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Real Options: Duopolistic Competition under Asymmetric Information

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Abstract

The real options approach to investment decisions has gained widespread popularity over the past decades for the ability to highlight the value of flexibility under uncertainty. Theory has been evolving rapidly, much of which has been concerned with increasing realism through incorporating characteristics of various real-world scenarios. This thesis seeks to contribute to this literature by studying exercise of real options when both asymmetric information and duopolistic competition are important factors to consider. Specifically, the aim is to compare exercise of real options in a duopoly with and without the need for external financing under asymmetric information. Two different models are developed, building on existing models on asymmetric information and duopolistic competition. Firms are assumed to have different growth prospects, and require external finance from uninformed outsiders to be able to invest. Equilibrium outcomes show that external financing under asymmetric information affect the trigger level for a good type of firm, but that the direction of distortion is unclear. When outsiders can deduce the type of follower, the trigger level of a good firm will always drop, while in the opposite case this is not necessarily true. Furthermore, the analysis shows that when outsiders are able to deduce the type of follower, good firms may actually choose to invest reactively in order to avoid underpricing. Finally, both models predict that the need for external finance under asymmetric information will erode the value of old shareholders in a good type of firm, either through distorted timing or underpricing arising from adverse selection. Old shareholders of bad firms are contrarily indifferent at worst, and may for some initial values be better under asymmetric information.

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Chapter 1

Introduction

Real options theory extends the application of option valuation methods to capital budgeting decisions under uncertainty. The theory asserts that under certain circumstances, opportunities to e.g. invest in a new project, expand on a current project or abandon operations, can be seen as analogies to financial call or put options. The fundamental idea is that there is a value associated with the flexibility to wait when the future is uncertain. Waiting allows to resolve some uncertainty, potentially enhancing value. By taking into account the value of flexibility under uncertainty, the real options approach is well suited to refine valuation and decision making in many situations. Particularly, when firms have an investment opportunity, the real options approach is a more sophisticated alternative to the traditional approach of using the net present value (NPV).

Briefly put, the traditional approach to investment decisions is to calculate the present value of expected future cash flows using a discount rate considered appropriate for the risk. This present value net of the investment cost constitutes the net present value, and investment is prescribed as long as NPV > 0. While simple to implement, this approach completely neglects the value of flexibility. Specifically, it fails to recognize that a positive NPV does not preclude the existence of an even better time to invest when the future is uncertain. This shortcoming highlights the key advantage of the real options approach. Under uncertainty, delaying investment effectively entails a gain of information. This information is valuable, as it can help firms reduce the probability of being invested in unfavourable states of the world. In essence, the real option approach is the use of option pricing techniques to incorporate the value of waiting.

The real options approach is however not suited for all investment opportunities. First, there must be some flexibility in terms of timing. If investment is temporarily deferred, the opportunity to invest must persist. Otherwise there would be no potential for waiting. In many real world scenarios, flexibility may be limited by competitive environments requiring firms to act quickly. In the extreme case where the opportunity to invest comes in the form of a one time offer, the real options approach is of no use. Second, the investment must be largely irreversible, with a fixed cost component. If the firm can opt out and regain expenses at any time, there is no value in the flexibility to wait. Consequently, investment in for example an office space is typically ineligible for the real options approach, as the office space can be sold at market value to recover costs. Finally, there must be some evolving uncertainty regarding future cash flows. Otherwise, there would again be no value of waiting, and decision makers would resort to the NPV-criterion. While the above conditions show that the real options approach is limited to certain situations, the widespread application by both academics and practitioners suggest that the theory still has great practical relevance.

The theory of real options spurred from the breakthrough in option pricing by Black and Scholes (1973). Myers (1977) coined the term ”real option”, introducing the analogy between real investment opportunities and options. Model development soon followed, with Brennan and Schwartz (1985) developing a model to determine the optimal policy for when to open and shut a copper mine, incorporating the option like features of real investments. McDonald and Siegel (1986) studies the optimal timing of investment in an irreversible project in continuous time, and developed what has come to be known as the standard real option model. Dixit and Pindyck (1994) discuss several extensions and implications of the early models in the first textbook on the subject.

The wide and flexible applicability of the real options approach has resulted in numerous branches of augmenting theory. Introducing imperfect competition leads to rich dynamics of strategic interaction, where the value of waiting must be balanced against strategic considerations. This has the potential to enhance realism, as firms typically have to factor in the response of competitors in real world situations. Game theory is often used to derive equilibrium solutions for real option exercise in an oligopoly, and there is a vast amount of literature on so called ”real option games,” well summarized in Azevedo and Paxson (2014). Asymmetric information is a key concept in corporate finance, and has proven to be highly relevant in real options theory as well. For example, when the decision maker has private information, exercise of a real option becomes a way of conveying this information, adding a signalling dimension to the real options framework. Examples of asymmetric information in the real options literature include agency conflicts, where a manager exercise on behalf of uninformed owners (Grenadier and Wang, 2005), and firms with private information seeking finance from uninformed outsiders (Morellec and Schürhoff, 2011). Other extensions of the standard model include parameter uncertainty (e.g. Décamps et al., 2005, 2009), the interaction of multiple real options (Trigeorgis, 1993), and the option to learn about the permanence of a shock (Grenadier and Malenko, 2010).

This thesis seeks to augment the standard real options model of McDonald and Siegel (1986) by introducing both asymmetric information and duopolistic competition, combining two existing branches of theory. The main objective is to facilitate analysis of real options exercise in the many real-world situations where both these concepts are of great importance. Specifically, the thesis aims to analyse how the combined impact of asymmetric information and duopolistic competition on real options exercise relates to the impact of each concept separately. Small tech-firms developing and adopting new technologies are of particular relevance to the analysis. Such firms typically have private information regarding their technology and R&D efforts, and are often hesitant to disclose this information to outsiders. Furthermore, such firms rarely have sufficient internal funds to support significant physical investment, and they often face oligopolistic competition. Analysing the dynamics of real options exercise for these types of firms can thus be seen as a specific motivation for the thesis.

The thesis is organized as follows: The standard real options model for a monopolist is presented as a point of reference in chapter 2. Chapter 3 reviews a model of asymmetric information, where firms of different quality seek financing from uninformed suppliers of equity. The chapter closely follows Morellec and Schürhoff (2011). Chapter 4 first reviews a standard model of duopolistic real option exercise where firms are assumed symmetric, building on Dixit and Pindyck (1994). The model is then augmented to cover the case of asymmetric firms. Chapter 5 combines the two concepts. Two different models are developed, followed by discussion and comparison of the results. Chapter 6 concludes.

Chapter 2

Monopolistic Benchmark

To better see how competition and market imperfections affect the real options approach, it is useful to first establish a benchmark case. Hence, this chapter briefly reviews what is considered to be the standard model of real options theory, first derived by McDonald and Siegel (1986). This model will also serve to highlight the potentially drastic disparity between the real options approach and the NPV approach. For a more thorough treatment including elaboration of the technical procedure see e.g. Dixit and Pindyck (1994, pp. 136-152).

There is a single firm with an opportunity to undertake an investment project at any time $t \in [0, \infty)$ for a fixed cost I. Investment is presumed to be irreversible, and will generate an observable cash flow X_t in eternity once undertaken.[1] Time is continuous, and X_t follows a geometric Brownian Motion:

$$dX_t = \alpha X_t dt + \sigma X_t dz_t, \tag{2.1}$$

where α is the expected instantaneous drift, σ is the instantaneous standard deviation and dz represents a standard Wiener process. Agents are risk neutral, the risk free rate is denoted r, and to ensure the existence of a solution in finite time we require $r > \alpha$.

[1]Alternatively, the stochastic variable can represent the value of the firm instead of the cash flow level, with a similar approach

The firm controls nothing but the time of investment. It has to solve an optimal stopping time problem, where X is the state variable and time the control variable. That is, the firm has to find the optimal cash flow level, denoted X_M, such that immediate exercise is optimal once $X \geq X_M$. Intuitively, this threshold is characterized by the option value of waiting exactly being offset by foregone profits from delaying investment. Let $V(X)$ denote the value of the investment opportunity. Using Itô's Lemma to obtain dV, substituting in for dX and rearranging, it can be shown that $V(X)$ must satisfy the differential equation

$$\frac{1}{2}\sigma^2 X^2 V''(X) + \alpha X V'(X) - rV = 0. \tag{2.2}$$

Furthermore, V(X) is subject to three boundary conditions:

$$V(X_M) = \frac{X_M}{r-\alpha} - I \tag{2.3}$$

$$V(0) = 0 \tag{2.4}$$

$$V'(X_M) = 1 \tag{2.5}$$

Here, eq. 2.3 is the value-matching condition, stating that once the trigger level X_M is reached, the option payoff is the profit flow in eternity less the investment cost. Eq. 2.4 establishes 0 as an absorbing level of the stochastic process, meaning that if X reaches zero, it will stay at zero and the option is worthless. Eq. 2.5 is a smooth-pasting condition to ensure trigger-optimality. Solving the homogenous differential equation subject to the boundary conditions gives the critical threshold

$$X_M = \frac{\beta}{\beta - 1} I(r-\alpha), \tag{2.6}$$

where

$$\beta = \frac{1}{\sigma^2}\left(-\left(\alpha - \frac{\sigma^2}{2}\right) + \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right) > 1. \tag{2.7}$$

The option value at some level X is given by:

$$V(X) = \begin{cases} \left(\frac{X_M}{r-\alpha} - I\right)\left(\frac{X}{X_M}\right)^{\beta}, & \text{for } X < X_M \\ \left(\frac{X_M}{r-\alpha} - I\right), & \text{for } X \geq X_M \end{cases} \tag{2.8}$$

Note that the factor $(X/X_M)^\beta$ has the useful interpretation as the price of an Arrow security that pays one unit once the critical trigger value is reached.

If X initially is above the critical threshold X_M, both the NPV-approach and the real options approach prescribe immediate investment. Focusing on the more interesting case where the level of X initially is below X_M, we can however see there is a difference. In this case, applying the simple NPV rule would yield a project value of $V_{NPV} = \frac{X}{r-\alpha} - I$. The corresponding critical level where a firm should invest immediately is given by

$$X_{NPV} = I(r-\alpha) < I(r-\alpha)\frac{\beta}{\beta-1} = X_M \tag{2.9}$$

since $\beta > 1$.

This shows how negligence of the value of waiting may result in premature investment, and thus deterioration of value. The NPV-approach dictates investment once the cash flow level at that time is sufficiently high to balance the investment cost. The real options approach conversely shows that the firm can do better by delaying investment. Intuitively, it can be useful to consider the real option value as an opportunity cost. By investing today the firm receives V_{NPV}, but at the same time it will "lose" its option to invest at a later point in time. Consequently, the firm should wait until the value received from investing today fully compensates for the loss of the option value. This results in a critical level for optimal exercise that in many cases is significantly higher than X_{NPV}. Figure 1 depicts how the real options approach may drastically change both timing and valuation of a project for different values of X

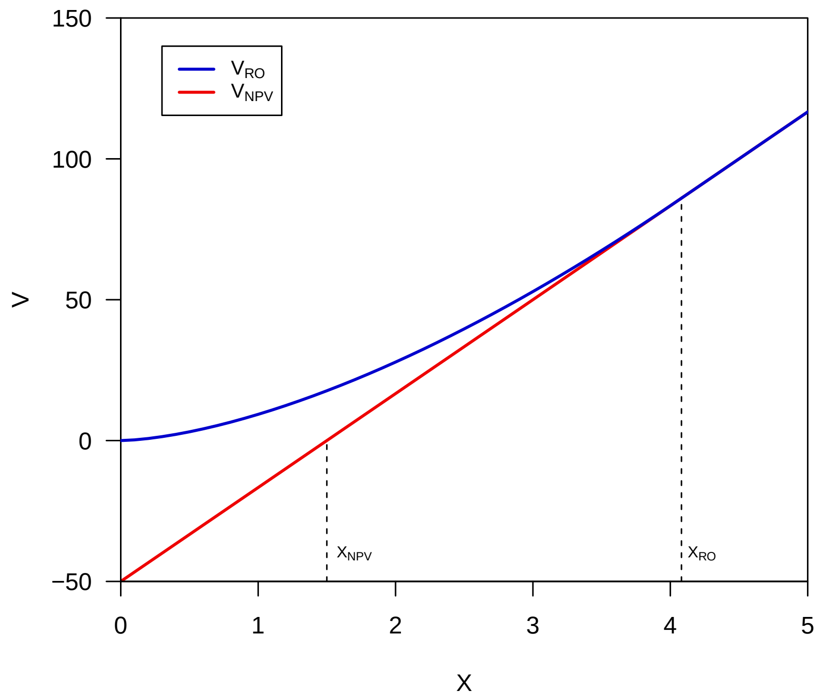

Figure 1: Real options approach vs. NPV approach. Here $\sigma = 0.2, I = 50, r = 0.05$ and $\alpha = 0.02$. When X is low, the NPV approach greatly understates the value of the investment opportunity. While the NPV-approach dictates investment once X hits X_{NPV}, the optimal trigger level found using the real option approach is more than two times higher.

Chapter 3

Asymmetric information

An important concept in many economic subfields, ignored in the standard real options model, is that the decision makers often have private information. Compared to uninformed outsiders, corporate insiders might have superior knowledge about firm characteristics such as growth prospects and success probabilities. In the presence of such informational asymmetries, investment decisions become a way to signal private information, and thus affect the beliefs of outsiders. Since the utility of informed insiders often depends on the beliefs of outsiders, asymmetric information can significantly alter investment and financing decisions. The real option literature typically treats financing as a trivial matter, either explicitly or indirectly assuming firms have sufficient internal funds or can borrow at the risk free rate. However, under asymmetric information, financing is not necessarily a trivial matter anymore. This chapter introduces both asymmetric information and the need for external finance in a real options framework, analysing how this affects the results obtained in chapter 2.

To motivate the relevance in a real option setting, consider for example a small start-up firm with an opportunity to invest in a risky, new project using patented technology. First, the firm will most likely be forced to rely on external finance. Second, the firm will most likely have superior information regarding success and growth probabilities, potentially creating an adverse selection problem. To devise an optimal exercise strategy in this case, it is necessary to address the signalling dynamics explicitly.

