The most useful application of the Nelson-Siegel model is our ability to model a continuous yield curve across both time and maturities. This allows us to compute a yield curve that matches instruments in real life, where prices fluctuate due to time-varying factors and decreasing time to maturity. Another possible approach is to model the interest rates themselves, which brings us to the stochastic short-rate model, where we identify characteristics for each maturity individually.

F. *Stochastic short-rate model*

Our second model will be an equilibrium-based framework. The choice of framework was introduced by Vasicek in 1977, and describes the future short-rate as a stochastic differential equation, which exhibits a set of properties: the future short-rate is only effected by a long term interest rate level θ, a decay parameter k, the volatility of the interest rate σ and a normally distributed Brownian Motion W_t. The Vasicek model is a special case of the Ornstein-Uhlenbeck process, and is both Gaussian (normally distributed) and Markovian ($E[r_t] = E[r_t|r_{t-1}] \; \forall \; t$). The stochastic differential equation for the model is

$$dr_t = k(\theta - r_t)dt + \sigma dW_t \tag{16}$$

This equation has an explicit solution for r_t which can be derived using stochastic calculus (and changing to more convenient notation). Let $dr(t) = k(\theta - r(t))dt + \sigma dW(t)$ and introduce the transformation $f(t, r(t)) = r(t)e^{kt}$. By Ito's lemma we can derive the stochastic differential equation which is done through several steps shown in appendix B.B. After deriving the expectation and variance, we can derive the corresponding yield curve

$$y(T) = \frac{1 - e^{-kT}}{kt} r_0 + \left(1 - \frac{1 - e^{-kT}}{kt}\right)\theta - \frac{1}{2}\left(\frac{\sigma}{k}\right)^2 \left(1 - 2\frac{1 - e^{-kT}}{kT} + \frac{1 - e^{-2kT}}{2kT}\right) \tag{17}$$

As mentioned, the Vasicek and Nelson-Siegel yield curves are surprisingly similar, however in the Vasicek model we are able to incorporate the dispersion element expressed by a Wiener process.

Using L'Hôpital's rule we can evaluate the limits of $y(T)$. $\frac{d}{dx}(1 - e^{-kT}) = ke^{-kT}$ and $\lim_T \to 0$ $y(T)$ is equal to r_0, and for $\lim_T \to \infty$ $y(T)$ is equal to $\theta - \frac{\sigma^2}{2k^2}$. Interestingly, the long term yield is not equal to the long term interest rate, but is dependent on the volatility of the short-rate and the decay parameter k.

G. *Time series*

When working with observations in the interest and bond market, we are working with time series. Time series require more caution than working with time invariant data, as violations to the Gauss-Markov theorem (Plackett, 1950) for linear estimation techniques for static observations is arguably not as problematic as violations to the fundamentals for time series, such as the same Gauss-Markov theorem for time series and stationarity.

A problem when working with time series is non-stationary data, which can lead to spurious results. A series can be characterized as stationary if we can conclude that $\rho < 1$ for $y_t = \rho y_{t-1} + \epsilon_t$. For yield estimation, this is not a problem, as the corresponding yield curves are estimated across maturities, not time. This must however be addressed for the factors β_1, β_2 and β_3 in the Nelson-Siegel model, and θ, k and σ in the Vasicek model as we want to understand the dynamics of these factors over time. A time series is non-stationary if it contains a unit root. Series containing *unit root* is characterised by being permanently effected by stochastic shocks. The most common tests for unit root is Augmented Dickey-Fuller test (1979) and the Phillips-Perron test (1988). Stationarity will be adressed using an unrestricted Augmented Dickey-Fuller test correcting for serial correlation with appropriate number of lags

$$\Delta y_t = \alpha + \beta y_{t-1} + \sum_{i=1}^{n} \delta_i \Delta y_{t-i} + \varepsilon_t. \tag{18}$$

A linear relationship between different time series elements is popular to approximate some real world representation of data. Linear frameworks are composed of one or many of the fundamental building blocks for time series analysis; a *moving average* process describes a stochastic process which is equal to a weighted sum of past stochastic elements. An $MA(q)$ process can be written as $y_t = y_0 + \sum_{i=1}^{q} A_i \varepsilon_{t-i} + \varepsilon_t$. An *autoregressive* process describes the current value of a time series y_t as a function of weighted past values *and* past stochastic elements. Thus an $AR(p)$ process is expressed as $y_t = y_0 + \sum_{i=1}^{p} B_j y_{t-j} + \varepsilon_t$. By combining these elements, we can express a time series as an ARMA(p,q) model. When working with a set of time series, the model can be constructed as a vectorized autoregressive model, *VARMA*, to account for indirect effects between the series, and the y_t becomes a vector. If we wish to incorporate additional exogenous factors in our VARMA model, we can express the relationship between current values as a general VARMAX model

$$Y_t = \alpha + \beta X_t + \sum_{i=1}^{p} A_i Y_{t-i} + \sum_{j=1}^{q} B_j \varepsilon_{t-j} + \varepsilon_t, \tag{19}$$

Where α is a constant, β is the the coefficent for the exogenous factor X_t, and each A_i is a 3×3 matrix with the autoregressive effects from lag Y_{t-i}, while each B_j is a 3×3 matrix for the moving average components for ε_{t-i}.

In addition to our theoretical frameworks for the short-rate and yield to evaluate the accuracy of the interbank rates, we need tools to evaluate the precision of the models themselves.

H. *Model evaluation*

There are several tools and techniques to evaluate the goodness of performance for a linear or non-linear model. R-squared and adjusted R-squared which is commonly used in econometrics is a naive approach to performance evaluation, as it does not penalize an increase in independent variables, and tends to be a poor indicator for the performance of the model. The two most common measures for forecast performance and accuracy are Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), and will be used to compare the accuracy of each model, and their relative performance (Hyndman u.a., 2006).

$$RMSE = \sum_{i=1}^{n} \sqrt{\frac{(\hat{y} - y)^2}{N}} \tag{20}$$

$$MAE = \sum_{i=1}^{n} \frac{|\hat{y} - y|}{N} \tag{21}$$

V. Chapter 1: Evaluating interbank rates in foreign and domestic currency

Recall that we wanted to determine whether NIBOR accurately represents the true lending rate for the panel banks. The Nelson-Siegel model will be used to estimate three currency specific factors and allows us to construct the entire yield curve. The yield curve can then be compared to historical bond prices in domestic and foreign currency. The Vaiscek model will be applied to understand dynamics of the interbank rates over time, and check if the they are consistent with current prices in the bond market.

A. *Applying the Nelson-Siegel model*

The first approach for estimating a continuous yield curve for each interest rate is the Nelson-Siegel model. To model the yield curve, we have estimated an ordinary least squares regression for each day given by

$$y_{t,INTERBANK} = y_t(T) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\hat{\lambda}_t T}}{\hat{\lambda}_t T}\right) + \beta_{3t}\left(\frac{1 - e^{-\hat{\lambda}_t T}}{\hat{\lambda}_t T} - e^{-\hat{\lambda}_t T}\right). \tag{22}$$

Randomly selected dates have been plotted with corresponding estimated yield curve which are available in appendix C.A. The sample of dates covers different yield curve structures and fits the data across time. The sample of yield curves shows that the Nelson-Siegel model is a good fit, and that we are able to reproduce a variety of yield curve structures.

Figure 6, 7 and 8 illustrates the estimated yield curve across both time and maturity. By applying the Nelson-Siegel model for each interbank rate separately, we end up with three factors, which are country (currency) specific. The exact estimated factors have been plotted in figure 32, 33 and 34 available in appendix C.B. Using the estimated factors, we represent the development of the yield curve across time for each interbank rate in figure 6, 7 and 8. From the figures, we observe that the changes in yield over maturity are not very large, however this is expected as the maturity horizon is only 6 months.

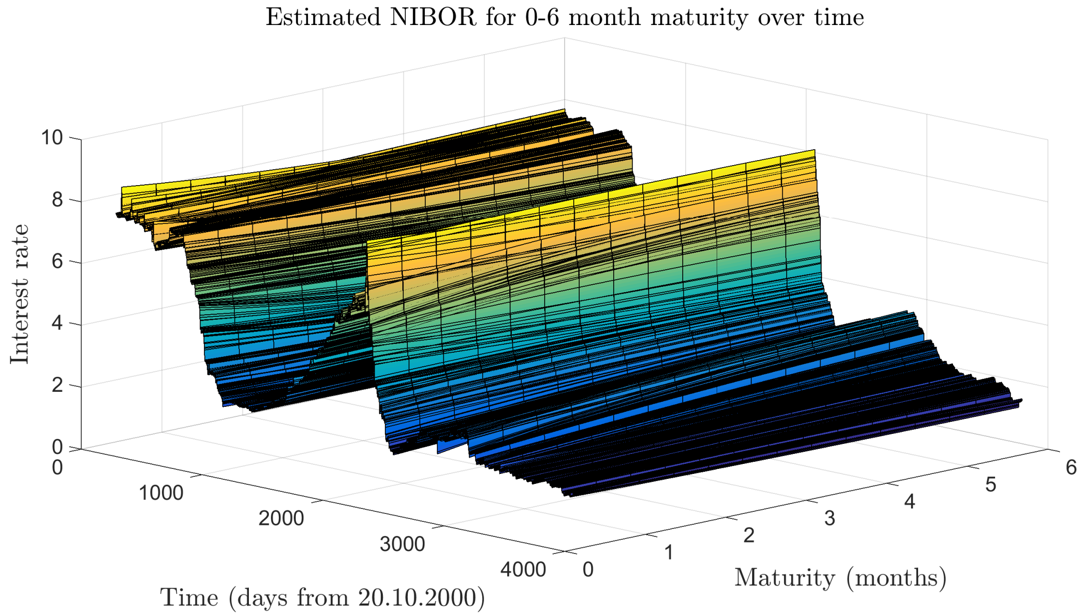

Figure 6. Estimated yield curve for NIIBOR over time and maturity.

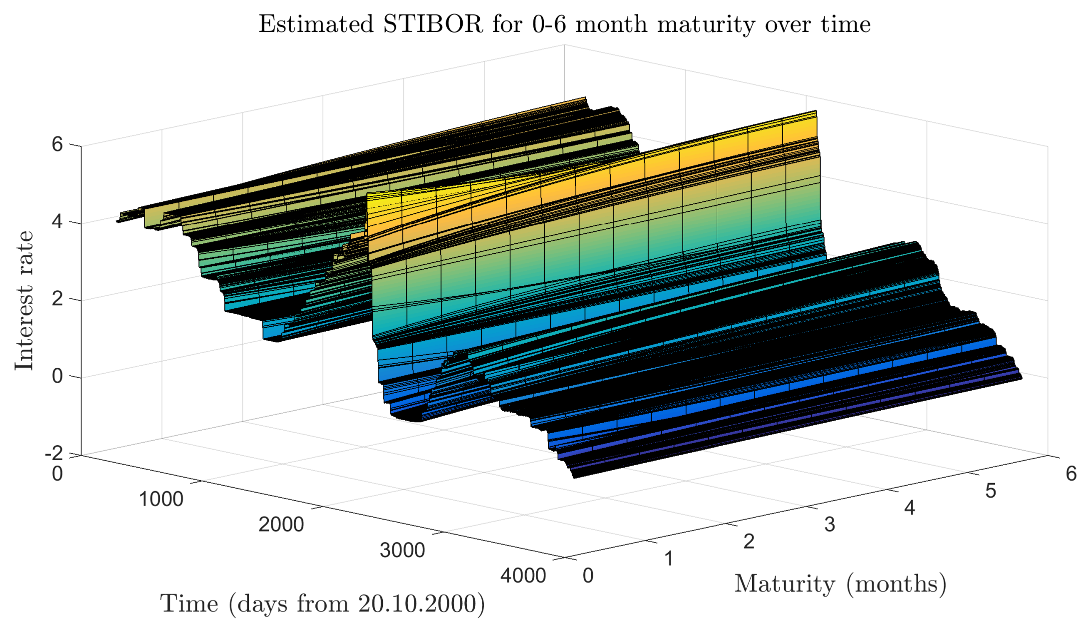

Figure 7. Estimated yield curve for STIBOR over time and maturity.

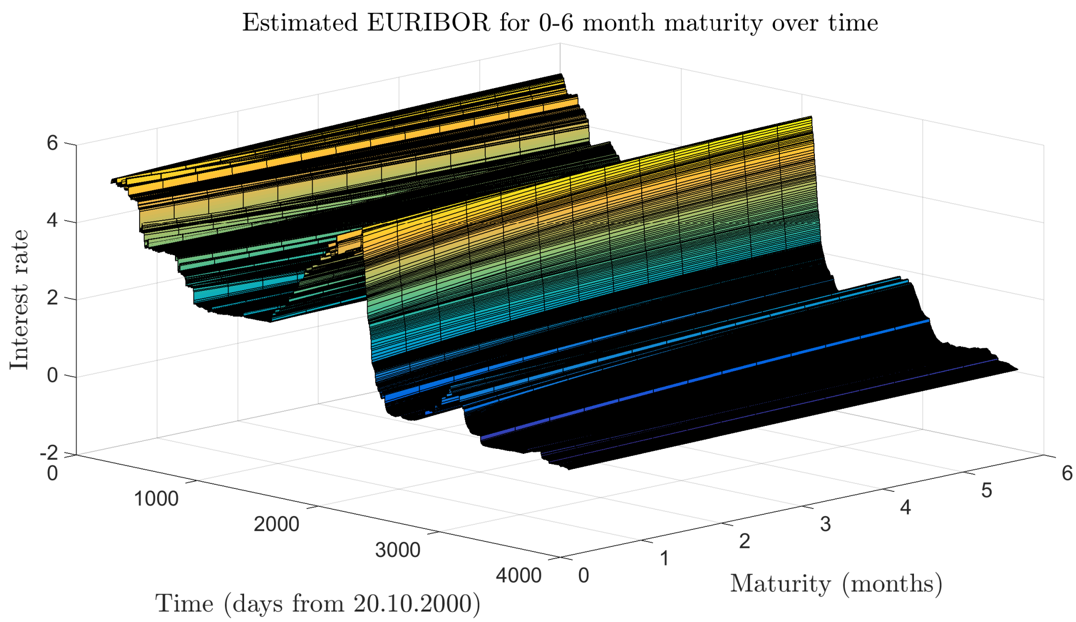

Figure 8. Estimated yield curve for EURIBOR over time and maturity.

The average factor loadings are summarized in Table VIII, and from the estimated β_1 coefficients we observe prominent variations in NIBOR compared to STIBOR and EURIBOR. The difference in β_1 tells us two things: the estimated average interest rate when maturity approaches zero ($\beta_1+\beta_2$) for NIBOR is almost 1.5% above EURIBOR, and the estimated average interest rate as maturity approaches infinity (β_1) is a 1.9% higher for NIBOR than EURIBOR. This is apparent by looking at the raw interest rates themselves. Our convexity factor, β_3, is different for NIBOR compared to the other interest rates, which implies that NIBOR is less sensitive to the duration to maturity.

Table VIII Descriptive statistics for estimated factors

Rate	Factor	Mean	Std.Dev.	$\rho(1)$	$\rho(7)$	$\rho(30)$
NIBOR	β_1	4,327	1,053	0,959	0,839	0,654
	β_2	-0,867	1,042	0,952	0,813	0,599
	β_3	-0,103	1,272	0,950	0,800	0,578
STIBOR	β_1	2,898	0,871	0,973	0,859	0,694
	β_2	-0,607	0,863	0,956	0,786	0,535
	β_3	0,595	1,041	0,959	0,781	0,545
EURIBOR	β_1	2,399	0,585	0,993	0,948	0,814
	β_2	-0,428	0,576	0,994	0,948	0,800
	β_3	0,881	0,715	0,994	0,950	0,825

Note: Sample is 20.10.2000-28.04.2015. $\rho(k)$ is k'th lagged autocorrelation.

The left graph in figure 9 displays how the interest rate at zero maturity ($\beta_1+\beta_2$) behave over time, where we observe the same deviation between NIBOR, STIBOR and EURIBOR as in figure 2 presented in the introduction. The right graph in figure 9 illustrates the average yield curve over the whole sample, and the deviation between yields for NIBOR and STIBOR and EURIBOR does not diminish over time. The figure is also consistent with our proposition that the average estimated yield curve is increasing and concave.

All models exhibit an R^2 between 87-97% which is expected due to the low number of observations (maturities available). In appendix C.C we dig deeper in the various interbank yield curve residuals to evaluate how the model performs for different maturities. When comparing the yield curve residuals to each other, we observe the Nelson-Siegel model consistently overestimates the shortest maturity, and underestimates the longest maturity. The variation in residuals (standard deviation) indicates a small difference between the accuracy of the NIBOR estimations and the other reference rates, but all the models exhibit satisfactory estimation results. From MAE and RMSE the tree models perform well, but in particular EURIBOR show signs of small prediction errors across all maturities.

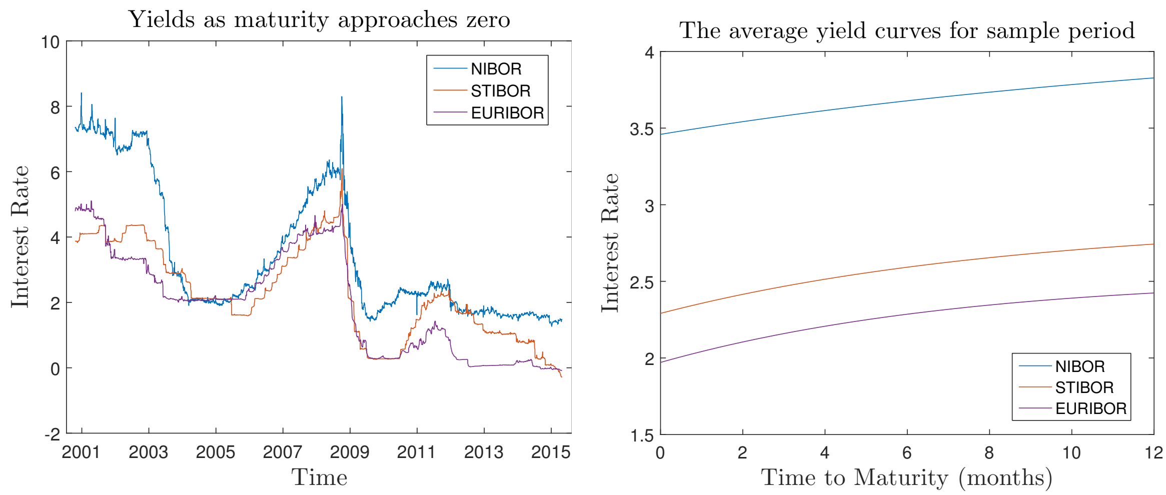

Figure 9. Estimated level for interest rate and average yield curve

The estimated β_1, β_2 and β_3 will serve as country specific factors which are directly comparable, and the estimation results supports that the model accurately reflects the continuous yield curve for the interbank rates across maturity. The accuracy of the model is vital, as the Nelson-Siegel model will be utilised to approximate interest rates between the observable maturities in the market. The model shows there is a significant difference between NIBOR, STIBOR and EURIBOR, but these can be justified if it also is reflected in their funding costs.

Using the estimated factors, we can check if the domestic interbank rates are based on the domestic lending costs. By matching the estimated yield curve for the interbank rate against historic lending costs, we can examine the relationship between lending cost and offered rates.

B. Evaluating the interbank rates in domestic currency using the Nelson-Siegel model

Recall our original definition of an interbank rate in a domestic economy using the risk-free rate and default premium

$$i_{C,C} = rf_C + rp_C. \tag{23}$$

In a frictionless market, $rf_C + rp_C$ reflects the funding cost for a panel bank in the same currency as the interbank rate is offered. Funding costs in neither the Norwegian, Swedish nor the European money market is directly observable for a set of maturities like the interbank rate, and thus the funding costs must be proxied. A suitable proxy is historical bond data for the panel banks, as the bond market is commonly used as a source for capital.

To evaluate and compare the funding costs to our interbank rates, we will use price data for various corporate bonds issued during the 15-year period from the panel banks presented earlier in the descriptive statistics.

One of the major problems when comparing interest rate data and market prices for bonds is that time to maturity is different for each observation in the bond data (matures at a fixed date), while the interest rates are expressed with the same maturity every day (matures at a variable date). For bonds where the only observable variable is prices, it is almost impossible to isolate what change in prices between days can be attributed to change in time to maturity, and what is due to time-varying effects. Time-varying effects are changes in the default risk for the bank, changes in the bond market, or fluctuations in the economy as a whole. It is not possible to address the time-varying effect in the bonds themselves, as bond prices are only observed once for each day. To address the time-varying effects, we must adjust the interbank yield curve using the Nelson-Siegel model. The model allows us at any given date to express a continuous yield curve. By using simple bond valuation to translate the bond prices into bond yields, we can directly compare them to the Nelson-Siegel adjusted yields for interbank rates.

If we theorize about the underlying risk, we know that the domestic interbank rate can be decomposed into both the risk-free rate and a risk premium. Identifying how these components behave over time is complex, and quantifying the short-rate risk, expected risk-free return and default risk is challenging. To avoid identifying the underlying risk factors, we propose that they are exactly the same for both the bonds and interbank rates. As the bonds are issued to banks in the interbank market, and the interbank rates are offered to the same banks, they are both unsecured loans to the same banks, and must have the same underlying risk.

In perfect frictionless markets, we know that equally risky assets must have the same price to avoid arbitrage opportunities, and thus

$$i_{C,C} = rf_C + rp_C := \overline{i_{C,C}} \tag{24}$$

where $i_{C,C}$ is the observable interbank rate, rf_C is the risk-free rate, rp_C is the risk premium for the banks, and $\overline{i_{C,C}}$ is the financing cost. It is bold to assume perfect frictionless markets, and thus we allow for a discrepancy between the financing cost and the offered rate. By hypothesizing that the interbank rate and the interest rate offered to the interbank market should never be lower than the markets yield on the banks' corporate bonds, we can check if

$$i_{C,C} \geq \overline{i_{C,C}}. \tag{25}$$

In terms of yields, this translates to

$$y_{t,INTERBANK} \geq y_{t,BONDS}. \tag{26}$$

By only imposing that the interbank rate, $y_{t,INTERBANK}$, is larger or equal to the bond yields, $y_{t,BONDS}$, we allow for a positive premium between the financing costs for the banks and their offered interest rate in the interbank market. To evaluate if our hypothesized relationship between interbank rates and financing cost holds, we approximate the interbank yield $y_{t,INTERBANK}$ using the Nelson-Siegel model

$$y_{t,INTERBANK} = y_t(T) = \hat{\beta}_{1t} + \hat{\beta}_{2t}\left(\frac{1-e^{-\hat{\lambda}_t T}}{\hat{\lambda}_t T}\right) + \hat{\beta}_{3t}\left(\frac{1-e^{-\hat{\lambda}_t T}}{\hat{\lambda}_t T} - e^{-\hat{\lambda}_t T}\right). \tag{27}$$

The bond yields $y_{t,BONDS}$ are calculated from observable bond prices in the market adjusted for accrued interest rate if the bonds are interest bearing instruments by

$$y_{t,BONDS} = \left(\frac{100 + r(T-t)/360}{P_t}\right)^{\frac{360}{T-t}} - 1. \tag{28}$$

In our bond valuation r denotes the bond coupon, and 360 is the most common day count convention for interest bearing bonds. Note that the adjustment for accrued interest rate, $r(T-t)$, is a simplification of bond valuation with interest bearing characteristics. As no bonds in the sample have higher frequency on interest payments than semi-annually, there are no payments within the lifespan of NIBOR (up to 6 months maturity), which is our evaluation horizon. Both t, time, and T, time to maturity, changes every day. Thus, we are able to incorporate both the time-varying effect and the change in yield due to change in maturity using our daily estimated factor loadings from the Nelson-Siegel model. This allows us to compare our estimated yield curve to the bond yields.

As we want to evaluate the interbank rates in the respective domestic currencies, our sample of banks evaluated in this section are determined by the participation in the fixing of NIBOR to get comparable values. As some of the panel banks of Norway are also participating in the fixing of EURIBOR and STIBOR, we can evaluate the funding costs in local currencies against the money market rate in that given currency. Table IX provides a brief summary of which banks that participate in the examined interest rates, and which currency that is underlying. For transparency, the calculated yields for the individual bonds and estimated interbank rates are plotted and available in appendix D. Price data for the 8 bonds available in NOK is compared to NIBOR, while the sample for STIBOR is 5 corporate bonds, and 8 for EURIBOR. The number of observations for each time series is 120-125, depending on number of weekends and public holidays. By aggregating our interbank and bond data, individual inconsistent pricings become less detrimental for our analysis.

Table IX Banks evaluated in the different domestic currencies.

Interbank rate	NIBOR	STIBOR	EURIBOR
Contributing banks	Nordea Swedbank Handelsbanken SEB DNB Danske Bank	Nordea Swedbank Handelsbanken SEB Danske Bank	Nordea
Currency	NOK	SEK	EUR
Bonds in sample	8	5	8

The average yield curve for NIBOR and bonds in NOK with the same time to maturity is plotted in the top left of figure 10. Similarly, STIBOR, EURIBOR and corresponding bond yields are plotted in the top right corner and bottom left corner of figure 10. The average yield curve is computed as the arithmetic mean of the yields in the sample.

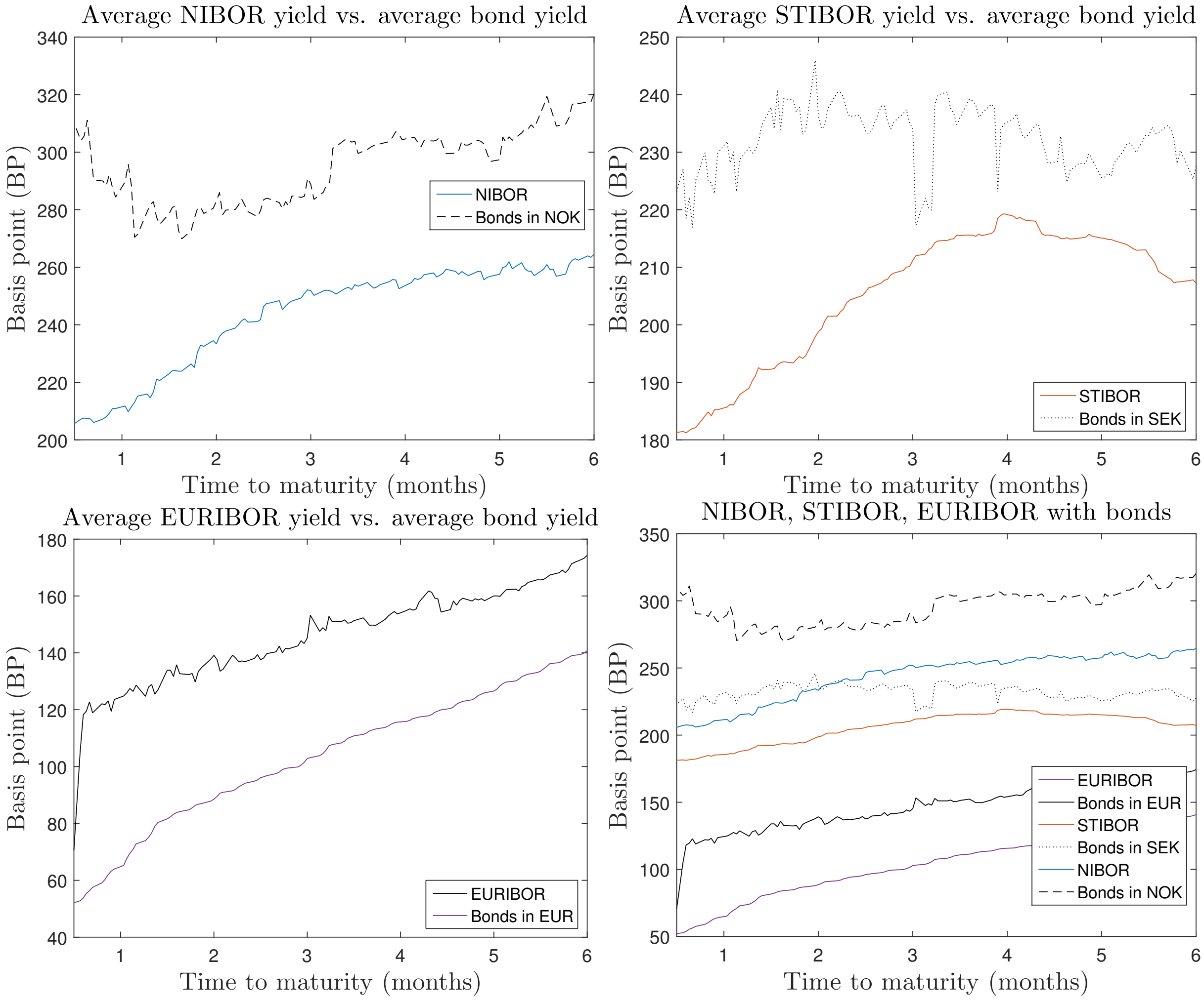

Figure 10. Comparing bond yields for swapped NIBOR/STIBOR and estimated bond yields.

The first notable feature visible from comparing bond yields in both NOK, SEK and EUR to corresponding interest rate yield is that bond yields are consistently higher than the interest rate yields. This systematic discrepancy which implies that the market price for corporate bonds are higher than their offered rate clearly contradicts that the interbank rates represent the true domestic lending rates in the interbank market. A plausible reason for this deviation for short maturities is incorrect pricing of corporate bonds as maturity approaches. There is no available data for volumes, but it is safe to assume that our root cause for this deviation is inconsistent pricing for short maturities. When looking at the raw bond data there is often little or no change in the prices for many bonds when there is fewer than 7 days to maturity. Due to inconsistent pricing patterns for low maturities, the analysis does not cover shorter maturities than 1 week.

In the last plot, NIBOR, STIBOR and EURIBOR are compared against each other. From the figures, it is straightforward to observe that the bond yields are higher than the interbank rates. We observe that the bond yields for the various interbank markets are not the same, and we conclude that the financing cost is not equal across domestic economies, which entails $i_{C,C} \neq i_{N,N}$ for $N \neq C$. The fact that domestic financing costs are unequal supports differences between the interbank rates NIBOR, STIBOR and EURIBOR.

As the financing costs are unequal, we evaluate each interbank rate separately against its own financing cost. The lack of consistent pricing as maturity approaches must be adjusted for before evaluating how the interbank and bond yields behave over maturity. The plot in the top left corner of figure 11 illustrates the sample deviation between bond yields and interbank rates, and the pricing patterns generate large deviation spikes for short maturities. When there is more than one month left to maturity, the pricing becomes more persistent, and we observe that the sample variations rapidly decrease after 1 month.

By adjusting the time frame of our sample for the liquidity issues in corporate bonds, we can evaluate the data between maturities longer than 1 month. When excluding the two shortest maturities (1 week and 1 month) it is possible to check how interbank and bond yields behave as time to maturity increases. We reformulate our hypothesized relationship between the bond yields and assume that over time, exchange-traded bonds will be priced correctly, and the bond yields will represent the true yield required for lending money to the relevant bank. We test if

$$y_{t,BOND} - y_{t,INTERBANK} = \beta_0 + \beta_1 t + \varepsilon, \tag{29}$$

Where we regress the difference between the yields on time to maturity, t. As time to maturity increases, the financing cost proxied by the bond yields, and the interbank rates approximated using the Nelson-Siegel model, should converge towards each other. We expect that if β_0 is larger than zero, β_1 should be negative. Estimating an ordinary least squares regression on the difference between the bond yields and the interbank yields on time gives us the best linear fit for the relationship between difference in yields and time.

The observed differences between bond and NIBOR yields are plotted in the two bottom and top right graphs in figure 11 along with the individual fitted regression lines. The regression results for NIBOR, STIBOR and EURIBOR over time are shown in table X.

Table X Descriptive statistics for structure of the interbank rates

	(NIBOR, robust)	(STIBOR, robust)	(EURIBOR, robust)
Time	2.0988*	-2.0538*	-3.9874***
	(0.94827)	(0.82962)	(0.84907)
Constant	36.158***	20.086***	55.107***
	(3.8084)	(3.3319)	(3.3633)
N	665	475	737
RMSE	31.3	23.2	29.1

Note: Standard errors in parentheses.
$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

As expected from the plotted interbank and bond yields, we have positive intercept (β_0). The interesting result is the difference in the magnitude for the time dependent coefficient (β_1). Both EURIBOR and STIBOR exhibit downward sloping time trends as we anticipated, which indicates that the deviation between bond yields and interbank yields are decreasing. For NIBOR, however, the coefficient is positive. This implies that the difference between the bond yields and NIBOR quotes are increasing as time to maturity increases. Even though the deviation is decreasing for EURIBOR and STIBOR, the magnitude of the coefficients are not large enough to impose that the bond yields match the interbank yields within the maturity time frame. The results point towards an inconsistency between the Norwegian interbank rate and the observed domestic funding cost. They also point towards the same inconsistencies in the Swedish and European money market for Norwegian banks. From analyzing the domestic financing cost against the interbank rates, we conclude that we cannot impose that NIBOR is more inconsistent with the domestic lending cost than other interbank rates.

The comparison of aggregate bond yields and interbank yields in the domestic economies gave inconclusive results regarding the accuracy of the Norwegian interbank rate against domestic financing cost. In order to further evaluate NIBOR, we must go back to our original definition of NIBOR as a swap rate. The European bond market is a more popular bond market with respects to number of bonds and liquidity. There will be a greater sample of bonds available for testing our hypothesized relationship between interbank rates and bond yields. In order to compare the interbank rates with a larger sample of bonds and under equal currency conditions, we swap NIBOR and STIBOR to euros.

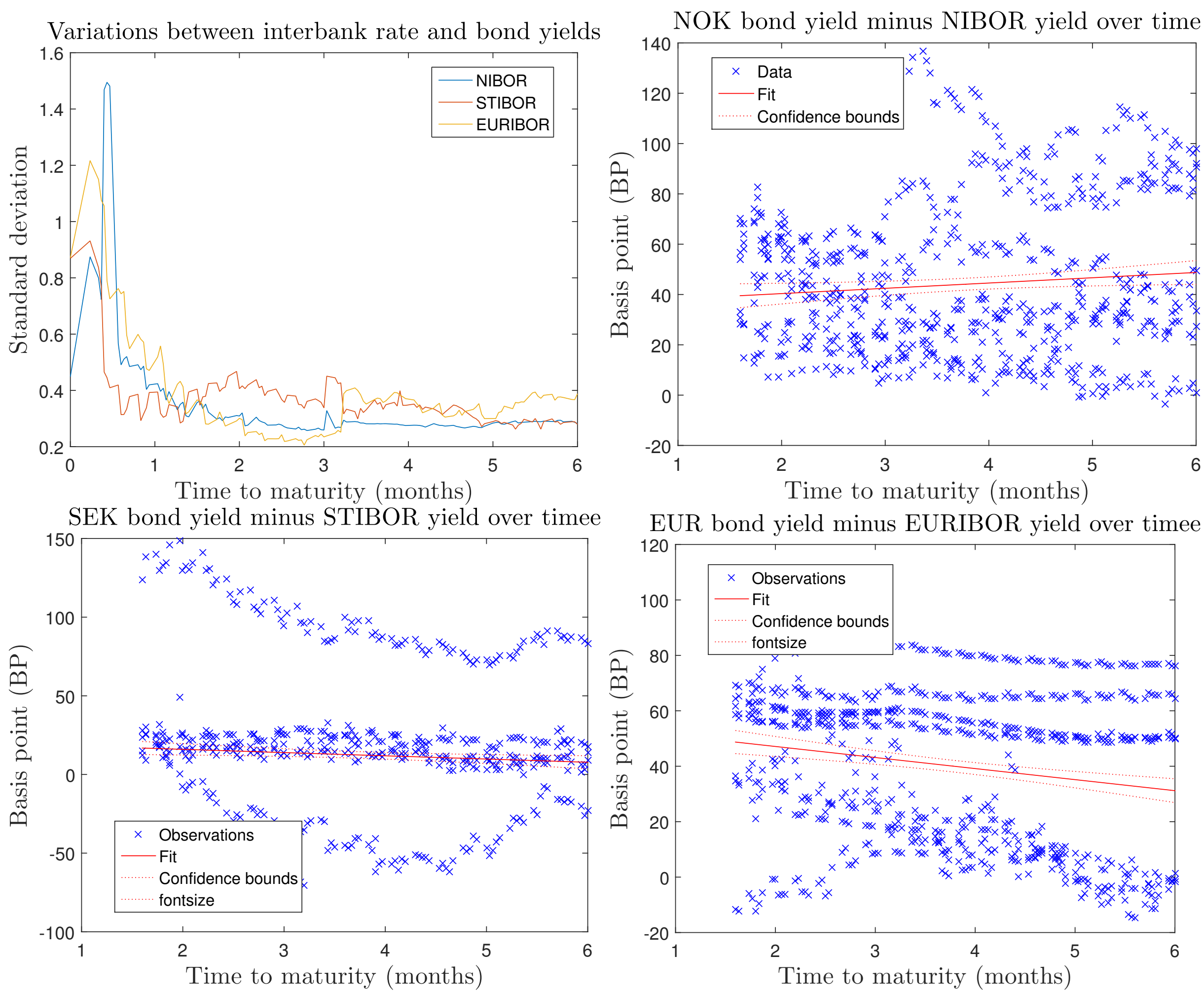

Figure 11. Deviation and estimated difference between bonds and interbank rates

C. *Evaluating the interbank rates in foreign currency using the Nelson-Siegel model*

We use the same technique to estimate factor loadings on swapped interest rates for NIBOR and STIBOR. This makes it possible to compare the implied swapped yields from our Nelson-Siegel model with a new sample of bonds to evaluate if the financing cost for Norwegian and Swedish banks are the same as the financing cost for European banks contributing to EURIBOR. If we observe that the financing cost for Norwegian banks in euros are higher than for European banks, we can justify the observed difference between NIBOR, STIBOR and EURIBOR.

