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Population ageing in extended Overlapping Generations Models

A numerical analysis of the financing of the Norwegian National Insurance Scheme

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Abstract

In recent years, a funding of the Norwegian National Insurance Scheme has been discussed as a sustainability-improving measure in the wake of the financial crisis and an ageing population due to baby boomers entering retirement age. Using an extended overlapping generations model, the thesis examines the long-run relationship between the defined-contribution rates of return of the current pay-as-you-go scheme and a fully funded alternative in 81 different scenarios. The scenarios differ with respect to demographic development, real return on pension savings, productivity growth, and retirement age. Projections of the real return for 2016-2100 suggests that the fully funded alternative is likely to yield a significantly larger rate of return, and therefore also a significantly smaller defined-benefit tax rate. A funding is found to increase the rate of return of the pension scheme in the long run in 51 of these 81 scenarios. Furthermore, funding is found to yield a higher rate of return than the current pay-as-you-go-financing in 7 of the 9 most probable scenarios. Additionally, for a given rate of return the current pay-as-you-go scheme implies an annual tax rate 22% larger than the alternate fully funded scheme in years 2044-2100 in the expected scenario. A gradual funding is discussed as a solution to the challenges related to transitioning from one form of financing to the other. Multiple approaches to a possible implementation of a gradual funding are examined, including examples from other countries that have starter similar systemic reforms.

Acknowledgements

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Contents

List of Figures

List of Tables

1. Introduction

1.1 Motivation

Due to decreasing mortality and declining fertility rates, population ageing is currently taking place in nearly all western countries, including Norway (United Nations, 2013). As the current Norwegian National Insurance Scheme is organized as a pay-as-you-go scheme, which means that benefits received by current pensioners are directly financed by contributions from current workers, changes in the age structure of the population impacts the sustainability of the system. Population ageing implies a decreasing ratio of workers per pensioner, and consequentially the contributions each worker must make in a pay-as-you-go scheme with defined benefits increases when a population is ageing.

A funding of the Norwegian National Insurance Scheme has been discussed as a possible solution to the sustainability-challenges caused by the ageing population (Thøgersen, 2010; Moum & Wold, 2001). In a funded scheme, individuals finance their own pension benefits through individual working-age contributions. Therefore, a fully-funded system is less susceptible to changes in population growth, and transitioning from pay-as-you-go to funding could increase the sustainability of the public pension system.

1.2 Methodology

To compare the projected rates of return of the current pay-as-you-go National Insurance Scheme and a potential fully-funded scheme, an extended overlapping generations model with 106-period lifetimes is developed. The analysis examines 81 projected scenarios, all covering years 2016 through 2100, that differs with respect to different combinations of projected demographic development, return on pension savings, productivity development, and retirement age.

Under the strict assumption that one should seek to maximize the rate of return of the pension system, and consequentially also minimize the defined-benefit tax rate, the thesis examines whether financing the National Insurance Scheme through funding should be deemed preferable to the current pay-as-you-go financing in the long run. The projected rates of return are compared annually in periods 2016 through 2100 in each scenario.

1.3 Contributions

In answering this research question, the thesis contributes to existing research in three ways.

First, a generalizable extended overlapping generations model that compares the rates of return of pay-as-you-go and fully funded pension schemes is developed. The model is generalizable in the sense that all nine variables can be adjusted for specific pension systems. These nine variables are as follows: Period durations, quantity of overlapping cohorts, fertility rates, mortality rates, effective labor market entry age, effective age of retirement, expected remaining lifetime at retirement age, real return on funded savings, and wage growth.

Second, the relationship between the rates of return in the current pay-as-you-go Norwegian National Insurance Scheme and a potential funded alternative is quantified using this model. Even though a potential funding of the Norwegian National Insurance scheme has been discussed in detail since the late 1990s (NOU, 1998), this thesis is the first objective numerical analysis directly comparing the projected long-run rates of return of the current scheme and a fully funded alternative using an extended overlapping generations model.

It is important to stress that the model simply compares the numerical values of the rate of return of pension schemes at any given time, and that it does not include any normative utility function. Moreover, the model does not consider time preferences or projected costs of transitioning from one system to a different system. For these reasons, the results yield limited grounds for concluding whether a funding should be implemented or not. With that said, the results should spur further discussions, as a higher rate of return is preferable ceteris paribus.

Third, the possible net gains/losses from a partial transition to a funded system is discussed in detail, as well as different implementation strategies.

1.4 Thesis outline

The thesis is structured as follows: Chapter 2 gives the reader additional background information regarding the projected challenges for the Norwegian National Insurance Scheme, with the main focus being on the impact of an ageing population. Furthermore, the extent of the population ageing is explained in further detail. Chapter 3 presents the theoretical foundation necessary to understand the subsequent chapters. The chapter consist of a brief overview of the current Norwegian pension system, an explanation of basic overlapping

generation models, and a short summary of how pension systems could differ with respect to financing and general design. In chapter 4, the extended overlapping generations model used in the analysis is developed. Chapter 5 presents the relevant data, and describes different projections of the key components of the model as well as the sources of this data. Chapter 6 presents, analyses, and discusses the results one gets when the data is combined with the model. Finally, chapter 7 concludes the thesis by summarizing the thesis and discussing the implications of the results as well as possible limitations and weaknesses.

2. A changing pension landscape

2.1 Demographic development

The landscape of pension arrangements across western economies has seen massive changes during the last few decades. As most OECD countries, Norway is facing an ageing population, and thus an increased old age dependency ratio. This partly due to decreased fertility and increased life expectancy, and partly due to the "Baby Boomer"-generation[1] entering pension age. Statistics Norway estimates that 22.3% of the total population will be aged 70 and over in year 2100, as compared to 11,0% in 2016 (Syse, Pham, & Keilman, 2016).[2]

This development is expected to put the current national insurance scheme under large amounts of pressure, as the system currently is financed as a pay-as-you-go scheme. This means that current pension benefits are directly financed through taxes and pension contributions provided by the working-age part of the population. [3] Thus, as the ratio of pensioners per worker in the system increases, benefits per pensioner must decrease and/or contributions per working-age individual must increase. While the employment rate for workers aged 55-64 in Norway is relatively large, and the average effective labor force exit ages are above the OECD average (OECD, 2014a), population ageing is still a major macroeconomic challenge that calls for a further increase in the employment of older individuals.

Figure 2.1 illustrates this population ageing, and shows that this development would lead to a large increase in government expenditures. As the rightmost panel of the figure illustrates, the bulk of net public transfers goes to people aged 67 and above, a group that based on these predictions will almost double in size.

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[1] "Baby boomers": Individuals born during the high fertility rates post World War II

[2] Similarly, United Nations predicts that 47,3% of the total population will be 60+ years in old in 2100 as compared to 26,0% in 2016 (United Nations, 2015, p. 29)

[3] Section 3.3 describes pay-as-you-go schemes and alternative forms of financing in further detail.
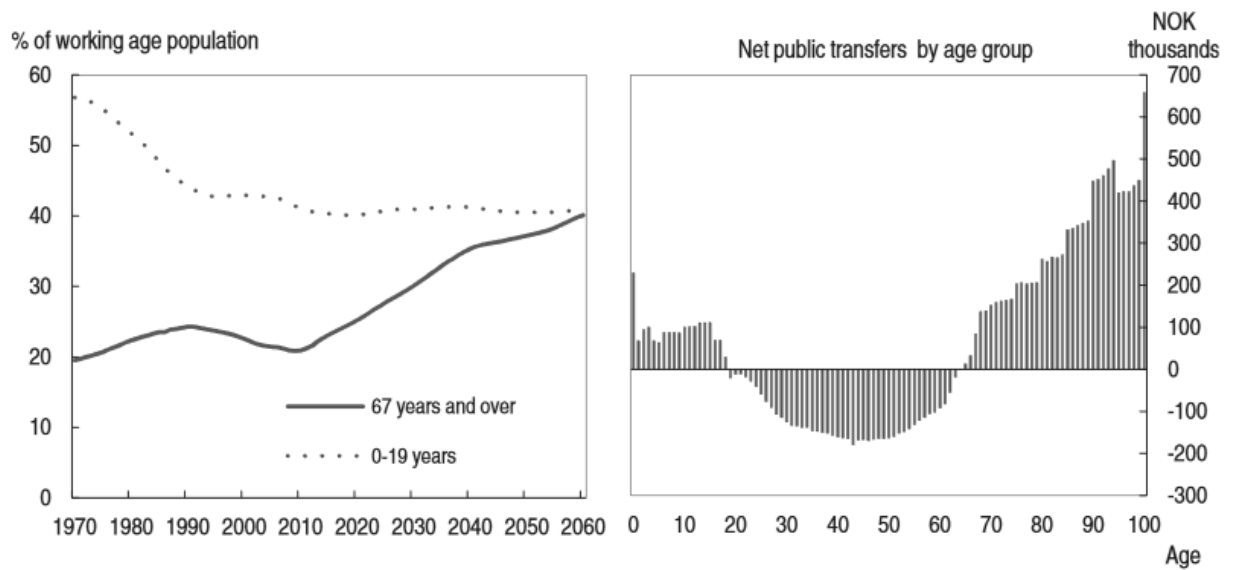
Figure 2.1: Population ageing will impose an increasing burden on public finances. Source: OECD (2010).

The population pyramids in figure 2.2 further illustrates the predicted demographic development. The figure breaks down the population in Norway by age and sex in five-year age groups for years 1990 and 2017, as well as corresponding projected levels for year 2040. In addition to a clear increase in the total population, these pyramids also show that the population is increasingly ageing. The 1990-pyramid is narrow for ages 46 and older, and a large share of the population is in working age. In 2017 and 2040 on the other hand, the distributions are increasingly taking the form of a pentagon due to the final "baby boomer" cohorts entering retirement age. The baby boomer bulge is moving upwards in the population pyramid, leaving the base relatively narrow. Clearly, the share of the population in working age is shrinking.
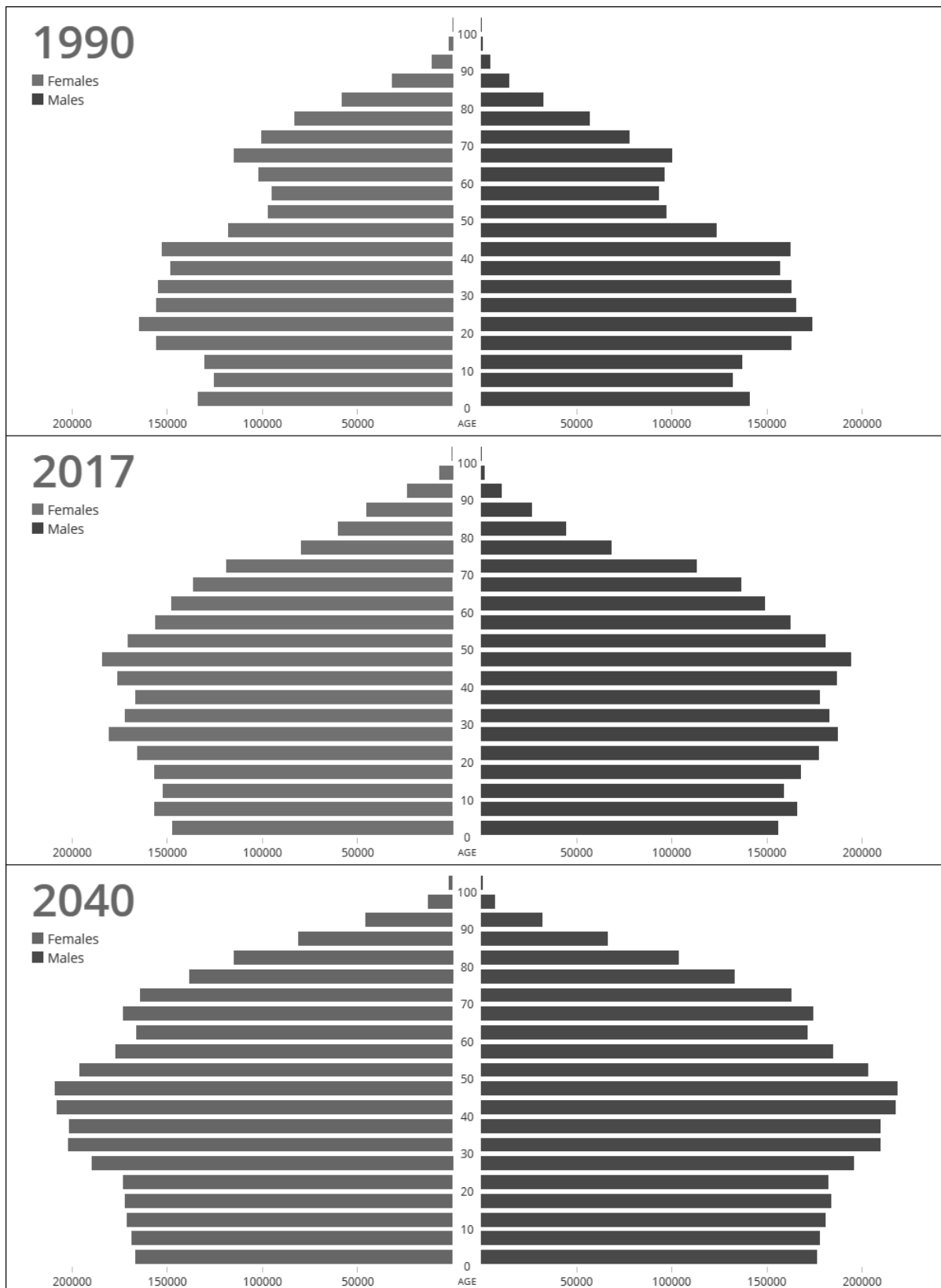
Figure 2.2: Population pyramids for years 1990, 2017, and 2040. Source: Statistics Norway (2017a).

2.2 Burden of care

The burden of care is defined as the population aged 0-19 years (children) plus the population 70 years and older (elderly), divided by the working-age population aged 20-69 years. An increasing burden of care implies that the working age population has an increased burden to provide for social expenditures required by children and the elderly.

Figure 2.3 illustrated the development in the burden of care in Norway divided into the burden of care for the elderly and the burden of care for children. The projections are based on the main population projection alternative provided by Statistics Norway, which assumes medium levels of fertility, life expectancy, domestic migration and immigration.[4] Towards the end of the century, the number of elderly is expected to surpass the number of children and teenagers, and the total burden of care is projected to rise continuously from a current level slightly below 0.6 to a level at around 0.8 in 2100. The increase can in its entirety be attributed to an increase in the burden of care for the elderly, as the burden of care for children is projected to remain relatively stable close to its current low level.

As previously mentioned; the current Norwegian National Insurance Scheme is organized as a pay-as-you-go scheme. Naturally, an increasing burden of care is damaging to a pay-as-you-go system, and reduces its sustainability. A funded scheme however, is not directly impacted by the demographic change, and could thus be a solution to the challenges related to an increasing burden of care.

Relatively large net migration helps slow down the projected ageing of the population, as most immigrants entering Norway are young (Cappelen, Skjerpen, & Tønnesen, 2016). However, as the estimated levels of immigration are highly uncertain, this number varies widely in the different population projection alternatives. While immigration helps curb the projected burden of care, immigrants also age, and consequently the projected level of immigration is not sufficiently large enough to stop the ageing completely even in the alternatives with high immigration and low ageing.

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[4] The population projections are explained in further detail in section 5.1.
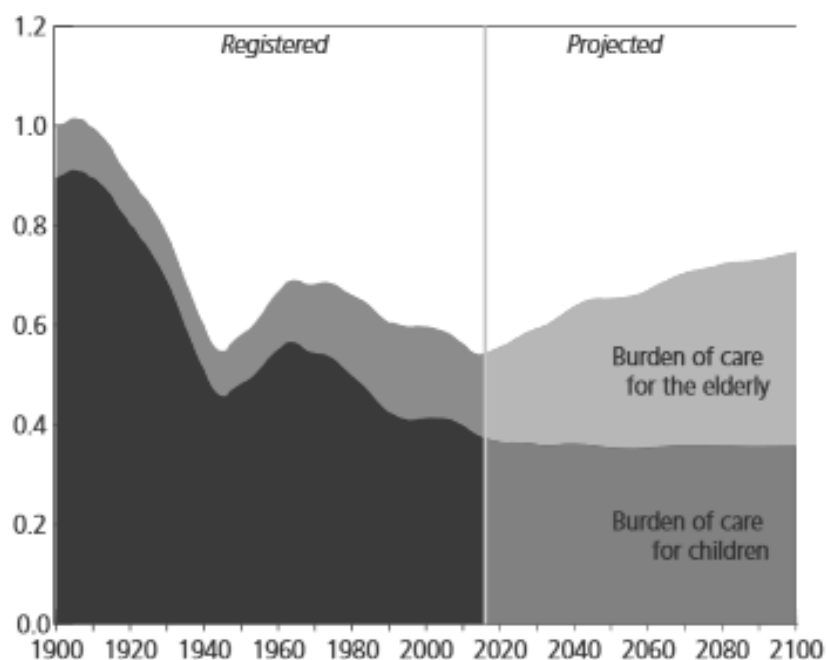
Figure 2.3: Registered and projected burden of care development. Source: Statistics Norway (2016).

2.3 Productivity growth

In a pay-as-you-go pension system where benefits move only with inflation, productivity growth could combat the adverse effects of an increasing burden of care. Contribution are a function of current wages, while benefits are a function of previous wages. Thus, the higher the productivity growth, the more sustainable a pay-as-you-go pension system is. On the other hand, falling productivity growth implies that an increased degree of funding might be optimal.

Following the financial crisis of 2007-2008, productivity growth in OECD countries has remained modest (Ollivaud & Turner, 2014), and global growth recovery is postponed as growth forecast continues to be revised down (IMF, 2017).

Productivity development in Norway is largely dependent on the development of other economies (NOU 2015:1). Therefore, it is worrying that productivity growth is projected to remain low in most western countries in the years ahead. Moreover, as the Norwegian economy adapts to a decreasing level of activity in the oil and gas sector, productivity growth might be further weakened going forward.

Productivity growth in Norway has gradually slowed down in recent years (Norges Bank, 2016a). Growth in trend productivity is predicted to remain low in coming years, and

productivity growth trend has fallen by more than half since the turn of the millennium (Norges Bank 2016b, p. 52).

Low productivity growth decreases the sustainability of the current Norwegian National Insurance scheme, and could thus be used as an argument as to why pension assets should be invested in funds.

2.4 Real return on Government Pension Fund Global

The thesis proposes a funding of the Norwegian National Insurance scheme by investing the fund as part of the Government Pension Fund Global,[5] while managing the accumulated values of the funds separately. Thus, the real rate of return of the fully-funded scheme would be equal to the real rate return of the Government Pension Fund Global.

Despite somewhat volatile real returns on the fund, the corresponding linear trend has remained relatively stable, as illustrated in figure 2.4. This stable trend can to some degree be attributed to the stable inflation rate slightly below the inflation target of 2.5% during the same period. Figure 2.5 illustrates the development of annual CPI growth, which can be used as a proxy for inflation (Cecchetti, Chu & Steindel, 2000). The sharp decline in real returns from 2007 to 2008 due to the financial crisis illustrates a potential weakness of a fully funded pension scheme. While demographic risks are strongly reduced when transitioning away from a pay-as-you-go scheme, financial risks are much more apparent in a funded scheme.

