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# **Asset Returns, Wage Rigidity and The Business Cycle**

*A Dynamic Stochastic General Equilibrium Approach*

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Master Thesis, MSc. Economics

Norwegian School of Economics

This thesis was written as a part of the Master of Science in Economics and Business Administration at NHH. Please note that neither the institution nor the examiners are responsible through the approval of this thesis for the theories and methods used, or results and conclusions drawn in this work.

# Preface

This thesis marks the end of my Master's degree in Economics and Business Administration taken at the Norwegian School of Economics. I would like to thank my supervisor, Professor Gernot Doppelhofer not just for his valuable feedback, comments and support for my thesis, but also for the methods taught in the course *ECS 502 Advanced Macroeconomics*.

# Asset Returns, Wage Rigidity and The Business Cycle

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## Abstract

This thesis extends the standard New Keynesian framework to incorporate asset pricing capabilities. An economic model which includes CRRA utility, nominal price rigidity, due to Calvo (1983), capital adjustment costs due to Jermann (1998) and monetary policy using a simple Taylor rule is calibrated to match the moments observed in US economy from 1955 to 2008. It also incorporates an equation for real wage rigidity that previously has not been used in such a framework. The thesis investigates the capability of the model to jointly replicate asset pricing and business cycle facts. It also investigates whether the model can provide a theoretical link between monetary policy and asset prices. Lastly, the thesis also studies whether the form of real wage rigidity used here could be useful for future work. I find that while the model is able to replicate business cycle moments for consumption, investment and output, it fails to match the moments for hours worked, wages, wage bill or labor share. The model also fails to capture important asset pricing moments. While the model dynamics and results fail to show that real wage rigidity can provide a direct theoretical link between monetary policy and asset prices, they do show that real wage rigidity is an important part of the model. Lastly, the results also show that the particular wage equation presented in this thesis may not be viable in the future because it does not break the link between wages and marginal product of labor.

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<sup>1</sup>Norwegian School of Economics (NHH), NO-5045 Bergen, Norway. The main programming for this thesis was done in Dynare. Code available from the author upon request.

# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
<b>2</b>	<b>Literature Review</b>	<b>8</b>
<b>3</b>	<b>Methodology</b>	<b>11</b>
<b>4</b>	<b>Economic Model</b>	<b>14</b>
4.1	Household Preferences . . . . .	14
4.2	Monopolistic Competition: Calvo Pricing . . . . .	16
4.3	Capital Adjustment Cost . . . . .	17
4.3.1	Tobin's Q . . . . .	17
4.4	Real Wage Rigidity . . . . .	18
4.5	Monetary Policy . . . . .	19
4.6	Exogenous Shocks . . . . .	20
<b>5</b>	<b>Data</b>	<b>22</b>
5.1	Macroeconomic Variables . . . . .	22
5.1.1	Hodrick-Prescott Filter . . . . .	23
5.2	Financial Moments . . . . .	24
5.2.1	Risk-free Rate of Interest . . . . .	24
5.2.2	Return on Equities . . . . .	24
<b>6</b>	<b>Calibration</b>	<b>26</b>
6.1	Technology . . . . .	26
6.2	Preferences . . . . .	27
6.3	Government . . . . .	27
<b>7</b>	<b>Quantitative Results</b>	<b>29</b>
7.1	Business Cycle Results . . . . .	29
7.2	Asset Pricing Quantitative Results . . . . .	30

<b>8</b>	<b>Model Dynamics</b>	<b>34</b>
8.1	One-Period Technology Shock . . . . .	34
8.1.1	Business Cycle Dynamics . . . . .	34
8.1.2	Asset Pricing Dynamics . . . . .	36
8.2	Long-Run Technology Shock . . . . .	38
8.2.1	Business Cycle Dynamics . . . . .	38
8.2.2	Asset Pricing Dynamics . . . . .	40
8.3	Government Spending Shock . . . . .	42
8.3.1	Business Cycle Dynamic . . . . .	42
8.3.2	Asset Pricing Dynamics . . . . .	45
8.4	Monetary Policy Shock . . . . .	46
8.4.1	Business Cycle Dynamics . . . . .	46
8.4.2	Asset Pricing Dynamics . . . . .	48
<b>9</b>	<b>Conclusion</b>	<b>51</b>
	<b>References</b>	<b>53</b>
	<b>Appendix A Impulse Response</b>	<b>59</b>
	<b>Appendix B Model Derivation</b>	<b>60</b>
B.1	Household . . . . .	60
B.2	Final Good Producer . . . . .	63
B.3	Intermediate Producers . . . . .	64
B.3.1	Calvo Price Setting . . . . .	66
B.4	Government . . . . .	67
B.5	Aggregation . . . . .	67
B.6	Equilibrium . . . . .	72
B.7	Steady State . . . . .	73

# Chapter 1

## Introduction

The financial crisis of 2008 and the subsequent weak recovery in many industrialized economies brought center stage important deficiencies in the macroeconomic literature at the time. In particular, issues related to the lack of a theoretical link between monetary policy and asset prices, and how the former can impact the latter were brought forth. While empirically this link is well established,<sup>1</sup> theoretical macroeconomic literature continues to face certain roadblocks. An important requirement for a macroeconomic model to be viable for such a study is that it needs to be able to jointly replicate business cycle and asset pricing facts. This has been problematic because incorporating frictions to reproduce one set of facts usually harms the ability of the model to replicate the other set of facts (Nath, 2016).

Attempts to reconcile the two sets of facts have had some success in the Real Business Cycle (RBC) literature (Jermann, 1998; Boldrin, Christiano, & Fisher, 2001) where incorporating habit persistence in consumption and capital adjustments costs allow the model to generate equity premia that match those observed in reality. The attempts to incorporate asset pricing capabilities in New Keynesian models have been relatively recent and quite promising (Wei, 2009; De Paoli, Scott, & Weeken, 2010; Challe & Giannitsarou, 2014; Nath, 2016). Following these attempts, in this thesis I incorporate asset pricing capabilities into the standard New Keynesian framework. The model includes Constant Relative Risk Aversion (CRRA) utility, nominal price stickiness, capital adjustment costs, real wage rigidity and four exogenous shocks. The model also includes endogenous capital, a feature that is not common among studies on New Keynesian Models. The novelty of the model is the incorporation of a wage equation which has previously not been used in such a framework to model real wage rigidity. Hence the purpose of this thesis is threefold. First, to investigate whether the model described above can generate realistic dynamics and moments for asset pricing and business cycle variables. Second, to analyze whether the

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<sup>1</sup>The impact of monetary policy on asset prices in papers like Rigobon and Sack (2004) and more recently Alessi and Kerssenfischer (2016) and the impact of asset prices on monetary policy in papers like Bernanke and Gertler (2000) and Rigobon and Sack (2003).

model is able to establish a strong theoretical link between monetary policy and asset prices. Third, whether the real wage rigidity used in this model holds any promise for future work in this literature.

Usually, in New Keynesian literature, the first step for solving the model is to log-linearize the optimality conditions. This does not consider the second order terms while approximating the solution of the model. Log-linearisation leaves behind the certainty equivalents of all assets and removes all premia due to asymmetries introduced by the concavity of the utility function. Hence the risk averse agent behaves risk neutral and is therefore, indifferent between risk-free and risky assets. To do away with the issues introduced by log-linearisation, perturbation methods in Dynare are utilized to obtain second order approximations of the model's solution.

To investigate the research questions, I follow a methodology that has now become standard in macroeconomic modelling. First, the simulated business cycle and asset pricing moments are compared with actual moments observed in the data. Second, the business cycle and asset pricing dynamics generated by the model in response to exogenous shocks are compared with those found in other closely related studies.

For the quantitative results, I find that the model performs well for some business cycle variables namely consumption, investment and output. The model produces unrealistically low volatility for hours worked, wages, real wages and labor share. The model generates countercyclical labor share and procyclical hours worked both of these are consistent with the data. Further the model is able to generate an equity premium but in line with other work, CRRA preferences are not able to generate sufficient equity premium to match the data. Moreover, the model is not able to produce reasonable volatility for either risk free bonds or equities.

In response to exogenous shocks, the model produces some interesting dynamics. The model produces consistent dynamics for a one-period technology shock. For a long-run technology shock, the behavior of inflation and a marginal drop in risk free rate is inconsistent with (Nath, 2016). Inflation dynamics are problematic even when the model is disturbed with a shock to government spending, while economic intuition suggests that inflation should rise at impulse, it falls. Similarly, for a contractionary monetary policy inflation should fall at impulse, but according to model, the inflation rises at impulse by a very large amount, and only very slowly returns to steady state levels. Overall, the model's dynamics are not remarkable and there are several inconsistencies with other studies.

Even though, the model's performance is not outstanding behavior of risk-free rate in response to a monetary policy shock also shows that labor side rigidity is necessary. It may not be a direct theoretical link between monetary policy and asset prices, but it ensures the correct behavior of risk-free rate in this model. Furthermore, the wage equation presented in this thesis does not produce consistent results. It is my conjecture that a wage equation that does not include output would produce better results. For asset

pricing facts, it is necessary to explore different kinds of preferences like Epstein-Zin type preferences or GHH preferences.<sup>2</sup>

The remainder of this thesis is structured as follows: chapter 2 provides a review of important studies on topics that are closely related to ours. Chapter 3 is dedicated to explaining the methodology that we have followed. Chapter 4 gives an in-depth description of the economic model. Chapter 5 describes the data that has been used to generate the stylized facts and the moments observed in the data. Chapter 6 describes the parameter calibrations used for the model. Chapter 7, presents the quantitative results while chapter 8 provides the model dynamics. Chapter 9 concludes.

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<sup>2</sup>See Nath (2016)



# Chapter 2

## Literature Review

First and foremost, this thesis belongs to the literature that addresses business cycle variables and asset prices in an inclusive framework. Rietz (1988), Abel (1990), Benninga and Protopapadakis (1990) and Constantinides (1990) were the first papers in this line of work and they were successful to a certain extent in explaining the *Equity Premium Puzzle*.<sup>1</sup> All of these papers were set in an endowment economy framework. Models which first incorporated non-trivial production sectors were less successful, for example, Rouwenhorst et al. (1991) and Danthine, Donaldson, and Mehra (1992). As Jermann (1998) notes, a frictionless production sector is used by the agent as a hedge against fluctuations in consumption. Jermann (1998) and Boldrin et al. (2001) overcame this issue by incorporating a cost for any adjustments in capital. By also incorporating habit formation in consumption, these papers were successful in generating reasonable equity premia. These papers are quite relevant to this work because this thesis also incorporates a non-trivial production sector and capital adjustment costs.<sup>2</sup> This line of research has continued with the incorporation of more sophisticated features like recursive preferences due to Epstein and Zin (1989) and Wei (2009), different kinds of shocks, long run risk due to Bansal and Yaron (2004) and time varying uncertainty. Recent work includes Jaimovich and Rebelo (2009), Kaltenbrunner and Lochstoer (2010), Campanale et al. (2010) and Croce (2014).

This thesis also contributes to another line of work that incorporates labor side frictions to simultaneously match asset pricing and business cycle facts. The argument for incorporating these frictions is similar to the argument for incorporating capital adjustment costs. Households tend to use wage as a hedge to protect themselves against fluctuations in consumption (Lettau & Uhlig, 2000; Guvenen et al., 2003; Krueger & Uhlig, 2006). This insurance does not allow the model to generate the equity premia observed in the data. Uhlig (2007) showed that a DSGE model can match both sets of facts by incorpo-

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<sup>1</sup>The observation that the difference between return on risk assets and risk-free assets is much higher than can be explained by standard macroeconomic models using reasonable calibrations, see Mehra and Prescott (1985)

<sup>2</sup>Cochrane (1991, 1996) provides a different perspective to production based asset pricing by evaluating the producer's first order conditions.

rating a real wage rigidity with habits in consumption. While the initial focus was incorporating wage rigidities through simple ad-hoc wage equations (Blanchard & Galí, 2007; Uhlig, 2007), these models have been extended to include search and matching frictions and wage bargaining.<sup>3</sup> Moreover, the role of wage rigidities in determining aggregate fluctuations has been shown by Hall (2005) and Shimer (2005). Blanchard and Galí (2007) also show that real wage rigidities are particularly important for generating a trade-off between inflation and output stabilization in a New Keynesian framework. Since this thesis also uses a New Keynesian model, I lay strong emphasis on real wage rigidity and use a wage equation similar to the one proposed in Blanchard and Galí (2007).

This thesis is also related to the literature on New Keynesian Models which incorporate asset pricing capabilities. Surprisingly, even though the usage of New Keynesian Models has become so widespread, the asset pricing implications of these models have been extensively studied in only a few papers.<sup>4</sup> Chan, Foresi, and Lang (1996) were the first to look at monetary policy and asset prices. They investigate the asset pricing implications of a cash-in-advance model based on Lucas Jr and Stokey (1985). They find that the estimates of the curvature parameter are lower in money based CAPM than those obtained in consumption based CAPM. Another important study is by Sangiorgi and Santoro (2005). They study whether price or wage rigidities are more important in explaining asset pricing facts in a New Keynesian model. They find that staggered wage setting is able to generate a higher equity premium than staggered price setting alone. But they do not have endogenous capital in the model. Moreover, in their solution methodology they log-linearise around a steady state.

While Wei (2009) does have endogenous capital in the model, log-linearisation is used to solve a canonical New Keynesian model. In this setting, Wei (2009) finds that under a standard monetary rule, the real impact of a monetary policy shock is too weak to generate a reasonable equity premium and that technology shocks have a very slight contribution to the equity premium. De Paoli et al. (2010) also study a similar research question, but they use a second order approximation. Their results suggest that incorporating real rigidities enhances the risk premia. They also find that in an economy with only technology shocks, nominal rigidities actually reduce the risk premia.

The results of De Paoli et al. (2010) are particularly interesting because the implications of real rigidity found in their work provide a direct motivation for this thesis. This thesis however, does not incorporate habit formation in either consumption or labor. Moreover, while De Paoli et al. (2010) use a money-in-utility formation, I use a standard CRRA set up with just contemporaneous consumption and hours worked. Challe and Giannitsarou (2014) also have some success in jointly replicating asset pricing

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<sup>3</sup>Important recent works include Kuehn, Petrosky-Nadeau, and Zhang (2012), Favilukis and Lin (2015), Weber (2015), Li and Palomino (2014) and Schmidt (2016).

<sup>4</sup>Some papers in this line of research focus on the term structure, and abstract from equity premium issues. For example Rudebusch and Wu (2008)

and business cycle facts by utilizing a present value approach for asset pricing variables which, according to them, preserves higher order properties. Their results hinge on the slow mean reversion of the real interest rates, as predicted by New Keynesian models and the smoothness of consumption process under habit formation.

The study by Nath (2016) is the closest to this thesis, but there are important differences. Nath (2016) uses nominal wage rigidities, nominal price rigidities à la Rotemberg (1982) (as opposed to staggered price resetting à la Calvo (1983) used in this thesis), capital adjustment costs and monetary policy. Another difference is that in Nath (2016), GHH preferences<sup>5</sup> are used instead of CRRA preferences used in this thesis. This is partially why Nath (2016) obtains better results than this thesis. The other reason is the incorporation of wage inflation and the relationship between wage inflation and price inflation both of which are absent in our model, due to which price inflation dynamics are inconsistent with those in Nath (2016). The model in Nath (2016) is approximated to the second order. While the quantitative results in Nath (2016) are a significant improvement over other similar work done previously, it is still unable to produce the levels of equity premia observed in the data.

Due to its similarities with this work, Nath (2016) is used as a benchmark at several points in this thesis to evaluate the consistency of results.

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<sup>5</sup>Greenwood-Hercowitz-Huffman preferences due to Greenwood et al. (1988)

# Chapter 3

## Methodology

Beginning with Kydland and Prescott (1982), dynamic stochastic general equilibrium (DSGE) modelling has become widely accepted over the years. Today, DSGE models are a standard tool for research in macroeconomics and international economics, to deal with problems relating to inflation stabilization, fiscal policies and business cycles. Blanchard (2008) notes that after many years of debate, a consensus on the accepted macroeconomic methodology has been reached.

As the name suggests, DSGE models are dynamic, referring to the importance of time paths of variables instead of static one-time variables. Stochastic refers to the fact that the model economy is affected by stochastic disturbances. General refers to explaining the aggregate movements in the whole economy and not just partial markets. Equilibrium alludes to the idea that the model is based on economic theory where there is a balance between the demand and the supply in the different sectors of the economy.

Withing DSGE models, there are three types of models, the Real Business Cycle (RBC) models, the New Keynesian models and Heterogenous Agent Models.<sup>1</sup> A Real Business Cycle model consists of a neoclassical model with flexible prices at its core which is augmented by real shocks to study business cycle fluctuations. New Keynesian Models build on RBC models by introducing prices and price stickiness into the framework as well as a monetary policy shocks. The models also incorporate monopolistic competition.

Due to the increasing complexity of DSGE models, both in terms of theoretical features and computation, solution techniques for DSGE models are in themselves a topic of research. While new techniques for dealing with different kinds of assumptions and complexities in the model are being introduced frequently, the overall DSGE modelling procedure has become standardized. After the model's assumptions have been identified, the first-order conditions need to be derived. The first order optimality conditions

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<sup>1</sup>Heterogeneity can be introduced along, for example, age in Overlapping Generations Models, or along policies in models which incorporate progressive marginal tax rates.

combined with the structural equations build a system of stochastic difference equations. The system is usually highly non-linear and needs an approximation method to transform the problem into a linear system and then reach an approximate solution. Finally, the method's performance is evaluated using impulse responses and/or by comparing second-order moments of the simulated data series to the moments of actual data series.

In this thesis, I follow the standard "recipe" for DSGE models:

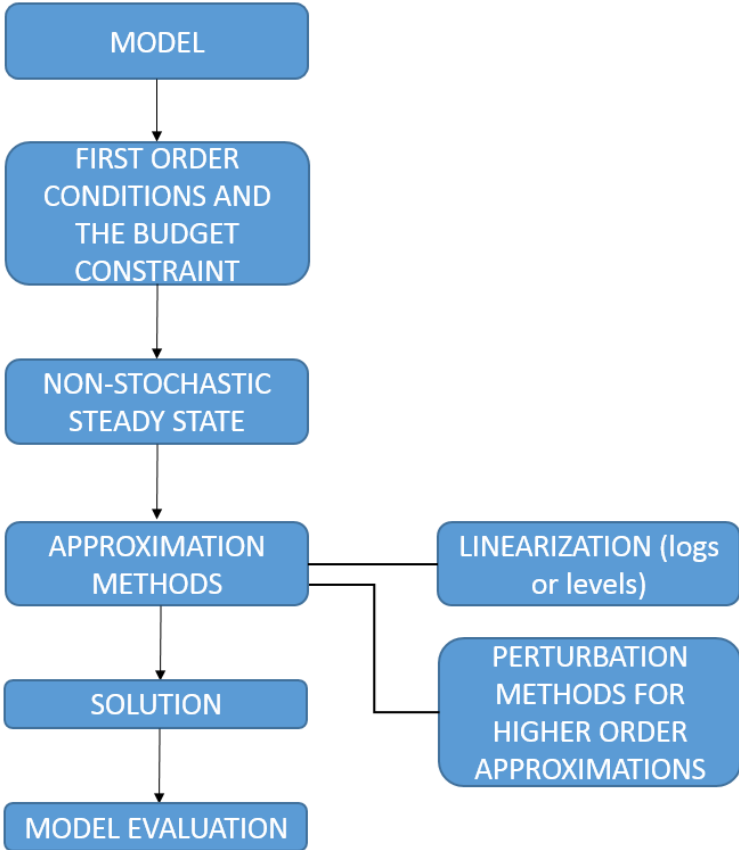
1. **Setting up the economic model:** This includes assuming the consumer's preferences for optimally choosing consumption and leisure over their lifetime given their budget constraint. Further, the firm's problem is also set up to maximize their profits given restriction on production technology.
2. **Derivation of the first-order conditions and removing all the redundant variables:** Combining the optimality conditions with the structural equations (the stochastic shocks), a system of non-linear stochastic difference equations is obtained.
3. **Calibration:** The structural parameters of the model are then calibrated to values suggested by seminal microeconomic studies.
4. **DYNARE:** At the next step, Dynare is used for solution and simulation of the model.<sup>2</sup>
5. **Model Solution:** Since the model is highly non-linear and does not have a closed analytical solution, the solution is approximated to the second order around a non-stochastic steady state.
6. **Model Evaluation:** Evaluation of the model by looking at how well simulated moments and dynamics match the observed moments and dynamics in real world data.

