Norwegian School of Economics
Bergen, Autumn, 2016

Managing Volatility: An Empirical Analysis of the Time-series Relation Between Risk and Return

Norwegian Evidence

Thomas André Johansen and Lars Kristian Eckhoff

Supervisor: Francisco Santos

Master of Science in Economics and Business Administration,

Finance

NORWEGIAN SCHOOL OF ECONOMICS

Acknowledgements

We would like to thank our supervisor, Francisco Santos, for always being available to answer our questions throughout the process. His insights and advice has been invaluable for us. Further, we wish to thank PhD student Jens Sørlie Kværner for introducing us to the topic and for helpful guidance. Lastly, we thank PhD student Erik Hetland Tvedt for useful assistance with the econometrics.

Bergen, December 2016.

Lars Kristian Eckhoff Thomas André Johansen

Abstract

In this paper, we examine the time-series relation between risk and return. We replicate the methodology of Moreira and Muir (2016a) and construct volatility managed portfolios that decrease the risk exposure when volatility is high, and vice versa. We implement the strategy on well-known risk factors in Norway and the UK, in addition to industry portfolios in Norway and in the U.S. The strategy in general produces large alphas and increased Sharpe ratios and the results are robust when controlling for exposure to other risk factors. We further show that using forecasted variance from sophisticated forecasting models rather than realized variance can improve the results of the volatility managed portfolios.

Contents

List of Figures

List of Tables

1 Introduction

An important assumption of traditional finance is the positive relation between risk and return, and early models such as the Capital Asset Pricing Model (CAPM), introduced by Sharpe (1964), predict a positive linear relationship between systematic risk and expected return. However, the relation does not seem to hold empirically. In fact, various research have found that the opposite may hold in practice. Historically, the majority of research has examined the cross-sectional relation between risk and return, which has led to the Low Volatility Puzzle[1]. In more recent years, empirical studies of the time-series relation between risk and return indicate that it is beneficial for investors to increase the risk exposure of systematic factors when volatility is low, and vice versa. Barroso and Santa-Clara (2015) find that managing the risk of the momentum factor eliminates the well-known momentum crashes and nearly doubles the Sharpe ratio, while Moreira and Muir (2016a) find that managing the volatility of several factors, including the well-known Fama-French factors, leads to increased Sharpe ratios and utility gains for mean-variance investors. These findings are interesting as they challenge the presumptively positive relation between risk and return in the time-series of returns.

In this thesis, we examine the time-series relation between risk and return. We replicate the methodology of Moreira and Muir (2016a) and construct volatility managed portfolios that decrease the risk exposure when volatility is high, and vice versa. Specifically, the volatility managed portfolios are constructed by monthly scaling each factor by the inverse of its realized variance in the previous month. If the realized variance is high one month, the managed portfolio will decrease the exposure to the factor the following month and the investor would rather allocate his wealth into a risk-free asset. Monthly realized variance is simply calculated using daily returns, and the method does only rely on return data; thus the strategy is easy to implement for investors in real time.

Our main contribution in this thesis is to manage the volatility of portfolios that have not been considered by Moreira and Muir (2016a). They manage the volatility of the well-known Fama-French factors in the U.S., momentum, profitability, return on equity, investment factors in equities and the currency carry trade. They also demonstrate the results for stock market indices of 20 OECD countries. We construct volatility managed portfolios of the Fama-French factors plus the momentum

[1] The Low Volatility Puzzle is that low-volatility stocks outperform high-volatility stocks

factor in Norway, which has not been done earlier, and for the UK, where only a few factors have been considered in previous research.[2] For the Norwegian analysis we also consider a liquidity factor. In addition to managing volatility of these pricing factors, we also manage volatility for Norwegian and U.S. industry portfolios. The industry portfolios consist of several single stocks within an industry, thus the portfolios are not as diversified as the systematic Fama-French factors. We find it interesting to see if it also is beneficial to manage the volatility of portfolios that do not consist of systematic diversified factors.

We run OLS regressions of the volatility managed portfolios on the non-managed portfolios in order to evaluate the performance of the managed portfolios.[3] A positive alpha from these regressions implies that the managed portfolios expand the mean-variance frontier relative to the non-managed portfolios. This assumption holds when we consider systematic factors that are well diversified. In general, we see large and significant alphas for the volatility managed portfolios. For the Norwegian factors, three out of five factors has a large and significant alpha, the highest for the momentum factor. For the U.K factors, only the momentum factor has a positive and significant alpha. The finding of large significant alphas for the momentum factor is consistent with the findings of Moreira and Muir (2016a); they find the largest alpha for the volatility managed momentum factor in the U.S. This is also consistent with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016). To show directly how much the managed portfolios increase the Sharpe ratio, we report the appraisal ratio (AR).

Even larger alphas and appraisal ratios are obtained when we construct three mean-variance efficient portfolios based on the Norwegian pricing factors, and then manage the volatility of these portfolios. More specifically, the mean-variance efficient portfolios are constructed using (1) the three Fama-French factors (”FF3”), (2) the three Fama-French factors plus momentum (”FF4”) and (3) the three Fama-French factors plus momentum and liquidity (”FF5”). The largest alpha is found for the ”FF4” managed portfolio. The mean-variance efficient portfolios are well diversified, and the large alphas and appraisal ratios we find implies that managing these portfolios expands the mean-variance efficient frontier.

[2]Moreira and Muir (2016a) consider a UK market indice excluding dividends, and Barroso and Santa-Clara (2015) only examine the momentum factor.

[3]As we point out later, this is exactly what Moreira and Muir do to evaluate the performance of the volatility managed portfolios.

The volatility managed industry portfolios also in general have large and significant alphas, where the results are most consistent for the U.S. industry portfolios. In fact, 8 out of 10 managed U.S. industry portfolios have large and significant alphas. Also for the Norwegian industry portfolios the majority of the managed portfolios have large and significant alphas, but 3 out of 7 portfolios we consider have insignificant alphas. Note that since the industry portfolios are not well diversified systematic factors, we can not conclude directly that the positive alphas imply that the managed portfolios expand the mean-variance efficient frontier, as we can for the managed Fama-French factors, the momentum factor, the liquidity factor and the mean-variance efficient portfolios.

We do the same analysis as Moreira and Muir (2016b) do to analyze the relation between risk and return more directly. We focus on the Norwegian factors in this analysis, and show for the majority of the factors that variance is able to predict future variance, but not future returns. This is in general the reason why managing the volatility of portfolios leads to higher risk-adjusted returns; if variance predicts future variance but not future returns, the risk-return trade-off will be less attractive when variance increases. An interesting finding is that for the HML factor, variance predicts positive future returns. The volatility managed HML factor in Norway and the UK has a negative alpha, which might be consistent with the fact that variance of the HML factor predicts positive returns. Another interesting finding is that for the market factor, variance predicts negative returns, and we find a positive and significant alpha for the volatility managed portfolio of this factor.

Using lagged realized variance to construct the managed portfolios is easy to implement for the average investor. More importantly, the realized variance in the previous month is a good proxy for the realized variance the following month.[4] We aim to find a better proxy for realized variance than its lagged value, and consider two different forecasting models in order to do so. First, we consider a forecasting model where we run an OLS regression of realized variance on lagged realized variance, and use the coefficients obtained in the regression to forecast monthly variance. We call this method the "OLS method", and we do both an in-sample forecast using the whole sample, and an out-of-sample forecast where we use a rolling estimation window of 24 months. Not surprisingly, the OLS method does not forecast out-of-sample variance in a particularly good way. Next, we consider a more sophisticated method to forecast variance, and use a GARCH (1,1) model to forecast monthly variance. The forecasted variance by the GARCH model in general seems to be a better

[4]Remember, variance is highly predictable at short horizons.

proxy for realized variance than both the forecasted variance by the OLS method and lagged realized variance.

We re-construct the volatility managed portfolios using both the OLS- and GARCH forecasted variance rather than lagged realized variance. When we manage portfolios with the out-of-sample forecasted variance from the OLS method, we see that the alphas and appraisal ratios decrease compared to scaling the managed portfolios with lagged realized variance. This is not surprising as the forecasted variance seems to be a weaker proxy for realized variance. Managing the portfolios with the GARCH forecasted variance leads to higher alphas and appraisal ratios for the majority of the portfolios that we consider, and we see a pattern that finding a better proxy for realized variance in general leads to stronger results.

The rest of the thesis is organized as follows. In section 2 we review the relevant literature. Section 3 describes the data. Section 4 shows the methodological approach. In section 5 we present our main results. Section 6 evaluates the performance of the forecasting models. In section 7 we discuss and analyze the results. Section 8 presents the conclusion.

2 Literature Review

In this section, we highlight the relevant literature for our thesis. Our main focus is on research examining the time-series relation between risk and return, and especially the findings of Moreira and Muir (2016a). In addition, we choose to focus on studies on the time-series relation between risk and return for the momentum factor. Since the majority of the research on risk and return historically has been in the cross-section, we also highlight research which has led to the "Low Volatility Puzzle".

2.1 Studies of the time-series relationship between risk and return

2.1.1 The findings of Moreira and Muir (2016)

Our thesis is to a large extent a replication of the study of Moreira and Muir (2016a), who examine the time-series behavior of risk and return. They find that managing volatility benefits both short- and long-term mean-variance investors and produces large utility gains. They construct volatility managed portfolios that increase the risk exposure when the volatility is low, and vice versa. Specifically, the volatility managed portfolios scale monthly factor returns by the inverse of the realized variance in the previous month. A remarkable feature is that the strategy is easy to implement for investors in real time.

Moreira and Muir (2016b) show that volatility predicts future volatility but not future returns; thus an increase in volatility leads to a weaker risk-return trade-off. This is proved by running regressions of future returns on realized volatility and regressions of future volatility on lagged volatility.[5] They further show that the conditional variance increases by far more than the expected return in response to a variance shock, where a variance shock leads to an immediate increase in future variance, while expected returns increases more slowly over time. The large increase in variance, however, decreases more quickly. The optimal strategy of an investor is therefore to reduce the risk exposure after a variance shock until the risk-return trade-off is favorable again due to mean-reverting volatility and a slow increase in expected returns. The volatility managed portfolios reduce risk after volatility spikes, while in traditional finance the common advice is to hold the risk exposure constant.

[5]Later in the thesis, we run the same regression using variance rather than volatility.

Moreira and Muir (2016a) find significant and positive alphas for the majority of the volatility managed portfolios. In fact, only the SMB factor has a negative alpha. As they point out, a positive alpha implies that the managed portfolios expand the mean-variance frontier and increase the Sharpe ratios compared to the non-managed portfolios.[6] The finding of positive alphas for the volatility managed portfolios is documented for the market factor, value, momentum, a profitability factor, return on equity, investment factors in equitites and the currency carry trade. The results are robust when examining 20 OECD countries, including a betting against beta (BAB) factor, using expected rather than realized variance and considering leverage constraints. The results are relatively unchanged when they control for other well-known risk factors. They also show that the strategy produces large utility gains for investors who are already invested across several factors by managing the volatility of mean-variance efficient portfolios.

2.1.2 Managing the Momentum factor

The momentum factor deserves special attention on its own. As Barroso and Santa-Clara (2015) state, the momentum factor is a pervasive anomaly in asset prices, but in addition to remarkable good performance, the momentum factor has large occasional crashes.[7] This makes the momentum factor very volatile and unattractive to investors who dislike left skewness and kurtosis. Barroso and Santa-Clara (2015) find that the risk of momentum is highly variable over time but predictable, and that managing the risk of momentum eliminates the crashes and nearly doubles the Sharpe ratio compared to the static momentum factor. They argue that the most important benefit of managing the volatility of the momentum factor comes from a reduction in crash risk, and they show that the excess kurtosis drops significantly and that the left skewness becomes less negative or even turns positive.[8]

Daniel and Moskowitz (2016) also study volatility timing related to momentum crashes. They find that momentum strategies are negatively skewed, and that the negative returns can be pronounced and persistent. Investigating the predictability of momentum crashes, they find that crashes tend to occur in times of market stress, when the market has fallen and ex-ante measures of volatility are high. Consistent

[6] Moreira and Muir point out that this holds if one consider systematic factors that summarize pricing information for a wide cross-section of assets and strategies.

[7] The momentum factor was discovered by Carhart (1997).

[8] Inspired by this analysis, we also analyze higher-order moments of our volatility managed portfolios in Section 7.

with Barroso and Santa-Clara (2015), they find that a volatility timing strategy significantly outperforms the standard static momentum strategy, more than doubling its Sharpe ratio and delivering significant positive alphas.

Both Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) manage the volatility of the momentum factor using a relatively similar methodology as Moreira and Muir (2016a). Barroso and Santa-Clara (2015) estimate the risk of momentum by the realized variance of daily returns, and they scale the momentum portfolio by its realized volatility in the previous six months, with a constant target volatility. To evaluate the performance of the managed portfolios, they run a regression on the Fama-French pricing factors and find positive alphas.[9] Daniel and Moskowitz (2016) evaluate a strategy which dynamically adjusts the weight on the basic WML strategy. The WML strategy is a zero-investment Winner-Minus-Loser portfolio, i.e. the difference between the winners and losers each period. The weights are depending on its forecast return and volatility, and the dynamic strategy is levered up or down over time to maximize the unconditional Sharpe ratio. To see the exact design of the strategy, see Section 5 in their paper.

2.2 The Low Volatility Puzzle: Studies of the cross-sectional relationship between risk and return

A positive linear relationship between risk and return has been the core of traditional finance and asset pricing since the introduction of the Modern Portfolio Theory (MPT) by Markowitz (1952) and the Capital Asset Pricing Model (CAPM) by Sharpe (1964). However, a large number of empirical studies have found that the relationship is flatter as predicted by the CAPM and also negative. The volatility puzzle is that low-volatility stocks outperform high-volatility stocks, and is considered to be one of the greatest anomalies in the field of finance, according to Baker et al. (2011). The vast majority of research has been about the cross-sectional relation between risk and return of volatility, thus the research is conceptually different from the time-series relation which we consider in this thesis. Anyhow, we believe it is relevant to give a brief overview of the findings which has led to what is known as the low volatility puzzle.

A wide amount of research has been conducted on the relationship between idiosyncratic risk and return. Ang et al. (2006) examine the pricing of aggregat volatil-

[9] They also find negative loadings on the risk factors, implying that momentum diversified risk in the sample.

ity risk in the cross-section of stock returns, and find that stocks with high idiosyncratic volatility relative to the Fama and French model have low average returns. They show that the results cannot be explained by exposure to aggregate volatility risk, size, book-to-market, momentum or liquidity effects. Ang et al. (2009) verify these results and show that the effect is significant in international markets. They find a strong co-movement in the low returns to high idiosyncratic volatility, implying that broad, not easily diversifiable factors may lie behind the anomaly. Chen and Petkova (2012) find that portfolios with high idiosyncratic volatility relative to the Fama-French model from 1993 have positive exposures to innovations in average stock variance and therefore produce lower expected returns. Note that several studies have found the opposite relation to hold, see for instance Goyal and Santa-Clara (2003) and Fu (2009) among others.

