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A Five Factor approach to the Low Volatility Anomaly

An Empirical Study of the Norwegian Stock Market

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ABSTRACT

In this thesis, we construct the Fama-French five-factor model (2015a) for the Norwegian stock market in order to examine the existence of the low volatility anomaly. We estimate risk as idiosyncratic volatility relative to the Fama-French three-factor model (1993) and total volatility defined as a stock's standard deviation with a trailing window of 24 months. Stocks are then sorted into value- and equally weighted quintile portfolios based on both risk measurements individually. Further, the excess portfolio returns are regressed on the Fama and French five-factor model to control for the systematic risk factors; market, size, value, operating profitability and investment. This lets us examine the existence of the low volatility anomaly by looking at monthly excess returns, Sharpe (1966) ratios and alphas for each of the quintile portfolios. We are unable to prove the existence of the anomaly through excess returns alone as we find a positive, but statistically insignificant, difference in excess return between the lowest and highest quintile portfolio. However, we are able to document the anomaly through the alphas. Regardless of volatility measurement and weighting scheme, we find statistically significant positive differences in alphas between the lowest and highest quintile portfolios. Our results are robust after controlling for different measurements of idiosyncratic volatility, subsamples, filtering process and return requirements. This leads us to the conclusion that the low volatility anomaly is present in the Norwegian stock market in the period August 1993 to December 2017.

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1 INTRODUCTION

By now it is well known that the traditional Capital Asset Pricing Model by Sharpe (1964), Lintner (1965) and Mossin (1966) does not hold as the reliant model it has been looked upon for almost three decades. There is evidence by Black, Jensen, and Scholes (1972), and Fama and MacBeth (1973) who all show that the relation between average return and market beta is flatter than predicted by CAPM. When Fama and French (1993) published their three-factor model they worked off several studies of market inefficiency, see Banz (1981), Basu (1983), Rosenberg, Reid and Lanstein (1985), and Bhandari (1988), according to CAPM. These studies show that when used alone, CAPM betas have little or no explanatory power for the cross-section of average returns. Fama and French (1992) found that there were two effects in particular that, when used together, could explain the cross-section of average returns for U.S. stocks in the time-period 1963-1990. These effects are size and book-to-market equity, and together with the market factor, they form the Fama-French three-factor model (FF-3). Since their paper in 1993 their model has been widely used and respected among economists, but there have also been documented market anomalies that the FF-3 model fails to explain. The most prominent anomalies are market beta (Black, Jensen, and Scholes, 1972), net share issues (Ikenberry, Lakonishok and Vermaelen, 1995 and Loughran and Ritter, 1995), accruals (Sloan 1996), momentum (Jegadeesh and Titman, 1993) and volatility (Ang et al., 2006). The latter anomaly is the focus of this thesis.

Since the highly recognized article by Ang et al. (2006) found a negative relation between idiosyncratic volatility and the cross-section of returns in the U.S. stock market, the existence of the low volatility anomaly has been highly debated. Some studies contradict Ang et al. (2006) and find a positive relation between idiosyncratic volatility and returns, see Fu (2009), others find no relation between the two, see Bali and Cakici (2008), and others support the findings of Ang et al. (2006), see Baker and Haugen (2012). The first study solely focusing on the Norwegian stock market was conducted by Hafskjær and Østnes (2013). In addition, two more recent studies examine the existence of the low volatility anomaly in the Norwegian stock market, see Arnesen and Borge (2017) and Bakøy and Huskic (2017). However, these papers show mixed results regarding the existence of the low volatility anomaly also in the Norwegian stock market.

In this thesis we estimate risk as idiosyncratic volatility relative to the Fama-French three-factor model (1993), and total volatility defined as a stock's standard deviation using a 24-month trailing window. Based on these estimates of risk we sort stocks into both value- and equally weighted quintile portfolios. In order to control for systematic risk factors, we construct the CAPM, Fama-French three- and Fama-French five-factor model (FF-5) for the Norwegian stock market using raw stock and accounting data. While previous studies of the low volatility anomaly in Norway have focused on the estimated FF-3 alphas augmented with the momentum and liquidity factors, the focus and contribution of this thesis is to estimate the alphas when controlling for systematic risk factors in the FF-5 model; market, size, value, operating profitability and investment. We find this approach intriguing as Fama and French (2015b) show that controlling for additional factors (profitability and investment) reduce the abnormal returns in the low volatility anomaly.

Based on the quintile portfolios we evaluate the existence of the low volatility anomaly by looking at monthly excess returns, Sharpe ratios and alphas when controlling for systematic risk given by the CAPM, FF-3 model and FF-5 model. Before starting this thesis, we expect to find the low volatility when controlling for both the CAPM and the FF-3 model. Further, we expect that including additional factors with the FF-5 model will contribute to reduce or abolish the existence of the low volatility anomaly in the Norwegian stock market.

Our findings show evidence of a positive difference in excess return between the low volatility portfolio and the high volatility portfolio. However, the difference is not statistical significant, and we can therefore not prove the existence of the low volatility anomaly based on excess return differences alone. This applies for both value- and equally weighted portfolios and both measurements of volatility. Additionally, we observe that all long portfolios produce negative alphas. Yet, high volatility portfolios yield significantly lower alphas than low volatility portfolios when controlling for CAPM, FF-3 and FF-5. Based on the sign and magnitude on the difference portfolios' alphas we conclude that the low volatility anomaly is existent in the Norwegian stock market in our sample period.

On the contrary to what we initially expected, we find that controlling for additional systematic risk factors increase the estimated alphas of the difference portfolios. Lastly, we observe that low volatility stocks have the characteristics of large value stocks with relatively robust operating profitability, on the contrary high volatility stocks are associated with small growth stocks with relatively weak operating profitability.

The structure for the rest of this thesis is as follows. In section 2, we present previous literature regarding the low volatility anomaly in international markets as well as the Norwegian market. In the same section, we also present studies regarding the construction of the asset pricing models used in this thesis. Our hypothesis is stated in detail in section 3. Following this, section 4 provides an overview of the data extraction and filtering methods. The methodology behind the construction of the right-hand side and left-hand side portfolios, and robustness tests are shown in section 5, before the results are provided in section 6. Lastly, we conclude our findings in Section 7.

2 THEORY AND BACKGROUND

One of the fundamental foundations in traditional finance has been the concept of demanding increased expected returns for taking on additional risk. This relationship between risk and return has been heavily studied in the literature, but in later years it has been revealed that the relation is not as straightforward as first thought. The findings of Ang et al. (2006) shook up the balance and contrasted the earlier understanding of risk-reward, showing that the relation could in fact be negative. Section 2.1 presents literature on the Beta Puzzle by Haugen and Heins (1975) who early documented a negative relationship between risk and return based on beta covariance as the volatility measure. Section 2.2 covers literature on the anomaly based on idiosyncratic volatility and presents studies that have vastly different conclusions on the relation between idiosyncratic volatility and the cross-section of returns. Section 2.3 covers the Total Volatility Puzzle where the standard deviation of stocks is the proxy for volatility, section 2.4 continues by presenting studies conducted in the Norwegian stock market. In section 2.5, we discuss the literature regarding potential explanations behind the anomaly. Lastly, section 2.6 introduces the Fama-French three-factor model and the Fama-French five-factor model.

2.1 THE BETA PUZZLE

One of the first documentations of the negative relation between risk and realized return was found by Haugen and Heins (1975). They look at the relation between risk and return in the U.S. stock market, with the risk of a stock defined as the beta covariance with the market. Due to their focus on beta, this version of the anomaly has been named the Beta Puzzle. They find that in bull markets there is a positive covariance between risk and return, and vice versa in bear markets. Their results find no indication of a risk premium and show that in the long run the relationship between risk and return are actually negative when you take the market performance of the preceding ten-year period into account. They suggest that researchers who find a positive relationship, find it due to the fact that they look at sample periods with bullish markets without accounting for the nature of these markets. The Beta Puzzle was revisited by Frazzini and Pedersen (2014) who document a strategy betting against beta (long low beta stocks – short high beta stocks), which achieves a higher Sharpe ratio than both the value and momentum factors. They also show how leverage constraints can lead investors to overinvest in stocks with beta larger than one. They argue that this overweighting, by unlevered agents, toward high beta stocks causes risky high beta assets to yield lower risk-adjusted returns than low beta assets that would require leverage.

2.2 IDIOSYNCRATIC VOLATILITY

There is a lot of research done on the relationship between idiosyncratic volatility (IVOL) and returns, but the findings are not concrete as researchers find vastly different results. Bali and Cakici (2008) summarizes the different findings among research done on the positive relation between IVOL and returns. If investors are not able to hold a substantial number of assets in their portfolios it is shown theoretically by Levy (1978) that idiosyncratic volatility has an effect on equilibrium asset prices. Merton (1987) reaches the same conclusion and states that undiversified investors should not only care about market risk, but also total volatility if they are unable to hold the fully diversified market portfolio. In this case, stocks with higher total (or idiosyncratic) volatility should require a risk premium to compensate for the lack of diversification. Tinic and West (1986) and Malkiel and Xu (1997) support the theoretical findings of Levy (1978) and Merton (1987) with empirical research and find that high IVOL portfolios in fact achieve higher average returns. Malkiel and Xu (2002) also find that the relation between IVOL and the cross-section of expected returns is significantly positive at the firm-level. However, they do not use an individual stock's idiosyncratic volatility. Rather, they calculate the idiosyncratic volatility of 200 size-beta sorted portfolios and assign the IVOL of a portfolio to each of the stocks belonging in the respective portfolio as the residual standard deviation. Lehmann (1990) uses residual variance in the cross-sectional firm-level regressions. He finds that the idiosyncratic volatility coefficient changes sign in specific econometric specifications, but overall in his full sample-period he finds a significant positive coefficient.

Ang et al. (2006) finds a negative relation between idiosyncratic volatility and the cross-section of returns, and are the leading authors in finding the low volatility anomaly based on idiosyncratic volatility. They argue that if market volatility is a missing factor of systematic risk, models such as the CAPM and Fama-French three-factor model should fail to price portfolios sorted on idiosyncratic volatility. This because they do not include factor loadings proxying the risk of market volatility. As CAPM fails to explain cross-sectional stock returns they focus on idiosyncratic volatility measured relative to the three-factor model.

Portfolios are formed as L/M/N strategies based on an estimation period of L months, a waiting period of M months and a holding period of N months, focusing on a 1/0/1 strategy. This results in sorting stocks into quintile portfolios based on their idiosyncratic volatility estimated using daily returns for the past month, with no waiting period. The portfolios are value-weighted and held for one month before rebalancing. Their results are statistically significant and show that the lowest quintile stocks outperform the highest quintile stocks by 1.06% per month in their sample period (July 1963 to December 2000), contradicting traditional literature such as Merton (1987). The FF-3 alphas are also highly significant and the difference between the lowest and highest quintile is 1.31%. This shows that the Fama-French three-factor model fails to price the portfolios correctly. They are also able to identify patterns which show that, in general, stocks with low idiosyncratic risk are large stocks with low book-to-market ratios (value stocks). Whilst stocks with high idiosyncratic risk are small stocks with high book-to-market ratios (growth stocks). They conduct robustness tests on their results for cross-sectional effects that have shown to be potential risk factors or anomalies. Their results turn out to be robust after controlling for size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, coskewness, dispersion in analyst's forecasts, momentum effects and different formation- and holding periods up to one year. Their results also show to be robust in different subsamples during their original time-period, in bull and bear markets, NBER recessions and expansions, and in volatile and stable periods.

Ang et al. (2009) is a follow-up paper where they look at the Idiosyncratic Volatility Puzzle as an international phenomenon, and thus increases the likelihood of an underlying economic source for the anomaly. In all G7 countries the pattern is the same as for the U.S. data and they find strong results in 23 other developed markets – including Norway. They sort stocks across all countries on past idiosyncratic volatility and find a statistically significant difference in FF-3 alpha between the lowest quintile portfolio and the highest quintile portfolio of 1.31% per month. These international results of low IVOL stocks outperforming high IVOL stocks suggests that the findings in Ang et al. (2006) is not a sample specific or country specific effect. The discovery they label as perhaps most interesting, is the fact that the negative spread between stocks with high and low idiosyncratic volatility in the international markets strongly co-move with the difference in returns between stocks in the U.S. market with high and low idiosyncratic volatility. In fact, the international IVOL effect is captured by a U.S. IVOL factor, making the alphas insignificant. Their finding of this co-movement suggests evidence of broad factors behind the phenomenon that are not easy to diversify against. They also conduct further research on the U.S. data and test for a set of new factors; trading or clientele structures, higher moments, information dissemination, the leverage interaction story of Johnson (2004), and future exposure to IVOL. None of these factors achieves to explain the low volatility anomaly and, according to the authors, the search for true economic sources for the phenomenon remains a puzzle.

