NHH

A Race to the Bottom?

A Game-Theoretic Approach to Monetary Policy Interdependence

Andreas Hole

Johannes Karlsen

Supervisor: Professor Øystein Thøgersen

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NORWEGIAN SCHOOL OF ECONOMICS

Preface

This thesis is submitted as a part of finalizing our Master of science degree in Finance at the Norwegian School of Economics NHH. It covers the topic of monetary policy interdependencies and game theory, and was motivated by the current low interest rate regime of most central banks in the world.

We are very grateful to our supervisor Øystein Thøgersen who has provided excellent guidance and kept motivating us to pursue a topic we find very interesting.

Executive summary

The recent low interest regime of most central banks in the world raises several questions. First, it appears that the interest rates may have been too low during periods of recessions. Second, the key policy rates seem to stay low even after signals of improved economic conditions. Considering this, we question whether monetary policy truly considers the optimal development of the domestic economy? Or are interdependencies between different central banks limiting the freedom for domestic optimal monetary policies? This thesis aim to highlight how monetary policy interdependencies and inflation targeting can be better understood using game theoretic intuition.

Our point of departure is a static New Keynesian framework for optimal monetary policy. Then, we illustrate how an economy mainly considering its domestic economy will react to foreign interest rates. By extending the framework to allow for interdependencies, we illustrate how Nash-behavior drive two central banks to respond more aggressively to each other's interest rate. This results in a "race to the bottom" in the key policy rate levels when the economies are hit by an adverse economic shock. Further, we argue that certain mechanisms might contribute to a prisoner's dilemma hindering key policy rates to return to "normal" levels after an economic shock. We also derive a theoretical cooperative interest rate equilibrium between the two central banks. Here we illustrate how the central banks can facilitate each other's monetary policy and thus reduce the aggregate loss of both economies.

To shed empirical light on these mechanisms, we then compare the key policy rate of nine inflation targeting economies to the Taylor rate. Our analysis find some evidence of key policy rates being systematically below the Taylor rate during and after the Dotcom bubble recession. During and after the Financial crisis recession, there are stronger evidence for key policy rates deviating significantly from the Taylor rate. This, suggest that the interest rate levels were too low. At last we find some evidence of a negatively sloped trend in the key policy rates compared to the Taylor rate. This, we argue, might indicate that the monetary policy strategy has become more aggressive over time. If this is true, it calls for a revision of how inflation targeting is carried out, incorporating game theoretic intuition. Further, the discussion concerning global monetary policy coordination might also have to be revised.

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1. Introduction

Over the last century internationalization has increased (Rogoff 2003). Firms have grown beyond borders. Technology for communication and transport has seen large innovations. Outsourcing is widely practiced, and imports and exports make up significant shares of countries' GDP. Because of this continuously ongoing trend, the world economies are becoming increasingly integrated and thus also more interdependent. Interdependence between economies describes not only that two or more economies are mutually dependent, but also to which degree they are dependent on each other at the margin (Cooper, 1985). Thus, what happens in one economy, whether some form of shock, or a political decision that affect the economy, might have various levels of impact on one, or several other economies. Evidence of this is clear from the recent financial crisis, which started in the US housing and financial markets, before it quickly spread to the worlds' financial markets causing both short and long term negative effects on many of the worlds' economies (Claessens, 2010; Blinder, 2013). Due to the consequences such interdependencies might have for the respective economies, it is of great interest to understand how these interdependencies are linked and which implications they have.

In our thesis, we will explore an area that has received less attention so far, at least in terms of formal analysis. We zoom in on the game theoretic aspect of monetary policy interaction between different central banks, conducting versions of flexible inflation targeting. Central banks generally have mandates from their governments to secure achievement of certain goals aiding a healthy development for the respective economies. Since each mandate considers aspects of the domestic economy, it might incentivize monetary policy strategies that are beneficial for one economy at the expense of another (Drazen, 2000). Known as beggar-thy-neighbor policies, this can for example come through a competitive devaluation of a country's currency, spurring growth in one economy at the cost of another (Corsetti et.al, 2000). Since each central bank might have incentive to either conduct such a policy or react to it, one can associate it with game theory, Nash-behavior and ultimately the prisoner's dilemma. The essence of the prisoner's dilemma is that each player has an incentive to conduct a strategy that under certain condition is individually optimal but is not optimal collectively (Gibbons,

1992). If Nash-behavior is present among central banks, it is likely that the world economy is experiencing a lower level of welfare than what is optimally possible.

Formerly Canzoneri (1991) illustrated a game between central banks under simple conditions, using money supply as the only monetary policy instrument. By means of a simple game-theoretic framework he proved that the outcome might be sub-optimal unless the game is coordinated. Our aim is to build a similar framework using a simple version of a static New Keynesian model, taking into consideration that the interest rate is the main monetary policy instrument. We will use the US, the Eurozone and the Norwegian economy as examples in the illustrated games. The US and the Eurozone, are large open economies and must take each other's monetary policy actions into considerations. Norway on the other hand, is a small open economy and thus assumed to be a potential follower in this game setting.

Using this game theoretic approach, we hope to add new insight into the implications of competitive devaluations when interest rate setting is the monetary policy instrument. We share the view of Stiglitz (2010) who claims that *"competitive devaluations engineered through low interest rates has become the preferred form of beggar-thy-neighbor policies"*. If this claim is true, one might expect that there are reasonable gains to be achieved through such a competitive devaluation through a low interest rate. Or, that there are significant losses associated with not following suit when another central bank does so.

Based on game theoretic intuition, it is reasonable to expect that central banks will try to counter interest rate engineered competitive devaluations. If one central bank lowers the key policy rate, interdependent central banks should therefore collectively lower their interest rate to avoid being the loosing part. This is a mechanism that we will refer to as "a race to the bottom". The rationale for such a mechanism are the potential negative implications of a relatively strong currency (Taylor, 2013). An interesting aspect that we will elaborate on, is assessing the effect various economic shocks has on the interest rate equilibrium in the game between two or several central banks. It might be reasonable for one central bank to lower the interest rate due to some form of adverse shock on inflation, GDP or risk premia. However, if

this economy has a large import and or export shares of GDP, this will have implications for other economies. Hence one could expect a similar response where all central banks lowers their interest rate.

Several central banks underbidding each other's interest rates, could possibly lead to a sub-optimal low interest rate level for the combined economies. A sub optimal level implies that their individual domestic economies would be better off with a higher level of interest rate. A potential consequence of deviation form an optimal interest rate level is the formation of unwanted asset bubbles that would otherwise not build up under a higher interest rate level, see for example Ahrend (2008) or Kahn (2010). This is an argument against keeping interest rates too low for too long.

We argue that the FED's current interest rate policy is a potential example of interest rate interdependence. In the FOMC meeting in December 2016, the FED claimed that the economy was not doing well enough to support a quicker elevation of the interest rate level. However, with decent inflation and a job market hitting unemployment levels lower than pre-recession, one might wonder if the low European interest rate levels were playing a role. A possible strengthening of the dollar relative to the euro and other currencies, in the presence of an interest hike, can potentially have large negative implications for the US exporting industry. If this effect is considered large enough, it might be hard for the FED to further raise the interest rate significantly without causing unwanted negative impact on their own economy.

Starting out, in the next chapter, we will present a brief introduction to modern monetary policy and the three central banks used as examples in our theoretical framework. Then we will dive into developing our formal framework, which is the backbone of our thesis. We will first present a simple new Keynesian model for optimal interest rate setting under flexible inflation targeting. Then, we will develop the game theoretic framework, which we will use to derive each central banks' optimal open economy interest rate response function. Using this framework, we will analyze the implications economic shocks and disequilibria have for the interest rate level in a stylized two-economy setting, given non-cooperation. We will also, as

a comparison, introduce a model that allows for cooperation in the monetary policy. By comparing these two outcomes, we will illustrate that there is a possible interest rate equilibrium that minimize the total loss for all involved economies. Following the theoretical model, we present the data and method used to conduct an empirical analysis. The goal of our empirical analysis is to shed light on potential Nash-behavior among inflation targeting central banks, emphasizing sub-optimal key policy rates during and after recessions. To indicate that key policy rates might be sub-optimal, we will use different versions of the Taylor rule as a proxy for an optimal key policy rate.

2. Theoretical framework

In this chapter, we will build a simple theoretical framework for our theoretical analysis. This will later work as a guidance for empirical analysis. As this thesis considers the topic of monetary policy interdependence, we have included a brief introduction to modern monetary policy. Then we follow up with a short overview of the monetary policy in three economies referred to in our theoretical framework: Norway, EU and the USA. As a point of departure, we will present a static New Keynesian model for interest rate setting under flexible inflation targeting. This model will then be expanded to also include macroeconomic interdependencies. This allows us to emphasize competitive devaluations under flexible inflation targeting regimes. Based on this we will assess the link between monetary policy interdependence and game theory.

Our formal analysis will start out with the derivation of optimal interest rate setting in a small open economy. Then, assuming a world economy consisting of two large open economies, we will allow for interdepencies between these two central banks. As both central banks will perform their individually optimal monetary policy, considering the other central bank, we will allow for a possible Nash-equilibrium to exist. Using game theoretic insights, we will discuss each central bank's incentives and thus also their ideal strategies under stylized conditions. The interest rate setting will be discussed under the assumption of equal monetary policy response functions and equal economies with respect to size, sectors and trade. We will also assume that the central banks have perfect information about each other.

Further, we will extend our theoretical discussion by an evaluation of the effects of differences in the two interdependent economies has for interest rates, focusing on synchronized and non-synchronized shocks. These results will also be compared to a model where the foreign interest rate is exogenous, assuming that one central bank does not strategically consider its effect on the other central bank's interest rate. We will also assess how this central bank game can be coordinated to obtain a possible optimal interest rate setting for both economies that minimizes both economies' aggregate loss. Challenges for policy coordination will then be addressed, before we introduce a quick discussion about potential prisoner's dilemma situations between central banks.

2.1 Introduction to monetary policy

Over the past few decades, price stability is increasingly viewed as the most important goal of monetary policy. To achieve low and stable inflation central banks adopt nominal anchors such as an inflation target, or intermediate targets like the money supply or the nominal exchange rate (Mishkin, 2013; Martínez, 2008). The monetary policy makers can influence the money supply and the interest rate through open market operations, standing facilities and reserve requirements (Mishkin, 2013). The money supply and the interest rates impacts economic activity through several monetary transmission mechanisms (Mishkin, 2013), and is thus important for the economic development and growth of an economy. Each country's monetary authority, further assumed to be the central bank, has typically a mandate to conduct a monetary policy that ensures price stability and contribute to an efficient utilization of available capital and labour resources.

2.2 Overview of the monetary policy in Norway, the EU and the US

In the 1970s, monetary targeting was adopted by several countries, using the money supply as a nominal anchor (Mishkin, 2013). It was assumed that increasing the amount of money would increase inflation directly, which builds on the long-term relationship between money growth and inflation (Hall, 2012). After realizing the possible lack of a stable relationship between monetary aggregates and inflation, several countries have in more recent decades turned to inflation targeting to achieve the goal of price stability (Mishkin, 2013). In the next three sections, we will present a short summary of the monetary policy goals adopted by the central bank of Norway, the European Central Bank and the Federal Reserve System. Then, in our empirical analysis we will also include 6 other central banks performing various versions of inflation targeting.

The monetary policy objective of the Central Bank of Norway (Norges Bank) is keeping inflation low and stable, and close to 2.5 % over time (Norges Bank, 2016). The inflation targeting strategy was adopted in 2001 (Gjedrem, 2008). The objective is flexible inflation targeting in the sense that both variations in inflation as well as in production and employment,

are accounted for. Due to time lags in the effects of monetary policy, the interest rate will be set to obtain their goal over the medium term (Norges Bank, 2016).

The European Central Bank (ECB) has since 1999 been responsible for conducting the monetary policy for the countries that have adopted the euro (Eurozone), which in 2017 consist of 19 countries[1]. ECB´s main objective is to maintain price stability, defined in ECB (2011) as *"inflation rates below, but close to, 2% over the medium term"*. To achieve this goal, the ECB operates with a two-pillar approach. The first pillar called "economic analysis" is essentially similar to the approach described over for Norway, which considers aspects of the real economy and long term price stability (ECB, 2011; Hall, 2012). Where ECB set itself apart from most major central banks is through the second pillar which is called "monetary analysis", weighting monetary targeting in the medium and long term (Hall, 2012). The second pillar is most used as a cross check for the shorter term "economic analysis" (ECB, 2011). There has in recent time been a discussion of whether the ECB has put considerable more weight on the first pillar, as the money demand appears to be rather unstable (Hall, 2012).

The Federal Reserve (FED) has a mandate to promote maximum employment, stable prices, and moderate long-term interest rates. To reach this goal, the FED has recently set a long run goal of 2 % inflation, while also focusing on the unemployment rate compared to the natural rate of unemployment (FOMC, 2013). The recent adoption of a specific inflation target by the FED suggests that they have adopted flexible inflation targeting. However, we will argue that this for all practical matters has been the case for a longer period despite the absence of a specific inflation target (Bernanke, 2004).

The monetary policy of the FED, the ECB and Norges Bank thus all share the goal of flexible inflation targeting at various levels. As further discussed in the following subchapters, inflation is influenced by changes in the real exchange rate and thus also by changes in the interest rate levels of its respective interdependent economies. This we will argue, might often

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[1] https://www.ecb.europa.eu/ecb/orga/escb/html/index.en.html

lead to high levels of correlation between the central banks' interest rates, as central banks will be sensitive to the effects of interest rate differences (Taylor, 2013). To further analyse this relationship under a flexible inflation targeting regime, we will use a New Keynesian model for interest rate setting as a starting point for formalizing the transmission mechanism. By using a model for an open economy, with floating exchange rates, we allow for an assessment of the exchange rate channel's role in the interaction between central banks.

2.3 A New-Keynesian framework

In this section, we will present a simple, static New-Keynesian framework for a small open economy. Our model is based on Røisland and Sveen's (2005) model for an open economy, which again is based on basic underlying theory developed by Lars Svensson and others[2]. We will first start with a short explanation of the basic model, which captures the case of a small open economy. Then we will derive the optimal interest rate response function for an open economy. Further, we will extend this function to allow for a possible Nash-equilibrium in a two-economy model. Based on the open economy model, we will also derive a theoretical cooperative response function for a two-economy framework.

The model is a static model, based on the underlying idea that the economy moves towards its equilibrium in the long run, but with fluctuations in the short run, which allows monetary policy to matter. Because of various economic shocks, it is also most likely that an equilibrium level will not be obtained for long periods of time. Further, the model accounts for the medium to long run result after monetary policy actions has been conducted (Røisland & Sveen, 2005). It is therefore given that the model does not mirror the dynamics of the economy, nor the time lags of monetary policy's effect on inflation, which can be long and variable (Svensson, 1997). It is possible to derive a dynamic, micro founded version of our model, which will be large and complex. However, to derive intuitive insight of the game theoretic aspect of monetary policy, we will argue that this simple model will be beneficial.

Our aim, using these models, is to assess theoretical interest rate equilibriums that occurs between interdependent open economies, and what factors that affect them in the medium to long run. To incorporate interdependencies into our models, we use intuition from various sources like Canzoneri (1991), Cooper (1985) and Drazen (2000). Further, by means of game theoretic insight we illustrate how these interdependent central banks will strategically relate to each other using the interest rate as a policy instrument. Hereunder, we will emphasize how

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[2] See Svensson (1997), Clarida (1999) or Carlin (2005)

the exchange rate channel potentially incentivizes central banks to either under or overbid other central banks' interest rates.

The following simple static New-Keynesian model for a small open economy will be the foundation for the extended models in the following subchapters. We therefore include a short but thorough explanation of the variables and formal relationships that the model describes. The basic model in the next section does not consider interdependencies, which is modelled by keeping the foreign interest rate exogenous. In the following subchapters, we will introduce interdependencies by formally allowing all central bank rates to be endogenous in the models.

2.3.1 A New-Keynesian model for a small open economy

Our formal point of departure is a central bank with the following monetary loss function:

(2.1) $\mathcal{L}_t = \frac{1}{2}((\pi - \pi^*)^2 + \lambda(y_t - y^*)^2)$

Here π is the inflation, y the natural log of output, and π^* is the economy's inflation target, and y^* the natural log of potential output. The magnitude of λ determines the degree in which the central bank emphasizes the real economy, and thus "the degree" of flexible inflation targeting. The square of each gap reflects that the central bank values stability in the economy and thus any deviation, negative or positive, from the target is undesired. The squared loss also illustrates that larger deviations are exponentially worse than small deviations. Figure 2.1 illustrates this by showing the central banks indifference curves, capturing the loss function given a λ of 1. The central bank thus has diminishing utility the further the levels of inflation and output are from their equilibrium levels.

Figure 2.1: Central bank loss function
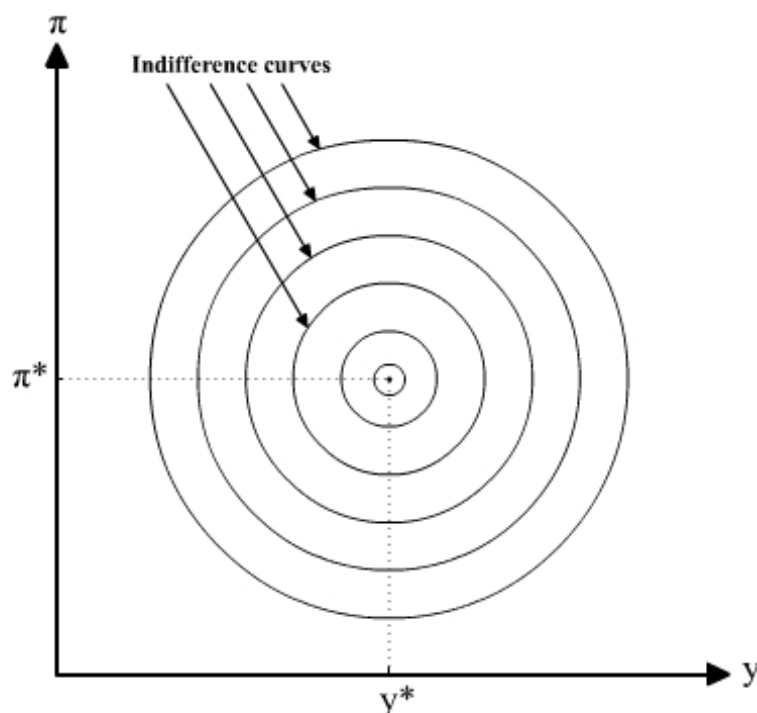
The basic model consists of three equations: one equation for the aggregate demand, one equation for the supply side of the economy and one equation for the currency channel.

2.3.2 The aggregate demand

The aggregate demand equation for the economy is an IS curve expanded to account for exchange rate effects on the real economy:

$$(2.2) \quad y = y^* - \alpha_1(i - \pi^e - r^*) + \alpha_2(e - e^*) + v, \quad e = s + p^f - p$$

Here, i is the nominal interest rate, π^e is the expected inflation in the economy. r^* represents the long-term equilibrium natural real interest rate (NRIR) of the economy. This is the level that the real interest rate tends to move towards over time. Hence, $(i - \pi^e - r^*)$ represent the deviation of the short term real interest rate relative to the NRIR. This assumes that the central bank through their key policy interest rate (i) can affect the real interest rate $(i - \pi^e)$ in the short run, indirectly assuming some degree of price stickiness. The coefficient a_1 thereby represent the short run effect of interest rate setting on the real economy. An increase in the

nominal interest rate, which causes the real interest rate to rise above the NRIR in the short to medium term, is thus assumed to have a negative effect on real output.

Another factor that can push output away from its equilibrium level, is the international competitiveness of the economy. This is captured by $(e - e^*)$ which is the difference between the natural log of the real exchange rate (e) and the equilibrium real exchange rate (e^*). e is a function of the nominal exchange rate in natural logs (s) and the price level difference relative to the foreign country $(p^f - p)$. A positive $(e - e^*)$ implies a depreciation of the currency relative to equilibrium levels, which will boost exports and increase output. v captures potential demand-side shocks that might push output away from its equilibrium level.

2.3.3 The supply side

The supply side of the economy is represented by an expanded Phillips curve, capturing the effect of the real exchange rate on the inflation:

(2.3) $$\pi = \pi^e + \gamma(y - y^*) + \beta(e - e^*) + u$$

Here π is inflation, π^e is expected inflation and u_i captures various supply side shocks in the economy. The coefficient γ accounts for the output gap's effect on inflation, while β captures the real exchange rate effect on inflation. A depreciated real exchange rate, represented by an increase in $(e - e^*)$ is related to imported inflation through more expensive imported goods and input factors. As we observe from graph 2.1, gross import as a share of GDP varies between countries, implying that the magnitude of β varies significantly between economies. In addition, (2.2) and (2.3) imply a more indirect effect of e on π because exchange rate depreciation boosts export sector activity and capacity utilization.

Graph 2.1: Imports of goods and services (% of GDP)

Legend: Australia, Canada, Euro area, Japan, Norway, New Zealand, Sweden, United Kingdom, United States

Source: World Bank national accounts data, and OECD National Accounts data files.
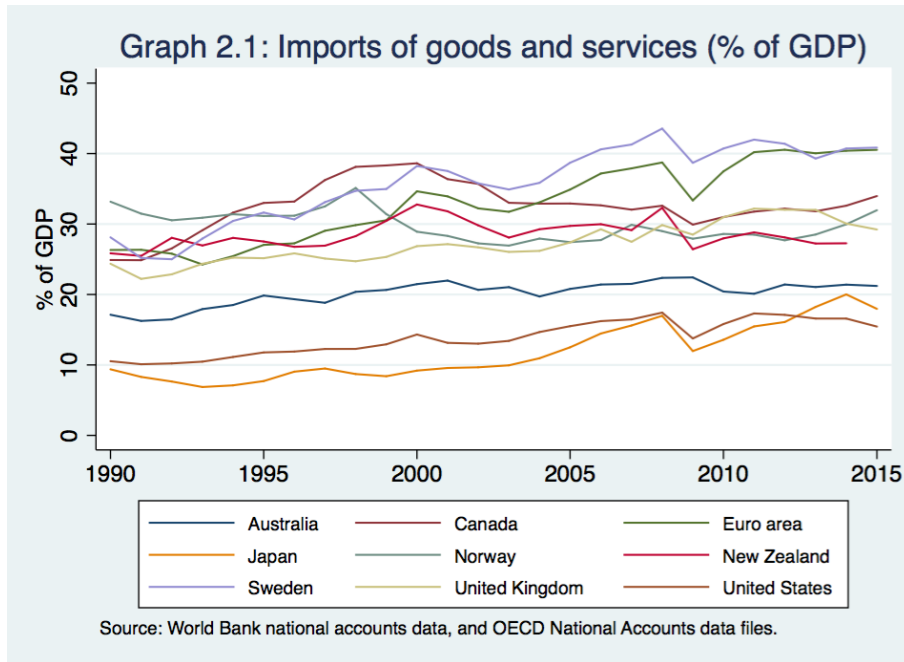
2.3.4 The currency market

Equation (2.4) expresses (s) as a function of uncovered interest rate parity $\left(s^e - (i - i^f)\right)$ and z, which is an exchange rate shock. z covers any deviations from UIP, which might be results of breaks with UIP, speculative attacks on the currency or some sort of risk premium (RP).

(2.4)
$$s = s^e - \left(i - i^f\right) + z$$

We rewrite (2.4) in terms of e rather than s, which is the specification we will use later:

(2.5)
$$e = e^e - [(i - \pi^e) - (i^f - \pi^{e,f})] + z$$

Here the expected inflation abroad can be expressed as: $\pi^{e,f} = p^{e,f} - p^f = \ln(P^{e,f}) - \ln(P^f)$.

2.3.5 First order condition (F.O.C)

The central bank determines the optimal i by minimizing (2.1) subject to (2.2), (2.3) and (2.5), as shown in appendix chapter 8.1.1. This leads to the following first-order condition:

(2.6)
$$(\pi - \pi^*) = -\frac{\lambda * (\alpha_1 + \alpha_2)}{[\gamma(\alpha_1 + \alpha_2) + \beta]} (y - y^*)$$

We observe that the loss function is minimized when both the inflation gap $(\pi - \pi^*)$ and the output gap $(y - y^*)$ equal zero, or alternatively when the gaps have different signs (Røisland & Sveen, 2005). When the gaps have equal signs, a more aggressive interest rate response is warranted because this will push both gaps towards zero, reducing the loss of the central bank.

2.3.6 Reaction function for a small open economy

From the F.O.C the central banks reaction function can be expressed by inserting (2.2), (2.3) and (2.5) in (2.6), solving the expression for i. The steps are shown in detail in the second part of appendix chapter 8.1.1. We obtain the following optimal interest rate response function:

$$(2.7) \quad i^{opt} = C\left((\pi^e - \pi^*) + A\alpha_1(\pi^e + r^*) + u + Av + B\left(i^f + (e^e - e^*) + (\pi^e - \pi^{e,f}) + z\right) \right)$$

$$where\ A = \left(\frac{\lambda*(\alpha_1+\alpha_2)}{[\gamma(\alpha_1+\alpha_2)+\beta]} + \gamma\right), B = (A\alpha_2 + \beta)\ and\ C = \frac{1}{A(\alpha_1+\alpha_2)+\beta}\ ,\ A, B, C > 0$$

To provide interpretations, we will in the following section elaborate on each part of the central banks reaction function. This will be helpful as the formal framework only grows more complex as we open-up for various forms of interdependencies in the following sub-chapters. It is worth mentioning that it follows from the economic theory, which the model is founded upon, that each of the coefficients are positive. Therefore, the aggregate coefficients A, B and C are also positive. An important property of this function, that will be elaborated on later, can be seen from the product of the coefficients $B * C$, which is the central banks reaction to the exchange rate channel. As all coefficients are positive, $BC = \left(\frac{A\alpha_2+\beta}{A(\alpha_1+\alpha_2)+\beta}\right) < 1$. This means that the central bank does not react by a one to one response to the foreign interest rate.

The first part of the reaction function $(\pi^e - \pi^*)$, is the difference between expected and target inflation, which can be viewed as an "inflation trust shock". Inflationary expectations are assumed to influence the economy's inflation. To minimize its losses by reducing inflation deviation, we see from (2.3) that the central bank will increase (reduce) the interest rate if expected inflation is higher (lower) than the target. A quick assessment of (2.7) shows that inflation expectation in the model is treated equally by the central bank as an inflation shock

given by the variable u. One can interpret this as the importance of the central banks' inflationary guidance for the economy.

Second, $(\pi^e + r^*)$ is the sum of expected inflation and the equilibrium real interest rate. This can be interpreted as the long run equilibrium nominal interest rate[3]. In the model, an increase in the long run equilibrium nominal interest rate $(\pi^e + r^*)$, should be responded to by an increase in the nominal interest rate set by the central bank, all else held constant.

Third, $\left(i^f + (e^e - e^*) + \left(\pi^e - \pi_f{}^e\right) + z\right)$ represents the central banks response to the factors that influences $(e - e^*)$. This equation contains several important relationships for an open economy, that we will consider separately. We highlight the importance of understanding these relationships, as the rest of this thesis mainly will consider the implications the exchange rate channel and competitive devaluations have for monetary policy. The exchange rate channel introduces several challenges to the central banks' monetary policy. Particularly, the fact that it must consider variables that are no longer only influenced by the home economy or its own interest rate, but also by foreign economic development and foreign monetary policy.

i^f is the foreign interest rate. A reduction in the foreign interest rate level, all else held constant, will according to (2.5) lead to a depreciation of the foreign currency. This reduces imported inflationary pressure domestically, as imported goods and input factors from the foreign economy becomes relatively cheaper (2.3). Further, this also contributes to decreasing the domestic export sector competitiveness, consequently leaving the foreign export sector more competitive. Thus, a relatively lower i^f thereby contributes to a reduction in aggregate demand (2.2), which will contribute towards a negative output gap in the domestic economy.

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[3] $(\pi^e + r^*) \equiv i^*$

To counter the effects from a lower i^f, and minimize its loss, the domestic central bank must reduce its own interest rate to avoid negative inflation and output pressure. To mathematically assess this response, one can see that $\frac{di^{opt}}{di_f} = B_i C_i < 1$. Thus, the central bank will have a response that is less than one for one with respect to the foreign interest rate, a natural result as the central bank must consider a trade-off with respect to its own economy when setting the interest rate.

$(e^e - e^*)$ is the difference between the expected and the equilibrium real exchange rate. In (2.3) through (2.5), a higher e^e relates an expected increase in imported prices, which contributes to increased inflation, all else held constant. Simultaneously, a higher e^e is assumed to spur activity, as the domestic export sector is expected to become increasingly competitive relatively to foreign export sectors (2.2). A negative development of the expected real exchange rate is naturally assumed to have the opposite effect. Thus, the central bank will respond by increasing (lowering) its interest rate if e^e increases (decreases) relative to the equilibrium rate.

$(\pi^e - \pi_f^e)$ is the difference in inflation in the domestic and foreign economy. An expected increase in domestic inflation compared to foreign expected inflation, will cause a relative reduction in the domestic real interest rate compared to the foreign, all else held constant. This will then cause the domestic currency to depreciate, which in the model causes the variable e to increase. Further, this causes an increase in demand and imported inflation. Thus, if domestic expected inflation increases (decreases) relative to the foreign, the central bank must increase (decrease) the interest rate to reduce pressure on inflation.

In the next section, we will introduce numerical applications of our framework in a stylized two-economy model. We will start out by graphically illustrating how one central bank adapts to an exogenously given i^f.

2.4 Numerical applications of the framework in a stylized two-country economy

So far in our model, i^f is given exogenously. Thus, the central bank's optimal monetary policy for a small open economy must adapt to i^f, but does not consider its influence on i^f. We will also argue that keeping i^f exogenous in our framework illustrates that the central bank will mainly consider a monetary policy directed towards the domestic economy whether the economy is large or small. We will come back to this last point in the game theory section.

By first illustrating a one-way dependence, where i is dependent on an exogenous i^f, we lay the foundation to extend our analysis to a two-way interdependence in the next subchapter. To open the model for analysing interdependencies, we need to allow for multiple economies. Therefore, we start out by generalizing equation (2.7), introducing economy specific denotations, which gives us:

(2.8) $\quad i_h^{Opt} = C_h \left((\pi_h^e - \pi_h^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + u_h + A_h v_h + B_h \big(i_f + (e_h^e - e_h^*) + (\pi_h^e -$

$\pi_f^e) + z_h \big) \right)$

$where\ h = home\ economy, f = foreign\ economy$

$A_h = \left(\frac{\lambda_h * (\alpha_{1,h} + \alpha_{2,h}) + \gamma_h [\gamma_h (\alpha_{1,h} + \alpha_{2,h}) + \beta_h]}{[\gamma_h (\alpha_{1,h} + \alpha_{2,h}) + \beta_h]} \right)$

$B_h = \left(A_h \alpha_{2,h} + \beta_h \right)$

$C_h = \left(\frac{1}{A_h (\alpha_{1,h} + \alpha_{2,h}) + \beta_h} \right)$

$A, B, C > 0$

We have in our generalized framework used denotations for the home (h) and the foreign (f) economy to capture that the reaction function is for the home economy which also considers the foreign economy. However, as we move further in studying interdepencies between two economies, both economies will naturally consider their own economy as the home economy. Therefore, to be more precise, we will address the two economies as economy 1 and 2 in the subsequent sections. In the following example, we can think of economy 1 being Norway, as we for now assume that the domestic economy is a small open economy.

Economy 1 is initially assumed to be in equilibrium. Here the expected inflation (π^e) equals the inflation target, implying that there is no "inflation trust shocks" in either economy. The inflation target is assumed to be 2 %[4]. The natural real interest rate (r^*) is assumed to be 2 % in line with Taylor (1993). Under these assumptions it follows from the relationship $i^* = \pi^e + r^*$ that the nominal equilibrium interest rate equals 4 %. We also exclude any inflationary pressure from the exchange rate channel as the expected real exchange rate equals the equilibrium real exchange rate, or formally that $(e^e - e^*) = 0$. In our first illustration, we start out with no shocks affecting the economies. The assumptions above are summarized in table 2.1.

Table 2.1: *Summary of assumptions part I*

Expected inflation	π_1^e	2%
Inflation target	π_1^*	2%
Real equilibrium interest rate	r_1^*	2%
Inflation shock	u_1	0
Demand shock	v_1	0
Exchange rate shock	z_1	0
Expected - equilibrium real exchange rate	$(e_1^e - e_1^*)$	0

Based on the assumptions in table 2.1, we can rewrite (2.8) for economy 1 on reduced form as (2.9), eliminating the variables $(\pi_1^e - \pi_1^*)$, $(e_1^e - e_1^*)$, $(\pi_1^e - \pi_1^e)$, u_1, v_1, z_1.

(2.9)
$$i_1^{opt} = C_1 B_1 i_2 + C_1 A_1 \alpha_{1,1}(\pi_1^e + r_1^*)$$

By inserting the expression for B_i and C_i, (2.10) can be expressed as:

(2.10)
$$i_1^{opt} = \frac{A_1 \alpha_{2,1} + \beta_1}{A_1 \alpha_{1,1} + A_1 \alpha_{2,1} + \beta_1} * i_2 + \frac{A_1 \alpha_{1,1}}{A_1 \alpha_{1,1} + A_1 \alpha_{2,1} + \beta_1} * (\pi_1^e + r_1^*)$$

---

[4] Noway currently has an inflation of 2.5%, but we will use 2 % for the rest of the examples in line with the inflation target of Taylor (1993).

From (2.10) we see that the sum of the coefficients for economy 1 adds up to one[5]. This implies that the optimal interest setting, given our strict assumptions, is a weighted function of economy 2's nominal interest rate (i_2) and the domestic nominal equilibrium interest rate $(\pi_1^e + r_1^*)$. To make a numerical analysis of this finding, we further need to make some assumptions for α_1, α_2, γ, β and λ from (2.1), (2.2) and (2.3). Table 2.2. summarizes the assumptions for these coefficients as well as the aggregate coefficients A_1, B_1 and C_1.

Table 2.2: *Summary of assumptions part II*

$\alpha_{1,1}$	0.15
$\alpha_{2,1}$	0.03
γ_1	0.03
β_1	0.06
λ_1	0.5
A_1	1.41
B_1	0.10
C_1	3.19

To assign values to these coefficients, the ideal solution would be calibrating the model for the respective economies. We have used the coefficient values from Røisland & Sveen (2006) who calibrated a simple dynamic version of this model to a small open economy without interdependencies. Using these values, it is unlikely that the model will yield correct interest rates in the various scenarios studied. On the other hand, our framework is mainly illustrating the mechanisms of interdependent monetary policy through the interest rate and the exchange rate mechanism.

By inserting the values for each of the coefficients, (2.11) can be expressed as:

$$i_1^{opt} = (3{,}1938 * 0{,}10218) * i_2 + (3{,}1938 * 1{,}40615 * 0{,}15) * (\pi_1^e + r_1^*)$$

This can be re-written as:

$$i_1^{opt} = 0{,}33 * i_2 + 0{,}67 * (4\%) = 2{,}7\% + 0{,}33 * i_2$$

---

[5] $\dfrac{A_1\alpha_{2,1}+\beta_1}{A_1\alpha_{1,1}+A_1\alpha_{2,1}+\beta_1} + \dfrac{A_1\alpha_{1,1}}{A_1\alpha_{1,1}+A_1\alpha_{2,1}+\beta_1} = \dfrac{A_1\alpha_{1,1}+A_1\alpha_{2,1}+\beta_1}{A_1\alpha_{1,1}+A_1\alpha_{2,1}+\beta_1} = 1$

The equations present a simple linear relationship for the optimal interest rate for economy 1, expressed as a weighted function of economy 2's interest rate and the nominal equilibrium interest rate. For economy 1 this relationship can be shown graphically in figure 2.2, where the y-axis is the interest rate in economy 1 and the x-axis the interest rate of economy 2. The slope of the line is the magnitude of the central bank 1's response with respect to the i_2. One clearly see that the response is less than a one to one, as the central bank must consider a trade-off between the domestic economy and the foreign interest rate.