As for the relation between systematic risk and return, early research as Black et al. (1972) found the CML[10] to be flatter than as predicted by the CAPM, see also Fama and MacBeth (1973) and Haugen and Heins (1975). The beta puzzle is the phenomenon that low beta stocks historically have performed better than high beta stocks. Frazzini and Pedersen (2014) construct a Betting against Beta (BAB) factor which is long low-beta assets and short high-beta assets, and show that it produces significant positive risk-adjusted returns. Among others, Baker and Haugen (1991), Baker and Haugen (1996), Baker et al. (2011) and Clarke et al. (2010) find similar results.

[10]The Capital Market Line (CML) is the allocation of capital between risk-free securities and risky securities for all investors combined.

3 Data description

The Norwegian factors and industry portfolios are downloaded from Bernt Arne Ødegaards website[11]. The Norwegian single factors consist of daily and monthly return data for the three original Fama French factors: Market, SMB and HML[12], plus the Fama French momentum factor UMD, and a liquidity factor, LIQ.[13] All the portfolios are value weighted and the data is available in the period July 1980 to December 2015. For information of exactly how these portfolios are constructed, see Ødegaard (2016b) and Ødegaard (2016a).

The Norwegian industry data consists of stocks traded on OSE stock exchange that are sorted into industry portfolios based on their Global Industry Classification Standard (GICS). See Ødegaard (2016b) for how the industry portfolios are constructed. We have used both daily and monthly return data in the period 1980-2015 for all the factors and industries. Note that for HML and SMB we have missing values for the first year.

For the American industry portfolios, we have used data from Kenneth French´s website.[14] The industry portfolios consist of stocks traded on NYSE, AMEX and NASDAQ that are sorted into industry portfolios based on their four digit Standard Industrial Classification (SIC) code. We have used both daily and monthly return data from the period 1926-2015 to construct volatility managed portfolios for the U.S. industry portfolios.

The U.K return data is collected from the University of Exeter's website.[15] This data consists of monthly and daily return data for the market, SMB, HML and the Carhart momentum factor. We use both daily and monthly return data from the period 1988-2015 to construct managed portfolios for the U.K factors. For more information, see Christidis et al. (2013).

[11]www.finance.bi.no/ bernt

[12]SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios. HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios

[13]See Næs et al. (2008) for more information on the liquidity factor.

[14]www.http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

[15]http://business-school.exeter.ac.uk/research/centres/xfi/famafrench/

4 Methodology

4.1 Portfolio construction

We replicate the methodology of Moreira and Muir (2016a) and construct volatility managed portfolios by scaling monthly factor returns by the inverse of their realized variance (RV_t) in the previous month.[16] Thus, the portfolios are rebalanced each month. The volatility managed portfolio of a given factor in month t+1 then look like:

$$f_{t+1}^{\sigma} = \frac{c}{RV_t} f_{t+1} \tag{1}$$

where c is a constant which controls the average risk exposure of the strategy.[17] More specifically, c is set such that the total unconditional standard deviation of the managed portfolio is equal to the total unconditional standard deviation of the non-managed portfolio. f_{t+1} is the return of the non-managed factor. Realized variance (RV) is calculated by squaring the monthly standard deviation, where the monthly standard deviation is calculated based on daily return data.

$$RV_t = \sigma_t^2 = \sum_{d=1/td}^{1} \left(f_{t+d} - \frac{\sum_{d=1/td}^{1} f_{t+d}}{td} \right)^2 \tag{2}$$

where td is the number of trading days in a given month.

In section 6.2, we re-construct the volatility managed portfolios using forecasted variance from more sophisticated forecasting models rather than using lagged realized variance. The managed portfolio at time t then looks like:

$$f_t^{\sigma} = \frac{c}{\hat{\sigma}_t} f_t \tag{3}$$

[16]The notation in this section is similar to the notation of Moreira and Muir (2016a). See the section "Portfolio formation" in their paper.

[17]We also tried different specifications of c, e.g. to target a volatility which is higher than the unconditional variance of the non-managed portfolio. This does not have eny effects on the Sharpe ratio; thus the fact that we use the full sample to compute c does not bias our results.

where $\hat{\sigma_t}$ is the forecasted variance in month t. Note that when we manage the portfolios with forecasted variance, we scale the monthly portfolio returns by the forecasted variance from the *same month*, compared to scaling with the lagged realized variance in the main results. Both forecasted variance and lagged realized variance is a proxy for realized variance, and both methods are implementable in real time.

4.1.1 Empirical regressions and performance evalution

Both the empirical regressions and the performance evaluation we consider in this section is similar to the approach of Moreira and Muir (2016a). We evaluate the portfolio performance from a mean-variance perspective, and focus on the mean-variance trade-off (risk-return trade-off):

$$\frac{E_t[R_{t+1}]}{Var_t[R_{t+1}]} \tag{4}$$

where R_{t+1} is the excess return. In order to evaluate the performance of the volatility managed portfolios, we run a regression of the volatility managed factor portfolios on the non-managed factor portfolios:[18]

$$f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \epsilon_{t+1} \tag{5}$$

A positive intercept from this regression implies that the volatility managed portfolios expand the mean-variance frontier relative to the non-managed portfolios[19], thus increasing the Sharpe ratio. The Sharpe ratio is defined as the excess portfolio return divided by the standard deviation of the portfolio.

In order to quantify directly how much the volatility managed portfolios increase the Sharpe ratio relative to the non-managed portfolios, we consistently report the

[18]The notation is similar to Moreira and Muir (2016a). See the section "Empirical Methodology" in their paper.

[19]As Moreira and Muir (2016a) point out, this holds when we consider systematic factors that summarize pricing information for a wide cross-section of assets and strategies.

annualized appraisal ratio (AR)[20]

$$AR = \sqrt{12}\frac{\alpha}{RMSE} \tag{6}$$

where α is the unconditional alpha and RMSE is the Root Mean Square Error of regression (5). Formally, RMSE is defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{y_n} - y_n)^2} \tag{7}$$

where y_n is the observed values and $\hat{y_n}$ is the predicted values from our model.

In the following section, we present a volatility timing strategy in the cross-section of returns, which is conceptually different from the main strategy we presented above.

4.1.2 Long-short strategy

We want to test a volatility timing strategy that exploits the variation in volatility between factors. As we show later in the thesis, volatility does not predict future returns, but volatility predicts future volatility. The risk-return trade-off will then in general be weak when volatility is high, and vice versa. The idea is that buying factors with low volatility (stronger risk-return trade-off), and selling factors with high volatility (weaker risk-return trade-off) will give higher risk-adjusted returns. Note that the methodology that replicates Moreira and Muir (2016a) scale in and out of a single factor, while in our "long-short strategy" we go long and short between different factors and do not consider any risk-free asset. Specifically, we construct portfolios that go long \$1 in the factor with the lowest realized variance in the previous month, and short \$1 in the factor with the highest realized variance in the previous month.[21]. The portfolios are rebalanced each month. Formally:

Consider a portfolio p which consists of factors x and y, with returns $r_{x_{t+1}}$ and $r_{y_{t+1}}$ at time t+1 and realized variance $\sigma^2_{x_t}$ and $\sigma^2_{y_t}$at time t. The long-short portfolio at time t+1 is then:

[20] In other words, the appraisal ratio is the excess Sharpe ratio of the volatility managed portfolios, as pointed out by Moreira and Muir (2016a).

[21] Realized variance is calculated as shown in equation 2

$$r_{p_{t+1}} = r_{x_{t+1}} - r_{y_{t+1}} \quad \text{if} \quad \sigma^2_{y_t} > \sigma^2_{x_t} \tag{8}$$

and

$$r_{p_{t+1}} = r_{y_{t+1}} - r_{x_{t+1}} \quad \text{if} \quad \sigma^2_{x_t} > \sigma^2_{y_t} \tag{9}$$

Note that we consider the long-short strategy both for two factors and several factors. When including several factors, we only go long in the factor with lowest realized variance in the previous month and short in the factor with highest realized variance in the previous month.[22]

[22]The factor(s) which do not have the highest or lowest variance in the previous month, are not invested in.

4.2 Forecasting variance

In our main results, we use lagged realized variance as a proxy for realized variance. We also consider two other sophisticated forecasting models to predict monthly variance. The hypothesis is that if forecasted variance is a better proxy for realized variance than its own lagged value, the risk-adjusted returns of the volatility managed portfolios will be higher.

An important feature of the volatility managed portfolios is that the strategy is easy implementable for investors in real time. We therefore initially consider a relatively simple forecasting model, less sophisticated than forecasting models such as the ARCH and GARCH models. We emphasize that our intention is not to find the optimal forecasting model, but to show that forecasted variance can be a better proxy for realized variance than its own lagged value, and subsequently improve the results for the volatility managed portfolios.

In the following, we will present our two forecasting models. We start by presenting a model which we call the "OLS method". This model is relatively easy to implement for the average investor.

4.2.1 OLS forecasting

As Ederington and Guan (2004) point out, several past studies have considered the forecasting ability of linear regression models.[23] We forecast variance using ordinary least squares regressions (OLS). In this method, we define realized variance in a month as the sum of the squared daily returns in that particular month.[24] We then run an OLS regression of the realized monthly variance on lagged realized monthly variance, and use the coefficients to forecast future variance. Formally:

We define the daily return for a given day d in a given month m as r_d. The realized variance (σ_m^2) in a given month m is then estimated as:

[23] These models have typically forecasted volatility as $\sigma_{t+1} = \alpha_0 + \sum_{n=0}^{N} \alpha_n r_{t-n}^2$, where the forecasted volatility is a function of the squared residuals.

[24] Note that this definition of realized variance is **only** used for the OLS forecasting. When we evaluate the performance of both the OLS forecasted variance and the GARCH forecasted variance, we compare the forecasted variance with realized variance as we defined it in section 4.1.

$$\sigma_m^2 = \sum_{d=1}^{d} r_d^2 \tag{10}$$

We then run an OLS regression of the realized monthly variance (σ_m^2) on the lagged monthly realized variance (σ_{m-1}^2):

$$\sigma_h^2 = \alpha + \beta\sigma_{m-1}^2 + \epsilon \tag{11}$$

We use the obtained coefficients explicitly in the forecasting of monthly variances. The forecasted variance in month m+1 is:

$$\hat{\sigma}_{m+1}^2 = \hat{\alpha} + \hat{\beta}\sigma_m^2 \tag{12}$$

The idea is to exploit the close relationship between realized variance and lagged realized variance, knowing that variance is highly forecastable in the short-term.[25] By using the beta coefficient explicitly to forecast variance, the forecasted variance reflects the linear relationship between realized variance and lagged realized variance.[26]

We initially include the whole sample in the regression to estimate the coefficients and call the forecasted variance "OLS Whole Sample". Note that because we use the whole sample, the method is not directly implementable for investors in real time. In order to make an out-of-sample forecast, i.e. making the method implementable in real time, we create a rolling window of 24 months when estimating the coefficients with the OLS regressions. The estimated alpha and beta coefficients used for the forecast at any given month m are dependent on the observations from the past 24 months.[27] The forecasted variance for month m is then:

[25] We show later in the thesis that variance predicts future variance. Also, Andersen and Bollerslev (1997) state that volatility is time-varying and predictable.

[26] In other words, it is an autoregressive model. The method is relatively similar to a *Simple Regression* method which, according to Granger and Poon (2003), forecasts volatility as a function of its past values and an error term, i.e. it is principally an autoregressive method.

[27] That is, we obtain the coefficients to forecast variance in month 25 by running an OLS regression of realized variance on lagged realized variance based on month 1 to 24. For month 26, we estimate the coefficients based on month 2 to 25, and so on. Note that we also tried different length on the rolling windows, but that 24 months in general gave the best results.

$\hat{\sigma}^2_m = \alpha|\theta_{m-i} + (\beta|\theta_{m-i})\sigma^2_{m-1}, i =\in [1, 24]$

where we define θ_{m-i} as the available information in month m; that is, the observation in the past 24 months. σ^2_{m-1} is the realized variance which we defined in equation 10.

In the following, we consider a more sophisticated GARCH model to forecast variance.

4.2.2 GARCH forecasting

In this section, we consider the GARCH model introduced by Bollerslev (1986) in order to forecast variance. As Starica (2003) points out, "the GARCH model is widely used and highly regarded in practice as well as in the academic discourse"(Starica, 2003, page 5). One of the reasons why there is a preference for the GARCH model, is that it generally requires less parameters to be estimated in order to capture the volatility process, compared to the ARCH model. Another important property of the GARCH model is that it treats heteroskedasticity as a variance to be modeled.[28] Furthermore, financial data often exhibits time-varying volatility, i.e. periods of high volatility and periods of low volatility. This is called volatility clustering and is also observed in our financial data.[29] GARCH models are designed to deal with volatility clustering, as pointed out by Engle (2001). The general GARCH (p,q) model looks like:[30]

$$\sigma^2_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon^2_{t-i} + \sum_{i=1}^{q} \beta_i \sigma^2_{t-i} \tag{13}$$

From equation (13), we see that the forecasted variance (σ^2_t) is dependent on a weighted average of past residuals (ϵ^2_{t-i}) and own previous lags (σ^2_{t-i}).

We use a GARCH (1,1) model when forecasting variance. The GARCH (1,1) model looks like:

[28]Heteroskedasticity is when variances of the error terms are not equal, and is often observed in financial data. As Engle (2001) points out, least squares models assumes that the expected value of all error terms when squared is the same at any given point. This is called homoskedasticity.

[29]See Figure 5 in Appendix C where we plot monthly volatility of each individual factor in Norway.

[30]See appendix B for more information on the methodology of the ARCH and GARCH models

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{14}$$

We do not consider any AIC/BIC criteria in order to find the optimal GARCH specification for our model, and we emphasize again that the intention of this thesis is not to find the optimal forecasting model. Various research have concluded that the GARCH (1,1) model forecasts volatility well. For instance, Akgiray (1989) concluded that a GARCH (1,1) model showed a better forecasting capacity when compared to other traditional models. Hansen and Lunde (2005) find no evidence that a GARCH (1,1) model is outperformed by more sophisticated models when they study various ARCH-type models in terms of their ability to describe the conditional variance.

Initially, we used monthly return data to forecast monthly variances, but this gave unsatisfying results. As Goyal (2000) points out, it is likely that volatility estimated from daily data is more precise than GARCH forecasted volatility based on monthly data, due to higher frequency of data. Figlewski (2004) comes to a similar conclusion, and found that the GARCH methodology performed better with daily data when the forecasting horizon is quite short.[31] He emphasises that the GARCH model can be improved by using daily data due to an increase in data points.

We forecast monthly variance as follows. First, we use daily return data and forecast daily volatility using a GARCH (1,1) model. We call the daily forecasted volatility $\hat{\sigma}_d$. The GARCH coefficients are obtained using an estimation window of 24 months.[32] We then convert the forecasted daily volatility into an estimation of monthly volatility by taking the square root of the number of trading days in the month multiplied with the average of the forecasted daily volatilities in the month[33]:

$$\hat{\sigma}_m = \sqrt{td}\, \frac{\sum_{i=1}^{td} \hat{\sigma}_d}{td} \tag{15}$$

Where we define td as the number of trading days in any given month.