In a general equilibrium setup, every sector of the economy is modelled independently. Consumers and firms together are combined in one block referred to as the private sector and Governments and monetary policy makers together referred to as the public sector of the economy. All assumptions relating to the behavior of agents in both sectors need to be described in analytically tractable mathematical formulae by assuming properties of functions which make the problem slightly easier. Even after using analytically tractable formulations, the model may still not be solvable and approximation techniques are necessary (Campbell, 1994).

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<sup>2</sup>Dynare is a set of MATLAB codes used for DSGE models, developed by Michael Julliard and collaborators.

Figure 3.1: Flowchart depicting the standardized sequence of steps involved in solving and evaluating DSGE models



# Chapter 4

## Economic Model

This chapter presents the features of the model economy and the mathematical formulae used to model various features. The full model and the derivation of the equilibrium conditions are provided in Appendix B.

### 4.1 Household Preferences

The households follow the following instantaneous utility function.

$$U(C_t, N_t^s) = \left( \frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} - \frac{(N_t^s)^{1+\omega}}{1+\omega} \right) \quad (4.1)$$

where  $C_t$  and  $N_t^s$  are respectively the consumption and hours of work supplied by the household at time  $t$  and  $\frac{1}{\psi}$  and  $\omega$  are respectively the degree of relative risk aversion and the Frisch elasticity of labor supply. These utility functions are also called Constant Relative Risk Aversion (CRRA) utility functions. To show the benefits of CRRA preferences consider first a simpler form of CRRA utility,

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad (4.2)$$

where  $\sigma$  is the relative risk aversion and  $\frac{1}{\sigma}$  is the intertemporal elasticity of substitution. The generalized form of the intertemporal Euler Equation is,

$$r = \rho - \underbrace{\left( \frac{u''(c) \cdot c}{u'(c)} \right)}_1 \cdot \frac{\tilde{c}}{c}$$

where  $r$  indicates the rate of interest,  $\tilde{x}$  indicates the derivative of  $x$  with respect to time and  $\rho$  is the rate of time preference.

For a DSGE model to be considered useful, it has to be able to replicate the behavior of the real world economy as well as possible. An important feature of the U.S. economy is a relatively stable growth

rate. At the steady state,  $r$  and  $\rho$  are constants. For the second term on the right hand side of the above equation to be constant, Part I needs to be a constant. CRRA utility ensures that the consumption growth is constant where

$$\sigma = -\left(\frac{u''(c).c}{u'(c)}\right)$$

where  $\sigma$  is the relative risk aversion and is a constant. Secondly, this functional form also implies that division between safe and risky asset is independent of initial level of wealth and the choice is scale independent (homothetic preferences).

CRRA preferences give rise to two kinds of effects, the income effect and the substitution effect. To understand these two mechanisms consider again the simplified utility function of equation (4.2). Maximizing the net present value subject to the budget constraint, we obtain the following,

$$C_t^{-\sigma} = \beta(1+r)C_{t+1}^{-\sigma}$$

where  $\beta$  is the subjective discount factor. Taking logarithms on both sides,

$$c_t = c_{t+1} - \frac{1}{\sigma} \log \beta - \frac{1}{\sigma} \log(1+r) \approx c_{t+1} - \frac{1}{\sigma}(r - \rho)$$

If  $r$  increases, the above equation implies that consumption growth ( $c_{t+1} - c_t$ ) increases as well. Essentially, this is the *substitution effect*. An increase in the interest rate makes consumption tomorrow relatively less expensive compared with consumption today. In other words, the consumer has to sacrifice less units of consumption today to have the same amount of consumption tomorrow. Additionally, the consumer also experiences the *income effect*. Higher interest rates imply higher income and this leads to a contemporaneous rise in consumption. The net effect on future consumption is always positive but the effect on consumption today depends on which effect dominates. The effect that dominates plays an important role in the transmission of exogenous shocks throughout the model.

In this thesis, the interaction between these two effects is important to understand the mechanism that generates the business cycle and asset pricing dynamics in the model. Wherever necessary an interpretation of the mechanism and the underlying effects is provided.

The utility function used in this thesis, shown in equation (4.1), also includes  $\omega$ , the Frisch elasticity of labor supply. It measures the percentage change in hours worked due to the percentage change in wages, holding constant the marginal utility of wealth. It is important to note that  $N_t^s$  is the number of hours supplied by the household in the utility function. So in this sense, the utility function is slightly different from those used to in similar studies where usually the leisure term is included in the utility and then the labor is total time, usually normalized to 1, minus the leisure time. The Frisch elasticity ignores the effect of wage shocks on the hours worked.

The value of Frisch elasticity is a slightly controversial issue in the literature. Some economists have argued against large aggregate elasticities (MaCurdy, 1981; Altonji, 1986). But others, notably Keane



and Rogerson (2012), argue that this view is flawed. Rogerson (1988) uses the idea of indivisible labor to show that small micro elasticities can be reconciled with large aggregate elasticities. Keane and Rogerson (2012) and Violante (2014) also show that the two can be reconciled. Accordingly, in this thesis when the model is calibrated a small value for  $\omega$  is assumed. An important distinction is that between intensive margin and extensive margin. The intensive margin is the number of hours the agent chooses to work given employment and the extensive margin is the binary choice between working and not working. This thesis focuses on the number of hours worked, i.e., the intensive margin.

## 4.2 Monopolistic Competition: Calvo Pricing

In standard RBC models, nominal shocks do not have real effects. New Keynesian models use an RBC model as their foundation along with price stickiness so that nominal shocks have real effects. To generate price stickiness, we need to incorporate monopolistic competition where firms have the power to set prices. Hence the production side is split into intermediate goods producers and final goods producers. Final goods producers aggregate intermediate goods into final goods and continue to act in a perfectly competitive setting. Intermediate goods producers act in monopolistic competition where each firm produces a different variety and has monopoly over that variety. The monopolists compete because different varieties are imperfect substitutes.

For price stickiness, the assumption is that all firms face a fixed probability with which they can change their prices in any given period. If the firm is allowed to change its price in a period, it sets its price bearing in mind that it might not be allowed to change its price for many periods in the future. This formulation was first proposed by Calvo (1983) and has become one of the two most common methods of introducing nominal price rigidities. The other method is the Rotemberg adjustment (Rotemberg, 1982). While the assumptions of Calvo pricing mechanism are slightly unrealistic, it facilitates aggregation over a continuum of firms.

Due to its impact on real and nominal variables, price stickiness allows for an active stabilization policy. Price indexation refers to a situation where even if the firm is not allowed to adjust its price, the price is updated by a certain factor. Price indexation has become a usual practice in the New Keynesian literature, but for sake of simplicity I abstract from it.

## 4.3 Capital Adjustment Cost

One of the most common stylized facts<sup>1</sup> in financial literature is that the return on equities is higher than the return on risk-free securities. This observation is at the heart of the *Equity Premium Puzzle*<sup>2</sup> where the predictions of standard DSGE models are not consistent with the data. Therefore, generating an excess return, i.e., return on equities less return on risk free bonds is important if a model is to jointly be consistent with asset pricing and business cycle facts. The simplest way to generate an excess return is to introduce a friction on the capital side. Capital owned by households then evolves according to the following law of motion:

$$K_{t+1} = (1 - \delta_K)K_t + I_t - G_t K_t$$

where  $\delta_K$  is the constant depreciation rate, and  $G_t$  allows for convex adjustment costs à la Jermann (1998). In particular,

$$G_t = \frac{I_t}{K_t} - \left[ \frac{\alpha_1}{1 - 1/\xi} \left( \frac{I_t}{K_t} \right)^{1-1/\xi} + \alpha_0 \right] \quad (4.3)$$

where  $\xi$  determines the degree of rigidity and  $\alpha_0$  and  $\alpha_1$  are chosen such that at the steady state  $\bar{G} = \bar{G}' = 0$ . While this is the most common formulation of capital adjustment costs, other formulations such as Sims (2011a) may also be useful. A consequence of this kind of rigidity is that the firm pays an increasing convex cost of net investments, i.e., all investment net of depreciation. In other words, the firms pays higher costs if it owns more capital. The direct impact that these costs have in a DSGE model is that Tobin's  $Q \neq 1$ . Unit Tobin's  $Q$  cannot generate an excess return.

### 4.3.1 Tobin's $Q$

Developed by Tobin (1969), the  $q$  theory of investment states that of the market value of physical capital owned by a firm is higher than the replacement cost than capital has more value inside the firm than outside it. Formally,

$$Q = \frac{\text{Market Value of Capital Owned by a firm}}{\text{Replacement Cost of Capital}}$$

Further, Tobin showed that if  $Q < 1$ , then the firm should reduce their capital and if  $Q > 1$  then the firm should acquire more capital. But Tobin's  $Q$  is an average value, whereas for DSGE models the marginal  $Q$ , i.e.,  $\frac{dV}{dk}$  is needed. Hayashi (1982) laid out the conditions under which the marginal  $Q$  is equal to the average  $Q$ . The conditions are the following:

1. The production function needs to be homogeneous of degree one.

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<sup>1</sup>See Chapter 5, section 5.2 of this thesis.

<sup>2</sup>See Mehra and Prescott (1985)

2. The adjustment cost function, defined as  $G(I_t, K_t)K_t$  above, needs to be homogeneous of degree one.

*Proof.* It is straightforward to show that the Cobb-Douglas production function is homogeneous of degree one. Proof can be found in (Romer, n.d., pages 10-11) and I eschew a formal proof.

For the second condition,

$$G_t(I_t, K_t)K_t = \frac{I_t}{K_t} - \left[ \frac{\alpha_1}{1 - 1/\xi} \left( \frac{I_t}{K_t} \right)^{1-1/\xi} + \alpha_0 \right] K_t$$

Therefore,

$$G_t(\lambda I_t, \lambda K_t) = \frac{\lambda I_t}{\lambda K_t} - \left[ \frac{\alpha_1}{1 - 1/\xi} \left( \frac{\lambda I_t}{\lambda K_t} \right)^{1-1/\xi} + \alpha_0 \right] \lambda K_t$$

Cancelling the  $\lambda$  from the numerator with  $\lambda$  in the denominator, we obtain

$$G_t(\lambda I_t, \lambda K_t)\lambda K_t = \lambda G_t(I_t, K_t)K_t$$

Hence, the second condition is also satisfied. □

## 4.4 Real Wage Rigidity

In a standard frictionless model, the real wage is equal to the marginal product of labor. In such a setting, the wages are highly correlated with output and are as volatile as the output. Moreover, Favilukis and Lin (2015) show that introducing real wage rigidity reduces the volatility of wages and even enhance the performance of the model for asset pricing variables. This is because under real wage rigidity the agents are not free to adjust their wages and hence wage cannot be used as a hedge by shareholders. The wage rigidity in this thesis is that the real wage in the current period depends on previous period's real wage and a fraction of the marginal product like the following:

$$wage_t = (1 - \theta_w)wage_{t-1} + \theta_w \left( (1 - \alpha) \frac{Y}{L} \right)^{\gamma_w} \quad (4.4)$$

where the real wage at time  $t$  is a weighted sum of the real wage in the previous time period and the marginal product which is dampened by a factor of  $\gamma_w$ . This wage equation has two benefits, it captures the persistence in real wages along with incomplete adjustment to the marginal product. The persistence of wages has been empirically well documented.<sup>3</sup> There are two main explanations for this persistence,

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<sup>3</sup>See Beaudry and DiNardo (1991), Baker, Gibbs, and Holmstrom (1994), McDonald and Worswick (1999), Grant (2003), Macis (2006) and Von Wachter and Bender (2008)

firstly, the wages are persistent due to implicit contracts that insure workers against large wage declines.<sup>4</sup> The second explanation is the differential human capital accumulation.<sup>5</sup> The second part of the wage equation on the right hand side includes the marginal product of labor dampened by a factor of  $\gamma_w$  where  $\gamma_w < 1$ . This allows firms to adjust wages in every period but not as much as the marginal product. This dampens the link between output volatility and wage volatility because in this case the wages do not adjust one-to-one with changes in output. The benefit of equation (4.4) is that it is independent of the precise mechanism of persistence. On the other hand, equation (4.4) is ad-hoc and has not been determined by an optimization process.

To my knowledge, this thesis is the first to incorporate such a wage equation into a New Keynesian asset pricing framework with endogenous capital and capital adjustment costs. As in Favilukis and Lin (2015), one of the motivations of this study is to investigate whether incorporating this wage equation into an asset pricing framework, under reasonable values of  $\theta_w$  and  $\gamma_w$ , has any bearing on the link between monetary policy and asset prices.

## 4.5 Monetary Policy

In the model, monetary policy is conducted through a simple Taylor-style rule (Taylor, 1993). In traditional New Keynesian models, the Central Bank reacts to changes in inflation and the output gap, i.e., the gap between output for the economy with rigidities less the output of flexible price economy. I use a simple rule in which the Central Bank responds changes in inflation and changes in output gap. Since the conduct of monetary policy is not the focus of this thesis, I opt for a simple formulation. Formally, the Central Bank in the model uses the following rule,<sup>6</sup>

$$i_t = \gamma_i i_{t-1} + \phi_\pi (\pi_t - \pi^*) + \phi_y (Y_t - Y^*) + u_t$$

where  $i_t$  is the nominal interest rate at time  $t$ ,  $\gamma_i < 1$ ,  $\pi_t$  is the inflation at time  $t$ ,  $Y_t$  is the output,  $Y^*$  is the natural level of output,  $\phi_\pi$  and  $\phi_y$  determine the size of the response to changes in inflation and the output, respectively,  $\pi^*$  is the inflation target interpreted in this model as the steady state level of inflation and  $u_t$  is the monetary policy shock process (discussed in the next section). In the model,  $\pi^*$ , is set such that the Central Bank strives to achieve an inflation target of 2%. The central bank responds to contemporaneous values of output and inflation.

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<sup>4</sup>See Harris and Holmstrom (1982), J. Thomas and Worrall (1988), Beaudry and DiNardo (1991) and J. P. Thomas and Worrall (2007).

<sup>5</sup>See Reder (1955), Okun et al. (1973) and Gibbons and Waldman (2006)

<sup>6</sup>The Taylor rule is implemented in Dynare for log-deviations in output, but since I use HP filtered output, this is equivalent to stabilizing around a natural level of output.

## 4.6 Exogenous Shocks

Besides incorporating nominal and real frictions, I also include four exogenous shocks<sup>7</sup> to the model economy. These shocks disturb the economy away from its steady state and play a crucial role in analyzing the performance of a DSGE model. The dynamics generated after a disturbance, as the economy returns to its steady state, should be consistent with those observed in other studies. Except the one-period technology shock, all other exogenous shocks are assumed to be  $AR(1)$  processes without unit roots.<sup>8</sup> Furthermore, all shocks are assumed to be positive.

$$z_t = \epsilon_t^z \quad (4.5)$$

Equation (4.5) is a one-period technology shock and the shock itself lasts only for one period. The innovation term,  $\epsilon_t^z$  is drawn from  $\mathcal{N}(0, \sigma_z)$ . The effects of these shocks might last for more than one period due to strong internal propagation mechanisms embedded in the model.

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \quad (4.6)$$

Equation (4.6) above is a long-run technology shock, where  $0.9 < \rho_a < 1$ . The innovation term,  $\epsilon_t^a$  is drawn from  $\mathcal{N}(0, \sigma_a)$ .

$$g_t = \rho_g g_{t-1} + \epsilon_t^g \quad (4.7)$$

Equation (4.7) captures the exogenous government spending shock in the economy. The economy-wide budget constraint then becomes:

$$\underbrace{Y_t}_{\text{Output}} + \underbrace{e^{g_t} T_t}_{\text{Government Spending}} = \underbrace{C_t}_{\text{Consumption}} + \underbrace{I_t}_{\text{Investment}}$$

$T_t$  receives the government spending shocks through  $\epsilon^g (\sim \mathcal{N}(0, \sigma_g))$  when, for example, the government discovers oil or any other such resource where the revenue can be costlessly channelled into the economy. This approach is similar to Mayer, Moyen, and Stahler (2010). The shock formulation is admittedly very ad-hoc and unrealistic, but since the focus of this thesis is not the fiscal side, an ad-hoc formulation suffices.<sup>9</sup>

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t} \quad (4.8)$$

where  $\epsilon_t^u \sim \mathcal{N}(0, \sigma_u)$ . The above equation (4.8) is a monetary policy shock, where  $\epsilon_t^u$  captures monetary policy news or announcements regarding the monetary policy by the Central Bank. Examples include

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<sup>7</sup>Dynare requires one shock per exogenous variable.

<sup>8</sup>The absence of unit roots assumption is standard in the literature on simulating DSGE models, see calibration used in Nath (2016). Also see Lafourcade and de Wind (2012) as an example of a model which incorporates shocks with unit roots.

<sup>9</sup>Note that the it is the government spending shock that is exogenous. Government spending is financed by bonds and lump sum taxation.

changes in interest rates made during the Federal Open Market Committee meetings (FOMC) by the U.S. Federal Reserve.

# Chapter 5

## Data

In this section, I present a description of the data used in this study. I use quarterly U.S. data starting from the first quarter of 1955 to the third quarter of 2008.<sup>1</sup> I select the time period between 1955: 1 and 2008: 3 for this study. Firstly, FRED,<sup>2</sup> BEA<sup>3</sup> and BLS<sup>4</sup> quarterly employment and wage data is only available from the start of 1947. Secondly, as explained by Favilukis and Lin (2015), during World War II there were large movements of labor in and out of the private sector which, due to their temporary nature, would add unusual volatility to the analysis. Thirdly, FRED 3-month U.S. Treasury Bill rates are only available from the start of 1955. Lastly, following Li and Palomino (2014), I exclude data starting from 2008: 4 because the Great Recession prompted the US Federal Reserve to use unconventional monetary policy tools (like quantitative easing) as interest rates were driven to the zero bound. The analysis of these tools is beyond the scope of this work.

### 5.1 Macroeconomic Variables

Nominal GDP, nominal consumption and nominal investment data from National Income and Product Accounts (NIPA) table 1.1.5 is used. As in Favilukis and Lin (2015), nominal consumption is defined as the sum of expenditures on non-durable consumption and services. Nominal investment includes fixed private investment and durable goods. The total wage bill in the economy is defined as the sum of compensation of employees in the private sector and the supplements to wages received, collected from NIPA table 2.1. Employment is defined as the total number of people employed in the private sector and has been obtained from US Bureau of Labor Statistics (BLS Data Series CES0500000001: Total private

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<sup>1</sup>Henceforth, I use the notation, YYYY: Q, where the numbers on the left of the colon represent the year and the number on the right represents the quarter.

<sup>2</sup>FRED: Federal Reserve Economic Data

<sup>3</sup>BEA: U.S. Government's Bureau of Economic Analysis

<sup>4</sup>BLS: U.S. Government's Bureau of Labor Statistics

**Table 5.1: Variables**

<i>Variables and Sources</i>	<i>Source</i>
<i>Nominal GDP</i>	NIPA Table 1.1.5
<i>Nominal Consumption</i>	NIPA Table 1.1.5
<i>Nominal Investment</i>	NIPA Table 1.1.5
<i>Employment</i>	BLS Data Series CES0500000001
<i>Nominal Total Wage Bill</i>	NIPA Table 2.1
<i>Nominal Wage</i>	Obtained by dividing Nominal Total Wage Bill by Employment
<i>Labor Share</i>	Obtained by dividing Nominal Total Wage Bill by output
<i>Consumer Price Index</i>	FRED Series: CPALTT01USQ661S

employees, seasonally adjusted).

Following Favilukis and Lin (2015), I only look at the impact of private sector labor frictions on asset prices. Inflation,  $\pi$ , is calculated using the Consumer Price Index (CPI)<sup>5</sup> in the following manner:

$$\pi_t = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} * 100$$

Further, I divide all nominal variables by the CPI to get their real counterparts, i.e, real GDP, real consumption, real investment, real total wage bill and real wage. I, then, take logarithmic transformations of all real variables. Finally, as has become standard in the literature, to remove the trend components of these series, I apply the Hodrick-Prescott Filter. I calibrate the H-P filter to match the quarterly frequency of the data, i.e, set the  $\lambda = 1600$ . The business cycle data moments are shown in the first column of table 7.1.

A short review of the Hodrick-Prescott Filter is presented below.