The effect of idiosyncratic risk is highly debated and a study by Fu (2009) show that idiosyncratic volatility varies significantly over time. Therefore, he suggests that research which fails to identify a positive IVOL effect does so because the conditional idiosyncratic risk used in their studies does not capture the time-varying property. He argues that the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) model is a more suiting estimate of conditional IVOL than the lagged realized IVOL used by Ang et al. (2006). Fu (2009) uses monthly data to provide in-sample estimates of the conditional idiosyncratic variance of stock returns based on the EGARCH model of Nelson (1991). His results indicate the existence of a significantly positive relation between IVOL and returns. Later on, Guo, Kassa, and Ferguson (2014) analyses the method used by Fu (2009) and show that his results are driven by a look-ahead bias and state that an EGARCH idiosyncratic volatility methodology requires caution.

Bali and Cakici (2008) highlight the varying choices of methodology as the reason behind the mixed conclusions regarding the relation between idiosyncratic volatility and the cross-section of expected returns. They apply different methodologies on two different samples (NYSE/ AMEX/NASDAQ and NYSE) and their results suggest that there is no robust evidence for a significant relation between the two. In their main analysis, using the NYSE/AMEX/NASDAQ sample Bali and Cakici (2008) have the following findings. When idiosyncratic volatility is estimated using daily data over the previous month, there is a negative and significant cross-sectional relation between risk and return only when the value-weighted portfolios are constructed on the CRSP breakpoints. This is in line with the research of Ang et al. (2006). However, they find no significant relation using different breakpoints based on NYSE or a 20% market share, nor when using equally weighted or inverse volatility-weighted portfolios. Furthermore, when idiosyncratic volatility is estimated using monthly data over the past two to five years they reach the same conclusion of no relation, regardless of weighting scheme or choice of breakpoint. Lastly, they find that focusing on differences in FF-3 alphas only shows a significant negative relation if the value-weighting scheme is used with CRSP breakpoints, and insignificant for the other methodologies. In order to check whether small and illiquid stocks are driving the results of Ang et al. (2006) they apply a filter excluding the smallest, lowest, priced and least liquid stocks. Their results show a statistically significant negative relationship between IVOL and the cross-section of expected returns on the small-stock portfolio and insignificant results for the large-stock portfolio.

2.3 Total volatility

Blitz and van Vliet (2007) recognize that efficient market theory has been challenged by the findings of return premiums such as size, value and momentum strategies. They create decile portfolios based on a three-year ranking of stocks' historical return volatility and find a clear volatility effect. Their findings show that portfolios created from stocks with the lowest historical volatility have significantly higher risk-adjusted returns (Sharpe ratio of 0.72) compared to the market portfolio (Sharpe ratio of 0.40). On the contrary, stocks with the highest historical volatility underperform compared to the market portfolio (Sharpe ratio of 0.05). The effect is documented in both international and regional markets and the alpha spread between the top versus bottom decile portfolio of 12% per annum for large-cap stocks on a global basis. They control for size, value and momentum effects using both global and local Fama and French (1993) regressions with double sorting, and discover the volatility effect as a separate effect of comparable magnitude to the more recognized effects. Their results show that low volatility portfolios are consistent with low beta stocks in how they underperform the market during up-market months, while outperform in down-market months. The underperformance is less than the outperformance, but they state that this effect is slightly countered by the fact that there are more up-months in total. In addition, they find that the low volatility portfolio yields a large reduction in maximum drawdown statistics. In their regional study of the effect they find evidence that the low volatility strategy may be able to avoid bubbles, as it seems to avoid both of the two main bubbles in their sample period (the Japan bubble and the dot-com bubble).

Baker and Haugen (2012) cover stocks in 21 developed countries and 12 emerging markets, among these is Norway represented, in the period 1990 to 2011. They estimate the volatility of total return for each company in each country over the previous 24 months. Stocks are ranked in each country by volatility and formed into deciles. They find that past volatility is a good predictor for future volatility and that low volatility stocks outperform in their global universe and in each individual country. Their results show that in some countries the low volatility portfolio outperforms the high volatility portfolio by almost 25%. The difference is even larger when risk-adjusted, with differences in Sharpe ratio of over 75% in some countries (>175% in Germany). It is important to mention that their Sharpe ratios are computed as the ratio of average return to standard deviation, and not excess returns due to not having risk-free rates for all countries.

2.4 EVIDENCE OF THE LOW VOLATILITY ANOMALY IN NORWAY

There have already been done empirical studies to test for the existence of the low volatility anomaly solely in the Norwegian stock market. The first study was done by Hafskjær and Østnes (2013) using data from 1981 to 2012 from the Oslo Børs Information Financial Database. They estimate idiosyncratic volatility based on Ang et al. (2006)'s approach and total volatility on a similar manner to Baker and Haugen (2012). Their findings show that when controlling for size, value, liquidity and momentum there is no idiosyncratic volatility puzzle in Norway. The authors argue that this conclusion holds when testing for different subsamples, methodological measure of volatility, industry exposure and data filters. A more recent study by Arnesen and Borge (2017) use data from Børsprosjektet NHH's database Amadeus in the period from January 1987 to December 2016. When utilizing a rolling window model to estimate IVOL they find evidence for the low volatility anomaly in Norway. As their focus is on the potential explanations for the anomaly, they further argue that the low average returns of high volatility stocks can be explained by firm characteristics (size, skewness and illiquidity) and that the low volatility anomaly can be explained by short-term reversals. They also argue that the existence of the low volatility anomaly is no longer present when using a GARCH model to estimate volatility. In the same year Bakøy and Huskic (2017) did a similar study, where they test the existence of the low volatility anomaly using stock data from Bloomberg in the period 1990 to 2016. They focus on idiosyncratic volatility and find that the anomaly is present in their sample, for both value- and equally weighted portfolios. It should be noted that their findings show that low volatility stocks outperform high volatility stocks but fail to outperform the market on a risk-adjusted basis, i.e. produce a negative FF-3 alpha.

2.5 POTENTIAL EXPLANATIONS BEHIND THE ANOMALY

Baker, Bradley and Wurgler (2010) believe that there are two reasons behind the low volatility anomaly; less than fully rational investor behavior, and underappreciated limits to arbitrage. They suggest that well-documented behavior such as the preference for lotteries, the representativeness bias, and the overconfidence bias leads individual investors to have an irrational demand for volatile stocks.

Preference for lotteries is a theory rooted in a study by the psychologists Kahneman and Tversky (1979). Baker, Bradley and Wurgler (2010) suggest that investors would not invest in high volatility stocks in fear of realizing a loss. When probabilities are evenly distributed between a loss of $100 and a gain of $110, most people would not take the bet, even though the expected payoff is positive. They continue describing how probability shifting changes loss aversion considerably. When investors are introduced to a choice between an almost certain small loss and an unlikely high pay-off most people would take the gamble, they illustrate this by introducing a new bet with the same expected payoff of $5. Now the investors get the choice between an almost certain loss of $1 and a 0.12% chance of winning $5,000. Even though the expected payoff is the same in both examples, people seem to be more willing to take their chances when introduced to a positive skewed gamble. Baker, Bradley and Wurgler (2010) state that buying a highly volatile stock with a low price is in principle the same as buying a lottery ticket, there is a small chance of the stock multiplying in value, and a higher chance of the stock losing value. This is backed up with evidence by Kumar (2009), who shows that some individual investors clearly prefer stocks that provide lottery-like payoffs. He finds them to invest much more money into low priced stocks with high idiosyncratic volatility and skewness, rather than investing in less risky stocks. Blitz and van Vliet (2007) also tie the preference for lotteries into their study. They look at the two-layer behavioral portfolio theory of Shefrin and Statman (2000), which identifies private investors to have a low aspiration layer designed to avoid poverty, and a high aspiration layer designed for a shot at getting rich. They suggest that investors will overpay for volatile stocks, as they are perceived as a chance of achieving big payoffs. This irrational behavior and deviation from risk-aversion may cause overpricing in high volatility stocks, and on the contrary, underpricing in low volatility stocks that could play a part in explaining the low volatility anomaly.

Representativeness is another concept by Kahneman and Tversky (1972). They show that individuals estimate probabilities based on the sample they have seen or experienced for themselves, and therefore believe to be more representative for the full population, and not on the actual characteristics of the full population. This is relevant to the low volatility anomaly when it comes to how investors define ‘good investment possibilities’. Baker, Bradley and Wurgler (2010) illustrate this with an example of how two individuals make different decisions based on their knowledge. An individual with no professional background draws the conclusion that buying speculative shares in Microsoft upon their IPO in 1986 and generally buying new technology stocks is the way to riches, blatantly ignoring the high base rate at which small and speculative investments fail. This leads the individual to overpay for volatile stocks, while an individual with a background in quantitative analysis would stay away from speculative technology stocks unless he could distinguish Microsoft from the many downfalls seen in his sample size.

Overconfidence in forecasting also plays a role in how individuals value stocks. Baker, Bradley and Wurgler (2010) tie the psychology studies of Fischhoff, Slovic, and Lichtenstein (1977), and Alpert and Raiffa (1982) on individuals' overconfidence into their reasoning behind how individuals prefer volatile stocks. They state that overconfident investors are more likely to disagree with forecasts and are inclined to sticking with their own (false) estimates. They also find the disagreement to increase parallel to the uncertainty of the outcome, leading to a wide range of opinions among investors on matters like defining growth stocks. Lastly, they note that for overconfidence to increase the demand for volatile stocks, pessimists must act less aggressively than the optimist in the market. For these optimists to set the price in the market there must also be a general reluctance or inability to short stocks in comparison to buying stocks. This is not an unrealistic assumption as short selling often comes with additional costs compared to the classical long position, also the inability and reluctance to short stocks is well-documented through empirical work. All of the above then leads stocks with a wide range of opinions (higher volatility) to have more optimists among their shareholders and therefore selling for higher prices, resulting in lower future returns.

Assuming these behavioral biases explain the reason for overpriced high volatility stocks among individuals does not cover the complexity of the low volatility anomaly. If it did, it should also explain why institutional investors do not take advantage of the low volatility anomaly, even when using sophisticated models and investing strategies. Baker, Bradley and Wurgler (2010) present several issues that hinder institutional investors in short selling the poor performing high volatility quintile. Firstly, the top quintile consists of small stocks that are costly to trade in large quantity, increasingly so for short selling. In addition, the number of shares available for borrowing is restricted and often comes with high borrowing costs. Secondly, they introduce the nature of benchmarking and the maximization of information ratio (IR) relative to the benchmark without using leverage. For an institutional investor to take advantage of the high returns from low volatility stocks the alpha must be substantial to make up for the loss of IR that comes with overweighting stocks with beta less than one. As an investor's performance is often measured by the IR relative to his benchmark, he would most likely not take advantage of such possibilities. Further, Baker, Bradley and Wurgler (2010) present a solution to the benchmark problem by introducing leverage to take advantage of the low volatility anomaly, however, most mutual funds are not allowed to use leverage in their portfolios. Therefore, in order for an institutional investor to underweight high volatility stocks with beta larger than one, the alpha would have to be substantially negative. They claim that investment managers are mostly concerned with exploiting mispricing in stocks close to market risk, beta around one, and overweight (underweight) positive (negative) alphas. This means that as the beta increases (decreases) the alpha has to decrease (increase) in order for the manager to underweight (overweight) these types of stock. This relates directly to the low volatility anomaly and shows that low risk is underpriced and high risk is overpriced, even in the institutional investors' eyes. Stambaugh, Yu and Yuan (2015) supports these findings regarding shorting constraints. Their results show that stocks with high idiosyncratic volatility has higher chances of being mispriced, and due to the constraints faced when being willing to short the risky stocks an arbitrage asymmetry occurs. This arbitrage asymmetry leads high IVOL stocks to be mispriced and to stay mispriced longer than low IVOL stocks, thus flattening the relation between risk and return.

2.6 Fama-French models

2.6.1 Fama-French three-factor model

Fama and French (1992) set out to build an asset pricing model designed to account for several of the contradictions to the CAPM. Among these are the size effect found by Banz (1981) who shows that a stock's market equity helps explain the cross-section of average returns provided by market beta. His main findings are that small stocks have too high average returns relative to their beta estimates, and vice versa for large stocks. Another contradiction to CAPM is that average returns of stocks are positively related to a firm's book-to-market (*B/M*) ratio documented by Stattman (1980), and Rosenberg, Reid and Lanstein (1985). Further, Basu (1983) finds earnings-to-price ratios to have explanatory power on the cross-section of average returns in the U.S. stock market. Lastly, Bhandari (1988) find leverage and average returns to be positively related. He shows that when leverage is tested alongside size and market beta, leverage helps explain the cross-section of average stock returns.