The formal relationship between NIBOR and foreign interbank rates are contingent on whether the financing cost for Norwegian banks are equal to the financing cost for the European, $\overline{i_{N,C}} = \overline{i_{C,C}}$. By swapping STIBOR and NIBOR to euros the interbank rates can be compared to a new sample of 20 bonds. For comparison, we introduce a new independent sample for EURIBOR containing corporate bonds for the large European banks contributing to EURIBOR, but not NIBOR or STIBOR. The European interbank yield is compared to 50 bonds from ING Bank, Deutsche Bank and Barclays, whom are fairly similar to the Nordic banks and will serve as the domestic financing cost in the eurozone, $i_{C,C}$. The banks have been chosen based on geographic location, financial similarities and default risk when using Standard Poor's and Moody's credit ratings as a risk benchmark. All banks except Deutsche Bank have the highest short-term rating, P-1, by Moody's (Deutsche Bank has P-2), while S&P's short-term rating varies from A-1 to A-1+ for Nordic banks, and from A-2 to A-1 for European banks.

We first examine if the banks contributing to NIBOR and STIBOR face the same financing conditions as the European banks contributing to EURIBOR, given by

$$\overline{i_{N,C}} = \overline{i_{C,C}}. \tag{30}$$

These financing costs can be compared with the corresponding interbank rates, $i_{C,C}$ and $i_{N,C}$. The swapped interbank rates, $y_{t,INTERBANK}$ are approximated with the Nelson-Siegel model

$$y_{t,INTERBANK} = y_t(T) = \hat{\beta}_{1t,S} + \hat{\beta}_{2t,S}\left(\frac{1-e^{-\hat{\lambda}_t T}}{\hat{\lambda}_t T}\right) + \hat{\beta}_{3t,S}\left(\frac{1-e^{-\hat{\lambda}_t T}}{\hat{\lambda}_t T} - e^{-\hat{\lambda}_t T}\right), \tag{31}$$

where $\hat{\beta}_{1t,S}$, $\hat{\beta}_{2t,S}$ and $\hat{\beta}_{3t,S}$ are estimated factors for swapped NIBOR and STIBOR to euros. The bond yields $y_{t,BONDS}$ are calculated from observable bond prices in the euro bond market adjusted for accrued interest rate, thereby

$$y_{t,BONDS} = \left(\frac{100 + r(T-t)/360}{P_t}\right)^{\frac{360}{T-t}} - 1. \tag{32}$$

During the evaluation of domestic financing costs in Norway, Sweden and Europe we concluded that these were unequal across currencies. We will start by examining if financing cost for the NIBOR and STIBOR banks are equal to the independent European banks.

Figure 12 shows how the average financing cost up to 6 months maturity for NIBOR, STIBOR and EURIBOR banks. The financing costs are fairly consistent, regardless of where the bank belongs. If we exclude the fluctuations during the first month, the difference between financing costs from STIBOR to EURIBOR is 4.6 basis points, and 4.8 basis points between NIBOR and EURIBOR. The average financing cost for NIBOR is 161 basis points and 155 for EURIBOR banks for 1-6 months maturity, and an average difference of under 5 basis points is considered to be insignificant. We conclude that banks contributing to NIBOR, STIBOR and EURIBOR face the same financing costs in the European money market, and $\overline{i_{N,C}} = \overline{i_{S,C}} = \overline{i_{C,C}}$.

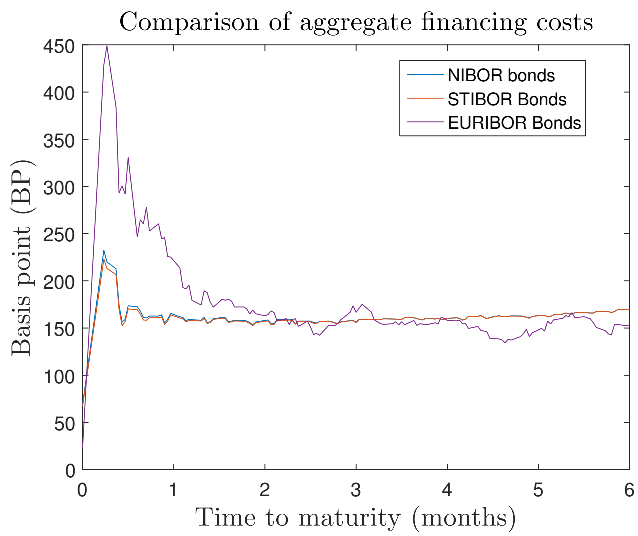

Figure 12. Comparing financing cost.

When the banks face the same financing cost, $\overline{i_{N,C}} = \overline{i_{C,C}}$, the relationship between financing cost in a foreign currency and the domestic offered rate is the swap rate

$$i_{N,N} = \overline{i_{N,C}} + f - s = \overline{i_{C,C}} + f - s. \tag{33}$$

In figure 13 both bond yields and interbank yields in euros have been plotted to compare the financing cost in euros against the swapped offered rate. Comparing the swapped Norwegian and Swedish interbank rate to the panel banks bond yields tells us a different story than what we observed for local currency bonds. In the last graph in the bottom right corner, we have compared the discrepancy between interbank rate and financing cost for NIBOR, STIBOR and EURIBOR. EURIBOR is consistently below the observed financing cost, and converges to a deviation of around -60 basis points. For STIBOR, the financing cost is surprisingly consistent with the banks' financing cost, and the offered interbank rate is on average 6 basis points above the same bond yields.

Swapped NIBOR is on average 53 basis points above the bond yields, even though the financing cost for Norwegian banks is about the same as European banks.

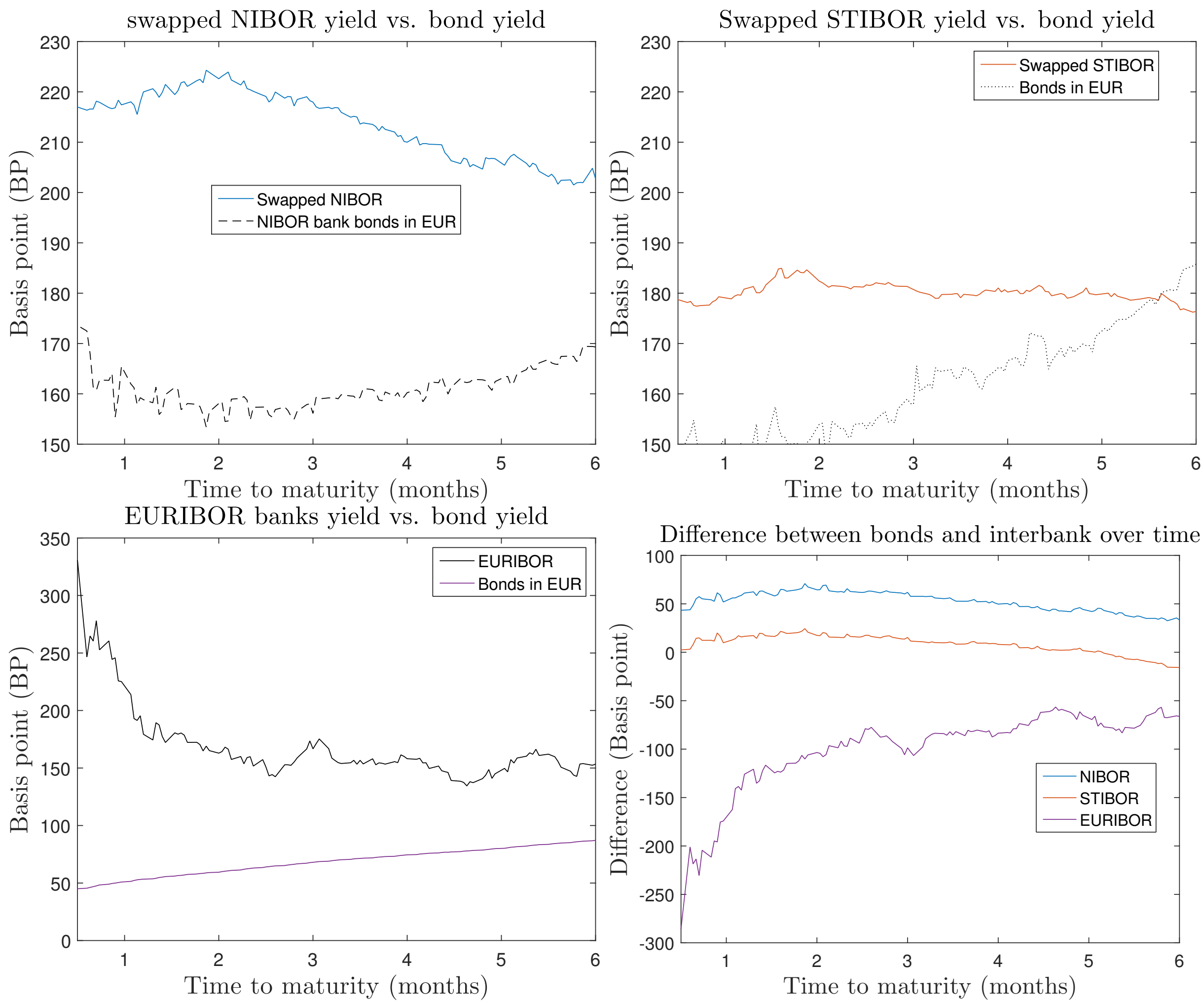

Figure 13. Comparing bond yields for swapped NIBOR/STIBOR and estimated bond yields.

The interbank rate and the aggregate bond yields have the same risk exposure, namely the exposure towards any risk associated with the banks present in the interbank market. We observe that there exists a premium in the Norwegian Interbank Offered Rate, which is not reflected in the panel banks' observed financing cost. This comparison of financing costs and offered rates implies that even though Norwegian banks face the same financing costs abroad, there is an added premium in the domestic interbank rate after adjusting for costs associated with neutralizing currency risk.

Before introducing the time dimension to the Nelson-Siegel model, we will apply the second model for evaluating the interbank rates, the Vasicek model. Applying a secondary framework based on different assumptions is a good way of evaluating if our results from the Nelson-Siegel model are consistent with other aspects of interest rate theory.

D. Evaluating the Nelson-Siegel model over time

The Nelson-Siegel model has only been evaluated in-sample with a separate estimation of the parameters β_1, β_2 and β_3 for each day. To evaluate if the dynamics of the estimated models are consistent out-of-sample, and to increase our confidence in the findings, we perform an out-of-sample forecast of the parameters. When introducing the time dimension to The Nelson-Siegel model, characteristics concerning autocorrelation and stationarity of the time series must be identified.

We use the unit root test introduced by Dickey-Fuller to test for the presence of unit root in our estimated factor and Treasury bill time series. The results for our Nelson-Siegel model are shown in table XI, where several of our time series contain unit root. As the test fails to reject our null hypothesis that unit root is present, they will be differentiated. After differentiating all the time series are stationary with p-values less than 0.01. Based on the results from the Augmented Dickey-Fuller test we use the differentiated series in the autoregressive framework.

Table XI Results from Augmented Dickey-Fuller (ADF) test for estimated factors

	β_1		β_2		β_3		Treasury Bill	
	H	P-value	H	P-value	H	P-value	H	P-value
NIBOR	0	0.1191	1	0.0045	1	0.0109	0	0.0693
EURIBOR	0	0.1034	1	0.0291	0	0.0940	0	0.0808
STIBOR	0	0.1164	1	0.0022	1	0.0071	0	0.1300

Note: Sample is 20.10.2000-28.04.2015. ADF performed with 0-150 lags.

By structuring the three factors for each interest rate in a vector autoregressive framework with exogenous factors (VARMAX), we are able to capture the interaction between the factors, the lagged factors and other exogenous factors. We will use 3 month bond yield in NOK, SEK and EUR to proxy a risk-free process, X_t. By incorporating the risk-free interest rate in the VARMAX model, we can at the same time identify how changes in risk-free return over time affects the factors, and thus the interbank rates.

To determine if an autoregressive model allows us to perform a more accurate out-of-sample foreacast, we specify two different forecasting models. The first model is a tailored VARMAX model, where each interbank rate is specified based on the autocorrelation function and partial autocorrelation function. Our second model is an approach where the future factors cannot be foreseen, and the variation is purely determined by stochastic shocks. The lack of a deterministic component implies that the factors are random walk, and a differentiated random walk series is a VARMAX(0) with no autoregressive components.

To determine how many lags we must include in the tailored model to capture the time series dynamics, we evaluate the factor's autocorrelation function and partial autocorrelation function. Plotted partial autocorrelation functions and autocorrelation functions for each time series is available in figure 46 to figure 51 in appendix E. The partial autocorrelation suggest autoregressive lags, and NIBOR, STIBOR and EURIBOR are specified separately. No moving average components are included as we suspect the small spikes in the autocorrelation plots will be addressed by the autoregressive component. Our unrestricted vector autoregressive exogenous model (VARMAX) for each interbank rate is

$$NIBOR: Y_t = \alpha + \beta X_t + \sum_i A_i Y_{t-i} + \varepsilon_t \quad \forall\, i = 1, 2, 3, 5, 6, 18, 46, 75, 95, 109 \tag{34}$$

$$STIBOR: Y_t = \alpha + \beta X_t + \sum_i A_i Y_{t-i} + \varepsilon_t \quad \forall\, i = 1, 2, 3, 8, 10, 11, 45, 59, 60, 80, 86, 89, 110 \tag{35}$$

$$EURIBOR: Y_t = \alpha + \beta X_t + \sum_i A_i Y_{t-i} + \varepsilon_t \quad \forall\, i = 1, 6, 13, 20, 45, 60, 65, 70, 74, 80 \tag{36}$$

Each Y_t is a vector of the three factors, β_1, β_2 and β_3 to ensure we capture indirect effects between factors. Coefficients for the VARMAX models were estimated using least squares linear optimization. The estimation results are summarized in table XII. When comparing MRSE and MAE for the two proposed structures in table XII, there is some performance improvement for the tailored model. Both models reject the null hypothesis in the Ljung-Box (1978) which indicates the residuals are not white noise. Testing squared errors confirm that the variances for the models are conditional. This indicates that a GARCH model with autoregressive variance would be appropriate to adjust for time dependent variance, but due to the vast increase in complexity and the scope of the paper, this will not be implemented.

Table XII Model 1, Model 2 and Random Walk model

	Model 1: Tailored			Model 2: Random walk		
	NIBOR	STIBOR	EURIBOR	NIBOR	STIBOR	EURIBOR
α_1	0,0006	-0,0014	-0,0017	0	0	0
α_2	-0,0019	0,0011	0,0013	0	0	0
α_3	-0,0026	0,0009	0,0016	0	0	0
β_1	0,5747	-1,3509	-0,0017	0	0	0
β_2	-0,5636	1,6145	0,0087	0	0	0
β_3	-0,7151	2,9581	0,0230	0	0	0
RMSE_1	0,1710	0,4077	0,1710	0,7683	0,4486	0,1767
RMSE_2	0,1547	0,3904	0,1547	0,7332	0,4280	0,1601
RMSE_3	0,2706	0,6306	0,2706	1,2263	0,6966	0,2811
MAE_1	0,0905	0,1968	0,0905	0,4167	0,1760	0,0924
MAE_2	0,0842	0,1901	0,0842	0,3955	0,1712	0,0864
MAE_3	0,1312	0,2994	0,1312	0,6622	0,2674	0,1353
LogLF	16590	9734	16590	2570	9058	15865

Note: Sample period is 20.10.2000-28.04.2015.

The most interesting elements from the VARMAX models is how the risk-free rate effects the change in factors. From the β coefficients in table XII, we see that the model predicts an increase in the risk-free rate does not effect EURIBOR, while STIBOR does not change in the short term level $(\beta_1 + \beta_2)$, but decreases the long term factor β_1. For NIBOR, the short term level is unchanged, but the long term factor β_1 is increased. We will not draw any further conclusions based on the magnitude of the coefficients as we have not corrected for conditional variance, but observing differences between the interbank factors is still noteworthy.

We forecast out-of-sample for 125 observations using model 1 and the random walk model. A summary of the results are presented in table XLII, XLIII and XLIV available in appendix E. The last 180 observations along with the forecasted interval, and actual factors are plotted in figure 14, 15 and 16. The forecast is able to incorporate some of the variations out-of-sample - but there are still large variations we are not able to capture. The forecasting accuracy for NIBOR is significantly better than random walk, but STIBOR and EURIBOR perform similarly to random walk.

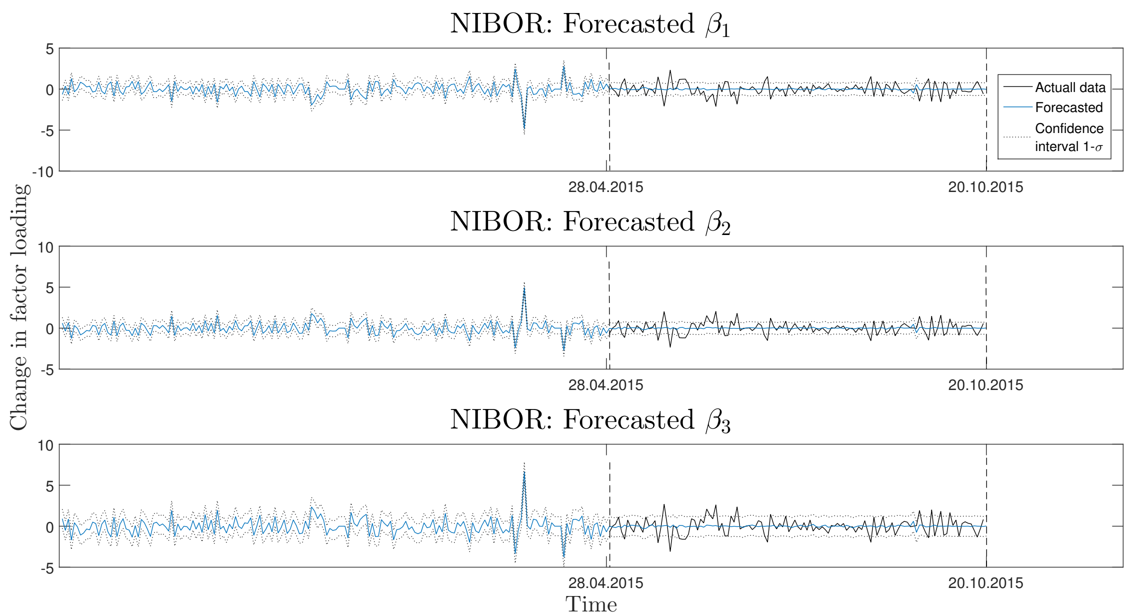

Figure 14. NIBOR out-of-sample forecasted factor changes.

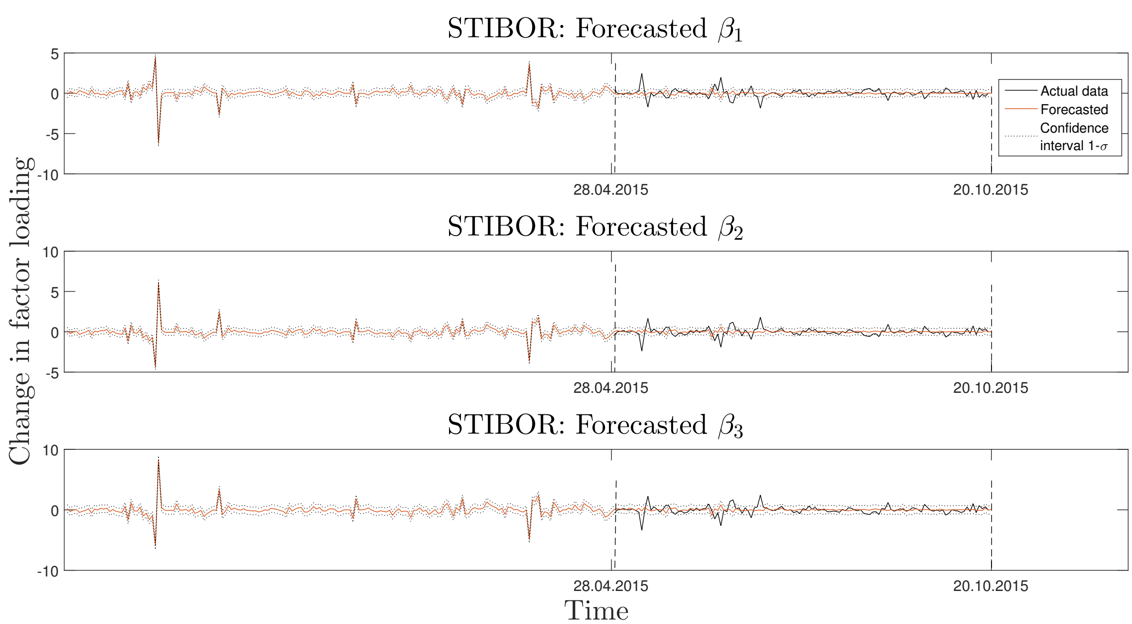

Figure 15. STIBOR out-of-sample forecasted factor changes.

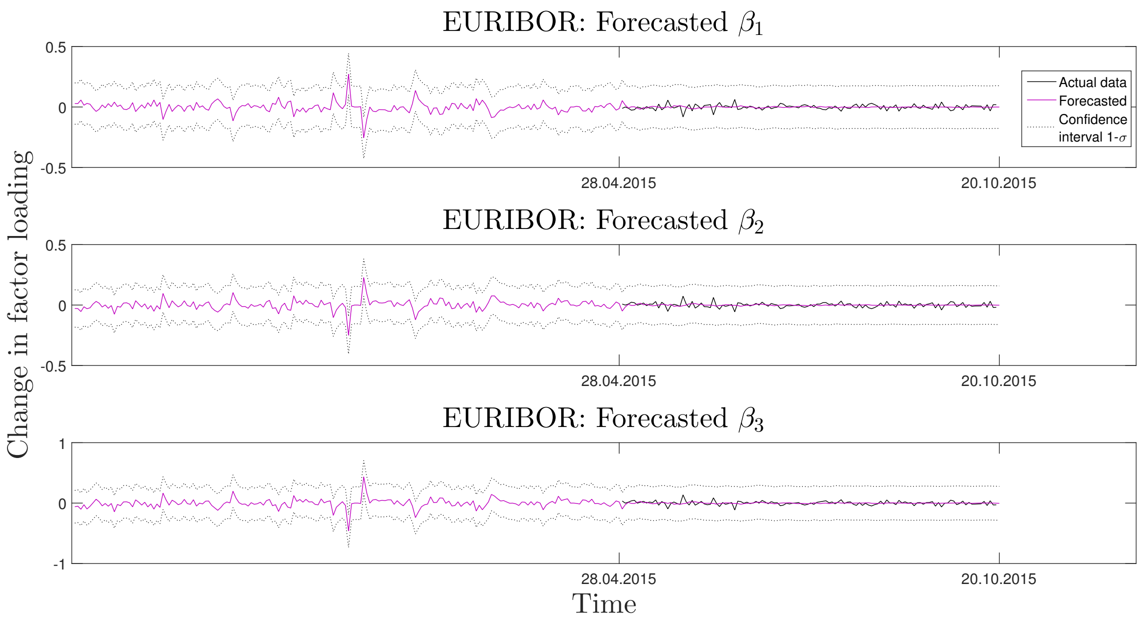

Figure 16. EURIBOR out-of-sample forecasted factor changes.

So what does this tell us about the dynamics of Nelson-Siegel model? Firstly, when we compare the forecasted betas to the actual interest rate data, they fit well to the interbank rates we observe, as shown when comparing forecasted interest rates against actual interest rates in appendix D. This implies that the factors we forecast are quite consistent with the interest rate we observe out-of-sample. The most important result from the out-of-sample forecast however, is that the model does not forecast any extreme deviations from the actual factor loadings we observe. This would indicate that the factors do not accurately represent the yield curve. Overall, the forecast provided useful information about the forecasting ability of the Nelson-Siegel model and the accuracy of the model on new observations.

E. Limitations of the Nelson-Siegel model

Even though the results indicated that the Nelson-Siegel model was able to fit the yield curves for interbank rates well, there are several limitations and weaknesses to the analysis.

For time period with few maturities, over-fitting is a clear weakness when estimating three parameters. It is challenging to quantify how serious over-fitting is for the time periods, but we do observe some high variations in the estimated factors, which are likely to be caused by small daily sample sizes. Fixing the decay parameter was one measures taken to reduce extensiveness of over-fitting. The issue is briefly discussed when introducing the model, and another possibility to address the over-fitting is to include other sources of data. As other sources will not have the same risk dynamics, this approach was discarded. Despite the advantages of fixing the decay parameter, this limits the Nelson-Siegel models ability to capture yield curve dynamics. The model was tested using a variety of decay parameters to evaluate how changes in the decay parameter changed the estimation results, but the changes were not systematically identified to get a complete picture of how a misspecified decay parameter would effect the overall results.

The bond sample used in the estimation of the Nelson-Siegel model was unsecured senior bonds, which may have biased the sample. As the distribution of bonds is not evenly spread across the panel banks, it is clear that there are variations in the number of bonds with stated guarantee type between panel banks. Bloomberg and Reuters might favor to check and verify guarantee type for panel banks conditional on how large the demand from their users is for a particular bank or type of bond. We were able to include both fixed and zero coupon bonds, but could not use the vast sample of bonds with other structures such as variable coupon and index-linked prices. These bond structures could be favored by some panel banks or the financial party issuing bonds on behalf of the banks, and thus make the bank underrepresented in our sample.

When evaluating if the financing costs were equal across currencies, the underlying bond distribution across years is different for the Norwegian and European banks. As the bond yields vary across time, this could make the average bond yields not have the same time-varying effects, which would effect our results. We were able to ensure that all panel banks were represented with corporate bonds in the paper, but not all banks are represented in every estimation of financing costs.

There are many limitations and complications in the estimation of the VARMAX model such as conditional variance, not sufficiently incorporating moving average dynamics and cross-checking our estimated model against similar models with different autoregressive components. As our results were not especially dependent upon the result from the VARMAX mode, these weaknesses have not influenced our results significantly.

F. The Vasicek model

In our second model, we assume the short-rate follows a stochastic process. Recall the stochastic expansion of the short-rate over time in equation (16). To determine the structure of the yield curve each day we must estimate the parameters θ, k and σ. There are two different approaches to applying a short-rate model: calibration and estimation.

Calibration of a short-rate model implies that the parameters θ, k and σ are calibrated according to available market data in an information set F_t. In our case, this implies estimating the expansion of the short-rate given the available interbank rates for different maturities at each point of time similarly to the Nelson-Siegel model. The underlying assumption for the calibration process is that the future short-rate from time $t = 0$ will follow the stochastic process in the Vasicek model. *Estimation* of the short-rate model involves modeling the best parameters for historical data across time for the individual maturities. The underlying assumption for the estimation technique is that each interest rate itself follows the stochastic process, and each interest rate has its own long term trend, mean reversion and volatility.

As the Nelson-Siegel model is approximated to fit the yield curve, we wish to gain additional insights in the accuracy of the dynamics in the interest rate by using the estimation approach. We have established that the financing cost in foreign currency is inconsistent with the interbank rate for past bond prices, but so far we have not incorporated any information for bonds in today's money market. By using this estimation approach, we can derive the interest rate dynamics to evaluate current bond prices, and we can cross check if our results for bonds maturing in the future are consistent with the evaluation of bonds whom matured in the past. To estimate our the parameters in the Vasicek framework, we model the innovation process over time as the Vasicek differential equation

$$dr(t) = k(\theta - r(t))dt + \sigma dW(t), \tag{37}$$

and the equation will be estimated as

$$y_t = \beta_1 + \beta_2 x_t + \varepsilon_t \tag{38}$$

y_t is equal to the change in interest rate, $dr(t)$, β_1 is the time invariant component, $k\theta$, and β_2 is the daily effect of the decay parameter, $-kd(t)$. The interest rate volatility is calculated based on the residuals from the regression, $\sigma = \sqrt{\frac{Var(\varepsilon_t)}{dt}}$. Our time increments are 1 day, we thereby convert NIBOR, STIBOR and EURIBOR yields to daily effective yields.

A summary of estimation results for selected maturities is presented in table XIII. The Vasicek model allows for negative interest rates, but after estimating the parameters, we observe that the results for STIBOR and EURIBOR become skewed. As the interest rates for STIBOR pass the zero threshold, the mean reversion parameter, k, becomes negative. For EURIBOR, the ordinary least squares application makes the mean reversion parameter k very small, and our long term trend θ becomes unproportionally scaled as $\hat{\theta} = \frac{\hat{\beta}_1}{\hat{k}}$. There are also problems with stationarity of the process when the decay parameter is negative. NIBOR has never been below zero, and thus we can still use the results from the Vasicek estimation to evaluate if the estimated parameters are consistent with market data we observe.

Table XIII Summary of estimated Vasicek paramters

	Maturity	k	θ	σ
Nibor	1 Month	0,00099	1,92487	0,00019
	3 Month	0,00067	1,23102	0,00013
EURIBOR	1 Month	0,00027	-2,66492	0,00006
	3 Month	0,00022	-3,72377	0,00004
STIBOR	1 Month	-0,00011	13,03425	0,00008
	3 Month	-0,00014	11,06119	0,00008

To evaluate if our estimated dynamics of NIBOR are consistent with the market's interpretation of dynamics of interest rates for unsecured loans, we apply the Vasicek model to calculate an explicit price for any given day. To see if current prices for bonds maturing in the future are consistent with the estimated prices, we use zero coupon bonds maturing after our sample period. As there is a limited sample of unsecured zero coupon bonds in Norway, and the results from the Nelson-Siegel model were inconclusive for the domestic market, we will utilise Vasicek on Norwegian banks in foreign currency to either confirm or contradict the results so far.

G. Evaluating the interbank rates in foreign currency using Vasicek

As the bonds are issued in euros, we must estimate new Vasicek parameters for the swapped NIBOR interbank rates. The adjusted parameters are presented in table XIV. There are minor differences in the estimated parameters k and σ, but the estimated long term interest rate, θ drops significantly between 1 month and 2 month maturity. As the difference between 1 month and 2 month NIBOR does not change this radically, it is caused by a change in the forward rates.

Table XIV Summary of estimated Vasicek paramters for NIBOR

	k	θ	σ	RMSE	MAE
1 Week	0,385965	1,850461	0,000762	0,150453	0,015893
1 Month	0,344612	1,628436	0,000682	0,126783	0,014226
2 Month	0,239203	0,87487	0,000493	0,104275	0,010274
3 Month	0,2297	0,733602	0,000445	0,10169	0,009271
6 Month	0,251888	0,793196	0,000457	0,106383	0,009525

To evaluate if these dynamics are consistent with the dynamics we observe in the bond market, we price bonds using the estimated dynamics, and compare to actual bond prices. The Vasicek model lets us explicitly calculate the bond price based on our assumptions about the future short-rate by first expressing the bond yield as

$$y(T) = \frac{1-e^{-\hat{k}T}}{kt}r_0 + \left(1 - \frac{1-e^{-\hat{k}T}}{kt}\right)\hat{\theta} - \frac{1}{2}\left(\frac{\hat{\sigma}}{\hat{k}}\right)^2\left(1 - 2\frac{1-e^{-\hat{k}T}}{\hat{k}T} + \frac{1-e^{-2\hat{k}T}}{2\hat{k}T}\right), \tag{39}$$

where we use the estimated $\hat{k}$, $\hat{\theta}$ and $\hat{\sigma}$ for each maturity. The Vasicek model has some similarities to the Nelson-Siegel model, but are fundamentally different, which makes it suitable for comparing results. Using the estimated yield, we can derive a continuous price path during the sample given by

$$P(t,T) = P(T,T)e^{y(T-t)}. \tag{40}$$

Where $P(T,T)$ is the payment received at time T when the bond matures. For simplicity and to make the results consistent with price data in the market, we use zero coupon bonds where $P(T,T) = 100$. Inserting our explicit solution for the yield lets us express the price at time t as

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}, \tag{41}$$

Where the price at time t, $P(t,T)$, is a function of $A(t,T)$ and $B(t,T)$ expressed by the estimated parameters from the Vasicek model

$$A(t,T) = exp[\left(\hat{\theta} - \frac{\hat{\sigma}^2}{2\hat{k}^2}\right)(B(t,T) - T + t) - \frac{\hat{\sigma}^2}{4k}B(t,T)^2], \tag{42}$$

$$B(t,T) = \frac{1}{\hat{k}}\left(1 - e^{-\hat{k}(T-t)}\right). \tag{43}$$

We hypothesize that if the dynamics are consistent with the dynamics in the bond market, the actual bond prices should lie within the span of the dynamics we have estimated. The estimated price path, $\hat{P}(t,T)$ will be compared to actual bond prices, and if the interest rate dynamics we derived using the Vasicek model is the underlying interest rate dynamics for the bonds, and thus the financing cost, they should be the same.

there were 13 unsecured bonds available in the European money market, and the complete list of relevant bond information is available in table XXXIV in appendix A.A. In figure 17, 18 and 19 we have plotted the bond prices against the 5 estimated prices paths for each set of interest rate dynamics. The figures compare the estimated price patterns given by the Vasicek dynamics to the actual bond prices observed during our 125 day period. The figures highlights two important aspects; the first is how consistently priced our bonds are. As many of the bonds mature close to each other, we expect that prices look similar during the period. The consistency in prices implies there is little market friction present to affect the bond prices. The second, and most important observation, is that bond prices are consistently higher than the estimated prices using the interbank rate dynamics. Only one out of 13 bonds lie within the price span given by the interbank dynamics for a majority of the time period. As the bond prices are higher than the estimated prices, they have a lower yield, which indicates there is an added premium on the interbank rates, which is not reflected in the financing cost. Higher interbank yields than bond yields is consistent with our previous result when we compared the financing cost against the interbank rates in euros using the Nelson-Siegel model.