### 5.1.1 Hodrick-Prescott Filter

The Hodrick-Prescott (H-P) Filter<sup>6</sup> is a statistical technique used to separate the trend and cyclical components of a time series. In particular,

$$y_t = y_t^g + y_t^c$$

<sup>5</sup>Data from FRED Series: CPALTT01USQ661S (seasonally adjusted to the value of 1 dollar in 2010).

<sup>6</sup>For a detailed description of the Hodrick-Prescott filter, the reader is referred to Hodrick and Prescott (1997) and Ravn and Uhlig (2002).



Where  $y_t$  is the original time series and  $y_t^g$  and  $y_t^c$  are the trend growth and cyclical components respectively. Given  $\lambda$ ,  $y_t^g$  minimizes the following loss function:

$$\sum_{t=0}^{\infty} (y_t - y_t^g)^2 + \lambda \sum_{t=0}^{\infty} ((y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g))^2$$

Hodrick and Prescott (1997) show that to match quarterly frequency of the data,  $\lambda$  must be set to 1600.

Table 5.1 outlines all the business cycle variables and their sources that have been used in this study.

## 5.2 Financial Moments

### 5.2.1 Risk-free Rate of Interest

To calculate the Excess Return and the Sharpe Ratio, a proxy of the risk-free rate is needed. I use 3-Month Treasury Bills issued by the U.S. Federal Reserve for this purpose.<sup>7</sup> The series, in its raw form shows the monthly values of annualized returns on 3-Month Treasury Bills. I convert it to quarterly frequency by calculating three month simple moving averages, in the following manner:

$$R_{firstquarter,year}^f = \frac{R_{January,year}^f + R_{February,year}^f + R_{March,year}^f}{3}$$

$$R_{secondquarter,year}^f = \frac{R_{April,year}^f + R_{May,year}^f + R_{June,year}^f}{3}$$

$$R_{thirdquarter,year}^f = \frac{R_{July,year}^f + R_{August,year}^f + R_{September,year}^f}{3}$$

$$R_{fourthquarter,year}^f = \frac{R_{October,year}^f + R_{November,year}^f + R_{December,year}^f}{3}$$

### 5.2.2 Return on Equities

To calculate the return on equities, I use monthly values of the Standard & Poor's (S&P) 500 index (1955 to 2008), collected from Bloomberg. The S&P 500 is based on the market values of 500 large companies. The S&P 500 is a widely used benchmark to study the evolution of asset prices.<sup>8</sup> In this study, I use the month-end values of S&P 500 to calculate the monthly returns:

$$R_t^E = \left( \frac{S\&P500_t - S\&P500_{t-1}}{S\&P500_{t-1}} \right) \times 100$$

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<sup>7</sup>Data Series: TB3MS (not seasonally adjusted) from Federal Reserve of St. Louis (Federal Reserve Economic Data (FRED))

<sup>8</sup>See for example Christiano, Eichenbaum, and Evans (1994, 2005) and Rogoff et al. (2006).

Where  $t$  is measured in months and  $R_t^E$  captures the monthly return on S&P 500. Next, I annualize the return on S&P 500, to make the equities data comparable with the Risk-free rate:

$$\tilde{R}_t^E = \left[ \left( 1 + \frac{R_t^E}{100} \right)^{12} - 1 \right] \times 100$$

Where  $t$  is measured in months and  $\tilde{R}_t^E$  is the annual return on S&P 500. Finally, I convert this monthly series into a quarterly series using the same procedure outlined in Section 5.2.1.

To calculate the Excess Return, I deduct the the Risk-free rate from the Return on Equities and calculate the Sharpe Ratio as shown in the first row of Table 7.2.

# Chapter 6

## Calibration

All parameters have been calibrated to a quarterly frequency. I evaluate three versions of the model. First, the fully blown model with CRRA utility, real wage rigidity, capital adjustment costs and nominal price rigidity which I will refer to, in the rest of the thesis, as the **baseline model**. Second, a model with CRRA utility, capital adjustment costs, nominal price rigidity and flexible wages ( $\theta_w = 1, \gamma_w = 1$ ) which I refer to as the **flexible wage model**. Third, the model with CRRA utility, real wage rigidity and nominal price rigidity with flexible capital ( $\frac{1}{\xi} = 0, \text{Tobin's } Q = 1$ ) referred to as the **flexible capital model**.

### 6.1 Technology

The technology parameters govern the production process, the depreciation and the technology shock processes. The calibrations for these parameters are quite standardized in the literature. I follow Croce (2014) and set the share of capital in production to 0.34. A quarterly depreciation rate of 0.025 is set to have an annual depreciation of 10 per cent. The degree of substitutability parameter,  $\eta$ , is set according to the assumption on the markup,  $\frac{\eta}{\eta-1}$ . Following Christiano et al. (2005), I set the markup equal to 1.2, which implies that  $\eta = 6$ . I follow Nath (2016), and calibrate the long run technology shock to have a standard deviation of 0.007 and an autocorrelation coefficient of 0.95. The transitory (one-period) shock is calibrated to have a standard deviation of 0.005 following Sims (2011b).

Possibly the most critical parameter in the model is the Calvo pricing parameter,  $\theta_p$ . The larger the value, the more sticky prices are. The probability that a firm can change its price in a given period is  $(1 - \theta_p)$ . If a firm updates its price in a period, then the expected duration that the firm is stuck with that price is  $\frac{1}{1-\theta_p}$  (Sims, 2011a). Bils and Klenow (2004) determine the average time between price changes through microeconomic data. While there are considerable differences across goods, they determine the average time period between price changes to be six to nine months, which correspond to

$0.5 \leq \theta_p \leq 0.66$ . Accordingly, I set the  $\theta_p$  to be 0.6.

For the convex cost, Jermann (1998) and Boldrin et al. (2001) set  $\frac{1}{\xi}$  to be 0.23. But as Nath (2016) points out that this value implies very strong adjustment costs. For the model in this thesis, I use a relatively small value of 0.01 to not make the model excessively rigid. Further, to ensure that the adjustment function is zero in the steady state, we have,

$$\alpha_1 = \delta^{\frac{1}{\xi}}$$

$$\alpha_0 = \frac{\delta}{1 - \frac{1}{\xi}}$$

## 6.2 Preferences

The subjective discount factor, also known as the rate of time preference,  $\beta$  is calibrated to match the long run risk free rate. Following Kehoe and Perri (2002) and Boldrin et al. (2001), I set  $\beta = \frac{1}{1.0518} \approx 0.96$  to match the stylized facts presented in the Data chapter of this thesis.

For the Frisch elasticity of labor supply, there are considerable differences across studies, while Smets and Wouters (2007) use  $\omega = 1.85$ , Schmitt-Grohé and Uribe (2012) use  $\omega = 0.4$ . I follow Li and Palomino (2014), who use  $\omega = 0.3$ . The only constraint imposed on the value of the Frisch elasticity of labor supply is by Chetty et al. (2011) who suggest that a value of  $\omega > 1$  is inconsistent with micro evidence.

The key feature of CRRA preferences is the marriage of the coefficient of relative risk aversion (RRA) and the inter-temporal elasticity of substitution (IES). The IES measures the aversion to variation in consumption across time periods and RRA measures the aversion to variation in consumption across states of nature. I use a calibration of 2 for the IES, which implies that the RRA is 0.5.

For the real wage rigidity, while no study has used the exact wage equation that is used in this thesis, Uhlig (2007) is the closest work. I calibrate the weight on the marginal product to be 0.6 close to the calibration used by Uhlig (2007). Secondly, I calibrate the exponent parameter to be 0.4 to achieve reasonable dampening of the marginal product, also in line with Romer (n.d.).

## 6.3 Government

For the parameters in the Taylor rule and the monetary policy process, I closely follow (Nath, 2016). The inflation response,  $\phi_\pi$ , and the output response,  $\phi_y$  have values of 1.5 and 0.5 respectively. The degree of interest rate smoothing is set at 0.85 and the standard deviation of the monetary policy shock is set at 0.0028. The autocorrelation coefficient is set to 0.65.

Following Sims and Wolff (2013), the government spending shock is calibrated to have a standard deviation of 0.005 and the autocorrelation coefficient is set to 0.9.

**Table 6.1: Calibration Values for Parameters in Baseline Model**

Parameter	Value	Description
$\alpha$	0.34	Share of Capital in Production
$\psi$	2	Intemporal Elasticity of Substitution
$\delta$	0.025	Quarterly Depreciation Rate
$\beta$	0.96	Subjective Discount Factor
$\rho$	0.99	Persistence of Long-Run Technology Shock
$\theta_w$	0.6	Real Wage Rigidity Parameter
$\gamma_w$	0.4	Real Wage Exponent Parameter
$\omega$	0.3	Frisch Elasticity of Labor Supply
$\frac{1}{\xi}$	0.01	Convex Adjustment Cost Coefficient
$\theta_p$	0.6	Probability that a firm can change its price in any given period
$\eta$	6	Degree of Substitutability
$\rho_g$	0.9	Autocorrelation Coefficient of Government Spending shock
$\rho_i$	0.65	Autocorrelation Coefficient of Monetary Policy Shock
$\rho_a$	0.95	Autocorrelation Coefficient of Long-run technology shock
$\phi_\pi$	1.5	Inflation Response in Taylor Rule
$\phi_y$	0.5	Output Response in Taylor Rule
$\gamma_i$	0.85	Degree of Interest Rate Smoothing
$\sigma_z$	0.007	Standard Deviation of the one-period shock
$\sigma_a$	0.007	Standard Deviation of the long-run shock
$\sigma_g$	0.005	Standard Deviation of the government shock
$\sigma_u$	0.0028	Standard Deviation of the monetary policy shock

# Chapter 7

## Quantitative Results

### 7.1 Business Cycle Results

The main quantitative results are shown in Table 7.1. Column 3 shows the results for the baseline model (i.e. CRRA utility, convex adjustment costs, real wage rigidity, monopolistic competition among firms and monetary policy via a simple Taylor-style rule with one period and long run technology shocks, a government spending shock and a monetary policy shock). The baseline model is able to closely match the volatility of output, consumption, investment and relative variance of consumption to output and the relative variance of investment to output. These business cycle results are basically due to forward looking consumers having a weaker response to the productivity shocks due to the presence of rigidities and the interplay of productivity and other shocks. For example, investment is more volatile as firms try to reduce their marginal costs in response to increasing wage bills by increasing investment.

Once the capital adjustment costs are turned off (flexible capital model), investment becomes very volatile, nearly twice as much as data. Interestingly, also for the model without wage rigidity (flexible wages) investment is more volatile than the data. In the baseline model, these two rigidities act together to bring investment volatility closer to the data. The results also show that rigidities are required for the investment side as well as the labor side to remove any opportunity for the agent to use either side as a hedge against fluctuations in consumption. All three models perform well in capturing the volatility of output.

Further, in the baseline model the real wage volatility is far too little compared with the data. A likely reason for this could be the functional form of real wages rigidity. In the current set-up, wages are not being determined by households and the wage equation is ad-hoc. It could be that there is excessive dampening as households are not setting the wages through an optimization process. Indeed, once the rigidity is turned off (Column 5, flexible wages), volatility of hours worked, wages and wage bill increases. In the flexible wage model, labour share is less volatile than the baseline model, likely reason could be

the volatility of output affects the volatility of labor share. Comparing the results for the flexible capital model and the baseline model, it is evident that capital adjustment costs do not have any major impact on wages, hours worked, wage bill and labor share. This observation further justifies the inclusion of labor side rigidities.

All three versions are also able to closely match the volatilities with respect to volatility of output for output, consumption and investment. But none of the models come close to the the relative volatilities for labor side variables. There is some evidence that the wage rigidity might be excessively dampening relative volatility (with respect to volatility of output) for hours worked as it becomes very large for the flexible wage model and too low for the baseline compared with the data. For relative volatilities also the capital adjustment costs don't seem to have much effect, when comparing the relative volatilities for labor side variables obtained from the flexible capital and baseline models. Turning capital adjustment costs on or off doesn't affect the labor side relative volatilities.

While the baseline model is not able to reproduce wage and labor quantitative facts well, it is able to generate a pro-cyclical in employment. This is an improvement over standard RBC models which produce counter-cyclical labor supply. In fact all three version are able to produce this result.

Further, all three versions of the model are able to generate a counter-cyclical labor share, a result that not all New Keynesian models are able to produce. One clear observation is that wages, hours worked and labor share are tightly correlated with the output. This problem has also been noted by Uhlig (2007). For the baseline model, these correlations are relatively the weakest, but still quite high when compared with the data. This again points the fact that while the real wage rigidity is required and does marginally improve correlations, wage rigidities that completely break the link between wage and marginal product might produce better results.<sup>1</sup>

An extension which can possibly better match the quantitative results in the data would endogenize the wage setting process and the economy-wide wage would result from the household's optimization. Successful models would include wage bargaining, search and matching or other techniques which realistically model the labor side.

## 7.2 Asset Pricing Quantitative Results

Table 7.2 shows the results for the financial variables produced by the model. The model is able to generate a small excess return, but is unable to match the excess return observed in the literature. The volatility of return on risk free assets is also an indication that the real wage rigidity is too high and at lower levels of real wage rigidity, hours worked, wages, wage bill and labor share become too

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<sup>1</sup>Li and Palomino (2014) show that nominal wage rigidities produce better results.

**Table 7.1: Data and Business Cycle Results**

	Data	Baseline Model	Flexible Capital $\frac{1}{\xi} = 0, \text{Tobin's } Q = 1$	Flexible Wages $\theta_w = 1, \gamma_w = 1$
$\sigma(y)$	1.66	1.733	1.80	1.55
$\sigma(c)$	0.98	1.137	1.070	0.76
$\sigma(i)$	4.27	5.461	7.07	6.68
$\sigma(n)$	1.48	0.196	0.223	3.39
$\sigma(w)$	1.03	0.091	0.01	0.38
$\sigma(w * n)$	1.80	0.422	0.47	0.92
$\sigma\left(\frac{w*n}{y}\right)$	1.02	0.057	0.056	0.0017
$\frac{\sigma(c)}{\sigma(y)}$	0.61	0.66	0.59	0.49
$\frac{\sigma(i)}{\sigma(y)}$	2.62	3.15	3.92	4.30
$\frac{\sigma(n)}{\sigma(y)}$	0.87	0.113	0.12	2.17
$\frac{\sigma(w)}{\sigma(y)}$	0.62	0.005	0.01	0.25
$\frac{\sigma(w*n)}{\sigma(y)}$	1.12	0.24	0.26	0.60
$\frac{\sigma\left(\frac{w*n}{y}\right)}{\sigma(y)}$	0.61	0.033	0.03	0.0011
$\rho(c, y)$	0.85	0.82	0.82	0.63
$\rho(i, y)$	0.92	0.78	0.71	0.67
$\rho(n, y)$	0.66	0.80	0.83	0.99
$\rho(w, y)$	0.56	0.92	0.93	0.99
$\rho(w * n, y)$	0.422	0.99	0.98	0.99
$\rho\left(\frac{w*n}{y}, y\right)$	-0.27	-0.98	-0.97	-0.95

The Data column represents the moments observed in the data. It represents HP-filtered values. The variables are following: log real output ( $y$ ), log real consumption( $c$ ), log real investment( $i$ ), log real employment( $n$ ), log real wage ( $w$ ), log real wage bill( $w*n$ ), labor share( $\frac{w*n}{y}$ ) and inflation( $\pi$ ). All volatilities are measured in per cent per quarter. The Baseline Model column shows the simulated moments for the model with real wage rigidity and convex adjustments costs. The Flexible Capital Column shows the simulated moments for the model without convex costs. The Flexible Wages column shows the simulated moments for the model without real wage rigidity. (Notation: the symbol  $\sigma(x)$  refers to the standard deviation of  $x$  used as a measure of volatility, the symbol  $\rho(x, b)$  refers to the correlation between  $x$  and  $b$ )



tightly correlated with the output. These results are consistent with the literature that shows a trade-off generated when attempting to satisfy both business cycle and asset pricing facts. If the real wage rigidity is lowered, equity volatility will be closer to the data but correlation of labor side variables with output becomes very high.

The model generates a very high Sharpe ratio, but this is because the model produces a very small volatility of equity returns. Overall, the model does not match the asset pricing moments very well. The unconditional mean of the stochastic discount factor,  $\mathbf{E}(m)$  produced by the model is 0.9819 while the volatility of the stochastic discount factor,  $\sigma(m)$  is 0.76.<sup>2</sup> A simple comparison with equity moments shows that the stochastic discount factor generated by the model violates the Hansen-Jagannathan (H-J) lower bound. Consider the following,

$$\frac{\sigma(m)}{\mathbf{E}(m)} \geq \frac{|\mathbf{E}(R^e)|}{\sigma(R^e)}$$

Substituting the moments generated by the model, to satisfy the H-J lower bound

$$\frac{0.76}{0.9819} \text{ should be greater than } \frac{2.39}{0.13}$$

but,

$$0.78 < 18.38$$

Hence the H-J lower bound is violated. This results is not surprising and has commonly been observed in models with CRRA preferences (Cochrane, 2009). The results obtained for asset pricing variables are consistent with those observed in other similar studies (Nath, 2016). The model with flexible capital is able to generate a risk-free return close to that observed in the data, but as discussed earlier, it is unable to generate an excess return and hence the Sharpe Ratio is zero. The volatility of risk free return is also very small. Further, for the model with flexible wages, it seems that the model generates negative risk-free rate and return on equities. This shows that the model needs a friction on the labor side.

While the baseline model is qualitatively able to match the moments, it is unable to match the quantitative observations in the data. Possible solutions include incorporating habit formation in the utility functions and/or using Epstein-Zin type preferences.

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<sup>2</sup>Both  $\mathbf{E}(m)$  and  $\sigma(m)$  are taken from the output generated by the Dynare code of the model.

**Table 7.2: Unconditional Financial Moments**

	$\mathbf{E}[R^F]$	$\sigma(R^F)$	$\mathbf{E}[R^E]$	$\sigma(R^E)$	$(\mathbf{E}[R^E] - \mathbf{E}[R^F])$	$(\frac{\mathbf{E}[R^E] - \mathbf{E}[R^F]}{\sigma(R^E)})$
<i>Data</i>	5.18	2.76	19.89	35.28	14.70	0.42
<i>Baseline</i>	0.97	0.10	2.39	0.13	1.42	10.92
<i>Flexible Capital</i>	5.25	0.09	5.25	0.09	0.0	0.0
<i>Flexible Wages</i>	-0.81	3.70	-0.65	0.43	0.16	0.37

The *Data* row represents the values observed in the data. The *Baseline* row shows the results for the model with real wage rigidity and convex adjustments costs. The *Flexible Capital* row shows the results for the model without capital adjustment costs. The *Flexible Wages* row shows the results for the model without real wage rigidity.  $\mathbf{E}[R^F]$  represents the unconditional mean of the return on risk-free bonds.  $\mathbf{E}[R^E]$  represents the unconditional mean of the return on equities.  $\sigma(R^F)$  represents the volatility of the return on risk-free bonds and  $\sigma(R^E)$  represents the volatility of return on equities.  $(\mathbf{E}[R^E] - \mathbf{E}[R^F])$  is the excess return and  $\frac{\mathbf{E}[R^E] - \mathbf{E}[R^F]}{\sigma(R^E)}$  is the Sharpe Ratio. All volatilities are measured in per cent per quarter.

