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Discussion paper

Seasonality matters: a multi-season, multi-state dynamic optimization in fishery

BY
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Seasonality matters: a multi-season, multi-state dynamic optimization in fishery

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Abstract

Many biological and economic processes in fishery happen seasonally. Most of the extant literature tends to neglect this fact. This work is an initial attempt to treat seasonality in a systematic and proper way. We apply a periodic Bellman approach to obtain the optimal feedback policy of each season. Our approach has rich potentials. It could deal with seasonal patterns of uneven lengths: some may span years and some within the year.

We find that, in some cases the equilibrium consists of one harvesting season followed by a moratorium period, indicating an optimal closure of the fishery that would be overlooked by a yearly model. Unlike a typical policy that enforces a moratorium to recover the stock, we find that many states first undergo harvesting all year round and later evolve into the seasonal moratorium. A rising group biomass could be the overshooting effect instead of a clear sign to increase harvest. We sometimes observe declining optimal harvest with increasing states ('valley'), which may relate to the unit profit difference between seasons. Fishing pressure on the mature elicits even heavier harvest in the next season on the same group. A protective moratorium of the immature seems to hinder the value of the whole stock.

Keywords

OR in natural resources; Seasonality; Dynamic programming; Feedback policy; Fisheries

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1. Introduction

1.1 Motivation behind seasonality

Numerous commercial fisheries exhibit periodic patterns, undergoing periods of feeding, reproduction, migration, harvesting and so on (Clark, 2010). However, most commercial fisheries are managed quintessentially on a yearly basis. For example, agencies like the International Council for the Exploration of the Sea (ICES) collect annual data and provide annual advice regarding stock status, reference points and Total Allowable Catches (TACs) (Fisheries - European Commission, 2016). Often, annual TACs are allotted to different vessel groups that target the resource in different seasons (Kvamsdal, Maroto, Morán, & Sandal, 2017). Therefore, we conduct this work believing that research and policymaking should incorporate seasonality in fisheries to a larger and deeper extent.

Our approach is an initial attempt that applies Dynamic Programming (DP) to obtain the optimal feedback control policy with both seasonality and multidimensional states in a bioeconomic model. It has rich potentials because this approach makes it possible to take into consideration longer seasonality that is spanning more than a single year. For instance, we may model El Niño conditions that happen at intervals spanning several years combined with a seasonal feature that takes place every year.

Many fisheries with notable seasonal migrations have been studied using diverse methodology. In the Northeast Atlantic, the mature part of the Northeast Arctic cod (NEA) migrates from the Barents Sea to the Norwegian coast to spawn in the spring, with a remarkable amount aggregated around Lofoten. Newly hatched larvae will drift north with the current and reach the Barents Sea by summer, where they stay and grow until mature enough to initiate their spawning migration (Durant, Hjermann, Sabarros, & Stenseth, 2008). Hannesson, Salvanes, & Squires (2010) analyzed the effect of technological change on labor and productivity in the seasonal Lofoten fishery using data over 130 years. Hermansen & Dreyer (2010) investigated how the regime aiming to shift the harvesting pattern towards the low season performed for Lofoten fishery. For the Texas shrimp fishery, there exists a tradeoff between catching the small shrimp in bays and harvesting the large shrimp in the Gulf of Mexico (Önal, McCarl, Griffin, Matlock, & Clark, 1991). Önal et al. (1991) developed a nonlinear mathematical programming model with multi-periods and size-structure to determine the optimal harvesting pattern and unveiled substantial gains from reallocating the effort throughout the season by improving the size composition of the catch.

Although many have approached seasonality in the literature, most built intra-seasonal bioeconomic models to analyze within-season harvesting in regulated open access fisheries (Smith, 2012). Larkin & Sylvia (1999) set up a nonlinear programming model to determine the time path of harvest by allocating effort across months (intra-season harvest pattern) and annual quotas between competing

industry sectors (catch rights) and found that intra-season quality generated higher net benefits than the allocation of catch rights. Holland (2011) simulated an intra-annual numerical model to optimize effort and catch over a one-year period in the Maine lobster fishery under seasonal variability in catchability. Such intra-season studies tend to assume that fishing takes place over a single season, with a frequently fixed season length (Kvamsdal et al., 2017).

Both the profound role of seasonality in fisheries and the inadequacy in the literature calls for further research in this direction. Seasonality in fisheries is a common phenomenon: both biological processes, such as spawning, and human activities, such as harvesting, are seasonal rather than smoothly distributed over time (Bjørndal & Munro, 2012; Clark, 2010). At the same time, seasonality is complex and difficult to cope with: when considering one-year time increments, discrete-time models tend to neglect seasonality; when considering autonomous optimal feedback policies, continuous-time models neglect it as well (Kvamsdal et al., 2017). Moreover, when seasonality is investigated instead of being disregarded, those intra-season studies that imply a single harvesting season can be an oversimplification of many real world fisheries.

We hereby argue that it is appropriate to turn to discrete-time models (Bjørndal & Munro, 2012) and we propose uneven time increments that are less than one year in order to incorporate seasonality. To fill the gap in the current literature, we introduce multiple harvesting seasons within each year. Each season length can be adjusted according to specific fisheries. The stock develops alternately according to seasonal dynamics and the cycle goes on in an infinite time horizon.

1.2 Remarks on the approach

A rich literature on fisheries management includes implementation of the DP approach (Clark, 1985; Lane, 1988; Mangel & Clark, 1983; Poudel, Sandal, Steinshamn, & Kvamsdal, 2012; Walters, 2002) but few addressed the issue of seasonality. With the recent finding on periodic Bellman approach (Kvamsdal, Maroto, Morán, & Sandal, 2016), we are able to simplify the problems that consist of multiple periods in one cycle and solve them with common DP technique. Nevertheless, it is widely recognized that discrete DP suffers from the curse of dimensionality (Anderson, Ben-Israel, Custis, & Sarabun, 1981). The skyrocketing computing time brought by additional dimension remains, therefore, we begin with the simplest case of two-dimensional states, seeking to preserve both solution efficiency and theoretic insights.

With multidimensional states, a bioeconomic model can describe either interacting species that often includes predator-prey relationships (Poudel et al., 2012; Sandal & Steinshamn, 2010) or a single stock that is categorized into age or size groups (Tahvonen, 2014). Age-structured models, which are widely used to assign catch quotas, have gained more popularity among economists as a consequence of the critics on the ‘oversimplified’ traditional biomass approach (Tahvonen, 2010; Townsend, 1986). Due to generally large cohort numbers, few of the age-structured studies employed DP approach.

To better understand the inner structure of a stock, we adopt a commonly accepted practice and divide a population into mature (adult) and immature (juvenile) individuals (Holden & Conrad, 2015; Sandal & Steinshamn, 2003). Though simple, this practice is intuitive and allows inclusion of critical biological processes such as recruitment, as well as other potential model expansions and specifications such as size-dependent prices.

We achieve the optimal feedback solution with the control variable (seasonal harvests) as a direct function of the state variable (mature and immature group biomasses). The feedback models take prevailing stocks as inputs and automatically respond to the changes in the states, and thus adapt to new situations instantaneously (Sandal & Steinshamn, 1997). In addition, the feedback approach can easily be extended with stochasticity since it is superior than the alternative time paths approach when faced with uncertainty (Agnarsson et al., 2008).

1.3 Highlights and guidance

Having proposed the motivation to investigate seasonality and the argumentations for our approach, we emphasize that this work, to our knowledge, is the first numerical study of seasonal optimal feedback policy with two states using DP. Some findings are intuitive while some others turn out to be less expected and novel.

We find that sometimes the optimal equilibrium cycle consists of one harvesting season followed by a moratorium period, implying a natural seasonal closure of the fishery that could be neglected by a yearly model. Unlike a typical policy that implements a moratorium to recover the stock, we find that many states undergo harvesting during the entire year first and later settle into the cycle of harvesting-moratorium. An increasing group biomass could be the overshooting effect instead of a decisive signal to harvest more. In some cases, a ‘valley’ effect, i.e. declining harvest with rising states, exists in the seasonal optimal harvesting policy. We find this effect closely related to the scale of the unit profit difference between seasons. The unit harvesting cost shapes the characteristics of the state space area where harvest in one season is more prominent than harvest in the rest of the year. Fishing pressure on the mature stock in one season can arouse even more aggressive harvesting on the same group in the next season. It seems that a regulatory moratorium of the immature group may stem from conservative objectives but in fact hinder the value of the whole stock. Therefore, when choosing which group to target, it can be beneficial to avoid only harvesting the mature in order to let them reproduce better in the spawning season. When the economic profit in one season increases, it takes longer time for the maximum state to decrease and leave the abundant region on the state space.

In Section 2, we first demonstrate the basic model structure with seasonality and two-dimensional states; we then specify the biological state transitions and economic rent functions; lastly, we establish the credibility of our method and results by referring to the recent theoretic findings on periodic Bellman approach. In Section 3, a simple depiction of the solution procedure is provided. Then we

elaborate on parameter specifications and introduce eight cases. Section 4 manifests the numerical approximation results of all the cases, analyzes the effects and provides possible explanations. Conclusions and discussions follow in Section 5.

2. The model

2.1 Model structure

As elaborated in Section 1, we choose a simple but representative model structure where we look at an infinite horizon, discounted, discrete-time dynamic optimization problem with two-dimensional states (mature and immature) and two seasons (S1 and S2), aiming to access both conceptual insights and solution feasibility. It is straightforward to depict the seasonal state transitions as follows:

$$\dots \xrightarrow{S2} \begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow[T_1(x, y, h_1)]{S1} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} \xrightarrow[T_2(\hat{x}, \hat{y}, h_2)]{S2} \begin{bmatrix} X \\ Y \end{bmatrix} \xrightarrow{T_1(X, Y, h_1)}{S1} \dots$$

Fig. 1. Two-dimensional two-season dynamic cycle.

The population is represented by the biomass of two age groups, namely mature (x) and immature (y). Seasonal harvest policy or control of the system is h_1 for S1 and h_2 for S2 respectively. There are two dynamic operators during a year noted as $T_1(x, y, h_1)$ and $T_2(\hat{x}, \hat{y}, h_2)$, driven by its seasonal transitions and policy. States \hat{x} and \hat{y} are the new states in the end of S1 after dynamic operator T_1 . States X and Y appear in the end of S2 as a result of the dynamic operator T_2 and are the states in the beginning of the next S1. The periodic cycle goes on infinitely as demonstrated in Fig. 1.