Figure 2.2: *Optimal interest setting for economy 1 with respect to economy 2's interest rate*
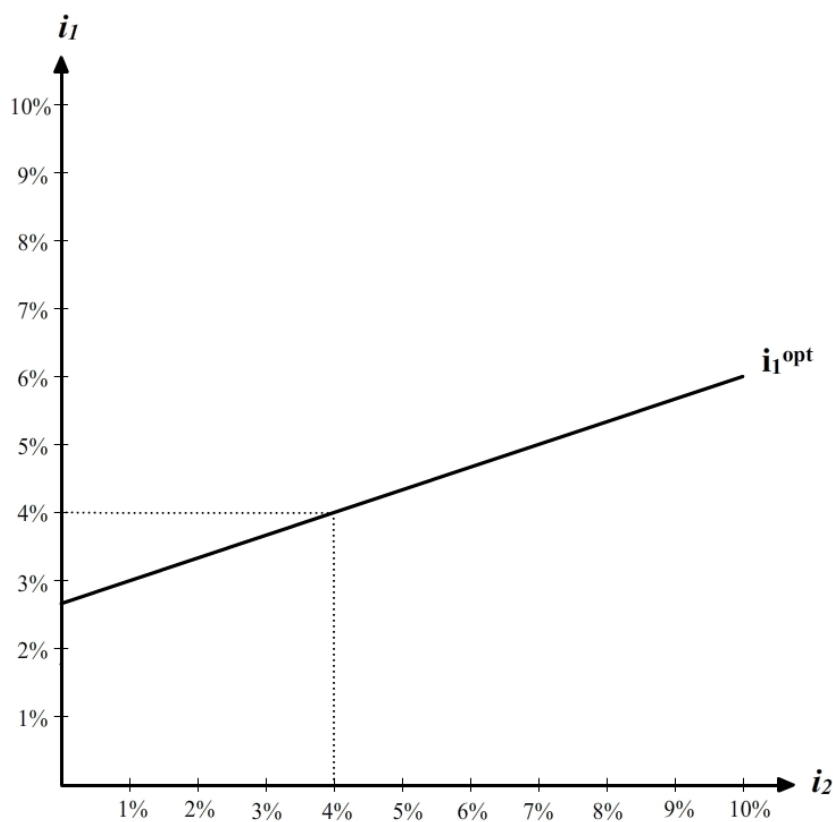
Given that economy 2 has a zero-interest rate, in this example, the optimal key policy rate for economy 1 would be 2,7%. At each point along the i_1^{opt} curve, central bank 1 minimizes its loss with respect to i_2, all else held constant. If the foreign interest rate is 4 % the resulting interest rate in economy 1 is also 4 %, which is consistent with our assumptions above. As central bank of economy 1 considers several other variables than the foreign interest rate, we will now see how shocks to economy 1 influence its reaction to the exogenous foreign interest rate. We refer to (2.9) for a full overview over the central banks response to the various shocks.

First, we start out with introducing a positive inflation shock to economy 1 of 0,42% ($u_1 = 0,42\%$). This arbitrary number is chosen for illustrative purposes. This, could for example be interpreted as an unexpected increase in nominal wages. In figure 2.3, the inflation shock causes a parallel upward shift in the response curve for economy 1, from i_1^{opt} to the blue dashed line $i_1^{opt\prime}$. Central bank 1's optimal response would now be a higher interest rate for any given level of i_2. From (2.9) we see that the various shocks in our framework would case similar parallel shifts to that of figure 2.2. However, the direction and magnitude of the shifts will depend on the magnitude and nature of the shocks, as well as the assumed coefficients.

Figure 2.3: *Optimal interest setting for economy 1 with respect to economy 2's interest rate with and without a positive inflation shock*
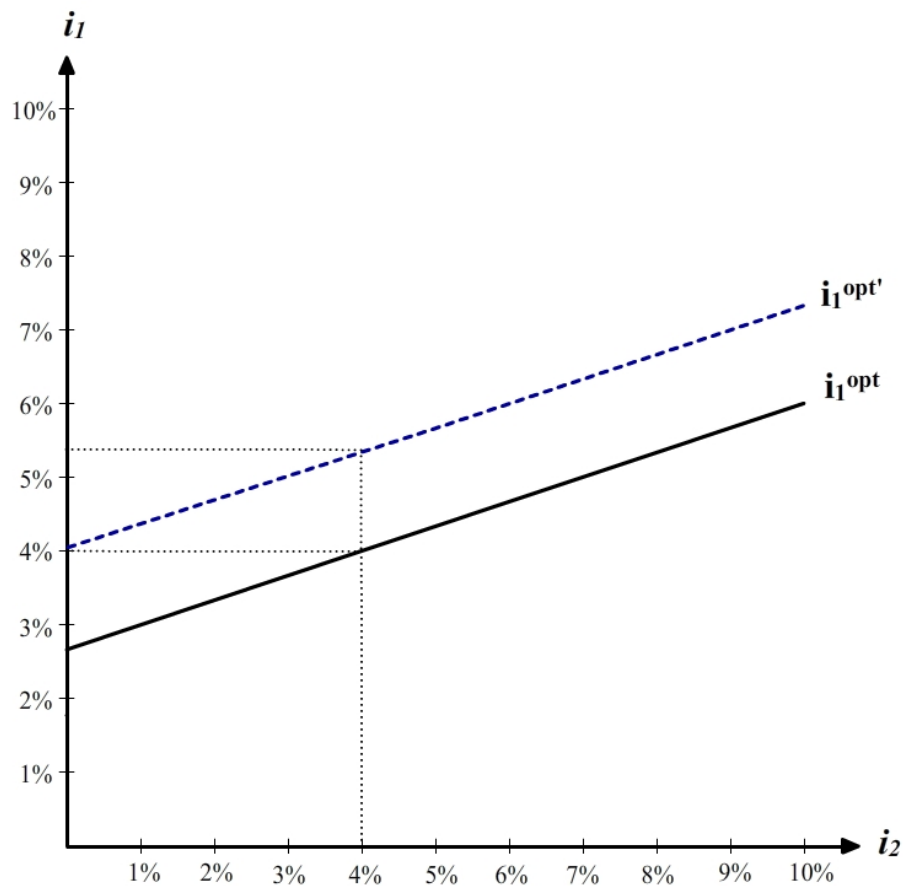
So far, we have considered a framework of two economies, where the interest rate of economy 2 (i_2) was exogenous. Thus, we have illustrated how the interest rate strategy of central bank 1 depends on the interest rate set by central bank 2. We will now extend this framework to enable both central banks to be interdependent, in the sense that we allow both central banks to simultaneously consider each other in their monetary policy strategy. This implies that both

i_1 and i_2 are endogenous, and that both central bank's interest rates therefore are functions of each other. Before we address this formally, we will introduce the game-theoretic concept for analysing such an interdependence.

2.5 Interdependence

Cooper (1985) defines "interdependence" as the degree of two-way influence of one economy on another at the margin. Interdependence can be divided into four different types. The first type is structural interdependence where economic events in one economy strongly influence events in another economy. The second type is interdependency among objectives of economic policy, which implies that *"one country will be concerned about attainment of policy targets in the other country"* (Cooper, 1985). A third type of interdependence is interdependence between exogenous disturbances or shocks, which with a high correlation implies that they will reinforce each other. The last form of interdependence is policy interdependence where central banks' optimal actions depends on the actions of other central banks and the other way around (Cooper, 1985)[6]. Consequently, policy interdependence arises from both structural interdependence and interdependence among objectives, and will also depend on the interdependence between exogenous disturbances or shocks.

As we have seen above, central banks of interdependent economies must consider the other central banks when pursuing an optimal monetary policy. As the different central bank's monetary policy influence the other central banks, interdependence introduce a strategic element for monetary policy, see for example Drazen (2000) and Canzoneri (1991). By analysing monetary policy using a game theoretic approach, it is possible to obtain insight in how this strategic element will alter monetary policy and its outcomes.

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[6] Further in this chapter we have limited central bank actions to mainly focus on setting the optimal interest rate.

2.6 The game aspect of policy interdependency

Within the realm of interdependence, several games or areas of potential conflict and cooperation exist, for example trade agreements, taxation of imports etc. Our aim is to explore the game that occurs between two or more central banks performing monetary policy under flexible inflation targeting, using the interest rate as their instruments. Monetary policy games have formerly been addressed using the money supply as instrument, see Cooper (1985), Drazen (2000) or Canzoneri (1991). It is generally assumed that central banks through managing the interest rate and money supply, can influence the real economy in the short to medium term, as well as inflation in the long term (ECB, 2011).

Game theory gives insight about how players relate to each other under situations of conflict or cooperation, usually under quite strict assumptions (Montet & Serra, 2003). We therefore find it useful to define some important assumptions from game theory that we will use to expand our formal framework. These assumptions will be key in our analysis and we will consider the implications changing these assumptions has for monetary policy strategy and interdependence.

First, for most of our analysis, we assume that the game between the central banks is non-cooperative. This means that each central banks' goal is to achieve its own self-interests. Formally this is assumed by each central bank minimizing its own loss function (2.1), taking the other central banks behavior into account. Starting out, we further assume that no central bank can commit itself to, or enforce, a certain strategy or threat (Montet & Serra, 2003). Later the results under these conditions, will be compared to those of a game where we allow for cooperation.

Second, the term "Nash equilibrium", which is well known within the realm of non-cooperative games, is a key concept in our analysis. Based on Gibbons (1992), we define a Nash equilibrium as when each central bank ($k = 1,2,...,n$) has an optimal interest rate strategy (i_k^{opt}) with a corresponding minimal total loss (L_k^{opt}), for every optimal interest rate

strategy of the other central banks. Then the combination of these optimal interest rate strategies for each central bank $(i_1^{opt}, i_2^{opt}, ..., i_n^{opt})$ is a Nash equilibrium, if each optimal interest rate has a corresponding loss value $(L_1^{opt}, L_2^{opt}, ..., L_n^{opt})$ which is equal to or lower than that of any other interest rate strategy available. For such a strong assumption to hold in its pure form, each central bank must ideally have complete information about the international economic development as well as the other central banks response strategies.

The assumption of complete information, as used in general game theory, will not likely hold in a "central bank game". However, the underlying criteria for complete information from game theory are valuable to discuss further. Montet & Serra (2003) define several underlying assumptions for complete information, that we will relate to a game between central banks. First, it is necessary that the development of the respective economies is common knowledge. Thus, each central bank has complete insight and information about all variables and shocks affecting the economy. This includes knowing the structure of the economies and how the economy responds to the level of the key policy rate. However, monetary policy works with time lags (George, 1999), which combined with the complexity of countries economic development yields uncertainty for central banks attempting to predict the future (Koenig & Emery, 1991). Therefore, it is unlikely that such an assumption will hold in its strict sense.

The central banks also need to have complete insight about the other central banks respective strategies and response functions. This is necessary to be able to carry out the correct monetary policy strategy in response to the other central banks strategies, without having to wait for their official response to be published. Further, each central bank also need to be a rational player aiming to minimize their economic loss function. As each central bank is rational, they can expect the others to be rational as well. Under these conditions, Nash-behavior is likely. Also, no unexpected surprises in monetary policy will happen, as each central bank will be able to understand and reproduce the other central banks behavior. However, central banks do not bind themselves to a specific mathematical rule, and use some discretion (Taylor, 2013), which arguably makes their behavior hard to always predict accurately. Therefore, this assumption will not likely hold in its strict sense.

Another specification used in game theory, that we will make use of, is assuming games are either static or dynamic. Static games are simultaneous one-shot games, where both, or all players choose their strategy at the same time (Gibbons, 1992). Monetary policy naturally introduces a dynamic game since the interest rate is assumed to influence the economy over time and work with considerable time lags (George, 1999). Further, each central bank normally announces their interest rate decisions at their interest rate meeting at set dates during the year. These meetings are not at the same dates for the three economies mainly referred to in our thesis. Our framework is based on a static monetary policy model and we will assume that the involved central banks simultaneously sets their interest rates. We also assume that it is a static game in the understanding that they do not consider interest rates from previous periods in their strategy. There are more dynamic versions of New-Keynesian monetary policy models. However, keeping the model static yields a better framework for illustrative purposes.

On a more general level, our framework allows for three different approaches to how the central banks relate to each other's monetary policy strategy. A central bank can perform an optimal monetary policy with respect to its own economy considering other central banks actions exogenous. This was modelled in subchapter 2.3. One would expect the pursuit of such a strategy to rely on the assumption that the central banks' action, and thus the domestic economy's development, does not affect other economies significantly. This is an assumption used for New Keynesian interest rate models for small open economies (Galí, 2008). One example of this is the Norwegian central bank monetary policy. Due to the relative small size of the Norwegian economy in a world economy setting, the impact of Norwegian interest rates on other economies and other interest rates are arguably smaller compared to that of the FED or the ECB. Hence, the Norwegian central bank will likely not consider its impact on other central banks monetary policy as significant.

Another approach is that each central bank pursues its objectives independently, but that one or several central banks tries to anticipate how the other central bank(s) will react in response to its actions. This is naturally based on the assumption that the respective central banks action's and the respective economies development have a significant effect on other economies. As a contrast to these approaches, it might also be possible, at least in principle,

that the central banks could cooperate to conduct an internationally optimal monetary policy through the interest rate level. The two last approaches will be formally analysed and discussed in detail in the next two subchapters.

2.7 Non-Cooperative behavior assuming complete information

Now, we will reintroduce the framework and assumptions from chapter 2.3 and 2.4. In the last section, we assumed that economy 1 was a small open economy where i_2 was exogenous. Now, we will assume that both economy 1 and 2 are large and open, and that both i_1 and i_2 are endogenous and interdependent. Formally, interest rate interdependence can be understood as the home economy's interest rate (i^h) being a function of the foreign interest rate (i^f) and vice versa:

(2.11)
$$i_t^h = i^h(i_t^f) \text{ and } i_t^f = i^f(i_t^h)$$

Both central bank's aim is to minimize their own loss function. The loss function is assumed to be identical for both central banks. Moreover, each of the economies are described by the same formal framework as presented in chapter 2.2. Thus, both economies and the corresponding monetary policy strategy is equal. Both economies are assumed to start out in equilibrium state. As we are now addressing two large economies, one can think of the economies being the Eurozone and the US economy. Correspondingly we assume that both economies have an inflation target of 2 %. We also assume that the long run natural real interest rate (NRIR) for economy 1 and 2 is 2 %, in line with Taylor (1993). Further, we use the same coefficient assumptions as before for both economies, summarized in table 2.3 and table 2.4.

Table 2.3: *Summary of assumptions part I*

Expected inflation	π_1^e, π_2^e	2%
Inflation target	π_1^*, π_2^*	2%
Real equilibrium interest rate	r_1^*, r_2^*	2%
Inflation shock	u_1, u_2	0
Demand shock	v_1, v_2	0
Exchange rate shock	z_1, z_2	0
Expected - equilibrium real exchange rate	$(e_1^e - e_1^*), (e_2^e - e_2^*)$	0

Table 2.4: *Summary of assumptions part II*

$\alpha_{1,1}, \alpha_{1,2}$	0.15
$\alpha_{2,1}, \alpha_{2,2}$	0.03
γ_1, γ_2	0.03
β_1, β_2	0,06
λ_1, λ_2	0.5
A_1, A_2	1.41
B_1, B_2	0.10
C_1, C_2	3.19

We know from sub-chapter 2.3 that the central banks' optimal monetary policy can be expressed as a weighted function of the nominal equilibrium interest rate and the foreign interest rate. Both economies and monetary policy strategies are assumed to be identical to the one derived in sub-chapter (2.3). We therefore start out with two identical equations for the optimal monetary policy for central bank 1 and 2:

$$i_1^{opt} = 0{,}33 * i_2 + 0{,}67 * (4\%) = 2{,}7\% + 0{,}33 * i_2$$

$$i_2^{opt} = 0{,}33 * i_1 + 0{,}67 * (4\%) = 2{,}7\% + 0{,}33 * i_1$$

Figure 2.4 illustrates these strategies. Since we mainly want to assess the implications of interdependence, we let each function be a response to the other economy's interest rate, keeping everything else constant. As these curves represent the optimal monetary policy, each point at the curves i_1^{opt} and i_2^{opt} minimizes the central banks' loss function with respect to the other central banks interest rate.

Figure 2.4: *Optimal interest setting for economy 1 and 2 with respect to the other economy's interest rate*
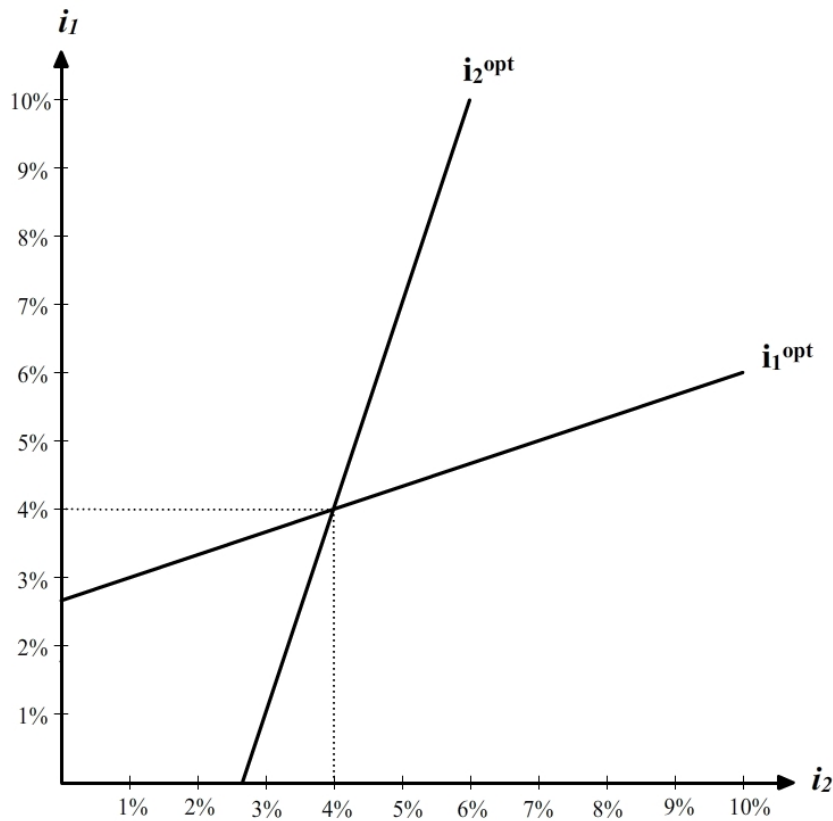
Both economies are assumed to be in an equilibrium state. Therefore, we end up with an equilibrium interest rate of 4 % corresponding to the intercept between the two interest rate functions. This is the only point where both central bank 1 and 2 can minimize their losses simultaneously and it follows that this is the Nash equilibrium. At this point, when not exposed to any shocks, both central banks loss functions are at zero loss, i.e. the inflation targets are fulfilled and the output gaps are zero. Thus, in this situation the Nash-equilibrium is also the optimal solution that minimizes the total loss for both economies. It is well known in economic theory that a Nash-equilibrium does not necessarily result in the solution that optimize the total welfare for all players. This will be clear when we introduce various shocks to our framework. In each scenario, as a comparison to the Nash equilibrium, we will include an interest rate equilibrium where the central banks cooperate in the interest rate setting. Before we analyse the various scenarios, we will therefore also derive the cooperative solution that minimize the aggregate loss for both economies.

2.8 Cooperative solution

To derive a cooperative solution, we let both central banks incorporate the effect their instrument incur on both the domestic and the foreign economy. Our method, is similar to the one used by Canzoneri (1991) where he derives various possible cooperative solutions that make both economies better off than a Nash solution. In his scenarios, he lets both central banks consider the effect their money supply has for both economies. Using this method, he creates an efficient frontier of the various combinations of the two economies' money supply that has a lower aggregate loss than the Nash solution. In our framework, one can imagine that the two central banks the FED and the ECB collaborates in their monetary policy. Such an assumption is strong, and would be hard to implement. However, it yields a valuable reference point for our analysis.

To formally allow for cooperation we let both central banks minimize the sum of their own and the foreign economy's loss with respect to their respective interest rates. This yields the following first-order conditions:

(2.12) $\frac{\partial L_h}{\partial i_h} + \frac{\partial L_f}{\partial i_h} = 0$

(2.13) $\frac{\partial L_f}{\partial i_f} + \frac{\partial L_h}{\partial i_f} = 0$

We use the same method of substitution as before. However, as we see from (2.12) and (2.13) we also solve for the home economy's interest rate effect on the loss function of the foreign economy, and vice versa. This is shown step by step in appendix 8.1.3 and the resulting optimal interest setting given cooperation can be expressed as:

(2.14) $i_h^{co} = M_h * \Big((\pi_h^e - \pi_h^*)E_h - (\pi_f^e - \pi_f^*)F_f + \alpha_{1,h}(\pi_h^e + r_h^*)G_h - \alpha_{1,f}(\pi_f^e +$

$r_f^*)H_f + u_h E_h - u_f F_f + v_h G_h - v_f H_f + \Big((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h \Big) * J_h - \Big((e_f^e -$

$e_f^*) + (\pi_f^e - \pi_h^e) + z_f \Big) * K_f + i_f N_f \Big)$

where $h = home\ economy, f = foreign\ economy$, co = cooperative optimal solution

$E_h = \gamma_h(\alpha_{1,h} + \alpha_{2,h}) + \beta_h$, $F_f = \gamma_f \alpha_{2,f} + \beta_f$

$G_h = \lambda_h(\alpha_{1,h} + \alpha_{2,h}) + \gamma_h E_h$, $H_f = (\alpha_{2,f}\lambda_f + \gamma_f F_f)$,

$J_h = \alpha_{2,i}G_h + \beta_h E_h$, $K_f = (\beta_f F_f + \alpha_{2,f}H_f)$,

$M_h = \frac{1}{J_h + \alpha_{1,h}G_h + K_f}$ *and* $N_f = (K_f + \alpha_{1,f}H_f + J_h)$

In (2.12) and (2.13) both central banks internalize the externality their interest rate causes for each other. This normally causes the optimal interest rate response to differ from the one derived in the non-cooperative framework, given that the economies are not in equilibrium. The cooperative solution will by default be a Pareto-optimal solution, where neither central bank can further minimize its loss without increasing the loss for the other central bank (Canzoneri & Gray, 1985).

Comparing the cooperative and non-cooperative solution, the main difference is how the central bank under cooperation considers its influence on the foreign economy. Comparing (2.14) with (2.8) for non-cooperation, we now see that the central bank of each economy responds differently to the other economy's "trust shock" $(\pi_f^e - \pi_f^*)$, real interest rate $(\pi_f^e + r_f^*)$, inflation shock (u_f), demand shock (v_f), and the exchange rate channel $\Big((e_f^e - e_f^*) + (\pi_f^e - \pi_i^e) + z_f \Big)$.

When assessing the coefficients of the foreign economy variables in (2.14), one thing stands out. Each of the exogenous foreign variables now require a negatively correlated response for the domestic interest rate, all else held constant. This implies that central bank 1 now would increase its interest rate to facilitate the monetary policy of central bank 2, in response to a

negative inflation shock in economy 2 ($u_2 < 0$). For the various shocks, this negatively correlated effect is to some degree offset when accounting for the positive correlation between the interest rates of the two economies ceteris paribus. Thus, the combined effect suggests a less aggressive interest rate response in cooperation, compared to that of a non-cooperative scenario. This eliminates potential losses from Nash-behavior.

This is in line with theory as the quadratic loss functions of both central banks result in exponentially growing losses when the economies deviate from the equilibrium levels. Therefore, instead of pushing the interest rate equilibriums to suboptimal levels, the central banks now respond to facilitate the monetary policy for the other economy in addition to their own economy. Although this incurs potential higher loss to an economy not hit by a shock (economy 1), the aggregate loss of both economies is reduced. Next time, when a shock hits economy 1, central bank 2 can return the favor. Thus, over time the cumulative aggregate loss of both economies will be reduced.

To illustrate the optimal interest rate function graphically we use the same assumptions as in table 2.3: No shocks and both economies in equilibrium state. We can then write (2.14) on reduced form as (2.15):

(2.15) $$i_h^{co} = M_h \left(\alpha_{1,h} G_h (\pi_h^e + r_h^*) - \alpha_{1,f} H_f \left(\pi_f^e + r_f^* \right) + N_f i_f \right)$$

Using the assumptions in chapter 2.4, equation (2.14) can be written as[7]:

$$i_h^{co} = 0.56 * (\pi_h^e + r_h^*) - 0.102 * \left(\pi_f^e + r_f^* \right) + 0.54 * i_f = 1.83\% + 0.54 * i_f$$

Using the same assumptions, the coefficient of i_f is larger in cooperation, compared to that of non-cooperation. However, this is offset by the dampening effect each of the respective foreign economy variables has on the domestic interest rate. Given identical optimal interest rate functions for economy 1 and 2 they can be expressed as:

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[7] See appendix chapter 8.1.4 for further calculation of aggregate coefficients not included in chapter 2.4.

$$i_1^{co} = 1.83\% + 0.54 * i_2$$

$$i_2^{co} = 1.83\% + 0.54 * i_1$$

Graphically this can be shown in figure 2.5, which also includes the optimal interest rate curves for the non-cooperative framework. The cooperative interest rate responses are illustrated by i_1^{co} and i_2^{co}, and the non-cooperative responses by i_1^{opt} and i_2^{opt}. As we are currently assuming equilibrium levels in both economies, the cooperative and non-cooperative curves both intercepts at an 4 % interest rate. Thus, when the economies are in equilibrium, the cooperative and non-cooperative framework suggest the same interest rate solution. This is consistent with our assumptions.

Figure 2.5: Cooperative- and Nash equilibrium
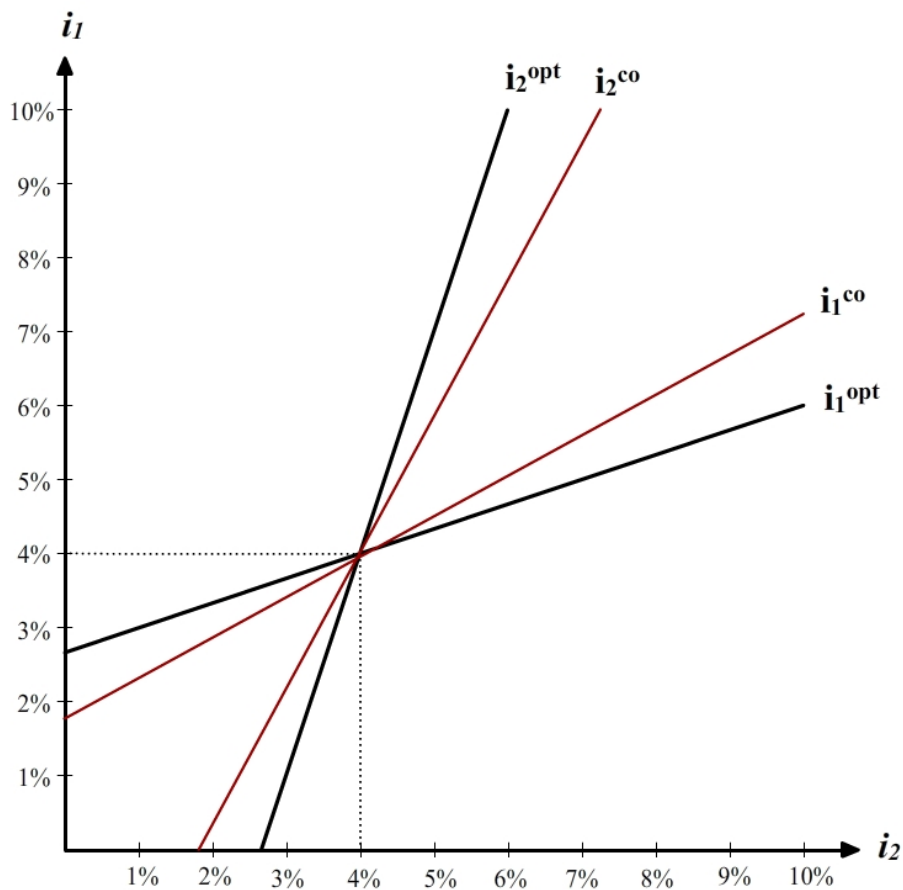
Having introduced the optimal reaction functions for the cooperative solution as well as the Nash-solution, we will next introduce various shocks and disequilibria to the economies to study the resulting effects for the interest rate level in both economies.

2.9 Economic disequilibria and shocks: A scenario analysis of Nash and cooperative behavior

In this section, we will formally and graphically analyse the central banks' behavior in response to disequilibria and shocks. Equation (2.8) gives a full overview over the central banks response to the different shocks assuming no cooperation. Since both economies and the monetary policies are symmetric, the central banks will naturally respond in the same way when hit by symmetric shocks (Canzoneri, 1991). It is important to notice that in each scenario we assume that the central banks default to non-cooperative behavior. However, to illustrate that this is not the optimal solution, we also include the cooperative solution from equation (2.14). Some of the shocks' magnitude in our examples might seem arbitrary and are only chosen for illustrative purposes.

Starting out, we introduce a scenario where a positive inflation shock hits both economies. Then we will move to various scenarios of adverse shocks and disequilibria that will receive a more thorough analysis. In each figure, point A illustrates the long run interest rate equilibrium, when no shocks or disequilibria are present. Point B illustrates the Nash-equilibrium and point C illustrates the cooperative equilibrium solution, given shocks. As a reference, the various equilibrium points are included in each figure. Note that the axis of Panel A and Panel B in each figure are different for illustrative purposes.

2.9.1 Positive inflation shock to both economies

We first start out with a scenario introducing a synchronized and equal positive inflation shock to both economies. At the start of our scenario, both central banks notice a factor creating sudden inflation pressure, potentially raising inflation above the target. Both central banks therefore increase their interest rate to reduce the inflation pressure. Central bank 1 and 2 knows that if one of them raises interest rates more, or earlier, it can appreciate its currency relative to the other currency and thus reduce imported inflation and its loss. Let us say central bank 2, aim to "overbid" central bank 1 to gain an advantage by increasing the rate more and earlier than central bank 1. This in turn, forces central bank 1 to increase its interest rate further to avoid increased inflation from a relatively depreciated currency. This drives both central

banks to increase their interest rates until both central banks has nothing more to gain individually from an interest rate hike. In figure 2.6 panel A, this is illustrated by introducing inflation shocks of 0,42 %, such that $u_1 = u_2 = 0,42\%$. Without cooperation, this causes a parallel upward shift in the response curve for both central banks with a resulting equilibrium interest rate shift from 4% (A) to 6% (B).

Figure 2.6: Equilibrium interest rate levels given a positive inflation shock, with and without cooperation
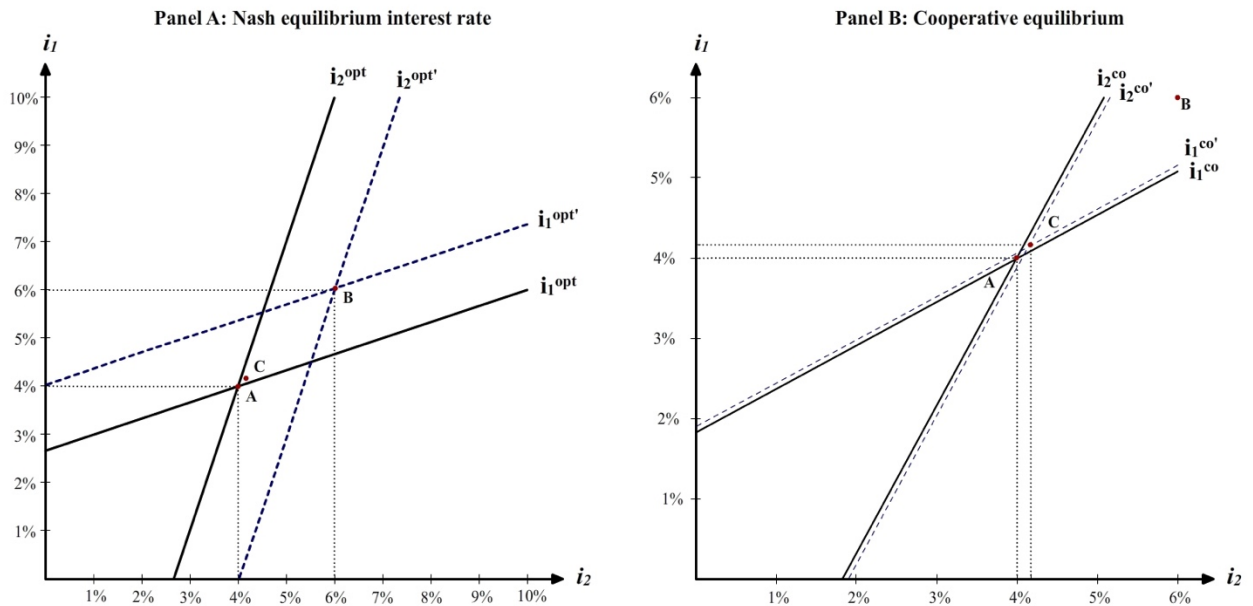
The same scenario under cooperation is different: Both central banks now consider the loss they incur on each other. Therefore, they will not behave aggressively and avoid the temptation to 'overbid' the other central bank's interest rate. Similarly as in the non-cooperative scenario, both central banks will increase the interest rate to respond appropriately to the shock. However, the increase will be much smaller, as they do not compete through the exchange rate channel. In our model, the outwards parallel shift in both response curves would only be of a magnitude of 0.13 percentage point. The resulting equilibrium interest rate, in figure 2.6 Panel B, shifts from 4% (A) to 4.16% (C) for both central banks. This is only a marginal change, compared to the non-cooperative equilibrium of 6 % in point B. Consequently, by cooperating, the combined loss for both central banks is lower.

2.9.2 Negative inflation shock to both economies (a potential race to the bottom)

When introducing a negative inflation shock to both economies the same mechanisms work in opposite direction. Both economies now experience a negative inflation shock, which require the central banks to counter the deflationary pressure. Both central bank 1 and 2 therefore responds by reducing their interest rate to increase inflation. However, as both central banks simultaneously reduce their interest rate, this offsets the exchange rate channel effect on inflation as both economies does not obtain a relative depreciation of their currency.

If one of the central banks first reduce its interest rate only marginally, the other central bank, let's say central bank 2, could take advantage of this. By reducing the interest rate further than central bank 1, central bank 2 can contribute to a relative depreciation of economy 2's currency, minimizing its loss. This creates a deflationary pressure for central bank 1 caused by a relative appreciation of economy 1's currency. Consequently, the loss of central bank 1 would increase. Therefore, central bank 1 has a strong incentive to reduce its interest rate further to avoid losses from the behavior of central bank 2. This will incentivize both central banks to compete for the lowest interest rate until they have nothing more to gain. Thus, this mechanism can be interpreted as a "race to the bottom" in the interest rate setting.

Under interest rate cooperation, none of the central banks would aggressively attempt to underbid the other central bank's interest rate to reduce their individual losses. Both central banks instead internalize their effect on the other central banks loss function. Therefore, they no longer have incentives to reduce their interest rate below the level of the other central bank. If they did, the central banks would increase their losses rather than reduce it, as they also consider their interdependent effects. Thus, the resulting cooperative interest rate equilibrium that minimize the aggregate loss of both economies is higher than under non-cooperation.