# Chapter 8

## Model Dynamics

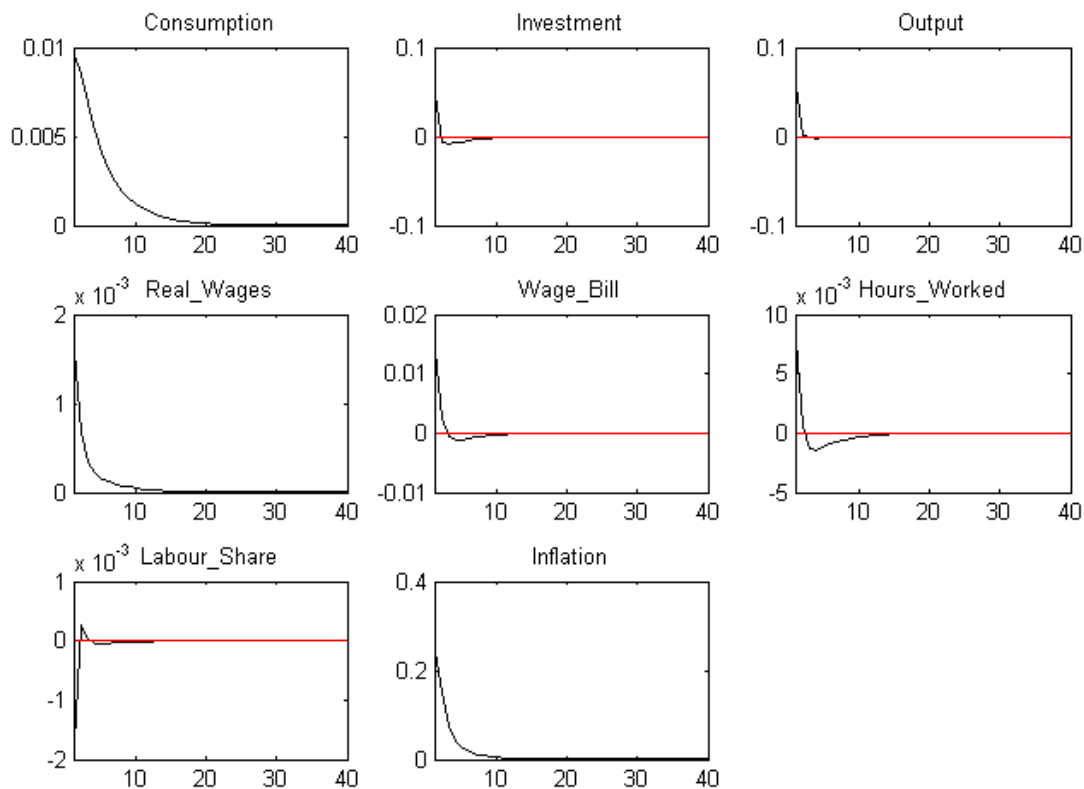
In this section, I present the business cycle and asset pricing dynamics following exogenous shocks. The Flexible Capital model does not generate an excess return and its dynamics are not useful for a study that tries to jointly replicate asset pricing and business cycle facts. Moreover, the focus of this study is on assessing whether the real wage rigidity presented has any scope for future use so I only present the dynamics for the Baseline and Flexible Wage models.

### 8.1 One-Period Technology Shock

#### 8.1.1 Business Cycle Dynamics

The impulse responses in figures 8.1 and 8.2 show how business cycle variables behave given a one-period technology shock. Figure 8.1 shows the responses for the real variables of the model with real wage rigidity and figure 8.2 shows the responses for the model without real wage rigidity. The first clear observation is that the real wage rigidity has an attenuating effect as the economy reacts less vigorously to the one-period shock. Further in figure 8.1, consumption and investment both increase at the same time as the representative agent tries to increase her contemporaneous consumption and smooth consumption over time. The technology shock raises the output through the production function and hence raises the wages. The latter is ensured by the formulation of the real wage rigidity. Increased income causes two conflicting effects: the substitution effect and the income effect. The substitution effect is the result of the agent's desire to accumulate capital for future consumption and therefore she increases investment and reduces consumption today. On the other hand, income effect is the desire to increase immediate consumption and feel richer today because of higher wages. For the model in this thesis, in the presence of a short-run shock, the income effect dominates the substitution effect which explains the rise in both consumption and investment. The increase in wages and hours worked lead to a contemporaneous rise in the economy's wage bill. Since the output and hours worked are directly driven by the shock but the

**Figure 8.1: Impulse Responses for Real Variables with Real Wage Rigidity**

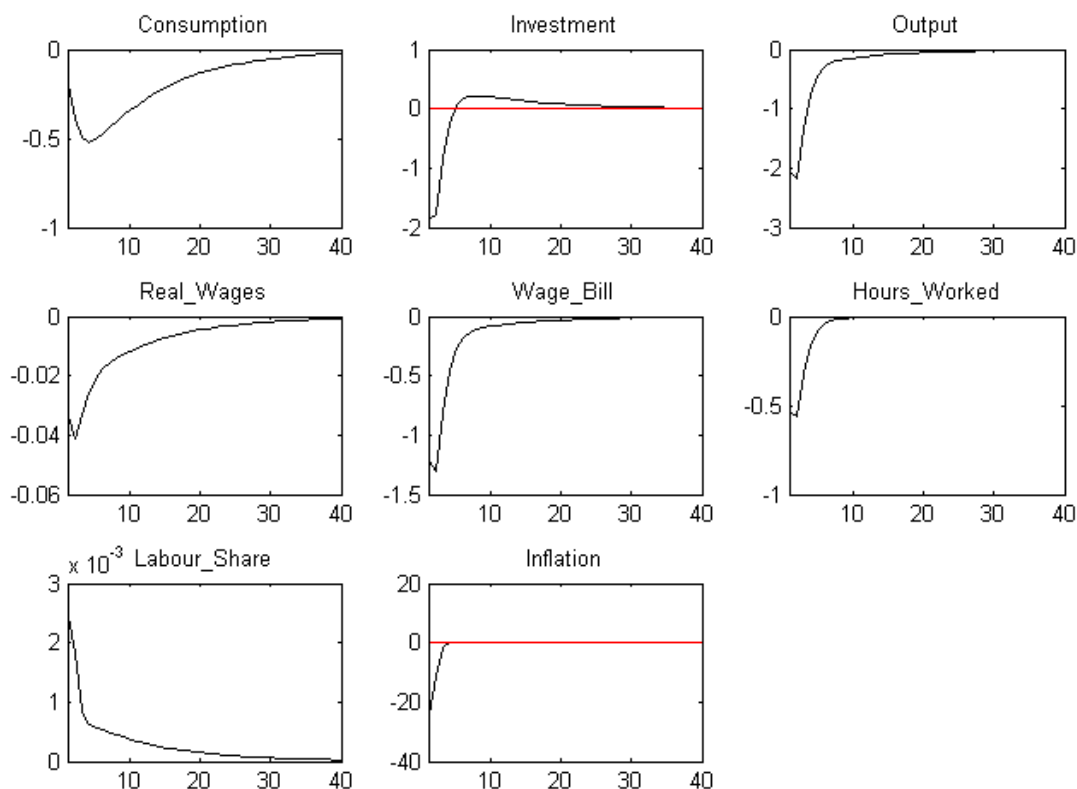


*The above impulse responses show the behavior of business cycle variables in response to a One-Period Technology shock for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

hours worked are attenuated due to real wage rigidity, the labour share drops.

The model generates inflation dynamics consistent with Nath (2016). One potential mechanism that explains the behavior of inflation could be that due increased consumption, there is an increase in the demand which motivates the intermediate goods producers increase prices in response to increased contemporaneous sales. On the other hand, when the real wage rigidity is turned off, the model generates counter-intuitive impulse responses. Figure 8.2 shows that consumption, investment, output and hours worked go down. The decline in inflation is also unreasonably high. These responses are inconsistent with theory. Therefore, this model needs frictions on the labour side to generate dynamics consistent with theory in response to a one-period technology shock.

**Figure 8.2: Impulse Responses for Real Variables without Real Wage Rigidity**



*The above impulse responses show the behavior of business cycle variables in response to a One-Period Technology shock for the model without real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

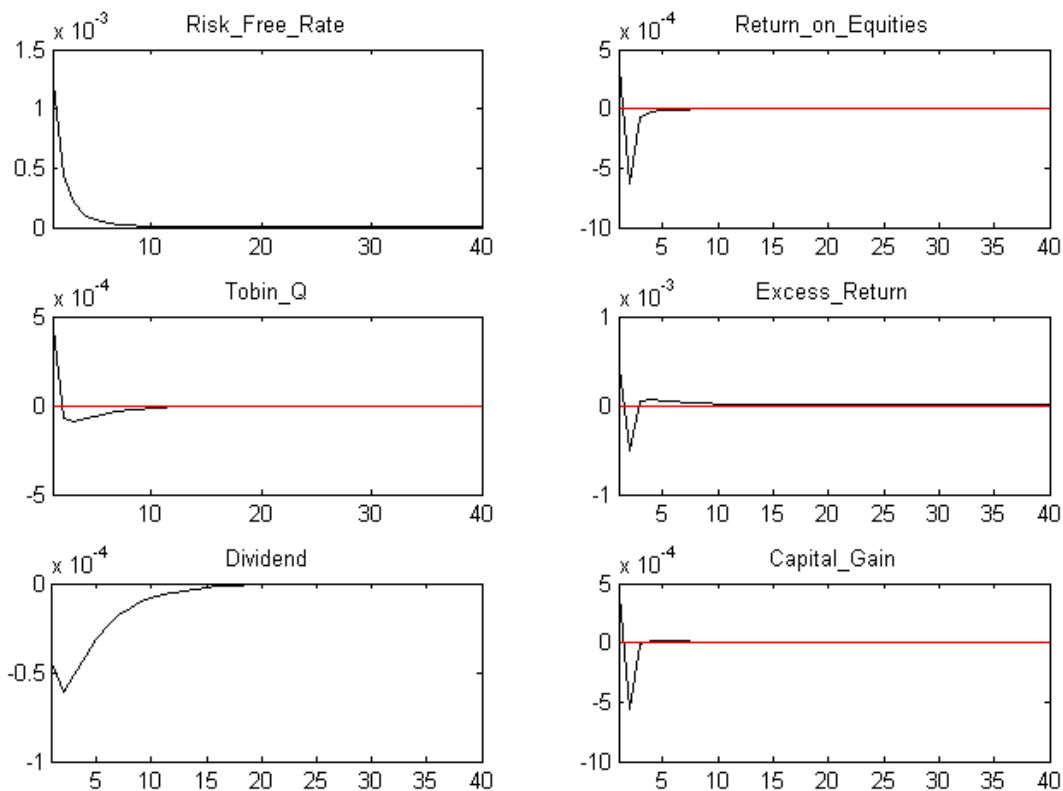
### 8.1.2 Asset Pricing Dynamics

Beginning with the risk-free rate, the model's dynamics show that the income effect is very strong as the agents need to be given higher rates on risk-free assets. Her desire to consume today is the key driver of the risk-free rate but as noted in the previous section, again the response is slightly muted in figure 8.3 compared with figure 8.4 due to demand inertia introduced by the real wage rigidity. Secondly, the Return on Equity has the characteristic zig-zag behavior. The Tobin's Q follows the dynamics of investment. The persistence in the wage formulation ensures that the deviation in Tobin's Q is less in figure 8.3 than in figure 8.4. Further, the model without real wage rigidity produces dynamics which are opposite to those predicted by Nath (2016). Here again, it has to be concluded that some form rigidity is necessary on the labor side for this model.

To understand the dynamics of return on equity, consider the following equation:

$$R_t^E = \frac{D_t}{Q_{t-1}} + \frac{Q_t}{Q_{t-1}}, \quad (8.1)$$

**Figure 8.3: Impulse Responses for Asset Pricing Variables with real wage rigidity**



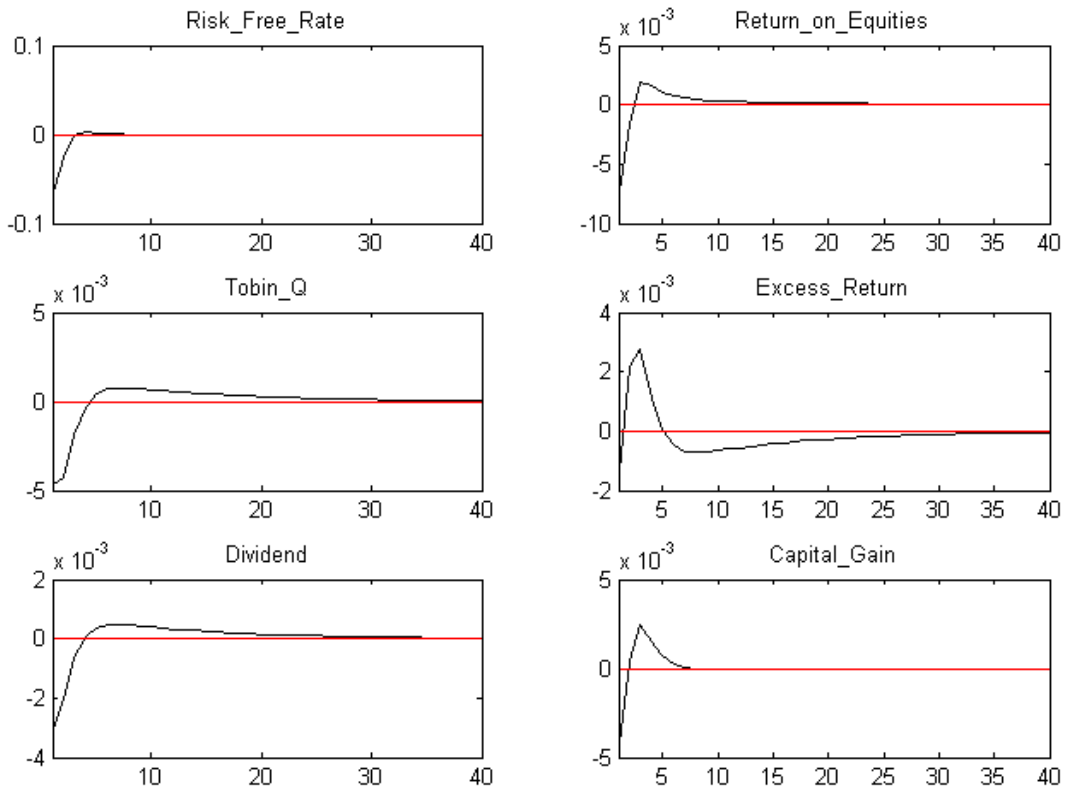
*The above impulse responses show the behavior of asset pricing variables in response to a One-Period Technology shock for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

where  $R_t^E$  is the real return on equities at time  $t$  and  $D_t$  is the dividend from production that goes to households and  $Q_t$  is the Tobin's Q.

The first part on the right hand side of the above equation is the dividend driver of return on equity and the second part of the right hand side is the capital gain driver of equity returns. The rise in the wage bill, as shown in the previous section, leads to a reduction in the real profits and hence the dividends falls. But as noted in the literature, this channel is rather small and plays a minor role in the evolution of return on equity. The other channel is the capital gain. Under a positive technology shock, the price of equity follows the price of capital, as captured in this model by the Tobin's Q. Therefore the price of equity rises and hence the return on equity rises at impulse. Subsequently, due to household over-investment in the equities, the return goes down. The excess return is the difference between the return on equities and the the risk-free rate and its dynamics are straightforward and follow the dynamics of the return on equities.

In the absence of wage rigidity, figure 8.4 shows that the risk-free rate drops but this is not consistent

**Figure 8.4: Impulse Responses for Asset Pricing Variables without real wage rigidity**



*The above impulse responses show the behavior of asset pricing variables in response to a One-Period Technology shock for the model without real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

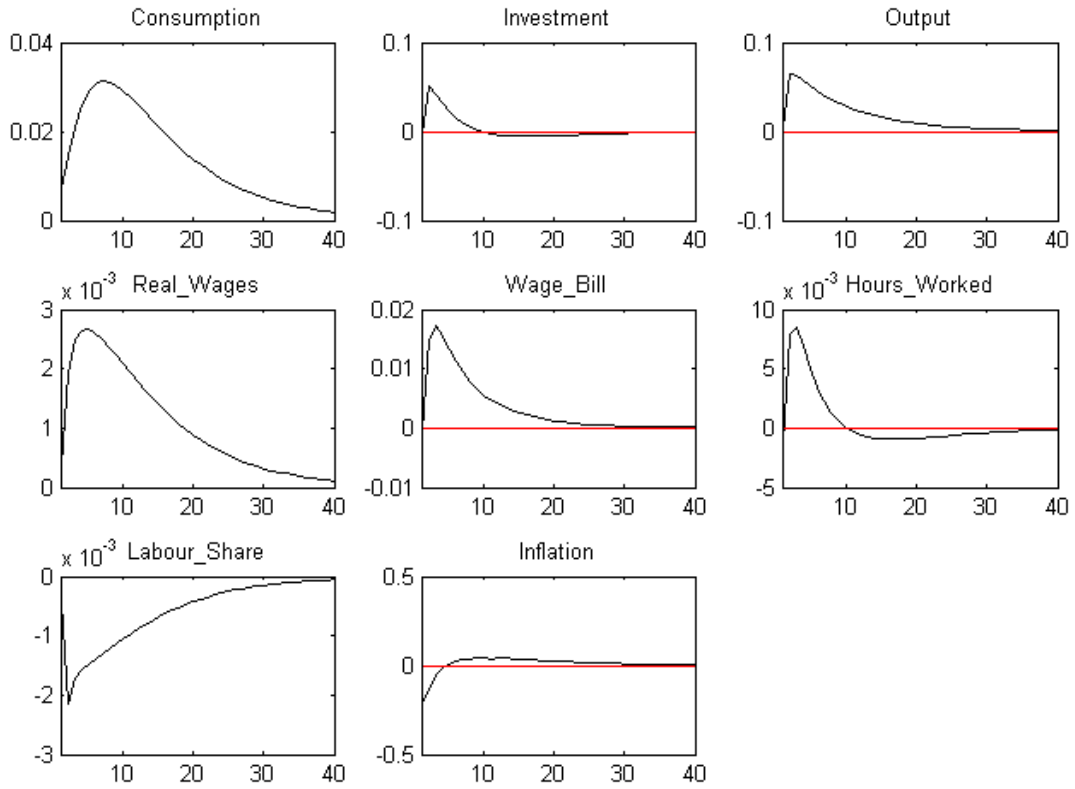
with the dynamics of investment, higher savings should imply higher investments at impulse. But this is not observed in figure 8.2. These dynamics also show that real wage rigidity acts as an important part of the model.

## 8.2 Long-Run Technology Shock

### 8.2.1 Business Cycle Dynamics

Figures 8.5 and 8.6 show the dynamics of business cycle variables in response to a long run technology shock with real wage rigidity and without real wage rigidity, respectively. Here again real wage rigidity has a dampening effect on the economy. As in the previous case, the output rises due to the technology shock. The primary mechanism is the impact of the shocks through the production function, as described previously. A higher output raises the real wage albeit less than in the model without real wage rigidity.

**Figure 8.5: Impulse Responses for Real Variables with Real Wage Rigidity**



The above impulse responses show the behavior of business cycle variables in response to a Long-Run Technology shock for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.

Higher wages again raise the issue of income effect versus the substitution effect and similar to the previous case, the income effect dominates and consumption and investment both rise. Comparing 8.5 with 8.1 shows that even a one-period shock has quite long lasting effects and that the internal propagation mechanism of the model is quite strong.