As a practice of tradition (Gordon, 1954; Scott, 1955), we believe it is legitimate to set our objective as to maximize the economic rent from the fishery resource. The core purpose of fisheries management is to take into account both the conservation of the resource base, as well as the exploitation of it (Bjørndal, Lane, & Weintraub, 2004). Profit maximization achieves many of the prevalent fishery objectives, for example, it implies conservation of fish stocks and marine environment to the extent that this conservation contributes to harvesting profits and conservation in itself is valuable (Arnason, 2009).

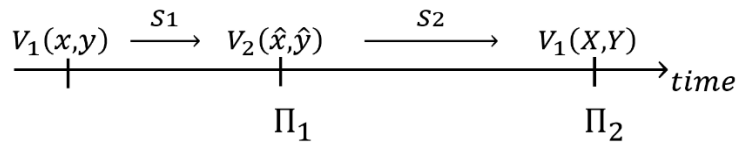


Fig. 2: Two value functions and seasonal economic gains.

Seasonal net economic gains are denoted as Π_1 and Π_2 . In each season, we assume that the stock first undergoes biological processes, such as growth and spawning, and then becomes available for the

human harvesting activities in the end of the season. Therefore, Π_1 is obtained at the end of S1 while Π_2 is realized at the end of S2 as shown in Fig. 2 with a timeline.

The value function manifests the best possible value of the objective and is non-autonomous, depending on which season the present time is. V_1 indicates the value in the beginning of S1 or in the end of S2 while V_2 represents the value when standing in the end of S1 or in the beginning of S2. Note that V_1 and V_2 are not the value of each separate season but rather of the infinite time horizon.

The value functions in our problem are:

$$\begin{aligned} V_1(x, y) &= \max_{h_1, h_2} \sum_{t=1}^{+\infty} [\beta_1 \beta^{t-1} \Pi_1(x, y, h_1) + \beta^t \Pi_2(\hat{x}, \hat{y}, h_2)] \\ V_2(\hat{x}, \hat{y}) &= \max_{h_1, h_2} \sum_{t=1}^{+\infty} [\beta_2 \beta^{t-1} \Pi_2(\hat{x}, \hat{y}, h_2) + \beta^t \Pi_1(X, Y, h_1)] \end{aligned} \quad (1)$$

where $\beta_1 = \beta^\theta$ and $\beta_2 = \beta^{1-\theta}$. The parameter θ indicates the length proportion of S1 in a whole year and β is the annual discount factor. A varying θ generates flexibility and allows the model to deal with species or ecological systems of various kinds.

2.2 A simplified fishery

Our bioeconomic model describes a single species stock in which the parameters for spawning ground condition (cost structure), selectivity and price function are modified. This simplified fishery model serves as a starting point to probe into the implications of modeling seasonality in such problems and to stimulate and support further research in this direction.

We assume that spawning or recruitment takes place during S1, setting S1 as the shorter season in this context. Harvesting is assumed to take place smoothly within the season. We introduce a major difference between seasons about harvesting: in S1, only the mature group lends itself to harvesting while in S2 the selection between x and y remains unfixed. This protection of the young in the spawning season can be advocated by multiple arguments: modeling-wise, it simplifies the decision variables; economically, it may be easier to take the mature group in S1 when they agglomerate in the spawning ground; biologically, sparing the immature at this critical time of the year is likely to advantage future prosperity of the stock. If the season is short enough, then one can choose to ignore the growth during the season (Mangel, 1984). Thus, we assume for simplicity that x has little change except for biomass loss due to harvesting as shown in Eq. (2).

$$\hat{x} = x - h_1 \quad (2)$$

$$\hat{y} = F(x, y) = \min [y_{max}, (b_1 + b_2 x)y + b_3 x] \quad (3)$$

Function $F(x, y)$ in Eq. (3) is linear in y with upper bound y_{max} and it effectively grasps the spawning dynamics: both the slope ($b_1 + b_2x$) and the intercept (b_3x) are dependent on x . As x escalates, $F(x, y)$ rises via both slope and intercept until it reaches the maximum biomass y_{max} . This structure implies that a stronger x produces more offspring creating higher population density for y and thus induce better fitness and faster growth rate of y according to ‘The Allee effect’ (Allee, 1931). When y becomes zero, x still contributes as much as (b_3x) to y through spawning, which maintains the immature group even without any to start with. When x is zero, y will grow to b_1y after S1. Here the ‘biomass growth’ counts for both individual weight gain and biomass loss caused by natural mortality, so b_1 is not necessarily larger than one.

$$X = D_x(\hat{x}, \hat{y}) - sh_2 \quad (4)$$

$$Y = D_y(\hat{x}, \hat{y}) - (1 - s)h_2 \quad (5)$$

$$D_x(\hat{x}, \hat{y}) = m\hat{x} + a\hat{y} \quad (6)$$

$$D_y(\hat{x}, \hat{y}) = G(\hat{y}) - a\hat{y} - c\hat{x}\hat{y} \quad (7)$$

$$G(\hat{y}) = \frac{g_1\hat{y}}{1+g_2\hat{y}} \quad (8)$$

During S2, both groups can lend themselves to harvesting where a gear selectivity parameter s indicates the proportion of h_2 that comes from the mature group. Having a parameter s enables the model to adjust according to fishing gear or policy. For example, $s = 1$ may represent a policy to spare all the immature individuals and $s = 0$ can signify a fishing gear type that perfectly select all the young fish. $D_x(\hat{x}, \hat{y})$ and $D_y(\hat{x}, \hat{y})$ are the dynamics of the mature and immature group respectively before harvest in S2.

On top of the natural growth and mortality within the mature group itself, which is assumed linear, every year a part of the immature matures and transforms to join the mature group. Thus, the dynamics of the mature group during S2 consists of three parts: linear growth ($m\hat{x}$), maturity transformation ($a\hat{y}$) and harvest (sh_2).

The function $G(\hat{y})$ has a form resembling that of the discrete-time Beverton-Holt recruitment function (Beverton & Holt, 1957). It provides a non-linear growth for the immature group during the longer season in which smaller states correspond to higher growth rates and as the state increases the immature biomass after S2 asymptotically approaches a fixed level. Note that the original Beverton-Holt function gives the number of population but we are working with stock biomass instead.

Cannibalistic behavior is expressed in the model as $c\hat{x}\hat{y}$. The multiplicative form means that only when both groups coexist, will there be biomass loss due to cannibalism. The dynamics of the immature group includes the following items: non-linear growth $G(\hat{y})$, maturity transformation ($a\hat{y}$),

cannibalistic behavior ($c\hat{x}\hat{y}$) and harvest $(1 - s)h_2$. Fig. 3 summarizes the state transitions of the modeled fishery and serves as an extension and specialization of Fig. 1.

The size and value of the fish stock as a renewable source can grow and change over time due to biological factors, regulatory decisions as well as market conditions (Clark & Munro, 1975). After elaborating on the biological facets, we now focus on the economic conditions, which can be decisive in determining the value functions.

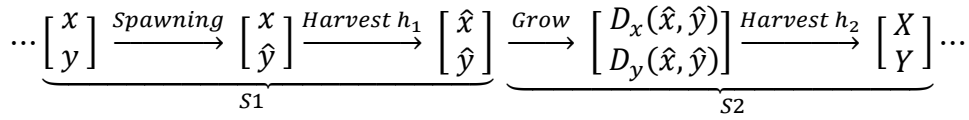


Fig. 3: Two-dimensional two-season dynamic cycle with harvest.

Inspired from the NEA cod stock as one of the most important fisheries worldwide with minimum prices set through negotiations (Pettersen & Myrland, 2016), we assume that the unit price is a typical downward sloping function of the seasonal harvest with a price floor. Both seasons share the same price function structure as in Eq. (9): when harvest is big, the unit price of the fish goes towards the minimum p_{min} and when harvest is small, the price approaches the maximum p_{max} . We can easily adjust parameter values to enable various price ranges for each season. In this work, we first employ the same price function for both seasons leaving costs as the only element contributing to the economic difference between seasons. Then, in order to enhance the realism of the model, we explore time-dependent price functions that elicit different prices of the fish depending on which season the harvest takes place in. Numerical details are elaborated in Section 3.3.

$$P(h) = p_{min} + (p_{max} - p_{min})e^{-ah} \quad (9)$$

$$\Pi_1(h_1, x) = P(h_1)h_1 - \frac{k_1 h_1}{x^l} \quad (10)$$

$$\Pi_2(h_2, \hat{x}, \hat{y}) = P(h_2)h_2 - \frac{k_2 h_2}{D_x(\hat{x}, \hat{y}) + D_y(\hat{x}, \hat{y})} \quad (11)$$

For the seasonal cost functions, there is an important concept in fisheries management: the sensitivity of harvesting cost to the biomass density, which is referred to as the marginal stock effect (MSE)¹ (Clark & Munro, 1975). This establishes a connection between stock size and harvesting cost, making it possible to account for density dependence and thereby analyze anything from completely uniformly distributed stock to perfectly schooling fish (Steinshamn, 2011).

We introduce stock-cost parameter l to include various levels of stock dependency in the harvesting cost. One critical value is $l = 0$, signifying that the unit harvest cost is completely independent of the

¹ A parameter usually referred to as the stock elasticity, or the schooling parameter in some studies, embodies the MSE either in the production function or in the unit harvest cost function.

aggregated stock size and the total harvesting cost is simply proportional to the harvest amount. This can be a valid assumption when the stock is highly densified and it may elicit overfishing because taking the last school of fish can remain profitable given a constant unit cost (Bjørndal & Conrad, 1987). For $l > 0$, the unit harvest cost depends on the stock size, via the corresponding density if the ground area is unchanged: less densified stocks are difficult to search and have higher unit harvest cost; more densified stocks are easy to find and have lower unit cost. Such stock dependency is quintessentially an economic protection of the weaker stocks.

In S1 when a population migrates into a compact enough spawning ground, the biomass density rises as a result of the gathering. We infer that this aggregation or agglomeration in S1 may effectively alter the value of l in Eq. (10) given various spawning ground characteristics: when they suggest a salient population densification, l is set to zero²; when they indicate a modest density increment, l is set to be positive. Having such season-specific cost structure makes it possible to capture more of the diverse implications of seasonality.

In S2, the stock is assumed to be dispersed in a vast enough area with an implicit stock-cost parameter of one: the seasonal unit harvest cost is dependent on the total stock size in Eq. (11). We choose the state right before harvesting as the reference for the stock size, which may underestimate the costs when the harvest amount is significant.

