

FOR 20 2016

ISSN: 1500-4066

November 2016

Discussion paper

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BY

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A bridge between continuous and discrete-time bioeconomic models: Seasonality in fisheries*

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*This research has been supported by a grant from Iceland, Liechtenstein and Norway through the EEA Financial Mechanism. Operated by Universidad Complutense de Madrid. Ref: 021-ABEL-CM-2013, and by the Research Council of Norway, project no. 257630.

*Part of the computations of this research was performed in EOLO, the HPC of Climate Change of the International Campus of Excellence of Moncloa (Universidad Complutense de Madrid), funded by MECD and MICINN.

Abstract

We develop a discretization method for continuous-time bioeconomic models. Based on this method, we develop a discrete-time bioeconomic model to analyze seasonality in fisheries. The discretization method consists of three steps: first, we estimate a proper growth function for the continuous-time model with the ensemble Kalman Filter. Second, we use the Runge-Kutta method to discretize the growth function. Third, we use the Bellman approach to analyze optimal management of seasonal fisheries in a discrete-time setting. We analyze both the case of quarterly harvest and the case of monthly harvest, and we compare these cases with the case of annual harvest. We find that seasonal harvesting is a win-win optimal solution with higher harvest, higher optimal steady state equilibrium, and higher economic value than obtained in the case of annual harvesting. We also demonstrate that the discretization method overcomes the errors and preserves the strengths of both continuous and discrete-time bioeconomic models.

Keywords: OR in natural resources; bioeconomic modeling; seasonal fisheries; discrete-time optimization.

1. Introduction

There is a fundamental choice to be made when developing a bioeconomic model for fisheries management: discrete or continuous-time modeling.

Continuous-time bioeconomic models are based upon the assumption that both biological processes, such as growth, and human activity, such as harvesting, are taking place continuously, while in discrete-time bioeconomic models, they are taking place at discrete-time steps (usually annual).

Continuous-time models have proved to be useful for analytical purposes (Clark 2010). In addition, continuous-time models may serve as a conceptual framework and guideline for discrete-time models, despite the difficulty they present in terms of estimation. However, continuous-time models are unable to encompass sequential effects, commonplace in real world fisheries, due to the fact that the response of fish stocks to external factors, such as harvesting, is instantaneous. Moreover, both biological processes, like spawning, and human activity, such as harvesting, are seasonal rather than continuous over time (Clark 2010; Bjørndal and Munro 2012). In addition, data are usually available on an annual basis. For these reasons, discrete-time models, which are generally an approximation of continuous-time models, are often used to analyze optimal management of commercial fisheries. As pointed out by Bjørndal and Munro (2012), it is often not only necessary, but also appropriate to turn to discrete-time models, especially when using empirical models. In addition, most modeling of economic time series works with discrete time, yet time is in fact continuous (Sims 2008).

Errors in mathematical modeling of the natural growth function of fish stocks are frequently found in the literature on bioeconomic modeling of fisheries in a continuous-time setting. In particular, most natural growth functions used in continuous-time models, which are inserted into differential equations, are, however, often estimated in discrete time, which uses difference equations (e.g. Agnarsson et al. 2008, for the cod fisheries), despite the well-known fact that the dynamical properties of discrete and continuous-time population dynamics are entirely different (Gyllenberg et al. 1997). Approximation errors involved in developing discrete-time models are commonly assumed by most economists to be trivially small, but the approximation error can be important in some conditions (Sims 2008).

From an economic point of view, discrete-time bioeconomic models neglect the change in stock during harvesting when considering that harvest costs depend on the stock at the start of the year, and consequently discrete-time models may incur systematic errors by underestimating or overestimating the harvest costs. A continuous cost function is the appropriate cost function when the harvesting process is dependent on stock density, and the fraction of the stock harvested within a season is significant (Maroto et al. 2012).

On the other hand, most commercial fisheries are managed on an annual basis where annual biological and fisheries data are used by discrete-time models to find that the solution converges to an annual optimal steady-state equilibrium where both biological processes, like the growth of the species, and fishing activity take place only once a year.

There are, however, various reasons why seasonality may be relevant in fisheries. One such reason, for example, may be the seasonal pattern of migration of most exploited fish stocks which leads to seasonal harvest (e.g. Pelletier and Mahévas 2005, and references therein; Hannesson et al. 2010, and Hermansen and Dreyer 2010, for the case of North-East arctic cod; Hannesson 2013, for the case of mackerel; Smith et al. 2016, for the case of cod, haddock, and saithe).

Optimal management of seasonal fisheries has become a hot topic in fisheries economics, particularly considering that both discrete and continuous-time models are not able to cope with the complex phenomenon of seasonality in fisheries. When considering one-year time increments, discrete-time models neglect seasonality. Continuous-time models also neglect it when considering time-independent optimal feedback policies.

The aim of this paper is to develop a discretization method for continuous-time bioeconomic models, based on the fourth order Runge-Kutta method, which allows us to build a bridge between continuous and discrete-time bioeconomic models by overcoming the errors and by preserving the strengths of both approaches. Specifically, in the discretization method developed in this paper, a continuous-time model is thought to be the limiting case of a discrete-time model in which the interval between times Δt in the discrete time frame $t + \Delta t$ becomes vanishingly small ($\Delta t \rightarrow 0$). This allows for time increments of less than one year ($\Delta t \leq 1$) and, consequently, it allows us to analyze seasonal fisheries. This is carried out using the Runge-Kutta method which is a well known robust numerical method used in temporal discretization to approximate solutions for differential equations (e.g. Press et al. 2007).

Using the North-East Arctic cod stock by way of illustration, the main contributions from this article are, first, in contrast to the natural growth function used in continuous-time models, which is often erroneously estimated in discrete-time, a proper continuous-time natural growth function, which is a differential equation, is estimated using data assimilation methods. In particular, we use the ensemble Kalman filter (EnK) to show that the proper natural growth function estimated in continuous-time is quite different to what is estimated in discrete-time. In brief, the EnK is a data assimilation method widely used in fields that study dynamic and chaotic systems. Examples include meteorology, oceanography, and fluid and reservoir dynamics; see Evensen (2003) for an account of the formal framework and a review of various applications. Recently, EnK has also successfully been applied to multidimensional fisheries models (Kvamsdal and Sandal 2015, Ekerhovd and Kvamsdal 2016). EnK has structural relationships with the classical Kalman filter for linear models, but extends to a large class of nonlinear models. Theoretically, the EnK is presented as an application of Bayes Law (Evensen 2003), where the propagation of the state vector probability density over time is a major challenge. EnK relies on a Markov Chain Monte Carlo method for this, which essentially solves the Fokker-Planck equation. Furthermore, the method facilitates parameter estimation by including them in the state space.