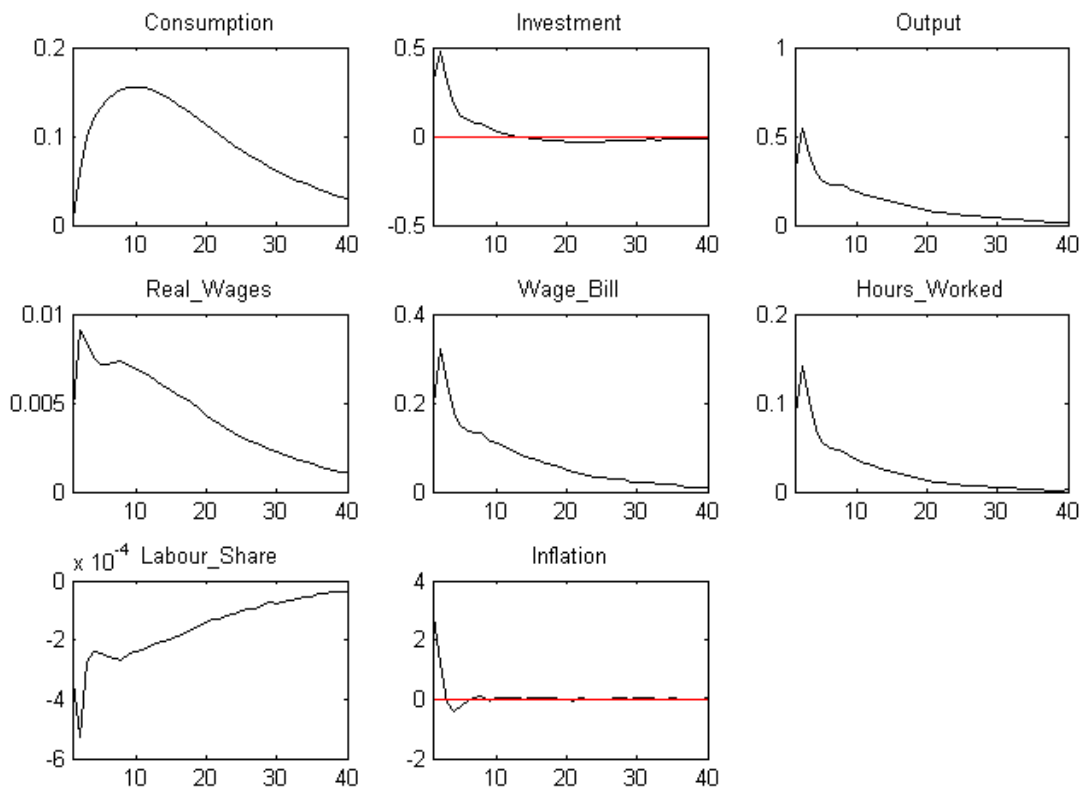
Qualitatively, consumption dynamics in figures 8.5 and 8.6 are quite similar with *hump-shaped* responses. The attenuated responses indicates that the real wage rigidity attenuates the income effect by not allowing the wages to increase as much as in the flexible economy. Consider the wage equation:

$$wage_t = (1 - \theta_w)wage_{t-1} + \theta_w((1 - \alpha)\frac{Y_t}{L_t}) \quad (8.2)$$

The second term on the right hand side consists of contemporaneous output and it rises due to the shock, raising the wages. But when the wages do not rise as much due to stickiness, consumption increases less. Hence the income effect is less strong due to real wage rigidity. The behavior of inflation is problematic,



**Figure 8.6: Impulse Responses for Real Variables without Real Wage Rigidity**



*The above impulse responses show the behavior of business cycle variables in response to a Long-Run Technology shock for the model without real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

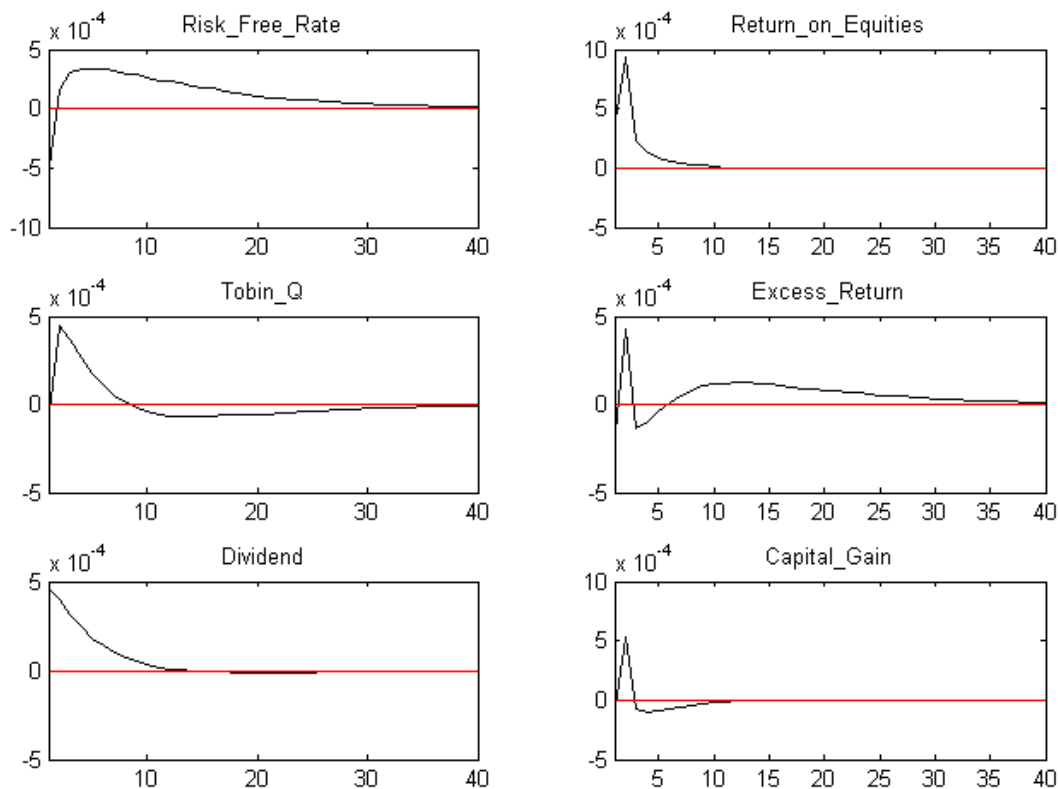
while economic intuition suggest that it should increase, it decreases. Interestingly, the model with flexible wages is able to capture the correct behavior of inflation. In fact, the model with flexible wages has the consistent dynamics for all business cycle variables compared with Nath (2016). With flexible wages, output is a very strong driver of all the variables and hence all other variables (except consumption, inflation and labor share) have dynamics similar to the dynamics of output.

The wage bill rises as the hours worked and wages both increase. The labor share again behaves similar to the one period technology shock case in figure 8.1, but here it goes much more slowly back to the steady state due to high persistence of the shock.

### 8.2.2 Asset Pricing Dynamics

Figures 8.7 and 8.8 show the dynamics that the model generates given real wage rigidity and without real wage rigidity, respectively. The asset pricing responses for long-run shocks are quite different from

**Figure 8.7: Impulse Responses for Asset Pricing Variables with Real Wage Rigidity**

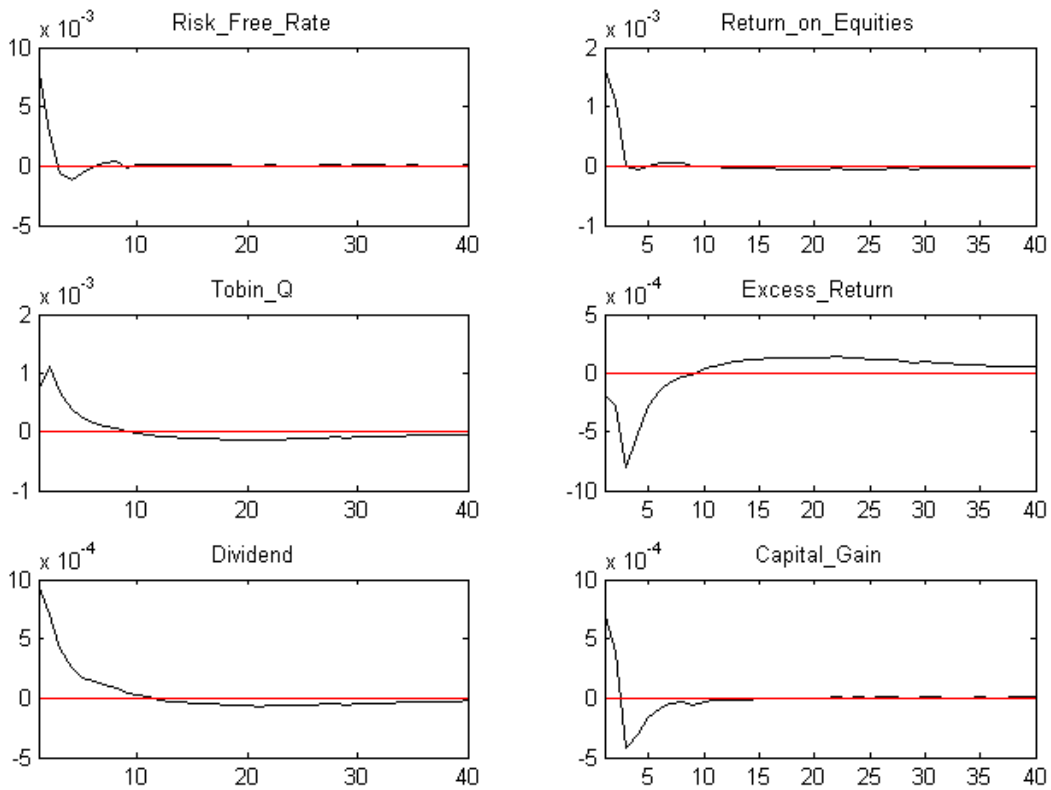


*The above impulse responses show the behavior of asset pricing variables in response to a Long-Run Technology shock for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

one-period shocks. First, the risk free rate falls and then rises after impulse rather than the immediate rise observed in figure 8.3. This is because at impulse, the agent is willing to substitute it to future periods at lower interest rates. But as contemporaneous consumption rises even further, the risk free rate goes up as the agent is less willing to substitute across periods at lower risk free rates. This response is inconsistent with Nath (2016).

The equity return, consistent with literature, has a zig-zag shape. The zig-zag can be better described as the lagged household response to the long-run shock. The internal mechanism driving the initial rise in equity return can once again be understood by the decomposition of returns. The price of equity (and hence the capital gain) rises because it follows the dynamics of output, which rises. When the shock hits, the return on equity rises and households tend to over-invest in equity which subsequently drives down the return on equity. The Tobin's Q, which is the price of capital in this model, acts as a multiplier to the capital and follows the dynamics of investment. Therefore, investment and Tobin's Q are qualitatively identical. Here again the wage rigidity has a dampening effect on variables including

**Figure 8.8: Impulse Responses for Asset Pricing Variables without Real Wage Rigidity**



*The above impulse responses show the behavior of asset pricing variables in response to a Long-Run Technology shock for the model with flexible wages. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

the return on equity.

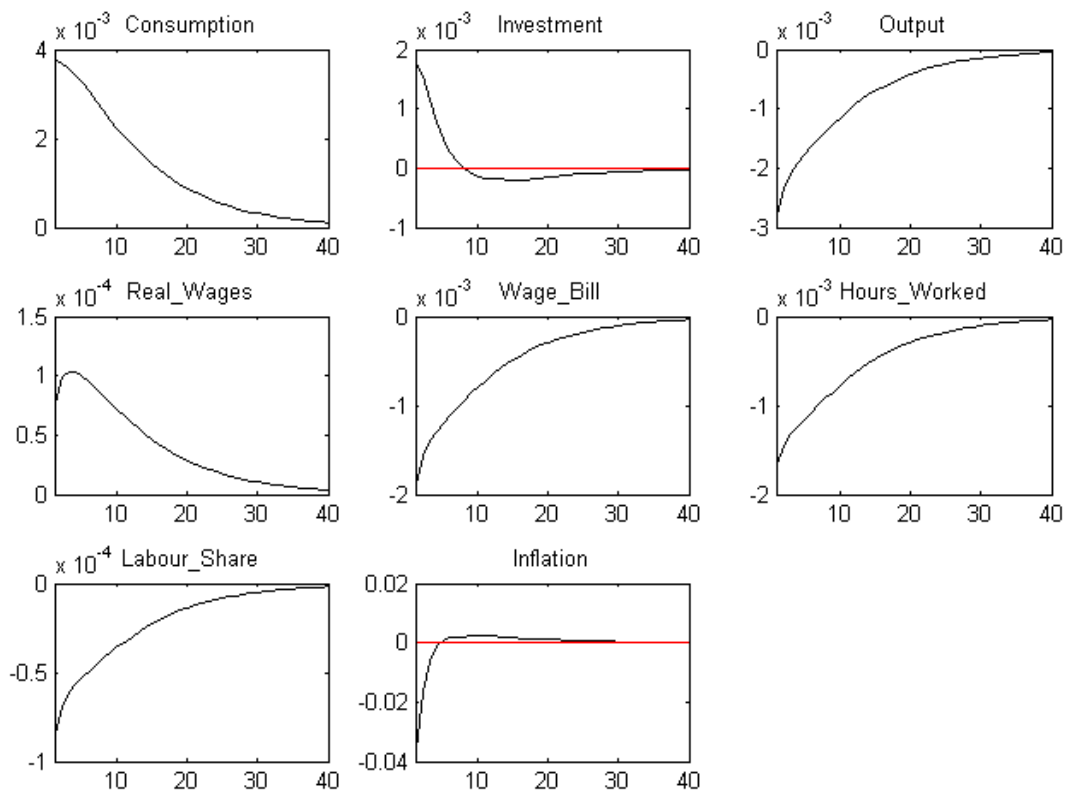
The model without sticky wages generates a higher risk free rate at impulse than return on equity which explains the negative excess return. This is again inconsistent with (Nath, 2016).

## 8.3 Government Spending Shock

### 8.3.1 Business Cycle Dynamic

Figures 8.9 and 8.10 show the dynamics of real variables in response to a government spending shock with real wage rigidity and without real wage rigidity, respectively. The first striking observation is that while both consumption and investment rise at impulse, the hours worked, the wage bill and even the output fall. Clearly, government spending has a very strong impact on the economy. The agent, on

**Figure 8.9: Impulse Responses for Real Variables with Real Wage Rigidity**



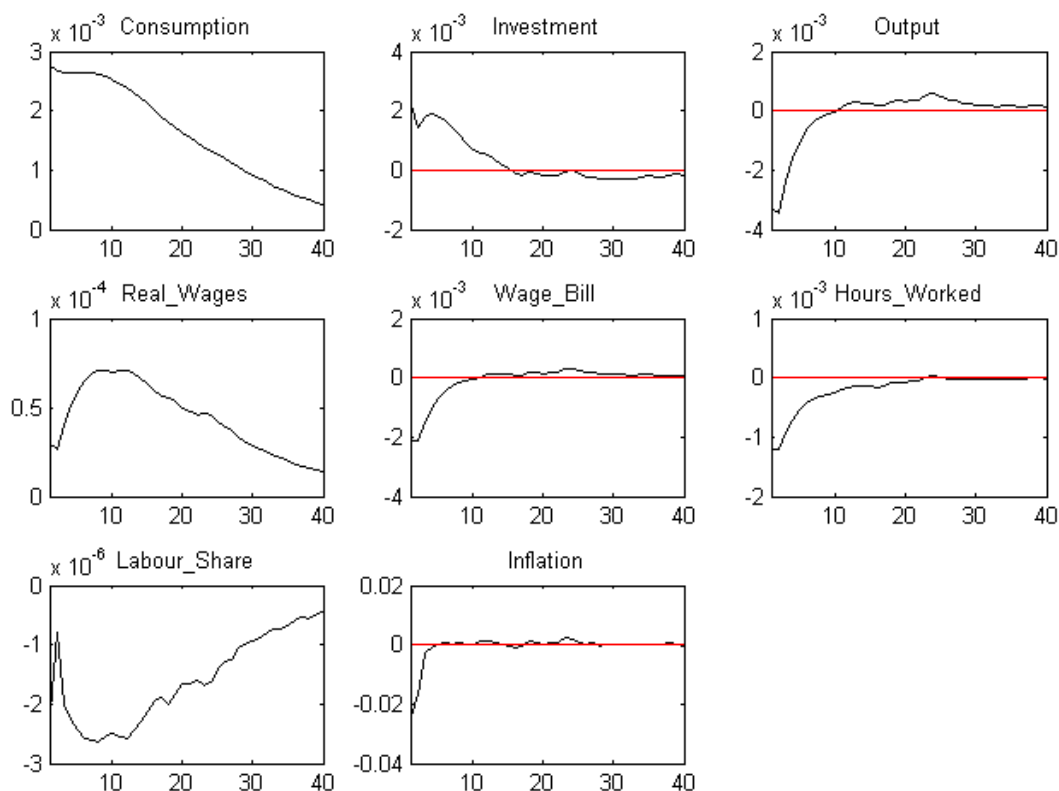
*The above impulse responses show the behavior of business cycle variables in response to a Government spending shock for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

receiving the government spending, uses it for consumption and investment. Further she decides to work fewer hours which reduces the wage bill and the labour share. This scenario causes the most distortion in the consumption-hours worked tradeoff, because of the costless nature of the government transfers. A more realistic government sector and fiscal policy would be able to better explain the mechanism behind this behavior.

Another difference between the impact of this shock and the technology shocks is that in this case for consumption, the real wage rigidity seems to be amplifying the impact of the government spending shock. This is because when the hours worked are reduced at impulse and real wages increased, with real wage rigidity the wage earnings of the household do not drop as much they do in the case without real wage rigidity. Adding to this the government transfer, the household has much more disposable income and since the income effect dominates, consumption earnings go up much more than the case without real wage rigidity.

The dynamics of inflation are problematic. In theory, we should see a sharp rise in inflation at

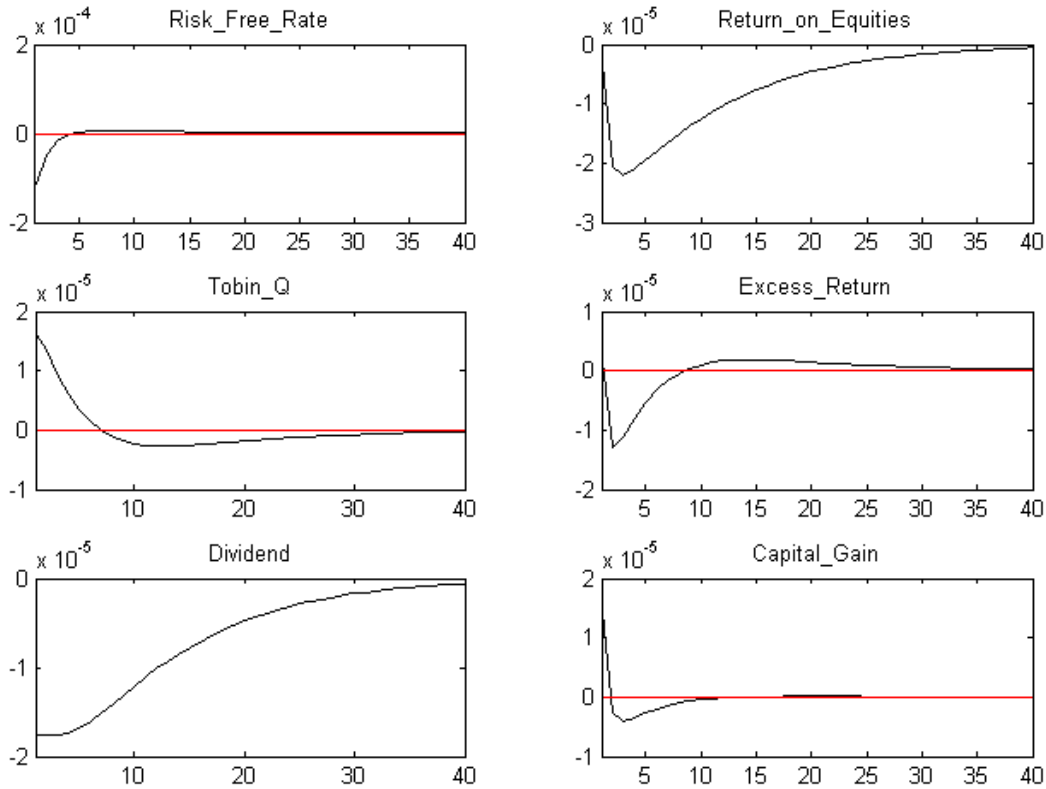
**Figure 8.10: Impulse Responses for Real Variables without Real Wage Rigidity**



*The above impulse responses show the behavior of business cycle variables in response to a Government spending shock for the model with flexible wages. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

impulse because the households receive government transfers and their disposable income goes up which shifts their demand curve and producers increase their prices. This behavior is also observed in the case without real wage rigidity. Therefore, the model generates counter-intuitive dynamics for inflation. When the wages are flexible, as shown in figure 8.10, we see the responses are qualitatively similar to those observed in 8.9 but they are much more volatile. This shows that the real wage rigidity acts to smooth the response of the economy to the government spending shock. If the government adopts a more realistic fiscal policy and the spending shock is endogenously driven, then it might be the case that the hours worked do not drop as much as this model. The specific mechanism would then depend on the kind of taxes (lump sum or ad valorem) and the manner of transfer (lump sum transfer, subsidies for specific products or food stamps etc.)