The more volatile the returns, the riskier a fund is. However, as long as the long run trend of the real returns remains relatively stable, the fund should remain sustainable.

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[5] Commonly referred to as "The Oil Fund".
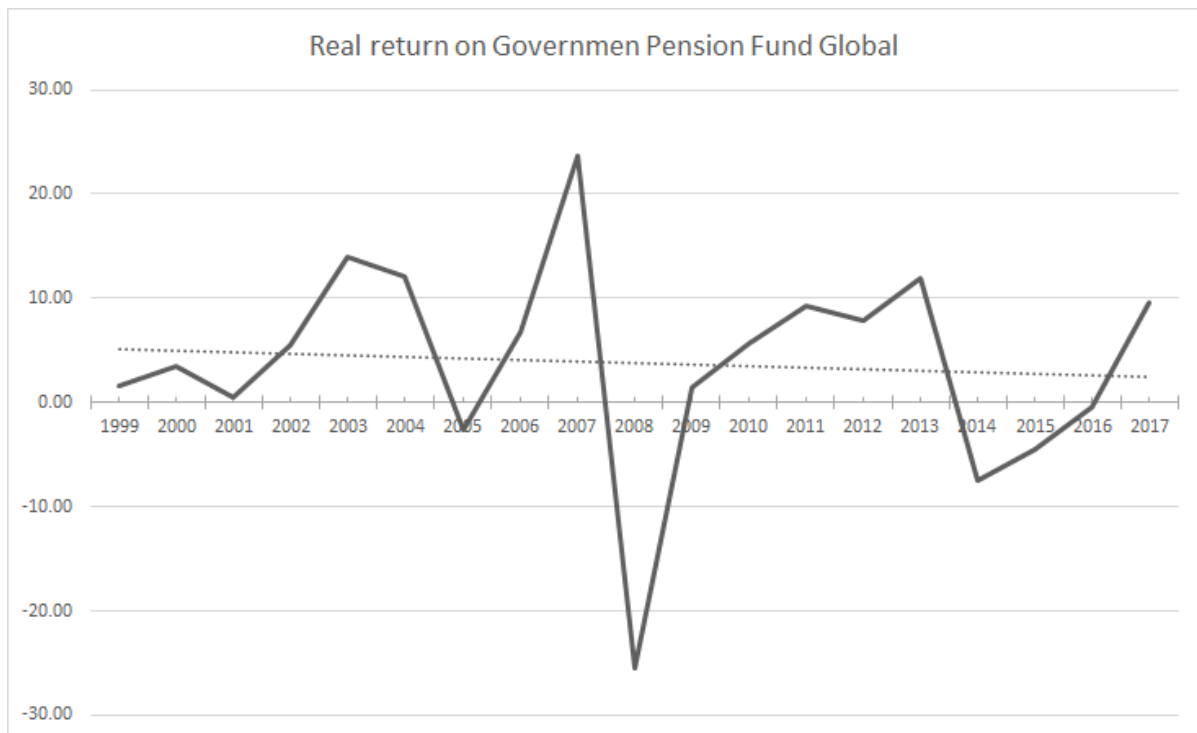
Figure 2.4: Real return on the Government Pension Fund Global and linear trend for periods 1999 to 2017.
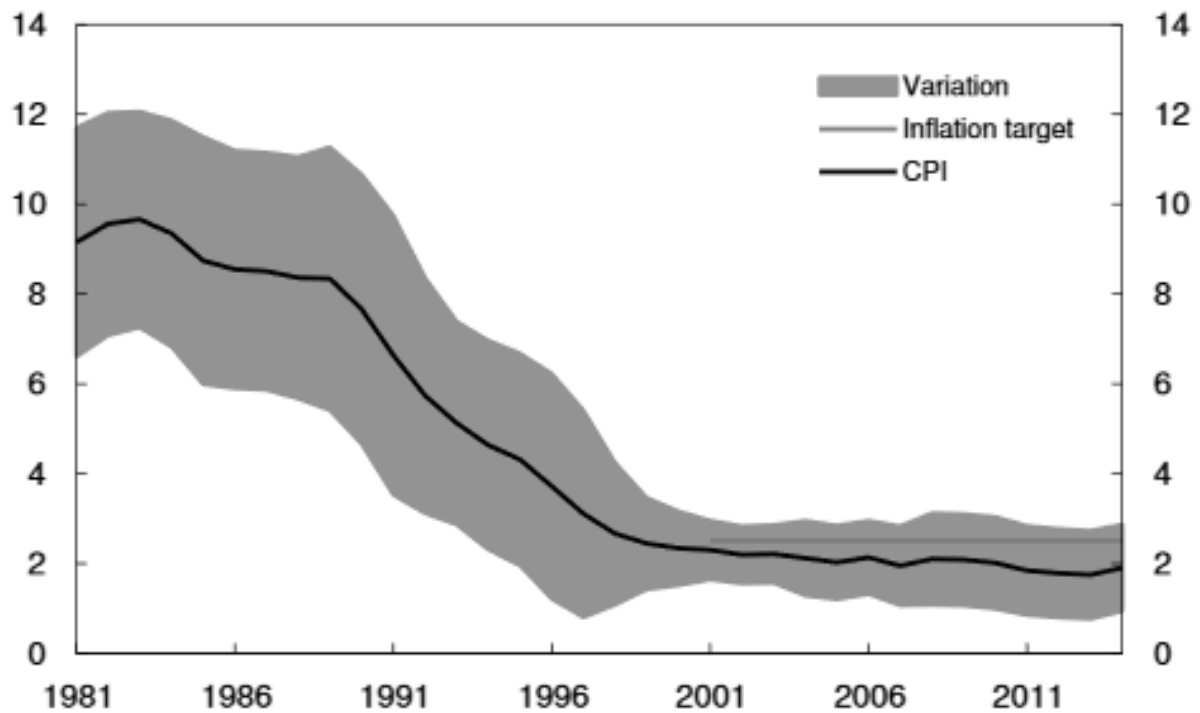
Figure 2.5: Moving 10-year average and variation in CPI. Annual growth. Percent. Source: Norges Bank (2015).

3. Theory

3.1 The Norwegian pension scheme

This section gives a brief overview of the current Norwegian pension scheme, mainly focusing on the mandatory public pension scheme: The National Insurance Scheme.

The primary purpose of the Norwegian pension scheme is to provide economic and social security in situations where individuals are unable to support themselves due to old age, disabilities or loss of dependents (NOU 2004:1). Pension benefits are determined as a ratio of working-age income, and consequentially the system has a consumption smoothing effect. The scheme also provides a guaranteed annual base rate to all pensioners, with the purpose of both preventing and reducing poverty.

Utility-maximizing individuals choose to save when in working-age to ensure the financing of an acceptable level of consumption after retiring. However, they do not know for certain how long they are going to live, and could therefore potentially outlive their own savings (Barr, 2012). Through a pooling of a large group, life expectancies can be used to reduce this risk, and the larger the group the lower the risk. This risk sharing is one of the main intuitions behind pension schemes, and illustrates how pension schemes could improve the well-being of individuals.

The Norwegian pension scheme is composed of three pillars; a public pay-as-you go scheme[6] known as the National Insurance Scheme, employment-based fully funded occupational pension schemes[7], and individual pension schemes/individual savings. The National Insurance Scheme consists of two statutory schemes: A guaranteed minimum pension, sometimes called a "zero pillar", and an earnings-related income scheme. To get the full guarantee pension, residence in Norway of at least 40 years is required. For every year below this, a proportional reduction is applied, and individuals with a period of residence of less than three years are not eligible to receive the guarantee pension. Furthermore, this guarantee pension cannot be claimed before the age of 67. The earnings-related scheme is a pay-as-you-go scheme financed

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[6]"Folketrygden"

[7] "Tjenestepensjon"

through a combination of taxation and social security contributions from both employers and employees.

In a first-best world with no market failures and perfectly altruistic individuals, all the objectives of pension schemes could be achieved by private schemes, and no public intervention would be necessary. However, as market distortions exists in the real world, there are several reasons why public involvement often is needed. Common challenges facing insurance systems includes moral hazard; insured individual can impact the liability of the insurance company without its knowledge, and adverse selection; individuals can conceal that they are a poor risk (Pauly, 1974). Mandatory pension schemes do not face problems related to asymmetric information, by the nature of being mandatory. Thus, mandatory public pension schemes are more likely than optional pension schemes to be sustainable in the long-run (Einav & Finkelstein, 2011).

The average composition of potential pension income at retirement in Norway is as follows, with all numbers representing the percentage of total pension income: 71 % pay-as-you-go public pensions, 21 % funded pensions based on rights, 3 % funded pensions based on assets, and 5 % other pensions (OECD, 2014b).

3.1.1 The 2011 pension reform

The Norwegian National Insurance Scheme was subject to a major reform in 2011. The primary goal of the reform was to improve the sustainability of the social security system. This subsection briefly explains how the new pension system differs from the old, why these changes were made, and how the reform impacts the fiscal sustainability of the pension system. Finally, the expected long-run impact of the reform will be discussed.

Prior to the 2011 pension reform, the Norwegian pension system consisted of a statuary retirement age of 67 and an early retirement age of 62, with the early retirement age only applying to specific groups of workers. Postponed pension claims were not subject to actuarily neutral adjustments, and labor earnings past a certain threshold lead to a reduction in pension benefits.[8]

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[8] The latter is often referred to as an earnings test.

The 2011 pension reform introduced fundamental changes to these three features. Namely, following the pension reforms; (i) individuals can start claiming pension benefits anytime between the age of 62 and 75, (ii) there are actuarially neutral adjustments for early and late pension benefits, and (iii) pension benefits are no longer earnings tested (Brinch, Vestad & Zweimüller, 2015). The actuarially neutral pension adjustments are based on life expectancy, and implies that individuals can retire anytime within the flexible retirement age-range without changing their expected social security wealth. Thus, the decision of when to start claiming pension benefits has less of an impact on the decision of when to permanently withdraw from the labor market as it is possible to combine pension and labor participation without any additional financial restrictions. The intuition behind this disentanglement, and the abolishment of the earnings test, is that these changes yields individuals further financial incentives to work past the early retirement age, and thus discourages early retirement. A complete explanation of the basis for calculation of pension benefits is provided by OECD (OECD, 2013a, p. 49) amongst others.

This pension reform aimed to improve the long run fiscal sustainability of the public pension system in the face of an increasingly ageing Norwegian population through stronger labor supply incentives past early retirement age. Studies show that the reform is likely yield a great fiscal impact, mainly through increasing employment, but that the reform alone is far from enough to solve the long run fiscal sustainability-challenges the Norwegian economy is facing (Fredriksen, Holmøy, Strøm & Stølen, 2015). However, the fiscal outlook would look far more worrying had the old system remained.

Due to no major changes in the old age pension for public sector employees and disabled workers, only about 40% of all new pensioners are significantly impacted by the changes in the pension system (OECD, 2014c). This is a potentially major shortcoming of the reform, and limits the potential impact of the reform as incentives to continue working past retirement age remain weak in the public sector, which accounts for about one third of employment in Norway.

The Contractual Early Retirement (AFP)[9] scheme in the private sector and occupational pensions were reformed in line with these changes to the public pension system. However, as the analysis is limited to the public pension system, this will not be discussed in further detail.

The pension system remains a part of the general public finances, and is therefore still financed as a pay-as-you-go scheme (Fredriksen & Stølen, 2011).

3.2 Overlapping Generations Model (OLG)

This section provides a brief overview of a basic overlapping generations (OLG) model based on Fehr & Thøgersen (1995) and Steigum (1993). The purpose of this brief overview is to ensure that all readers know the intuition behind OLG-models, and the foundation they build upon, before and extended OLG model is developed in chapter 4. Further, the basic model allows one to illustrate the difference between the two main forms of pension scheme financing with relatively simple and intuitive figures. There are numerous possible applications of OLG models, including impacts of long-term fiscal policy, social security analysis, and effects of demographic development amongst others.

OLG models are a type of equilibrium growth models in which the lives of agents are finite and there is a constant stream of new agents arriving. Agents born at different times overlap in the sense that they simultaneously exist within the economy at a given time, but are in different stages of life. Cohorts lives consist of multiple life stages, and thus their life-span overlap with the life-span of other cohorts. In each life stage, agents face different choices and could have different preferences.

The model framework was initially devised by Allais (1947) and Samuelson (1958), and later extended and developed further by several other economists such as Diamond (1965).

In the basic OLG model agents live for two periods, with each cohort denoted as a generation. All members of a generation are in the same age-cohort and all agents in all generations move to the next stage of life at the same time. Additionally, as the oldest generations dies out, a new generation enters the economy. Thus, at any given time the economy consists of a young

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[9] Norwegian: "Avtalefestet pensjon".

generation and an old generation. In the first life stage, agents are assumed to be workers making decisions regarding labor, saving and investment. In the second life stage, agents are retirees consuming their savings and return on investments. This two-period time structure is presented in figure 3.1. The arrows indicated interaction between the generations at given time periods.

While the basic OLG model can be applied to provide several theoretical insights, the transition of age cohorts and generations through life-stages is far from perfectly synchronized in real modern economies, implying that one should use caution when applying the model.

Although each generation dies after living for two periods, the economy is ongoing due to the continuous introduction of new generations each period.

3.3 Pay-as-you-go versus funding

Broadly speaking, the financing of pension benefits can be organized as either a pay-as-you-go (PAYGO) scheme or a funded scheme. This section aims to explain these contrasting forms a financing, and relate them to the basic OLG model.
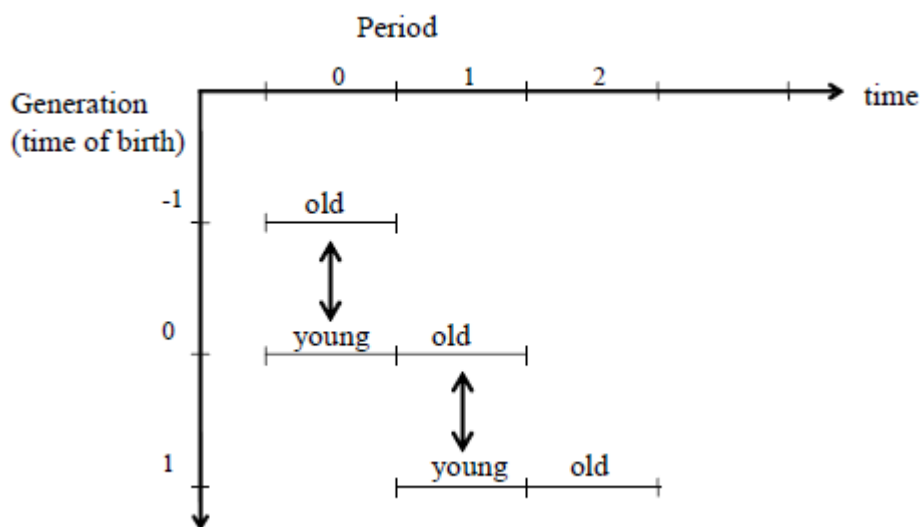
Figure 3.1: The two-period model time structure. Source: Groth (2015).

Pension benefits in a PAYGO system are directly financed through pension contributions from current workers. Workers finance the pension benefits of current pensioners in trade for a promise that the next generation will do the same. In a funded system on the other hand, pensioners finance their own pension through their own working age contributions. Some of their labor income is paid into a fund consisting of financial assets accumulating returns, which eventually finances their own pension benefits. Thus, the sustainability of a funded pension system relies on capital markets, and is not directly impacted by ageing populations or other relevant demographic changes (Barr, 2012).

Figure 3.2 based on Fehr and Thøgersen (2007) illustrates these differences in a simple overlapping generations framework with exactly two generations present at any time-period. The arrows indicate financing. This figure helps clarify intergenerational risk sharing effects of a PAYGO-scheme, and why this is not a factor in a funded scheme. To explain intergenerational risk sharing, one can assume that a representative generation are exposed to a wage shock in the current period, and a capital returns shock in the following period. If the contribution were to remain unchanged, some of the wage shock is transferred to the previous generation. Additionally, the following generation is exposed to the wage shock, and there is an intertemporal sharing of wage income risks. If the benefits were to remain unchanged however, the wage shock determines both the wage in the current period and the pension benefits. Thus, this intergenerational risk sharing perspective is not relevant in the case where benefits are predefined by the system (Thøgersen, 1998).

As pension benefits in PAYGO are directly financed by the contributions made by the working age population, changes in the population profile impacts the sustainability of the system. An ageing population leads to an increase in the ratio of pensioners per worker, and therefore leads to each individual worker having to pay larger pension contributions to keep benefits unchanged. Thus, PAYGO schemes face demographic risks. The demographic risk of pay-as-you-go schemes is one of the main reasons why a funding of the Norwegian pension system has been discussed as a potential method of combating the negative impacts of an ageing population (NOU, 1998:10).
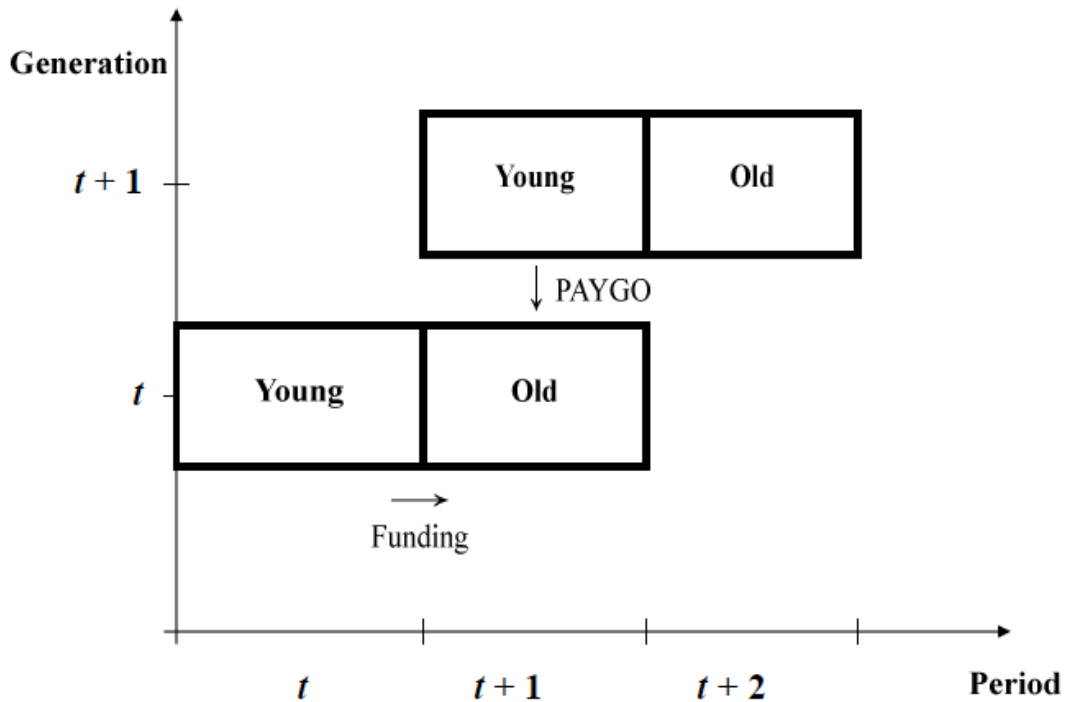
Figure 3.2: Pension financing in a basic OLG-model. Based on figure 2.1 in Fehr and Thøgersen (2007).