To illustrate this graphically, we introduce a synchronized and equal inflation shock to both economies of -0,42 %, such that $u_1 = u_2 = -0,42\%$. This causes a parallel shift downwards in both response curves. From figure 2.7 panel A we see the new Nash-equilibrium interest

rate of 2% at point B. With cooperation in panel B, the equilibrium interest rate only shifts from 4% (A) to 3.83% (C), which is the point that minimizes the aggregate loss of the two economies. As expected, in cooperation the equilibrium interest rate represents a smaller change from long run levels, compared to the Nash-equilibrium change, which is of a significantly larger magnitude.

Figure 2.7: Equilibrium interest rate levels given a negative inflation shock, with and without cooperation
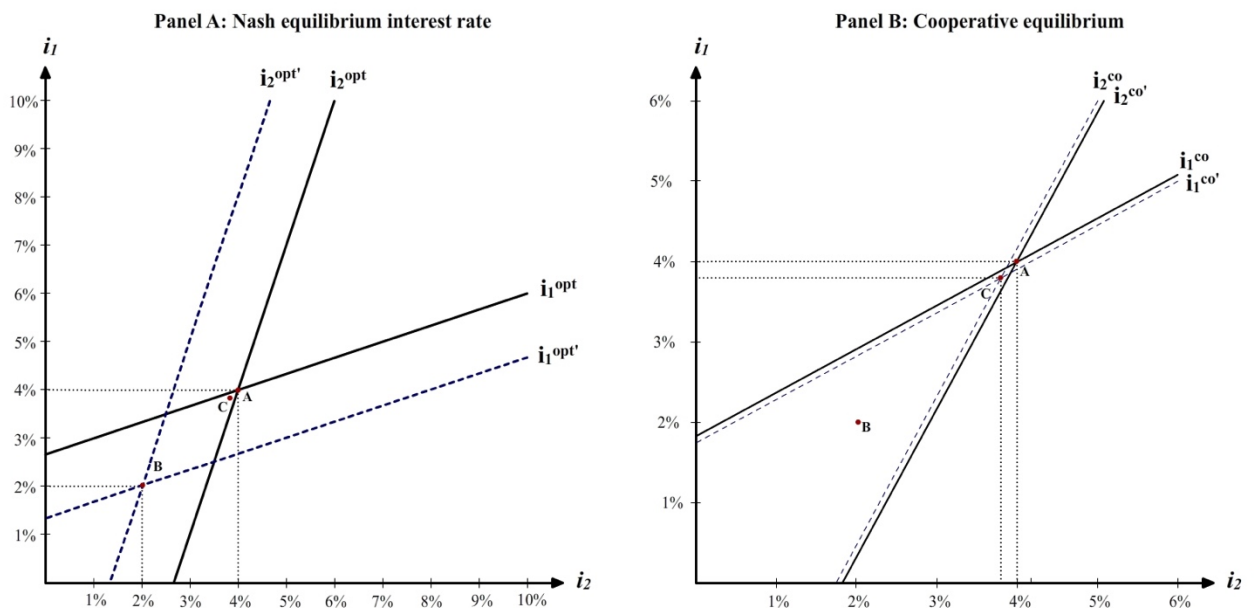
2.9.3 Negative inflation shock to only one economy

In this scenario, we assume that a negative shock hits economy 1. The shock requires a monetary policy response by central bank 1. Economy 2, on the other hand, experiences no shock. Central bank 1 correspondingly reduces its interest rate to counter the falling inflation. This causes a depreciation of the currency, which contributes to increased activity in the export sector, as well as imported inflation through more expensive input factors. In our two-economy framework, the increase in inflation through the exchange rate channel is at the direct expense of economy 2. Economy 2's currency appreciates relative to economy 1's currency. Economy 2 therefore experience reduced inflation from cheaper imported input factors as well as reduced activity in the export sector, as the export sector becomes less competitive. Without economy 2 being directly exposed to the shock in economy 1, central bank 2 experiences a loss from the monetary policy strategy of central bank 1.

Thus, as an appropriate response, given that the two central banks do not cooperate, central bank 2 reduces its interest rate to minimize its individual loss from an appreciated currency. This counter-measure by central bank 2 reduces the interest rate difference and limits the relative appreciation of its currency. This again reduces the effect of central banks 1 monetary policy by reducing the exchange rate channel effect on economy 1. To keep losses at a minimum, central bank 1 then has a new incentive to reduce its interest rate further. Aiming to lower its own loss, central bank 2 is left with little choice, but to again follow suit and lower the interest rate further. This mechanism will drive down the interest rate in both economies until none of the central banks has anything to gain in their individual loss function from a further reduction in the interest rate. In this scenario the shock only directly affected economy 1. Thus, central bank 1 will have an incentive to lower its interest rate more than central bank 2, which only responds to the monetary policy strategy of central bank 1 through the exchange rate channel. Thus, we will have different interest rates in the resulting equilibrium.

If both central banks cooperate, the result is quite different. Central bank 1 will naturally seek to increase inflation by reducing the interest rate and depreciate its currency. The response from central bank 2 is to increase the interest rate to facilitate the expansionary policy of central bank 1. Thereby, central bank 2 is incurring a larger loss for itself, while central bank

1 reduces its loss significantly. In this way, both central banks share the loss of a shock to one economy.

To illustrate these two outcomes graphically, we let $u_1 = -0{,}42\%$ and $u_2 = 0$. In figure 2.8 Panel A, we see a parallel shift in the response curve of central bank 1, while central bank 2's response curve stays the same. The resulting new equilibrium is 2.5% and 3.5% for central bank 1 and 2 respectively, as seen in point B. When allowing for cooperation, we see from Panel B that the response curve of central bank 1 shifts inwards, while the response curve of central bank 2 shifts outwards. The new cooperative equilibrium is given by point C, with an interest rate equilibrium of 3.2% and 4.6 % for central bank 1 and 2 respectively.

Figure 2.8: Equilibrium interest rate levels given a negative inflation shock to economy 1, with and without cooperation
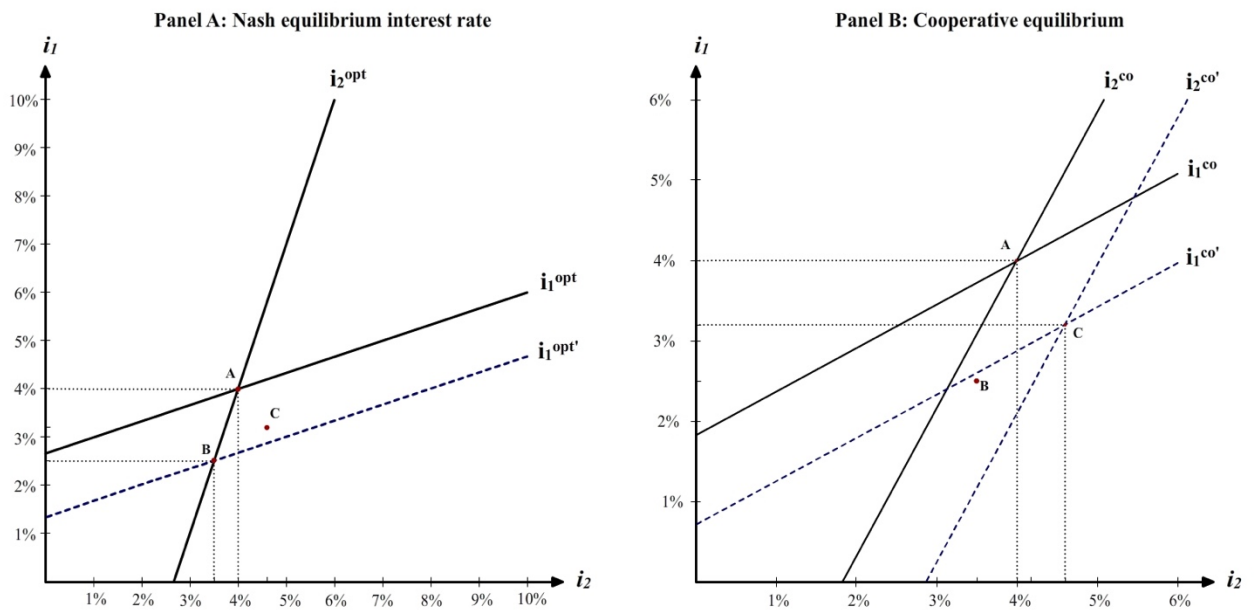
2.9.4 Changes in the expected inflation

In this scenario, we will analyse the effect of a drop in expected inflation from target level of 2 % to 1.5% for both economies. This could for example be due to a trust issue between the private sector and the central bank after a recession. Both central banks aim to lower their interest rate to drive up expectations and gain trust again. As previously, the same mechanism drives both central banks to lower their interest rate further than what is optimal for the aggregate loss. However, if both central banks can commit themselves to cooperation, they refrain from competing the interest rate further, and end up at an equilibrium interest rate level that is substantially higher than the Nash-equilibrium interest rate.

In figure 2.9 panel A, both economies experience an expected inflation of 1.5%, 0.5 % below the target. This results in a significant drop in the interest rates equilibrium from 4 % (A) to point 1 % (B) for both economies. This significant monetary policy response, might be a striking illustration of the importance inflation expectation has for the central banks' ability to minimize their loss functions. In panel B, we see the cooperative equilibrium rate at 3.3 % (C) for both economies.

Figure 2.9: Equilibrium interest rate levels given a change in expected inflation in both economies, with and without cooperation
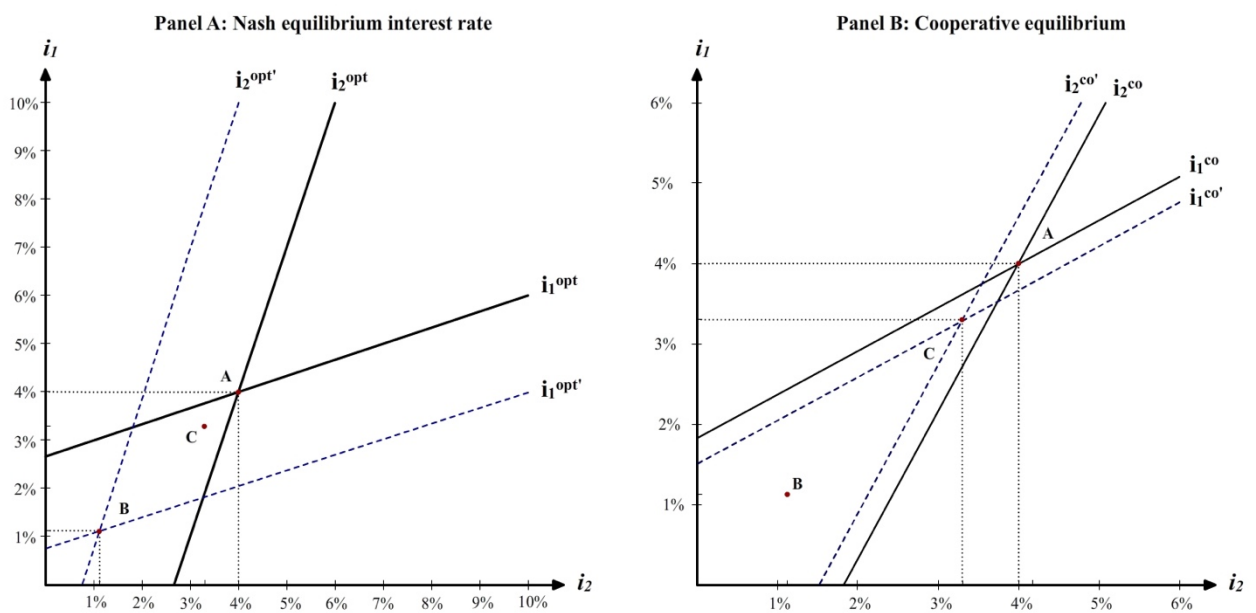
Next, we see how a drop in the inflation expectations in one economy influences the other central bank. We only introduce an inflation expectation drop of 0.5 percentage point below target for economy 1. Central bank 1 accordingly reduces its interest rate to drive up inflation expectations, attempting to mitigate the "trust shock". The following Nash-behavior from both central banks results in an interest rate Nash-equilibrium of 1.7% and 3.4% for economy 1 and 2 respectively, as seen in point B. Whereas the cooperative solution results in an interest rate equilibrium at point C of 2.6 % and 4.7 % for central bank 1 and 2 respectively. Again, we see that central bank 2 when cooperating, increases the interest rate to support central bank 1's expansionary policy. Consequently, the resulting aggregate loss for both economies is lower than that of the Nash-equilibrium. Figure 2.10 panel A and B respectively illustrates the Nash and cooperative equilibrium interest rates for this scenario.

Figure 2.10: Equilibrium interest rate levels given a change in expected inflation in economy 1, with and without cooperation
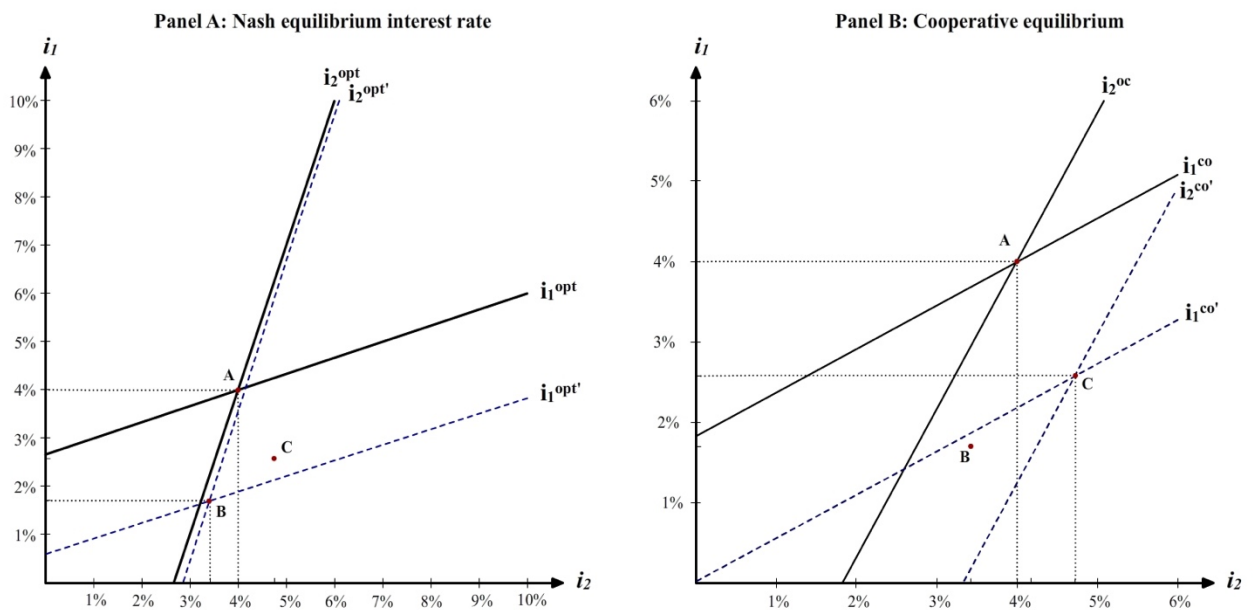
2.10 Deriving the formal Nash and cooperative equilibriums

We have graphically illustrated that interdependence and Nash behavior might lead central banks to aggressive interest rate responses when facing disequilibria or shocks. As a comparison, central banks considering the foreign interest rate exogenous will react less aggressive. This is a result that will hold independent of the size of the economies coefficients, given that the coefficients in the basic model equations are greater than zero, which is generally consistent with economic theory.

In this section, we will assess the graphically studied Nash behavior of central bank 1 and 2 formally. A formal analysis of these frameworks requires attention to detail as the mathematical reaction functions are quite complex. Further, the step by step process of deriving these expressions are extensive and tedious. These are therefore only included in appendix chapter 2.2 and 2.3.

Here, we will allow formally for each central bank to have complete information. We therefore incorporate the foreign central bank's reaction function in the domestic central banks reaction function by substituting for the foreign interest rate variable. Thus, the foreign interest rate is no longer exogenous. Instead each central bank now considers all the exogenous variables that makes up both economies, as well as the other central banks' reaction function coefficients.

To formally derive the non-cooperative Nash Equilibrium in the New Keynesian framework, we start out with (2.8). Here, we substitute in the foreign central banks reaction function for the formerly exogenous variable i_f and solve for the interest rate in the home economy (i_h) again. We are thus assuming the aggregate world-economy to consist only of two economies. In chapter 8.1.2, we have done this substitution step by step, resulting in equation (2.16). This equation is on a general form, and formally shows how a central bank in addition to its home

economy also considers the condition of the foreign economy. Here N^* denotes the Nash equilibrium interest rate for the home economy.

(2.16) $i_h^{N*} = D_h \left((\pi_h^e - \pi_h^*) + B_h C_f (\pi_f^e - \pi_f^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + B_h C_f A_f \alpha_{1,f} (\pi_f^e + r_f^*) + \right.$

$u_h + B_h C_f u_f + A_h v_h + B_h C_f A_f v_f + B_h \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h \right) + B_h C_f B_f \left((e_f^e - e_f^*) + \right.$

$\left. \left. (\pi_f^e - \pi_h^e) + z_f \right) \right)$

$for\ h = home\ economy, f = foreign\ economy$

$A_h = \left(\frac{\lambda_h*(\alpha_{1,h}+\alpha_{2,h})+\gamma_h[\gamma_h(\alpha_{1,h}+\alpha_{2,h})+\beta_h]}{[\gamma_h(\alpha_{1,h}+\alpha_{2,h})+\beta_h]} \right), \quad A_f = \left(\frac{\lambda_f*(\alpha_{1,f}+\alpha_{2,f})+\gamma_f[\gamma_f(\alpha_{1,f}+\alpha_{2,f})+\beta_f]}{[\gamma_f(\alpha_{1,f}+\alpha_{2,f})+\beta_f]} \right)$

$B_h = (A_h \alpha_{2,h} + \beta_h), \quad B_f = (A_f \alpha_{2,f} + \beta_f)$

$C_h = \left(\frac{1}{A_h(\alpha_{1,h}+\alpha_{2,h})+\beta_h} \right), \quad C_f = \left(\frac{1}{A_f(\alpha_{1,f}+\alpha_{2,f})+\beta_f} \right)$

$D_h = \frac{C_h}{(1-C_h B_h C_f B_f)}$

The formal Nash-equilibrium solution for two economies is a complex composition of many variables as well as coefficients. This is not surprising when formally analysing interdependencies. To get a more intuitive solution, most coefficients are aggregated in the seven aggregate coefficients $A_h, B_h, C_h, D_h, A_f, B_f, C_f$. Following economic theory, it can be proved that all the coefficients in our framework have positive values.

Solving for each of the exogenous variables in both equations will give us a suggested Nash-equilibrium interest rate level. This yields the same solutions as the intercepts in our illustrations, given the same assumptions. Compared to optimal open economy interest rate function in (2.8), the formal Nash solution (2.16) has two important distinctions. First, the bank now directly considers the development of the foreign economy. Second, the coefficients which determines the magnitude of response to development in the domestic economy have changed.

When considering the home economy, a valuable insight can be derived by comparing equation (2.8) and (2.16). When the foreign interest rate was considered exogenous, the central bank reacted to any variable in the model by the size of the coefficient C_h in combination with potential other coefficients. When it also considers the game with the opposite central bank, this coefficient is now D_h, which can be proved larger than C_h. To prove this, we first start out with defining $D_h = \left(\frac{C_h}{1-(B_h B_f C_h C_f)}\right)$. For D_h to be greater than C_h the denominator for D_h must be smaller than 1 but greater than zero. We have previously proved that that $B_h C_h < 1$ and positive for all positive coefficients. This is also the case for $B_f C_f$. Therefore, the sum of these four variables is also less than one, but positive. Formally this can be expressed as $B_h B_f C_h C_f < 1$. Then $\frac{1}{1-(B_h B_f C_h C_f)} > 1$. Thus, it follows that $D_h = \left(\frac{C_h}{1-(B_h B_f C_h C_f)}\right) > C_h$ or simply that $D_h > C_h$.

This result is important, and can be examined using a scenario where a negative inflation shock hits economy 1. Formally we have that $u_1 < 0$. Central bank 1 want to lower its interest rate, but must also consider that the central bank 2 will lower its interest rate in response. When central bank 2 lowers its interest rate, it indirectly reduces the marginal influence of central bank 1 on its own economy's inflation through the exchange rate channel. To account for this, central bank 1 must further reduce its interest rate by a factor that is the difference between D_h minus C_h times the magnitude of the inflation shock.

To derive the formal cooperative equilibrium, we use the same method of substitution as for the Nash equilibrium. We start out with the optimal cooperative interest rate for each economy (2.14). Then we substitute for the foreign interest rate for the two respective countries and solve for the domestic interest rate. The resulting equation (2.17) for the cooperative equilibrium interest rate yields the exact cooperative equilibriums that are given by graphical illustration above. "co*" denotes cooperative equilibrium interest rate for the home economy given the foreign economy's situation. Due to the intricate nature of this equation, it is hard to

derive any intuitive insight from the coefficients without assuming their values, and any attempt to do so, results in quite messy algebra.

(2.17) $\quad i^{co*}{}_h = P_h * \Big((E_h - M_f N_f F_h)(\pi_h^e - \pi_h^*) - (F_f - M_f N_f E_f)(\pi_f^e - \pi_f^*) + (\alpha_{1,h} G_h -$

$M_f N_f \alpha_{1,h} i_h)(\pi_h^e + r_h^*) - (\alpha_{1,f} H_f - M_f N_f \alpha_{1,f} G_f)(\pi_f^e + r_f^*) + (E_h - M_f N_f F_h) u_h - (F_f -$

$M_f N_f E_f) u_f + (G_h - M_f N_f i_h) v_h - (H_f - M_f N_f G_f) v_f + (J_h - M_f N_f K_h)\Big((e_h^e - e_h^*) +$

$(\pi_h^e - \pi_f^e) + z_h \Big) - (K_f - M_f N_f J_f)\Big((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f \Big) \Big)$

$E_i = \gamma_h(\alpha_{1,h} + \alpha_{2,h}) + \beta_h \, , E_f = \gamma_f(\alpha_{1,f} + \alpha_{2,f}) + \beta_f$

$F_f = \gamma_f \alpha_{2,f} + \beta_f \, , F_h = \gamma_h \alpha_{2,h} + \beta_h$

$G_h = \lambda_h(\alpha_{1,h} + \alpha_{2,h}) + \gamma_h E_h, G_f = \lambda_f(\alpha_{1,f} + \alpha_{2,f}) + \gamma_f E_f$

$H_f = \alpha_{2,f} \lambda_f + \gamma_f F_f$

$J_h = \alpha_{2,h} G_h + \beta_h E_h$

$K_f = \beta_f F_f + \alpha_{2,f} H_f$

$M_i = \dfrac{1}{J_h + \alpha_{1,h} G_h + K_f} \, , M_f = \dfrac{1}{J_f + \alpha_{1,f} G_f + K_h}$

$N_f = K_f + \alpha_{1,f} H_f + J_h$

$P_h = \dfrac{M_h}{1 - M_h M_f N_f N_h}$

2.11 Obstacles to economic coordination and empirical gains from cooperation

The framework we have developed identifies several externalities between interdependent economies, which can be reduced with cooperation. This corresponds to older literature and frameworks which uses money supply as the monetary policy tool (Canzoneri, 1985). It is however some disputing research related to the topic of policy coordination. Rogoff (1985) found very small, if not negative, effects of cooperation. In his article, he points out that *"international monetary cooperation may actually be counterproductive"* when facing a credibility problem. He further argues that cooperation might lead to a higher inflation bias, because the incentives for wage setters to inflate is greater in a cooperative regime (Drazen, 2000; Rogoff, 1985).

Canzoneri (1985) points out that empirical research up until that time found little or no gains from cooperation and further argues that the gains from cooperation is limited to large shocks. This can be related to our framework where cooperation only leads to better solutions when the economy is not in an equilibrium state or is hit by shocks. Canzoneri's also argue that from a game theoretic point of view the central banks must resolve the issue of being able to make credible commitments. Drazen (2000) argues that policy competition between central banks might solve credibility issues to some degree, as this opens for the need to use reputation and other commitment tools for the central banks. He further argues that the effect of cooperation might be welfare reducing if those commitment mechanisms where to be removed with cooperation.

Cooper (1985) argues that the main reasons for the lack of coordination comes from the different country-specific objectives among the central banks, even if all participating central banks have recognized the potential gain from coordination. This relates to several obstacles: First, central banks may differ in the objectives of cooperation. An example of this is the weight each central bank puts on stimulating growth and employment (or in our framework the output-gap), versus the weight they put on keeping inflation low and stable. Second, central banks might disagree on the structure of the economy, which can lead to disagreements about the appropriate policy response. Third, to achieve successful coordination the central banks

need to trust each other, and the lack of trust between central banks undermines the possibility of cooperation. Fourth, to achieve cooperation, the central bank must give up some of its national autonomy, which goes against the preference to retain their freedom of action (Drazen, 2000).

The difficulty of implementing policy coordination, combined with the research in the early 1980s, which found little or no gains from cooperation, made "getting the domestic policy right" the preferred solution (Taylor, 2013; Taylor, 1985). Taylor's (1985) definition of "getting the domestic policy right" was to follow a monetary policy rule where each central bank optimizes its own economy's domestic macroeconomic performance assuming other central banks would do the same. As, we have argued before, this can be related to our example in subchapter 2.4. Here we kept i^f exogenous, assuming that the central bank did not consider international spillover effects in its optimal monetary policy. It is clear from comparing this illustration with the Nash behavior, that an emphasis on the domestic economy, without considering spillover effects, results in less aggressive interest rate behavior.

Taylor (2013) argues that "getting the domestic policy right" was a good fit to the monetary policy from the 1980's until the period of 2003 – 2006, when Taylor (2013) and Kahn (2010) claim interest rates were too accommodative in the US. This resulted in large deviation from different specifications of the Taylor rule. Until then, there was supposedly little concern about adverse spillover effects, and a corresponding balance in the international monetary system (Taylor, 2013). In recent years, it seems like this balance have disappeared, and Taylor (2013) points out that the "1980s view about monetary policy coordination needs to be reexamined". It thereby opens for the possibility that the research of the early 1980s, who found little or no gains from cooperation, might no longer be valid.

2.12 Prisoners dilemma – returning to equilibrium levels

So far we have developed a static framework, assuming complete information and no cooperation by default. Under such strict assumptions, it is natural to expect that the interest rate levels will gradually return to their long run equilibrium levels in the absence of additional shocks. However, it is obviously unrealistic that each central bank has complete information about the developments of the domestic economy or the foreign interdependent economies. Further, the interpretation of various leading and lagging indicators of the business cycles are subject to a lot of uncertainty and monetary policy works with time lags (George, 1999; Koenig, 1991). Moreover, shocks are not equally distributed across economies, which often demand different responses from the various central banks with respect to their domestic economy. These factors combined, create a highly uncertain environment for central banks when carrying out their monetary policies. In turn, this might hinder a gradual return to normal key policy rate levels.

Such a situation might be related to a possible prisoner's dilemma, hindering central banks to return to "normal" interest rate levels in the period after a shock. To examine this conceptually: let's say that a negative shock of some sort hits several of economies to various extents, and with uncertainty around the results and recovery time for each economy. As before, the central banks end up reducing their interest rates to a Nash-equilibrium. When the effect of the shock is successfully counteracted, the central banks would ideally return to their normal equilibrium interest rate levels. However, assessing the right timing might be difficult and potentially disagreed upon between the various central banks. If one central bank moves to increase the interest rate, others might, for one of the various reasons mentioned above, keep their interest rates low.

This might result in negative spillover effects for this central bank, like increased deflationary and contractionary pressure, as the interest rate gap between the economies would increase (Taylor, 2013). To avoid such a situation, it is often likely that all the central banks will wait (too) long to increase their interest rates again to avoid the potential losses of being the first-mover. This might contribute to a lack of international coordination when moving back to equilibrium interest rates after shocks. We will refer to such a situation as a prisoner's

dilemma, as all the players (central banks) according to our framework have an individual optimal policy of non-cooperation, even if this is not the aggregate optimal solution.

2.13 Motivation for empirical analysis

So far we have built a theoretical framework of interdependencies between central banks' monetary policy under flexible inflation targeting. Our framework suggests that central banks are incentivized to take part in non-optimal Nash behavior when facing economic shocks or other forms of disequilibria in the domestic or foreign economies. Considering the current international monetary policy situation, we will in the next part of our thesis focus mainly on the effects of negative economic shocks and the corresponding key policy rate levels. We have illustrated that the effect of adverse shocks to the economies leads to sub-optimally low interest rates, or what we refer to as "a race to the bottom" between central banks. In the following chapters, we will study the interest rate policies of various central banks in search for evidence of the suggested Nash-behavior. Particularly, we will assess how adverse economic shocks influence the following interest rate equilibrium between central bank's key policy interest rates.

To shed empirical light on the hypothesis that the interest rates are lower than optimal in certain periods, we will use various Taylor rate as a proxy for the optimal interest rate for each economy. Our expectation, based on our theoretical framework, is that actual short term central bank rates will be significantly lower than the suggested Taylor rate during periods of adverse economic shocks, even when controlling for variations of the underlying assumptions. As we also have argued, we suspect that there might be evidence of a prisoner's dilemma situation when central banks ideally should have returned to their long run equilibrium interest rates. We therefore incorporate into our analysis the possibility that the interest rates will be lower than the Taylor rate also after periods of negative economic shocks.

The theoretical model's prediction is limited to economies using flexible inflation targeting, keeping their currency floating. It is also based on an underlying assumption of interdepencies between the respective economies. To test the theoretical models empirical validity, we have selected data from nine developed economies currently performing various levels of flexible inflation targeting: Australia, Canada, Eurozone, Japan, New Zealand, Norway, Sweden, UK and USA. Table 2.5 summarize the official date of adopting inflation targeting as well as their current target for the respective economies. An official inflation target does not exist for a few

of the economies early in our sample period. However, according to Bernanke (2004) several of the economies, including the US, did arguably change their monetary policies towards inflation targeting in the 1990's. For all practical matters, we therefore assume implicit inflation targets of 2% for the periods where we are lacking an explicit inflation target. A further discussion is included in the inflation data section.

Table 2.5: Official date of adopting inflation targeting and current target

Economy	Date of officially introducing inflation targeting	Current inflation target
Australia	June 1993	3 %
Canada	February 1991	2 %
Eurozone	October 1998*	2 %
Japan	January 2013	2 %
New Zealand	Dec 1989	2 %
Norway	March 2001	2.5 %
Sweden	January 1993	2 %
UK	October 1992	2 %
USA	January 2012	2 %

Sources: Hammond (2012) and Johnson (2002). Current inflation target: April 2017.

To assess the central banks' behavior during and after shocks, we use an event study analysis method. Here we set up dummies for the periods of the respective shocks, as well as a period of three consecutive years after the trough. This aftermath period is to account for potential a potential prisoners' dilemma. The "shocks" selected for analysis are the US recession after the Dotcom bubble as well as the Great recession that occurred in the US after the Financial crisis. Using these two large and easily verifiable shocks, reduces potential ambiguity to the definition of the "shocks". Due to variation in our data series, as well as the natural limitations of few observations in quarterly data, it will arguably be easier to estimate the effects of shocks that have implications for several of our sample economies. The basis for our event analysis is the National Bureau of Economic Research's (NBER) classifications of the peaks and troughs for USA, see table 2.6. Using the recession period definitions for only one of the economies in the sample, a large economy at that, is based on the important role interdependencies plays in our theoretical framework.

Table 2.6: NBER definition of the last 2 recessions for the US

Business Cycle Reference Dates		Duration in months			
		Contraction	Expansion	Cycle	
Peak	Trough	Peak to Trough	Previous trough to this peak	Trough from Previous Trough	Peak from Previous Peak
March 2001 (I)	March 1991 (I)	8	120	128	128
December 2007 (IV)	June 2009 (II)	18	73	91	81

Source: NBER

In the next chapter, we will start out with a definition of the data used in our analysis. Here the choice of data as well as necessary assumptions will be discussed in detail. Then, a chapter about our choice of empirical method will follow, before we move on to the empirical analysis and our findings in chapter 5.

3.   Data and Taylor rate estimations

In this section, we will present the data used for panel data analysis. Our aim is to analyze potential deviations between central banks' key policy rates and various Taylor rates. To carry out our analysis we have gathered data on key policy rates, GDP, inflation, inflation targets, and the natural real interest rate (NRIR) for nine sample economies.

3.1 Key policy rates

Table 3.1 summarizes the name of the key monetary policy rates for the economies in our sample and the current level of these rates. The key policy rate, is the interest rate used by the central bank to carry out its monetary policy. In appendix chapter 8.2.1, the developments of these key policy rates are shown graphically.

Table 3.1: *Key policy rate for our sample economies*

Economy	Key policy rate	Current level
Australia	Cash Rate	1.50 %
Canada	Policy interest rate	0.50 %
Eurozone	Marginal refinancing rate (refi)	0.25 %
Japan	Basic Loan Rate	0.30 %
New Zealand	Official Cash Rate	1.75 %
Norway	Key policy rate	0.50 %
Sweden	The repo rate	-0.50 %
United Kingdom	Official Bank Rate	0.50 %
United States	Federal funds target rate	1.00 %

The data were obtained on the 27th of April. Source: Central banks official cites

3.2 Data on inflation

3.2.1 Headline inflation

For the headline inflation data, we have used consumer price data from Bank for International Settlements (BIS). This database contains long series on consumer prices for 60 countries, including all 9 countries in our sample. The data has monthly frequency, and we have used the 12 month % change (Year-on-year changes, in per cent) as an estimate for the monthly headline inflation. Next we converted the data to quarterly data by taking the average of the monthly data in each quarter. In appendix chapter 8.2.2 we have included the original source BIS used to generate the consumer price index, and a graphical presentation of the data.

3.2.2 Core inflation

Core inflation, also called underlying inflation, lack a clear definition. We have chosen to use the inflation rate which the central bank has mentioned in their monetary reports or published as underlying inflation. In appendix chapter 8.2.2, we have summarized the name of the data series used, together with the source of the data. Several of the inflation series used are based on CPI excluding volatile components such as energy prices and food prices. However, New Zealand uses core inflation estimates based on a factor model and Sweden uses the underlying inflation defined by trimmed series[8]. Headline inflation, on the contrary, is equally defined across the economies and might therefore be easier to compare. However, using the underlying inflation estimates considered by each central bank, arguably yields a better comparison for our Taylor rate deviations.

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[8] The trimmed series excludes the least and most volatile components each month.

3.2.3 Inflation target adjustments and inflation gaps

The inflation gap is defined as the deviation between actual inflation and target inflation. In line with Taylor (1993) our base scenario use an inflation target of 2%, defining the inflation gap as any deviation from the 2 % target. In appendix chapter 8.2.2 we have also derived series where the inflation target is adjusted for the official inflation target of the central bank. However, changing the inflation target from 2 % to official targets only resulted in marginal differences in our analysis. We will therefore only present the results using a 2% inflation target for all the sample economies, while the results using the adjusted series are in appendix chapter 8.4. Under in figure 3.1 and 3.2, we have plotted the inflation gap both with an inflation target of 2% and an adjusted inflation target using the headline and core inflation respectively. Both figures, includes shaded areas for NBER defined US recessions for each economy. This is to provide an illustration of the recession periods we will use in our analysis.