Second, we show that seasonal harvesting is a win-win optimal solution by achieving higher annual harvest, higher steady state equilibrium, and higher economic value than any obtained in the case of annual harvesting. These results can be explained by the combined effects of the actualization of the growth rates of the stock, decrease of harvesting costs, and more frequent discounting which all take place if the stock is harvested seasonally (quarterly or monthly).

Third, we show that the discrete-time solution converges fast to that obtained in continuous-time. In particular, convergence is achieved if the stock is harvested quarterly ($\Delta t = 0.25$), which seems to confirm the robustness of the discretization method.

2. Background

Most of the literature on fisheries economics has approached the problem of seasonality in fisheries by developing intra-seasonal bioeconomic models in order to analyze within-season harvesting behavior (effort decisions) in regulated open access fisheries (See Smith 2012, and references therein). In these models, fishing is assumed to take place over a single season, with a frequently fixed length, during which natural population processes are frequently ignored.

In Holland (2011), an intra-annual numerical bioeconomic model is simulated to optimize effort and catch over a one-year period in order to analyze congestion and in-season depletion externalities in the Maine lobster fishery under seasonal variability in catchability. This model does not take into account discounted economic profits and effects on stock size in future years, since optimization is over a year. While considerable progress has been made in understanding the role of within-season externalities in regulated open access fisheries, less attention has been given to seasonal fisheries which consist of several seasons during the year, and which are optimally managed by a sole owner who would regulate seasonal harvest optimally over time by maximizing the discounted economic profits from the fishery. The reason for this deficiency may be that the seasonal dynamic optimization problem unquestionably becomes much harder to solve, especially if non-linearities in the objective functional are taken into account.

In Flaaten (1983), a linear continuous-time bioeconomic model is developed to analyze the optimal harvest with seasonal growth by introducing a time-dependent intrinsic growth rate in the growth function of the resource. Such cyclical variations in growth are frequently observed in zooplankton and krill. Assuming constant prices, and costs linear in the harvest rate, the solution to the problem will be to use a combination of 'bang-bang' and 'singular' control (Clark and Munro 1975). In this setting, the optimal harvest is derived to be seasonal with a shorter fishing season than would occur in an open access fishery.

Linear bioeconomic models are based on highly simplified economic and biological conditions. In real life fisheries, prices may depend on the supply level (downward sloping demand), and harvesting costs may be non-linear in harvest (increasing marginal costs). Under non-linearity in harvest, and in the absence of seasonality, a feedback rather than an open-loop policy is generally adopted in continuous-time models. A feedback model refers to a model where the optimal control (harvest) is a direct

function of the state variable (stock) and it is not found by forecasting (time-independent harvest). In this setting, an optimal feedback policy, as opposed to the bang-bang solution, is characterized by an asymptotic approach to steady state (Sandal and Steinshamn 2001).

The discretization method developed herein is substantially different from those developed in the literature. In particular, the discrete-time model obtained by the discretization method is a more general model for optimal management of seasonal fisheries by including several seasons during the year, general specifications of demand and cost relationships (non-linearities), and population dynamics properly estimated in a continuous-time setting. The discretization method allows us to obtain the socially optimal solution in seasonal fisheries. In addition, constructing a bridge between continuous and discrete-time models, the discretization method allows us to both overcome the errors and preserve the strengths of both approaches. Thus, the discretization method provides a theoretically-grounded, practical starting point for seasonal fisheries management.

3. Discretization method of continuous-time bioeconomic models

In this section we describe the discretization method which takes a standard continuous-time model as a starting point, given by:

$$\begin{aligned} \max_h \int_0^{\infty} e^{-\beta t} \Pi(h, x) dt \\ \text{s.t. } \dot{x}(t) = F_C(x) - h \\ x(0) = x_0, \end{aligned} \quad (1)$$

where x represents the fish stock biomass, h the harvest rate, Π net revenues, β the discount rate, and $\dot{x}(t) = dx/dt$ the population dynamics where $F_C(x)$ represents the natural growth function.

The discretization method consists of several stages:

i) In contrast to the standard continuous-time model, the natural growth function

$$\dot{x}(t) = F_C(x), \quad (2)$$

is properly estimated in a continuous-time setting by using the ensemble Kalman filter.

ii) The natural growth function estimated in *i)* is discretized by using the fourth order Runge-Kutta method. A brief description of the Runge-Kutta algorithm is given in the Appendix.

Given a temporal interval $[t, t + \Delta t]$, the Runge-Kutta method (see Appendix) allows us to obtain the stock value at period $t + \Delta t$, $x_{t+\Delta t}$, as a function of the stock value at period t

$$x_{t+\Delta t} = f_{RK}^{(\Delta t)}(x_t), \quad (3)$$

where $f_{RK}^{(\Delta t)}(x_t)$ is the proper discrete-time approximation, for incremental time Δt , derived from the continuous-time growth function $F_C(x)$, as defined in (2). In this way, the discrete-time approximation of the continuous-time population dynamics, as defined in (1), is given by:

$$x_{t+\Delta t} = f_{RK}^{(\Delta t)}(x_t) - h_t, \quad (4)$$

where h_t represents harvest quantity at the end of period t .

It should be noted that equation (4) can be rewritten as $h_t = f_{RK}^{(\Delta t)}(x_t) - x_{t+\Delta t}$. This implies a constraint on the escapement (stock after harvesting) which we call y . This constraint in the optimization problem described below is given by:

$$y \leq f_{RK}^{(\Delta t)}(x_t), \quad (5)$$

which implies $h = f_{RK}^{(\Delta t)}(x_t) - y \geq 0$.

iii) The net revenue function from the fishery $\Pi(h, x)$, as defined in (1), is discretized by considering the temporal interval $[t, t + \Delta t]$, as described in ii). In particular, a generic non-linear net revenue function

$$\Pi(h, x) = p(h)h - C(h, x), \quad (6)$$

which is formulated in a continuous-time setting (h represents the harvest rate), and where both the inverse demand function $p(h)$ and the cost function $C(h, x)$ have been estimated on an annual basis, is reformulated to contemplate the temporal interval $[t, t + \Delta t]$ under consideration. In this way, as described below, the discrete-time approximation of the net revenue function, as defined in (6), will be given by:

$$\Pi(h_t, x_t, \Delta t) = p(h_t, \Delta t)h_t - C(h_t, x_t, \Delta t), \quad (7)$$

where h_t represents harvest quantity at the end of period t , as described in (4).

It should be noted that, using (4), equation (7) can be rewritten as $\Pi(x_t, x_{t+\Delta t}, \Delta t)$.