[31]Figlewski (2004) examined using daily data to forecast both daily and monthly volatility.

[32]That is, the GARCH coefficients used for the forecast of any given day is obtained from the previous 24 months.

[33]Diebold et al. (1997) point out that there are no known analytic methods for computing h-day volatilities from 1-day volatilities. Our method is therefore not very sophisticated and we are aware of this.

To convert monthly volatility to monthly variance, we simply square the monthly volatility and obtain the forecasted monthly variance, $\hat{\sigma}_m^2$, which is our proxy of realized variance.[34]

In the next section, we report our main results.

[34] We use $\hat{\sigma}_m^2$ when re-constructing the volatility managed portfolios later in the thesis.

5 Main Results

In this section, we report the results of the volatility managed portfolios where we use lagged realized variance. First, we report the results for the Norwegian single factors and mean-variance efficient portfolios. Next, we report the results for the U.K. single factors. We then show the results for industry portfolios both in Norway and the U.S. Lastly, we show the results from the long-short portfolios.[35]

5.1 Results for the Norwegian factors

Before reporting our main results, we refer to appendix A where we show the results of replicating Moreira and Muir (2016a) for the U.S. factors MKT, SMB, HML and MOM. We do this to verify that the method we use is the same as their method. We obtain very similar results.[36]

Table 1, Panel A reports the results for the Norwegian single factors. The results are obtained from running a regression of the volatility managed factors on the non-managed factors. We see positive and significant alphas for the market factor, SMB and the momentum factor. The liquidity factor is positive and marginally significant[37], while HML is negative and insignificant. The largest alpha is for the momentum factor, with an annualized alpha of 8,2 percent. This is consistent with previous research on managing volatility. Moreira and Muir (2016a) also find the largest alpha for the momentum factor, while Barroso and Santa-Clara (2015) find that managing momentum almost doubles the Sharpe ratio compared to the non-managed factor. See also Daniel and Moskowitz (2016). As Moreira and Muir (2016a) point out, a positive alpha implies that the managed portfolios expand the mean-variance frontier relative to the non-managed portfolios. In other words, the risk-adjusted performance is better for the managed factors than for the non-managed factors. Note that we find a positive alpha for SMB and negative alpha for HML, while Moreira and Muir (2016a) find the opposite.

[35] Note that Moreira and Muir (2016a) do not consider this long-short strategy, thus this strategy is not a replication of their work.

[36] For the market factor, we get an annualized alpha of 4,87 (Moreira and Muir (2016a) got 4,86), for momentum 12,53 (12,51), for SMB -0,51 (-0,58), for HML 1,88 (1,97). Note that we get the same R^2 and very similar RMSE. To see the results of Moreira and Muir (2016a), see Table 1 in their paper.

[37] $P - value < 0.10$

We also report the appraisal ratio (AR), which measures how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed factors, i.e. the increase in Sharpe ratios. Specifically, the appraisal ratio is calculated as $\frac{\alpha}{RMSE}\sqrt{12}$ where multiplying with 12 annualizes the ratio and RMSE is the Root Mean Square Error. The appraisal ratios indicate large utility gains for the significant factors, and range from 0,30 to 0,60. Momentum has the highest appraisal ratio of 0,60 while the other appraisal ratios are: SMB (0,41), Market (0,35), Liquidity (0,30). HML, which is not significant, has an appraisal ratio of (-0,22).

Figure 4 in Appendix D plots the monthly weights of the volatility managed portfolio for the market factor versus the realized monthly variance.[38] We see clearly that following a spike in realized variance, the weight of the market factor goes substantially down. This is consistent with the theoretical properties of the strategy, i.e. that the portfolios decrease the risk exposure when volatility is high and vice versa. In figure 5 in Appendix D we plot the monthly volatility of each individual factor in Norway. We see signs of volatility clustering, and also that volatility for the factors follow the same cycle. When volatility is high for one factor, it is in general more likely that the volatility is also high for other factors.

In panel B we show the results when testing for subsamples. We split the sample into two periods, 1981-1998 and 1998-2015. Moreira and Muir (2016a) test for subsamples only for mean-variance efficient portfolios, but we find it appropriate to test the single factors to see if the positive alphas still remain for shorter time intervals. In the first subperiod 1981-1998, the market has a significant annualized alpha of 7,36, while UMD is marginally significant and has an annualized alpha of 5,92. SMB has an insignificant alpha, while HML has a negative and significant alpha. In the subsample 1998-2015, the momentum factor has a large significant alpha of 10,6, while SMB has a marginally significant alpha of 4,0. The market has a positive alpha but insignificant alpha. Note that HML in this subperiod has a positive insignificant alpha, compared to a negative significant alpha in the subperiod 1981-1998.

Compared to using the whole sample, the results in the subperiods are not as consistent. We emphasize that we have a relatively small sample compared to Moreira and Muir (2016a) where the sample is much larger. A possible explanation could be that the sample is too small when testing for subperiods.

[38] We only show this for the market factor, but the pattern is the same for all single factors.

Table 1: **Results, single factors, Norway.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increase the Sharpe ratio. The sample is 1981-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

Panel A: Full sample portfolios

	σMKT	σSMB	σHML	σUMD	σLIQ
MKT	0,76				
	(14.5)				
SMB		0,78			
		(12.9)			
HML			0,72		
			(12.8)		
UMD				0,74	
				(14.4)	
LIQ					0,78
					(15.7)
α	4,9	3,9	-2,6	8,2	3,2
t-stat	1,96	2,42	-1,3	3,75	1,79
R^2	0,57	0,62	0,52	0,55	0,61
N	431	412	413	418	419
RMSE	48,48	32,76	41,71	47,00	36,34
AR	0,35	0,41	-0,22	0,60	0,30

Panel B: Sub-periods portfolios

1981-1998

	σMKT	σSMB	σHML	σUMD
α	7,36	2,61	-8,55	5,92
t-stat	1,97	1,3	-2,43	1,79

1998-2015

	σMKT	σSMB	σHML	σUMD
α	2,1	4,0	2,8	10,6
t-stat	0,61	1,78	1,07	3,76

In table 2 we show the results of managing volatility of mean-variance efficient portfolios, similar to what Moreira and Muir (2016a) do.[39] The mean-variance efficient portfolios are constructed as follows. We first combine several factors[40] and find constant weights so that the multifactor portfolio is mean-variance efficient, using the whole sample of monthly return data and the average risk-free rate. We then find the monthly return of the mean-variance efficient portfolio by multiplying the constant weights with the monthly returns. We define this as the static multifactor portfolio, which we volatility manage using the methodology from section 4.1:

$$f_{t+1}^{MVE_\sigma} = \frac{c}{RV_t} f_{t+1}^{MVE} \tag{16}$$

where $f_{t+1}^{MVE_\sigma}$ is the volatility managed multifactor portfolio and f_{t+1}^{MVE} is the static multifactor portfolio. Note that the relative weights of single factors within both the static and volatility managed multifactor portfolio are held constant. Thus, the managed multifactor portfolio only scales in and out of the portfolio and a risk-free asset. Also note that in order to find realized variance, RV_t, which is used to scale the managed multifactor portfolio on a monthly frequency, we use daily multifactor returns.[41] The daily multifactor returns are calculated by multiplying the constant multifactor weights with daily return data. Because we use the whole sample to

[39] See Table 2 in their paper.

[40] FF3 is the original Fama-French factors, where we included Momentum in FF4 and Liquidity in FF5.

[41] This is consistent with how we calculate realized variance for single factors, look at section 4.1

estimate the weights of the mean-variance efficient portfolios, the strategy is not implementable for investors in real time. However, the intention of managing volatility of mean-variance efficient portfolios is to analyze whether the volatility managing strategy expands the mean-variance frontier for investors who are already invested in multiple factors, as Moreira and Muir (2016a) point out. Investors could easily construct mean-variance efficient portfolios in real time based purely on historical information.[42]

We see large and significant alphas and high appraisal ratios for all of the multifactor portfolios. In fact, the appraisal ratios for the managed multifactor portfolios are in general higher than for the managed single factors. This is consistent with the findings of Moreira and Muir (2016a).[43] FF4 has the highest annualized alpha of 5,02, while FF3 and FF5 have alphas of 4,96 and 4,70, respectively. The appraisal ratio is 0,68 for FF5, 0,63 for FF3 and 0,60 for FF4. This implies large utility gains relative to the static mean-variance efficient portfolios. This might be an indication that the utility gains of the strategy we consider in general is linked to the time-series of returns and not the cross-section of returns; i.e. that it is not the single risk of an asset that makes the strategy beneficial to investors but the risk of the assets over time.

As a robustness check, we run regressions of both the managed Norwegian single factors and the multifactor portfolios on well-known risk factors:

$$f^{\sigma}_{t+1} = \alpha + \beta_{Mkt} Mkt_{t+1} + \beta_{HML} HML_{t+1} + \beta_{SMB} SMB_{t+1} + \beta_{UMD} UMD_{t+1} + \beta_{LIQ} LIQ_{t+1} + \epsilon_{t+1}$$

We test if the alphas remain significant when controlling for exposure to well-known risk factors. The results can be seen in Table 3. We see that all single factors remain significant except for the market factor. This indicates that the high Sharpe ratio of the volatility managed market factor can be explained by other risk factors. As for the other managed portfolios, both single factors and the multifactor portfolios, the alphas remain significant when controlling for risk exposure. The significance of the alphas does not change when running a regression using only the original Fama-French 3-factors as independent variables.

[42] In fact, Moreira and Muir (2016a) argue that the original mean-variance efficient Sharpe ratios are likely to be overstated relative to the truth due to the in-sample bias, thus the potential increase in Sharpe ratios are likely to be understated.

[43] Again: See Table 2 in their paper.

Table 2: **Results mean-variance efficient portfolios, Norway.** The table reports the regression results of the volatility managed mean-variance efficient portfolios on the non-managed mean-variance efficient portfolios: $f_{t+1}^{MVE_\sigma} = \alpha + \beta f_{t+1}^{MVE} + \epsilon_{t+1}$. The mean-variance efficient portfolios are formed in-sample and represent the relevant information set for a given investor. The volatility managed mean-variance efficient portfolios are constructed based on monthly data: $f_{t+1}^{MVE_\sigma} = \frac{c}{RV_t} f_{t+1}^{MVE}$, where RV_t is the realized variance in month t, calculated based on daily data. The factors are annualized in percent by multiplying factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed mean-variance portfolios expand the slope of the MVE frontier compared to the non-managed mean-variance portfolios.

	σMKT	σFF3	σFF4	σFF5
MKT	0,76			
	(14.5)			
FF3		0,75		
		(14.9)		
FF4			0,76	
			(15.9)	
FF5				0,77
				(15.4)
α	4,9	5,0	5,0	4,7
t-stat	1,96	3,46	3,75	3,22
R^2	0,57	0,56	0,57	0,59
N	431	413	413	413
RMSE	48,48	27,10	25,52	27,07
AR	0,35	0,63	0,68	0,60

Table 3: **Exposure to Fama-French factors, volatility managed single factors and mean-variance efficient portfolios.** The tables reports the regression results of the volatility managed portfolios on the Fama-French 5-factor model: $f^{\sigma}_{t+1} = \alpha + \beta_{Mkt}Mkt_{t+1} + \beta_{HML}HML_{t+1} + \beta_{SMB}SMB_{t+1} + \beta_{UMD}UMD_{t+1} + \beta_{LIQ}LIQ_{t+1} + \epsilon_{t+1}$. The sample is 1981-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily return data. The factors are annualized in percent by multiplying monthly factors by 12. We report betas with t-stats in parantheses. The betas represent the managed portfolios' exposure to the Fama-French factors.

	σMKT	σSMB	σHML	σUMD	σLIQ	σFF3	σFF4	σFF5
MKT	0,82	-0,04	-0,06	0,14	-0,05	0,44	0,42	0,43
	(12.95)	(-1.14)	(-1.52)	(3.37)	(-1.37)	(13.45)	(14.35)	(13.78)
SMB	0,03	0,81	-0,05	0,07	-0,03	0,34	0,31	0,37
	(0.53)	(12.79)	(-0.87)	(1.15)	(-0.74)	(9.23)	(9.52)	(10.17)
HML	-0,02	-0,03	0,72	0,01	0,03	-0,07	-0,06	-0,05
	(-0.54)	(-0.73)	(12.9)	(0.29)	(0.89)	(-2.9)	(-2.76)	(-2.23)
UMD	0,02	-0,02	0,00	0,77	-0,03	0,04	0,07	0,07
	(0.47)	(-1.14)	(0.00)	(15.37)	(-1.23)	(1.56)	(3.35)	(2.97)
LIQ	0,11	-0,08	-0,01	0,03	0,77	-0,06	-0,04	-0,11
	(1.68)	(-1.48)	(-0.27)	(0.53)	(12.41)	(-1.44)	(-1.13)	(-3.03)
α	3,14	5,00	-0,78	4,44	5,07	5,20	5,36	5,04
t-stat	1,17	2,81	-0,44	2,12	2,41	3,61	4,05	3,5
R^2	0,55	0,62	0,52	0,56	0,6	0,64	0,61	0,59
N	414	412	413	414	414	413	413	413

5.2 Results for the UK factors

In this section, we report the results of the volatility managed portfolios for the UK factors. Moreira and Muir (2016a) do not include UK factors when implementing the strategy on various countries. Thus, we want to analyze whether the strategy of managing volatility gives similar results for the UK. We also considered other countries, but we choose to only implement the strategy in the UK where the data is more reliable and avoiding possible issues with outliers, missing values and smaller samples.[44]

Table 4 shows the results from the three original Fama-French factors plus the Carhart momentum factor. We see that the momentum factor is the only factor that has a positive and significant alpha. This is consistent with the findings of both Moreira and Muir (2016a) and various other research. For instance, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) show similar results. The other factors are insignificant. The HML factor is the only factor with a negative alpha for the UK factors, similar to what we found for the Norwegian factors.

Due to the general weak results for the UK factors, we choose not to include them in any further analysis. We emphasize that the sample is 1988-2015, i.e. an even smaller sample than for the Norwegian factors. We do not know whether the small sample affects our results in any particular way.

[44]We considered implementing the strategy for factors in Germany, France and Japan, but these countries have even smaller samples than the UK.

