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The Equity Risk Premium: A Solved puzzle

An emperical study of the recursive utility model with estimates for the wealth portfolio

Shan Asghar & Andrea Mortensen

Supervisor: Knut Kristian Aase

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Abstract

In this thesis, we calibrate recursive utility models in discrete and continuous time, and find a range of plausible preference parameters for the utility discount rate (β), the relative risk aversion (α) and the elasticity of intertemporal substitution in consumption (EIS). When challenging the consumption-based asset pricing model based on expected utility with our collected data, we provide evidence for an ongoing equity premium puzzle in The United States. Our results indicate that deriving the risk-free rate and risk premium by using recursive utility, rather than expected utility, is a promising way to resolve the puzzle. We consider the market portfolio (M) to be an unfavourable proxy for wealth (W), argued by the low stock participation as a consequence of inequality. Instead, we use our own estimates for the wealth portfolio.

Acknowledgements

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The process of writing this thesis has been interesting, educational and at the same time challenging. Working with this topic has given us broad insight in the literature linked to the equity premium puzzle, and given us the knowledge we need to perform empirical research on financial subjects.

We would like to express our sincere gratitude to our supervisor, Knut Kristian Aase, for introducing us to the topic, as well as sharing knowledge, useful comments and constructive guidance throughout our working process.

Finally, we would also like to thank our family for encouragement and support throughout the process of finishing our Master of Science degree.

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Andrea Mortensen & Shan Asghar

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1. Introduction

Rajnish Mehra and Edward Prescott published a paper[1] in 1985 challenging financial theory with empirical findings in historical US equity premium. They discovered the equity premium to be an order of magnitude greater than what can be rationalized in the context of standard neoclassic paradigm of financial economics. No reasonable parameters for the utility discount rate or the relative risk aversion was found. This mystery, commonly referred to as "The Equity Premium Puzzle", has spawned a plethora of research efforts to explain and resolve the puzzle. See, for example, Rietz (1988), MacGrattan and Prescott (2003) and Constantinides (1990).

During our sample period from 1960-2015, we find the average equity risk premium (ERP) in United States to be 6,16%, with a risk-free rate of 0,91% and a return on S&P500 of 7,07%. This ERP is approximately the same as in Mehra and Prescott's findings.

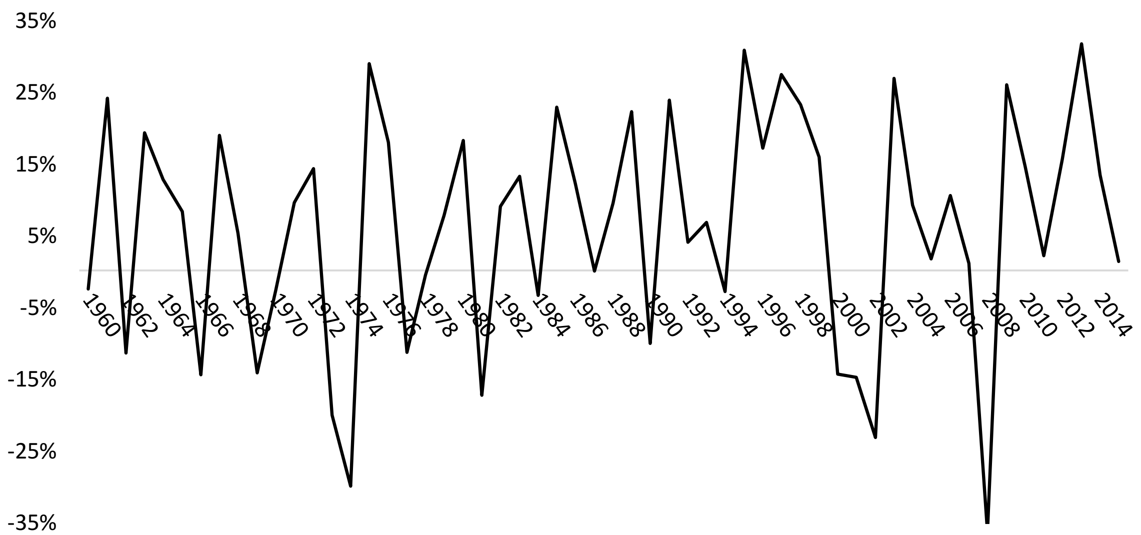


Figure 1: The yearly real equity risk premium in the period between 1960 and 2015

By calibrating recent US data to the consumption-based asset pricing model with additive expected utility preferences, we provide evidence for an ongoing equity premium puzzle in this thesis.

[1] Mehra, R. and Prescott, E C. 1985. *The Equity Premium: A Puzzle*. Journal of Monetary Economics, p.145-161.

We thereafter calibrate Aase's (2016) model based on recursive utility in both discrete and continuous time to our data. In his paper "Recursive utility using the stochastic maximum principle", where he studies and further develop the Epstein-Zin model with recursive utility to solve the equity premium puzzle, Aase presents promising results both with Mehra and Prescott's data from 1985 and more recent Norwegian data[2]. Other researchers have also considered recursive utility as a solution to the puzzle[3], but Aase has made important alterations, such as developing explicit expressions for the risk premium and the equilibrium interest rate, and using estimates for the wealth portfolio instead of the market portfolio as a proxy.

We estimate the wealth portfolio and describe why the market portfolio is not necessarily a good proxy for the wealth portfolio by looking at the inhabitant's low stock participation, and connecting it to the inequality in the United States. Vissing-Jørgensen (1999) has analysed whether the limited stock market participation improve the performance of the consumption-based asset pricing model.

When testing the continuous time model with estimates for wealth, we get reasonable parameters like a relative risk aversion (α) around 2, an impatience rate (β) just under 1 and the elasticity of intertemporal substitution in consumption (EIS) to be marginally larger then 1. We get similar results calibrating the discrete time model, with the exception that (β) is slightly over 1. Although it is not reasonable that $\beta > 1$, we see improved results compared to the traditional expected utility model where we get $\beta > 5$ and $\alpha > 135$. These values are implausible.

We will in chapter 2 give an overview of the theoretical aspects used to explain the traditional consumption-based asset pricing model with expected utility, before we in section 2.3 present equations which is used to calculate the equity premium and risk-free rate. Further in section 2.4, we derive the theory behind recursive utility. In chapter 3, we account for Mehra and Prescott's findings in 1985, and look at Aase's (2016) recursive model used to solve the puzzle. The next chapter (Ch. 4) contains a presentation of US historical data between 1960 and 2015, which is used in chapter 5 to calibrate the models. In chapter 6 we discuss the

[2] The Norwegian data set used in Aase's working paper can be found in Hjortlands (2015) paper "The Equity Premium Puzzle in Norway".

[3] Weil (1989) claims that recursive utility did not give a better explanation than the standard theory – expected utility. Bansal and Yaron (2004) provide empirical support to that Epstein and zin's preferences can explain key asset markets phenomena.

robustness of our results and in chapter 7 we conclude. The thesis contains an Appendix with a more complementary data set.

2. Theoretical Framework

Economic theory is the very complicated mechanism of prices and production, and of the gaining and spending of income. An approach to this vast problem is gained by the analysis of the behavior of the individuals which constitute the economic community (Neumann & Morgenstern, 2007). In the theoretical framework, we look at how the expected utility and the recursive utility models try to explain how people makes decisions.

2.1 Expected utility and risk aversion

Decision theory deals with choice under uncertainty. The perspective approach used in expected utility theory assumes a perfect decision-maker who has all the information and is able to decide with full rationality. This theory is in contrasts to the descriptive approach where the theory tries to describe how individuals actually makes decisions.

If we consider the traditional expected utility theory, there is a minimum set of conditions an individual must hold for us to assume that he/she wants to maximize utility - and therefore make rational and reasonable decisions:

Comparability. An individual can prefer x to y $(x > y)$, y to x $(x < y)$ or the individual is indifferent between the two outcomes $(x = y)$.

Consistency. If the individual prefers x to y $(x > y)$ and y to z $(y > z)$, then x is preferred to z $(x > z)$.

Strong independence. If the individual is indifferent as to x and y, then he/she will also be indifferent as to a first gamble, set up between x with probability α and a mutually exclusive outcome, z, and a second gamble, set up between y with probability α and the same mutually exclusive outcome, z.

Measurability. If outcome y is preferred less than x but more than z, then there is a unique α (probability) where the individual will be indifferent between y and a gamble between x with a probability α and z with a probability $(1 - \alpha)$.

Ranking. If $x > y > z$ and $x > u > z$ and an individual is indifferent between *y* and a gamble between *x* (with probability α_1) and *z,* while also indifferent between u and another gamble between *x* (with probability α_2) and *z*. If α_1 is greater then α_2, *y* is preferred to *u*.

These axioms of behavior used by economists give us the following assumptions; All individuals always make completely rational decisions and people are assumed to be able to make these rational choices among thousands of alternatives. The individual acts as if he/she maximizes expected utility.

We can use expected utility to rank combinations of risky outcomes and the expected utility of the stochastic outcome x can be written as:

$$E[U(x)] = \sum_{i=1}^{n} p_i U(x) \qquad 1.1.1$$

for $i = 1,2, \dots, n$, with probabilities p_i.

Rational individuals prefer more wealth then less, and therefore the utility function is growing $U'(w) > 0$. In addition, an individual is risk-averse if, at any wealth level *w*, he or she dislikes every lottery with an expected payoff of zero. Since it is reasonable to assume that more risk-averse individuals are willing to pay more to get rid of the gamble with a given risk, you can compare the degree of risk-aversion by looking at the risk premium.

Many of the utility functions is concave and growing, but by adding a third assumption we can eliminate most of them. This assumption is documented by Arrow, and is called constant relative risk aversion (CRRA):

$$CRRA = U(c) = \frac{w^{1-\alpha} - 1}{1 - \alpha} \text{ where } \alpha > 0 \text{ og } \alpha \neq 1 \qquad 1.1.2$$

α is a measure in risk aversion.