Fama and French (1992) argue that all these contradictions are different variations of scaling stock prices. They therefore set out to evaluate the joint roles of market beta, size, earnings to price, leverage and book-to-market equity in the cross-section of returns on NYSE, AMEX and NASDAQ stocks in the period 1962 to 1989. Their findings do not support the CAPM and they conclude that average stock returns are not positively related to market betas. However, using Fama-MacBeth (1973) regressions their main findings are that the two factors size and book-to-market equity capture the cross-sectional variation in average stock returns associated with size, earnings-to-price, book-to-market equity and leverage. This is because book-to-market equity seems to capture the effects of leverage, and the relationship between earnings and price are explained by the combination of size and B/M ratio. Fama and French (1993) expand on their 1992 findings and conclude the FF-3 model for stock returns containing a value-weighted market portfolio (*MKT*), the size factor (*SMB*) and the book-to-market equity factor (*HML*) to explain the cross-section of stock returns. *SMB* is the difference in average returns between small and big stock portfolios with identical weighted-average of book to market equity, while *HML* is the difference between the average returns of portfolios containing firms with high *B/M* ratios and low *B/M* ratios regardless of size.

2.6.2 Fama-French five-factor model

In 2015 Fama and French (2015a) published their five-factor model where they include two additional factors in which they call operating profitability (*RMW*) and investment (*CMA*). They show that the five-factor model explains the cross-section of returns better than the three-factor model of Fama and French (1993). Their choice of adding investment and profitability to the three-factor model to form the five-factor model, is based on Miller and Modigliani (1961) who show that the market value of a firm's stock at time t, is implied by

$$\mathrm{M}_t = \sum_{\tau=1}^{\infty} \frac{\mathrm{E}(\mathrm{Y}_{t+\tau} - \mathrm{dB}_{t+\tau})}{(1+\mathrm{r})^{\tau}} \tag{2.1}$$

Where, $Y_{t+\tau}$ is equity earnings in the period $t + \tau$, $dB_{t+\tau}$ is the change in book equity, and r is the internal rate of return on expected cashflows to shareholders, used as a proxy for the long-term expected stock return. They then show that dividing by book equity at time t, gives the following equation (2.2)

$$\frac{\mathrm{M}_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \mathrm{E}(\mathrm{Y}_{t+\tau} - \mathrm{dB}_{t+\tau})/(1+\mathrm{r})^{\tau}}{B_t} \tag{2.2}$$

With equation (2.2) they show that both an increase in the book-to-market equity ratio, and an increase in expected earnings implies higher expected returns. It also shows that an increase in expected growth in book equity implies a lower expected return. The fact that higher expected earnings are due to higher expected profitability, and growth in book equity occurs when a firm invests, proves that operating profitability and investment are both natural choices to be added to their evolving asset pricing model.

Fama and French (2015a) show that the value factor is redundant and explained by the combination of the operating profitability and the investment factors for U.S. data in the 1963-2013 period. They therefore conclude that in applications where the interest is solely to evaluate abnormal returns, the four-factor model (*HML* excluded) performs as well as the five-factor model. However, the five-factor model is a useful tool if one also has the interest of estimating the factor loadings to size, value, operating profitability, and investment premiums. An alternative is to construct a *HMLO* (orthogonal *HML*) factor, which is defined as the sum of the intercept and residual from a regression of *HML* on the remaining four factors in the five-factor model. This is done by Fama and French (2015b) where they dissect anomalies using their five-factor model for the U.S. stock market. In this paper they study the implication of the FF-5 on anomalies regarding market beta, net share issues, volatility, accruals and momentum. When constructing volatility portfolios Fama and French define IVOL equal to Ang et al. (2006)'s method. However, they use 60 days of lagged returns instead of 30 days lagged returns to estimate both idiosyncratic and total volatility. Their study looks at the U.S. stock market in the period of July 1963 to December 2014. Focusing on the idiosyncratic volatility measure, Fama and French show that stocks with positive (negative) exposure to the operating profitability and investment factors explain the high (low) average returns of stocks with low (high) volatility. In other words, by controlling for additional factors their findings demonstrate that the abnormal returns are reduced as low volatility stocks are associated with firms that have relatively robust operating profitability and a conservative investment approach. The five-factor model, however, fails to completely capture average returns and the low volatility anomaly is still present in their study.

3. HYPOTHESIS

In this thesis we investigate the existence of the low volatility anomaly for the Norwegian stock market in the time period August 1993 to December 2017. More specifically, we evaluate whether the low volatility anomaly can be explained by the FF-5 model. This is partly because the low volatility investment strategy at certain moments overlap with the value investment strategy (Blitz, 2016), and the FF-5 model augments the FF-3 model with the operating profitability and investment factors. Both of these factors are related to the value factor (Fama and French, 2015a) as shown and explained in subsection 2.6.2. Another reason for why we find this approach interesting and relevant is the findings of Fama and French (2015b) where they dissect the low volatility anomaly using the FF-5 model. Our approach is the following. We start by constructing three asset pricing models: CAPM, FF-3 and FF-5. These are referred to as right-hand side portfolios as they are used as regressors on the excess returns of the volatility portfolios in order to test for abnormal returns (alphas). The volatility portfolios are referred to as left-hand side portfolios, and we make two sets of portfolios based on two different measurements of volatility; idiosyncratic volatility and total volatility. In order to measure the idiosyncratic volatility for each stock we regress excess stock returns on the FF-3 model over a 24-month trailing window. Idiosyncratic volatility is then defined as the standard deviation of the error terms from these regressions. Total volatility, however, is defined as the standard deviation of each stock's return over the same trailing window. The construction of the left-hand side portfolios is done by sorting stocks into five quintile portfolios, ranging from lowest to highest volatility, based on both volatility measures individually. Further, we estimate average excess return, standard deviation, calculate Sharpe (1966) ratio and market share for each quintile portfolio. This is also done for the difference portfolio[1], which is given by the lowest minus the highest quintile portfolio. In addition, we regress the excess portfolio returns on all three asset pricing models, allowing us to evaluate the estimated alphas and factor loadings.

In order to test for the existence of the low volatility anomaly we conduct two different tests of statistical significance. In the first test we evaluate whether the excess return between the extreme portfolios (lowest and highest quintile portfolio) is statistically significant different

[1] Throughout the thesis the terms difference portfolio, long-short portfolio and Q1-Q5 will be used interchangeably.

from each other. The second test evaluates, for every asset pricing model, if the estimated alphas for the difference portfolio is statistically significant different from zero. Our conclusion on whether we find the existence of the low volatility anomaly, for both tests, is based on estimated robust *t*-statistics[2] (Newey and West, 1987). Robust *t*-statistics are also used in order to evaluate the factor loadings and characteristics of the different portfolios.

According to classical economic theory, we would expect to find no relation between idiosyncratic volatility and return, or if we find a relation, it should be positive. Meaning that if investors are not fully diversified and take on more idiosyncratic volatility, they should be compensated for this in terms of higher expected returns. However, our hypothesis differs from the classical view as several studies document the low volatility anomaly, also for the Norwegian stock market, when controlling for the CAPM and the FF-3 model. Therefore, our hypothesis is to find a negative relation between risk and return in the Norwegian stock market when controlling for the systematic risk factors given in the CAPM and FF-3. Meaning that we expect to find the existence of the low volatility anomaly. However, when controlling for the FF-5 model we expect to find (if not an abolishment) at least a reduction in the low volatility anomaly. This is based on the empirical findings of Fama and French (2015b), who find the abnormal returns to decrease when controlling for additional systematic risk factors (given in the FF-5). Nevertheless, it is not given that the findings of Fama and French (2015b) are true in our data sample.

[2] Using a two-sided t-test we require a significance level of five percent for our results to be statistically significant, i.e. an estimated robust t-statistics above 1.96.

4. DATA

In the following section we describe both our stock and accounting dataset used to construct the Fama-French five-factors and the volatility portfolios. We explain how our data was obtained and introduce the different filtering methods applied to clean up the samples. Lastly, we describe the combined dataset containing the needed elements from both samples. The technical uses and calculations in the dataset is explained in the methodology (Section 5).

4.1 STOCK SAMPLE

We obtain monthly stock data for the time-period January 1990 to December 2017 from Amadeus 2.0, this is NHH Børsprosjektet's client which provides financial data from the Oslo Stock Exchange through Oslo Børs Informasjon. The following variables are extracted: *TradeDate, SecurityID, Symbol, ISIN, SecurityName, SecurityType, IsStock, Last (Price), AdjLast (Price)* and *ShareIssued*. Monthly NIBOR rates are used as a proxy for the risk free-rates throughout our time period and are obtained from Ødegaard's database[3].

4.1.1 FILTERING THE STOCK SAMPLE

In order to prepare the stock data for our analysis we apply various filtering methods. The focus of this thesis is on common stocks, and therefore we start by filtering out all other security issues. Further, all observations in our dataset in which a company has zero shares issued are removed. We follow the example of Ødegaard (2018) and continue our filtering by excluding penny stocks. This is because small absolute changes in price for low priced stocks can lead to extreme relative changes in returns. We define penny stocks as stocks worth less than NOK 1, unlike Ødegaard (2018) who define them as stocks with a price less than NOK 10. We choose to ease up on the constraint to keep more observation in our dataset as the NOK 10 requirement led to a reduction in observations of 33%. This is also in line with the Oslo Børs (2018) delisting rules, which states the following: "the market value of a stock shall not be less than NOK 1, if it is lower than this for a period longer than 6 months the board shall take action to satisfy the requirement". Defining penny stock as less than NOK 1 also reduces our chances of ignoring survivorship bias. A similar filtering is done with market capitalization, calculated as *last stock*

[3] http://finance.bi.no/~bernt/financial_data/ose_asset_pricing_data/index.html

price times *total shares issued*, here we follow Ødegaard (2018) and require a stock to have a total value of market capitalization of minimum NOK 1 million.

4.1.2 Return Calculations

Simple returns are calculated on a monthly basis by extracting end of month adjusted last prices, which are adjusted for dividends, stock splits and other corporate events.

$$r_t^i = \frac{AdjLast_t^i - AdjLast_{t-1}^i}{AdjLast_{t-1}^i} \tag{4.1}$$

To make sure that the price observations are not too old, end of month observations are defined as adjusted last price observations within the last six days of a month. We choose six days as we see from our observations that this is a re-occurring breaking point. In order to avoid extreme return observations affecting our results the top and bottom 0.1% quantile of returns are trimmed before we apply the filters described in subsection 4.1.1. This reduces our period's maximum and minimum monthly return observation from 796.00% and -95.71% to 139.43% and -72.88% respectively.

4.2 Accounting sample

Accounting data for Norwegian firms in the period 1989 to 2017 are obtained from the Compustat Global database through Wharton Research Data Services[4]. Extracted accounting variables are showcased in table 4.1. The total dataset contains accounting data for 371 companies over the period, however the number of annual observations vary as Compustat was less comprehensive on Norwegian data before 1997.

4.2.1 Filtering accounting data

In order to construct the Fama-French five-factor model we clean up and filter our dataset. As this thesis follows the methodology of Fama and French (2015a), financial firms (SIC between 6000 and 7000) are excluded from our sample. They exclude financial firms due to the abnormal amount of leverage in their capital structure, which in non-financial firms would most likely indicate financial distress. Not excluding these types of firms could therefore bias our results

[4] https://wrds-web.wharton.upenn.edu/wrds/

as they could get categorized as something they are not. To assure that we only keep Norwegian listed firms, and not Norwegian firms traded on foreign exchanges, we exclude all firms not listed with the Oslo Stock Exchange code 228 and 229 (over the counter). Lastly, we exclude firms with negative or zero book equity and firms with zero assets. This is to avoid letting extreme observations or insufficient reporting distort our results as they are used to calculate operating profitability and investment respectively. Not excluding negative book equity observations would lead to categorizing a firm with negative operating profit as a firm with positive operating profitability, see equation (5.3). Furthermore, not excluding zero assets observations would distort the investment measure strictly by the lack of observation.