Table 4: **Results single factors, UK.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f^{\sigma}_{t+1} = \alpha + \beta R_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increase the Sharpe ratio. The sample is 1988-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily return data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

	σMKT	σSMB	σHML	σUMD
MKT	0,72			
	(11.44)			
SMB		0,79		
		(11.03)		
HML			0,65	
			(7.76)	
UMD				0,53
				(7.93)
α	1,01	0,98	-0,99	13,09
t-stat	0,54	0,73	-0,58	4,97
R^2	0,52	0,63	0,42	0,29
N	320	320	320	320
RMSE	34,61	24,11	30,23	47,08
AR	0,10	0,14	-0,11	0,96

5.3 Results for the industry portfolios

Table 5 and 6 report the results of managing volatility for the Norwegian and U.S. industry portfolios, respectively.[45] Moreira and Muir (2016a) do not consider industry portfolios, and we would like to see if the strategy achieves excess return for other portfolios than systematic factors. In general, using alpha as a measure of portfolio performance assumes that the portfolios are well diversified, thus eliminating unsys-

[45]See section 3 for description of the industry portfolios.

tematic risk. The industry portfolios are less diversified than systematic factors such as the Fama-French factors, but are on the other hand more diversified than single securities.[46] Because the industry portfolios are not as well diversified as the pricing factors, we find it hard to compare the alphas and appraisal ratios directly. Even though there are short-comings with the analysis of the industry portfolios, we still find it interesting to examine them.

Panel A in Table 5 shows the results of running regressions of the volatility managed industry portfolios on the non-managed industry portfolios for Norwegian industries from 1980-2015. We would like to emphasize that some industry portfolios in Norway occasionaly consist of a small number of individual securities, and these portfolios are therefore less diversified than both the systematic factors and the U.S. industry portfolios.[47]

We see positive and significant alphas for the majority of the managed industry factors in Panel A. Material, Consumer Discretionary (ConsDisc), Consumer Staples (ConsStapl) and Finance all have large positive and significant alphas. The largest is found for Material, with a significant alpha of 16,72 percent. Energy, Industry and Health all have positive alphas, but they are not statistically significant. The highest appraisal ratio is found for Finance. As a robustness check, we also run regressions of the managed portfolios on the three original Fama-French factors plus momentum to see whether the excess returns can be explained by exposure to well-known risk factors. From Panel B Only ConsStapl and Health now show a significant alpha. We see that the alpha of Material goes from 16,72 to -2,68 and that the excess return can be explained by a large exposure to the market factor. ConsDisc still has a positive alpha, but it is insignificant.

Table 6 reports the results for the volatility managed U.S industry portfolios from 1926-2015. In Panel A, we see that the majority of the managed industry portfolios have large significant alphas. Out of the 10 managed portfolios, only Consumer NonDurables (NoDurbl) and Health have insignificant alphas. The positive and significant alphas range from 5,25 percent to 7,23 percent, where Other has the highest alpha of 7,23 percent. The appraisal ratios are also substantial for the majority of the managed portfolios. The finding of large alphas and appraisal ratios for the in-

[46]A positive alpha only implies that the portfolios expand the efficient frontier if we consider well-diversified portfolios that summarize pricing information for a wide cross-section of assets.

[47]The data for the Norwegian industries also has several missing values. We do not know whether the missing values affect the results in any particular way.

dustry portfolios indicates that managing the volatility for less diversified portfolios than systematic factors also leads to higher risk-adjusted returns and utility gains for mean-variance investors.

We test for exposure to well-known risk factors and report the results in Panel B. We see that now all of the 10 managed portfolios have positive and significant alphas. Thus, the excess return can not be explained by exposure to the risk factors. Some factors, such as the Consumer Durables (Durbl), Hitec, Manufacturing (Manuf) and Others, have a large exposure to the market factor but still has a significant alpha.

Table 5: **Results Industry Portfolios, Norway.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f^{\sigma}_{t+1} = \alpha + \beta R_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increase the Sharpe ratio. The sample is 1981-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily return data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

Panel A: Industry Portfolios, Norway

	σEnergy	σMaterial	σIndustry	σConsDisc	σConsStapl	σHealth	σFinance
Energy	0,8						
	(16.01)						
Material		0,49					
		(2.76)					
Industry			0,76				
			(13.48)				
ConsDisc				0,67			
				(8.54)			
ConsStapl					0,73		
					(13.68)		
Health						0,75	
						(12.09)	
Finance							0,6
							(11.15)
α	4,41	16,72	2,58	12,55	8,90	3,60	11,57
t-stat	1,55	2,91	0,88	3,09	3,39	1,18	3,76
R^2	0,65	0,24	0,58	0,46	0,53	0,56	0,37
N	431	431	431	431	431	431	431
RMSE	56,78	126,16	60,42	91,09	60,76	68,75	65,82
AR	0,27	0,46	0,15	0,48	0,51	0,18	0,61

Panel B: Exposure to Fama-French factors. The tables reports the regression results of the volatility managed portfolios on the Fama-French 4-factor model: $f^{\sigma}_{t+1} = \alpha + \beta_{Mkt}Mkt_{t+1} + \beta_{HML}HML_{t+1} + \beta_{SMB}SMB_{t+1} + \beta_{UMD}UMD_{t+1} + \epsilon_{t+1}$. The sample is 1981-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily return data. The factors are annualized in percent by multiplying monthly factors by 12. We report betas with t-stats in parantheses. The betas represent the managed portfolios' exposure to the Fama-French factors.

	σEnergy	σMaterial	σIndustry	σConsDisc	σConsStapl	σHealth	σFinance
MKT	0,87	1,15	0,83	0,93	0,63	0,50	0,6
	(12.27)	(9.63)	(11.37)	(9.97)	(10.06)	(7.08)	(11.12)
SMB	0,047	0,19	0,04	0,435	0,07	-0,06	0,2
	(0.72)	(1.62)	(0.58)	(3.91)	(0.70)	(-0.74)	(3.05)
HML	0,10	0,22	0,06	0,03	-0,00	-0,22	0,09
	(1.59)	(2.24)	(1.08)	(0.34)	(-0,07)	(-2.83)	(2.09)
MOM	0,01	-0,08	0,03	0,02	0,13	0,04	-0,02
	(0.12)	(-1.13)	-0,69	(0.2)	(2.17)	(0.81)	(-0.42)
α	-0,44	-2,68	0,80	5,01	11,27	9,27	3,48
t-stat	-0,11	-0,46	-0,23	0,97	3,07	2,17	1.,02
R^2	0,44	0,34	0,47	0,26	0,25	0,16	0,31
n	414	414	414	414	414	414	414

Table 6: **Results Industry Portfolios, U.S.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f^{\sigma}_{t+1} = \alpha + \beta R_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increase the Sharpe ratio. The sample is 1926-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily return data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

Panel A: Managed Industry Portfolios, U.S.

	σNoDurbl	σDurbl	σManuf	σEnergy	σHitec	σTelecom	σShops	σHealth	σUtils	σOthers
NoDurbl	0,6									
	(9.16)									
Durbl		0,63								
		(8.4)								
Manuf			0,6							
			(9.18)							
Energy				0,61						
				(14.43)						
Hitec					0,62					
					(11.8)					
Telecom						0,35				
						(11.76)				
Shops							0,61			
							(11.28)			
Health								0,67		
								(11.11)		
Utils									0,44	
									(9.66)	
Others										0,59
										(8.9)
α	3,03	5,82	6,49	5,25	6,70	6,64	6,84	3,08	6,83	7,23
t-stat	1,85	2,78	3,67	3,19	3,32	4,46	3,87	1,83	3,74	3,90
R^2	0,36	0,39	0,35	0,37	0,38	0,12	0,37	0,45	0,19	0,34
N	1081	1081	1081	1081	1081	1081	1081	1081	1081	1081
RMSE	44,20	72,23	60,42	57,97	68,75	51,74	55,79	49,92	59,93	62,72
AR	0,24	0,28	0,37	0,31	0,34	0,44	0,42	0,21	0,39	0,40

Panel B: Exposure to Fama-French factors The tables reports the regression results of the volatility managed portfolios on the Fama-French 4-factor model: $f^{\sigma}_{t+1} = \alpha + \beta_{Mkt} Mkt_{t+1} + \beta_{HML} HML_{t+1} + \beta_{SMB} SMB_{t+1} + \beta_{UMD} UMD_{t+1} + \epsilon_{t+1}$. The sample is 1926-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily return data. The factors are annualized in percent by multiplying monthly factors by 12. We report betas with t-stats in parantheses. The betas represent the managed portfolios' exposure to the Fama-French factors.

	σNoDurbl	σDurbl	σManuf	σEnergy	σHitec	σTelecom	σShops	σHealth	σUtils	σOthers
MKT	0,47	0,87	0,76	0,53	0,866	0,295	0,63	0,63	0,41	0,738
	(9.6)	(14.3)	(15.99)	(12.74)	(15.32)	(10.76)	(12.43)	(12.42)	(11.02)	(15.17)
SMB	0,06	-0,006	-0,02	-0,17	0,0137	-0,07	0,058	-0,009	-0,12	0,04
	(1.11)	(-0.08)	(-0.31)	(-2.87)	(0.19)	(-2.32)	(0.92)	(-0.17)	(-2.40)	(0.53)
HML	0,002	-0,07	-0,067	0,22	-0,215	0,009	-0,13	-0,13	-0,005	0,06
	(0.06)	(-0.99)	(-1.24)	(3.58)	(-3.60)	(0.27)	(-2.7)	(-2.79)	(-0.13)	(0.99)
MOM	0,16	0,22	0,26	0,2	0,288	0,1	0,17	0,21	0,127	0,26
	(4.87)	(4.32)	(6.6)	(4.61)	(6.99)	(4.42)	(4.85)	(5.73)	(3.69)	(5.95)
α	4,91	5,94	6,12	6,36	6,72	7,2	8,4	5,88	7,56	5,33
t-stat	(2.87)	(2.48)	(3.25)	(3.31)	(3.09)	(4.2)	(4.37)	(3.1)	(3.77)	(2.72)
R^2	0.29	0.32	0.37	0.209	0.36	0.09	0.31	0.32	0.13	0.35
N	1069	1069	1069	1069	1069	1069	1069	1069	1069	1069

5.4 Results for the Long-short strategy

In the following, we report the results from the long-short strategy for the single factors in Norway. This strategy is conceptually different from the original volatility timing strategy: We form self-financed two- and multifactor portfolios where we go long $1 in the factor with the lowest volatility in the previous month, and short $1 in the factor with highest volatility.[48] In this strategy, we time volatility in the cross-section of returns and not in the time-series of returns. We do this to see if it

[48]See section 4.1.2.

is possible to capture abnormal returns by timing volatility between factors rather than scaling in and out of single factors and a risk-free asset as Moreira and Muir (2016a) do. Previous research on the cross-section of risk and returns has typically examined the low volatility puzzle and formed quantile portfolios based on volatility that rebalance every month, see for instance Baker and Haugen (2012). They argue that the low volatility puzzle is present in Norway.

Table 5 reports the results for the long-short strategy where we run regressions of the long-short strategy returns on the factors used in the respective strategy. MKT-UMD is a strategy where we go long in the market if the volatility of the market was lower than the volatility of UMD in the previous month, this position is financed by going short in the UMD factor. If the volatility of the market was higher previous month, we go short in the market and long in UMD. The portfolio is rebalanced every month. The strategies SMB-UMD and SMB-MKT follow the same principle. In "ALL" we consider all the Fama French factors plus the liquidity factor in Norway. Note however that we only go long in one factor and short in one factor each month, where we go long in the factors with the lowest volatility in the previous month and short in the factor with highest volatility.

We see positive but insignificant alphas for all the long-short strategies, and we are not able to conclude that this strategy is able to increase the wealth of investors. We have also implemented the strategy for the U.S. single factors as well as both U.S and Norwegian industries, these results are very similar to the one we report on the Norwegian single factors. Therefore, we choose not to present them.[49]

[49] We considered other long-short strategies with other specifications, but as the results were weak we choose not to report these strategies.

Table 7: **Results Long-short strategy, single factors, Norway.** The table reports the regression results of the volatility managed long-short portfolios on the non-managed factors: $f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. The sample is 1981-2015. The managed portfolios are constructed based on monthly data and goes long (short) in the factor(s) that has the lowest (highest) realized variance in the previous month. The realized variance in month t is calculated based on daily return data. The factors are annualized in percent by multiplying monthly factors by 12.

	σMKT-UMD	σSMB-UMD	σSMB-MKT	σ SMB-UMD-MKT	σAll
α	4,14	5,48	7,28	8,16	-2,99
t-stat	0,71	1,34	1,09	1,26	-0,60
R^2	0,10	0,09	0,01	0,09	0,07
N	413	413	413	413	413

6 The Performance of Forecasting Models

We want to improve the results of the volatility managing portfolios by using forecasted variance rather than the previous month's realized variance.[50] The hypothesis is that if forecasted volatility is a better proxy for realized variance than its own lagged value, the performance of the volatility manged portfolios will increase. In this section, we consider the forecasted variance by the OLS method and the GARCH method. First, we evaluate the forecasted variance based on its ability to predict realized variance. We compare the relationship between forecasted variance and realized variance with the relationship between realized variance and its lagged value. Next, we re-construct volatility managed portfolios using the forecasted variance rather than lagged realized variance.

6.1 Evaluating the Variance Forecasts

As Lopez (2001) points out, the mean-square error (MSE) criterion is commonly used to evaluate out-of-sample forecasts of volatility.[51] We focus on the root mean square error (RMSE) to evaluate the forecasting models. In our analysis, the RMSE is the root of the mean squared differences between the realized variance, σ_n^2, and the forecasted variance, $\hat{\sigma}_n^2$.[52] Formally, RMSE is calculated as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\sigma_n^2 - \hat{\sigma}_n^2)} \tag{17}$$

We obtain the RMSE by running an OLS regression of realized variance on the forecasted variance. In general, the smaller the RMSE, the better the forecasting ability. In addition to RMSE, we report the correlation between the forecasted variance and the realized variance, and compare it to the correlation between the realized variance and its lagged value. A small RMSE and high correlation indicates a better proxy for realized variance, which is what we try to find. We do not report the R^2

[50] Moreira and Muir (2016a) show in their appendix that the strategy can be improved through the use of sophisticated models of variance forecasting. They use three lags of realized log variance to form a forecast, i.e. an AR (1) model.

[51] For instance, Medeiros et al. (2004) recommend the RMSE for the purpose of comparing volatility forecasts.

[52] Realized variance is as before calculated as shown in section 4.1.

from the regressions, but a lower RMSE generally yields a higher R^2 in our regressions.[53]

6.1.1 The OLS Method forecast

In Table 9 we show the correlation and RMSE between the forecasted variance from the OLS method and realized variance, and we compare it to the correlation and RMSE between the realized variance and lagged realized variance. We show this for the Norwegian single factors. Panel A reports the results from using the whole sample to estimate the forecasting coefficients, i.e. an in-sample forecast. As one would expect, the results of the OLS method using the whole sample are very similar to the results of lagged realized variance. Using the whole-sample beta as a constant weight to forecast monthly variance, would necessarily lead to a quite similar proxy for realized variance as its lagged value.

Panel B shows the results when using a rolling window of 24 months to estimate the forecasting coefficients, i.e. the out-of-sample forecast.[54] The results imply that the forecasted variance by the OLS rolling method is a less satisfying proxy for realized variance than lagged realized variance. The correlation is lower and the RMSE is higher for all factors, except for the HML factor where the RMSE is equal as when using lagged realized variance. The results are not very surprising, because the forecasting model is not very sophisticated.[55] We do not report the forecasted variance using the OLS method for the U.S. and Norwegian industry factors, because they give similar results. We are aware that the OLS forecasting model is slightly naive, and that a more sophisticated forecasting model in general would give better results.