2.3 Periodic Bellman approach

Kvamsdal et al. (2016) have shown that classical Bellman approach can be extended to periodic problems. Instead of formulate the problem using one high-dimensional single equation with the annual contraction factor, we can work directly with the coupled dynamic programming equations and legitimately obtain the same unique solution. This extended periodic version of the Bellman approach is proved able to avoid cumbersome numerical routines and provide reliable results at the same time, offering a feasible and practical solution for such problems.

If the optimal control $\{h_1^*, h_2^*\}$ is known, the value functions in Eq. (12) are the fixed points of the Bellman operators, for which β_1 and β_2 are the seasonal contraction factors respectively.

$$\begin{aligned} V_1(x, y) &= \beta_1 \Pi_1(x, y, h_1^*) + \beta_1 V_2(T_1(x, y, h_1^*)) \\ V_2(\hat{x}, \hat{y}) &= \beta_2 \Pi_2(\hat{x}, \hat{y}, h_2^*) + \beta_2 V_1(T_2(\hat{x}, \hat{y}, h_2^*)) \end{aligned} \tag{12}$$

In our two-season setting, the coupled dynamic programming equations are as follows:

$$V_1(x, y) = \max_{h_1} \{\beta_1 \Pi_1(h_1, x, y) + \beta_1 V_2(\hat{x}, \hat{y})\}$$

² Though empirically zero is hardly a realistic number, a stock independent cost structure can provide profound implications from our seasonal model.

$$V_2(\hat{x}, \hat{y}) = \max_{h_2} \{\beta_2 \Pi_2(h_2, \hat{x}, \hat{y}) + \beta_2 V_1(X, Y)\} \quad (13)$$

We establish our numerical work based on the theoretical findings about periodic Bellman approach summarized above. The procedure here is to utilize both of the coupled equations in Eq. (13) at the same time and solve for control h_1 from the first contraction process and h_2 from the second. We demonstrate further numerical algorithm details in Section 3.1.

3. Numerical approach

3.1 Solution procedure

We discretize the two-dimensional state space evenly with N1 grid points along x and N2 grid points along y . All the state transitions before control are $F(x, y)$, $D_x(\hat{x}, \hat{y})$ and $D_y(\hat{x}, \hat{y})$, which can be calculated once for all. For any state ending up outside the grid, it will be set to the upper bounds of the grid: x_{max} and y_{max} . Lower bounds for both dimensions are zero.

In the numerical scheme, we use the escapements after each season as the decision variable of that season. The policy is bounded so that the states remain non-negative at all times.

Starting with a rough guess of the value functions, we seek to utilize them to arrive at the optimal policies. For each grid point, we find the corresponding upper and lower bounds for the policy and then discretize this feasible part of the escapement space evenly using a number of points that is proportional to the length of the feasible range. Hence, the step length of the discretized escapement is comparable for all grid points. For each grid point, we interpolate within the grid and obtain all the feasible values according to Eq. (12) then pick the maximum value and update the current value function on this grid point, completing Eq. (13). Subsequently, we locate the escapement level that provides the highest value and update it as the optimal escapement on this grid point, which in turn gives us the optimal harvest, which is more straightforward to interpret. One policy iteration is accomplished when all the grid points are updated with the newest values and policies for both S1 and S2.

Then a number of coupled value iterations follow, contracting both value functions without updating the policies. When the value functions have moved closer to the true fixed points, they are put into another round of policy iteration. This iterating process continues until the change in the value functions and policies are negligible.

3.2 Phenomenological parameter specifications

We construct a reasonable and stylized population to work with, referring to the NEA cod stock. The focus and purpose of the numerical illustration is modeling seasonality instead of empirical ones. The first season S1 lasts from January to April representing the shorter and more intense harvest season

(Hermansen & Dreyer, 2010). In this work, we conduct the numerical approximation with an annual discount factor of 0.97 and accordingly the seasonal discount factors are $\beta_1 = 0.97^{\frac{1}{3}}$ and $\beta_2 = 0.97^{\frac{2}{3}}$. Referring to several stock assessments from ICES, the state space is set to be 4000 (x_{max}) by 4000 (y_{max}) with 200 (N1 and N2) points distributed evenly along each dimension. All parameters introduced earlier are positive numbers. The unit of the states as well as other state-related concepts such as harvest and biomass difference is kiloton (10^6kg). The unit of the profits and costs is million Norwegian kroner (10^6NOK).

The biological parameters mimicking the scale of the population and its dynamics without harvest are specified below:

$$\hat{y} = F(x, y) = \min[4000, (0.92 + 0.00005x)y + 0.2x] \quad (14)$$

$$D_x(\hat{x}, \hat{y}) = 0.75\hat{x} + 0.2\hat{y} \quad (15)$$

$$D_y(\hat{x}, \hat{y}) = \frac{1.2\hat{y}}{1+0.00005\hat{y}} - 0.2\hat{y} - 0.00001\hat{x}\hat{y} \quad (16)$$

The inherent equilibrium cycle of the stock lies well inside the stock space: see the yellow diamond and the black diamond in Fig. 4(a). If left alone, the stock will not collapse with low biomass level so it never exhibits critical depensation (Clark, 2010). Yet, the biomass recovery in such situations is very slow as shown in Fig. 4(b). If either group is wiped out every year consecutively in whichever season possible, the population eventually collapses to zero. This phenomenon goes in line with the single species setting here that neither group can be sustained by itself.

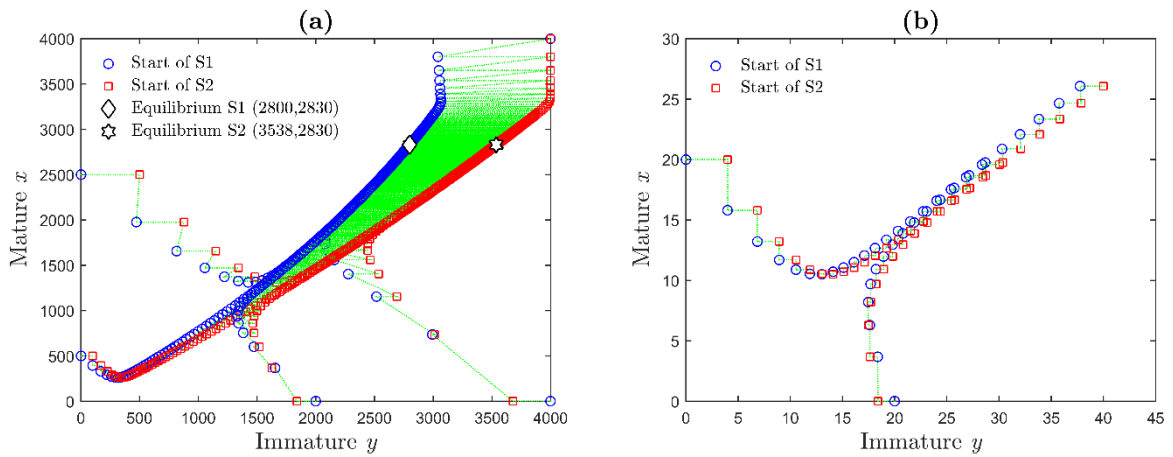


Fig. 4. Dynamic paths without harvesting:

(a) Full state space in 200 years; (b) Partial state space in 20 years.

For state transitions, there exists more nonlinearity and complexity for the immature than the mature, so we demonstrate the immature group visually. A message from Fig. 5 is the different level of sensitivity and susceptibility brought by the states between seasons. When spawning activities occur in

S1, the new immature state depends heavily on the mature group while the link is not as strong in the other case in S2. This is intuitive in the sense that spawning season may display the uttermost direct impact from the mature to the immature group.

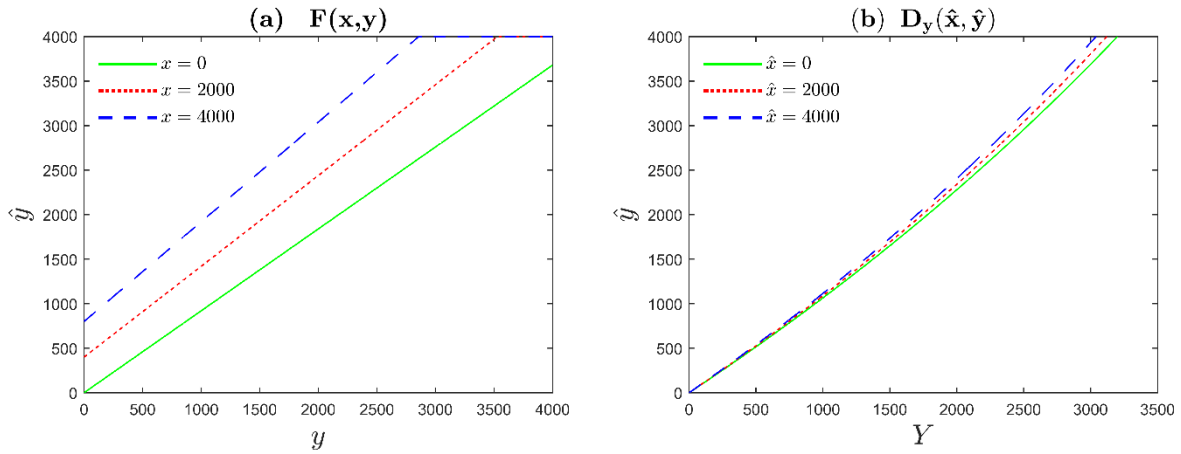


Fig. 5. Immature group state transitions without harvesting:

(a) $F(x, y)$ in S1; (b) $D_y(\hat{x}, \hat{y})$ in S2.

3.3 Cases

Having determined the biological parameters, we now proceed to elaborate on the possibilities of parameter values that address harvest activities and economic drives.

We look at two special values of selectivity parameter s : zero and one. For any in between values, the optimization results are analyzed but not included here because they lie between the results when s equals to zero and one. When $s = 0$, the whole stock is harvested separately timewise: in S1 only the mature is harvested and in S2 only the immature is harvested; when $s = 1$, only the mature group lends itself to harvesting during the whole year. For $s = 0$, the harvest range in S2 is $h_2 \in [0, D_y(\hat{x}, \hat{y})]$; for $s = 1$, it is $h_2 \in [0, D_x(\hat{x}, \hat{y})]$. The following equation summarizes the escapement policy constraints in both cases:

$$\begin{aligned}
 u_1 &\in [0, x] \\
 u_2 &\in \begin{cases} [0, D_y(\hat{x}, \hat{y})], & s = 0 \\ [0, D_x(\hat{x}, \hat{y})], & s = 1 \end{cases} \quad (17)
 \end{aligned}$$

Regarding the price, we first propose the same price function $P_L(h)$ as in Eq. (18) for both seasons (Fiskeridirektoratet, 2015). To go a bit further with sophistications, we argue that the longer period S2 may elicit less intensive harvesting, lower average harvesting rate and limited supply to the market, which in turn increases prices especially when the demand is not necessarily dropping in S2.