The discretization method described above gives us a discrete-time approximation of the continuous-time model, as defined in (1). In particular, the discrete-time model obtained by the discretization method is given by:

$$\begin{aligned} & \max_{\{x_{t+\Delta t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \Pi(x_t, x_{t+\Delta t}, \Delta t) \\ & \text{s.t. } 0 \leq x_{t+\Delta t} \leq f_{RK}^{(\Delta t)}(x_t), \quad t = 0, 1, \dots, \\ & \quad x_0 > 0 \text{ given, } \Pi(x_t, x_{t+\Delta t}, \Delta t) \geq 0, \quad t = 0, 1, \dots, \end{aligned} \quad (8)$$

where the objective functional is the present value of net revenues from the fishery. Π is as defined in (7), $\delta \in (0, 1)$ is an annual discount factor, and x_0 is the initial stock level. If $f_{RK}^{(\Delta t)}(x_t)$ is as defined in (4), then $f_{RK}^{(\Delta t)}(x_t) - x_{t+\Delta t} = h_t$ in problem (8).

Using the dynamic programming approach (Stokey et al. 1989), we can define the following Bellman equation associated with (8):

$$V(x) = \max_{0 \leq y \leq f_{RK}^{(\Delta t)}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)], \quad (9)$$

where the constraint on the escapement y is as defined in (5).

The discretization method described above allows us to construct a bridge between continuous and discrete-time models due to the fact that a continuous-time model, as defined in (1), may be thought of

as the limiting case of the discrete-time model, as defined in (8), in which the interval between times Δt in the discrete-time frame $t + \Delta t$ becomes vanishingly small. In other words, the discrete-time model, as defined in (8), converges to the continuous-time model, as defined in (1), when $\Delta t \rightarrow 0$.

4. Bioeconomic model

In order to make numerical experiments realistic while keeping their scope of application wide enough, we take the North-East arctic cod (NEAC) as our starting point which is the largest cod stock in the world and, consequently, one of the most important species in Norwegian fisheries (e.g. Armstrong et al. 2014). The NEAC fishery is a clear example of a seasonal fishery due to its seasonal migration pattern, where 80% of stock is harvested in the high season (the winter quarter) in a limited geographical area (Lofoten islands) where the stock has gathered and migrated from the Barents Sea to spawn. This seasonal migration pattern gives rise to seasonal harvesting (e.g. Hannesson et al. 2010, Hermansen and Dreyer 2010).

In this section, the discretization method is applied to the NEAC fishery which has been analyzed extensively in the literature on fisheries economics by using continuous-time models, as defined in (1) (e.g. Arnason et al. 2004; Agnarsson et al. 2008).

4.1 Natural growth function

Using the ensemble Kalman filter (see *i*) above), the natural growth function for the NEAC, as defined in (2), is properly estimated in a continuous-time setting¹. In particular, we estimate a modified logistic function

$$F_C(x) = r_C x^2 \left(1 - \frac{x}{K_C}\right), \quad (10)$$

where growth is measured in 1,000 tons, $r_C = 0.00045371$ is the intrinsic growth rate, and $K_C = 3,703$ (1,000 tons) is the carrying capacity of the environment. The estimated parameter values are based on the same data set that was used in Kvamsdal and Sandal (2015), who provide a detailed description of technical and implementation details.

Using the Runge-Kutta method (see *ii*) above), the discrete-time approximation, as defined in (3), of the continuous-time growth function $F_C(x)$, as defined in (10), is given by:

$$x_{t+\Delta t} = f_{RK}^{(\Delta t)}(x_t). \quad (11)$$

Figure 1 represents the discrete-time approximation $f_{RK}^{(\Delta t)}(x_t)$, as defined in (11), for incremental time $\Delta t = 1$, and the growth dynamics (discontinuous line) which reflects a gradual approximation to K_C from a high initial stock value $x_0^{RK} = 4,700$ (1,000 tons).

In most of the literature on fisheries economics, a discrete-time natural growth function is estimated even in the case of continuous-time models. This is the case of NEAC where the natural growth function was estimated by Agnarsson et al. (2008)

$$F_D(x_t) = r_D x_t^2 \left(1 - \frac{x_t}{K_D}\right), \quad (12)$$

where growth is measured in 1,000 tons, $r_D=0.000665$, and $K_D=2,473$ (1,000 tons).

It should be noted that, as expected, the above parameter estimations are very different from those obtained in a continuous-time setting (see $F_C(x)$, as defined in (10)).

It should also be noted that the natural growth function, as defined in (12), was estimated in a discrete-time setting, which implies that the proper population dynamics of the stock is described by the difference equation

$$x_{t+1} = x_t + F_D(x_t) = f_D(x_t). \quad (13)$$

Figure 1 also represents the growth function of the NEAC $f_D(x_t)$ estimated in a discrete-time setting, as defined in (13), and the growth dynamics (discontinuous line) from a high initial stock value $x_0^D = 3,375$ (1,000 tons), which is obtained through the path

$$x_0^D \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow K_D.$$

We can observe in Figure 1 that the discrete-time approximation $f_{RK}^{(\Delta t)}(x_t)$, derived from the continuous-time natural growth function $F_C(x)$, is quite different from the discrete-time growth function $f_D(x_t)$. Consequently, we can see in Figure 1 that the growth dynamics are also quite different in both².

Thus, these numerical experiments show that a significant error occurs when inserting discrete-time natural growth functions into differential equations, as is often done in the literature on fisheries economics. The discretization method allows us to overcome this error through a proper discrete-time approximation derived from the continuous-time natural growth function.

4.2 The objective functional

Using annual data, a generic non-linear net revenue function, as defined in (6), from the NEAC fishery has been estimated by Agnarsson et al. (2008)

$$\Pi(h, x) = p(h)h - C(h, x), \quad (14)$$

where $p(h) = (p_1 - p_3 h)$, $p_1 = 12.65$ and $p_3 = 0.00839$; $C(x, h) = c \frac{h^{1.1}}{x}$, $c = 5,848.1$. Stock x and harvest h are measured in 1,000 tons, net revenue $\Pi(h, x)$ is measured in million NOK, and prices are measured in NOK/kg.

Using the discretization method (see *iii*) above), the discrete-time approximation, as defined in (7), of the net revenue function (14) is given by:

$$\begin{aligned}\Pi(x_t, x_{t+\Delta t}, \Delta t) &= (p_1 - p_3(h_t / \Delta t))h_t - c\Delta t \frac{(h_t / \Delta t)^{1.1}}{x_t} = \\ &= (p_1 - p_3((f_{RK}^{(\Delta t)}(x_t) - x_{t+\Delta t})\Delta t^{-1}))(f_{RK}^{(\Delta t)}(x_t) - x_{t+\Delta t}) - c\Delta t^{-0.1} \frac{(f_{RK}^{(\Delta t)}(x_t) - x_{t+\Delta t})^{1.1}}{x_t},\end{aligned}\quad (15)$$

where $h_t = f_{RK}^{(\Delta t)}(x_t) - x_{t+\Delta t}$, as defined in (8), and $f_{RK}^{(\Delta t)}(x_t)$, as defined in (11).