3.1 Theoretical Background

Asymmetric information has been a well-studied topic in many fields of economics. Akerlof (1970) analyses the potential implications of asymmetric information in a market with products of varying quality. Specifically, when buyers cannot observe the true quality of a product, they will fear buying a product of bad quality, a ”lemon.” With the linear utility functions applied by Akerlof, buyers will resort to the average quality of products in the market when forming demand. This disincentivizes sellers of good quality to participate in the market, leading to a case of adverse selection. Spence (1973) shows how informed agents (job applicants) of good quality may willingly incur costs (education) in order to signal their quality to uninformed outsiders (employers). This is made possible as long as the cost of signal is negatively correlated with the quality of the agent.

Majluf and Myers (1984) builds on the paper of Akerlof (1970) in a more complex setting of corporate finance, where firms of varying quality need external financing from uninformed outsiders to invest. They show how the inherent incentive for overvalued (bad) firms to issue (overpriced) equity cause undervalued (good) firms to suffer dilution when issuing claims. This might in turn render an otherwise positive NPV investment opportunity unprofitable. In equilibrium, firms may thus rationally pass on positive NPV-projects due to asymmetric information. These examples highlight two key properties of asymmetric information. First, agents of good quality are disposed to suffer information costs, as outsiders are unable to appreciate their true quality. Second, such good agents will look for ways to credibly signal their quality, even if it is costly. For a more thorough treatment of asymmetric information in corporate finance see Tirole (2006).

In a real option framework, asymmetric information turns the exercise of a real option into a potential mechanism for transmitting information. Through the investment timing (including the decision not to invest), informed insiders may signal firm characteristics to uninformed outsiders. This introduces a new dimension to real options exercise, and decision-makers consequently have to balance the potential effects of signalling against the deterioration in value arising by deviating from the first best trigger level.

The literature on real options under asymmetric information is not overly extensive, but there is nevertheless ample evidence of how informational asymmetries can affect timing decisions.

Grenadier and Wang (2005) consider a case of moral hazard, where a manager set to invest on behalf of a principal has private information about parts of the real option value. They find that informational asymmetry in this way will cause delayed investment compared to the first best solution. Grenadier and Malenko (2010) analyse real option exercise under asymmetric information in four corporate finance settings. They demonstrate the potential distorting effect of signalling incentives, and that the direction of this effect is situationally dependent. Bustamante (2012) studies a real options model where the timing of an IPO can serve as a signalling mechanism. She finds that whether a separating or pooling equilibrium ensues depends on the fraction of good type of firms in the market. Specifically, when the fraction of good firms is relatively low, such that perceived average quality in a pooling equilibrium also is low, good firms will deviate from their first best timing and speed up the IPO to signal their type.

Morellec and Schürhoff (2011) form the basis of this chapter. Building on Majluf and Myers (1984), they too find that the existence of a pooling equilibrium depend on the fraction of good firms in the market (along with other parameter values). Good firms will be inclined to opt for a separating equilibrium and invest prior to the first best threshold when the proportion of bad firms is sufficiently high to make the cost of pooling higher than the distortion cost of separating. They furthermore analyse the difference between debt and equity financing, and find that good firms in some situations may prefer to separate by issuing debt.

3.2 Setup

Following Morellec and Schürhoff (2011), there is real investment opportunity where firms again have the option to invest at any time $t \in [0, \infty)$ for a fixed cost I. Firms have zero internal funds, and thus have to obtain external finance to invest. While Morellec and Schürhoff consider both equity and risky debt as means of financing, the scope of this chapter is mainly limited to equity. Firm managers have private information regarding growth prospects of the firm. Firms have either good (γ_g) or bad (γ_b) growth prospects, hereafter interchangeably referred to as good or bad quality, or simply good or bad firms. Type of firm is indexed by $k = g, b$, and the fraction of good firms in the economy is given by $p \in (0, 1)$. Uninformed outsiders know the proportion of good firms and the magnitude of γ_g and γ_b, but cannot observe directly the type of any given firm. To facilitate a separating equilibrium, operating expenses will be modelled explicitly as a constant cost f which arises post-exercise. Growth prospects act multiplicatively on cash flow, such that profit flow after exercise is given by $\gamma_k X_t - f$, where X_t evolve as geometric Brownian motion as in chapter 2 and $\gamma_g > \gamma_b > 0$. Managers maximize the value of old shareholders, and are free to exercise the option to invest at any time. The initial capital structure is assumed to consist of $n_g = n_b = 1$ share, and old shareholders are ineligible to buy new shares upon issuing.

3.3 External financing under perfect Information

If firm types were publicly known, investment triggers and project values are obtained just as in chapter 2. Defining $F \equiv f/r$ as the present value of operating expenses, we get critical trigger levels for $k = g, b$ using the same approach as in the previous chapter:

$$X_k = (I + F)\frac{\beta}{\beta - 1}\frac{r - \alpha}{\gamma_k}, \tag{3.1}$$

where β once again is given by

$$\beta = \frac{1}{\sigma^2}\left(-\left(\alpha - \frac{\sigma^2}{2}\right) + \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right) > 1. \tag{3.2}$$

Since $\gamma_g > \gamma_b$, we must have $X_G < X_B$. This means that under perfect information, good firms will invest earlier than bad firms. Intuitively, the profit lost from postponing investment is relatively larger for good quality firms, inducing a lower trigger level. The option value $V_k(X)$ for some level X is again expressed as in the monopolistic benchmark case:

$$V_k(X) = \left(\gamma_k \frac{X_k}{r - \alpha} - I - F\right)\left(\frac{X}{X_k}\right)^\beta, \tag{3.3}$$

where we naturally have $V_g > V_b$.

Since firms are assumed to lack internal funds, they need to obtain I through issuing equity. Under perfect information equity claims must be fairly priced. The project value held by new shareholders post-exercise will therefore equal the investment cost, giving the budget constraint:

$$\frac{\Delta n_k}{1 + \Delta n_k}\left(\gamma_k \frac{X_k}{r - \alpha} - F\right) = I, \tag{3.4}$$

where Δn_k denote the number of shares issued to finance investment. Rearranging, we have

$$\Delta n_k = \frac{I}{\gamma_k \frac{X_k}{r-\alpha} - F - I} = \left(\frac{I}{V_k(X)}\right)\left(\frac{X}{X_k}\right)^{\beta} \tag{3.5}$$

. Note that for a given trigger level X, the number of new shares issued is negatively correlated with the growth prospects of a firm. Intuitively, better growth prospects imply a higher value of the real option, which again leads to less dilution of old shareholders as outsiders require a smaller fraction of the firm to invest I.

3.4 External financing under asymmetric Information

When outside investors no longer can observe the type of firm, bad firms may have an incentive to mimic the investment behaviour of good firms. If successful, outsiders will be unable to distinguish good from bad firms, analogous to the scenario outlined in Akerlof (1970). The perceived quality of bad firms will thus be higher than their actual quality, allowing them to sell overpriced shares. To see how bad firms can take advantage of the information asymmetry in this way, rewrite the budget constraint under asymmetric information. If bad firms invest at X_G and successfully mimic good firms, we get in equilibrium

$$\Delta n = \frac{I}{\gamma \frac{X_G}{r-\alpha} - F - I} \tag{3.6}$$

Here, γ is the perceived value of an average firm by outside investors. The magnitude of γ depends on the relative frequency of good firms, but we have for certain $\gamma_b < \gamma < \gamma_g$. This implies that dilution of old shareholders of bad firms is reduced compared to exercising at X_G under perfect information.

Let V_{bp}^{old} denote the value of old shareholders in a bad firm when both types of firms pool at the first best trigger of good firms. This value is given by

$$
\begin{aligned}
V_{bp}^{old}(X;X_G) &= \frac{\left(\gamma_b \frac{X_G}{r-\alpha} - F\right)\left(\frac{X}{X_G}\right)^{\beta}}{1+\Delta n} \\
&= \frac{\left(\gamma_b \frac{X_G}{r-\alpha} - F\right)\left(\frac{X}{X_G}\right)^{\beta}\left(\gamma \frac{X_G}{r-\alpha} - F - I\right)}{\gamma \frac{X_G}{r-\alpha} - F} \\
&= V_b(X;X_G) + \frac{I\left(\frac{X_0}{X_G}\right)^{\beta}(\gamma-\gamma_b)\frac{X_G}{r-\alpha}}{\gamma \frac{X_G}{r-\alpha} - F} > V_b(X;X_G),
\end{aligned}
\tag{3.7}
$$

where the inequality holds since $\gamma - \gamma_b$ is positive. This shows how reduced dilution ceteris paribus increase the value of old shareholders of the bad firm. The additional value gained by old shareholders mimicking good firms is given by the term:

$$
\frac{I\left(\frac{X}{X_G}\right)^{\beta}(\gamma-\gamma_b)\frac{X_G}{r-\alpha}}{\gamma \frac{X_G}{r-\alpha} - F}
\tag{3.8}
$$

It is important to notice that this increase in value is relative to investing at X_G under perfect information. However, bad firms would under perfect information invest at the threshold X_B rather than X_G. Mimicking the timing of good firms thus additionally leads to a loss in option value from distortion of the first best timing. Specifically, this cost is given by

$$
V_b(X;X_B) - V_b(X;X_G),
\tag{3.9}
$$

where the second argument indicates the trigger level.

Deviating from the perfect information trigger X_B to mimic good firms consequently has two opposing effects for old shareholders of bad firms. The optimal exercise strategy is devised by weighing these two effects against each other. More precisely, as long as the marginal gain from mimicking, $\frac{\partial V_b}{\partial \gamma}$, is higher than the marginal cost of distortion, $\frac{\partial V_b}{\partial X_B}$, bad firms will be inclined to lower their exercise trigger to mimic good firms.

Good firms on the other hand want outsiders to know their true quality. In an attempt to successfully separate themselves from bad firms they will try to make mimicking more costly. This can be achieved by speeding up investment, hence increasing the distortion cost for bad firms. However, speeding up investment cause distortion costs for good firms as well. Again, the optimal exercise strategy will have to weigh the cost of distortion against the benefit from separating and reducing dilution. In general, there are three possible types of equilibria. These can be characterized as follows:

(i) *First best separating equilibrium.* If the distortion cost from mimicking is higher than the gain from selling overpriced shares at X_G, bad firms will stick to the same investment scheme as under perfect information. This is the case when

$$\begin{aligned} V_b(X;X_B) &> V_{bp}^{old}(X;X_G) \\ \iff V_b(X;X_B) - V_b(X);X_G) &\geq \frac{I\left(\frac{X}{X_G}\right)^{\beta}(\gamma-\gamma_b)\frac{X_G}{r-\alpha}}{\gamma\frac{X_G}{r-\alpha}-F} \end{aligned} \tag{3.10}$$

In this event, good firms do not have to speed up investment to avoid mimicking, and the first-best solution under perfect information persists.

(ii) *Second-best separating equilibrium.* If bad firms prefer to mimic the first best timing of good firms, but good firms are able to successfully speed up investment sufficiently to keep bad firms from mimicking, we get a separating equilibrium where good firms overinvest and bad firms stick to their first-best timing. Good firms thus suffer a cost of the information asymmetry.

(iii) *Pooling equilibrium.* If imposing mimicking costs on bad firms prove too expensive to good firms, a pooling equilibrium where both types of firms invest simultaneously ensues. Firms are here undistinguishable to outsiders at the time of investment. New shareholders hence lose on their investment in bad firms ex-post and profit from investment in good firms.

The first case is trivially solved as under perfect information. Hence, the reminder of this section concentrate on the latter two cases, and bad firms are henceforth assumed to prefer mimicking good firms unless good firms impose further mimicking costs. Before moving on to analyse the existence and characteristics of separating- and pooling equilibria under this assumption, first note that for good firms to successfully impose mimicking costs and facilitate a separating equilibrium, distorting investment must be relatively more expensive for bad firms than for good firms. If this is not the case, threatening to speed up investment would not be a credible signal, and good firms would rather settle for a pooling equilibrium. As shown by Morellec and Schürhoff (2011, p.

283), the single crossing property,

$$\frac{\partial}{\partial\gamma_k}\left(\frac{\frac{\partial V_k(X;\bar{X},\gamma)}{\partial\gamma}}{\frac{\partial V_k(X;\bar{X},\gamma)}{\partial\bar{X}}}\right) > 0, \tag{3.11}$$

holds globally in the given setup as long as $f > 0$. That is, in a signalling scenario where firms invest at some threshold $\bar{X}$ and perceived quality is γ, the marginal value of increased perceived quality relative to the marginal cost of distortion is increasing in firm quality. This essentially means that firms with good growth prospects ceteris paribus find it more desirable to speed up investment, as the cost of distortion relative to the benefit of reduced dilution will be less severe to good firms.

Two incentive compatibility constraints characterize the boundaries of a separating equilibrium. First, good firms must invest at a trigger sufficiently low for bad firms to rationally prefer investing at their first best threshold X_B. Assuming good firms speed up investment and exercise at a trigger level $X < X_G$, the incentive compatibility constraint for bad firms not to prefer mimicking the timing of good firms is given by

$$\left(\frac{\gamma_b X_B}{r-\alpha} - F - I\right)\left(\frac{X}{X_B}\right)^{\beta} \geq \frac{\left(\gamma_b\frac{X}{r-\alpha} - F\right)\left(\gamma_g\frac{X}{r-\alpha} - F - I\right)}{\gamma_g\frac{X}{r-\alpha} - F} \tag{3.12}$$

Intuitively, bad firms will not mimic when the real option value from following the first best investing scheme is higher than the value to old shareholders when mimicking. Note that in the constraint above, bad firms are assumed to be perceived as good firms if they mimic. In equilibrium however, such a strategy would lead to all firms being perceived as average. By solving for X when the constraint is binding, we get an upper threshold X_B^{ICC}. This is the threshold at which good firms must invest prior to in order to avoid being mimicked.