Following this argument, we keep the minimum price 10 and raise the maximum price from 18 to 25, roughly doubling the min-max price difference, arriving at a shifted and higher price function $P_H(h)$.

This can reflect the unmet demand when harvesting happens less intensively during S2.

$$P_L(h) = 10 + (18 - 10)e^{-0.002h} \quad (18)$$

$$P_H(h) = 10 + (25 - 10)e^{-0.002h} \quad (19)$$

Stock independent cost function in S1, i.e. $l = 0$, is a critical and interesting scenario. Except this, we are also curious to investigate a moderate level of economic protection, or stock dependency in the cost function, that may arise in the case of a crowded spawning ground. Therefore, we employ $l = 0.5$ as the second option. In order to achieve reasonable results and comparable scales, we calibrate the cost parameters and employ the following numerical expressions for the seasonal economic gains:

$$\Pi_1(h_1) = P_L(h_1)h_1 - 6h_1 \quad (20)$$

$$\Pi_1(h_1, x) = P_L(h_1)h_1 - \frac{255h_1}{x^{0.5}} \quad (21)$$

$$\Pi_2(h_2, \hat{x}, \hat{y}) = P_L(h_2)h_2 - \frac{24000h_2}{D_x(\hat{x}, \hat{y}) + D_y(\hat{x}, \hat{y})} \quad (22)$$

$$\Pi_2(h_2, \hat{x}, \hat{y}) = P_H(h_2)h_2 - \frac{24000h_2}{D_x(\hat{x}, \hat{y}) + D_y(\hat{x}, \hat{y})} \quad (23)$$

We display Π_1 and Π_2 in Fig. 6 with various state values to offer a brief visualization of the economic drive behind the optimization. For most states, the function is monotonically increasing in seasonal harvest, while for some states it is concave e.g. when $x = 2400$ in Fig. 6(b). As the states grow, Eq. (22) and Eq. (23) begin to merge due to drastically decreasing price differences. However, this is not explicitly displayed since high harvest levels are of little relevance in our representation.

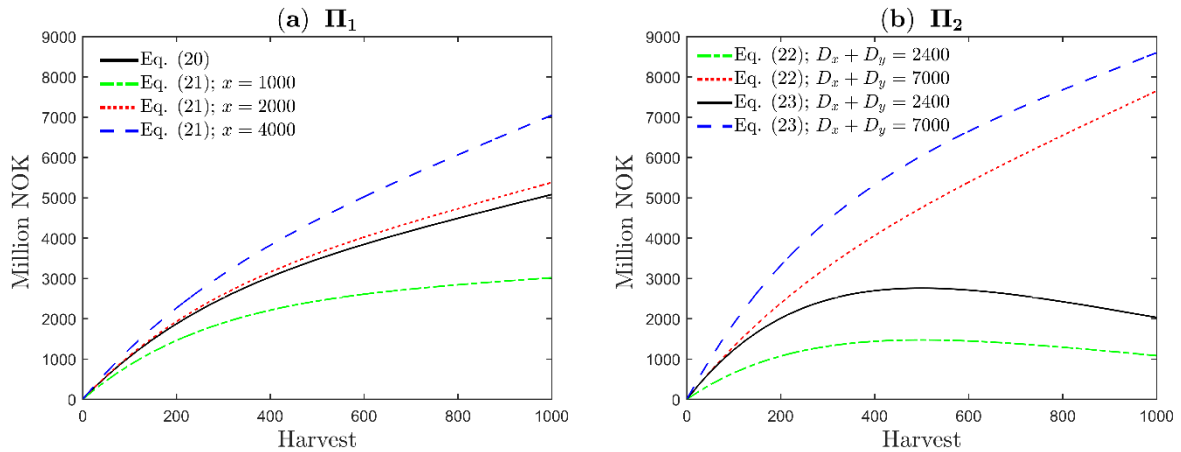


Fig. 6. Seasonal net economic gains: (a) Π_1 in S1; (b) Π_2 in S2.

The graphical comparison of the unit harvest cost is illustrated in Fig. 7. The red surface manifesting the costs in S2 writes as $\frac{24000}{D_x(\hat{x}, \hat{y}) + D_y(\hat{x}, \hat{y})}$. The blue surface representing S1 is 6 when $l = 0$ in Fig. 7(a) and $\frac{255}{x^{0.5}}$ when $l = 0.5$ in Fig. 7(b). In both Fig. 7(a) and (b), the red surface surpasses the blue surface

on roughly the same percentage of the state space area, allowing them to be more suitable for comparison and discussion.

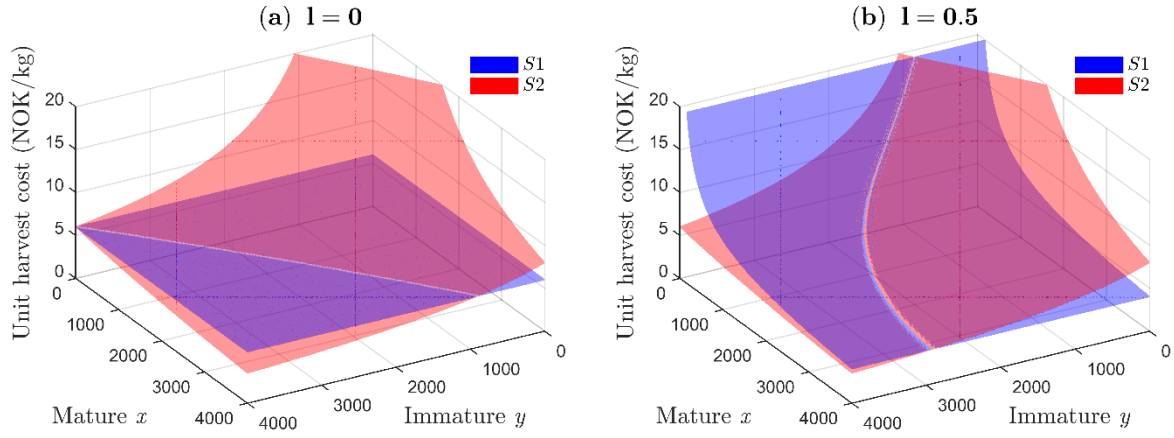


Fig. 7. Unit harvest costs: (a) $l = 0$; (b) $l = 0.5$.

Table 1 displays the overview of eight combinations of parameter values. We present the cases that we believe are necessary, representative and informative. All the cases are independent and can potentially represent specific fisheries around the world. We argue that it is legitimate to compare among them thanks to properly established numerical scales.

Table 1. Parameter values for all cases.

	$l = 0$	$l = 0.5$	$s = 0$	$s = 1$	$P_l(h_1); P_l(h_2)$	$P_l(h_1); P_H(h_2)$	$\Pi_1; \Pi_2$
Case 1	✓		✓		✓		Eq. (20); Eq. (22)
Case 2	✓		✓			✓	Eq. (20); Eq. (23)
Case 3	✓			✓	✓		Eq. (20); Eq. (22)
Case 4	✓			✓		✓	Eq. (20); Eq. (23)
Case 5		✓	✓		✓		Eq. (21); Eq. (22)
Case 6		✓	✓			✓	Eq. (21); Eq. (23)
Case 7		✓		✓	✓		Eq. (21); Eq. (22)
Case 8		✓		✓		✓	Eq. (21); Eq. (23)

4. Results

Table 2 summarizes the most interesting general findings of our results. We elaborate and provide more detailed observation and analysis for each finding.

Table 2. Highlights of general findings.

General Findings	Explanation & Implications
Moratorium in S2 at the equilibrium for some cases.	Equilibrium cycle acts as a target escapement policy. Emerges naturally instead of being enforced.
Some states begin with positive year round harvests and later evolve into harvesting (S1)-closure (S2).	Typically, moratorium is enforced to recover the stock.
Overshooting potentially caused by contraction process that is stronger in one direction.	A rising group biomass could be the overshooting effect instead of a clear signal to increase harvest.
A ‘valley’ effect (declining harvest with rising states) exists for some h_1 .	We find it closely related to the unit profit difference between seasons.

Starting from various initial states, we follow the seasonal optimal harvesting policies and reach the ‘equilibrium’ that consists of two seasonal states in a yearly cycle as in Fig. 8. One novel observation is that the equilibrium h_2 in Cases 1-5 and 7 are all zero³, signifying that the optimal harvesting includes a seasonal closure of the fishery, which is not forced by regulations but naturally emerge instead. For example, in Cases 1-4 it is optimal to harvest 57 kilotons of the mature fish during S1 and leave the whole stock to itself during S2. This cycle acts as a target escapement policy. In the equilibrium cycle of Cases 6 and 8, harvesting takes place all year round but the rate and amount differ between seasons. A typical yearly model may simply overlook such important implications provided by our model with uneven season lengths and multiple harvesting seasons.

Harvest moratorium is not a new topic or finding from other fisheries models (Clark, Clarke, & Munro, 1979; Kasperski & Wieland, 2009; Kennedy, 1992). Nonetheless, it is novel to bring seasonality into such models and reach a moratorium during a part of the annual cycle as a long-term optimal steady situation. This coincides with practices in real fishery: harvesting is often assumed to take place during the whole year but many fleets only harvest seasonally.

Typically, the stock first goes through moratorium to recover the biomass and then lends itself to harvesting when it becomes abundant enough. Our results contradict this pattern and indicate that many initial states begin with positive harvests in both seasons but later evolve and settle down in an annual equilibrium cycle of harvesting (S1)-closure (S2). Visual examples of such initial states in Fig. 8 include the maximum states and the one with the maximum immature group and zero mature stock in Cases 1-5 and 7.

³ A consistency check for the validity of our numerical results is that all three pairs, i.e. Cases 1-3, 2-4, 5-7, with different values of selectivity parameter s end up in the same equilibrium cycle with the same equilibrium values since selectivity in S2 makes no difference when we don’t harvest in S2 at all.

