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Relational Incentive Contracts and Performance Measurement

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Abstract

This paper analyzes relational contracts under moral hazard. We first show that if the available information (signal) about effort satisfies a *generalized* monotone likelihood ratio property, then irrespective of whether the first-order approach (FOA) is valid or not, the optimal bonus scheme takes a simple form. The scheme rewards the agent a fixed bonus if his performance index exceeds a threshold, like the FOA contract of Levin (2003), but the threshold can be set differently. We next derive a sufficient and necessary condition for non-verifiable information to improve a relational contract. Our new informativeness criterion sheds light on the nature of an ideal performance measure in relational contracting.

KEYWORDS: Relational contracts, non-verifiable performance measures, first-order approach, bonus scheme, informativeness criterions

1. Introduction

In many organizations, managerial incentives are frequently implicit. Recent empirical studies report that firms have since the 1990s increasingly been adopting a practice of using non-financial measures such as customer satisfaction scores, leadership, or other subjective evaluations, to assess and pay for managerial performance.[1] Although being relatively easier to obtain than objective indicators, such non-verifiable measures cannot be used in incentive contracts enforced by external parties. Nevertheless, if contracting parties repeatedly transact over time, a wide array of contracts can be self-enforced by the value of an ongoing relationship. Such relational contracts between firms were observed several decades ago by legal scholars (Macaulay (1963)), and

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[1]For instance, Murphy and Oyer (2003) and Gillan, Hartzell and Parrino (2009) found that more than one-half of their sample firms base employees' annual bonus at least in part on non-financial measures of individual performance.

have since been extensively analyzed and applied in economics and other areas.[2] However, the related literature has mainly focused on the problem of designing an optimal contract with non-verifiable information (i.e., how to pay) but paid little attention to the problem of choosing an ideal performance measure among many alternatives (how to evaluate), although both aspects – an appropriate performance measure and well-designed incentive contract – are key ingredients to successful long-term relations.

In this paper, we analyze both of these aspects. We consider an infinitely repeated principal-agent relationship where the parties are risk-neutral and the agent provides hidden effort that is valuable to the principal. We assume that all available performance measures (or signals), including the principal's objective, are imperfect and non-verifiable, but observable to the contracting parties.[3] We formulate this agency problem as a two-part mechanism, where the principal chooses a performance evaluation system at the outset and then designs an incentive contract based on the system. By virtue of Levin (2003), our analysis of optimal contracts focuses on the stationary contract where the principal offers a time-invariant base salary and discretionary bonus every period. Within this contracting environment, we provide a novel criterion for one measurement system to be more informative than another in the spirit of Holmström (1979).

The main contribution of this paper is thus two-fold. First, we extend the characterization of optimal relational bonus schemes to a wider class of multivariate measurement systems than those for which the standard first-order apptoach (FOA) can be applied, and show that the simple structure of these schemes prevails in this wider class. To be precise, we prove that as long as the measurement system satisfies a generalized version of the monotone likelihood ratio property (MLRP), the optimal bonus scheme retains a simple hurdle structure, as in the optimal FOA contract characterized by Levin (2003). The agent is then awarded a bonus if his performance, measured by an index given by the signal's likelihood ratio, surpasses a threshold.[4] This characterization is of interest in its own right, but also allows us to obtain a more robust informativeness criterion by relaxing several of the conditions on the available measurement system and contracting environment that must be imposed to validate FOA.

To illustrate our first main result, suppose the performance measure is a univariate non-verifiable output which is affected additively by effort and noise, and suppose for concreteness that the noise is normally distributed. Under this specification, it is natural to think that as the standard deviation (σ) of the noise gets lower, the principal would be able to alleviate the agency cost and hence elicit higher effort. In fact, under the assumption that FOA is valid, a straightfor-

[2]Seminal contributions include Klein and Leffler (1981), Bull (1987), MacLeod and Malcomson (1989) and Levin (2003). See also Malcomson (2012) for a review.

[3]Much non-verifiable information in practice may not be observable by the agent, in particular when information is gathered by the principal's subjective appraisal. A standard example of non-verifiable but observable measures in organizations is a performance evaluation by other human resource divisions or customer's satisfaction scores. Hence we abstract away interesting problems of subjective measures such as leniency bias (MacLeod (2003)), favoritism (Prendergast and Topel (1996)), or influence activities (Milgrom (1988)).

[4]That is, our aim is not to provide a condition that ensures validity of FOA in the stationary environment of relational contracting, but to provide a condition under which the optimal bonus scheme has a simple hurdle structure as the FOA contract. A recent paper by Hwang (2016) establishes a condition in the same environment as ours under which FOA is justified.

ward comparative static analysis of the optimal contract in Levin (2003) confirms this presumption. However, this local approach is not applicable when σ is sufficiently small: the optimal effort identified by FOA is then a stationary point of the agent's expected utility, but does not maximize his utility.[5] As a result, the agent would deviate to lower effort, and the FOA bonus scheme would not implement the desired effort. A characterization of the optimal contract has been lacking for this case, and a ranking of measurement systems based on FOA has therefore been incomplete even in this most natural and simple setting. Even if one system is a garbling of another in the sense of Blackwell (1951, 1953), the existing approach cannot tell which one is more informative in relational contracting.

We fill in this gap by providing an alternative approach for characterizing the optimal bonus scheme. Our approach does not call for the so-called Mirrlees-Rogerson conditions on the measurement system and can thus be applied to a large class of signals, even multivariate ones.[6] In Section 3, we provide a sufficient condition—the generalized MLRP—under which our approach is justified. As its name suggests, the condition is more general than MLRP, and thus implies that our approach can be applied to the normally distributed noisy signal in the example above, irrespective of its standard deviation. We then show that as long as the measurement system satisfies this condition, the optimal bonus scheme takes a hurdle form for the likelihood ratio: the agent is awarded a bonus if the likelihood ratio clears a hurdle. In contrast with the FOA optimal contract, this hurdle is no longer necessarily set at zero.

To understand why a non-zero hurdle arises, it is instructive to see why FOA does not solve the optimal contract problem in the example above. In relational contracts with two risk-neutral parties, the optimal contract is designed so as to provide the agent with the strongest incentive for effort. When only the local incentive compatibility condition is relevant, a simple way to provide the strongest incentive is to maximize the agent's marginal gain from effort, given the constrained monetary incentives in a relational contract. Since the sign of marginal incentives is determined by the sign of the likelihood ratio, the FOA contract pays a maximal bonus for all outcomes where the likelihood ratio exceeds the value of zero. Given this hurdle-form contract, as the performance measure becomes more precise about the hidden effort, marginal incentives are strengthened in the aspect that by exerting additional effort, the agent can considerably increase the probability of clearing the hurdle. However, this local approach concerns only the marginal incentives in the neighborhood of the target effort and overlooks the incentives at low effort distant from the target, where extra effort has little impact on the agent's payoff, thereby undermining incentives to work. Overall, the impact of a decrease in σ on the agent's total payoff is therefore ambiguous. If the total gain from exerting the target effort cannot cover the corresponding cost, then the agent would respond by choosing a minimal level of effort and thus the FOA contract cannot implement the target effort.[7]

[5] Kvaløy and Olsen (2014) pointed out that FOA is valid only if the output shock is sufficiently diffuse in this specific setting.

[6] In the static environment of contracting with multivariate verifiable measures, Conlon (2009) and Jung and Kim (2015) derive conditions under which FOA is justified. See also Kirkegaard (2017).

[7] A similar discussion can be found in the tournament literature stemming from Lazear and Rosen (1981), where

Our discussion demonstrates that when FOA is invalid, the optimal hurdle reflects a trade-off between providing on the one hand strong incentives for effort on the margin (locally) and preventing on the other hand deviations to distinctly lower effort. Depending on the agent's inclination to deviate from the optimal effort, the hurdle is adjusted in the optimal bonus scheme. In Section 3, we revisit the example above and illustrate how the trade-off affects the optimal hurdle. It turns out that when σ is sufficiently small, the optimal bonus scheme features a negative hurdle, put another way, a more lenient threshold than the FOA contract.[8] Furthermore, we show that the optimal contract, equipped with an adjusted hurdle, implements higher effort as σ decreases. As a result, our approach provides not only a full characterization of optimal contracts, but a complete (and intuitive) ranking of available measurement systems in the example.

We use our characterization of optimal bonus schemes to derive our second main result, where we examine the principal's problem of choosing a performance measurement system: Between two (multivariate) measurement systems, which one does always lead to a higher surplus in the optimal relational contract and thus a more successful relationship for the parties? With the simple hurdle structure of the optimal contract and its applicability to a broad class of signals satisfying the generalized MLRP, we establish a *robust* criterion for a more informative system. That is, our criterion can be applied to determine a binary ranking of non-verifiable signals for a wide class of relational contracting environments.

The previous example suggests that, like objective measures in explicit contracts, non-verifiable signals about hidden effort in relational contracts can be ranked by a standard statistical order. It is intuitive that an improvement of the measurement system in the sense of Blackwell garbling alleviates the agency cost and results in a more efficient contract. However, there is a notable difference between the two types of contracts. While the agency costs arise from moral hazard in explicit contracts with risk-averse agents, the costs arise from the constrained monetary incentives due to the enforcement problem in relational contracts, as we know that without such constraints the first-best is implementable in risk-neutral environments. For this reason, it is unclear whether the standard results on information structures, for example the sufficient statistic theorem in Holmström (1979), can be applied to rank non-verifiable signals.

In Section 4 we present a new criterion, the *likelihood ratio order*, which delivers a tight condition for one signal to be more informative than another in relational contracts. Our criterion rests on the distribution of the signal's likelihood ratio, which follows naturally from the fact that this ratio plays a key role as a performance index in the optimal contract. As the ratio is a unidimensional information variable, the criterion provides a unified treatment for a comparison of multivariate (noninclusive) signals, as long as the signals satisfy the generalized MLRP. Simply put, the likelihood ratio order compares the variability of likelihood ratios. If one signal's likelihood ratio is more variable with regard to the agent's choice of effort than another, then it contains more

unless the shock to individual output is sufficiently diffuse, the objective function of each agent is not globally concave so that FOA is invalid.

[8] We also provide a sufficient condition under which the optimal relational contract has a nonpositive hurdle in our general model.

information about his potential deviations so that the principal can more effectively control the hidden effort by designing a bonus plan based on that signal. Conversely, the criterion is also necessary for the principal to induce higher effort from the agent.[9] Consequently, our result provides a complete characterization of informativeness for a class of relational contracting problems.

To utilize our result in applications, it would be useful if there is a simple way to check for the likelihood ratio order. We find that our criterion of ranking signals is closely related to the notion of precision introduced by Lehmann (1988). Compared to the notion of Blackwell garbling (or sufficiency), Lehmann's criterion is not just easier to check, but also provides a link to the existing signal orders developed in the standard agency problems. The link sheds light on how ideal performance measures differ between explicit and relational contracts.

Related Literature

This paper is related to two strands of literature in contract theory, in that it develops an alternative approach for the optimal design of incentive contracts, and provides a new criterion for an ideal performance measure in relational contracting environments.

Our first main result on optimal bonus schemes in relational contracting complements the seminal work by Levin (2003), which characterizes an optimal incentive contract in the environment where FOA is valid and the univariate performance measure is exogenously given by the principal's objective (output). A recent paper by Hwang (2016) allows the principal to use alternative multivariate measures and establishes a sufficient condition on the signal's distribution and the agent's cost function under which the agent's expected payoff is globally concave and thus FOA is justified. Our approach is different from his in the aspect that instead of providing conditions that justify FOA, we seek conditions that ensure the optimal bonus scheme to take a simple form.[10] In the same spirit as this paper, Poblete and Spulber (2012) analyzed a static model of financial contracting between two risk-neutral parties but with two-sided limited liability, and provided a condition under which debt-style contracts are optimal regardless of the validity of FOA. As has been pointed out by Levin (2003), self-enforcement imposes a lower and upper bound on monetary incentives, much like limited liability does. In Appendix B we further discuss and compare the analysis in Poblete and Spulber (2012) with ours.

Our second result on performance measurement extends a line of research initiated by Holmström (1979). The existing literature on comparison of information structures in agency models is mostly restricted to verifiable signals in the standard formal contracting problem with a risk-averse agent. [11] The classic results, including Holmström (1979), Gjesdal (1982) and Grossman and

[9] More precisely, the necessary part can be established by showing that if one signal (say X) does not dominate another (Y) in the likelihood ratio order, there exists a model of relational contracting, represented by the principal's objective and the agent's cost function from effort, in which the principal prefers to design an incentive contract based on Y rather than X.

[10] It is worthwhile to note that the generalized MLRP (GMLRP) is complementary to the condition of Hwang (2016), the local convexity of distribution function condition (LCDFC). As we have seen in the example above, when the additive noise has a small σ, the distribution of output does not satisfy LCDFC but obeys GMLRP. On the contrary, there is a set of signals satisfying LCDFC but not GMLRP.