[53]The R^2 values are available on request.

[54]Note that in the rolling OLS, due to the estimation window, we do not forecast any variance the first 24 months of the sample, and we therefore report the lagged realized variance in Panel B for the same period as the OLS rolling forecasted variance.

[55]Ederington and Guan (2004) point out that linear regression models empirically do not forecast out-of-sample volatility in a particularly good way.

Table 8: **OLS forecasted variance vs. Lagged Variance, Norway single factors.** The table reports the correlation and RMSE between the realized variance of single factors and 1) lagged realized variance (Lagged variance) and 2) the OLS forecasted variance (OLS full sample). The values give us a sense for the relation between realized variance and the proxies of realized variance, where a high correlation and low RMSE in general indicates a close relation.

Panel A: OLS whole sample

Lagged Variance	MKT	SMB	HML	UMD	LIQ
Correlation	0,61	0,75	0,32	0,38	0,55
RMSE	0,28	0,14	0,23	0,23	0,22

OLS	MKT	SMB	HML	UMD	LIQ
Correlation	0,62	0,78	0,32	0,37	0,57
RMSE	0,24	0,14	0,23	0,24	0,22

Panel B: OLS Rolling

Lagged Variance	MKT	SMB	HML	UMD	LIQ
Correlation	0,61	0,76	0,27	0,35	0,56
RMSE	0,28	0,14	0,23	0,22	0,22

GARCH	MKT	SMB	HML	UMD	LIQ
Correlation	0,42	0,72	0,22	0,25	0,52
RMSE	0,31	0,16	0,23	0,23	0,23

6.1.2 The GARCH method forecast

In Table 10 we evaluate the forecasting ability of the GARCH method, as we did for the OLS method in Table 9. Again, we calculate RMSE and correlation between the forecasted variance and the realized variance. For the GARCH method, we report the Norwegian single factors, the Norwegian industry portfolios and the U.S. industry portfolios. We choose to include the industry portfolios when examining the GARCH method, because the method gave more satisfying results than the OLS method. Panel A shows the Norwegian single factors. We see that the GARCH forecasted variance has a higher correlation with realized variance for four out of five factors, if we compare to lagged realized variance; only for the liquidity factor the GARCH forecasted variance has a higher relative RMSE. These results indicate that the GARCH forecasted variance is a good proxy for realized variance for these factors.

The results are less consistent for the Norwegian industry portfolios, as shown in panel B. The GARCH forecasted variance has a lower correlation for 4 out of 7 industries, if we compare to lagged variance. The RMSE is lower or equal for the GARCH forecasted variance compared to lagged realized variance for 3 out of 7 industries. For the U.S. industry portfolios the results are more consistent, shown in panel C. The GARCH forecasted variance has a high correlation for the majority of the factors, and generally a lower RMSE than lagged realized variance. In fact, the forecasted variance has a higher correlation with realized variance compared to lagged variance in 8 out of 10 industries. This indicates that the GARCH forecasted variance is a good proxy for realized variance for the U.S. industry portfolios. Note that the sample of the U.S. industry portfolios is much larger than for the Norwegian single factors and Norwegian industry portfolios.

Table 9: **GARCH Predicted Variance vs. Lagged Variance.** The table reports the correlation and RMSE between the realized variance of single factors and 1) lagged realized variance (Lagged Variance) and 2) the GARCH forecasted variance (GARCH). The values give us a sense for the relation between realized variance and the proxies of realized variance, where a high correlation and low RMSE in general indicates a close relation.

Panel A: Norway Single Factors

Lagged Variance	MKT	SMB	HML	UMD	LIQ
Correlation	0,61	0,76	0,27	0,35	0,56
RMSE	0,28	0,14	0,23	0,22	0,22

GARCH	MKT	SMB	HML	UMD	LIQ
Correlation	0,63	0,79	0,39	0,39	0,47
RMSE	0,26	0,14	0,22	0,22	0,23

Panel B: Norwegian industries.

Lagged Variance	Energy	Material	Industry	ConsDisc	ConsStapl	Health	Finance
Correlation	0,65	0,78	0,59	0,58	0,58	0,32	0,65
RMSE	0,34	4,98	0,31	0,55	0,34	0,6	0,34

GARCH	Energy	Material	Industry	ConsDisc	ConsStapl	Health	Finance
Correlation	0,67	0,76	0,59	0,60	0,55	0,23	0,64
RMSE	0,32	5,15	0,31	0,55	0,6	0,6	0,35

Panel C: U.S. industries.

Lagged Variance	No durbl	Durbl	Manuf	Energy	Hi-Tec	Telecom	Shops	Health	Utility	Others
Correlation	0,44	0,65	0,60	0,61	0,64	0,57	0,55	0,52	0,63	0,70
RMSE	1,34	3,12	2,29	2,53	2,97	1,8	1,79	1,83	2,23	2,19

GARCH	No Durbl	Durbl	Manuf	Energy	Hi-Tec	Telecom	Shops	Health	Utility	Others
Correlation	0,54	0,63	0,65	0,63	0,70	0,55	0,64	0,57	0,69	0,74
RMSE	1,26	3,19	2,17	2,47	2,75	1,82	1,65	1,8	2,08	2,03

6.2 Results Managed Portfolios Using Forecasted Variance

In the following, we re-construct the volatility managed portfolios using forecasted variance rather than lagged realized variance. Note that the method still is similar to the original method, the difference is that we now replace lagged realized variance with the forecasted variance. The volatility managed portfolio at time t is then $\frac{c}{\hat{RV_t}^{OLS}} f_t$ when using the OLS forecasted variance and $\frac{c}{\hat{RV_t}^{GARCH}} f_t$ when using the GARCH forecasted variance.[56]

6.2.1 Results Using OLS Forecasted Variance

First, we show the results of re-constructing the volatility managed portfolios for the Norwegian single factors using the OLS forecasted variance. Because the forecasted variance using the OLS method indicated a worse proxy for realized variance than the GARCH method, we *do not* re-construct the industry portfolios using OLS forecasted variance. Table 11 reports the results when scaling the volatility managed

[56]See equation 3 in section 4.1.

portfolios with the OLS forecasted variance. Panel A reports the results of using the OLS forecasted variance when we use the whole sample to obtain the coefficients, while Panel B shows the results when using a rolling estimation window of 24 months to obtain the coefficients. We see in Panel A that when using the whole-sample OLS forecasted variance, the results are very similar to when we use lagged realized variance.[57] This is not surprising given the fact that the forecasted variance seems to be a similar proxy for realized variance as lagged realized variance.[58] As we mentioned before, using the whole sample OLS method can not be implemented directly by investors because the forecast is in-sample.

In Panel B, we see that the managed portfolios using the OLS rolling method exhibit substantial lower alphas and appraisal ratios compared to using the whole-sample OLS method and the main strategy using lagged realized variance. This is consistent for all factors which we consider. For instance, the appraisal ratio of the managed market factor decreases from 0,33 to 0,19 when using the OLS whole sample method, and the momentum factor decreases from 0,60 to 0,56. An important finding is that the market has the largest decrease in the appraisal ratio when using the OLS rolling method. Looking at Table 9, we see that the correlation between the OLS rolling forecasted variance and realized variance for the market factor is 0,42, compared to 0,62 for the whole-sample OLS method; thus indicating that the forecasted variance for the market factor is the least satisfying proxy for realized variance among all factors. The largest decrease in the appraisal ratio for the volatility managed market factor is therefore consistent with the hypothesis that a better proxy for realized variance would improve the results of the volatility managed portfolios.[59]

[57] See Table 1 for comparison.

[58] See Table 9, Panel A.

[59] In other words, the least satisfying proxy for realized variance yields the largest decrease in Sharpe ratio.

Table 10: **Volatility Managed Portfolios using OLS forecasted variance for Norway single factors.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increase the Sharpe ratio. The sample is 1981-2015. The portfolios are constructed based on monthly data: $f_t^\sigma = \frac{c}{\hat{\sigma}_t^2 OLS} f_t$, where $\hat{\sigma}_t^2 OLS$ is the OLS forecasted variance in month t. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

Panel A: Results OLS whole sample

	σMKT	σSMB	σHML	σUMD	σLIQ
MKT	0,76				
	(14.42)				
SMB		0,79			
		(12.88)			
HML			0,72		
			(12.82)		
UMD				0,74	
				(14.37)	
LIQ					0,79
					(15.67)
α	4,50	3,80	-2,70	8,16	3,18
t-stat	1,77	2,37	-1,30	3,75	1,79
R^2	0,58	0,62	0,52	0,55	0,62
RMSE	47,82	32,83	40,97	47,00	36,34
N	419	413	413	418	419
AR	0,33	0,40	-0,23	0,60	0,30

Panel B: Results OLS Rolling

	σMKT	σSMB	σHML	σUMD	σLIQ
MKT	0,165				
	(8.09)				
SMB		0,55			
		(13.79)			
HML			0,86		
			(17.55)		
UMD				0,86	
				(20.96)	
LIQ					0,75
					(20.03)
α	3,97	4,30	-2,91	5,61	2,28
t-stat	1,24	2,10	-1,90	3,23	1,24
R^2	0,03	0,30	0,73	0,75	0,57
N	396	390	390	396	396
RMSE	72,70	43,49	30,20	34,62	37,01
AR	0,19	0,34	-0,33	0,56	0,21

6.2.2 Results Using Garch Forecasted Variance

We report the results from the volatility managed portfolios using the forecasted variance by the GARCH method in Table 12. The results for the Norwegian single factors are shown in Panel A; Norwegian industry portfolios in Panel B and the U.S. industry portfolios in Panel C.

We see large and significant alphas for the Norwegian single factors in Panel A, and the appraisal ratios increase relative to the main results for all factors except for the liquidity factor.[60] As mentioned in section 6.1.2, only for the liquidity factor the GARCH forecasted variance has a higher RMSE if we compare to lagged realized

[60]When using the GARCH forecasted variance to manage the volatility of the portfolios, the two first years of the original sample are not included due to the estimation period of 24 months. Therefore, in Appendix D we report the volatility managed portfolios using lagged realized variance excluding the first two years of data.

variance. Thus it is consistent that the liquidity factor has the largest decrease in the appraisal ratio when we use the GARCH forecasted variance to manage the portfolios. It is also worth noticing that HML still has a negative and insignificant alpha, but is less negative and has a higher t-stat when applying the GARCH forecasted variance. Generally, the higher appraisal ratios are driven by both an increase in alpha and a decrease in RMSE.

In figure 2 in appendix C, we plot the accumulated return of the volatility managed market portfolio using both lagged realized variance and the GARCH forecasted variance, as well as the accumulated return for non-managed portfolio. We see clearly that the managed portfolios outperform the non-managed portfolio, where the managed portfolio using GARCH forecasted variance has the highest accumulated return. The managed portfolios avoid the largest declines in returns compared to the non-managed portfolio. A sudden drop in returns is associated with high volatility, and our volatility managed portfolios avoid these drops as they scale out of the factors.

Panel B reports the results of the volatility managed U.S. industry portfolios using the GARCH forecasted variance. As for the Norwegian single factors, we see substantial alphas and appraisal ratios for the U.S. industry portfolios. 5 out of 10 portfolios gets a higher alpha when using the GARCH forecasted variance compared to lagged realized variance, and 8 out of 10 portfolios has higher appraisal ratios. The higher appraisal ratios are due to generally lower RMSE's for the managed portfolios using GARCH forecasted variance. In fact, 9 out of 10 managed portfolios have a lower RMSE when using GARCH forecasted variance rather than lagged realized variance. It is worth mentioning that the Utility portfolio has a lower appraisal ratio when using the GARCH forecasted variance compared to using lagged realized variance, while at the same time the forecasted variance seems to be a better proxy for realized variance than lagged realized variance.[61]

The results for the Norwegian industry portfolios can be seen in panel C. Only 2 out of 8 managed portfolios exhibit higher appraisal ratios when using the GARCH forecasted variance, compared to using lagged realized variance, while four managed portfolios exhibit lower appraisal ratios. The managed Health portfolio has a higher appraisal ratio when using the GARCH forecasted variance, but the GARCH forecasted variance also has a lower correlation and higher RMSE compared to lagged

[61]For the Utility portfolio, the GARCH forecasted variance has a higher correlation and lower RMSE relative to realized variance, if we compare to lagged realized variance; thus indicating that the GARCH forecasted variance is a better proxy for realized variance.

realized variance. Note however that the differences are marginal, and that both the forecasted variance and lagged variance is a very similar proxy for realized variance.

Table 11: **Volatility Managed Portfolios using GARCH forecasted variance.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f_t^\sigma = \alpha + \beta f_t + \epsilon_t$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increase the Sharpe ratio. The sample is 1981-2015 for Norway single factors and industry portfolios, and 1926-2015 for U.S. industry portfolios. The portfolios are constructed based on monthly data: $f_t^\sigma = \frac{c}{\hat{\sigma}_t^2 GARCH} f_t$, where $\hat{\sigma}_t^2 GARCH$ is the GARCH forecasted variance in month t. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

Panel A: Norway single factors

	σMKT	σSMB	σHML	σUMD	σLIQ
MKT	0,86				
	(22.34)				
SMB		0,84			
		(15.11)			
HML			0,82		
			(17.6)		
UMD				0,88	
				(22.7)	
LIQ					0,85
					(16.44)
α	6,42	4,05	-1,51	8,55	2,10
t-stat	3,44	2,75	-0,9	5,03	1,39
R^2	0,74	0,7	0,68	0,76	0,71
N	408	390	390	395	396
RMSE	37,62	28,7	33,25	33,36	30,36
AR	0,59	0,49	-0,16	0,89	0,24

Panel B: U.S. industry factors.

	σNoDurbl	σDurbl	σManuf	σEnergy	σHitec	σTelecom	σShops	σHealth	σUtility	σOthers
NoDurbl	0,67									
	(13.85)									
Durbl		0,68								
		(8.91)								
Manuf			0,67							
			(9.51)							
Energy				0,74						
				(17.84)						
Hitec					0,7					
					(12.43)					
Telecom						0,58				
						(16.52)				
Shops							0,69			
							(12.75)			
Health								0,75		
								(12.23)		
Utility									0,39	
									(9.46)	
Others										0,66
										(8.97)
α	3,76	5,52	6,99	5,09	8,04	7,18	6,69	4,76	5,21	7,06
t-stat	2,97	2,79	4,18	3,39	4,23	5,19	4,12	3,32	2,65	4,00
R^2	0,44	0,47	0,44	0,55	0,49	0,33	0,47	0,56	0,16	0,44
N	1059	1059	1059	1058	1058	1058	1058	1058	1058	1059
RMSE	41,36	67,62	56,52	49,42	62,77	45,35	51,07	44,78	61,44	58,57
AR	0,32	0,28	0,43	0,36	0,44	0,55	0,45	0,37	0,29	0,42

Panel C: Norway industry factors.