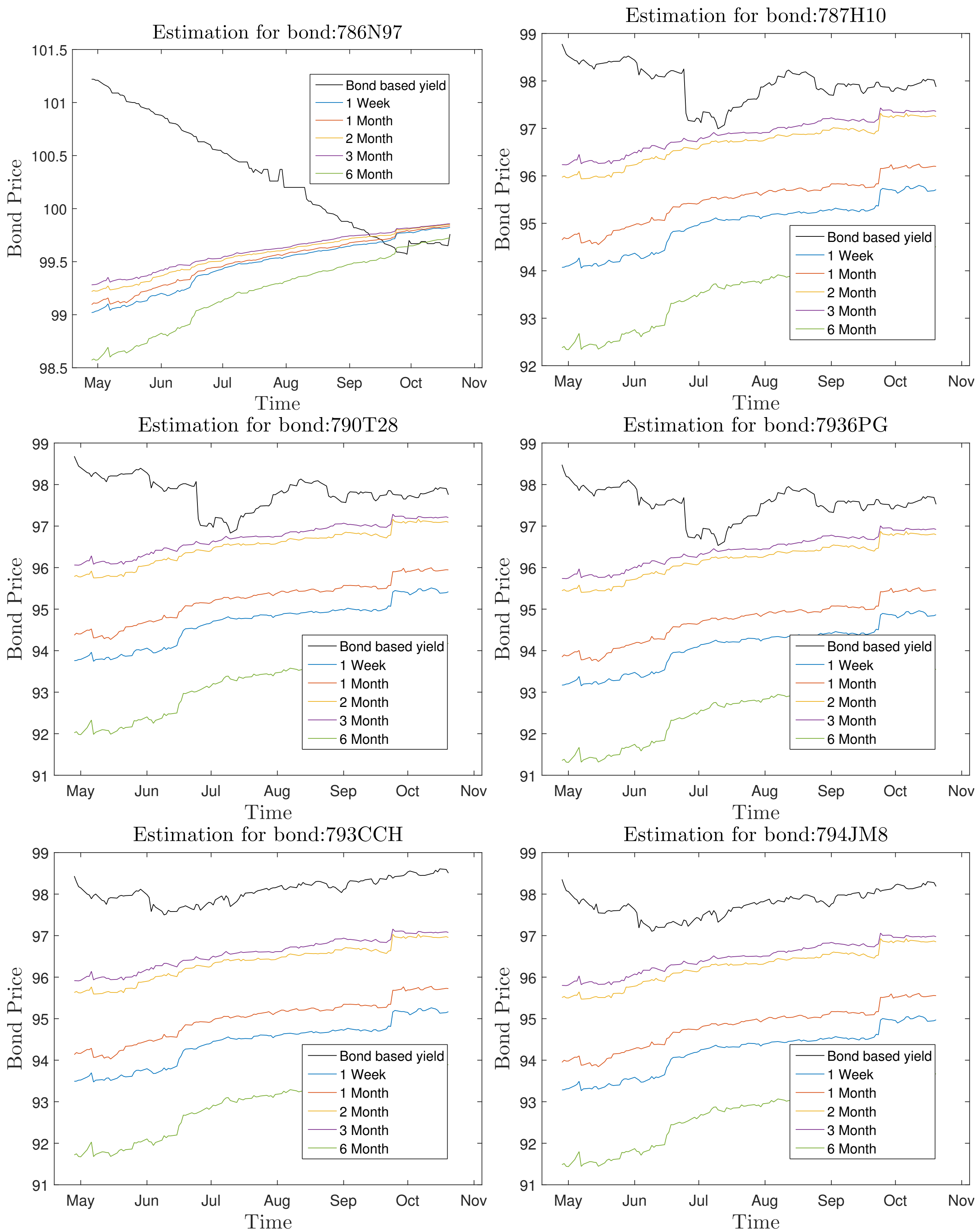

Figure 17. Comparing estimated price with actual bond price.

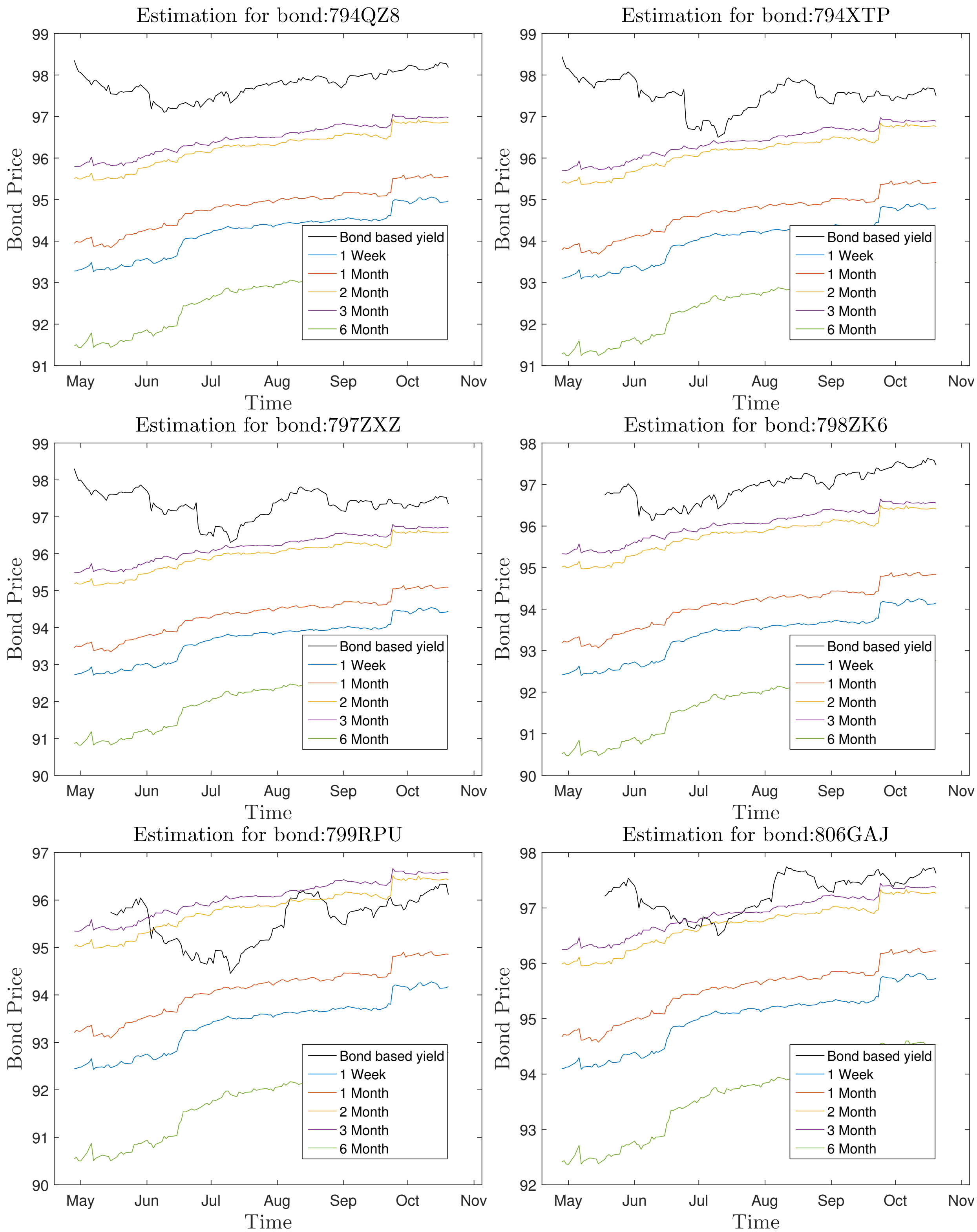

Figure 18. Comparing estimated price with actual bond price.

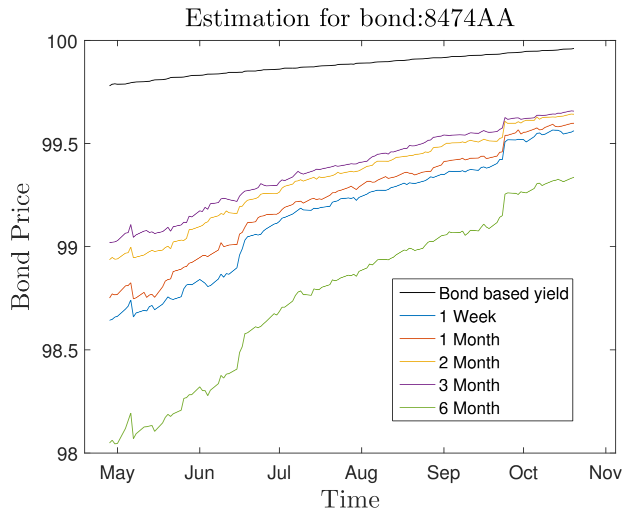

Figure 19. Comparing estimated price with actual bond price.

H. Limitations to the Vasicek model

As the Vasicek model assumes a deterministic long term trend θ, decay parameter k and the short rate volatility σ, there are limitations to the flexibility of the model. We are unable to incorporate fundamental changes to the interest rate market, and the future estimated values are influenced by all the historic data we utilise during the estimation. It can be argued that a sample of 15 years is not representative for the current interest rate landscape, or that a more dynamic model which allows for time varying factors is a better fit.

Utilising the calibration method for the Vaicek model is arguably more directly comparable to the Nelson-Siegel approach, and might provide more useful information than the estimation technique. This approach would, however, suffer from over-fitting similarly to the Nelson-Siegel model as there are more parameters to estimate for each day in the Vasicek model.

The bond sample used in the estimation of the Vasicek model was unsecured senior bonds, like in the Nelson-Siegel model, and thereby face the same weaknesses. In addition to this, a potential sample bias is the use of only zero coupon bonds. Some banks may favor issuing zero coupon bonds opposed to using other coupon structures.

I. Partial conclusion

The aim of this section was to determine if NIBOR represents the true lending cost in the Norwegian interbank market. We have applied two different theoretical frameworks in order to provide insights to the accuracy of the offered interbank rates compared to the observed lending costs in the market.

We have compared the yield curves for interbank rates with corresponding financing cost in both domestic and foreign currency. The interbank rates were transformed to a continuous yield curve using the Nelson-Siegel model while the financing costs were estimated using senior unsecured corporate bonds. Comparing domestic financing cost against domestic interbank rates showed inconsistencies between the lending cost in the Norwegian money market compared to NIBOR. The results are however inconclusive as we found similar results in the Swedish and European money market for Norwegian banks. In all three cases, we observed higher financing costs than the offered interbank rate for corresponding maturities, and observed that financing costs in the domestic currencies are not equal. Differences in financing cost across currencies can justify differences between interbank rates.

To further evaluate the foreign lending cost for Norwegian banks, we used the European money market by swapping NIBOR and STIBOR to euros. We introduced a new sample of banks in Europe with similar characteristics to determine if the financing cost for Norwegian and European banks in the European monkey market is consistent. The results from the Nelson-Siegel model showed that the Norwegian Interbank Offered Rate has been higher than both the Swedish and the European interbank rate, after adjusting for currency risk, even though banks face equal financing cost. This discrepancy was again evaluated using the the Vasicek model for each individual interest rate by estimating interest rate dynamics for each swapped NIBOR maturity. The Vasicek model was used to compute a price range for corporate bonds consistent with the interest rate dynamics observed in the interbank market. Using a new sample of senior unsecured corporate bonds, we compared the estimated prices with actual price data. Comparison showed that bond prices were consistently higher than prices estimated by the model. These results are in line with results from the Nelson-Siegel model, which indicated that there exists a discrepancy between the offered interbank rate in Norway and the financing cost in foreign currency.

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VI. Chapter 2: The microstructure of NIBOR

In the previous chapter we examined the dynamics of the NIBOR fixing with respect to maturities and time in a cross-country perspective by evaluating NIBOR in both domestic and foreign currency. In the following chapter we extend the analysis by taking a closer look at the microstructure of NIBOR. We define the microstructure of NIBOR as the underlying quotes reported by the individual banks of the NIBOR panel.

This section analyses the quotes that constitute the rate along two dimensions: the first dimension will be looking the long term dynamics of the quotes for the 12:00 pm daily fixing relative to underlying credit risk proxied by credit default swap spreads. The second dimension will utilise the high frequency indicative deposit rates (IDR) to examine and identify short term patterns in the quotes. Note that the IDRs are not covered by the NIBOR rules, and are not regarded as actual NIBOR fixings. However, the quoted IDR from each bank is used for the NIBOR fixing at 12:00 pm, and it will serve as a good indicator of how the individual reported quotes, and thereby how NIBOR fluctuate within a given day. By examining the microstructure along these two dimensions, we seek to answer the fundamental research question of how the structure and governance of NIBOR affect the participants' contribution to the interest rate. Our concluding remarks from chapter 1 indicated that even though Norwegian banks face equal financing conditions as European banks, NIBOR is quoted significantly higher. In the extension of this, we seek to give further insight on whether there exist any systematic similarities or differences in the rate reporting across panel banks.

A. Choice of framework

The approach and theoretical frameworks will be different for the two dimensions of interest. Part one decomposes the NIBOR rate based on the six underlying quotes that constitute the 12:00 pm fixing. This entails having both a cross-sectional dimension and a time-series dimension, which allows us to study bank characteristics across time. Utilising these inherent properties, we will construct a model allowing for unobserved individual effects across banks and time. The model utilises credit default swap spreads on corporate senior bonds as a proxy for bank credit risk. It examines long term dynamics by comparing the quoted interest rates to underlying credit risk.

The second part explores the short term patterns of the high frequency IDRs. High-frequency data have the inherent property of arriving at irregularly spaced intervals, which significantly limits the number of suitable econometric models which often requires fixed interval data. The availability of high-frequency data has increased simultaneously with increasing computational power the last decades. This has resulted in a new research area, which has grown substantially. What is considered the start of this fast growing area of research was first presented by Robert F. Engle in

1996 and published in 2000. This pioneering work has later resulted in several econometric models applicable for irregularly spaced financial time series, in which many of them are developments of models that originally was intended for regular time series. Many of these frameworks seek to capture the non-linearities, intraday seasonalities, and dynamics of the duration between each observation (Hautsch, 2012). However, being interested in examining short term patterns of the IDRs we have chosen to apply a hidden markov model. The application of hidden markov models is more uncommon within economics, but is widely used in computer science on sequential data, and especially within speech recognition (Rabiner, 1989). In this paper, the model will be used in a similar fashion to recognize and hence capture patterns of the IDRs.

B. *Unobserved individual effects*

In order to address the unobserved individual effects, let us postulate a standard multiple regression model with K explanatory variables. Denote each individual bank with i, the time period t, and extend the model by allowing for unobserved effects a_i. This coefficient represents the unobserved effects within the sample, which in our case is between the individual panel banks.

$$y_{it} = \sum_{k=0}^{K} x_{itk}\beta_k + a_i + u_{it}, \ t = 1, 2, ..., T. \tag{44}$$

Note that we define $x_{it0} = 1$ and that u_{it} is idiosyncratic error as it changes across both dimensions, t and i. Unless we are able to isolate and incorporate the unobserved effects in the model, a_i is a part of the error term. When not addressing the presence of unobserved effects in our regression model, the standard regression model is

$$y_{it} = \sum_{k=0}^{K} x_{itk}\beta_k + v_{it}, \quad v_{it} = a_i + u_{it} \ \wedge \ t = 1, 2, ..., T. \tag{45}$$

Although disagreed on by some econometricians, modern econometric parlance often refer to the unobserved effect as either fixed or random effects(Wooldridge, 2010). Having unobserved *random effects* is identical to stating that there is zero correlation between the explanatory variables and the unobserved effect, $Cov(x_{itk}, a_i) = 0, \ t = 1, 2, ..., T$. A *fixed effects* model on the other hand allows for correlation between the unobserved effect and the explanatory variables, $cov(x_{itk}, a_i) \neq 0, \ t = 1, 2, ..., T$. Knowing the proper definitions, we may now draw up models that take the preceding statements into account, and explore the limitations and advantages of utilising one approach over the other. Lastly, we present a selection criterion comparing the estimation results from fixed and random effects to help us decide the most appropriate model according to the data we have. Note that for both models to yield unbiased, and hence consistent results, we need strict exogeneity in the explanatory variables, i.e $E(u_{it}|x_{itk}, a_i) = 0$ (Wooldridge, 2012).

For the fixed effect model, there are several approaches. The most common ones are the least square dummy variable approach (LSDV), the within group estimator (WG) and the first difference approach (FD). While the LSDV and WG yield equal results for all numbers of time periods, the FD approach is only equal to the other two approaches when $T = 2$. Let us start of by presenting the models, and thereafter select the fitting model for our desired purpose.

The LSDV approach remove individual specific effects from the error term by including dummy variables. Generally, a multiple regression model with dummies to correct for effects is expressed as

$$y_{it} = \sum_{k=0}^{K} x_{itk}\beta_k + \sum_{i=1}^{N-1} a_i\delta_i + u_{it}, \quad t = 1, 2, ..., T. \tag{46}$$

Where δ_i is a dummy variable indicating each individual. We include $N-1$ dummies, as having N dummies would imply perfect multicollinearity. A very important detail worth noting is the fact that we have kept the individual effects a_i in the equation.

The WG estimator relies on a transformation to correct for fixed effects. Define the following variables:

$$\begin{aligned} y^*_{it} &= y_{it} - \bar{y}_i, \\ x^*_{it} &= x_{it} - \bar{x}_i, \\ u^*_{it} &= u_{it} - \bar{u}_i. \end{aligned}$$

Let us expand y^*_{it} using a multiple regression model as in equation (44) for a given time in a one explanatory variable case for simplification. This entails

$$y^*_{it} = (\beta_0 + \beta_1 x_{it1} + a_i + u_{it}) - (\beta_0 + \beta_1 \bar{x}_i + a_i + \bar{u}_i). \tag{47}$$

By simplifying terms, we are left with our WG model. Note that the model in order to address the unobserved effects have removed a_i from the equation:

$$y^*_{it} = \beta_1 x^*_{it} + u^*_{it}. \tag{48}$$

Lastly, the FD approach transforms the equation by taking the first difference of the multiple regression model in equation (44). Similarly to the WG model, a_i disappears as it does not fluctuate across time, and

$$\Delta y_{it} = \sum_{k=0}^{K} \Delta x_{itk}\beta_k + \Delta u_{it}, t = 1, 2, ..., T. \tag{49}$$

The three approaches all correct the multiple regression for fixed effects, but in different ways. Both the WG and the FD approach address unobserved effects by removing the time invariant effect from the equation. This correction prohibits us to make any inference about variation between the groups, which is the variation between panel banks we want to examine. The LSDV approach is the only approach to address unobserved effects and still keep it in the modelled equation. We will thereby only use the LSDV approach of the fixed effect models.

As we briefly addressed in the introductory discussion, we distinguish between fixed and random effects. Deriving the random effects model is quite similar to the WG estimation, however it is a bit more tedious (Wooldridge, 2010). Let us again start out at the multiple regression in equation (44) and impose some assumptions regarding the error term and the individual effects (Wooldridge, 2012).

$$y_{it} = \sum_{k=0}^{K} x_{itk}\beta + v_{it}, t = 1, 2, ..., T \tag{50}$$

where

$$v_{it} = a_i + u_{it}, \tag{51}$$

$$a_i \sim N(0, \sigma_a^2), \tag{52}$$

$$u_{it} \sim N(0, \sigma_u^2). \tag{53}$$

Under these assumptions, we must have serially correlated v_{it} across time:

$$corr(v_{it}, v_{is}) = \frac{\sigma_a^2}{(\sigma_a^2 + \sigma_u^2)}, t \neq s. \tag{54}$$

We now have to use generalized least squares (GLS) to solve the serial correlation problem. The GLS transformation itself is quite simple, although deriving it is time and space consuming. Let us thereby define the following GLS transformation:

$$\lambda = 1 - [\frac{\sigma_a^2}{\sigma_a^2 + T\sigma_u^2)}]^{\frac{1}{2}}, \quad 0 \leq \lambda \leq 1, \tag{55}$$

$$y_{it} - \lambda \bar{y}_i = \sum_{k=0}^{K} (x_{itk} - \lambda \bar{x}_{ik})\beta_k + (v_{it} - \lambda \bar{v}_i). \tag{56}$$

The random effects model hence subtracts a fraction, λ, of the average dependent on the variance of the error terms σ_a^2, σ_u^2. Observe now that the random effects transformation in accordance with the LSDV method allow for time invariant unobserved effects. On a further note the random effects model is equal to the within group model when $\lambda = 1$. This entails that the random effects model must be more efficient as it consumes less degrees of freedom, contingent on the models producing the same results.

C. *Hausman test*

Next we have to determine whether to apply the LSDV approach or the random effects model. Hausman proposed a specification test that is applicable for our particular problem (1978). The basic idea of the Hausman test is to check whether the random effect and fixed effect estimates lie close enough such that it does not matter which one we use. We would preferably use the random effects model, as it is more efficient, and as such, it tests whether we have enough evidence to reject the random effects model.

Let us first define the following variables, where VC is the variance-covariance matrix, and the βs are the regression coefficients of the model.

$$VC(\hat{q_1}) = VC(\hat{\beta}^{FE}) - VC(\hat{\beta}^{RE}), \tag{57}$$

$$\hat{q_1} = \hat{\beta}^{FE} - \hat{\beta}^{RE}. \tag{58}$$

Utilising these variables, the Hausman test is specified as

$$m = \hat{q_1}'[VC(\hat{q_1}]^{-1}\hat{q_1}, \tag{59}$$

where m is assumed to follow a chi-square distribution $\sim \chi_{df=p}$, and p is the number of endogenous variables. The null hypothesis and alternative hypothesis being

$$H_0 : \hat{\beta}^{FE} \cong \hat{\beta}^{RE}, \tag{60}$$

$$H_A : \hat{\beta}^{FE} \neq \hat{\beta}^{RE}, \tag{61}$$

which entails that if we reject the null hypothesis, the fixed effects model would be preferred. The Hausman test serves an arguably more important role than determining if we can use the more efficient random effects model: it allows us to test if there are fixed effects present in the data.

The introduction of the Hausman test concludes the frameworks used to quantify the relation between the interest rate and underlying credit risk. In the following section, we will introduce the hidden markov model that will be applied to examine the short term patterns of the indicative deposit rates.

D. *Hidden markov model*

For analysing the time-varying high frequency data, we can evaluate a sequence of observed data as the result of an underlying Hidden Markov process. Hidden markov models (hereby HMM) are often referred to as probabilistic functions of Markov chains, and the earliest work on theory of probabilistic functions of Markov chains originates from work published by Leonard E. Baum and his colleagues at the Institute for Defense Analyses(1966; 1967; 1970; 1972). A Markov chain has the following property:

Definition: Let $X = X_1, X_2, ...$ be a random process in discrete state space $\mathbb{R}$. It is called a Markov chain if the conditional probabilities between the outcomes at different times satisfy the *Markov property* for every sequence $x_1, x_2, .., x_{t+1} \in \mathbb{R}$:

$$P(X_{t+1} = x_{t+1} | X_t = x_t, X_{t-1} = x_{t-1}, ..., X_1 = x_1) = P(X_{t+1} = x_{t+1} | X_t = x_t),$$

and the probability of what we observe is only conditional on the previous observation. The theory underlying HMMs has been known for decades, however, due to it only being published in mathematical journals, its availability and application was for a long time somewhat unknown to other disciplines. With an increasing amount of tutorials on implementation and theory throughout the 1980s, popularity began to rise, especially within the field of speech recognizing. There are numerous different versions of the HMM, adapted to different situations. For this particular case, we limit ourselves to look at the discrete case original HMM, as this approach is suitable for our problem at hand. The application in this paper has been programmed in MATLAB® based on theory from the article published by Lawrence R. Rabiner(1989)

Hamilton (1987) popularized application of Markov chains on autoregressive time series. This development spans the foundation for application on financial time series. Recent work with HMMs on financial data seek to forecast values, like for instance Hassan and Nath (2005) which forecast stock prices of airline companies. Other applications use HMMs to capture dynamics of the volatility of financial time series. Zhuang and Chan (2008) use it to capture structural volatility changes that regular GARCH models do not capture.

The complete derivation of the HMM is quite tedious and somewhat complex. We will thereby only present the intuition behind each step in the following section. The interested reader may refer to appendix G for the complete mathematical derivation of the procedures utilised during the implementation. To get a firm grip on the intuition of the HMM, we will first introduce its elements, their properties and relationship. Thereafter we will look at some of the implementation issues, which arise, and how they can be solved.

E. Elements of the hidden markov model

The HMM contains five different elements, which span the basis of the model:

1. N, the number of states in the model. These states follow a Markov process and are by definition hidden, we do hence not know which state the observation has been extracted from. Denote the states by $S = S_1, S_2, ..., S_N$, and the state at time t as q_t.

2. M, the number of distinct observation symbols per state. We denote the individual symbols as $V = v_1, v_2, ..., v_k$.
3. The state transition probability distribution $A = a_{ij}$, where a_{ij} is the probability to transition from state S_i at time t, to state S_j at time t+1:

$$a_{ij} = P[q_{t+1}] = S_j | q_t = S_i], 1 \leq i, j \leq N$$

4. The observation symbol probability distribution in state j, $B = \{b_j(k)\}$, where $b_j(k)$ is the probability of observing v_k at time t given state S_j:

$$b_j(k) = P[v_k \ at \ t | q_t = S_j], 1 \leq j \leq N, 1 \leq k \leq M$$

5. The initial state distribution $\pi = \{\pi_i\}$ where π_i is the probability of starting in state S_i:

$$\pi_i = P[q_1 = S_i], 1 \leq i \leq N$$

To apply the HMM, we must specify N, M, the number of observation, a discrete distribution for the observations, and the probability matrices A, B and π. We will similarly to the article by Rabiner (1989) use the following notation for the HMM:

$$\lambda = (A, B, \pi),$$

where λ is a collective notation for the chosen A, B and π.

F. Implementing and optimising the hidden markov model

Given the preceding framework, three problems must be addressed in order to efficiently implement the model:

1. Given the observation sequence $O = O_1 O_2 \cdots O_T$, and a model $\lambda = (A, B, \pi)$, how do we in an efficient manner compute $P(O|\lambda)$ - the probability of the observation sequence, given the model?

2. Given the observation sequence $O = O_1 O_2 \cdots O_T$, and a model $\lambda = (A, B, \pi)$, how do we choose a corresponding state sequence $Q = q_1 q_2 \cdots q_T$ which best explains the observations?

3. How do we adjust the model parameters of $\lambda = (A, B, \pi)$ in order to maximize $P(O|\lambda)$?

1. The intuitive approach to calculating $P(O|\lambda)$ would be to enumerate through every possible state sequence of length $"T"$. The probability of the observations sequence given the model is obtained by summing over the joint probability over all possible state sequences:

$$P(O|\lambda) = \sum_{Q} P(O|Q,\lambda)P(Q,\lambda) = \sum_{q_1,q_2,\cdots,q_T} \pi_{q1} b_{q_1}(O_1) a_{q_1 q_2} b_{q_2}(O_2) \cdots a_{q_{T-1} q_T} b_{q_t}(O_T)$$

Using the brute force method would entail a total of $2T \cdot N^T$ calculations, as there are N possible states that could be reached at any given time. Which would be unfeasible for N=6 states even with quite few time steps. It would for instance imply $2 \cdot 50 \cdot 6^{50} \approx 8 \cdot 10^{40}$ given T=50 - an unfeasible amount of computations. The solution to the preceding problem is a *forward-backward procedure.* The forward-backward procedure consists, as the name suggests of two different variables, the forward and the backward variable. Technically, we only need the forward variable for this particular problem, but we will introduce the concept of the backward variable as it is used to solve the other two implementation issues.

The forward variable $\alpha_t(i)$ is the probability of observing a given observation sequence until time t, and state S_i at time t. The probability of observing the observation sequence given the model must hence be the sum of $\alpha_t(i)$ across all states at time t. Using this method instead of the brute force procedure reduces the number of calculations to N^2T. This entails that we have reduced the number of calculations from $8 \cdot 10^{40}$ to $6^2 \cdot 50 = 1800$ for our preceding example. The backward variable $\beta_t(i)$ is on the other hand the probability of the observation sequence from $t+1$ to time T, given state S_i at time t. They will both be utilised to solve our next two problems.

2. The second problem may be solved in several different manners, whereas the most dynamic one would be to apply a *Viterbi algorithm*, which takes into account limitations concerning possible states to transcend to. However, since we might reach any state s_j from state s_i, the problem may be solved straightforward utilising the forward and backward variables. In order to find the the optimal state sequence, we define a new variable $\gamma_t(i)$, which is a product of our forward and backward variable, as the probability of being in state S_i at time t. The most likely state would hence be uncovered by using gamma to find the most likely state at for all t.

3. As for problem 3, there is no known way to analytically solve for the model that maximises the probability of the observation sequence. We are however able to choose $\lambda = (A, B, \pi)$ such that the probability of the observation sequence given the model is *locally* maximized by using an algorithm such as the Baum-Welch(1970). Applying the Baum-Welch algorithm is known as *training* the model as it iteratively improves the likelihood of $P(O|\lambda)$ with each iteration.

The algorithm uses our forward and backward variables to generate a new variable $\xi_t(i,j)$, which is the probability of being in state S_i at time t, and state S_j at time t+1. It then re-estimates our initial parameters A, B and π by using both $\gamma_t(i)$ and $\xi_t(i,j)$. These re-estimation equations originate from the intuition brought to the table by Baum and his colleagues(1970). Define the new model as $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$. They then proved that if you maximize a given likelihood function, using $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$ leads to an increase in the likelihood of the model. Utilising their result, we can replace λ with $\bar{\lambda}$ and repeat the re-estimation procedure. This re-estimation is then repeated until we reach the nearest local maximum of the likelihood function. Note that this iterative technique will only converge towards the nearest local maximum of the likelihood function, which hence gives us no guarantee of having the global maximum of the likelihood function.

Because the iterative technique only converges locally, the initial values of the model will have an impact on our solution considering that the optimization surface is complex and has multiple maximum. Rabiner(1989) points to the fact that there are no straightforward answer to selecting initial state. He further states that for A and π, experience shows that either random or uniform initial estimates are adequate. For B on the other hand, good initial estimates are helpful in the discrete case, and more essential for the continuous case. He further points out that initial estimates can be obtained in a number of ways, such as manual segmentation of sequences into states with averaging of observations within state, maximum likelihood segmentation of observations with averaging and segmental k-means segmentation with clustering.

As we wanted to examine short term patterns of the Indicative Deposit Rates, training the hidden markov model on the entire sequence of observations would yield no insight with regards to the question at hand. Hence we would have to modify our training of the model such that it maximizes $P(O|\lambda)$ for each defined interval. Addressing the problem in such a manner, we can start out by dividing our entire data sample into k sequences of data. Assuming that the observation sequences are independent of each other, we would now seek to maximize the product of the probability of observing each sequence, given the model. We are then able to utilise the individual model for each observation sequence to create the model that is on average the most likely for a given day across the entire set of observations.

VII. Chapter 2: Evaluating the microstructure of NIBOR

So far, our results have indicated there seems to be some discrepancies between NIBOR and the lending cost of panel banks relative to other countries. In the following section, we seek to uncover characteristics of the microstructure of NIBOR that might be cause for the observed discrepancy across countries. We will evaluate the discrepancy along our two dimensions; long term dynamics and short term patterns. The first part investigates whether the individual NIBOR quotes reflects the panel banks individual underlying credit risk. Although NIBOR is quoted as an offered rate to an unspecified counterparty in the interbank market, it should still reflect the individual banks' credit risk. The second part examines the short term development of the interest rate, and seeks to uncover whether panel banks have the same perception of the short term development of the interest rate.

A. The relationship between NIBOR quotes and underlying credit risk

Recall that we wanted to examine how the structure and governance of NIBOR affected the participants' contribution to the interest rate. From a theoretical point of view, the credit default swap (CDS) is a financial derivative for a specific instrument (bond), which are to cover unspecified debt from a specific counterparty (bank). The swaps prices are quoted as the spread between the return on a senior corporate bond and the risk-free rate, and should hence be a good reflection of the credit risk within a certain bank. As the CDS spread reflects the credit risk across the panel banks, our data serves as a proxy of the credit risk present in the Norwegian interbank market for each individual bank. In Chapter 1, we derived the relationship between the interbank rates, financing cost, the risk-free interest rate and default premium. The financing cost in a domestic economy is given by

$$\overline{i_{N,N}} = rf_N + rp_N, \tag{62}$$

and the financing cost is again lower than the offered interest rate, $i_{N,N} \leq \overline{i_{N,N}}$. Having proxied the credit risk in the interbank market, a reasonable assumption to impose on the result would be that increased credit risk within the Norwegian interbank market should have an effect on both higher borrowing costs in both foreign and domestic currency, and thus increasing the cost of domestic lending. Theory would hence predict that increased credit risk would increase NIBOR. Recognizing the connection between the credit default swap spreads and credit risk is the foundation for our upcoming model.

Before applying our framework for identifying individual characteristics across banks and time, we must transform our data to ensure unbiased results. As the lowest available credit default swap maturity is one year, there is no interbank rate which coincide perfectly with the maturity of the credit default swaps. Though this is not optimal, and provides some unpredictability in the results, we will utilise the 6 month NIBOR rate in order to get the time horizons as coinciding as possible.

To isolate and compare the default premium for the banks to the individual quotes, we subtract the domestic risk-free rate from the NIBOR quotes. We proxy the risk-free rate by using Norwegian government 6 month Treasury bill, and consider the difference between NIBOR and the risk-free rate as the NIBOR domestic premium. Finally, we swap the CDS spreads from USD to NOK to eliminate currency risk, and convert from basis points to percentages to conveniently have the same unity across the variables. Observe furthermore that our panel has relatively few individuals compared to the length of the time frame. This entails that we have to address the issue of having stationary series to avoid problems with spurious regression, i.e providing relations that does not really exist. We used the Augmented Dickey-Fuller test to check for stationarity in the panel data, and conclude that all our time series are stationary with non-zero mean. The number of lags were determined using the Akaike Information Criterion. The critical values from our tests may be found in table XLV appendix F.

In figure 20 the average NIBOR domestic premium along with the CDS spreads has been plotted for the time period October 2012 to November 2015. The relationship within each individual bank is available in figure 52 appendix F. There seem to be some mutual tendencies between the series, it is however quite difficult to tell whether there exist a relationship between the CDS spread and the domestic premium by just looking at the graph.

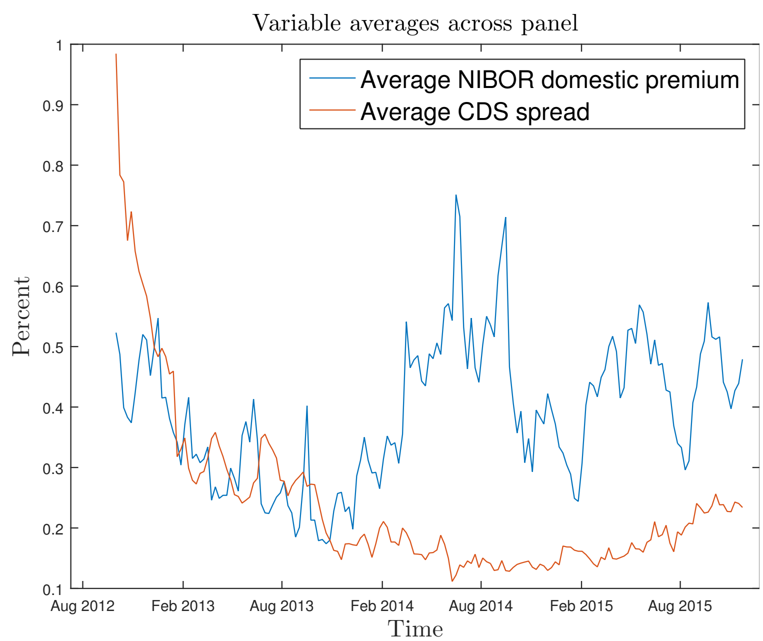

Figure 20. Average of 6M NIBOR domestic premium and 1 year maturity CDS spread

We might increase our understanding further by looking at the relationship between the variables within each bank across time. Figure 21 plots the domestic premium divided by the CDS spread within each bank for the entire sample. The notches indicate whether we may reject on a 95% significance level that the medians are different. The edge of the boxes indicate the 25% and 75% percentiles, while the end of the stapled lines roughly corresponds to the 99% percentile. Everything marked by red crosses are thereby defined as outliers. Observe by figure 21 that the median rate to spread ratio varies to some extent across banks. There is furthermore an extensive spread across banks in the volatility of the ratio. Although we recognize the fact that there should not be a one-to-one relationship between the spreads and the interest rate for reasons such as not coinciding maturities, there should be some consistency between the two. This consistency seems to vary both across and within banks.