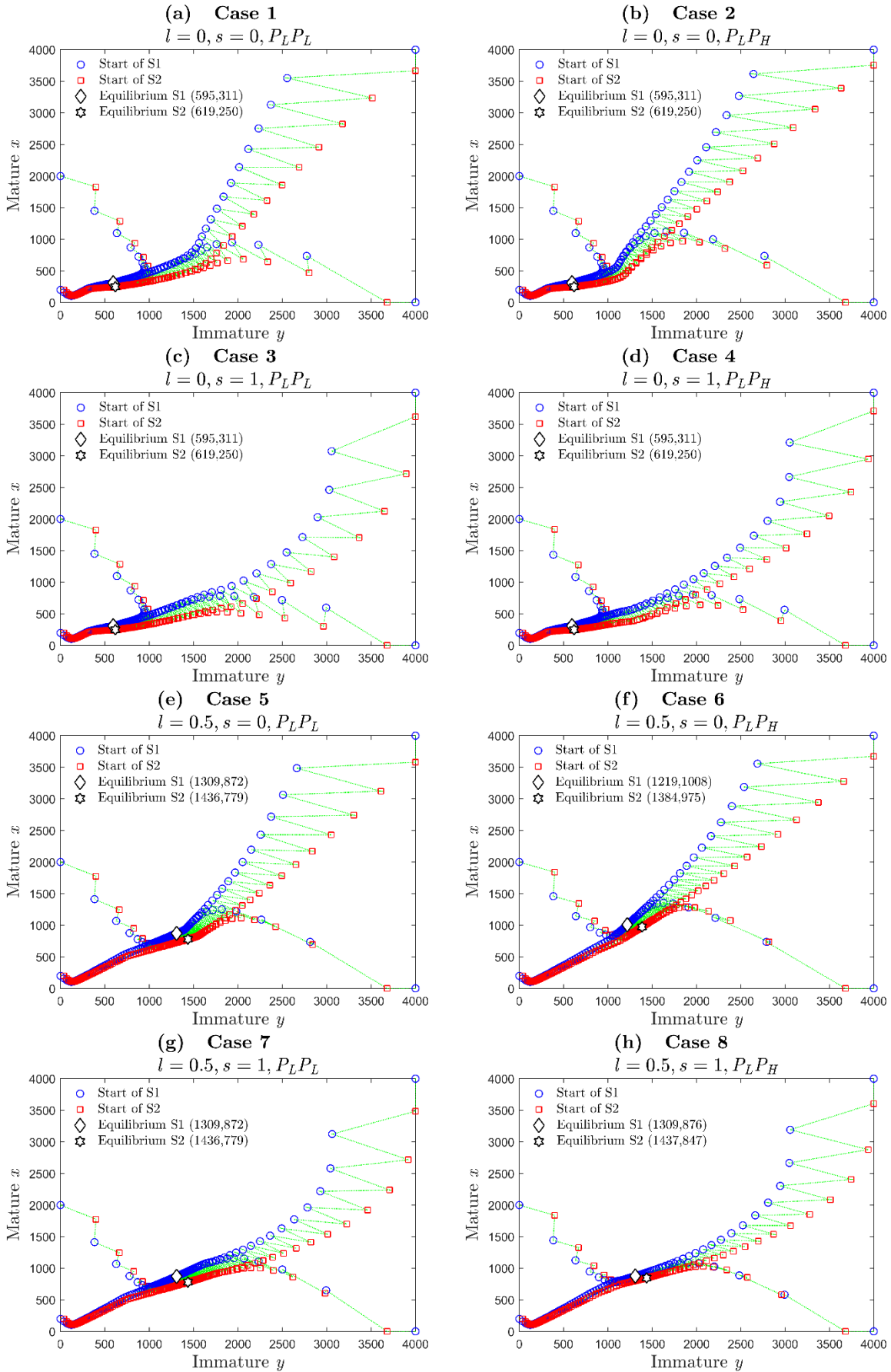


Fig. 8. Dynamic paths with optimal policy in 200 years.

Moreover, even when the stock undergoes moratorium first and harvesting later, our results exhibit diversified paths towards the equilibrium cycle depending on the initial state and the case-specific parameters. In Cases 1-5 and 7 when the initial state is 200 kilotons of the mature, an annual moratorium first applies, and then h_1 becomes positive but S2 remains a moratorium period. In Cases 6 and 8, the same initial state will undergo a similar process but eventually lend itself to harvest in S2 as well. When the initial state enlarges to 2000 kilotons of the mature in Cases 1-5 and 7, S1 is the only harvesting season and this situation maintains for the entire trajectory. Nevertheless, in Cases 6 and 8, S2 will later turn into a harvesting season as well. When the initial state is 4000 kilotons of the immature in Cases 6 and 8, S1 becomes the temporary moratorium period in the earlier years.

In each case, various initial states in Fig. 8 seem to join a certain path converging to the equilibrium. The hallmarks of the trajectories imply that the contraction can be strong in one direction so that the states move quickly towards the path and weak in another so that they progress slowly along the path. Taking Case 1 for example, we find that the contraction is much faster along direction 1⁴ than direction 2⁵ in the neighborhood of the equilibrium states. Though our calculation is valid closely around the equilibrium, the features seem rather global on the state space.

Overshooting in some cases could come as a result of the speed and direction of the contraction process. Given an initial state far from the equilibrium, it takes a long time to approach the steady state. Together with a faster speed along a certain direction, overshooting can emerge. For example, starting with the maximum immature state in Case 1, the mature stock quickly rises above the equilibrium then slowly drops towards the steady state. Therefore, observing an increase of group biomass could potential be the overshooting effect instead of a clear indication to harvest more.

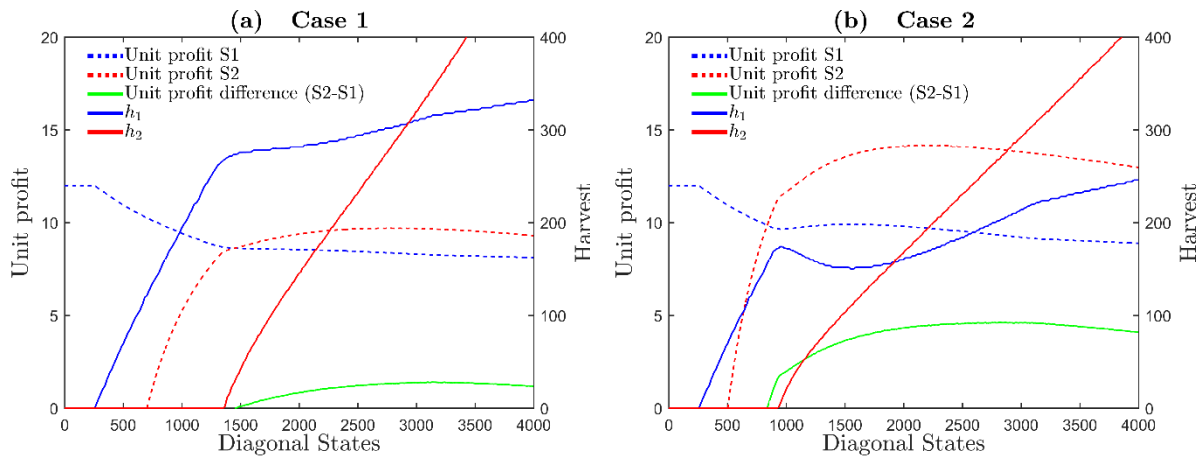


Fig. 9. Unit profit and harvest along the diagonal of the state space.

⁴ The eigenvalue is 0.58 for direction 1 with eigenvector of (-0.82, 0.58). A smaller eigenvalue indicates a stronger and faster contraction.

⁵ The eigenvalue is 0.97 for direction 2 with eigenvector of (0.48, 0.88).

We identify decreasing harvest with increasing states (‘valley’) on some h_1 surfaces in Fig. 10. To investigate closely, we demonstrate seasonal unit profit of harvest and seasonal harvest along the diagonal⁶ of the state space, taking Cases 1 and 2 as an example in Fig. 9. According to the solid blue line in Fig. 9(b), it is optimal to harvest 174 kilotons of the mature on top of the ‘valley’ (diagonal states 960) and only 152 in the bottom (diagonal states 1500) even though both states are more prolific in the latter case.

It is counterintuitive to diminish harvest given stocks that are more abundant. Since Π_1 is monotonically increasing with harvest, the key must lie in the benefits from later periods. When h_2 remains zero, h_1 seems to be rising with the states at a fast rate. When h_2 is positive, the fishing pressure is diverted partly to S2, consequently h_1 increases less drastically. Moreover, when the economic advantage of S2 (solid green line in Fig. 9) is significant such as in Case 2, h_1 not only rises slower with harvest but also decreases in order to exploit higher profits from S2.

In fact, this phenomenon has traces in all cases: the slope of h_1 drops a little ahead of the end of S2 moratorium, preparing for better stock conditions for harvest in S2; when h_2 becomes positive, h_1 either continues increasing at a lower speed or decreases creating the dent depending on the scale of the unit profit difference. The ‘valley’ is an interesting and robust effect as a result of the trade-off, or competition, between harvesting seasons.

Table 3 lists the most important findings that refer to specific parameters.

Table 3. Highlights of parameter-specific findings.

Parameter-specific Findings	Explanation & Implications
$l = 0$ can originate from S1 but work on both seasons in the same direction (elicit heavier harvesting) when x is small.	One consequence is that the equilibrium states are smaller for Cases with $l = 0$.
h_2 occupies bigger share in the annual harvest when the unit harvest cost is lower in S2.	Difference of the unit harvest cost partially determines the state space area where one seasonal harvest dominates the other.
Fishing pressure on x in S2 arouses even more aggressive harvesting on x in S1.	It is more efficient to gain biomass by having a large amount of y rather than x in the beginning of S1.
x is pressed to lower levels along its optimal path towards the equilibrium when $s = 1$.	With $s = 1$, y is spared and can fully utilize the growth, thus x can be harvested to a lower level.
$s = 1$ in S2 results in a lower value of the resource in the long term.	Higher efficiency of transferring x to y via spawning. When determining s , we should avoid too much pressure on x in S2 so that they reproduce better in S1.
When Π_2 is higher, the maximum state moves towards the equilibrium at a slower pace.	The stock suffers more natural biomass loss when staying in the abundant states region. With higher Π_2 , it becomes more affordable to remain there.

⁶ For example, diagonal states of 1000 means that both mature and immature states are 1000 kilotons.