2.2 Asset pricing

Asset pricing theory tries to understand the prices or values of claims to uncertain payments (Cochrane, 2001). An important contribution to asset pricing is the capital asset pricing model (CAPM) of William Sharpe (1964) and John Lintner (1965). The model states that the expected return on any asset is proportional to the amount of non-diversifiable risk. The expected rate of return for any asset can then be presented as:

$$E(R^i) = R_f + \beta_i[E(R^m) - R_f] \qquad 2.2.1$$

Where:

$$\beta_i = \frac{COV(R^i, R^m)}{VAR(R^m)}$$

In the model, $E(R^i)$ represents the expected return of the asset and β_i represents the non-diversifiable risk.

In CAPM, the only source of risk in the economy is the uncertainty regarding the return on the market portfolio, but it does not provide any tools to help us identify what causes the market portfolio to be risky. To be able to draw connections between the return of companies and a proxy for overall economic activity, such as consumption, we look to the consumption-based capital asset pricing model (CCAPM) of Lucas (1978) and Breeden (1979).

The consumption-based asset pricing model is a financial model that extends the capital asset pricing model to include the amount individuals or firms seek to consume in the future. The model is based on the idea that an agent prefers investments which gives more dividends or has a value increase when consumption falls. Such an investment gives an agent the possibility to maintain the consumption level. Therefore, if an investment is positively correlated with consumption it is not as attractive as if the investment is negative correlated.

Assuming a multi-period endowment economy which goes into infinity, this gives us the following utility function:

$$E_t[U_t] = U(c_t) + E_t\left[\sum_{k=1}^{\infty} \beta^k U(c_{t+k})\right] \qquad 2.2.2$$

The agent will maximize its utility by maximizing consumption in the current period t, and in the future periods $t + k$. The intuitively pleasing implication is that our agent will value consumption in the near future higher than consumption in the more distant future. Empirically the value of β is set slightly less than 1, which implies that consumption in consecutive time periods are close substitutes.

2.3 Equity premium and risk-free rate

In today's changing and evolving financial playground the equity risk premium (ERP) remains a fundamental component in asset pricing. It is a key input in estimating the cost of equity, cost of capital, individual savings decisions and government budgeting plans.

The realized returns on a security is:

$$R_{t+k}^{i} = E\left(R_{t+k}^{i}\right) + \mathrm{error}_{\mathrm{t+k}}$$

R_{t+k}^{i} is realized returns between time t and $t + k$ and $E\left(R_{t+k}^{i}\right)$ is the returns that were expected from t to $t + k$ using information available at time t. The variable $\mathrm{error}_{\mathrm{t+k}}$ is a random variable that is unknown at time t and realized at time $t + k$.

The equity risk premium at time t for horizon k is defined as:

$$E(R_{t+k}^{e}) = E\left(R_{t+k}^{i}\right) - R_{f\ t+k}$$

$R_{f\ t+k}$ is the risk-free rate for investing from t to $t + k$.

Three important aspects of the equity risk premium appear. First, future expected returns and future ERP are stochastic, since expectations depend on the arrival of new information that has a random component unknown at time t. Second, ERP has an investment horizon k embedded in it, since we can consider expected excess returns over, say, one month, one year or five years from today. If we fix t, and let k vary, we trace the term structure of the equity risk premium. Third, if expectations are rational, because the unexpected component $\mathrm{error}_{\mathrm{t+k}}$ is stochastic and orthogonal to expected returns, ERP is always less volatile than realized excess return. We express the equity premium as:

$$E(R^{e}) = -\rho_{m,R^{e}} \frac{\sigma(m)}{E(m)}\, \sigma(R^{e}) = -\frac{cov(R^{e}, m)}{E(m)}$$

Any asset which has a negative covariance with the stochastic discount factor (m) will lead to a higher risk premium. Assets with this characteristic will yield high returns when marginal utility is low and consumption is high. A risk averse agent desire an even consumption over time, so an asset like this will require a high risk premium to make it desirable.

ERP can also be expressed as:

$$E(R^e) \approx \alpha \sigma_c \sigma(R^e) \qquad 2.3.1$$

A rising α represents an increasingly risk averse representative agent, whom will require a higher risk premium. ERP is also growing with the standard deviation of consumption and the standard deviation of the risky asset, as more volatility regarding consumption and returns makes it more uncertain.

Turning our attention back to the risk-free rate and applying the first order condition of a utility maximizing representative customer we get:

$$r_f \approx \ln R_f = -\ln \beta + \alpha E(\mathrm{c}) - \frac{1}{2}\alpha^2 \sigma_c^2 \qquad 2.3.2$$

The risk-free rate (R_f) is high when the impatience rate (β) is low. The marginal utility of future consumption is reduced when the agents is impatient and the agent will require a high risk-free rate to move current consumption to the future.

When expected consumption growth is high, R_f is high. The expected marginal utility of consumption will be lower in the future period because the growth is expected to be high. Our representative agent will therefore require a high risk-free rate to be willing to substitute present consumption for future consumption.

R_f is low when conditional consumption volatility is low due to lack of need for precautionary savings. As future consumption becomes more certain, our representative agent will not have to save as much to achieve even consumption over time. The agents relative risk aversion (α) is accounted for twice, so the net effect is uncertain.

2.4 Recursive utility

In the standard life-cycle model with additive and separable utility it is not possible to separate between risk aversion and elasticity of intertemporal substitution (EIS) in consumption (Aase, 2016). This is considered a weakness, since there are different aspects of an individual's preferences. The notion of using expected utility theory to analyze risk aversion has come under criticism from behavioral finance, and other important theories has risen. Such as recursive utility, where the combination between time and uncertainty are important elements.

Recursive utility was first introduced by Kreps and Porteus (1978) as a model in discrete time. It was further developed by Epstein and Zin (1989-91) whom suggested a special parametrization which makes the model applicable to numerous financial topics. "Recursivity" is here a central axiom, which is identical to the idea of dynamic consistency explored by Johnsen and Donaldson (1985). Epstein and Zin (1989-91) presented two underlying assumptions for their specification of intertemporal utility. First, it is assumed that the agent forms a certainty equivalent of random future utility using his risk preferences. The second assumption is that to obtain current-period lifetime utility, this certainty equivalent is combined with deterministic current consumption via an aggregator function. A utility form like this generalizes the recursive structure introduced by Koopmans (1960).

We assume a risk-free economy with dynamic consistency, irrelevance of past consumption and state independence. With consumption preferences $(c_0 + c_1,\cdots, c_T)$ characterized with $U(c_0 + c_1,\cdots, c_T)$ where $V_t = U_t(c_t + c_{t+1},\cdots, c_T)$, consequently:

$$V_t = f(u(c_t), CE_{t+1}) = \left((1-\beta)c_t^{1-p} + \beta(E_t(V_{t+1}^{1-\alpha}))^{\frac{1-p}{1-\alpha}}\right)^{\frac{1}{1-p}} \qquad 2.4.1$$

The function f represents the utility of current and the future consumption simultaneously. The alternation from Koopmans (1960) is uncertainty in the model, where CE_{t+1} represents the certainty equivalent of all possible future consumption sequences. We can see that recursive utility leads to separation of risk aversion from the elasticity of intertemporal substitution (EIS) in consumption, within a time-consistent model framework. The separation holds several advantages over the traditional model, as the agent is less nearsighted and instead considers a longer time span in his decision-making.

β $(0 < \beta < 1)$ is the utility discount factor with relative impatience rate $\delta = \ln\left(\frac{1}{\beta}\right)$. p is the time preference parameter, the inverse to EIS-parameter, where $EIS = \frac{1}{p}$ is the elasticity of intertemporal substitution to consumption i.e. the agent's willingness to substitute consumption intertemporally. A consumer who saves more with a high interest rate is characterized by a high EIS. More formally, EIS is defined as the negative ratio of changes in log consumption growth and log growth of marginal utility of consumption:

$$EIS = -\partial log\left(\frac{C_{t+1}}{C_t}\right) / \partial log\left(\frac{\partial U/\partial C_{t+1}}{\partial U/\partial C_t}\right)$$

Where U represent the utility function of the consumer.

The time preference parameter p is here accepted to be different from the risk aversion (α). If $\alpha > p$ the individual prefers early clarification of uncertainty rather than late. If $\alpha < p$ the individual prefers late clarification of uncertainty rather than early.

3. The Equity Premium Puzzle

Historically the average return on equity has far exceeded the average return on short-term virtually default-free debt. Over the ninety-year period 1889-1978 the average real annual yield on the Standard and Poor 500 (S&P500) Index was seven percent, while the average yield on short-term debt was less than one percent.

Many intuitively answer to why stocks have been such an attractive investment relative to bonds is that since stocks are "riskier" than bonds, investors require a larger premium for bearing additional risk. And indeed, the standard deviation of the return on stocks is larger than the return on T-bills, so obviously, they are considerably riskier than bills. Or are they?

First, we explain Mehra and Prescott's findings in their famous paper from 1985. Afterwards, we present equations from Aase (2016) for the risk premium and the risk-free rate based on recursive utility in both discrete and continuous time, which we consider as a promising solution to the well-established puzzle.

3.1 A Puzzle?

What Mehra and Prescott concluded with in their paper "The Equity Premium; A Puzzle" was that stocks and bonds pay off in approximately the same states of nature or economic scenarios and they should command approximately the same rate of return.

In their paper, Mehra and Prescott employ a variation of Lucas (1978) pure exchange model, and assumes the growth rate of the endowment to follow Markov process[4] with defined states:

$$\lambda_1 = 1 + \mu + \delta, \qquad \lambda_2 = 1 + \mu - \delta$$

And probabilities:

$$\phi_{11} = \phi_{22} = \phi, \qquad \phi_{12} = \phi_{21} = 1 - \phi$$

They then fit sample values of historical data from the time-period and search for the parameters α and β. The restrictions and assumptions used in the model, results in the highest

[4] Markov process is a random process whose future possibilities are determined by its most recent values. A stochastic process $x(t)$is called Markov if for every n and $t_1 < t_2 \dots t_n$, we have $P[x(t_n) \le x_n | x(t_{n-1}), \dots, x(t_1)] = P[x(t_n) \le x_n | x(t_{n-1})]$.

obtainable value of equity premium being 0,35%. Actual observed sample risk premium was 6,18% and could not be explained unless implausible values where used. Mehra & Prescott concluded that the preference parameters, coefficient of relative risk aversion (α) and the time discount factor (β), was way too high. Their results gave birth to the entrancing equity premium puzzle.