	σEnergy	σMaterial	σIndustry	σConsDisc	σConsStapl	σHealth	σFinance
Energy	0,81						
	(15.4)						
Material		0,48					
		(2.7)					
Industry			0,77				
			(13.52)				
ConsDisc				0,67			
				(8.2)			
ConsStapl					0,72		
					(21.2)		
Health						0,76	
						(11.4)	
Finance							0,66
							(10.93)
α	4,75	16,4	1,92	12,49	9,06	3,84	9,67
t-stat	1,63	2,7	0,66	2,94	2,92	1,27	3,12
R^2	0,65	0,23	0,59	0,44	0,53	0,58	0,43
N	408	408	408	408	408	408	408
RMSE	56,22	129,24	54,36	93,00	64,80	67,27	63,46
AR	0,29	0,44	0,12	0,47	0,48	0,20	0,53

7 Discussion

In this section, we discuss our main results and link the analysis to Moreira and Muir (2016b). First, we present a possible explanation for why it is beneficial to decrease the risk exposure when volatility is high, and vice versa. Next, we consider whether leverage constraints and transaction costs may affect our results. Lastly, we link our results to higher order moments, similar to Barroso and Santa-Clara (2015), who emphasize that their volatility managing strategy for the momentum factor improves higher order moments such as left skewness and kurtosis.

7.1 The risk-return trade-off and the persistence of volatility

Moreira and Muir (2016b) show theoretically that the alpha of their strategy is tightly linked to the strength of the risk-return trade-off in the time series, and point out that the all conditional alphas are zero. By taking unconditional expectations they decompose the unconditional alpha into a component that captures the ability of the strategy to time volatility and a component that captures the co-movement of the strategy with expected returns. They further point out that if the strategy takes less risk when conditional variance is low, then the unconditional beta will be lower than its average beta.

Further, they point out that when there are no time-series relation between expected returns and variance, the alpha increases in the ability to forecast volatility. On the other hand, when the relationship between variance and expected returns is strong, the unconditional alpha is approximately zero. A strong relationship between variance and expected returns intuitively leads to sacrificing high expected returns due to less exposure when the variance is high.

Similar to Moreira and Muir (2016b), we study the relationship between risk and return directly. We run the same regressions as they do on our sample: First, we show that realized volatility for each factor predicts future volatility to a large extent. Specifically, we run a regression of log realized variance on log lagged realized variance and report the coefficients in Table 12.[62] We see that the variance of the factors is persistent, and the average coefficient is approximately 0,6. We can conclude that there is a significant positive relationship between lagged realized variance

[62]The results do not change if we run the regressions on realized variance rather than log realized variance.

and variance for our sample, which is important in explaining the results.

Next, we show that realized volatility does not predict future returns by running a regression of monthly returns on previous month's realized volatility for each factor. The regression results can be seen in Table 13. The results are consistent with the findings of Moreira and Muir (2016b), that find both positive and negative coefficients, but in general the coefficients were not significant. An interesting finding is that the HML factor has a significant and positive relation to future returns, which is consistent with our finding that managing the volatility for the HML factor does not yield positive and significant alphas. Another interesting finding is that the market factor has a significant negative coefficient, implying that there is a negative relationship between variance and future returns. This indicates that managing volatility should lead to high risk-adjusted performance, which is in line with our results in Table 1. For the other factors, the coefficients are negative and insignificant, and we can conclude that variance does not predict returns for these factors.

We have also run regressions of variance on both variance and return for the U.S. industry portfolios. The results are similar. We focus mainly on the Norwegian single factors in the analysis, and report the results for the U.S. industries in appendix E.

Table 12: **Variance predicting variance, Norway single factors.** The table reports the regression results of log realized variance on log lagged realized variance: $lnRV_t = \alpha + \beta lnRV_{t-1} + \epsilon_t$. The beta coefficients represent the relationship between realized variance and lagged realized variance, i.e. if variance predicts variance. The sample is 1981-2015 and we use monthly data. T-stats are in paranthesis.

	$lnRV_t^{Mkt}$	$lnRV_t^{SMB}$	$lnRV_t^{HML}$	$lnRV_t^{PR1YR}$	$lnRV_t^{MOM}$	$lnRV_t^{LIQ}$
$lnRV_{t-1}^{Mkt}$	0,55					
	(11.49)					
$lnRV_{t-1}^{SMB}$		0,68				
		(17.56)				
$lnRV_{t-1}^{HML}$			0.59			
			(14.76)			
$lnRV_{t-1}^{PR1YR}$				0,62		
				(16.20)		
$lnRV_{t-1}^{MOM}$					0,56	
					(12.50)	
$lnRV_{t-1}^{LIQ}$						0,59
						(13.74)
N	419	413	413	418	418	419
R^2	0,3	0,46	0,35	0,39	0,32	0,35

Table 13: **Variance predicting returns, Norway single factors.** The table reports the regression results of future returns on realized variance: $R_{t+1} = \alpha + \beta RV_t + \epsilon_t$. The beta coefficients represent the relationship between future returns and realized variance, i.e. if realized variance predicts future returns. The sample is 1981-2015 and we use monthly data. T-stats are in paranthesis.

	R_t^{Mkt}	R_t^{SMB}	R_t^{HML}	R_t^{MOM}	R_t^{LIQ}
RV_{t-1}^{Mkt}	-0,27				
t-stat	(-2.1)				
RV_{t-1}^{SMB}		-0,18			
t-stat		(-1.59)			
RV_{t-1}^{HML}			0,26		
t-stat			(2.17)		
RV_{t-1}^{MOM}				-0,25	
t-stat				(-1.87)	
RV_{t-1}^{LIQ}					-0,16
t-stat					(-1.49)
N	419	413	413	418	419
R2	0,02	0,01	0,01	0,01	0,01

7.2 Leverage Constraints

We do not consider the impact of leverage constraints empirically, but refer to the empirical tests which Moreira and Muir (2016a) do. Then, we compare their findings to the properties of our managed portfolios, in order to examine if investors under tight leverage constraints can benefit from the strategy. Moreira and Muir (2016a) point out that several research has showed that the risk-return trade-off in the cross-section of returns can be weakened when considering leverage constraints. They further point out that high beta assets may be attractive to leverage constrained investors, and argue that low volatility periods are analogous to low beta assets. One could think that investors are not able to take on leverage in periods of low volatility, thus not being able to increase the risk exposure as the strategy suggests.

Moreira and Muir (2016a) consider a strategy that only updates the portfolio when volatility is above its mean value, i.e. they do not use leverage in low volatility periods where the risk exposure of the strategy would increase majorly. They also construct volatility managed portfolios with weights constraints of 1 and 1.5, i.e. strategies that do not include leverage and portfolios with maximum leverage of 50 percent. Note that a leverage of 50 percent is consistent with standard margin requirements. A third test they run is to implement the strategy using options, where they use in-the-money call options instead of leverage when the strategy demands high risk exposure. They find positive and significant alphas for all leverage constraint tests, indicating that the strategy can be implemented by investors in real time.

We have analyzed the upper distribution of the weights for the volatility managed market factor when using lagged realized variance, similar to what Moreira and Muir (2016a) do. The mean weight is approximately 0,92, where the 70th, 90th, 95th, and 99th percentile is 3,6, 4,09, 4,18 and 4,48, respectively. Our strategy uses modest leverage in the majority of the time, but the amount of leverage increases in the upper part of the distribution. Increased leverage occur when the volatility is low, as the strategy then increases the risk exposure to the factor.

When using GARCH forecasted variance the extreme weights decrease, and the 75th, 90th, 99th percentile is reduced to 3, 3,19 and 3,95, respectively. This indicates that using a sophisticated forecasting model makes the strategy less dependent on leverage. Moreira and Muir (2016a) find that using realized volatility[63] decreases the extreme weights of the strategy, thus making it less levered. We construct managed portfolios using both lagged realized volatility and forecasted volatility from the GARCH model to see if we obtain similar results. In Table 14, we show the maximum weights and alphas of the managed market factor using four different proxies of realized volatility to manage the portfolios. We find that the maximum weight of the managed portfolio is reduced to 2,41 when we use lagged realized volatility, compared to 4,48 when using lagged realized variance. The maximum weight is further reduced to 2,13 when we use GARCH forecasted volatility. Note that the weights of our managed portfolios are lower than the weights of Moreira and Muir's managed portfolios. This may be an indication that the strategy can be implemented by investors with leverage constraints.

7.3 Transaction costs

Moreira and Muir (2016a) evaluate the volatility timing strategy for the market portfolio when including transaction costs. They consider strategies that capture

[63]Realized volatility is defined as the standard deviation.

the idea of volatility timing but reduce the trading activity. The strategies considered are among others using (1) realized volatility and (2) expected variance, rather than realized variance when constructing the volatility managed portfolios. They find that the strategies survive transaction costs at 1bps, 4bps and 10bps, which is consistent with the findings of Barroso and Santa-Clara (2015).

We do not perform a robustness check by including transaction costs explicitly, but we analyze important properties of the volatility managed market portfolio in Norway and compare it to the managed U.S. market portfolio of Moreira and Muir (2016a). In Table 14, we report the alphas, average absolute change in monthly weights, maximum weights and mean weights using four different proxies for risk for the volatility managed market portfolio in Norway. We find that the volatility managed portfolio of the Norwegian market factor using lagged realized variance has a monthly average change in weights of 0,52, compared to 0,73 of Moreira and Muir.[64] When applying the GARCH forecasted variance, the average change in weights is 0,25, indicating that the transaction costs go further down when using a sophisticated forecasting model to forecast variance.[65] When using the GARCH forecasted volatility, the monthly average change in weights goes further down to 0,14, which is substantially lower than Moreira and Muir found when considering expected volatility (0,37). In general, the monthly average change in weights for our managed portfolios is lower compared to the managed portfolios which Moreira and Muir (2016a) consider. This may indicate that the strategy survives transaction costs.

[64]To see Moreira and Muir (2016a) analysis of transaction costs, see Table 4 in their paper.

[65]We also tested this for other factors than the market factors, and the weigths consistently go down when using the GARCH forecasted variance and volatility. These results are available on request.

Table 14: **Weights managed portfolios for the market factor in Norway.** The table reports the annualized alpha, t-stat of the alpha, the average change in weight, the maximum weight and the mean weight of the volatility managed portfolios for the market factor in Norway. Lagged Realized Variance is the managed portfolio using lagged realized variance, Lagged Realized Volatility is the managed portfolio using realized volatility. GARCH Variance is the managed portfolio using the GARCH forecasted variance, and GARCH volatility is the managed portfolio using GARCH forecasted volatility.

Proxy	Alpha	t-stat	$\Delta weight$	Max Weight	Mean Weight
Lagged Realized Variance	4,94	1,96	0,52	4,48	0,92
Lagged Realized Volatility	3,35	2,01	0,31	2,41	1,01
GARCH Variance	6,42	3,44	0,25	3,95	1,07
GARCH Volatility	3,36	2,73	0,14	2,13	1,06

7.4 Skewness and Kurtosis

We find it relevant to link the properties of our managed portfolios to the findings of Barroso and Santa-Clara (2015), who investigate momentum crashes and volatility time the momentum factor. As mentioned before, their method is quite similar to the one of Moreira and Muir (2016a) which we replicate, and they find that managing the volatility of the momentum factor improves the Sharpe ratios of the non-momentum factor substantially.

What we find particularly interesting, is that Barroso and Santa-Clara (2015) argue that the most important benefit from their volatility timing strategy is that it reduces the excess kurtosis and the left skewness of the original factor. Kurtosis is a statistical measure used to describe the distribution, or the skewness of observed data around the mean, sometimes referred to as the volatility of volatility. Intuitively, a distribution with a high/low kurtosis tend to have heavy tails/light tails. Skewness is a measure of the symmetry or lack of symmetry of the distribution. Positive skewness, or right-skewed distribution, indicates a distribution with an asymmetric tail extending towards positive values. Negative skewness, or left skewed distribution, similarly indicates a distribution with an asymmetric tail towards negative values. A decrease in these two higher order moments can be viewed as reducing the downside risk, as pointed out by Barroso and Santa-Clara (2015).

In figure 1, we show the distribution of the return for the market, and the volatility managed market factor using both lagged realized variance and GARCH forecasted variance. The figure shows that the market has a fat left tail; it is left skewed. When we manage volatility for the market factor, the left tail is removed and the distribution gets positively skewed. Note however that the managed factor also shows increased kurtosis relative to the non-managed factor, which indicates a distribution with heavy tails or outliers. The excess kurtosis for the market factor increases by 3,12, and the reason may be that managing volatility yields a strategy with a highly levered risk exposure when the volatility is low.

By using GARCH forecasted variance to manage portfolios, kurtosis is reduced compared to using lagged realized variance. When we compare the managed portfolios using the GARCH forecasted variance with the non-managed portfolios, left skewness is turned to positive skewness, while kurtosis stays relatively unchanged. See Table 15. One possible explanation is that the exposure to risk is lower when applying the GARCH forecasted variance, and that the portfolios in general use less leverage. The lower kurtosis and the right skewness may indicate that using GARCH forecasted variance to manage the portfolios reduces downside risk compared to using the lagged realized variance.

It is worth mentioning that our distribution is not directly comparable to the one of Barroso and Santa-Clara (2015). They experience a decline in excess kurtosis from 18,42 to 2,68 when managing the momentum factor, while the skewness drops from -2,47 to -0,42. Our non-managed momentum factor in Norway has a substantially lower kurtosis of 4,86 and it increases to 6,54 when managing the portfolio with lagged realized variance. In other words, the momentum factor we consider is more observed around the mean, thus indicating lower downside risk. When managing the portfolios with the GARCH forecasted variance, the kurtosis goes down compared to using lagged realized variance. We believe this is because these portfolios are less exposed to risk due to less extreme weights, indicating a reduced downside risk.