TABLE 4.1: EXTRACTED VARIABLES FROM COMPUSTAT

Variable	Description
Identification variables	
datadate	Date
conm	Company name
gvkey	Global Company Key
exchg	Stock Exchange Code
fic	Incorporation Country Code
sic	Standard Industrial Classification Code
ISIN	International Securities Identification Number
Accounting variables	
curcd	Currency Code
fyear	Fiscal Year
fyrc	Current Fiscal Year End Month
at	Total Assets
cogs	Cost of Goods Sold
lt	Total Liabilities
revt	Total Revenue
seq	Stockholders' Equity
txdb	Total Deferred Taxes
txditc	Deferred Taxes and Investment Tax Credit
xint	Total Interest Expense
xintd	Interest Expense Long-Term Debt
xopr	Total Operating Expenses
xopro	Other Operating Expenses
xsga	Selling, General and Administrative Expense

4.3 COMBINING THE DATA

In order to calculate the five factors in the FF-5 model the stock and accounting data is combined into one dataset. This procedure reduces our number of observations as we impose a restriction where an observation needs to have both stock and accounting data to be included in the construction of the FF-5 model. We merge market capitalization from the stock data with book equity, profitability and investment from the accounting data, this enables us to calculate book-to-market ratios. As a result, our final dataset contains factor data for a total of 236 firms over our period.

It is worth to mention that in our stock sample financial institutions are included in contrast to the filtering of the accounting sample. This is done to keep our investment universe as complete as possible but at the same time keeping the Fama-French five-factors unaffected from the characteristics of the capital structure in these institutions. In the end our total investment universe includes 607 stocks.

5. METHODOLOGY

In this section, we will go through the definition of the factors, construction of the right-hand side (RHS) portfolios, the different measures used to estimate risk and construction of the left-hand side (LHS) portfolios. Figure 5.1 at the end of subsection 5.2.2 showcase an intuitive overview of the portfolio construction for both the RHS and LHS portfolios. Lastly, the end of this section provides the methodology used to test the robustness of our results.

5.1 THE RIGHT-HAND SIDE PORTFOLIO – THE FIVE-FACTOR MODEL

We construct the Fama and French five-factor model (2015a) where the market, size and value factors are known from the Fama and French three-factor model (1993) and the additional factors are operating profitability and investment. Given our data sample we have to make some adjustments, however, if not stated otherwise we follow Fama and French (2015a)'s method when constructing the factors.

5.1.1 VARIABLE DEFINITION

The size variable is given by the market capitalization of each individual firm, defined as *last closing price* times *number of shares issued*, see equation (5.1).

$$Size_t = Market\ Cap_t \tag{5.1}$$

The book-to-market factor is the ratio of book equity to market capitalization, where book equity is given at the end of the fiscal year ending in year t-1 and market capitalization at the end of December in year t-1. Since not all firms end their fiscal year in December there may be some time difference, still we ignore this potential time difference as Fama and French (1992) find that correcting for this does not affect their results significantly. Book equity is defined as the sum of *stockholders' equity* and *deferred taxes* from the balance sheet. However, in our sample there are two observations with missing values for *stockholders' equity* and in both cases we use *total assets* minus *total liabilities* as a proxy for stockholders' equity.

$$B/M_t = \frac{Book\ Equity_{t-1}}{Market\ Cap_{Dec.\ t-1}} \tag{5.2}$$

Fama and French (2015a) define operating profitability at the end of June in year t as *total revenue (revt)* minus *cost of goods sold (cogs)* minus *selling, general, and administrative expenses (xsga)* minus *total interest expense (xint)*, all divided by book equity and all from accounting data at fiscal year ending in year t-1. As a consequence of missing values regarding *cost of gods sold* and *selling, general and administrative expenses values*, the variable *aggregate total operating expenses (xopr)* is used instead, as shown in equation (5.3).

$$OP_t = \frac{revt_{t-1} - xopr_{t-1} - xint_{t-1}}{Book\ Equity_{t-1}} \tag{5.3}$$

The last factor, investment, is defined in year t as the difference in *total assets (at)* from the fiscal year end in year t-1 and year t-2 over *total assets* from the fiscal year end in year t-2, see equation (5.4).

$$Inv_t = \frac{at_{t-1} - at_{t-2}}{at_{t-2}} \tag{5.4}$$

5.1.2 FACTOR SORTING AND BREAKPOINTS

In their three-factor model Fama and French (1993) use a 2 x 3 sorting mechanism to construct the factors, where they sort stocks independently into two size groups and three value (book-to-market) groups. They point out that the 2 x 3 sorting was arbitrary and therefore revise their sorting mechanism by comparing it with two alternative methods in Fama and French (2015a). However, they conclude that the two alternative sorting methods are not significantly better than the 2 x 3 approach, and we therefore follow this method when constructing the five-factor model for the Norwegian stock market.

An illustration of the 2 x 3 sorting approach with breakpoints and factor construction is provided in table 5.1. We sort independently on two *Size* groups where the sample median is the breakpoint and three independent sorts where breakpoints is given by the 30^{th} and 70^{th} sample percentile for the *B/M*, *Op* and *Inv.* As shown in column 2 in table 5.1 the *SMB* factor (small minus big) is given by the average of the three different portfolios constructed by first sorting on *Size* and then a second sort on *B/M*, *OP* and *Inv* individually. The intuition behind this construction mechanism is to make the *Size* factor independent of firm value, operating profitability and investment. Further, the *HML* factor (high book-to-market minus low book-to-market) is given by the difference in average return of the two high *B/M* portfolios and the average return of the two low *B/M* portfolios. We use a similar approach to construct the *RMW* (robust minus weak operating profitability) and the *CMA* (conservative minus aggressive investment), but where the second sort is operating profitability and investment, respectively. Based on this we end up with four different portfolios which are four of the five factors in the FF-5 model. The fifth and last factor, the market (*MKT*), is constructed as the monthly excess return over risk-free rate for the value-weighted average of all the stocks in our sample based on market capitalization. The FF-5 model (RHS portfolio) is constructed at the end of June in year t and held for one year before rebalancing.

TABLE 5.1: 2 X 3 SORTS ON *SIZE* AND *B/M*, *SIZE* AND *OP*, AND *SIZE* AND *INV*.

Factor breakpoints and construction. The portfolios are labeled with two letters referring to their sorting, where the first one refers to Size group, small (S) or big (B), the second letter refers to the second independent sorting mechanism: high (H), neutral (N) or low (L) book-to-market, robust (R), neutral (N) or weak (W) operating profitability, conservative (C), neutral (N) or aggressive (A) investment. SMB (small minus big), HML (high minus low B/M), RMW (robust minus weak operating profitability) and CMA (conservative minus aggressive investment) are the factor portfolios.

Breakpoints	Factors and their components
Size: Sample median	$SMB = (SMB_{BM} + SMB_{OP} + SMB_{Inv}) /3$
	$SMB_{B/M} = (SH + SN + SL)/3 - (BH + BN+ BL)/3$
	$SMB_{OP} = (SR + SN + SW)/3 - (BR + BN+ BW)/3$
	$SMB_{Inv} = (SC + SN + SA)/3 - (BC + BN+ BA)/3$
B/M: 30th and 70th sample percentiles	HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2
OP: 30th and 70th sample percentiles	RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2
Inv: 30th and 70th sample percentiles	CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2

5.2 LEFT-HAND SIDE PORTFOLIOS – VOLATILITY PORTFOLIOS

5.2.1 IDIOSYNCRATIC VOLATILITY

In our analysis we estimate idiosyncratic volatility based on the error terms from the Fama-French three-factor model, which is consistent with the method of Ang et al. (2006).

$$r_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{i,t} MKT_t + s_{i,t} SMB_t + h_{i,t} HML_t + \varepsilon_{i,t} \tag{5.5}$$

Where $r_{i,t}$ in equation (5.5) is the return of stock i at time t, we generate the excess return by subtracting the risk-free rate (r_f). The FF3-alpha ($\alpha_{i,t}$) is the pricing error, MKT_t is the excess market return, SMB_t is the excess return of small minus big stocks, HML_t is the excess return of high *B/M* minus low *B/M* stocks, and ε_t is the error term. Factor loadings for stock i at time t is given by $\beta_{i,t}$, $s_{i,t}$ and $h_{i,t}$ for *MKT*, *SMB* and *HML,* respectively.

Instead of using daily returns similar to Ang et al. (2006), we use monthly returns in order to estimate IVOL. The reason for this is that Bali and Cakici (2008) find monthly returns instead of daily returns to provide a better characterization of expected future volatility and to be a more robust estimate. This also leads us to adopt their 24-month trailing window estimation of IVOL. More specifically, we estimate IVOL at time t based on the standard deviation of the error terms from the FF-3 model using the preceding 24 months, where we require return data for at least 12 months. The estimation given a formation period of N months is shown in equation (5.6).

$$\widehat{IVOL}_t = \sqrt{\frac{1}{N-1}\sum_{k=0}^{N-1}(\varepsilon_{t-k} - \bar{\varepsilon})^2} \tag{5.6}$$

5.2.2 Total volatility

In the estimation of total volatility, we follow Baker and Haugen (2012)'s methodology where we use a 24-month trailing window with the same requirements of return data as in the case with IVOL. However, we now look at the standard deviation of the stock return and do not control for any systematic risk. Given a formation period of N months the estimation of TVOL at time t is given in equation (5.7), where *R* represents the excess return over the risk-free rate.

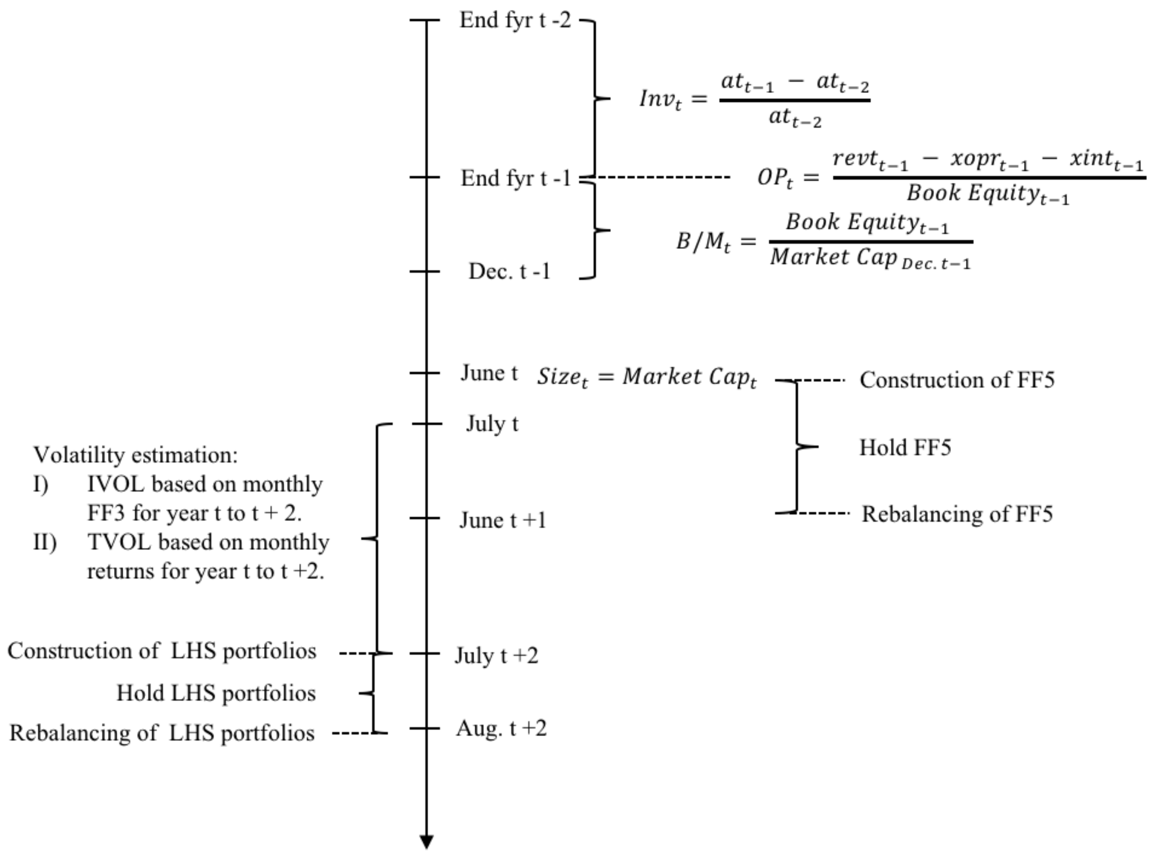
$$\widehat{TVOL}_t = \sqrt{\frac{1}{N-1}\sum_{k=0}^{N-1}(R_{t-k} - \bar{R})^2} \tag{5.7}$$

5.2.3 Construction of the left-hand side portfolios

At the end of each month, we sort all sample stocks on IVOL and TVOL into quintile portfolios ranging from lowest to highest volatility. We label the lowest volatility portfolio Q1 and the highest volatility portfolio Q5. To test for the low volatility anomaly, we also construct a long-short portfolio that involves buying Q1 and selling Q5. This enables us to evaluate the return difference between the extreme portfolios. Each portfolio is held for one month and we calculate both the value- and equally weighted excess return each month, where the value-weighting is based on market capitalization at the beginning of the holding period. This gives us the first formation period from July 1991 to July 1993 and we construct the first portfolios at the end of July 1993, which is held until the end of August 1993 before rebalancing. By repeating this procedure until December 2017, we end up with 293 months of return observations for each portfolio.