After Mehra and Prescott (1985) opened the debate of the equity premium puzzle, there has been many attempts to provide an explanation for this puzzle. It cannot be dismissed lightly, because much of our economic intuition is based on this class of models that fall short so dramatically when confronted with financial data (Mehra, 2003).

The puzzle has become a major research impetus in finance and economics over the past 30-years. Researchers have proposed several theories to account for the puzzle. Rietz (1988) came with a solution based on disaster insurance. McGrattan and Prescott (2003) proposed a possible solution for the high equity returns in the period after the Second World War based on the declining marginal tax rate. Benartzi and Thaler (1995) presented a solution to the equity premium puzzle related to the behavioral finance literature – and called their attempt myopic loss aversion. Initiated by Constantinides (1990), a solution based on habit formation was presented.

All of these attempts to resolve the puzzle appear unsatisfactory. Mehra (2008) states that no explanation has fully resolved the mystery, but considerable progress has been made and the equity premium is a lesser puzzle today than it was 25 years earlier.

3.2 Aase's recursive utility model – The solution

One possible solution is presented by Aase (2016), where he further develops the Epstein-Zin model based on recursive utility, and use the stochastic maximum principle to analyze the model. He develops explicit expressions for the risk premium and the equilibrium interest rate. When calibrating his adjusted recursive utility model to Mehra and Prescott's data from 1985, and more recent Norwegian data, he presents compelling results.

Earlier attempts to resolve the puzzle through recursive utility includes the well know paper Weil (1989), where the author wrongfully concludes that recursive utility cannot explain the equity premium and instead finds a new puzzle "the risk-free rate puzzle" which is also invalidated in the light of Aase's research.

Another significant alteration from previous attempts to solve the puzzle using recursive utility is assuming that the market portfolio is an unsatisfactory proxy for the wealth portfolio. We use the definition of wealth which includes current consumption, so the gross real rate of return on the wealth portfolio over the period $(t, t+1)$ is:

$$R_{t+1}^{W} = \frac{W_{t+1}}{w_t - c_t}$$

Aase (2016) argues that the W_t represents the wealth portfolio of the representative agent and should not be confined to the market portfolio, but instead include exogenous income streams which could be viewed as dividends of some shadow asset.

Using the development in Aase (2016) we end up with the following expression for the equity risk premium and the risk-free rate in discrete time:

$$\begin{aligned} &E_t(\ln R_{t+1}^{R}) - \ln R_{t+1}^{f} \\ &\qquad = \frac{p(1-\alpha)}{1-p} cov_t\left(\ln\left(\frac{c_{t+1}}{c_t}\right), \ln R_{t+1}^{R}\right) + \frac{a-p}{1-p} cov_t(\ln R_{t+1}^{W}, \ln R_{t+1}^{R}) \end{aligned} \qquad 3.2.1$$

$$\begin{aligned} \ln R_{t+1}^{f} = {} & \frac{1-\alpha}{1-p} ln\left(\frac{1}{\beta}\right) + \frac{p(1-\alpha)}{1-p} E_t\left(\ln\left(\frac{c_{t+1}}{c_t}\right)\right) - \frac{1}{2}\frac{p^2(1-\alpha)^2}{(1-p)^2} var_t\left(\ln\left(\frac{c_{t+1}}{c_t}\right)\right) \\ & + \frac{(\alpha-p)}{1-p} E_t(\ln R_{t+1}^{W}) - \frac{1}{2}\frac{(p-\alpha)^2}{(1-p)^2} var_t(\ln R_{t+1}^{W}) \\ & + p\frac{1-\alpha}{1-p}\frac{p-\alpha}{1-p} cov_t\left(\left(\ln\left(\frac{c_{t+1}}{c_t}\right)\right), \ln R_{t+1}^{W}\right) \end{aligned} \qquad 3.2.2$$

What distinguishes the model from the old framework is the preference representation of the representative agent. If we set $\alpha = p$, we end up with the conventional consumption-based capital asset pricing models from equation 2.3.1 and 2.3.2.

And in continuous time the expressions for the equity risk premium and the risk-free rate is:

$$E_t(R_t^R) - R_t^f = \frac{p(1-\alpha)}{1-p} cov_t(R_t^C, R_t^R) + \frac{\alpha - p}{1-p} cov_t(R_t^W, R_t^R) \qquad 3.2.3$$

$$R_t^f = ln\frac{1}{\beta} + p * E(R_t^C) - \frac{1}{2}\frac{p(1-\alpha p)}{1-p} var_t(R_t^C) + \frac{1}{2}\frac{p-\alpha}{1-p} var_t(R_t^W) \qquad 3.2.4$$

R is the return on any risky asset in the market while W is the return on the wealth portfolio. If there are no estimates for the wealth portfolio, we use S&P500 (M) as a proxy. As mentioned earlier Aase argue that using W instead of M might be more realistic as earlier research has shown stock market participation to be low. When calibrating the model, R is set equal to M.

4. Data description

In this chapter, we present historical data from the United States needed to calibrate the models. For the data set we have chosen a 55-year long time period from 1960 to 2015. This period makes it possible for us to identify if there has been an equity premium puzzle in the US market the recent years. We have calculated the period-average, standard deviations and covariance estimates from the historical data for consumption growth, return on the market portfolio, risk-free rate, national wealth and the equity premium.

4.1 Key data

	Expectation	Standard dev.	Covariances	Corr. coeff.
Return S&P500	7,07%	16,36%	$\hat{\sigma}_{W,M} = 0{,}000881$	0,1818
Consumption growth	2,89%	1,59%	$\hat{\sigma}_{M,c} = 0{,}000253$	0,0975
Government bills	0,91%	2,09%	$\hat{\sigma}_{c,Rf} = 0{,}000156$	0,4692
Equity Premium	6,16%	16,01%		
Real National wealth	1,11%	2,96%	$\hat{\sigma}_{c,W} = 0{,}000333$	0,7084

Table 4-1: key historical US-data (1960-2015) in real terms

	Expectation	Standard dev.	Covariances	Corr. coeff.
Return S&P500	5,56%	16,62%	$\hat{\sigma}_{W,M} = 0{,}001048$	0,2145
Consumption growth	2,85%	1,55%	$\hat{\sigma}_{M,c} = 0{,}000345$	0,1341
Government bills	0,89%	2,07%	$\hat{\sigma}_{c,Rf} = 0{,}000152$	0,4708
Equity Premium	4,67%	16,38%		
Real National wealth	1,06%	2,94%	$\hat{\sigma}_{c,W} = 0{,}000324$	0,7124

Table 4-2: Key historical US-data (1960-2015) in real log-terms

	Expectation	Standard dev.	Covariances	Corr. coeff.
Return S&P500	6,83%	16,24%	$\hat{\sigma}_{W,M} = 0{,}000880$	0,1671
Consumption growth	2,86%	1,59%	$\hat{\sigma}_{M,c} = 0{,}000253$	0,0931
Government bills	0,91%	2,08%	$\hat{\sigma}_{c,Rf} = 0{,}000156$	0,3847
Equity Premium	5,92%	14,85%		
Real National wealth	1,10%	2,95%	$\hat{\sigma}_{c,W} = 0{,}000333$	0,5355

Table 4-3: Key historical US-data (1960-2015) in real terms, continuous-time compounding

4.2 The market portfolio

The Standard and Poor's 500 (S&P500) measures performance of the broad domestic economy through changes in the aggregate market value of 500 stocks representing all the major industries in the US. We considered it to be the best representation of the US stock market and therefore the best way to represent the market portfolio. We use year-end adjusted closing prices, and retrieved the data through Bloomberg[5].

To get S&P500 in real terms we adjust the data by using the Consumer Price Index – CPI, (Appendix, table A.1) obtained from Bureau of Labor Statistics. Thru the 55-year period we calculated the real average market return to be 7,07%, and the complete data set can be found in appendix, table A.2. In the figure below we present yearly growth on the Standard and Poor's 500 real return:

[5] Bloomberg is a famous global provider of 24-hour financial news and information, including real-time and historical price data and trading news.

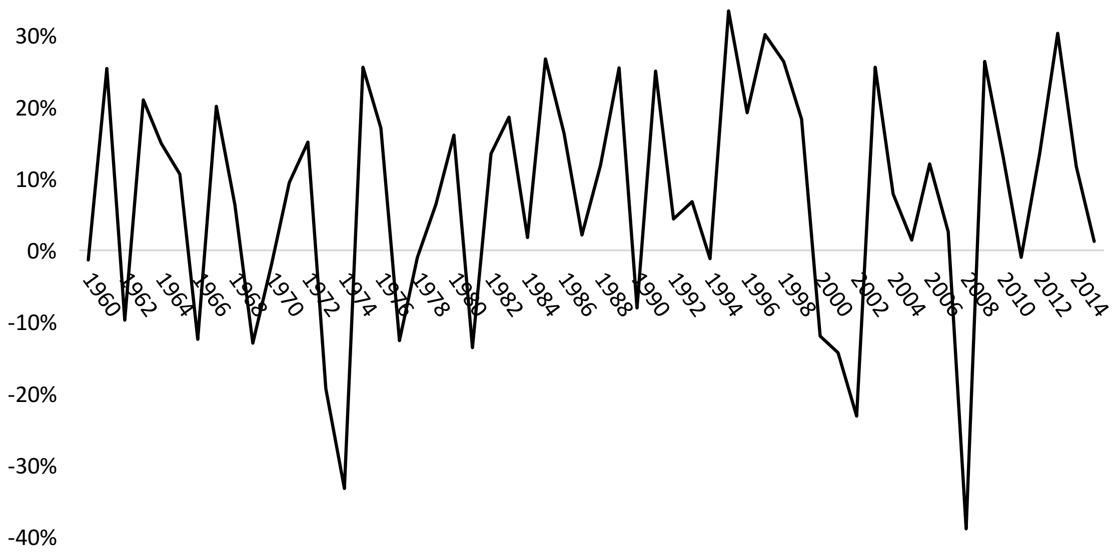


Figure 2: Real Standard and Poor's 500 return (%) in the period from 1960 to 2015

From Figure 2 we can see that the return in the US market is highly volatile with a standard deviation exceeding 16%. The highest return is in 1995 of 33,41%, and an extreme low point in 2008 with a negative return of 38,90%.