It should be noted in (14) that both the inverse demand function $p(h)$ and the cost function $C(x, h)$ are formulated in a continuous-time setting (h represents the harvest rate), and both have been estimated on an annual basis. In order to discretize the net revenue function (14), the annual harvest rate h has been replaced in (15) with $h_t / \Delta t$, where h_t denotes harvested quantity. In this way, the inverse demand function is properly expressed as quantity per unit of time.

It should be also noted in (15) that the annual cost parameter c has been replaced with $c\Delta t$, in order to properly discretize the costs function.

Using (15), the discrete-time bioeconomic model, as defined in (8), can be established and, consequently, the Bellman equation, as defined in (9), for the NEAC fishery is given as:

$$V(x) = \max_{0 \leq y \leq f_{RK}^{(\Delta t)}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)], \quad (16)$$

where the constraint on the escapement y is as defined in (5).

5. Numerical analysis of seasonal fisheries

The discretization method described above allows for time increments of less than one year which is the standard time interval, $\Delta t = 1$, considered in discrete-time models. Specifically, if the one-year interval is divided into N periods, then this can be done by defining $\Delta t = \frac{1}{N} \leq 1$ in (3) and (9) ((11) and (16) in the case of NEAC). In this way, the discretization method allows us to analyze the seasonality phenomenon in fisheries for different values of $\Delta t < 1$. In particular, taking the NEAC fishery as an illustration, in this section we analyze both the case of quarterly harvest, $\Delta t = 0.25$ ($N=4$), and the case of monthly harvest, $\Delta t = 0.083$ ($N=12$), and we compare these cases with the case of annual harvest, $\Delta t = 1$ ($N=1$).

All data in the numerical experiments described below were performed using code written in standard FORTRAN.

Figure 2 represents the optimal harvest H^* (1,000 tons) which corresponds to the solution of (16) with the discrete-time approximation $f_{RK}^{(\Delta t)}(x_t)$ for the NEAC, as defined in (11), for a discount factor $\delta = 0.95$, and for different values of Δt . In particular, this figure represents the annual optimal harvest if the stock is harvested annually, $AH(x_t)$, harvested quarterly, $AQH(x_t)$, and harvested monthly $AMH(x_t)$, where x_t (1,000 tons) represents the stock value at the beginning of the year.

It should be noted that, while $AH(x_t)$ can be obtained directly by solving (16) with the discrete-time approximation $f_{RK}^{(\Delta t)}(x_t)$, for $\Delta t = 1$, a previous step is needed to obtain both $AQH(x_t)$ and $AMH(x_t)$.

For example, in the case of AQH(x_t), the optimal harvest in any quarter Q_i of the year $Q_i H(x_t)$, must first be obtained by solving (16) with the discrete-time approximation $f_{RK}^{(\Delta t)}(x_t)$, for $\Delta t = 0.25$. Once $Q_i H(x_t)$ is obtained for each stock value x_t at the beginning of the quarter Q_i , the optimal quarterly harvest path over the year is given by:

$$Q_1 H(x_t) = f_{RK}^{(\Delta t)}(x_t) - y_1 \rightarrow Q_2 H(y_1) = f_{RK}^{(\Delta t)}(y_1) - y_2 \rightarrow Q_3 H(y_2) = f_{RK}^{(\Delta t)}(y_2) - y_3 \rightarrow Q_4 H(y_3) = f_{RK}^{(\Delta t)}(y_3) - y_4, \quad (17)$$

Where y_1 is the optimal stock level at the beginning of the second quarter (after harvesting) if x_t is the stock level at the beginning of the current year, $y_i; i=2,3$, is the optimal stock level at the beginning of the quarter Q_{i+1} if y_{i-1} is the optimal stock level at the beginning of the quarter Q_i , and y_4 is the optimal stock level at the beginning of the next year if x_t is the stock level at the beginning of the current year. As described below, the latter is the annual optimal policy function if the stock is harvested quarterly. In this way, AQH(x_t) can be obtained for each stock level x_t at the beginning of the year

$$AQH(x_t) = Q_1 H(x_t) + Q_2 H(y_1) + Q_3 H(y_2) + Q_4 H(y_3). \quad (18)$$

Based on the above, AMH(x_t) can be obtained for each stock level x_t at the beginning of the year

$$AMH(x_t) = M_1 H(x_t) + M_2 H(y_1) + \dots + M_{12} H(y_{11}), \quad (19)$$

where $M_i H(\cdot)$ is the optimal harvest in any month M_i of the year.

We can observe in Figure 2 that, in all cases, the optimal harvest follows the same pattern as obtained in the continuous-time model described in Agnarsson et al. (2008), which consists of harvest moratorium at low stock levels $x \leq x^m = 900$, with a gradual increase in harvest at high enough stock values, $x \geq x^m$. It seems to confirm the robustness of the discretization method due to the fact that the discrete-time solution converges to the solution obtained in continuous-time. In addition, we can also see in Figure 2 that AQH is similar to AMH which implies that the discrete-time solution converges fast to the solution obtained in continuous-time, where convergence is achieved if the stock is harvested quarterly ($\Delta t = 0.25$).

We can also observe in Figure 2 that both AQH and AMH are greater than AH at high enough stock values, a somewhat counterintuitive result which can be explained by the actualization of the growth rates (AGR) of the resource which takes place if the stock is more frequently harvested, on a monthly or quarterly basis, instead of waiting for growth over the whole year, as is the case of annual harvest. If the stock is more frequently harvested, then the AGR gives rise to greater growth rates of the resource which in turn implies higher annual harvest.

Figure 3 represents the optimal policy function and the optimal policy dynamics which correspond to the solution of (16) with the discrete-time approximation $f_{RK}^{(\Delta t)}(x_t)$, for the NEAC, as defined in (11), for a discount factor $\delta = 0.95$, and for different values of Δt . In particular, this figure represents the annual optimal policy functions, AP and APQ, if the stock is harvested annually and quarterly,

respectively. In both cases, the annual optimal policy function represents the optimal stock level in the next period (year), x_{t+1}^* (after optimal harvesting), as a function of the stock level at the beginning of the current year x_t (1,000 tons), where the optimal harvest is represented in Figure 2, AH and AQH, respectively.

We can see in Figure 3 that, if the stock is harvested annually, then the optimal stock level $x_{t+1}^* = x_1^*$, is obtained from the initial stock level $x_t^* = x_0 = 1,000$. In this way, the annual optimal policy dynamics (AD) from the initial stock level $x_t = x_0$ is obtained through the optimal path

$$x_0 \rightarrow x_1^* \rightarrow x_2^* \rightarrow x_3^* \rightarrow x_4^* \rightarrow \dots \rightarrow x_A^*. \quad (20)$$

This means that the solution converges to optimal steady state equilibrium $x_A^* = 3,065$.