Figure 3.1: Inflation gap using headline inflation (HI)

Gap HI with 2% inflation target   Gap HI with adj. inflation target

Red lines indicates zero inflation gap and shaded area indicates NBER US Reccessions.
Sample period: 2001Q1 - 2015Q4

Figure 3.2: Inflation gap using core inflation (CI)

Red lines indicates zero inflation gap and shaded area indicates NBER reccessions.
Sample period: 2001Q1 - 2015Q4

3.3 Natural real interest rate (NRIR)

One important input factor in the Taylor formula is the natural real interest rate (NRIR). In Taylors' original formula this rate is assumed to be 2 % (Taylor, 1993). However, keeping in mind the current low interest rate regime, it is likely that this rate has declined in recent time. Changes in the natural rate as an input factor will alter the suggested Taylor rate. Thus, to make our estimates more robust, we control for possible changes in the natural real interest rate. The natural rate cannot be directly observed. Therefore, we base our estimated for the natural interest rate on the findings of Holston, Laubach & Williams (2017). They have estimated the natural real interest rate for the four large economies; the US, Canada, the Eurozone and the United Kingdom in 1990, 2007 and 2016, as well as the change between these years.

To avoid any structural breaks, we have assumed a linear trend between these data points for each of the respective economies. For the other five economies in our sample, we use the average of the above estimates from Holston, Laubach & Williams (2017), also assuming a linear trend between the datapoints. Table 3.2 summarizes the estimates. To see how this fits

with our own data series, we have in figure 3.3 plotted the NRIR estimates and the real key policy rate. The real key policy rate is estimated both using core and headline inflation.

Figure 3.3: Natural real interest rate (NRIR)
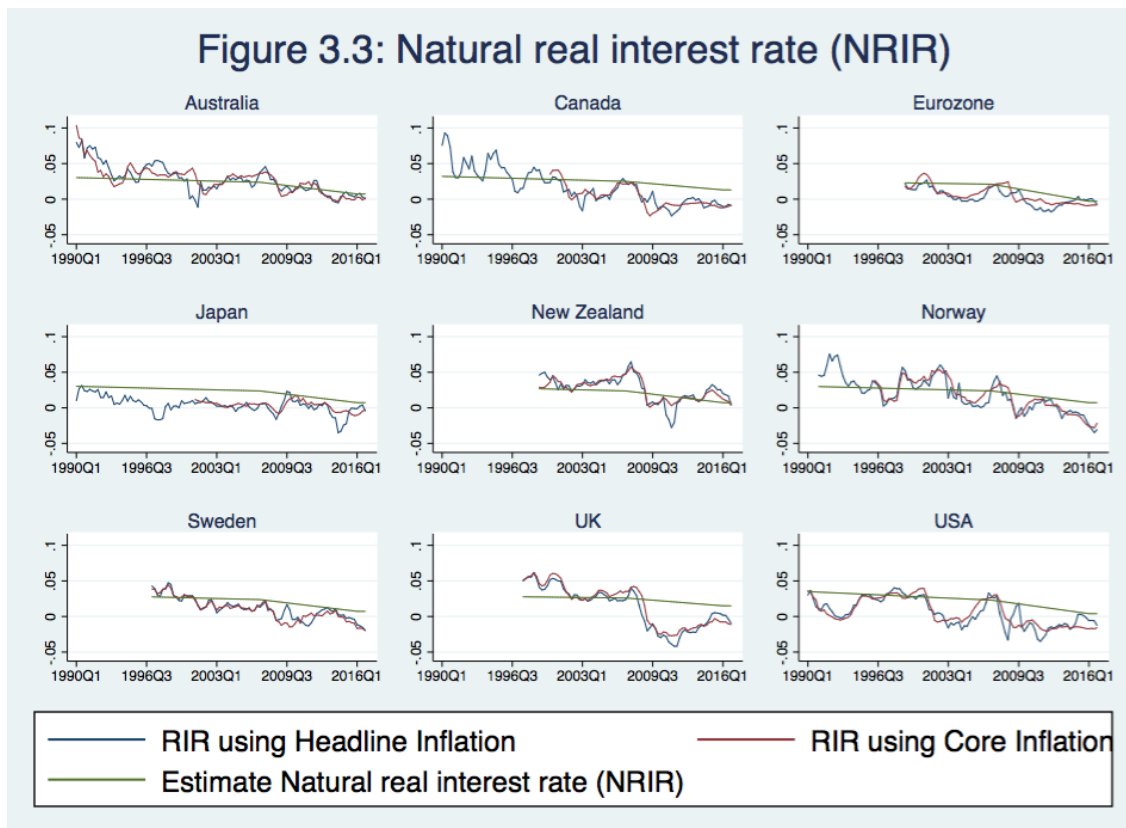
Table 3.2: *Estimates Natural real interest rate (NRIR)*

| | Natural real interest rate | | | Change in the natural real interest rate | | | |
| | | | | 1990-2007 | | 2007-2016 | |
Economy	1990	2007	2016	Total chg	Chg per quarter	Total chg	Chg per quarter
United states	3.5%	2.3%	0.4%	-1.2%	-0.018%	-1.9%	-0.053%
Canada	3.2%	2.5%	1.3%	-0.7%	-0.010%	-1.2%	-0.033%
Euro Area	2.5%	2.1%	-0.3%	-0.4%	-0.006%	-2.4%	-0.067%
United Kingdom	2.9%	2.6%	1.5%	-0.3%	-0.004%	-1.1%	-0.031%
Average	3.0%	2.4%	0.7%	-0.7%	-0.010%	-1.7%	-0.046%

Source: Holston, Laubach & Williams, 2017

3.4 Gross domestic product (GDP)

The original Taylor rule assumes that the central bank adjusts its short-term interest rate target in response to fluctuations in inflation and output gaps (Taylor, 1993). This is consistent with flexible inflation targeting, which is based on minimization of a loss function, see equation (2.1). To derive the Taylor rule, we have collected quarterly, real prices, seasonally adjusted, expenditure based GDP data for each of the nine economies in our sample. Using seasonally adjusted data we reduce volatility in the data that merely results from seasonal variety and not variation in the underlying GDP trend.

3.4.1 Potential GDP estimates and deviation

The Taylor rate uses GDP deviations from trend growth as input. To derive the GDP trend, we use the standard HP filter developed by Hodrick & Prescott (1997). The foundation of their applied method, is the understanding that the real GDP over time (y_t) consists of a trend growth component (g_t) and a cyclical component (c_t), see equation (3.1). The trend component of GDP is determined by structural factors that change slowly over time. The HP filter equation (3.2) minimizes the sum of two components: The sum of the square of the cyclical component over time, and the deviation of the trend growth change from period $t - 1$ to t subtracted the trend change between period $t - 2$ and $t - 1$, squared. The last component, weighted by lambda (λ), penalizes variability in the trend component. λ is therefore defined as the smoothing parameter where $\lambda = 0$ would result in a GDP trend equal to the actual GDP, while a high value or $\lambda \approx \infty$ would result in a linear trend.

(3.1) $$y_t = g_t + c_t$$

(3.2) $$\min_{\{g_t\}_{t=-1}^T} \left\{ \sum_{t=1}^T c_t^2 + \lambda \sum_{t=1}^T [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\}$$

Hodrick & Prescott (1997) suggests using $\lambda = 1600$ for quarterly GDP data, which is also the standard practice in literature for all countries when working with quarterly GDP data (Marcet

& Ravn, 2004). Some suggest using $\lambda = 40000$ for Norwegian quarterly GDP data due to the large volatility in the data, compared to the actual growth trend (Bjørnland, Brubak & Jore, 2005). However, for comparison across many economies, we base our analysis on the international acknowledge standard of $\lambda = 1600$ for quarterly data for all economies in our sample. Figure 3.4 illustrates the gap between the actual GDP and the potential GDP for all nine sample economies. In appendix chapter 8.2.3 we have included a graphical presentation of both GDP and potential GDP.

Figure 3.4: GDP deviation from potential GDP (GDP gap)

Potentail GDP estimated using HP filter with Lambda = 1600. Red lines indicates zero deviation between actual and potential GDP. Shaded areas indicates NBER US reccessions. Sample period: 2001Q1 - 2015Q4
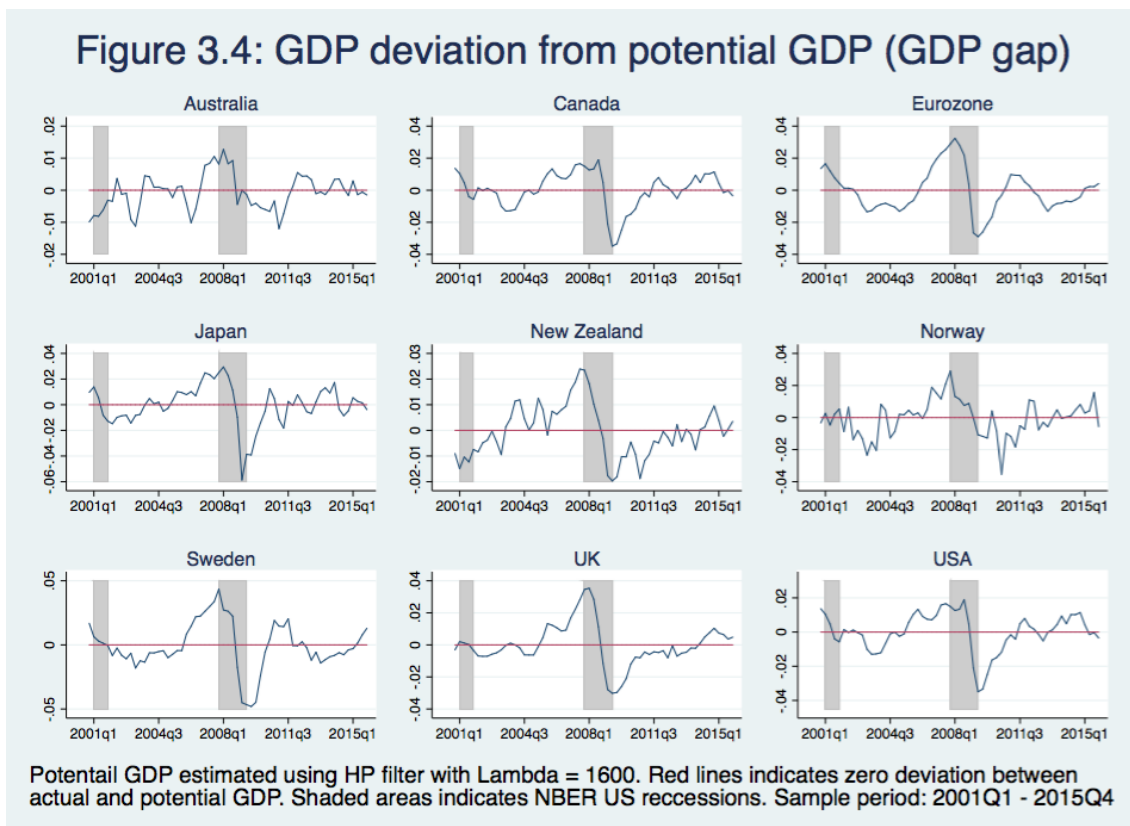
3.5 Taylor rates

Taylor's (1993) original monetary policy rule is given by equation (3.3) and generalized in equation (3.4).

(3.3) $$i_t = \pi_t + 2\% + 0.5(y_t - y_t^*) + 0.5(\pi_t - 2\%)$$

(3.4) $$i_t = \pi_t + r_t^* + a_\pi(\pi_t - \pi_t^*) + a_y(y_t - y_t^*)$$

where;
i_t is the short term key policy rates
π_t is the rate of inflation, over the previous four quarters
r_t^* is the equilibrium natural real interest rate (NRIR)
π_t^* is the target inflation
y_t is the logarithm of the real GDP
y_t^* is the logarithm of potential output
Originally, Taylor proposed that $a_\pi = a_y = 0.5$

The formula implies a constant inflation target and equilibrium real interest rate both at 2%. When both gaps are zero, the implied equilibrium nominal interest rate is 4% (Taylor, 1993). Woodford (2001) argues that the Taylor rate has several features of an optimal monetary policy, while also stating the importance of further research into the underlying input factors. Kahn (2010) states that central banks might respond to other factors, including forward looking indicators, not incorporated in the simple Taylor rule. Therefore, short and small deviations from the Taylor rate, can indicate appropriate monetary policy responses. However, there are some evidence suggesting that deviating significantly from the Taylor rate over time may result in increased asset prices and potential financial imbalances, see for example Ahrend (2008) or Kahn (2010). Thus, significant deviations over time might be evidence of sub-optimal monetary policy. In our analysis, we focus mainly on detecting negative gaps resulting from key policy rates being lower than what the Taylor rate suggests.

As a robustness test, our analysis uses various versions of the Taylor rate which are summarized in table 3.3. This allows us to control for the possible influence variation in inflation measure, and changes in the natural real interest rate (NRIR) might have. These four

rates, including the original rate (Taylor rate 1) are estimated for each of the nine economies of our sample[9]. We have in figure 3.5 and figure 3.6 illustrated the effect of adjusting for variation in NRIR using headline and core inflation respectively. By means of graphic illustration, one might argue that there is small variation in the suggested Taylor rates when controlling for the natural real interest rate (NRIR). However, for the Eurozone, the variation in recent years seem to be larger than that of other economies in our sample. This is mainly due to the current negative natural real interest rate estimate for the Eurozone (Holson, Laubach & Williams, 2017).

Table 3.3: *Taylor rates definition*

Name	Inflation type	Natural real interest rate (NRIR)
Taylor rate 1	Headline	2%
Taylor rate 2	Headline	Adjusted
Taylor rate 3	Core	2%
Taylor rate 4	Core	Adjusted

From figure 3.5 and figure 3.6, we see that the Taylor estimates based on headline inflation and core inflation differ significantly, where headline inflation based rates are more volatile. Kahn (2010) argues that headline inflation has been the best measure to keep the Taylor rate consistent with the federal funds rate, and that it is closest to what Taylor originally proposition of using the GDP deflator. On the other hand, core inflation is more stable and assumed to be a better indicator of future inflation in the medium term (Yellen, 2015). This argues that core inflation is emphasized by the central bank when carrying out its monetary policy strategy. We therefore include both headline and core inflation based estimates. In table 3.4 we have defined the different Taylor gaps.

Table 3.4: *Taylor gap definitions*

Name	Definition
Taylor gap 1 (T-gap 1)	Central bank policy rate - Taylor rate 1
Taylor gap 2 (T-gap 2)	Central bank policy rate - Taylor rate 2
Taylor gap 3 (T-gap 3)	Central bank policy rate - Taylor rate 3
Taylor gap 4 (T-gap 4)	Central bank policy rate - Taylor rate 4

See table 3.3 for Taylor rate definitions

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[9] Series including adjusted inflation targets are included in appendix chapter 8.2.4

Figure 3.5: Taylor rates based on headline inflation

Range of Taylor rates based on headline inflation

Taylor rate 1 (Headline inflation with 2% NRIR)
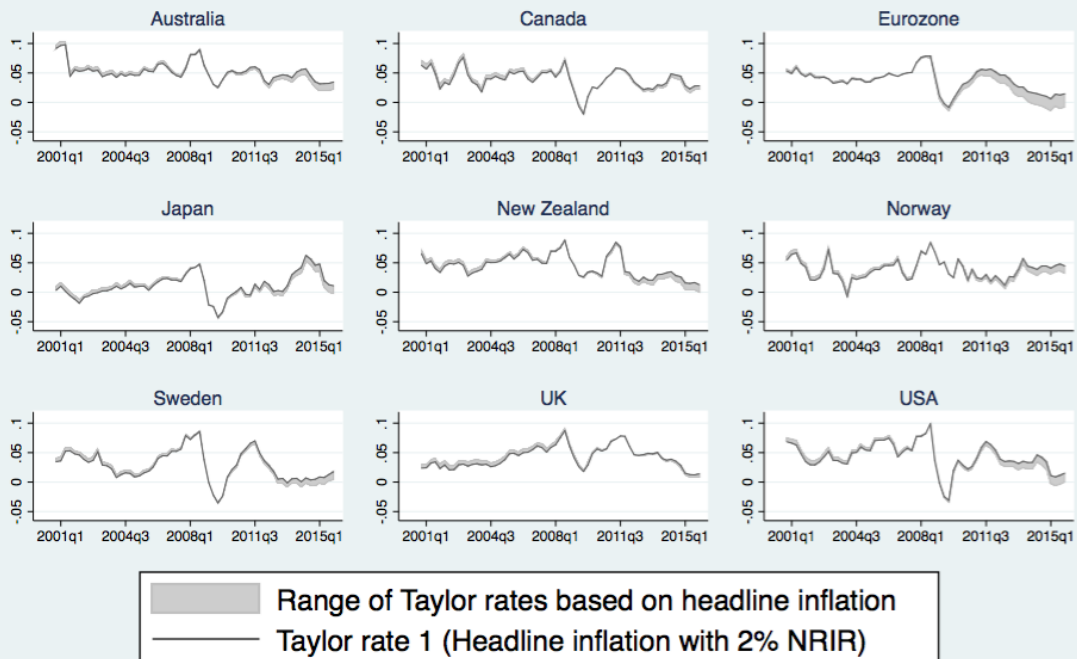
The range of Taylor rates using headline inflation is defined as the minimum and maximum levels of Taylor rate 1 and 2 for each point in time. Sample period: 2001Q1 - 2015Q4

Figure 3.6: Taylor rates based on core inflation

Range of Taylor rates based on core inflation

Taylor rate 3 (Core inflation with 2% NRIR)
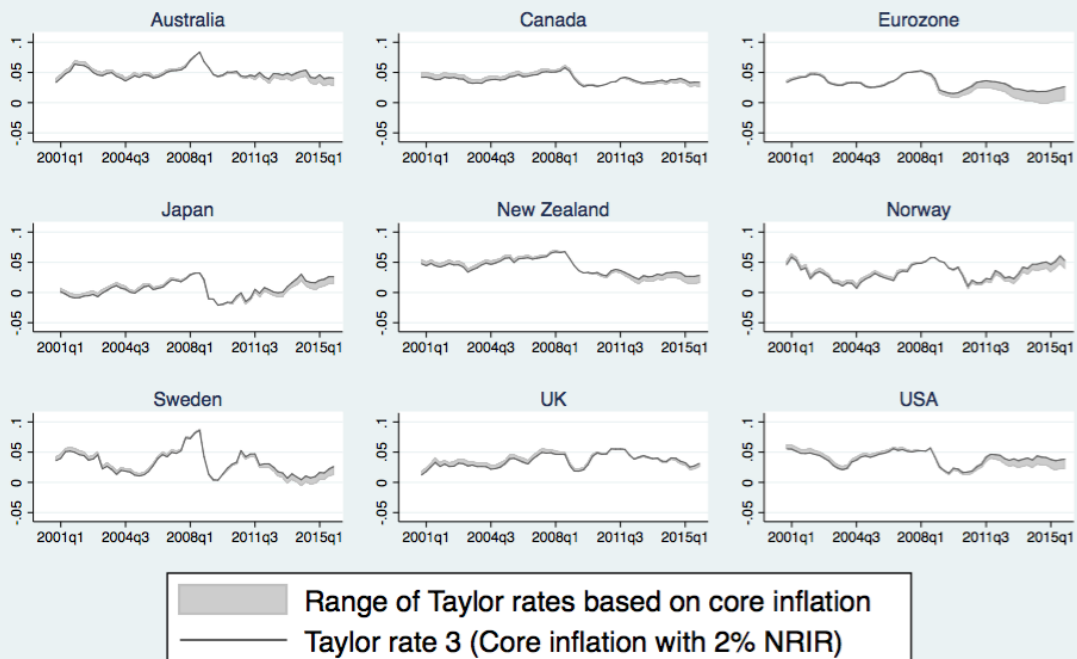
The range of Taylor rates using core inflation is defined as the minimum and maximum levels of Taylor rate 3 and 4 for each point in time. Sample period: 2001Q1 - 2015Q4

3.6 Length of time series

The Levin, Lin, Chu unit root test for panel data, which will be discussed in chapter 4.3.2, requires a strongly balanced panel data set. To achieve a strongly balanced panel data set, we start our sample in 2001 Q1 for all series, as this corresponds with some of the shorter core inflation series. The Hodrick-Prescott filter (HP) has poor reliability of the end of sample estimates (ECB, 2000), as is uses two-sided filtering (Bjørnland, Brubakk, & Jore, 2005). We have therefore omitted the four end quarters of the GDP gap estimates to avoid potential biased results, ending our time series in 2015 Q4. The selected length of our time series used in our analysis is therefore from 2001 Q1 to 2015 Q4.

4.   Method

4.1 Panel data methods

Our panel data set contains quarterly time series observations for nine economies. Using panel data poses several advantages. First, it allows for the possibility to increase the sample size. Second, because there most likely are unobserved country specific effects in the panel data set, we cannot assume that the observations are independently distributed across time. However, panel data methods allow us to use different estimators like the fixed effects (FE) estimator to control for such unobserved effects (Wooldrige, 2009). In addition to FE, we will also use first differenced (FD) time series, which also controls for time constant economy specific effects. Third, using panel data times series methods, can also, under certain conditions, mitigate for unit root problems in the individual time series, see Levin, Lin & Chu (2002) or Hsiao (2013).

4.1 Fixed effects estimator (FE)

The economies of our samples are of various size and composition with respect to GDP, imports, exports, industries etc. This might alter each central banks' incentive to carry out a more, or less aggressive monetary policy when compared to an optimal monetary policy proxy. If some of these attributes do not change over time, one might expect that there are economy specific time invariant effects (a_i) in the sample data. If these effects are correlated with one or several of the explanatory variables, one can use the fixed effects (FE) transformation to control for these effects. If they are not correlated with the explanatory variables, one can use the Random effects (RE) estimator. However, as the time dimension (T) gets large, the FE and RE estimates tend to be very similar (Wooldridge, 2009). Therefore, we will default to the FE transformation when controlling for possible economy specific fixed effects. See appendix chapter 4.1 for a more detailed presentation of the FE transformation.

When N is small and T is large, which is the case for our sample, one must use the fixed effect estimator with caution. In such cases inference is highly sensitive to violations of the

underlying assumptions, particularly unit root problems, which might lead to spurious regression (Wooldridge, 2009). However, as we are attempting to assess the average Taylor interest rate gap in certain periods by using time-period dummies, we argue that the interpretation of our model estimates might be more robust, even in the light of possible unfulfilled assumptions.

4.2 First differencing (FD)

Using the first difference (FD) can also remove time invariant economy specific effects in the same manner as using the FE estimator (Wooldridge, 2009). Further, if a time series is integrated of order one I(1), one can apply the first difference method to mitigate a unit root problem. In such cases the first difference of the process is weakly dependent and often stationary (Wooldridge, 2009). In our model, the constant term is interpreted as the time trend, as this is the estimated constant difference between two periods across the entire time series.

4.3 Assumptions for unbiased and valid estimates

4.3.1 Serial correlation and heteroskedasticity

The presence of heteroskedasticity or serial correlation does not cause the coefficient values to be biased, however it results in invalid error terms and test statistics (Wooldridge, 2009). To avoid potential heteroskedasticity or serial correlation, we use clustered error terms in the FE estimates, and robust standard errors in first difference models (Hoechle, 2007). By applying the first difference method, one can reduce problems of serial correlation of order one AR(1) in the error term (Wooldridge, 2009).

4.3.2 Unit root

An important assumption for applying OLS is that the time series are stationary. Violating this assumption might lead to spurious regression problems[10]. To test for unit roots in the individual time series we use the Augmented Dickey Fuller test (ADF). When considering unit root tests for panel data time series, we use a panel data unit root test developed by Levin, Lin & Chu (2002). According to Levin, Lin & Chu (2002) the power of panel based unit root tests are much higher compared to tests for unit roots in each individual time series, as one can draw on the central limit theorem, when increasing the number of observations. Some panel data unit root tests such as Harris & Tzavalis (1996), uses the central limit theorem by increasing number of individual series N, while keeping the time dimension T constant. This will not work with our sample of 9 economies.

Levin, Lin & Chu's (2002) shows how increasing the time dimension T, even for moderate sample sizes N, also yields a powerful unit root test. If the model allows for individual specific fixed effects, which is the case with our model, then sample sizes of $N \approx 10$ and $T \approx 50$ yield a sufficient test strength. One of the assumptions underlying their findings is that the sample statistics are independently distributed around individuals i for each T. According to our theoretical framework this might not be the case, which potentially makes the test results invalid. However, by including time specific intercepts, this assumption can be somewhat relaxed to allow for a limited degree of dependence (Levin, Lin & Chu 2002).

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[10] See appendix chapter 8.3.2 for a more detailed discussion of unit root.

5. Analysis

In the previous chapters, we defined the data and described the methods we applied. Before we move to the empirical analysis, we will present a graphical illustration of the data series of interest, showing the dependent variable time series for each economy. Then in the following section, we will present our models and the corresponding results as well a discussion of our findings.

The dependent variable in our analysis is the "Taylor gap", which we have defined as the difference between the central bank key policy rate and the calculated Taylor rate. Thus, if the central bank has a lower rate than the Taylor rate, it results in a negative Taylor gap. To control for the underlying input factors, we have included four Taylor rates. Taylor rate 1 and 2 uses headline inflation, where Taylor rate 2 is adjusted for variation in the natural real interest rate (NRIR). Taylor rate 3 and 4 uses core inflation, where Taylor rate 4 includes adjustments for variation in NRIR.
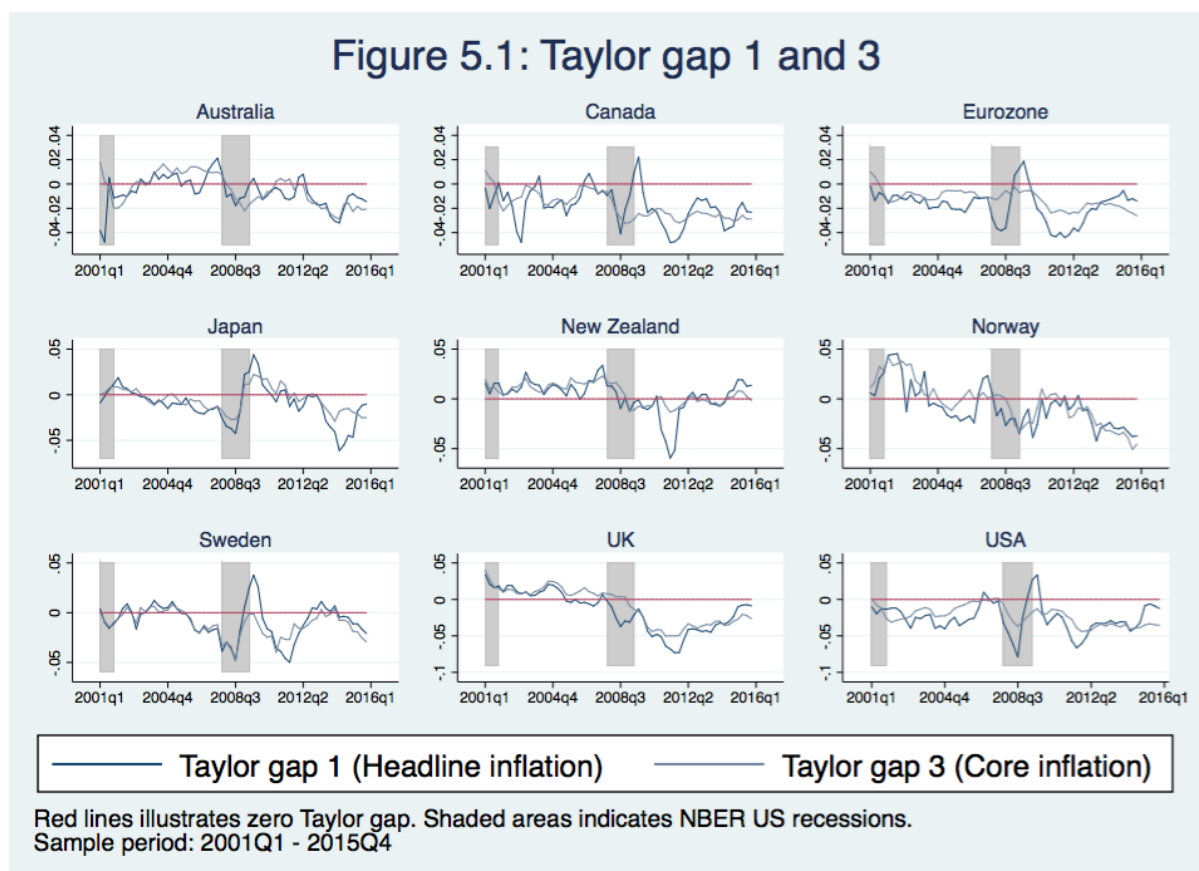

Figure 5.1: Taylor gap 1 and 3

Taylor gap 1 (Headline inflation)   Taylor gap 3 (Core inflation)

Red lines illustrates zero Taylor gap. Shaded areas indicates NBER US recessions.
Sample period: 2001Q1 - 2015Q4

Figure 5.1 presents the Taylor gaps based on the original Taylor rate using headline inflation (Taylor gap 1) and core inflation (Taylor gap 3)[11]. The shaded areas represent the two most recent NBER defined US recession, and is included for all sample economies. The rationale for this is to provide a visual illustration of possible key rate interdependence during the US recessions. From the figure, we see that there are indications of a drop below zero in Taylor gap 1 (Headline inflation) and Taylor gap 3 (Core inflation) during and after the recessions for several of the economies.

5.1 The models

From our theoretical framework, we expect that the magnitude of the Taylor gap will increase when a shock hits the economy. Thus, if an adverse shock hits the economy, we expect that the key rates will go significantly below the suggested Taylor rate. Correspondingly, this causes the magnitude of a negative Taylor gap to increase during a recession. The adverse shock might also lead to a prisoners' dilemma effect as described above, causing the negative Taylor gap to persist over time. We therefore include time dummies for the actual recession as well as the aftermath of three consecutive years. Further, graphical analysis of the cross-sectional Taylor gap time series in figure 5.1, indicates that there has been a negative time trend. The time trend appears less clear when controlling for variation in NRIR, see appendix chapter 8.4. To test our hypothesis, including a potential time trend, we used the following model:

Model 1: ***Fixed effects model***
(5.1) $$tgap_{i,t}^{j} = \beta_0 + \beta_1 D_1 + \beta_2 D_2 + \beta_3 D_3 + \beta_4 D_4 + \beta_5 Quarter + \alpha_{i,t} + \varepsilon_i$$

$$i = economy\ 1, \dots, 9 \quad t = quarter\ 1, \dots, 60 \quad j = Tgap\ version\ 1, \dots, 4$$

$D_1 = Dummy\ for\ the\ Dotcom\ Bubble\ (2001Q1 - 2001Q4)$

$D_2 = Dummy\ for\ the\ after\ math\ of\ the\ Dotcom\ Bubble\ (2002Q1 - 2004Q4)$

$D_3 = Dummy\ for\ the\ Financial\ Crisis\ (2007Q4 - 2009Q2)$

$D_4 = Dummy\ for\ the\ after\ math\ of\ the\ Financial\ Crisis\ (2009Q3 - 2012Q2)$

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[11] In appendix chapter 8.4, we have included another version of Figure 5.1, which includes adjustment for changes in the natural real interest rate (NRIR).

In our model the dummies are supposed to pick up any possible deviation across the economies that cannot be explained by a time trend. The dummies included are for the US recession following the Dotcom Bubble and the Financial Crisis, and the aftermath of those two recessions. The aftermath is defined as the three years following the trough of the recession.

To control for potential economy specific effects in model 1, we use the fixed effects estimator (FE). Applying clustered robust standard errors avoids potential problems of heteroskedasticity and serial correlations in the error term. As there is some evidence of non-stationarity in the individual series, we have also used a first difference (FD) approach to our analysis. Using FD also controls for potential time fixed effects. Further, FD naturally omits the time trend variable (quarter), as the trend variable in FD is represented by the constant term.

Our FD model given by (5.3) is slightly modified compared to the FE model (5.1). We exchange the period dummies in (5.1) for individual quarterly dummies in (5.3), which allows us to get a better assessment of the time varying Taylor gap dynamics from quarter to quarter. Thus, we start out with the following model which we apply the FD method to:

(5.2) $$tgap_{k,t}^{j} = \beta_1 D_1 + \cdots + \beta_{34} D_{34} + u_{it}$$

Model 2:

(5.3) $$\Delta tgap_{k,t}^{j} = \beta_0 + \beta_1 \Delta D_{1i} + \cdots + \beta_{34} \Delta D_{34} + \Delta u_{it}$$

$$k = 1, \dots, 4, \quad i = 1, \dots, 9, \quad t = 1, \dots, 540$$

In the FD model, β_0 is included to account for a potential time trend in the Taylor gaps. Dummy $1 - 16$ accounts for the Dotcom bubble recession and the aftermath of three years, a total of 16 quarters. Dummy $17 - 34$ accounts for the financial crisis and the aftermath of three years, a total of 18 quarters.

5.2 Empirical evidence

Table 5.1 presents the results from model 1 (5.1) and table 5.2 presents the results from model 2 (5.3). These results are graphically illustrated in figure 5.2 and 5.3. The discussion of the estimates is divided into three sections: The Dotcom bubble recession, the Financial crisis recession and a discussion about a possible time trend.

Tests statistics for unit root tests are included in appendix chapter 8.3. Estimating Taylor gap 3 and 4 based on core inflation, using fixed effects (FE) estimation, shows some evidence of a unit root with a p-value of 0.12 and 0.05 respectively. The Taylor gap using headline inflation rejects the null hypothesis of a unit root at a 1 % level. This is congruent with our expectation that the core inflation based Taylor gaps are less volatile and thus might tend to be slightly more persistent over time. Applying First Difference (FD), the null hypothesis of a unit root is clearly rejected for all versions of the Taylor gap. Therefore, the estimates from model 2 (5.3) in table 5.2 using FD estimation are considered somewhat more robust than those of model 1 (5.1) in table 5.1, especially when considering Taylor gap 3 and 4.