FIGURE 5.1: TIMELINE OF THE PORTFOLIO CONSTRUCTION FOR BOTH THE RIGHT-HAND SIDE (RHS) AND LEFT-HAND SIDE PORTFOLIOS (LHS)

The RHS portfolio is constructed at the end of June and held for one year before rebalancing. The size variable is defined as the market cap at June t. B/M is given by the ratio of book equity at fiscal year-end over market cap for December at time t-1. OP is given by total revenue minus aggregate total operating expenses and total interest expenses over book to equity. Inv at time t is given by the growth rate of investment from time t-2 to t-1. The RHS portfolio gives us the FF-3 model which is used to estimate monthly IVOL from year t to t+2. For the same time period we use monthly stock return to estimate TVOL. Based on these estimates we form LHS portfolios (quintile portfolios) at the end of July t+2 which are held for one month before rebalancing.



5.2.4 Evaluation of the portfolio returns

In order to evaluate the performance of the portfolios we estimate Sharpe ratios, which is given by the excess return over the standard deviation (Sharpe, 1966). We also evaluate abnormal returns for every quintile portfolio by controlling for systematic risk factors. This is done by regressing the excess portfolio returns on the CAPM, Fama-French three-factor model and Fama-French five-factor model. Equation (5.8) illustrates the regression of excess return for portfolio i at time t when controlling for systematic risk factors given by the Fama-French five-factor model.

$$r_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{i,t}MKT_t + s_{i,t}SMB_t + h_{i,t}HML_t + r_{i,t}RMW_t + c_{i,t}CMA_t + \varepsilon_{i,t} \quad (5.8)$$

5.3 Robustness tests

5.3.1 Alternative measurements of idiosyncratic volatility

In our analysis we have defined idiosyncratic volatility relative to the FF-3 model, which is consistent with previous studies, e.g. Ang et al. (2006). In order to test our result for robustness regarding choice of asset pricing model used to estimate idiosyncratic volatility, we conduct the same analysis where IVOL is defined relative to the CAPM and the FF-5 model. Based on this we can evaluate if the measurement of idiosyncratic volatility has an effect on our results, we exclude factors (size and value) in the CAPM and include factors (operating profitability and investment) in the FF-5 model relative to the FF-3 model.

5.3.2 Different subsamples

Our full sample analysis examines quintile portfolio returns in the period of August 1993 to December 2017. To account for the fact that our results may be driven by effects specific to our sample period, we conduct the same analysis for subsample periods. We divide our full sample data in two where the first subsample contains portfolio returns from August 1993 to December 2007 and the second subsample is from January 2008 to December 2017. We name the first subsample pre-financial crisis and the latter one post-financial crisis. In addition, we also test for the relation between volatility and returns since the new millennium, i.e. from January 2000 to December 2017.

5.3.3 Penny stocks

To control for robustness regarding our choice of filtering the raw data, we conduct the same analysis where we use a stricter definition of penny stocks. We initially defined penny stocks as stocks with (last) price below NOK 1 and in our robustness test we change our definition to less than NOK 5.

5.3.4 Return requirements

Initially we require a stock to have at least 12 months of return data in a 24-month trailing window to be included in the construction of the volatility portfolios. Using daily return observations Fama and French (2015b) require return observations for 20 days over a 60-day period when dissecting the low volatility anomaly. In a similar manner, we relax our constraint and require only eight months of return observations when using a 24-month trailing window, i.e. a return constraint of one-third.

6. RESULTS

6.1 EXCESS RETURNS, SHARPE RATIOS AND ABNORMAL RETURNS

6.1.1 PORTFOLIOS SORTED BY IDIOSYNCRATIC VOLATILITY

Table 6.1 reports results for value-weighted portfolios in panel A and equally weighted portfolios in table B. For both weighting schemes we observe no clear pattern on average excess return going from the low to the high volatility portfolio (Q1 to Q5). However, in both panels the average excess return falls dramatically for the highest volatility portfolio relative to the other portfolios. This gives a positive average excess return for the long-short portfolio of 0.46% per month for the value-weighted and 0.53% per month for the equally weighted portfolio, but as shown by the *t*-statistic in brackets the difference is not statistically significant. The standard deviation falls from the lowest quintile to the highest quintile portfolio, which is in line with our expectations. Interestingly, when portfolios are value-weighted the long-short portfolio performs poorly based on the Sharpe ratio measurement, whilst the equally weighted long-short portfolio yields the second-best Sharpe ratio. We also observe that the market share decrease going from portfolio Q1 to Q5, indicating a negative relation between size and volatility.

When controlling for systematic risk factors all long portfolios, surprisingly, reports negative alphas, however not all are statistically significant different from zero. Focusing on the difference portfolios, we observe that for both weighting schemes the long-short portfolio returns a positive alpha which is highly statistical significant for every model except the CAPM alpha in the value-weighted scenario[5]. We also note that the alphas for the long-short portfolios increase when controlling for more systematic risk factors. More specifically, for the value- (equally) weighted portfolio the alphas per month are 0.9% (1.0%), 1.7% (1.8%) and 1.8% (1.8%) when controlling for risk according to CAPM, FF-3 and FF-5 respectively. The reason for the increase in alphas when controlling for more factors is that the abnormal return for portfolio Q5 is more affected than the abnormal return of portfolio Q1. Even though our findings report negative alphas, they show that the lowest quintile portfolio sorted on IVOL has

[5] When portfolios are value-weighted and sorted by idiosyncratic volatility controlling for the CAPM yields a slightly insignificant alpha (at a five percent significant level) for the difference portfolio with a robust t-statistic (Newey and West, 1987) of 1.824.

TABLE 6.1: VALUE- AND EQUALLY WEIGHTED PORTFOLIOS SORTED BY IVOL

We form value- and equally weighted portfolios every month sorted by idiosyncratic volatility relative to FF-3 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust *t*-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.406	5.543	0.073	60.32	-0.005	-0.004	-0.004
					[-4.540]	[-3.431]	[-3.487]
Q2	0.372	7.318	0.051	18.80	-0.007	-0.009	-0.009
					[-2.104]	[-2.885]	[-3.007]
Q3	0.427	7.841	0.054	10.33	-0.006	-0.01	-0.01
					[-1.970]	[-3.277]	[-3.465]
Q4	0.882	9.992	0.088	6.40	-0.004	-0.009	-0.01
					[-1.072]	[-2.382]	[-2.367]
Q5	-0.051	11.060	-0.005	4.16	-0.014	-0.021	-0.022
					[-3.189]	[-4.741]	[-5.108]
Q1 - Q5	0.457	9.006	0.051		0.009	0.017	0.018
	[0.907]				[1.824]	[3.519]	[3.948]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.683	5.143	0.133	60.32	-0.001	-0.003	-0.003
					[-0.616]	[-1.779]	[-1.777]
Q2	0.422	6.730	0.063	18.80	-0.006	-0.01	-0.011
					[-2.039]	[-3.727]	[-3.822]
Q3	0.660	7.369	0.090	10.33	-0.003	-0.009	-0.01
					[-1.036]	[-3.433]	[-3.533]
Q4	0.580	8.767	0.066	6.40	-0.006	-0.013	-0.013
					[-1.647]	[-3.682]	[-3.656]
Q5	0.150	10.165	0.015	4.16	-0.011	-0.021	-0.021
					[-2.507]	[-5.059]	[-5.168]
Q1 - Q5	0.533	7.597	0.070		0.01	0.018	0.018
	[1.162]				[2.242]	[3.951]	[4.082]

a higher abnormal return than the highest quintile portfolio. The latter finding is in line with Ang et al. (2006). Thus, based on portfolios sorted by idiosyncratic volatility estimated relative to the FF-3 model using a 24-month trailing window, we document that the low volatility anomaly is present in our sample data for the period August 1993 - December 2017. The evidence of the low volatility anomaly holds for both value- and equally weighted portfolios when controlling for systematic risk factors; market, size, value, operating profitability and investment. The output from the FF-5 regressions is shown and discussed in more detail in subsection 6.2.2.

6.1.2 PORTFOLIOS SORTED BY TOTAL VOLATILITY

In subsection 6.1.1 we examined the relation between returns and idiosyncratic volatility, in this subsection we consider the relation between returns and total volatility as defined in subsection 5.2.2. The results for the value- and equally weighted portfolios sorted on total volatility is provided in table 6.2. We observe that for both value- and equally weighted portfolio Q1 yields the second-best average excess return of 0.53% and 0.73% per month. In addition, the standard deviation increases monotonically going from Q1 to Q5. Consequently, portfolio Q1 outperforms the other quintile portfolios when using Sharpe ratio as a performance measurement. The long-short portfolio also yields a relatively good Sharpe ratio at 0.071 and 0.094 for value- and equally weighted. As in the case with portfolios sorted by idiosyncratic volatility, we observe no clear pattern in the excess return for portfolios sorted by total volatility. However, the excess return difference per month between portfolio Q1 and Q5 is positive and yields 0.60% and 0.73% when portfolios are value- and equally weighted, but the difference is not statistically significant.

Considering the alpha columns, we observe the following pattern – for every asset pricing model the alphas decrease going from Q1 to Q5 (with the exception of portfolio Q4 being equal or larger than portfolio Q3). Indicating a negative relationship between total volatility and abnormal returns. For the difference portfolios (Q1-Q5) the alphas are positive and statistical significant for all asset pricing models. More specifically, the abnormal returns per month for the value-weighted (equally weighted) long-short portfolios are 1.0% (1.3%), 1.9% (2.1%) and 2.0% (2.1%) when controlling for systematic risk according to CAPM, FF-3 and FF-5 respectively. In other words, we find the same qualitative results on the portfolios sorted by total volatility as with the portfolios sorted by idiosyncratic volatility. Meaning that we find a negative relation between volatility and estimated alphas, and evidence of the low volatility anomaly in our data sample when we sort portfolios by total volatility. This is the case for both value- and equally weighted portfolios and present when controlling for the market, size, value, operating profitability and investment as systematic risk factors. The FF-5 regression output and factor loadings for the portfolios sorted by total volatility is shown and discussed in subsection 6.2.3.

TABLE 6.2: VALUE- AND EQUALLY WEIGHTED PORTFOLIOS SORTED BY TVOL

We form value- and equally weighted portfolios every month sorted by total volatility estimated using a 24-month rolling window. The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust *t*-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Total Volatility							
Q1	0.532	5.507	0.097	52.96	-0.004	-0.002	-0.002
					[-2.555]	[-1.535]	[-1.465]
Q2	0.332	6.760	0.049	23.45	-0.006	-0.009	-0.009
					[-2.390]	[-3.204]	[-3.285]
Q3	0.285	8.310	0.034	12.16	-0.009	-0.012	-0.013
					[-2.972]	[-4.017]	[-4.191]
Q4	0.669	9.480	0.071	6.75	-0.007	-0.01	-0.011
					[-1.905]	[-2.970]	[-2.848]
Q5	-0.064	10.429	-0.006	4.68	-0.014	-0.021	-0.022
					[-3.228]	[-4.952]	[-5.502]
Q1 - Q5	0.596	8.370	0.071		0.01	0.019	0.02
	[1.248]				[2.140]	[3.808]	[4.348]
Panel B: Equally Weighted Portfolios Sorted by Total Volatility							
Q1	0.731	4.918	0.149	52.96	0.000	-0.002	-0.002
					[0.010]	[-1.167]	[-1.174]
Q2	0.765	6.349	0.121	23.45	-0.001	-0.006	-0.006
					[-0.439]	[-2.635]	[-2.691]
Q3	0.381	7.719	0.049	12.16	-0.007	-0.012	-0.013
					[-2.049]	[-3.858]	[-3.945]
Q4	0.660	8.950	0.074	6.75	-0.006	-0.012	-0.013
					[-1.535]	[-3.707]	[-3.663]
Q5	0.005	10.205	0.000	4.68	-0.013	-0.023	-0.023
					[-3.181]	[-5.862]	[-6.084]
Q1 - Q5	0.726	7.740	0.094		0.013	0.021	0.021
	[1.600]				[3.237]	[4.923]	[5.241]

6.2 Factor loadings

6.2.1 Factor returns and correlation

Before evaluating the factor loadings of the different portfolios, we first look at returns for the systematic risk factors in the same period as the return observations for the volatility portfolios, August 1993 to December 2017. Table 6.3 reports the monthly average returns, standard deviations and *t*-statistics for the factors in the Fama-French five-factor model for our data sample. Our summary statistics differ from Fama and French (2015a) who find all factor returns to be positive and statistical significant in their sample data (U.S. stocks from July 1963 to December 2013). We observe that both the market and size factors have positive and statistically significant average monthly returns. More specifically, on average the market outperforms the risk-free rate with 1.05% per month and a diversified portfolio of small stocks outperforms a diversified portfolio of large stocks with 1.46% per month in the period August 1993 to December 2017. On the other hand, a portfolio long stocks with robust operating profitability and short stocks with weak operating profitability yields a statistically significant and negative return of -1.45% per month in our sample period. The high book-to-market minus low book-to-market portfolio gives a negative return, whilst the conservative minus aggressive portfolio yields a positive return. Both these returns are, however, statistically insignificant.