[11] To our best knowledge, one exception is the paper by Dewatripont, Jewitt and Tirole (1999) which compares the

Hart (1983), were developed by applying Blackwell's theorem. Kim (1995) subsequently showed that provided FOA is valid, the signal having a more dispersed likelihood ratio distribution (in terms of mean-preserving spread) is more informative in the standard model.[12] Our informativeness criterion has a similar flavor to Kim's in that both criteria pertain to the variability of the likelihood ratio and thus provide a unified treatment of comparison of signals regardless of their dimension. In addition to different notions of variability, one notable difference is that the MPS criterion is based on the variability of the ratio at each effort level, whereas our criterion is on the variability in response to the agent's possible effort deviations. This highlights the different sources of the agency costs in formal and relational contracts.

The rest of this paper is organized as follows. In Section 2 we present the model and formulate the optimal stationary contract problem. We also address further the motivating example and demonstrate that the conditions given in the literature are not sufficient for validating FOA. In Section 3 we introduce the generalized MLRP, illustrate its implications, and characterize the optimal bonus scheme. In Section 4 we compare measurement systems and derive a tight condition for a more efficient system. Section 5 concludes. All omitted proofs are relegated to Appendix A and more details on the generalized MLRP can be found in Appendix B.

2. The Model

We consider a repeated transaction between a risk-neutral principal and agent on an infinite time horizon, as in e.g. Levin (2003). At the outset of each period $t = 1, 2, \cdots$, the principal offers the agent a compensation scheme that consists of a base salary w_t and a discretionary bonus β_t. The agent, if he accepts the offer, privately chooses a level of effort e_t from $[0, \bar{e}] \subset \Re$ by incurring a cost of $c(e_t)$. If he rejects, nothing happens until the next period. The effort e_t results in gross expected benefits $v(e_t)$ accruing to the principal in that period, and also generates a set of commonly observable but unverifiable outcomes (or performance) $\mathbf{x}_t = (x_t^1, \cdots, x_t^n) \in \mathbb{X} \subset \Re^n$ according to a time-invariant cumulative distribution function (CDF) $F(\mathbf{x}_t, e_t)$ conditional on the agent's choice of effort. [13] We assume that both v and c are increasing and continuously differentiable functions over $[0, \bar{e}]$, and that $v - c$ is increasing for $e < e^{FB} = \text{argmax}_{e' \in [0,\bar{e}]}(v(e') - c(e'))$. We also assume that $F(\mathbf{x}_t, e_t)$ is twice continuously differentiable with respect to both arguments, and we denote by f the density function of $\mathbf{x}_t$. We shall call this outcome-generating process a *signal* hereafter.[14] Throughout the paper we use a capital letter for a random vector and a small letter for its realiza-

market signals about the agent's unknown talent in the career concern model. Their paper finds that an improvement of signals (even in the sense of Blackwell sufficiency) may strengthen or undermine incentives to work.

[12]Recently, Chi and Choi (2018) established that Kim's mean-preserving spread (MPS) criterion is also necessary for a verifiable measure to be more informative in the standard agency model, under the assumption that FOA is valid. They also showed that for univariate signals satisfying MLRP, the MPS criterion is equivalent to the Lehmann (1988) order.

[13]In standard agency models with a univariate signal, the principal's objective is given by the expected value of the signal; i.e. $v(e_t) = E(X_t|e_t)$. In our model, the realized benefit in period t need not be part of $\mathbf{x}_t$, that is, the exact benefit may or may not be observed by both parties when the bonus is paid. We discuss more details in Section 4.

[14]A signal is therefore defined by a set of distributions $F(\mathbf{x}, e)$ for each $e \in [0, \bar{e}]$. In contract theory literature, this is often referred to as a performance measurement system or an information system.

tion. A bold letter represents a vector, whereas a normal letter represents a scalar.

After observing an outcome vector $\mathbf{x}_t$, the principal pays the fixed salary w_t as agreed initially and decides which bonus β_t to pay. Here w_t is a legally enforceable payment that the principal can commit to, but the bonus $\beta_t : \mathbb{X} \rightarrow \Re$ is a discretionary payment that can be conditioned on the observed performance. Subsequent to the payment stage, the ex post payoff in period t of each party is determined. The principal obtains a payoff of the realized benefit minus $w + \beta_t(\mathbf{x}_t)$, and the agent obtains $w_t + \beta_t(\mathbf{x}_t) - c(e_t)$. Finally, each party decides whether to continue their relationship in the future or separate. If at least one party decides to walk away, the game ends. Let $\overline{\pi}$ and $\overline{u}$ denote the principal's and agent's reservation payoff, respectively. Both discount future payoffs by a common factor $\delta \in (0, 1)$.

Following Levin (2003), we confine ourselves to stationary contracts for characterization of the optimal contract. In a stationary contract, the principal offers the same base salary $w_t = w$ and bonus scheme $\beta_t = \beta$ every period, in anticipation that such payments induce the agent to make effort $e_t = e$. The key intuition of stationary contracts lies in the fact that the two instruments for providing incentives—the promised utility to the agent and the bonus scheme—are equally effective under risk-neutrality. Accordingly, we can think of such a stark form of contracts where the agent's promised utility remains constant over time and incentives are created by the instantaneous bonus only. Dropping the time index, we represent a stationary contract by (w, β, e) from now on.

In order for a contract (w, β, e) to be sustainable, its implicit part (β, e) must respect the following two conditions. First, the payment scheme should provide a proper incentive for the agent to put forth the desired effort e, so that e must maximize the agent's expected payoff. Abstracting away the fixed payment w that is unrelated the agent's choice of effort, this condition can be written as

$$e \in \underset{e' \in [0, \bar{e}]}{\operatorname{argmax}} \int_{\mathbb{X}} \beta(\mathbf{x})\, f(\mathbf{x}, e') d\mathbf{x} - c(e'). \tag{G-IC}$$

On top of this incentive compatibility constraint, the voluntary bonus scheme must be self-enforcing because there is no legal obligation to pay β. The bonus will be paid as promised only if both parties wish so, put another way, only if the expected payoffs from on-going relationship to each party are higher than those from reneging on the payment. Assuming that each party responds by terminating future transactions to breach of contracts, we can write the self-enforcement constraints as follows: for all possible realizations $\mathbf{x} \in \mathbb{X}$,

$$-\beta(\mathbf{x}) + \frac{\delta}{1-\delta}\Big(v(e) - w - \mathbb{E}[\beta(\mathbf{X})|e]\Big) \geq \frac{\delta}{1-\delta}\overline{\pi}$$
$$\beta(\mathbf{x}) + \frac{\delta}{1-\delta}\Big(w + \mathbb{E}[\beta(\mathbf{X})|e] - c(e)\Big) \geq \frac{\delta}{1-\delta}\overline{u}.$$

Denoting by $s(e) \equiv v(e) - c(e) - \overline{\pi} - \overline{u}$ the net per-period expected surplus from the on-going relationship, it is well known (e.g. Levin (2003)) that there are bonuses and payments that satisfy the two enforcement conditions if and only if the following aggregate enforcement condition holds:

$$0 \leq \beta(\mathbf{x}) \leq \frac{\delta}{1-\delta}s(e) \quad \forall\, \mathbf{x} \in \mathbb{X}. \tag{EC}$$

An optimal contract maximizes the expected surplus $s(e)$ subject to (G-IC) and (EC). The standard approach to this problem is to replace the global condition (G-IC) with the local stationary condition and check that the solution obtained is indeed optimal. In this procedure, the solution maximizes $s(e)$ subject to (EC) and

$$\int_{\mathbb{X}} \beta(\mathbf{x}) l(\mathbf{x}, e) f(\mathbf{x}, e) d\mathbf{x} \; - \; c'(e) \; = \; 0, \tag{L-IC}$$

where $l(\mathbf{x}, e) \equiv \partial \log f(\mathbf{x}, e)/\partial e = f_e(\mathbf{x}, e)/f(\mathbf{x}, e)$ denotes the likelihood ratio of signal $\mathbf{X}$.[15] The information variable $l(\mathbf{x}, e)$ captures how likely it is that the agent has chosen the desired effort e rather than other nearby effort given outcome $\mathbf{x}$.

Taking this first-order approach (FOA), we have the associated Lagrangian linear in β. As a result, the optimal bonus scheme $\beta^\dagger$ is bang-bang with $\beta^\dagger(\mathbf{x}) = 0$ if $l(\mathbf{x}, e^\dagger) < 0$ and $\beta^\dagger(\mathbf{x}) = b^\dagger \equiv \frac{\delta}{1-\delta}s(e^\dagger)$ if $l(\mathbf{x}, e^\dagger) \geq 0$, where the dagger superscript "†" of each contractual term stands for the FOA optimal contract. Intuitively, incentives for effort are maximized by paying a bonus for those outcomes where $f_e(\mathbf{x}, e^\dagger) > 0$, i.e. for outcomes which are made more likely with higher effort.

To conclude that this solution is indeed an optimal contract, we need to verify that $(\beta^\dagger, e^\dagger)$ satisfies the global IC constraint for the agent:

$$b^\dagger \Pr(l(\mathbf{x}, e^\dagger) > 0 \,|\, e) - c(e) \;\leq\; b^\dagger \Pr(l(\mathbf{x}, e^\dagger) > 0 \,|\, e^\dagger) - c(e^\dagger), \quad \forall\, e \in [0, \bar{e}]. \tag{1}$$

$(\beta^\dagger, e^\dagger)$ satisfies this constraint, and FOA is then justified, if the agent's expected payoff function is globally concave in his choice of effort for the given bang-bang structure of $\beta^\dagger$. A recent paper by Hwang (2016) establishes one sufficient condition for such global concavity that requires $\Pr(l(\mathbf{x}, e^\dagger) \leq 0 | c^{-1}(z))$ to be convex in z, and shows that this condition (named LCDFC by the author, the local convexity of distribution function condition) is less restrictive than CDFC (the convexity of distribution function condition) first introduced by Mirrlees (1979).[16] However, as we will show shortly by an example, LCDFC does not hold and neither does the global constraint (1) in several interesting applications. In such cases, FOA is no longer valid and thus the obtained solution is not optimal.[17]

Before turning to the example, we note that the above analysis is relevant as long as the first-best effort (denoted e^{FB}), at which the expected surplus $s(e)$ takes its maximal value, cannot be

[15] In accordance with custom, we use the subscript of a multivariable function to denote its partial derivative.

[16] Mirrlees (1979) also assumed the monotone likelihood ratio property (MLRP) which, for a univariate signal X, requires $l(x, e)$ to be monotone increasing in x for all e. In our setting MLRP plays no role in validating FOA; the property is used to guarantee the optimal bonus scheme being monotone in x.

[17] Kirkegaard (2017) proposes an alternative approach for examining whether local incentive compatibility implies global incentive compatibility in the agency model. In the setting considered here, with risk neutral parties, his sufficient condition (Proposition 1) is equivalent to LCDFC.

implemented. Throughout the paper, we will assume that this is the case. More precisely, we assume that there exists no contract implementing effort $e \geq e^{FB}$. A sufficient condition for this is

$$\frac{\delta}{1-\delta} s(e) \; < \; c(e) \qquad \forall \; e \geq e^{FB}.$$

The left-hand side of the inequality is the maximal bonus that can be paid under the enforcement condition (EC), and thus the inequality implies that there exists no bonus plan covering the agent's effort cost for $e \geq e^{FB}$.

An Illustrative Example

Consider a unidimensional signal $X \sim N(e, \sigma^2)$, for which we have likelihood ratio $l(x,e) = (x-e)/\sigma^2$. Under this specification, the optimal FOA contract awards the agent a maximal bonus $b^\dagger$ in case of $x > e^\dagger$. Hence the probability of obtaining the bonus can be written as

$$\Pr\left(l(X, e^\dagger) > 0 \middle| e \right) \; = \; \Pr\left(\frac{X - e^\dagger}{\sigma} > 0 \middle| e \right) \; = \; 1 - \Phi\left(\frac{e^\dagger - e}{\sigma} \right),$$

where $\Phi(\cdot)$ indicates the standard normal CDF. Being offered this FOA contract $(w, \beta^\dagger, e^\dagger)$, the agent's marginal net gain from exerting effort is $b^\dagger \, \Phi'(\frac{e^\dagger - e}{\sigma}) \frac{1}{\sigma} - c'(e)$. In equilibrium, effort $e = e^\dagger$ must satisfy the agent's first-order condition, and (as can be easily verified) the EC constaint must bind for the bonus $b^\dagger$. The optimal effort $e^\dagger$ can then be obtained by solving the following equation:

$$\frac{\delta}{\sigma(1-\delta)} s(e) \Phi'(0) \; = \; c'(e).$$

From this condition, it is straightforward to see that a more precise signal about the agent's effort (with lower σ) would elevate the agent's marginal revenue and thus allow higher effort to be implemented. Given that effort is below first best, this will in turn allow for a higher bonus, and hence equilibrium effort $e^\dagger$ and surplus $s(e^\dagger)$ must unambiguously increase. Whenever this local approach is valid, therefore, a simple comparative static analysis confirms the idea that a better signal alleviates the loss to the principal from being unable to observe the agent's action and hence improves efficiency.