Table 5.1: *Panel data regression of key policy rates deviation from the Taylor rule using Fixed Effects estimator with clustered SE*

	(1) T-gap 1	(2) T-gap 2	(3) T-gap 3	(4) T-gap 4
Dummy for the Dotcom Bubble (2001Q1 - 2001Q4)	-0.00256 (0.00468)	0.00333 (0.00481)	-0.00707* (0.00371)	-0.00118 (0.00384)
Dummy for the aftermath of the Dotcom Bubble (2002Q1-2004Q4)	-0.000343 (0.00336)	0.00302 (0.00344)	-0.00671 (0.00386)	-0.00335 (0.00389)
Dummy for the Financial Crisis (2007Q4 - 2009Q2)	-0. 00669*** (0.00176)	-0.00761*** (0.00174)	-0.00560** (0.00226)	-0.00652** (0.00223)
Dummy for the aftermath of the Financial Crisis (2009Q3-2012Q2)	-0.00756 (0.00592)	-0.00804 (0.00591)	-0.00321 (0.00451)	-0.00369 (0.00450)
Quarter	-0.000382*** (0.0000905)	0.0000296 (0.000113)	-0.000641*** (0.000118)	-0.000230 (0.000132)
Constant	0.00239 (0.00300)	-0.0104** (0.00380)	0.0137** (0.00436)	0.000930 (0.00483)
R^2	0.178	0.057	0.395	0.107
Adjusted R^2	0.170	0.048	0.389	0.099
Observations	540	540	540	540

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, Sample period: 2001Q1 – 2015Q4.
T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR.
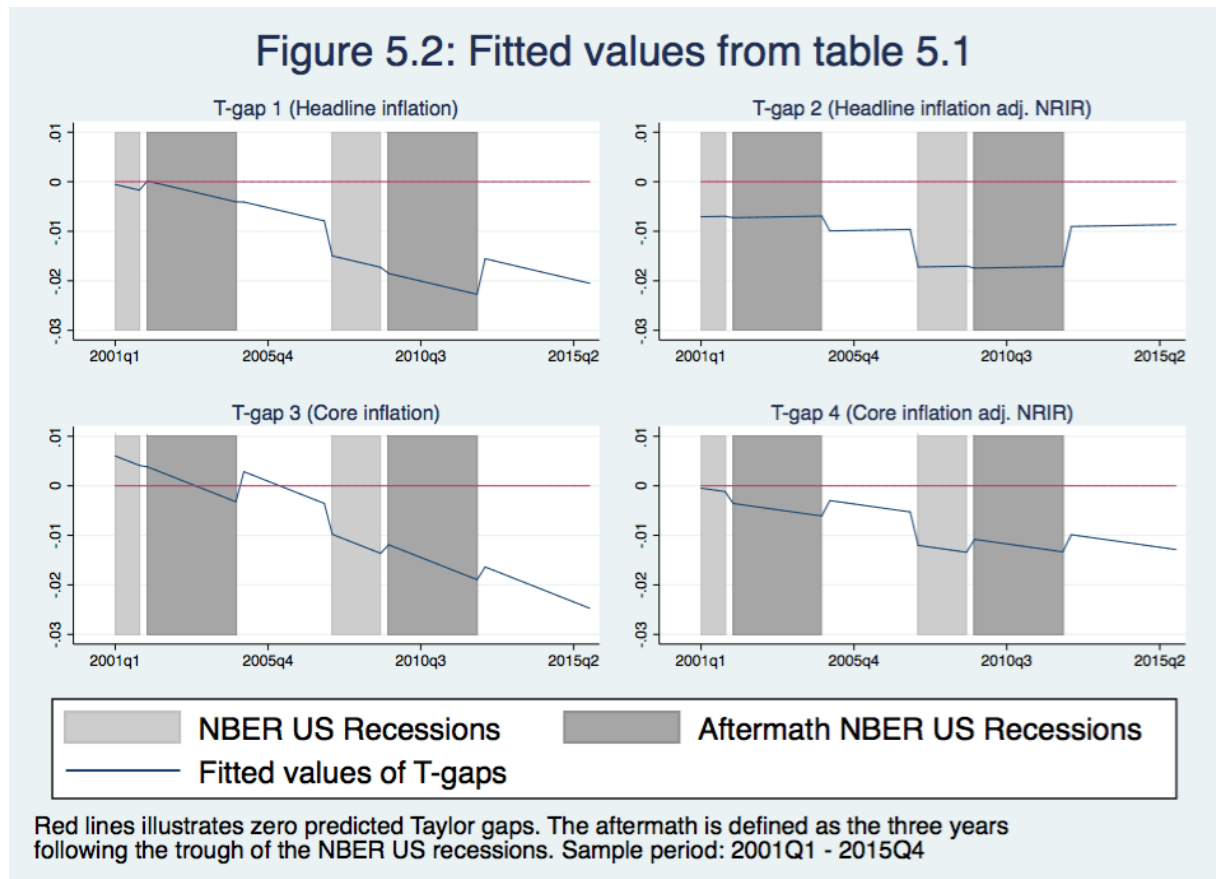

Figure 5.2: Fitted values from table 5.1

Red lines illustrates zero predicted Taylor gaps. The aftermath is defined as the three years following the trough of the NBER US recessions. Sample period: 2001Q1 - 2015Q4

Table 5.2: *Panel data regression of key policy rates deviation from the Taylor rule using first differenced series*

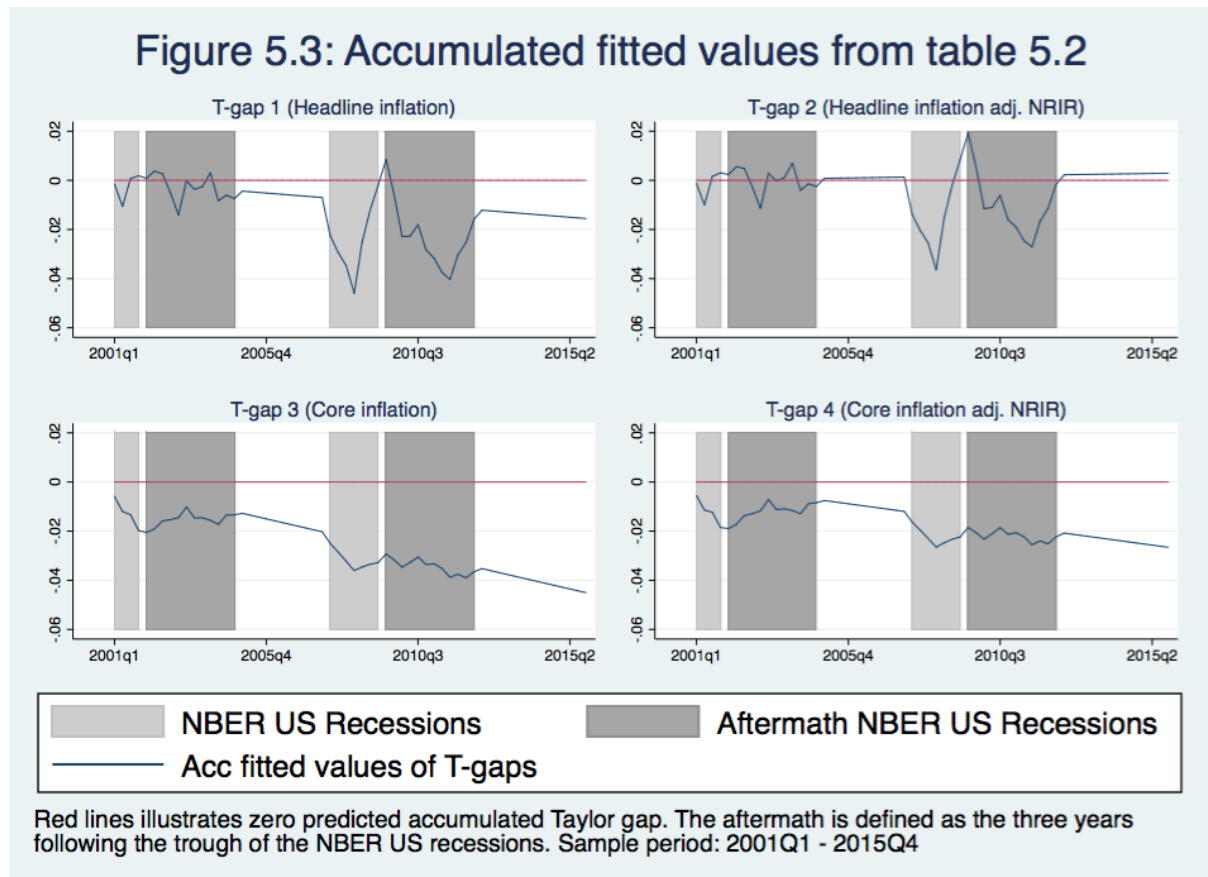|	(1) T-gap 1	(2) T-gap 2	(3) T-gap 3	(4) T-gap 4
Constant	-0.000259 (0.000387)	0.0000484 (0.000387)	-0.000750*** (0.000240)	-0.000443* (0.000240)
Dotcom Bubble (2001Q1 – 2001Q4)				
D.Dummy 2001Q1	-0.00132 (0.00259)	-0.00132 (0.00259)	-0.00515*** (0.00154)	-0.00512*** (0.00155)
D.Dummy 2001Q2	-0.0101*** (0.00342)	-0.0101*** (0.00342)	-0.0105*** (0.00255)	-0.0105*** (0.00256)
D.Dummy 2001Q3	0.00149 (0.00577)	0.00149 (0.00577)	-0.0110*** (0.00326)	-0.0110*** (0.00327)
D.Dummy 2001Q4	0.00287 (0.00592)	0.00287 (0.00592)	-0.0168*** (0.00363)	-0.0168*** (0.00364)
The aftermath of the Dotcom Bubble (2002Q1-2004Q4)				
D.Dummy 2002Q1	0.00214 (0.00618)	0.00214 (0.00618)	-0.0168*** (0.00381)	-0.0167*** (0.00382)
D.Dummy 2002Q2	0.00528 (0.00613)	0.00528 (0.00614)	-0.0146*** (0.00376)	-0.0145*** (0.00377)
D.Dummy 2002Q3	0.00448 (0.00608)	0.00448 (0.00608)	-0.0106*** (0.00367)	-0.0106*** (0.00368)
D.Dummy 2002Q4	-0.00309 (0.00607)	-0.00309 (0.00607)	-0.00934*** (0.00361)	-0.00932** (0.00362)
D.Dummy 2003Q1	-0.0118** (0.00596)	-0.0118** (0.00596)	-0.00779** (0.00353)	-0.00778** (0.00354)
D.Dummy 2003Q2	0.00239 (0.00556)	0.00239 (0.00557)	-0.00267 (0.00336)	-0.00266 (0.00337)
D.Dummy 2003Q3	-0.000855 (0.00519)	-0.000855 (0.00519)	-0.00637** (0.00315)	-0.00636** (0.00316)
D.Dummy 2003Q4	0.000573 (0.00493)	0.000573 (0.00494)	-0.00558* (0.00298)	-0.00557* (0.00298)
D.Dummy 2004Q1	0.00640 (0.00448)	0.00640 (0.00448)	-0.00584** (0.00268)	-0.00584** (0.00268)
D.Dummy 2004Q2	-0.00471* (0.00276)	-0.00471* (0.00277)	-0.00673*** (0.00221)	-0.00672*** (0.00222)
D.Dummy 2004Q3	-0.00214 (0.00211)	-0.00214 (0.00211)	-0.00221 (0.00189)	-0.00221 (0.00190)
D.Dummy 2004Q4	-0.00330** (0.00158)	-0.00330** (0.00159)	-0.00135 (0.00128)	-0.00135 (0.00128)

The Financial Crisis (2007Q4 – 2009Q2)

D.Dummy 2007Q4	-0.0153***	-0.0153***	-0.00398*	-0.00398*
	(0.00254)	(0.00252)	(0.00219)	(0.00218)
D.Dummy 2008Q1	-0.0219***	-0.0219***	-0.00678**	-0.00678**
	(0.00414)	(0.00412)	(0.00273)	(0.00273)
D.Dummy 2008Q2	-0.0270***	-0.0270***	-0.00975***	-0.00975***
	(0.00488)	(0.00487)	(0.00312)	(0.00312)
D.Dummy 2008Q3	-0.0379***	-0.0379***	-0.0128***	-0.0128***
	(0.00533)	(0.00533)	(0.00346)	(0.00346)
D.Dummy 2008Q4	-0.0168**	-0.0168**	-0.0106**	-0.0106**
	(0.00661)	(0.00661)	(0.00442)	(0.00442)
D.Dummy 2009Q1	-0.00353	-0.00353	-0.00869	-0.00869
	(0.00757)	(0.00757)	(0.00543)	(0.00544)
D.Dummy 2009Q2	0.00669	0.00669	-0.00729	-0.00729
	(0.00777)	(0.00777)	(0.00538)	(0.00539)

The aftermath of the Financial Crisis (2009Q3- 2012Q2)

D.Dummy 2009Q3	0.0176**	0.0176**	-0.00301	-0.00301
	(0.00773)	(0.00773)	(0.00529)	(0.00530)
D.Dummy 2009Q4	0.00351	0.00351	-0.00469	-0.00469
	(0.00789)	(0.00788)	(0.00525)	(0.00525)
D.Dummy 2010Q1	-0.0133*	-0.0133*	-0.00690	-0.00690
	(0.00776)	(0.00775)	(0.00520)	(0.00520)
D.Dummy 2010Q2	-0.0129*	-0.0129*	-0.00421	-0.00421
	(0.00759)	(0.00758)	(0.00513)	(0.00513)
D.Dummy 2010Q3	-0.00796	-0.00796	-0.00127	-0.00127
	(0.00737)	(0.00736)	(0.00497)	(0.00497)
D.Dummy 2010Q4	-0.0180**	-0.0180**	-0.00357	-0.00357
	(0.00703)	(0.00702)	(0.00476)	(0.00476)
D.Dummy 2011Q1	-0.0210***	-0.0210***	-0.00251	-0.00251
	(0.00660)	(0.00659)	(0.00430)	(0.00430)
D.Dummy 2011Q2	-0.0266***	-0.0266***	-0.00369	-0.00369
	(0.00628)	(0.00626)	(0.00416)	(0.00416)
D.Dummy 2011Q3	-0.0292***	-0.0292***	-0.00651*	-0.00651*
	(0.00593)	(0.00592)	(0.00381)	(0.00381)
D.Dummy 2011Q4	-0.0188***	-0.0188***	-0.00452	-0.00452
	(0.00463)	(0.00462)	(0.00327)	(0.00326)
D.Dummy 2012Q1	-0.0136***	-0.0136***	-0.00524**	-0.00524**
	(0.00312)	(0.00310)	(0.00214)	(0.00214)
D.Dummy 2012Q2	-0.00395	-0.00395	-0.00198	-0.00198
	(0.00264)	(0.00263)	(0.00192)	(0.00192)
R^2	0.391	0.391	0.149	0.148
Adjusted R^2	0.349	0.349	0.089	0.089
Observations	540	540	539	539

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, Sample period: 2001Q1 – 2015Q4.
T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR.

Figure 5.3: Accumulated fitted values from table 5.2

Red lines illustrates zero predicted accumulated Taylor gap. The aftermath is defined as the three years following the trough of the NBER US recessions. Sample period: 2001Q1 - 2015Q4

5.2.1 Early 2000 recession: The Dotcom bubble recession

NBER's dating of US business cycles defines the recession in the early 2000 to span from March 2001 (2001 Q1) to November 2001 (2001 Q4). We refer to this recession as the dot-com bubble recession, as it is "*associated with the bursting of the dot-com bubble in the US*" (Aastveit, Jore & Ravazzolo, 2016). The dot-com bubble in the US burst late 2000 (2000 Q4) after the NASDAQ index, a major index for the US stock market, peaked in march 2000 and underwent a "huge and bloody bath" in the following years (DeLong & Magin, 2006). The recession followed shortly after the peak for the US stock market and spanned over a relatively short time frame.

From table 5.1 and figure 5.2 we see that there is hardly any evidence for a significant drop in central bank rates below the Taylor rate during and after the Dotcom bubble recession using FE estimation. This can be evaluated further in table 5.3 below, which displays the fitted values. Here the deviations are mostly negative, but close to zero. Only for core inflation (T-gap 3), we get significant result on a 10 % level, however, the estimates suggest a positive gap

which contrary to our expectations. This indicate that there is not sufficient evidence to conclude about a potential suboptimal Nash behavior between central banks in this recession using our FE model. On the other hand, this estimate might be affected by the constant term, which is positive, potentially affecting the estimates early in the time period. Further, as discussed before, even though panel data unit root test give good results, our fixed effects estimates might be prone to violations of the tests assumptions, especially for Taylor gap 3. Thus, evidence of unit roots in the individual series might bias the coefficients size and statistical inference using the model.

Table 5.3: *Fitted values for the Dotcom bubble, FE (Model 1)*

	T-gap 1	T-gap 2	T-gap 3	T-gap 4
Dotcom bubble				
Peak (2001 Q1)	-0,06 %	-0,70 %	0,60 %[*]	-0,05 %
Trough (2001 Q4)	-0,17 %	-0,70 %	0,41 %[*]	-0,12 %
Dotcom aftermath (3 Years)				
Start (2002 Q1)	0,01 %	-0,72 %	0,38 %	-0,36 %
End (2004 Q4)	-0,41 %	-0,69 %	-0,33 %	-0,61 %

Source: Table 5.1. T-gaps estimated by multiplying the constant coefficient with quarter number, then adding the respective dummy coefficients. [*] indicate dummy significance level where * p < 0.10, ** p < 0.05, *** p < 0.01. See figure 5.2 for graphical illustration. T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR.

T**able 5.4***: Accumulated difference for Dotcom bubble recession using FD*

Quarter	T-gap 1	T-gap 2	T-gap 3	T-gap 4
2001 Q1	-0.16%	-0.13%	-0.59%[***]	-0.56%[***]
2001 Q2	-1.06%[***]	-1.00%[***]	-1.2%[***]	-1.14%[***]
2001 Q3	0.07%	0.16%	-1.33%[***]	-1.23%[***]
2001 Q4	0.18%	0.31%	-1.98%[***]	-1.86%[***]

Source: Table 5.2 [*] indicate dummy significance level where [*] $p < 0.10$, [**] $p < 0.05$, [***] $p < 0.01$. T-gaps estimated by multiplying the constant coefficient with quarter number, then adding the respective dummy coefficients. T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR.

By applying first difference (FD) and quarterly dummies, we see from table 5.2 that there is more evidence for a significant negative gap from the Taylor rate when using core inflation based rates, compared to headline based Taylor rates. It is natural to assume that most central banks put more weight on core inflation, as it is believed to be a better indicator of future inflation in the medium term (Yellen, 2015). Therefore, the significance of the negative gaps using core inflation is a potential argument for sub-optimal key policy rates in this period. Table 5.4 above presents the accumulated fitted values using first differenced series. This is

estimated by accumulating the constant time trend and adding the respective quarter dummies for the respective quarters. Here, we see that the magnitude of the negative Taylor gap increased from -0.6 to nearly -2.0 percentage points using core inflation (T-gap 3 and 4). All the dummy coefficients for these two series are significant on a 1 % level. This indicates that the central banks have lowered their interest rates more aggressively during the Dotcom bubble recession than what the Taylor rate suggests. Using headline inflation based Taylor rates gives no clear indication of a negative gap across our sample economies for the period, except for 2001 Q1, where the negative interest rate drop was significant. Including NRIR adjustments does not change the quarterly dummy coefficients in table 5.2, nor the accumulated difference in table 5.4 significantly.

Overall there are weak signals regarding a sub-optimal key policy rate level during the Dotcom recession. Further, the signals are mixed when comparing headline and core inflation based estimates. We find no significant Taylor gaps using headline inflation. However, when using first differenced series (FD) and core inflation, there are significant negative Taylor gaps for the Dotcom recession. We have argued that our FD estimates are more reliable, especially concerning core inflation estimates. Therefore, this strengthens an argument for sub-optimal key policy rates during the Dotcom recession. Thus, we carefully conclude that there might have been sub-optimal key policy rates during the Dotcom recession.

The aftermath of the Dotcom bubble using fixed effects (FE) estimation show no evidence of a significant negative Taylor gap, see table 5.1 or 5.3. Further, first differenced (FD) series, gives no clear indication of negative Taylor gaps based on headline inflation (T-gap 1 and T-gap 2), see table 5.2 above and table 5.5 below. However, core inflation based Taylor gaps (T-gap 3 and T-gap 4) in table 5.2 and table 5.5 clearly indicate that the interest rates were below Taylor rates. As we have argued before, core inflation estimates might be weighted more than headline estimates by the central bank. If one take such a position, there is a strong indication that key policy rates were lower than the Taylor rate also after the Dotcom recession. Thus, one might claim that this is potential evidence of a suboptimal prisoners' dilemma among the central banks. If so, this might have hindered them from returning to higher levels after the trough of the recession.

Table 5.5: *Accumulated difference for Dotcom bubble recession aftermath using FD*

Quarter	T-gap 1	T-gap 2	T-gap 3	T-gap 4
2002 Q1	0.08%	0.24%	-2.06%***	-1.89%***
2002 Q2	0.37%	0.56%	-1.91%***	-1.72%***
2002 Q3	0.27%	0.48%	-1.56%***	-1.37%***
2002 Q4	-0.52%	-0.27%	-1.53%***	-1.29%**
2003 Q1	-1.41%**	-1.14%**	-1.45%**	-1.18%**
2003 Q2	-0.02%	0.29%	-1.02%	-0.71%
2003 Q3	-0.37%	-0.03%	-1.46%**	-1.12%**
2003 Q4	-0.25%	0.12%	-1.46%**	-1.09%*
2004 Q1	0.30%	0.70%	-1.56%***	-1.16%**
2004 Q2	-0.83%*	-0.4%*	-1.72%***	-1.29%***
2004 Q3	-0.60%	-0.14%	-1.35%	-0.89%
2004 Q4	-0.74%**	-0.25%**	-1.34%	-0.84%

Source: Table 5.2 * indicate dummy significance level where * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. T-gaps estimated by multiplying the constant coefficient with quarter number, then adding the respective dummy coefficients. T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR

Combined, we will argue that these results give somewhat unclear evidence for a significant negative Taylor gaps in the period after the Dotcom bubble recessions. However, if one bases the argument solely on core inflation Taylor rates, the evidence toward sub-optimal key rates appears stronger. One possible reason for the mixed signals can be visually derived from our graphs presenting the inflation and the GDP time series. It might seem as there are small or no negative gaps in GDP and inflation for several of our sample economies during and after the Dotcom bubble recession. This might, according to our theoretical framework, reduce incentives for the economies in our sample to lower their interest rates below the Taylor rate. Thus, we carefully conclude that there is only weak evidence supporting an argument about a prisoners' dilemma in this period.

5.2.2 Financial crisis

NBER defined the US recession following the financial crisis to start at the peak in December 2007 (2007 Q4) and ended at with the trough in June 2009 (2009 Q2). Table 5.1 presents the FE estimates, and table 5.2 the FD estimates for both the financial crisis recession and the aftermath for the various Taylor gaps. Using FE estimation, the dummy coefficients are negative and significant for the financial crisis recession. By assessing the coefficients, the Taylor gap magnitude based on headline inflation (T-gap 1 and T-gap 2) increased negatively

by approximately –0.7 percentage points. When controlling for variation in the natural interest rate (T-gap 2) the magnitude of the negative response is marginally smaller relative to the Taylor rate. The core inflation based gaps (T-gap 3 and T-gap 4) are also negative and significant on a 5 % level, indicating that the magnitude of the negative Taylor gap increased by approximately -0.6 percentage points.

To calculate the FE fitted values for the period, we have included the constant and the trend component in Table 5.6. Here the estimated average Taylor gap based on headline inflation, range from -1.5 and -1.7 percentage points for T-gap 1 and T-gap 2 respectively. This suggests that the average key rates were significantly below the suggested Taylor rates. When using core inflation (T-gap 3 and T-gap 4), the magnitude of the negative gaps was somewhat smaller. This can seem intuitively wrong as they exclude energy prices which dropped significantly due to the oil price in the period. This might indicate that during the period of the crisis the central banks pushed the interest rate below what was optimal, which is in line with the expectations of our theoretical framework.

Table 5.6: *Fitted values for the Financial crisis, FE (model 1)*

	T-gap 1	T-gap 2	T-gap 3	T-gap 4
Financial Crisis				
Peak (2007 Q4)	-1,50 %[***]	-1,72 %[***]	-0,98 %[**]	-1,20 %[**]
Trough (2009 Q2)	-1,73 %[***]	-1,70 %[***]	-1,37 %[**]	-1,34 %[**]
Financial Crisis AM (3 Years)				
Start (2009 Q3)	-1,85 %	-1,74 %	-1,19 %	-1,08 %
End (2012 Q2)	-2,27 %	-1,71 %	-1,90 %	-1,33 %

Source: Table 5.1. T-gaps estimated by multiplying the constant coefficient with quarter number, then adding the respective dummy coefficients. [*] indicate dummy significance level where * p < 0.10, ** p < 0.05, *** p < 0.01. See figure 5.2 for graphical illustration. T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR

Table 5.2 presents the result from the FD analysis for the financial crisis. Here we see that the FD coefficients for the financial crisis recession dummies are all negative and mostly significant at a 1 % level, except for the last two quarters, 2009 Q1 and Q2. The magnitude of the negative gaps is slightly smaller considering a core inflation based rate compared to headline inflation. The magnitude of the estimated negative gaps increases from 2007 Q4 and reached a peak in 2008 Q3.

In table 5.7 the accumulate fitted values from the first differences series are displayed. We estimate that the average central bank rates in our sample went from approximately -2.3 to -4.6 percentage points below the suggested Taylor rate based on headline inflation (T-gap 1) between 2007 Q4 to 2008 Q3. Using core inflation based Taylor rates (T-gap 3 and T-gap 4) the estimated magnitude of the gaps were significant and only marginally lower than the corresponding gaps using headline inflation. By accounting for natural real interest rate (NRIR) adjustments in T-gap 2 and T-gap 4, relative to T-gap 1 and T-gap 3 respectively, the magnitude of the gaps was reduced by approximately 1 percentage point, but still significantly negative. The gap dummies in 2009 Q1 and Q2 are not significantly different from zero when considering headline nor core inflation.

Table 5.7: *Cumulative difference for Financial crisis recession using FD estimates*

Quarter	T-gap 1	T-gap 2	T-gap 3	T-gap 4
2007 Q4	-2.26%***	-1.39%***	-2.5%*	-1.64%*
2008 Q1	-2.94%***	-2.05%***	-2.85%**	-1.96%**
2008 Q2	-3.48%***	-2.55%***	-3.23%***	-2.3%***
2008 Q3	-4.59%***	-3.64%***	-3.61%***	-2.65%***
2008 Q4	-2.51%**	-1.53%**	-3.46%**	-2.48%**
2009 Q1	-1.21%	-0.19%	-3.34%	-2.33%
2009 Q2	-0.21%	0.83%	-3.28%	-2.24%

Source: Table 5.2. * indicate dummy significance level where * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. T-gaps estimated by multiplying the constant coefficient with quarter number, then adding the respective dummy coefficients. T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR.

One can notice from table 5.7 that the magnitude of the negative Taylor gap for 2009 Q1 and Q2 is larger for T-gap 3 and T-gap 4, compared to T-gap 1 and T-gap 2 respectively. This is a result of a significant time trend for core inflation, which is negative and larger in magnitude compared to headline inflation. Thus, resulting in a larger negative gap, despite the lack of significant response dummies. If the central banks were to base their key rates solely on core inflation, this might indicate that they have been continuously moving their rates further and further below the Taylor rate. We will discuss the time trend in more detail below.

The aftermath period after the Financial crisis recession is defined as the three years after the trough. In this period, the FE estimates suggest that the key policy rates did not drop

significantly below the Taylor rate, when controlling for a time trend. This might indicate that the aftermath key rates were not suboptimal compared to the various versions of the Taylor rate. On the other hand, it might be related to the sudden positive spike in interest rates relative to the Taylor rate right after the financial crisis for many of our sample economies. One possible explanation is that our Taylor rates are not based on forward looking estimates, which might cause a natural delay between our Taylor rates and the central bank responses. Another potential explanation is that the monetary policy rates were dramatically lowered in two rounds, while the Taylor rate caught up in between rounds, resulting in the positive spike.

Table 5.8: *Cumulative difference for Financial crisis recession aftermath using FD estimates*

Quarter	T-gap 1	T-gap 2	T-gap 3	T-gap 4
2009 Q3	0.85%**	1.93%**	-2.93%	-1.85%
2009 Q4	-0.58%	0.50%	-3.17%	-2.06%
2010 Q1	-2.29%*	-1.15%*	-3.47%	-2.33%
2010 Q2	-2.27%*	-1.11%*	-3.27%	-2.10%
2010 Q3	-1.81%	-0.61%	-3.05%	-1.85%
2010 Q4	-2.84%**	-1.61%**	-3.36%	-2.13%
2011 Q1	-3.16%***	-1.90%	-3.33%	-2.07%
2011 Q2	-3.75%***	-2.46%***	-3.52%	-2.23%
2011 Q3	-4.03%***	-2.71%***	-3.88%*	-2.56%*
2011 Q4	-3.02%***	-1.67%***	-3.75%	-2.40%
2012 Q1	-2.53%***	-1.14%***	-3.9%*	-2.52%*
2012 Q2	-1.59%	-0.17%	-3.65%	-2.24%

Source: Table 5.2. * indicate dummy significance level where * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Accumulated difference is estimated using the constant first difference accumulated over time plus the respective quarter dummy coefficients. T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR

The cumulative fitted values for the financial crisis aftermath, using FD estimation, is displayed in table 5.8. The dummies for the five first quarters, 2009 Q3 to 2010 Q3, show little or no evidence of significant negative Taylor gaps. The following six quarters, 2010 Q4 to 2012 Q1 on the other hand show clear evidence of a negative interest rate response in the period relative to the Taylor rate based on headline inflation. This result is slightly reduced but still significant when controlling for variations in the natural real interest rate in T-gap 2. Using core inflation based Taylor rates (T-gap 3 and T-gap 4), we find almost no significant key policy response after the financial crisis recession.

For the headline inflation based estimates, a significant negative response comes first nearly a year after the trough. This can possibly be explained by the fact that we use actual inflation and GDP estimates for our Taylor rates, which have been revised and updated, while the various central banks use forward looking estimates (Koenig & Emery, 1991). This might cause our estimates to be somewhat uncorrelated in certain periods with the actual central bank rates. A possible explanation for the lack of significance for the quarterly dummies using core inflation during the financial crisis aftermath is that the volatility of core inflation is less than for headline inflation. In turn, this might explain a significant trend component and less significant key policy rate movement compared to the Taylor rate. Moreover, it is clear from table 5.8 that the trend component largely influences the core inflation based gaps (T-gap 3 and 4), which ends at -3.6 and -2.2 percentage points respectively in 2012 Q2.

Assessing the FE and the FD estimates for the Financial crisis recession, we find clear evidence for a key policy rate response below the suggested Taylor rate. This suggest that the central banks of the sample economies have carried out potential suboptimal monetary policy strategies during the period. For the aftermath period of three years after the trough, there are mixed evidence of significant negative responses, especially when considering core inflation based Taylor gaps (T-gap 3 and T-gap 4). However, when assessing the cumulative differences in table 5.8 for the whole period, the estimated Taylor gaps using core inflation are around -2 and -3 percentage point below the suggested Taylor rates. This can be explained by a negative time trends in the Taylor gaps. The magnitudes of these cumulative negative gaps or time trends are quite large, and might suggest that the monetary policy have become more sub-optimal over time. This will be discussed in the next section.

5.2.3 Trend component – monetary policy development

Some of the data for the various Taylor gaps gives an indication of a possible negative time trend. If present, such a trend suggests that the central bank key policy rates for our sample economies are moving further below the Taylor rate over time. In the FE estimates seen in table 5.1, the time trend (quarter) coefficient is significant for both headline and core inflation. On average, by summing up four quarters, the trend is approximately -0.15 and $-$ 0.26

percentage point per year for headline and core inflation respectively. When accounting for natural real interest rate (NRIR) variation, the trend estimates are no longer significant. This suggests vague evidence for a time trend using FE estimation.

When applying FD estimation, the time trend is now represented by the constant term, which is equivalent to the average linear change in the gaps between all periods in our sample. The FD estimates in table 5.2 yield significant evidence of a time trend using core inflation, also when using adjusted NRIR. Here the trend coefficients are approximately -0.3 percentage point per year using a fixed NRIR and -0.17 percentage point per year using adjusted NRIR. It is interesting to notice that even when controlling for NRIR variation, the core inflation based Taylor gap trend is significantly negative. This might indicate a downward biased trend in the key policy rates and thus also the monetary policy strategy of the sample economies central banks. When using headline inflation based Taylor gaps, we find no significant evidence of a time trend with and without NRIR adjustments. This might be an argument in disfavor of a downward sloping trend. On the other hand, headline inflation is more volatile which naturally might reduce the significance of a potential trend coefficient.

Combined, these results leave unclear evidence for the significance of a possible negative time trend in the key policy rates relative to the Taylor rate. The FE model are less conclusive, but also more prone to violations of the assumptions required for statistical inference. The first differences model (FD) estimates is arguably a more robust model for statistical inference, and are also more clear when considering a trend. Here three out of four T-gaps show signs of a possible negative trend, which argues in favor of a negative trend. Further, both core inflation based Taylor gap trends are significant and negative, even when controlling for NRIR. This further strengthens an argument for a downward biased trend in the monetary policies relative to the Taylor rate. However, The Taylor gap using total inflation adjusted for NRIR gives a positive trend close to zero, which makes it difficult to reach a clear conclusion.

5.3 Summarizing the results of our analyses

The Dotcom bubble recession and aftermath response dummies are mostly negative and significant when considering a core inflation based Taylor rule. Using a headline-inflation based Taylor rule, there is unclear evidence regarding negative gaps between the key policy rate and the Taylor rate. The financial crisis estimates are more clear as all the various Taylor gaps are significant and negative, suggesting that the interest rates responses for this period was suboptimal compared to the Taylor rate. This is consistent with a potential Nash-behavior of the various central banks. The financial crisis recession aftermath results in insignificant dummy coefficients for the first 1.5 years, but significant negative coefficients the last 1.5 years using total inflation, even when using adjusted NRIR.

The time trends coefficients are mostly negative and to some degree significant. With exception of Taylor gap 2, based on headline inflation and adjusted NRIR, which is positive but close to zero. To further examine a time trend in the interest rate responses, we also look at the response dummies over time. In table 5.1 and 5.2 the response dummy coefficients become significantly more negative and larger in magnitude over time. One potential explanation is that the financial crisis imposed stronger incentives for suboptimal monetary policy compared to the Dotcom bubble. On the other hand, when combined with the estimated negative time trends, we argue that there still is some evidence of central bank policy rates being more aggressive and negatively biased over time, compared to the suggested Taylor rate. This indicates that monetary policies have become more aggressive in the sense that central banks are more willing to underbid each other, causing a potential suboptimal Nash equilibrium in the global key policy rates.

Taylor (2013) argues that rule based monetary policies which emphasized the domestic economies, yielded successful results for monetary policies from the 1980's until lately. One reason for this, he argues, is that more rule based domestically focused monetary policies led to indirect international monetary policy coordination, since each central bank were committed to their rule. However, since early 2000's, he argues that that monetary policies have been less successful since interest rates are increasingly moving below what simple rule based rates would suggest. One reason for this he states, is that central banks have moved away from this

domestically focused rule-based monetary policies. This is in line with our findings and supports an argument that the potential lack of optimal coordination between central banks results in more aggressive monetary policies, Nash behavior and negative spillover effects between the various economies.

This naturally leads to a discussion of monetary policy cooperation and coordination which we have modelled and discussed in chapter 2. As we have illustrated in our simple framework it is possible to reduce the combined loss of two or several central banks of interdependent economies when they cooperate on their interest rate setting. For example, central banks less affected by economic shocks can facilitate the monetary policy of more affected central banks by moving their interest rate in the opposite direction. In this way, a central bank can by incurring a slightly higher loss onto oneself, facilitate another central banks' monetary policy, reducing the aggregate loss. One challenge for such coordination are difficulties of credibility, trust and aligning of incentives discussed in chapter 2.11, making it hard for various central banks to commit to a coordination of their monetary policies. For such coordination to take place, the central bank incurring a higher loss to itself must have an incentive to do so, for example that another central bank would return the favor. This could potentially be solved by reputation. However, in a complex world with many interdependent economies and various shocks, it is likely that it would be very hard to determine which central banks would facilitate each other's monetary policy.

From game theory, another possible solution is allowing for a Stackelberg leadership equilibrium in international monetary policy, where e.g. the US or another large economy would be the Stackelberg leader. It is known that the Stackelberg solution will be more efficient than the non-cooperative Nash-equilibrium, given that the Stackelberg leader can commit himself (Canzoneri, 1991). When one country commits itself to a certain policy, other economies (Stackelberg followers) can position themselves optimally relative to the Stackelberg leader, which accordingly can reduce the aggregate loss for the economies involved (Canzoneri, 1991). Taylor (2013) also argues that the lack of coordination might come from reduced central bank independence as governments demand more from central banks than what is optimal. If such a claim is true, it calls for a review of central bank independence.