Second, for good firms to prefer separating at some level X, we have another incentive compatibility constraint:

$$\left(\frac{\gamma_g X}{r-\alpha} - F - I\right) \geq \frac{\left(\gamma_g\frac{X_B}{r-\alpha} - F\right)\left(\frac{X}{X_B}\right)^{\beta}\left(\gamma_b\frac{X_B}{r-\alpha} - F - I\right)}{\gamma_b\frac{X_B}{r-\alpha} - F} \tag{3.13}$$

Again solving for X when the condition is binding, we now obtain a lower threshold, X_G^{ICC}. This threshold forms the lower bound of a potential separating equilibrium, such that when $X < X_G^{ICC}$, good firms will rather prefer to pool with bad firms. These two thresholds arising from the incentive compatibility constraints consequently form an upper and lower bound for the separating trigger. As we concentrate on the cases where bad firms would want to mimic X_G, a separating threshold must also lie below the first best trigger of good firms X_G.[1] It is straightforward to see that good firms will prefer to distort their first best timing as little as possible when separating. Thus, in a least cost separating equilibrium good firms invest at the critical value $X_{G,LC} = X_B^{ICC}$, while bad firms invest at their first best threshold X_B.

The existence of a separating equilibrium does however not rule out the possibility of a pooling equilibrium that pareto-dominates the least cost separating equilibrium. The value of old shareholders in a pooling equilibrium where firms invest simultaneously at X_{pool} is given by

$$V_k^{old} \frac{\left(\gamma_k \frac{X_{pool}}{r-\alpha} - F\right)\left(\frac{X}{X_B}\right)^{\beta}\left(\gamma \frac{X_{pool}}{r-\alpha} - F - I\right)}{\gamma \frac{X_{pool}}{r-\alpha} - F} \tag{3.14}$$

Old shareholders of good firms prefer pooling at some X if

$$\frac{\left(\gamma_g \frac{X_{pool}}{r-\alpha} - F\right)\left(\frac{X}{X_{pool}}\right)^{\beta}\left(\gamma \frac{X_{pool}}{r-\alpha} - F - I\right)}{\gamma \frac{X_{pool}}{r-\alpha} - F} \geq \left(\frac{\gamma_g X_{G,LC}}{r-\alpha} - F - I\right)\left(\frac{X}{X_G, LC}\right)^{\beta} \tag{3.15}$$

Similarily, old shareholders of bad firms prefer pooling if

$$\frac{\left(\gamma_b \frac{X_{pool}}{r-\alpha} - F\right)\left(\frac{X}{X_{pool}}\right)^{\beta}\left(\gamma_p \frac{X_{pool}}{r-\alpha} - F - I\right)}{\gamma_p \frac{X_{pool}}{r-\alpha} - F} \geq \left(\frac{\gamma_b X_B}{r-\alpha} - F - I\right)\left(\frac{X}{X_B}\right)^{\beta} \tag{3.16}$$

A pooling equilibrium that pareto-dominates the least-cost separating equilibrium exists if there is some threshold X_{pool} that satisfies both of the above incentive compatibility constraints when good and bad firms invest simultaneously. As shown in Morellec and Schürhoff (2011), this depends on the fraction of good firms in the economy for a given set of parameter values. Specifically, if the fraction of good firms surpasses some parameter-dependent threshold[2], there exists a pooling equilibrium where firms invest at a threshold $X_G < X_{pool} < X_B$. Intuitively, a higher fraction

[1] Technically, the separating is only bounded upwards by X_B, but if the trigger lies between X_G and X_B we simply get the trivial solution of perfect information

[2] See Morellec and Schürhoff (2011, p. 285) for analytical expressions.

of good firms makes the underpricing to each good firm in a pooling equilibrium less severe, as the perceived average quality across all firms will increase. At the same time, the distortion cost from separating remains the same for each firm, such that pooling becomes more attractive. Furthermore, the critical frequency of good firms to result in a pooling equilibrium is increasing in the difference between good and bad quality, as this makes separating relatively cheaper for good firms.

To summarize, we see that asymmetric information in the form of adverse selection affects the timing of real options exercise found in chapter 2. Good firms may be inclined to speed up investment in order to reveal their type to outsiders. In this way, adverse selection erodes the option value for old shareholders of good firms due to distortion of the first best trigger. On the other hand, information asymmetry may also lead to a pooling equilibrium where good firms delay investment and bad firms speed up. For old shareholders of good firms, adverse selection in this way causes a loss in value from underpricing compared to the first best case. Good firms consequently suffer a cost in both instances, while old shareholders of bad firms actually profit from the information asymmetry in the pooling case.

Chapter 4

Strategic Interaction

In most real world scenarios firms do not operate under monopolistic circumstances. A competitive environment may render the standard real options model unsatisfactory, as firms under these circumstances have to factor in potential strategic interaction when deciding how to optimally exercise a real option. Game theory is often used to derive equilibrium outcomes when oligopolistic competition is introduced in real options models, and literature on such ”option games” has been rapidly growing over the past two decades. In general, the first best timing will often be distorted by the presence of strategic considerations, but the direction of distortion is situationally dependent. In some scenarios it can be advantageous to be the first firm to exercise, e.g. to seize a favourable market position or to avoid some negative externality experienced by not investing first. Such option games with a first mover advantage are typically referred to as preemtpion games, and is the most common case in the literature. In other situations there may rather be a second-mover advantage. In such games, often labelled a war of attrition, firms delay investing in order to attain valuable information. This can be the case when the investment of one firm leads to an information gain by both firms, e.g. in the search for natural resources. This chapter first reviews the equilibrium outcome for a preemption game in a symmetric duopoly, following Dixit and Pindyck (1994, Chapter 9), before extending the model to the case asymmetric firms.

4.1 Theoretical background

The literature on real option games spawned with Smets (1991), summarised in a slightly simplified form in Dixit and Pindyck (1994, Chapter 9). Here, two symmetric firms compete in continuous time for a first mover advantage, with an equilibrium in mixed strategies where each firm becomes the leader with equal probability. Huisman and Kort (1999) extend the model to firms already active in the given market, and show that for certain initial values there is a positive probability for a coordination mistake where both firms invest simultaneously. Both models predict that investment will be speeded up by at least one firm compared to a non-strategic case. Grenadier (1996) develops a model of strategic real option exercise which also predicts earlier investment compared to the single firm case. He specifically analyses the real estate market, and shows how this model can explain building booms in a situation of declining demand. Weeds (2002) looks at R&D policies through a real option lens, and while she too finds that competition might lead to preemption and early investment, she shows that delayed investment of both firms may occur as well under certain circumstances.

Lambrecht and Perraudin (2003) analyse an extreme case of preemption where there is only room for one firm to invest. Firms are shown to speed up investment, but through modelling asymmetric costs as private information, the difference to the monopolistic case depends entirely on the distribution of competitors, and optimal investment strategies will still yield a (distribution dependent) positive expected payoff in most cases. Dias (1997) on the other hand analyses a case of war of attrition in discrete time, where firms are inclined to delay exploration for oil in order to see the results of neighbouring firms.

Applying game theoretic models to real option theory means that instead of maximizing expected utility, agents maximize the value of their option to invest. This has several complicating implications, since the real option value typically is modelled in continuous time. Specifically, modelling a mixed strategy equilibrium in continuous time is not straightforward. Fudenberg and Tirole (1985) identify two problems with simply specifying strategies through a function $G_i(t)$.[1] First, this does not allow strategies specified for every potential subgame (what if a firm still have not invested if $G_i(t) = 1$), preventing the concept of a subgame perfect Nash equilibrium to be used.

[1] Interpreted as the cumulative probability that player i has invested at time t.

Second, solely specifying strategies through $G_i(t)$ entails a risk of loss of information for mixed strategies compared to the limit of the analogous discrete case[2] (Fudenberg and Tirole, 1985, pp. 389-392). I.e., mixed strategy equilibriums in discrete time where firms invest in each period with a given probability will in in continuous time not necessarily be properly represented as the limit of discrete time by $G_i(t)$ alone.

To avoid this loss of information, Fudenberg and Tirole (1985) first alter the definition of $G_i(t)$ to the cumulative probability that player i has invested by time t *conditional* on the other player not having invested. Furthermore, they enlarge the strategy space by introducing a so called "atomic" function $h_i(t)$ to track the instantaneous intensity of investment immediately after $G_i(t)$ hits one. A useful intuitive interpretation of this intensity function (see e.g. Thijssen et al., 2012)is that it is the probability of which a firm i, playing mixed strategies, would invest in a discrete representation of the given subgame at time t. This hypothetical discrete game takes zero time, and can be repeated infinitely, thus allowing the discrete-time limit to be represented in continuous time. Using the methodology of Fudenberg and Tirole (1985), the strategy of a player will hence be represented by the two functions G_i and h_i, where $h_i(t) > 0$ by definition implies that $G_i(t) = 1$.[3]

4.2 Symmetric duopoly

This section closely follows the basic duopoly case first modelled by Smets (1991) and summarized in Dixit and Pindyck (1994, chapter 9). Two identical firms, indexed a and b, compete in a duopoly. Both firms have an option to invest, and the investment opportunity is characteristics as in chapter 2 except for the addition of operating costs as in chapter 3. There are four different states of the economy depending on which firm(s) have invested. These states of the economy is modelled through a competition factor $D_i(N_a, N_b)$, where $i = a, b$, and $N_i = 0$ means that firm i has not yet exercised while $N_i = 1$ indicates that firm i has exercised. Intuitively, $D(N_a, N_b)$ can be

[2]See Fudenberg and Tirole (1985, p. 390) for examples.

[3]For technical purposes, Fudenberg and Tirole (1985) furthermore define $\tau_i(t)$ as the time where h first is positive for player i. Defining $\tau(t) \equiv \min(\tau_a(t), \tau_b(t))$ we have that for a subgame starting at time t there is surely investment by time $\tau(t)$.

thought of as describing the profitability of being active in the market for a the given state of the economy.

Furthermore, impose $D_a(1,0) > D_a(1,1) > D_a(0,1) = D_a(0,0) = 0$ and $D_b(0,1) > D_b(1,1) > D_a(1,0) = D_b(0,0) = 0$. This simply means that the profitability of firm i is zero as long as it has not yet invested, positive when both firms have invested, but even more positive when firm i is the only active firm. Firms are hence assumed to be inactive in the given product market prior to exercise, but this assumption can be relaxed as in Huisman and Kort (1999) where $D_a(0,0) > D_a(0,1)$. The properties of $D_i(\cdot)$ implies there is a potential first mover advantage if the other firm is preempted, such that for some levels of X firms may be inclined to speed up investment to try seize the leader position. The first mover advantage is however gone once the other firm enters, meaning that if both firms have invested at some time t they get the same profit stream independently of which firm invested first.

Since firms are ex-ante symmetric, equilibrium strategies will be symmetric as well. It will thus be useful to adopt the terminology of a leader and follower rather than distinguishing between firm a and b. $D(1,0)$ will hereafter thus imply that the leader is the only active firm. The leader here refers to any firm investing first, while a follower refers to any firm being the second to invest. To solve the game we have to work backwards and first calculate the optimal investment threshold of a follower, X_F. That is, given that the other firm has already invested, what is the optimal threshold to invest for the remaining firm? Since a follower takes the competition factor $D(1,1)$ as given, this trigger value is calculated as in the benchmark case of chapter 2:

$$X_F = \frac{\beta}{\beta - 1}\frac{(I+F)(r-\alpha)}{D(1,1)} \tag{4.1}$$

This has the following implication: If $X > X_F$ when the leader invests, the follower will reply by investing immediately. Otherwise, the follower will wait and invest the moment X reaches X_F. At any level X, the expected value of a follower is thus given by

$$V_F(X) = \begin{cases} \left(\frac{D(1,1)X_F}{r-\alpha} - I - F\right)\left(\frac{X}{X_F}\right)^{\beta}, & \text{for } X < X_F \\ \left(\frac{D(1,1)X}{r-\alpha} - I - F\right), & \text{for } X \geq X_F \end{cases} \tag{4.2}$$

Having established the optimal response of a follower, we move on to calculate the expected value of a firm investing as a leader, V_L. If $X \geq X_F$, the leader and follower profit will be the same, as the follower will invest immediately after. When $X < X_F$, the leader will receive a profit stream $\left(\frac{D(1,0)X}{r-\alpha}\right)$ as long as X remains below the follower trigger, and $\left(\frac{D(1,1)X}{r-\alpha}\right)$ if the follower trigger is reached. The total value can be expressed in terms of expectations:

$$\begin{aligned} V_L(X) &= E\left[\int_{t=0}^{T} D(1,0)X_t e^{-rt}dt + \int_{t=T}^{\infty} D(1,1)X_t e^{-rt}dt - I\right] \\ &= E\left[\int_{t=0}^{T} D(1,0)X_t e^{-rt}dt\right] + E\left[e^{-rT}\right] D(1,1)\frac{X_F}{r-\alpha} - I \end{aligned} \tag{4.3}$$

where T denotes the time where X first hits the level that triggers investment from the follower. The second expectation yields the usual expression $(X/X_F)^{\beta}$, while the first expectation is given by [4] $\left(\frac{D(1,0)X}{r-\alpha}\right)\left(1-\left(\frac{X}{X_F}\right)^{\beta-1}\right)$. Rearranging, the leader value can be expressed as

$$V_L(X) = \begin{cases} \left(\frac{D(1,0)X}{r-\alpha}\right) + \left(\frac{[D(1,1)-D(1,0)]X_F}{r-\alpha}\right)\left(\frac{X}{X_F}\right)^{\beta} - I - F, & \text{for } X < X_F \\ \left(\frac{D(1,1)X}{r-\alpha} - I - F\right), & \text{for } X \geq X_F \end{cases} \tag{4.4}$$

The next question is at which threshold a leader would consider to invest in a symmetric equilibrium. This unique threshold X_L is defined such that firms are indifferent between investing at X_L to become the leader and waiting until X_F, as depicted in figure 2. To see why this must be true, consider the case of both firms initially set to invest at X_F. If one firm deviates and invests at $X_F + \epsilon$, for an infinitely small ϵ, this firm will increase its value by earning the leader income for a brief interval. This potential gain from preempting the other firm persists as long as the leader value is greater than the follower value, and the hypothetical preemption game thus continues until X_L, where firms are indifferent between adopting the role of a leader and a follower. Intuitively, the expected cost of overinvestment here exactly balances the expected gain from the leader position.