The above analysis suggests that signals in this example can be ranked by their variance. However, there is a caveat, because the first-order approach is only valid in this setting if the variance is not too small. In particular, let $\overline{\sigma}$ be such that the FOA conditions hold for $e^\dagger = e^{FB}$, indicating that first best effort can be implemented. But under our standard assumption this cannot be the case, hence FOA can not be valid for variance $\overline{\sigma}^2$. In fact, in equilibrium the probability of obtaining a bonus is $1/2$, so the agent's net payoff from effort e^{FB} is at most $\frac{1}{2} \frac{\delta}{1-\delta} s(e^{FB}) - c(e^{FB})$, which is certainly negative under our assumptions.

LCDFC is not fulfilled in this case, and for a low enough variance the global IC conditions are violated. Figure 1 provides an illustration.

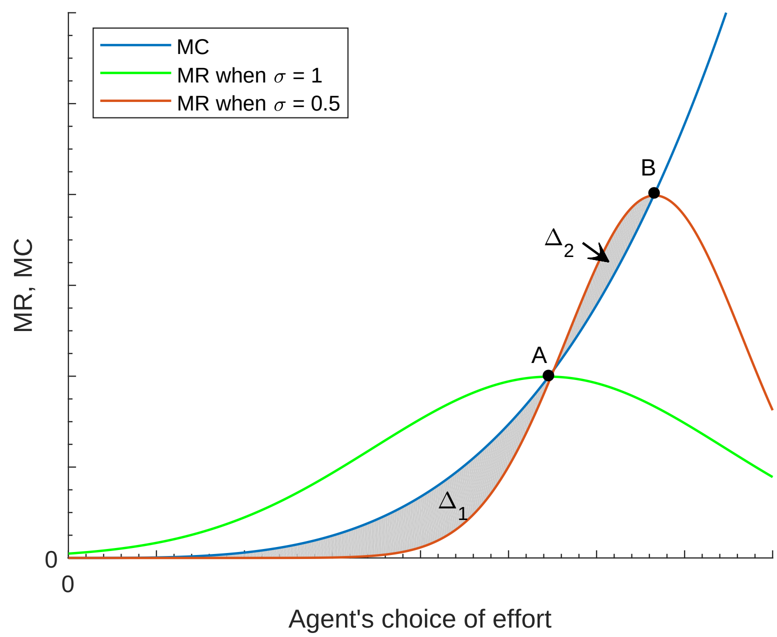


Figure 1: Illustrative example where the first-order approach is not valid.

As we have just seen, the agent's marginal revenue from effort follows the normal density, and while $e^{\dagger}$ is a local maximum for the agent, it is not a global one if σ is sufficiently small. When σ is large, the agent's marginal gain from effort (the green-colored curve in Figure 1) intersects with the corresponding marginal cost (the blue-colored curve) at a single point, where the agent's expected payoff is in fact maximized. Put differently, the local stationary condition implies the global IC condition for a large σ. For a relatively small σ, however, taking the local approach and solving the problem leads us to point B, at which the marginal revenue is maximal and equal to marginal cost. But the level of effort at B is not implemented unless the shaded area Δ_2 is larger than Δ_1, for otherwise the agent would deviate and instead choose the minimum level of effort.

This example raises two questions, first, what is an optimal bonus scheme in such cases where FOA breaks down, and second, will lower σ also in these cases be beneficial?

The normal distribution does not generally satisfy LCDFC but does satisfy MLRP. In the following we will show that MLRP is sufficient to characterize the optimal bonus scheme, and we will see that for this bonus scheme, a lower σ is indeed beneficial.

3. Optimal Relational Contracts

The discussion in the previous section suggests that we need to develop an alternative approach to characterize an optimal contract for cases where the FOA is not valid. It turns out that, under a condition that generalizes the MLRP, the optimal bonus scheme always takes a hurdle form like the FOA contract, in the sense that $\beta(\mathbf{x})$ is either maximal or minimal, depending on whether the likelihood ratio $l(\mathbf{x}, e)$ exceeds a hurdle. In contrast with the FOA contract, this hurdle is not necessarily zero.

We first introduce a generalized version of MLRP, which plays a key role in the subsequent analysis.

DEFINITION 1. *Signal* $\mathbf{X}$ *is said to possess the generalized monotone likelihood ratio property* (GMLRP) *if its likelihood ratio satisfies the following two conditions:*

(i) **(Regularity)** *for any* $\kappa \in \Re$ *and* $e, e' \in [0, \bar{e}]$, *there exists a* $\kappa' \in \Re$ *such that*

$$\{\mathbf{x} \in \mathbb{X} | l(\mathbf{x}, e) > \kappa\} \;=\; \{\mathbf{x} \in \mathbb{X} | l(\mathbf{x}, e') > \kappa'\}$$

(ii) **(Stochastic Dominance)** *for all* e *and* κ,

$$\Pr\left(l(\mathbf{X}, e) > \kappa | e'\right) \quad \textit{is increasing in } e'.$$

The second condition has a natural interpretation: the distribution of the likelihood ratio $l(\mathbf{X}, e)$ conditioned on the agent's choice of effort e' can be ordered by first-order stochastic dominance. That is, for all $e \in [0, \bar{e}]$, high effort e' generates a higher value of the likelihood ratio on average. It is well-documented (e.g. Milgrom (1981)) that if a univariate signal obeys MLRP (that is, $l(x, e)$ is increasing in x for all e), then the corresponding CDF satisfies $F(x, e'') \leq F(x, e')$ for all $e'' > e'$. This in turn implies the first-order stochastic dominance of $l(x, e)$. As a result, a univariate signal with MLRP satisfies the second condition of GMLRP.

The first condition essentially requires that every upper level set of $l(\mathbf{x}, e)$ can be duplicated by the upper set of $l(\mathbf{x}, e')$ with an adjusted level. Analogous to classic consumer theory, this condition endows the likelihood ratio with an ordinal property: if $l(\mathbf{x}', e) \geq l(\mathbf{x}, e)$ for some $(\mathbf{x}', \mathbf{x})$ and e, then $l(\mathbf{x}', e') \geq l(\mathbf{x}, e')$ for all $e' \in [0, \bar{e}]$. That is, if outcome $\mathbf{x}$ is less likely to occur than $\mathbf{x}'$ at effort e, then $\mathbf{x}$ remains less likely than $\mathbf{x}'$ at other efforts. If the likelihood ratio $l(\mathbf{x}, e)$ satisfies this ordinal property, we say that $l(\mathbf{x}, e)$ is regular. Observe that for scalar x the regularity condition holds if $l(x, e)$ is increasing, decreasing or constant in x for all e. Hence our condition is literally a generalized version of MLRP.[18]

The next result provides a simple characterization of the regular likelihood ratio.

PROPOSITION 1. *The likelihood ratio* $l(\mathbf{x}, e)$ *is regular if and only if for each* e *and* e', *there exists an order-preserving transformation* $\Psi : \Re \rightarrow \Re$ *satisfying* $l(\mathbf{x}, e') = \Psi\left(l(\mathbf{x}, e)\right)$ *for all* $\mathbf{x} \in \mathbb{X}$.

PROOF OF PROPOSITION 1: See Appendix A.1. □

In what follows, we shall be concerned with signals satisfying GMLRP. As a leading example, the most natural case $\mathbf{X} = \mu e + \epsilon$, where $\mu = (\mu_1, \cdots, \mu_n)$ and the random noise vector

[18] For example, $X \sim N(0, \sigma^2)$ with $\sigma = \sigma(e)$ increasing in e has a likelihood ratio $l(x, e)$ that is U-shaped in x and yet satisfies the regularity condition.

$\epsilon = (\epsilon_1, \cdots, \epsilon_n)$ follows a multivariate normal distribution with mean zero vector and covariance matrix $\Sigma = [\sigma_{ij}]$, satisfies the GMLRP. Straightforward algebra shows that the likelihood ratio can be written as

$$l(\mathbf{x}, e) = \sum_{i=1}^{n} m_i (x_i - \mu_i e), \quad \text{where } m_i = \sum_{j=1}^{n} \sigma_{ij}^{-1} \mu_j,$$

and σ_{ij}^{-1} are the elements of Σ^{-1}. The upper set $\{\mathbf{x} \in \mathbb{X} | l(\mathbf{x}, e) > \kappa\}$ is thus a half-space of the form

$$\left\{ \mathbf{x} \in \mathbb{X} \;\middle|\; \sum_{i=1}^{n} m_i x_i > e \sum_{i=1}^{n} m_i \mu_i + \kappa \right\}.$$

It is therefore obvious that $\mathbf{X}$ possesses the GMLRP. Another noteworthy class of such signals includes the case where $\mathbf{X}$ consists of an n-tuple of independent random variables X_is with likelihood ratios $l_i(x_i, e) = a(e) l(x_i) + \alpha_i(e)$, $a(e) > 0$. Then the likelihood ratio for $\mathbf{X}$ takes the form

$$l(\mathbf{x}, e) \;=\; a(e) \sum_{i=1}^{n} l(x_i) + \sum_{i=1}^{n} \alpha_i(e),$$

and thus it is regular. This class includes as a special case X_i being negative exponential with mean $EX_i = e$, and thus $l_i(x_i, e) = x_i/e^2 - 1$. In this case $l(\mathbf{x}, e) \;= \sum_{i=1}^{n} x_i/e^2 + n$, where $W = \sum_{i=1}^{n} X_i$ for effort e' has a gamma distribution with mean $h = ne'$, which implies that the second condition in GMLRP is also satisfied.[19]

DEFINITION 2. *A bonus scheme β is a hurdle scheme for the likelihood ratio at effort $e \in [0, \bar{e}]$ with hurdle $\kappa \in \Re$ if the scheme β takes the form*

$$\beta(\mathbf{x}) = \begin{cases} b\,(> 0) & \text{if } \; l(\mathbf{x}, e) > \kappa \\ 0 & \text{otherwise.} \end{cases}$$

An interpretation of this scheme is that the agent is rewarded on the basis of a performance index computed from the outcomes $\mathbf{x}$. The relevant index is the likelihood ratio $l(\mathbf{x}, e)$, and the bonus scheme is to reward the agent with a one-step bonus b for all outcomes having index value higher than a hurdle κ.

Our main result in this section states that the optimal bonus scheme maximizing the joint surplus $s(e)$ is of this type whenever the likelihood ratio is regular. The optimal scheme derived under the FOA is therefore a special case with hurdle zero ($\kappa = 0$).

PROPOSITION 2. *Assume that signal $\mathbf{X}$ has regular likelihood ratios and that no relational contract can implement effort $e \geq e^{FB}$. Then the optimal bonus scheme is a hurdle scheme for the likelihood ratio at the optimal effort e^*.*

PROOF OF PROPOSITION 2: See Appendix A.2. □

[19] If W has CDF $G(w; n, h)$, then $G_h < 0$ and hence G is decreasing in e'.

As long as the likelihood ratio is regular, Proposition 2 allows us to focus on a set of hurdle-type bonus schemes in characterizing optimal relational contracts, regardless of whether the FOA is justified or not. This greatly simplifies the analysis. The underlying intuition for this result is straightforward. Whenever $e \geq e^{FB}$ is not implementable due to the issues of unverifiable performance measures and unobservable effort, the contract between two risk-neutral parties should be designed so that it provides the agent with the strongest incentive for effort. [20] A way to achieve the goal under FOA is to offer a bonus scheme $\beta(\mathbf{x})$ that maximizes the marginal gain from effort at the optimal effort e^*:

$$\int_{\mathbb{X}} \beta(\mathbf{x})\, l(\mathbf{x}, e^*)\, f(\mathbf{x}, e^*) d\mathbf{x},$$

resulting in the hurdle scheme for $l(\mathbf{x}, e^*)$ with hurdle zero being optimal. But as we have discussed in the previous section, this local approach can be justified only if the global IC constraints are satisfied at the target effort e^*. If not, the scheme must be modified, and Proposition 2 shows that under regularity, the appropriate modification is simply to adjust the hurdle (and of course the target effort). As illustrated below, this adjustment reflects a trade-off between on the one hand inducing strong marginal incentives at the target effort, and on the other, preventing deviatons to distinctly lower effort.

The formal proof of Proposition 2, given in Appendix A.2, proceeds in two steps. We first show that if a non-hurdle scheme β satisfying (EC) implements a level of effort e^*, then there is a hurdle scheme β^* for the likelihood ratio $l(\mathbf{x}, e^*)$, with $\beta^* \neq \beta$ for some positive measure, such that β^* yields the same expected payoff for the agent as β, but a higher marginal gain from effort at e^*. Such a hurdle scheme β^* can be found for any distribution. If this scheme, which provides stronger marginal incentives for effort at e^*, also discourages the agent from deviating to any lower effort (i.e. satisfies all downwards IC constraints), then it will dominate the non-hurdle scheme β by implementing a higher effort than e^*. In the second step of the proof, we show that the downwards IC constraints are indeed satisfied if the likelihood ratio is regular. Consequently, a hurdle scheme is more efficient than others in that the scheme provides the strongest incentives for effort to the agent.