At last, we will discuss Taylor's (2013) argument that domestically focused rule based monetary policy leads to better coordination. In our simple static framework, we have illustrated that the interest rate responses are less aggressive when the central bank's emphasis is on the domestic economy keeping the foreign interest rate as an exogenous variable. However, we have also proved that the optimal interest rate policy and the nature of the loss function incentivize Nash behavior under certain conditions. This leads to strategic interest rate positioning relative to other central banks. Then by under or overbidding other central banks, each central bank can perform an individually optimal strategy at the cost an increased aggregate loss for the economies combined. To potentially achieve the coordination towards more domestically focused rule based monetary policy, we believe it needs to be an agreement between central banks of at least several large economies to commit to such a policy. These central banks could e.g. be the FED and the ECB which we have referred to in our theoretical framework. These economies could also at least in theory use some form of punishment for economies unwilling to cooperate. However, determining whether a central bank is too aggressive in its monetary policy might be challenging.

6. Concluding remarks

In this paper, we have developed a static framework proving that central bank interest rates, under certain conditions, will reach suboptimal levels when one or several interdependent economies are hit by shocks. Further, we extended our framework to prove that this solution was suboptimal compared to a possible cooperative solution between the central banks. We have also illustrated how policy coordination can reduce the aggregate loss of interdependent central banks. This is achieved when central banks less affected by economic shocks facilitates the monetary policy of the central banks more affected by economic shocks. In addition, we saw how central banks emphasizing the domestic economy reacted less aggressive to foreign key policy rate changes. This is in line with Taylor's (2013) emphasis on "getting the domestic policy right". Our framework also illustrated the importance of expected inflation for the central banks' ability to carry out its monetary policy. In the same manner, as with economic shocks, cooperation could also here vastly reduce the magnitude of the needed interest rate response from the central banks.

The empirical analysis compared the central bank interest rates of nine flexible inflation targeting economies to various versions of the Taylor rate. The sample period spanned from year 2001 Q1 to 2015 Q4. We find some evidence of central bank rates moving below the Taylor rates during the Dotcom bubble recession, indicating that the central banks in our sample reacted by lowering their interest rates too much during the recession. In the aftermath period of the Dotcom recession, there is some evidence of lower interest rates compared to Taylor rates. This suggests that there might to some degree have been a prisoner's dilemma, hindering the return of normal interest rate levels.

During the financial crisis recession, there is clear evidence of central banks lowering their interest rates below the various suggested Taylor rates, supporting our hypothesis of Nash-behavior among central banks. The aftermath period, using headline inflation based Taylor rates, shows evidence for a potential prisoner's dilemma hindering the return to long term equilibrium interest rates. However, the significant negative Taylor gaps responses occurs 1.5 years after the trough of the financial crisis recession. This time lag makes it hard to conclude

whether this was due to the prisoners' dilemma situation described in our paper. On the other hand, the lack of significance the first 1.5 years can be explained by a positive spike in interest rate levels relative to the Taylor rate at the end of the financial crisis. This might relate to the fact that the Taylor rate uses historical data, while central banks base their monetary policy partly on forecasts. Considering core inflation based Taylor gaps, there are almost no significant response dummies after the financial crisis. However, the trend coefficient still suggest that the key policy rates clearly were below the Taylor rate in the period after the financial crisis. Combined, this suggest a possible prisoners' dilemma, hindering key policy rates from returning to higher levels.

The are some evidence for a negative trend in key policy rates relative to the Taylor rate, even when controlling for core inflation and adjusted natural real interest rate (NRIR). This suggests that central banks over time may have moved towards a more suboptimal global monetary policy. When combining the trend and the development of the response dummies studied, we argue that monetary policy strategies over time have become more aggressive. Implying that central banks' willingness to underbid other central banks' key policy rates has increased, resulting in potential suboptimal key policy rate levels. Over time, this might lead to negative consequences like financial imbalances (Kahn, 2010). If our arguments are true, inflation targeting might cause central banks to part take in what we refer to as "a race to the bottom" when hit by adverse shocks. If such a trend continues, it might call for a revision of how monetary policy under flexible inflation targeting is carried out, opening for a revision of the discussion around policy cooperation. Further, our conclusion is that monetary policy under inflation targeting need to incorporate insights from game theory as this might extend the understanding of policy interdependencies.

One limitation of our theoretical framework is that it only covers a static Nash-equilibrium. In comparison, monetary policy interdependencies is a repeated game where central banks continuously adapt their monetary policy to economic development as well as other central banks strategies. Further development of a complex, micro-founded, dynamic model, aiming to study central banks behavior under inflation targeting, would possibly better highlight the intricate nature of the repeated game our thesis describes.

Our method of event studies using time dummies does not give a concluding answer to what exactly causes the interest rates to drop below the Taylor rate. Although our findings are in line with our theoretical framework, we suggest that further research develops a model to empirically examine which factors that affect the magnitude of the interest rate deviations from the Taylor rates. One particular factor it would be interesting to look at is the real exchange rate.

7. References

Aastveit, K. A., Jore, A. S., & Ravazzolo, F. (2016). Identification and real-time forecasting of Norwegian business cycles. *International Journal of Forecasting*, 32(2), 283-292.

Ahrend, R., Cournède, B., & Price, R. (2008). Monetary policy, market excesses and financial turmoil.

Akaike, H. (1987). Factor analysis and AIC. *Psychometrika*, 52(3), 317-332.

Bernanke, B. S., & Woodford, M. (2004). Introduction to" The Inflation-Targeting Debate". *In The Inflation-Targeting Debate* (pp. 1-10). University of Chicago Press.

Bjørnland, H. C., Brubakk, L., & Jore, A. S. (2005). The output gap in Norway-a comparison of different method. Norges Bank. *Economic Bulletin*, 76(2), 90.

Blinder, A. S. (2013). *After the music stopped: The financial crisis, the response, and the work ahead*. Penguin Books.

Canzoneri, M. B., & Gray, J. A. (1985). Monetary policy games and the consequences of non-cooperative behavior. *International Economic Review*, 547-564.

Canzoneri, M. B., & Henderson, D. W. (1991). *Monetary policy in interdependent economies: A game-theoretic approach*. MIT press.

Carlin, W., & Soskice, D. (2005). *Macroeconomics: imperfections, institutions, and policies*. OUP Catalogue.

Claessens, S., Dell'Ariccia, G., Igan, D., & Laeven, L. (2010). Cross-country experiences and policy implications from the global financial crisis. *Economic Policy*, 25(62), 267-293.

Clarida, R., Gali, J., & Gertler, M. (1999). *The science of monetary policy: a new Keynesian perspective* (No. w7147). National bureau of economic research.

Cooper, R (1985), "Economic Interdependence and Coordination of Economic Policies," in R. Jones and P. Kenen, eds., *Handbook of International Economics vol. II*, Amsterdam: North-Holland, 1195-1234

Corsetti, G., Pesenti, P., Roubini, N., & Tille, C. (2000). Competitive devaluations: toward a welfare-based approach. *Journal of International Economics*, 51(1), 217-241.

DeLong, J. B., & Magin, K. (2006). *A short note on the size of the dot-com bubble* (No. w12011). National Bureau of Economic Research.

Drazen, A (2000). *Political Economy In Macroeconomics*. Princeton University Press, 559-580

European Central Bank. (2000). Potential Output Growth and Output Gaps: Concept, Uses and Estimates. *ECB Monthly Bulletin October 2000*.

European Central Bank. (2011). *The Monetary Policy of The ECB* (Third edition, May 2011). Frankfurt: European Central Bank, 2001. ISBN 978-92-899-0778-1.

Federal Open Market Committee. (2013). Statement on longer-run goals and monetary policy strategy. *amended on January,* 29, 2013. as amended effective January 26, 2016.

Galí, J. (2008). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*.

George, E., King, M., Clementi, D., Budd, A., Buiter, W., Goodhart, C., ... & Vickers, J. (1999). The transmission mechanism of monetary policy. *Bank of England*, 1-12.

Gibbons, R. (1992). *A primer in game theory*. Harvester Wheatsheaf.

Hall, S. G., Swamy, P. A. V. B., & Tavlas, G. S. (2012). Milton Friedman, the demand for money, and the ECB's monetary policy strategy. *Federal Reserve Bank of St. Louis Review*, 94(3), 153-85.

Hammond, G. (2012). State of the art of inflation targeting. *Handbooks*.

Harris, R., & Tzavalis, E. (1996). *Inference for Unit Roots in Dynamic Panels*. Exeter University, Department of Economics.

Hodrick, R. J., & Prescott, E. C. (1997). Postwar US business cycles: an empirical investigation. *Journal of Money, credit, and Banking*, 1-16.

Hoechle, D. (2007). Robust standard errors for panel regressions with cross-sectional dependence. *Stata Journal*, 7(3), 281.

Hsiao, C. (2013). *Panel Data Analysis-Advantages and Challenges* (No. 2013-10-14). Wang Yanan Institute for Studies in Economics (WISE), Xiamen University.

Johnson, D. R. (2002). The effect of inflation targeting on the behavior of expected inflation: evidence from an 11 country panel. *journal of monetary economics*, 49(8), 1521-1538.

Kahn, G. A. (2010). Taylor rule deviations and financial imbalances. *Economic Review-Federal Reserve Bank of Kansas City*, 95(2), 63.

Koenig, E. F., & Emery, K. M. (1991). Misleading indicators? Using the composite leading indicators to predict cyclical turning points. *Economic Review-Federal Reserve Bank of Dallas*, 1.

Levin, A., Lin, C. F., & Chu, C. S. J. (2002). Unit root tests in panel data: asymptotic and finite-sample properties. *Journal of econometrics*, 108(1), 1-24.

Liew, V. K. Sen,(2004)" Which Lag Length Selection Criteria Should We Employ?.". *Economics Bulletin,* 3(33), 1-9.

Marcet, A., & Ravn, M. O. (2004). *The HP-Filter in Cross-Country Comparisons* (No. 4244). CEPR Discussion Papers.

Martínez, G. O. (2008). Inflation targeting. *Bank of Canada a festschrift in honor of David Dodge, Bank of Canada, November*.

Mishkin, F. S., Matthews, K., & Giuliodori, M. (2013). *The economics of money, banking and financial markets*: european edition.

Montet, C., & Serra, D. (2003). *Game theory and economics*. New York: Palgrave macmillan.

Norges Bank. (2016). Monetary Policy Report with financial stability assessment 4/2016. *Norges Bank*, Oslo. ISSN: 1894-9259

Rogoff, K. (1985). Can international monetary policy cooperation be counterproductive?. *Journal of International Economics*, 18(3-4), 199-217.

Rogoff, K. (2003). Globalization and global disinflation. *Economic Review-Federal Reserve Bank of Kansas City*, 88(4), 45.

Røisland, Ø., & Sveen, T. (2006). Pengepolitikk under et inflasjonsmål: en dynamisk analyse. *Norsk Økonomisk Tidsskrift*, 120, 90-103.

Røisland, Ø. og T. Sveen (2005): «Pengepolitikk under et inflasjonsmål», *Norsk Økonomisk Tids- skrift* 119, 16-38.

Stiglitz, Joseph. (2010). Why Easier Money Won* t Work. *Wall Street Journal*, 23, 2010.

Svensson, L. E. (1997). Inflation forecast targeting: Implementing and monitoring inflation targets. *European economic review*, 41(6), 1111-1146.

Taylor, J. B. (1985). International coordination in the design of macroeconomic policy rules. *European Economic Review*, 28(1-2), 53-81.

Taylor, J. B. (1993, December). Discretion versus policy rules in practice. In Carnegie-Rochester conference series on public policy* (Vol. 39, pp. 195-214). North-Holland.

Taylor, J. B. (2013). International monetary coordination and the great deviation. *Journal of Policy Modeling*, 35(3), 463-472.

Wooldridge, J. M. (2009). *Introductory Econometrics: A modern approach.* Canada: South-Western Cengage Learning.

Yellen, J. L. (2015). Inflation Dynamics and Monetary Policy. *speech delivered at the Philip Gamble Memorial Lecture, University of Massachusetts, Amherst, Amherst, Mass., September, 24.*

8. Appendix

8.1 Appendix chapter 2: Theoretical framework

8.1.1 Deriving optimal interest rate setting in a static New Keynesian model for a small open economy

In this appendix, we will show step by step how we derived the first order condition in line with Røisland & Sveen (2005), and how we used this to solve for an optimal interest rate setting for a small open economy.

Deriving the first order condition (F.O.C)

We start by setting the partial derivative of the central bank's loss function with respect to the interest rate equal zero.

1. $\dfrac{dL_t}{di} = \dfrac{d\frac{1}{2}((\pi-\pi^*)^2+\lambda(y_t-y^*)^2)}{di} = 0$

2. $\dfrac{dL_t}{di} = (\pi - \pi^*)\dfrac{d\pi}{di} + \lambda(y - y^*)\dfrac{dy}{di} = 0$

Solve for various relationships:

3. $\dfrac{de}{di} = \dfrac{de}{ds} * \dfrac{ds}{di} = -1$

4. $\dfrac{dy}{di} = -\alpha_1 + \alpha_2 * \dfrac{de}{di} = -\alpha_1 + \alpha_2 * -1 = -(\alpha_1 + \alpha_2)$

5. $\dfrac{d\pi}{di} = \gamma\dfrac{dy}{di} + \beta\dfrac{de}{di} = \gamma * -[\alpha_1 + \alpha_2] + \beta * (-1) = -[\gamma(\alpha_1 + \alpha_2) + \beta]$

Insert solutions from (3), (4) and (5) into (2) yields:

6. $(\pi - \pi^*) * -[\gamma(\alpha_1 + \alpha_2) + \beta] + \lambda(y - y^*) * -(\alpha_1 + \alpha_2) = 0$

7. $(\pi - \pi^*) * -[\gamma(\alpha_1 + \alpha_2) + \beta] = \lambda(y - y^*) * (\alpha_1 + \alpha_2)$

$(\pi - \pi^*) = \dfrac{\lambda(y-y^*)*(\alpha_1+\alpha_2)}{-[\gamma(\alpha_1+\alpha_2)+\beta]} = -\dfrac{\lambda*(\alpha_1+\alpha_2)}{[\gamma(\alpha_1+\alpha_2)+\beta]}(y - y^*)$

This yields the following first order condition (F.O.C) presented as equation (2.6) in chapter 2.3.5:

8. $(\pi - \pi^*) = -\frac{\lambda*(\alpha_1+\alpha_2)}{[\gamma(\alpha_1+\alpha_2)+\beta]} (y - y^*)$

Reaction function

Substitute for the endogenous variables (y), (π) and (e) by inserting equation (2.2), (2.3) and (2.5) from chapter 2.3 into the F.O.C.:

First insert equation (2.3) for (π),

9. $(\pi^e + \gamma(y - y^*) + \beta(e - e^*) + u - \pi^*) = -\frac{\lambda*(\alpha_1+\alpha_2)}{[\gamma(\alpha_1+\alpha_2)+\beta]} (y - y^*)$

10. $(\pi^e + \beta(e - e^*) + u - \pi^*) = -\frac{\lambda*(\alpha_1+\alpha_2)}{[\gamma(\alpha_1+\alpha_2)+\beta]} (y - y^*) - \gamma(y - y^*)$

11. $(\pi^e + \beta(e - e^*) + u - \pi^*) = -\left(\frac{\lambda*(\alpha_1+\alpha_2)}{[\gamma(\alpha_1+\alpha_2)+\beta]} + \gamma\right) (y - y^*)$

12. $(\pi^e + \beta(e - e^*) + u - \pi^*) = -\left(\frac{\lambda*(\alpha_1+\alpha_2)+\gamma[\gamma(\alpha_1+\alpha_2)+\beta]}{[\gamma(\alpha_1+\alpha_2)+\beta]}\right) (y - y^*)$

13. $(\pi^e + \beta(e - e^*) + u - \pi^*) = -A(y - y^*)$
    where $A = \left(\frac{\lambda*(\alpha_1+\alpha_2)+\gamma[\gamma(\alpha_1+\alpha_2)+\beta]}{[\gamma(\alpha_1+\alpha_2)+\beta]}\right)$

Insert equation (2.2) for (y) in (13):

14. $(\pi^e + \beta(e - e^*) + u - \pi^*) = -A(y^* - \alpha_1(i - \pi^e - r^*) + \alpha_2(e - e^*) + v - y^*)$

15. $(\pi^e + u - \pi^*) = -A(-\alpha_1(i - \pi^e - r^*) + \alpha_2(e - e^*) + v) - \beta(e - e^*)$

16. $(\pi^e - \pi^*) + u = A\alpha_1 i + A\alpha_1(-\pi^e - r^*) - A\alpha_2(e - e^*) - Av - \beta(e - e^*)$

17. $(\pi^e - \pi^*) + u = A\alpha_1 i + A\alpha_1(-\pi^e - r^*) - A\alpha_2 e + A\alpha_2 e^* - Av - \beta e + \beta e^*$

18. $(\pi^e - \pi^*) + u + Av = A\alpha_1 i + A\alpha_1(-\pi^e - r^*) - (A\alpha_2 + \beta)e + (A\alpha_2 + \beta)e^*$

Insert equation (2.5) for (e) in (18):

19. $(\pi^e - \pi^*) + u + Av = A\alpha_1 i + A\alpha_1(-\pi^e - r^*) - (A\alpha_2 + \beta)(e^e - i + \pi^e + i^f - \pi^{e,f} + z) + (A\alpha_2 + \beta)e^*$

20. $(\pi^e - \pi^*) + u + Av - A\alpha_1(-\pi^e - r^*) - (A\alpha_2 + \beta)e^* = A\alpha_1 i - (A\alpha_2 + \beta)(e^e - i + \pi^e + i^f - \pi^{e,f} + z)$

21. $(\pi^e - \pi^*) + u + Av - A\alpha_1(-\pi^e - r^*) - (A\alpha_2 + \beta)e^* = A\alpha_1 i - (A\alpha_2 + \beta)(e^e - i + \pi^e + i^f - \pi^{e,f} + z)$

22. $(\pi^e - \pi^*) + u + Av + A\alpha_1(\pi^e + r^*) - (A\alpha_2 + \beta)e^* = A\alpha_1 i - (A\alpha_2 + \beta)(e^e - i + \pi^e + i^f - \pi^{e,f} + z)$

23. $(\pi^e - \pi^*) + u + Av + A\alpha_1(\pi^e + r^*) - Be^* = A\alpha_1 i - B(e^e - i + \pi^e + i^f - \pi^{e,f} + z)$

 where $B = (A\alpha_2 + \beta)$

24. $(\pi^e - \pi^*) + u + Av + A\alpha_1(\pi^e + r^*) - Be^* + B(e^e + \pi^e + i^f - \pi^{e,f} + z) = A\alpha_1 i + Bi$

25. $(\pi^e - \pi^*) + u + Av + A\alpha_1(\pi^e + r^*) + B(i^f + (e^e - e^*) + (\pi^e - \pi^{e,f}) + z) = A\alpha_1 i + Bi$

Solve for i:

26. $i(A\alpha_1 + B) = (\pi^e - \pi^*) + u + Av + A\alpha_1(\pi^e + r^*) + B(i^f + (e^e - e^*) + (\pi^e - \pi^{e,f}) + z)$

27. $i = \frac{1}{(A\alpha_1 + B)}\left((\pi^e - \pi^*) + u + Av + A\alpha_1(\pi^e + r^*) + B(i^f + (e^e - e^*) + (\pi^e - \pi^{e,f}) + z)\right)$

28. $i^{opt} = C\left((\pi^e - \pi^*) + A\alpha_1(\pi^e + r^*) + u + Av + B(i^f + (e^e - e^*) + (\pi^e - \pi^{e,f}) + z)\right)$

 $Where\ C = \frac{1}{(A\alpha_1 + B)} = \frac{1}{(A\alpha_1 + A\alpha_2 + \beta)} = \frac{1}{A(\alpha_1 + \alpha_2) + \beta}$

This gives us the optimal interest setting for a small open economy (29) as shown in equation 2.7 in section 2.3.6. Note, "opt" denotes optimal interest rate response function:

29. $i^{opt} = C\left((\pi^e - \pi^*) + A\alpha_1(\pi^e + r^*) + u + Av + B(i^f + (e^e - e^*) + (\pi^e - \pi^{e,f}) + z)\right)$

 $where\ A = \left(\frac{\lambda*(\alpha_1 + \alpha_2) + \gamma[\gamma(\alpha_1 + \alpha_2) + \beta]}{[\gamma(\alpha_1 + \alpha_2) + \beta]}\right), B = (A\alpha_2 + \beta)\ and\ C = \frac{1}{A(\alpha_1 + \alpha_2) + \beta}$

 $A, B, C > 0$

To generalize our model to look at different central banks optimal interest rate setting, we include denotations (h) for the home economy and (f) for the foreign economy, expressed as (30) as shown in equation 2.8 in chapter 2.4:

30. $i_h^{Opt} = C_h \left((\pi_h^e - \pi_h^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + u_h + A_h v_h + B_h \left(i^f + (e_h^e - e_h^*) + \left(\pi_h^e - \pi^{e,f} \right) + z_h \right) \right)$

$where\ h = home\ economy, f = foreign\ economy$

$A_h = \left(\dfrac{\lambda_h * (\alpha_{1,h} + \alpha_{2,h}) + \gamma_h [\gamma_h (\alpha_{1,h} + \alpha_{2,h}) + \beta_h]}{[\gamma_h (\alpha_{1,h} + \alpha_{2,h}) + \beta_h]} \right)$

$B_h = \left(A_h \alpha_{2,h} + \beta_h \right)$

$C_h = \left(\dfrac{1}{A_h (\alpha_{1,h} + \alpha_{2,h}) + \beta_h} \right)$

$A, B, C > 0$

8.1.2 Deriving the Nash equilibrium interest rate in a static New Keynesian model

Deriving the general Nash Equilibrium for a two-country economy

Using the optimal interest rate setting in a small open economy in chapter 8.1.1 equation (30), we insert the reaction function for the opposite economy's central bank as a substitute for the foreign interest rate. This is done by inserting equation (30) with opposite denotations for i_f into the home economy's reaction function. In this way, we assume that each central bank has complete information about the other central banks interest rate setting function. We will in this general model, derive it for the home economy denoted h, where the foreign country is denoted f. The optimal interest setting for economy h, when substituting for the optimal interest rate of economy f, can be written as:

1. $i_h^{opt} = C_h \left((\pi_h^e - \pi_h^*) + A_h \alpha_{1,h}(\pi_h^e + r_h^*) + u_h + A_h v_h + B_h \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + \right.\right.$

   $\left. z_h \right) + B_h C_f \left((\pi_f^e - \pi_f^*) + A_f \alpha_{1,f}(\pi_f^e + r_f^*) + u_f + A_f v_f + B_f (i_h + (e_f^e - e_f^*) + (\pi_f^e - \right.$

   $\left.\left.\left. \pi_h^e) + z_f) \right)\right)$

   where $h = home\ economy, f = foreign\ economy$

   $A_h = \left(\frac{\lambda_h * (\alpha_{1,h} + \alpha_{2,h}) + \gamma_h [\gamma_h(\alpha_{1,h} + \alpha_{2,h}) + \beta_h]}{[\gamma_h(\alpha_{1,h} + \alpha_{2,h}) + \beta_h]} \right), A_f = \left(\frac{\lambda_f * (\alpha_{1,f} + \alpha_{2,f}) + \gamma_f [\gamma_f(\alpha_{1,f} + \alpha_{2,f}) + \beta_f]}{[\gamma_f(\alpha_{1,f} + \alpha_{2,f}) + \beta_f]} \right),$

   $B_h = \left(A_h \alpha_{2,h} + \beta_h \right), B_f = \left(A_f \alpha_{2,f} + \beta_f \right),$

   $C_h = \left(\frac{1}{A_h(\alpha_{1,h} + \alpha_{2,h}) + \beta_h} \right) and\ C_f = \left(\frac{1}{A_f(\alpha_{1,f} + \alpha_{2,f}) + \beta_f} \right)$

Next we extract the optimal interest (i_h) rate for economy h:

2. $i_h = (B_h B_f C_h C_f) * i_h + C_h \left((\pi_h^e - \pi_h^*) + A_h \alpha_{1,h}(\pi_h^e + r_h^*) + u_h + A_h v_h + B_h \left((e_h^e - \right.\right.$

   $\left. e_h^*) + (\pi_h^e - \pi_f^e) + z_h \right) + B_h C_f \left((\pi_f^e - \pi_f^*) + A_f \alpha_{1,f}(\pi_f^e + r_f^*) + u_f + A_f v_f + B_f \left((e_f^e - \right.\right.$

   $\left.\left.\left. e_f^*) + (\pi_f^e - \pi_h^e) + z_f) \right)\right)$

Move (i_h) to the left

3. $i_h(1 - B_h B_f C_h C_f) = C_h \Big((\pi_h^e - \pi_h^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + u_h + A_h v_h + B_h \big((e_h^e - e_h^*) +$

$(\pi_h^e - \pi_f^e) + z_h \big) + B_h C_f \big((\pi_f^e - \pi_f^*) + A_f \alpha_{1,f} (\pi_f^e + r_f^*) + u_f + A_f v_f + B_f \big((e_f^e - e_f^*) +$

$(\pi_f^e - \pi_h^e) + z_f \big) \big) \Big)$

Solve for (i_h)

4. $i_h^N = \frac{C_h}{(1 - B_h B_f C_h C_f)} \Big((\pi_h^e - \pi_h^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + u_h + A_h v_h + B_h \big((e_h^e - e_h^*) + (\pi_h^e -$

$\pi_f^e) + z_h \big) + B_h C_f \big((\pi_f^e - \pi_f^*) + A_f \alpha_{1,f} (\pi_f^e + r_f^*) + u_f + A_f v_f + B_f \big((e_f^e - e_f^*) + (\pi_f^e -$

$\pi_h^e) + z_f \big) \big) \Big)$

Compress the first coefficient

5. $i_h^N = D_h \Big((\pi_h^e - \pi_h^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + u_h + A_h v_h + B_h \big((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) +$

$z_h \big) + B_h C_f \big((\pi_f^e - \pi_f^*) + A_f \alpha_{1,f} (\pi_f^e + r_f^*) + u_f + A_f v_f + B_f \big((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) +$

$z_f \big) \big) \Big)$

where $D_h = \frac{C_h}{(1 - C_h B_h C_f B_f)}$

Solve the inner parenthesises yields the following solution where (N*) denotes Nash-equilibrium, presented in chapter 2.10 as equation (2.16):

6. $i_h^{N*} = D_h \left((\pi_h^e - \pi_h^*) + B_h C_f (\pi_f^e - \pi_f^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + B_h C_f A_f \alpha_{1,f} (\pi_f^e + r_f^*) + \right.$

$u_h + B_h C_f u_f + A_h v_h + B_h C_f A_f v_f + B_h \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h \right) + B_h C_f B_f \left((e_f^e - \right.$

$\left. \left. e_f^*) + (\pi_f^e - \pi_h^e) + z_f \right) \right)$

$for\ h = home\ economy, f = foreign\ economy$

$A_h = \left(\frac{\lambda_h*(\alpha_{1,h}+\alpha_{2,h})+\gamma_h[\gamma_h(\alpha_{1,h}+\alpha_{2,h})+\beta_h]}{[\gamma_h(\alpha_{1,h}+\alpha_{2,h})+\beta_h]} \right), \ A_f = \left(\frac{\lambda_f*(\alpha_{1,f}+\alpha_{2,f})+\gamma_f[\gamma_f(\alpha_{1,f}+\alpha_{2,f})+\beta_f]}{[\gamma_f(\alpha_{1,f}+\alpha_{2,f})+\beta_f]} \right)$

$B_h = \left(A_h \alpha_{2,h} + \beta_h \right), \ \ B_f = \left(A_f \alpha_{2,f} + \beta_f \right)$

$C_h = \left(\frac{1}{A_h(\alpha_{1,h}+\alpha_{2,h})+\beta_h} \right), \ C_f = \left(\frac{1}{A_f(\alpha_{1,f}+\alpha_{2,f})+\beta_f} \right)$

$D_h = \frac{C_h}{(1-C_h B_h C_f B_f)}$

8.1.3 Deriving the cooperative equilibrium interest rate in a static New Keynesian model

In this part of the appendix, we will derive the cooperative equilibrium interest rate model. The method follows Canzoneri (1991), which derived the equilibrium based on a simple model with money supply as instrument. We will on the other hand derive the equilibrium based on the framework developed in chapter 2.3, assuming the world consists of two economies.

The goal of this model is to mathematically prove that there exists a cooperative interest rate equilibrium for both economies. This interest rate equilibrium is more optimal than the Nash-solution, in the sense that it reduces the total loss for both economies, when exposed to shocks or deviations from equilibrium levels. Here both central banks minimize their own, as well as the foreign central banks loss function with respect to their own interest rate.

We start out with the following two simplified first order conditions where h denotes the home economy and f denotes the foreign economy:

$$\left[\frac{\partial L_h}{\partial i_h}\right] + \left[\frac{\partial L_f}{\partial i_h}\right] = 0$$

$$\left[\frac{\partial L_f}{\partial i_f}\right] + \left[\frac{\partial L_h}{\partial i_f}\right] = 0$$

As both central bank's reaction function is assumed to be equal, we only solve for one central bank and then change denotations for the other central bank optimal interest rate function.

Deriving the first order condition(F.O.C):

We will first start solving the first condition for i_h. In doing so, we first need to make expressions for each part of the condition:

From chapter 8.1.1, we can express $[\partial L_h / \partial i_h]$ as:

1. $\left[\frac{\partial L_h}{\partial i_h}\right] = -(\pi_h - \pi_h^*) * \left[\gamma_h(\alpha_{1,h} + \alpha_{2,h}) + \beta_h\right] - \lambda_h(y_h - y_h^*) * (\alpha_{1,h} + \alpha_{2,h})$

$[\partial L_f / \partial i_h]$ can be expressed as:

2. $\left[\frac{\partial L_f}{\partial i_h}\right] = \frac{d\left(0.5\left(\left(\pi_f - \pi_f^*\right)^2 + \lambda_f\left(y_f - y_f^*\right)^2\right)\right)}{di_h} = \left(\pi_f - \pi_f^*\right) * \frac{d\pi_f}{di_h} + \lambda_f\left(y_f - y_f^*\right) * \frac{dy_f}{di_h}$

The following relationships also needs to be defined:

3. $\frac{de_f}{di_h} = \frac{de_f}{ds_f} * \frac{ds_f}{di_h} = \frac{d(s_f + p_h - p_f)}{ds_f} * \frac{d\left(s_f^e - (h_f - i_h) + z_f\right)}{di_h} = 1, \ or \ \frac{de_f}{di_h} = \frac{d\left(e_f^e + \left(\pi_f^e - \pi_f^e\right) + (i_h - h_f) + z_f\right)}{di_h} = 1$

4. $\frac{dy_f}{di_h} = \frac{d\left(y_f^* - \alpha_{1,f}\left(h_f - \pi_f^e - r_f^*\right) + \alpha_{2,f}\left(e_f - e_f^*\right) + v_f\right)}{di_h} = \alpha_{2,f} * \frac{de_f}{di_h} = \alpha_{2,f}$

5. $\frac{d\pi_f}{di_h} = \frac{d\left(\pi_f^e + \gamma_f\left(y_f - y_f^*\right) + \beta_f\left(e_f - e_f^*\right) + u_f\right)}{di_h} = \frac{d\left(\pi_f^e + \gamma_f\left(y_f - y_f^*\right) + \beta_f\left(e_f - e_f^*\right) + u_f\right)}{di_h} = \gamma_f * \frac{dy_f}{di_h} + \beta_f *$

$\frac{de_f}{di_h} = \gamma_f \alpha_{2,f} + \beta_f$

With this we can rewrite $[\partial L_f / \partial i_h]$ as;

6. $\left[\frac{\partial L_f}{\partial i_h}\right] = \left(\pi_f - \pi_f^*\right) * \left(\gamma_f \alpha_{2,f} + \beta_f\right) + \lambda_f\left(y_f - y_f^*\right) * \alpha_{2,f}$

By combining the expression for $\left[\frac{\partial L_h}{\partial i_h}\right]$ and $\left[\frac{\partial L_h}{\partial i_h}\right]$, we can express $\left[\frac{\partial L_h}{\partial i_h}\right] + \left[\frac{\partial L_h}{\partial i_f}\right] = 0$ as:

7. $\left(\pi_h - \pi_h^*\right) * \left[-\gamma_h\left(\alpha_{1,h} + \alpha_{2,h}\right) + \beta_h\right] + \lambda_h\left(y_h - y_h^*\right) * \left[-\left(\alpha_{1,h} + \alpha_{2,h}\right)\right] + \left(\pi_f - \pi_f^*\right) *$
$\left(\gamma_f \alpha_{2,f} + \beta_f\right) + \lambda_f\left(y_f - y_f^*\right) * \alpha_{2,f} = 0$

Substitute for the following coefficients

8. $\left(\pi_h - \pi_h^*\right) * \left[-E_h\right] + \lambda_h * \left(y_h - y_h^*\right) * \left[-\left(\alpha_{1,h} + \alpha_{2,h}\right)\right] + \left(\pi_f - \pi_f^*\right) * F_f + \lambda_f *$
$\left(y_f - y_f^*\right) * \alpha_{2,f} = 0$
$E_h = \gamma_h\left(\alpha_{1,h} + \alpha_{2,h}\right) + \beta_h$
$F_f = \gamma_f \alpha_{2,f} + \beta_f$

Move the supply side and demand side gaps to opposite sides and we obtain the following first order condition (F.O.C):

9. $\lambda_h * (y_h - y_h^*) * [-(\alpha_{1,h} + \alpha_{2,h})] + \lambda_f * (y_f - y_f^*) * \alpha_{2,f} = (\pi_h - \pi_h^*) * E_h +$
$(\pi_f - \pi_f^*) * (-F_f)$

Reaction function:

Insert equation (2.3) chapter 2.3 for π_h and π_f

10. $\lambda_h * (y_h - y_h^*) * [-(\alpha_{1,h} + \alpha_{2,h})] + \lambda_f (y_f - y_f^*)\alpha_{2,f} = ((\pi_h^e - \pi_h^*) + \gamma_h (y_h - y_h^*) +$
$\beta_h (e_h - e_h^*) + u_h) * E_h + ((\pi_f^e - \pi_f^*) + \gamma_f (y_f - y_f^*) + \beta_f (e_f - e_f^*) + u_f) * (-F_f)$

Solve for $(y_h - y_h^*)$ and $(y_f - y_f^*)$

11. $\lambda_f (y_f - y_f^*)\alpha_{2,f} + \gamma_f (y_f - y_f^*)F_f + \lambda_h * (y_h - y_h^*) * [-(\alpha_{1,h} + \alpha_{2,h})] - \gamma_h (y_h - y_h^*)E_h =$
$((\pi_h^e - \pi_h^*) + \beta_h (e_h - e_h^*) + u_h)E_h + ((\pi_f^e - \pi_f^*) + \beta_f (e_f - e_f^*) + u_f) * (-F_f)$

Solve for $(y_h - y_h^*)$ and $(y_f - y_f^*)$,

12. $(y_f - y_f^*) * (\alpha_{2,f}\lambda_f + \gamma_f F_f) - (y_h - y_h^*) * (\lambda_h (\alpha_{1,h} + \alpha_{2,h}) + \gamma_h E_h) = ((\pi_h^e - \pi_h^*) +$
$\beta_h (e_h - e_h^*) + u_h) * E_h + ((\pi_f^e - \pi_f^*) + \beta_f (e_f - e_f^*) + u_f) * (-F_f)$

Combine the coefficients

13. $(y_h - y_h^*)(-G_h) + (y_f - y_f^*)H_f = ((\pi_h^e - \pi_h^*) + \beta_h (e_h - e_h^*) + u_h) * E_h +$
$((\pi_f^e - \pi_f^*) + \beta_f (e_f - e_f^*) + u_f) * (-F_f)$
$G_h = \lambda_h (\alpha_{1,h} + \alpha_{2,h}) + \gamma_h E_h$
$H_f = \alpha_{2,f}\lambda_f + \gamma_f F_f$

Insert equation (2.2) from chapter 2.3 for y_h and y_f

14. $\left((y_h^* - y_h^*) + \alpha_{1,h}\left((\pi_h^e + r_h^*) - i_h\right) + \alpha_{2,h}(e_h - e_h^*) + v_h\right)(-G_h) + \left((y_f^* - y_f^*) + \right.$

$\alpha_{1,f}\left((\pi_f^e + r_f^*) - h_f\right) + \alpha_{2,f}(e_f - e_f^*) + v_f\right)H_f = \left((\pi_h^e - \pi_h^*) + \beta_h(e_h - e_h^*) + u_h\right) *$

$E_h + \left((\pi_f^e - \pi_f^*) + \beta_f(e_f - e_f^*) + u_f\right) * (-F_f)$

Remove y_h^* and y_f^*

15. $\left(\alpha_{1,h}\left((\pi_h^e + r_h^*) - i_h\right) + \alpha_{2,h}(e_h - e_h^*) + v_h\right)(-G_h) + \left(\alpha_{1,f}\left((\pi_f^e + r_f^*) - h_f\right) + \right.$

$\alpha_{2,f}(e_f - e_f^*) + v_f\right)H_f = \left((\pi_h^e - \pi_h^*) + \beta_h(e_h - e_h^*) + u_h\right) * E_h + \left((\pi_f^e - \pi_f^*) + \right.$

$\beta_f(e_f - e_f^*) + u_f\right) * (-F_f)$

Solve for $(e_h - e_h^*)$ og $(e_f - e_f^*)$

16. $-(e_h - e_h^*)(\alpha_{2,h}G_h + \beta_h E_h) + \left(\alpha_{1,h}\left((\pi_h^e + r_h^*) - i_h\right) + v_h\right)(-G_h) + \left(\alpha_{1,f}\left((\pi_f^e + r_f^*) - \right.\right.$

$\left.h_f\right) + v_f\right)H_f = \left((\pi_h^e - \pi_h^*) + u_h\right) * E_h + \left((\pi_f^e - \pi_f^*) + u_f\right) * (-F_f) - (e_f - e_f^*)(\beta_f F_f + $

$\alpha_{2,f}H_f)$

Insert for endogenous variable (e) by inserting equation (2.5) from chapter 2.3.