Figure 3 also represents the quarterly optimal policy function (QP) which represents the optimal stock level in the next quarter (after harvesting, as defined in (17)) as a function of the stock level in the current quarter. We can see in Figure 3 that, as defined in (17) and (18), if the stock is harvested quarterly, then the quarterly optimal policy dynamics (QD) from the initial stock level $x_t = x_0 = 1,000$ is obtained through the optimal path

$$x_0 \rightarrow y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow \dots \rightarrow x_Q^*. \quad (21)$$

This means that the solution converges to optimal steady state equilibrium $x_Q^* = 3,266$.

Figure 3 also represents the annual optimal policy function if the stock is harvested quarterly (APQ), as defined in (17) and (18). For example, y_4 is the (harvested quarterly) optimal stock level at the beginning of the next year if x_0 is the stock level at the beginning of the current year. This figure also represents the annual optimal policy function if the stock is harvested monthly (APM), as defined in (19), which is similar to APQ due to the fact that, as described above (see Figure 2), AMH is similar to AQH.

We can see in Figure 3 that, if the stock is harvested more frequently, on a monthly or quarterly basis, then the resource is preserved at higher stock values (higher steady state equilibrium) than if the stock is harvested annually, despite the lower annual harvest in the latter case (see Figure 2). In particular, we can see in Figure 3 that $x_Q^* = 3266$ is about 6% greater than $x_A^* = 3065$. This somewhat counterintuitive result can also be explained by the AGR, as explained above.

Figure 4 represents the annual costs C^* (million NOK) incurred in optimal harvesting if the stock is harvested annually, $AC(x_t)$, and harvested quarterly, $ACQ(x_t)$, where x_t (1,000 tons) represents the stock value at the beginning of the year.

It should be noted that $AC(x_t)$ can be obtained directly by using the cost function, as defined in (12), for $\Delta t = 1$, and for $AH(x_t)$, as defined above (see Figure 2)

$$AC(x_t) = c \frac{AH(x_t)^{1.1}}{x_t}. \quad (22)$$

It should also be noted that a previous step is needed to obtain $ACQ(x_t)$. In particular, the quarterly costs incurred in optimal harvesting $Q_i C(x_t)$, must first be obtained by using both the cost function, as defined in (12), for $\Delta t = 0.25$, and $Q_i H(x_t)$, as defined in (17)

$$Q_i C(x_t) = c(0.25)^{-0.1} \frac{Q_i H(x_t)^{1.1}}{x_t}, \quad (23)$$

where x_t represents the stock value at the beginning of the quarter Q_i . Using both (17) and (23), the quarterly costs path incurred in optimal harvesting over the year is given by:

$$Q_1 C(x_t) \rightarrow Q_2 C(y_1) \rightarrow Q_3 C(y_2) \rightarrow Q_4 C(y_3). \quad (24)$$

In this way, $ACQ(x_t)$ can be obtained for each stock level x_t at the beginning of the year

$$ACQ(x_t) = Q_1 C(x_t) + Q_2 C(y_1) + Q_3 C(y_2) + Q_4 C(y_3). \quad (25)$$

We can see in Figure 4 that, there is a stock value $x' \approx x_4^* \approx 2800$, which corresponds to x_4^* in Figure 3, such that, for stock values $x_t \leq x'$, the annual costs incurred in optimal harvesting if the stock is harvested quarterly are lower than obtained if the stock is harvested annually, $ACQ(x_t) \leq AC(x_t)$, while the opposite is the case for high stock values $x_t \geq x'$. The reason for this is that, for stock values $x_t \leq x' \approx x_4^*$ at the beginning of the year, the optimal stock at the beginning of each quarter y_i , which is the denominator of the quarterly cost function, as defined in (23), is greater than obtained in the previous quarter y_{i-1} , and all of them are greater than the stock level at the beginning of the year x_t , which is the denominator of the annual cost function, as defined in (22), $x_t < y_1 < y_2 < y_3 < y_4 < x_{t+1}$ (see Figure 3), which implies that $Q_2 C(y_1) < Q_3 C(y_2) < Q_4 C(y_3) < AC(x_t)$, and consequently $ACQ(x_t)$, as defined in (23) and (25), are lower than $AC(x_t)$, as defined in (22) (see Figure 4). We can also see in Figure 3 that the opposite is the case for high stock values $x_t \geq x' \approx x_4^*$.

Thus, discrete-time bioeconomic models on an annual basis neglect the change in the stock during harvesting when considering that the harvest costs depend on the stock at the start of the year, and consequently they may incur systematic errors by overestimating or underestimating the harvest costs (Maroto et al. 2012). The above results suggest that there is less overestimation or underestimation in costs if the stock is harvested more frequently.

Figure 5 represents the value function V (million NOK), as defined in (16), for a discount factor $\delta = 0.95$, and for different values of Δt . In particular, this figure represents the annual value function if the stock is harvested annually, $AV(x)$, harvested quarterly, $AVQ(x)$, and harvested monthly, $AVM(x)$, where x represents the stock value at the beginning of the year.

It should be noted that $AV(x)$ can be obtained directly by solving (16) for $\Delta t = 1$, while $AVQ(x)$ and $AVM(x)$ are given by:

$$AVQ(x) = \Pi(Q_1H(x), x) + \delta^{0.25}\Pi(Q_2H(y_1), y_1) + \delta^{0.5}\Pi(Q_3H(y_2), y_2) + \delta^{0.75}\Pi(Q_4H(y_3), y_3) + \delta AVQ(y_4), \quad (26)$$

$$AVM(x) = \Pi(M_1H(x), x) + \delta^{\frac{1}{12}}\Pi(M_2H(y_1), y_1) + \dots + \delta^{\frac{11}{12}}\Pi(M_{12}H(y_{11}), y_{11}) + \delta AVM(y_{12}),$$

where $Q_iH(\cdot)$ and $M_iH(\cdot)$ have been defined in (17) and (19), respectively.

We can see in Figure 5 that $AVQ(x)$ is similar to $AVM(x)$ due to the fact that, as mentioned above, the annual optimal harvest is similar in both cases, $AQH(x) \approx AMH(x)$ (see Figure 2).

We can also see in Figure 5 that both $AVQ(x)$ and $AVM(x)$ are greater than $AV(x)$. The lower annual costs incurred in optimal harvesting if the stock is harvested quarterly or monthly, as described above (see Figure 4), could be an explanation for this counterintuitive result for stock values $x_t \leq x'$. However, based on this argument, the opposite should be the case, $AVQ(x) \approx AVM(x) < AV(x)$, for stock values $x_t \geq x'$, which does not happen (see Figure 5). This means that, besides the costs, other factors like the AGR, and discounting, influence the economic value of the problem. In particular, the AGR, as described above, is a kind of compound interest where the effect of a smaller Δt is to increase the growth rate of the species over a unit of time which in turn implies, as described above, both higher annual harvest and higher steady state equilibrium if the stock is harvested seasonally (quarterly and/or monthly). This effect is similar to that obtained in the economic value of the problem if it is more frequently discounted by using $\delta^{\Delta t}$, as described in (26), which implies higher economic value.