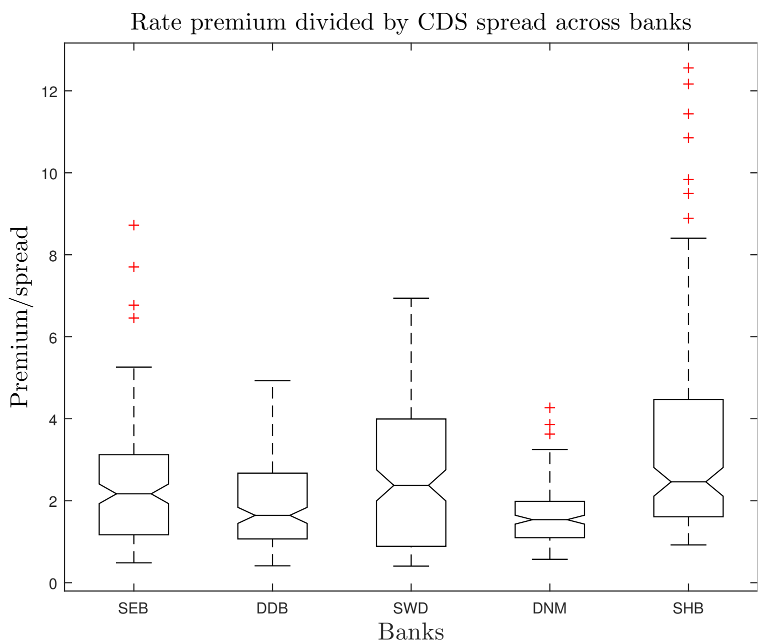

Figure 21. Rate premium divided by CDS spread across banks

Let us now postulate a model that quantifies the relationship between the market price for underlying credit risk within the panel banks and their corresponding NIBOR premium relative to the 6-month Norwegian government Treasury bill. We used the premium as our dependent variable and the CDS spread as our independent variable. Observing differences across banks, we also control for unobserved effects utilising both a random effects and a fixed effects model. This postulated model did however not indicate any significant relationship between the CDS spread. By looking at the dynamics of figure 20, we then tried to account for time fixed effects through including both year and monthly dummies, which gave quite different results. The Hausman test suggests that the individual unobserved effects are fixed for our first model, and random for our second and third. The models are presented in table XV. Results from the Hausman test may be found in figure XLIX appendix F. Equation (63), (64) and (65) shows the model specifications:

$$Ratepremium_{it} = \beta_1 CDS_{it} + a_i + u_{it},\ t = 1, 2, ..., T. \tag{63}$$

$$Ratepremium_{it} = \beta_1 CDS_{it} + \sum_{k=2013}^{2015} \beta_k \delta_k + a_i + u_{it},\ t = 1, 2, ..., T. \tag{64}$$

$$Ratepremium_{it} = \beta_1 CDS_{it} + \sum_{k=2013}^{2015} \beta_k \delta_k + \sum_{k=1}^{12} \beta_k \delta_k + a_i + u_{it},\ t = 1, 2, ..., T. \tag{65}$$

Note that both 1 month and year 2012 is omitted to avoid multicollinearity, such that January 2012 is the constant coefficient for the third model in table XV. When implementing the seasonal adjustments, which are all statistically significant, the coefficient of the CDS spread transcends from being almost equal to zero and statistically insignificant to being statistically significant. As theory would predict, the credit default swap spreads is positively correlated with the NIBOR premium. However, from the magnitude of the CDS coefficient, there is a surprisingly small relationship between the banks' CDS and the domestic premium in the interbank market. Our third model, adjusting for both yearly and monthly seasonal effects predicts that a 1% increase in the CDS spread only increases the domestic premium with 0.10%.

When adjusting for time-varying factors, our two last models confirm that there does exist a relationship between the market price for underlying credit risk for the banks and their submitted quotes. The relationship between the two variables are however quite small relative to what one would expect. Another interesting result from the model is the size of the constant coefficient, which relative to the effect of time-varying factors and increased CDS is quite large. The magnitude of the constant indicates that regardless of the CDS level, there is always some added premium between the default premium in the financing cost, and the offered interbank rate.

Regarding the causation, one can only speculate whether credit risk as a whole is reflected in the banks CDS spreads, and thereby reflects current information on credit risk that panel banks utilise in their calculation of the quote. Regardless of causality, there should be a correlation between the CDS spreads and domestic premium.

	(Model 1,FE)	(Model 2,RE)	(Model 3,RE)
	Rate premium	Rate premium	Rate premium
CDS	-0.0210	0.0857*	0.100***
	(0.0293)	(0.0336)	(0.0296)
2013.Year		-0.157***	-0.194***
		(0.0166)	(0.0155)
2014.Year		0.0211	-0.0146
		(0.0190)	(0.0176)
2015.Year		0.00517	-0.0320
		(0.0181)	(0.0170)
2.Month			0.0621***
			(0.0142)
3.Month			0.0986***
			(0.0137)
4.Month			0.0955***
			(0.0140)
5.Month			0.157***
			(0.0143)
6.Month			0.185***
			(0.0137)
7.Month			0.0663***
			(0.0140)
8.Month			0.0659***
			(0.0140)
9.Month			0.139***
			(0.0135)
10.Month			0.0282*
			(0.0138)
11.Month			0.0538***
			(0.0137)
12.Month			0.0573***
			(0.0141)
constant	0.397***	0.413***	0.360***
	(0.00791)	(0.0229)	(0.0243)
N	825	825	825
R^2			
$R^2 within$	0.0006	0.4160	0.5885
$R^2 between$	0.7690	0.7690	0.7690
$R^2 overall$	0.0006	0.4118	0.5801

Standard errors in parentheses

$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Table XV Unobserved effects Models 1-3

The previous model adjusted for constant time-varying effects, but assumed a constant relationship between CDS spread and domestic premium throughout our entire sample. Recognizing that the marginal contribution of CDS spreads might change across time allows us to include date specific variations in the correlation between the CDS and domestic premium. This allows us to examine the effects of changes to governance structure of NIBOR, as we know when major governance changes were made. In the following section, we will try to examine the effect of structural changes utilising our preceding model as a foundation, while allowing for the marginal contribution of the CDS spread to fluctuate across time.

B. *Evaluating effect of changes to governance structure*

To incorporate governance effects in the model, we identify important changes to the regulatory framework of NIBOR. The last major change in the NIBOR regulations was implemented at the end of 2013. The important changes were increased transparency and requirements for documentation, reducing the number of maturities from ten to five, implementing guidelines for determination of quotes, and the introduction of the IDRs. To evaluate if the changes have influenced the relationship between default premium for the banks and the domestic premium, and if the banks behave differently subsequent to the changes of governance, we modify the model to allow for variations in the independent variable.

By examining figure 20 again, we notice increasing difference between the domestic premium and the CDS as of the end of 2013. As the two variables starts drifting apart, there might have been a structural change in the relationship between our variables. This hypothesis may be subject to further investigation by statistical testing. We tested for structural changes by using a Wald test on a regression of the individual bank rate premium on their CDS spread. The test suggest that there is a structural break in the relationship in the start of 2014 for all our series. Results of the test may be found in table LI appendix F. Let us analyse these two periods separately by utilising the same box plot illustration as we did for the entire sample in section VII.A. For simplicity and in accordance with the structural break tests we will operate with the periods being preceding and subsequent to 2014. Figure 22 plots the rate divided by the spread for each bank for the periods before and after the structural break. The blue squares represents data for a given bank before 2014, while the black squares represents the period subsequent to the change.

Figure 22 illustrates how the ratio between the domestic premium and the CDS spread seem to be quite consistent for our first period, while it for the second period seem to differ quite widely both across and within banks. To allow the relationship between CDS and domestic premium to vary contingent on the structural break, we include the dummy *change* which is equal to 1 for the period subsequent to the structural change, along with an interaction term between this dummy and the CDS spread variable. In order to examine our hypothesis of the structural difference being caused by the change of governance and regulations, we also present a model which allows for the

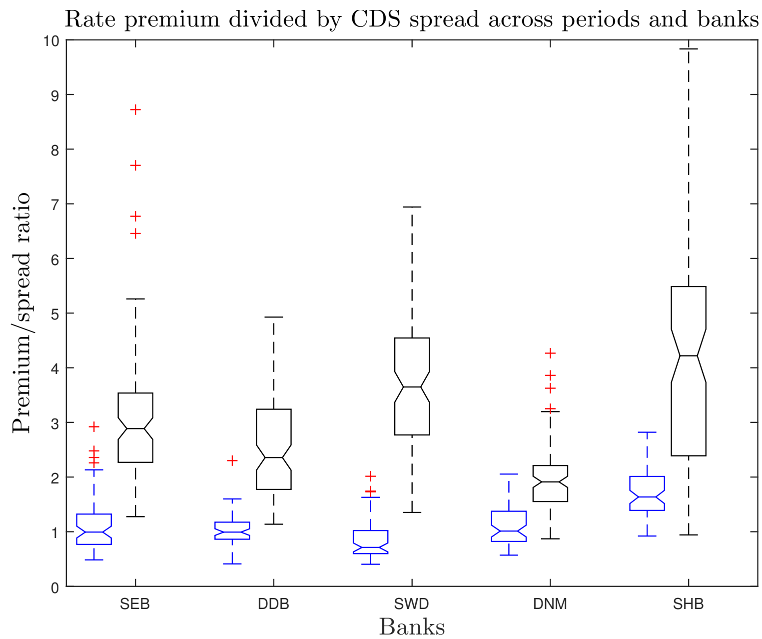

Figure 22. Rate premium divided by CDS spread ratio across banks and periods

change of dynamics across each of our four years to allow for gradual changes after the new governance structure. The second model contains a dummy for each given year, along with an interaction term between the CDS and the given year. For our first model, the Hausman test suggests that we cannot reject the effects being random, while the second model rejects the effects being random. Results from the Hausman test may be found in table L appendix F. We thereby represent the least square dummy variable approach for the second model, which quantifies differences across banks of the panel. The regressions may be found in table XVI. For illustration purposes we have left the coefficients for each month out of the regression, but note that these were also adjusted for in the regression with quite similar results as in table XV. Equation (66) and (67) shows the model specifications:

$$Ratepremium_{it} = \beta_1 CDS_{it} + \beta_2 change_{it} + \beta_3 change \times CDS_{it} + a_i + u_{it},\ t = 1, 2, ..., T. \tag{66}$$

$$Ratepremium_{it} = \beta_1 CDS_{it} + \sum_{k=2013}^{2015} \beta_k year.k_{it} + \sum_{k=2013}^{2015} \beta_k year_k \times CDS_{itk} + a_i + u_{it},\ t = 1, 2, ..., T. \tag{67}$$

	(Model 4, RE)	(Model 5, FE)
	Rate_6M_premium	Rate_6M_premium
CDS	0.379***	-0.00472
	(0.0270)	(0.0416)
change	0.236***	
	(0.0142)	
change$\times CDS$	-0.333***	
	(0.0632)	
2013		-0.285***
		(0.0283)
2013$\times CDS$		0.236***
		(0.0660)
2014		-0.0339
		(0.0292)
2014$\times CDS$		-0.154
		(0.0991)
y2015		-0.107***
		(0.0305)
2015$\times CDS$		0.190*
		(0.0825)
2.Bank		0.0455***
		(0.00889)
4.Bank		0.00800
		(0.00874)
5.Bank		0.0171
		(0.00911)
6.Bank		-0.0136
		(0.00887)
constant	0.0965***	0.406***
	(0.0159)	(0.0291)
N	825	825
R^2		
$R^2 within$	0.5173	0.6013
$R^2 between$	0.5207	
$R^2 overall$	0.5171	

Standard errors in parentheses

$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Table XVI Unobserved effects models 4-5

Model 4 examines the marginal contribution of the CDS spread before and after the last major change to the NIBOR governance and structure. This model allows the market price for credit risk to be reflected differently in the rate dependent on time. The model clearly suggests that the relationship between changes across our panel as CDS positive while $change \times CDS$ is negative. Observe that model 4 suggests that the coefficient for the CDS in our first period is as high as 0.379, which is almost four times as large as what model 3 predicted. What is even more interesting however, is the fact that the marginal contribution of the CDS for our second period is $0.379 - 0.333 = 0.047$, which is very small. This entails that the relationship between the banks' proxied underlying credit risk and their reported interest rate quotes, although significant, is almost equal to zero. These results are surprising as it suggests that the influence of credit risk varies across periods, and for periods that are more recent, the relationship between CDS and domestic premium is almost nonexistent.

One hypothesis causing the preceding could be that the introduction of the indicative deposit rates that makes information on other banks quotes available is taken into account when submitting their own quote. This might well be involuntary, but available market information will still be incorporated in their submission. These factors might reduce the importance of underlying credit risk. Although observing these changes to the marginal correlation could certainly favour our hypothesis of it being caused by changes done to the NIBOR regulations, it is not anywhere clear that this is the main driver for the observed changes. Another hypothesis could be that our proxy does not correctly capture the dynamics of the interbank market risk in Norway. Then there must be some sort of discrepancy between the market and the banks' conception of interbank risk within certain time frames. Another explanation might be that there are other factors than Norwegian interbank credit risk that explains the development of NIBOR. What seems quite clear however is that the market price for risk should at least reflect some of the total risk in the interbank market, and there should thereby at least be a consistent relationship between the domestic premium and the credit risk. This does not seem to be the case.

In model 5, our last model, we incorporate yearly dynamics by allowing for changes in CDS for each year in the sample. The coefficient for the CDS, which is the baseline in 2012, is now not significant. A likely explanation is the low number of observations for the first year (from October to December) compared to preceding years. Few observations within each interval is also a weakness of model 5, as we only have a complete year for 2013 and 2014. The relationship between CDS and domestic premium for 2014 is not significantly different from zero, and indicates the lack of a systematic relationship between CDS and domestic premium from January 2014 to December 2014. As only half of our interaction terms are significantly different from zero, the model suggests there are significant differences in the persistency of the relationship between an increase in the banks' CDS, and the increased domestic premium in the interbank rates. In addition, the Hausman test fails to reject that the variations between banks are random which indicates fixed effects across the NIBOR panel banks. A joint hypothesis of all the individual bank coefficients being equal to zero is rejected, which indicate that there are systematical differences across banks within our panel with respect to the relationship between their underlying credit risk and their quoted rate. This is however, the first indication we have of there being systematic differences across banks with regards to the relationship between their underlying credit risk and their reported interest rate.

Our results does not collectively allow us to either reject or accept our hypothesis that changes in CDS spread and the domestic premium is linked to changes in NIBOR governance and regulations. Our results however suggests that the relationship between the rate and underlying credit risk in the Norwegian interbank market is not systematic across time. Furthermore, we only have very small indications on there being systematic differences across the banks of the panel, which indicates a mutual view on how individual credit risk affects the domestic premium. This could however be a function of small differences between the rate submission from each bank, and the fact that the underlying credit risk within each bank is mutually effected by conditions of the economy as a whole. The results do indicate that the relationship between credit risk within the Norwegian interbank market and the domestic premium is inconsistent over time.

C. Limitations of the unobserved effects model

Looking past the theoretical assumptions of our econometric models, which are addressed during the introduction of our frameworks, there are some limitations with regards to applying the framework on our underlying data. The first is that as we only have access to weekly data on the credit default swap spreads we might lose some of the dynamics that happens within a given week. It might enrich the analysis to apply the same framework to daily observations to increase precision even further.

One could argue that our proxies do not adequately capture the features we want. The Treasury bill would for instance not be completely risk-free, and thereby subtracting it might capture some of the premium. This property is hard to bypass, and would in that case reduce the size of the NIBOR domestic premium. Furthermore, it could be that our credit default swaps do not capture credit risk in the underlying bank. However, studies such as the one by Chiramonte and Casu (2010), who examined a wide panel of European banks suggest that CDS spreads reflect risk captured by balance sheet ratios.

D. The hidden markov model

Previous results indicate that NIBOR is higher than the financing cost in foreign currency, and that the relationship between the domestic premium and credit default risk is inconsistent over time. By uncovering characteristics of the interest rate in the shortest time frame possible, we can see whether there are consistencies between the banks perception of interest rate movements. If there is a consistent opinion among the panel banks, there must be a mutual perception on the development of the rate. In order to investigate the short term patterns, we use the HMM on the indicative deposit rates (IDR). The idea is that the HMM recursively trains forth the best model based on what it observes within the specified time frame.

To implement the HMM, we need to define which part of our data that corresponds to the five elements of the model. We will start by assuming that the hidden states represent the individual banks, denoting the states $S = [S_1, S_2, S_3, S_4, S_5, S_6]$. This assumption allows for the real model on long term to differ from what we see within our sample. We will utilise the empirical information on the state distribution when addressing initialization of the model. By this definition the state transition matrix $A = a_{ij}$ is a 6×6 matrix, where we allow for the possibility to transition from any given bank S_i to any S_j, and hence $a_{ij} > 0 \;\; \forall \;\; i, j$. This implies that after Bank 1 has submitted an indicative quote, either Bank 1 can submit the next indicative quote, or we can transition to one of the other banks to submit an indicative quote. Matrix π denotes the probability of a bank to be the first observation within a sequence. We further define the observation symbols V to be the change between the current quote, and the previous quote from the same bank. We choose to use two bins, which symbolises changing the rate either up or down. By this representation we have $V = [V_1, V_2] = [UP, DOWN]$, which entails that our symbol probability distribution matrix $B = b_j(k)$ by definition is a 6×2 matrix.

Before moving forward to the iterative part we need to address the choice of initial values for the model $\lambda = (A, B, \pi)$. Given that we know the empirical distribution of both the states and the observations within our sample, we will try three different paths for our initial estimates. Our first approach will be to initiate A, B and π according to our empirical distribution. The second approach is based on practical experience in the literature, and we initiate with uniformly distributed A and π matrices, while B is still based on the empirical distribution. For our third approach, we will utilise the empirical distribution for A and π, but for B we will assume that all the banks behave in a similar fashion, and are thus uniformly distributed. In order to determine which model fits our sample the best we will compare the likelihoods of the models. Note however as mentioned that this method does not guarantee a global maximum.

In order to ensure an efficient implementation, we examined the empirical properties and compared it to the properties of the fixing. Transcending to the frequency of changes distributions in appendix A.C, we observed that there within a given day on average seem to be substantially more activity preceding the fixing, and especially during the first two hours of the day. Based on the observed nature of the NIBOR fixing, we decided to apply the model on two different samples within each day. *Model 1* will be trained on the 10 last observations preceding the fixing each day, while *Model 2* will be trained on the 10 first observations within a given day.

Model 1 is important because the NIBOR fixing is based on prevailing quotes from each bank at 12:00 pm, and the model examines the last behaviours before fixing. The second model is interesting because of the observed frequency distribution, as there seems to be significantly higher activity within the first two hours of the day. As the model require fixed length observations sequences, we removed days that did not have at least 10 observations before 12:00 pm from the sample before applying the model. In addition to looking at two different samples, we applied the model across three different maturities within each sample: 1 week, 3 month and 6 month. Applying the models across maturities will bring another dimension to the analysis, and the choice of maturities are based on wanting to have a broad variety with respect to the length of the maturity of the interest rate.

Before introducing the models, let us recall some of the tendencies from the descriptive statistics as this may help us get a better economic intuition on the results of the models. We observed that the interest rate had fallen within our time frame, which would most likely lead to the majority of the observations being negative as the aggregate change is less than zero. Furthermore there was a clear trend that the majority of changes done to the interest rate is either one basis point up or down. This is likely caused by banks fluctuating around two basis points, and keeps reporting one basis point up or down, dependent on which side it tilts towards.

From the summary of frequencies for the individual banks in table XXXV in appendix F we see that there are distinct differences in the relative frequency of changes across banks. The aggregate data will help us identify discrepancies between the aggregated dynamics and dynamics within our selected samples. Each model will be presented in its own section and compared to empirical tendencies. The results from our two models will be utilised to draw up a broader picture on the patterns of the indicative deposit rates. Lastly, we will comment on some of the limitations and assumptions that follow the implementing the HMM on the indicative deposit rates.

E. Model 1: The last 10 quotes preceding the fixing

The logarithmic likelihoods of our three different approaches suggest that initiating with an empirical distribution of matrix A and π and a uniform B matrix gives the best fitting model. A summary of the likelihoods along with a complete presentation of the model across maturities may be found in appendix G.B. Note that the tables are read from left to right (a_{12} is the probability of transitioning from Bank 1 to Bank 2, while $b_1(1)$ is the probability of observing a reduced rate from bank 1). There are two important components to highlight; the transition matrix between banks, and the probabilities of the interest rate to go either up or down.

For the transition matrix, we know from the descriptive statistics that there are significant frequency differences among banks, which is reflected in the model. Table XVII compares the empirical aggregate frequency of a bank against the probability of transitioning to that bank predicted by the model. In order to convert the latter to a probability measure, we sum a given banks column in the transition matrix and divide by the number of banks, and the model is able to incorporate that some banks change their interest rate more than others.

Table XVII Probability of observing given bank

Maturity/Bank	SEB	DDB	NDA	SWD	DNM	SHB
1 week						
Model	0.1480	0.1357	0.2294	0.1752	0.1288	0.1828
Empirical	0.1316	0.2107	0.2806	0.066	0.1001	0.2103
Difference	0.0164	-0.0750	-0.0512	0.1092	0.0287	-0.0275
3 month						
Model	0.1658	0.1390	0.2315	0.1360	0.1327	0.1950
Empirical	0.1619	0.1193	0.3170	0.0619	0.1220	0.2177
Difference	0.0039	0.0197	-0.0855	0.0741	0.0107	-0.0227
6 month						
Model	0.1310	0.2310	0.1954	0.1025	0.1750	0.1652
Empirical	0.0926	0.2471	0.3204	0.0459	0.1444	0.1498
Difference	0.0384	-0.0161	-0.1250	0.0566	0.0306	0.0154

There is mostly marginal differences between the predicted bank contribution frequency and the empirical distribution of our entire sample, except for NDA and SWD. NDA, although still quite dominant, seem to be less active in the last period before fixing relative to their average contribution during the whole day. SWD's behaviour is opposite as their participation during the last 10 is significantly higher than their average day. This difference suggests that the frequency of bank changes are more evenly distributed preceding the fixing compared to the aggregate empirical data over the whole day. The dominance of NDA is also reflected in table XVIII, which shows the initial state distribution across maturities.

Table XVIII Initial state distribution

Maturity / Bank	SEB	DDB	NDA	SWD	DNM	SHB
1 Week	0.0789	0.0239	0.6192	0.0318	0.0520	0.1941
3 Month	0.0853	0.0360	0.5438	0.0203	0.0947	0.2199
6 Month	0.0439	0.0253	0.4708	0.0098	0.3971	0.0531

Looking more specifically at the transition matrices in appendix G.B there seems to be a relatively high probability of back-to-back observations from the same bank. This seems to be a consequence of the two decimal precision of the data, which as mentioned in the introduction leads to a bank reporting a changed interest rate when it fluctuates between two basis points. This indicates that the last changes before fixing are relatively small in size, and is supported by the symbol probability distributions across maturities in table XIX, which are all very close to being uniformly distributed across all maturities.

Table XIX Symbol probability distributions across maturities

Table XX 1 Week

Bank	Rate down	Rate up
SEB	0.5226	0.4774
DDB	0.4977	0.5023
NDA	0.5135	0.4865
SWD	0.4987	0.5013
DNM	0.5066	0.4934
SHB	0.5211	0.4789

Table XXI 3 Month

Rate down	Rate up
0.4902	0.5098
0.4988	0.5012
0.4801	0.5199
0.5016	0.4984
0.4971	0.5029
0.4774	0.5226

Table XXII 6 Month

Rate down	Rate up
0.4796	0.5204
0.5104	0.4896
0.4929	0.5071
0.5056	0.4944
0.4880	0.5120
0.4827	0.5173

For the part of the sample preceding the fixing, there seems to be quite a coinciding opinion across banks regarding the development of the rate. This claim mostly is based on the uniformity of the distribution of changes, which indicate that the participants on average have the same opinion regarding the last development of the interest rate before fixing. Although it can not be excluded that they always move in the opposite direction, and that they thereby on average have the same distribution, it is very unlikely. The similarities are likely to be caused by small changes within the last quotes preceding as the panel banks are fluctuating between two basis points. This indicates that the last quotes before the fixing does not cause for radical changes in the interest rate, and the behaviour of the panel banks last change before fixing is homogeneous.

F. Model 2: The first 10 quotes of the day

The logarithmic likelihoods for model 2 suggests the second approach, initiating with uniformly distributed A and π matrix, and an empirical B matrix. For this particular model, we will also give note to our π matrix, as it now yields some interesting information regarding which bank has the highest probability of having the first interest rate change of the day. A complete presentation of the likelihoods as well as the preferred models across maturities may be found in appendix G.C.

Looking at the properties of the transition matrices in a similar fashion as the previous section, observe from table XXIII that there seems to be quite significant differences between what the model predicts and the underlying empirical distribution for the whole sample. Especially SWD, which is the bank of the panel that empirically is the least frequent changer of their interest rate, is predicted by the model to be the most likely bank to be observed both for the shortest and longest maturity. Looking at the relative activity across both banks and maturities, the model indicates the activity level being quite evenly distributed across our panel banks. High activity among all the banks subsequent to the opening is also intuitive, as these early quotes would have incorporated information generated since the closing on the previous day.

Table XXIII Probability of observing given bank

Maturity/Bank	SEB	DDB	NDA	SWD	DNM	SHB
1 week						
Model	0.1691	0.1559	0.1512	0.2349	0.1446	0.1442
Empirical	0.1316	0.2107	0.2806	0.066	0.1001	0.2103
Difference	0.0375	-0.0548	-0.1294	0.1689	0.0445	-0.0661
3 month						
Model	0.1828	0.1818	0.1882	0.1494	0.1472	0.1507
Empirical	0.1619	0.1193	0.3170	0.0619	0.1220	0.2177
Difference	0.0209	0.0625	-0.1288	0.0875	0.0252	-0.0670
6 month						
Model	0.1503	0.1514	0.1496	0.2167	0.1901	0.1419
Empirical	0.0926	0.2471	0.3204	0.0459	0.1444	0.1498
Difference	0.0577	-0.0903	-0.1708	0.1708	0.0457	-0.0079

The state transition matrices in appendix G.C suggest that the probability of having repeated observations from a given bank is not more likely than observing two different banks, but quite the contrary. This is another indication which supports the distribution in table XXIII where all banks have a high probability of being observed during the first 10 observations. The initial state distribution matrix also supports the distribution, as there is seemingly no consistency in which bank has the first quote each day across maturities. Observe from table XXIV that the probability of a given bank being first differs widely across maturities, and that all our banks except SHB seem to frequently have the first observation for at least one maturity.

Table XXIV Initial state distribution

Maturity / Bank	SEB	DDB	NDA	SWD	DNM	SHB
1 Week	0.3331	0.0834	0.0561	0.4694	0.0285	0.0294
3 Month	0.2312	0.2963	0.3914	0.0157	0.0298	0.0357
6 Month	0.0268	0.0273	0.0155	0.4345	0.4825	0.0135

Assuming that there is sufficient activity across all banks in the opening of a given day, we can investigate how the different banks interpret the changes in the interest rate from yesterday's closing to the first contrition. This is an indicator of whether the banks perceive the short term development of the interest rate over night in a similar fashion. Table XXV shows the rate change distribution across the banks of the panel. This table has some interesting characteristics across the respective maturities. Observe especially that the relative probabilities of changing the rate either up or down varies widely across banks.

Table XXV Symbol probability distributions across maturities

Table XXVI 1 Week

Bank	Rate down	Rate up
SEB	0.8199	0.1801
DDB	0.7469	0.2531
NDA	0.6904	0.3096
SWD	0.0788	0.9212
DNM	0.4425	0.5575
SHB	0.4761	0.5239

Table XXVII 3 Month

Rate down	Rate up
0.8911	0.1089
0.9056	0.0944
0.1272	0.8728
0.5771	0.4229
0.3476	0.6524
0.3065	0.6935

Table XXVIII 6 Month

Rate down	Rate up
0.7185	0.2815
0.7263	0.2737
0.3339	0.6661
0.0918	0.9082
0.9014	0.0986
0.4312	0.5688

This observation is quite interesting, as there seems to be little consistency among the banks regarding which way the rate should move. Having observed that the frequencies vary quite significantly across banks within a given maturity could be an indication of the banks not having the same perception of how new information should be incorporated in the interest rate. However, it could also be that the banks face different information or lending costs within that given day and that it thereby varies significantly across the participants. There are some cases of banks having quite significantly different distributions across maturities. This seems odd, but might be explained by the bank having a different perspectives on the risk across the maturities. It might also be caused by limitations of the model, which will be addressed in section VII.H.

G. Comparative model analysis

So far, we have looked separately at two empirically and structural interesting parts of the day which have proven to have very different characteristics. In the following section, we will draw on the observed attributes from our two models to acquire a broader picture on the patterns of the indicative deposit rates.

The first interesting observation is the different distribution of changes between the two periods. The perception of what direction the interest rate should evolve differs widely between the banks at the start of the day, while it preceding to the fixing seem to be more of an agreement. These very differences might be explained by incorporation of all the information that has come in hand subsequent to the closing the previous day being incorporated in the start of the day. It is also an indication that information seems to be quite instantly incorporated in the interest rate level. Iterating even further, the information does not only influence the interest rate, but it seems to influence the interest rate differently with respect to direction across our banks.

Examining this difference even further, we review the empirical properties which may enrich the understanding of the model results. What is highly relevant is the relative size at which the information effects the interest rate. Observe by figure 23 that among the 10 first observations there are almost twice as many changes larger than 1 basis point across all maturities compared to the 10 observations preceding the fixing. The empirical distribution thereby suggest that there are the rate is more volatile during the start of the day. Incorporating this with the results of the previous paragraph suggests not only volatility, but that the differences across banks are not only coincidences based on fluctuating between two basis points. Knowing that the distribution of changes differ across banks during the start of the day, and that the changes are of significant sizes, we have results indicating that the perception of the short term development of the interest rate differs across our panel.

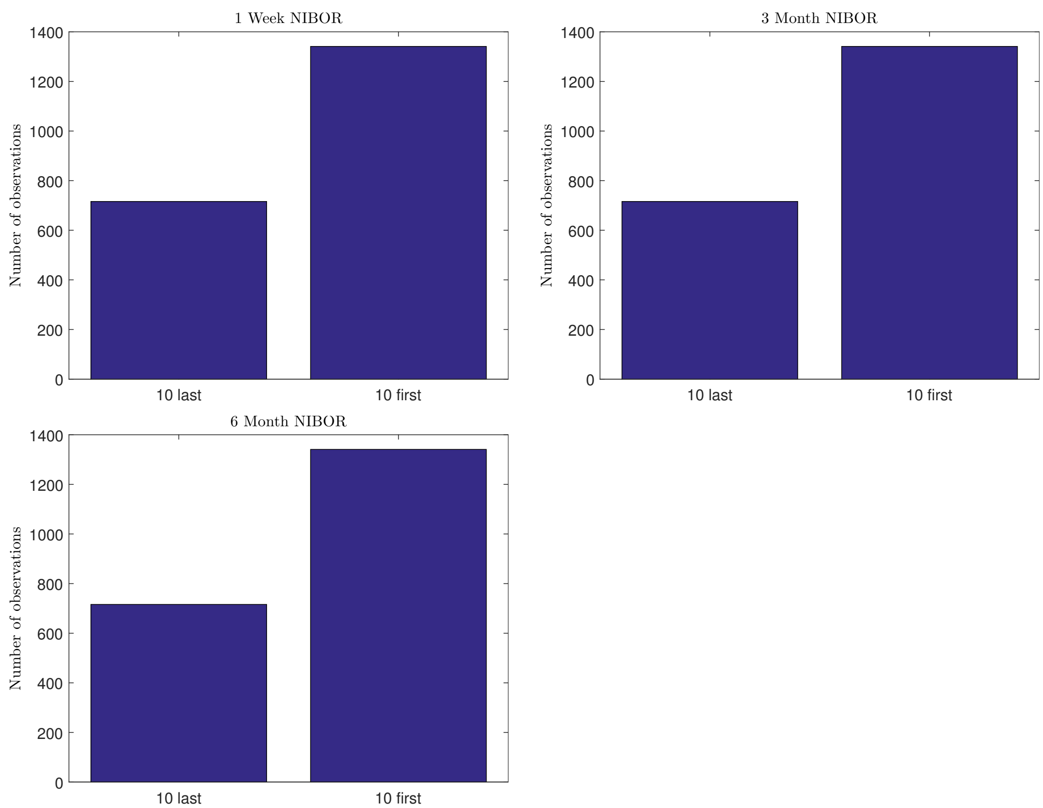

Figure 23. Number of changes larger than 1 basis points, by sample

Another interesting result is that the relative impact on the interest rate for the periods seem quite different. While for the period preceding the fixing there seems to be mutual agreement concerning the direction and development of the interest rate, there is a wide difference in the opening of the day. This suggest that a vast majority of the factors influencing is incorporated during the start of the day, and indicates that the first quotes, regardless of contributor, will indicate the baseline for the interest rate that day. This period is thus decisive for the outcome of the NIBOR fixing.

Based on these results, it is peculiar that even though information seem to be incorporated in such a different manner across the panel banks, the rates seem to follow each other quite accurately in the long run. Observing such a tendency could be an indication that banks continuously include previously submitted quotes in their information set. Taking that information into account is in fact reasonable, as it contains relevant information concerning other banks.

The causation of the observations found by utilising the HMM are not clear, but what seems evident is that banks perceive the short term development of the interest rate differently. This could entail that they either misinterpret the information, or that they have a different perception of what and how a given piece of information effects the rate. As they all follow the same long term trend however, these differences seem to diminish, which is peculiar if they react different to influencing factors.

H. Limitations of the hidden markov model

There are some obvious limitations concerning the application of the HMM on the indicative deposit rates. Although the states are dependent on previous states, the observed values are assumed independent of each other. Assuming that all our banks consider the same information when quoting their interest rate, it is highly unlikely that this assumption holds. Even if they did not consider the same information, it is still likely that they take information on other quotes into account, which in the end would lead to the observations being dependent. Another assumption which might be violated, is that there is dependence among the order of banks quoting their interest rate. It is not unlikely that the banks of the NIBOR panel report only based on their own information. However, one could infer that taking information on other banks quotes into account would make their rate change dependent on how the other participants behave in the market.

The most important limitation to emphasize is however, the fact that we have no guarantee for the model being the global maximum of our likelihood function. As its surface is complex and contains multiple maximum, iterating to the nearest local maximum does not anyway guarantee a globally optimal solution. Although we try three different initial estimates, this would only limit our error to some extent. It will only make sure that we are at least not selecting the local maximum with the lowest likelihood value. As we estimate 54 parameters, the surface of the likelihood function is highly complex, which makes it very difficult to verify whether we have selected the global optimum. The results are thereby dependent on our three selected approaches.

From a practical point of view, the precision of the data with respect to only being reported with a two decimal accuracy also limits some of the possible insights from the model. If the precision was better, the number of bins could have been expanded, and thereby accrue even richer dynamics in the model. The reported data also suffers from low precision, as if one of the panel banks fluctuate between two basis points, it keeps reporting one basis point up or down dependent on which side it tilts towards. Had the reporting contained even more decimals, the analysis could be even more precise.

Although there are some limitations concerning the application the HMM on the IDRs, the intent of the model was to apply a framework that allowed us to quantify short term market dynamics. Other presentations of such high frequency IDRs has earlier only been done in a descriptive manner, which obviously also has certain limitations concerning getting a good understanding of the dynamics of the data. One needs to exercise caution and be critical to observed results, but the quantification is still heavily based on underlying data, regardless of whether it is a local or a global maximum.

I. Partial conclusion

This section has investigated the microstructure of NIBOR in order to determine how the structure and governance of NIBOR effect the participants' contribution to the interest rate. We address the question along two dimensions; a long term dimension using an unobserved effects framework on the individual quotes that constitute the fixing, and a short term dimension using a hidden markov model on the high frequency indicative deposit rates.

Our first results suggest that the relationship between NIBOR and underlying credit risk in the Norwegian interbank market is inconsistent. This inconsistent relationship is mutual across the panel banks as there are no fixed effects present. We can neither reject nor accept the hypothesis of whether the inconsistency across time has been caused by the last structural change. A mutual perception of the inconsistent relationship is however peculiar as the rate should reflect conditions in the Norwegian interbank market which we proxy with the market price for credit risk.