While PAYGO and funding are two theoretically contrasting schemes, it is possible to have a pension system consisting of a mix of the two schemes. As explained in chapter 2, the Norwegian pension systems is composed of a mandatory public PAYGO scheme, a mandatory occupational pension primarily financed through funding (Ponds, Severinson & Yermo, 2011), and voluntary private pension savings. Thus, a mix of PAYGO and funding allows for diversifying of risk (Diamond, 2002).

3.4 Transitioning from PAYGO to funding

In this section examines the transition dynamics when moving from pay-as-you-go financing to funding.

Transitioning from one system to the other is far from straightforward in practice. In general, a Pareto-improving shift to funding is not possible (Brunner, 1993). Moreover, if a shift from PAYGO to funding is found to be desirable, several other challenges arises. An immediate shift from pure PAYGO to full funding would result in one generation having contributed to the system as young, while not receiving any benefits financed by others as old (Kuné, 2001).

This problem is illustrated in figure 3.2. This figure assumes that an immediate shift from a pure PAYGO scheme to full funding is implemented as generation t transitions from young to old. Since generation t faces a PAYGO scheme as workers and a funded scheme as retirees, they must finance the pension benefits of the previous generation in addition their own pension funds. Thus, generation t are clear losers, and there are no clear short-run winners.

While this could be a theoretically optimal way of implementing the transition in the long run, it is obvious that this solution would be highly unpopular, and thus far from viable in a democracy. Any political party suggesting to force the current generation of workers to pay pension contributions twice would realistically not be able to reach a position where they could have the power to implement these suggestions.

Potential solutions include a smoother transition, reducing the burden on current labor force participants by gradually increasing the degree of funding, or spreading the burden through a reduction of the generosity of the scheme to current retirees. There would however still be some degree of negative short term distortions, while the long-run gains are realized by future generations.

Challenges related to a potential transition from pay-as-you-go to funding, and potential solutions, are discussed in further detail in section 6.5.
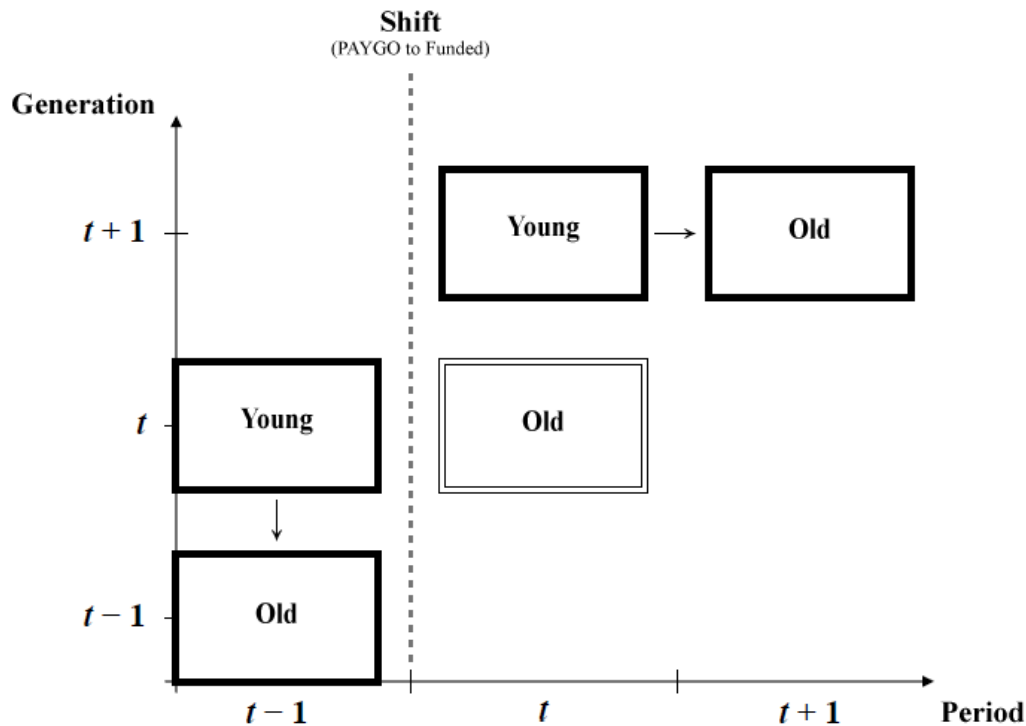
Figure 3.3: The losing generation when transitioning from pay-as-you-go to a fully funded scheme.

3.5 Defined benefit (DB) and defined contribution (DC)

A separate question to the financing of pension schemes is how benefits are related to working-age contributions. Typically, two approaches are distinguished between: Defined benefit and defined contribution (Bodie, Marcus & Merton, 1988).

In a defined benefit (DB) plan the pension benefits retirees receives are determined by a specific formula taking into account years of employment and earnings. The benefits are clearly defined, and the work-force contributions becomes the endogenous variable. Thus, those who contribute face all the risk. In a PAYGO scheme these are individuals who are part of the labor force, while in a public funded scheme this is the government.

One major advantage of defined benefit plans from the workers point of is the stable replacement rates of income these plans provide. The insurance against real wage risks workers receive implies that the risks are borne by firms. However, larger employers are able to diversify real wage risks, unlike employees, and thus the stable replacement rate should be regarded as an advantage of defined benefit plans.

In a defined contribution (DC) plan the level of pension benefits is determined by total contributions to a personal account, and investment earnings on the accumulations in the account. Thus, DC-plans are fully funded by definition. As contribution are clearly defined, only beneficiaries carry risks related to varying returns on savings or changes in future earnings. It is possible to share these risks more broadly through a guaranteed minimum pension.

Advantages of defined contribution plans are mainly related to inflation uncertainty. The value of the pension wealth is predictable at any time, and workers can more easily determine the true present value of the annual pension benefits they earn. Moreover, by being fully funded, the advantages of funding mentioned in section 3.3 apply to DC plans.

In a defined contribution plan, benefits face an exposure to possible wage shocks in the succeeding period, implying uncertain benefits. By definition, defined benefit plans are not exposed to these shocks, since the benefits are certain. Thus, when adopting an interim risk sharing ex poste perspective, defined benefit plans should be favored (Wagener, 2003).

In practice, most pension systems are a combination of defined benefit and defined contribution, and the Norwegian pension system is example of such a hybrid system. While OECD has chosen to define the Norwegian National Insurance Scheme as a traditional pay-as-you-go defined benefit plan (OECD, 2012), the system fulfils several criteria for a defined contribution scheme through the actuarial elements introduced by the 2011 pension reform. Thus, neither benefits nor contributions are completely exogenous, and the new public system should rather be characterized as a quasi-actuarial (Fredriksen & Stølen, 2011). The mandatory occupational plan (OTP)[10] is a defined contribution plan, and voluntary occupational pension plans can be either DB or DC. Workers in the public sector and individuals who are employed by large companies typically have defined benefit pensions, but following the introduction of OTP, defined contribution plans have gotten increasingly popular (OECD, 2008).

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[10] Norwegian: «Obligatorisk tjenestepensjon».

4.   Model Framework

4.1 Introduction to the model

To quantify the impact of population ageing on the optimal financing of public pension schemes, the thesis develops a dynamic extended overlapping generations model. This model is based on the same basic principles as the simplified OLG-model presented in section 3.2. It is extended to include 106 overlapping cohorts, and introduces mortality as an additional dynamic variable. While modelling population projections endogenously through fertility and mortality might seem unnecessary in the Norwegian case, as Statistic Norway provides direct population estimations, doing so improves the generalizability of the model.

The model does not build on a normative utility- or welfare function. For this reason, it is important to specify which variable one is either seeking to maximize or minimize when searching for optimality. Optimizing with respect to different variables might lead to different conclusions. Thus, it is crucial to always specify which variable one is taking into consideration when one discusses optimality by using a OLG-model with no normative social welfare function.

The model presented in this chapter is based on the Norwegian economy and pension system. The generalized version of the model is presented in appendix 1.

4.1.1 Demographics

Demographic development is handled with similar annotations as Andrews et al (2016). Following their approach, probabilistic ageing is introduced through an exogenous marginal probability of reaching the next life stage.[11] Additionally, the model developed in this thesis is extended to include more overlapping cohorts and consists of shorter periods.

During each 1-year period, the household sector consists of 106 overlapping cohorts of ages between 0 and 105, where $j \in \{0,1,2, ... ,105\}$ denotes the age of the cohort. Heterogeneity is assumed to be intercohort only, implying a representative household j for each period t.

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[11] Alternatively, one could introduce probabilistic ageing through an exogenous conditional probability, depending on the data one has available. Whether the probability is marginal or conditional does not impact the model in any other way than changing the annotation of the relevant variable.

Individuals are assumed to earn real wages $w_{j,t}$ when in working age. Neither children nor retirees work. Wages at a given time t are assumed to be equal for all working age cohorts.

The size of the representative household is given by $N_{j,t}$, which represents the size of cohort j in period t. A new generation aged $j = 0$ is born each period, and all other current generations shift forward one life stage. The fertility rate represents the exogenous population growth rate of the newly introduced generation at period t, and is denoted as n_t. Each household at a given age has an exogenous marginal probability $m_{j,t}$ of reaching the next life stage in the next period. As the oldest generation deterministically dies out in the subsequent period, $m_{j,t} = 0$ for $j \geq 105$, and population in period t can be expressed as:

$$N_{j,t} = \begin{cases} (1 + n_t)N_{0,t-1} & if\ j = 0 \\ m_{j-1,t-1}N_{j-1,t-1} & if\ j \in \{1,2,\dots,105\} \\ 0 & if\ j > 105 \end{cases}$$

However, Statistics Norway provides direct population projections for years 2016 through 2100 with the population separated into one-year age groups. Thus, substituting for demographic development is not necessary when analyzing the Norwegian National Insurance Scheme as projections of the size of all representative households $N_{j,t}$ are provided directly. Substitution for demographic development is presented in the generalized model in appendix 1.

4.2 Rate of return

To find out whether PAYGO or funding is the optimal form of financing, the rate of return of the schemes yield is compared. The model does not incorporate a normative utility function or consider time preferences, and thus it is important to once again stress that, in this case, stating that one of the schemes is optimal means that the scheme is viewed as preferable over the other scheme with respect to the relevant choice of measurement.

4.2.1 Pay-as-you-go

In a pay-as-you-go scheme contributions made by workers at time t equals benefits received by retirees at time t. Working age is assumed to consist of ages 20 through 66. Thus, the relationship between contributions and benefits in a PAYGO-scheme can be written as:

$$\sum_{j=20}^{66} N_{j,t}\tau_t w_{j,t} = \left(\sum_{j=67}^{105} N_{j,t}\right)\left[\theta_t\left(\sum_{j=20}^{66} w_{j,t-67+j}\right)\right]$$

Where τ represents share of working-age income contributed to the pension scheme through taxes and θ represents the annual pension benefits received as a share of total lifetime working income; the rate of return of the pension scheme.

4.2.2 Funding

In a funded scheme, each generation finance their own pension through savings in working age. Savings are placed in a fund that accumulating returns, and is used to finance annual pension benefits after reaching pension age. In this framework, the relationship between contributions and benefits in a funded scheme can be written as:

$$\sum_{z=1}^{47}(1+r)^z\,\tau_{t-z}w_{j,t-z} = \varphi_t\theta_t\left(\sum_{j=20}^{66} w_{j,t-67+j}\right)$$

Where r represents real rate of return on the funded savings and φ represent expected remaining lifetime at retirement age. Hence, savings needs to finance φ annual payments of pension benefits for a representative individual. The long-run real rate of return on savings is assumed to be relatively stable, despite short-run volatility, and thus this variable is independent of time. In the case of the Norwegian National Insurance Scheme, a funding as part of the relatively stable Government Pension Fund Global is proposed, as discussed in section 2.4. For other economies with more volatile inflation levels, this assumption holds true to a lesser degree. θ represents the rate of return of the pension scheme, which is defined as the annual pension benefits received as a share of total lifetime working income, just as in the pay-as-you-go case.

4.2.3 Comparison

Rearranging the PAYGO-equation by dividing both sides of the equality by $\left(\sum_{j=67}^{105} N_{j,t}\right)$ and rearranging the funded-equation by multiplying both sides of the equality with $\frac{1}{\varphi_t}$, makes it simple to directly compare the rates of return as the RHS is equal in the two equations:

$$\frac{\sum_{j=20}^{66} N_{j,t} \tau_t w_{j,t}}{\left(\sum_{j=67}^{105} N_{j,t}\right)} >=< \frac{1}{\varphi_t} \sum_{z=1}^{47} (1+r)^z \tau_{t-z} w_{j,t-z}$$

The equation is simplified further by assuming that the tax rate; the share of income contributed to the pensions scheme, does not depend on period t: $\tau_x = \tau_t$ for all x.[12]

$$\frac{\sum_{j=20}^{66} N_{j,t} w_{j,t}}{\sum_{j=67}^{105} N_{j,t}} >=< \frac{1}{\varphi_t} \sum_{z=1}^{47} (1+r)^z w_{j,t-z}$$

Productivity growth, λ, is used as a proxy for wage growth,[13] and wages at a given time t are assumed to be equal for all working age cohorts: $w_{j,t} = w_t$ for $j \in \{20,21,\dots,66\}$. Thus, wages in period t can be expressed as:

$$w_{j,t} = \begin{cases} 0 & if\ j < 20 \\ (1+\lambda_t) w_{j,t-1} & if\ j \in \{20,21,\dots,66\} \\ 0 & if\ j \geq 67 \end{cases}$$

Thus:

$$\left(\frac{\sum_{j=20}^{66} N_{j,t}}{\sum_{j=67}^{105} N_{j,t}}\right) w_t >=< \frac{1}{\varphi_t} \sum_{z=1}^{47} (1+r)^z w_{t-z}$$

Substitution yields:

$$\left(\frac{\sum_{j=20}^{66} N_{j,t}}{\sum_{j=67}^{105} N_{j,t}}\right)$$

$$>=<$$

$$\frac{1}{\varphi_t} \left[\frac{(1+r)}{(1+\lambda_t)} + \frac{(1+r)^2}{(1+\lambda_t)(1+\lambda_{t-1})} + \dots + \frac{(1+r)^{47}}{\prod_{b=1}^{47}(1+\lambda_{t-b+1})} \right]$$

---

[12] This is based on a defined-contribution real rate of return maximizing approach. An alternative, defined-benefit tax-minimizing approach is explained in section 4.3.

[13] The relationship between productivity growth and wage growth is discussed by Feldstein (2008) and Fehr & Thøgersen (2007) among others.

This equation is simplified by creating the following aggregate variable:

$$\bar{\Lambda}_x = \prod_{x=1}^{47}(1 + \lambda_{t-x+1})$$

Thus, the relationship between the rate of return of the two pension schemes is given by:

$$\left(\frac{\sum_{j=20}^{66} N_{j,t}}{\sum_{j=67}^{105} N_{j,t}}\right) >=< \frac{1}{\varphi_t}\sum_{s=1}^{47}\left[\frac{(1+r)^s}{\bar{\Lambda}_s}\right]$$

When the left-hand-side of this equation is larger than the right-hand-side, PAYGO yields a larger rate of return than funding in the model. Similarly, when the right-hand-side of the equation is larger than the left-hand-side, funding yields a larger rate of return than PAYGO in the model.

To compare the rate of return in a PAYGO scheme to the rate of return in a fully funded scheme, the relative rate of return (RRR) is estimated. We define the relative rate of return as the rate of return of a PAYGO scheme divided by the rate of return of a funded scheme in an identical framework with identical assumptions, minus one.

$$RRR_t = \frac{\left(\dfrac{\sum_{j=20}^{66} N_{j,t}}{\sum_{j=67}^{105} N_{j,t}}\right)}{\dfrac{1}{\varphi_t}\sum_{s=1}^{47}\left[\dfrac{(1+r)^s}{\bar{\Lambda}_s}\right]} - 1$$

In other words, when the relative rate of return is positive (negative) the rate of return of a PAYGO scheme is larger (smaller) than the rate of return of a funded scheme, and when the gap is zero the two schemes yields equal rates of return. A relative rate of return equal to 1 (negative 1) implies that the rate of return in a PAYGO scheme (funded scheme) is twice as large as the rate of return in a funded scheme (PAYGO scheme). Likewise, a relative rate of return of 0.5 implies that a PAYGO scheme yields a rate of return that is 50% larger than the alternate fully-funded scheme.

4.3 Tax rate

Alternatively, one could consider the optimal scheme to be the one that minimizes the tax rate for a given rate of return, rather than the one that maximizes the rate of return for a given tax rate. This is a defined-benefit tax-minimizing approach to the problem.

Taxes are likely to lead to efficiency losses, and the magnitude of the loss is assumed to be an increasing convex function of the tax rate. Thus, in a welfare state with a high tax rate one would prefer to avoid the need further tax-increases and sharp jumps in tax rates (Kydland & Prescott, 1980).

The two fundamental equations of the model representing the relationship between contributions and benefits remain unchanged with one exception: The variable representing the share of income contributed to the pension scheme through taxes is not necessarily equal in the two equation. Thus, the two equations are written as:

$$\sum_{j=20}^{66} N_{j,t} \tau^{PAYGO} w_t = \left(\sum_{j=67}^{106} N_{j,t} \right) \left[\theta_t \left(\sum_{j=20}^{66} w_{t-67+j} \right) \right]$$

$$\sum_{z=1}^{47} (1+r)^z \tau^{FUND} w_{t-z} = \varphi_t \theta_t \left(\sum_{j=20}^{66} w_{t-67+j} \right)$$

Rearranging both equation to have the tax-variable isolated on the LHS of the equality allows one to directly compare the necessary tax-rates of the two schemes for a given rate of return.

$$\tau^{PAYGO} = \frac{\left(\sum_{j=67}^{106} N_{j,t} \right) \left[\theta_t \left(\sum_{j=20}^{66} w_{t-67+j} \right) \right]}{\sum_{j=20}^{66} N_{j,t} w_t}$$

$$\tau^{FUND} = \frac{\varphi_t \theta_t \left(\sum_{j=20}^{66} w_{t-67+j} \right)}{\sum_{z=1}^{47} (1+r)^z w_{t-z}}$$

These two equations make it possible to directly compare the defined benefit tax-rates in PAYGO and funded schemes with the following equation:

$$\frac{\left(\sum_{j=67}^{106} N_{j,t} \right)}{\sum_{j=20}^{66} N_{j,t} w_t} >=< \frac{\varphi_t}{\sum_{z=1}^{47} (1+r)^z w_{t-z}}$$

This equation tells us that the necessary tax-rate is larger in a PAYGO-scheme than in a funded scheme when the left-hand-side is larger than the right-hand-side, and that the necessary tax-rate is larger in a funded-scheme than in a PAYGO-scheme when the right-hand-side is larger than the left-hand side.

Clearly this equation is the inverse of the equation comparing the rate of return of the two systems. Therefore, if the model predicts that the rate of return is larger in one system than the other with equal tax-rates, it also predicts that the necessary tax rate for a given rate of return is smaller in that system than the other. The scheme that maximizes rate of return in a defined contribution system also minimizes the tax rate in a defined benefit system.