**Figure 8.11: Impulse Responses for Asset Pricing Variables with Real Wage Rigidity,**  
 $\sigma_g = 7 \times 10^{-5}$

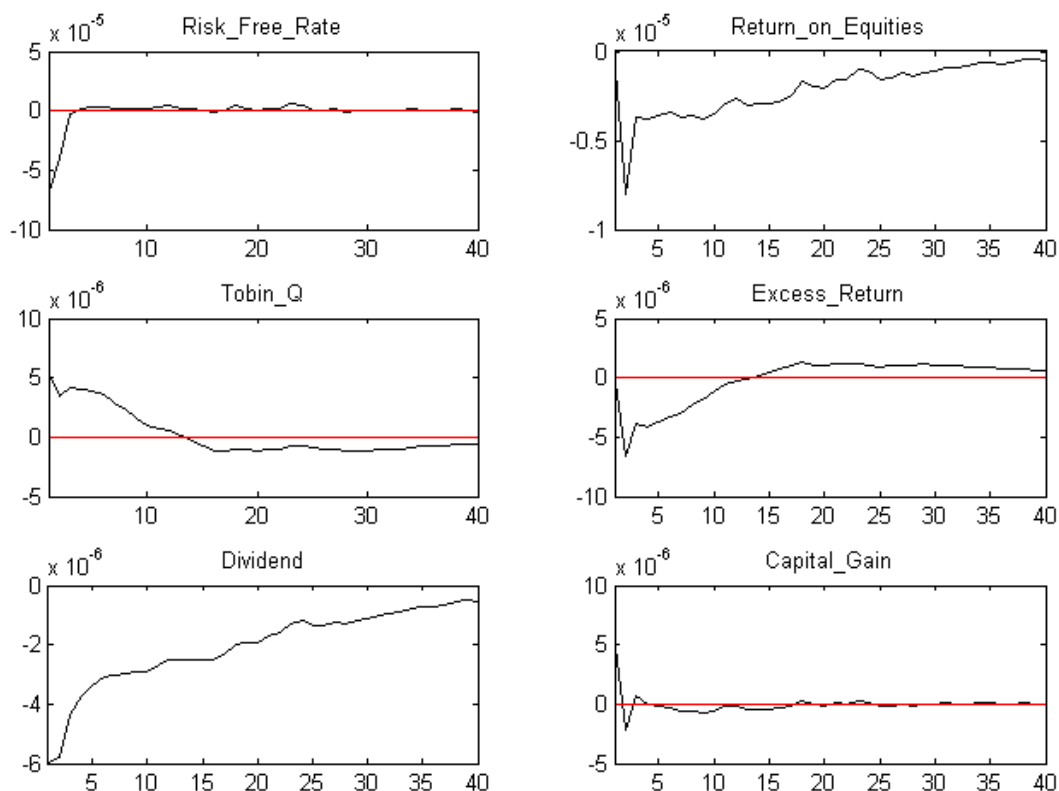


The above impulse responses show the behavior of asset pricing variables in response to a Government spending shock ( $\sigma_g = 7 \times 10^{-5}$ ) for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.

### 8.3.2 Asset Pricing Dynamics

The government transfers allow for higher investment contemporaneously with lower labour supply. This increases the price of capital as captured by the Tobin's Q. The impulse responses provide evidence of household over-investment in both risk free bonds and equities. Another possible explanation could be that the government spending shock has been improperly calibrated. But as figure A in Appendix A, shows that even at lower levels of government shock volatility, the responses are qualitatively similar. Higher levels of volatility lead to explosive impulse responses and are therefore ruled out for the purpose of this thesis. Here the real wage rigidity acts to smooth the response of asset pricing variables.

**Figure 8.12:** Impulse Responses for Asset Pricing Variables without Real Wage Rigidity



The above impulse responses show the behavior of asset pricing variables in response to a Government spending shock for the model with flexible wages. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.

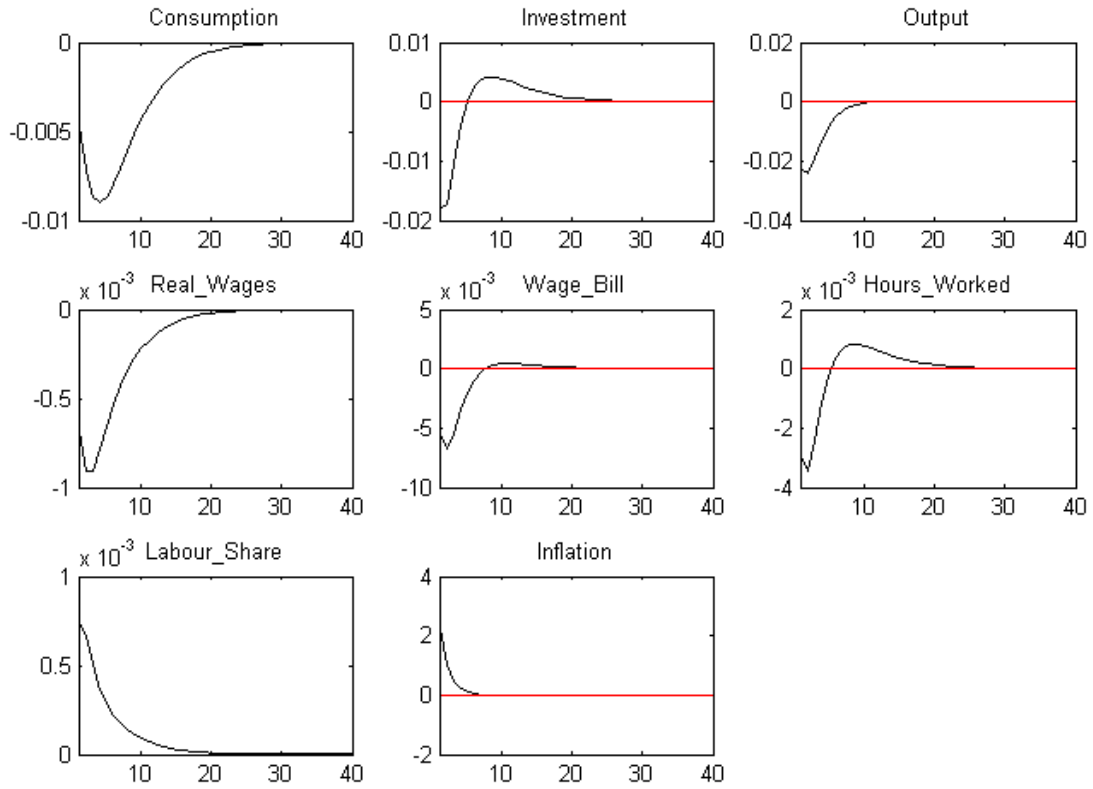
## 8.4 Monetary Policy Shock

### 8.4.1 Business Cycle Dynamics

A contractionary monetary policy shock which raises the return on the nominal interest rate makes risk free bonds more attractive and households reduce their consumption. This shift toward nominal bonds also has a drastic impact on return on equities and consumption falls. This impact on consumption is consistent with the literature. The shift in the demand forces producers to reduce their wage bills by reducing the number of hours of labor they utilize. Hence, the hours worked go down. This reduction in the number of hours worked reduces output and the wage bill.

Since, the real wage is dependent on the contemporaneous output, the fall in real wage follows the fall in output. This fall in real wage is inconsistent with Nath (2016) and this is due to differences in the assumed wage function. The labor share rises but this can be explained by the huge slowdown in consumption and output and real wage rigidity does not allow the real wage to shift a by the required

**Figure 8.13: Impulse Responses for Real Variables with Real Wage Rigidity**



*The above impulse responses show the behavior of business cycle variables in response to a contractionary monetary policy shock for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

amount. Consider the following definition of the labor share at time  $(t - 1)$ ,

$$\tau_{t-1} = \frac{w_{t-1} * n_{t-1}}{y_{t-1}}$$

At time  $t$  we have,

$$\tau_t = \frac{w_t * n_t}{y_t}$$

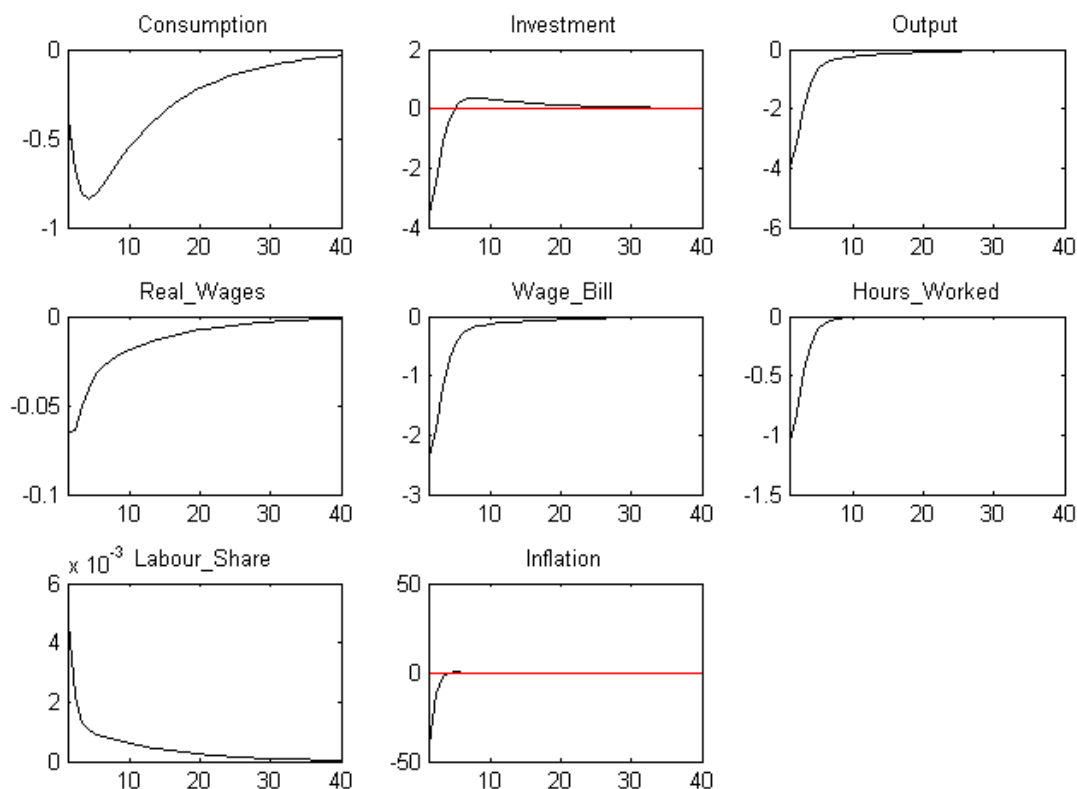
The change in labour share is,

$$\Delta\tau_t = \frac{w_t * n_t}{y_t} - \frac{w_{t-1} * n_{t-1}}{y_{t-1}}$$

Due to the shock at  $t$ , there is a fall in labour demand and output which implies  $n_t < n_{t-1}$  and  $w_t < w_{t-1}$ .  $y_t$  is relatively smaller than  $y_{t-1}$  and the difference is so large that the first part of the equation, due to a smaller denominator, becomes larger than the second and hence the impulse response shows an increase in the labor share in the economy.



**Figure 8.14: Impulse Responses for Real Variables without Real Wage Rigidity**



*The above impulse responses show the behavior of business cycle variables in response to a contractionary monetary policy shock for the model with flexible wages. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

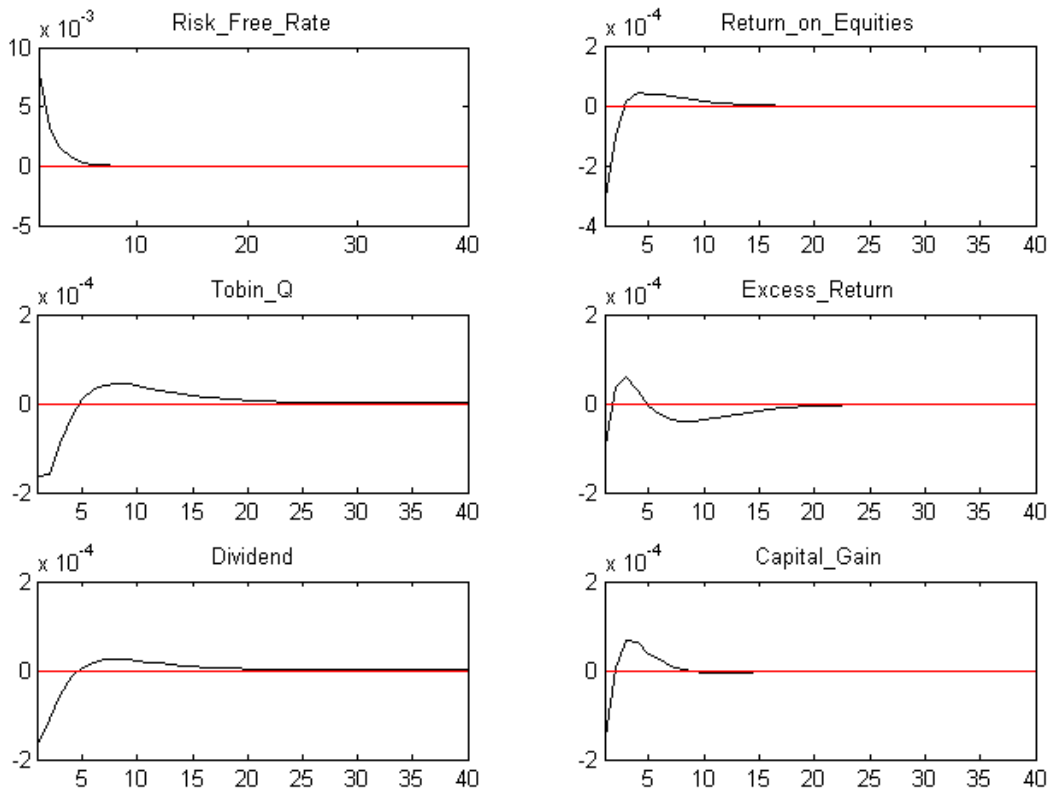
The only problematic response is the behavior of inflation. In fact comparing 8.13 with 8.14, it is evident that the real wage rigidity actually reverses the behavior of inflation and in the case without real wage rigidity the dynamics of inflation are consistent with Nath (2016).

## 8.4.2 Asset Pricing Dynamics

Almost all of the asset pricing dynamics are consistent with those observed in Nath (2016). In the presence of a contractionary monetary policy shock, in the model without real wage rigidity, shown in figure 8.16, at impulse the equity returns and risk free rate go down because the transmission mechanism is too weak. This transmission mechanism is strengthened by the real wage rigidity. As shown in figure 8.15, once real wage rigidity is introduced contractionary monetary policy news, i.e. a move to restrict the supply of money, raise the risk-free interest rates.

Contractionary monetary policy news reduce the return on equities. Again this fall can be explained

**Figure 8.15: Impulse Responses for Asset Pricing Variables with Real Wage Rigidity**

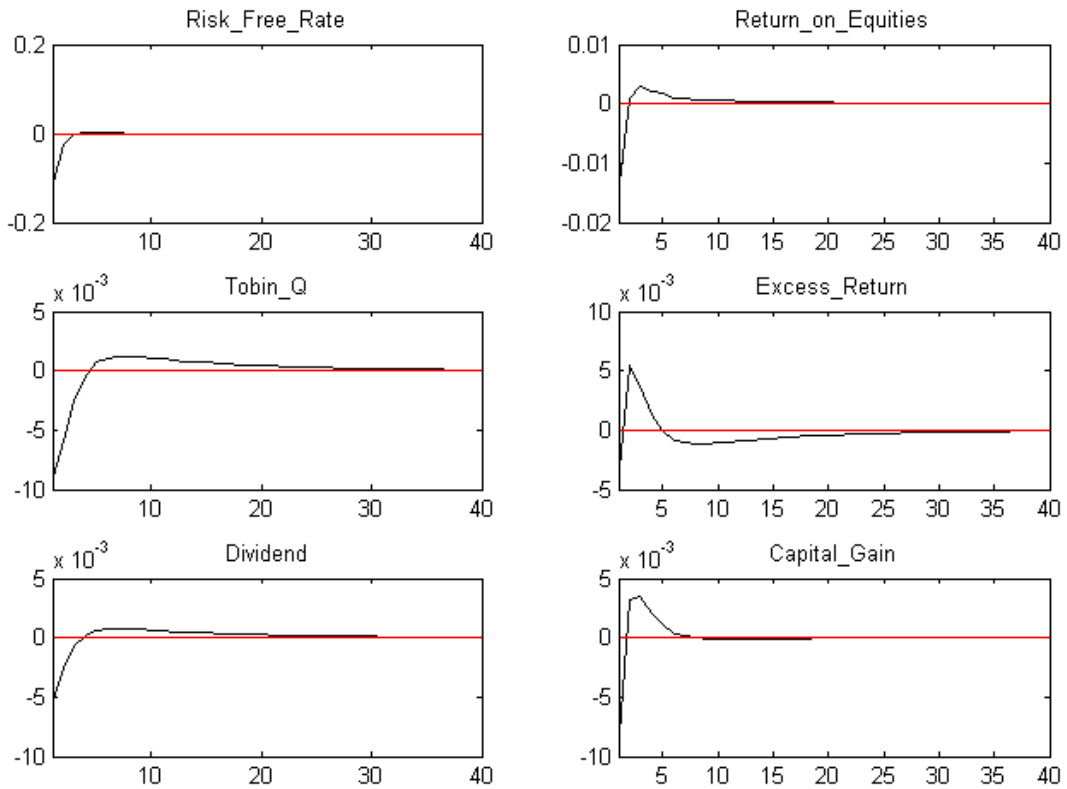


*The above impulse responses show the behavior of asset pricing variables in response to a contractionary monetary policy shock for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

by looking at the dividend channel and the capital gain channel. The dividend that household's receive goes down as the wage bill goes down, but due its size this channel is not the sole determinant of all the dynamics of equity returns. The other channel, capital gain falls as evidenced by the fall of Tobin's Q. This mechanism is consistent with the response observed in Nath (2016). Since both the dividend and the capital gain do down at impulse, an fall in the equity returns is observed. Intuitively, when the return on risk-free assets goes up, the risk free securities become more attractive. Subsequently, there is a substitution away from investment in risky securities and an over-investment in risk-free securities. Hence we see that in future periods, the risk free rate goes down as agents bid down the return on risk free bonds. A similar mechanism operates in the reverse direction for the return on equities. At impulse, the equity returns fall and households move away from these securities, but as risk-free rates are bid down, risky securities become attractive once again.

The strength of the real wage rigidity transmission mechanism can be observed by comparing figures 8.16 and 8.15. Barring the behavior of the risk free rate, all other asset pricing variables are qualitatively

**Figure 8.16: Impulse Responses for Asset Pricing Variables without Real Wage Rigidity**



*The above impulse responses show the behavior of asset pricing variables in response to a contractionary monetary policy shock for the model with flexible wages. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

similar in the two cases. This shows that while real wage rigidity in this model is not necessarily affecting the theoretical link between monetary policy and asset prices, it does allow the model to generate the correct dynamics for the risk-free rate of return.

# Chapter 9

## Conclusion

This thesis investigates whether a New Keynesian model with asset pricing capabilities and a real wage rigidity can jointly replicate both business cycle and asset pricing facts. Further, it investigates whether the model can establish a theoretical link between monetary policy and asset prices. Lastly, it studies whether the real wage equation used has any promise for future research. To analyse these questions, I use a standardized DSGE *recipe* to, firstly, develop a medium scale DSGE model which incorporates various frictions and exogenous disturbances, secondly, generate simulated moments which are compared with those observed in the data and, lastly, generate the dynamics of variables which are then analyzed for consistency with other studies. This methodology has become one of the pillars of modern quantitative macroeconomics. Another important component of this thesis is that it uses non-linear methods and is able to relate equity return with the real economy without log-linearization. Further, very few models in the New Keynesian literature incorporate endogenous capital which is a part of this thesis.

I find that the model produces moments that match those observed in the data for consumption, investment and output. It is not able to reproduce the moments for any of the labor side variables including the hours worked, the wages, the wage bill or the labor share. The model is able to overcome the problem of high wage volatility but the problem of high correlation with output persists (Uhlig, 2007). While the model does better than the model with flexible wages, a wage function that does not depend on marginal product of labor is necessary. For asset pricing variables, the model produces very small values of equity volatility and equity premium volatility.

To further evaluate the performance of the model, I analyze the business cycle and asset pricing dynamics generated by the model. For the business cycle dynamics, I find that in response to a one-period technology shock, the model with the wage rigidity generates dynamics which are consistent with those observed in other studies. Further, in response to a long-run technology shock, the model dynamics are consistent for all variables except the behavior of inflation which are not consistent with Nath (2016) but is consistent with Sims (2011a). A government spending shock, raises consumption and investment

but lowers hours worked so it reduces output. This is due to costless transmission of government spending into the economy. Finally, in response to a contractionary monetary policy shock output, consumption, investment and hours worked decline, indicated a slowdown in economic activity which is consistent with Nath (2016).

For the asset pricing dynamics, a one-period technology shock raises the risk free rate, equity return and the excess return, which are consistent with Croce (2014). An inconsistency arises with a long-run shock, where the risk free rate actually decreases at impulse. Further, the responses to government spending and monetary policy shocks are consistent with those observed in other studies.

Overall, the model performance with the real wage rigidity cannot be considered outstanding. Other forms of real wage rigidity which produce superior dynamics and results have been proposed, for example by Favilukis and Lin (2015). The results in this thesis indicate that a reliable form of wage rigidity must not be dependent on output directly. Staggered wage setting à la Li and Palomino (2014) or infrequent wage setting à la Favilukis and Lin (2015) could solve the problems raised in this thesis. Other modelling devices like a CES production function, fixed costs in production, idiosyncratic productivity would also produce better results but increase computational complexity.