17. $-\left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h + h_f - i_h\right) * J_h + \left(\alpha_{1,h}\left((\pi_h^e + r_h^*) - i_h\right) + v_h\right)(-G_h) + $

$\left(\alpha_{1,f}\left((\pi_f^e + r_f^*) - h_f\right) + v_f\right)H_f = \left((\pi_h^e - \pi_h^*) + u_h\right) * E_h + \left((\pi_f^e - \pi_f^*) + u_f\right) *$

$(-F_f) - \left((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f + i_h - h_f\right) * K_f$

$J_h = \alpha_{2,i}G_h + \beta_h E_h$

$K_f = \beta_f F_f + \alpha_{2,f}H_f$

Solve for i_h and i_f

18. $i_h\left(J_h + \alpha_{1,h}G_h + K_f\right) - \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h\right) * J_h + \left(\alpha_{1,h}(\pi_h^e + r_h^*) + \right.$

$\left. v_h\right)(-G_h) + \left(\alpha_{1,f}\left((\pi_f^e + r_f^*)\right) + v_f\right)H_f = \left((\pi_h^e - \pi_h^*) + u_h\right) * E_h + \left((\pi_f^e - \pi_f^*) + u_f\right) *$

$(-F_f) - \left((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f\right) * K_f + h_f\left(K_f + \alpha_{1,f}H_f + J_h\right)$

Move i_h to the left side

19. $i_h\left(J_h + \alpha_{1,h}G_h + K_f\right) = \left((\pi_h^e - \pi_h^*) + u_h\right) * E_h + \left((\pi_f^e - \pi_f^*) + u_f\right) * (-F_f) +$

$(\alpha_{1,h}(\pi_h^e + r_h^*) + v_h)(G_h) - \left(\alpha_{1,f}\left((\pi_f^e + r_f^*)\right) + v_f\right)H_f + \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + \right.$

$\left. z_h\right) * J_h - \left((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f\right) * K_f + h_f\left(K_f + \alpha_{1,f}H_f + J_h\right)$

Express i_h

20. $i_h = \frac{1}{(J_h + \alpha_{1,h}G_h + K_f)} * \left((\pi_h^e - \pi_h^*)E_h - (\pi_f^e - \pi_f^*)F_f + \alpha_{1,h}(\pi_h^e + r_h^*)G_h - \right.$

$\alpha_{1,f}(\pi_f^e + r_f^*)H_f + u_h E_h - u_f F_f + v_h G_h - v_f H_f + \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h\right) * J_h -$

$\left. \left((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f\right) * K_f + h_f\left(K_f + \alpha_{1,f}H_f + J_h\right)\right)$

Combine coefficients into optimal cooperative interest rate function

21. $i_h = M_h * \left((\pi_h^e - \pi_h^*)E_h - (\pi_f^e - \pi_f^*)F_f + \alpha_{1,h}(\pi_h^e + r_h^*)G_h - \alpha_{1,f}(\pi_f^e + r_f^*)H_f + \right.$

$u_h E_h - u_f F_f + v_h G_h - v_f H_f + \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h\right) * J_h - \left((e_f^e - e_f^*) + \right.$

$\left. (\pi_f^e - \pi_h^e) + z_f\right) * K_f + h_f N_f\Big)$

$M_h = \frac{1}{J_h + \alpha_{1,h}G_h + K_f}$

$N_f = K_f + \alpha_{1,f}H_f + J_h$

Deriving the general cooperative equilibrium for a two-country economy

Insert for the foreign interest rate to find the cooperative equilibrium rate for the respective economies. We assume that both economies have the same optimal response function. Note that denotations for the inserted functions are changed from $h\ to\ f$ and f to h.

22. $i_h = M_h * \Big((\pi_h^e - \pi_h^*)E_h - (\pi_f^e - \pi_f^*)F_f + \alpha_{1,h}(\pi_h^e + r_h^*)G_h - \alpha_{1,f}(\pi_f^e + r_f^*)H_f +$

$u_h E_h - u_f F_f + v_h G_h - v_f H_f + \Big((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h \Big) * J_h - \Big((e_f^e - e_f^*) +$

$(\pi_f^e - \pi_h^e) + z_f \Big) * K_f + N_f \Big(M_f \Big((\pi_f^e - \pi_f^*)E_f - (\pi_h^e - \pi_h^*)F_h + \alpha_{1,f}(\pi_f^e + r_f^*)G_f -$

$\alpha_{1,h}(\pi_h^e + r_h^*)i_h + u_f E_f - u_h F_h + v_f G_f - v_h i_h + \Big((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f \Big) * J_f -$

$\Big((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h \Big) * K_h + i_h N_h \Big) \Big) \Big)$

Solve for i_h and the other exogenous variables

23. $i_h(1 - M_h M_f N_f N_h) = M_h * \Big((E_h - M_f N_f F_h)(\pi_h^e - \pi_h^*) - (\pi_f^e - \pi_f^*)(F_f - M_f N_f E_f) +$

$(\pi_h^e + r_h^*)(\alpha_{1,h} G_h - M_f N_f \alpha_{1,h} i_h) - (\pi_f^e + r_f^*)(\alpha_{1,f} H_f - M_f N_f \alpha_{1,f} G_f) +$

$u_h(E_h - M_f N_f F_h) - u_f(F_f - M_f N_f E_f) + v_h(G_h - M_f N_f i_h) - v_f(H_f - M_f N_f G_f) +$

$\Big((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h \Big) * (J_h - M_f N_f K_h) - \Big((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f \Big) *$

$(K_f - M_f N_f J_f) \Big)$

Cooperative equilibrium interest rate for each country, presented in chapter 2.10 as equation (2.17):

24. $i^{co*}{}_h = P_h * \Big((E_h - M_f N_f F_h)(\pi_h^e - \pi_h^*) - (F_f - M_f N_f E_f)(\pi_f^e - \pi_f^*) + (\alpha_{1,h} G_h -$

$M_f N_f \alpha_{1,h} i_h)(\pi_h^e + r_h^*) - (\alpha_{1,f} H_f - M_f N_f \alpha_{1,f} G_f)(\pi_f^e + r_f^*) + (E_h - M_f N_f F_h)u_h -$

$(F_f - M_f N_f E_f)u_f + (G_h - M_f N_f i_h)v_h - (H_f - M_f N_f G_f)v_f + (J_h - M_f N_f K_h)\Big((e_h^e -$

$e_h^*) + (\pi_h^e - \pi_f^e) + z_h \Big) - (K_f - M_f N_f J_f)\Big((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f \Big) \Big)$

$P_h = \frac{M_h}{1 - M_h M_f N_f N_h}$

8.1.4 Practical examples of Nash and cooperative equilibriums

In this part of the appendix, we will show formally the values of the equilibriums graphically illustrated in the figures in sub-chapter 2.9, based on the formulas derived in chapter 8.1.2 and 8.1.3. To recap, the equation for the Nash equilibrium can be shown as in equation (1):

(1) $\quad i_h^{N*} = D_h \left((\pi_h^e - \pi_h^*) + B_h C_f (\pi_f^e - \pi_f^*) + A_h \alpha_{1,h} (\pi_h^e + r_h^*) + B_h C_f A_f \alpha_{1,f} (\pi_f^e + \right.$

$r_f^*) + u_h + B_h C_f u_f + A_h v_h + B_h C_f A_f v_f + B_h \left((e_h^e - e_h^*) + (\pi_h^e - \pi_f^e) + z_h \right) + B_h C_f B_f \left((e_f^e - \right.$

$\left. e_f^* \right) + (\pi_f^e - \pi_h^e) + z_f \left. \right) \right)$

The cooperative equilibrium interest rate can be expressed as equation (2):

(2) $\quad i^{co*}{}_h = P_h * \left(\left(E_h - M_f N_f F_h \right)(\pi_h^e - \pi_h^*) - \left(F_f - M_f N_f E_f \right)(\pi_f^e - \pi_f^*) + \left(\alpha_{1,h} G_h - \right. \right.$

$M_f N_f \alpha_{1,h} i_h)(\pi_h^e + r_h^*) - \left(\alpha_{1,f} H_f - M_f N_f \alpha_{1,f} G_f \right)(\pi_f^e + r_f^*) + \left(E_h - M_f N_f F_h \right) u_h - \left(F_f - \right.$

$M_f N_f E_f \right) u_f + \left(G_h - M_f N_f i_h \right) v_h - \left(H_f - M_f N_f G_f \right) v_f + \left(\quad {}_h - M_f N_f K_h \right) \left((e_h^e - e_h^*) + \right.$

$\left. (\pi_f^e - \pi_f^e) + z_h \right) - \left(K_f - M_f N_f J_f \right) \left((e_f^e - e_f^*) + (\pi_f^e - \pi_h^e) + z_f \right) \right)$

The assumptions about the economy, coefficients and the aggregate coefficients are shown in table 8.1-8.3. Next we will show the equilibrium interest rates given the different shocks introduced in chapter 2.4.

Table 8.1: *Summary of assumptions part I*

Expected inflation	π_1^e, π_2^e	2%
Inflation target	π_1^*, π_2^*	2%
Real equilibrium interest rate	r_1^*, r_2^*	2%
Inflation shock	u_1, u_2	0
Demand shock	v_1, v_2	0
Exchange rate shock	z_1, z_2	0
Expected - equilibrium real exchange rate	$(e_1^e - e_1^*), (e_2^e - e_2^*)$	0

Table 8.2: *Summary of assumptions part II*

$\alpha_{1,1}, \alpha_{1,2}$	0.15
$\alpha_{2,1}, \alpha_{2,2}$	0.03
γ_1, γ_2	0.03
β_1, β_2	0.06
λ_1, λ_2	0.5

Table 8.3: *Summary of assumptions part III*

A_1, A_2	1.4061
B_1, B_2	0.1022
C_1, C_2	3.1938
D_1, D_2	3.5745
E_1, E_2	0.0654
F_1, F_2	0.0609
G_1, G_2	0.0920
H_1, H_2	0.0168
J_1, J_2	0.0067
K_1, K_2	0.0042
M_1, M_2	40.5911
N_1, N_2	0.0134
P_1, P_2	57.5219

Positive inflation shock to both economies

$(u_1 = u_2 = 0{,}42\%, \text{shown in figure 2.6})$

Nash equilibrium:

$I_1^{N*} = I_2^{N*} = 3.5745 * (1.4061 * 0.15 * 4\% + 0.32635 * 1.41061 * 0.15 * 4\% + 0.42\% + 0.32635 * 0.42\%) = 6\%$

Cooperative equilibrium:

$i_1^{co*} = i_2^{co*} = 57.5219 * \big(0.012425 * (4\%) - (-0.00495975) * (4\%) + 0.03236 * (0.42\%) - 0.025418 * (0.42\%)\big) = 4.16\%$

Negative inflation shock to both economies

$(u_1 = u_2 = -0{,}42\%,$ shown in figure 2.7)

Nash equilibrium:

$I_1^{N*} = I_2^{N*} = 3.5745 * (1.4061 * 0.15 * 4\% + 0.32635 * 1.41061 * 0.15 * 4\% + (-0.42\%) + 0.32635 * (-0.42\%)) = 2\%$

Cooperative equilibrium:

$i_1^{co*} = i_2^{co*} = 57.5219 * (0.012425 * (4\%) - (-0.00495975) * (4\%) + 0.03236 * (-0.42\%) - 0.025418 * (-0.42\%)) = 3.83\%$

Negative inflation shock to one economy

$(u_1 = -0{,}42\%,$ shown in figure 2.8)

Nash equilibrium:

$I_1^{N*} = 3.5745 * (1.4061 * 0.15 * 4\% + 0.32635 * 1.41061 * 0.15 * 4\% + (-0.42\%)) = 2.5\%$

$I_2^{N*} = 3.5745 * (1.4061 * 0.15 * 4\% + 0.32635 * 1.41061 * 0.15 * 4\% + 0.32635 * (-0.42\%)) = 3.5\%$

Cooperative equilibrium:

$i_1^{co*} = 57.5219 * (0.012425 * (4\%) - (-0.00495975) * (4\%) + 0.03236 * (-0.42\%)) = 3.218\%$

$i_2^{co*} = 57.5219 * (0.012425 * (4\%) - (-0.00495975) * (4\%) - 0.025418 * (-0.42\%)) = 4.614\%$

Changes in the expected inflation

Sudden drop in expected inflation relative to equilibrium inflation of 0.5%, such that $\pi^e = 1.5\%$ (Inflation trust shock of -0.5%) for both economies (Shown in figure 2.9):

Nash equilibrium:

$$I_1^{N*} = I_2^{N*} = 3.5745 * \Big((1.5\% - 2\%) + 1.4061 * 0.15 * (1.5\% + 2\%) + 0.32635 *$$

$$\big((1.5\% - 2\%) + 1.4061 * 0.15 * (1.5\% + 2\%)\big)\Big) = 1.1\%$$

Cooperative equilibrium:

$$i_1^{co*} = i_2^{co*} = 57.5219 * \big(0.03236 * (-0.5\%) - 0.025418 * (-0.5\%) + 0.012425 *$$

$$(3,5\%) - (-0.00495975) * (3,5\%)\big) = 3.3\%$$

Sudden drop in expected inflation relative to equilibrium inflation of 0.5%, such that $\pi^e = 1.5\%$ (Inflation trust shock of 0.5%) in economy 1 (shown in figure 2.10):

Nash equilibrium:

$$I_1^{N*} = 3.5745 * \Big(0.0681 * (-0.5\%) + (1.5\% - 2\%) + 1.4061 * 0.15 * (1.5\% + 2\%) +$$

$$0.32635 * \big(1.4061 * 0.15 * (2\% + 2\%)\big)\Big) = 1.7\%$$

$$I_2^{N*} = 3.5745 * \Big(0.0681 * (0.5\%) + 1.4061 * 0.15 * (2\% + 2\%) + 0.32635 * \big((1.5\% -$$

$$2\%) + 1.4061 * 0.15 * (1.5\% + 2\%)\big)\Big) = 3.4\%$$

Cooperative equilibrium:

$$i_1^{co*} = 57.5219 * \big(0.03236 * (-0.5\%) + 0.012425 * (3.5\%) - (-0.00495975) *$$

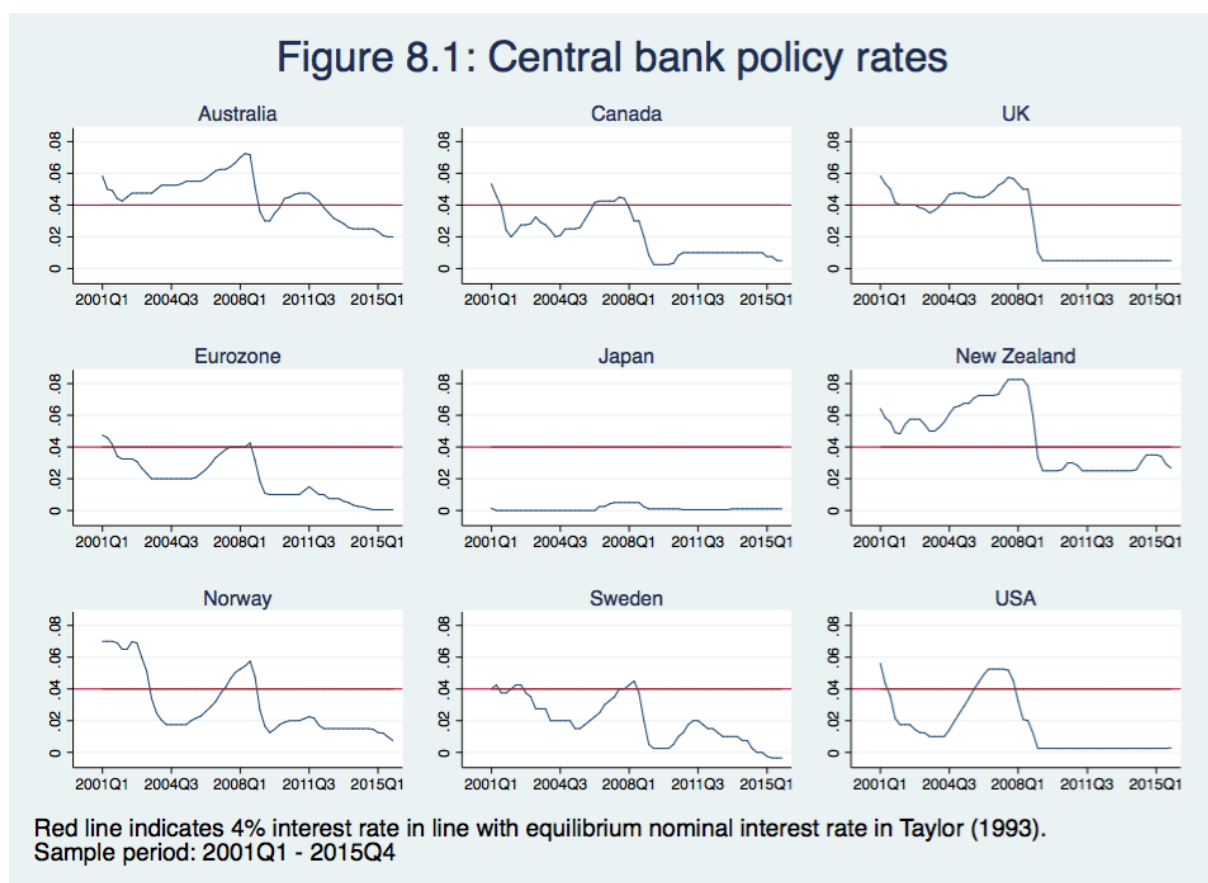$$(4\%) + 0.00442658 * (-0,5\%) - 0.000533167 * (0,5\%)\big) = 2.569\%$$

$$i_2^{co*} = 57.5219 * \big(-0.025418 * (-0.5\%) + 0.012425 * (4\%) - (-0.00495975) *$$

$$(3.5\%) + 0.00442658 * (0,5\%) - 0.000533167 * (-0,5\%)\big) = 4.731\%$$

8.2 Appendix chapter 3: Data and Taylor rate estimations

In this part of the appendix we graphically present the development in the key policy rates, headline and core inflation, actual and potential GDP and estimated Taylor rates. We also include the inflation and GDP data sources referred to in chapter 3.

8.2.1 Key policy rates

Figure 8.1, shows the development of the key policy rates for our sample economies.

Figure 8.1: Central bank policy rates

Red line indicates 4% interest rate in line with equilibrium nominal interest rate in Taylor (1993).
Sample period: 2001Q1 - 2015Q4

8.2.2 Inflation

Adjusted inflation target

The inflation gap is defined as the deviation between actual inflation and target inflation. In line with Taylor (1993) we have as a base scenario used an inflation target of 2%, defining inflation gap as any deviation from the 2 % target. We have also derived a series where the inflation target is adjusted for the official inflation target of the central bank. In table 2.5 in chapter 2.13 we summarized the official date of adopting inflation targeting and the current inflation target.

For the time series where an official inflation target is missing for parts of the period, we have assumed a 2% inflation target, as this is in line with the specified equilibrium inflation in the original Taylor rule (Taylor, 1993). Sudden changes in the inflation target will result in structural breaks. This is not a problem for Canada, the Eurozone, Sweden and the US which all have had an inflation target of 2%. However, to avoid structural breaks in the other series, we have introduced inflation target changes linearly over four quarters. Some countries such as Japan, UK and New Zealand have changed their inflation target within the sample period. As these changes are mostly only 0,5 percentage points, we have not included any smoothed adjustments for such small changes.

Inflation data

In table 8.4 we have summarized the original source BIS has used to generate the consumer price index, which is the basis for our headline inflation. Table 8.5 summarizes the name of data series used, together with the data source for core inflation. Figure 8.2 and 8.3 shows the development of the headline and core inflation for our sample economies in the sample period. Note that we have included the adjusted inflation target described above, together with the base line inflation target of 2%.

Table 8.4: *Headline inflation*

Reference area	Publication Source
Australia	Australian Bureau of Statistics
Canada	Statistics Canada
United Kingdom	Office for National Statistics and Bank of England calculation
Japan	Statistics Bureau (Japan)
Norway	Statistics Norway
New Zealand	Statistics New Zealand
Sweden	National Central Bureau of Statistics
United States	Bureau of Labor Statistics (United States)
Eurozone	Eurostat

Source: https://www.bis.org/statistics/cp/cp_long_documentation.pdf

Table 8.5 *Core inflation*

| Reference area	Publication Source	Name of data series
Australia	Reserve Bank of Australia	CPI 12mth: Excluding volatile items
Canada	Bank of Canada	CPI 12mth: Excluding eight of the most volatile components as well as the effect of changes in indirect taxes on the remaining components
United Kingdom	Office for National Statistics	CPI 12mth: Excluding energy, food, alcohol & tobacco
Japan	Bank of Japan	CPI 12mth: Excluding fresh food and energy
Norway	Statistics Norway	CPI 12mth: adjusted for tax changes and excluding energy products. Pre-2003: CPI 12mth: Excluding energy products
New Zealand	Reserve Bank of New Zealand	Factor model created by the Reserve Bank
Sweden	The Central Bank of Sweden (Sveriges Riksbank)	Trim85: A measure of the 85 % weighting of the CPI 12mth excluding the 7.5% of the highest and lowest price change rates for the sub-groups included in the CPI.
United States	Bureau of Labor Statistics (United States)	CPI 12mth: Excluding food and energy
Eurozone	Eurostat	HICP 12mth: Excluding energy and food

Figure 8.2: Headline inflation
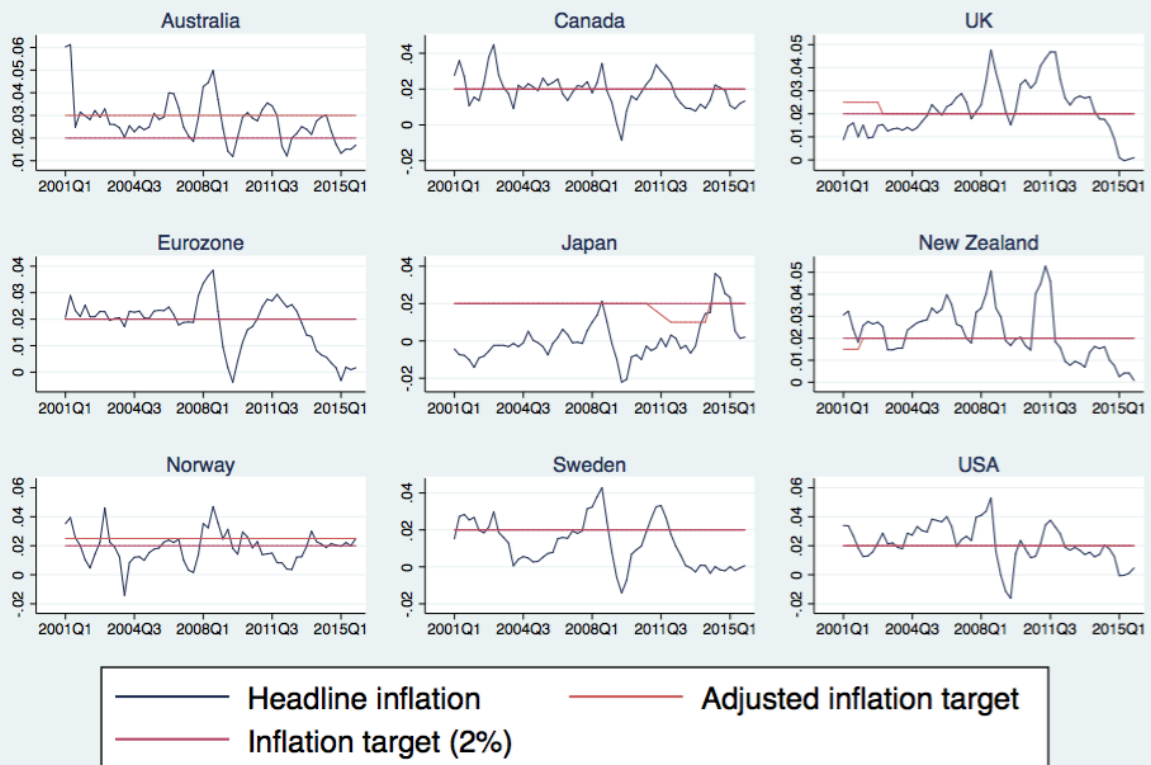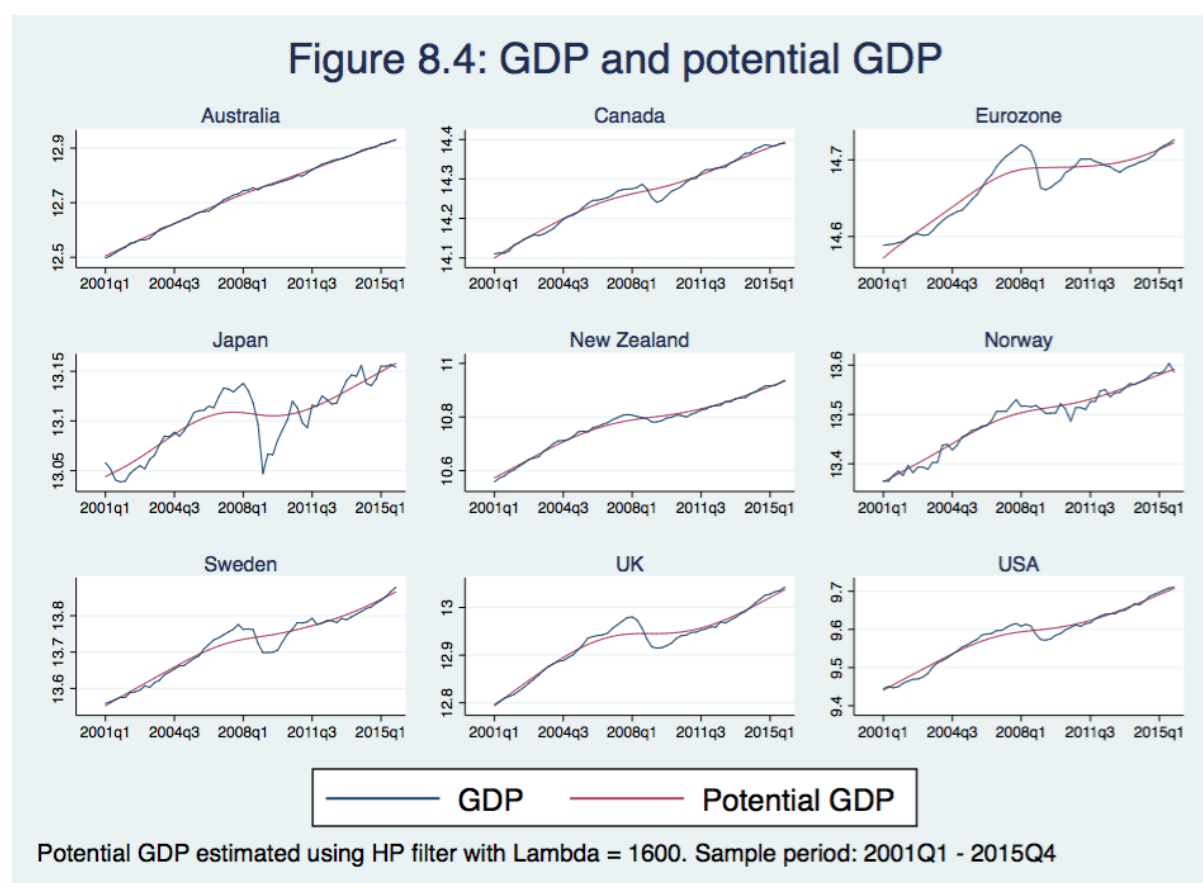

Figure 8.3: Core inflation
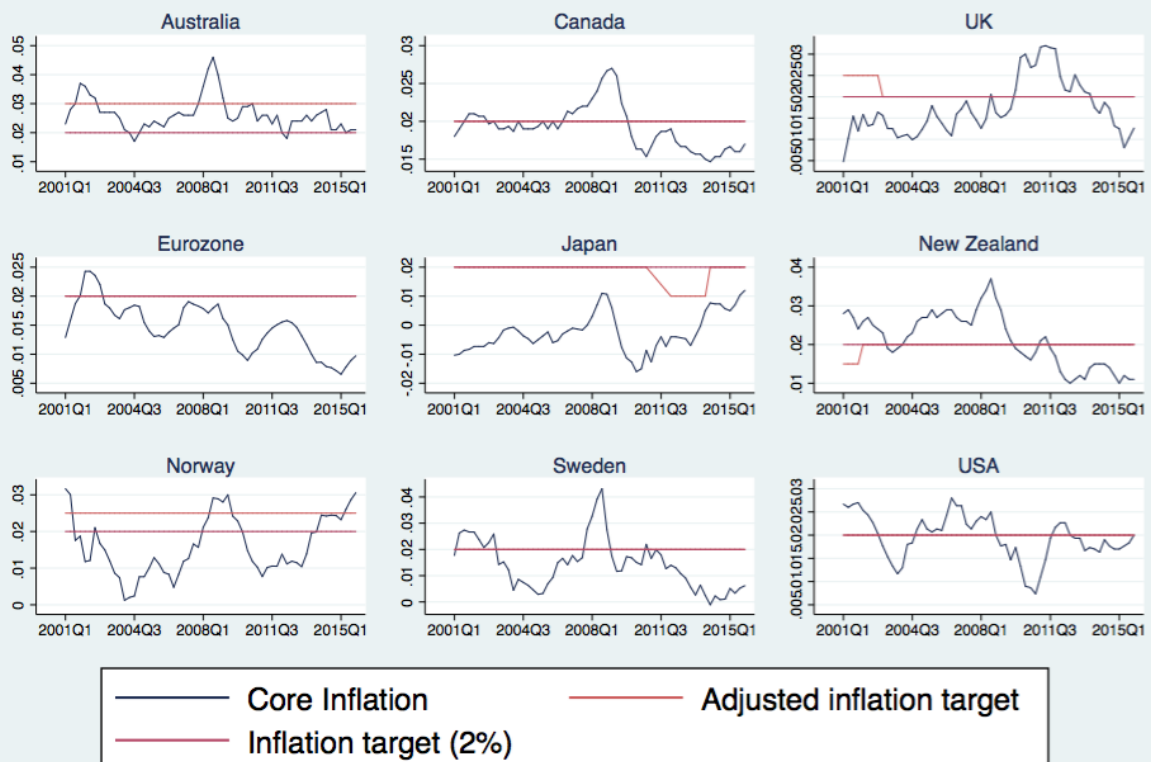
8.2.3 Gross domestic product (GDP)

Table 8.6 summarizes the data sources for GDP. Figure 8.4 present the natural log of GDP for our sample economies, including potential GDP estimates.

Table 8.6: Sources GDP data

Economy	Source
Australia	Australia Bureau of Statistics
Canada	Statistics Canada
Eurozone	Statistical Data Warehouse, European Central Bank
Japan	Economic and Social Research Institute, Cabinet Office
New Zealand	Stats NZ
Norway	Statistics Norway
Sweden	Statistics Sweden
United Kingdom	Office for National Statistics
United states	Federal Reserve Bank of St. Louis

Figure 8.4: GDP and potential GDP

Potential GDP estimated using HP filter with Lambda = 1600. Sample period: 2001Q1 - 2015Q4

8.2.4 Taylor rates

Table 8.7 is an extended version of table 3.3 in chapter 3.5. The table now also includes Taylor rates adjusted for the official central bank inflation target defined above. Figure 8.5 − 8.8 present the estimated Taylor rates compared to the central bank key policy rate. To illustrate the effect of adjusting the inflation targets are Taylor rate 1-4 compared to the corresponding series with adjusted inflation target. Note that for Canada, Eurozone, Sweden and the US there is no difference in the two series.