Another meaningful insight on the regularity condition can be found by linking it to another strand of contract theory literature. As Levin (2003) has observed, the stationary relational contract environment is similar to the static environment with two-sided limited liability, in the aspect that both environments impose a lower and upper bound on the payment scheme. In the context of financial contracts between a risk-neutral investor and entrepreneur, Innes (1990) has shown that under the FOA, the additional constraints on liability lead to debt-style contracts being optimal within the class of monotonic contracts. This result has been extended by Poblete and Spulber (2012) to a more general model where the FOA is not valid. To establish the optimality of debt contracts (in a setting where the slope of the payment scheme is constrained to be between 0 and 1), they introduced a critical ratio, defined as the marginal return to the principal from increasing the slope of the payment scheme, and assumed this ratio to be regular in a similar vein as the

[20] In fact, this part of the intuition is exactly the same as in Levin (2003), which assumed FOA to be valid.

regularity condition introduced here for the likelihood ratio.[21] Under this assumption plus the signal X being univariate, they showed that the optimal contract has slope one if the critical ratio exceeds a hurdle but has slope zero otherwise.

When the performance measure X is unidimensional, and the principal's value is the mean $\mathbb{E}[X|e]$, the likelihood ratio can be interpreted as the corresponding critical ratio in relational contracts. To see this, suppose without loss of generality that the agent's promised utility in the stationary optimal contract is fixed at $\overline{u}$.[22] In this case, an increment in bonus $\beta(x)$ by Δ over $[x, x+dx]$ would increase the principal's benefit by $\Delta \cdot f_e(x,e)dx$ through the agent's marginal incentive, but at the same time increase the principal's cost by $\Delta \cdot f(x,e)dx$ in order to maintain the continuation value $\overline{u}$. Therefore, the likelihood ratio indicates the marginal returns to the principal from increasing the bonus.

While Proposition 2 only relies on the regularity part of GMLRP, our next result also relies on the stochastic dominance part.

PROPOSITION 3. *If GMLRP holds and no $e \geq e^{FB}$ can be implemented in a relational contract, then*

(i) the maximal bonus is $b^* = \frac{\delta}{1-\delta}s(e^*)$ in an optimal contract,

(ii) if the likelihood ratio decreases with e, then $\kappa \leq 0$ in an optimal contract.

PROOF OF PROPOSITION 3: See Appendix A.3 □

It now follows that, under the assumptions in Proposition 3, an optimal contract can be found by solving for the highest effort $e^* \in [0, e^{FB}]$ that satisfies all downward IC constraints:

$$b^*\Pr(l(\mathbf{x}, e^*) > \kappa | e') - c(e') \leq b^*\Pr(l(\mathbf{x}, e^*) > \kappa | e^*) - c(e^*), \quad \forall\, e' \leq e^*, \tag{2}$$

for some hurdle κ and $b^* = \frac{\delta}{1-\delta}s(e^*)$.

An illustration and some intuition for the optimal negative hurdle $\kappa < 0$ in Proposition 3 can be gained from the example in the previous section.

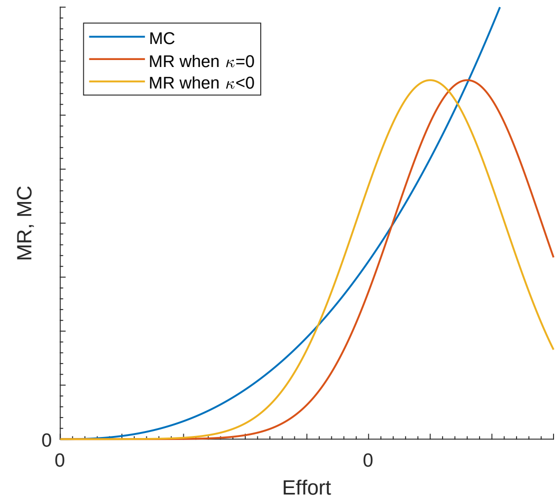
In Figure 2-(a), the red and blue curves depict the agent's marginal gain and marginal cost from effort, respectively, for the case of a signal $X \sim N(e, \sigma^2)$, where the bonus hurdle has been set at $\kappa = 0$ in accordance with FOA.[23] In the case depicted, the signal variance is small, and the FOA solution for effort (given by the intersection point where marginal revenue is maximal) is a local but not a global optimum for the agent, given the bonus scheme. Given this scheme, the agent would thus deviate to a smaller effort.

Here a variation of the hurdle κ will entail a horizontal shift of the marginal revenue curve (for a given bonus level b). The yellow-colored curve corresponds to some negative hurdle $\kappa < 0$ for

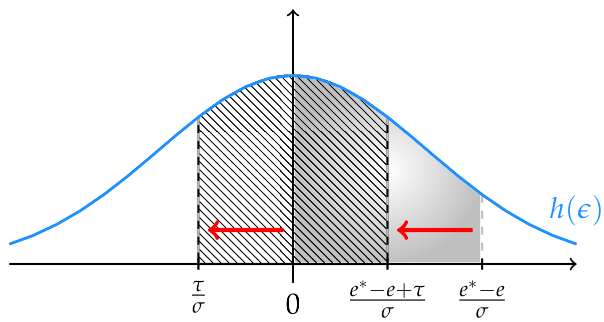
[21] In Appendix B, we formally derive the critical ratio and compare their regularity condition with ours in more detail. The MLRP is sufficient for both regularity conditions, but in general there is no direct connection between them.

[22] It follows by Theorem 1 in Levin (2003) that the way to split the joint surplus has no influence on the optimal bonus scheme and thus the agent's choice of effort because of the fixed wage.

[23] This bonus hurdle for the likelihood ratio corresponds to a hurdle $x > e^\dagger$ for the signal outcome x, and the marginal revenue is then proportional to the normal density $\Phi'(\frac{e^\dagger - e}{\sigma})$.



(a) Marginal Gains



(b) Total Gains

Figure 2: The effect of lowering hurdle κ on the agent's marginal and total gains from effort

the likelihood ratio, and thus a lower bonus hurdle for the outcome x than the FOA hurdle ($e^\dagger$), say a hurdle $e^\dagger - \tau$. The effect of this lower hurdle for obtaining the bonus, is to reduce the agent's marginal incentives for "high" efforts (near $e^\dagger$), but also to increase his total payoff for such efforts. Effort $\hat{e}$ at the highest intersection of the marginal revenue (yellow) curve and the marginal cost (blue) curve is now a global optimum for the agent.

The example illustrates that by relaxing the bonus hurdle, the agent's downwards incentive constraints will be relaxed, but the agent's marginal incentives for "high" effort will be reduced. The optimal contract must find the right balance between these two effects.

3.1. Example

To illustrate how to characterize the optimal hurdle scheme, consider a unidimensional noisy signal of effort $X = e + \epsilon\sigma$, where ϵ has a log-concave density $h(\cdot)$ with a unique mode at zero: $0 = \text{argmax}_\epsilon\, h(\epsilon)$. Then the likelihood ratio of X at e can be written as

$$l(x,e) \;=\; -\frac{1}{\sigma}h'\left(\frac{x-e}{\sigma}\right)/h\left(\frac{x-e}{\sigma}\right),$$

which is increasing in x but decreasing in e. Hence for each hurdle κ, there exists a unique τ such that (i) $l(x,e) > \kappa$ iff $x - e > \tau$; and (ii) $\kappa = 0$ iff $\tau = 0$. This enables us to write the distribution of l as

$$1 - \Pr(l(X,e) \le \kappa | e') \;=\; \Pr\left(X - e > \tau | e'\right) \;=\; 1 - H\left(\frac{e - e' + \tau}{\sigma}\right),$$

where $H(\cdot)$ is the CDF of ϵ.

Suppose that the principal offers hurdle scheme β with $\beta(x) = b^* \equiv \frac{\delta}{1-\delta}s(e^*)$ if $l(x,e^*) > \kappa$

and $\beta(x) = 0$ otherwise, and that the first-order approach is valid, that is,

$$u(\beta, e^*) \ \geq \ u(\beta, e) \ \text{ for all } e \in [0, \bar{e}] \quad \Leftrightarrow \quad b^* h\left(\frac{\tau}{\sigma}\right) = \sigma c'(e^*).$$

In order to induce the highest effort e^* under this local constraint, the principal must set $\tau = 0$, or equivalently $\kappa = 0$ in the hurdle scheme, which provides the strongest marginal incentive to the agent.

When the first-order approach is not valid, on the other hand, the optimal compensation scheme β must induce the highest effort under the global downward constraint: $u(\beta, e^*) \geq u(\beta, e)$ for all $e \leq e^*$, or

$$b^* \left[H\left(\frac{e^* - e + \tau}{\sigma}\right) \ - \ H\left(\frac{\tau}{\sigma}\right) \right] \ \geq \ c(e^*) \ - c(e) \quad \forall \, e \leq e^*.$$

Figure 2 illustrates how to design the optimal scheme when the first-order approach is not valid. The scheme corresponding to $\tau = 0$ provides the strongest marginal incentives at the desired effort e^* but may not induce the agent to choose e^*. This is indeed the case if the shaded area in Figure 2-(b) is smaller than $\frac{c(e^*)-c(e)}{b^*}$, resulting in a deviation from e^*. The way to resolve this incentive problem is to lower the hurdle: by setting $\tau < 0$, the principal can feasibly increase the net gain from making e^* to the agent as is displayed in Figure 2. Such a lower hurdle relaxes the global downward constraints, thereby implementing higher effort than the scheme with $\tau = 0$.

At an optimal non-zero solution for the hurdle τ (and equivalently for κ), some downwards IC constraint must be binding, and the corresponding effort, say e_0, must be a local optimum for the agent. Thus we must have

$$b^* \left[H\left(\frac{e^* - e_0 + \tau}{\sigma}\right) \ - \ H\left(\frac{\tau}{\sigma}\right) \right] \ = \ c(e^*) \ - c(e_0), \quad \ e_0 < e^*,$$

and

$$b^* h\left(\frac{e^* - e_0 + \tau}{\sigma}\right) \frac{1}{\sigma} \ \leq \ c'(e_0), \quad \ e_0 \geq 0,$$

where the last two inequalities hold with complementary slackness at the local optimum e_0. In addition, e^* must be a local (and interior) optimum fior the agent, and EC must hold, so we must have

$$b^* h\left(\frac{\tau}{\sigma}\right) \frac{1}{\sigma} \ = \ c'(e^*), \qquad b^* \ = \ \frac{\delta}{1-\delta} s(e^*).$$

These are necessary conditions. If in addition we know, say that the agent's payoff has at most two local maxima (as is the case when ϵ is normal and $c'(e)$ is linear), the conditions will also be sufficient to determine τ, e^* and e_0.

4. Value of Information

In the previous section, we studied the properties of an optimal bonus scheme in the stationary environment for a *given* signal, i.e., a given performance measurement system. We now turn to a problem of ranking non-verifiable signals satisfying GMLRP, say $\mathbf{X}$ with support $\mathbb{X} \subset \Re^n$ and $\mathbf{Y}$ with support $\mathbb{Y} \subset \Re^m$, and seek a criterion for their ranking in terms of the agency costs they generate in relational contracts. As we have seen, these costs arise from underprovision of effort, and the higher ranked signal will thus be the one that allows a higher level of effort to be implemented. The latter signal is more informative in the sense that it conveys information that supports a better contract. It turns out that the simple hurdle structure of an optimal bonus scheme enables us to establish a tight condition for one signal to be more informative than another signal in this sense.

There are a few papers investigating the nature of a more informative signal in a principal-agent framework. However, most attention has been devoted to explicit (or formal) contracts, that is, to models of contracting with a contractible signal and risk-averse agent, where the agency cost arises from moral hazard. The existing literature has developed criteria for a signal to be more informative and thus better alleviate agency costs in this environment; for instance, the informativeness criterion by Holmström (1979) and the mean-preserving spread (MPS) criterion by Kim (1995), among others. In a relational contract with a risk-neutral agent, on the other hand, it is the enforcement problem rather than moral hazard that hinders a contract from implementing the first-best, as moral hazard alone does not induce any agency cost in a risk-neutral environment. The different source of the agency cost suggests that a direct application of the existing criteria to relational contracts is inappropriate.