To model symmetric equilibrium strategies, we must use the approach and enlarged strategy space of Fudenberg and Tirole (1985). Specifically, when $X \geq X_L$ firms can be thought of as contesting in a game of discrete time to determine roles. As previously mentioned, playing the game takes zero time, and hence represents the limit of discrete time. If both firms play ”invest”, there will be joint investment. If only one firm plays ”invest”, it will become leader while the

[4] see Dixit and Pindyck (1994, p.316) for computation of the expectations

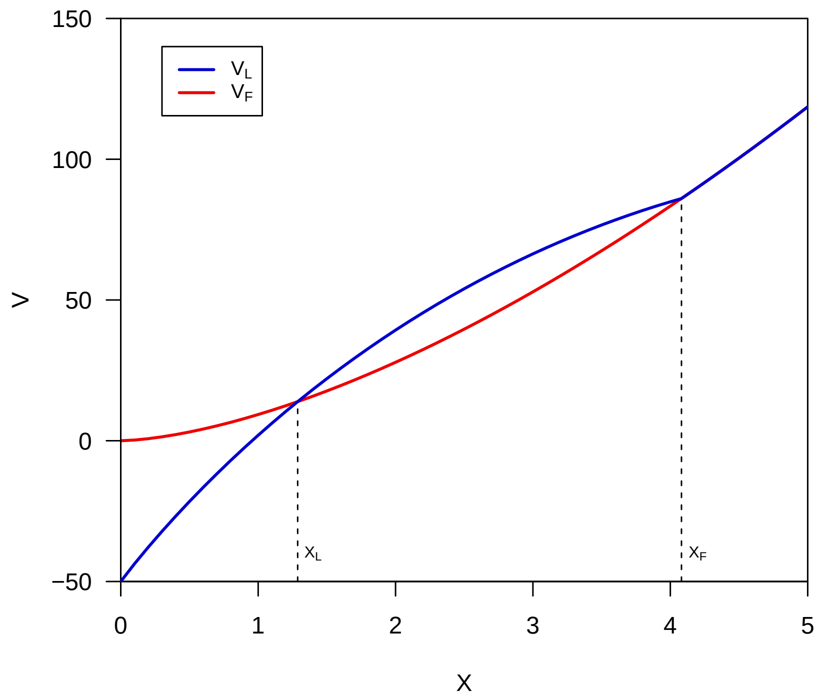

Figure 2: Leader and follower values in symmetric duopoly. Here, $\sigma = 0.2$, $I = 30$, $F = 20$, $r = 0.05$, $\alpha = 0.02$, $D(1,0) = 2$ and $D(1,1) = 1$.We see that when $X = X_L$, leader and follower values are identical, and firms are indifferent about their role. In the intermediate region however, when $X_L \leq X < X_F$, the leader value is higher, and there ensues a fight for the leader position.

other becomes follower. If neither play "invest", the game is repeated until one firm has invested. Each firm play mixed strategies to mitigate the potential for miscoordination, and the intensity function $h_i(t) = h(t)$ corresponds to the probability of which a firm will invest each time this game is repeated. Specifically, $h(t)$ is specified through maximizing the payoff of each firm in the hypothetical game of discrete time. This is shown in appendix A.1. We have

$$h(t) = \begin{cases} 0, & \text{for } t < T_L \\ \frac{V_L(X(t)) - V_F(X(t))}{V_L(X(t)) - V_J(X_(t))} & \text{for } T_L \leq t < T_F \\ 1, & \text{for } t \geq T_F, \end{cases} \tag{4.5}$$

where T_L denotes the time when X hits X_L, T_F denotes the time when X hits X_F and V_J is the value of joint investment. The probability of firm i being leader is now given by

$$
\begin{aligned}
p_i &= h(1-h) + (1-h)(1-h)p_i \\
\iff \quad p_i &= \frac{1-h}{2-h},
\end{aligned} \tag{4.6}
$$

The probability that either one of the two firms attain the leader role while the other invests as a follower is consequently given by $2(1-h)/(2-h)$. The probability of a coordination error in the form of joint investment is similarly given by

$$
\begin{aligned}
p_J &= h^2 + (1-h)(1-h)p_J \\
\iff \quad p_J &= \frac{h}{2-h}.
\end{aligned} \tag{4.7}
$$

To characterize equilibrium outcomes more clearly, it is useful to distinguish between three subcases for varying initial values of X.

(i) $X < X_L$

Here, one firm will invest once X_L is reached while the other will wait and invest at X_F. Firms are indifferent about being a leader or follower, as both roles will yield the same payoff. They do however want to avoid joint investment, which would reduce the payoff of both firms. We have

$$
G_i(t) = G(t) = \begin{cases} 0, & \text{for } t < T_L \\ 1, & \text{for } t \geq T_L, \end{cases} \tag{4.8}
$$

where subscripts are dropped due to the symmetric nature of the game. Furthermore, at X_L firms are per definition indifferent about adopting a leader role or waiting to invest as a follower. I.e., $V_L(X_L) = V_F(X_L)$. Thus, from 4.5 we have $h(T_L) = 0$. From 4.6 and 4.7, we thus get $p_i = 1/2$ and $p_J = 0$. Consequently, we see that at the trigger X_L, there is zero chance of a coordination error where both firms invest simultaneously. Each firm will invest first with the probability of one half, while the other firm will wait and invest at X_F. Intuitively, since there is no first mover advantage at this level, firms will not be willing to risk investing simultaneously, so there will never be joint investment.

$X_L \leq X < X_F$

Here, the leader position will result in a superior payoff, so there is a first mover advantage. Again, firms would like to avoid joint investment, but in this case there is an incentive fight more aggressively for the leader position. We have

$$G(t)) = 1 \quad \forall\, t \tag{4.9}$$

Since $X \geq X_L$, we have $V_L > V_F \implies h(t) \neq 0$ from 4.5. From 4.7 we thus see that $p_J > 0$. That is, the first mover advantage will result in a positive probability for a coordination error in this range[5] Intuitively, the probability for joint investment is such that firms are indifferent about being a follower for sure and taking the risk to become a leader. In equilibrium, each firm immediately invests and becomes leader with probability $(1-h)/(2-h)$, with the other firm waiting to X_F, while there is a probability $h/(2-h) > 0$ for immediate joint investment.

$\boldsymbol{X \geq X_F}$

In this scenario we have a trivial equilibrium where both firms immediately invest simultaneously.

To summarize, competition in the form of a symmetric duopoly with potential first-mover advantages will cause at least one firm to speed up exercise of the real option in an attempt to preempt the other firm. The option value will consequently be partly eroded compared to a non-strategic case. Intensity functions show that the undesirable result of joint investment will never occur for low initial values of X. However, for $X \in (X_L, X_F)$, the first-mover advantage is increasing in X, and for these initial values we have a positive, increasing probability of a coordination mistake.

4.3 Asymmetric duopoly

Now consider the same setup as in the previous section, but with one firm having superior growth prospects like in chapter 3. That is, there is an asymmetric duopoly where one firm has good growth prospects and gets the cash flow $\gamma_g X D(\cdot)$ upon exercise, and one firm has bad growth prospects and earns $\gamma_b X D(\cdot)$ when exercising, where $\gamma_g > \gamma_b$ as in chapter 3. The analysis is analogous to Chevalier-Roignant and Trigeorgis (2011), but differs in that they model asymmetry

[5]The possibility for a coordination mistake is not acknowledged in Smets (1991) or Dixit and Pindyck (1994), but pointed out in e.g. Huisman and Kort (1999)

in variable costs.

If there were no competition, we know from chapter 3 that a good firm would optimally invest at a lower threshold than a bad firm. This means that the good firm is naturally predisposed to attain the leader position. However, the roles may be reversed in some scenarios, and equilibrium solutions depend heavily on initial values and the magnitude of difference in firm quality. Prior to further analysis, it is useful to single out the case where a good firm surely will invest first for any $X < X_{F,B}$, where $X_{F,B}$ denotes the optimal investment threshold for a bad firm investing as a follower. That is, we want to establish under which circumstances a bad firm never would prefer to compete for the leader position. This is the case when the difference in quality between the good and bad firm is relatively large. Through the usual approach, we can calculate the optimal investment threshold for a bad firm acting as a follower:

$$X_{F,B} = \frac{\beta}{\beta - 1} \frac{(I + F)(r - \alpha)}{\gamma_b D(1, 1)}. \tag{4.10}$$

The expected value of a bad firm acting as a follower is consequently given by

$$V_{F,B}(X) = \begin{cases} \left(\frac{\gamma_b D(1,1) X_{F,B}}{r-\alpha} - I - F \right) \left(\frac{X}{X_{F,B}} \right)^{\beta}, & \text{for } X < X_{F,B} \\ \left(\frac{\gamma_b D(1,1) X}{r-\alpha} - I - F \right), & \text{for } X \geq X_{F,B}, \end{cases} \tag{4.11}$$

while the leader value of a bad firm is given by

$$V_{L,B}(X) = \begin{cases} \left(\frac{\gamma_b D(1,0) X}{r-\alpha} \right) + \left(\frac{[D(1,1) - D(1,0)] \gamma_b X_{F,G}}{r-\alpha} \right) \left(\frac{X}{X_{F,G}} \right)^{\beta} - I - F, & \text{for } X < X_{F,G} \\ \left(\frac{\gamma_b D(1,1) X}{r-\alpha} - I - F \right), & \text{for } X \geq X_{F,G} \end{cases} \tag{4.12}$$

When $V_{F,B}(X) \geq V_{L,B}(X)\ \forall\ X$, the bad type will never invest as a leader. The good type can in this scenario invest without the fear of being preempted. Define $X^*_{L,G}$ as the threshold that maximizes the value of a good leader who can invest without any strategic considerations (as if roles had been exogenously determined). If $X < X^*_{L,G}$, the good firm will wait and invest once the threshold is reached, while if $X \geq X^*_{L,G}$, the good type of firm will invest immediately. Equilibrium outcomes for cases without the risk of a bad leader are summarized in table 1:

Table 1: Equilibrium strategies in asymmetric duopoly when the bad firm prefers to always act as a follower

X-Range	**Equilibrium strategies**
$X < X^*_{L,G}$	Good firm invests when X reaches $X^*_{L,G}$. Bad firm invests at $X_{F,B}$
$X^*_{L,G} \le X < X_{F,B}$	Good firm invests immediately. Bad firm invests at $X_{F,B}$
$X \ge X_{F,B}$	Both firms immediately invest simultaneously

Next, consider the case where the bad firm will prefer to have the leader role for some X. This is the case when the disparity of growth prospects is not too big. Define $X_{P,G}$ as the threshold where a bad firm is indifferent about being a leader and a follower in this case. That is, for a good firm to seize the leader role with certainty it will have to invest at $X \le X_{P,G}$. Following Chevalier-Roignant and Trigeorgis (2011), it is useful to distinguish two subcases for clarity purposes. [6]

$X_{P,G} > X^*_{L,G}$

In this case, a good firm can keep investing at the non-strategic leader trigger for sufficiently low values of X. When $X^*_{L,G} \le X < X_{P,G}$, a good firm will invest immediately and still preempt the bad firm. The interesting case occurs when $X_{P,G} \le X < X_{F,G}$. In this region the bad firm will profit from the leader role, i.e., $V_{L,B}(X) \ge V_{F,B}(X)$. Consequently, the bad firm will invest with a positive probability for initial values of X in this region. The equilibrium outcome is analogous to the intermediate region in the symmetric case of section 4.2: Each firm play mixed strategies and invest immediately with intensity $h_g(t)$ and $h_b(t)$ respectively. The difference to the symmetric case is that these intensities - and thus equilibrium probabilities - are no longer

[6]Note however that the portrayal of these subcases in Chevalier-Roignant and Trigeorgis (2011, pp. 386-388) is both incoherent and erroneous. E.g., on p. 387 they consider the case of $X_{P,G} < X_{P,B}$, implying there exists a situation where a good (low cost) firm must invest earlier to preempt a bad (high cost) firm than vice versa. The corresponding figure on p. 388 does not match either, clearly showing $X_{P,B} < X_{P,G}$

symmetric. The good firm profits more from being the leader, and will hence invest with a higher intensity. Intensity functions are derived as in the symmetric case, giving:

$$h_g(t) = \begin{cases} 0, & \text{for } t < T^*_{L,G} \\ 1, & \text{for } T^*_{L,G} \leq t < T_{P,G} \\ \frac{V_{L,B}(t)-V_{F,B}(t)}{V_{L,B}(t)-V_{J,B}(t)}, & \text{for } T_{P,G} \leq t < T_{F,G} \\ 1, & \text{for } t \geq T_{F,G} \end{cases} \tag{4.13}$$

and

$$h_b(t) = \begin{cases} 0, & \text{for } t < T_{P,G} \\ \frac{V_{L,G}(t)-V_{F,G}(t)}{V_{L,G}(t)-V_{J,G}(t)}, & \text{for } T_{P,G} \leq t < T_{F,G} \\ 0, & \text{for } T_{F,G} \leq t \geq T_{F,B} \\ 1, & \text{for } t \geq T_{F,B} \end{cases} \tag{4.14}$$

Notice how the intensity of one firm depends on the potential payoffs of the other firm. Intuitively, equilibrium requires intensity functions such that each firm get the same expected payoff from each pure strategy.

Equilibrium outcome probabilities are given as in eq. 4.6 and 4.7 except for the lack of symmetry. We have

$$p_{GL} = h_g(1-h_b) + (1-h_g)(1-h_b)p_{GL} = \frac{h_g(1-h_b)}{h_g + h_b - h_g h_b} \tag{4.15}$$

$$p_{BL} = \frac{h_b(1-h_g)}{h_g + h_b - h_g h_b} \tag{4.16}$$

$$p_J = \frac{h_b h_g}{h_g + h_b - h_g h_b}, \tag{4.17}$$

where p_{GL} is the probability of the good firm becoming leader, p_{BL} is the probability of the bad firm becoming leader, and p_J is the probability of miscoordination in the form of joint investment. When $X_{P,G} \leq X_{F,G}$, these probabilities characterize the equilibrium. When $X_{F,G} \leq X < X_{F,B}$, the good firm will surely invest immediately, since X is above the optimal follower threshold. With this credible "threat" in mind, the bad firm will in this region invest as a follower, exercising at the trigger $X_{F,B}$. Equilibrium strategies are summarized in table 2.