Regarding the short term dimension, we focus on two important periods of the day. We analyse the changes in interest rates during opening hours, and the last changes preceding the fixing. Our results suggest that changes during the beginning of the day has the highest impact on the outcome of the daily fixing. This is likely a consequence of new information overnight being directly incorporated in the interest rate. Analysing the first daily changes indicates that information is being incorporated in very different ways across banks. This could very well be caused by misinterpreting the effect of the new information set, but it does however suggests that there is not a mutual agreement on how economic factors should effect the rate within a very short time frame. Although there are short term inconsistencies with respect to the development of the rate, the rate seems to have the same long term trend across the banks of the panel. This suggests that the differences that occur during the morning are diminishing. A possible reason could be bias from observing quotes from other participants.

We find inconsistencies in the microstructure of NIBOR that might suggest that the structure and governance of NIBOR does not provide sufficient guidelines to ensure that the banks have the same perception of factors effecting the interest rate.

VIII. Conclusion

This paper has examined the nature and causes of the Norwegian Interbank Offered Rate. By studying NIBOR in a domestic and cross-country perspective, we have investigated whether NIBOR has similar characteristics as comparable money market rates, such as EURIBOR and STIBOR. Digging deeper into the structure and governance of NIBOR has enabled us to get a broader understanding of the underlying process that determines the outcome of the interest rate. In the concluding remarks of the paper, we utilise the results from our analysis to describe the dynamics of NIBOR. Based on these results, we furthermore present possible changes to the NIBOR regulations. We believe these changes may address some of the issues highlighted in the paper, and increase the quality and accuracy of the interbank rate. Lastly, we discuss and suggest interesting future research on the subject.

A. The structure and behaviour of NIBOR

We evaluate whether NIBOR accurately reflects the lending cost in the Norwegian interbank market by comparing it to other interbank rates in both foreign and domestic currencies. To provide sufficient answers to this question, we used the yield curve model introduced by Nelson-Siegel. To confirm that the model is consistent with the behaviour of the interest rate, and fits observations out of the estimation sample, we performed an out-of-sample forecast. Interesting result from the Nelson-Siegel model were evaluated again using the Vasicek short-rate model to ensure that different aspects of interest rate theory produce coinciding results.

When comparing domestic money markets, we observed that domestic financing cost in currencies NIBOR banks are present are not the same, which supports differences between the interbank rates. The results regarding the relationship between the domestic financing cost in the Norwegian money market and the offered interbank rate was inconclusive. We observed similar discrepancies between the domestic interbank rates and domestic financing costs for the Norwegian banks in Sweden and the eurozone. This made us unable to determine if banks contributing to NIBOR are more or less accurate in the Norwegian interbank market, compared to other interbank markets they are present in.

When exploring the behaviour of interbank rates in the same foreign currency, euros, the Norwegian banks contributing to NIBOR and STIBOR were compared to an independent sample of European banks contributing to EURIBOR. The financing cost, regardless of the banks' origin were observed as the same, and we concluded that banks face equal financing costs in the European money market. When comparing currency adjusted interbank rates to the European financing cost, we observed a discrepancy between the interbank rates. NIBOR has consistently been quoted higher than both STIBOR and EURIBOR, this difference can not be justified by higher financing cost.

To check the validity of the results regarding the discrepancy between foreign financing cost and swapped interbank rates, we estimated the Vasicek model on a new sample of bonds maturing in the future. When estimating the Vasicek model on a new sample of bonds out of the Nelson-Siegel estimation period, we observed coinciding results. Estimated bond prices based on interbank rate dynamics were lower than the actual bond prices. This indicates that the underlying interest rate in the banks' bonds are lower than what we observe in the interbank market.

By applying the unobserved effects model on the microstructure of NIBOR we found inconsistencies across time in the relationship between the domestic premium and underlying credit risk using credit default swaps for each individual bank. This inconsistency is mutual across all the banks in the panel. The lack of a consistent relationship between default risk and domestic premium is one possible explanation for the observed discrepancy between the banks' financing cost and the interbank rate. It is peculiar that the domestic premium in the Norwegian money market does not have a consistent relationship with the underlying risk of default. There are no apparent patterns in the inconsistency, the relationship seems inconclusive.

Digging deeper into the short term development of NIBOR using the hidden markov model, we find differences in behaviour when facing new information during opening hours. These differences seem to be diminishing in the long run, which could possibly be caused by bias from observing quotes from other participants. Taking quotes from other participants into account is reasonable, as they provide useful information regarding other participants in the Norwegian interbank market. The results from analysing intraday development of the interbank rate uncovers a possible weakness in the governance structure, as the interpretation of information varies among the participants. Furthermore, we found that the magnitude of the changes closer to fixing are decreasing, which entails that the the first quotes are most important for the daily fixing of NIBOR.

B. Suggestions for improving NIBOR

What seems to be the most evident weakness of NIBOR is that it opens for individual opinions from all the contributing participants. These individual opinions do not only allow for misinterpretation of what the interest rate should reflect, but also makes evaluating the correctness of quotes a very difficult task. In the following section we propose changes that we, based on our results, believe could improve trustworthiness, correctness and transparency of NIBOR. We will suggest a variety of changes ranked by their extensiveness.

The first and least radical suggestion would be to make the IDRs invisible. The findings from applying the hidden markov model indicated that participants seem to have quite different perceptions regarding impacts of information accumulated since the previous day, which has a tendency to diminish over time. Banks might be influenced by the observable rates form other panel banks when submitting own quotes, which makes their contribution biased by other things than their own financing cost. Removing the IDRs would be one solution to ensure that quotes at least within a given day are independent of quotes from other participants.

To facilitate the evaluation of the historical quotes' correctness, another suggestion would be to require panel banks to document their lending cost. This would simplify the process of separating funding costs and premium, which makes it easier for the compliance committees' job to evaluate the accuracy of NIBOR.

Transcending to a more radical change, transitioning to an interbank regime where the banks are financially accountable for their submitted quotes would increase the consequences of quoting incorrect or inaccurate interest rates. A financial binding regime could be implemented in a variety of ways, such as introducing a specified monetary amount the banks are obliged to borrow, or lend to, within a span of their submitted quote. As it is unfair to expect that banks are able to quote their lending cost within one basis point, there should be some wiggle room for the precision of the quote.

The most radical and drastic change we present to increase the trustworthiness of the interbank rate is based on limiting the number of individual opinions involved with fixing NIBOR. We observed the lack of a common perception of what effects the short term development NIBOR, and a discrepancy between NIBOR and other interbank rates, even though the bank financing cost is the same. This discrepancy was not consistently driven by underlying credit risk. Based on this, it seems reasonable to claim that individual opinions and market interaction reduce the correctness of NIBOR. A solution to the problem could then be to incorporate an independent third party in the fixing process. The third party could either calculate an official benchmark directly comparable to NIBOR, or be assigned to calculate specific components of NIBOR, such as the premium exceeding financing cost.

C. Future research

Within the scope of evaluating the accuracy of interest rates, other interest rate models with time-dependent volatility, two and three-factor models are applicable on interbank rates to gain further insights in the dynamics of interest rates for unsecured loans. This paper has only evaluated if there exists a discrepancy between the financing cost for banks and their offered interest rate, not elaborated further on the magnitude or causes for an added premium across currencies. Topics involving identifying *why* this deviation exists is arguably more interesting than pointing out their existence, but has not been possible within the boundaries of this paper.

Iterating even further on the perspective of *why*, it would have enriched the analysis with respect to understanding the effects of governance changes if we had examined the individual quotes even further back in time. This data has however proven to be very difficult to accrue. This would enable a more comprehensive analysis of the effects of previous changes to the governance structure. Having benchmarks on changes would entail a deeper understanding of how changes to structure and governance has affected the participants.

For the last part of our second chapter, we combined elements from interest rate quotes and pattern recognition models on high frequency data. Pattern recognition has been applied on equity prices with mixed results, but is still an underdeveloped research area within financial economics. Interesting topics for future research within pattern recognition is identifying human behaviour in financial markets, such as in this paper with the indicative deposit rates. There has been increasing interest for behavioral finance and human decision making during the 21st century, but has yet to be approached using more advanced pattern recognition models.

Appendix A. Descriptive Statistics

Appendix A. Interbank rates

Table XXIX Descriptive statistics for STIBOR

Maturity(months)	Time Period (obs)	Mean	Std. dev.	Minimum	Maximum
0.03	20.10.2000 - 28.04.2015 (3788)	2.27	1.41	-0.30	6.48
0.23	20.10.2000 - 28.04.2015 (3788)	2.32	1.40	-0.28	6.08
1	20.10.2000 - 28.04.2015 (3788)	2.37	1.40	-0.28	5.4
2	20.10.2000 - 28.04.2015 (3788)	2.42	1.40	-0.25	5.45
3	20.10.2000 - 28.04.2015 (3788)	2.48	1.40	-0.22	5.60
6	20.10.2000 - 28.04.2015 (3788)	2.58	1.38	-0.16	5.60
9	20.10.2000 - 01.03.2013 (3226)	2.99	1.22	0.90	5.64
12	20.10.2000 - 01.03.2013 (3226)	3.08	1.22	0.95	5.78

Note: Number of observations in parentheses.

Table XXX Descriptive statistics for EURIBOR

Maturity(months)	Time Period (obs)	Mean	Std. dev.	Minimum	Maximum
0.23	20.10.2000 - 28.04.2015 (3788)	1.98	1.57	-0.09	5.17
0.47	15.10.2001 - 28.04.2015 (3532)	1.81	1.44	-0.07	5.04
0.70	15.10.2001 - 31.10.2013 (3144)	2.04	1.39	0.09	5.05
1	20.10.2000 - 28.04.2015 (3788)	2.05	1.57	-0.03	5.20
2	20.10.2000 - 28.04.2015 (3788)	2.11	1.57	-0.02	5.25
3	20.10.2000 - 28.04.2015 (3788)	2.18	1.55	-0.01	5.39
4	20.10.2000 - 31.10.2013 (3442)	2.44	1.45	0.22	5.42
5	20.10.2000 - 31.10.2013 (3442)	2.48	1.43	0.26	5.43
6	20.10.2000 - 28.04.2015 (3788)	2.28	1.50	0.06	5.45
7	20.10.2000 - 31.10.2013 (2282)	2.54	1.40	0.32	5.46
8	20.10.2000 - 31.10.2013 (2282)	2.56	1.38	0.35	5.47
9	20.10.2000 - 28.04.2015 (3226)	2.36	1.47	0.11	5.48
10	20.10.2000 - 31.10.2013 (2282)	2.61	1.36	0.42	5.49
11	20.10.2000 - 31.10.2013 (2282)	2.64	1.35	0.44	5.50
12	20.10.2000 - 28.04.2015 (3788)	2.44	1.44	0.17	5.53

Note: Number of observations in parentheses.

Reuters ID	ISIN	Type	NAME	Currency	Coupon	Issued	Maturity
6308T4	XS0598824570	ZERO	NORDEA BANK AB 2011 ZERO 01/09/11	EUR	0	01.03.2011	10.09.2011
6065XV	FI4000008669	FIX	NORDEA BANK FINLAND 2010 1.8% 01/02/12 4344	EUR	1,8	01.02.2010	10.02.2012
6022U6	FI4000005962	FIX	NORDEA BANK FINLAND 2009 1.8% 05/10/11 4311	EUR	1,8	05.10.2009	05.10.2011
6001V3	FI0003030890	FIX	NORDEA BANK FINLAND 2009 2.35% 09/02/12 4225	EUR	2,35	02.09.2009	09.02.2012
5915U4	XS0266020097	ZERO	DNB BANK ASA. 2006 ZERO 15/09/11	EUR	0	15.09.2006	15.09.2015
5915L5	XS0264984906	ZERO	DNB BANK ASA. 2006 ZERO 01/09/11	EUR	0	01.09.2006	10.09.2011
3837N1	XS0430768332	FIX	DNB BANK ASA. 2009 4 1/2% 29/05/14 503	EUR	4,5	29.05.2009	29.05.2014
3897P5	XS0443210090	FIX	NORDEA BANK AB 2009 3% 06/08/12 REG.S	EUR	3	06.08.2009	06.08.2012
3932KQ	XS0406936533	ZERO	NORDEA BANK AB 2009 ZERO 16/08/12	EUR	0	16.02.2009	16.08.2012
5905L4	XS0249057992	FIX	DNB BANK ASA. 2006 3 3/4% 14/04/11	EUR	3,75	28.03.2006	14.04.2011
5900Q8	XS0240290691	FIX	DNB BANK ASA. 2006 3 3/8% 10/10/11 302	EUR	3,375	10.01.2006	10.10.2011
5921FN	XS0276500153	FIX	DNB BANK ASA. 2006 4% 14/04/11	EUR	4	24.11.2006	14.04.2011
5925LQ	XS0284685046	ZERO	DNB BANK ASA. 2007 ZERO 31/01/12 392	EUR	0	31.01.2007	31.01.2012
5975DE	XS0372019017	FIX	DNB BANK ASA. 2008 5.9% 27/06/11	EUR	5,9	27.06.2008	27.06.2011
5975EC	XS0372646959	FIX	DNB BANK ASA. 2008 5.84% 27/06/11	EUR	5,84	27.06.2008	27.06.2011
5984JF	XS0392020938	FIX	DNB BANK ASA. 2008 5.9% 27/06/11 498	EUR	5,9	10.10.2008	27.06.2011
6019RH	XS0456656668	FIX	DNB BANK ASA. 2009 3 3/8% 08/10/14 REG.S	EUR	3,375	08.10.2009	08.10.2014
6040QU	XS0487501768	FIX	DNB BANK ASA. 2010 3% 16/02/15 REG.S	EUR	3	16.02.2010	16.02.2015
6465FR	XS0632507835	FIX	NORDEA BANK AB 2011 2.81% 20/02/15 S	EUR	2,81	31.05.2011	20.02.2015
6009XH	FI0003032268	FIX	NORDEA BANK FINLAND 2009 2 1/4% 05/06/12 4272	EUR	2,25	05.06.2009	05.06.2012
5996CJ	NO0010496557	FIX	NORDEA BANK NORGE 2009 4% 27/02/12 NODA79	NOK	4	27.02.2009	27.02.2012
5924DN	NO0010345051	FIX	NORDEA BANK FINLAND 2006 4.85% 31/10/11	NOK	4,85	04.12.2006	31.10.2011
5787WP	NO0010133135	FIX	SKAND.ENSKILDA 2002 6.85% 06/02/12	NOK	6,85	06.02.2002	06.02.2012
6015R8	NO0010534399	FIX	SWEDBANK 2009 5 3/4% 02/09/14 SWB08 P	NOK	5,75	02.09.2009	02.09.2014
5796PK	NO0010195670	FIX	DANSKE BANK A/S 2003 5.4% 26/08/13	NOK	5,4	26.08.2003	26.08.2013
5923L6	NO0010345739	ZERO	SVENSKA HANDBKN.AB 2006 ZERO 19/12/11	NOK	0	19.12.2006	19.12.2011
6003V1	NO0010512171	FIX	SKAND.ENSKILDA 2009 5.8% 15/05/14 SEB04 P	NOK	5,8	15.05.2009	15.05.2014
5931FW	SE0001969221	ZERO	NORDEA BANK AB 2007 ZERO 13/04/12 1141	SEK	0	13.04.2007	13.04.2012
5931JJ	SE0001969197	FIX	NORDEA BANK AB 2007 4% 13/04/12 1143	SEK	4	13.04.2007	13.04.2014

Note: All bonds are senior unsecured.

Table XXXI List of bonds used in the Nelson-Siegel model (1/3).

Reuters ID	ISIN	Type	NAME	Currency	Coupon	Issued	Maturity
5898FT	SE0001556036	FIX	SWEDBANK 2005 3.29% 28/10/13 106	SEK	3,29	28.10.2005	28.10.2013
5916NQ	SE0001803370	ZERO	SKAND.ENSKILDA 2006 ZERO 05/04/11 177	SEK	0	04.09.2006	05.04.2011
6015VQ	SE0002978072	FIX	SKAND.ENSKILDA 2009 2.9% 26/08/11 449	SEK	2,9	26.08.2009	26.08.2011
6038JH	XS0483821186	ZERO	ING BANK 2010 ZERO 08/02/11	EUR	0	08.02.2010	08.02.2011
6509M1	XS0643356420	FIX	ING BANK 2011 2.96% 01/07/14 REG.S	EUR	2,96	01.07.2011	01.07.2014
6320PF	XS0602697699	ZERO	ING BANK 2011 ZERO 06/03/12	EUR	0	07.03.2011	06.03.2012
6018N2	XS0454656751	FIX	ING BANK 2009 4.1% 04/10/12	EUR	4,1	01.10.2009	04.10.2012
5963Q5	XS0355972802	ZERO	ING BANK 2008 ZERO 02/06/11	EUR	0	30.05.2008	02.06.2011
7861X1	XS0770194487	FIX	ING GROEP 2012 4% 18/09/13 REG.S	EUR	4	04.04.2012	18.09.2013
2316PW	XS0385699029	FIX	ING GROEP 2008 5 5/8% 03/09/13	EUR	5,625	03.09.2008	03.09.2013
6270KK	XS0591666119	FIX	ING BANK 2011 4.7% 14/02/12 REG.S	EUR	4,7	14.02.2011	14.02.2012
24903P	DE0003933420	FIX	DEUTSCHE BANK 2003 2 3/4% 15/07/11 342	EUR	2,75	06.06.2003	15.07.2011
59360K	DE0003933727	FIX	DEUTSCHE BANK 2004 2 1/2% 17/11/10 372	EUR	2,5	26.10.2004	17.11.2010
59360P	DE0003933834	FIX	DEUTSCHE BANK 2004 2 3/4% 30/12/10 383	EUR	2,75	14.12.2004	30.12.2010
5609V4	DE000DB7XHA5	FIX	DEUTSCHE BANK 2010 2 7/8% 13/04/15	EUR	2,875	13.04.2010	13.04.2015
5972PJ	DE000DB5S659	FIX	DEUTSCHE BANK 2008 4 3/8% 02/06/14 S65	EUR	4,375	02.06.2008	02.06.2014
59360R	DE0003933859	FIX	DEUTSCHE BANK 2004 2.2% 01/02/11 385	EUR	2,2	20.12.2004	01.02.2011
59360Q	DE0003933842	FIX	DEUTSCHE BANK 2004 3% 30/12/11 384	EUR	3	14.12.2004	30.12.2011
49885M	DE0003933974	FIX	DEUTSCHE BANK 2005 2.15% 15/07/11 397	EUR	2,15	03.05.2005	15.07.2011
6065CT	XS0523096039	ZERO	BARCLAYS BANK PLC 2010 ZERO 08/04/13 REG.S	EUR	0	28.07.2010	08.04.2013
789EQ6	XS1018576030	FIX	BARCLAYS BANK PLC 2014 0.02% 20/03/15 11492	EUR	0,02	15.01.2014	20.03.2015
788MH5	XS1008749621	FIX	BARCLAYS BANK PLC 2013 0.02% 20/03/15 11492	EUR	0,02	20.12.2013	20.03.2015
6695XN	FR0011130228	FIX	BARCLAYS BANK PLC 2011 3 1/2% 13/04/15 REG.S	EUR	3,5	12.10.2011	13.04.2015
5906CL	XS0249956664	ZERO	BARCLAYS BANK PLC 2006 ZERO 05/07/12 REG.S	EUR	0	05.07.2006	05.07.2012
66553Q	XS0255546045	ZERO	BARCLAYS BANK PLC 2006 ZERO 03/08/12 REG.S	EUR	0	03.08.2006	03.08.2012
91388Q	XS0294450878	FIX	BARCLAYS BANK PLC 2007 4.185% 04/04/12 REG.S	EUR	4,185	04.04.2007	04.04.2012
5986R4	XS0398871185	FIX	BARCLAYS BANK PLC 2008 5.055% 11/11/11 REG.S	EUR	5,055	11.11.2008	11.11.2011
2025HL	XS0347689357	ZERO	BARCLAYS BANK PLC 2008 ZERO 16/04/13 11985	EUR	0	16.04.2008	16.04.2013
5990V1	XS0363856690	ZERO	BARCLAYS BANK PLC 2008 ZERO 24/02/14 SN12751	EUR	0	14.05.2008	24.02.2014
6002HK	XS0426423801	FIX	BARCLAYS BANK PLC 2009 4 1/2% 19/06/14 REG.S	EUR	4,5	12.06.2009	19.06.2014

Note: All bonds are senior unsecured.

Table XXXII List of bonds used in the Nelson-Siegel model (2/3).

Reuters ID	ISIN	Type	NAME	Currency	Coupon	Issued	Maturity
6001VN	XS0425636171	ZERO	BARCLAYS BANK PLC 2009 ZERO 30/04/14 REG.S	EUR	0	30.04.2009	30.04.2014
3708CM	XS0411153355	ZERO	BARCLAYS BANK PLC 2009 ZERO 19/12/13	EUR	0	30.01.2009	19.12.2013
5992K6	XS0406933191	ZERO	BARCLAYS BANK PLC 2009 ZERO 16/08/12	EUR	0	16.02.2009	16.08.2012
6064NE	XS0531411295	FIX	BARCLAYS BANK PLC 2010 2.3% 13/08/13 CSN2712	EUR	2,3	13.08.2010	13.08.2013
5547EF	IT0006711862	FIX	BARCLAYS BANK PLC 2010 3 1/4% 13/01/15	EUR	3,25	13.01.2010	13.01.2015
6051QR	XS0500113641	ZERO	BARCLAYS BANK PLC 2010 ZERO 05/11/14	EUR	0	07.04.2010	05.11.2014
6190XE	XS0571096378	FIX	BARCLAYS BANK PLC 2010 1.86% 18/12/13 REG.S	EUR	1,86	22.12.2010	18.12.2013
6034JE	XS0476055982	FIX	BARCLAYS BANK PLC 2010 0.715% 20/01/12 REG.S	EUR	0,715	04.01.2010	20.01.2012
6002U9	XS0427568141	FIX	BARCLAYS BANK PLC 2009 4% 29/05/14 REG.S	EUR	4	29.05.2009	29.05.2014
6239RD	XS0530230456	ZERO	BARCLAYS BANK PLC 2010 ZERO 30/01/12 REG.S	EUR	0	30.07.2010	30.01.2012
7880M2	XS0743102815	FIX	BARCLAYS BANK PLC 2012 1/2% 01/12/14 REG.S	EUR	0,5	12.04.2012	01.12.2014
75109U	DE000DB2J7Z7	FIX	DEUTSCHE BANK 2006 3.45% 01/09/10 J7Z	EUR	3,45	05.07.2006	01.09.2010
75109V	DE000DB2J705	FIX	DEUTSCHE BANK 2006 3 5/8% 01/09/11 J70	EUR	3,625	05.07.2006	01.09.2011
75109W	DE000DB2J713	FIX	DEUTSCHE BANK 2006 3 3/4% 31/08/12 J71	EUR	3,75	05.07.2006	31.08.2012
60053K	DE000DB2D9Y2	FIX	DEUTSCHE BANK 2005 3 1/8% 31/01/13 D9Y	EUR	3,125	16.11.2005	31.01.2013
60053L	DE000DB2D9Z9	FIX	DEUTSCHE BANK 2005 3 1/4% 31/01/14 D9Z	EUR	3,25	16.11.2005	31.01.2014
60053J	DE000DB2D9X4	FIX	DEUTSCHE BANK 2005 2 7/8% 31/01/11 D9X	EUR	2,875	16.11.2005	31.01.2011
55544F	DE000DB2D9B0	FIX	DEUTSCHE BANK 2005 1.95% 17/11/10 D9B	EUR	1,95	10.06.2005	17.11.2010
482886	DE0003933735	FIX	DEUTSCHE BANK 2004 3.1% 17/11/14 373	EUR	3,1	26.10.2004	17.11.2014
46787T	DE0003934469	FIX	DEUTSCHE BANK 2004 3.05% 15/09/10 46	EUR	3,05	15.09.2004	15.09.2010
59361E	DE0003934600	FIX	DEUTSCHE BANK 2004 3% 15/12/10 60	EUR	3	15.12.2004	15.12.2010
59360T	DE0003934527	FIX	DEUTSCHE BANK 2004 3 1/4% 27/10/10 52	EUR	3,25	27.10.2004	27.10.2010
46626X	DE0003934444	FIX	DEUTSCHE BANK 2004 2.95% 01/09/10 44	EUR	2,95	01.09.2004	01.09.2010
7069LC	XS0700834699	FIX	ING BANK 2011 4.4% 05/01/15 REG.S	EUR	4,4	30.10.2011	05.01.2015
7232CP	XS0708778054	FIX	DEUTSCHE BANK 2011 1.4% 23/11/13	EUR	1,4	23.11.2011	23.11.2013
20812N	NL0000085975	FIX	ING BANK 2002 5 1/2% 14/03/11	EUR	5,5	14.03.2002	14.03.2011

Note: All bonds are senior unsecured.

Table XXXIII List of bonds used in the Nelson-Siegel model (3/3).

Reuters ID	ISIN	Type	NAME	Currency	Coupon	Issued	Maturity
794JM8	XS1074393668	ZER	DANSKE BANK A/S 2014 ZERO 08/07/19	EUR	0	-	08.07.2019
794QZ8	XS1075640471	ZER	DANSKE BANK A/S 2014 ZERO 09/07/19	EUR	0	-	09.07.2019
798ZK6	XS1120658908	ZER	DANSKE BANK A/S 2014 ZERO 15/01/20	EUR	0	-	15.01.2020
793CCH	XS1063338310	ZER	DANSKE BANK A/S 2014 ZERO 23/05/19	EUR	0	-	23.05.2019
806GAJ	XS1025417822	ZER	SKAND.ENSKILDA 2014 ZERO 09/01/19	EUR	0	-	09.01.2019
799RPU	XS1127834825	ZER	SKAND.ENSKILDA 2014 ZERO 09/01/20	EUR	0	-	09.01.2020
786N97	XS0988100136	ZER	SKAND.ENSKILDA 2013 ZERO 23/12/15	EUR	0	-	23.12.2015
787H10	XS0996936141	ZER	NORDEA BANK FINLAND 2013 ZERO 15/01/19	EUR	0	-	15.01.2019
8474AA	SE0005040227	ZER	NORDEA BANK FINLAND 2013 ZERO 21/03/16 4790	EUR	0	-	21.03.2016
7936PG	XS1072257659	ZER	NORDEA BANK FINLAND 2014 ZERO 02/08/19	EUR	0	-	02.08.2019
797ZXZ	XS1108708170	ZER	NORDEA BANK FINLAND 2014 ZERO 08/11/19	EUR	0	-	08.11.2019
794XTP	XS1078019954	ZER	NORDEA BANK FINLAND 2014 ZERO 16/08/19	EUR	0	-	16.08.2019
790T28	XS1039387243	ZER	NORDEA BANK FINLAND 2014 ZERO 25/03/19	EUR	0	-	25.03.2019

Note: All bonds are senior unsecured.

Table XXXIV List of bonds used in the Vasicek model.

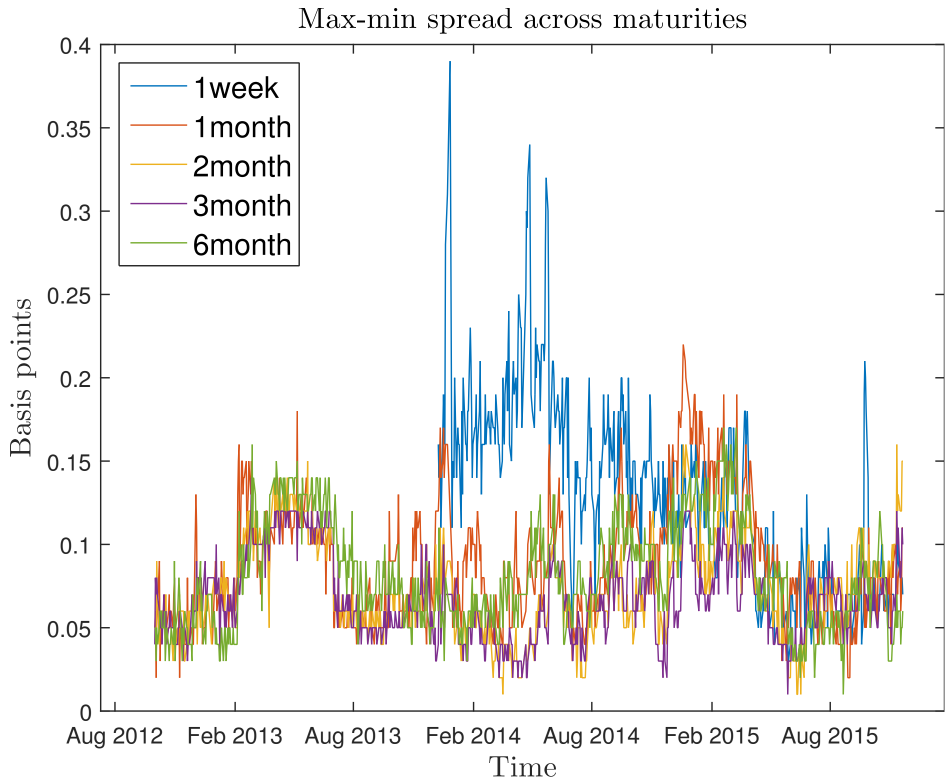

Figure 24. Spread between highest and lowest quote for the NIBOR fixings

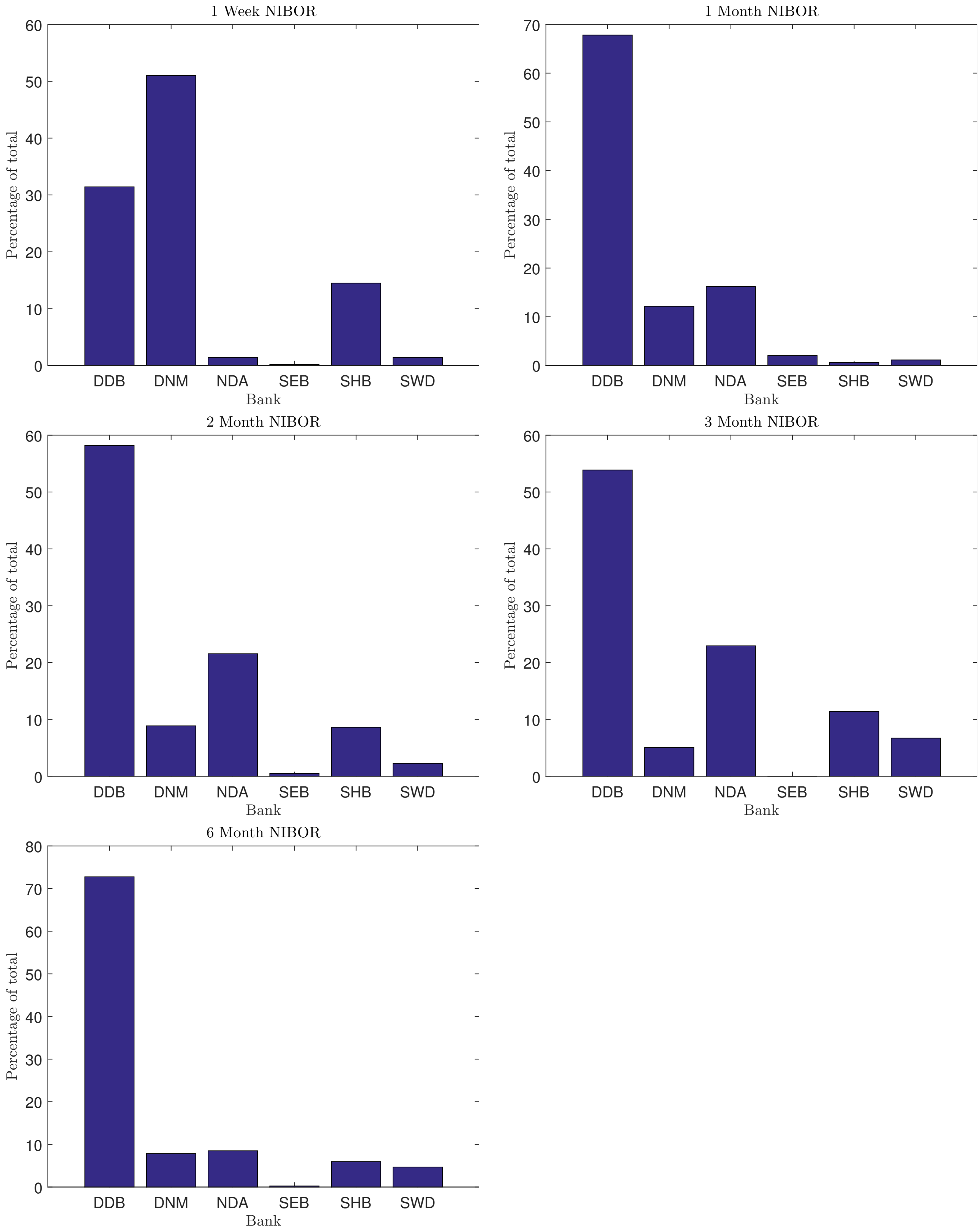

Figure 25. Distribution of banks with highest quote

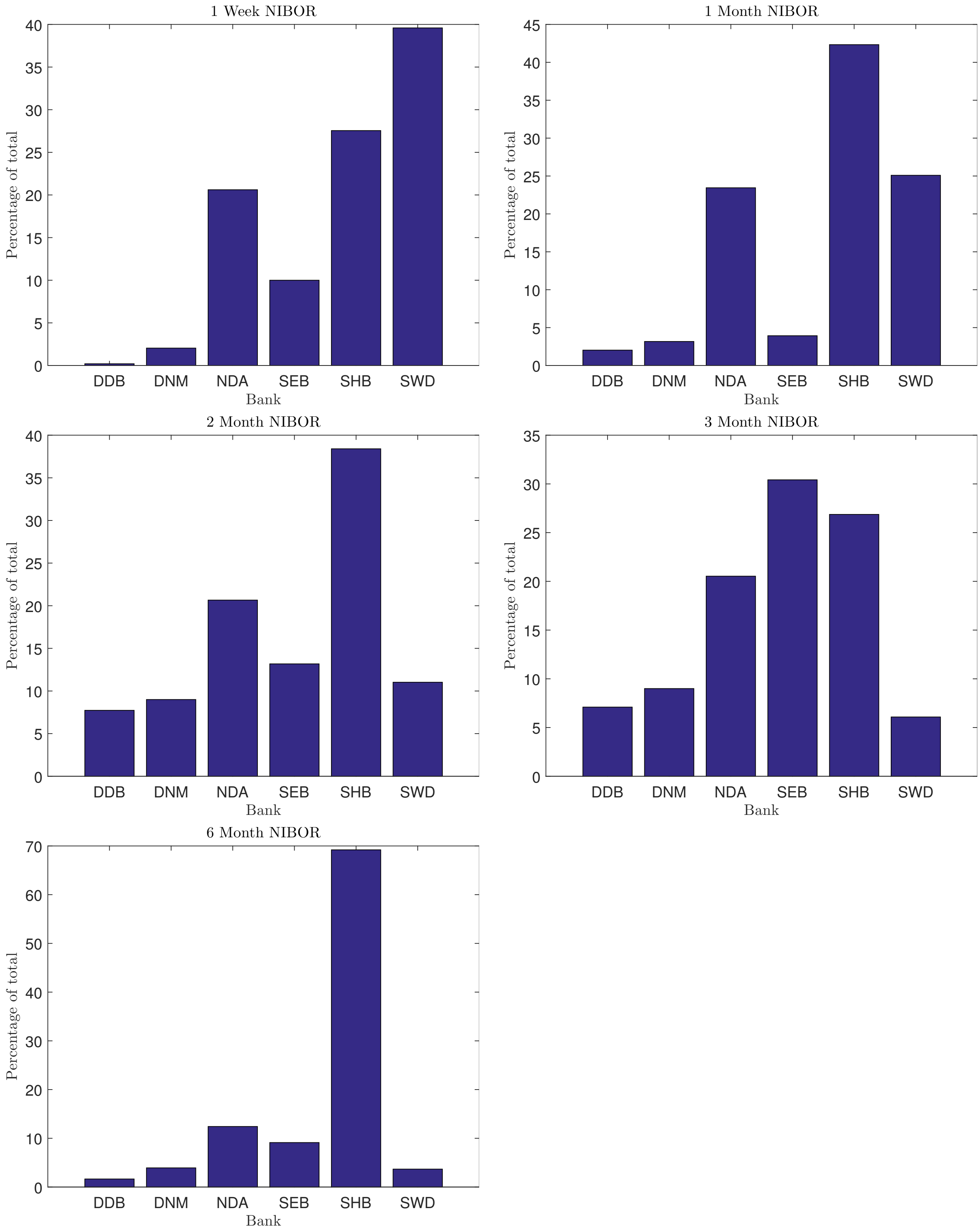

Figure 26. Distribution of banks with lowest quote

Table XXXV Panel bank relative frequency

interest rate	Time period	SWD	SHB	SEB	NDA	DNM	DDB
1 week	09.12.2013 - 22.09.2015	0.066	0.2103	0.1316	0.2806	0.1001	0.2107
	(21213)	(1413)	(4461)	(2792)	(5953)	(2124)	(4470)
1 month	09.12.2013 - 22.09.2015	0.0594	0.2125	0.1356	0.2826	0.1137	0.1961
	(21967)	(1305)	(4669)	(2979)	(6208)	(2498)	(4308)
2 month	09.12.2013 - 22.09.2015	0.0627	0.2266	0.1333	0.3097	0.1314	0.1362
	(20420)	(1281)	(4628)	(2722)	(6324)	(2684)	(2781)
3 month	09.12.2013 - 22.09.2015	0.0619	0.2177	0.1619	0.3170	0.1220	0.1193
	(20649)	(1279)	(4496)	(3344)	(6546)	(2520)	(2464)
6 month	09.12.2013 - 22.09.2015	0.0459	0.1498	0.0926	0.3204	0.1444	0.2471
	(27737)	(1272)	(4154)	(2568)	(8886)	(4004)	(6853)
Aggregate	09.12.2013 - 22.09.2015	0.0585	0.2001	0.1286	0.3029	0.1235	0.1864
	(111986)	(6550)	(22408)	(14405)	(33917)	(13830)	(20876)