TABLE 6.3: SUMMARY OF FACTOR RETURNS

Average monthly factor returns (mean) and the standard deviation of monthly factor returns (Std. Dev.) both measured in percentage. The last row gives *t*-statistics for the monthly factor return. The market portfolio (*MKT*) is the value-weighted portfolio of all stocks within our sample data excess the risk-free rate (monthly NIBOR). *SMB*, *HML*, *RMW* and *CMA* represents the size, value, profitability and investment factor, respectively. Factor returns are based on sample stocks in the period August 1993 to December 2017, 293 months.

	MKT	SMB	HML	RMW	CMA
Mean	1.05	1.46	-0.33	-1.45	0.07
Std. Dev.	5.89	6.05	6.81	8.22	7.04
t-Statistics	3.04	4.13	-0.83	-3.02	0.17

The correlation matrix between the factor returns are reported in table 6.4. We observe that the value, profitability and investment factors have a negative or low correlation with the market and the size factor, which is in line with the findings of Fama and French (2015a). The size factor seems to have a small but positive correlation with the market factor, which is a fair implication that small stocks have a larger market exposure (market beta) than large stocks. Further, the correlations between the value factor and the two augmented factors, operating profitability and investment, is positive but low relative to Fama and French (2015a)'s stock sample. This could indicate that the value factor in fact is not redundant in our stock sample, in contrast to Fama and French (2015a) who find that the *HML* factor is explained by the combination of the *RMW* and *CMA* factors.

TABLE 6.4: CORRELATION MATRIX BETWEEN FACTOR RETURNS

The market portfolio (*MKT*) is the value-weighted portfolio of all stocks within our sample data over the risk-free rate (monthly NIBOR). *SMB*, *HML*, *RMW* and *CMA* represents the size, value, profitability and investment factor, respectively. Correlation matrix is based on sample stocks in the period August 1993 to December 2017.

	MKT	SMB	HML	RMW	CMA
MKT	1.00	0.05	-0.03	-0.17	-0.02
SMB	0.05	1.00	-0.33	-0.41	0.05
HML	-0.03	-0.33	1.00	0.04	0.16
RMW	-0.17	-0.41	0.04	1.00	-0.09
CMA	-0.02	0.05	0.16	-0.09	1.00

6.2.2 FACTOR LOADINGS FOR PORTFOLIOS SORTED BY IVOL

Panel A and B in table 6.5 reports the coefficient estimates from the FF-5 regressions on value- and equally weighted portfolios sorted on idiosyncratic volatility. Examining the results, we see that the market exposure increases going from portfolio Q1 to Q5 for both value- and equally weighted portfolios. Meaning that systematic market risk has a positive relation to idiosyncratic risk, which is what we expected. Further, we observe the same positive relation implies for the size factor, which is in line with the decreasing market share going from Q1 to Q5 as shown in table 6.1. In addition, for both value- and equally weighted portfolios, table 6.5 reports a negative relation between idiosyncratic volatility for both the value and the profitability factors. Lastly, there seems to be no clear pattern between the quintile portfolios and the investment factor.

Focusing on the value-weighted extreme portfolios, Q1 and Q5, we observe that the low volatility portfolio returns have significant positive loadings on the value and profitability factors and a significant negative loading on the size factor. This indicates that low volatility stocks are large value stocks with relatively robust operating profitability. On the other hand, portfolio Q5 loads significantly positive on the size factor and significantly negative on the value factor, meaning that high volatility stocks have the characteristics of small growth stocks. Based on this the long-short portfolio (shown in column 6) have negative and significant loadings to both the market and size factors, while the portfolio loads significantly positive on the value and profitability factors. Controlling for the Fama-French five-factor model yields a positive and significant alpha of 1.8% per month.

TABLE 6.5: FF-5 REGRESSIONS ON VALUE- AND EQUALLY WEIGHTED PORTFOLIOS SORTED BY IVOL

The portfolio with lowest (highest) IVOL is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by column Q1-Q5. MKT is the value-weighted portfolio of all stocks within our sample data over the risk-free rate (monthly NIBOR). *SMB*, *HML*, *RMW* and *CMA* represents the size, value, profitability and investment factor, respectively. Coefficients are measured in decimals. Sample period is August 1993 to December 2017, 293 months. Robust *t* statistics Newey and West (1987) in brackets. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Panel A: FF-5 regressions on value-weighted portfolios sorted by IVOL

	(1) Q1	(2) Q2	(3) Q3	(4) Q4	(5) Q5	(6) Q1 - Q5
MKT	0.907***	0.956***	0.957***	1.186***	1.270***	-0.363***
	[39.141]	[11.939]	[15.788]	[13.869]	[11.366]	[-2.781]
SMB	-0.054**	0.072	0.164***	0.191**	0.363***	-0.417***
	[-2.354]	[1.292]	[2.736]	[2.553]	[4.041]	[-4.492]
HML	0.075***	-0.179***	-0.127**	-0.227***	-0.142**	0.218***
	[2.693]	[-3.752]	[-1.971]	[-2.623]	[-2.494]	[3.611]
RMW	0.050***	-0.089**	-0.120***	-0.189***	-0.135*	0.185**
	[3.626]	[-2.485]	[-3.066]	[-4.277]	[-1.820]	[2.488]
CMA	0.025	0.028	0.024	0.036	0.014	0.011
	[1.204]	[0.422]	[0.521]	[0.701]	[0.206]	[0.150]
Alpha	-0.004***	-0.009***	-0.010***	-0.010**	-0.022***	0.018***
	[-3.487]	[-3.007]	[-3.465]	[-2.367]	[-5.108]	[3.948]

Panel B: FF-5 regressions on equally weighted portfolios sorted by IVOL

Portfolio	(1) Q1	(2) Q2	(3) Q3	(4) Q4	(5) Q5	(6) Q1 - Q5
MKT	0.754***	0.903***	0.889***	1.081***	1.147***	-0.393***
	[17.564]	[16.554]	[19.946]	[16.484]	[19.215]	[-4.528]
SMB	0.150***	0.316***	0.360***	0.400***	0.608***	-0.457***
	[3.486]	[5.383]	[5.296]	[5.573]	[6.617]	[-5.721]
HML	-0.006	-0.031	-0.115***	-0.094*	-0.114*	0.109
	[-0.224]	[-0.909]	[-2.624]	[-1.945]	[-1.810]	[1.462]
RMW	-0.013	-0.043	-0.083	-0.098**	-0.114	0.102
	[-0.457]	[-1.113]	[-1.602]	[-2.425]	[-1.484]	[1.254]
CMA	0.000	0.005	0.043	0.092**	0.001	-0.001
	[0.017]	[0.140]	[1.071]	[2.357]	[0.022]	[-0.015]
Alpha	-0.003*	-0.011***	-0.010***	-0.013***	-0.021***	0.018***
	[-1.777]	[-3.822]	[-3.533]	[-3.656]	[-5.168]	[4.082]

For the equally weighted portfolio, the low volatility portfolio loads significantly positive on the size factor, indicating that it contains small stocks rather than large stocks. However, the low volatility portfolio loads relatively less on small stocks when comparing it to the other quintile portfolios. Further, the difference portfolio (Q1-Q5) loads only statistically significantly (and negative) on the market and size factors and yields a significant abnormal return of 1.8% per month.

6.2.3 Factor loadings for portfolios sorted by TVOL

We conduct the same analysis for factor loadings based on value- and equally weighted portfolios sorted by total volatility. The results for value- and equally weighted portfolios are reported in table 6.6 panel A and B, respectively. We observe the same patterns as with the portfolios sorted on idiosyncratic volatility – when going from the low to the high volatility portfolio the loadings on the market and size factors increase, but the loadings on the value and profitability factors decrease. Panel A in table 6.6 shows that the value-weighted low volatility portfolio sorted by total volatility does not load significantly on the value factor, which is in contrast to when sorted by idiosyncratic volatility. However, it still exhibits characteristics of large stocks with relatively lower market beta. The value-weighted high volatility portfolio shows features of small growth stocks with relatively weak profitability and high market exposure.

When equally weighted the factor loadings for the low volatility portfolio sorted by total volatility is similar to the low volatility portfolio sorted by idiosyncratic volatility. It loads statistically significantly and positive on both the market and size factor, but relatively less than the other quintile portfolios. Indicating that the portfolio contains relatively large stocks with relatively less market exposure. The loadings on the remaining factors is not statistically significant. The Q5 portfolio exhibits the same qualitative loadings as in the case with value-weighted portfolios – small value stocks with relatively high market beta and weak operating profitability.

Although our results are not totally consistent for every measurement of volatility and weighting scheme, we conclude that the low and high volatility portfolio exhibit opposites. The low volatility portfolio often contains large value stocks with relatively low market exposure and robust operating profitability, whilst the high volatility portfolio has the characteristics of small growth stocks with relatively high market exposure and weak operating profitability.

TABLE 6.6: FF-5 REGRESSIONS ON VALUE- AND EQUALLY WEIGHTED PORTFOLIOS SORTED BY TVOL

The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by column Q1-Q5. MKT is the value-weighted portfolio of all stocks within our sample data over the risk-free rate (monthly NIBOR). *SMB*, *HML*, *RMW* and *CMA* represents the size, value, profitability and investment factor, respectively. Coefficients are measured in decimals. Sample period is August 1993 to December 2017, 293 months. Robust *t* statistics Newey and West (1987) in brackets. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Panel A: FF-5 regressions on value-weighted portfolios sorted by TVOL						
	(1)	(2)	(3)	(4)	(5)	(6)
	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
MKT	0.868***	0.879***	1.075***	1.205***	1.227***	-0.359***
	[29.360]	[13.544]	[17.076]	[19.516]	[15.359]	[-3.729]
SMB	-0.070***	0.166**	0.123***	0.087	0.362***	-0.432***
	[-3.185]	[2.306]	[2.619]	[1.169]	[5.118]	[-6.067]
HML	0.030	-0.043	-0.168**	-0.235***	-0.172***	0.202***
	[1.511]	[-0.685]	[-2.521]	[-4.049]	[-3.080]	[3.414]
RMW	0.033**	-0.001	-0.147***	-0.199***	-0.161***	0.194***
	[2.459]	[-0.029]	[-3.220]	[-3.617]	[-2.734]	[3.182]
CMA	0.002	0.048	-0.010	0.051	0.015	-0.013
	[0.078]	[1.035]	[-0.241]	[0.808]	[0.261]	[-0.196]
Alpha	-0.002	-0.009***	-0.013***	-0.011***	-0.022***	0.020***
	[-1.465]	[-3.285]	[-4.191]	[-2.848]	[-5.502]	[4.348]
Panel B: FF-5 regressions on equally weighted portfolios sorted by TVOL						
Portfolio	(1)	(2)	(3)	(4)	(5)	(6)
	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
MKT	0.688***	0.817***	0.975***	1.123***	1.165***	-0.478***
	[20.646]	[17.165]	[14.955]	[17.777]	[20.293]	[-6.476]
SMB	0.166***	0.347***	0.317***	0.375***	0.626***	-0.460***
	[3.387]	[5.352]	[4.267]	[4.661]	[8.459]	[-7.293]
HML	0.005	-0.044	-0.058	-0.140***	-0.118**	0.123*
	[0.153]	[-1.482]	[-0.886]	[-3.081]	[-2.036]	[1.712]
RMW	0.001	-0.006	-0.100**	-0.119**	-0.124**	0.125**
	[0.052]	[-0.133]	[-2.115]	[-1.999]	[-2.009]	[2.182]
CMA	-0.016	0.040	0.030	0.067	0.014	-0.030
	[-0.618]	[0.978]	[0.803]	[1.548]	[0.291]	[-0.610]
Alpha	-0.002	-0.006***	-0.013***	-0.013***	-0.023***	0.021***
	[-1.174]	[-2.691]	[-3.945]	[-3.663]	[-6.084]	[5.241]

6.3 Robustness tests

6.3.1 Alternative measurements of idiosyncratic volatility

In our analysis we have defined idiosyncratic volatility relative to the Fama-French three-factor model, which is consistent with previous studies e.g. Ang et al. (2006). In order to test the robustness of our results, we conduct the same analysis where idiosyncratic volatility is defined relative to CAPM and Fama-French five-factor model. The results for excess returns, standard deviations, Sharpe ratios and estimated alphas where idiosyncratic volatility is estimated relative to CAPM and FF-5 is given in tables A.1 and A.2, respectively, in the appendix. Comparing with our main results in table 6.1 we find our results to be consistent – we still observe a positive but insignificant difference in the excess return between the extreme portfolios. There is still evidence of the low volatility anomaly for all asset pricing models, meaning that all long low volatility and short high volatility portfolios produce a positive and statistically significant alpha. Based on these findings we conclude that our results are robust to different measurements of idiosyncratic volatility.