In this section, we establish a new criterion for a more informative signal tailored to relational contracts. In general, a signal $\mathbf{X}$ is more informative than another $\mathbf{Y}$ if writing a contract based on $\mathbf{X}$ is more effective in reducing the agency costs than doing so based on $\mathbf{Y}$. In our framework, such a cost reduction would lead to higher effort in the optimal contract. Our objective is to obtain a robust condition with respect to the characteristics of the model, under which signal $\mathbf{X}$ induces higher effort than signal $\mathbf{Y}$. For this purpose, we represent a relational contract problem by five elements $\langle (v,\overline{\pi}),(c,\overline{u}),\delta\rangle$: the principal's objective and reservation payoff, the agent's effort cost and reservation payoff, and their common discount factor. We denote by Ω the class of contracting problems of our interest:

$$\Omega \equiv \left\{ \langle (v,\overline{\pi}),(c,\overline{u}),\delta\rangle \;\middle|\; v,c:[0,\bar{e}] \to \Re \; C^1 \text{ and increasing; } \overline{\pi},\overline{u} \in \Re_+;\; \delta \in (0,1); \right.$$
$$\left. e \geq e^{FB} \text{ not implementable, } \; s(0) \leq 0 < s(e^{FB}), \; \text{ and } \; s(e) \text{ increasing over } [0,e^{FB}] \right\}.$$

To put it in a nutshell, the class Ω is a collection of contracting parties such that their transaction is valuable ($s(e) > 0$ for some e) but the efficient outcome $e^{FB} = \operatorname{argmax}_e s(e)$ is not possible. For each problem $\omega \in \Omega$, denote by $e_{\mathbf{X}}(\omega)$ and $e_{\mathbf{Y}}(\omega)$ the level of effort implemented by the optimal contract based on signal $\mathbf{X}$ and $\mathbf{Y}$, respectively. The notion of a more informative signal in our framework can be stated as follows:

DEFINITION 3. *Signal $\mathbf{X}$ is more informative than signal $\mathbf{Y}$ within class Ω if $e_{\mathbf{X}}(\omega) \geq e_{\mathbf{Y}}(\omega)$ for all $\omega \in \Omega$.*

One important feature of this notion is that the principal's objective v is not directly affected by her choice of signals but only indirectly affected through the agent's choice of effort. Irrespective of whether she designs a contract with signal $\mathbf{X}$ or $\mathbf{Y}$, her expected returns from the agent's costly effort are generated by the relationship with the agent itself rather than by the information structure.[24] By ruling out its direct effect on v, we can focus on the signal's effect on incentives.

We present a statistical criterion that characterizes a more informative signal. The results in the previous section suggest that the criterion pertains to the likelihood ratio of a signal. Given e, let $\mathbf{X} \sim F(\cdot, e)$ and $\mathbf{Y} \sim G(\cdot, e)$, where F and G are the respective CDFs, and let $f(\mathbf{x}, e)$ and $g(\mathbf{y}, e)$ denote the respective densities of each signal. For ease of notation, given fixed e^*, which we will call the target effort hereafter, we define the CDF of the likelihood ratio $l(\mathbf{x}, e^*)$ conditional on the agent's choice of effort e as

$$L_{\mathbf{X}}(\kappa, e) \;\equiv\; \Pr\Big(l(\mathbf{X}, e^*) \leq \kappa \,\Big|\, e\Big) \;=\; \int_{\mathbb{X}} \mathbf{1}_{\{l(\mathbf{x}, e^*) \leq \kappa\}}(\mathbf{x}) f(\mathbf{x}, e) d\mathbf{x}.$$

$L_{\mathbf{Y}}(\kappa, e) \equiv \Pr\left(l(\mathbf{Y}, e^*) \leq \kappa \,|\, e\right)$ can be defined in a similar way.[25]

With this notation, our informativeness criterion and main result of this section can be stated as follows:

DEFINITION 4. *Signal $\mathbf{X}$ dominates signal $\mathbf{Y}$ in the likelihood ratio order if for every $\kappa \in \Re$ and target effort $e^* \in [0, \bar{e}]$, there exists an κ' such that*

$$L_{\mathbf{Y}}(\kappa, e) - L_{\mathbf{Y}}(\kappa, e^*) \;\leq\; L_{\mathbf{X}}(\kappa', e) - L_{\mathbf{X}}(\kappa', e^*) \quad \textit{for all} \;\; e < e^*. \tag{L}$$

If (L) holds between $\mathbf{X}$ and $\mathbf{Y}$, then we write $\mathbf{X} \succ_L \mathbf{Y}$.

PROPOSITION 4. *Suppose that two signals $\mathbf{X}$ and $\mathbf{Y}$ satisfy* GMLRP. *Then $\mathbf{X}$ is more informative than $\mathbf{Y}$ within class Ω if and only if $\mathbf{X} \succ_L \mathbf{Y}$.*

PROOF OF PROPOSITION 4: See Appendix A.4. □

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[24] This assumption can be easily justified in two cases: (1) the realized returns are not observed by the parties and hence not part of $\mathbf{x}$ or $\mathbf{y}$, or (2) the returns are observable and determined by part of $\mathbf{x}$ or $\mathbf{y}$ (for instance, the first element of each signal), but both signals have the same marginal distribution on that part.

[25] Observe that under the regularity condition of GMLRP, the distribution of $l(\mathbf{x}, e')$ with another target effort $e' \neq e^*$ can be easily transformed into the distribution of $l(\mathbf{x}, e^*)$ with an adjusted κ', that is, $\Pr(l(\mathbf{X}, e') < \kappa | e) = L_{\mathbf{X}}(\kappa', e)$. This ordinal property allows us to drop the target effort e^* in the notation of $L_{\mathbf{X}}$.

Roughly speaking, the statistical order (L) compares the variability of the likelihood ratios with respect to the agent's choice of effort. The difference on the left-hand side of (L) represents the change in the distribution of $l(\mathbf{y}, e^*)$ when the agent does not follow the instruction e^* but deviates to some lower effort $e < e^*$. Accordingly, the difference can be interpreted as the amount of information regarding the agent's possible deviations conveyed by the informational variable $l(\mathbf{y}, e^*)$. Therefore, the inequality (L) implies that the maximal amount of information contained in $l(\mathbf{y}, e^*)$ is outweighed by that contained in $l(\mathbf{x}, e^*)$ regardless of the desired effort e^*, and thereby the principal can more effectively control the agent's hidden action by designing a contract based on signal $\mathbf{X}$ rather than $\mathbf{Y}$.

In order to clarify the significance of condition (L), suppose that the target effort e^*, bonus b^* and hurdle κ constitute an optimal contract under signal $\mathbf{Y}$, and consider the downward IC constraints (2), which now can be written as

$$b^* \left(L_{\mathbf{Y}}(\kappa, e) - L_{\mathbf{Y}}(\kappa, e^*) \right) \ \geq \ c(e^*) - c(e), \quad \forall\, e \leq e^*.$$

Note that $1 - L_{\mathbf{Y}}(\kappa, e)$ is the probability that the agent will clear the hurdle κ and receive the bonus b^* with choice of effort e. The left-hand side of the inequality above is thus the expected loss of bonus when the agent deviates from e^* to a lower effort level e. The inequality states that this loss should exceed the effort costs saved by any such deviation.

If signal $\mathbf{X}$ dominates $\mathbf{Y}$ in the likelihood ratio order, then there is a hurdle scheme with hurdle κ' for signal $\mathbf{X}$ such that a deviation to lower effort $e < e^*$ entails a larger reduction in the probability to pass the relevant hurdle under $\mathbf{X}$ than under $\mathbf{Y}$. Given the same bonus level b^*, downwards effort deviations are then even less attractive under signal $\mathbf{X}$, and this implies that effort e^* can be implemented also under this signal. Condition (L) thus guarantees that optimal effort under $\mathbf{X}$ is no smaller than optimal effort under $\mathbf{Y}$.

Proposition 4 tells us that the condition (L) is also necessary for $\mathbf{X}$ to be more informative than $\mathbf{Y}$ within the class Ω. To be specific, if the two signals $\mathbf{X}$ and $\mathbf{Y}$ cannot be ranked by the likelihood ratio order, then there exists a pair of contracting parties $\langle (v, \overline{\pi}), (c, \overline{u}), \delta \rangle \in \Omega$ for whom it is more efficient to evaluate the agent's performance based on $\mathbf{Y}$ rather than on $\mathbf{X}$. Consequently, $\mathbf{X} \succ_L \mathbf{Y}$ is the right informativeness criterion for ranking signals in relational contracting.

To see how the likelihood ratio order is related to the existing informativeness criteria in the literature, consider the notion of precision of a signal, first introduced by Lehmann in the field of statistical decision theory.[26]

DEFINITION 5. (LEHMANN (1988)) *Univariate signal $X \sim F$ is more precise about unknown parameter $e \in [0, \overline{e}]$ than another univariate signal $Y \sim G$ if for every outcome $y \in \mathbb{Y}$, there exists an increasing*

[26] For univariate signals X and Y satisfying MLRP, Lehmann (1988) found that X is more informative than Y in a statistical decision problem (with some restrictions on the decision maker's underlying payoff function) if and only if $X \succ_P Y$. Comparing with the statistical order in terms of sufficiency developed by Blackwell (1951, 1953), (P) is a more complete and intuitive order, and moreover, is easier to check so that it has been applied to several economic environments since Persico (2000). Refer to Chi (2014) for more details on Lehmann's order.

function $T_y : [0, \bar{e}] \to \mathbb{X}$ *such that*

$$F(T_y(e),\, e) = G(y,\, e) \quad \text{for all } e. \tag{P}$$

If (P) *holds between* X *and* Y*, we write* $X \succ_P Y$*.*

The essential requirement for one signal to be statistically more precise in Lehmann's notion is the monotone property of the function T_y in the unknown parameter for each y. To see its role, consider the two uniform distributions $X \sim U[e - \sigma/2,\ e + \sigma/2]$ and $Y \sim U[e - 1/2,\ e + 1/2]$. Equating their CDFs, we can compute the associated T-transformation:

$$T_y(e) \;=\; \sigma(y - e) + e \quad \text{for } \; y \in \left[e - \frac{1}{2},\ e + \frac{1}{2}\right].$$

We see that the obtained T_y is increasing in e if and only if $\sigma < 1$, i.e., the density of X is more clustered around e. For a signal that is exposed to an additive shock, Lehmann's order provides a more intuitive and complete ranking than Blackwell's sufficiency.[27]

With this order we have the following result:

PROPOSITION 5.

(i) Suppose that $l(\mathbf{X}, e^*)$ *is more precise than* $l(\mathbf{Y}, e^*)$ *for all* $e^* \in [0, \bar{e}]$*. That is, for every* $\kappa \in \Re$ *in the support of* $l(\mathbf{Y}, e^*)$*, there exists an increasing function* $T_\kappa(e)$ *such that*

$$L_{\mathbf{X}}\Big(T_\kappa(e),\, e\Big) \;=\; L_{\mathbf{Y}}(\kappa,\, e) \quad \forall\, e.$$

Then $\mathbf{X} \succ_L \mathbf{Y}$*.*

(ii) For a comparison of univariate signals satisfying strict MLRP, signal X *is more precise than* Y *if and only if* $l(X, e^*)$ *is more precise than* $l(Y, e^*)$ *for every* e^**. Consequently,*

$$X \succ_P Y \quad \textit{implies} \quad X \succ_L Y.$$

PROOF OF PROPOSITION 5: See Appendix A.5. □

By establishing a link to Lehmann's order, Proposition 5 generalizes our finding in the previous example of Section 3. On top of that, it provides an intuitive interpretation of the likelihood ratio order and a simple method to check whether signals can be ranked in the likelihood ratio order.[28]

The result has two implications. First, a signal with a more precise likelihood ratio dominates in the likelihood ratio order, and hence allows higher effort to be implemented in a relational

[27] The given example is due to Lehmann (1988). Comparing with Blackwell's sufficiency, he showed that X is sufficient for Y only if $\sigma = 1/k$ where $k \geq 1$ is a natural number.

[28] The condition (P) can be put in a number of alternative ways. For example, the condition holds if and only if (1) the difference $G(y, e) - F(x, e)$ obeys the single-crossing property in e for every pair of x and y; (2) for all $e_1 < e_2$, $F(F^{-1}(\kappa, e_2), e_1) \geq G(G^{-1}(\kappa, e_2), e_1)$.

contract. The principal uses the likelihood ratio as a key indicator to decide whether to pay a bonus in the optimal contract, and thus she prefers to evaluate the agent's performance with a more precise likelihood ratio.

Second, for univariate signals satisfying strict MLRP, the notion of a more precise likelihood ratio is equivalent to Lehmann's original notion of a more precise signal. To gain some intuition for this equivalence, observe that for such signals, the distribution of the signal's likelihood ratio is isomorphic to that of the signal itself: if $l(y, e^*)$ is strictly increasing in y, then $L_Y(\kappa, e) = \Pr(l(Y, e^*) \leq \kappa \,|\, e) = G(y, e)$, where y is the unique solution to $l(y, e^*) = \kappa$. As a result, $X \succ_{\mathrm{P}} Y$ implies $X \succ_{\mathrm{L}} Y$. By virtue of this implication, we can tell which is an informative signal in relational contracting just by comparing the underlying distributions, without further computation work for the likelihood ratio and its distribution, thereby greatly simplifying the whole analysis of an informative signal. The next example illustrates how Proposition 5 can be applied to the previous example in Section 3.