The responses to a monetary policy shock also show that while real wage rigidity in this model is not necessarily impacting the theoretical link between monetary policy and asset prices, it does allow the model to generate the correct dynamics for the risk-free rate of return. Furthermore, without real wage rigidity the model produces unreasonable asset pricing moments. Therefore, real wage rigidity is part of the story and more ingredients are necessary to explain the link between monetary policy and asset prices. The results obtained also suggest that while there is a role for wage rigidity, the functional form used in this thesis might not be the way forward for this literature. The behavior of wages in this model suggests that there are problems with assuming ad-hoc wage equations. The problem arises with establishing the relationship of wages with output. Wages that are too tightly correlated with output then cause problems with hours worked because of the intra-temporal Euler equation. Hence, we see that none of the simulated labor side variables match the data. Favilukis and Lin (2015) overcome this problem by assuming a wage equation which depends on number of workers hired.

The model in this thesis can be extended to include wages that are determined through a process of household optimization or wage bargaining which would, regardless of performance, be more realistic. Firm-level heterogeneity could also produce useful results. Few models have incorporated these features in a New Keynesian framework along with asset returns. Computational complexity and reasonable calibrations are two major roadblocks for more complex models. These questions are left for future research. While many believe that New Keynesian models can jointly replicate business cycle and asset pricing facts, the results obtained in this thesis force me to take a conservative position on this view.

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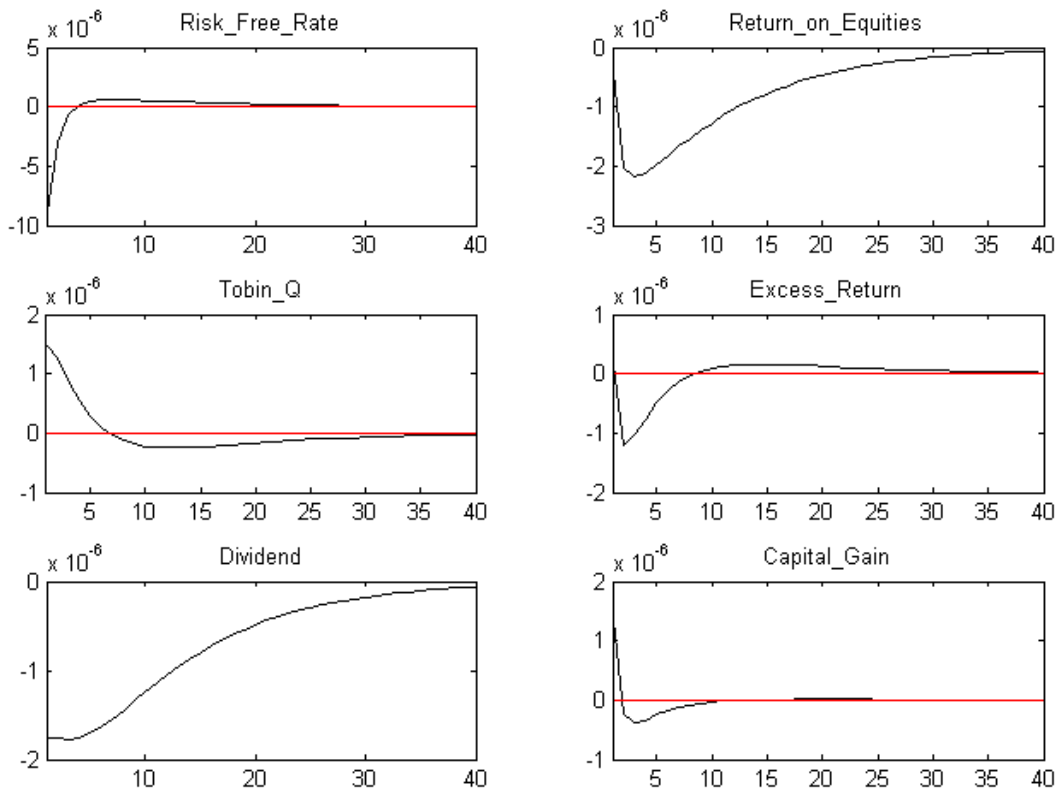
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# Appendix A

## Impulse Response

**Figure A.1:**

Impulse Responses for Asset Pricing Variables with Real Wage Rigidity,  $\sigma_g^2 = 7 \times 10^{-7}$



*The above impulse responses show the behavior of asset pricing variables in response to a government spending shock ( $\sigma_g^2 = 7 \times 10^{-7}$ ) for the model with real wage rigidity. The horizontal axis depicts the time period (measured in quarters) and the vertical axis depicts log-deviations from steady state. All parameters of the model are calibrated according to Table 6.1.*

# Appendix B

## Model Derivation

### B.1 Household

The model economy consists of a representative agent with CRRA utility who chooses amount of consumption goods,  $C_t$ , and hours worked,  $N_t^s$ , to maximize the following utility function:

$$U(C_t, N_t^s) = \left( \frac{C_t^{1-\psi}}{1-\psi} - \frac{(N_t^s)^{1+\omega}}{1+\omega} \right)$$

where  $\beta$  ( $>0$ ) is the subjective discount factor,  $\psi$  ( $>0$ ) determines the coefficient of risk aversion  $\frac{1}{\psi}$  ( $>0$ ) is the elasticity of intertemporal substitution (EIS) and  $\omega$  ( $>0$ ) is the Frisch elasticity of labor supply. An important feature of CRRA preferences is that the EIS and coefficient of risk aversion are determined by one single parameter,  $\psi$ .

Further, under the assumption that the households are the owners of capital and have access to complete asset markets, the budget constraint becomes:

$$C_t + I_t + B_t + \tau_t = w_t N_t^s + r_t^K K_t + R_{t-1}^F \frac{B_{t-1}}{\pi_t} + D_t$$

where  $C_t$  denotes the household's consumption,  $I_t$  the investment,  $B_t$  is the stock of real bond holdings,  $\tau_t$  is the lump sum tax,  $w_t$  is the real wage,  $N_t^s$  is the labor hours,  $r_t^K$  is the rental rate of capital,  $K_t$  is the capital,  $R_{t-1}^F$  is the risk free rate of interest,  $\pi_t$  is the inflation rate  $\frac{P_t}{P_{t-1}}$ , and  $D_t$  is the aggregate real dividend (profits).

Capital owned by households evolves according to the following law of motion:

$$K_{t+1} = (1 - \delta_K)K_t + I_t - G_t K_t$$

where  $\delta_K$  is the constant depreciation rate, and  $G_t$  allows for convex adjustment costs à la Jermann (1998) where  $G_t$  is defined as:

$$G_t = \frac{I_t}{K_t} - \left[ \frac{\alpha_1}{1 - 1/\xi} \left( \frac{I_t}{K_t} \right)^{1-1/\xi} + \alpha_0 \right] \quad (\text{B.1})$$

and at the steady state satisfies  $\bar{G} = \bar{G}' = 0$ . For analytical purposes, as in the literature,  $G_t$  is assumed to be a function of  $\frac{I_t}{K_t}$ . This is important for the derivation of the optimality conditions for Tobin's Q. Formally, the household's problem is:

$$\underset{C_t, N_t^s, B_t, K_{t+1}}{\text{maximize}} \quad \left( \frac{C_t^{1-\psi}}{1-\psi} - \frac{(N_t^s)^{1+\omega}}{1+\omega} \right)$$

subject to:

$$C_t + I_t + B_t + \tau_t = w_t N_t^s + r_t^K K_t + R_{t-1}^F \frac{B_{t-1}}{\pi_t} + D_t$$

$$K_{t+1} = (1 - \delta_K) K_t + I_t - G_t K_t$$

where  $G_t$  evolves according to (B.1). The next step is to derive the first order conditions for which the Lagrangian is set up as follows:

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\psi}}{1-\psi} - \frac{(N_t^s)^{1+\omega}}{1+\omega} \right) + \sum_{t=0}^{\infty} \lambda_t \left[ w_t N_t^s + r_t^K K_t + R_{t-1}^F \frac{B_{t-1}}{\pi_t} + D_t - C_t - I_t - B_t - \tau_t \right] \\ + \sum_{t=0}^{\infty} \Xi_t \left[ (1 - \delta_K) K_t + I_t - G_t K_t - K_{t+1} \right] \end{aligned}$$

where  $\lambda_t$  and  $\Xi_t$  are the time-varying Lagrangian multipliers. First order conditions imply:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \beta^t C_t^{-\psi} = \lambda_t \quad (\text{B.2})$$

$$\frac{\partial \mathcal{L}}{\partial N_t^s} : \beta^t (N_t^s)^\omega = \lambda_t w_t \quad (\text{B.3})$$

$$\frac{\partial \mathcal{L}}{\partial B_t} : \lambda_t = \lambda_{t+1} \frac{R_t^F}{\pi_{t+1}} \quad (\text{B.4})$$

$$\frac{\partial \mathcal{L}}{\partial I_t} : -\lambda_t + \Xi_t \left[ 1 - G' \left( \frac{I_t}{K_t} \right) \right] = 0 \quad (\text{B.5})$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} : \lambda_{t+1} r_{t+1}^K + \Xi_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] - \Xi_t = 0 \quad (\text{B.6})$$

Dividing (B.3) by (B.2), we obtain the intratemporal Euler equation:

$$(N_t^s)^\omega = w_t C_t^{-\psi} \quad (\text{B.7})$$

Substituting (B.2) in (B.4): which becomes:

$$\frac{1}{R_t^F} = \frac{\beta C_{t+1}^{-\psi}}{\pi_{t+1} C_t^{-\psi}} \quad (\text{B.8})$$

The basic pricing equation is:

$$1 = \mathbf{E}_t[M_{t,t+1}R_{t+1}]$$

where  $M_{t,t+1}$  is the stochastic discount factor. Since the risk free rate is known ahead of time, the above can be rewritten as:

$$\frac{1}{R_t^F} = \mathbf{E}_t[M_{t,t+1}] \quad (\text{B.9})$$

Hence, the nominal stochastic discount factor becomes:

$$M_{t,t+1} = \beta \left[ \frac{C_{t+1}}{C_t} \right]^{-\psi} \pi_{t+1}^{-1} \quad (\text{B.10})$$

To obtain the expression for Tobin's Q, we rewrite equation (B.5) as:

$$\begin{aligned} \lambda_t &= \Xi_t \left[ 1 - G' \left( \frac{I_t}{K_t} \right) \right] \\ Q_t &= \frac{\Xi_t}{\lambda_t} = \frac{1}{1 - G' \left( \frac{I_t}{K_t} \right)} \end{aligned} \quad (\text{B.11})$$

where  $Q_t$  is the Tobin's Q, the marginal rate of transformation between new capital and consumption.

Lastly, rewriting equation (B.6) as:

$$\lambda_{t+1} r_{t+1}^K + \Xi_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] = \Xi_t$$

Substituting equation (B.5) above:

$$\lambda_{t+1} \left[ r_{t+1}^K + \frac{\Xi_{t+1}}{\lambda_{t+1}} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] \right] = \frac{\lambda_t}{1 - G' \left( \frac{I_t}{K_t} \right)}$$

Using equation (B.11) above:

$$\lambda_{t+1} \left[ r_{t+1}^K + Q_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] \right] = \lambda_t Q_t$$

Using equation (B.4) above, we have the following:

$$\lambda_{t+1} \left[ r_{t+1}^K + Q_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] \right] = \lambda_{t+1} \frac{Q_t R_t^F}{\pi_{t+1}}$$

which can be written as:

$$\frac{1}{R_t^F} \left[ \frac{\left[ r_{t+1}^K + Q_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] \right] \pi_{t+1}}{Q_t} \right] = 1$$

Using equation (B.9):

$$1 = \mathbf{E}_t \left[ M_{t,t+1} \left( \frac{\left[ r_{t+1}^K + Q_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] \right] \pi_{t+1}}{Q_t} \right) \right] \quad (\text{B.12})$$

Therefore, the equilibrium conditions for the household's problem are:

$$(N_t^s)^\omega = w_t C_t^{-\psi} \quad (\text{B.13})$$

$$\frac{1}{R_t^F} = \mathbf{E}_t[M_{t,t+1}] \quad (\text{B.14})$$

$$M_{t,t+1} = \beta \left[ \frac{C_{t+1}}{C_t} \right]^{-\psi} \pi_{t+1}^{-1} \quad (\text{B.15})$$

$$Q_t = \frac{1}{1 - G'(\frac{I_t}{K_t})} \quad (\text{B.16})$$

$$1 = \mathbf{E}_t \left[ M_{t,t+1} \left( \frac{\left[ r_{t+1}^K + Q_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] \right] \pi_{t+1}}{Q_t} \right) \right] \quad (\text{B.17})$$

## B.2 Final Good Producer

The final good producer operates in a perfectly competitive environment and uses CES aggregation technology, due to Dixit and Stiglitz (1977), to combine a continuum of intermediate goods indexed by  $j \in [0, 1]$ :

$$Y_t = \left( \int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \quad (\text{B.18})$$

Here  $\eta$  represents the elasticity of substitution for each intermediate good and is assumed to be strictly greater than unity. As the input prices  $P_t(j)$  and price of the final good  $P_t$  are taken as given by the final good producer, its profit maximization problem becomes:

$$\underset{Y_t(j)}{\text{maximize}} \quad P_t \left( \int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} - \int_0^1 P_t(j) Y_t(j) dj$$

The first order condition for the  $j^{\text{th}}$  variety of intermediate goods is:

$$P_t \frac{\eta}{\eta-1} \left( \int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{1}{\eta-1}} \left( \frac{\eta-1}{\eta} \right) Y_t(j)^{-\frac{1}{\eta}} - P_t(j) = 0$$

Solving further, the following is obtained:

$$\left( \int_0^1 Y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{1}{\eta-1}} Y_t(j)^{-\frac{1}{\eta}} = \frac{P_t(j)}{P_t} \quad (\text{B.19})$$

Using equations (B.18) and (B.19), the demand for the  $j^{\text{th}}$  intermediate good as a function of the relative price is given as:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t \quad (\text{B.20})$$



The next step is to obtain an expression for the relationship between the price of the final good and the prices of the intermediate goods. I define the aggregate nominal output as the summation of the prices times the quantities of intermediate goods, in the following manner:

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj \quad (\text{B.21})$$

Substituting the demand for the  $j^{\text{th}}$  variety from equation (B.20) in equation (B.21):

$$P_t Y_t = \int_0^1 P_t(j)^{1-\eta} P_t^\eta Y_t dj$$

On the RHS,  $P_t$  and  $y_t$  are independent of  $j$ . Therefore the following can be written:

$$P_t Y_t = P_t^\eta Y_t \int_0^1 P_t(j)^{1-\eta} dj$$

Simplifying, the aggregate price level is given as:

$$P_t = \left( \int_0^1 P_t(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}} \quad (\text{B.22})$$

### B.3 Intermediate Producers

Intermediate goods are produced in a monopolistically competitive environment, where the  $j^{\text{th}}$  variety is produced by a monopolist using the following Cobb-Douglas production function, with aggregate labor-augmenting productivity  $Z_t$  which is affected by transitory,  $z_t$ , and permanent productivity shocks,  $a_t$ :

$$Y_t(j) = K_t^d(j)^\alpha [e^{x_t} Z_t N_t^d(j)]^{(1-\alpha)}, x \in \{a, z\}$$

$$A_t \equiv e^{x_t} Z_t$$

Intermediate producers face an exogenous common wage and may not be able to adjust prices each period due to Calvo (1983) staggered price setting, hence they always minimize the total nominal cost of hiring workers and renting capital, with the constraint that they produce enough to meet the final goods producer's demand.

$$\underset{N_t^d(j), K_t^d(j)}{\text{minimize}} \quad W_t N_t^d(j) + R_t^K K_t^d(j)$$

subject to:

$$(K_t^d(j))^\alpha (A_t N_t^d(j))^{(1-\alpha)} \geq \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t$$

where  $W_t (= P_t w_t)$  is the nominal wage and  $R_t^K (= P_t r_t^k)$  is the nominal rental rate of capital. I assume that firms adjust wages according to the following exogenous wage function:

$$w_t = \frac{W_t}{P_t} = (1 - \theta_w) w_{t-1} + \theta_w \left( (1 - \alpha) \frac{Y}{L} \right)^{\gamma_w}$$

where  $(1 - \theta_w)$  is the weight on previous periods wage and  $\gamma_w$  is the dampening factor for the marginal product. Setting up the Lagrangian for this problem:

$$\mathcal{L} = -W_t N_t^d(j) - R_t^K K_t^d(j) + \varphi_t(j) \left( (K_t^d(j))^\alpha [A_t N_t^d(j)]^{1-\alpha} - \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t \right)$$

Substituting the wage function, I obtain:

$$\mathcal{L} = -W_t N_t^d(j) - R_t^K K_t^d(j) + \varphi_t(j) \left( (K_t^d(j))^\alpha [A_t N_t^d(j)]^{1-\alpha} - \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t \right)$$

$$\frac{\partial \mathcal{L}}{\partial K_t^d} : R_t^K = \varphi_t(j) \alpha (K_t^d(j))^{\alpha-1} (A_t N_t^d(j))^{1-\alpha} \quad (\text{B.23})$$

$$\frac{\partial \mathcal{L}}{\partial N_t^d} : W_t = \varphi_t(j) (1 - \alpha) A_t (K_t^d(j))^\alpha (A_t N_t^d(j))^{-\alpha} \quad (\text{B.24})$$

Dividing (B.24) by (B.23), the following is obtained:

$$\frac{W_t}{R_t^K} = \frac{(1 - \alpha) K_t^d(j)}{\alpha N_t^d(j)} \quad (\text{B.25})$$

The production function can be rearranged to obtain the expression for  $N_t^d$  in the following manner:

$$N_t^d(j) = \frac{Y_t(j)}{A_t^{(1-\alpha)}} \left[ \frac{N_t^d(j)}{K_t^d(j)} \right]^\alpha \quad (\text{B.26})$$

$$C_t(j) = W_t N_t^d(j) + R_t^K K_t^d(j)$$

Using (B.25) the above equation becomes:

$$C_t(j) = W_t N_t^d + \frac{\alpha}{1 - \alpha} \frac{P_t \Phi Y_t^\chi N_t^d(j)}{K_t^d(j)} K_t^d(j)$$

Which becomes:

$$C_t(j) = \frac{W_t N_t^d}{1 - \alpha}$$

Now using (B.26):

$$C_t(j) = \frac{W_t}{1 - \alpha} \frac{Y_t(j)}{A_t^{(1-\alpha)}} \left[ \frac{N_t^d(j)}{K_t^d(j)} \right]^\alpha$$

Substituting (B.25) above:

$$C_t(j) = \frac{W_t}{1 - \alpha} \frac{Y_t(j)}{A_t^{(1-\alpha)}} \left[ \frac{(1 - \alpha) R_t^K}{\alpha W_t} \right]^\alpha$$

Finally, the nominal cost function is:

$$C_t(j) = \left[ \frac{W_t}{1 - \alpha} \right]^{1-\alpha} \frac{Y_t(j)}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha \quad (\text{B.27})$$

### B.3.1 Calvo Price Setting

Under the Calvo staggered price setting regime, while each firm has monopolistic power to set the price for its own output,  $P_t(j)$ , only a fraction  $(1 - \theta)$  are allowed to exercise full adjustment in any given period. Therefore, the fraction of firms not allowed to change prices is  $\theta$  and the average duration of a price is  $(1 - \theta)^{-1}$ . We have,

$$P_j(t) = \begin{cases} P_{t-1}(j) & \text{with probability } \theta \\ P_t^*(j) & \text{with probability } 1 - \theta \end{cases}$$

where  $P_t^*(j)$  denotes the optimal price if the producer were allowed to optimize in period  $t$ . The pricing problem for the intermediate goods producer is dynamic; if given the chance to adjust her price in a particular period, there is a probability that she might not get a chance to adjust the price of her good for multiple periods. Formally,

$$\text{maximize}_{P_t^*(j)} \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( P_t^*(j) Y_{t+k}^*(j) - C_{t+k}^*(j) \right) \right] \quad (\text{B.28})$$

subject to:

$$Y_{t+k}^*(j) = \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} Y_{t+k} \quad (\text{B.29})$$

for  $k = 0, 1, 2, \dots$  and  $C_t^*(j)$  is the total real cost of production at period  $t$  given output  $Y_t^*(j)$ .  $M_{t,t+k}$  is obtained by successive application of the one period stochastic discount factor:

$$M_{t,t+k} = M_{t,t+1} M_{t+1,t+2} \dots M_{t+k-1,t+k}$$

Since output will equal demand, we substitute (B.29) in (B.28) to get the following:

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( P_t^*(j) \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} Y_{t+k} - C_{t+k}^*(j) \right) \right]$$

The first order condition becomes:

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( (1 - \eta) \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} Y_{t+k} - \frac{\partial C_{t+k}^*(j)}{\partial Y_{t+k}^*(j)} \frac{\partial Y_{t+k}^*(j)}{\partial P_t^*(j)} \right) \right] = 0$$

Which can be rearranged as:

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( (1 - \eta) \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} Y_{t+k} + \eta \mathcal{M} C_{t+k}^*(j) \left( \frac{P_t^*(j)^{-\eta-1}}{P_{t+k}^{-\eta}} \right) Y_{t+k} \right) \right] = 0 \quad (\text{B.30})$$

Which can be written as:

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} Y_{t+k} - \frac{\eta}{\eta - 1} \mathcal{M} C_{t+k}^*(j) \left( \frac{P_t^*(j)^{-\eta-1}}{P_{t+k}^{-\eta}} \right) Y_{t+k} \right) \right] = 0 \quad (\text{B.31})$$

Now using (B.29), I rewrite (B.31) as:

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( Y_t^*(j) - \frac{\eta}{\eta - 1} \mathcal{M} C_{t+k}^*(j) \left( \frac{Y_t^*(j)}{P_t^*(j)} \right) \right) \right] = 0$$

Rearranging:

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} Y_t^*(j) \left( P_t^*(j) - \frac{\eta}{\eta - 1} \mathcal{M} C_{t+k}^*(j) \right) \right] = 0 \quad (\text{B.32})$$

## B.4 Government

The government determines both fiscal and monetary policy. It issues one period riskless nominal bond  $B_t$  that pays the nominal gross return  $R_t$  and collects lump-sum tax  $\tau_t$ . Assuming the government runs a balanced budget every period, we have the following budget constraint for the government:

$$R_{t-1}B_{t-1} + P_t T_t = B_t + \tau_t \quad (\text{B.33})$$

Monetary policy is described by simple Taylor rule of the form:

$$i_t = \gamma_i i_{t-1} + \phi_\pi \left( \frac{\pi_t}{\pi^*} \right) + \phi_y (Y_t - Y^*) + u_t \quad (\text{B.34})$$

where  $i_t$  is the nominal interest rate at time  $t$ ,  $\gamma_i < 1$ ,  $\pi_t$  is the inflation at time  $t$ ,  $Y_t$  is the output and  $Y^*$  is the output gap,  $\phi_\pi$  and  $\phi_y$  determine the size of the response to changes in inflation and the output, respectively,  $\pi^*$  is the inflation target interpreted in this model as the steady state level of inflation and  $u_t$  is the monetary policy shock process.