4.3 The Risk-free rate

We use the average of four 3-Month Treasury Bills to find the annual risk-free rate. The market considers it to be virtually no chance for the government defaulting on its obligations. The numbers are provided from the Economic Research Division by the Federal Reserve Bank of St. Louis, and we consider the data highly credible. After we adjust it to CPI, we found the average of the real risk-free rate (appendix, table A.3) to be 0,91%. The standard deviation is 2,09%, which means that the real risk-free rate has not been as volatile as the return on the market portfolio.

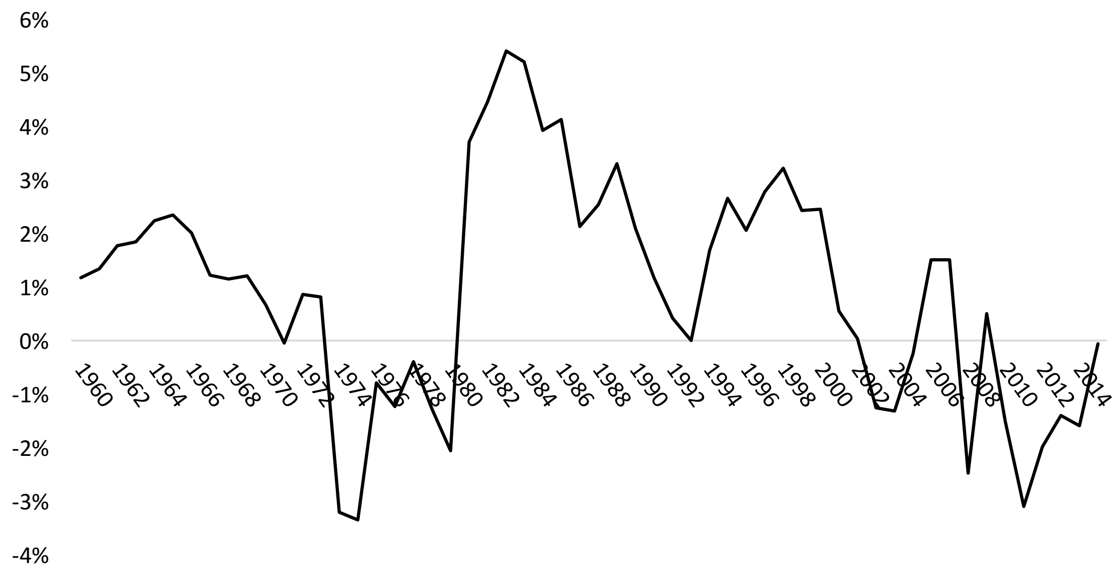


Figure 3: Real Risk-free rate in the period from 1960 to 2015

4.4 The realized Equity Risk Premium

As stated in theoretical framework chapter 2.3, the equity risk premium (ERP) is a result of the return on market portfolio subtracting the risk-free rate. We calculate the average ERP to be 6,16%, and can ascertain the fact that investing in the US stock market has been very lucrative.

To illustrate the difference, imagine investors placing $100 in the US stock market or in a risk-free investment in 1960. An average investor who placed $100 in the US stock market would have $4,283 in 2015. In contrast, the investors who made a risk-free investment would have $164.

As we see in Figure 4, if you invested in the US stock market with a 20-year horizon, and with a diversified portfolio, the average annual return to the market portfolio has exceeded the risk-free rate. Historical data provide a wealth of evidence documenting that for more than 50-years, US stock returns have been considerably higher than returns for T-bills:

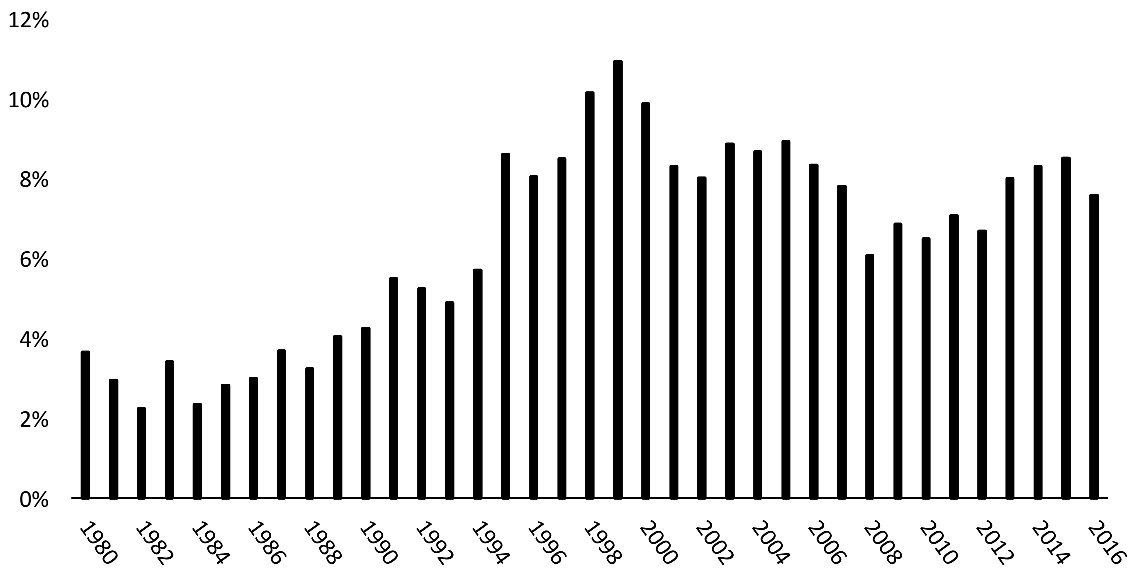


Figure 4: Realized annual average on real equity premium (vertical) for a 20-year holding period ending in (horizontal)

If the time horizon is shorter, you can find periods with a negative equity premium. In this period, the investor would have had a negative return on the investment. We can see this negative trend in a few periods with a ten-year horizon:

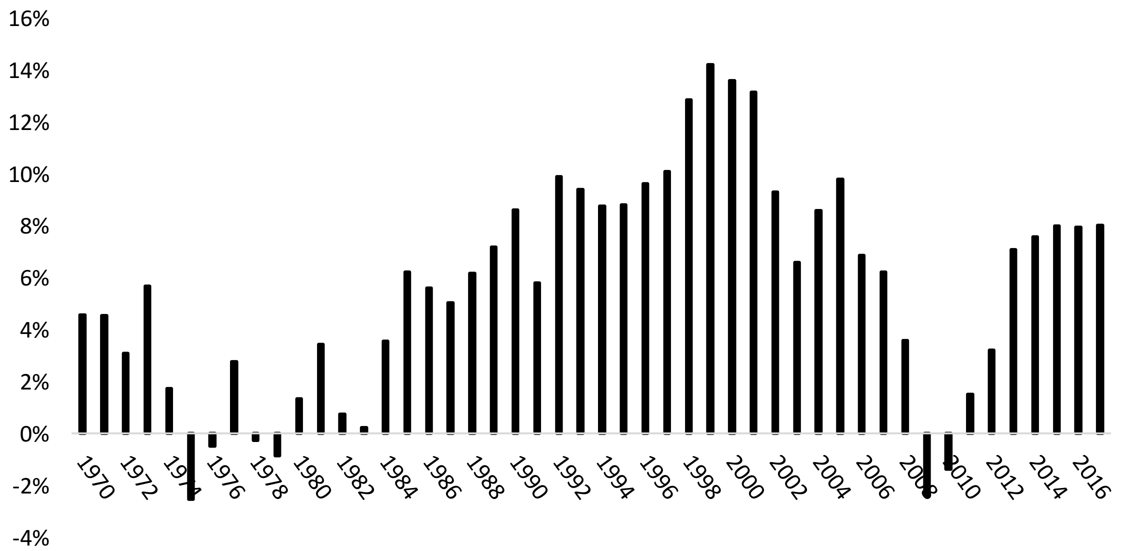


Figure 5: Realized annual average on real equity premium (vertical) for a 10-year holding period ending in (horizontal)

The 20-year and 10-year average ERP peaked during the bull market of the 1990s and more recently after the end of the global financial crisis in 2008. The average risk premium was particularly low during the oil price shocks in the 1970s, coupled with US economy experiencing a period of low growth and high inflation, it resulted in especially low equity returns and large short-term Treasury rates. Also, the financial crisis in 2008 contributed to a low 10-year average equity risk premium.