Note: Number of observations in parentheses

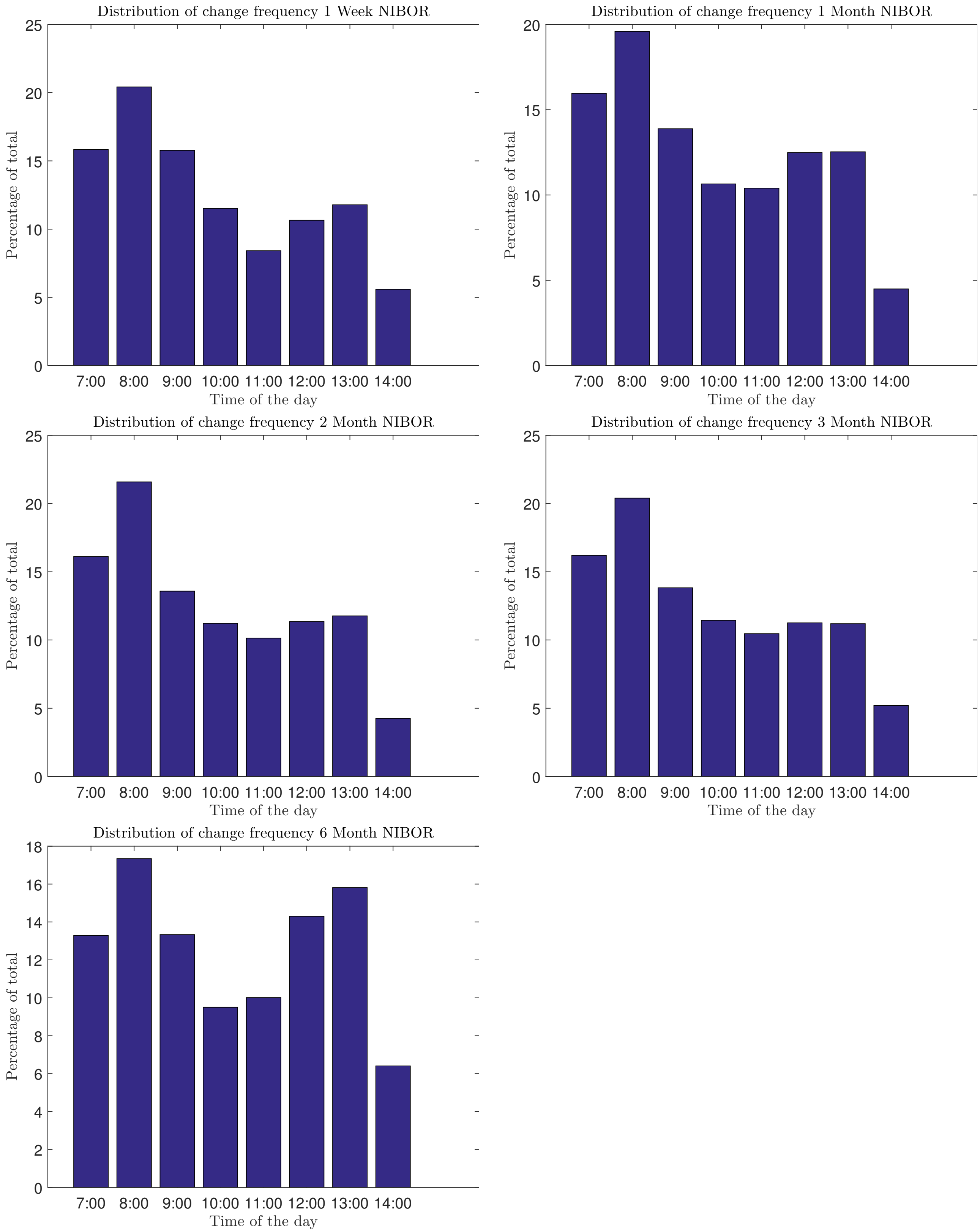

Figure 27. Distribution frequency of changes

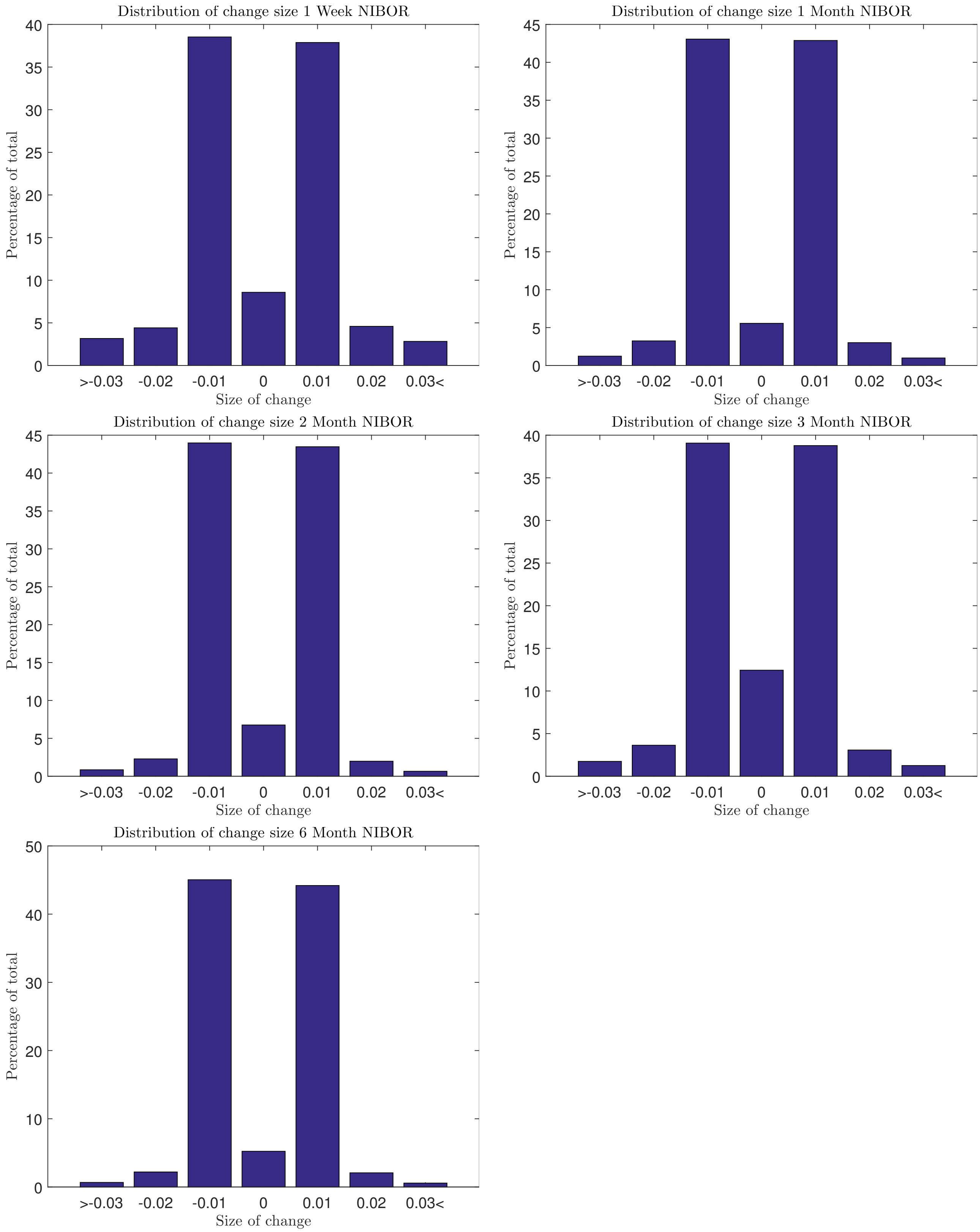

Figure 28. Distribution interest rate change size

Table XXXVI Descriptive statistics CDS

Table XXXVII Across time

CDS spread	mean	std.dev	max	min
2012	55.77	24.11	133.28	19.45
2013	24.82	9.76	59.46	8.77
2014	14.281	6.10	28.78	4.36
2015	18.377	7.62	35.57	6.28

Table XXXVIII Across banks

CDS spread	mean	std.dev	max	min
SHB	13.71	8.13	35.57	4.35
DNM	25.05	7.50	59.76	16.44
SEB	21.29	15.17	83.83	6.13
SWD	23.84	20.15	101.74	7.06
DDB	26.42	15.38	133.28	9.62

Appendix B. Mathematical proofs

Appendix A. Solution to non-linear least squares for Nelson-Siegel model

Let our objective function be $f(\theta, \beta_1, \beta_2, \beta_3)$ that satisfy the condition of a C^2 function (thus non-linear). The corresponding non-linear least squares problem is

$$\min \sum_{i=1}^{n} \left(y_i - f(\theta, \beta_1, \beta_2, \beta_3)\right)^2 \tag{B1}$$

As this is an unrestricted optimization problem it will not have a set of solutions that satisfy (for simplicity, let the parameters be noted g_i:

$$\sum_{i=1}^{n} \frac{\partial f(g_i)}{\partial g_i} = 0 \tag{B2}$$

In order to find a solution we use Newtons Method using a first order Taylor expansion for some initial value

$$f(g_i) \approx f(g_i^0) + \sum_{i=1}^{n} \frac{\partial f(g_i^0)}{\partial g_i} \left(g_i - g_i^0\right) \tag{B3}$$

Using this approximation technique our solution wil be the closes local/global minimum in the neighbourhood of g_i^0. To determine if the solution is local or global we evaluate the Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial \theta^2} & \frac{\partial^2 f}{\partial \theta \partial \beta_1} & \frac{\partial^2 f}{\partial \theta \partial \beta_2} & \frac{\partial^2 f}{\partial \theta \partial \beta_3} \\ \frac{\partial^2 f}{\partial \beta_1 \partial \theta} & \frac{\partial^2 f}{\partial \beta_1^2} & \frac{\partial^2 f}{\partial \beta_1 \partial \beta_2} & \frac{\partial^2 f}{\partial \beta_1 \partial \beta_3} \\ \frac{\partial^2 f}{\partial \beta_2 \partial \theta} & \frac{\partial^2 f}{\partial \beta_2 \partial \beta_1} & \frac{\partial^2 f}{\partial \beta_2^2} & \frac{\partial^2 f}{\partial \beta_2 \partial \beta_3} \\ \frac{\partial^2 f}{\partial \beta_3 \partial \theta} & \frac{\partial^2 f}{\partial \beta_3 \partial \beta_1} & \frac{\partial^2 f}{\partial \beta_3 \partial \beta_2} & \frac{\partial^2 f}{\partial \beta_3^2} \end{bmatrix}$$

Appendix B. Expectation for Vasicek model

Let $dr(t) = k(\theta - r(t))dt + \sigma dW(t)$ and introduce the transformation $f(t, r(t)) = r(t)e^{kt}$. By Ito's lemma we can derive the stochastic differential equation for f:

$$df(t, r(t)) = \left(r(t)ke^{kt} + e^{kt}k(\theta - r(t))\right) dt + e^{kt}\sigma dW(t) \tag{B4}$$

$$r(t)e^{kt} = \int_0^t r(t)ke^{ks} ds + \int_0^t e^{ks}k(\theta - r(t))ds + \int_0^t \sigma e^{ks} dW(t) \tag{B5}$$

With boundary condition $r(0) = r$ we derive the explicit solution for the integral

$$r(t) = r(0)e^{-kt} + (1 - e^{-kt})\theta + \sigma e^{-kt} \int_0^t e^{ks} dW(t) \tag{B6}$$

From this we can derive the expectation and variance for r_t. As $E[W_t] = 0$ and $\text{Var}[W_t] = t$ by construction:

$$E_0[r_t] = E_0\left[r(0)e^{-kt} + (1-e^{-kt})\theta + \sigma e^{-kt}\int_0^t e^{ks}dW(t)\right] = r(0)e^{-kt} + (1-e^{-kt})\theta \tag{B7}$$

$$\text{Var}[r_t] = \text{Var}\left[\sigma e^{-kt}\int_0^t e^{ks}dW(t)\right] = \sigma^2 e^{-2kt}\int_0^t e^{2ks}ds = \frac{\sigma^2}{2k}\left(1-e^{-2kt}\right) \tag{B8}$$

the expectation and variance is useful when we want to derive a price for instruments with interest rate as the underlying. Recall the formal definition of a zero coupon bond value at time t

$$P(t) = P(T)e^{-y_{tT}(T-t)}. \tag{B9}$$

The value of receiving P_T at time T is equal to $E_t[P_t] = E_t[P_T e^{-\int_t^T r(s)ds}]$. From our original stochastic differential equation we know that W_t has Gaussian increments, and thus r_t is normally distributed - and $E[e^{r_t}]$ follows a log-normal distribution $e^{\mu+\frac{1}{2}\sigma^2}$. For $P_T = 0$ our expectation for the bond price is

$$E_t[e^{-\int_t^T r(s)ds}] = e^{E_t[-\int_t^T r_s ds]+\frac{1}{2}\text{Var}_t[-\int_t^T r_s ds]} \tag{B10}$$

For $t = 0$ we can derive the price of a bond by finding $E_0[-\int_0^T r_s ds]$ and $\text{Var}_0[-\int_0^T r_s ds]$.

$$E_0\left[-\int_t^T r_s ds\right] = E_0[(r_0-\theta)\frac{1}{k}(1-e^{-kT} + \theta T + \int_0^T \frac{\sigma}{k}(1-e^{-k(T-s)}dW_s)$$

$$E_0\left[-\int_t^T r_s ds\right] = (r_0-\theta)\frac{1}{k}(1-e^{-kt}) + \theta T \tag{B11}$$

$$\text{Var}_0\left[-\int_t^T r_s ds\right] = \text{Var}_0\left[\int_0^T \frac{\sigma}{k}(1-e^{-k(T-s)})dW_s\right]$$

$$\text{Var}_0\left[-\int_t^T r_s ds\right] = \left(\frac{\sigma}{k}\right)^2\int_0^T (1-e^{-k(T-s)})^2 ds$$

$$\text{Var}_0\left[-\int_t^T r_s ds\right] = \left(\frac{\sigma}{k}\right)^2\left(\int_0^T ds - 2\int_0^T e^{-2k(T-s)}ds + \int_0^T e^{-2k(T-s)}ds\right)$$

$$\text{Var}_0\left[-\int_t^T r_s ds\right] = \left(\frac{\sigma}{k}\right)^2\left(T - 2\frac{1}{k}(1-e^{-kT}) + \frac{1}{2k}(1-e^{-2kT})\right) \tag{B12}$$

Now we can derive the solution for the bond price

$$E_0[e^{-\int_0^T r_s ds}] = \exp\left(-(r_0-\theta)\frac{1}{k}\left(1-e^{-kT}\right) - \theta T + \frac{1}{2}\left(\frac{\sigma}{k}\right)^2\left(T - \frac{2}{k}\left(1-e^{-kT}\right) + \frac{1}{2k}\left(1-e^{-2kT}\right)\right)\right) \tag{B13}$$

Appendix C. The Nelson-Siegel model

Appendix A. Fitted samples of the yield curve

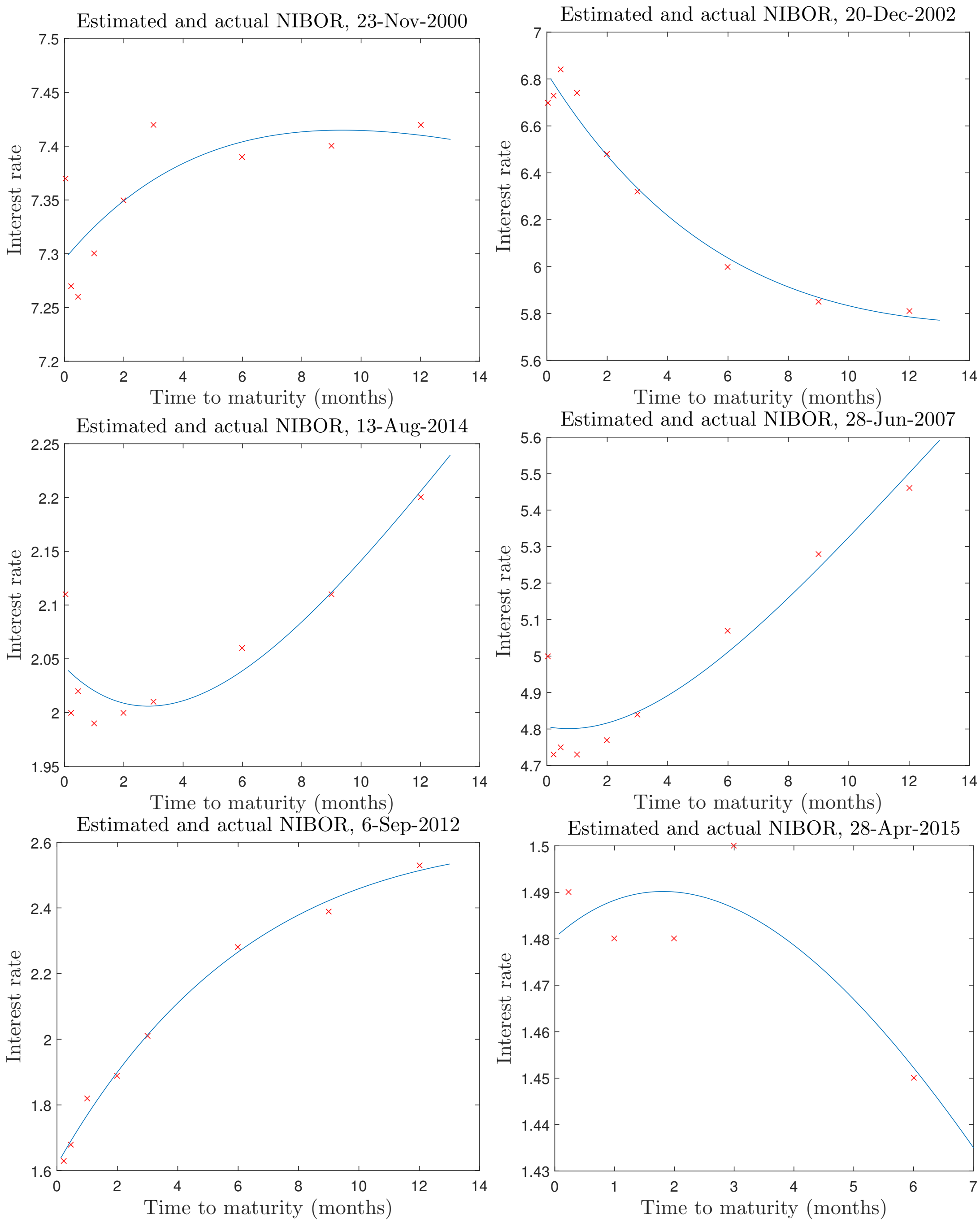

Figure 29. Estimated and actual yield for selected dates, NIBOR.

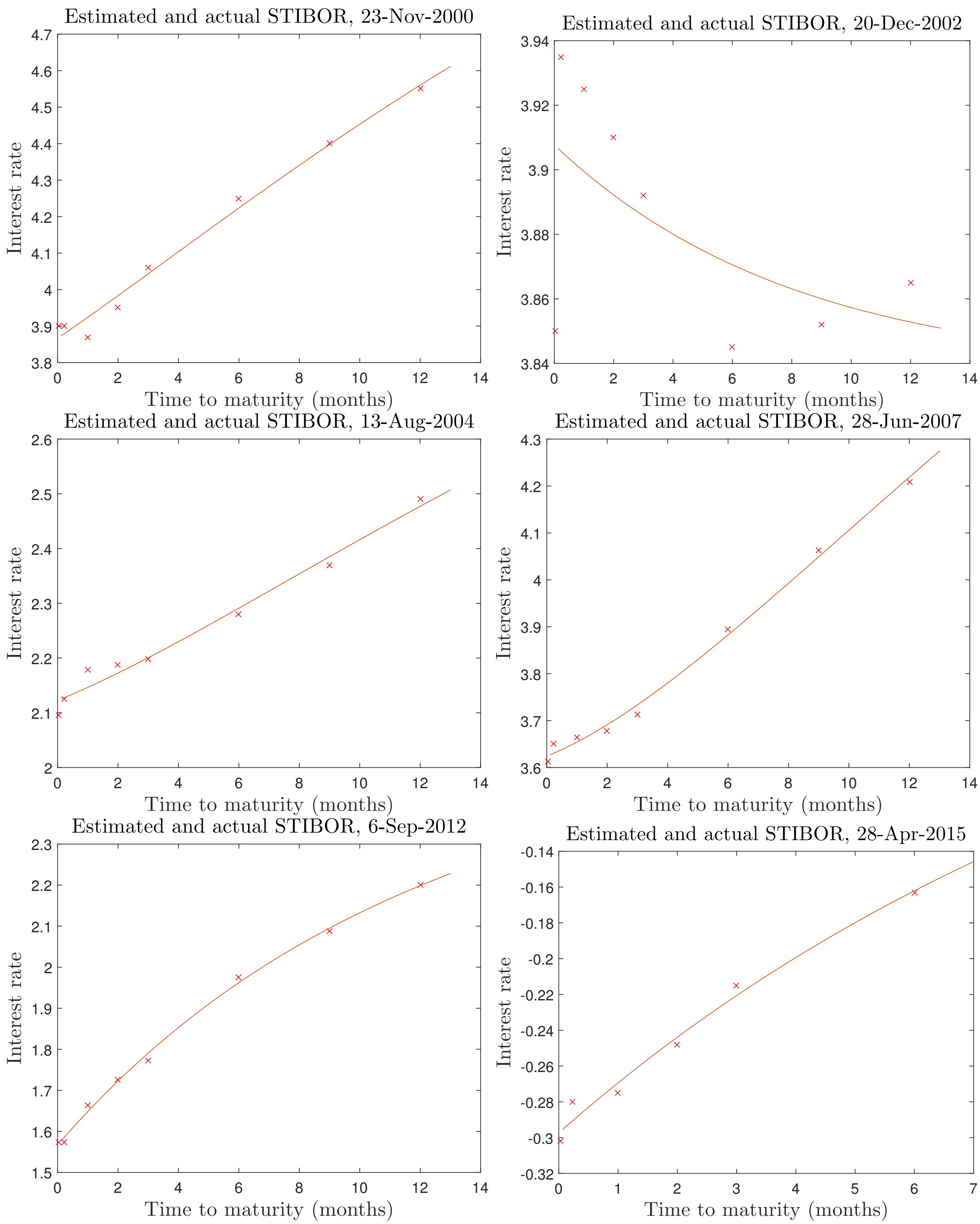

Figure 30. Estimated and actual yield for selected dates, STIBOR.

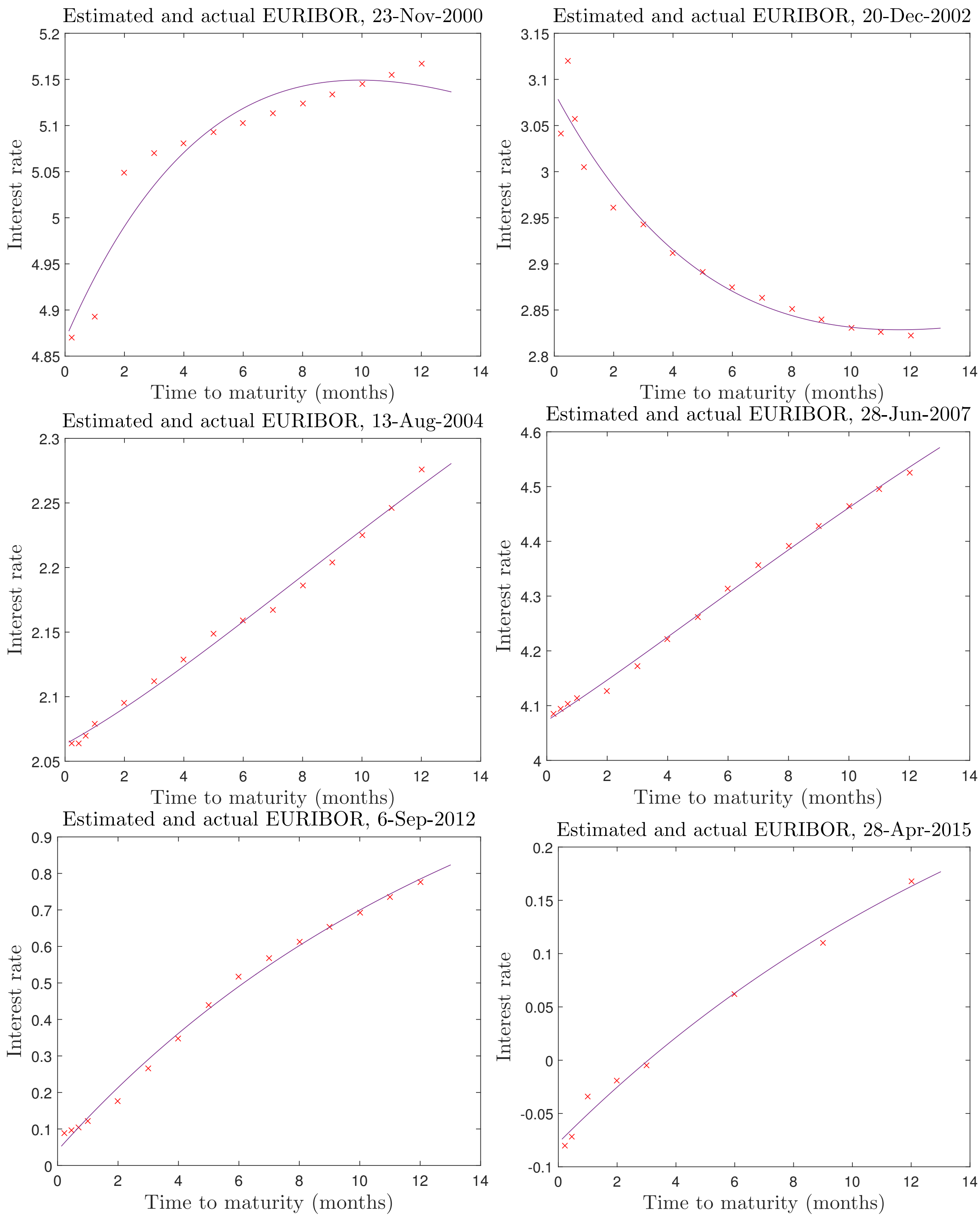

Figure 31. Estimated and actual yield for selected dates, EURIBOR.

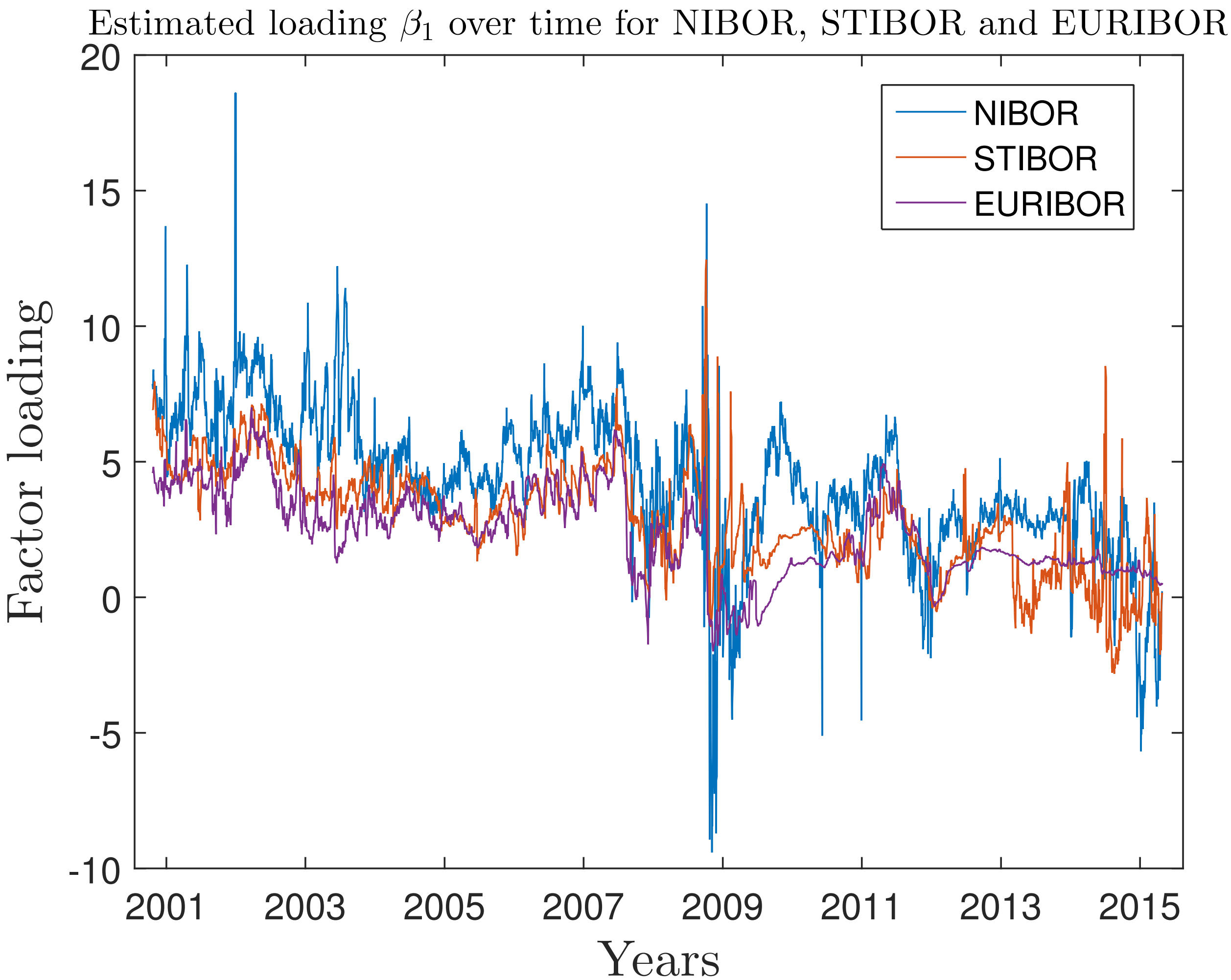

Figure 32. Estimated loading for factor β_1

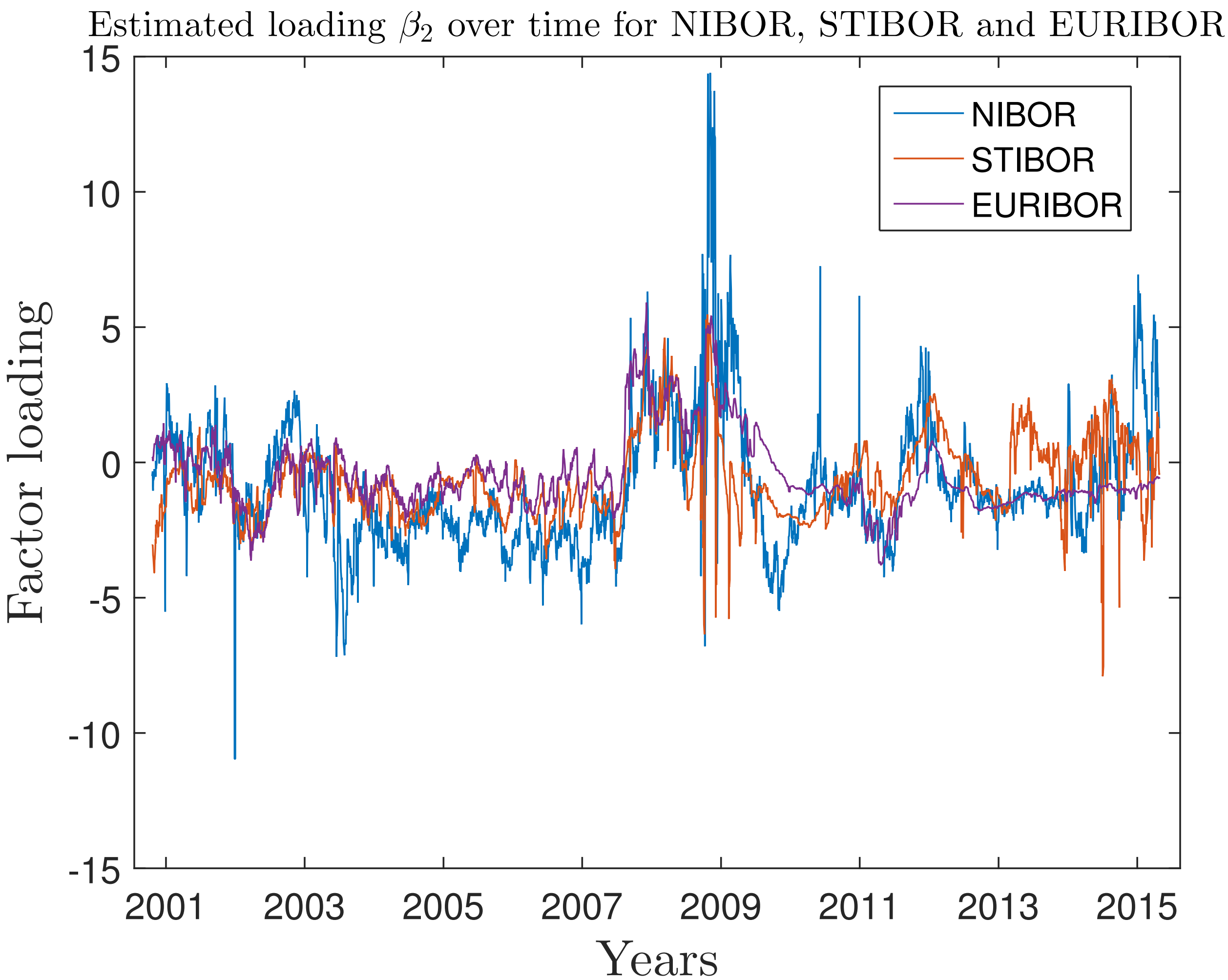

Figure 33. Estimated loading for factor β_2

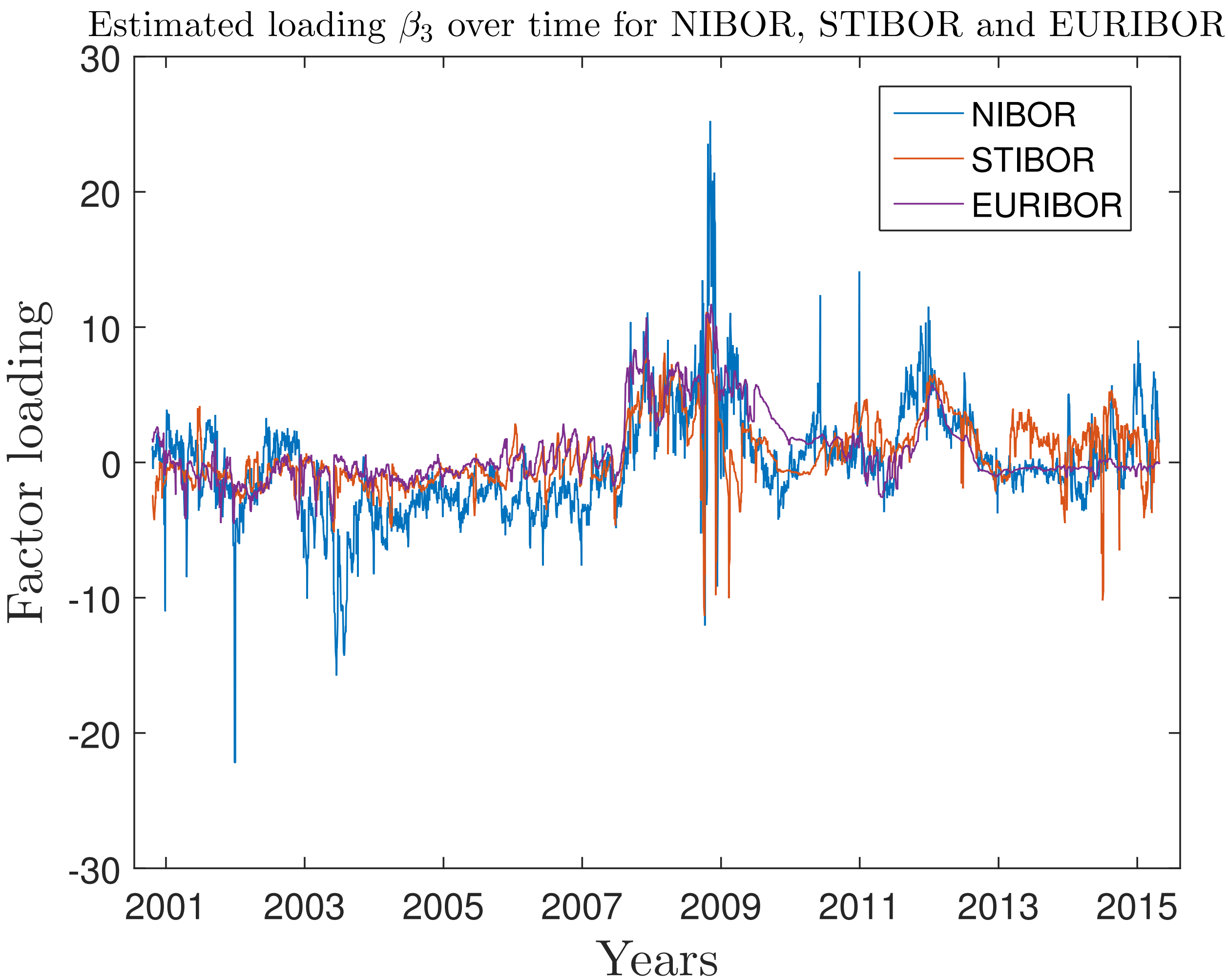

Figure 34. Estimated loading for factor β_3

Table XXXIX Descriptive statistics for NIBOR

Maturity	Mean	StdDev	Min	Max	MAE	RMSE
0.23	0.015	0.0664	-0.7592	0.8	0.015	0.068
0.47	0.0049	0.0571	-0.4416	0.7278	0.0049	0.0573
1	0.0046	0.0743	-1.2802	0.6542	0.0046	0.0744
2	0.0068	0.0398	-0.3467	0.2667	0.0068	0.0404
3	-0.0121	0.0355	-0.4282	0.3937	0.0121	0.0375
6	-0.0038	0.0424	-0.3191	0.2955	0.0038	0.0426
9	0.01	0.0375	-0.2415	0.2715	0.01	0.0388
12	-0.0042	0.033	-0.2247	0.2158	0.0042	0.0333