## B.5 Aggregation

The first step toward aggregation is removing heterogeneity using Calvo pricing properties. I start with equation (B.22):

$$P_t^{1-\eta} = \int_0^1 P_t(j)^{1-\eta} dj$$

Now  $(1 - \theta)$  fraction of firms update their price to  $P_t^*(j)$  and because all firms are identical,  $P_t^*(j) = P_t^*$ . Breaking up the integral from equation (B.22):

$$P_t^{1-\eta} = \int_0^{1-\theta} (P_t^*)^{1-\eta} dj + \int_{1-\theta}^1 P_{t-1}(j)^{1-\eta} dj$$

which becomes:

$$P_t^{1-\eta} = (1 - \theta)P_t^* + \int_{1-\theta}^1 P_{t-1}(j)^{1-\eta} dj$$

Since there are a continuum of firms, the above can be simplified as:

$$P_t^{1-\eta} = (1 - \theta)P_t^* + \underbrace{\theta \int_0^1 P_{t-1}(j)^{1-\eta} dj}_{\theta P_{t-1}^{1-\eta}}$$

The aggregate pricing behavior can then be written as:

$$P_t^{1-\eta} = (1 - \theta)P_t^* + \theta P_{t-1}^{1-\eta}$$

Or:

$$1 = (1 - \theta)(p_t^*)^{1-\eta} + \theta\pi_t^{\eta-1} \quad (\text{B.35})$$

Where  $p_t^* = \frac{P_t^*}{P_t}$  captures the relative price of the intermediate good to the final good and  $\pi_t = \frac{P_t}{P_{t-1}}$  denotes the gross inflation of the final good. Now consider equation (B.25).

$$\frac{W_t}{R_t^K} = \frac{(1 - \alpha) K_t^d(j)}{\alpha N_t^d(j)} \quad (\text{B.36})$$

$dj$  is then multiplied to the numerator and denominator on the RHS and integrated over the interval  $[0, 1]$  to obtain:

$$\frac{W_t}{R_t^K} = \frac{(1 - \alpha) \int_0^1 K_t^d(j) dj}{\alpha \int_0^1 N_t^d(j) dj} \quad (\text{B.37})$$

Let aggregate demand for labor be  $N_t^d = \int_0^1 N_t^d(j) dj$  and for capital  $K_t^d = \int_0^1 K_t^d(j) dj$ . Equation (B.37) becomes:

$$\frac{W_t}{R_t^K} = \frac{(1 - \alpha) K_t^d}{\alpha N_t^d} \quad (\text{B.38})$$

Equation (B.38) can be expressed with real costs by diving both the numerator and denominator on the RHS by  $P_t$ :

$$\frac{W_t/P_t}{R_t^K/P_t} = \frac{(1 - \alpha) K_t^d}{\alpha N_t^d} \quad (\text{B.39})$$

$$\frac{w_t}{r_t^K} = \frac{(1 - \alpha) K_t^d}{\alpha N_t^d} \quad (\text{B.40})$$

The aggregate nominal cost can be written as:

$$\begin{aligned} C_t &= \int_0^1 C_t(j) dj \\ &= \int_0^1 \left[ \frac{W_t}{1 - \alpha} \right]^{1-\alpha} \frac{Y_t(j)}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha dj \\ &= \left[ \frac{W_t}{1 - \alpha} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha \int_0^1 Y_t(j) dj \end{aligned}$$

Using equation (B.20):

$$C_t = \left[ \frac{W_t}{1 - \alpha} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t(j) dj$$

Which can be simplified to:

$$C_t = \Omega_t Y_t s_t \quad (\text{B.41})$$

where:

$$\Omega_t = \left[ \frac{W_t}{1 - \alpha} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha$$

and

$$s_t = \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\eta} dj$$

where  $s_t$  is the price dispersion. In absence of pricing frictions,  $s_t = 1$ . If prices differ, it can be shown that the that price dispersion has a lower bound of unity, or  $s_t \geq 1$ . Note that:

$$\mathcal{MC}_t(j) = \Omega_t = \left[ \frac{W_t}{1-\alpha} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha \quad (\text{B.42})$$

Using (B.41) marginal cost of aggregate production becomes:

$$\mathcal{MC}_t = \frac{\partial \mathcal{C}_t}{\partial Y_t} = \mathcal{MC}_t(j) s_t$$

Therefore, the real marginal cost of aggregate production is:

$$mc_t = \frac{\mathcal{MC}_t(j)}{P_t} s_t$$

Using (B.36), the explicit relationships between real marginal cost of aggregate production and the factor prices can be derived as:

$$\frac{mc_t}{s_t} = \frac{\mathcal{MC}_t(j)}{P_t}$$

Using (B.42):

$$\begin{aligned} \frac{mc_t}{s_t} &= \frac{1}{P_t} \left[ \frac{W_t}{1-\alpha} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha \\ &= \left[ \frac{W_t}{P_t(1-\alpha)} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{P_t \alpha} \right]^\alpha \\ &= \left[ \frac{w_t}{1-\alpha} \right] \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{r_t^K (1-\alpha)}{\alpha W_t} \right]^\alpha \end{aligned}$$

Using (B.36):

$$\begin{aligned} \frac{mc_t}{s_t} &= \left[ \frac{w_t}{1-\alpha} \right] \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{N_t^d}{K_t^d} \right]^\alpha \\ \frac{mc_t}{s_t} A_t^{(1-\alpha)} (1-\alpha) \left[ \frac{K_t^d}{N_t^d} \right]^\alpha &= w_t \end{aligned} \quad (\text{B.43})$$

Similarly, for  $r_t^K$ : Using (B.42):

$$\begin{aligned} \frac{mc_t}{s_t} &= \frac{1}{P_t} \left[ \frac{W_t}{1-\alpha} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{\alpha} \right]^\alpha \\ &= \left[ \frac{W_t}{P_t(1-\alpha)} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{R_t^K}{P_t \alpha} \right]^\alpha \\ &= \left[ \frac{\alpha w_t}{(1-\alpha) r_t^K} \right]^{1-\alpha} \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{r_t^K}{\alpha} \right]^\alpha \end{aligned}$$

Using (B.36):

$$\begin{aligned} \frac{mc_t}{s_t} &= \left[ \frac{K_t^d}{N_t^d} \right] \frac{1}{A_t^{(1-\alpha)}} \left[ \frac{r_t^K}{\alpha} \right]^\alpha \\ \frac{mc_t}{s_t} A_t^{(1-\alpha)} (\alpha) \left[ \frac{K_t^d}{N_t^d} \right]^{(\alpha-1)} &= r_t^K \end{aligned} \quad (\text{B.44})$$

Equation (B.31) states that:

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} Y_{t+k} - \frac{\eta}{\eta-1} \mathcal{MC}_{t+k}^*(j) \left( \frac{P_t^*(j)^{-\eta-1}}{P_{t+k}^{-\eta}} \right) Y_{t+k} \right) \right] = 0$$

which can be written as:

$$\begin{aligned} \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} Y_{t+k} - \frac{\eta}{\eta-1} \frac{\mathcal{MC}_{t+k}^*(j)}{P_{t+k}} \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta-1} Y_{t+k} \right) \right] &= 0 \\ \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( \frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta-1} Y_{t+k} \left( \frac{P_t^*(j)}{P_{t+k}} - \frac{\eta}{\eta-1} \frac{\mathcal{MC}_{t+k}^*(j)}{P_{t+k}} \right) \right] &= 0 \end{aligned}$$

Using the expression for the real aggregate marginal cost calculated above and the fact that  $P_t^*(j) = P_t^*$ :

$$\mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\eta-1} Y_{t+k} \left( \frac{P_t^*}{P_{t+k}} - \frac{\eta}{\eta-1} \frac{mc_{t+k}}{s_{t+k}} \right) \right] = 0$$

I define:

$$\xi_t \equiv \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\eta} Y_{t+k} \right]$$

and

$$\zeta_t \equiv \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\eta-1} Y_{t+k} \frac{mc_{t+k}}{s_{t+k}} \right]$$

Using the definition of  $\xi_t$ , we can write the following:

$$\xi_t = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\eta} Y_t + \mathbf{E}_t \left[ \sum_{k=1}^{\infty} \theta^k M_{t,t+k} \left( \frac{P_t^*}{P_{t+k}} \right)^{-\eta} Y_{t+k} \right]$$

We let  $k \equiv k-1$  and substitute it above along with the definition of relative price of the intermediate good to the final good:

$$\xi_t = (p_t^*)^{-\eta} Y_t + \theta \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+1} M_{t+1,t+k+1} \left( \frac{P_t^*}{P_{t+1}} \right)^{-\eta} \left( \frac{P_{t+1}^*}{P_{t+k+1}} \right)^{-\eta} Y_{t+k+1} \right]$$

I multiply and divide by  $P_t^{-\eta}$  and  $P_{t+1}^{-\eta}$  in the following manner:

$$\xi_t = (p_t^*)^{-\eta} Y_t + \theta \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+1} M_{t+1,t+k+1} \left( \frac{P_t^*}{P_t} \right)^{-\eta} \left( \frac{P_{t+1}}{P_{t+1}^*} \right)^{-\eta} \left( \frac{P_t}{P_{t+1}} \right)^{-\eta} \left( \frac{P_{t+1}^*}{P_{t+k+1}} \right)^{-\eta} Y_{t+k+1} \right]$$

To obtain:

$$\xi_t = (p_t^*)^{-\eta} Y_t + \theta \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \theta^k M_{t,t+1} M_{t+1,t+k+1} (p_t^*)^{-\eta} (\pi_{t+1} p_{t+1}^*)^{\eta} \left( \frac{P_{t+1}^*}{P_{t+k+1}} \right)^{-\eta} Y_{t+k+1} \right] \quad (\text{B.45})$$

Iterating the definition of  $\xi_t$  forward by one period:

$$\xi_{t+1} = \mathbf{E}_{t+1} \left[ \sum_{k=0}^{\infty} \theta^k M_{t+1,t+k+1} \left( \frac{P_{t+1}^*}{P_{t+k+1}} \right)^{-\eta} Y_{t+k+1} \right]$$

Substituting  $\xi_{t+1}$  in (B.45), the following recursive form of  $\xi_t$  is obtained:

$$(p_t^*)^{\eta} \xi_t = Y_t + \theta \mathbf{E}_t \left[ M_{t,t+1} (\pi_{t+1} p_{t+1}^*)^{\eta} \xi_{t+1} \right] \quad (\text{B.46})$$

Through a similar process, the recursive expression for  $\zeta_t$  can be obtained:

$$(p_t^*)^{\eta+1} \zeta_t = Y_t \frac{mC_t}{s_t} + \theta \mathbf{E}_t \left[ M_{t,t+1} (\pi_{t+1} p_{t+1}^*)^{\eta+1} \zeta_{t+1} \right] \quad (\text{B.47})$$

Moreover:

$$\xi_t = \frac{\eta}{\eta - 1} \zeta_t \quad (\text{B.48})$$

As defined above, the price dispersion is:

$$s_t = \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\eta} dj$$

Which can be broken into the following:

$$s_t = \int_0^{1-\theta} \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} dj + \int_{1-\theta}^1 \left( \frac{P_{t-1}(j)}{P_t} \right)^{-\eta} dj$$

Which can be written as:

$$s_t = \int_0^{1-\theta} \left( \frac{P_t^*}{P_t} \right)^{-\eta} dj + \left( \frac{P_{t-1}}{P_t} \right)^{-\eta} \int_{1-\theta}^1 \left( \frac{P_{t-1}(j)}{P_{t-1}} \right)^{-\eta} dj$$

Which becomes:

$$s_t = (1 - \theta) p_t^{*-\eta} + \theta \pi_t^\eta s_{t-1} \quad (\text{B.49})$$

The final set is to aggregate the production function:

$$Y_t(j) = K_t^d(j)^\alpha [A_t N_t^d(j)]^{(1-\alpha)}$$

Using (B.20):

$$\left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t = K_t^d(j)^\alpha [A_t N_t^d(j)]^{(1-\alpha)}$$

Integrating both sides with respect to  $dj$ :

$$Y_t \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\eta} dj = A_t^{(1-\alpha)} \int_0^1 \left( \frac{K_t^d(j)}{N_t^d(j)} \right)^\alpha N_t^d(j) dj$$

Using (B.36), the following is obtained:

$$Y_t \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\eta} dj = A_t^{(1-\alpha)} \int_0^1 \left( \frac{\alpha W_t}{(1-\alpha) R_t^K} \right)^\alpha N_t^d(j) dj$$

Using the definition that  $N_t^d = \int_0^1 N_t^d(j) dj$  and equation (B.38):

$$Y_t \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\eta} dj = A_t^{(1-\alpha)} \left( \frac{K_t^d}{N_t^d} \right)^\alpha N_t^d$$

Using the definition of  $s_t$ :

$$Y_t s_t = [K_t^d]^\alpha [A_t N_t^d]^{(1-\alpha)} \quad (\text{B.50})$$



## B.6 Equilibrium

The equilibrium can be characterized as follows:

$$K_t^d = K_t \quad (\text{B.51})$$

$$N_t^d = N_t^s = N_t \quad (\text{B.52})$$

$$K_{t+1} = (1 - \delta_K)K_t + I_t - G_t K_t \quad (\text{B.53})$$

$$N_t^\omega = w_t C_t^{-\psi} \quad (\text{B.54})$$

$$\frac{1}{R_t^F} = \mathbf{E}_t[M_{t,t+1}] \quad (\text{B.55})$$

$$M_{t,t+1} = \beta \left[ \frac{C_{t+1}}{C_t} \right]^{-\psi} \pi_{t+1}^{-1} \quad (\text{B.56})$$

$$Q_t = \frac{1}{1 - G'(\frac{I_t}{K_t})} \quad (\text{B.57})$$

$$1 = \mathbf{E}_t \left[ M_{t,t+1} \left( \frac{\left[ r_{t+1}^K + Q_{t+1} \left[ (1 - \delta) - G' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right] \right] \pi_{t+1}}{Q_t} \right) \right] \quad (\text{B.58})$$

$$i_t = \gamma_i i_{t-1} + \phi_\pi (\pi_t - \pi_{ss}) + \phi_y (Y_t - Y^*) + u_t \quad (\text{B.59})$$

$$1 = (1 - \theta)(p_t^*)^{1-\eta} + \theta \pi_t^{\eta-1} \quad (\text{B.60})$$

$$w_t = (1 - \theta_w)w_{t-1} + \theta_w \left( (1 - \alpha) \frac{Y_t}{N_t} \right)^{\gamma_w} \quad (\text{B.61})$$

$$\frac{m c_t}{s_t} A_t^{(1-\alpha)} (1 - \alpha) \left[ \frac{K_t}{N_t} \right]^\alpha = w_t \quad (\text{B.62})$$

$$\frac{m c_t}{s_t} A_t^{(1-\alpha)} (\alpha) \left[ \frac{K_t}{N_t} \right]^{(\alpha-1)} = r_t^K \quad (\text{B.63})$$

$$(p_t^*)^\eta \xi_t = Y_t + \theta \mathbf{E}_t \left[ M_{t,t+1} (\pi_{t+1} p_{t+1}^*)^\eta \xi_{t+1} \right] \quad (\text{B.64})$$

$$(p_t^*)^{\eta+1} \zeta_t = Y_t \frac{m c_t}{s_t} + \theta \mathbf{E}_t \left[ M_{t,t+1} (\pi_{t+1} p_{t+1}^*)^{\eta+1} \zeta_{t+1} \right] \quad (\text{B.65})$$

$$\xi_t = \frac{\eta}{\eta - 1} \zeta_t \quad (\text{B.66})$$

$$s_t = (1 - \theta)(p_t^*)^{-\eta} + \theta \pi_t^\eta s_{t-1} \quad (\text{B.67})$$

$$Y_t s_t = [K_t]^\alpha [A_t N_t]^{(1-\alpha)} \quad (\text{B.68})$$

$$Y_t = C_t + I_t + e^{g_t} T_t \quad (\text{B.69})$$

$$z_t = \epsilon_t^z \quad (\text{B.70})$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \quad (\text{B.71})$$

$$g_t = \rho_g g_{t-1} + \epsilon_t^g \quad (\text{B.72})$$

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t} \quad (\text{B.73})$$

## B.7 Steady State

The non-stochastic steady of the model given  $N$  is given below:

$$\begin{aligned} \bar{G} &= 0 \\ \bar{G}' &= 0 \\ I &= -\delta_K K \\ M &= \beta \pi_{ss}^{-1} \\ Q &= 1 \\ r_{t+1}^K &= (1 - \delta) - \frac{1}{M_{t,t+1} \pi_{ss}} \\ p^* &= \left[ \frac{1 - \theta \pi_{ss}^{\eta-1}}{1 - \theta} \right]^{\frac{1}{1-\eta}} \\ w_t &= \left( (1 - \alpha) \frac{Y}{L} \right)^{\gamma_w} \\ mc &= A^{(1-\alpha)} (1 - \alpha) \left[ \frac{K}{N} \right]^\alpha s \\ \frac{Y}{\xi} &= (p^*)^\eta (1 - \theta M \pi_{ss}) \\ \frac{Y}{\zeta} \frac{mc}{s} &= (p^*)^{\eta+1} (1 - \theta M \pi_{ss}) \\ \xi &= \frac{\eta}{\eta - 1} \zeta \\ s &= \frac{(1 - \theta)(p^*)^{-\eta}}{(1 - \theta \pi_{ss}^\eta)} \\ Y &= \frac{K^\alpha (AN)^{(1-\alpha)}}{s} \\ C &= Y - \delta_K K + T \\ A &= Z \\ i &= \frac{\phi_\pi + \phi_y Y}{1 - \gamma_i} \\ z &= a = g = u = 0 \end{aligned}$$