4.5 Consumption

We found the data for consumption (appendix, table A.6) through the Bureau of Economic Analysis website, and includes consumption of non-durable goods and services. We divided the numbers by population size to find consumption growth per capita:

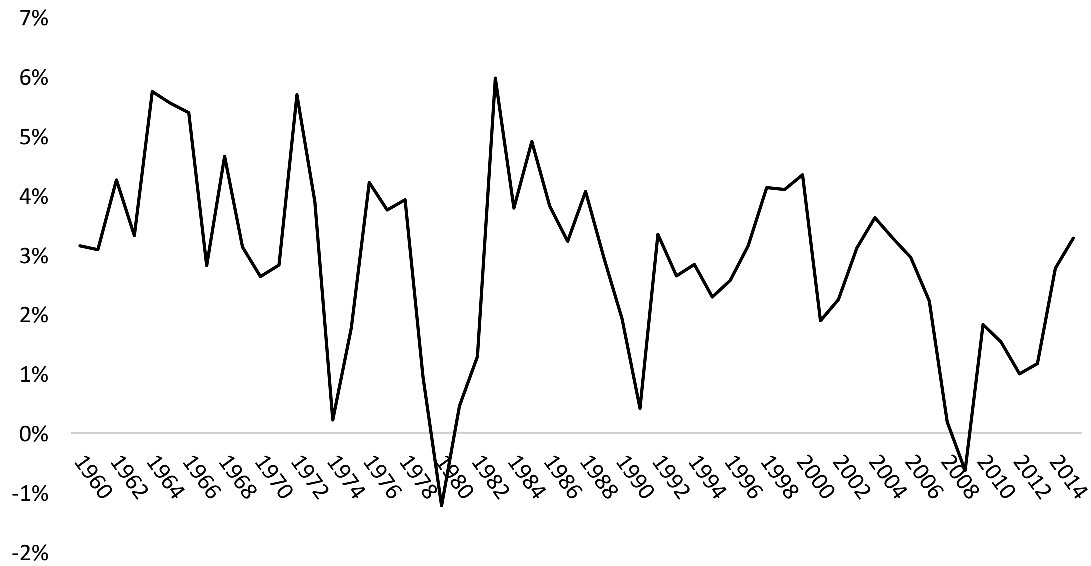


Figure 6: Consumption growth per-capita in the period from 1960 to 2015

The average growth has been 2,89%, with the highest growth in 1983 on 6,0% and the lowest in 1980 with a negative growth on 1,2%.

	Average
1960-1969	4,1 %
1970-1979	3,0 %
1980-1989	2,9 %
1990-1999	2,7 %
2000-2009	2,3 %
2005-2015	1,8 %

Table 4-4: 10-year average consumption growth

From our data set we can see that the consumption growth has declined from an average of 4,1% in the 1960-1969 period to an average on 1,8% in the 2005-2015 period.

4.6 National wealth

From our calculations, we find the average growth in national wealth to be 1,1% with a standard deviation on 2,96%:

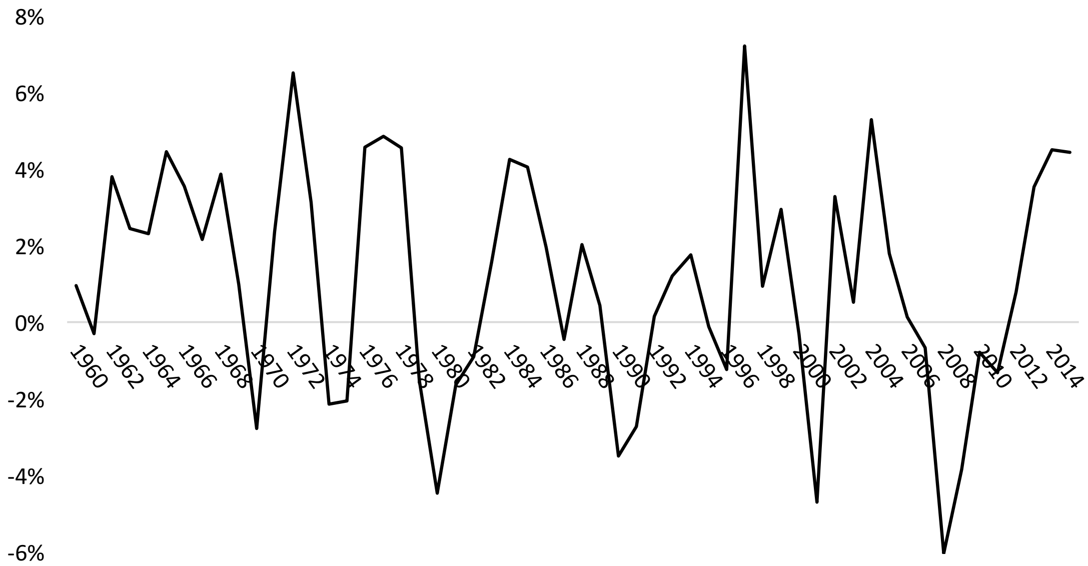


Figure 7: Real annual per-capita growth in the US national wealth (1960-2015)

The national wealth is highly affected by crisis in the market. For example, the financial crisis in 2008 leads to a decline in national wealth of 6,0%. Also, we see a huge drop in 2000-2001 following the dot-com bubble, resulting in a 4,7% decline.

In our work to calculate The US national wealth we have used Statistics Norways approach. In principle, everything that affects welfare should be included in the calculation of national wealth such as natural resources, biodiversity and the populations health and more. In practice, many of these variables is difficult to quantify. Because of this, our calculations are limited to real capital, net financial capital, human capital and natural resources.

The real capital includes equipment and machines etc. which is used to produce goods and buildings. **Net financial capital** is the US foreign debt and the national equity. **The natural resources** are materials provided by the earth that humans can use to make more complex, human-made, products. It can be both renewable and non-renewable, and includes for example trees, minerals, soils, water, oil etc. These three elements together constitute 20% of the total national wealth.

Human capital is the fourth component in the US national wealth, and is defined in the Oxford English Dictionary as "the skills the labor force possesses and is regarded as a resource or an asset." In theory, there are three different approaches to measuring human capital: Indicators, cost method and income method. We use the income method to estimate the human capital by finding the present value of future income.

To calculate the human capital, we have used the expression:

$$W = \sum_{t=1}^{\infty} \left(\frac{1+p}{1+r}\right)^{t} w_0 h_0 N_0 = w_0 h_0 N_0 \frac{1+p}{r-p}$$

Where,

W = Present value of future income

w_0 = Wage level per hour

p = Annual growth in labor productivity

r = Discount rate

h_0 = Numbers of hour worked per capita in output year

N_0 = Number of inhabitants in output year

The long-run annual growth for labor productivity since world war II has been 2,2%. We choose to adjust it to 2,0%, as the growth rate has been declining in the more recent years. The yearly rates are provided from International Monetary Fund.

The discount rate we use in our calculations is 4,0%. It reflects the risk adjusted return on labor, based on the perceived risk on a 55-year time period. The World Bank (1998) estimated the discount rate in industrial countries to be between 2% - 4%. The rate used is often referred to as the social discount rate, and it reflects a society's relative valuation on today's well-being versus well-being in the future.

According to advanced economic growth theory, human capital is considered to play a decisive role in determining a country's economic prosperity. Our calculations indicate that human capital is the most important component of the US national wealth, accounting for approximately 80% of the total national wealth.

5. Calibration and validation of models

Calibration and validation of models is required to test how well the models results matches the actual measured data. In this chapter, we analyse how well the data in the previous chapter fits in the traditional model based on expected utility. This makes us able to state if there still is an equity premium puzzle. We calibrate the recursive utility models with our calculated data by using the market portfolio as a proxy for wealth. We argue why this is an unsuitable proxy, and present results using our own wealth estimates. To be able to conclude, we first discuss which preference parameters we consider reasonable in this thesis.

5.1 Reasonable preference parameters

As we explained in chapter 3.1, the values for β and α in the standard model for risk premium and risk-free rate is meaningless. Mehra and Prescotts got a time discount factor (β) higher than 1 (1,1) and a coefficient of relative risk aversion (α) of approximately 22. They stated that this was too high, and their conclusion is verified by many others.

The preference parameters we consider to be reasonable in this thesis is;

$0{,}95 < \beta < 1$

$1 < \alpha < 5$

$0{,}5 < p < 2 \quad \rightarrow \quad 0{,}5 < EIS < 2$

$\alpha \neq p \quad \rightarrow \quad \alpha > p$

The subjective time discount factor (β) should, according to Mehra & Prescott (1985), be between 0 and 1 ($0 < \beta < 1$). A $\beta > 1$ is unreasonable, since it leads to a negative impatience rate. A $\beta < 0$ implies that consumption in the future is unwanted, an assumption that is highly implausible. Researchers that treat the impatience parameter as exogenous, use values close to 1. Kocherlakota (1990) and Mehra (2003) used a value of 0,99, Weil (1989) used a value of 0,95 and 0,98. We will consider values between 0,95 and 1 as plausible in this thesis.

In the standard model CCAPM, the coefficient of relative risk aversion (α), must be inexplicable large to explain the observed risk premium in the market. By accepting people to be more risk averse than expected, another problem arises. If we use a high α in solving the problem with a high risk premium, we see from expression 2.3.2 that we will also get a high risk-free rate. Mehra & Prescott (1985) considered the parameter α, in CRRA, to be a maximum of ten and presented many studies which implied it to be between 1 and 2 ($1 < \alpha < 2$). Weil (1989) state that α is usually estimated to be in the range 1 to 5 ($1 < \alpha < 5$). According to Kocherlakota (1996) a value above five implies implausible behavior.

To find a reasonable value for elasticity of intertemporal substitution (EIS), we look at what other researchers consider as plausible. Kydland and Prescott (1982) and Jones *et al*. (2000) set up equilibrium business cycle models and argue that an EIS between 0.8 and 1 ($0{,}8 < EIS < 1$) gives the best fit to the data. Lucas (1990) rules out an EIS lower than 0,5 ($EIS > 0{,}5$). Epstein and Zin (1991) estimated EIS to be between 0,17 and 0,86 ($0{,}17 < EIS < 0{,}86$). Thimme and Völkert (2015) estimates EIS to be 0,78 ($EIS = 0{,}78$). Chen et al. (2013) estimate EIS to be in the range 1,67 to 2 ($1{,}67 < EIS < 2$). Dagvik, Strøm and Jia (2006) estimate EIS for the Norwegian population to be between 1 and 1,5 ($1 < EIS < 1{,}5$). The magnitude of the EIS-parameter is debated, and it is not obvious what value should be considered reasonable.

Solutions where $\alpha > p$ is more plausible, since it is reasonable to believe that agents want to resolve uncertainty as soon as possible.