Table 8.7: *Taylor rates definition (extended to include adjusted inflation target)*

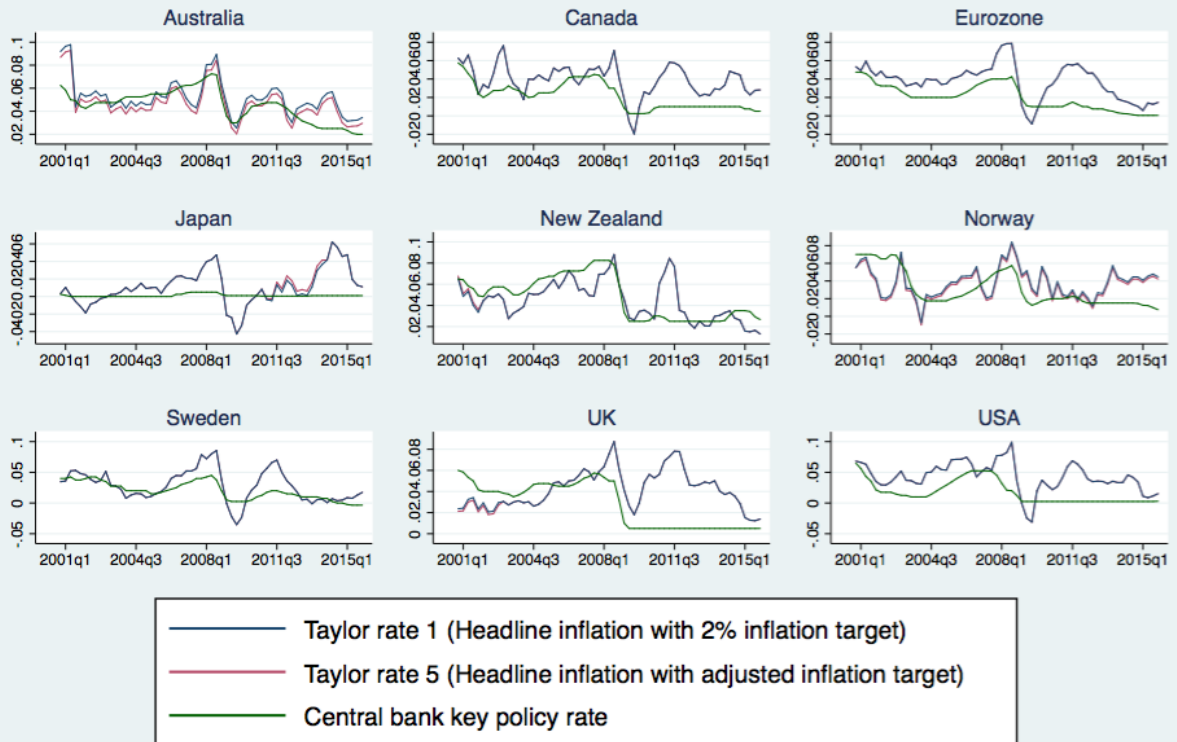| Name	Inflation type	Natural real interest rate (NRIR)	Inflation target
Taylor rate 1	Headline	2 %	2 %
Taylor rate 2	Headline	Adjusted	2 %
Taylor rate 3	Core	2 %	2 %
Taylor rate 4	Core	Adjusted	2 %
Taylor rate 5	Headline	2 %	Adjusted
Taylor rate 6	Headline	Adjusted	Adjusted
Taylor rate 7	Core	2 %	Adjusted
Taylor rate 8	Core	Adjusted	Adjusted

From figure 8.5 − 8.8 we see that the difference of adjusting for official inflation target is marginal for all economies except Australia and Norway. In table 8.8, we have defined the various Taylor gaps. As the results is only marginally different when adjusting for the official inflation targets, will the results using T-gap 5 − 8 only be shown in appendix chapter 8.4.

Table 8.8: *Taylor gap definitions*

Name	Definition
Taylor gap 1	Central bank policy rate - Taylor rate 1
Taylor gap 2	Central bank policy rate - Taylor rate 2
Taylor gap 3	Central bank policy rate - Taylor rate 3
Taylor gap 4	Central bank policy rate - Taylor rate 4
Taylor gap 5	Central bank policy rate - Taylor rate 5
Taylor gap 6	Central bank policy rate - Taylor rate 6
Taylor gap 7	Central bank policy rate - Taylor rate 7
Taylor gap 8	Central bank policy rate - Taylor rate 8

See table 8.7 for Taylor rate definitions

Figure 8.5: Taylor rate 1 and 5

Taylor rate 1 (Headline inflation with 2% inflation target)
Taylor rate 5 (Headline inflation with adjusted inflation target)
Central bank key policy rate

Sample period: 2001Q1 - 2015Q4

Figure 8.6: Taylor rate 2 and 6

Taylor rate 2 (Headline inflation with 2% inflation target with adjustments for NRIR)
Taylor rate 6 (Headline inflation with adjustemtns for inflation target and NRIR)
Central bank key policy rate
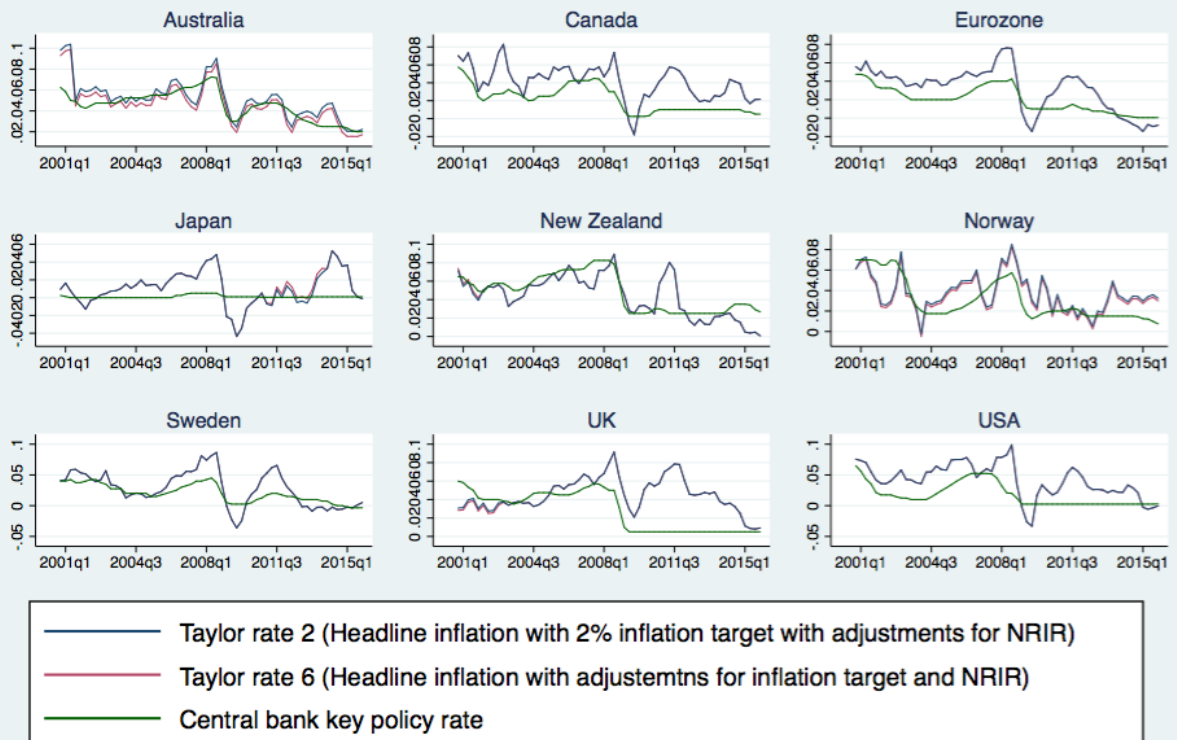
Sample period: 2001Q1 - 2015Q4

Figure 8.7: Taylor rate 3 and 7

Taylor rate 3 (Core inflation with 2% inflation target)
Taylor rate 7 (Core inflation with adjusted inflation target)
Central bank key policy rate
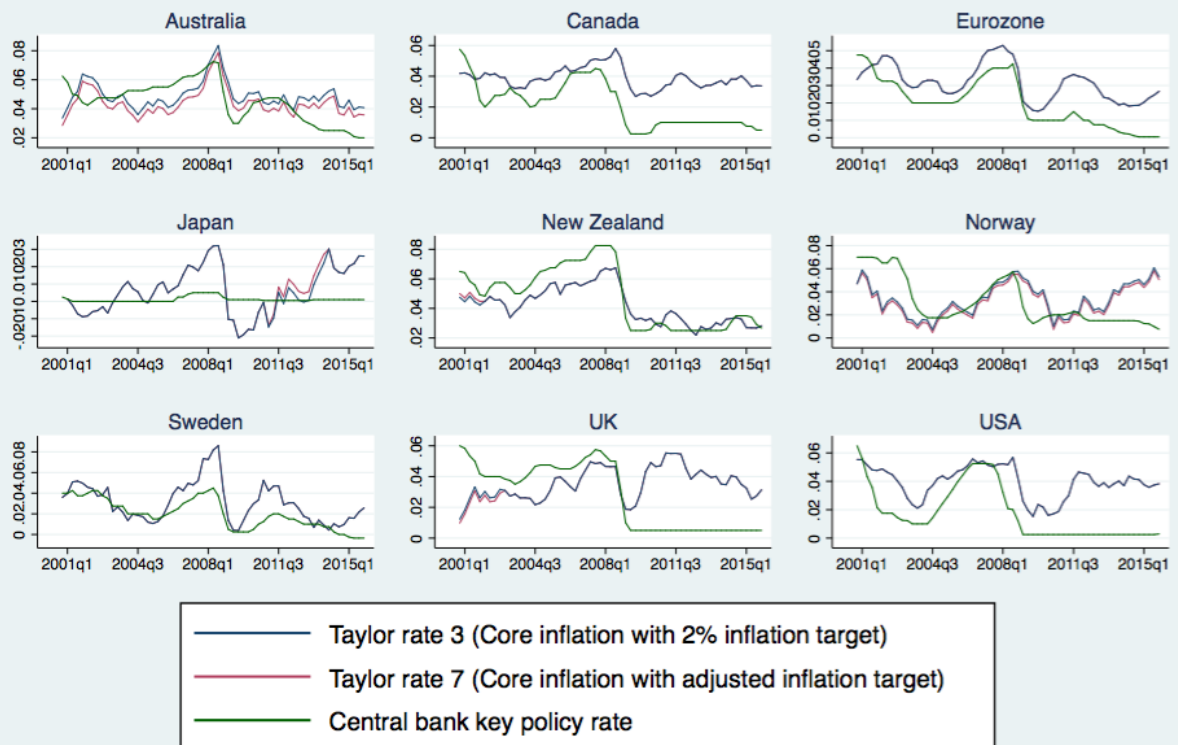
Sample period: 2001Q1 - 2015Q4

Figure 8.8: Taylor rate 4 and 8

Taylor rate 4 (Core inflation with 2% inflation target with adjustments for NRIR)
Taylor rate 8 (Core inflation with adj. inflation target and adjustments for NRIR)
Central bank key policy rate
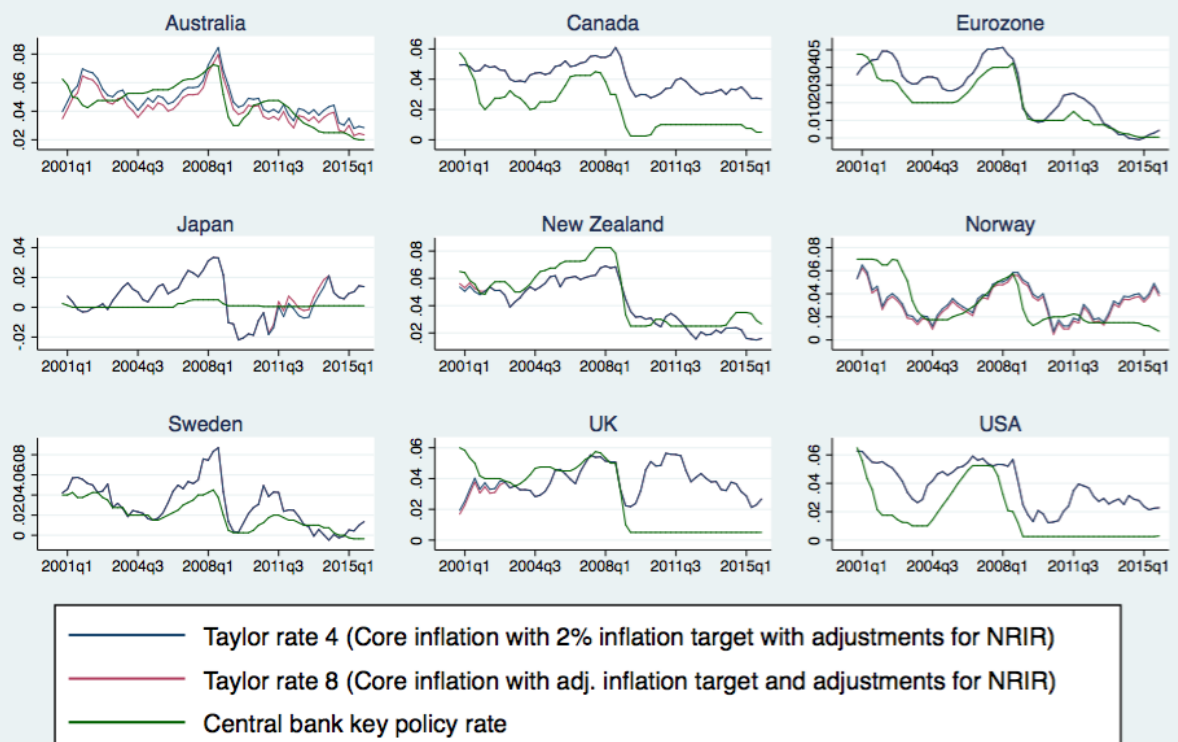
Sample period: 2001Q1 - 2015Q4

8.3 Appendix chapter 4: Method

8.3.1 Fixed effects estimator

This section is derived from Wooldridge (2009). To analyze our panel data, we use a within group estimator that looks at the variation within each group. Which in our dataset is variation in the key policy rate subtracted the Taylor rate for each economy. This allows us to eliminate possible fixed effects, also referred to as unobserved country specific effects (a_i). In the following section, we will introduce the method used for the fixed effects transformation in a model including several explanatory variables.

(8.1) $$y_{i,t} = \beta_1 x_{i,t,1} + \beta_2 x_{i,t,2} + \cdots + \beta_k x_{i,t,k} + a_i + u_{i,t}, \quad t = 1,2,\dots,T$$

Here i denotes the economy, t denotes the time-period given by quarters, and k denotes the explanatory variable. In the model, the unobserved factor affecting the dependent variable $(v_{i,t})$ is the sum of a constant term (a_i) and a term that vary over time $(u_{i,t})$. Next, for each i, the average of each dependent variable over time is given by equation (8.2).

(8.2) $$\bar{y}_i = \beta_1 \bar{x}_{i,1} + \beta_2 \bar{x}_{i,2} + \cdots + \beta_k \bar{x}_{i,k} + a_i + \bar{u}_i,$$

$\bar{y}_i = T^{-1} \sum_{t=1}^{T} y_{i,t}$ and so on.

To complete the transformation, the average of each group (8.2) is subtracted from each group (8.1) for each time t. This implies that a_i is eliminated, as it is assumed to be constant over time resulting in (8.3), which is further compressed in (8.4). Model (8.4) can be estimated using pooled OLS (Wooldridge, 2009).

(8.3) $$y_{i,t} - \bar{y}_i = \beta_1\left(x_{i,t,1} - \bar{x}_{i,1}\right) + \cdots + \beta_k\left(x_{i,t,k} - \bar{x}_{i,k}\right) + u_{it} - \bar{u}_i \quad t = 1,2,\dots,T$$

(8.4) $$\ddot{y}_{i,t} = \beta_1 \ddot{x}_{i,t,1} + \beta_2 \ddot{x}_{i,t,2} + \cdots + \beta_k \ddot{x}_{i,t,k} + \ddot{u}_{i,t,k}, \quad t = 1,2,\dots,T$$

8.3.2 Unit root

As discussed in section 4.3.2, is stationarity in the time series an important assumption for the interpretation of our regressions. A non-stationary times series that follows a unit root process is said to be integrated of order one I(1). Testing for a unit root in (8.5) tests the null hypothesis that $\{y_t\}$ is I(1), and thus a random walk, against the alternative hypothesis that $\{y_t\}$ is integrated of order zero I(0). If the correlation with the last observation y_{t-1} is less than one $\rho < 1$, it makes $\{y_t\}$ a weakly dependent time series, and might be stationary (Wooldridge, 2009).

(8.5) $$y_t = \rho y_{t-1} + e_t, t = 1, 2, ...,$$

(8.6) $$E(e_t | y_{t-1}, y_{t-2}, ..., y_0) = 0$$

Here e_t denotes a process that has zero mean.

We will now test the series used in our analysis for unit root. Since the models in the analyses uses both Taylor-gaps with and without using first difference (FD), both forms are tested for stationarity. In section 8.3.2.1 the series are tested for unit root using a unit root test for panel data (LLC), and in section 8.3.2.2 each individual economy´s time series are tested using the Augmented Dickey Fuller (ADF) test. In section 8.3.2.3 we will comment on the overall result of the unit root tests, and the implications it has for the interpretation of the result and the robustness of our analysis.

8.3.2.1 LLC (Levin, Lin & Chu) panel data unit root test.

In table 8.9 the result of the unit root test for panel data (LLC) is presented for the various Taylor gaps. The number of lags included for the ADF regression are determined by Akaike Information Criterion (AIC), see for example Akaike (1987) or Liew (2004). The null hypothesis that panels contain unit roots, can be rejected at a 1% level for Taylor-gap 1, 2, 5, 6. For Taylor gap 3, 4, 7 and 8 the test reviles indications for unit root, especially for Taylor-gap 3 and 7 which have p-values of 12.38% and 11.81%.

Table 8.9: *Levin-Lin-Chu unit-root test*

Ho: Panels contain unit roots
Ha: Panels are stationary
AR parameter: Common
Panel means: Included
Time trend: Not included
Number of panels = 9
Number of periods = 60
LR variance: Bartlett kernel, 16.00 lags average (chosen by LLC)

	Unadjusted t Statistic	*Adjusted t* Statistic*	*Adjusted t* p-value*	*Average lags chosen by AIC for ADF regression*
T-gap 1	-6.6848	-2.4729	0.0067	4.44
T-gap 2	-7.5391	-3.1165	0.0009	4.00
T-gap 3	-4.9892	-1.1561	0.1238	3.89
T-gap 4	-6.3984	-1.6126	0.0534	3.78
T-gap 5	-6.6746	-2.5014	0.0062	4.44
T-gap 6	-7.5322	-3.1449	0.0008	4.00
T-gap 7	-4.9878	-1.1844	0.1181	3.89
T-gap 8	-6.3924	-1.6512	0.0494	3.78

Note: Sample period: 2001Q1 - 2015Q4

Using FD for the series the result of the unit root test for panel data is presented in table 8.10.

When using FD the null hypothesis of panels containing unit roots can be rejected for all series.

Table 8.10: *Levin-Lin-Chu unit-root test FD*

Ho: Panels contain unit roots
Ha: Panels are stationary
AR parameter: Common
Panel means: Included
Time trend: Not included
Number of panels = 9
Number of periods = 60
LR variance: Bartlett kernel, 16.00 lags average (chosen by LLC)

	Unadjusted t Statistic	*Adjusted t* Statistic*	*Adjusted t* p-value*	*Average lags chosen by AIC for ADF regression*
d.T-gap 1	-12.9187	-4.7357	0.0000	5.56
d.T-gap 2	-12.8782	-4.7140	0.0000	5.56
d.T-gap 3	-15.4071	-10.2441	0.0000	4.33
d.T-gap 4	-15.5150	-10.3893	0.0000	4.33
d.T-gap 5	-12.9017	-4.7513	0.0000	5.56
d.T-gap 6	-12.8606	-4.7286	0.0000	5.56
d.T-gap 7	-15.3119	-10.1643	0.0000	4.33
d.T-gap 8	-15.4175	-10.3060	0.0000	4.33

Note: Sample period: 2001Q1 - 2015Q4

8.3.2.2 Augmented Dickey-Fuller (ADF) test for unit root.

To test the series for each individual economy, we have used the Augmented Dickey-Fuller test for unit roots. The results are presented in table 8.11 and 8.12, with the Dickey–Fuller test statistic including the MacKinnon approximate p-value in parentheses. From table 8.11 we see clear indications that Taylor-gap 1-4 for UK and Norway have unit root present. In line with the result of the LLC unit root test for panel data, it is especially Taylor-gap 3 that suffer from potential unit root, with p-values above 10% for UK, Japan, New Zealand and Norway. For the series for Taylor-gap 1 and 2, only UK have unit root with p-values above 5%.

Table 8.11: *Augmented Dickey-Fuller test for unit root for our sample economies*

Ho: Unit root is present in a time series sample
Ha: Time series are stationary
ADF regression lags: 3
Number of obs = 60

	Australia	Canada	UK	Eurozone	Japan	New Zealand	Norway	Sweden	USA
T-gap 1	-3.433	-3.730	-1.878	-4.006	-3.830	-2.797	-2.221	-4.513	-4.166
	(0.0099)	(0.0037)	(0.3425)	(0.0014)	(0.0026)	(0.0587)	(0.1988)	(0.0002)	(0.0008)
T-gap 2	-4.013	-4.440	-1.916	-3.571	-4.250	-3.040	-2.871	-4.458	-4.141
	(0.0013)	(0.0003)	(0.3247)	(0.0063)	(0.0005)	(0.0313)	(0.0488)	(0.0002)	(0.0008)
T-gap 3	-3.023	-3.121	-2.059	-3.297	-2.382	-2.035	-1.273	-3.115	-2.995
	(0.0328)	(0.0251)	(0.2611)	(0.0150)	(0.1469)	(0.2716)	(0.6413)	(0.0255)	(0.0353)
T-gap 4	-4.082	-4.151	-2.269	-4.554	-2.552	-2.795	-1.887	-3.385	-4.344
	(0.0010)	(0.0008)	(0.1822)	(0.0002)	(0.1033)	(0.0590)	(0.3383)	(0.0115)	(0.0004)

Note: Sample period: 2001Q1 - 2015Q4. MacKinnon approximate p-value for Z(t) in parentheses.

In table 8.12 the ADF test for unit root on the FD of the Taylor-gaps shows no clear indications of unit root, with p-values under 5% for each individual economy for all series.

Table 8.12: *Augmented Dickey-Fuller test for unit root of FD for our sample economies*

Ho: Unit root is present in a time series sample
Ha: Time series are stationary
ADF regression lags: 4
Number of obs   = 56

	Australia	Canada	UK	Eurozone	Japan	New Zealand	Norway	Sweden	USA
d.T-gap 1	-4.168	-4.292	-3.322	-4.959	-4.870	-4.278	-4.266	-4.072	-3.890
	(0.0007)	(0.0005)	(0.0139)	(0.0000)	(0.0000)	(0.0005)	(0.0005)	(0.0011)	(0.0021)
d.T-gap 2	-4.247	-4.295	-3.291	-4.873	-4.829	-4280	-4.250	-4.057	-3.880
	(0.0005)	(0.0005)	(0.0153)	(0.0000)	(0.0000)	(0.0005)	(0.0005)	(0.0011)	(0.0022)
d.T-gap 3	-4.337	-4.725	-3.096	-4.959	-3.957	-3.949	-3.301	-4.764	-4.065
	(0.0004)	(0.0001)	(0.0269)	(0.0000)	(0.0017)	(0.0017)	(0.0148)	(0.0001)	(0.0011)
d.T-gap 4	-4.438	-4.757	-3.105	-4.960	-3.913	-4.037	-3.295	-4.732	-4.132
	(0.0003)	(0.0001)	(0.0262)	(0.0000)	(0.0019)	(0.0012)	(0.0151)	(0.0001)	(0.0009)

Note: Sample period: 2002Q1 - 2015Q4. MacKinnon approximate p-value for Z(t) in parentheses

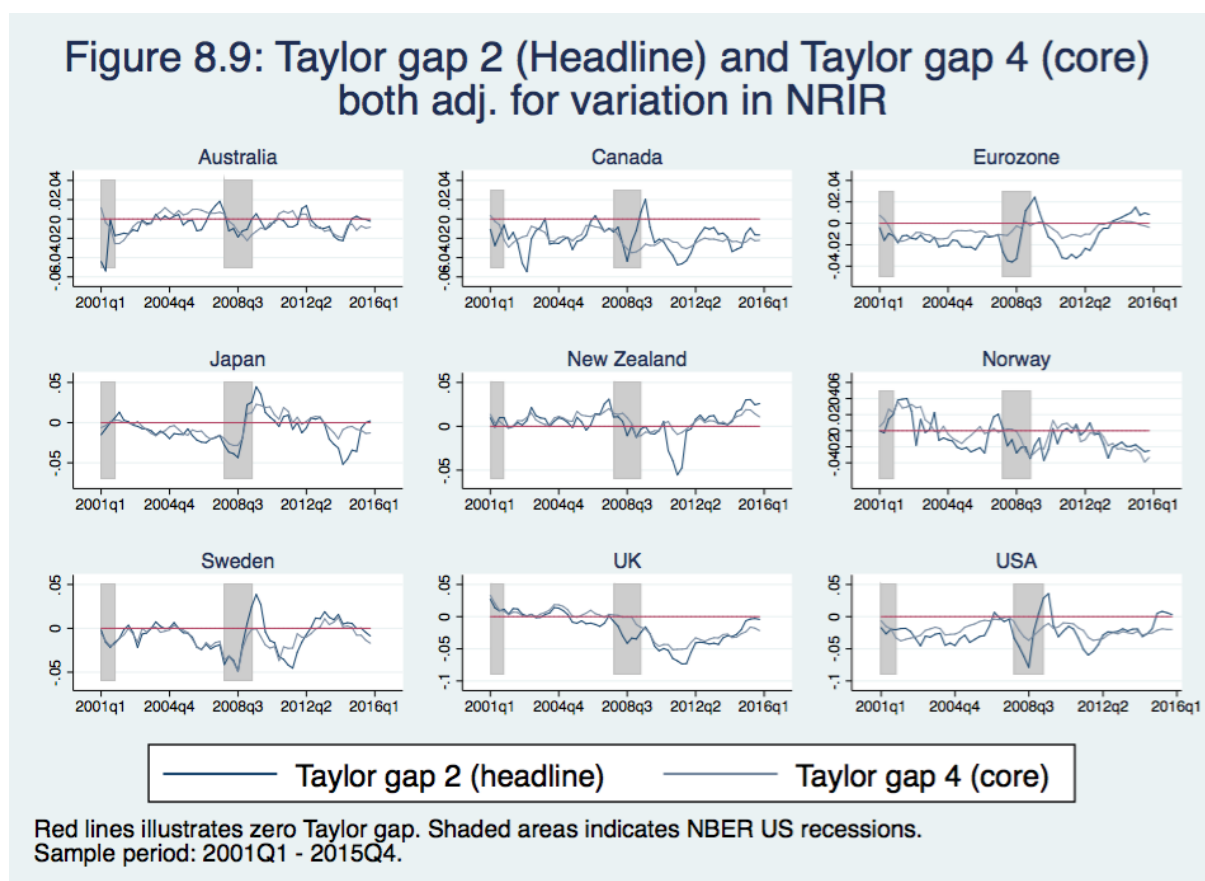## 8.3.2.3 Summary of test for unit root.

In section 8.3.2.1 and 8.3.2.2 we have presented unit root test both for panel data and for the individual economies for the series used in the analysis. The result of LLC unit root test for panel data shows some evidence of unit root for Taylor gap 3 and 4 (using core inflation), with a p-value of 0.12 and 0.05 respectively, whereas most other Taylor gap series rejects the null hypothesis of a unit root at a 1 % level. This is congruent with our expectation that the core inflation based Taylor gaps are less volatile and thus might tend to be slightly more persistent over time. Using first difference the LLC unit root test for panel data rejected the null hypothesis of unit root at a 1% level for all Taylor-gaps.

Similar results were obtained using the Augmented Dickey-Fuller test for unit root for each individual economy. Where some of the economies had unit root present in level form, here especially Taylor gap 3 and 4, using FD the null hypothesis of a unit root could be rejected at a 5 % level for all series for each economy.

The implication of these findings is that when using level, we must be careful when interpreting the results based on Taylor gap 3 and 4. When using the FD of the Taylor gaps, the estimation is somewhat more robust.

8.4 Appendix chapter 5: Analysis

In chapter 5, figure 5.1 presented the Taylor gaps based on the original Taylor rate using headline inflation (T-gap-1) and core inflation (T-gap-3). Figure 8.9 present the same series adjusted for variation in the natural real interest rate (NRIR). By comparing figure 5.1 and 8.9, the time trend appears less clear when controlling for variation in NRIR.

Figure 8.9: Taylor gap 2 (Headline) and Taylor gap 4 (core) both adj. for variation in NRIR

Red lines illustrates zero Taylor gap. Shaded areas indicates NBER US recessions.
Sample period: 2001Q1 - 2015Q4.

We will now show how the result are only marginally different using Taylor rate adjusted for the official inflation target. In table 8.13 and 8.14, we display both the FE and FD estimates using Taylor gaps 5 – 8. Note that Taylor gap 1 should be compared to Taylor gap 5 and so on (see table 8.8 in section 8.2.4). When comparing the result to that of table 5.1 and 5.2, we see that there are only marginal differences in the estimates when adjusting for the official inflation target. This can be explained by official inflation targets close to 2% for our sample economies (See section 8.2.4 and 3.2.3).

Table 8.13: *Panel data regression of key policy rates deviation from the Taylor rule using Fixed Effects estimator with clustered SE*

	(5) T-gap 5	(6) T-gap 6	(7) T-gap 7	(8) T-gap 8
Dummy for the Dotcom Bubble (2001Q1 - 2001Q4)	-0.00268 (0.00477)	0.00321 (0.00488)	-0.00719* (0.00374)	-0.00130 (0.00386)
Dummy for the after math of the Dotcom Bubble (2002Q1-2004Q4)	-0.000314 (0.00337)	0.00305 (0.00345)	-0.00668 (0.00384)	-0.00332 (0.00387)
Dummy for the Financial Crisis (2007Q4 - 2009Q2)	-0.00661*** (0.00180)	-0.00754*** (0.00178)	-0.00552** (0.00227)	-0.00645** (0.00224)
Dummy for the after math of the Financial Crisis (2009Q3-2012Q2)	-0.00761 (0.00589)	-0.00809 (0.00588)	-0.00325 (0.00448)	-0.00374 (0.00447)
Quarter	-0.000388*** (0.0000923)	0.0000229 (0.000115)	-0.000648*** (0.000117)	-0.000237 (0.000131)
Constant	0.00336 (0.00297)	-0.00946** (0.00378)	0.0147*** (0.00425)	0.00190 (0.00473)
R^2	0.181	0.057	0.400	0.111
Adjusted R^2	0.173	0.049	0.395	0.103
Observations	540	540	540	540

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, Sample period: 2001Q1 – 2015Q4.
T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR.

Table 8.14: *Panel data regression of key policy rates deviation from the Taylor rule using first differenced series*

	(5) T-gap 5	(6) T-gap 6	(7) T-gap 7	(8) T-gap 8
D.Dummy 2001Q1	-0.00106 (0.00251)	-0.00106 (0.00251)	-0.00485*** (0.00143)	-0.00483*** (0.00143)
D.Dummy 2001Q2	-0.00985*** (0.00337)	-0.00985*** (0.00337)	-0.0102*** (0.00250)	-0.0102*** (0.00250)
D.Dummy 2001Q3	0.00172 (0.00574)	0.00172 (0.00574)	-0.0108*** (0.00322)	-0.0107*** (0.00323)
D.Dummy 2001Q4	0.00308 (0.00589)	0.00308 (0.00589)	-0.0165*** (0.00360)	-0.0165*** (0.00361)
D.Dummy 2002Q1	0.00261 (0.00613)	0.00261 (0.00613)	-0.0163*** (0.00379)	-0.0162*** (0.00379)
D.Dummy 2002Q2	0.00574 (0.00609)	0.00574 (0.00610)	-0.0141*** (0.00374)	-0.0141*** (0.00375)
D.Dummy 2002Q3	0.00492 (0.00604)	0.00492 (0.00605)	-0.0101*** (0.00365)	-0.0101*** (0.00366)
D.Dummy 2002Q4	-0.00266 (0.00604)	-0.00266 (0.00604)	-0.00889** (0.00360)	-0.00888** (0.00361)
D.Dummy 2003Q1	-0.0117* (0.00593)	-0.0117* (0.00593)	-0.00764** (0.00352)	-0.00763** (0.00353)
D.Dummy 2003Q2	0.00251 (0.00554)	0.00251 (0.00554)	-0.00254 (0.00335)	-0.00253 (0.00336)
D.Dummy 2003Q3	-0.000757 (0.00517)	-0.000757 (0.00517)	-0.00626** (0.00315)	-0.00625** (0.00316)
D.Dummy 2003Q4	0.000655 (0.00492)	0.000655 (0.00492)	-0.00549* (0.00297)	-0.00548* (0.00298)
D.Dummy 2004Q1	0.00646 (0.00447)	0.00646 (0.00447)	-0.00577** (0.00267)	-0.00576** (0.00268)

D.Dummy 2004Q2	-0.00467*	-0.00467*	-0.00667***	-0.00667***
	(0.00276)	(0.00276)	(0.00221)	(0.00222)
D.Dummy 2004Q3	-0.00211	-0.00211	-0.00218	-0.00217
	(0.00210)	(0.00211)	(0.00189)	(0.00190)
D.Dummy 2004Q4	-0.00328**	-0.00328**	-0.00133	-0.00133
	(0.00158)	(0.00159)	(0.00127)	(0.00128)
D.Dummy 2007Q4	-0.0152***	-0.0152***	-0.00396*	-0.00396*
	(0.00254)	(0.00252)	(0.00219)	(0.00218)
D.Dummy 2008Q1	-0.0218***	-0.0218***	-0.00673**	-0.00673**
	(0.00414)	(0.00412)	(0.00273)	(0.00273)
D.Dummy 2008Q2	-0.0269***	-0.0269***	-0.00967***	-0.00967***
	(0.00488)	(0.00487)	(0.00312)	(0.00312)
D.Dummy 2008Q3	-0.0378***	-0.0378***	-0.0127***	-0.0127***
	(0.00534)	(0.00533)	(0.00346)	(0.00346)
D.Dummy 2008Q4	-0.0167**	-0.0167**	-0.0104**	-0.0104**
	(0.00661)	(0.00661)	(0.00442)	(0.00442)
D.Dummy 2009Q1	-0.00336	-0.00336	-0.00852	-0.00852
	(0.00757)	(0.00757)	(0.00543)	(0.00544)
D.Dummy 2009Q2	0.00689	0.00689	-0.00710	-0.00710
	(0.00777)	(0.00777)	(0.00538)	(0.00539)
D.Dummy 2009Q3	0.0178**	0.0178**	-0.00279	-0.00279
	(0.00774)	(0.00773)	(0.00530)	(0.00530)
D.Dummy 2009Q4	0.00376	0.00376	-0.00444	-0.00444
	(0.00790)	(0.00789)	(0.00525)	(0.00525)
D.Dummy 2010Q1	-0.0130*	-0.0130*	-0.00662	-0.00662
	(0.00777)	(0.00776)	(0.00520)	(0.00520)
D.Dummy 2010Q2	-0.0126*	-0.0126*	-0.00390	-0.00390
	(0.00760)	(0.00759)	(0.00513)	(0.00513)
D.Dummy 2010Q3	-0.00762	-0.00762	-0.000938	-0.000938
	(0.00738)	(0.00737)	(0.00498)	(0.00498)
D.Dummy 2010Q4	-0.0176**	-0.0176**	-0.00321	-0.00321
	(0.00704)	(0.00703)	(0.00477)	(0.00477)
D.Dummy 2011Q1	-0.0207***	-0.0207***	-0.00223	-0.00223
	(0.00662)	(0.00661)	(0.00432)	(0.00432)
D.Dummy 2011Q2	-0.0264***	-0.0264***	-0.00350	-0.00350
	(0.00632)	(0.00630)	(0.00419)	(0.00418)
D.Dummy 2011Q3	-0.0291***	-0.0291***	-0.00640*	-0.00640*
	(0.00596)	(0.00594)	(0.00382)	(0.00382)
D.Dummy 2011Q4	-0.0188***	-0.0188***	-0.00449	-0.00449
	(0.00467)	(0.00465)	(0.00329)	(0.00328)
D.Dummy 2012Q1	-0.0137***	-0.0137***	-0.00529**	-0.00529**
	(0.00312)	(0.00311)	(0.00214)	(0.00214)
D.Dummy 2012Q2	-0.00398	-0.00398	-0.00201	-0.00201
	(0.00264)	(0.00263)	(0.00192)	(0.00192)
Constant	-0.000254	0.0000530	-0.000745***	-0.000438*
	(0.000387)	(0.000387)	(0.000240)	(0.000240)
R^2	0.392	0.392	0.148	0.147
Adjusted R^2	0.350	0.350	0.088	0.088
Observations	540	540	539	539

Standard errors in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, Sample period: 2001Q1 – 2015Q4.
T-gap 1: Headline inflation, T-gap 2: Headline inflation adj. NRIR, T-gap 3: Core inflation and T-gap 4: Core inflation adj. NRIR.