Table XL Descriptive statistics for yield curve residuals, STIBOR

Maturity	Mean	StdDev	Min	Max	MAE	RMSE
0.03	0.0207	0.0511	-0.868	0.2568	0.0207	0.0551
0.23	-0.008	0.035	-0.1521	0.6522	0.008	0.0359
1	-0.0122	0.044	-0.3433	0.4382	0.0122	0.0456
2	-0.0006	0.0285	-0.1303	0.2167	0.0006	0.0285
3	-0.0111	0.0267	-0.1026	0.0447	0.0111	0.0289
6	0.011	0.0288	-0.2117	0.1541	0.011	0.0308
9	0.009	0.0292	-0.1146	0.1127	0.009	0.0306
12	-0.0087	0.0239	-0.1109	0.1357	0.0087	0.0255

Table XLI Descriptive statistics for EURIBOR

Maturity	Mean	StdDev	Min	Max	MAE	RMSE
0.23	0.0072	0.0427	-0.1976	0.6065	0.0072	0.0433
0.47	0.0063	0.027	-0.241	0.3847	0.0063	0.0277
0.7	0.0047	0.0266	-0.3965	0.2267	0.0047	0.027
1	-0.0035	0.04	-0.442	0.1265	0.0035	0.0401
2	-0.0065	0.0407	-0.2737	0.087	0.0065	0.0412
3	-0.0199	0.0349	-0.1815	0.0366	0.0199	0.0402
4	-0.007	0.0141	-0.0725	0.0202	0.007	0.0157
5	0.0019	0.0093	-0.0261	0.0605	0.0019	0.0095
6	0.0015	0.0192	-0.041	0.1027	0.0015	0.0193
7	0.0095	0.0208	-0.0408	0.1173	0.0095	0.0229
8	0.0123	0.0192	-0.0376	0.1039	0.0123	0.0228
9	0.009	0.0134	-0.0237	0.0685	0.009	0.0162
10	0.0049	0.0056	-0.007	0.026	0.0049	0.0074
11	-0.0032	0.0093	-0.05	0.0192	0.0032	0.0099
12	-0.0141	0.0231	-0.1364	0.0481	0.0141	0.0271

Appendix D. Corporate bonds

Appendix A. Bonds in NOK

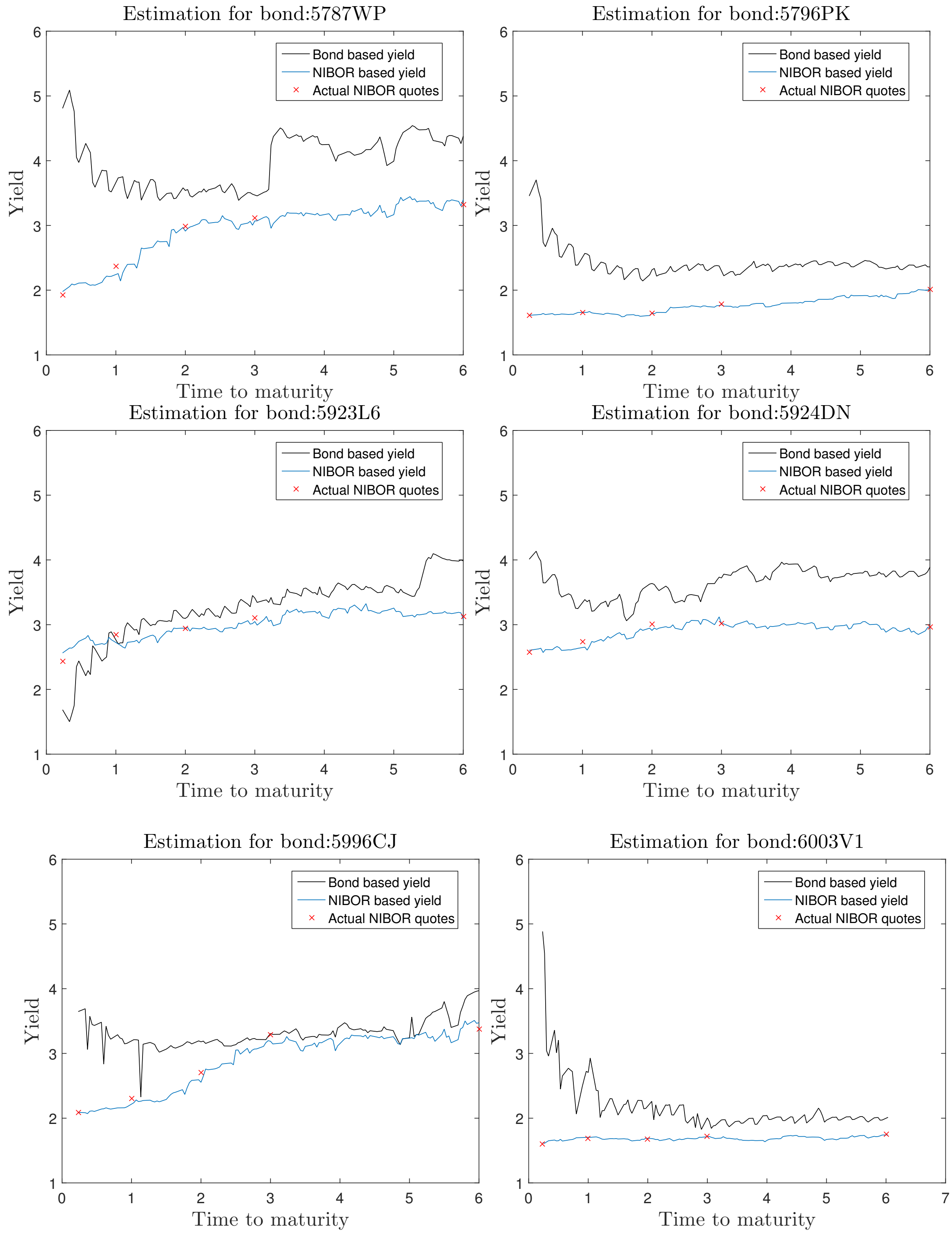

Figure 35. Comparison of bonds in NOK to estimated NIBOR.

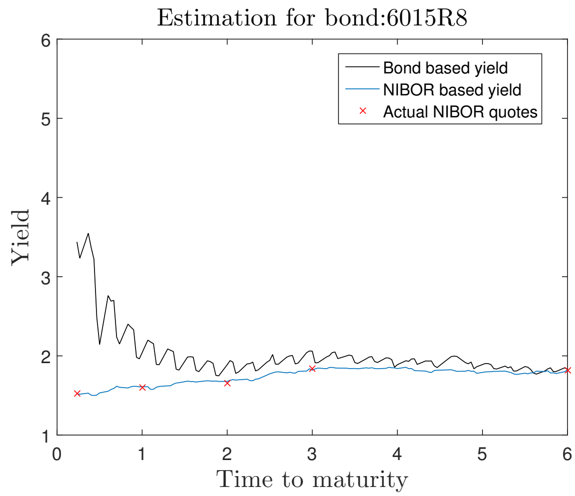

Figure 36. Comparison of bonds in NOK to estimated NIBOR.

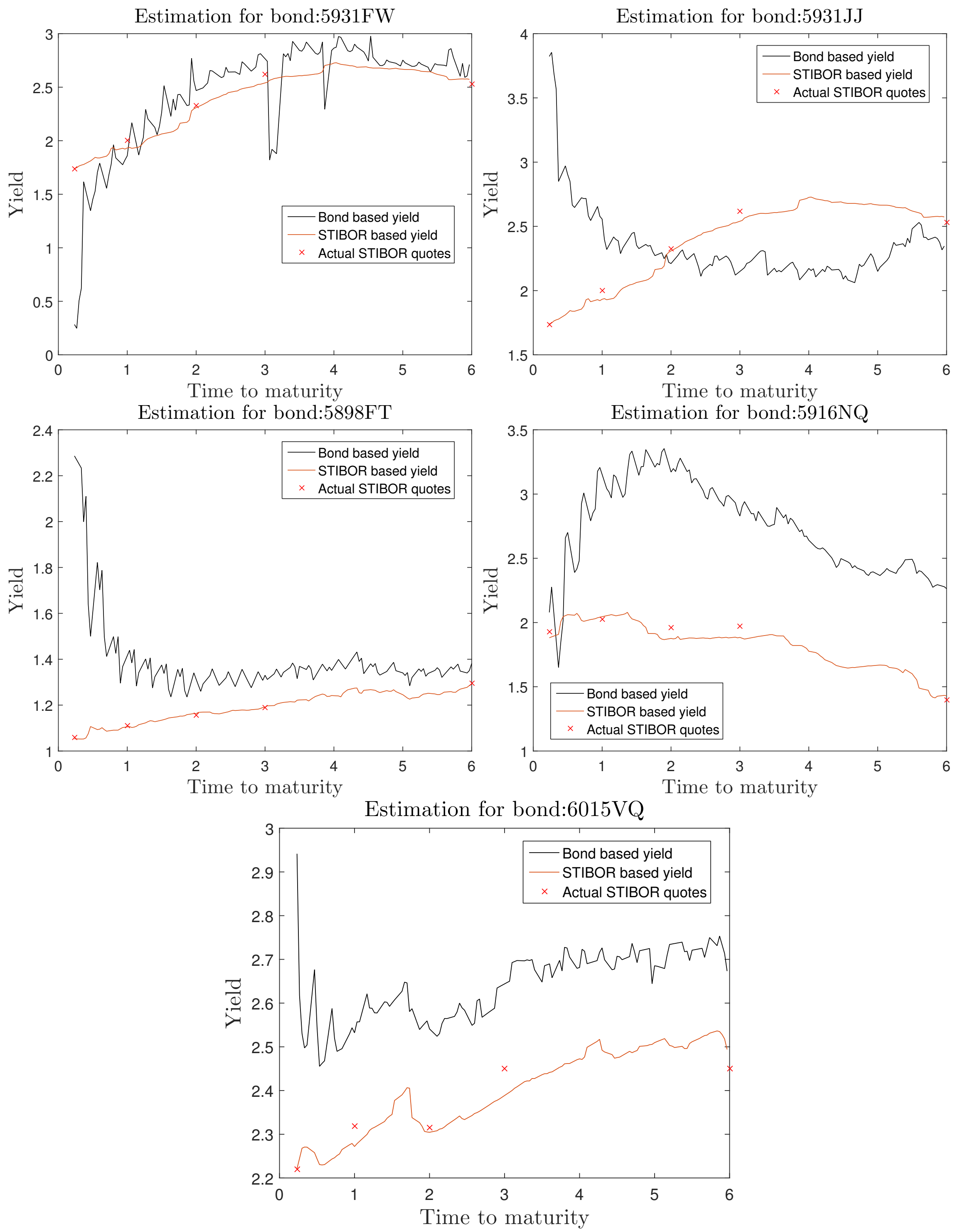

Figure 37. Comparison of bonds in NOK to estimated STIBOR.

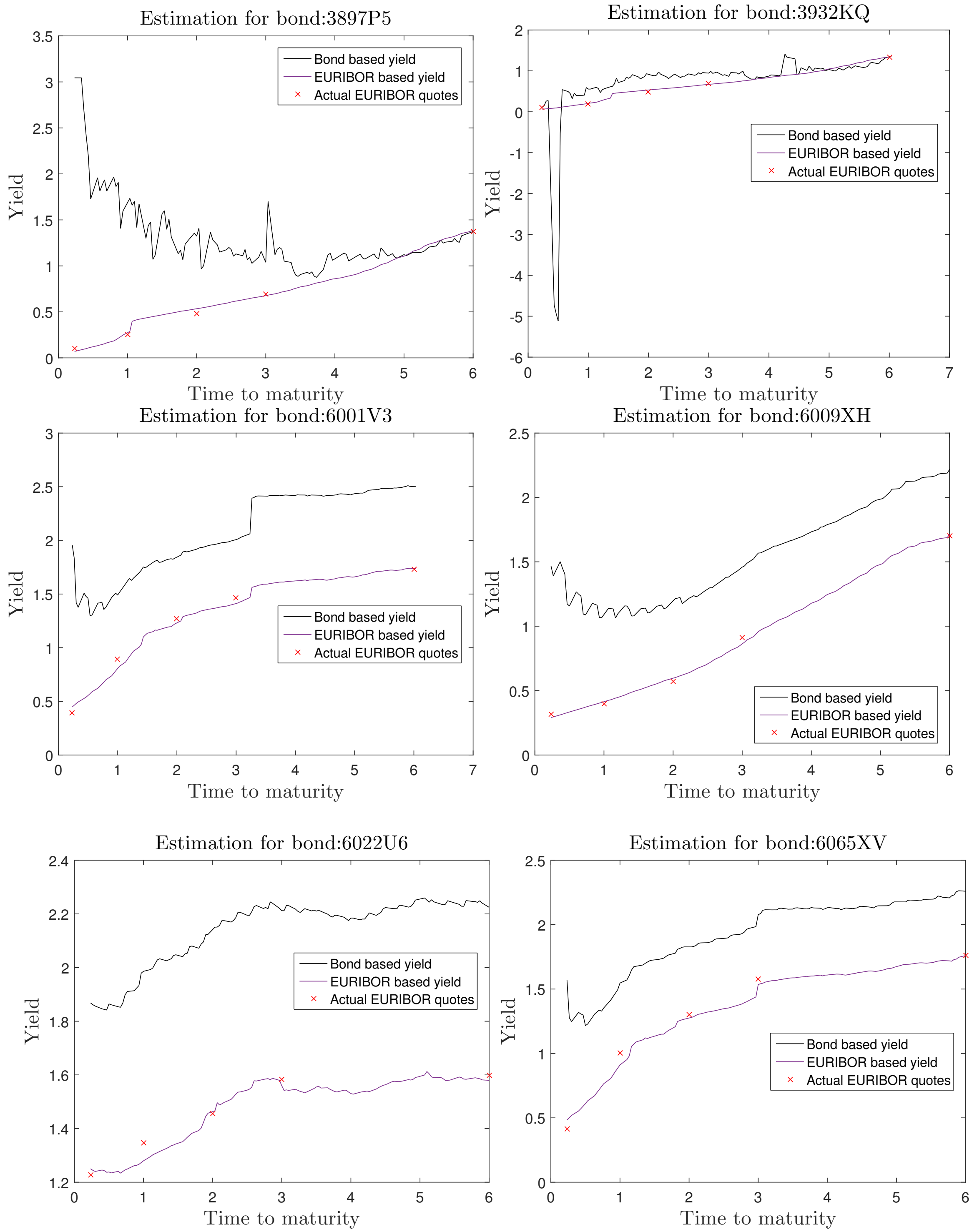

Figure 38. Comparison of bonds in EUR to estimated EURIBOR.

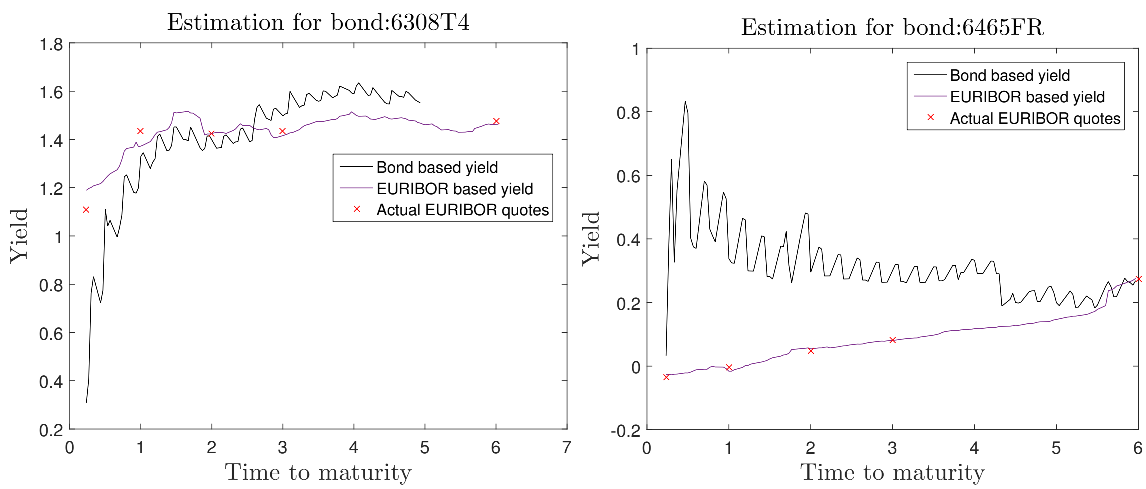

Figure 39. Comparison of bonds in EUR to estimated EURIBOR.

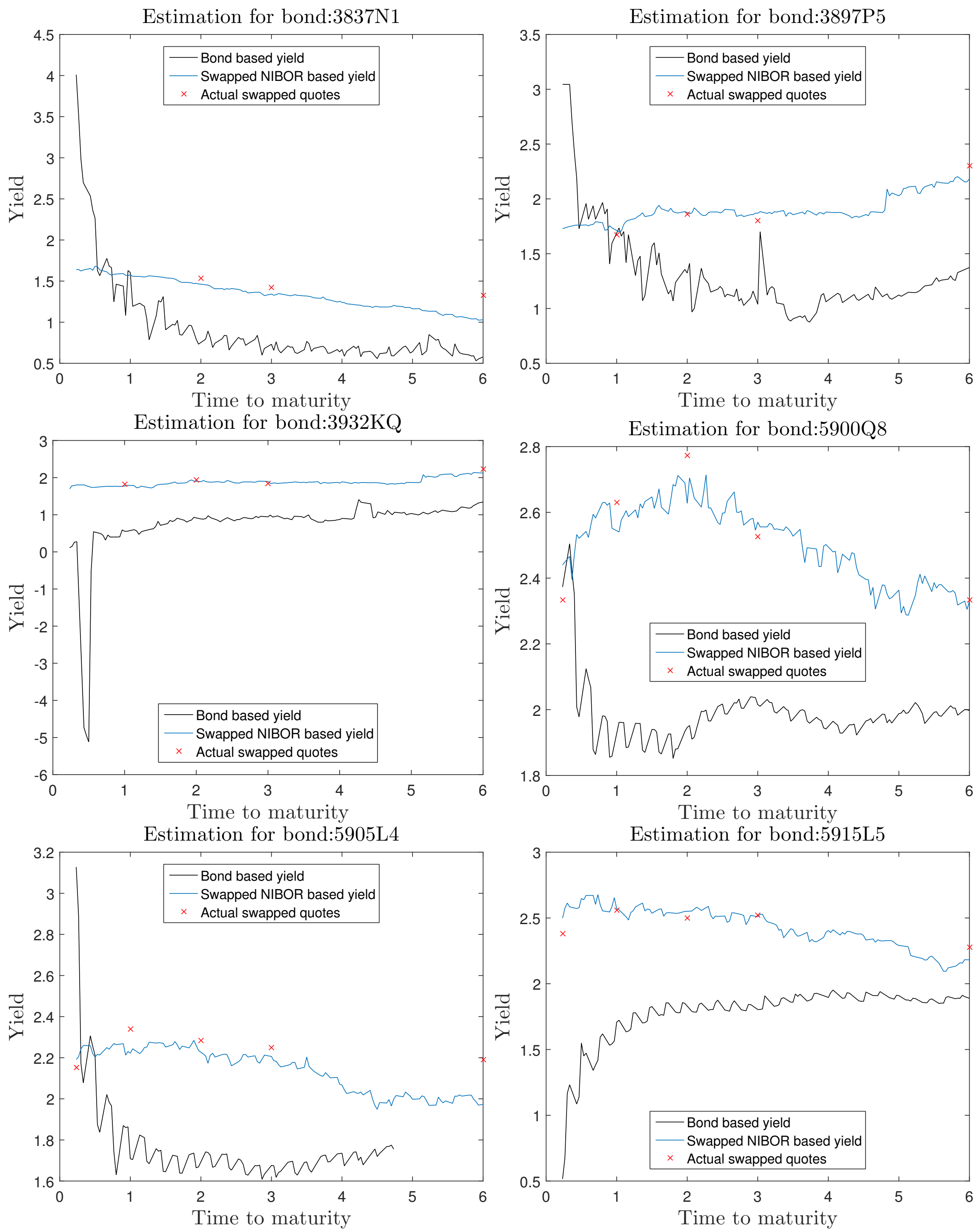

Figure 40. Comparison of bonds in EUR to estimated swapped NIBOR.

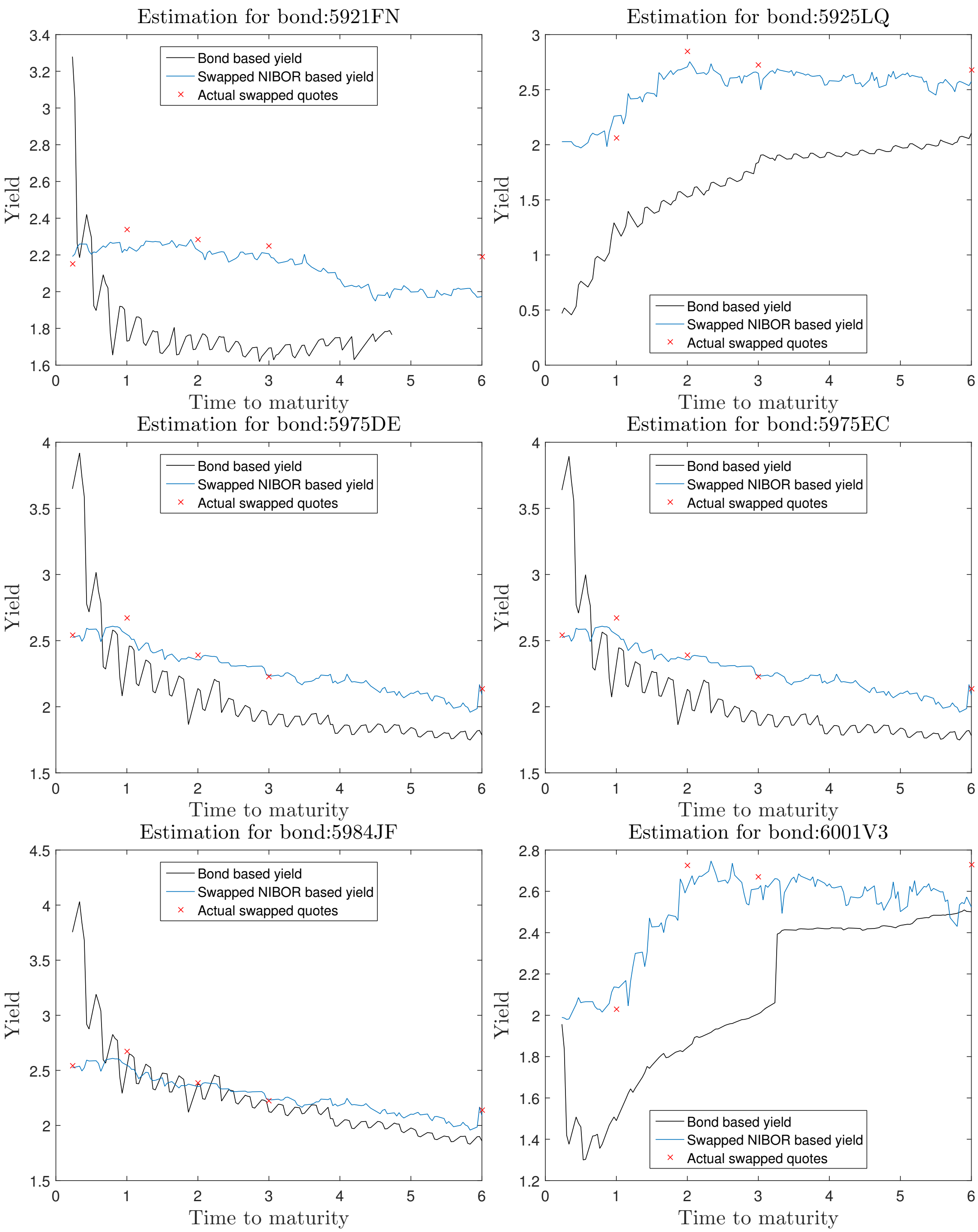

Figure 41. Comparison of bonds in EUR to estimated swapped NIBOR.

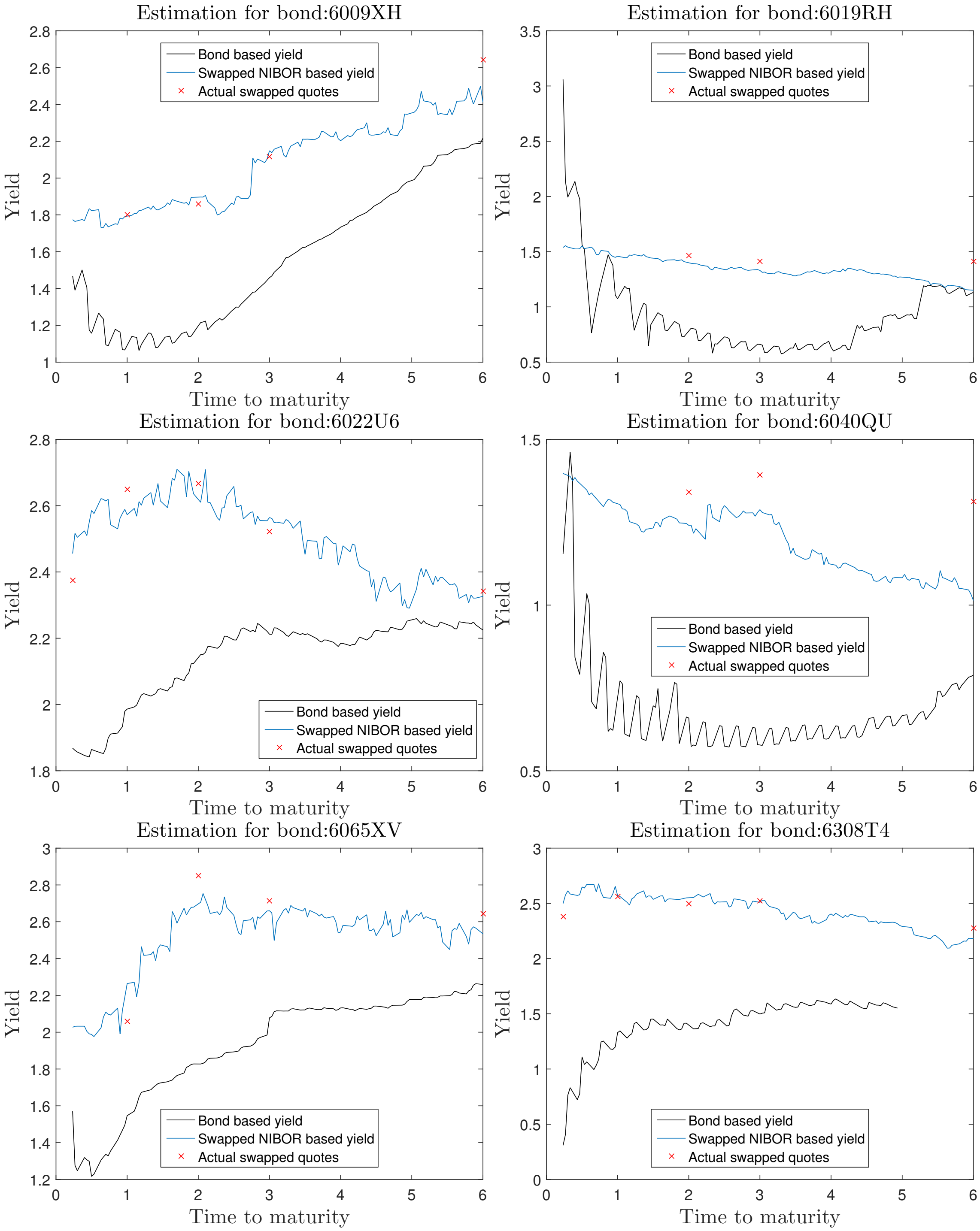

Figure 42. Comparison of bonds in EUR to estimated swapped NIBOR.

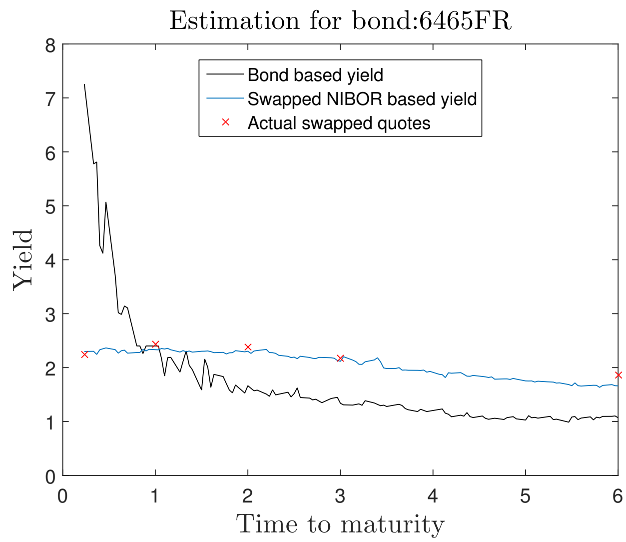

Figure 43. Comparison of bonds in EUR to estimated swapped NIBOR.

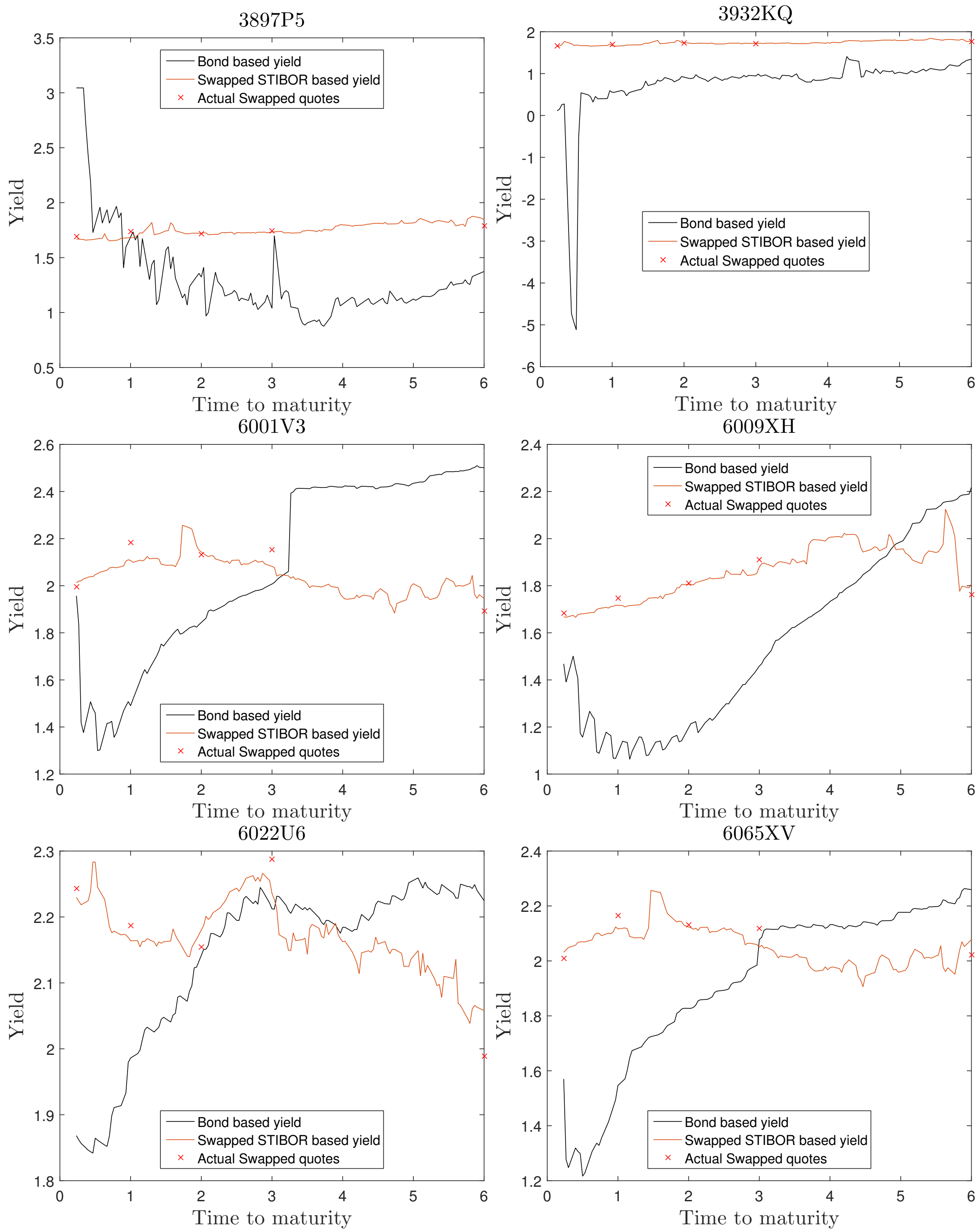

Figure 44. Comparison of bonds in EUR to estimated swapped STIBOR.