EXAMPLE 1. *Let $X = e + \epsilon\sigma_1$ and $Y = e + \epsilon\sigma_2$, where the additive noise ϵ has CDF $H(\cdot)$ as in the example in Section 3.1. We can write the distribution of each signal as $F(x, e) = H\left(\frac{x-e}{\sigma_1}\right)$ and $G(y, e) = H\left(\frac{y-e}{\sigma_2}\right)$. The associated T-transformation is therefore*

$$T_y(e) \;=\; \frac{\sigma_1}{\sigma_2} \cdot y + \frac{\sigma_2 - \sigma_1}{\sigma_2} \cdot e,$$

which is increasing in e whenever $\sigma_2 > \sigma_1$. Consequently, X is more precise than Y, and hence more informative than Y if $\sigma_1 < \sigma_2$. This implication holds regardless of whether FOA can be applied or not.

In case of univariate signals satisfying MLRP, if signal X is more precise than signal Y in the sense of (P), then X dominates Y in the likelihood ratio order so that X is a more informative signal in relational contracting. The converse is generally not true, but we have the following result.

COROLLARY 1. *Given two univariate signals X and Y satisfying strict MLRP, suppose that the associated T-transformation satisfying $F(T_y(e), e) = G(y, e)\ \forall\, e$ is additively separable. Then $X \succ_L Y$ if and only if $X \succ_P Y$.*

If the T-transformation is additively separable, then $X \succ_L Y$ implies $X \succ_P Y$ so that the two stochastic orders are equivalent. To see this, observe that in case of univariate signals satisfying MLRP, the likelihood ratio order (L) can be written in the following fashion: for all $y \in \mathbb{Y}$, there exists a $x \in \mathbb{X}$ such that $G_e(y,\, e) \geq F_e(x, e)$. Taking the derivative of both sides of the identity $F(T_y(e), e) = G(y, e)$ with respect to e, we have

$$G_e(y,\, e) \;=\; f(T_y(e),\, e) \cdot \frac{\partial T_y(e)}{\partial e} + F_e(T_y(e), e). \tag{3}$$

Since $T_y(e)$ is a bijective function of y for each e, for the existence of x satisfying $G_e(y, e) \geq F_e(x, e)$ for all y, $\partial T_y(e)/\partial e$ must be nonnegative at least for some y. Hence if T is additively separable, we have a nonnegative derivative of T_y for every y, leading to $X \succ_P Y$.

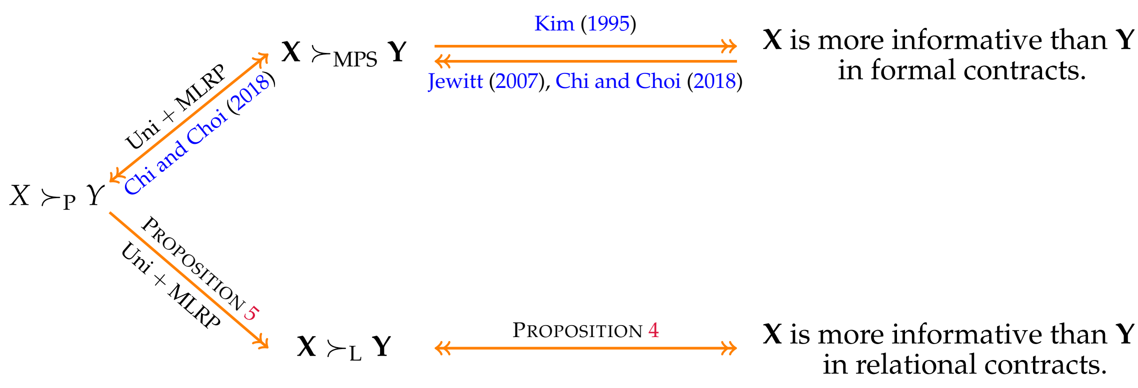

Figure 3: Link to Alternative Signal Orderings

For signals generated by an additive shock as in the previous example, the associated T-transformation is indeed additively separable, and consequently X dominates Y in the likelihood ratio order if and only if $\sigma_2 > \sigma_1$.

Proposition 5 also presents a link to the MPS criterion proposed by Kim (1995) for verifiable signals in legally enforced contracts. With our notation, Kim's criterion $\mathbf{X} \succ_{\text{MPS}} \mathbf{Y}$ can be stated as follows: the distribution $L_{\mathbf{X}}(\kappa, e) \equiv \Pr(l(\mathbf{X}, e) \leq \kappa | e)$ is more dispersed than the distribution $L_{\mathbf{Y}}(\kappa, e) \equiv \Pr(l(\mathbf{Y}, e) \leq \kappa | e)$ for all e in the sense of a mean-preserving spread. Just as the likelihood ratio order in relational contracts, $\mathbf{X} \succ_{\text{MPS}} \mathbf{Y}$ serves as an informativeness criterion for ranking verifiable signals in a standard formal contract (but in the latter case under the assumption that FOA is justified).[29] The key insight is similar: a more dispersed likelihood ratio conveys more information about the agent's possible deviations, and writing a contract based on such a signal is thus more effective in mitigating the agency problem.

The likelihood ratio order is in line with the MPS criterion proposed by Kim (1995) in the aspect that both criteria are based on the variability of the likelihood ratio. In particular, for a comparison of univariate signals satisfying MLRP, these two criteria are connected through Lehmann's order. A recent working paper by Chi and Choi (2018) shows that for univariate signals obeying MLRP, the MPS criterion is equivalent to Lehmann's order. Hence it follows from Proposition 5-(ii) that $X \succ_{\text{MPS}} Y$ implies $X \succ_{\text{L}} Y$. Figure 3 displays this linkage. However, in general, the two criteria do not mutually imply each other, since they adopt different notions of variability.[30] This distinction brings forth a new approach to ranking signals in relational contracts, enlightening the different sources of the agency costs in the two types of contracts.

[29] Kim (1995) establishes the sufficiency part of the MPS criterion for informativeness. Jewitt (2007) and more recently Chi and Choi (2018) establish the necessity of the criterion by using the dual approach.

[30] Another notable difference is that the MPS criterion hinges upon FOA so that it compares the variability of the likelihood ratios in response to the agent's *local* deviation from each target effort, whereas our criterion compares the variability in response to *all* possible downward deviations.

5. Conclusion

Performance measurement and design of incentive schemes are central issues in agency theory. The main purpose of this paper has been to develop a new criterion to characterize a better measurement system in relational contracts. Relational incentive contracts with non-verifiable information are subject to the enforcement problem, which is the main source of agency costs when the contracting parties are risk neutral. This results in a distinctive criterion, compared to the existing informativeness criteria for verifiable signals in formal contracts with risk averse agents. For its application to a wide class of signals, we also developed a condition (GMLRP) under which the optimal bonus scheme takes a simple hurdle form, regardless of whether FOA is applicable. In such settings the likelihood ratio order can be employed to tell which is a more efficient measurement system.

Our criterion is especially useful in the aspect that in contrast with objective measures, a wide array of non-verifiable (subjective) measures are available in most organizations. As long as both contracting parties have the same beliefs on such measures, our criterion can be used to address the problem of how to evaluate and reward employees' performances.

A limitation of our model is that it is confined to one-dimensional effort. Many agents are involved in multi-tasking, and the issues analyzed here are of course also relevant for such settings. When FOA is valid in a multi-task problem, the likelihood ratios on the agent's various tasks will play a key role in the optimal bonus scheme for a relational contract (Kvaløy and Olsen (2017)). It would be interesting to see if the results developed in this paper can be extended also to a multi-task setting.

While this paper is confined to relational contracting with only non-verifiable performance measurements, in reality there is often a combination of verifiable and non-verifiable measures available. A recent paper Miller, Olsen and Watson (2018) develops a general framework to analyze relational contracting in such settings, where contracts will have self-enforced as well as externally enforced elements. Extending our results regarding optimal bonus schemes and ranking of performance measurement systems to such environments would also be interesting and useful.

A. Omitted Proofs

A.1. Proof of Proposition 1

The sufficiency part can be easily done by setting $\kappa' = \Psi^{-1}(\kappa)$. To prove the other direction, suppose the likelihood ratio $l(\mathbf{x}, e)$ is regular. Observe that the regularity condition implies the existence of $\kappa' \in \Re$ satisfying

$$\{\mathbf{x} \in \mathbb{X} | l(\mathbf{x}, e) = \kappa\} = \{\mathbf{x} \in \mathbb{X} | l(\mathbf{x}, e') = \kappa'\} \quad \text{for every } e, e' \in E \text{ and } \kappa, \tag{4}$$

because the upper set of $L(\mathbf{x}, e)$ can be written as

$$\{\mathbf{x} \in \mathbb{X} | l(\mathbf{x}, e) = \kappa\} = \bigcap_{n=1}^{\infty} \left\{ \mathbf{x} \in \mathbb{X} \middle| l(\mathbf{x}, e) > \kappa - \frac{1}{n} \right\}.$$

Consider a partition $P_N \equiv \{\kappa_0, \cdots, \kappa_N\}$ of the image of $L(\mathbf{x}, e)$ made by an increasing sequence of $\{\kappa_n\}_{n=0}^{N}$ with $\kappa_{n+1} > \kappa_n$. Then the likelihood ratio can be approximated by a simple function:

$$\sum_{n=0}^{N} \kappa_n \mathbb{1}_{\{l(\mathbf{x},e)=\kappa_n\}}(\mathbf{x}) \xrightarrow{N \uparrow \infty} l(\mathbf{x}, e).$$

To a sequence of $\{\kappa_n\}$, we obtain a corresponding sequence $\{\kappa'_n\}$ satisfying (4) for each n. Observe that the obtained sequence is also increasing in n. Define a real-valued function Ψ that transforms κ_n to κ'_n, i.e., $\Psi(\kappa_n) = \kappa'_n$. It is self-evident that the constructed Ψ is an order-preserving transformation.

Then using the regularity condition, we can write the next simple function as

$$\sum_{n=0}^{N} \kappa'_n \mathbb{1}_{\{l(\mathbf{x},e')=\kappa'_n\}}(\mathbf{x}) = \sum_{n=0}^{N} \Psi(\kappa_n) \mathbb{1}_{\{l(\mathbf{x},e)=\kappa_n\}}(\mathbf{x}).$$

Then the result follows from the limiting argument that the expression on each side approaches $l(\mathbf{x}, e')$ and $\Psi(l(\mathbf{x}, e))$, respectively, as N increases. □

A.2. Proof of Proposition 2

Given β and $e \in [0, \bar{e}]$, define by $u(\beta, e)$ the agent's expected payoff:

$$u(\beta, e) = \int_{\mathbb{X}} \beta(\mathbf{x}) f(\mathbf{x}, e) d\mathbf{x} - c(e),$$

and by $u_e(\beta, e) \equiv \partial u(\beta, e)/\partial e$ the marginal incentive at e given β. Observe that we abstracted away the base salary w in the definitions above as it does not affect the agent's choice of effort.

The next lemma shows that for an arbitrary bonus scheme that implements effort e^0, there exists a bonus scheme taking a hurdle form such that this new scheme yields the same expected payoff but provides a higher marginal incentive than the original scheme at the target effort e^0.

LEMMA 1. *Let $\beta_N : \mathbb{X} \to \Re$ be a bounded bonus scheme with $0 \leq \beta_N(\mathbf{x}) \leq b$ for all $\mathbf{x} \in \mathbb{X}$. Then for each admissible effort e^0, there exists a hurdle scheme β such that*

$$u(\beta, e^0) = u(\beta_N, e^0) \quad \textit{but} \quad u_e(\beta, e^0) > u_e(\beta_N, e^0).$$

PROOF OF LEMMA 1: Given admissible effort $e^0 \in [0, \bar{e}]$, construct a hurdle scheme for likelihood ratio $l(\mathbf{x}, e^0)$ with hurdle $\kappa \in \Re$ as follows:

$$\beta(\mathbf{x}) \; = \; \begin{cases} 0 & \text{if } \; l(\mathbf{x}, e^0) < \kappa \\ b & \text{otherwise,} \end{cases}$$

where κ is set to yield the same expected payoff as the payoff from the given non-hurdle scheme β_N at $e = e^0$: $b(1 - \Pr(l(\mathbf{X}, e^0) \leq \kappa | e^0) - c(e^0) = u(\beta_N, e^0)$. The intermediate value theorem guarantees the existence of such a κ.