Table 2: Equilibrium strategies in asymmetric duopoly when the bad firm will contest for leader role for some X, but where the threshold for the good firm to preempt the bad firm is *above* the non-strategic optimal threshold

X-Range	**Equilibrium strategies**
$X < X^*_{L,G}$	Good firm invests when X reaches $X^*_{L,G}$. Bad firm invests at $X_{F,B}$
$X^*_{L,G} \leq X < X_{P,G}$	Good firm invests immediately. Bad firm invests at $X_{F,B}$
$X_{P,G} \leq X < X_{F,G}$	With probability p_g the good firm invests immediately and becomes leader while the bad firm invests at $X_{F,B}$. With probability p_b the bad firm invests immediately while the good firm invest at $X_{F,G}$. With probability p_m there is a coordination mistake and both firms invest simultaneously.
$X_{F,G} \leq X < X_{F,B}$	The good firm will invest immediately, independent of the action of the bad firm. The bad firm thus wait and invests at $X_{F,B}$.
$X \geq X_{F,B}$	Both firms immediately invest simultaneously

$\boldsymbol{X_{P,G} < X^*_{L,G}}$

Compared to the previous subcase it is now relatively harder for the good firm to preempt the bad firm. The only difference in equilibrium strategies is however that the range of where both firms may become leader in equilibrium begin at $X_{P,G}$ rather than $X^*_{L,G}$. Intensity functions and equilibrium probabilities for the intermediate case is derived as in the previous subsection. Table 3 summarizes the results.

Table 3: Equilibrium strategies in asymmetric duopoly when the bad firm will contest for leader role for some X, and the threshold for the good firm to preempt the bad firm is *below* the non-strategic optimal threshold

X-Range	**Equilibrium strategies**
$X < X_{P,G}$	Good firm invests when X reaches $X_{P,G}$. Bad firm invests at $X_{F,B}$
$X_{P,G} \leq X < X_{F,G}$	With probability p_g the good firm invests immediately and becomes leader while the bad firm invests at $X_{F,B}$. With probability p_b the bad firm invests immediately while the good firm invests at $X_{F,G}$. With probability p_m there is a coordination mistake and both firms invest simultaneously
$X_{F,G} \leq X < X_{F,B}$	Good firm invests immediately. Bad firm invests at $X_{F,B}$
$X \geq X_{F,B}$	Both firms immediately invest simultaneously

In an asymmetric duopoly, we thus see that exercise still will be speeded up by the good firm compared to a non-strategic case, but to a lesser degree than in a symmetric duopoly. Intuitively, asymmetry reduces the intensity of competition, allowing the good firm to invest as a leader without having to distort timing as much as in the symmetric case. The result highlights how the value of a real option generally is decreasing in the intensity of competition.

Chapter 5

Duopolistic competition and external finance under asymmetric information

Thus far, we have seen how important real world concepts in the form of asymmetric information and competition affect the standard real options case. Both concepts will typically accelerate investment of at least a good type of firm, either to seize a first-mover advantage or to avoid underpricing. Through distortion of the first best timing, the value of the investment opportunity is hence partly eroded. This chapter seeks to merge the two branches of theory. Of particular interest is whether the eroding effects of each concept will act additively, and whether the need for external financing under asymmetric information further speeds up exercise in a duopoly. Section 5.1 introduces the need for external finance in the asymmetric duopoly of section 4.3, while section 5.2 considers an alternative setup where the composition of firm quality in the duopoly is unknown.

5.1 Asymmetric duopoly

The setup is as in section 4.3. We have a duopoly where two firms both have an option to invest. Firms are ex-ante identical apart from their growth prospects, which are either good or bad as in chapter 3. There is exactly one good firm in the duopoly, meaning that firms know the type of their competitor. To finance investment firms need to issue equity. Uninformed outsiders are ex-ante unable to distinguish between the two types of firms, so there is asymmetric information. To enhance realism, outsiders are assumed to correctly deduce the type of firm investing as a follower. This is motivated by the opportunity to observe cash flows of the leader ex-post. The revelation of follower quality is assumed to take place instantly to enable a more tractable analysis.

Introducing the dynamics of signalling from chapter 3 into the asymmetric duopoly of section 4.3. is not without complications. First, the determination of trigger values and exercise strategies will depend on the beliefs of outsiders, but these beliefs again depend on exercise strategies and trigger values. This mutual dependence precludes an analytical solution, as values will have to be determined simultaneously. Second, the concept of outsider beliefs is particularly difficult to treat in a consistent manner when it comes to exercise off any equilibrium path. Specifically, if one firm invests as a leader off the equilibrium path, it is hard to provide any rationale for the belief outsiders will assign to the quality of that firm, especially since these beliefs again will affect the incentive for each firm to invest off the equilibrium path.

Despite these complications, the model provides qualitative insight regarding potential equilibrium outcomes. Denote outsiders' beliefs about the quality of a follower and leader γ_F and $\gamma_L(X)$ respectively. As stated initially, γ_F will equate to the true quality of the follower, while the belief concerning leader quality will depend on the level of X. The level-dependence of this belief arises since the incentive for each firm to invest being dependent on the level of X. For levels where only a good firm would ever invest, we have $\gamma_L(X) = \gamma_g$, while $\gamma_L(X) < \gamma_g$ for levels of X where the bad firm in equilibrium invest as a leader with positive probability. If there is joint investment, perceived quality of both firms will be $\gamma = [\gamma_b + \gamma_g]/2$.

To begin with, define

$$V_L^{out}(X) = \begin{cases} \frac{\gamma_L X D(1,0)}{r-\alpha} - F - I - \frac{\gamma_L (D(1,0)-D(1,1)) X_F^{out}}{r-\alpha} \left(\frac{X}{X_F^{out}}\right)^{\beta} & \text{for } X < X_F^{out} \\ \frac{\gamma X D(1,1)}{r-\alpha} - F - I & \text{for } X \geq X_F^{out} \end{cases} \tag{5.1}$$

as the value of a leader perceived by outsiders, where X_F^{out} denotes the threshold at which outsiders believe a follower will invest at the time of leader investment. The perceived value of a follower is similarly given by

$$V_F^{out}(X) = \begin{cases} \left(\frac{\gamma_F X D(1,1)}{r-\alpha} - F - I\right)\left(\frac{X}{X_F^{out}}\right)^{\beta} & \text{for } X < X_F^{out} \\ \frac{\gamma_F X D(1,1)}{r-\alpha} - F - I & \text{for } X \geq X_F^{out} \end{cases} \tag{5.2}$$

where X_F^{out} now will equate the true follower trigger.

The value of old shareholders is given as in eq.3.7. Let $V_{F,k}^{old}$ denote the ex-post value of old shareholders in a firm with quality $k = G, B$ investing as a follower. We have:

$$V_{F,k}^{old}(X) = \begin{cases} \left(\frac{\frac{\gamma_k X D(1,1)}{r-\alpha} - F}{1+\Delta n(F)}\right)\left(\frac{X}{X_{F,k}}\right)^{\beta} & \text{for } X < X_{F,k} \\ \frac{\frac{\gamma_k X D(1,1)}{r-\alpha} - F}{1+\Delta n(F)} & \text{for } X \geq X_{F,k}, \end{cases} \tag{5.3}$$

where Δn is given as in 3.6, and can be written as

$$\Delta n(F) = \frac{I}{1 + \frac{I}{V_F^{out}}} \tag{5.4}$$

Since outsiders accurately will deduce the growth prospects of the follower, the value of a follower k will be the same as in section 4.3.

For a leader on the other hand, outsiders' belief may drastically impact the value of old shareholders compared to section 4.3. We have:

$$V_{L,k}^{old}(X) = \begin{cases} \frac{\frac{\gamma_k X D(1,0)}{r-\alpha} - F - \frac{\gamma_k (D(1,0)-D(1,1)) X_{F,c}}{r-\alpha}\left(\frac{X}{X_{F,c}}\right)^{\beta}}{1+\Delta n(L)} & \text{for } X < X_{F,c} \\ \frac{\frac{\gamma_k X D(1,1)}{r-\alpha} - F}{1+\Delta n(L)} & \text{for } X \geq X_{F,c}, \end{cases} \tag{5.5}$$

where $X_{F,c}$ denotes the follower trigger of the competitor, and where

$$\Delta n(L) = \frac{I}{1 + \frac{I}{V_L^{out}}} \tag{5.6}$$

This shows that whenever the perceived value of a leader, determined through $\gamma_L(X)$, is higher (lower) than its actual quality, the value of old shareholders of a firm investing as a leader will be higher (lower) than when firms did not have to rely on external finance. Consequently, the bad firm will have further incentives to speed up investment compared to section 4.3; It would now additionally benefit from selling overpriced shares, just as when mimicking in chapter 3.

The good firm wants to preempt and separate from the bad firm. To achieve this, it will have to exercise at a trigger such that $V_{L,B}^{old}(X) \leq V_{F,B}^{old}(X)$. Following eq. 3.11, we know that such a threshold exists. The higher leader profit gives the good firm additional incentive to separate from the bad firm compared to chapter 3, so when a separating threshold exists there, it must also exist here. $V_{L,B}^{old}$ is increasing in $\gamma_L(X)$. Since $\gamma_L(X) \geq \gamma_b$, we can conclude that the preemptive threshold under the need for external finance cannot be higher than the preemptive threshold $X_{P,G}$ without the need for external finance. Furthermore, the preemptive threshold under external finance cannot be below the leader trigger of a symmetric duopoly with two good firms, denoted X_L^g, as below this threshold a good firm would strictly prefer to wait and invest as follower. Concludingly, the preemptive threshold must be located somewhere between the preemptive threshold of an asymmetric duopoly and the preemptive threshold of a symmetric duopoly with good firms.

Define a ”pessimistic” preemptive threshold of a good firm $X_{P,G}^{EP}$. This threshold is characterized by $V_{L,B}^{old}(X_{P,G}^{EP}) \leq V_{F,B}^{old}(X_{P,G}^{EP})$ while simultaneously imposing $\gamma_L = \gamma_g$. I.e., this is the threshold that would preempt a bad firm even if the bad firm were sure to be perceived as good if investing as a leader. This is pessimistic in the sense that if the bad firm were to invest as a leader with a positive probability, γ_L would have to be lower than γ_g in equilibrium. Thus, there can be no equilibrium where the bad firm invests with positive probability at $X_{P,G}^{EP}$, as with consistent beliefs of outsiders this strategy would be strictly dominated by investing as a follower. Intuitively, the incentive for a bad firm to contest for the leader role is decreasing in the probability of actually becoming a leader.

Next, define an ”optimistic” preemptive threshold of a good firm $X_{P,G}^{EO}$. This threshold is characterized by the bad firm preferring the pure strategy of competing for the leader role to waiting and investing as a follower for all $X > X_{P,G}^{EO}$, *conditional* on γ_L being consistent with the ensuing probability of becoming a leader *and* the good firm investing with intensity $h_g(t) > 0$. That is, above this threshold, the bad firm would like to immediately invest with positive intensity given by $h_b(t)$, even when outsider beliefs are consistent with the ensuing probabilities of leader types.

For tractability, the analysis is limited to the assumption that $X_{L,G}^* > X_{P,G}^{EP}$. This means that the good firm would like to delay investment as long as possible while still preempting the bad firm. Now consider the following set of strategies: The good firm invests at $X_{P,G}^{EO}$. The bad firm, knowing that it would not like to compete for the leader position given the ensuing value ofγ_L, invests at $X_{F,B}$. For these strategies to be consistent with the beliefs of outsiders, we must have $\gamma_L = \gamma_g$, since the good firm always invests first. However, given those beliefs, the bad firm would have an incentive to compete for the leader position. Hence, this cannot constitute an equilibrium. Moreover, the same reasoning can be applied to any $X_{P,G}^{EP} < X \leq X_{P,G}^{EO}$: There cannot be an equilibrium in this range where the good firm successfully preempts the bad firm, as corresponding consistent beliefs of outsiders would induce the bad firm to invest. Consequently, in equilibrium the good firm must invest at $X_{P,G}^{EP}$ to preempt the bad firm. The bad firm will accordingly be investing at the first best follower trigger $X_{F,B}$, and the strategies are hence consistent with $\gamma_L = \gamma_g$.

This defines equilibrium investment strategies when $X < X_{P,G}^{EP}$. For any initial value $X_{P,G}^{EP} < X \leq X_{P,G}^{EO}$, we are however left with the problem of inconsistency outlined above. Both firms investing immediately with a positive intensity is not consistent with the ensuing beliefs of outsiders, under which the bad firm rather would like to invest as a follower. At the same time, the good firm investing immediately with the bad firm waiting is not consistent with the resulting outsider beliefs either, as belief consistency gives $\gamma_L = \gamma_g$, under which the bad firm would like to compete for the leader position after all. Therefore, to facilitate an equilibrium, we will assume that the bad firm will exercise according to a mixed strategy. While being an unattractive feature reducing tractability, this allows for consistent beliefs. Specifically, the bad type of firm will compete for the leader position with probability $p_{bc}(X)$ (thus investing immediately with positive intensity $h_b(t)$), and wait and surely invest at $X_{F,B}$ with probability $p_{bw}(X) = 1 - p_{bc}(X)$. $p_{bc}(X)$ will be increasing in X. It will be close to 0 for X-values close to $X_{P,G}^{EP}$, since the bad firm would here only be willing to compete for the leader role when γ_L is very high. Similarly, p_{bc} will converge to 1 as X approaches $X_{P,G}^{EO}$, since the bad firm here would be willing to compete for the leader even at the lower γ_L ensuing if it surely invests with positive intensity.