6.3.2 Different subsamples

We conduct the same analysis for different time-periods to check if the low volatility anomaly is still present in shorter timespans. The subsamples are pre-financial crisis (August 1993 to December 2007), post-financial crisis (January 2008 to December 2017) and since the start of the new millennium (January 2000 to December 2017). Result tables for value- and equally weighted portfolios sorted by idiosyncratic volatility and total volatility for all subsample periods are provided in appendix B. A natural consequence when comparing different subsamples is a difference in excess returns. We observe that the excess returns vary and sometimes becomes negative. However, our subsamples still show the same qualitative results – the long-short portfolio yields a positive return for all subsamples, but the difference is never statistically significant. Further, we observe that for almost all periods, all value-weighted and equally weighted difference portfolios yield positive and statistically significant alphas regardless of asset pricing model. The exception is a slightly insignificant estimated alpha (at a five percent significant level) when only controlling for CAPM, which also was the case in our main sample. As the results from the subsamples are in line with our main findings, we consider our findings robust and are not driven by effects in our original sample period.

6.3.3 PENNY STOCKS

Results when penny stocks are defined as less than NOK 5 rather than less than NOK 1 is provided in table C.1 and C.2 in Appendix C. We notice that the market shares of the low volatility portfolio decrease with 2.4 and 3.1 percentage points for portfolios sorted on idiosyncratic volatility and total volatility. This is distributed between portfolio Q2 to Q4 as the market share of the high volatility portfolio stays, more or less, the same. By comparing tables 6.1 and 6.2 with tables C.1 and C.2, we observe that the excess return difference between the low volatility portfolios and the high volatility portfolios increase when penny stocks are defined as below NOK 5. However, the difference in excess returns is never statistically significant. Further, the estimated alphas and the robust *t*-statistics for the long-short portfolio also increase. Meaning that the low volatility anomaly is still, if not more, present when imposing a stricter definition of penny stocks.

6.3.4 RETURN REQUIREMENTS

Tables D.1 and D.2 in Appendix D report the results for value- and equally weighted portfolios sorted by idiosyncratic volatility and total volatility, where we reduce our requirement to eight months of return observations to be included in the construction of the volatility portfolios. Relaxing this return requirement yields the same qualitative results, leading us to conclude that the low volatility anomaly is still present when only requiring return observations for one-third of the 24-month trailing window.

Tables 6.7 and 6.8 report a summary of FF-5 alphas for portfolios sorted by idiosyncratic volatility and total volatility for different robustness tests. We have only tabulated the extreme portfolios (Q1 and Q5) and the difference portfolio (Q1 – Q5). From the tables we observe that the FF-5 alpha for the difference portfolio is positive and statistical significant for all robustness tests. For the value- (equally) weighted portfolios sorted on idiosyncratic volatility the abnormal return for the long-short portfolio range between 1.8% (1.7%) to 2.2% (2.0%) per month. When portfolios are sorted by total volatility and value- (equally) weighted the abnormal return of the long-short portfolio range between 1.9% (2.0%) and 2.5% (2.5%) per month. Based on this we find that the abnormally higher returns for the low volatility portfolio relative to the high volatility portfolio are robust to the measurement of idiosyncratic volatility, subsamples, filtering process and return requirements.

TABLE 6.7: FF-5 ALPHAS FOR PORTFOLIOS SORTED BY IVOL

The table reports FF-5 alphas in decimals, with robust *t*-statistics Newey and West (1987) in brackets. We form value- and equally weighted portfolios every month sorted by idiosyncratic volatility relative to FF-3 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Panel A: FF-5 Alphas for Value-Weighted Portfolios Sorted by IVOL			
	Q1	Q5	Q5-Q1
Main results	-0.004	-0.022	0.018
	[-3.487]	[-5.108]	[3.948]
IVOL defined by CAPM	-0.003	-0.024	0.021
	[-2.624]	[-6.068]	[5.237]
IVOL defined by FF-5	-0.003	-0.024	0.021
	[-3.221]	[-5.804]	[4.841]
Aug. 1993 - Dec. 2007	-0.003	-0.025	0.022
	[-2.173]	[-4.395]	[3.804]
Jan. 2008 - Dec. 2017	-0.005	-0.02	0.015
	[-4.135]	[-3.875]	[3.005]
Jan. 2000 - Dec. 2017	-0.003	-0.024	0.021
	[-2.482]	[-4.932]	[3.946]
Penny stocks > 5	-0.003	-0.023	0.020
	[-3.170]	[-5.841]	[4.872]
Return requirements	-0.003	-0.022	0.020
	[-2.744]	[-5.395]	[4.361]

Panel B: FF-5 Alphas for Equally Weighted Portfolios Sorted by IVOL			
	Q1	Q5	Q5-Q1
Main results	-0.003	-0.021	0.018
	[-1.777]	[-5.168]	[4.082]
IVOL defined by CAPM	-0.004	-0.022	0.018
	[-1.915]	[-5.240]	[4.247]
IVOL defined by FF-5	-0.004	-0.022	0.018
	[-1.923]	[-5.502]	[4.175]
Aug. 1993 - Dec. 2007	-0.001	-0.021	0.020
	[-0.531]	[-3.734]	[3.427]
Jan. 2008 - Dec. 2017	-0.006	-0.022	0.017
	[-1.853]	[-3.993]	[3.765]
Jan. 2000 - Dec. 2017	-0.003	-0.022	0.019
	[-1.347]	[-4.386]	[3.598]
Penny stocks > 5	-0.003	-0.022	0.019
	[-1.637]	[-5.376]	[4.207]
Return requirements	-0.002	-0.02	0.018
	[-1.010]	[-5.042]	[4.796]

TABLE 6.8: FF-5 ALPHAS FOR PORTFOLIOS SORTED BY TVOL

The table reports FF-5 alphas in decimals, with robust *t*-statistics Newey and West (1987) in brackets. We form value- and equally weighted portfolios every month sorted by total volatility estimated using a 24-month rolling window. The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Panel A: FF-5 Alphas for Value-Weighted Portfolios Sorted by TVOL			
	Q1	Q5	Q5-Q1
Main results	-0.002	-0.022	0.020
	[-1.465]	[-5.502]	[4.348]
Aug. 1993 - Dec. 2007	-0.002	-0.022	0.019
	[-1.598]	[-5.827]	[4.639]
Jan. 2008 - Dec. 2017	0.000	-0.024	0.025
	[0.294]	[-4.589]	[4.719]
Jan. 2000 - Dec. 2017	-0.003	-0.023	0.021
	[-1.405]	[-4.993]	[3.761]
Penny stocks > 5	-0.002	-0.022	0.019
	[-1.598]	[-5.827]	[4.639]
Return requirements	-0.002	-0.022	0.019
	[-1.598]	[-5.827]	[4.639]

Panel B: FF-5 Alphas for Equally Weighted Portfolios Sorted by TVOL			
	Q1	Q5	Q5-Q1
Main results	-0.002	-0.023	0.021
	[-1.174]	[-6.084]	[5.241]
Aug. 1993 - Dec. 2007	0.000	-0.024	0.025
	[0.294]	[-4.589]	[4.719]
Jan. 2008 - Dec. 2017	0.000	-0.025	0.025
	[-0.147]	[-4.763]	[4.699]
Jan. 2000 - Dec. 2017	-0.003	-0.023	0.021
	[-1.246]	[-5.022]	[4.180]
Penny stocks > 5	-0.002	-0.022	0.020
	[-0.853]	[-6.127]	[5.168]
Return requirements	-0.002	-0.022	0.020
	[-0.853]	[-6.127]	[5.168]

6.4 Caution regarding our findings

6.4.1 Profitable investment strategy?

Our findings document the existence of the low volatility anomaly for the Norwegian stock market in our sample period. However, there is a difference between documenting the anomaly and it being a profitable investment strategy. In our analysis, we do not account for transaction costs associated with the strategy and we can therefore not make any implications on whether it would be profitable to exploit the anomaly. As shown in table 6.1 when portfolios are sorted by idiosyncratic (total) volatility the high volatility portfolio only consists of 4.16% (4.68%) of total market capitalization, in other words the strategy involves shorting a great number of small stocks. There is also evidence showing that stocks included in the high volatility portfolio are relatively illiquid; see Bali and Cakici (2008). This means that there may occur relatively large transaction costs if one tries to exploit the low volatility anomaly, and they could leave the investment strategy unprofitable in reality.

6.4.2 Choice of asset pricing model

Another critical element in our analysis is our choice of asset pricing model. Our focus is to estimate alphas when controlling for systematic risk relative to the FF-5 model, but we also control for the CAPM and the FF-3 model. We document the existence of the low volatility anomaly for all three asset pricing models. Nevertheless, we cannot be certain if the anomaly truly exists or if we are using incorrect asset pricing models, because we are unable to distinguish between them. It could be that the models we use exclude factors that should be included in order to explain the relation between risk and return[6], and an extension of our analysis could be to augment the FF-5 model with additional documented risk factors such as; liquidity (Pàstorand and Stambaugh, 2003) and momentum (Jegadeesh and Timan, 1993).

[6] This is an issue in econometrics known as omitted variable bias, however this is normally not discussed in the asset pricing literature. Derivation of the omitted variable bias is shown in appendix E.

7. CONCLUSION

In this thesis, we look at the relationship between risk and return for the Norwegian stock market in the period of August 1993 to December 2017. More specifically, we use a 24-month trailing window in order to estimate idiosyncratic volatility relative to the Fama-French three-factor model and total volatility defined as the standard deviation of stock returns. Based on these estimates of volatility we form both value- and equally weighted quintile portfolios every month. In order to control for systematic risk factors, we construct CAPM, FF-3 and FF-5 for Norway using raw stock and accounting data. This enables us to control for the systematic risk factors; market, size, value, operating profitability and investment. We evaluate the existence of the low volatility anomaly by looking at monthly excess returns and estimated alphas when controlling for systematic risk relative to the CAPM, FF-3 model and FF-5 model.

Our findings show that in our sample period there is a positive difference in excess return between the low volatility portfolio and the high volatility portfolio. More specifically, when portfolios are value- (equally) weighted and sorted by idiosyncratic volatility the low volatility portfolio outperforms the high volatility portfolio with 0.46% (0.53%) per month. Similarly, when sorted by total volatility the difference between the extreme portfolios are 0.60% (0.73%) when portfolios are value- (equally) weighted. However, the difference is not statistical significant. Hence, even though our results indicate a negative relation between volatility and returns we cannot conclude the existence of the low volatility anomaly on the basis of excess return differences.

However, we are able to make statistical inferences when evaluating performance based on estimated alphas. Even though all long portfolios report negative alphas, we find that high volatility portfolios yield statistically significant lower alphas than low volatility portfolios when controlling for CAPM, FF-3 and FF-5. More specifically, when portfolios are sorted by idiosyncratic volatility the estimated FF-5 alpha for the long-short portfolio is 1.8% per month for both value- and equally weighted. The estimated FF-5 alphas are 2.0% and 2.1% per month when portfolios are value- and equally weighted and sorted by total volatility. All estimated FF-5 alphas are statistical significant. Based on the estimated alphas we therefore document the existence of the low volatility anomaly. This holds for value- and equally weighted portfolios, estimation method of volatility and when controlling for the market, size, value, operating profitability and investment as systematic risk factors. Our results are also robust when testing for different measurements of idiosyncratic volatility, subsamples, filtering process and return requirements.