5.2 The expected utility model

If we use the expressions from chapter 2.3 and our log data from table 4.2 we get:

Expression 2.3.1: $$0{,}046682 = \alpha \cdot 0{,}000345$$

Expression 2.3.2: $$0{,}008867 = -ln\beta + \alpha \cdot 0{,}028459 - \frac{1}{2}\alpha^2 \cdot 0{,}015479^2$$

By solving these equations, we get a coefficient of relative risk aversion (α) of 135,38. This is markedly higher than what Mehra and Prescott (1985) got ($\alpha = 22$) when they stated that the values was way too high to be reasonable. There is no way people are that risk averse. The impatience factor (β) is with this model 5,20. As stated in the previous section, this value means that the agent has negative impatience - the agent prefers consumption tomorrow over consumption today. These preference parameters indicate that the equity premium puzzle is still a reality in United States. In our sample period, we observe decreasing volatility. The expected utility model perform worse as time passes, and the above calculations makes it obvious that this model does not reflect reality.

5.3 The recursive utility model

First, we calibrate the models where we use the market portfolio (M) as a proxy for the wealth portfolio (W) both in discrete and continuous time, before we turn our attention to the models with our wealth estimates.

In the equations below, we have three unknown parameters. In the models where we use M as a proxy, we use variations of the impatience rate (β) to find estimates of the implied coefficient of risk aversion (α) and EIS-parameters $\left(\frac{1}{p}\right)$. In the models where we use our estimates for the wealth portfolio we fix variations of p to find estimates for α and β.

To calibrate the discrete-time models, we use the data from table 4.2, and to calibrate the continuous-time models, we use the data from table 4.3.

5.3.1 The model with the market portfolio as a proxy for the wealth portfolio

In discrete time

From expression 3.2.1:

$$0{,}0467 = \frac{p(1-\alpha)}{1-p}0{,}000345 + \frac{\alpha - p}{1-p}0{,}0276$$

From expression 3.2.2:

$$0{,}00887 = \frac{1-\alpha}{1-p}ln\left(\frac{1}{\beta}\right) + \frac{p(1-\alpha)}{1-p}0{,}0285 - \frac{1}{2}\frac{p^2(1-\alpha)^2}{(1-p)^2}0{,}000240 + \frac{(\alpha-p)}{1-p}0{,}05556 - \frac{1}{2}\frac{(p-\alpha)^2}{(1-p)^2}0{,}0276 + p\frac{1-\alpha}{1-p}\frac{p-\alpha}{1-p}0{,}000345$$

β	α	p	EIS	δ
0,950	1,34	0,51	1,95	0,051
0,955	1,21	0,70	1,44	0,046
0,960	1,07	0,88	1,14	0,041
0,970	0,83	1,24	0,81	0,030
0,980	0,58	1,59	0,63	0,020
0,990	0,33	1,94	0,51	0,010

Table 5-1: Implied preference parameters when the recursive model in discrete time is calibrated with the market portfolio as a proxy for the wealth portfolio

In continuous time

From expression 3.2.3:

$$0{,}0592 = \frac{p(1-\alpha)}{1-p}0{,}000253 + \frac{\alpha - p}{1-p}0{,}0264$$

From expression 3.2.4:

$$0{,}00908 = ln\frac{1}{\beta} + p \cdot 0{,}0286 - \frac{1}{2}\frac{p(1-\alpha p)}{1-p}0{,}000252 + \frac{1}{2}\frac{p-\alpha}{1-p}0{,}0264$$

β	α	p	EIS	δ
0,950	2,78	-0,44	-2,27	0,051
0,960	2,33	-0,08	-13,33	0,041
0,970	1,89	0,29	3,47	0,030
0,975	1,66	0,47	2,13	0,025
0,980	1,44	0,65	1,54	0,020
0,985	1,21	0,83	1,21	0,015
0,989	1,03	0,97	1,03	0,011
0,990	0,99	1,01	0,99	0,010

Table 5-2: Implied preference parameters when the recursive model in continuous time is calibrated with the market portfolio as a proxy for the wealth portfolio

In table 5-1 and 5-2 we see the implied parameters after calibrating the models in discrete and continuous time using market portfolio as a proxy for wealth, and by analyzing the results we see that both models have a range where all the preference parameters are within what is reasonable. Even so, the market portfolio seems to be an unacceptable substitute for the wealth portfolio.

5.3.2 Stock participation and inequality

We argue that using S&P500 as a proxy for the wealth portfolio might not be the way to go, as many Americans do not participate in the stock market. High inequality in both income and wealth has been present in our sample period. We see that the richest 10% ("top 10%"), have increased their income and wealth significantly more than the mean. Even more alarming is the observed trend, that even among "top 10%" the wealthiest 0,01% ("top 0,01%") are the ones increasing their wealth and income the most.

According to Vissing-Jørgensen (1999) stock participation was approximately 23% in the period 1982-1995. Gallup states that just over half of the adults Americans invests in stocks (on average 58,3%) in the period between 1998-2016. In the paper by Wolff (2010) stock ownership is around 44% in the period 1989-2007.

The poorest 90% ("bottom 90%") on average invested 8,5% of their wealth in equities. In comparison, "top 0,01%" invested 50,1% of their wealth in equities. Wolff (2010) found that in 2007 the "top 10%" had 91,1% of the stocks. To illustrate the difference, "top 0,01%" invested 12 821 times more than "bottom 90%" in 2012.

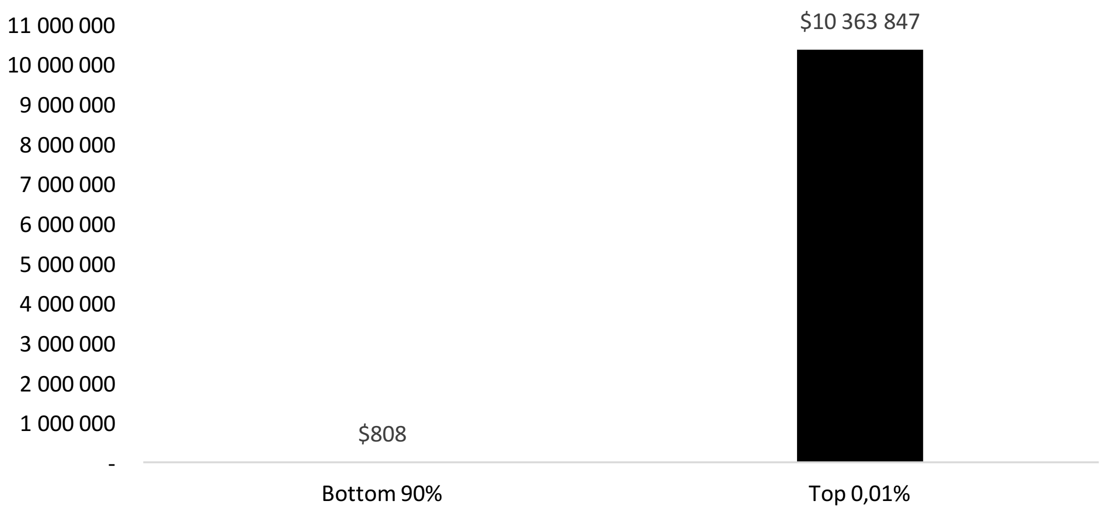


Figure 8: Average invested in equities per-capita in wealth classes in 2012

The growing income inequality is easily recognizable when we examine the Gini coefficient[6], which has grown from 0,397 to 0,479. The average real wage growth has in our sample period been slightly less than 1 percent. If we look exclusively at "bottom 90%", whom account for 50% of the total income, the average falls below 0,5%. In comparison, "top 0,1%", which in 2014 constituted of 10,26% of the total income, had an average growth of 3,15%, which is 7 times higher than "bottom 90%".

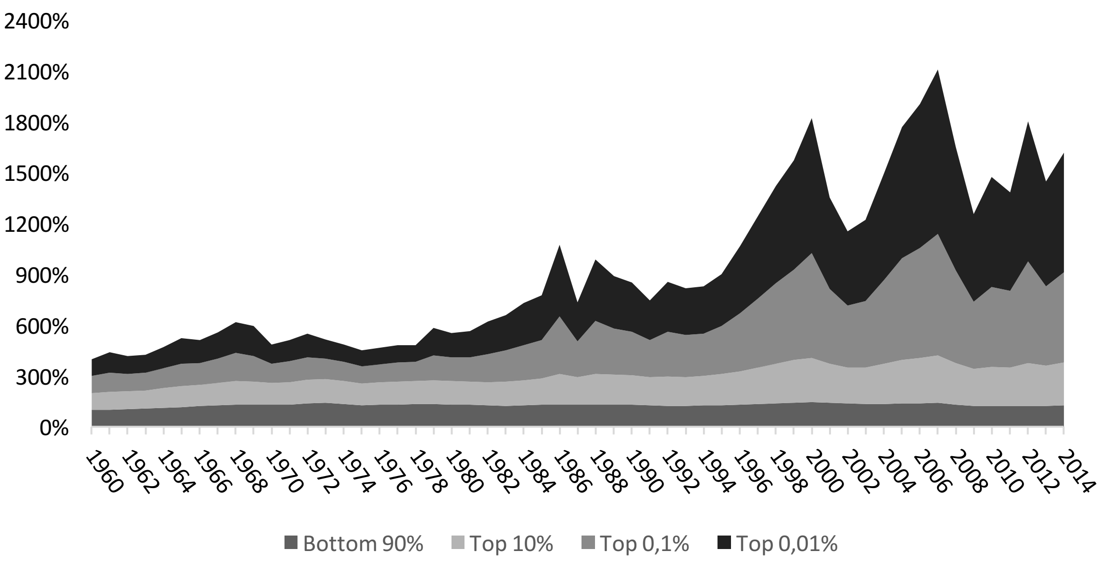


Figure 9: Real Cumulative income growth between 1960-2014

[6] The Gini coefficient is derived from a Lorentz curve, where the cumulative percentage of total income received against the cumulative number of participants, starting with the poorest household. It can theoretically range from 0 (maximum equality) to 1 (maximum inequality), where a value close to 1 is very unlikely in practice for large groups.

"Bottom 90%" does not have the opportunity to save any notable portion of their income. In the period from 1991-2011, they have been net borrowers. Therefore, the opportunity to invest in the stock market is limited if they wish to maintain consumption.

We believe that the nation's high inequality is one of the main reasons for the low stock participation, which makes it an argument against using the equity market as a proxy for return on the wealth portfolio.