Note that this alone does not resolve the inconsistency issue. If $p_{bc}(X)$ is low, the perceived value of a leader will be such that $V_{L,B}^{old}(X) \geq V_{F,B}^{old}(X_{P,G}^{EP})$. Given these beliefs, the bad firm would yet again have an incentive to deviate and compete more aggressively. To work around this inherent incentive to deviate, restrict the belief of outsiders in this region to $\gamma_L \in [\gamma_b, \bar{\gamma}(X)]$, where $\bar{\gamma}(X)$ is the perceived value of a leader such that $V_{L,B}^{old}(X) = V_{F,B}^{old}(X)$. Essentially, this means that

$p_{bc}(X)$ will be bounded from below, as there will be no point in further lowering the probability of competing when this no longer has any effect on outsiders beliefs. The bad firm will hence select $p_{bc}(X)$ such that $V_{L,B}^{old}(X) = V_{F,B}^{old}(X)$, and consequently have no incentive to deviate given consistent beliefs of outsiders $\gamma_L = \bar{\gamma}(X)$. While the restriction of outsider beliefs might seem like a strong assumption, it can be motivated by the following line of thought: If outsiders believe leader quality to be above $\bar{\gamma}$ and supply external finance accordingly for $X \in [X_{P,G}^{EO}, X_{P,G}^{EP})$, they will always be expected to lose on the investment on average, since the bad firm under these beliefs will have an incentive to exercise more aggressively.

In section 4.3., the bad firm would for any $X < X_{F,B}$ prefer being a follower to joint investment. With signalling incentives, there is conversely a threshold $X_{J,B}$ where the bad firm will prefer joint investment to investing as a follower for all $X > X_{J,B}$. Intuitively, this stems from the opportunity to sell overpriced share. Since this feature will alter the equilibrium dynamics, we limit the first region of intermediate initial values to $X_{P,G}^{EP} < X \leq X_{J,B}$. To summarize, we here have the following equilibrium outcome: (i) If $X < X_{F,G}$, the good firm invests immediately with intensity $h_g(t)$. The bad firm immediately invests with intensity $h_b(t)$ with probability $p_{bc}(X)$, and with probability $1 - p_{bc}(X)$ waits and invest at $X_{F,B}$. The probability of a good leader is thus given by

$$p_{GL} = 1 - p_{bc} + p_{bc}\frac{h_g(1 - h_b)}{h_g + h_b - h_g h_b} \tag{5.7}$$

I.e., the probability of the bad firm not competing plus the probability of being a leader when the bad firm competes. Similarly, the respective probabilities of the bad firm being leader and joint investment is given by

$$p_{BL} = p_{bc}\frac{h_b(1 - h_g)}{h_g + h_b - h_g h_b} \tag{5.8}$$

$$p_J = p_{bc}\frac{h_b h_g}{h_g + h_b - h_g h_b} \tag{5.9}$$

(ii) If $X \geq X_{F,G}$, the good firm will invest immediately with intensity 1. The bad firm prefers being a follower to joint investment in this region, and will hence wait and invest at $X_{F,B}$ as in section 4.3.

When both firms would like to compete for the leader position, exercise has thus far been assumed to be coordinated through intensity functions $h_k(t)$ without addressing the derivation of these intensity under asymmetric information. In general, the underlying principle will be the same as in chapter 4: Firms again maximize the payoff from the zero-time discrete game that represent the limit of discrete time at the time of investment. However, under asymmetric information firms must additionally factor in the strategic effect on outsiders' beliefs. Outsiders' beliefs must be consistent with the set of $h_k(t)$ in equilibrium. This can be enforced by imposing the self-fulfilling assumption that outsiders know which intensity each firm plays. This gives

$$\frac{dV_k}{dh_k} = \frac{\partial V_k}{\partial h_k} + \frac{\partial V_k}{\partial \gamma_L}\frac{d\gamma_L}{dh_k}, \tag{5.10}$$

where all values are given for old shareholders. Through the first order condition we get intensities implicitly defined

$$\frac{h_g^2(V_{L,B} - V_{J,B}) - h_g(V_{L,B} - V_{F,B})}{(h_g + h_b - h_g h_b)^2} = \frac{\partial V_B}{\partial \gamma_L}\frac{d\gamma_L}{dh_b} \tag{5.11}$$

$$\frac{h_b^2(V_{L,G} - V_{J,G}) - h_b(V_{L,G} - V_{F,G})}{(h_g + h_b - h_g h_b)^2} = \frac{\partial V_G}{\partial \gamma_L}\frac{d\gamma_L}{dh_g} \tag{5.12}$$

Intensities, beliefs and values must still be determined simultaneously. However, we see that the strategic effect on outsiders' beliefs ceteris paribus makes the bad firm invest with relatively higher intensity compared to section 4.3: The left side of 5.12 is increasing in h_b, while the right side is positive. Contrarily, the good firm invests with relatively lower intensity. This counter-intuitive result stems from the property of a mixed strategy equilibrium discussed in chapter 4, where each player must chose a strategy that equates the payoff to the other player from wither pure strategy. The good firm now has an extra incentive to increase h_g, as it increases the leader payoff, and consequently the bad firm must increase intensity to make it less desirable for the good firm to increase h_g.

Next, consider what happens when $X > X_{J,B}$. Here, the level of X is such that the bad firm prefers joint investment to being a follower. Note that equilibrium in this case requires $h_b \geq h_g$. If $h_g > h_b$, consistent beliefs of outsiders would make increasing h_b a dominant strategy to the bad firm. To see this, assume $h_g > h_b$. Consistent beliefs yield $\gamma_L > \gamma$, which allows the bad firm to sell even more overpriced shares as a leader than if there is joint investment. Consequently, the bad firm would strictly prefer being a leader to joint investment. Since the bad firm per definition prefer joint investment to being a follower in this region, raising investment intensity would always be a dominant strategy. For the sake of clarity, define two subcases:

$X > X^{EO}_{P,G}$

In this case, the value of the bad firm is per definition greater when investing as a leader than as a follower as long as $h_g > 0$. Since the bad firm prefers both joint investment and being a leader to being a follower, immediate investment with intensity $h_b = 1$ is a dominant strategy as long as the good firm also invests immediately with $h_g > 0$. However, the value of being a follower will always be greater than the value under joint investment for the good firm. Taking into account the dominant strategy of the bad firm, the good firm will thus prefer to delay exercise. Specifically, in order to avoid selling underpriced shares the good firm will prefer a reactive strategy where it invests at $max(X_{F,G}, X_B + \epsilon)$ conditional on the bad firm having invested, where X_B denotes the level of investment for the bad firm. I.e., if $X > X_{F,G}$, the good firm will invest immediately after the bad firm, otherwise it will wait and invest at the first-best follower trigger $X_{F,G}$. Realizing this, the bad firm will rationalize that there is no potential for selling overpriced shares, as outsiders will know the first firm to invest to be bad. If $X > X_{F,G}$, there is no leader profit to be earned either, and the bad firm will thus wait and invest at $X_{F,B}$, which is the first-best trigger in the absence of leader profit and signalling incentives. If $X < X_{F,G}$, there is a possibility that the leader profit makes immediate investment desirable. Since there is no signalling, we have the simple condition $V_{L,B}(X) > V_{F,B}(X)$ that must hold for the bad firm to prefer immediate investment, otherwise the bad firm will delay investment until $X_{F,B}$. Conditional on outsiders believing the first firm to be bad (a logical consequence of the reactive strategy employed by the good firm), the good firm will not have any incentive to deviate from this investment scheme, such that strategies constitute an equilibrium.

$X < X^{EO}_{P,G}$

In this case, there is not necessarily any dominant strategy for the bad firm. If the good firm chooses to invest immediately with positive intensity, we have $\gamma_L \leq \gamma$ due to $h_g \leq h_B$. Both leader- and joint investment thus impose a significant cost of dilution to old shareholders of the good firm. Specifically, if $V^{old}_{L,G}(X, \gamma_L) < V_{F,G}(X + \epsilon)$, we cannot have an equilibrium where both firms invest with positive intensity, since reducing h_g would always be preferred to the good firm. Here, the potential gain from earning leader profit is dominated by the cost of dilution, and we get the same equilibrium solution as in the previous subcase, where the good firm rather exercise reactively. If $V^{old}_{L,G}(X, \gamma_L) \geq V_{F,G}(X + \epsilon)$, the good firm would like to compete for the leader position, and equilibrium outcomes are determined through the intensity functions, imposing $h_g \leq h_b$.

Finally, if $X > X_{F,B}$, the good firm will commit to the same reactive strategy outlined above, investing immediately after the bad firm. The bad firm will in this region invest with intensity 1 whenever it invests. This prevents any equilibrium where both firms invest jointly, as the good firm will always have an incentive to delay and reveal its type. The bad firm facing the reactive strategy exercise immediately, and outsiders will correctly perceive its type.

Equilibrium strategies are summarized in table 4 for $X_{P,G}^{EO} \leq X_{J,B}$ and in table 5 for $X_{P,G}^{EO} > X_{J,B}$:

Table 4: Equilibrium strategies in asymmetric duopoly when firms seek external finance from uninformed outsiders and $X^{EO}_{P,G} \leq X_{J,B}$

X-Range	Equilibrium strategies
$X < X^{EP}_{P,G}$	The good firm invests preemptively as a leader when X reaches $X^{EP}_{P,G}$. The bad firm invests as a follower at $X_{F,B}$. Outsiders correctly deduce the type of firm at the time of investment.
$X^{EP}_{P,G} \leq X < X_{J,B}$	(i) If $X < X_{F,G}$, the good firm will invest immediately with intensity given by $h_g(t)$. With probability $p_{bc}(X)$, the bad firm will invest immediately with intensity $h_b(t)$, and with probability $1-p_{bc}(X)$ it will surely wait and invest as a follower at $X_{F,B}$. For $X > X^{EO}_{P,G}$, we have $p_{bc} = 1$. Probabilities for equilibrium outcomes are given by eq. 5.7 - 5.9. Outsiders are unable to deduce the type of leader at the time of investment, and hence resort to the their belief regarding the average quality of a leader γ_L when supplying finance. (ii) If $X \geq X_{F,G}$, the good firm will invest immediately with intensity 1. The bad firm prefers being a follower to joint investment in this region, and thus waits and invest as a follower at $X_{F,B}$. In this case outsiders correctly deduce the type of leader.
$X_{J,B} \leq X < X_{F,B}$	Since $X^{EO}_{P,G} \leq X_{J,B}$, investing immediately with intensity 1 is a dominant strategy for the bad firm conditional on immediate investment by the good firm. The good firm will consequently adopt a reactive strategy, exercising at $max(X_{F,G}, X_B + \epsilon)$ once the bad firm has invested, for an infinitely small ϵ. If $V_{L,B}(X) > V_{F,B}(X)$, the bad firm will invest immediately, while if $V_{L,B}(X) \leq V_{F,B}(X)$ it will wait and invest at $X_{F,B}$. Outsiders will correctly deduce the type of leader due to the reactive strategy of the good firm.
$X \geq X_{F,B}$	The good firm will again exercise reactively, right after the bad firm. The bad firm thus invests immediately to minimize distortion costs. Outsiders will again be able to correctly deduce the type of leader.

Table 5: Equilibrium strategies in asymmetric duopoly when firms seek external finance from uninformed outsiders and $X^{EO}_{P,G} > X_{J,B}$

X-Range	Equilibrium strategies
$X < X^{EP}_{P,G}$	As in table 4, the good firm invests when X reaches $X^{EP}_{P,G}$ and obtains the leader role. The bad firm invests as a follower at $X_{F,B}$. Outsiders correctly deduce the type of firm at the time of investment.
$X^{EP}_{P,G} \leq X < X_{J,B}$	(i) If $X < X_{F,G}$, the good firm will invest immediately with intensity given by $h_g(t)$. With probability $p_{bc}(X)$, the bad firm will invest immediately with intensity $h_b(t)$, and with probability $1 - p_{bc}(X)$ it will surely wait and invest at $X_{F,B}$. $p_{bc}(X)$ is increasing in X. Probabilities for equilibrium outcomes are again given by eq. 5.7 - 5.9. (ii) If $X \geq X_{F,G}$, the good firm will invest immediately with intensity 1. The bad firm prefers being a follower to joint investment in this region, and will thus wait and invest at $X_{F,B}$. In this case outsiders correctly deduce the type of leader.
$X_{J,B} \leq X < X^{EO}_{P,G}$	If $V^{old}_{L,G}(X, \gamma_L) < V_{F,G}(X + \epsilon)$, the good firm will exercise reactively. In the absence of signalling incentives, the bad firm will delay investment and invest as a leader at $X_{F,B}$. If $V^{old}_{L,G}(X, \gamma_L) \geq V_{F,G}(X + \epsilon)$, both firms will invest with positive intensity given by $h_k(t)$.
$X^{EO}_{P,G} \leq X < X_{F,B}$	Here we have the same dynamics as in table 4. Immediate investment with intensity 1 is a dominant strategy for the bad firm conditional on immediate investment by the good firm. The good firm will consequently adopt a reactive strategy, exercising at $max(X_{F,G}, X_B + \epsilon)$. If $V_{L,B}(X) > V_{F,B}(X)$, the bad firm will invest immediately, while if $V_{L,B}(X) \leq V_{F,B}(X)$ it will wait and invest at $X_{F,B}$. Outsiders will correctly deduce the type of leader due to the reactive strategy of the good firm.
$X \geq X_{F,B}$	The good firm exercises reactively. Anticipating this, the bad firm invests immediately. Outsiders correctly deduce the type of leader

Discussion

The treatment of outsider beliefs complicates the model. Specifically, the need for simultaneously determining beliefs, intensities and values makes it difficult to assert any definite insight for intermediate initial values of X. There is also situations where there is no logical way to specify consistent out of equilibrium beliefs, as the way outsiders perceive a firm investing first, off the equilibrium path, will affect the probability of each firm doing so. To make things even more difficult, there is a myriad of potential subcases for various configurations of key thresholds and parameters, negatively affecting tractability. Despite this, there are several key insights to highlight.