5. Data, parametrization and calibration

5.1 Demographics

Statistics Norway (2016) provide population projections based on predicted levels of fertility, life expectancy, internal migration, and immigration. The different projections are described using four letters, with each letter representing the development of the four different components, in the following order:

- Fertility
- Life expectancy
- Internal migration
- Immigration

For fertility, life expectancy and immigration three different projected levels are created: Low, medium, and high. The letter L denotes a low alternative, the letter M denotes a medium alternative, and the letter H denotes a high alternative. Assumptions are combined in a variety of ways. For example, the HHMH alternative describes population development with high fertility, high life expectancy, medium internal migration and high immigration. For internal migration, high and low alternatives are not created. In addition, the letter K is used to denote constant[14] immigration or life expectancy while zero is used to denote no internal or international migration or zero net migration.

Internal migration is included by Statistics Norway in their population projections to analyze projected centralization of the Norwegian population, and examine how municipalities differ in growth and ageing. Changes in the component does not impact the national population. Thus, changes in internal migration does not impact the analysis at a national level. For this reason, the medium alternative of internal migration in all alternatives in this thesis.

Changes in life expectancy will clearly also impact expected remaining lifetime at retirement age; the φ-variable. As the annuity divisor used in the current Norwegian system is equal for both genders, one needs to look at life expectancy at retirement for men and women combined.

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[14] "Konstant" in Norwegian.

In the main alternative, expected remaining lifetime for a 60-year Norwegian is expected to rise to 29.8 years in 2060 and 33.0 years in 2100. Expected lifetime is two to three years lower in the low alternative, and three to four years higher in the high alternative. Since there are no clear projections of this variable available, we create our own estimates by combining these projections with the current expectation of lifetime for a 67-year old Norwegian equal to 18.76 (Statistics Norway, 2017b), and smooth missing estimates. The projections are made comparable by assuming that expected remaining lifetime at age 67 is approximately equal to the expected remaining lifetime at age 60 minus 7. Figure 5.1 illustrates the development of expected remaining lifetime at retirement age in the different alternatives assuming effective retirement age equal to 67 years.

While several population projection alternatives are developed using different combinations of these four components, the thesis limits its focus to three: The main alternative (MMMM), the alternative with strong ageing (LHML), and the alternative with weak ageing (HLMH). The succeeding subsections describe these three alternatives and their assumptions in further detail.
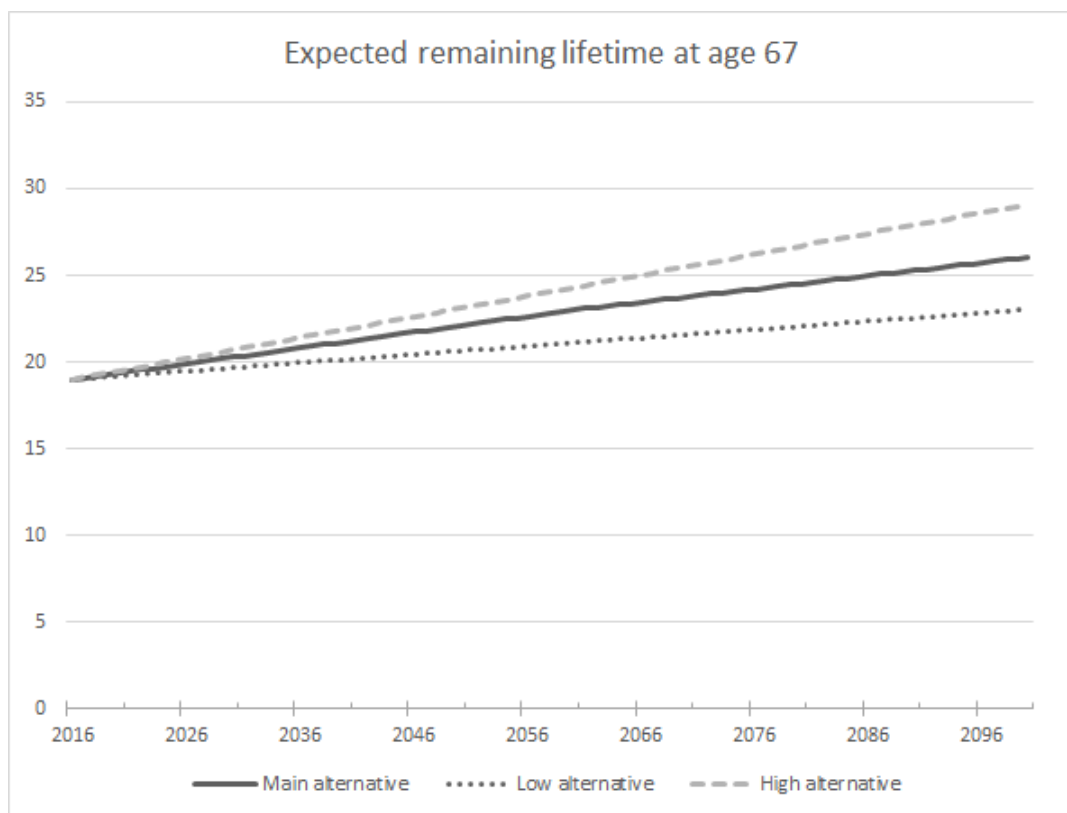
Figure 5.1: Projected expected remaining lifetime for both genders combined at age 67 in three different alternatives.

5.1.1 Main alternative (MMMM)

The main alternative assumes medium development in all four components. These are the assumptions that Statistics Norway considered to be the most plausible. In this alternative, the number of individuals aged 70 years or over is predicted to double in less than three decades, while the number of individuals aged 80 years or over is predicted to double even quicker.

Fertility is assumed to remain at approximately 1.69 children per woman, life expectancy is assumed to increase by close to seven years for men and five years for women by 2060, the internal migration is predicted to follow the same pattern as in recent decades, and long run net migration is assumed to be between 25,000 and 30,000.

If this alternative proves to be accurate, a population of six million will be passed around year 2030, equating to a growth of a million inhabitants in less than 20 years. This would be the quickest million ever in Norwegian history.

5.1.2 Strong ageing (LHML)

The alternative with strong ageing assumes low fertility, high life expectancy, and low immigration.

Fertility is assumed to quickly fall to a level 13 percent lower than the 2015 level, equating a total fertility rate of 1.48. This is slightly below the average EU total fertility rate in 2014 (Eurostat, 2016). Life expectancy is assumed to increase by ten years for men and eight years for women by 2060, and long run net migration is assumed to be slow down before stabilizing at somewhere between 5,000 and 10,000.

Low birth rates, strongly increased life expectancy and low net migration all lead to an ageing population, and thus the total effect in this alternative is a strong ageing of the population.

The net impact of this development in these three components is a projected increase in the burden of care for the elderly from 0.17 in 2016 to 0.58 in 2100. This implies a ratio of 1.73 individuals in working-age per individual in retirement age in 2100, in comparison to 5.86 in 2016. A development like this would put large amounts of pressure on a pure PAYGO-scheme as the payments workers would have to make to keep pension benefits unchanged would increase sharply, ceteris paribus.

5.1.3 Weak ageing (HLMH)

The alternative with weak ageing assumes high fertility, low life expectancy, and high immigration.

Fertility is assumed to rise to a level 13 percent larger than the 2015 level relatively quickly, equating a total fertility rate of 1.91, which is close to the Norwegian total fertility rate in 2009. Life expectancy is assumed to increase by three years for men and two years for women by 2060, and long run net migration is assumed to be grow rapidly, reaching a level above 90,000 by the end of the century.

High birth rates, weakly increased life expectancy and high net migration all helps slow down the ageing population, and thus the total effect in a situation like this would be a relatively weak ageing of the population.

The net impact of this development in these three components is a projected increase in the burden of care for the elderly from 0.17 in 2016 to 0.25 in 2100. This implies a ratio of 4.06 individuals in working-age per individual in retirement age in 2100, in comparison to 5.86 in 2016. While this decreasing ratio is far from ideal in a pay-as-you-go pension scheme, it does not necessarily decrease to attractiveness of a PAYGO-scheme relative to a funded scheme, and reforms incentivizing more people to continue working despite ageing could be enough to prevent a potential "old age crisis".

The projected development of the burden of care for the elderly in these three alternatives is illustrated in figure 5.2. All three alternatives imply a gradual increase in the burden of care, but they differ in magnitude.
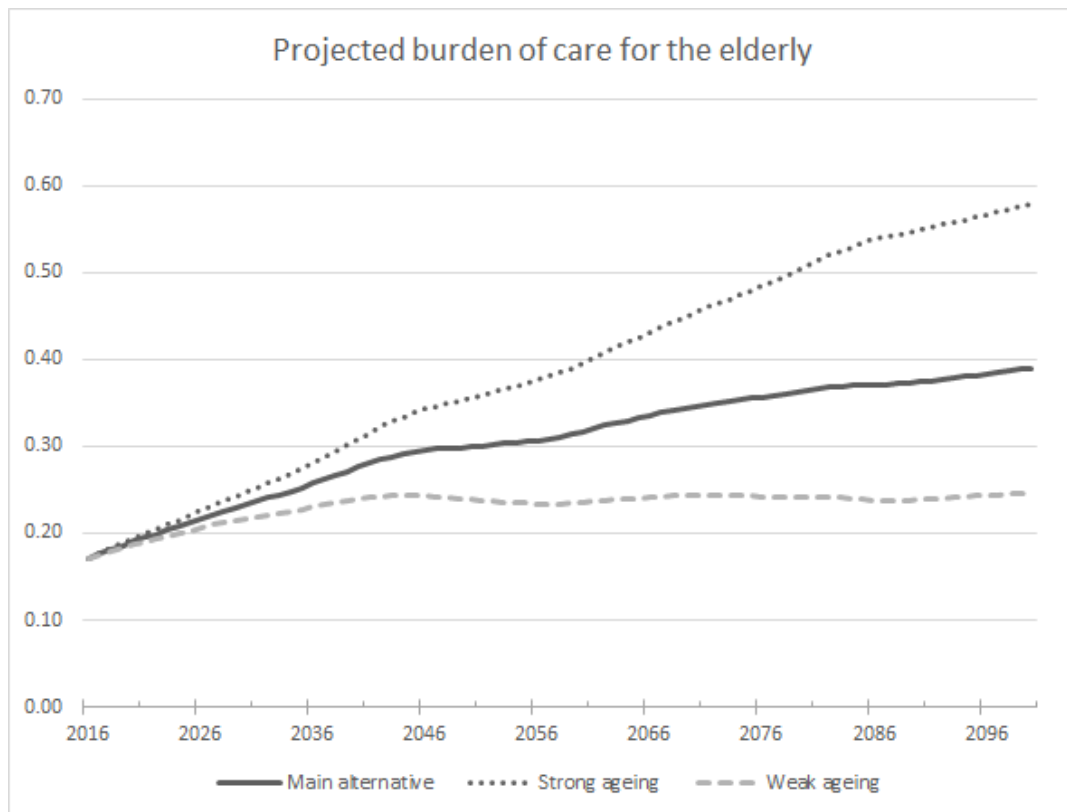
*Figure 5.2: Projected development of the burden of care for the elderly in
three alternatives for demographic development.*

5.2 Real return on savings

In the model developed in chapter 4, real return on savings is defined as the difference between the nominal return on funded pension savings and the domestic inflation-level. In the fully funded alternative proposed in this thesis, it is assumed that the pension fund is invested by Norges Bank Investment Management (NBIM) together with the Government Pension Fund Global while de facto being considered a separate fund. The two funds will be invested as one, while being considered as two different funds. This implies that the pension fund would have the same real return as the Government Pension Fund Global.

The model assumes a real rate of return equal in all periods from 2016 to 2100. While this simplification implies no volatility in the return of the fund, and thus weakens the results the model yields, the accumulated annualized real return of the fund has been relatively stable from 2005 to the first quarter of 2017 (Norges Bank Investment Management, 2017).

To account the large uncertainty of the real return, the thesis discusses and analyses three different scenarios: Expected-, high- and low real return on savings.

5.2.1 Expected return: Fiscal rule and inflation targeting

The Norwegian fiscal rule is based on a permanent income hypothesis. The 'rule' states that no more than the real return of total wealth of the fund should be allocated to the annual government budget. Norway aims to transform oil revenue into permanent income by limiting spending to the real return of financial wealth, and the petroleum-income is gradually phased into the domestic economy in parallel with expected real return of the fund.

The expected real rate of return on the Government Pension Global was set equal to 4 percent at the inception of the fiscal rule in 2001. However, in 2017 the expected real rate of return was reduced to 3 percent based on analysis from the Norwegian central bank (Meld. St. 29 (2016-2017), 2017).

The operational implementation of monetary policy of the Norwegian central bank is oriented towards a low and stable inflation target, with annual consumer price inflation at around 2.5 percent over time. Because of this operational flexible inflation targeting, sharp jumps in actual inflation are highly unlikely.

Based on this an alternative with an expected real return on pension savings equal to the expected real return on the Government Pension Fund global of 3 percent in every period is introduced:

$$r_t^E = 3\% \ for \ all \ t$$

5.2.2 High real rate of return

It could be argued that assuming a time-independent real rate of return on equal to three percent is overly pessimistic. In recent years, the difference between the rate of return on the Government Pension Fund Global (GPFG) and the inflation, measured as the year-on-year rise in the consumer price index (CPI), has one average been slightly larger than this predicted level. Table 5.1 below illustrates the real rate of return over the past 18 years in percent, measured the difference between the annual return on the GPFG in percent and inflation in percent. It is also worth noting that the CPI often exaggerates the actual inflation (Blinder, Triplett, Denison, & Pechman, 1980).

	Return on GPFG	Inflation	r
1999	3.78	2.20	1.58
2000	6.92	3.50	3.42
2001	2.74	2.30	0.44
2002	7.58	2.10	5.48
2003	15.95	2.00	13.95
2004	13.42	1.40	12.02
2005	-2.54	0.10	-2.64
2006	9.62	2.80	6.82
2007	25.62	2.00	23.62
2008	-23.31	2.20	-25.51
2009	4.26	2.80	1.46
2010	7.92	2.20	5.72
2011	11.09	1.80	9.29
2012	8.94	1.10	7.84
2013	12.59	0.60	11.99
2014	-4.74	2.70	-7.44
2015	-2.47	2.10	-4.57
2016	2.49	3.00	-0.51
2017	12.94	2.80	9.64
Average	**5.94**	**2.09**	**3.82**

Table 5.1: Real rate of return on Government Pension Fund Global over the past years, percent. Source: Statistics Norway and Norges Bank

In monetary the first policy report of 2017 from the Norwegian central bank (Norges Bank, 2017), they predict that the CPI will continue to fall until a level of 1.2 percent in 2019, and then slowly start climbing towards the inflation target of two and a half percent. Inflation lower than the target in recent years implies a higher real rate of return on pension savings for given levels of nominal return. Thus, a real rate of return on pension savings higher than three percent annually could be a possible scenario.

Based on these arguments, an alternative with a projected real rate of return on funded pension savings equal to 4 percent in every period is introduced:

$$r_t^H = 4\% \; for \; all \; t$$

5.2.3 Low real rate of return

While data from previous years show that the real return on the Government Pension Fund Global has been higher than the fiscal rule level of 3 percent, that does not necessarily mean that the returns will remain this high. Some argue that it appears that the real return on the

fund will be relatively low for the next 10 to 15 years, mainly due to low long-term interest rates (NOU 2015:9, 2015). If equity returns were to weaken as well, the real return of the fund could be as low as 2 percent in future periods.

While the Norwegian fiscal rule seems to be robust in the face of volatile returns, and the possible 15-year period of low returns is expected to be followed by periods with higher returns, a return to a long-term level of three or four percent annually is far from guaranteed.

Based on this argument, a projected real rate of return on funded pension savings equal to 2 percent in every period is introduced:

$$r_t^L = 2\% \; for \; all \; t$$

While this might seem like a highly unlikely scenario, it is still useful when analyzing the relative return of a pension scheme with respect to potential worst- or best-case scenarios.

5.3 Productivity

The model developed in chapter 4 endogenizes wage growth from one period to the next. Most available wage-data is however not generalizable or representative for the entirety of the Norwegian economy, and there are no official projections of national wage-growth. While one could make assumptions of future wage growth using historic data for a selected pool of seemingly representative groups in combination with projected macroeconomic development, using a proxy for productivity growth to be preferable.

Fehr and Thøgersen (2007) argues that, in the long run setting, wage growth reflects productivity growth. Productivity growth can be estimated through total factor productivity growth, which represents technological progress. In other words, total factor productivity is the portion of output that is not explained by differences in capital per worker (Prescott, 1998). GDP per worker has a relatively high correlation with total factor productivity (Miller & Upadhyay, 2002), and can thus be used as an estimate of productivity growth.

Productivity growth in periods 1970 through 2016 is calculated using existing data for GDP in constant 2005-prices and annual changes in mainland GDP volume (Statistics Norway, 2017c) in combination with the population projections described in section 5.1.

To account for the large uncertainty related to productivity growth, three different scenarios are examined: Expected-, declining- and increasing growth. Annual productivity growth for all three scenarios are presented in appendix 3 as well as in figure 5.3.

5.3.1 Expected productivity growth

In the expected growth path, productivity growth is assumed to be in line with national projections from Statistics Norway (Cappelen, Eika, & Prestmo, 2013). These projections assume a relatively low GDP-growth per capita following declining demand from the petroleum sector from 2020 to 2040.

Projected average annual GDP growth is presented in table 5.2. By combining GDP growth in Mainland Norway with data for demographic development one can find projected growth in GDP per capita, which is used as a proxy for productivity growth.

	2016-2020	2021-2025	2026-2030	2031-2035	2036-2040
GDP	1.9	1.5	1.2	1.3	1.4
Non-oil GDP	2.2	2.0	1.7	1.8	1.8
GDP Mainland Norway	2.3	2.1	1.8	1.9	1.8

Table 5.2: Projected average annual growth of selected main economic indicators.

In the years following 2040, the oil sector is assumed to be a small part of the Norwegian economy, and GDP growth is predicted to rise slightly before stabilizing at a level close to two percent annually.

5.3.2 Declining productivity growth

Most economic theory following the seminal work of Robert Solow in the 1950s assumes that economic growth is a continuous process that will persist forever. However, the economic growth prior to 1750 was weak.

Some argue that the rapid technological progress made during the last 250 years might be unique, and that endless productivity improvements is no guarantee (Gordon, 2012). The slowed down productivity growth in recent years could represent a new trend, and might not necessarily be a negative cycle. Even though the arguments Gordon presents focus on the US, they also hold true for Norway and other western economies to some degree.

While the 20th and 21st century have seen some short spurts of large technological growth, the first and second industrial revolution were far more ground-breaking than what has followed, and economies were still realizing the gains from these industrial revolutions 100 years later. In comparison, the impact on productivity growth from the third industrial revolution, lasting from the early-1960s to the late-1990s, seems to have withered during the mid-2000s. Despite large quantities of resources being invested in research and development, inventions after the turn of the century has mainly focused on entertainment and communication rather than fundamental changes in labor productivity.

Based on these arguments, in addition to several other US-specific challenges, Gordon develops a hypothetical growth rate path. This path is presented as a smoothly curved line gradually decreasing towards an annual growth in real GDP per capita of 0.2% by the end of the 21st century. While not all arguments are relevant to the Norwegian economy, the critical view on the future development of productivity growth is an important one to take into consideration.