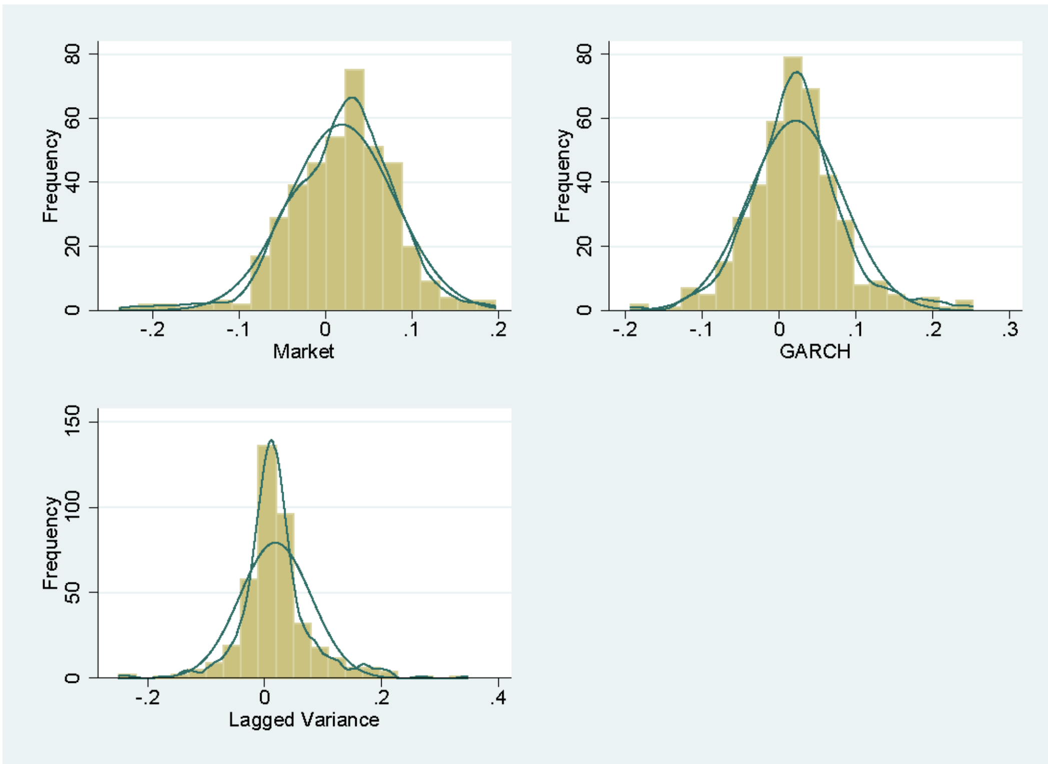
Table 15: **Statistic summary, non-managed and managed portfolios.** The Table reports the statistical properties of the portfolio returns distribution for the non-managed portfolios (Non-Managed) and volatility managed portfolios using (1) lagged realized variance (Managed L.var) and (2) GARCH forecasted variance (Managed GARCH) for single factors in Norway. Mean is the mean value of the portfolio returns, St.dev is the standard deviation of the portfolio returns, Skewness is measuring the symmetry of the distribution, while Kurtosis is a measure of the skewness of observed data around the mean. The sample is 1981-2015.

Non-managed	MKT	SMB	HML	UMD
Mean	1,90	0,80	0,28	0,58
St.dev	6,12	4,35	4,89	5,70
Skewness	-0,54	0,53	-0,14	-0,19
Kurtosis	4,64	6,4	4,22	4,86

Managed L.var	MKT	SMB	HML	UMD
Mean	1,88	0,98	0,02	1,26
St.dev	6,12	4,35	4,89	5,70
Skewness	0,62	1,08	-1,03	0,95
Kurtosis	7,76	10,91	14,89	6,54

Managed GARCH	MKT	SMB	HML	UMD
Mean	2,18	1,03	0,10	1,24
St.dev	6,12	4,35	4,89	5,70
Skewness	0,37	0,73	0,22	0,19
Kurtosis	4,68	5,78	5,91	4,69

Figure 1: **Distribution non-managed market factor portfolios.** The figure shows the distribution of the market factor for the non-managed portfolio, the volatility managed portfolio using lagged realized variance and the volatility managed portfolio using GARCH forecasted variance.



8 Conclusion

In this thesis, we replicate the methodology of Moreira and Muir (2016a), and we are the first to analyze whether it is beneficial to manage the volatility of well-known risk factors in Norway and the UK, and of industry portfolios in Norway and the U.S. The method is simple, and the volatility managed portfolios scale monthly returns by the inverse of the realized variance in the previous month; when volatility is high the managed portfolios take less risk, and vice versa.

Our results indicate that it is beneficial to manage the volatility of both well-diversified risk factors and less diversified industry portfolios. The volatility managed single factors in Norway generally have large and significant alphas, where the highest is for the momentum factor. For the UK. factors, the managed momentum factor is large and significant, but the other factors are insignificant. The finding of large alphas for the momentum factor is consistent with other empirical studies, such as Moreira and Muir (2016a) and Barroso and Santa-Clara (2015).

We find similar qualitatively results for the volatility managed industry portfolios, and the majority of the managed industry portfolios exhibit large significant alphas. The results are more consistent for the U.S. industry portfolios than the Norwegian. Because the industry portfolios are less diversified than the systematic factors, we are not able to conclude directly that the managed industry portfolios expand the mean-variance efficient frontier.

Because variance is highly predictable at short horizons, the previous month's realized variance is a good proxy for the realized variance the next month. We show that finding a better proxy for realized variance using a sophisticated forecasting model, such as the GARCH model, can improve the results of the volatility managed portfolios.

Appendices

A Replication of Moreira and Muir

Table 16: **Volatility Managed Portfolios, replication of Moreira and Muir for U.S. factors** The table shows our replication of Moreira and Muir's Volatility Managed Portfolios for the Fama-French three factors plus the momentum factor. The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increase the Sharpe ratio. The sample is 1926-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

	σMKT	σSMB	σHML	σMOM
MKT	0,61			
	(11.27)			
SMB		0,62		
		(25.2)		
HML			0,57	
			(7.58)	
MOM				0,47
				(7.15)
α	4,87	-0,51	1,88	12,53
t-stat	3,13	-0,54	1,85	7,33
R^2	0,37	0,37	0,32	0,22
N	1065	1065	1065	1060
RMSE	51,41	30,62	34,75	50,38
AR	0,33	-0,06	0,19	0,86

B ARCH/GARCH models

The mean value and the variance of a return of an asset or a portfolio is defined relative to a past information set. The return in the present will be equal to the mean value plus the standard deviation times the error term for the present period (Engle (2001), page 5):

$$r_t = \mu + \sqrt{\sigma_t^2}\epsilon_t \tag{18}$$

where μ is the expected value of r dependent on past information and $\sqrt{\sigma_t^2}$ is the standard deviation.

The econometric challenge is to specify how the information is used to forecast the variance of the return, conditional on only past information.(Engle (2001), page 5)

Before the introduction of the ARCH model by Engle (2001), virtually no methods were available to forecast the variance(Engle (2001), page 5). The general ARCH (p) model looks like:

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + ... + \alpha_p\epsilon_{t-p}^2 \tag{19}$$

which can be rewritten into

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i\epsilon_{t-i}^2 \tag{20}$$

The ARCH(1) model specifies an equation for the conditional variance:

$$\sigma_t^2 = E[\epsilon_t^2|\theta_{t-1}] = \alpha_0 + \alpha_1\epsilon_{t-1}^2 \tag{21}$$

where θ is the information set in which the expected squared residual ϵ_t^2 is dependent on. We see that the model is an autoregressive model, and the predicted volatility for the next period is conditional on information in this period. Note that $\alpha_0 > 0$ and $\alpha_1 \geq 0$ in order to ensure positive variance and stationarity Reider (2009). Intuitively, if the observed residual ϵ^2 is large one month, the forecasted variance for the subsequent month will also be large. The ARCH model estimated variance is only dependent on the previous period's squared residual, unlike the GARCH models.

The General Autoregressive Conditional Heteroskedastic (GARCH) model was first introduced by Bollerslev (1986). The GARCH model adds a moving average term, thus it is similar to an ARMA process. The moving average term allows a slower decay in variance from random shocks. In the simplest version of the GARCH model, the GARCH (1,1), the variance is dependent on last period's squared return (similar to the ARCH model) in addition to last period's forecast of the variance.:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{22}$$

where $\alpha_0 > 0$, $\alpha_1 > 0$ and $\beta_1 > 0$ in order to ensure that σ_t^2 is positive.

Note that a GARCH(1,1) model can be written as an ARCH(∞) model (Reider, 2009):

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1(\alpha_0 + \alpha_1 a_{t-2}^2 + \beta_1 \sigma_{t-2}^2) \\ &= \alpha_0 + \alpha_1 a_{t-1}^2 + \alpha_0 \beta_1 + \alpha_1 \beta_1 a_{t-2}^2 + \beta_1^2 \sigma_{t-2}^2 \end{aligned} \tag{23}$$

The generalized GARCH (p,q) model looks like:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \tag{24}$$

where p is the order of GARCH terms (σ^2) and q is the order of ARCH terms (ϵ^2).

C Figures and graphs

Figure 2: **Accumulated returns, market factor Norway.** The figure plots the accumulated return of the non-managed market factor (market), the volatility managed market factor using lagged realized variance (managed_var), and the volatility managed market factor using GARCH predicted variance (managed_garch).

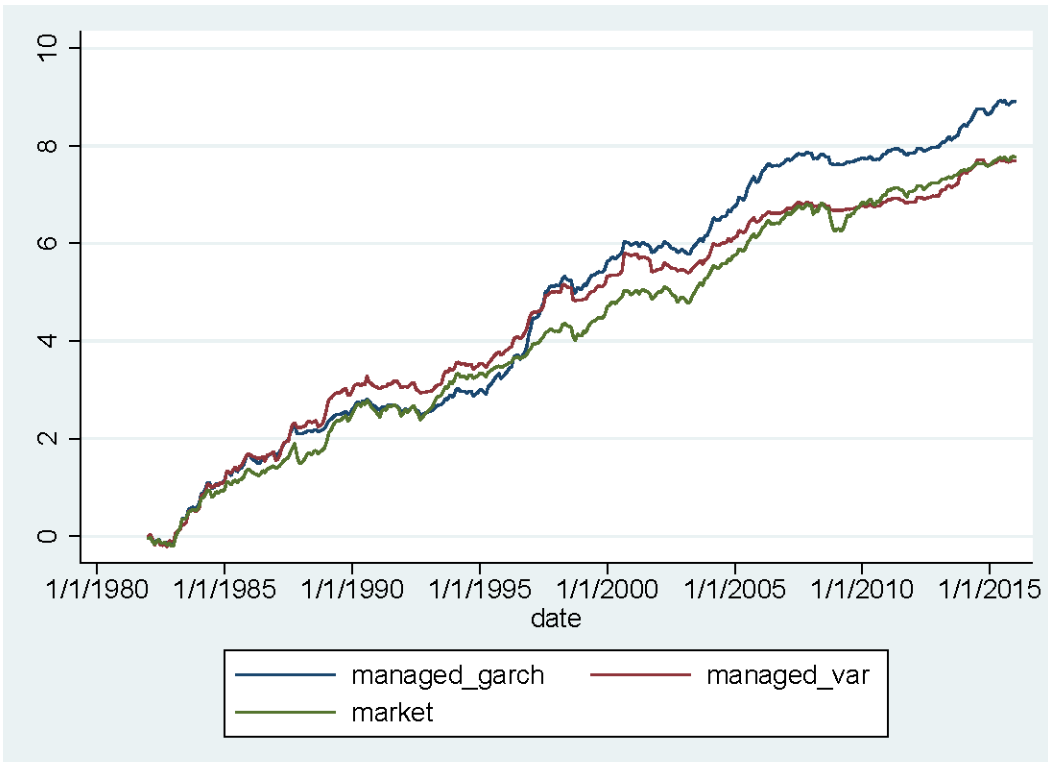


Figure 3: **Risk Exposure Managed Portfolios, GARCH vs. Lagged Realized Rariance.** The figure plots the weights of the managed portfolios using GARCH forecasted variance and lagged realized variance. We see that the risk exposure of both managed portfolios follow the same pattern, but that when applying GARCH, the exposure to the market factor tends to have lower spikes than when applying lagged realized variance. Note that the average total risk of both strategies is equal.

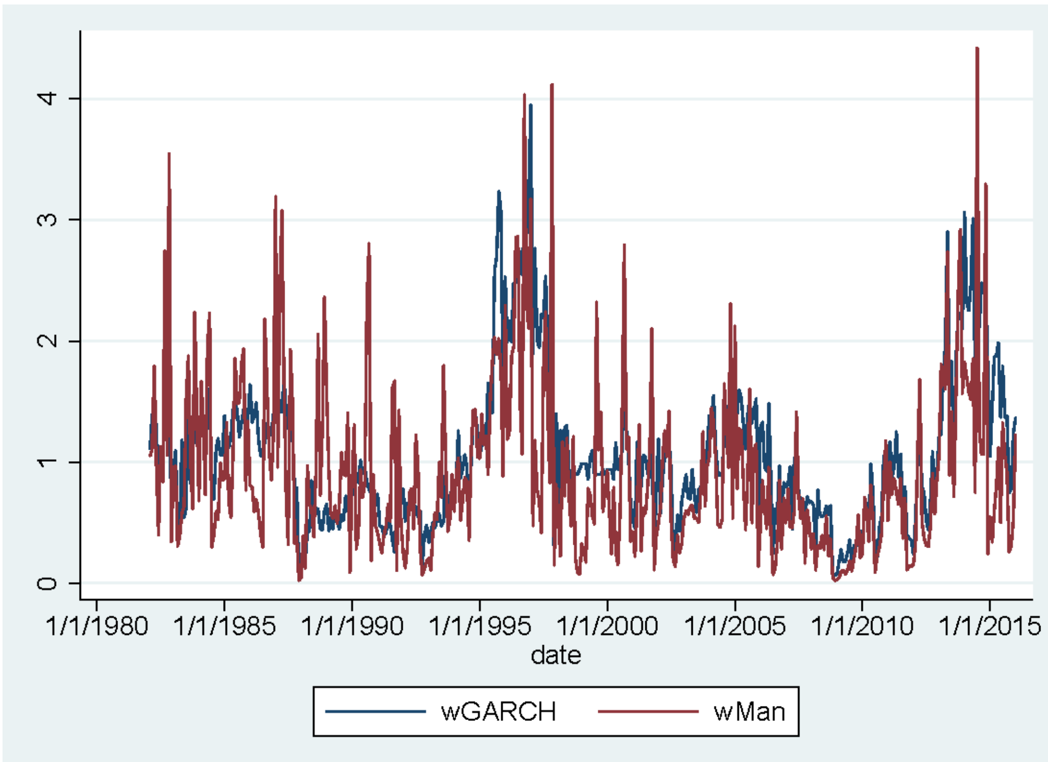


Figure 4: **Weight vs. Realized Variance, Norway.** The figure plots the monthly variance of each market factor (var) in Norway and the monthly portfolio exposure (weight) to the factor for the volatility managed portfolio of the market factor in Norway. We see that when variance is high, the weight lowers, and vice versa. After the variance shocks in the late 80s and in the recent financial crisis in the late 2000s, we see clearly that the exposure to the market factor decreases towards zero, i.e. the managed portfolio weights out of the factor.