First, the need for external finance from uninformed equity markets requires the good firm to invest at a lower threshold than in section 4.3 in order to preempt and separate from the bad firm. For low initial levels of X, this implies that the good firm will speed up investment. Compared to the asymmetric duopoly of section 4.3, the option value of a good firm is thus further eroded through the presence of asymmetric information. Whenever the good firm preemptively invests at $X_{P,G}^{EP}$, the bad firm will invest as a follower at the same threshold as in section 4.3, $X_{F,B}$. For low initial levels of X, the value of the bad firm will hence be unaffected by information asymmetry. Equity markets will for such low initial levels rationally be able to deduce the type of leader. This means that there is no cost of dilution, and the additional cost for the good firm comes from distortion.

Second, and perhaps more surprisingly, investment can also be delayed compared to section 4.3. Since outsiders are assumed able to correctly deduce the type of follower through observing the leader, the need for external finance creates an incentive for the good firm to wait and invest as a follower in order to avoid costs of dilution. Thus, for relatively high initial levels of X, where the good firm would invest immediately in section 4.3, it will under asymmetric information rationally wait and invest after the bad firm. Asymmetric information will in this way again impose a cost of distortion on the good firm, although in the opposite direction. This also means that for sufficiently high initial levels of X there will be no cost or gain from dilution, and the bad firm will again be indifferent (at worst) about the presence of asymmetric information.

Finally, for intermediate initial levels of X both types of firm may generally invest immediately as a leader with positive probability. Outsiders will consequently be unable to deduce the type of leader at the time of investment. Compared to section 4.3, asymmetric information thus impose a cost of dilution to good firms in this region, while bad firms contrarily do better than in the asymmetric duopoly without the need for external finance.

Concluding, under asymmetric information the need for external finance will in this model always deteriorate the value of the good firm's investment opportunity, either through dilution or further distortion of the first-best timing. The bad firm will on the other hand be no worse off, profiting from the information asymmetry for intermediate initial levels of X. The trigger level of a good firm investing as a leader will positively be lower than in section 4.3, such that investment will be speeded up for low initial levels of X. However, for high initial levels the reactive strategy of the good firm may lead to delayed investment.

5.2 Duopoly of unknown composition

The setup in the previous section rests on the premise that both firms know the type of the other firm, while equity markets are uninformed. This is potentially hard to motivate in a real world scenario. While the model still serve to shed light on the combined effect of duopolistic competition and adverse selection, it would be preferable to have a setup more aligned with real world circumstances. This section introduces one such alternative duopolistic setup where firm type is private information, and discuss how the need for external finance would alter the results.

Setup

Firms have either good or bad growth prospects as in chapter 3. They are assumed to know their own type, but only the probability of the other firm type being good. Denoted the probability of a good competitor p_g, with the probability of a bad competitor given by $p_b = 1 - p_g$. Probabilities are independent of a firm's own type, meaning that the likelihood of facing a good competitor is the same whether a firm is good or bad itself. No firm have any internal funds, and have to issue equity in order to finance their investment if they choose to exercise. Suppliers of external finance and the competing firm form rational beliefs based on the distribution of growth prospects and observed behaviour. As firm type is private information, firms do not know whether they operate in a symmetric duopoly as that in section 4.2, or an asymmetric duopoly as that in section

4.3.

No external finance

Consider first equilibrium outcomes when firms do not have to rely on external finance in order to invest. In order to find investment thresholds we once again start backwards and find the optimal follower response of a good and bad firm respectively. Conveniently, these are same as in chapter 4. When acting as a follower firms are indifferent about the type of competitor.

Now assume one firm is of good type and contemplate exercising as a leader at some level X. If the other firm is good (bad), we get follower trigger from section 4.2 (4.3). The expected value of a leader with good growth prospects is thus given by

$$V_{L,G}(X) = \begin{cases} p_g \left[\left(\frac{D(1,0)X\gamma_g}{r-\alpha} \right) + \left(\frac{[D(1,1)-D(1,0)]X_{F,G}\gamma_g}{r-\alpha} \right) \left(\frac{X}{X_{F,G}} \right)^{\beta} \right] & \\ +(1-p_g) \left[\left(\frac{D(1,0)X\gamma_g}{r-\alpha} \right) + \left(\frac{[D(1,1)-D(1,0)]X_{F,B}\gamma_g}{r-\alpha} \right) \left(\frac{X}{X_{F,B}} \right)^{\beta} \right] - I - F, & \text{for } X < X_{F,G} \\ p_g \left[\frac{D(1,1)X\gamma_g}{r-\alpha} \right] & \\ +(1-p_g) \left[\left(\frac{D(1,0)X\gamma_g}{r-\alpha} \right) + \left(\frac{[D(1,1)-D(1,0)]X_{F,B}\gamma_g}{r-\alpha} \right) \left(\frac{X}{X_{F,B}} \right)^{\beta} \right] - I - F & \text{for } X_{F,G} \leq X < X_{F,B} \\ \frac{D(1,1)X\gamma_g}{r-\alpha} - I - F, & \text{for } X \geq X_{F,B} \end{cases} \tag{5.13}$$

Similarly, for a bad firm contemplating to invest as a leader we have:

$$V_{L,B}(X) = \begin{cases} p_g \left[\left(\frac{D(1,0)X\gamma_b}{r-\alpha} \right) + \left(\frac{[D(1,1)-D(1,0)]X_{F,G}\gamma_b}{r-\alpha} \right) \left(\frac{X}{X_{F,G}} \right)^{\beta} \right] & \\ +(1-p_g) \left[\left(\frac{D(1,0)X\gamma_b}{r-\alpha} \right) + \left(\frac{[D(1,1)-D(1,0)]X_{F,B}\gamma_b}{r-\alpha} \right) \left(\frac{X}{X_{F,B}} \right)^{\beta} \right] - I - F, & \text{for } X < X_{F,G} \\ p_g \left[\frac{D(1,1)X\gamma_b}{r-\alpha} \right] & \\ +(1-p_g) \left[\left(\frac{D(1,0)X\gamma_b}{r-\alpha} \right) + \left(\frac{[D(1,1)-D(1,0)]X_{F,B}\gamma_b}{r-\alpha} \right) \left(\frac{X}{X_{F,B}} \right)^{\beta} \right] - I - F & \text{for } X_{F,G} \leq X < X_{F,B} \\ \frac{D(1,1)X\gamma_b}{r-\alpha} - I - F, & \text{for } X \geq X_{F,B} \end{cases} \tag{5.14}$$

First define $X^p_{L,G}$, (where superscript p denote the case of firm quality being private information), as the trigger value where a good firm in equilibrium will invest when X hits the trigger from below. By deviating from this strategy and instead investing at $X^p_{L,G} - \epsilon$, a good firm would experience two opposing effects on profitability. First, it would seize the leader role with certainty in case of facing another good firm. For any $X^p_{L,G} > X^g_L$, this would lead to an increase in the option value if the competitor is a good firm. Specifically, since no bad firm would compete for the leader role at $X^p_{L,G}$, intensity functions $h_g(T^p_{L,G})$ will be as in the symmetric case of 4.2. This means that the gain from preempting another good firm is given by the first mover advantage in a symmetric duopoly with good firms: $V^g_L(X) - V^g_F(X)$. Second, by investing at $X^p_{L,G} - \epsilon$ the good firm would be exercising farther from the optimal non-strategic threshold for a leader in case of facing a bad firm. This reduces the option value in case the competitor is a bad firm.

$X^p_{L,G}$ is characterized by the probability weighted gain of the first effect exactly being offset by the probability weighted loss of the second effect, such that no good firm has any incentive to deviate. It is however easy to see that the first effect will dominate the second as long as there is a first-mover advantage in the case of two good firms. For an infinitely small ϵ, the loss from distortion will be infinitely small as well, while there will be a jump in the value when facing a good firm. Thus, we must have $X^p_{L,G} = X^g_L$. Intuitively, the potential to encounter another good firm is sufficient to ensure full preemption.

Next, consider the threshold where a bad firm is indifferent about being a leader and a follower. Note that this threshold will depend on the initial value of X, as for some initial values a bad firm will be able to deduce the type of its competitor by the time X reaches the threshold. Specifically, if $X < X^g_L$, no investment by the competitor when X hits X^g_L will signal that the other firm is bad as well. In this case, the threshold where a bad firm is indifferent about being a leader and a follower is found the same way as in section 4.2. Denote this X^b_L. On the other hand, if we have an initial level $X > X^g_L$, a bad firm must decide whether to exercise immediately without knowing the type of competitor. Define $X^p_{L,B} > X^b_L$ as the threshold under these circumstances. That is, at $X^p_{L,B}$ a bad firm is indifferent about investing as leader (facing follower investment at $X_{F,G}$ with probability p_g and at $X_{F,B}$ with probability 1-p_g), and waiting to invest as a follower at $X_{F,B}$.

We are now ready to analyse potential equilibrium outcomes and strategies for various initial levels of X.

$X < X_L^g$

Here, no firm will exercise prior to X_L^g. When X reaches this threshold, we have three potential outcomes: If both firms are bad, neither will invest. Both firms will consequently deduce the type of the other firm, and a symmetric equilibrium ensues where firms invest at X_L^b and $X_{F,B}$ with probability one half each, with zero probability of a coordination error. If exactly one firm has good growth prospects, this firm will invest immediately, while the bad firm again waits and invest as a follower at $X_{F,B}$. If both firms are good they will both attempt to invest with intensity as in section 4.2. When one firm successfully invests as a leader, the other will wait and invest at $X_{F,G}$. Note that a good firm knows that only another good firm would like to compete for the leader role at X_L^g. Hence, the intensity functions will be given as in eq. 4.5, and the probability of a coordination error will be zero.

$X_{L,G}^p \leq X < X_L^b$

If both firms are bad we get the same outcome as above. Neither will invest immediately, and both will consequently deduce the type of the other firm. Hence, the same symmetric equilibrium as outlined above ensues, with zero probability for a coordination error. If exactly one firm is good, that firm will invest immediately. The bad firm again waits and invest at $X_{F,B}$. If both firms are good, they will compete to invest immediately. Since only a good firm would consider to invest in this range, intensity functions will still be given as in section 4.2. However, in this region the leader role is relatively more valuable. Firms will therefore invest with a higher intensity, leading to a positive probability for coordination error similar to the intermediate case of section 4.2.If $X < X_{F,G}$ the good firm that does not attain the leader role will wait and invest at $X_{F,G}$, while if $X > X_{F,G}$ both good firms will invest immediately and we get joint investment.

$X_L^b \leq X < X_{L,B}^p$

If both firms are bad in this region we have the following situation: If each firm knew that the other was bad, they would invest immediately with a positive intensity as there is a first mover advantage. However, they do not know this until they observe that there is no immediate investment, signalling the absence of a good firm. Not getting caught up in technicalities, we can model this as a symmetric equilibrium arising at $X + \epsilon$ for some infinitely small ϵ. Both firms contest for the leader role after learning that the competitor is also bad, and we get a positive probability for coordination error similar to the intermediate case of the symmetric duopoly in section 4.2. If exactly one firm is good, that firm will invest immediately. The bad firm again waits and invest at $X_{F,B}$. If both firms are good we have the same situation as in the previous subcase.

$\boldsymbol{X^p_{L,B} \leq X < X_{F,B}}$

For this range of X-values a bad firm will per definition contest for the leader role immediately without knowing the type of competitor. No matter firm composition, both firms will thus invest with positive intensity. Unlike the previous subcases, firms will not know what type of competitor they are up against when deciding on investment intensity. This uncertainty affect how firms maximize payoff, as discussed in Appendix A.2. Denote intensity functions under uncertainty $h^u_k(t)$.

If there are two bad firms, both invest immediately with intensity $h^u_b(t)$. The probabilities of potential equilibrium outcomes are given as in eq. 4.6 and 4.7. If there is exactly one good firm, we have immediate investment with intensity $h^u_b(t)$ and $h^u_g(t)$ respectively, where $h^u_g(t) = 1$ if $X > X_{F,G}$. Probabilities of potential equilibrium outcomes are given as in eq. 4.15 - 4.17. If there are two good firms, we have the same symmetric case as with two bad firms, only with different intensities.

$\boldsymbol{X > X_{F,B}}$

Here both types will invest immediately with intensity one, since $X_{F,G} < X_{F,B}$. We thus have joint investment no matter the composition of firm quality. Equilibrium strategies summarized from the point of view of a good and bad firm are summarized in table 6 and table 7 respectively:

Table 6: Equilibrium strategies for firm with good growth prospects in duopoly when the type of competitor is unknown

X-Range	Equilibrium strategy, good growth prospects
$X < X_L^g$	Wait until X hits X_L^g and invest with intensity $h_g(T_L^g) = 0$. If the other firm is good and manages to invest as a leader, wait and invest as a follower at $X_{F,G}$
$X_L^g \leq X < X_L^b$	Invest immediately with intensity $h_g(t) > 0$. If $X < X_{F,G}$ and the other (good) firm manages to invest as a leader, wait and invest as a follower at $X_{F,G}$
$X_L^b \leq X < X_{L,B}^p$	Invest immediately with intensity $h_g(t) > 0$. If $X < X_{F,G}$ and the other (good) firm manages to invest as a leader, wait and invest as a follower at $X_{F,G}$
$X_{L,B}^p \leq X < X_{F,B}$	Invest immediately with intensity $h_g^u(t) > 0$. If $X < X_{F,G}$ and the other firm manages to invest as a leader, wait and invest as a follower at $X_{F,G}$
$X \geq X_{F,B}$	Invest immediately with intensity $h_g(t) = 1$

Table 7: Equilibrium strategies for firm with bad growth prospects in duopoly when the type of competitor is unknown

X-Range	Equilibrium strategy, bad growth prospects
$X < X_L^g$	See if there is investment when X hits $X_{L,G}^p$. If true, wait and invest as follower at $X_{F,B}$. If false, conclude that competitor is bad and invest with intensity $h_b(T_L^b) = 0$ at X_L^b. If the other firm becomes leader, invest at $X_{F,B}$ as a follower.
$X_L^g \leq X < X_L^b$	See if there is immediate investment. If true, wait and invest as follower at $X_{F,B}$. If false, conclude that competitor is bad and invest the same way as above.
$X_L^b \leq X < X_{L,B}^p$	Wait for an infinitely small time ϵ to see if there is immediate investment. If true, wait and invest as follower at $X_{F,B}$. If false, conclude that the competitor is bad and invest immediately with intensity $h_b(t) > 0$.
$X_{L,B}^p \leq X < X_{F,B}$	Invest immediately with intensity $h_b^u(t) > 0$.
$X \geq X_{F,B}$	Invest immediately with intensity $h_b(t) = 1$

External finance

Now consider what happens if firms in this setup are required to obtain external finance from uninformed outsiders. Due to the vast amount of subcases and complex nature of beliefs in this model, the analysis is limited to certain qualitative observations and comparisons to previous sections.