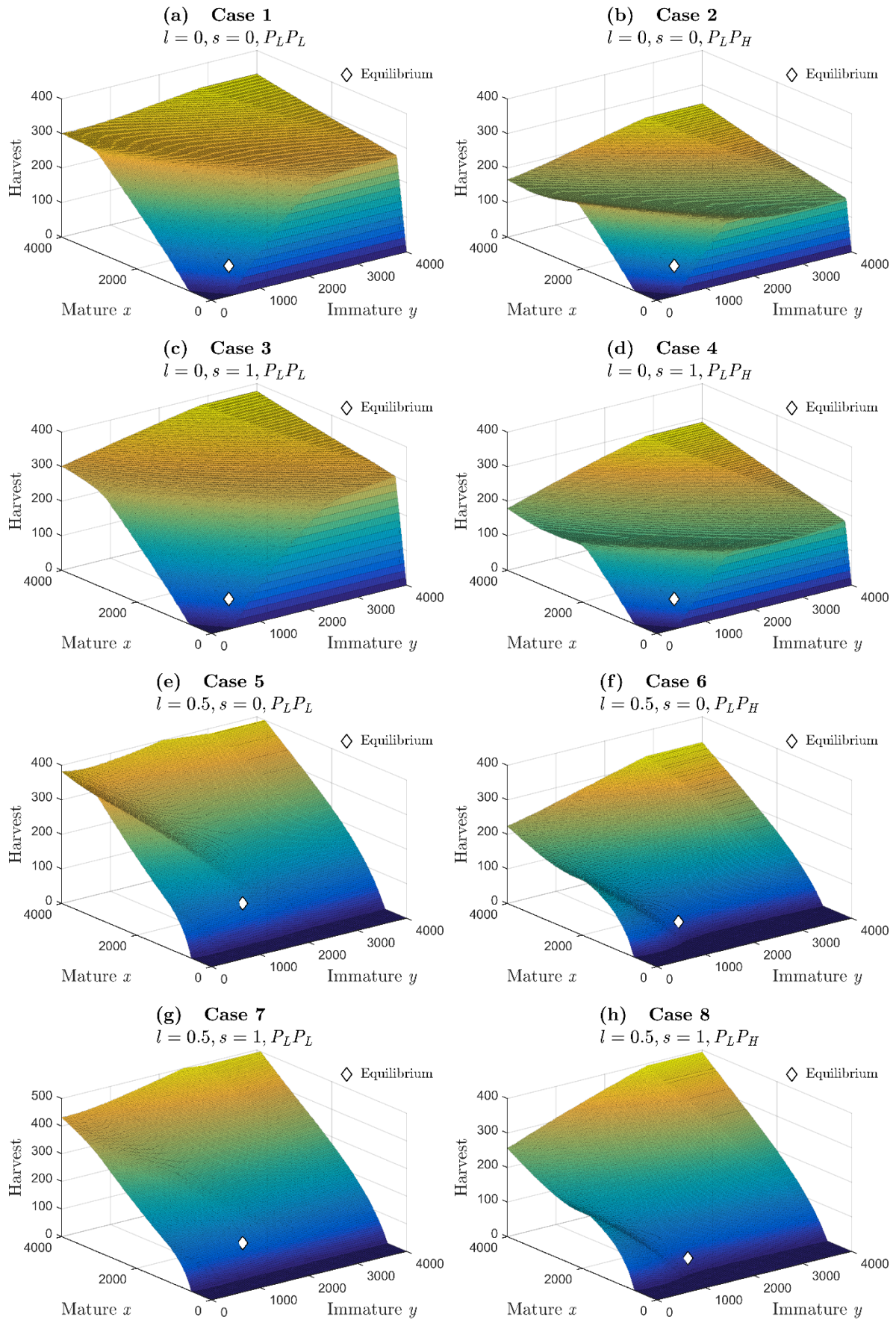


Fig. 10. Optimal harvest policy h_1 in S1.

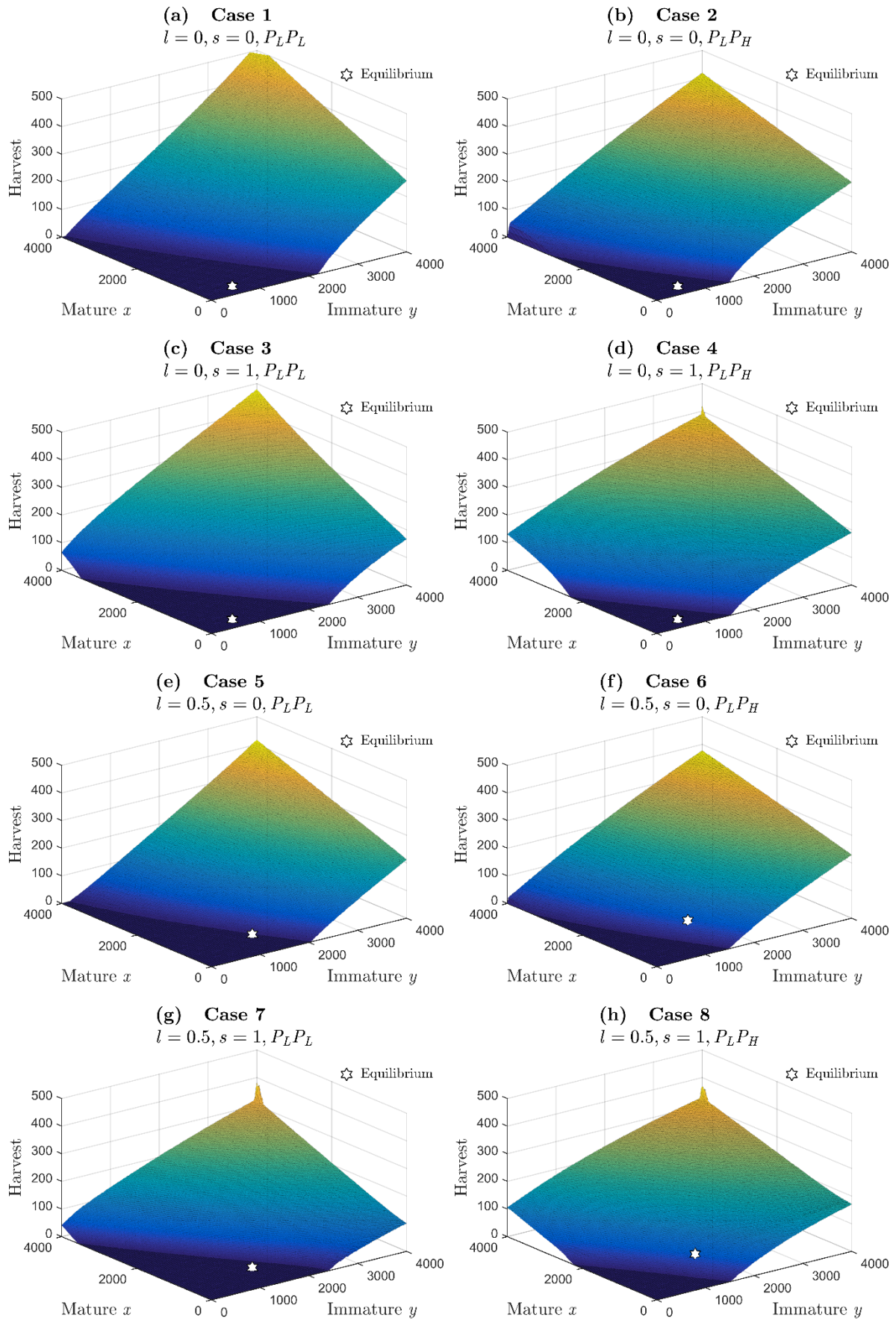


Fig. 11. Optimal harvest policy h_2 in S2.

Focusing on the distinctions triggered by the stock-cost parameter l , we compare Cases 1-4 where $l = 0$ with Cases 5-8 where $l = 0.5$. Looking at h_1 in Fig. 10, one of the most obvious disparities is the natural moratorium: in Cases 1-4, the moratorium zone covers roughly from zero to 500 kilotons for both states; in Cases 5-8, it spreads along the immature axis occupying much bigger state space area. These traits go in line with the model assumptions and parameter values. When $l = 0.5$, harvests are limited due to extremely high costs when the mature group is weak despite the state of the immature. Without this protective mechanism i.e. $l = 0$, the natural moratorium in S1 exists for much lesser states. Once the states fall out of this puny area, h_1 becomes positive.

Referring to Eq. (21) and the blue surfaces in Fig. 7, the unit harvesting cost in S1 with $l = 0.5$ escalates over the one with $l = 0$ when the mature group is smaller than 1806 kilotons. This gives rise to some features exemplified in Fig. 12(a): with smaller mature states, $l = 0$ (Case1) evokes relatively heavier harvesting; at higher mature stock levels, $l = 0.5$ (Case5) generates bigger harvests instead. The former feature is reflected in the optimal path of an initial state where the mature stock is zero and the immature stock is at its maximum (see Fig. 8): the trajectories reveal salient drops of the mature stock level after S1 along the path when $l = 0$ (Cases 1-4) while the counterparts (Cases 5-8) are much smoother. In addition, we observe that some part of the optimal h_1 surface when $l = 0$ even reaches the myopic solution, in which merely the present season's net revenue is valued. In the myopic part, it is economically justified to wipe out the mature group in S1 when the immature is abundant because the immature is protected against harvesting in S1 and a considerable part of them will transit to join the mature in the next season S2, maintaining the whole stock biomass level.

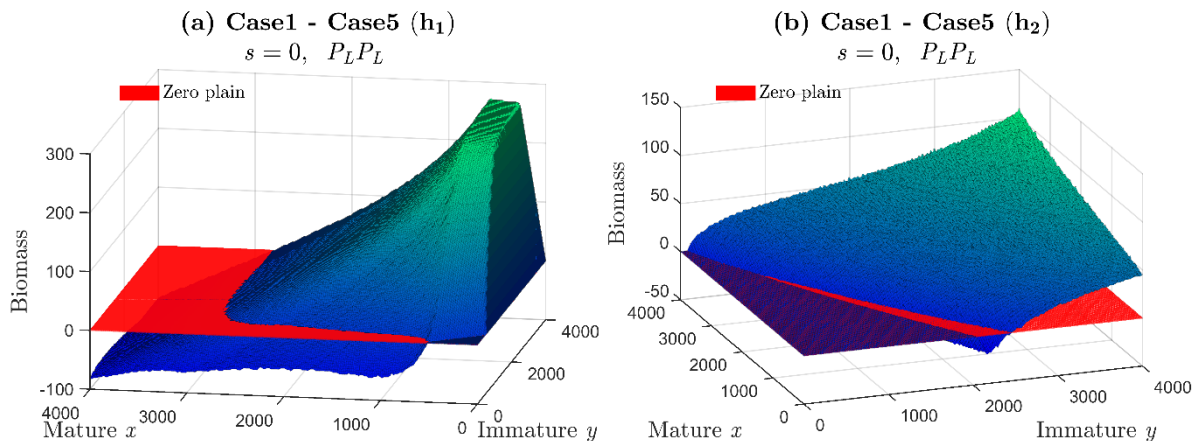


Fig. 12. Difference of h_1 and h_2 between Case 1 and Case 5: (a) h_1 ; (b) h_2 .