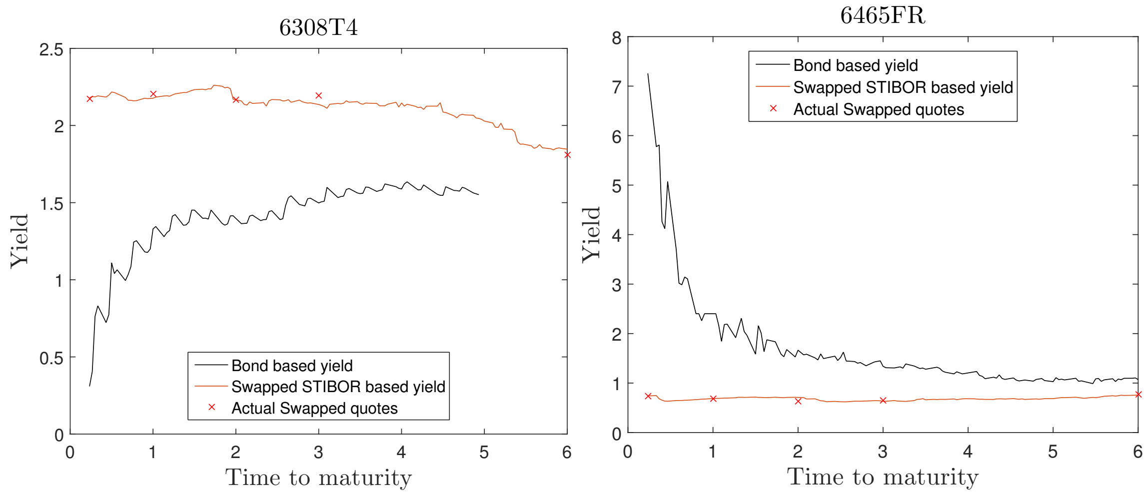

Figure 45. Comparison of bonds in EUR to estimated swapped STIBOR.

Appendix E. Autoregressive model

Appendix A. Autocorrelation and partial autocorrelation

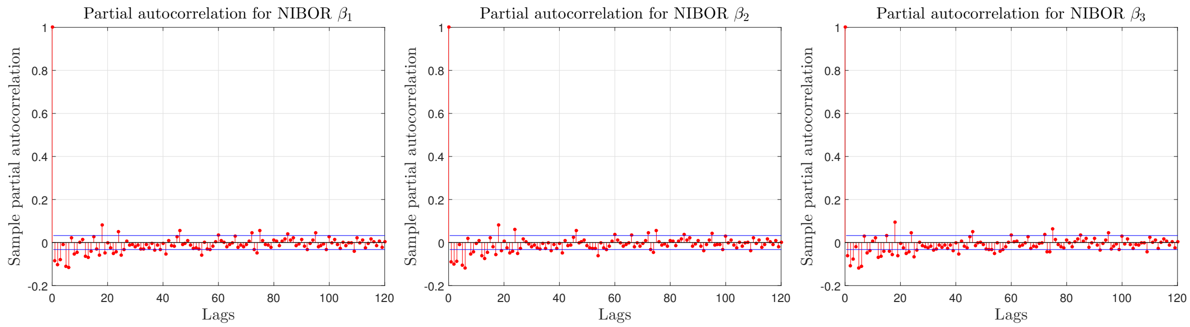

Figure 46. Partial autocorrelation function for NIBOR.

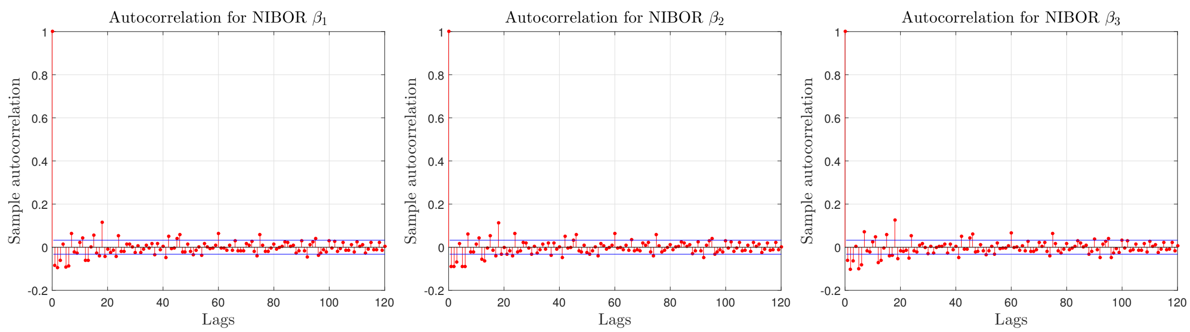

Figure 47. Autocorrelation function for NIBOR.

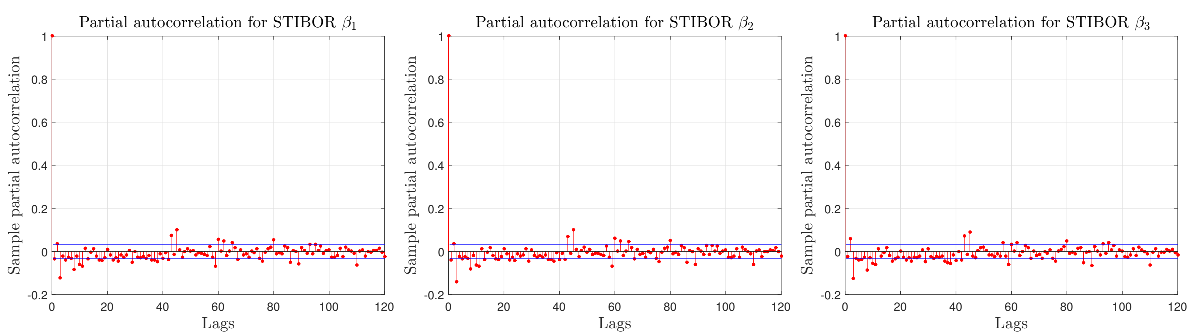

Figure 48. Partial autocorrelation function for STIBOR.

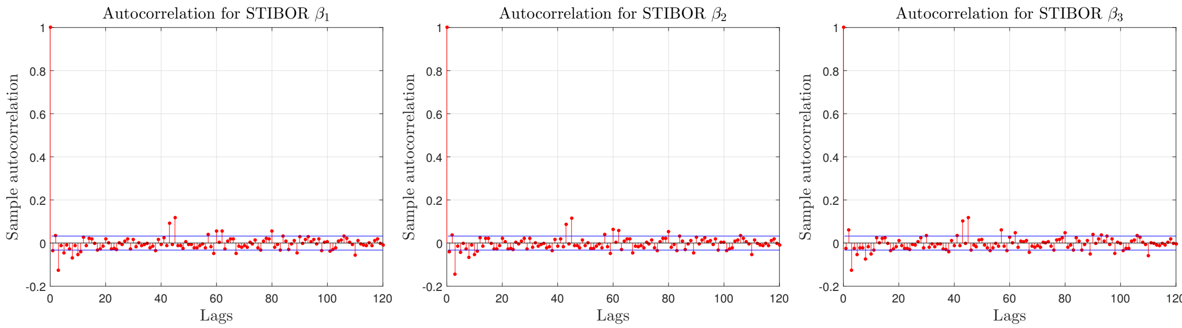

Figure 49. Autocorrelation function for STIBOR.

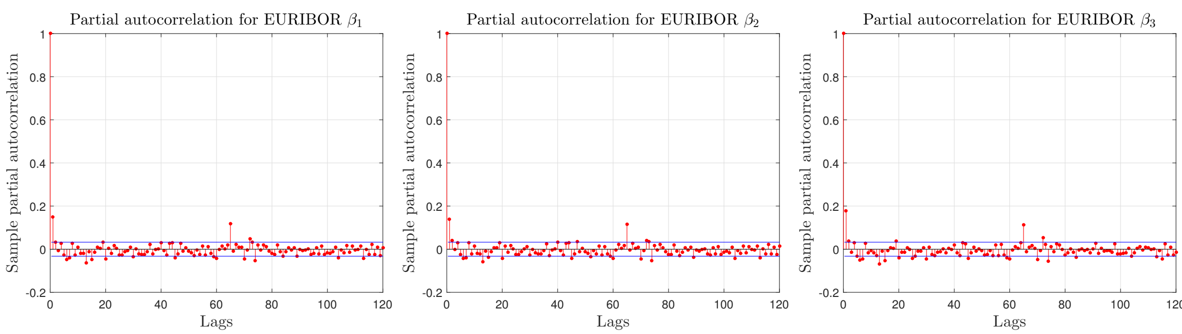

Figure 50. Partial autocorrelation function for EURIBOR.

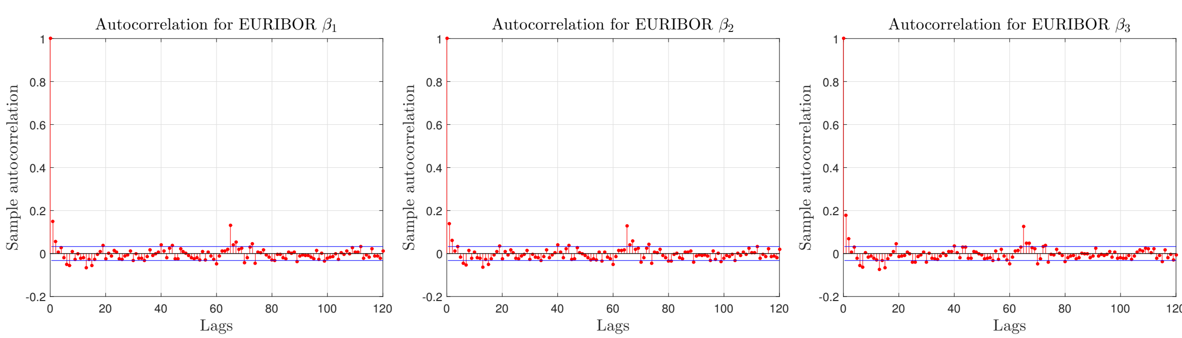

Figure 51. Autocorrelation function for EURIBOR.

Table XLII Forecasting results for NIBOR

	Mean	Std.dev	MAE	RMSE
Tailored model				
1 Week	-0.46837	0.074989	0.143988	1.603378
1 Month	-0.39918	0.073676	0.088888	0.986538
2 Month	-0.35777	0.072777	0.06607	0.689955
3 Month	-0.32208	0.072184	0.072775	0.748038
6 Month	-0.25977	0.071458	0.077461	0.807563
Random Walk				
1 Week	-0.306	-	0.041266	0.20466
1 Month	-0.255	-	0.055879	0.61901
2 Month	-0.227	-	0.068815	0.766286
3 Month	-0.2	-	0.055694	0.611376
6 Month	-0.142	-	0.046669	0.503883
Actual data				
1 Week	1.199274	0.174646	-	-
1 Month	1.218387	0.162012	-	-
2 Month	1.2375	0.144592	-	-
3 Month	1.274194	0.12954	-	-
6 Month	1.285242	0.120753	-	-

Table XLIII Forecasting results for STIBOR

	Mean	Std.dev	MAE	RMSE
Tailored model				
1 Week	-0.1678	0.043736	0.037676	0.40226
1 Month	-0.11715	0.044275	0.037652	0.419181
2 Month	-0.10152	0.044854	0.054079	0.602195
3 Month	-0.08481	0.045307	0.060364	0.672187
6 Month	-0.01926	0.046067	0.06378	0.71022
Random Walk				
1 Week	-0.089	-	0.042677	0.475236
1 Month	-0.038	-	0.041508	0.462214
2 Month	-0.022	-	0.025444	0.283327
3 Month	-0.005	-	0.019444	0.216514
6 Month	0.061	-	0.016645	0.183557
Actual data				
1 Week	-0.13168	0.014043	-	-
1 Month	-0.07951	0.022615	-	-
2 Month	-0.04744	0.015645	-	-
3 Month	-0.02444	0.012907	-	-
6 Month	0.044516	0.010224	-	-

Table XLIV Forecasting results for EURIBOR

	Mean	Std.dev	MAE	RMSE
Tailored model				
1 Week	-0.1678	0.043736	0.037676	0.40226
1 Month	-0.11715	0.044275	0.037652	0.419181
2 Month	-0.10152	0.044854	0.054079	0.602195
3 Month	-0.08481	0.045307	0.060364	0.672187
6 Month	-0.01926	0.046067	0.06378	0.71022
Random Walk				
1 Week	-0.089	-	0.042677	0.475236
1 Month	-0.038	-	0.041508	0.462214
2 Month	-0.022	-	0.025444	0.283327
3 Month	-0.005	-	0.019444	0.216514
6 Month	0.061	-	0.016645	0.183557
Actual data				
1 Week	-0.13168	0.014043	-	-
1 Month	-0.07951	0.022615	-	-
2 Month	-0.04744	0.015645	-	-
3 Month	-0.02444	0.012907	-	-
6 Month	0.044516	0.010224	-	-

Appendix F. Unobserved effects model

Table XLV Dickey-Fuller test statistics

Table XLVI NIBOR domestic premium

Bank	lags	test statistics	p-value
SHB	2	-2.417	0.0084
DNM	2	-2.831	0.0026
SEB	2	-2.822	0.0027
SWD	2	-2.740	0.0034
DDB	2	-2.912	0.0021

Table XLVII Credit default swap spreads

Bank	lags	test statistics	p-value
SHB	3	-2.289	0.0117
DNM	2	-3.668	0.0002
SEB	3	-3.488	0.0003
SWD	2	-5.042	0.0000
DDB	8	-3.874	0.0001

Table XLVIII Hausman test statistics

Table XLIX NIBOR domestic premium

Bank	test statistics	p-value
Model 1	13.75	0.0002
Model 2	11.23	0.5097
Model 3	6.91	0.9600

Table L Credit default swap spreads

Bank	test statistics	p-value
Model 4	2.58	0.9996
Model 5	29.45	0.0431

Table LI Wald test for structural break

Bank	test statistics	p-value	Estimated break date
SHB	142.34	0.0000	2014w11
DNM	120.03	0.0000	2014w12
SEB	137.48	0.0000	2014w11
SWD	138.27	0.0000	2014w12
DDB	127.70	0.0000	2014w12

Figure 52. NIBOR domestic premium and CDS spreads across banks

Appendix G. Hidden markov model

Appendix A. Derivation of the hidden markov model

Appendix A.1. Foundation of the hidden markov model

For analysing the time varying high frequency data, we can evaluate a sequence of observed data as the result of an underlying Hidden Markov process. hidden markov models (hereby HMM) are often referred to as probabilistic functions of Markov chains, and the earliest work on theory of probabilistic functions of Markov chains originates from work published by Leonard E. Baum and his colleagues at the Institute for Defense Analyses(1966; 1967; 1970; 1972). A Markov chain has the following property:

Definition: Let $X = X_1, X_2, ...$ be a random process in discrete state space $\mathbb{R}$. It is called a Markov chain if the conditional probabilities between the outcomes at different times satisfy the *Markov property* for every sequence $x_1, x_2, .., x_{t+1} \in \mathbb{R}$:

$$P(X_{t+1} = x_{t+1}|X_t = x_t, X_{t-1} = x_{t-1}, ..., X_1 = x_1) = P(X_{t+1} = x_{t+1}|X_t = x_t),$$

and the probability of what we observe is only conditional on the previous observation. The theory underlying HMMs has been known for decades, however, due to it only being published in mathematical journals, its availability and application was for a long time somewhat unknown to other disciplines. With an increasing amount of tutorials on implementation and theory throughout the 1980s popularity began to rise, especially within the field of speech recognizing. The application in this paper is programmed in MATLAB® and based on theory from the article published by Lawrence R. Rabiner (1989)

To get a firm grip of the HMM, we will first introduce its elements, their properties and relationship. Thereafter we will look at the implementation issues which arise. The HMM contains five different elements, which span the basis of the model:

1. N, the number of states in the model. These states follow a Markov process and are by definition hidden, we do hence not know which state the observation has been extracted from. Denote the states by $S = S_1, S_2, ..., S_N$, and the state at time t as q_t.
2. M, the number of distinct observation symbols per state. We denote the individual symbols as $V = v_1, v_2, ..., v_k$.
3. The state transition probability distribution $A = a_{ij}$, where a_{ij} is the probability to transition from state S_i at time t, to state S_j at time t+1:

$$a_{ij} = P[q_{t+1}] = S_j|q_t = S_i], 1 \leq i, j \leq N$$

4. The observation symbol probability distribution in state j, $B = \{b_j(k)\}$, where $b_j(k)$ is the

probability of observing v_k at time t given state S_j:

$$b_j(k) = P[v_k \ at \ t|q_t = S_j], 1 \leq j \leq N, 1 \leq k \leq M$$

5. The initial state distribution $\pi = \{\pi_i\}$ where π_i is the probability of starting in state S_i:

$$\pi_i = P[q_1 = S_i], 1 \leq i \leq N$$

To apply a HMM, we must specify N, M, the number of observation, a discrete distribution for the observations, and the probability matrices A, B and π. We will similarly to the article by Rabiner (1989) use the following notation for the HMM:

$$\lambda = (A, B, \pi),$$

where λ is a collective notation for the chosen A, B and π.

Appendix A.2. Implementation of the hidden markov model

Given the preceding framework, three problems must be addressed in order to efficiently implement the model:

1. Given the observation sequence $O = O_1O_2 \cdots O_T$, and a model $\lambda = (A, B, \pi)$, how do we in an efficient manner compute $P(O|\lambda)$ - the probability of the observation sequence, given our model?

2. Given the observation sequence $O = O_1O_2 \cdots O_T$, and a model $\lambda = (A, B, \pi)$, how do we choose a corresponding state sequence $Q = q_1q_2 \cdots q_T$ which best explains the observations?

3. How do we adjust the model parameters of $\lambda = (A, B, \pi)$ in order to maximize $P(O|\lambda)$.

1. The intuitive approach to calculating $P(O|\lambda)$ would be to enumerate through every possible state sequence of length *"T"*. Still assuming the underlying process being a *Markov chain*, the probability of observing any given sequence O is:

$$P(O|Q, \lambda) = \prod_{t=1}^{T} P(O_t|q_t, \lambda) = b_{q_1}(O_1) \cdot b_{q_2}(O_2) \cdots b_{q_T}(O_T)$$

The probability of a state sequence Q can be written as:

$$P(Q|\lambda) = \pi_{q_1} a_{q_1 q_2} a_{q_2 q_3} \cdots a_{q_{T_1} q_T}$$

The joint probaility of O and Q occuring at the same time must then be the product of the two preceding terms:

$$P(O, Q|\lambda) = P(O|Q, \lambda)P(Q, \lambda)$$

This entails the probability of the observation sequence given the model to be obtained by summing over the joint probability over all possible state sequences:

$$P(O|\lambda) = \sum_{Q} P(O|Q, \lambda)P(Q, \lambda) = \sum_{q_1, q_2, \cdots, q_T} \pi_{q1} b_{q_1}(O_1) a_{q_1 q_2} b_{q_2}(O_2) \cdots a_{q_{T-1} q_T} b_{q_t}(O_T)$$

Observe from the preceding equation that we would need a total of $2T \cdot N^T$ calculations as there are N possible states that could be reached at any given time. It is then fairly easy to see that this would be unfeasible for even quite few time steps with N=6 states. It would for instance imply $2 \cdot 50 \cdot 6^{50} \approx 8 \cdot 10^{40}$ given T=50 - an unfeasible amount of computations. The solution to the preceding problem is a *forward-backward procedure*. The procedure consists of a forward and a backward iteration. Only the forward variable is needed to compute $P(O|\lambda)$, but we will introduce the backwards variable in this section, as it will be utilized to solve the second implementation problem.

Define the forward variable $\alpha_t(i)$ as the probability of the observation sequence until time t, and that state S_i occurs at time t:

$$\alpha_t(i) = P(O_1 O_2 \cdots O_t, q_t = S_i|\lambda)$$

We might then solve the equation for $\alpha_t(i)$ inductively by first initializing:

$$\alpha_1(i) = \pi_i b_i(O_1)$$

and then by forward induction find $\alpha_{t+1}(j)$ iteratively:

$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij} \right] b_j(O_{t+1}), 1 \leq t \leq T-1, 1 \leq j \leq N$$

$P(O|\lambda)$ must then be determined by alpha at time t=T:

$$P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i)$$

Observe now with reference to our discussion regarding the problem of having unfeasible amounts of calculations. By performing the preceding iteration, we now only have N^2T calculations to perform. This entails that we have reduced the number of calculations from $8 \cdot 10^{40}$ to $6^2 \cdot 50 = 1800$ computations for our preceding example.

Then define the backward variable $\beta_t(i)$ as the probability of the observation sequence from t+1 to time T, given state S_i at time t and λ.

$$\beta_t(i) = P(O_{t+1}O_{t+2}\cdots O_T|q_t = S_i, \lambda)$$

Utilizing the inductive procedure for $\alpha_t(i)$ starting at time T and going backwards, we might find $\beta_t(i)$ by first initializing:

$$\beta_T(i) = 1, \;\; 1 \leq i \leq N$$

And then by backwards induction find $\beta_t(i)$:

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij}b_j(O_{t+1})\beta_{t+1}(j), t = T-1, T-2, \cdots, 1, 1 \leq i \leq N$$

2. The problem may be solved in several different manners, whereas the most dynamic one would be to apply a *Viterbi algorithm*, which takes into account limitations concerning possible states to transcend to. However, since we might reach any state s_j from state s_i, the problem may be solved straightforward utilizing the forward and backward variables found during the solution to problem 1. In order to find the the optimal state sequence, lets define a variable as the probability of being in state S_i at time t, given the observation sequence and the model:

$$\gamma_t(i) = P(q_t = S_i|O, \lambda)$$

Now observe that this probability might be written as a function of the forward and backward variable. The explanation being that $\alpha_t(i)$ accounts for the sequence $O_1O_2\cdots O_t$ and $\beta_t(i)$ accounts for the sequence $O_{t+1}O_{t+2}\cdots O_T$ - both given that the state at time t is S_i:

$$\gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{\sum\limits_{i=1}^{N}\alpha_t(i)\beta_t(i)} = \frac{P(O_1O_2\cdots O_t, q_t = S_i|\lambda)P(O_{t+1}O_{t+2}\cdots O_T|q_t = S_i, \lambda)}{P(O|\lambda)} = P(q_t = S_i|O, \lambda)$$

The denominator serves as a normalization factor, such that $\gamma_t(i)$ is a probability satisfying the following property:

$$\sum_{i=1}^{N}\gamma_t(i) = 1$$

We now know that the matrix $\gamma_t(i)$ shows the probability of each state at each given point in time, and might thereby solve for the individually most likely state q_t at any given time t using the following argument:

$$q_t = \text{argmax}_{1\leq i\leq N}[\gamma_t(i)], 1 \leq t \leq T$$

3. As for problem 3, there is no known way to analytically solve for the model that maximizes the probability of the observation sequence. We are however able to choose $\lambda = (A, B, \pi)$ s.t the probability of the observation sequence given our model is *locally* maximized by using an algorithm such as the Baum-Welch(1970). Applying the Baum-Welch algorithm is known as *training* the model as it iteratively improves the likelihood of $P(O|\lambda)$ with each iteration.

To put the Baum-Welch algorithm to work, we will start by defining the probability of being in state S_i at time t, and state S_j at time t+1 given the model and the observation sequence:

$$\xi_t(i,j) = P(q_t = S_i, q_{t+1} = S_j | O, \lambda) = \frac{P(q_t = S_i, q_{t+1} = S_j, O|\lambda)}{P(O|\lambda)}$$

Observe now with similar reasoning as with $\gamma_t(i)$ we may write $\xi_t(i,j)$ as follows while noting that this variable satisfies the desired probability:

$$\xi_t(i,j) = \frac{P(q_t = S_i, q_{t+1} = S_j | O, \lambda)}{P(O|\lambda)} = \frac{\alpha_t(i) a_{ij} b_j(O_{t+1}) \beta_{t+1}(j)}{\sum\limits_{i=1}^{N} \alpha_t(i)\beta_t(i)}$$

It is now fairly easy to see the relation between $\gamma_t(i)$ and $\xi_t(i,j)$:

$$\gamma_t(i) = \sum_{j=1}^{N} \xi_t(i,j)$$

Observe that if we sum the two preceding variables across time, we by (G1) get the expected number of transitions from S_i and by (G2) the expected number of transitions from S_i to S_j:

$$\sum_{i=1}^{T-1} \gamma_t(i) \tag{G1}$$

$$\sum_{i=1}^{T-1} \xi_t(i,j) \tag{G2}$$

We might then utilize these two values to calculate new values of the model parameters $\lambda = (A, B, \pi)$ by the following set of re-estimation equations:

$$\bar{\pi} = \gamma_1(i) \tag{G3}$$

$$\bar{a}_{ij} = \frac{\sum\limits_{t=1}^{T-1} \xi_t(i,j)}{\sum\limits_{t=1}^{T-1} \gamma_t(i)} \tag{G4}$$

For the observation symbol probability distribution, we need the numerator to be the expected number of times in state S_j observing symbol v_k, hence:

$$\bar{b}_j(k) = \frac{\sum\limits_{s.t.O_t=v_k} {}^{T-1}_{t=1}\gamma_t(j)}{\sum\limits_{t=1}^{T-1} \gamma_t(j)} \tag{G5}$$

These re-estimation equations originate from the intuition brought to the table by Baum and his colleagues (1970). Define our new model as $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$. They then proved that if you maximize the likelihood function G6, this leads in an increase in the likelihood of the model.

$$\max_{\bar{\lambda}} Q(\lambda, \bar{\lambda}) = \sum_{Q} P(Q|O,\lambda) log[P(O,Q|\lambda)] \implies P(O|\bar{\lambda}) > P(O|\lambda) \tag{G6}$$

Utilizing this result, we might hence use $\bar{\lambda}$ instead of λ and repeat the re-estimation procedure to bring forth an estimate that could improve the probability of the observation sequence even further. Note however that this iterative technique will only converge towards the nearest local maximum of the likelihood function. Hence, the initial values of our model will have a great impact on our solution considering that the optimization surface is complex and has multiple maximum.

Rabiner (1989) points to the fact that there are no straightforward answer to selecting initial state. He further states that for A and π, experience shows that either random or uniform initial estimates are adequate. For B on the other hand, good initial estimates are helpful in the discrete case, and more essential for the continuous case. He further points out that initial estimates can be obtained in a number of ways, such as manual segmentation of sequences into states with averaging of observations within state, maximum likelihood segmentation of observations with averaging and segmental k-means segmentation with clustering.

Appendix A.3. Optimizing the hidden markov model

As we wanted to examine short term patterns of the Indicative Deposit Rates, training the hidden markov model on the entire sequence of observations would yield no insight with regards to the question at hand. Hence we would have to modify our training of the model, such that it maximizes $P(O|\lambda)$ for each defined interval. Addressing the problem in such a manner, we might start out by dividing our entire data sample into k sequences of data:

$$O = [O^{(1)}, O^{(2)}, O^{(3)}, ..., O^{(k)}] \tag{G7}$$

and each observation sequence has the same length

$$O^{(k)} = O_1, O_2, O_3, ..., O_T \tag{G8}$$

Assuming that the observation sequences are independent of each other, we would now seek to maximize

$$P(O|\lambda) = \prod_{k=1}^{K} P(O^{(k)}|\lambda) \tag{G9}$$

and thereby maximize the probability of observing each individual observation sequence given our model.

Having done this, we are then able to utilize the individual model for each observation sequence to create the model that is on average the most likely for a given day across the entire set of observations. We might then write $\bar{a}_{ij}$, $\bar{b}_j(l)$ and $\bar{\pi}_i$ as:

$$\bar{a}_{ij} = \frac{1}{K}\sum_{k=1}^{K} \bar{a}_{ij}^{k} \tag{G10}$$

$$\bar{b}_i(l) = \frac{1}{K}\sum_{k=1}^{K} \bar{b}_j^{k}(l) \tag{G11}$$

$$\bar{\pi}_i = \frac{1}{K}\sum_{k=1}^{K} \bar{\pi}_i^{k} \tag{G12}$$

Note that the maximisation criteria entails that we can compare the models using a natural logarithm transformation. This will help us surpass the problem with having probabilities converging to zero as the number of sequences increases. Comparing different initial estimates might then be done by evaluating the following for the trained models λ_1 and λ_2:

$$\lambda^*(A, B, \pi) = argmax(\sum_{k=1}^{K}(lnP(O^{(k)}|\lambda_1), \sum_{k=1}^{K}(lnP(O^{(k)}|\lambda_2)) \tag{G13}$$

This concludes our derivation of the hidden markov model.

Appendix B. HMM Model 1: last 10 quotes preceding the fixing

Table LII Log likelihoods 10 last

Maturity / Approach	A,π ,B emp	A,π uni, B emp.	A,π emp, B uni.
1 week	-6314.64	-10862.76	-6309.28
3 month	-6273.48	-10251.71	-6266.60
6 month	-6891.66	-10097.37	-6871.63

Table LIII 1 Week NIBOR

Table LIV State transition matrix A

Bank	SEB	DDB	NDA	SWD	DNM	SHB
SEB	0.3015	0.0927	0.1559	0.2145	0.1368	0.0986
DDB	0.1101	0.3255	0.2725	0.0658	0.0721	0.1541
NDA	0.0895	0.1085	0.3137	0.2995	0.1135	0.0754
SWD	0.1362	0.1038	0.3002	0.1182	0.0782	0.2635
DNM	0.1480	0.0783	0.2394	0.1145	0.2636	0.1562
SHB	0.1028	0.1057	0.0950	0.2387	0.1086	0.3492

Table LV symbol prob. distr. matrix B

Bank	Rate down	Rate up
SEB	0.5226	0.4774
DDB	0.4977	0.5023
NDA	0.5135	0.4865
SWD	0.4987	0.5013
DNM	0.5066	0.4934
SHB	0.5211	0.4789

Table LVI Initial state distribution matrix π

Bank	SEB	DDB	NDA	SWD	DNM	SHB
Probability	0.0789	0.0239	0.6192	0.0318	0.0520	0.1941

Table LVII 3 Month NIBOR

Table LVIII State transition matrix A

Bank	SEB	DDB	NDA	SWD	DNM	SHB
SEB	0.3629	0.1023	0.1528	0.1689	0.1013	0.1119
DDB	0.1153	0.2740	0.2631	0.0638	0.1173	0.1665
NDA	0.0970	0.1307	0.3583	0.1902	0.1272	0.0966
SWD	0.1787	0.1018	0.2617	0.1234	0.1363	0.1981
DNM	0.1364	0.1124	0.2392	0.0950	0.1941	0.2230
SHB	0.1044	0.1130	0.1138	0.1749	0.1200	0.3739

Table LIX symbol prob. distr. matrix B

Bank	Rate down	Rate up
SEB	0.4902	0.5098
DDB	0.4988	0.5012
NDA	0.4801	0.5199
SWD	0.5016	0.4984
DNM	0.4971	0.5029
SHB	0.4774	0.5226

Table LX Initial state distribution matrix π

Bank	SEB	DDB	NDA	SWD	DNM	SHB
Probability	0.0853	0.0360	0.5438	0.0203	0.0947	0.2199

Table LXI 6 Month NIBOR

Table LXII State transition matrix A

Bank	SEB	DDB	NDA	SWD	DNM	SHB
SEB	0.3669	0.1810	0.1263	0.0927	0.1153	0.1178
DDB	0.0497	0.2571	0.3245	0.0354	0.2498	0.0835
NDA	0.0846	0.2897	0.2893	0.0873	0.1645	0.0846
SWD	0.1271	0.1456	0.1804	0.1330	0.3016	0.1123
DNM	0.0844	0.3710	0.1477	0.1510	0.1267	0.1192
SHB	0.0731	0.1416	0.1044	0.1154	0.0919	0.4737

Table LXIII symbol prob. distr. matrix B

Bank	Rate down	Rate up
SEB	0.4796	0.5204
DDB	0.5104	0.4896
NDA	0.4929	0.5071
SWD	0.5056	0.4944
DNM	0.4880	0.5120
SHB	0.4827	0.5173

Table LXIV Initial state distribution matrix π

Bank	SEB	DDB	NDA	SWD	DNM	SHB
Probability	0.0439	0.0253	0.4708	0.0098	0.3971	0.0531

Appendix C. HMM Model 2: the first 10 quotes of the day

Table LXV Log likelihoods 10 last

Maturity / Approach	A,π ,B emp	A,π uni, B emp.	A,π emp, B uni.
1 week	-10994.62	-7745.51	-11173.64
3 month	-10742.59	-7842.42	-10508.60
6 month	-9937.13	-9074.48	-9152.55

Table LXVI 1 Week NIBOR

Table LXVII State transition matrix A

Bank	SEB	DDB	NDA	SWD	DNM	SHB
SEB	0.1414	0.1360	0.1321	0.2845	0.1558	0.1503
DDB	0.1369	0.1346	0.1370	0.2741	0.1607	0.1568
NDA	0.1390	0.1377	0.1391	0.2716	0.1575	0.1550
SWD	0.2171	0.1813	0.1708	0.1858	0.1208	0.1242
DNM	0.1936	0.1751	0.1662	0.1897	0.1366	0.1388
SHB	0.1866	0.1707	0.1623	0.2038	0.1366	0.1399

Table LXVIII symbol prob. distr. matrix B

Bank	Rate down	Rate up
SEB	0.8199	0.1801
DDB	0.7469	0.2531
NDA	0.6904	0.3096
SWD	0.0788	0.9212
DNM	0.4425	0.5575
SHB	0.4761	0.5239

Table LXIX Initial state distribution matrix π

Bank	SEB	DDB	NDA	SWD	DNM	SHB
Probability	0.3331	0.0834	0.0561	0.4694	0.0285	0.0294

Table LXX 3 Month NIBOR

Table LXXI State transition matrix A

Bank	SEB	DDB	NDA	SWD	DNM	SHB
SEB	0.1598	0.1587	0.2127	0.1424	0.1598	0.1667
DDB	0.1641	0.1612	0.2092	0.1418	0.1587	0.1650
NDA	0.2014	0.1992	0.1728	0.1508	0.1364	0.1393
SWD	0.1638	0.1661	0.2157	0.1439	0.1529	0.1576
DNM	0.2022	0.2012	0.1618	0.1577	0.1382	0.1390
SHB	0.2055	0.2041	0.1568	0.1597	0.1372	0.1367

Table LXXII symbol prob. distr. matrix B

Bank	Rate down	Rate up
SEB	0.8911	0.1089
DDB	0.9056	0.0944
NDA	0.1272	0.8728
SWD	0.5771	0.4229
DNM	0.3476	0.6524
SHB	0.3065	0.6935

Table LXXIII Initial state distribution matrix π

Bank	SEB	DDB	NDA	SWD	DNM	SHB
Probability	0.2312	0.2963	0.3914	0.0157	0.0298	0.0357

Table LXXIV 6 Month NIBOR

Table LXXV State transition matrix A

Bank	SEB	DDB	NDA	SWD	DNM	SHB
SEB	0.1322	0.1319	0.1722	0.2572	0.1473	0.1592
DDB	0.1318	0.1318	0.1723	0.2571	0.1480	0.1590
NDA	0.1751	0.1767	0.1297	0.1583	0.2248	0.1353
SWD	0.1651	0.1676	0.1254	0.1949	0.2279	0.1190
DNM	0.1323	0.1329	0.1615	0.2563	0.1741	0.1429
SHB	0.1655	0.1677	0.1363	0.1761	0.2185	0.1360

Table LXXVI symbol prob. distr. matrix B

Bank	Rate down	Rate up
SEB	0.7185	0.2815
DDB	0.7263	0.2737
NDA	0.3339	0.6661
SWD	0.0918	0.9082
DNM	0.9014	0.0986
SHB	0.4312	0.5688

Table LXXVII Initial state distribution matrix π

Bank	SEB	DDB	NDA	SWD	DNM	SHB
Probability	0.0268	0.0273	0.0155	0.4345	0.4825	0.0135

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