6. Discussion and conclusions

Errors in mathematical modeling of the population dynamics for fish stocks are frequently found in continuous-time models by inserting a natural growth function estimated in discrete-time into differential equations.

Approximation errors involved in developing discrete-time models, which are commonly assumed by most economists to be trivially small, can be important under some conditions.

In this paper we have developed a discretization method to construct a bridge between continuous and discrete-time bioeconomic models which allows us to overcome errors in both approaches by properly estimating the population dynamics of fish stocks in a continuous-time setting, and by achieving fast convergence from discrete to continuous-time models.

Most of the literature on bioeconomic modeling of fisheries uses both discrete and continuous-time models indistinctly without a clear biological or economic justification. However, it is not obvious how discrete and continuous-time models are related to each other and, consequently, this is not a trivial choice, especially in fisheries economics, since they can provide different policy advice, with significant implications for fish stock sustainability.

Knowledge of the relationship between continuous and discrete-time models is crucial in order to avoid biologically and economically meaningless models that can lead to erroneous (suboptimal) policy advice, with the consequent uncertainty regarding the appropriate bioeconomic model which should be used to ensure long term sustainability.

In regards to the above, the discretization method could be a useful tool for overcoming the biological and economic weakness of continuous and discrete-time models, and for preserving the strengths of both approaches, by developing a proper discrete-time model which guarantees fish stock sustainability. Research is required in this direction.

Current bioeconomic models are designed on an annual basis despite the well-known fact that most fisheries are seasonal due to the seasonal pattern of migration of most exploited fish stocks. The underlying assumption in the annual steady state equilibrium achieved in standard bioeconomic models is that both biological processes, like the growth of the species, and fishing activity take place only once a year.

In this paper we have shown that the discretization method is able to deal with seasonal fisheries by providing a theoretically-grounded, practical starting point for optimal management of seasonal fisheries. In the case of the NEAC fishery, we have shown that seasonal harvesting is a win-win optimal solution by achieving higher annual harvest, higher steady state equilibrium, and higher economic value than obtained in the case of annual harvesting. These results can be explained by the combined effects of the actualization of the growth rates, decrease of harvesting costs, and more frequent discounting which all take place if the stock is harvested seasonally (quarterly or monthly). From the point of view of policy advice, most commercial fisheries are managed on an annual basis where collection and management of annual biological and fisheries data are used by management agencies, like the International Council for the Exploration of the Sea (ICES), to provide annual advice regarding stock status, reference points, and total allowable catches (TACs). In addition, in real world fisheries, annual TACs are allotted to different vessel groups which, however, target the resource in different seasons. Moreover, they are usually based on political decisions, rather than optimal bioeconomic criteria, with the consequent considerable effects upon the health of the stock and the economics of the fishery (e.g. Armstrong and Sumaila 2001, and Armstrong et al. 2014, for the case of NEAC). In this sense, the discretization method could provide a useful tool for providing science based seasonal regulation measures, like seasonal closures and (or) seasonal TACs, for testing the safety of limits reference points by analyzing the risk of collapse of seasonal fisheries, and for allotting an annual TAC to each season optimally.

Therefore, there is clearly a need for future research and improvement of seasonal data to exploit the wide range of possible applications for the discretization method in optimal management of seasonal fisheries.

Research is in progress in these directions.

Notes

1. See Sims (2008) for other techniques to estimate explicitly continuous-time models, using discrete data.
2. It is well known that the discrete-time homologue of the standard continuous-time logistic growth function is not the discrete-time logistic growth function but the Beverton–Holt growth function which is non-decreasing at high population sizes (Gyllenberg et al. 1997).

Appendix

Taking the differential equation $\dot{x}(t) = F_C(x)$, as a starting point as defined in (2), where x represents the fish stock biomass, and $F_C(x)$ the natural growth function of the species in a continuous-time setting, a brief description of the fourth order Runge-Kutta algorithm (RKA) is as follows (see, e.g., Press et al., 2007, for further details):

For each initial stock value $x = x_t \in X$, where X is the discrete-time state-space, the temporal interval $[t, t + \Delta t]$ is partitioned in M subintervals (number of iterations) where M should be high enough to ensure enough accuracy. Then, the following formula of the RKA is evaluated in each iteration

$$\begin{aligned} k_1 &= hF_C(x) \\ k_2 &= hF_C\left(x + \frac{k_1}{2}\right) \\ k_3 &= hF_C\left(x + \frac{k_2}{2}\right) \\ k_4 &= hF_C(x + k_3) \\ x &= x + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \end{aligned}$$

where $h = \frac{\Delta t}{M}$ is the step size.

Thus, for each stock value at period t x_t , the RKA computes an approximate solution for the stock value at period $t + \Delta t$, $x_{t+\Delta t}$, that is

$$x_{t+\Delta t} = f_{RK}^{(\Delta t)}(x_t) = x + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

where $f_{RK}^{(\Delta t)}(x_t)$ is the discrete-time approximation of the continuous growth function $F_C(x)$, as defined in (2).