5.3.3 The model with our estimates for the wealth portfolio

In discrete time

From expression 3.2.1:

$$0{,}0467 = \frac{p(1-a)}{1-p}0{,}000345 + \frac{a-p}{1-p}0{,}00105$$

From expression 3.2.2:

$$0{,}00887 = \frac{1-\alpha}{1-p}ln\left(\frac{1}{\beta}\right) + \frac{p(1-\alpha)}{1-p}0{,}0285 - \frac{1}{2}\frac{p^2(1-\alpha)^2}{(1-p)^2}0{,}000240 + \frac{(\alpha-p)}{1-p}0{,}0106 - \frac{1}{2}\frac{(p-\alpha)^2}{(1-p)^2}0{,}000865 + p\frac{1-\alpha}{1-p}\frac{p-\alpha}{1-p}0{,}000234$$

p	α	β	EIS	δ
0,50	27,06	1,020	2,00	-0,020
0,60	22,70	1,023	1,67	-0,022
0,70	17,97	1,025	1,43	-0,025
0,80	12,82	1,028	1,25	-0,027
0,90	7,19	1,031	1,11	-0,030
0,99	1,65	1,033	1,01	-0,033
1,10	-5,82	1,037	0,91	-0,036

Table 5-3: Implied preference parameters when the recursive model in discrete time is calibrated with estimates for the wealth portfolio

In continuous time

From expression 3.2.3:

$$0{,}0592 = \frac{p(1-\alpha)}{1-p}0{,}000253 + \frac{\alpha - p}{1-p}0{,}000880$$

From expression 3.2.4:

$$0{,}00908 = ln\frac{1}{\beta} + p \cdot 0{,}0286 - \frac{1}{2}\frac{p(1-\alpha p)}{1-p}0{,}000252 + \frac{1}{2}\frac{p-\alpha}{1-p}0{,}000876$$

p	α	β	EIS	δ
0,80	18,22	0,969	1,25	0,031
0,90	9,94	0,969	1,11	0,032
0,95	5,56	0,968	1,05	0,033
0,96	4,66	0,968	1,04	0,033
0,97	3,76	0,968	1,03	0,033
0,98	2,85	0,968	1,02	0,033
0,99	1,93	0,967	1,01	0,033
1,01	0,07	0,967	0,99	0,033

Table 5-4: Implied preference parameters when the recursive model in continuous time is calibrated with estimates for the wealth portfolio

When we calibrate the model with our estimates for wealth, we end up with the parameters in table 5-3 and 5-4. Even though the discrete time model provides us with significantly better results than the expected utility model, we get a β higher than 1. In section 5.1 we state that it should be $0{,}95 < \beta < 1$. On the contrary, the model in continuous time shows promising results.

6. Robustness

All our collected and calculated data presented in chapter 4 may be subject to estimation or sampling errors. The data material used in this study is mainly collected from the Bureau of Economic Analysis, Federal Reserve Bank of St. Louis and Bloomberg. Even though we consider the sources credible, there may be deviations.

We have used a 55-year sample period from 1960 to 2015. In Mehra and Prescotts original paper from 1985, they used an 89-year period from 1889. It can be argued that we should have analysed a longer period of time, but we believe that the chosen period makes us able say more about the recent past and todays situation on this subject.

The most sensitive estimate is the wealth portfolio, which contains uncertainty because it is based on net present value and contains many different elements. In our work to calculate the US national wealth we use Statistics Norway's approach, and must point out that different approaches may lead to different results. In principle, everything that affects welfare should be included in the calculation of national wealth, but in practice, many of these variables is difficult to quantify. Because of this, our calculations are limited to real capital, net financial capital, natural resources and human capital.

Human capital, which accounts for 80% of the national wealth, is sensitive to the estimates of a growth rate and a discount rate. We have done a sensitivity analysis to see how the value change with different rates, where we believe the growth on labor productivity can range from 1,0% - 2,75% and the discount rate from 2% - 6%:

	1,00%	**1,25%**	**1,50%**	**2,00%**	**2,25%**	**2,50%**	**2,75%**
2,0%	2 810 789	3 673 274	5 398 244	-	-	-	-
3,0%	1 529 745	1 716 376	1 965 216	2 836 157	3 707 097	5 448 979	10 674 624
4,0%	1 102 731	1 182 676	1 278 610	1 542 429	1 730 871	1 982 127	2 333 886
5,0%	889 223	933 616	984 350	1 111 186	1 191 900	1 288 757	1 407 137
6,0%	761 119	789 423	820 873	895 565	940 381	991 598	1 050 695

Table 6-1: Sensitivity analysis of the US national wealth in 2015 with different discount rate (vertical) and growth rate (horizontal)

This sensitivity analysis shows us that with a low discount rate, the national wealth is more sensitive to different growth rates in labor productivity. The growth in labor productivity can't exceed the discount rate in the analysis, because that would make national wealth per capita negative.

We have chosen to use a discount rate on 4% in our calculations. We do believe a discount rate in the range of 2% - 4% to be realistic, but considers rates larger than 4% to be plausible as the literature on this subject is inconclusive. Different discount rates affect the human capital considerably, but the rate is set after reading discussions in the literature by Statistics Norway (2015) and Zhuang & Liang et al (2007). A too high discount rate could preclude many socially desirable public projects from being undertaken, while setting a too low rate makes a lot of economically indifferent investments.

There is criticism against using S&P500 as a benchmark for market portfolio. This index is disproportionately weighted towards larger companies. The top 50 companies account for around 50% of the index's value, so these 50 companies have a larger impact on the index calculation. It is also one of the most successful indexes in the world.

Some argue that in practice the risk-free rate does not exist because even the safest investments carry a small amount of risk. We decided to use the average of four 3-Month Treasury Bills to find the annual risk-free rate. By using the same proxy as Mehra and Prescott, we believe it makes our findings easier to compare. The result of this operation is that our risk-free rate is not considered to be risk free in a one year perspective, as the short rate is a stochastic process.

In Aase's article "Heterogeneity and limited stock market participation", he divides the inhabitants in two groups, where one group participate in the stock market and the other does not. We have chosen to analyze the stock participation by dividing the inhabitants in "top 0,01%", "top 10%" and "bottom 90%". We met some challenges in the process of finding concrete and credible data on direct and indirect stock participation, since the data generally stems from surveys. By generalizing this to the population, the data is affected by sampling errors. Both our and Aase's method leads to the conclusion that estimates for the wealth portfolio should be used.

7. Conclusion

In this study, we have found that models with recursive utility is a possible solution for the equity premium puzzle in our sample period from 1960-2015. The model with additive and separable expected utility falls short when presented with our empirical data, there is no plausible parameters to be found. We have accounted for the shortcomings in the traditional model, and described how recursive utility provides us with a more correct picture of how a representative agent acts.

When applying Aase's (2016) alterations to the Epstein-zin model, we use wealth estimates for the wealth portfolio instead of the market portfolio as a proxy. Aase (2016) states that when stock participation is limited, letting the market portfolio represent the wealth portfolio is not necessarily reasonable. By taking a closer look at the stock market participation, we discover a low participation rate, which we believe derive from high inequality, as the average of the 90% poorest do not have the possibility to invest in equities.

We have demonstrated how both the discrete- and continuous-time models based on recursive utility fits our empirical data better than the model based on expected utility, and provides more plausible parameters considering the relative risk aversion (α), impatience rate (β) and elasticity of temporal substitution (EIS).

Appendix I

Table A.1: Consumer price index – CPI

Year	CPI	Year	CPI
1960	29,6	**1988**	118,3
1961	29,9	**1989**	124
1962	30,2	**1990**	130,7
1963	30,6	**1991**	136,2
1964	31	**1992**	140,3
1965	31,5	**1993**	144,5
1966	32,4	**1994**	148,2
1967	33,4	**1995**	152,4
1968	34,8	**1996**	156,9
1969	36,7	**1997**	160,5
1970	38,8	**1998**	163
1971	40,5	**1999**	166,6
1972	41,8	**2000**	172,2
1973	44,4	**2001**	177,1
1974	49,3	**2002**	179,88
1975	53,8	**2003**	183,96
1976	56,9	**2004**	188,9
1977	60,6	**2005**	195,3
1978	65,2	**2006**	201,6
1979	72,6	**2007**	207,34
1980	82,4	**2008**	215,30
1981	90,9	**2009**	214,54
1982	96,5	**2010**	218,06
1983	99,6	**2011**	224,94
1984	103,9	**2012**	229,60
1985	107,6	**2013**	232,96
1986	109,6	**2014**	236,74
1987	113,6	**2015**	237,01

Table A.2: Return on S&P500

Year	Return on S&P500	Year	Return on S&P500
1960	0,34	**1988**	16,54
1961	26,64	**1989**	31,48 %
1962	-8,81	**1990**	-3,06
1963	22,61	**1991**	30,23
1964	16,42	**1992**	7,49
1965	12,40	**1993**	9,97
1966	-9,97	**1994**	1,33
1967	23,80	**1995**	37,20
1968	10,81	**1996**	22,68
1969	-8,24	**1997**	33,10
1970	3,56	**1998**	28,34
1971	14,22	**1999**	20,89
1972	18,76	**2000**	-9,03
1973	-14,31	**2001**	-11,85
1974	-25,90	**2002**	-21,97
1975	37,00	**2003**	28,36
1976	23,83	**2004**	10,74
1977	-6,98	**2005**	4,83
1978	6,51	**2006**	15,61
1979	18,52	**2007**	5,48
1980	31,74	**2008**	-36,55
1981	-4,70	**2009**	25,94
1982	20,42	**2010**	14,82
1983	22,34	**2011**	2,10
1984	6,15	**2012**	15,89
1985	31,24	**2013**	32,15
1986	18,49	**2014**	13,52
1987	5,81	**2015**	1,36