This growth path is included in the model as a declining productivity growth alternative.

5.3.3 Increasing productivity growth

A contrasting train of thought is that future technological progress possibly might be able to transform lives in ways even surpassing the ways the first and second industrial revolution did.

Automation of and computerization will lead to a reduction of the demand for unskilled manufacturing workers, and move the employment share further towards high skill occupational groups (Frey & Osborne, 2013). While the short-term impact of the increase in technological innovation disrupts labor markets, and could lead to increased unemployment, the long-term impact could be an increased growth path. Thus, one would expect an increase in the quantity of human capital allocated to productivity-improving research and development. In addition, automation is expected to lead to increased productivity in the computerized sectors.

The pace of technological innovation in OECD economies is increasing (Brynjolfsson & McAffe, 2011), and does not only impact routine intensive manufacturing tasks: Tasks requiring a higher degree of adaptability are also impacted. For example, the introduction of

autonomous cars is predicted to lead to a decreased demand for non-automated transportation, and a large-scale automation of the transportation sector would likely lead to a reduction in transportation time. Similarly, several manual logistics tasks are automated by autonomous computerization.

Increased productivity following technological progress can be transferred internationally at relative ease, unlike increased domestic labor productivity. While prices might act as a barrier to global adoption in the short run, prices of information and communications technology hardware fall relatively quickly as more competitors enter the market, often inspired by the first innovators. Absorbing existing technology is easier and happens more rapidly than changes in domestic workers' direct labor productivity.

With these arguments in mind, an increasing productivity growth path following a smooth curve moving towards a stable level of 4.1% per year growth in real GDP by year 2100 is estimated. While this most likely is an overly optimistic and widely unrealistic estimation (Johansson et al, 2013), it does not seem to be beyond the realm of possibilities.

The three productivity growth scenarios are illustrated in figure 5.3. The growth paths of the two scenarios with non-expected productivity growth are considered highly unlikely, and should primarily be used to analyze potential best- and worst-case outcomes.
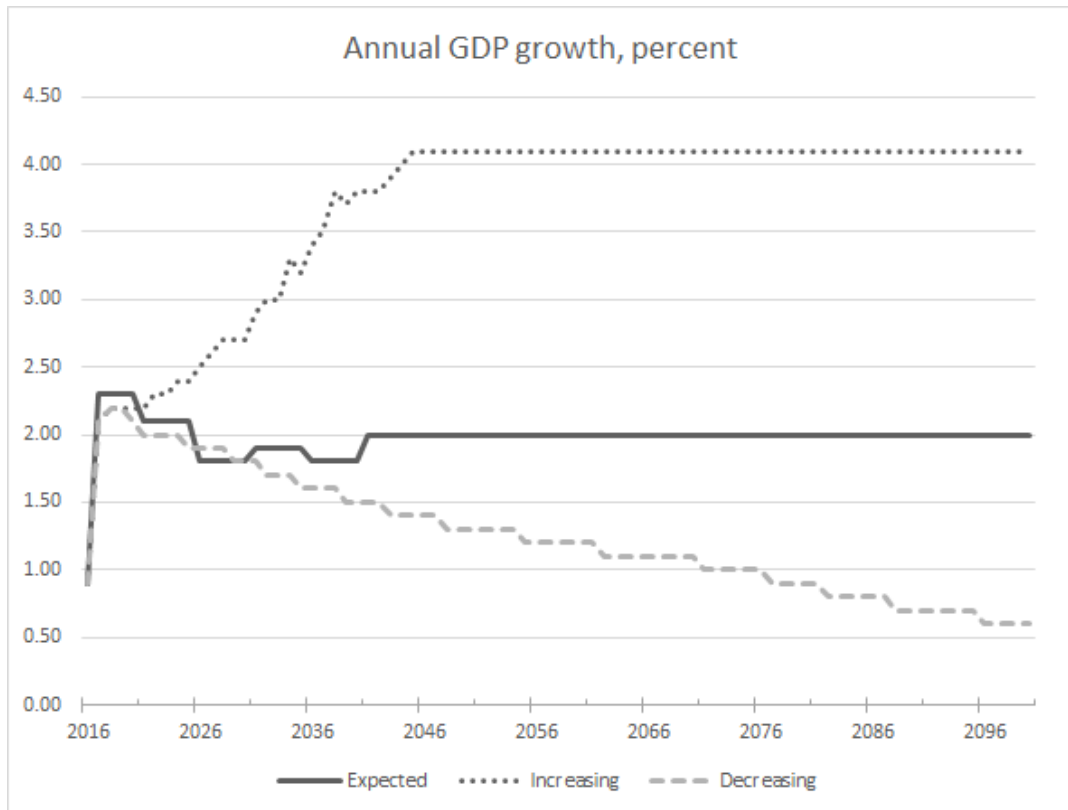
Figure 5.3: Projected annual GDP growth in three scenarios.

5.4 Retirement age and labour market entry age

The final components the extended overlapping generations model considers are the generalized effective retirement age and labor market entry age of the population. The higher the retirement age is, the more periods of a lifetime are spent in working age. An increase (decrease) in the effective retirement age impacts the burden of care both by increasing (decreasing) the working-age population and decreasing (increasing) the population in retirement age. Thus, pension reforms making early retirement less attractive could be a potential solution to combating the negative impacts of an ageing population.

Similarly, one could analyze the labor market entry age. Changes in labor market entry age would impact the quantity of working-age individuals, and thus the burden of care. However, successfully implementing policies that impacts the labor market entry age is far more challenging than implementing pension reforms incentivizing postponed retirement. Additionally, technological progress and automation might create upward pressure on the labor market entry age, as the demand for highly educated workers increase.

In selected comparable OECD countries,[15] the estimated average age of labor market entry stood at around 22 years in 2013 (OECD, 2015, p. 79). This number was impacted by the high unemployment rates in that period, as young individuals to a larger degree was impacted by employment losses than older workers. Therefore, the long-run generalizable labor market entry age is expected to be lower than the current level, and a labor market entry age of 20 is assumed for all scenarios. Arguments against the choice of generalized labor market entry age could be made, as only three fourths of individuals age 20-24 years were in employment in 2008, and the share has been declining ever since to a level of 62 percent in 2016 (Statistics Norway, 2017d). However, changes in market entry age impacts the rate of return through a reduction in working-age periods, and does impact the number of periods spent in retirement age. Thus, the changes in market entry age impacts the rates of return of a pay-as-you-go and a funded scheme in similar fashion. The choice of labor market entry age has a minimal impact on the relative rate of return of the two schemes.

5.4.1 Expected retirement age

The effective retirement age in Norway was 65.2 years for men and 64.3 for women in 2014 (OECD, 2015, p. 165). As explained in section 3.1 the current normal retirement age in the Norwegian public pension scheme is 67 years, but some workers are still encouraged to retire earlier by the pension system.

Effective retirement age is expected to increase slightly following the gradual introduction of new public pension scheme implemented in 2011. Early retirement is becoming less attractive following the pension reform, and effective retirement age is projected to move towards the normal retirement age of 67 years.

In the expected alternative, an effective retirement age of 67 years is assumed in all periods 2016 through 2100.

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[15] These countries consist of: United Kingdom, Sweden, Netherlands, Austria, Germany, Portugal, Estonia, Finland, Spain, Denmark, Slovenia, and France.

5.4.2 Increasing retirement age

As a consequence of the recent changes in the Norwegian pension system, and expected future reforms further increasing the incentives to postpone retirement, effective retirement age could potentially rise above the current normal retirement age in the future.

When expected lifetime of representative individuals increases, and their general health during old age improves, it seems logical to assume that retirement age should increase as well. Healthy ageing could mitigate some of the challenges increased longevity poses. To make this happen, changes must be made to the pension system such that the benefits of extra years of work are preferable to an extra year of retirement. Simply increasing the statutory retirement age would not be an effective policy instrument (Staubli & Zweimüller, 2013).

To analyze the impacts an increasing retirement age has on the relative return of a pension scheme, alternative where effective retirement age equals 70 years in all periods is introduced.

5.4.3 Decreasing retirement age

With the current Norwegian pension scheme, workers are strongly encouraged to retire at age 62 by the early retirement scheme in the public sector (OECD, 2014a).

Furthermore, as illustrated in figure 5.4, the average effective age of retirement in Norway from 1970 to 2014 had a negative linear trend for both men and women. Thus, it seems like one should expect a decreasing retirement age in future periods if there are made no further changes to the pension scheme. Average market exit age in most other OECD countries has followed a similar path with a negative linear trend since 1970 (OECD, 2013b).

The sharp decline in average effective age of retirement in Norway from the early 1980s to the early 1990s coincides with the introduction of the AFP[16] early retirement scheme in the private sector, which since has been completely revised as part of the 2011 pension reform explained in subsection 3.1.1. This illustrates how the presence of a pension scheme in the private sector can impact the effective retirement age in a public scheme, and why it is important to analyze them simultaneously rather than separately.

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[16] "Avtalefestet pensjon".
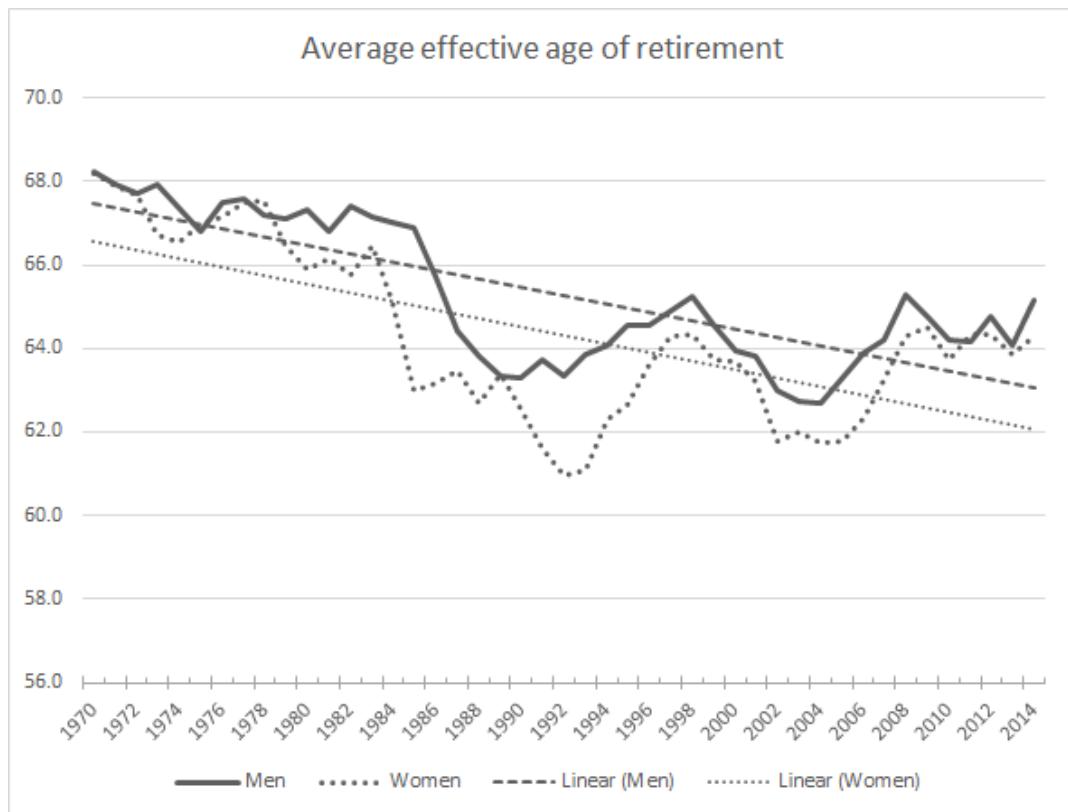
Figure 5.4: Average effective age of retirement in Norway 1970-2014.
Source: OECD.[17]

In this alternative, the negative trend in average effective age of retirement is assumed to stabilize itself as a result of the pension reform from 2011. This new stable level is however assumed to be somewhat lower than the normal retirement age of 67 years.

To analyze the impacts a decreasing retirement age has on the relative return of a pension scheme, an alternative where effective retirement age equals 64 years in all periods is introduced.

It is also worth mentioning that the sharp decline in effective retirement following the introduction of the AFP-scheme significantly impacts the trends. One could argue that using a single linear trend for the whole period might create misleading data, seeing as the pension system in the 1970s and early 1980s was completely different from the system after the introduction of the AFP-scheme. In fact, the linear trend from 1988 through 2014 is increasing

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[17] Data available online at http://www.oecd.org/els/emp/Summary_1970%20values.xls

for both men and women. This argument weakens the probability of the decreasing retirement age alternative.

6.   Results and analysis

In this chapter, the data presented in chapter 5 is combined with the extended overlapping-generations model developed in chapter 4 to analyze the projected relative rate of return of the Norwegian pension scheme in different alternatives.

Different aggregate projections are described using four letters, with each letter representing the development of four different components in the following specific order.

- Ageing
- Real return on savings
- Productivity growth
- Retirement age

To differentiate these aggregate projections from the population projections, different letters and terms are used for most components. An expected level is created for all components, denoted by the letter E. For ageing this represents the projected main alternative, for real rate of return it represents the fiscal rule level, for productivity growth it represents a stable path, and for retirement age it represents the current level of effective retirement age. In addition, ageing can be either strong (S) or weak (W), real return on savings can be either high (H) or low (L), productivity growth can be either increasing (I) or decreasing (D), and effective retirement age could be increasing (I) or decreasing (D).

Following this approach, there are 81 possible different combinations of the four components. [18] However, the probabilities of these projections differ, and the scenario where all components assume their expected level (EEEE) is the most likely scenario.

One should expect some degree of correlation between these components, which could impact some of the results. The three net migration projections explained in section 5.1 are based on three different predictions of Norwegian and international economic development. If domestic growth is larger than international growth, net migration is projected to increase (fall), which would lead to weaker (stronger) ageing (Cappelen, Skjerpen & Tønnesen, 2016). Thus, there is some degree of correlation between domestic productivity growth and net migration.

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[18] Four components where each component can assume three different values: $3^4 = 81$

Additionally, one would expect return on international investments to be correlated with international productivity growth, which in turn is correlated with domestic productivity growth due to technology sharing (Baumol, 1986). Thus, as The Government Pension Fund Global is invested globally in its entirety, one should expect some degree of correlation between the real return of the funded pension savings and productivity growth, which correlates with net migration. Consequently, if the real return is high (low), one would expect the net migration to be lower (higher) ceteris paribus.

These correlations between the relevant components are however not necessarily strong. The productivity growth component consists of domestic productivity growth, not its relative level. Moreover, the real rate of return on The Government Pension Fund Global explains the international economic development to some degree, not the international development relative to domestic. Thus, while the real rate of return on The Government Pension Fund Global and domestic productivity growth relative to international growth indirectly correlates with the population projections through their impact on net migration, this correlation does not necessarily exclude any possible combinations of components. It does however impacts their probabilities. Because of these correlations, the six cases consisting of strong (weak) ageing, low (high) real rate of return, and increasing (decreasing) productivity growth are considered as the least probable projections.

The aggregate projections are analyzed using the model developed in chapter 4, and the relative rate of return in all scenarios is compared. In this chapter, a selection of probable key combinations is analyzed in further detail. In addition, the two cases that theoretically should yield the largest differences in the rate of return in the systems is discussed, as both are potential worst-case scenarios depending on the financing form of the pension scheme. The key combination discussed in detail are the following: The expected alternative (EEEE), the eight alternatives where three of the components are given their expected values, and two non-expected alternatives with predicted correlation between the three first components (WLDE) and (SHIE).

A summary of the result of all 81 scenarios for years 2016 to 2100 is presented in appendix 2. Of the nine alternatives where at least three components follow their expected paths, seven of them deem a fully funded scheme preferable to a PAYGO-scheme in the long run. These alternatives are discussed in further detail in sections 6.1 and 6.2 below.

6.1 Expected alternative (EEEE)

The expected alternative consists of a demographic development following the main alternative, real return on savings equal to three percent, productivity growth following the expected path, and generalized effective retirement age equal to 67. Of all simulated 81 scenarios, this is the alternative one would expect to be closest to the actual economic development.

The relative rate of return (RRR) in the expected alternative is illustrated in figure 6.1. RRR starts out at 0.28 in 2016, falls to -0.19 in 2044, and then stabilizes at a level between -0.19 and -0.18 in all future periods leading up to 2100. The intersection with the x-axis is between 2028 and 2029, meaning that a pure PAYGO-scheme yields a higher rate of return from 2016 to 2028, while a fully funded scheme yields a higher from 2029 to 2100.

Following the defined-benefit approach of section 4.3, the model states that the tax-rate in a pay-as-you-go scheme must be around 22% larger than the tax rate in the funded alternative each period years 2044 through 2100 to yield equal benefits in this alternative.

In the early periods, pay-as-you-go yields a relatively high rate of return due to most baby boomer cohorts still being in working age, and partly due to the high productivity growth in the 1970s and early 1980s. The sharp decline from 2016 to around 2040 is mainly a consequence of the ageing population leading to an increasing burden of care as the baby boomer cohorts start entering retirement age. The projected growth rate of the burden of care for the elderly equals around 3.51% in 2017, and remains relatively large until the early 2040s. After 2040 however, average annual growth rate equals 0.58. This decreasing pace of the ageing process explains why the relative rate of rate return is projected to stabilize after year 2044.

The model does not consider time preference or impatience; two parameters that would impact the attractiveness of the two possible schemes. Thus, one cannot use these results in isolation to conclude that a funding of the Norwegian public pension scheme is preferable in the long run. They do however illustrate that there could be significant purely economic opportunity costs if the Norwegian National Insurance Scheme were to remain as a pay-as-you-go scheme.
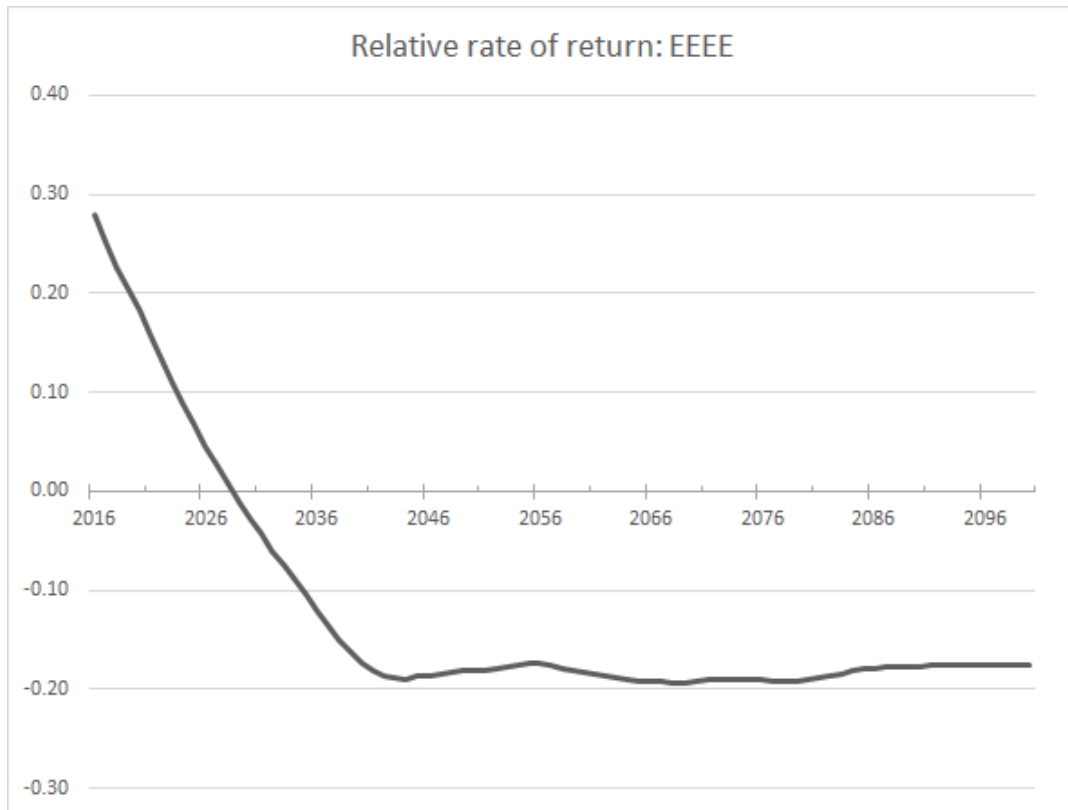
Figure 6.1: Estimated relative rate of return in the expected alternative.