To begin with, observe that contrarily to section 5.1, outsiders will no longer be able to deduce the type of follower just from observing the leader. Conditional on being a follower, this creates an incentive for a bad type of firm to speed up exercise in order to mimic a good type of follower. To separate from bad followers, a good firm investing as a follower may thus have to speed up investment. As a matter of fact, investment behaviour analogous to that in chapter 3 will arise after one firm has invested as a leader. To see this, first note that there will no longer be any excessive leader profit to be obtained. Hence, any follower will only be concerned with signalling incentives as in chapter 3. Furthermore, while there will be no competition per se once the leader has invested (as there will only be one firm left to invest), beliefs of outsiders at the time a follower invests will depend on the equilibrium strategies for followers. Good and bad firms can therefore be considered to compete and committing ex-ante when devising follower strategies.

We know that the fear of preemption will cause a good type of firm to invest as a leader at the low threshold $X_L^g < X_{P,G}$. A separating trigger for a good follower thus exists for low initial levels of X. For intermediate initial levels of $X > X_{P,G}$, where a leader will invest immediately, there is however no separating threshold for a good follower. In these cases there will instead be a pooling equilibrium of follower exercise, where both types of firms invest at the same threshold. Note that while Morellec and Schürhoff (2011) find that a pooling equilibrium might ensue even if a separating trigger exist, this is hard to motivate in the current setup. Specifically, it would require strong restrictions on outsider beliefs to prevent an incentive for good firms to deviate. Consequently, assume that a separating equilibrium of follower strategies ensues if there exists a separating trigger.

The incentive for a good type of follower to speed up exercise in turn affects the threshold where a good firm will invest as a leader. Without the need for external finance, this threshold X_L^g is characterized by a good firm being indifferent about adopting a leader or follower position conditional on the other firm being good as well. When a good type of follower is induced to speed up exercise in order to separate from a bad follower, the value of a good follower is eroded. At the same time, this erodes the leader value as well, since the leader now can expect to earn the superior leader profit for a shorter period of time before a good follower invests.

If the decrease in follower value is greater than the decrease in leader value, there would be a first-mover advantage at X_F^g. A good firm investing as a leader would therefore have an incentive to speed up exercise in order to preempt a potential good competitor. The threshold where a good firm invests as a leader would consequently drop below X_F^g. On the contrary, if the decrease in leader value is greater than the decrease in follower value, no firm would consider investing at X_F^g, and the threshold where a good firm invests as a leader would be above X_F^g. To see which effect dominates, consider the leader and follower value of a symmetric duopoly. From eq. 4.2

and 4.4, we have for $X < X_F$

$$V_F(X) = \left(\frac{D(1,1)X_F}{r-\alpha} - I - F\right)\left(\frac{X}{X_F}\right)^{\beta} \tag{5.15}$$

$$V_L(X) = \left(\frac{D(1,0)X}{r-\alpha}\right) + \left(\frac{[D(1,1) - D(1,0)]X_F}{r-\alpha}\right)\left(\frac{X}{X_F}\right)^{\beta} - I - F \tag{5.16}$$

The marginal impact of a good follower speeding up investment is given by

$$-\frac{\partial V_F(X)}{\partial X_F} = -\left(\frac{X}{X_F}\right)^{\beta}\left(\frac{\beta(I+F)}{X_F} - \frac{D(1,1)(\beta-1)}{r-\alpha}\right) \tag{5.17}$$

$$-\frac{\partial V_L(X)}{\partial X_F} = -\left(\frac{X}{X_F}\right)^{\beta}\left(\frac{[D(1,0) - D(1,1)](\beta-1)}{r-\alpha}\right) \tag{5.18}$$

If $D(1,0)(\beta-1)/(r-\alpha) > \beta(I+F)/X_F$, we see that the leader value on the margin will deteriorate more than the follower value. When the leader profit is sufficiently high, we thus see that under asymmetric information, the need for external finance may in fact cause good firms to invest at a higher threshold than before. This counter-intuitive result iis striking, and in stark contrast to the result in section 5.1, where the need for external finance always cause the trigger level of a good type of leader to drop. Specifically, it shows how outsiders' ability to deduce the type of follower effects not only the timing of followers, but also the threshold for leader investment.

When the leader profit is less substantial, we however get the same effect as in the previous section, where the need external finance under asymmetric information cause a good firm to speed up investment. Also, notice that while the effect on the timing of a good firm is ambiguous, the presence of asymmetric information unambiguously reduces the value of the investment opportunity for the good firm.

Next, consider how the need for external finance under asymmetric information affect investment behaviour for high initial values of X. In section 5.1 there were situations where a good firm would adopt a reactive strategy to avoid selling underpriced shares, thus investing at a later point in time than under perfect information. For this to be the case in the current model, a reactive strategy would have to credibly signal the quality of a follower. This will not the case: If good firms adopt a reactive strategy for all X above some arbitrary threshold, a bad follower in the case of two bad firms will have an incentive to mimic the reactive exercise scheme employed by good firms. Outsiders would hence not be able to deduce the type of follower, and good firms would in turn prefer not to exercise reactively.

For intermediate initial levels of X, bad firms might as already mentioned successfully mimic the follower timing of good firms. This will in turn affect the intensity of investment, as investing as a follower will be relatively more (less) desirable to a bad (good) firm. While the exact dynamics will depend on the specific nature of each subcase, good firms will generally experience a greater deterioration of the real option value compared to similar intermediate levels in the model of section 5.1.

To summarize, the impact of external finance under asymmetric information differs somewhat between the setup discussed here and in the previous section. In section 5.1, the need for external finance forces a good firm to lower its investment trigger in order to still preempt the bad firm. In this section however, the impact of external financing on the leader-trigger of a good firm will be unclear. Furthermore, uncertainty regarding the type of competitor enables bad firms to mimic even when investing as a follower. This prevents reactive exercise by a good firm, and increases the transfer of value from good firms to bad firms for intermediate levels of X. Common for both models is however the fact that when firms are required to seek external finance from uninformed investors, the option value of a good firm will ex-ante is further eroded compared to section 4.3

Chapter 6

Conclusion

The aim of this thesis was to facilitate analysis of real options exercise in situations where both imperfect competition and asymmetric information are important factors. Of particular interest was how the combined impact on optimal exercise timing related to the impact of each concept in isolation. One existing model on each concept were first reviewed, and these models formed the basis for subsequent model development. Specifically, a setup of Morellec and Schürhoff (2011) where firms of different quality seek external finance from uninformed capital markets was combined with an extended version of the duopolistic competition model in Dixit and Pindyck (1994). To further enhance realism, an alternative model was developed as well, where firm quality is presumed unknown to both equity markets and competitors.

Both models predict that a good type of firm will invest at a lower threshold compared to the standard real options model of McDonald and Siegel (1986). This is in line with existing models on each separate concept: Smets (1991), Huisman and Kort (1999) and Lambrecht and Perraudin (2003) all analyse real options exercise in a duopoly with a (potential) first-mover advantage and find that the incentive to preempt induce firms to invest at a lower threshold, while Morellec and Schürhoff (2011) show how adverse selection may induce good firms to speed up investment in order to credibly signal their quality to outsiders.

More interesting are the predictions regarding exercise timing relative to a duopoly where firms do not have to rely on external finance from uninformed equity markets. When firms know their type of competitor, the analysis shows that the need for external finance unambiguously lowers the threshold where a good type of firm first will invest in equilibrium. Furthermore, this model shows that a good type of firm will choose to adopt a reactive strategy for high initial levels of X, resulting in delayed investment compared to the case duopolistic competition with internal finance. When firms do not know the type of their competitor, analysis on the other hand shows that the threshold where a good type of firm invests in equilibrium can be distorted in both directions. Under these circumstances, there are conversely no room for a reactive strategy in equilibrium. Finally, both models predict that the option value of old shareholders in a good firm will be further eroded than when firms have sufficient internal funds.

Concluding, the thesis shows that the impact of external financing under asymmetric information in a duopolistic model of real options exercise is non-trivial. The trigger level of a good firm may be distorted in both directions, depending on parameter values and whether equity markets are able to deduce the type of the second firm to invest. Moreover, investment may be both speeded up or delayed for high initial levels of X, again depending on whether outsiders are able to deduce the type of follower. The value of the investment opportunity to old shareholders of a good firm is however further eroded for all initial levels of X. Contrarily, old shareholders of the bad firm are generally indifferent for low and high initial levels, while they gain from selling overpriced shares for intermediate levels.

As suggested initially, the results are relevant to industries where both asymmetric information and oligopolistic competition affect investment decisions. Investment of small, high-tech firms typically fulfil this criterion, and is hence of particular relevance. The lack of operating income often force this type of firms to seek external finance when investing. Furthermore, investment based on R&D efforts and innovations is inherently hard for suppliers of external finance to value, especially since firms often will prefer secrecy. This gives rise to informational asymmetries. Finally, competition is often seen between a small number of firms, with the potential for substantial first-mover advantages.

An interesting property of these types of R&D-intensive firms is that asymmetric information in many instances emerge from firms' unwillingness to transmit information to outsiders even when transmission of information is possible. In this respect, the results of this thesis highlight the implicit cost of secrecy to good types of firms with a flexible investment opportunity. By maintaining the information asymmetry, a good type of firm will effectively contribute to erosion of its own option value. Strategic considerations may of course still make secrecy the preferred choice of action, but it is anyway useful to be aware of the opportunity cost from a real options perspective.

The modelling approach has several limitations. First, there are no quantitative predictions. This is a consequence of the complexity introduced by the mutual dependence of outsider beliefs, intensity functions and option values, preventing analytical solutions. A natural extension would therefore be to estimate the models using numerical methods for various parameter values. Second, there is a lack of tractability for intermediate initial levels of X. Here, equilibrium outcomes depend on the intensity functions of investment, which have little informative value due to the mutual dependence on outsider beliefs. Moreover, exercise dynamics vary depending on the initial sub-region of X, and since the specification of sub-regions differ between models it is hard to facilitate any proper comparison. Finally, the scope of the thesis is limited to equity financing only. Introducing the possibility of debt financing as well would provide for a more complete analysis. This could serve as a good extension as well, but would most likely require a refinement of the modelling approach in order to limit the amount of subcases and promote tractability.

Appendix A

Appendix

A.1 Intensity functions with known type of competitor

To represent the limit of a mixed-strategy equilibrium in discrete time, firms are assumed to play a game in discrete time that takes zero time. If firm a plays invest while firm b does not, firm a will get the leader value while firm b gets the follower value, and vice versa. If both firms play invest, both get the inferior value of joint investment. If no firm play invest, the game is immediately repeated, potentially infinitely many times. The intensity functions correspond to the probability of investment that maximizes firm value in this game.

Expected payoff for firm a is given by

$$V_a = \frac{h_a h_b V_{J,a} + h_a(1-h_b)V_{L,a} + (1-h_a)h_b V_{F,a}}{h_a + h_b - h_a h_b} \tag{A.1}$$

The FOC is given by

$$\begin{aligned}
\frac{\partial V_a}{\partial h_a} &= \frac{[h_a(1-h_b)+h_b][V_{L,a}+h_b(V_{J,a}-V_{L,a}-V_{F,a})] - [h_a(1-h_b)][V_{L,a}+h_b(V_{J,a}-V_{L,a}-V_{F,a})]}{(h_a+h_b-h_a h_b)^2} \\
&\quad + \frac{V_F h_b(1-h_b)}{(h_a+h_b-h_a h_b)^2} \overset{!}{=} 0 \\
\iff \ & h_b[V_{L,a}+h_b(V_{J,a}-V_{L,a}-V_{F,a})] = V_{F,a} h_b(h_b-1) \\
\iff \ & h_b = \frac{V_{L,a}-V_{F,a}}{V_{L,a}-V_{J,a}} \qquad \text{(A.2)}
\end{aligned}$$

Following the same approach for firm b, we get

$$h_a = \frac{V_{L,b} - V_{F,b}}{V_{L,b} - V_{J,a}} \tag{A.3}$$

Note that the intensity function of each firm depends on the relative attractiveness to compete for a leader role for the other firm. This is a basic property of a mixed strategy equilibrium; each firm must select a probability of investment such that the other firm is indifferent about playing either pure strategy.

A.2 Intensity functions with unknown type of competitor

In this case the payoff function depend on the probability of facing each type of firm. E.g., if a good firm plays invest in the discrete zero-time game outlined in A.1, expected payoff is given by $p_g[(1 - h_g^u)V_L^g + h_g^u V_{J,G}] + p_b[(1 - h_b^u)V_{L,G} + h_b^u V_{J,G}]$. One could imagine that if neither firm played invest, each firm would update their beliefs regarding the type of the other firm using Bayes' theorem before the next iteration of the game. However, since the game takes zero time, this seems unrealistic. We thus assume each firm keep the initial beliefs regarding the type of the other firm until one firm has invested.

The payoff to be maximized by a good firm is thus given by:

$$\begin{aligned} V_g &= \frac{h_g^u\Big(p_g[(1 - h_g^u)V_L^g + h_g^u V_{J,G}] + p_b[(1 - h_b^u)V_{L,G} + h_b^u V_{J,G}]\Big)}{h_g^u + (1 - h_g^u)(p_g h_g^u + p_b h_b^u)} \\ &+ \frac{(1 - h_g^u)\Big(p_g h_g^u V_{F,G} + p_b h_b^u V_{F,G}\Big)}{h_g^u + (1 - h_g^u)(p_g h_g^u + p_b h_b^u)} \end{aligned} \tag{A.4}$$

with a similar expression for a bad firm. Expressions for $h_k^u(t)$ is found through the first order conditions as in A.1. However, the expressions get very long and have little informative value, and are therefore omitted.

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