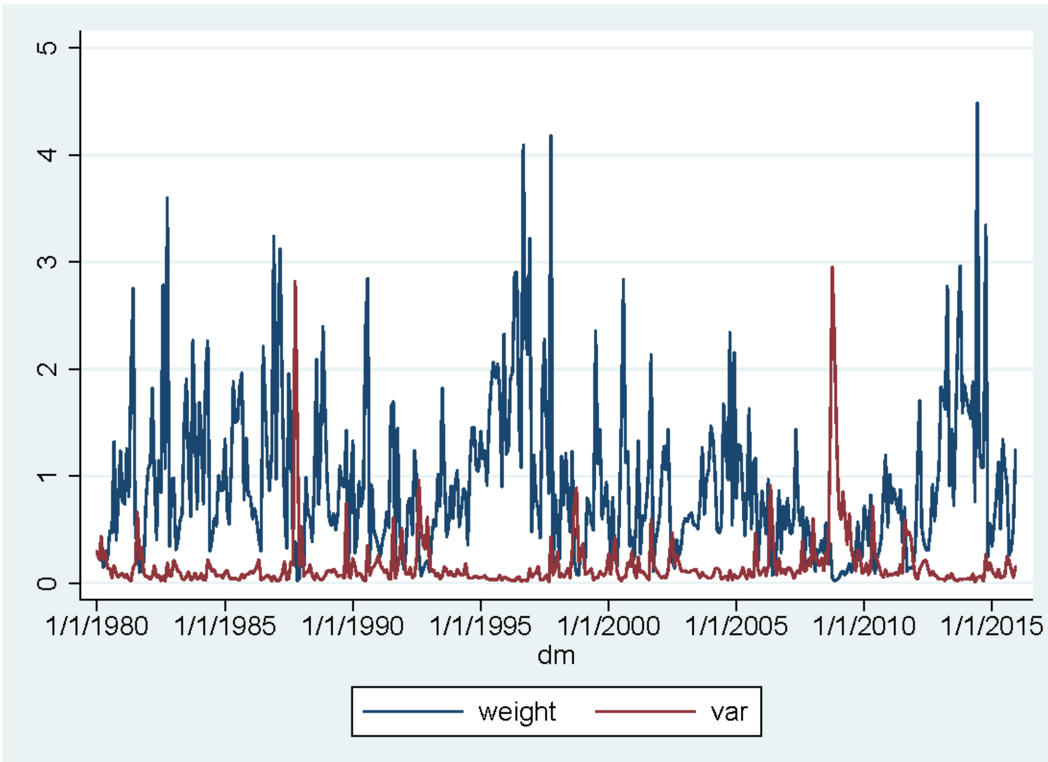
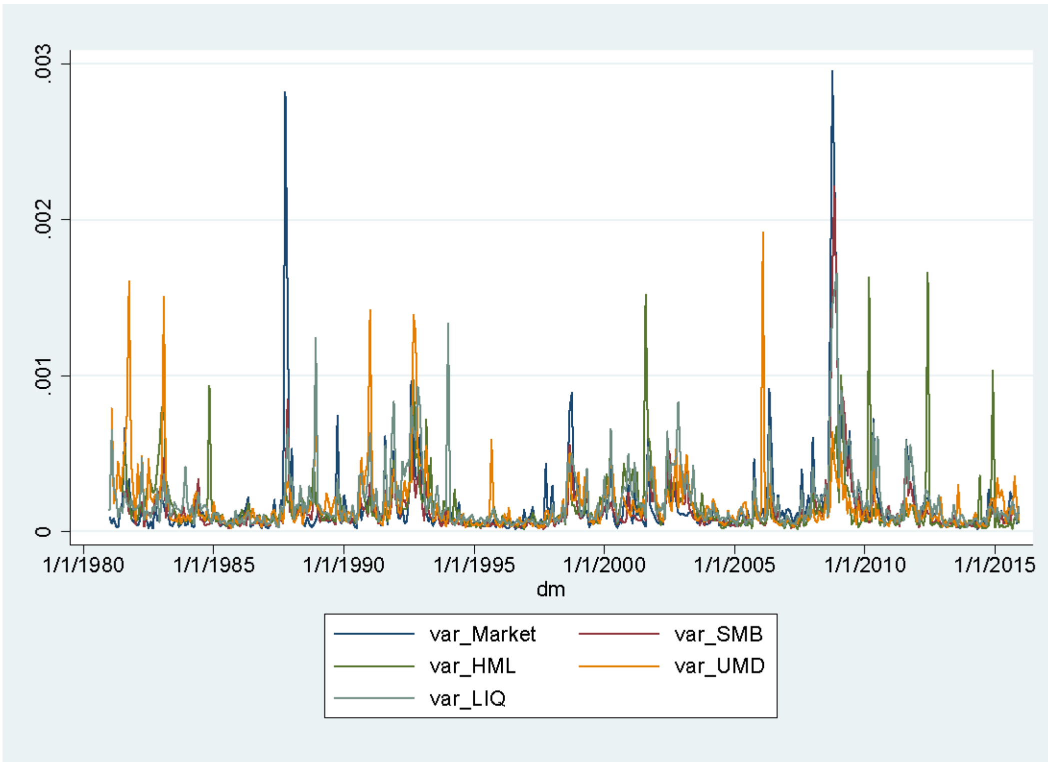


Figure 5: **Volatility by factor, Norway.** The figure plots the monthly volatility of each individual factor in Norway. We see that the movement in volatility follows the same pattern across factors, i.e. that the business cycle risk is relatively similar. We also see that volatility increases substantially during bad times, especially in the bank crisis during the late 80s/early 90s and the latest financial crisis starting in 2008.



D Managed portfolios, comparable period GARCH

Table 17: **Results Managed Portfolios Lagged Variance, Norway single factors.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f^{\sigma}_{t+1} = \alpha + \beta R_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increases the Sharpe ratio. The sample is 1983-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

	Mkt	SMB	HML	UMD
β	0,76	0,79	0,73	0,77
t-stat	14,3	12,1	12,4	14,5
α	5,03	4,04	-2,13	9,6
t-stat	2	2,47	-1,03	4,46
R^2	0,58	0,62	0,53	0,59
N	408	390	390	395
RMSE	47,47	32,30	40,31	44,02
AR	0,37	0,43	-0,18	0,75

Table 18: **Results Managed Portfolios Lagged Variance, Norway industry factors.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f_{t+1}^{\sigma} = \alpha + \beta R_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increases the Sharpe ratio. The sample is 1983-2015. The portfolios are constructed based on monthly data: $f_{t+1}^{\sigma} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

	Energy	Material	Industry	ConsDisc	ConsStapl	Health	Finance
β	0,81	0,48	0,77	0,67	0,72	0,76	0,66
t-stat	15,4	2,7	13,52	8,2	21,2	11,4	10,93
α	4,75	16,4	1,92	12,49	9,06	3,84	9,67
t-stat	1,63	2,7	0,66	2,94	2,92	1,27	3,12
R^2	0,65	0,23	0,59	0,44	0,53	0,58	0,43
N	408	408	408	408	408	408	408
RMSE	56,22	129,24	54,36	93,00	64,80	67,27	63,46
AR	0,29	0,44	0,12	0,47	0,48	0,20	0,53

Table 19: **Results Managed Portfolios Lagged Variance, U.S industry factors.** The table reports the regression results of the volatility managed portfolios on the non-managed portfolios: $f^{\sigma}_{t+1} = \alpha + \beta R_{t+1} + \epsilon_{t+1}$. A positive alpha implies that the managed portfolios expand the mean-variance frontier, thus increases the Sharpe ratio. The sample is 1928-2015. The portfolios are constructed based on monthly data: $f^{\sigma}_{t+1} = \frac{c}{RV_t} f_{t+1}$, where RV_t is the realized variance in month t, calculated based on daily data. The factors are annualized in percent by multiplying monthly factors by 12. The appraisal ratio (AR) is calculated as $\sqrt{12}\frac{\alpha}{RMSE}$ and is a measure of how much the managed portfolios expand the slope of the MVE frontier compared to the non-managed portfolios.

	Nodurbl	Durbl	Manuf	Energy	Hi-Tec	Telecom	Shops	Health	Utility	Others
β	0,60	0,62	0,59	0,61	0,61	0,35	0,60	0,66	0,43	0,58
t-stat	9	8,32	9,07	14,27	11,65	11,58	11,12	10,97	9,52	8,85
α	2,76	5,56	6,36	5,34	6,4	6,11	6,92	2,71	6,9	6,95
t-stat	1,66	2,59	3,53	3,17	3,1	4,02	3,86	1,6	3,72	3,67
R^2	0,35	0,38	0,35	0,37	0,37	0,12	0,37	0,45	0,19	0,34
N	1059	1059	1059	1058	1058	1058	1058	1058	1058	1059
RMSE	44,57	72,89	60,96	58,57	69,41	52,16	56,16	50,33	60,29	63,30
AR	0,21	0,26	0,36	0,32	0,32	0,41	0,43	0,19	0,40	0,38

E Variance predicting variance/return U.S. Industries

Table 20: **Variance predicting variance, U.S. industries.** The table reports the regression results of realized variance on lagged realized variance: $RV_t = \alpha + \beta RV_{t-1} + \epsilon_t$. The beta coefficients represent the relationship between realized variance and lagged realized variance, i.e. if variance predicts variance. The sample is 1926-2015 and we use monthly data. T-stats are in paranthesis.

	NoDurbl	Durbl	Manuf	Energy	Hitec	Telecom	Shops	Health	Utils	Others
β	0,69	0,71	0,72	0,72	0,74	0,75	0,74	0,67	0,82	0,72
t-stat	31.87	31.37	34.34	34.49	35.83	36.15	36.26	29.21	46.83	34.67
R^2	0,49	0,50	0,52	0,52	0,55	0,56	0,55	0,45	0,67	0,53
N	1081	1081	1081	1081	1081	1081	1081	1081	1081	1081

Table 21: **Variance predicting returns, U.S. industries.** The table reports the regression results of future returns on realized variance: $f_{t+1} = \alpha + \beta RV_{t+1} + \epsilon_{t+1}$. The beta coefficients represent the relationship between realized variance and future returns, i.e. if realized variance predicts future returns. The sample is 1926-2015 and we use monthly data. T-stats are in paranthesis.

	NoDurbl	Durbl	Manuf	Energy	Hitec	Telecom	Shops	Health	Utils	Others
β	0,05	-0,18	0,11	-0,07	-0,02	-0,18	0,04	-0,01	0,02	-0,05
t-stat	(0.19)	(-1.14)	(1.66)	(-1.18)	(-0.14)	(-1.87)	(0.18)	(-0.04)	(0.19)	(-0.29)
R^2	0,00	0,01	0.0025	0,00	0,00	0,01	0,00	0,00	0,00	0,00
N	1081	1081	1081	1081	1081	1081	1081	1081	1081	1081

References

Akgiray, V. (1989). Conditional heteroscedasticity in time series of stock returns: Evidence and forecasts. *The Journal of Business*, 62(1):55.

Andersen, T. G. and Bollerslev, T. (1997). Intraday periodicity and volatility persistence in financial markets. *Journal of Empirical Finance*, 4:115–158.

Ang, A., Hodrick, R., Xing, Y., and Zhang, X. (2009). High idiosyncratic volatility and low returns: International and further u.s. evidence. *Journal of Financial Economics*, 91(1):1–23.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. *The Journal of Finance*, 51:259–299.

Baker, M., Bradley, B., and Wurgler, J. (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. *Financial Analysts Journal*, 67(1):40–54.

Baker, N. L. and Haugen, R. A. (1991). The efficient market inefficiency of capitalization–weighted stock portfolios. *The Journal of Portfolio Management*, 17(3):35–40.

Baker, N. L. and Haugen, R. A. (1996). Commonality in the determinants of expected stock returns. *Journal of Financial Economics*, 41:401–439.

Baker, N. L. and Haugen, R. A. (2012). Low risk stocks outperform within all observable markets of the world. *SSRN Electronic Journal*.

Barroso, P. and Santa-Clara, P. (2015). Momentum has its moments. *Journal of Financial Economics*, 116(1):111–120.

Black, F., Jensen, M. C., and Scholes, M. S. (1972). The capital asset pricing model: Some empirical tests. *Studies in the Theory of Capital Markets*, pages 79–121.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31:307–327.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance*, 52(1):57–82.

Chen, Z. and Petkova, R. (2012). Does idiosyncratic volatility proxy for risk exposure? *Review of Financial Studies*, 25(9):2745–2787.

Christidis, A., Gregory, A., and Tharyan, R. (2013). Constructing and testing alternative versions of the fama-french and carhart models in the UK. *Journal of Business Finance & Accounting*, 40(1-2):172–214.

Clarke, R., de Silva, H., and Thorley, S. (2010). Minimum variance portfolio composition. *SSRN Electronic Journal*.

Daniel, K. and Moskowitz, T. J. (2016). Momentum crashes. *Journal of Financial Economics*, 122(2):221–247.

Ødegaard, B. A. (2016a). Empirics of the oslo stock exchange: Asset pricing results. 1980–2015. Paper.

Ødegaard, B. A. (2016b). Empirics of the oslo stock exchange. basic, descriptive, results 1980-2015. Paper.

Diebold, F., Hickman, A., Inoue, A., and Schuermann, T. (1997). Converting 1-day volatility to h-day volatility: Scaling by $\sqrt{h}$ is worse than you think. Working Paper.

Ederington, L. H. and Guan, W. (2004). Forecasting volatility. *SSRN Electronic Journal*.

Engle, R. (2001). Garch 101: An introduction to the use of arch/garch models in applied econometrics. *Journal of Economic Perspectives*, 15(4):157–168.

Fama, E. F. and MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *The Journal of Political Economy*, 81(3):607–636.

Figlewski, S. (2004). Forecasting volatility. Working paper.

Frazzini, A. and Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, 111(1):1–25.

Fu, F. (2009). Idiosyncratic risk and the cross-section of expected stock returns. *Journal of Financial Economics*, 91(1):24–37.

Goyal, A. (2000). Predictability of stock return volatility from garch models. Preliminary and Tentative - Comments Solicited.

Goyal, A. and Santa-Clara, P. (2003). Idiosyncratic risk matters! *The Journal of Finance*, 58(3):975–1007.

Granger, C. W. J. and Poon, S.-H. (2003). Forecasting volatility in financial markets: A review. *Journal of Economic Literature*, 41(2):478–539.

Hansen, P. R. and Lunde, A. (2005). A forecast comparison of volatility models: does anything beat a GARCH(1,1)? *Journal of Applied Econometrics*, 20(7):873–889.

Haugen, R. A. and Heins, J. A. (1975). Risk and the rate of return on financial assets: Some old wine in new bottles. *The Journal of Financial and Quantitative Analysis*, 10(5):775–784.

Lopez, J. A. (2001). Evaluating the predictive accuracy of volatility models. *Journal of Forecasting*, 20(2):87–109.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1):77–91.

Medeiros, M. C., Souza, L., and Veiga, A. (2004). Evaluating the forecasting performance of garch models using white's reality check. *Brazilian Review of Econometrics*, 25(1):43–66.

Moreira, A. and Muir, T. (2016a). Volatility managed portfolios. *SSRN Electronic Journal, October 25, 2016.*

Moreira, A. and Muir, T. (2016b). Volatility managed portfolios. *Working Paper. April 11, 2016.*

Næs, R., Skjeltorp, J. A., and Ødegaard, B. A. (2008). Liquidity at the oslo stock exchange. Paper.

Reider, R. (2009). Volatility forecasting i: Garch models. Working paper.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3):425–442.

Starica, C. (2003). Is garch(1,1) as good a model as the accolades of the nobel prize would imply? *SSRN Electronic Journal.*