6.2 Alternatives with a single non-expected component

This subsection presents a one-at-a-time sensitivity analysis of the expected alternative. The components are assigned different projected levels on at a time, keeping all other components at their expected level, and the impact on the relative rate of return is analyzed ceteris paribus. The projected levels of components are not necessarily unexpected, but they do not take their expected variables, and thus they are denote as "non-expected".

The results of the sensitivity analysis are illustrated in figures 6.2a-d.

6.2.1 Non-expected ageing

As one would expect, changes in the ageing components takes some time to impact the rate of return. There is next to no impact before year 2036. In the long run, stronger ageing leads to a decrease in the relative rate of return due to an increasing burden of care, while weaker ageing leads to an increase. Thus, in a scenario with stronger (weaker) ageing, a funded scheme (PAYGO-scheme) is relatively more attractive than in the expected alternative.

After year 2029 the relative return is negative in all three alternatives, implying that a funded system yields a larger rate of return. While weaker ageing reduces the size of the negative relative return, the impact is not large enough to make a PAYGO scheme yield higher return than a fully funded scheme in this scenario. Even in the alternative with weak ageing, the ageing is too strong for financing through pay-as-you-go to remain optimal.

Result 1: The stronger the ageing of the population, the less the relative rate of return.

6.2.2 Non-expected real return on savings

As the different alternatives for real return on savings assume immediate changes, the impact of changes in this component is immediate as well. A change in the real return on savings impacts the rate of return of a funded scheme, but does not impact the rate of return of a pure PAYGO-scheme. Thus, the geometrical shape of the relative rate of return remains unchanged, and the analysis is relatively simple.

An increase in the real return on pension savings leads to an increase in the rate of return of a funded scheme, while a decrease in the real return on pension savings leads to a decrease in the rate of return of a funded scheme. Thus, since these changes does not impact a pure PAYGO-scheme, the sign of the change in relative return is the opposite of the sign of the change in the rate of return on pension savings.

As figure 6.2b illustrates, changes in the real return on savings pushes the RRR-curve either north or south, and the shape of the curve remains unchanged. In the alternative with high real return on savings the relative rate of return is negative in all periods while in the alternative with low real return on savings the relative rate of return is positive in all periods. This implies that when real return on savings is low a PAYGO-scheme yields the highest rate of return in all periods, while when real return on savings is high a fully funded scheme yields the highest rate of return in all periods.

Result 2: The higher the real return on savings, the less the relative rate of return.

6.2.3 Non-expected productivity growth

Changes in productivity growth takes some time to impact the relative rate of return in the model, and there is next to no difference in the three alternatives before 2026. A change in the

productivity growth impacts the denominator of the relative rate of return equation, but does not impact the numerator.

An increasing productivity growth leads to an increasing real relative of return through a decreasing denominator, while a decreasing productivity growth leads to an decreasing rate of return of a funded scheme in the model. Thus, the sign of the impact on the relative return is equal the sign of the impact on the rate of return of a fully funded scheme.

The projected decreasing productivity growth path leads to a gradually decreasing relative rate of return with a negative RRR in all periods after 2027. The projected increasing productivity growth path leads to a cubic curve with a RRR that is negative in periods 2033 through 2041 and a positive in all other periods.

In the current Norwegian National Insurance pay-as-you-go scheme, benefits move only with inflation. Contribution are a function of current wages, while benefits are a function of the previous wage-level. Thus, productivity growth helps combat the adverse effects of ageing, and the higher the productivity growth, the more sustainable the current National Insurance Scheme is.

Result 3: The lower the productivity growth, the less the relative rate of return.

6.2.4 Non-expected effective retirement age

Of the four components, the net impact of a change in the effective retirement on the relative rate of return seems to be the most difficult to quantify. The immediate impact of an increase (decrease) in the effective retirement age is an increase (decrease) in the relative rate of return. However, the long-run impacts have the opposite sign: In the long-run, an increase (decrease) in the effective retirement age leads to a decrease (increase) in relative rate of return.

Increasing the retirement age reduces the burden of care for the elderly, and thus increases the rate of return of a PAYGO-scheme. Furthermore, increasing the retirement age increases the quantity of periods where savings accumulate returns in a funded scheme while reducing the expected remaining lifetime at retirement age, and thus increases the rate of return of a funded scheme as well. In the short run, the impact on the rate of return of a PAYGO-scheme is larger than the impact on the rate of return of a funded scheme. On the other hand, the impact on the rate of return of a funded scheme is larger than the impact on the rate of return of a PAYGO-scheme in the long run.

While the impact of changes in the effective retirement on the rate of return of a pension scheme is large in the model, the net impact on the relative rate of return between two different schemes is small. As figure 6.2d illustrates, all three alternatives follow a similar path where a PAYGO-scheme yields the larger rate of return prior to 2029, while a fully funded scheme yields the larger rate of return in 2029 and all following periods.

This inconclusive figure is a consequence of the ageing population and changing productivity growth. If one were to assume no population ageing and stable productivity growth equal, the isolated impact of a change in the effective retirement age becomes much more clear. This is illustrated in figures 6.2e-f which illustrates estimates KEKE, KEKI and KEKD, with K denoting a constant component, for two different levels of constant population. In these figures, productivity growth is assumed to be 1.70% every period, which is the average annual productivity growth in 1967 through 2100 in the expected alternative. Demographics is set equal to the demographics in 2016 for all periods in the 6.3a, and equal to the demographics in 2100 in the expected alternative for all periods in figure 6.3b. The curves are upward sloping due to the expected increase in life expectancy. Clearly, increasing retirement age increases the absolute value of the relative rate of return, and decreasing retirement age decreases the relative rate of return.

The intuition behind is as follows: The larger the ratio of working-age individuals in a population, and thus the larger the ratio of contributors relative to beneficiaries in the pension scheme, the more financing of the scheme matters.[19] Increasing effective retirement age increases the ratio of contributors per beneficiary, and consequently increases the importance of the choice of financing.[20]

Result 4: The higher the effective retirement age, the larger the absolute value of the relative rate of return.

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[19] Assuming a ratio ≥ 1

[20] To simplify the intuition further, one can imagine two different demographic scenarios: In the first scenario, the population consist of 1 worker and 1 pensioner at any time. In the second scenario on the other hand, the population consist of 10 workers and 1 pensioner at any time. The worker in the first scenario must either finance his own pension (funding) or the entire pension of the pensioner (pay-as-you-go). No matter what, he carries all of the financing burden. In the second scenario, the workers must either finance their own pension funds (funding) or on average finance one-tenth of the pension of the pensioner (pay-as-you-go). Thus, the financing burden is not given in the second scenario, and can be altered through the choice of financing.
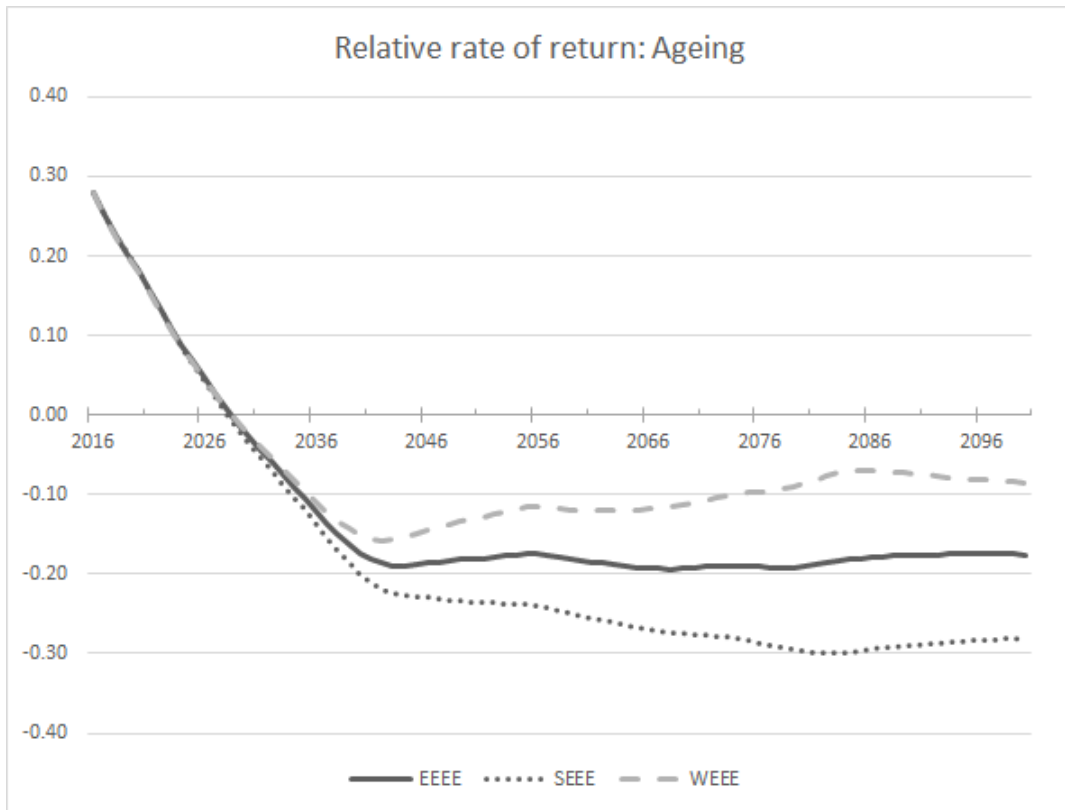
Figure 6.2a: Estimated relative rate of return, ageing scenarios.
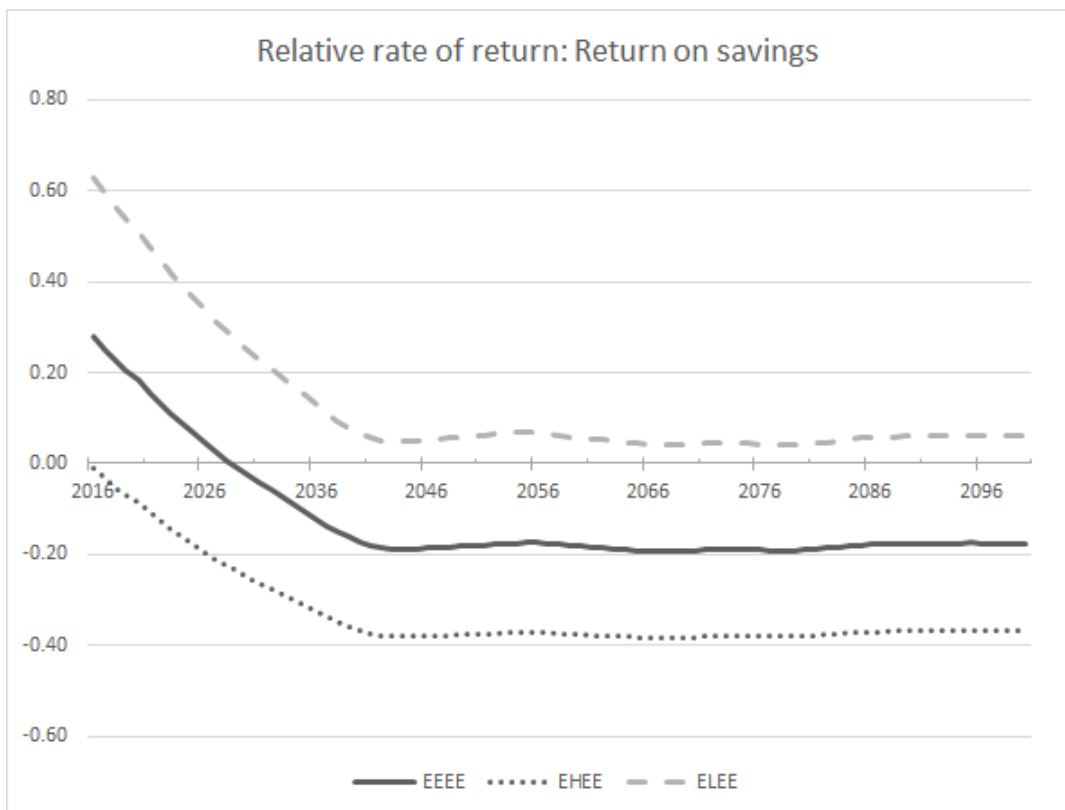
Figure 6.2b: Estimated relative rate of return, return on savings scenarios.

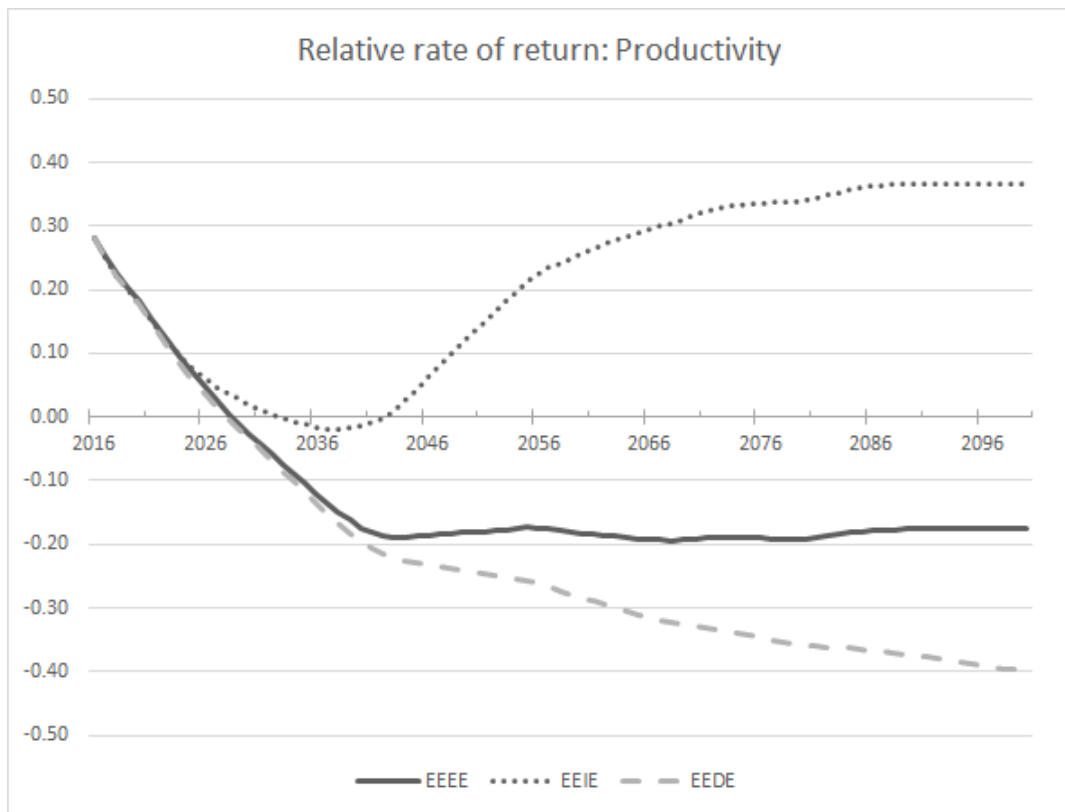*Figure 6.2c: Estimated relative rate of return, productivity scenarios.*
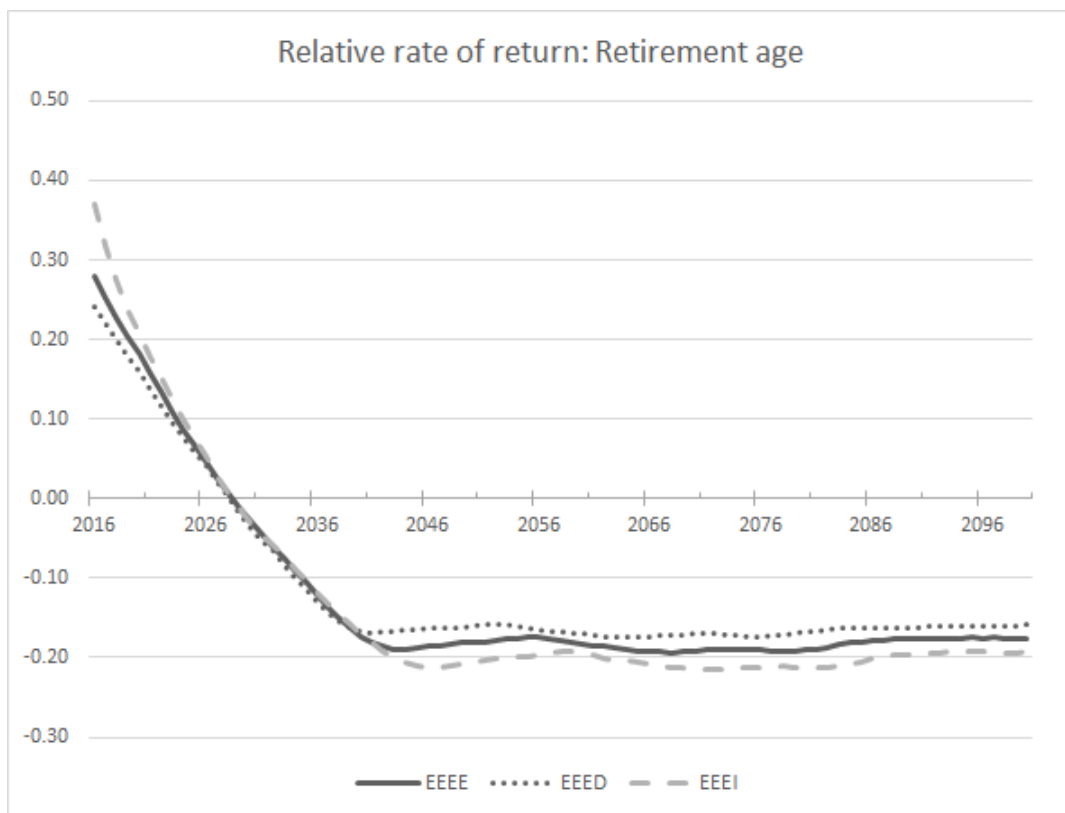
Figure 6.2d: Estimated relative rate of return, retirement age scenarios.
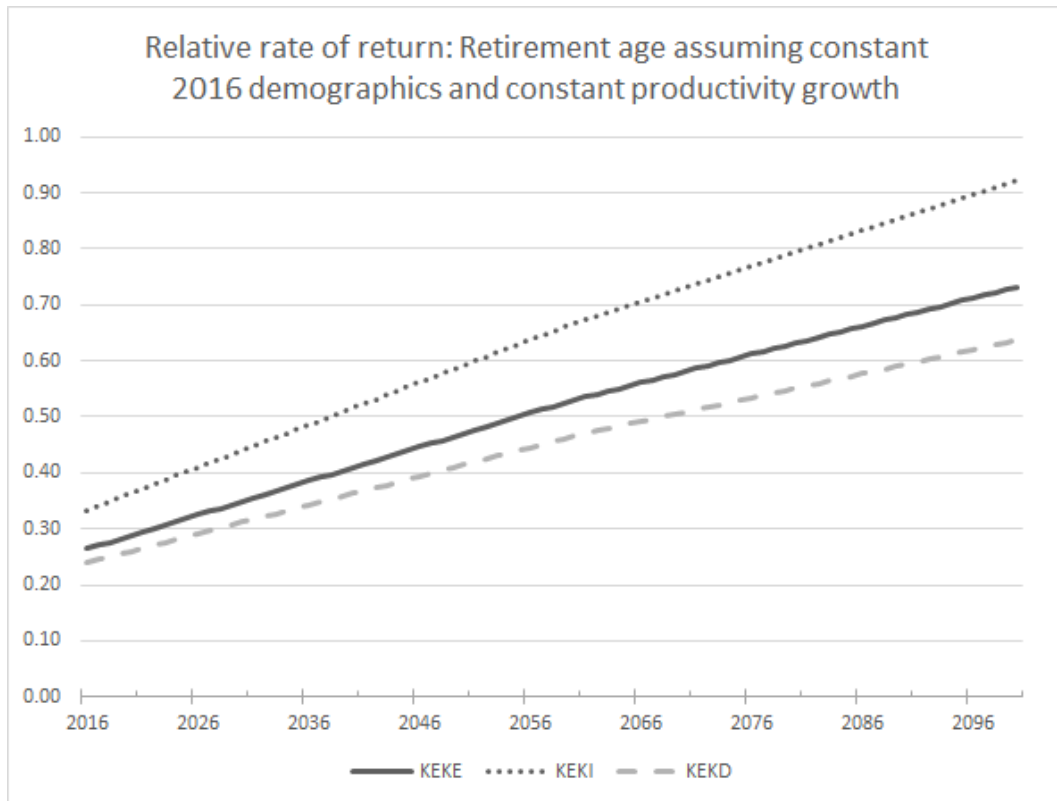
Figure 6.2e: Estimated relative rate of return, retirement age with constant 2016 demographics and constant productivity growth.
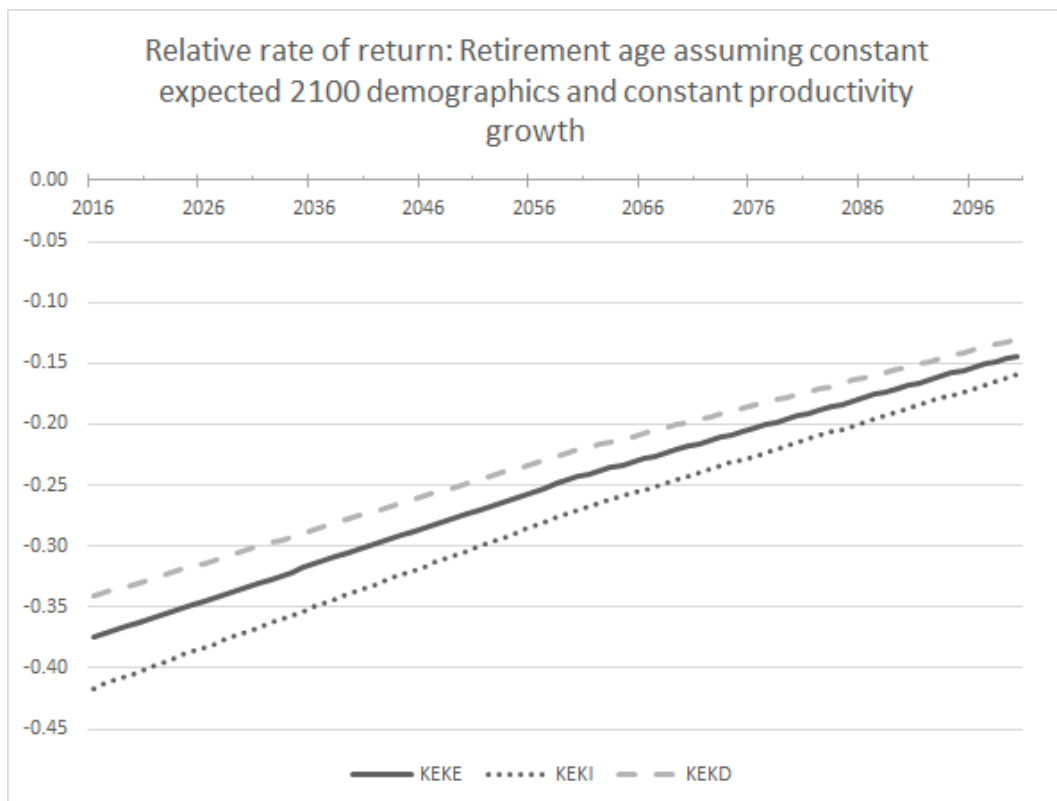
Figure 6.2f: Estimated relative rate of return, retirement age with constant expected 2100 demographics and constant productivity growth