Table A.3: Risk-free rate

Year	Risk-free rate	Year	Risk-free rate
1960	2,88	**1988**	6,67
1961	2,35	**1989**	8,11
1962	2,77	**1990**	7,49
1963	3,16	**1991**	5,38
1964	3,55	**1992**	3,43
1965	3,95	**1993**	3,00
1966	4,86	**1994**	4,25
1967	4,31	**1995**	5,49
1968	5,34	**1996**	5,01
1969	6,67	**1997**	5,06
1970	6,39	**1998**	4,78
1971	4,33	**1999**	4,64
1972	4,07	**2000**	5,82
1973	7,03	**2001**	3,39
1974	7,83	**2002**	1,60
1975	5,78	**2003**	1,01
1976	4,97	**2004**	1,37
1977	5,27	**2005**	3,15
1978	7,19	**2006**	4,73
1979	10,07	**2007**	4,35
1980	11,43	**2008**	1,37
1981	14,03	**2009**	0,15
1982	10,61	**2010**	0,14
1983	8,61	**2011**	0,05
1984	9,52	**2012**	0,09
1985	7,48	**2013**	0,06
1986	5,98	**2014**	0,03
1987	5,78	**2015**	0,05

Table A.4: Population

Year	Population	Year	Population
1960	180 671 000	**1988**	245 021 414
1961	183 691 000	**1989**	247 341 697
1962	186 538 000	**1990**	250 131 894
1963	189 242 000	**1991**	253 496 870
1964	191 889 000	**1992**	257 037 358
1965	194 303 000	**1993**	260 448 665
1966	196 560 000	**1994**	263 662 439
1967	198 712 000	**1995**	266 821 440
1968	200 706 000	**1996**	269 943 686
1969	202 677 000	**1997**	273 202 960
1970	205 052 000	**1998**	276 416 680
1971	207 661 000	**1999**	279 609 242
1972	209 896 000	**2000**	282 737 852
1973	211 909 000	**2001**	285 550 120
1974	213 854 000	**2002**	288 211 775
1975	215 973 000	**2003**	290 699 578
1976	218 035 000	**2004**	293 402 444
1977	220 239 000	**2005**	296 119 275
1978	222 585 000	**2006**	298 988 427
1979	225 055 000	**2007**	301 845 537
1980	227 726 463	**2008**	304 714 134
1981	229 966 237	**2009**	307 397 158
1982	232 187 835	**2010**	309 977 938
1983	234 307 207	**2011**	312 357 356
1984	236 348 292	**2012**	314 752 677
1985	238 466 283	**2013**	317 142 995
1986	240 650 755	**2014**	319 507 332
1987	242 803 533	**2015**	322 024 263

Table A.5: National wealth per-capita

Year	National wealth per-capita	Year	National wealth per-capita
1960	107 350	**1988**	679 200
1961	108 115	**1989**	715 019
1962	113 350	**1990**	727 366
1963	117 659	**1991**	737 283
1964	121 953	**1992**	760 662
1965	129 440	**1993**	792 952
1966	137 883	**1994**	827 626
1967	145 223	**1995**	850 094
1968	157 183	**1996**	864 372
1969	167 410	**1997**	948 122
1970	172 085	**1998**	972 042
1971	183 872	**1999**	1 022 882
1972	202 146	**2000**	1 053 957
1973	221 474	**2001**	1 033 024
1974	240 673	**2002**	1 083 781
1975	257 239	**2003**	1 114 236
1976	284 512	**2004**	1 204 853
1977	317 767	**2005**	1 268 000
1978	357 479	**2006**	1 310 729
1979	391 997	**2007**	1 339 136
1980	425 067	**2008**	1 306 510
1981	461 255	**2009**	1 251 723
1982	485 310	**2010**	1 262 368
1983	508 854	**2011**	1 285 110
1984	553 463	**2012**	1 322 047
1985	596 462	**2013**	1 388 860
1986	619 479	**2014**	1 475 139
1987	639 237	**2015**	1 542 429

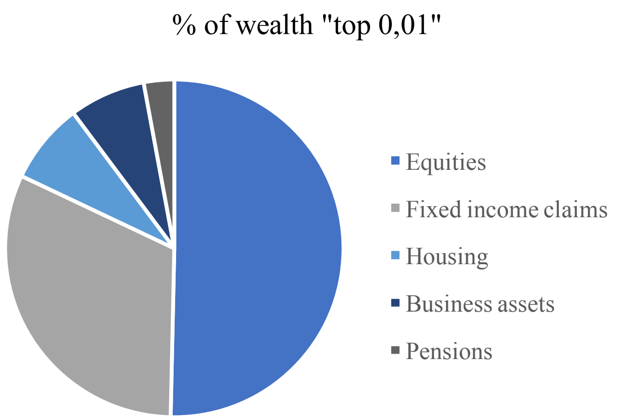
Table A.6: Consumption per-capita

Year	Non-durable goods	Year	Non-durable goods
1960	131,4	**1988**	862,3
1961	134,6	**1989**	929,5
1962	139,5	**1990**	994,2
1963	143,9	**1991**	1020,2
1964	152,7	**1992**	1055,2
1965	163,3	**1993**	1090,8
1966	177,9	**1994**	1139,4
1967	185,0	**1995**	1179,8
1968	199,8	**1996**	1241,4
1969	214,2	**1997**	1291,2
1970	228,8	**1998**	1329,4
1971	239,7	**1999**	1431,2
1972	257,4	**2000**	1540,3
1973	286,1	**2001**	1583,7
1974	321,4	**2002**	1613,2
1975	349,2	**2003**	1704,0
1976	377,7	**2004**	1820,4
1977	408,4	**2005**	1953,0
1978	450,2	**2006**	2079,7
1979	511,6	**2007**	2177,0
1980	573,4	**2008**	2273,4
1981	625,4	**2009**	2175,1
1982	646,3	**2010**	2292,1
1983	678,8	**2011**	2471,1
1984	721,6	**2012**	2547,2
1985	757,2	**2013**	2592,8
1986	774,2	**2014**	2675,7
1987	814,3	**2015**	2656,9

Year	Services	Year	Services
1960	154,6	**1988**	2009,6
1961	163,2	**1989**	2169,0
1962	174,1	**1990**	2334,3
1963	184,4	**1991**	2462,7
1964	199,0	**1992**	2652,4
1965	213,9	**1993**	2828,7
1966	231,0	**1994**	2994,5
1967	248,4	**1995**	3168,6
1968	272,8	**1996**	3350,4
1969	299,8	**1997**	3554,0
1970	328,9	**1998**	3794,3
1971	358,9	**1999**	4020,3
1972	395,6	**2000**	4339,5
1973	434,5	**2001**	4577,9
1974	480,5	**2002**	4785,5
1975	541,4	**2003**	5044,0
1976	603,9	**2004**	5359,8
1977	676,3	**2005**	5713,8
1978	762,6	**2006**	6068,2
1979	851,6	**2007**	6388,9
1980	954,8	**2008**	6638,0
1981	1068,1	**2009**	6648,6
1982	1174,6	**2010**	6839,4
1983	1312,8	**2011**	7092,9
1984	1434,5	**2012**	7311,5
1985	1585,1	**2013**	7526,7
1986	1702,8	**2014**	7892,9
1987	1835,8	**2015**	8271,6

Appendix II: Inequality & Stock participation

Figure B.1: Composition of wealth



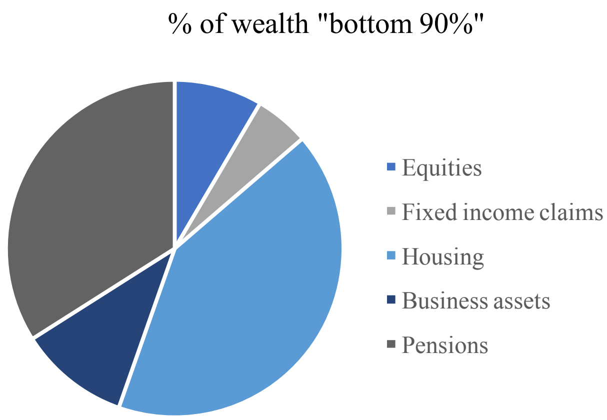


Figure B.2: Income inequality defined by gini-coefficient

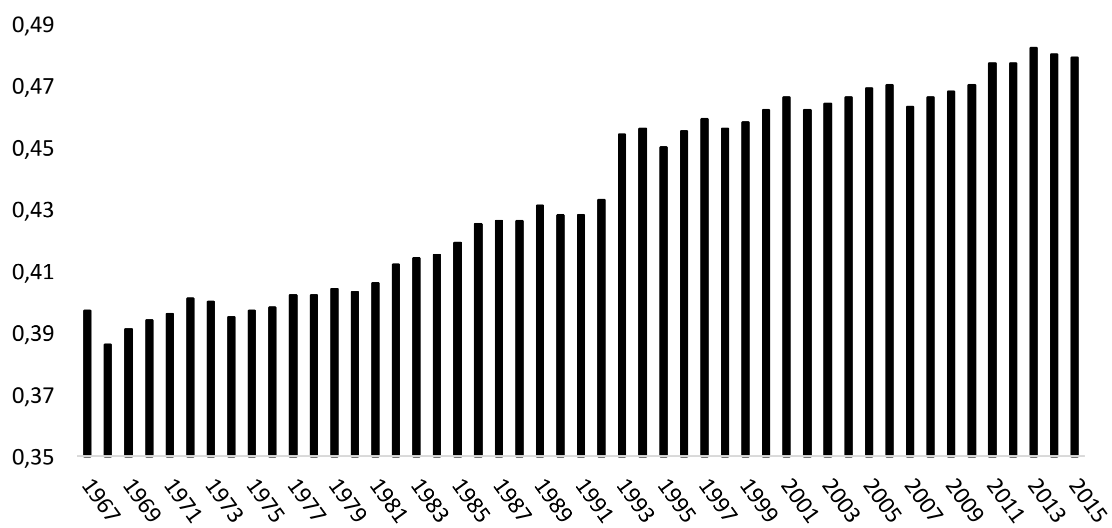


Figure B.3: Private savings

	Bottom 90%	***Top 10%***	***Top 1%***	***Top 0,1%***
Average (1960-2011)	3 %	26 %	35 %	47 %
20-year average (1991-2011)	-1 %	21 %	34 %	49 %
10-year average (2001-2011)	-3 %	21 %	36 %	54 %

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