The results of h_2 in Fig. 11 evince that the cost in S1 effectively influences the optimal harvest in S2, presumably through biological transitions. Altering the value of the stock-cost parameter l mainly shifts the h_2 surface, not its general shape. Such a shift typically lends itself to more aggressive h_2 when $l = 0$. The natural moratorium tends to remain its shape and size, except for minor shift on the edge in some cases. For instance, the sharp red area in the middle of the zero plain in Fig. 12(b) implies that the moratorium zone on the state space in Case 1 is slightly larger.

In addition to Case 1 and Case 5, the other three case pairs demonstrate similar attributes as in Fig. 12. It seems that the state-independent harvesting cost induced by $l = 0$ in S1 can originate from one season but work on both seasons in the same direction, i.e. to elicit heavier harvesting, when the mature group is below a certain level. This level, which is about 3000 in Fig. 12, can signify a sizable state space area, which includes the equilibrium cycle. Therefore, with more aggressive harvesting throughout the year, equilibrium cycle states are remarkably smaller for Cases 1-4 with $l = 0$.

Having examined the seasonal harvests separately, we probe into the tradeoff between harvesting in S1 and in S2. Fig. 13 manifests the proportion of h_1 to the total annual harvest if we follow the optimal policy during the year. The gray surface depicts this proportion with each state on the stock space as the starting state of S1. The blue part at the bottom displays the natural moratorium zone where it is optimal to leave the stock to itself throughout the year. The red plain highlights the area where the proportion is smaller than 50%, which indicates that h_2 is more dominant in the annual harvest. We find that the red plains in Fig. 13 tend to appear at locations on the state space that correspond to the area where the red surface (unit harvest cost in S2) lies beneath the blue surface (unit harvest cost in S1) in Fig. 7. In Cases 1-4 where $l = 0$, h_2 dominates when both states are ample while in Cases 5-8 where $l = 0.5$, h_2 is more weighted when the immature group is abundant and the mature state is smaller. We learn that the difference in the unit harvest cost determines, to some extent, the hallmarks of the state space area where harvest in one season dominates the other.

We proceed to investigate how selectivity parameter s alters the results. The key divergences of h_2 between cases, where s is the only varying parameter, lie along the two zero axes as shown in Fig. 11. On one hand, when the mature group is zero i.e. $\hat{x} = 0$, what left available for harvesting in S2 is $D_x(0, \hat{y}) = a\hat{y}$ of the mature and $D_y(\hat{x}, \hat{y}) = G(\hat{y}) - a\hat{y}$ of the immature. Referring to Fig. 5(b), we deduce that there is less harvesting potential when we select the mature group D_x with $s = 1$. Consequently, h_2 along this axis exhibits stronger conservation when $s = 1$. This decrease in h_2 is more visible when seasonal price functions are identical because otherwise the increment of economic reward in S2 brings up h_2 and alleviates this decrease effect. On the other hand, when the immature group is zero i.e. $\hat{y} = 0$, what left available to take is $D_x(\hat{x}, 0) = m\hat{x}$ of the mature and $D_y(\hat{x}, 0) = 0$ of the immature. Hence, for Cases 1,2,5 and 6 when $s = 0$, h_2 along this axis is all zero while for Cases 3,4,7 and 8 when $s = 1$, h_2 becomes positive as the mature group prospers.

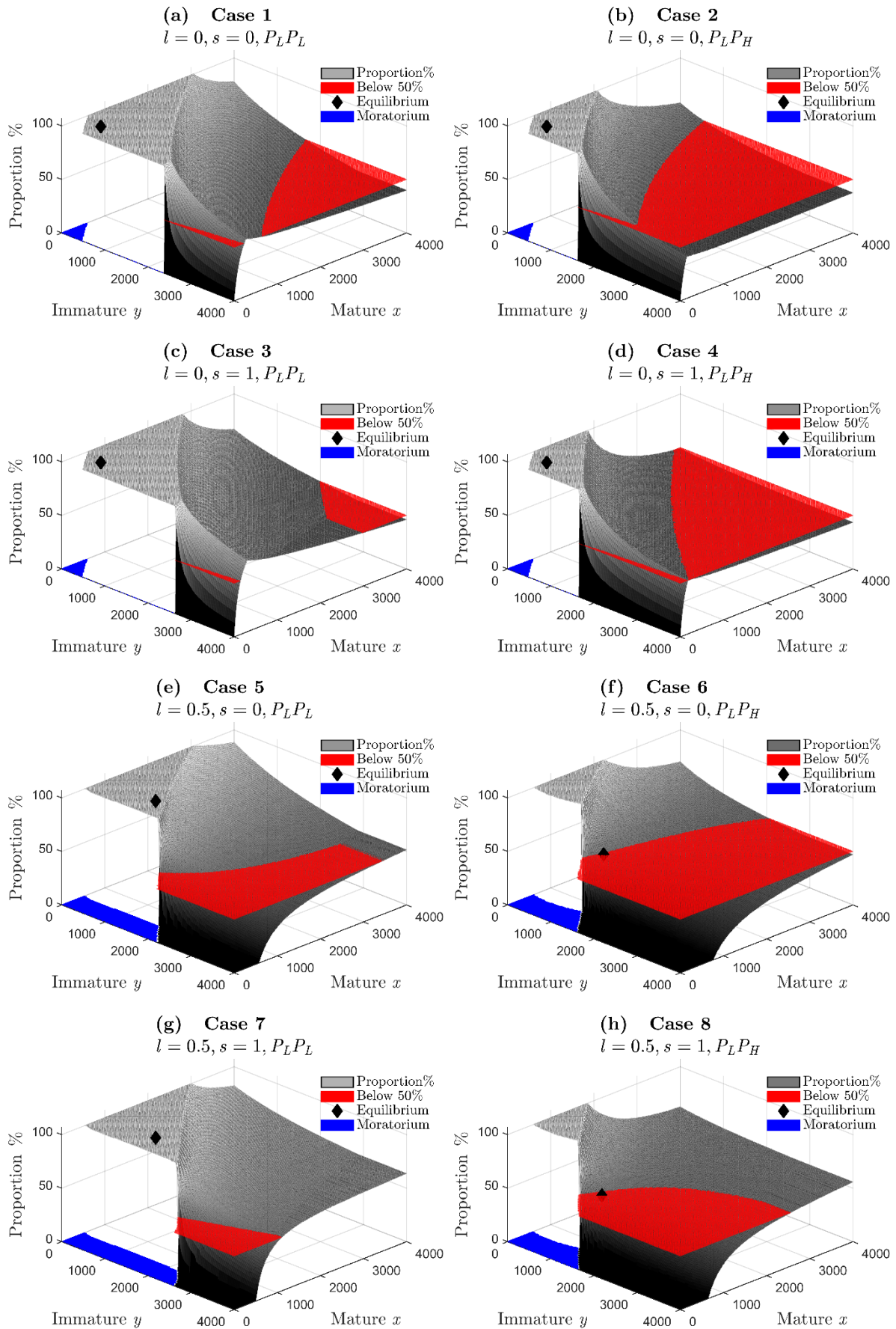


Fig. 13. Proportion of h_1 to annual harvest.

Selecting only the mature in S2 ($s = 1$) enhances h_1 on most of the stock space for all four case pairs. In addition, it shrinks the area represented by the red plain in Fig. 13, which means that h_1 occupies a larger share of the annual harvest on more state space area when $s = 1$. It may seem counterintuitive that the fishing pressure on the mature in S2 arouses even more aggressive harvesting in S1 on the same group.

This thought provoking phenomenon may be explained from the biological perspective: the strongest growth of the stock is the stepwise linear transition of the immature during S1, which is dependent on both states. The marginal biomass increment incurred by one more state unit is $(0.00005y + 0.2)$ of the mature and $(0.00005x + 0.92)$ of the immature. Apparently, it is more efficient to gain biomass by having a copious amount of the immature rather than the mature in the beginning of S1. When $s = 0$, we are forced to diminish the immature in the beginning of S1, hence it is crucial to retain the mature at a higher level in order to exploit the growth. When $s = 1$, the immature group is free from being harvested and can fully utilize the growth, so the mature group can lend itself to harvesting towards a lower level. One evidence is the optimal path of the maximum states as the initial point in Fig. 8: Cases 3,4,7 and 8 ($s = 1$) tend to press the mature group to lower levels along its path towards the equilibrium cycle.

The surfaces of both value functions are almost identical across cases. Therefore, we calculate the average value on the state space as an indicator for its general level. As illustrated in Table 4, both V_1 and V_2 are lower on average when $s = 1$, suggesting that the compulsory selection targeting the mature group in S2 results in a lower value of the resource in the long term. This may relate to the high efficiency of transferring mature biomass to the immature through spawning and the relatively low efficiency of transferring immature to the mature via natural maturation. Hence, when deciding on the selectivity s , it can be worthy to avoid putting all the fishing pressure on the mature group in S2 and to allow it to produce more offspring in the spawning season S1. From the standpoint of policymaking, it seems that a complete ban on harvesting a certain part of a stock may stem from conservative purposes but can also hinder the value and potential of the whole stock.

Additionally, this level drop of the value functions turns out to be particularly evident when price is P_H in S2 and when $l = 0.5$ in S1. First, when price is higher in S2, setting $s = 1$ implies a better reward for catching the mature while having $s = 0$ signifies a greater return on taking the immature.

Considering that sparing the mature during S2 can be beneficial according to the previous arguments on biomass transferring efficiency, diminishing the mature ($s = 1$) surely incurs a bigger value loss. Second, when the cost in S1 is dependent on the mature group, it is better to avoid landing at a low mature state in the beginning of S1. Consequently, with $s = 1$ we are constrained to lessen the mature group during S2, which in turn weakens the mature group in the beginning of S1 and thus elicits a wider gap of the values.

Table 4. Differences of the average V_1 and V_2 between cases with $s = 0$ and $s = 1$.