We now demonstrate that the constructed hurdle scheme provides a higher marginal incentive at e^0. For this purpose, observe that the difference of marginal incentives at $e = e^0$ can be written

$$\begin{aligned} u_e(\beta, e^0) - u_e(\beta_N, e^0) &= \int_{\mathbb{X}} (\beta(\mathbf{x}) - \beta_N(\mathbf{x}))\, l(\mathbf{x}, e^0)\, f(\mathbf{x}, e^0) d\mathbf{x} \\ &= -\int_{\{l(\mathbf{x}, e^0) < \kappa\}} \beta_N(\mathbf{x}) l(\mathbf{x}, e^0)\, f(\mathbf{x}, e^0) d\mathbf{x} \\ &\qquad\qquad + \int_{\{l(\mathbf{x}, e^0) \geq \kappa\}} (b - \beta_N(\mathbf{x})) l(\mathbf{x}, e^0)\, f(\mathbf{x}, e^0) d\mathbf{x} \\ &> \kappa \left[-\int_{\{l(\mathbf{x}, e^0) < \kappa\}} \beta_N(\mathbf{x})\, f(\mathbf{x}, e^0) d\mathbf{x} + \int_{\{l(\mathbf{x}, e^0) \geq \kappa\}} (b - \beta_N(\mathbf{x}))\, f(\mathbf{x}, e^0) d\mathbf{x} \right], \end{aligned}$$

where the last inequality holds in a strict form whenever the set $\{\mathbf{x} \in \mathbb{X} \,|\, \beta_N(\mathbf{x}) \neq \beta(\mathbf{x})\}$ has positive measure. Since the braketed expression is simply $u(\beta, e^0) - u(\beta_N, e^0) = 0$, we obtain the desired result. □

We use this lemma to prove the desired result by contradiction. Suppose to the contrary that the optimal bonus scheme takes a non-hurdle form β_N and elicits effort e^* from the agent. Due to the enforcement condition, we have $0 \leq \beta_N(\mathbf{x}) \leq \frac{\delta}{1-\delta} s(e^*)$ for all $\mathbf{x}$. Then it follows from the previous lemma that there exists a hurdle scheme β, $\beta(\mathbf{x}) = 0$ if $l(\mathbf{x}, e^*) < \kappa$ and $\beta(\mathbf{x}) = \frac{\delta}{1-\delta} s(e^*)$ otherwise, such that $u(\beta, e^*) = u(\beta_N, e^*)$ but $u_e(\beta, e^*) > u_e(\beta_N, e^*)$.

Observe that if β satisfies

$$u(\beta, e) \leq u(\beta_N, e) \quad \text{for all } \; e \leq e^*, \tag{5}$$

then it implements higher effort than e^*, contradicting that β_N is the optimal contract. We shall prove that (5) is indeed true whenever the GMLRP holds.

Suppose that $u(\beta, e') > u(\beta_N, e')$ for some $e' < e^*$. Along with $u(\beta, e^*) = u(\beta_N, e^*)$ and $u_e(\beta, e^*) > u_e(\beta_N, e^*)$, we can infer from the mean value theorem that there exists an $e^0 \in (e', e^*)$ satisfying $u(\beta, e^0) = u(\beta_N, e^0)$ and $u_e(\beta, e^0) < u_e(\beta_N, e^0)$. But by the regularity condition, β is also a hurdle scheme for the likelihood ratio $l(\mathbf{x}, e^0)$ with adjusted hurdle κ', leading to a contradiction. □

A.3. Proof of Proposition 3

Suppose the optimal bonus level is $b < b^*$, $b^* = \frac{\delta}{1-\delta} s(e^*)$. Since e^* is interior, the agent's FOC must hold, hence

$$0 = b \int_{\kappa < l(\mathbf{x}, e^*)} l(\mathbf{x}, e^*) f(\mathbf{x}, e^*) d\mathbf{x} - c'(e^*),$$

where we have used $f_e = lf$. A higher bonus $b' > b$ will strictly increase the marginal incentive for effort at e^*. Moreover, due to property (ii) of GMLRP, the higher bonus will relax all downwards IC constraints. (See (2), and note that a bonus $b' < b^*$ will tighten the constraints.) Bonus $b' > b$ will then induce higher effort than b, thus the latter cannot be optimal.

Suppose $\kappa > 0$. Optimal effort is the largest effort e^* that satisfies the downwards incentive constraints (2). Moreover, since e^* is interior, the agent's FOC must hold as above, with $b = b^*$.

Now replace κ with $\kappa' \in (0, \kappa)$. Then the agent's marginal revenue at effort e^* will strictly increase, and as shown below all downwards IC constraints (2) will be strictly relaxed when $l_e < 0$. Hence, by reducing κ a higher effort can be implemented. Then $\kappa > 0$ cannot be optimal.

Note that the claim regarding the IC constraints will hold true if the following claim is true:

Claim. $l_e(\mathbf{x}, e) < 0$ implies that $\Pr(l(\mathbf{X}, e) > \kappa | e) - \Pr(l(\mathbf{X}, e) > \kappa | e')$ is strictly decreasing in κ for $\kappa > 0$ and $e' < e$.

To verify the claim we show that $e' < e$ and $0 < \kappa' < \kappa$ imply

$$\Pr(l(\mathbf{X}, e) > \kappa' | e') - \Pr(l(\mathbf{X}, e) > \kappa | e') < \Pr(l(\mathbf{X}, e) > \kappa' | e) - \Pr(l(\mathbf{X}, e) > \kappa | e)$$

This is verified by showing that the expression on the LHS is strictly increasing in e'. The derivative of this expression with respect to e' satisfies

$$\int_{\kappa' < l(\mathbf{x}, e) \leq \kappa} f_e(\mathbf{x}, e') d\mathbf{x} > \kappa' \int_{\kappa' < l(\mathbf{x}, e) \leq \kappa} f(\mathbf{x}, e') d\mathbf{x} > 0,$$

where the first inequality follows from $f_e(\mathbf{x}, e') = l(\mathbf{x}, e') f(\mathbf{x}, e')$ with $l(\mathbf{x}, e') > l(\mathbf{x}, e) > \kappa'$ due to $e' < e$, $l_e < 0$, and the lower limit of integration being κ'. This verifies the claim and completes the proof.

A.4. Proof of Proposition 4

To establish the sufficiency part, it is enough to show that if $\mathbf{X} \succ_L \mathbf{Y}$, then the optimal effort under signal $\mathbf{Y}$ is implementable by a contract based on signal $\mathbf{X}$.

Choose a problem $\omega = \langle (v, \overline{\pi}), (c, \overline{u}), \delta \rangle$ from the set Ω and denote by $e_{\mathbf{Y}}$ the optimal effort under $\mathbf{Y}$ in this problem. In light of Proposition 2 and 3, we see that there exists a hurdle scheme

$$\beta(\mathbf{y}) = \begin{cases} 0 & \text{if } \; l(\mathbf{y}, e_{\mathbf{Y}}) < \kappa_{\mathbf{Y}} \\ b_{\mathbf{Y}} & \text{otherwise,} \end{cases}$$

with $b_{\mathbf{Y}} \equiv \frac{\delta}{1-\delta} s(e_{\mathbf{Y}})$, which implements $e_{\mathbf{Y}}$. Since $e_{\mathbf{Y}}$ is the optimal effort under $\mathbf{Y}$, the given contract $(w, \beta, e_{\mathbf{Y}})$ must satisfy the global IC constraint: $u(\beta, e_{\mathbf{Y}}) \geq u(\beta, e)$ for all e, or equivalently

$$L_{\mathbf{Y}}(\kappa_{\mathbf{Y}}, e) - L_{\mathbf{Y}}(\kappa_{\mathbf{Y}}, e_{\mathbf{Y}}) \;\geq\; \frac{c(e_{\mathbf{Y}}) - c(e)}{b_{\mathbf{Y}}} \quad \text{for all } \; e \in [0, \overline{e}].$$

Then it follows from $\mathbf{X} \succ_L \mathbf{Y}$ that there exists a hurdle κ' for signal $\mathbf{X}$ such that

$$L_{\mathbf{X}}(\kappa', e) - L_{\mathbf{X}}(\kappa', e_{\mathbf{Y}}) \;\geq\; \frac{c(e_{\mathbf{Y}}) - c(e)}{b_{\mathbf{Y}}} \quad \text{for all } \; e \leq e_{\mathbf{Y}}.$$

The last inequality implies that $e_{\mathbf{Y}}$ is also implementable by hurdle scheme β' for the likelihood ratio $l(\mathbf{x}, e_{\mathbf{Y}})$, which awards fixed bonus $b_{\mathbf{Y}}$ iff $l(\mathbf{x}, e_{\mathbf{Y}}) > \kappa'$. This proves that $\mathbf{X} \succ_L \mathbf{Y}$ is sufficient for $\mathbf{X}$ to be a more informative signal within Ω.

To prove the converse, suppose to the contrary that $\mathbf{X}$ does not dominate $\mathbf{Y}$ in the likelihood ratio order. This means that there exists a κ and e^* such that for all $\kappa' \in \Re$,

$$L_{\mathbf{Y}}(\kappa, e_\kappa) - L_{\mathbf{Y}}(\kappa, e^*) \;>\; L_{\mathbf{X}}(\kappa', e_\kappa) - L_{\mathbf{X}}(\kappa', e^*) \quad \text{for some } \; e_\kappa < e^*. \tag{6}$$

Below we demonstrate that if the two signals are not ranked in the likelihood ratio order, then there exists a pair of contracting parties with $\langle (v, \overline{\pi}), (c, \overline{u}), \delta \rangle \in \Omega$ for which e^* is implementable by a hurdle scheme under signal $\mathbf{Y}$, whereas no effort $e \geq e^*$ is implementable under $\mathbf{X}$. This contradicts that $\mathbf{X}$ is more informative than $\mathbf{Y}$ within class Ω.

The proof is by construction. To construct the agent's cost function from effort, denote by Γ the set of decreasing C^1-functions defined on the compact set $[0, e^*]$ such that $\gamma(e^*) = 0$ for all $\gamma \in \Gamma$. With κ and e^* that are specified in (6), note that $L_{\mathbf{Y}}(\kappa, e) - L_{\mathbf{Y}}(\kappa, e^*)$, regarding as a function of e and restricting its domain to $[0, e^*]$, becomes an element of Γ due to the stochastic dominance of GMLRP. Similarly, for every $\kappa \in \Re$, $L_{\mathbf{X}}(\kappa', e) - L_{\mathbf{X}}(\kappa', e^*) \in \Gamma$. Observe that if (6) holds, we can choose a function γ from the set Γ satisfying

$$L_{\mathbf{Y}}(\kappa, e) - L_{\mathbf{Y}}(\kappa, e^*) \;\geq\; \gamma(e) \;\; \text{for all } \; e \leq e^* \quad \text{and} \tag{7}$$

$$L_{\mathbf{X}}(\kappa', e_\kappa) - L_{\mathbf{X}}(\kappa', e^*) \;<\; \gamma(e_\kappa) \;\; \text{for } \; e_\kappa < e^*. \tag{8}$$

To construct the function γ on the remaining domain $[e^*, \overline{e}]$, fix $\gamma(e^*) = 0$ for its continuity and choose a continuous decreasing function satisfying

$$\gamma(e) \leq \min\left\{L_{\mathbf{Y}}(\kappa', e) - L_{\mathbf{Y}}(\kappa', e^*),\ \min_{\kappa \in \Re} L_{\mathbf{X}}(\kappa, e) - L_{\mathbf{X}}(\kappa, e^*)\right\} \tag{9}$$

with the inequality being strict except at $e = e^*$, where the min operator inside the curly bracket indicates the point minimization at each e.

We are now ready to construct the cost function from effort. We first assign one positive value to $c(e^*) \geq \frac{\delta}{1-\delta}s(e^*)\gamma(0)$ so that the cost function below takes a nonnegative value everywhere on $[0, \bar{e}]$ and set

$$c(e) \;=\; \begin{cases} c(e^*) - \frac{\delta}{1-\delta}s(e^*)\,\gamma(e) & \text{for } \ e \in [0, e^*] \\ c(e^*) - \frac{\delta}{1-\delta}s(e^{FB})\,\gamma(e) & \text{for } \ e \in [e^*, \bar{e}], \end{cases}$$

where the principal's objective v, an increasing and C^1 function, and the parties' reservation payoffs are chosen such that (i) the expected surplus $s(e) = v(e) - c(e) - \overline{\pi} - \overline{u}$ attains its maximum at $e^{FB} > e^*$, (ii) $s(e^*) > 0$, and (iii) the constructed cost function is differentiable at $e = e^*$, that is,

$$\lim_{e \downarrow e^*} c'(e) \;=\; -\frac{\delta}{1-\delta}s(e^{FB}) \lim_{e \downarrow e^*} \gamma'(e) \;=\; -\frac{\delta}{1-\delta}s(e^*) \lim_{e \uparrow e^*} \gamma'(e) \;=\; \lim_{e \uparrow e^*} c'(e).$$

Then the constructed problem is an element of Ω.

We demonstrate that within this contracting environment, e^* is implementable under signal $\mathbf{Y}$ but neither e^* nor any higher effort than e^* is implementable under signal $\mathbf{X}$. For signal $\mathbf{Y}$, consider the hurdle scheme $\beta(\mathbf{y}) = 0$ if $l(\mathbf{y}, e^*) < \kappa$ and $\beta(\mathbf{y}) = \frac{\delta}{1-\delta}s(e^*)$ otherwise, where κ and e^* are defined as in (6). Then it follows from (7) that the agent would find any downward deviation $e \leq e^*$ nonprofitable. No upward deviation $e > e^*$ is profitable either, since (9) yields

$$u(\beta, e) - u(\beta, e^*) \;=\; \frac{\delta}{1-\delta}s(e^*)\Big[L_{\mathbf{Y}}(\kappa, e^*) - L_{\mathbf{Y}}(\kappa, e)\Big] + \frac{\delta}{1-\delta}s(e^{FB})\gamma(e) \;<\; 0.$$

Hence e^* is indeed implementable with a feasible hurdle scheme under $\mathbf{Y}$.