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Figures

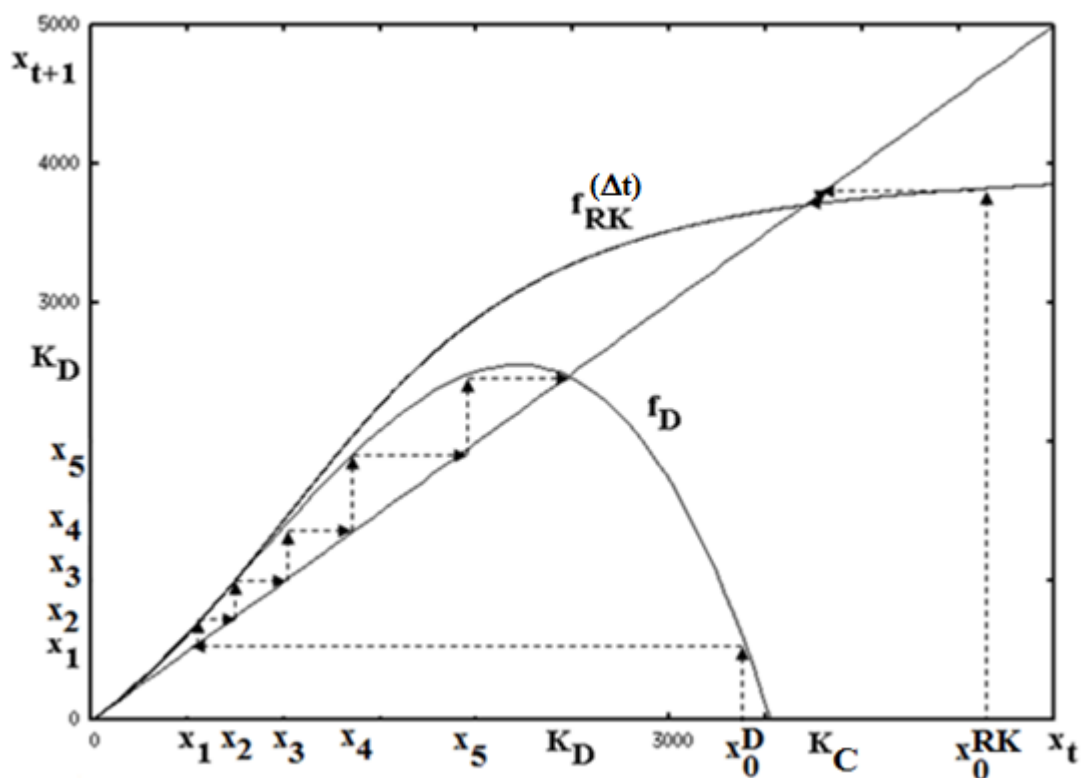


Figure 1.

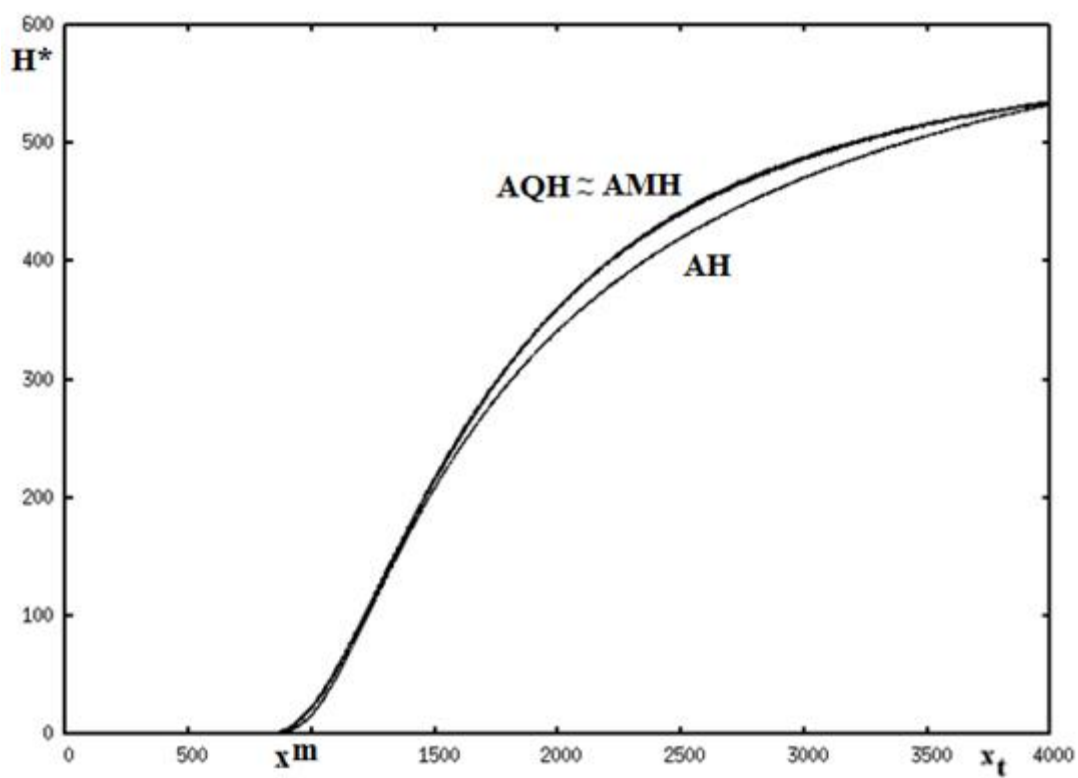


Figure 2.

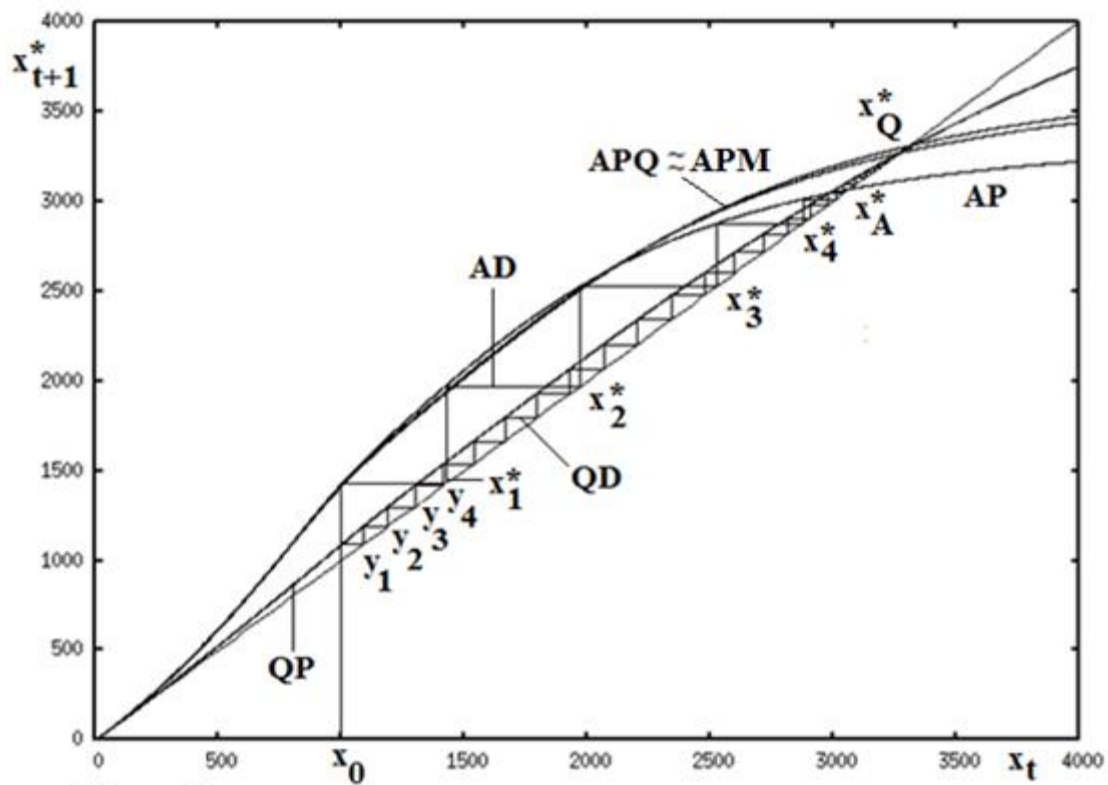


Figure 3.