6.3 Other alternatives with predicted correlation

This section further examines the two non-expected alternatives where ageing, real return on pension savings and productivity growth correlates as expected.

The development of the relative rate of return in these two alternatives is illustrated in figure 6.3, with the expected alternative included as a reference.

6.3.1 Correlation with high return on Government Pension Fund Global (SHIE)

If the real returns on the Government Pension Fund Global and the funded pension savings are larger than expected, one would expect that to be a consequence of increasing international productivity growth. Assuming some degree of technology sharing, the increasing international productivity growth is predicted to lead to an increase in domestic productivity growth as well. However, since not all technological progress is shared (Baumol, 1986), the international productivity growth is likely to exceed domestic productivity growth, which would lead to less domestic net migration and therefore stronger domestic ageing.

As show in section 6.2, stronger ageing and a higher return on pension savings reduces the relative rate of return in comparison to its level in the expected alternative, while increasing productivity growth leads to a gradual increase in the relative rate of return.

In this alternative, the model estimates that the immediate impact of the increased return on pension savings leads to a negative RRR. The net impact of higher return on pension savings and the gradual impact of strong ageing is decreased by the gradual impact of increasing productivity growth in the long-run, but the RRR remains negative all periods in the long run.

Thus, the rate of return of a fully funded scheme exceeds the rate of return of a pure PAYGO scheme in all periods in this alternative.

6.3.2 Correlation with low return on Government Pension Fund Global (WLDE)

If the real returns on the Government Pension Fund Global and the funded pension savings are lower than expected, one would expect that to be a consequence of decreasing international productivity growth. Assuming some degree of technology sharing, the decreasing international productivity growth is predicted to imply decreasing domestic productivity

growth. However, since not all technological progress is shared, the decline in international productivity growth is likely to be stronger than the decline in domestic productivity growth, which would lead to more domestic net migration and therefore weaker domestic ageing.

Weaker ageing and a lower return on pension savings increases the relative rate of return in comparison to the expected alternative, while decreasing productivity growth leads to a decrease in the relative rate of return.

The curve representing this alternative follows a similar development to the curve representing the expected alternative, but with a larger positive constant due to the immediate impact of the reduced real return on pension savings. Pay-as-you-go yields a higher rate of return than funding in all periods prior to 2062, while funding yields the highest rate of return in periods 2062 through 2100.
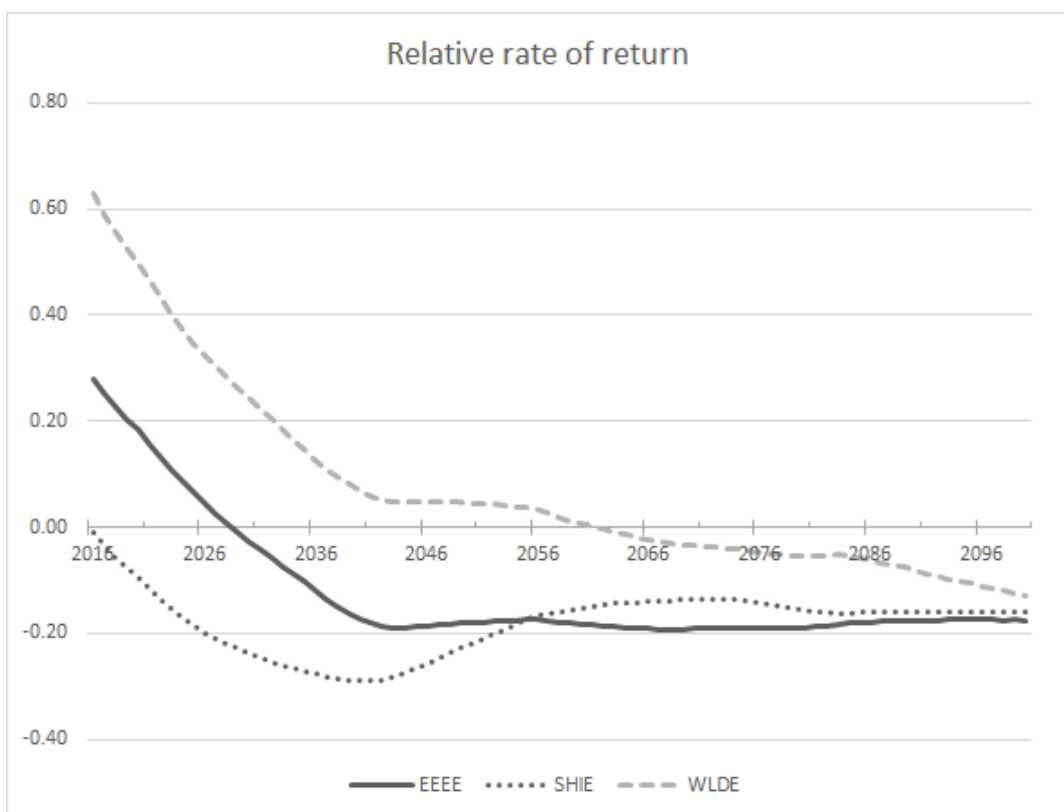
Figure 6.3: Estimated relative rate of return of alternatives with predicted correlation between components

6.4 Alternatives WLII and SHDD

In this section, the scenarios with the largest absolute relative rates of return are examined in further detail. These are potential best- and worst-case scenarios, depending on the choice of pension financing.

The model yields the largest relative rate of return when ageing is weak, real return on pension savings is low, productivity growth is increasing and retirement age is increased (WLII). In this alternative, PAYGO is preferable to funding in all periods, and increasingly preferable following the late 2030s. A decision to fund the public pension scheme followed by this development would lead to massive opportunity costs, and thus the probability of this alternative would need to be estimated before deciding to transition to funding. Similarly, not transitioning towards funding would lead to large opportunity costs in the alternative where the relative rate of return is as negative as possible. This is the case when ageing is strong, real return on pension savings is high, productivity growth is decreasing and retirement age is decreased (SHDD).

The correlation between ageing, real return on investments and productivity growth in these scenarios is not as one would expect, and none of four components follow their expected path. Thus, these scenarios seem to be two of the least probable. Their probabilities would however need to be evaluated further before one were to decide to fund the pension scheme, as they could lead to alternative costs.

The development of the relative rate of return in these alternatives is illustrated in figure 6.4 below, with the expected alternative included as a reference.
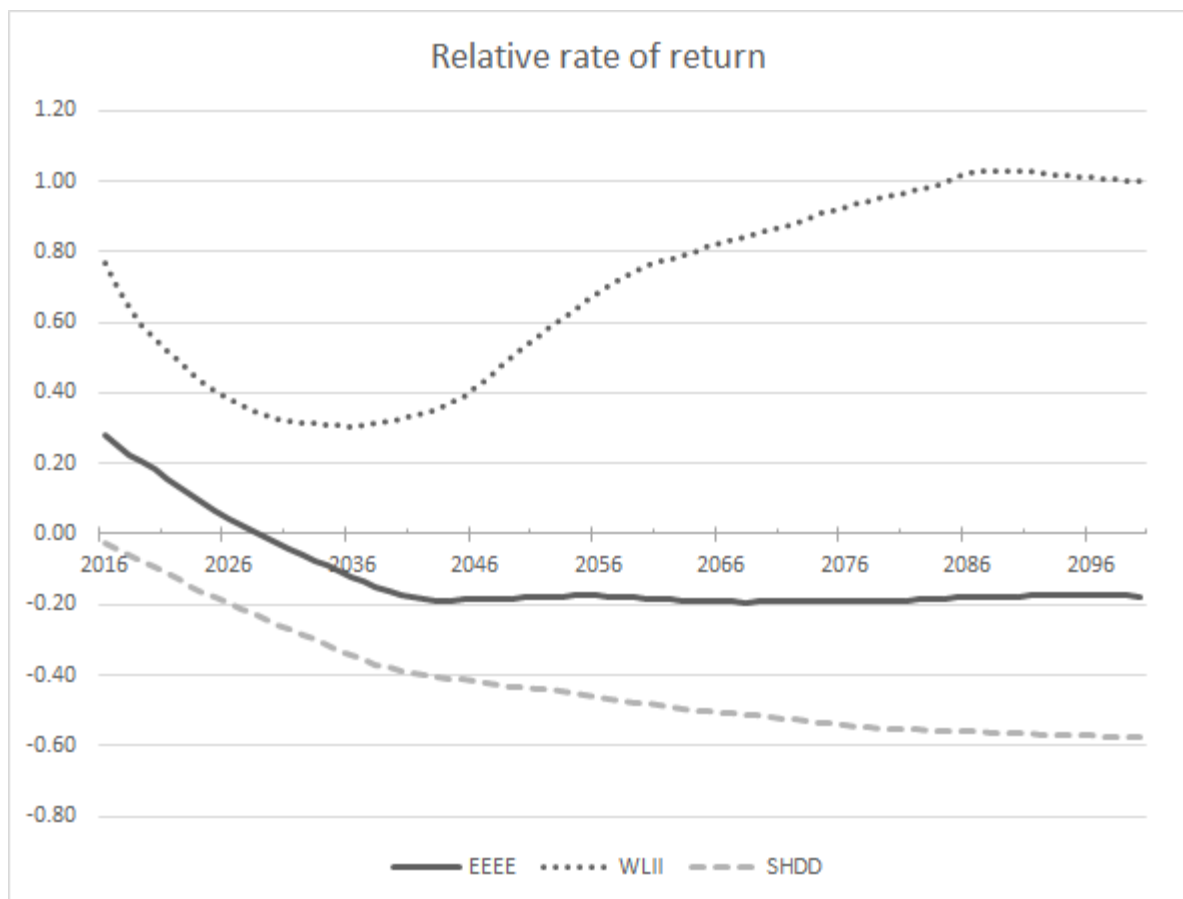
Figure 6.4: Estimated relative rate of return of alternatives WLII and SHDD compared to the expected alternative.

6.5 Assesment and evaluation

While additional pension reforms further discouraging early retirement would increase the sustainability of the current pension system, changes in effective retirement age does not necessarily improve the optimality of the current pay-as-you-go financing ceteris paribus, as illustrated in subsection 6.2.4 and figure 6.2d-f. Thus, actions mobilizing inactive older people should not be considered alternatives to an increased degree of funding, but rather sustainability-improving measures that could be implemented independent the decision regarding financing.

The analysis shows that a funding of the Norwegian National Insurance Scheme is optimal in 51 of 81 aggregate projections in the long run. Additionally, seven of the nine alternatives consisting of at least three components following their expected paths deems a fully funded scheme preferable to a PAYGO-scheme in the long run. These nine alternatives are the most probable scenarios, and should thus be given the most attention. Of these nine, a pure PAYGO-

scheme is only preferable in the alternative assuming a constant low real return of the Government Pension Global of 2 percent annually and the alternative assuming a long-term stable productivity growth above 4 percent annually. However, these two alternatives break the assumption of a dynamically efficient economy, as the real interest rate is less than the rate of growth of GDP in several periods in both alternatives (Diamond, 1965).

A comparison of the current pay-as-you-go scheme and a potential funded scheme cannot assume a blank slate. When evaluating the funded alternative, one must also consider the costs related to a potential transition. As explained in the section 3.4, transitioning from pay-as-you-go financing to funding results in some individuals being clear losers. A gradual transition would smooth the stream of compensatory payments these individuals needs to receive, but it would not eliminate this 'transition double burden' (World Bank, 2005). The larger the size of the pension scheme and the quicker the transition to funding is, the larger the transitional costs related to financing existing pay-as-you-go liabilities when transitioning to funding. Thus, a gradual funding should be preferred to an instantaneous transition.

Possible methods of introducing gradual funding include offering funded pensions only to workers younger than a specific cut-off age, or providing incentives to ensure that older workers remain in the old PAYGO-scheme. Alternatively, one could introduce the funded scheme through a small contribution to funded accounts at the time of introduction, and gradually increase the contributions. The latter approach share several similarities with the processes currently ongoing in Australia (Feldstein, 1997) and Hungary (Palacios & Rocha, 1998; Rocha & Vittas, 2002). In most cases, the size of the fund being small in early periods would result in large administering costs relative to accumulated returns. However, assuming that a potential Norwegian funded pension scheme will be invested as part of the Government Pension Fund Global, the new fund would not face this challenge.

Regarding the financing of a transition, there are two main options: Budgetary financing through either tax increases or reduced spending, or debt financing through issuing of government bonds. Debt financing transfer some of the transition costs onto future generations, who are expected to gain from the transition to funding, and the double burden current workers face is reduced. Further, debt financing allows tax rates to remain relatively unchanged. Financial markets might however not approve of the debt accumulation, and demand higher interest rates due to a perceived increase in risk in the public sector.

A potential gradual funding the public pension scheme would raise new challenges that the model does not consider. In cases where the rate of return of a fully funded scheme surpasses the rate of return of a PAYGO-scheme in the long-run, but not in the short-run, one cannot conclude that one form of financing is optimal as the model does not consider time-preference or impatience. Secondly, while transitioning from PAYGO to funding reduces the demographic risk of the pension scheme, it also increases the financial risk. 24 % of total pension income in Norway comes from funded pensions (OECD, 2014b), and the ratio of assets to GDP in funded private pensions is growing (OECD, 2016). Thus, an increased degree of funding could improve the overall risk diversification of the Norwegian National Insurance Scheme. However, while funding could improve the diversification of the pension system, it could also reduce the risk diversification of the economy as a whole due to the dependence on the real returns on the Government Pension Fund Global. A complete analysis of a potential transition from PAYGO to funding would need to take this reduced diversification into account when evaluating alternatives.

Lastly, a possible advantage of funding that the analysis does not consider is that premium payments are more likely to be regarded as own savings rather than tax-payments. Thus, funding could reduce the negative labor market distortions of taxation (Kuné 2001).