Differences of the average values	$P_L(h_1); P_L(h_2)$	$P_L(h_1); P_H(h_2)$
$l = 0$	Diff V_1 (Cases 1,3) = 722	Diff V_1 (Cases 2,4) = 1274
	Diff V_2 (Cases 1,3) = 559	Diff V_2 (Cases 2,4) = 972
	Diff V_1 (Cases 5,7) = 1403	Diff V_1 (Cases 6,8) = 2578
$l = 0.5$	Diff V_2 (Cases 5,7) = 1149	Diff V_2 (Cases 6,8) = 2187

Lastly, we look into the two price options i.e. Eq. (22) and Eq. (23) in S2. The relevant Π_2 with harvests less than 1000 kilotons are significantly higher for Cases 2,4,6 and 8 than Cases 1,3,5 and 7 as shown in Fig. 6(b). As exhibited in Fig. 10, there is a clear shift of h_1 towards lower harvest levels for all case pairs when Π_1 becomes comparatively less attractive. In addition, it is not surprising to observe evident declines of the proportion surfaces with enlarged Π_2 in Fig. 13, which denotes that as the profit in S2 improves the optimal policy tends to postpone and harvest more in S2.

However, this simple economic incentive explanation, i.e. take more when the return is better, is not valid for h_2 on the entire state space. A higher Π_2 seems to shrink the moratorium zone of h_2 , contributing to the positive part of the surfaces in Fig. 14, but propose a much more cautious harvesting policy in S2 as the states escalate, producing the negative part beneath the red zero plain when both stocks are plentiful. Consequently, in the abundant region on the state space, both seasonal harvests are smaller for cases with higher Π_2 . This is in line with the observations from Fig. 8: taking Case 1 and 2 as an example, starting from the maximum point in the state space, it takes only six years in Case 1 but eight years in Case 2 for both stocks to halve. The trajectories hint that when the economic return is higher in S2, the stocks spend longer time in the abundant area and move towards the equilibrium cycle at a slower pace.

This phenomenon can be motivated by the biological transition for the immature group in S2: on one hand, a greater mature stock diminishes the immature biomass more severely due to cannibalism as in Eq. (7), though on a trivial scale; on the other hand, the nonlinear curves, manifested in Fig. 5(b), elicit bigger biomass drops at larger immature states. These traits suggest that the stock suffers heavier biomass losses due to natural reasons when staying in the plentiful region on the stock space. When the profit increases in S2, it becomes more affordable to remain in the prolific region and enjoy benefits brought by ample biomasses, which leads to a comparatively prolonged process towards the equilibrium cycle.

When $s = 0$, harvesting in S2 directly brings down the immature stock, alleviating the natural biomass loss next year. When $s = 1$, reducing the mature stock merely affects the immature indirectly through spawning. Since the loss alleviation is stronger and more direct with $s = 0$, we speculate that the prolongation derived from the affordability to stay in the abundant state area with an increased Π_2 is

also more prominent when $s = 0$, thanks to the less urgent biomass losses. One confirmatory evidence lies in the prolongation effect of the maximum initial state. It takes longer time (ten years) in Case 4 than in Case 3 (nine years) for both states to halve. The time difference that stands for the prolongation is one year when $s = 1$ (between Case 3 and Case 4) but two years when $s = 0$ (between Case 1 and Case 2). Since the level of negativity in the abundant region in Fig. 14 is associated with the scale of the prolongation, another evidence is that Fig. 14(a) and (c) demonstrate more evident decline of the h_2 difference in the plentiful region than that in Fig. 14(b) and (d) respectively.

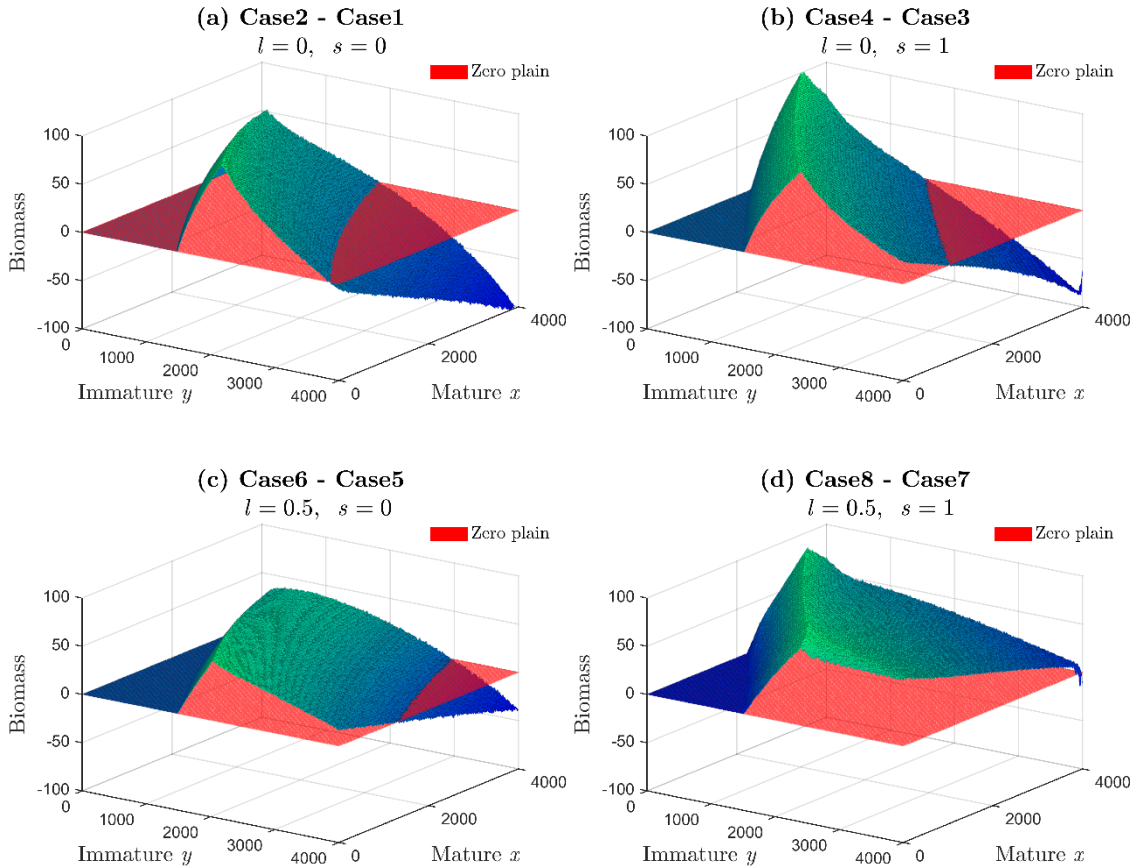


Fig. 14. Difference of h_2 between cases with $P_H(h_2)$ and $P_L(h_2)$.

Referring to Fig. 6(a), $l = 0.5$ leads to higher Π_1 than $l = 0$ when the mature stock is plentiful. Thus in the abundant region, the incremental affordability brought by a higher Π_2 seems less significant when Π_1 is already larger. Accordingly, the prolongation effect is supposed to be weaker when $l = 0.5$: the time difference is one year between Case 5 and Case 6 with $l = 0.5$, which is shorter than two years between Case 1 and Case 2 with $l = 0$. In addition, Fig. 14(c) and (d) display a lesser decline of h_2 difference in the plentiful area than that in Fig. 14(a) and (b) respectively.

5. Conclusions and discussions

As an initial attempt, we employ DP technique in a periodic Bellman approach to present a seasonal model numerically. We obtain the optimal feedback harvesting policies given multiple uneven seasons and two-dimensional states. Our method provides a useful tool for seasonal regulation measures, such as periodic moratorium, locational closure, seasonal TACs, fleet-specific TACs or seasonal target escapement and the like. The tradeoff of harvesting between seasons is driven by multifold mechanisms, which leads to profound implications for bioeconomic modeling and policymaking.

Key findings are listed in Table 2 and 3 and the most intriguing ones are rather counterintuitive. We find it sometimes optimal to adopt a seasonal closure of the fishery in the equilibrium cycle, which would hardly be discovered from an annual model. Unlike a typical policy that imposes a moratorium to recover the stock, we find that many states first undergo harvesting all year round and then develop into the seasonal moratorium. A growing group biomass could be the overshooting effect instead of a clear signal to increase harvest. In some cases on part of the state space, the optimal seasonal harvest declines with rising states creating a ‘valley’, which may originate from the unit profit difference between seasons. Fishing pressure on the mature in one season leads to even heavier harvesting on the same group in the next season. It seems that sparing the immature may derive from conservative purposes but in fact diminish the value of the whole stock.

Our approach can potentially adapt for various biological and economic seasonality in fishery. Some forage fish, such as herring, make vast migrations between spawning, feeding and nursery grounds. In this case, the season number and season lengths can be adjusted according to behaviors that are shaped by ocean currents, plankton abundance, tradeoff between predator avoidance and growth (Brönmark, Skov, Brodersen, Nilsson, & Hansson, 2008). Although the curse of dimensionality is inevitable, the multidimensional states may instead represent predator-prey multispecies relationships, such as cod and capelin in the Barents Sea. This allows for more focus on ecological analysis and food web studies, which are gaining more attention as a research topic. For single species models, the biological dynamics could include depensation so that the model becomes more realistic for species that are particularly vulnerable at lower states. The decision variables, i.e. harvesting policies, may be extended to season-state-specific instead of simplified to season-specific as in our presentation.

Possible modeling extensions exist in various directions. It is interesting to employ size(group)-dependent price functions (Zimmermann, Steinshamn, & Heino, 2011). The model can be in continuous time and also apply the periodic Bellman approach to investigate seasonality. Stochasticity can be added to the biological transitions via different fluctuations in such as the ocean temperature, food availability, cannibalistic behavior, predator distribution etc. Randomness may also be incorporated in the price and cost functions. We are currently working on establishing stochastic models with seasonality.

Furthermore, the big potentials and rich implications of our approach goes beyond the model we present in this work. This approach enables us to take into consideration longer seasonality that spans more than a single year. For example, the El Nino conditions impact the ocean temperature drastically at intervals spanning several years, affecting numerous ocean lives as well as commodity prices in different countries. In addition, the flexibility to employ uneven season lengths makes it possible to combine more than a single seasonal pattern in the model: for instance, we can model El Nino as a condition that happens once every several years together with a seasonal feature, such as upwelling in the spring, which takes place annually. Except for natural phenomenon, such combined seasonal patterns may potentially be extended to market price, technological breakthrough, economic trend and countless other aspects in natural resource modeling.

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