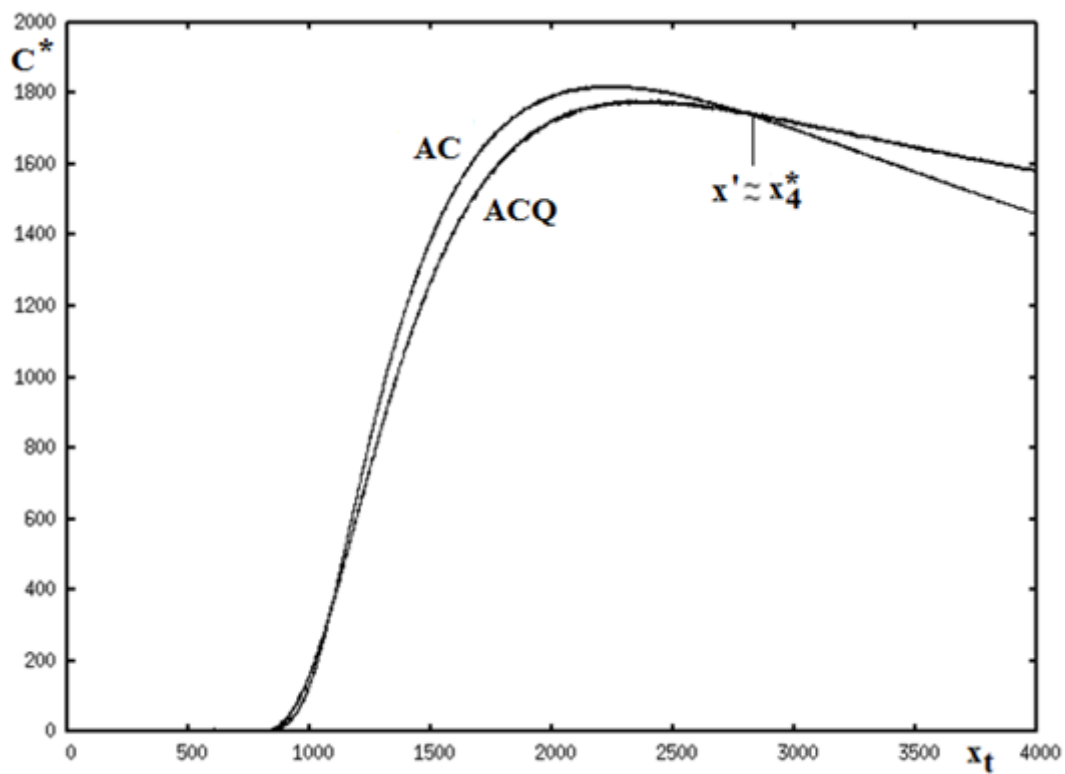


Figure 4.

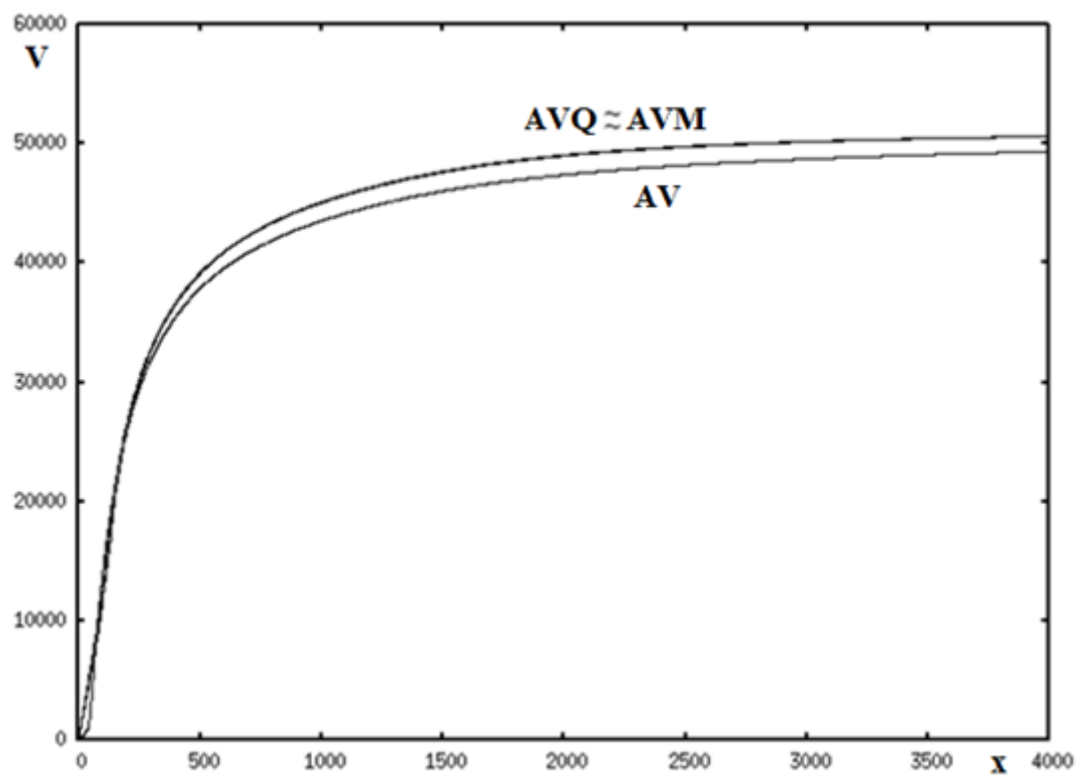


Figure 5.

Figure captions

Figure 1. Growth function $f_D(x_t)$ estimated in a discrete-time setting, discrete-time approximation

$f_{RK}^{(\Delta t)}(x_t)$ obtained by the discretization method for $\Delta t = 1$, and the 45 degree line.

Figure 2. Annual optimal harvest if the stock is: harvested annually, $AH(x_t)$, harvested quarterly

$AQH(x_t)$, and harvested monthly, $AMH(x_t)$.

Figure 3. Annual optimal policy functions if the stock is: harvested annually, $AP(x_t)$, harvested

quarterly $APQ(x_t)$, and harvested monthly, $APM(x_t)$. Annual optimal policy dynamics, AD. Quarterly optimal policy function $QP(x_t)$, and quarterly optimal policy dynamics, QD.

Figure 4. Annual costs incurred in optimal harvesting if the stock is harvested annually $AC(x_t)$, and

harvested quarterly, $ACQ(x_t)$.

Figure 5. Annual value function if the stock is: harvested annually, $AV(x)$, harvested quarterly

$AVQ(x)$, and harvested monthly, $AVM(x)$.