In the same manner as above, it can be shown that e^* is not implementable with any hurdle scheme under $\mathbf{X}$. To see that any higher effort than e^* is subject to deviations, suppose to the contrary that some $e_{\mathbf{X}} \in (e^*, e^{FB})$ is implementable under $\mathbf{X}$. By Proposition 2 and 3, amongst possible payment schemes implementing $e_{\mathbf{X}}$, we can restrict attention to a hurdle scheme, $\beta(\mathbf{x}) = 0$ for $l(\mathbf{x}, e_{\mathbf{X}}) < \kappa_{\mathbf{X}}$, equivalently by the regularity condition of GMLRP, $\beta(\mathbf{x}) = 0$ for $l(\mathbf{x}, e^*) < \kappa^*$, and $\beta(\mathbf{x}) = \frac{\delta}{1-\delta}s(e_{\mathbf{X}})$ for the other case. But given any schemes taking this form, a deviation to e^* is profitable to the agent because

$$u(\beta, e^*) - u(\beta, e_{\mathbf{X}}) \;=\; \frac{\delta}{1-\delta}s(e_{\mathbf{X}})\Big[L_{\mathbf{X}}(\kappa^*, e_{\mathbf{X}}) - L_{\mathbf{X}}(\kappa^*, e^*)\Big] - \frac{\delta}{1-\delta}s(e^{FB})\gamma(e_{\mathbf{X}}) \;>\; 0,$$

where the inequality follows from $s(e_{\mathbf{X}}) \leq s(e^{FB})$ and the inequality put in (9), which holds with a strict inequality at $e = e_{\mathbf{X}} > e^*$. Therefore, any feasible schemes under signal $\mathbf{X}$ cannot implement effort $e \geq e^*$ in the constructed contracting problem. This contradiction establishes that $\mathbf{X} \succ_L \mathbf{Y}$ is necessary for implementation of higher effort under X within class Ω. The proof is now complete. □

A.5. Proof of Proposition 5

Given a pair of multivariate signals $\mathbf{X}$ and $\mathbf{Y}$, suppose that the likelihood ratio of $\mathbf{X}$ is more precise than the ratio of $\mathbf{Y}$ in the sense of (P). That is, for each $\kappa \in \Re$, there exists an increasing function $T_\kappa : [0, \bar{e}] \to \Re$ such that

$$L_{\mathbf{X}}\big(T_\kappa(e), e\big) = L_{\mathbf{Y}}(\kappa, e) \qquad \forall\, e \in [0, \bar{e}]. \tag{10}$$

For a κ and e^*, let $\kappa' = T_\kappa(e^*)$. Then by definition of the function T_κ in (10), for every $e < e^*$ we have

$$\begin{aligned} L_{\mathbf{Y}}(\kappa, e) - L_{\mathbf{Y}}(\kappa, e^*) &= L_{\mathbf{X}}(T_\kappa(e), e) - L_{\mathbf{X}}(\kappa', e^*) \\ &\leq L_{\mathbf{X}}(\kappa', e) - L_{\mathbf{X}}(\kappa', e^*), \end{aligned}$$

where the inequality follows from $T_\kappa(e) \leq T_\kappa(e^*) = \kappa'$. As such κ' exists for arbitrarily chosen κ and e^*, we have $\mathbf{X} \succ_L \mathbf{Y}$. This proves the first statement of Proposition 5.

To prove the second statement, let $X \sim F$ and $Y \sim G$ be univariate signals satisfying strict MLRP. Regarding this proof, in order to avoid confusions, we attach subscript X and Y to represent their likelihood ratio. The strict MLRP implies that their likelihood ratios at e^*, $l_X(\cdot, e^*)$ and $l_Y(\cdot, e^*)$, are strictly increasing in the first argument. We show that in this case, $l_X(\cdot, e^*) \succ_P l_Y(\cdot, e^*)$ if and only if $X \succ_P Y$.[31] For the remaining implication, the same proof as above can be applied.

Suppose X is more precise than Y, i.e., for each y there exists an increasing function $T_y(e)$ such that $F(T_y(e), e) = G(y, e)$ for all e. Define by λ the inverse function of $l_Y(\cdot, e^*)$ with respect to the first argument, and put $\kappa = l_Y(y, e^*)$ for some y in the support of signal Y so $y = \lambda(\kappa)$. With this notation, we then have

$$L_Y(\kappa, e) = \Pr(l_Y(Y, e^*) \leq \kappa |\, e) = G(\lambda(\kappa), e) = F\left(T_{\lambda(\kappa)}(e), e\right). \tag{11}$$

Define $H_\kappa(e) \equiv l_X(T_{\lambda(\kappa)}(e), e^*)$. As its inner function $T_\lambda(\kappa)$ is increasing in e and its outer function l_X is increasing in the first argument, we see that their composition $H_\kappa(e)$ is increasing in e. Then by definition of L_X and $H_\kappa(e)$, we have

$$L_X\left(H_\kappa(e), e\right) = \Pr\left(l_X(X, e^*) \leq l_X(T_{\lambda(\kappa)}(e), e^*) \,\Big|\, e\right) = F\left(T_{\lambda(\kappa)}(e), e\right).$$

Hence it follows from (11) that $L_Y(\kappa, e) = L_X(H_\kappa(e), e)$, along with H_κ increasing, implying that the likelihood ratio of X is more precise than the ratio of Y.

[31] A similar proof can be used to establish a more general statement: X is more precise than Y if and only if $m(X)$ is more precise than $n(Y)$ with $m(\cdot)$ and $n(\cdot)$ strictly increasing.

To prove the converse, suppose $l_X(\cdot, e^*)$ is more precise than $l_Y(\cdot, e^*)$, i.e. for every κ in the support of $l_Y(Y, e^*)$, there exists an increasing function $H_\kappa(e)$ such that $L_X(H_\kappa(e), e) = L_Y(\kappa, e)$ for all e. Choose a sample y from the support of Y, and let $\kappa(y) = l_Y(y, e^*)$. Then we have

$$G(y,e) \;=\; L_Y\left(l_Y(y,e^*),e\right) \;=\; L_Y(\kappa(y),e) \;=\; L_X\left(H_{\kappa(y)}(e),e\right). \tag{12}$$

Define by μ the inverse of $l_X(\cdot, e^*)$. Then with μ and a constant $\kappa' \in \Re$, we can write

$$L_X(\kappa',e) \;=\; \Pr(l_X(X,e^*) \le \kappa' \,|\, e) \;=\; F(\mu(\kappa'),e). \tag{13}$$

Putting (12) and (13) together leads us to

$$G(y,e) \;=\; F\left(\mu(H_{\kappa(y)}(e)),e\right) \quad \forall\, e.$$

Since both μ and H_κ are increasing in its argument, their composition $T_y(e) \equiv \mu(H_{\kappa(y)}(e))$ must increase with e. This proves the existence of an increasing transformation equating the two distribution functions, and thus we have $X \succ_P Y$. The proof is now complete. $\square$

B. The regularity condition

In this section, we derive the critical ratio developed by Poblete and Spulber (2012) in the contracting environment with a verifiable unidimensional signal and compare their regularity condition with our regularity condition of GMLRP.

Assume that the outcome $x \sim F(\cdot, e)$ with support $[\underline{x}, \overline{x}]$ indicates the principal's objective, and denote by $s(x)$ the sharing rule between the two risk-neutral parties. After carrying out a contract, the principal obtains a payoff of $x - s(x)$ and the agent obtains $s(x) - c(e)$. The critical ratio $\rho(x, e)$ is defined as the ratio of the net expected benefit to cost from increasing the slope of $s(x)$ by Δ over $[x, x + dx]$. The increment of $s'(x)$ increases the principal's expected benefit by $-\Delta F_e(x, e)dx$ through the agent's marginal incentive, and at the same time aggravates her expected cost by $\Delta(1 - F(x, e))dx$.[32] Therefore, the critical ratio reduces into

$$\rho(x,e) \;=\; \frac{-\Delta F_e(x,e)dx}{\Delta(1-F(x,e))dx} \;=\; -\frac{F_e(x,e)}{1-F(x,e)}.$$

The critical ratio is regular if $\rho(x', e) \ge \rho(x, e)$ for some x and x' and for some e implies $\rho(x', e') \ge \rho(x, e')$ for all e'. Observe that the notion of regularity is the same as what we used for GMLRP.

To examine the relation between the regularity condition of $\rho(x, e)$ and $l(x, e)$, recall that if

[32] Integrating by parts, the agent's expected payoff from $s(x)$ can be written as

$$\int_{\underline{x}}^{\overline{x}} s(x)\, f(x,e)dx \;=\; \int_{\underline{x}}^{\overline{x}} s'(x)\left(1-F(x,e)\right)dx.$$

Therefore, the increment of $s'(x)$ by Δ over $[x, x + dx]$ would strengthen the agent's marginal incentives by $-\Delta F_e(x, e)dx$ and total incentives by $\Delta(1 - F(x, e))dx$, respectively.

$f(x,e)$ is log-supermodular, then the hazard rate $f(x,e)/[1-F(x,e)]$ is decreasing in e for all x. This in turn is equivalent to the increasing critical ratio property for all e. As a result, if $l(x,e)$ is increasing in x for all e, then $\rho(x,e)$ is increasing in x for all e as well. The reverse is not true, so MLRP is not always necessary for $\rho(x,e)$ to be increasing. See Poblete and Spulber (2012) for a counterexample. Hence the regularity condition of $l(x,e)$ is seemingly more restrictive than the regularity condition of $\rho(x,e)$. However, the following example shows the existence of a signal for which the likelihood ratio is regular but the critical ratio is not.

EXAMPLE 2. *Consider a signal $X \sim N(0, \sigma(e)^2)$, where the agent's effort does not affect the mean of X but affects its variance. This signal has likelihood ratio*

$$l(x,e) \;=\; \frac{\partial}{\partial e}\ln f(x,e) \;=\; \left[-1+\left(\frac{x}{\sigma(e)}\right)^2\right]\frac{\sigma'(e)}{\sigma(e)}.$$

Suppose $\sigma'(e) > 0$ for all e, implying that as the agent exerts higher effort, the output distribution is more diffuse. Then for every admissible e, $l(x,e)$ is increasing in x for all $x \geq 0$ but decreasing for all $x < 0$, and moreover, it satisfies the condition for regularity in Definition 1.[33]

On the other hand, the signal X has the critical ratio as

$$\rho(x,e) \;=\; \frac{z\phi(z)}{1-\Phi(z)}\cdot\frac{\sigma'(e)}{\sigma(e)}, \qquad z=\frac{x}{\sigma(e)},$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ indicate the c.d.f and p.d.f of the standard normal distribution, respectively. Then

$$\frac{\partial}{\partial x}\rho(x,e) \;=\; \rho_x(x,e) \;=\; \frac{d}{dz}\left(\frac{z\phi(z)}{1-\Phi(z)}\right)\frac{\sigma'(e)}{\sigma(e)^2},$$

where

$$\begin{aligned}\frac{d}{dz}\left(\frac{z\phi(z)}{1-\Phi(z)}\right) \;&=\; \frac{1}{(1-\Phi(z))^2}\Big[(z\phi'(z)+\phi(z))(1-\Phi(z)) - z\phi(z)(-\phi(z))\Big]\\ &=\; \frac{\phi(z)}{(1-\Phi(z))^2}\Big[(-z^2+1)(1-\Phi(z))+z\phi(z)\Big], \qquad \textit{since } \phi'(z) = -z\phi(z),\\ &\equiv\; \Psi(z).\end{aligned}$$

Note that the derivative $\Psi(z)$ takes a strictly positive value at $z=0$ but a negative value at $z=-1$. Hence there exist $z_1 < z_2 < 0$ such that $\Psi(z_1) < 0$ and $\Psi(z_2) > 0$.

For given e_1, let $x_1 = z_1\sigma(e_1)$ and let $e_2 > e_1$ denote the level of effort at which $\sigma(e_2)z_2 = x_1$. Then we have

$$\rho_x(x_1,e_1) \;=\; \Psi(z_1)\cdot\frac{\sigma'(e_1)}{\sigma(e_1)^2} < 0 \quad \textit{and} \quad \rho_x(x_1,e_2) \;=\; \Psi(z_2)\cdot\frac{\sigma'(e_2)}{\sigma(e_2)^2} \;>\; 0.$$

[33]However, the given signal does not satisfy GMLRP as it violates the stochastic dominance condition.

That is, the critical ratio is decreasing in x for effort e_1 but increasing for e_2. Consequently, $\rho(x, e)$ is not regular. □

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