Overall, it seems like the demographic factors that made pay-as-you-go plans preferable to funding no longer holds (World Bank, 1994). In most realistic scenarios, funding appears to be the best instrument to securing pension incomes in the ageing Norwegian society.

7. Conclusions

The thesis develops an extended overlapping generations model to analyze the impact of population ageing on the financing of the Norwegian National Insurance Scheme. Additionally, different projections of domestic productivity growth, real rate of return on a possible funded alternative, and retirement age are taken into consideration. The relative rate of return of the pension scheme, defined as negative one plus the rate of return of the current pay-as-you-go scheme divided by the rate of return of a funded scheme in an identical framework with identical assumptions, is examined in 81 scenarios based on different projected levels of the four relevant components. A funding is found to increase the rate of return in the long run in 51 of these 81 scenarios. Furthermore, funding is found to yield a higher rate of return than the current pay-as-you-go-financing in 7 of the 9 most probable scenarios.

A gradual funding is discussed as a solution to the challenges related to transitioning from one form of financing to the other. As several other countries have starter a transition from pay-as-you-go to funding, Norway can learn the experiences of these countries. The transition could either be financed through budgetary financing or debt financing, and the decision should depend on the expected response of Norwegian financial markets. However, country-specific considerations for Norway are not discussed in detail, and would need to be further analyzed before a possible implementation.

In the model, the tax rate is assumed to remain constant in all periods, and wages are assumed to be equal for all working age individuals each period. Moreover, the real return on funded pension savings is assumed to be constant in all periods. While these three assumption simplify the model framework, they also weaken the explanatory power of the results. Endogenizing possible developments and differences in these variables would strengthen the model. Also, the relative rate of return calculated by the model does not consider time preferences and risk aversion.

Finally, as the model does not incorporate a normative utility- or welfare function, the results are purely economic. For this reason, one cannot conclude that a form of financing is socially optimal, just that it yields the highest rate of return or lowest defined benefit contribution rate.

Further research should seek to estimate the impact a funding of the National Insurance Scheme would have on the overall risk profile of the economy, and the impact on the utility level of different cohorts. Moreover, the viability of alternative reforms and their impact on the sustainability of the pension scheme should be analyzed further.

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Appendix 1: Generalized OLG-model with γ overlapping cohorts

This appendix presents a generalized version of the model developed in chapter 4. Rather than assuming 106 overlapping cohorts, retirement age equal to 67 and labor market entry age equal to 20, these values are denoted as γ, $\beta + 1$ and α respectively. Additionally, we substitute for the population-parameter N. A part from that, everything in this appendix is equal to the model framework presented in chapter 4.

During each 1-year period, the household sector consists of γ overlapping cohorts of ages between 0 and $\gamma - 1$, where $j \in \{0,1,2,\dots,\gamma - 1\}$ denotes the age of the cohort. Heterogeneity is assumed to be intercohort only, allowing us to use a representative household j for each period t. Individuals are assumed to earn real wages $w_{j,t}$ when in working age. Neither children nor retirees work. Wages at a given time t are assumed to be equal for all working age cohorts.

The size of the representative household is given by $N_{j,t}$, which represents the size of cohort j in period t. A new generation aged $j = 0$ is born each period, and all other current generations shift forward one life stage. The fertility rate represents the exogenous population growth rate of the newly introduced generation at period t, and is denoted as n_t. Each household at a given age has an exogenous probability $m_{j,t}$ of reaching the next life stage in the next period. Since the oldest generation deterministically dies out in the subsequent period, we have that $m_{\gamma-1,t} = 0$, and population in period t can be expressed as:

$$N_{j,t} = \begin{cases} (1 + n_t)N_{0,t-1} & if\ j = 0 \\ m_{j-1,t-1}N_{j-1,t-1} & if\ j \in \{1,2,\dots,\gamma-1\} \\ 0 & if\ j > \gamma - 1 \end{cases}$$

To find out whether PAYGO or funding is the optimal form of financing, we compare the rate of return of the schemes yield. The model does not incorporate a normative utility function or consider time preferences, and thus it is important to once again stress that, in this case, stating that one of the schemes is optimal means that the scheme is viewed as preferable over the other scheme with respect to the relevant choice of measurement.

In a pay-as-you-go scheme contributions made by workers at time t equals benefits received by retirees at time t. Working age consists of ages α through β. Thus, the relationship between contributions and benefits in a PAYGO-scheme can be written as:

$$\sum_{j=\alpha}^{\beta} N_{j,t}\tau_t w_{j,t} = \left(\sum_{j=\beta+1}^{\gamma-1} N_{j,t} \right) \left[\theta_t \left(\sum_{j=\alpha}^{\beta} w_{j,t-(\beta+1)+j} \right) \right]$$

Where τ represents share of income contributed to the pension scheme through taxes and θ represents the annual pension benefits received as a fraction of total lifetime working income. This θ is also known as the rate of return of the pension scheme.

In a funded scheme, each generation finance their own pension through saving in working age. Their savings are placed in a fund and is used to finance annual pension benefits after reaching pension age. The relationship between contributions and benefits in a funded scheme can therefore be written as:

$$\sum_{z=1}^{\beta-\alpha+1} (1+r)^z \tau_{t-z} w_{j,t-z} = \varphi_{t,\beta+1}\theta_t \left(\sum_{j=\alpha}^{\beta} w_{j,t-(\beta+1)+j} \right)$$

Where r represents real rate of return on savings and φ represent expected remaining lifetime at retirement age. The real rate of return on savings is assumed to be independent of the time since the difference between the rate of return on the Norwegian oil fund is predicted to remain relatively stable and the inflation target for the monetary policy in Norway. For a representative individual, savings needs to finance φ annual payments of pension benefits.

Rearranging the PAYGO-equation by dividing both sides of the equality by $\left(\sum_{j=67}^{105} N_{j,t} \right)$ allows us to directly compare the rate of return of the two systems as the right-hand side of the equality is equal in the two equations.

$$\frac{\sum_{j=\alpha}^{\beta} N_{j,t}\tau_t w_{j,t}}{\left(\sum_{j=\beta+1}^{\gamma-1} N_{j,t} \right)} >=< \frac{1}{\varphi_{t,\beta+1}} \sum_{z=1}^{\beta-\alpha+1} (1+r)^z \tau_{t-z} w_{j,t-z}$$

We simplify the equation further by assuming that the tax rate; the share of income contributed to the pensions scheme, does not depend on period t: $\tau_x = \tau_t$ for all x.

$$\frac{\sum_{j=\alpha}^{\beta} N_{j,t} w_{j,t}}{\sum_{j=\beta+1}^{\gamma-1} N_{j,t}} >=< \frac{1}{\varphi_{t,\beta+1}} \sum_{z=1}^{\beta-\alpha+1} (1+r)^z w_{j,t-z}$$

Productivity growth, λ, is used as a proxy for wage growth, and wages at a given time t are assumed to be equal for all working age cohorts: $w_{j,t} = w_t$ for $j \in \{\alpha, \alpha + 1, \ldots, \beta\}$. Thus, wages in period t can be expressed as:

$$w_{j,t} = \begin{cases} 0 & if\ j < \alpha \\ (1 + \lambda_t)w_{j,t-1} & if\ j \in \{\alpha, \alpha + 1, \ldots, \beta\} \\ 0 & if\ j > \beta \end{cases}$$

Thus:

$$\left(\frac{\sum_{j=\alpha}^{\beta} N_{j,t}}{\sum_{j=\beta+1}^{\gamma-1} N_{j,t}} \right) w_t >=< \frac{1}{\varphi_{t,\beta+1}} \sum_{z=1}^{\beta-\alpha+1} (1 + r)^z w_{t-z}$$

Substitution yields:

$$\left(\frac{\sum_{j=\alpha}^{\beta} N_{j,t}}{\sum_{j=\beta+1}^{\gamma-1} N_{j,t}} \right)$$

$$>=<$$

$$\frac{1}{\varphi_{t,\beta+1}} \left[\frac{(1 + r)}{(1 + \lambda_t)} + \frac{(1 + r)^2}{(1 + \lambda_t)(1 + \lambda_{t-1})} + \cdots + \frac{(1 + r)^{(\beta-\alpha+1)}}{\prod_{b=1}^{\beta-\alpha+1}(1 + \lambda_{t-b+1})} \right]$$

We simplify this equation by creating the following aggregate variable:

$$\overline{\Lambda}_x = \prod_{x=1}^{\beta-\alpha+1} (1 + \lambda_{t-x+1})$$

Thus, relationship between the rate of return of the two pension schemes is given by:

$$\left(\frac{\sum_{j=\alpha}^{\beta} N_{j,t}}{\sum_{j=\beta+1}^{\gamma-1} N_{j,t}} \right) >=< \frac{1}{\varphi_{t,\beta+1}} \sum_{s=1}^{\beta-\alpha+1} \left[\frac{(1 + r)^s}{\overline{\Lambda}_s} \right]$$

When the left-hand-side of this equation is larger than the right-hand-side, PAYGO yields a larger rate of return than funding in the model, while when the right-hand-side of the equation is larger than the left-hand-side, funding yields a larger rate of return than PAYGO in the model.

For $j \in \{1, 2, \dots, \gamma - 1\}$ we have that:

$$N_{j,t} = \left(\prod_{x=1}^{j} m_{j-x,t-x} \right) N_{0,t}$$

Substituting for $N_{0,t}$:

$$N_{j,t=} \begin{cases} \left(\left(\prod_{x=1}^{j} m_{j-x,t-x} \right) \left(\prod_{y=0}^{z-j-1} (1 + n_{t-j-y}) \right) N_{0,t-z} \; if \; z > j \\ \left(\prod_{x=1}^{j} m_{j-x,t-x} \right) N_{0,t-z} \; if \; z = j \end{cases}$$

This yields $N_{0,t-\gamma-1}$ as the earliest common multiplicator for all $N_{j,t}$. Similarly to how we simplified the equation after substituting for productivity growth, we once again simplify by creating new aggregate variables:

$$\bar{N}_j = \prod_{y=0}^{z-j-1} (1 + n_{t-j-y})$$

$$\bar{M}_j = \prod_{x=1}^{j} m_{j-x,t-x}$$

Thus, the left-hand side of the equation can be written as:

$$\frac{\left(N_{0,t-\gamma-1} \right) \sum_{j=\alpha}^{\beta} \left(\bar{N}_j \bar{M}_j \right)}{\left(N_{0,t-\gamma-1} \right) \sum_{j=\beta+1}^{\gamma-2} \left(\bar{N}_j \bar{M}_j \right) + \bar{M}_{\gamma-1}}$$

Common multiplicator in both the numerator and denominator allows us to simplify further:

$$\frac{\sum_{j=\alpha}^{\beta} \left(\bar{N}_j \bar{M}_j \right)}{\sum_{j=\beta+1}^{\gamma-2} \left(\bar{N}_j \bar{M}_j \right) + \bar{M}_{\gamma-1}}$$

Thus, the comparison of the rate of return of the two pension schemes can also be written as:

$$\frac{\sum_{j=\alpha}^{\beta} \left(\bar{N}_j \bar{M}_j \right)}{\sum_{j=\beta+1}^{\gamma-2} \left(\bar{N}_j \bar{M}_j \right) + \bar{M}_{\gamma-1}} >=< \frac{1}{\varphi_{t,\beta+1}} \sum_{s=1}^{\beta-\alpha+1} \left[\frac{(1+r)^s}{\bar{\Lambda}_s} \right]$$

In a steady state where the exogenous population growth rate equals zero, the exogenous probability of reaching age $j + 1$ in period $t + 1$ equals 1 for $j \in \{1,2, \dots, \gamma - 1\}$ and 0 for $j = \gamma$, and each cohort consists of equal amounts of individuals, the PAYGO side of the equation will always equal $\frac{\beta - \alpha + 1}{\gamma - \beta}$. This is the case because the working-age includes $\beta - \alpha + 1$ cohorts, while pensioners include $\gamma - \beta$ cohorts. In comparison, the funded side of the equation will always equal $\frac{\beta - \alpha + 1}{\varphi_{t,\beta+1}}$ in a steady state with zero rate of return on savings and zero productivity growth.

To compare the rate of return in a PAYGO scheme to the rate of return in a fully funded scheme, we estimate the relative rate of return (RRR). We define the relative rate of return as the rate of return of a PAYGO scheme divided by the rate of return of a funded scheme in an identical framework with identical assumptions, minus one.

$$
RRR = \frac{\left(\dfrac{\sum_{j=\alpha}^{\beta} N_{j,t}}{\sum_{j=\beta+1}^{\gamma-1} N_{j,t}} \right)}{\dfrac{1}{\varphi_{t,\beta+1}} \sum_{s=1}^{\beta-\alpha+1} \left[\dfrac{(1+r)^s}{\bar{\bar{\Lambda}}_s} \right]} - 1
$$

In other words, when the relative rate of return is positive (negative) the rate of return from a PAYGO scheme is larger (smaller) than in a funded scheme, and when the gap is zero the two schemes yield the same rate of return. A relative rate of return equal to 1 (-1) implies that the rate of return in a PAYGO scheme (funded scheme) is twice as large as the rate of return in a funded scheme (PAYGO scheme).

Appendix 2: Rate of return in 81 scenarios

	$\theta^{PAYGO} > \theta^{Funding}$	$\theta^{PAYGO} < \theta^{Funding}$
EEEE	2016-2028	2029+
SEEE	2016-2028	2029+
WEEE	2016-2028	2029+
EHEE	-	2016+
ELEE	2016+	-
EEIE	2016-2032 and 2042+	2033-2041
EEDE	2016-2027	2028+
EEEI	2016-2028	2029+
EEED	2016-2028	2029+
EEII	2016-2033 and 2043+	2034-2042
EEID	2016-2031 and 2041+	2032-2040
EEDI	2016-2028	2029+
EEDD	2016-2027	2028+
EHEI	2016-2017	2018+
ELEI	2016+	-
EHED	-	2016+
ELED	2016+	-
EHIE	2063+	2016-2062
EHDE	-	2016+
ELIE	2016+	-
ELDE	2016-2045	2046+
SEIE	2016-2029 and 2049+	2030-2048
SEDE	2016-2027	2028+
WEIE	2016-2035 and 2037+	2035-2036
WEDE	2016-2028	2029+
SEEI	2016-2028	2029+
SEED	2016-2027	2027+
WEEI	2016-2028	2029+
WEED	2016-2028	2029+
SHEE	-	2016+
SLEE	2016-2043	2044+
WHEE	-	2016+
WLEE	2016+	-
EHII	2016-2017 and 2067+	2018-2066
EHID	2060+	2016-2059
EHDI	2016-2017	2018+
EHDD	-	2016+
ELII	2016+	-
ELID	2016+	-
ELDI	2016-2043	2044+
ELDD	2016-2048	2049+
SEII	2016-2031 and 2048+	2032-2047
SEID	2016-2030 and 2044+	2031-2043
SEDI	2016-2027	2028+
SEDD	2016-2027	2028+
WEII	2016-2035 and 2037+	2036
WEID	2016-2032 and 2039+	2033-2038
WEDI	2016-2027	2028+

WEDD	2016-2027	2028+
SHEI	2016-2017	2018+
SHED	-	2016+
SLEI	2016-2043	2044+
SLED	2016-2047	2048+
WHEI	2016-2017	2018+
WHED	-	2016+
WLEI	2016+	-
WLED	2016+	-
SHIE	-	2016+
SHDE	-	2016+
SLIE	2016+	-
SLDE	2016-2040	2041+
WHIE	2055+	2016-2054
WHDE	-	2016+
WLIE	2016+	-
WLDE	2016-2061	2062+
SHII	2016-2017	2018+
SHID	-	2016+
SHDI	2016-2017	2018+
SHDD	-	2016+
SLII	2016+	-
SLID	2016+	-
SLDI	2016-2040	2041+
SLDD	2016-2039	2040+
WHII	2016-2017 and 2058+	2018-2057
WHID	2053+	2016-2052
WHDI	2016-2017	2018+
WHDD	-	2016+
WLII	2016+	-
WLID	2016+	-
WLDI	2016-2060	2061+
WLDD	2016-2062	2063+

Appendix 3: Projected annual productivity growth

	Declining	Expected	Increasing
2017	2.1	2.3	2.1
2018	2.2	2.3	2.2
2019	2.2	2.3	2.2
2020	2.1	2.3	2.2
2021	2.0	2.1	2.2
2022	2.0	2.1	2.3
2023	2.0	2.1	2.3
2024	2.0	2.1	2.4
2025	1.9	2.1	2.4
2026	1.9	1.8	2.5
2027	1.9	1.8	2.6
2028	1.9	1.8	2.7
2029	1.8	1.8	2.7
2030	1.8	1.8	2.7
2031	1.8	1.9	2.9
2032	1.7	1.9	3.0
2033	1.7	1.9	3.0
2034	1.7	1.9	3.3
2035	1.6	1.9	3.2
2036	1.6	1.8	3.4
2037	1.6	1.8	3.5
2038	1.6	1.8	3.8
2039	1.5	1.8	3.7
2040	1.5	1.8	3.8
2041	1.5	2.0	3.8
2042	1.5	2.0	3.8
2043	1.4	2.0	3.9
2044	1.4	2.0	4.0
2045	1.4	2.0	4.1
2046	1.4	2.0	4.1
2047	1.4	2.0	4.1
2048	1.3	2.0	4.1
2049	1.3	2.0	4.1
2050	1.3	2.0	4.1
2051	1.3	2.0	4.1
2052	1.3	2.0	4.1
2053	1.3	2.0	4.1
2054	1.3	2.0	4.1
2055	1.2	2.0	4.1
2056	1.2	2.0	4.1
2057	1.2	2.0	4.1
2058	1.2	2.0	4.1
2059	1.2	2.0	4.1

2060	1.2	2.0	4.1
2061	1.2	2.0	4.1
2062	1.1	2.0	4.1
2063	1.1	2.0	4.1
2064	1.1	2.0	4.1
2065	1.1	2.0	4.1
2066	1.1	2.0	4.1
2067	1.1	2.0	4.1
2068	1.1	2.0	4.1
2069	1.1	2.0	4.1
2070	1.1	2.0	4.1
2071	1.0	2.0	4.1
2072	1.0	2.0	4.1
2073	1.0	2.0	4.1
2074	1.0	2.0	4.1
2075	1.0	2.0	4.1
2076	1.0	2.0	4.1
2077	0.9	2.0	4.1
2078	0.9	2.0	4.1
2079	0.9	2.0	4.1
2080	0.9	2.0	4.1
2081	0.9	2.0	4.1
2082	0.8	2.0	4.1
2083	0.8	2.0	4.1
2084	0.8	2.0	4.1
2085	0.8	2.0	4.1
2086	0.8	2.0	4.1
2087	0.8	2.0	4.1
2088	0.7	2.0	4.1
2089	0.7	2.0	4.1
2090	0.7	2.0	4.1
2091	0.7	2.0	4.1
2092	0.7	2.0	4.1
2093	0.7	2.0	4.1
2094	0.7	2.0	4.1
2095	0.7	2.0	4.1
2096	0.6	2.0	4.1
2097	0.6	2.0	4.1
2098	0.6	2.0	4.1
2099	0.6	2.0	4.1
2100	0.6	2.0	4.1