In our sample data, we find that the low volatility portfolio are large value stocks with relatively low market exposure and robust operating profitability. On the other hand, the high volatility portfolio contains small growth stocks with relatively high market exposure and weak operating profitability. Our results also show that controlling for more systematic risk factors increase the estimated alphas for the difference portfolio. This is in contrast to our hypothesis, where we initially believed that controlling for additional systematic risk factors would help explain the low volatility anomaly. Our findings for the Norwegian stock market are therefore not in line with the findings of Fama and French (2015b), who document a reduction of the low volatility anomaly when using the FF-5 model.

The results from our thesis are in line with previous literature, and show that the low volatility anomaly seems to be robust when testing for a variety of systematic risk factors known to explain the cross-section of returns. We therefore conclude that the low volatility anomaly exists in the Norwegian stock market for our sample period.

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APPENDIX A DEFINITION OF IDIOSYNCRATIC VOLATILITY

TABLE A.1: IDIOSYNCRATIC VOLATILITY RELATIVE TO CAPM

We form value- and equal-weighted portfolios every month sorted on idiosyncratic volatility relative to CAPM estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017,

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to CAPM							
Q1	0.492	5.576	0.088	61.204	-0.005	-0.003	-0.003
					[-3.866]	[-2.660]	[-2.624]
Q2	0.401	7.131	0.056	18.434	-0.006	-0.008	-0.008
					[-2.179]	[-2.942]	[-3.112]
Q3	0.074	8.434	0.009	10.208	-0.011	-0.014	-0.015
					[-3.349]	[-4.264]	[-4.281]
Q4	1.221	9.607	0.127	6.009	0	-0.005	-0.006
					[-0.118]	[-1.377]	[-1.449]
Q5	-0.222	11.205	-0.020	4.145	-0.017	-0.024	-0.024
					[-3.860]	[-5.713]	[-6.068]
Q1-Q5	0.713	8.940	0.080		0.012	0.021	0.021
	[1.389]				[2.676]	[4.755]	[5.237]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to CAPM							
Q1	0.625	5.214	0.120	61.204	-0.002	-0.004	-0.004
					[-0.900]	[-1.922]	[-1.915]
Q2	0.595	6.511	0.091	18.434	-0.003	-0.008	-0.008
					[-1.124]	[-2.907]	[-3.044]
Q3	0.498	7.588	0.066	10.208	-0.006	-0.012	-0.012
					[-2.094]	[-4.584]	[-4.687]
Q4	0.586	8.691	0.067	6.009	-0.006	-0.012	-0.013
					[-1.544]	[-3.665]	[-3.629]
Q5	0.188	10.244	0.018	4.145	-0.011	-0.021	-0.022
					[-2.481]	[-5.092]	[-5.240]
Q1-Q5	0.436	7.650	0.057		0.009	0.017	0.018
	[0.929]				[2.107]	[4.043]	[4.247]

TABLE A.2: IDIOSYNCRATIC VOLATILITY RELATIVE TO FF-5

We form value- and equal-weighted portfolios every month sorted by idiosyncratic volatility relative to FF-5 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-5							
Q1	0.499	5.698	0.088	59.97	-0.005	-0.003	-0.003
					[-4.508]	[-3.159]	[-3.221]
Q2	0.110	7.141	0.015	18.68	-0.009	-0.011	-0.012
					[-3.233]	[-4.139]	[-4.214]
Q3	0.582	7.744	0.075	10.33	-0.005	-0.008	-0.008
					[-1.662]	[-3.068]	[-3.364]
Q4	0.952	9.995	0.095	6.86	-0.003	-0.009	-0.009
					[-0.746]	[-2.205]	[-2.232]
Q5	-0.171	11.079	-0.015	4.16	-0.016	-0.023	-0.024
					[-3.699]	[-5.439]	[-5.804]
Q1-Q5	0.670	8.957	0.075		0.011	0.02	0.021
	[1.398]				[2.376]	[4.370]	[4.841]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-5							
Q1	0.656	5.423	0.121	59.97	-0.002	-0.004	-0.004
					[-0.934]	[-1.931]	[-1.923]
Q2	0.411	6.515	0.063	18.68	-0.005	-0.01	-0.01
					[-2.224]	[-4.421]	[-4.538]
Q3	0.676	7.322	0.092	10.33	-0.003	-0.009	-0.009
					[-0.856]	[-2.728]	[-2.811]
Q4	0.634	8.720	0.073	6.86	-0.005	-0.012	-0.013
					[-1.374]	[-3.536]	[-3.575]
Q5	0.154	10.089	0.015	4.16	-0.011	-0.021	-0.022
					[-2.658]	[-5.452]	[-5.502]
Q1-Q5	0.501	7.293	0.069		0.009	0.017	0.018
	[1.174]				[2.193]	[4.050]	[4.175]

APPENDIX B SUBSAMPLES

TABLE B.1: SUBSAMPLE PRE-FINANCIAL CRISIS, AUGUST 1993 TO DECEMBER 2007, PORTFOLIOS SORTED BY IVOL

We form value- and equal-weighted portfolios every month sorted by idiosyncratic volatility relative to FF-3 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2007, whilst return calculations are from the period of August 1993 to December 2007, 173 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.710	5.754	0.123	56.95	-0.005	-0.003	-0.003
					[-2.888]	[-2.995]	[-2.173]
Q2	0.221	7.159	0.031	18.13	-0.01	-0.013	-0.013
					[-2.112]	[-2.876]	[-2.776]
Q3	0.614	8.081	0.076	12.02	-0.007	-0.011	-0.01
					[-1.650]	[-3.378]	[-2.887]
Q4	1.935	9.657	0.200	7.88	0.004	0	0.001
					[0.849]	[-1.437]	[0.333]
Q5	0.488	12.569	0.039	5.03	-0.016	-0.025	-0.025
					[-2.694]	[-5.462]	[-4.395]
Q1 - Q5	0.222	10.290	0.022		0.011	0.023	0.022
	[0.299]				[1.652]	[4.376]	[3.804]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	1.051	4.914	0.214	56.95	-0.001	-0.001	-0.001
					[0.588]	[-0.658]	[-0.531]
Q2	0.677	7.001	0.097	18.13	-0.006	-0.012	-0.011
					[-1.409]	[-2.801]	[-2.582]
Q3	1.120	7.821	0.143	12.02	-0.002	-0.009	-0.008
					[-0.442]	[-2.366]	[-2.297]
Q4	1.202	9.181	0.131	7.88	-0.003	-0.01	-0.009
					[-0.671]	[-2.068]	[-1.810]
Q5	0.795	11.120	0.071	5.03	-0.010	-0.021	-0.021
					[-1.572]	[-3.608]	[-3.734]
Q1 - Q5	0.256	8.581	0.030		0.011	0.02	0.02
	[0.384]				[1.727]	[3.239]	[3.427]

TABLE B.2: SUBSAMPLE PRE-FINANCIAL CRISIS, AUGUST 1993 TO DECEMBER 2007, PORTFOLIOS SORTED BY TVOL

We form value- and equal-weighted portfolios every month sorted by total volatility estimated using a 24-month rolling window. The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2007, whilst return calculations are from the period of August 1993 to December 2007, 173 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Total Volatility							
Q1	0.950	5.596	0.170	51.11	-0.002	0.000	-0.002
					[-1.267]	[0.331]	[-1.598]
Q2	0.168	6.750	0.025	21.66	-0.009	-0.013	-0.008
					[-3.243]	[-4.242]	[-3.629]
Q3	0.537	8.736	0.061	13.69	-0.010	-0.014	-0.011
					[-2.212]	[-3.167]	[-3.540]
Q4	1.609	9.303	0.173	7.86	0.000	-0.002	-0.01
					[0.068]	[-0.608]	[-2.674]
Q5	0.364	11.501	0.032	5.68	-0.016	-0.025	-0.022
					[-2.712]	[-4.462]	[-5.827]
Q1 - Q5	0.587	9.345	0.063		0.014	0.025	0.019
	[0.824]				[2.331]	[4.478]	[4.639]
Panel B: Equally Weighted Portfolios Sorted by Total Volatility							
Q1	1.045	4.886	0.214	51.11	0.000	-0.002	0
					[-0.057]	[-0.839]	[0.294]
Q2	1.163	6.422	0.181	21.66	-0.002	-0.006	-0.012
					[-0.939]	[-2.776]	[-3.901]
Q3	0.854	8.404	0.102	13.69	-0.005	-0.011	-0.013
					[-1.655]	[-3.357]	[-3.199]
Q4	1.480	9.439	0.157	7.86	-0.005	-0.01	-0.001
					[-1.480]	[-3.142]	[-0.273]
Q5	0.388	11.056	0.035	5.68	-0.013	-0.022	-0.024
					[-3.361]	[-5.889]	[-4.589]
Q1 - Q5	0.656	8.723	0.075		0.013	0.02	0.025
	[1.006]				[3.212]	[4.956]	[4.719]

TABLE B.3: SUBSAMPLE POST-FINANCIAL CRISIS, JANUARY 2008 TO DECEMBER 2017, PORTFOLIOS SORTED BY IVOL

We form value- and equal-weighted portfolios every month sorted by idiosyncratic volatility relative to FF-3 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of December 2006 to December 2017, whilst return calculations are from the period of January 2008 to December 2017, 120 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	-0.033	5.217	-0.006	65.22	-0.006	-0.005	-0.005
					[-4.471]	[-3.905]	[-4.135]
Q2	0.591	7.566	0.078	19.77	-0.001	-0.002	-0.003
					[-0.325]	[-0.759]	[-0.915]
Q3	0.158	7.507	0.021	7.87	-0.005	-0.008	-0.01
					[-1.011]	[-1.714]	[-2.089]
Q4	-0.637	10.307	-0.062	4.25	-0.015	-0.02	-0.022
					[-2.815]	[-4.370]	[-4.001]
Q5	-0.829	8.412	-0.099	2.90	-0.014	-0.017	-0.02
					[-2.208]	[-2.896]	[-3.875]
Q1 - Q5	0.796	6.767	0.118		0.008	0.011	0.015
	[1.283]				[1.280]	[1.966]	[3.005]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.151	5.434	0.028	65.22	-0.006	-0.006	-0.006
					[-1.254]	[-2.163]	[-1.853]
Q2	0.055	6.330	0.009	19.77	-0.005	-0.009	-0.008
					[-1.511]	[-2.532]	[-1.888]
Q3	-0.004	6.640	-0.001	7.87	-0.005	-0.01	-0.009
					[-1.166]	[-2.655]	[-2.237]
Q4	-0.316	8.088	-0.039	4.25	-0.010	-0.016	-0.017
					[-1.792]	[-3.566]	[-3.871]
Q5	-0.780	8.566	-0.091	2.90	-0.014	-0.021	-0.022
					[-2.293]	[-4.095]	[-3.993]
Q1 - Q5	0.932	5.911	0.158		0.010	0.015	0.017
	[1.655]				[1.993]	[3.461]	[3.765]

TABLE B.4: SUBSAMPLE POST-FINANCIAL CRISIS, JANUARY 2008 TO DECEMBER 2017, PORTFOLIOS SORTED BY TVOL

We form value- and equal-weighted portfolios every month sorted by total volatility estimated using a 24-month rolling window. The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of December 2006 to December 2017, whilst return calculations are from the period of January 2008 to December 2017, 120 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Total Volatility							
Q1	-0.072	5.342	-0.013	55.65	-0.002	0.000	0.000
					[-1.267]	[0.331]	[0.294]
Q2	0.567	6.796	0.084	26.04	-0.009	-0.013	-0.012
					[-3.243]	[-4.242]	[-3.901]
Q3	-0.078	7.676	-0.010	9.95	-0.010	-0.014	-0.013
					[-2.212]	[-3.167]	[-3.199]
Q4	-0.686	9.606	-0.071	5.13	0.000	-0.002	-0.001
					[0.068]	[-0.608]	[-0.273]
Q5	-0.682	8.661	-0.079	3.24	-0.016	-0.025	-0.024
					[-2.712]	[-4.462]	[-4.589]
Q1 - Q5	0.610	6.759	0.090		0.014	0.025	0.025
	[0.824]				[2.331]	[4.478]	[4.719]
Panel B: Equally Weighted Portfolios Sorted by Total Volatility							
Q1	0.278	4.950	0.056	55.65	0.002	0.000	0.000
					[0.795]	[-0.173]	[-0.147]
Q2	0.191	6.225	0.031	26.04	0.001	-0.006	-0.005
					[0.266]	[-1.568]	[-1.324]
Q3	-0.300	6.581	-0.046	9.95	-0.006	-0.012	-0.011
					[-1.472]	[-2.780]	[-2.597]
Q4	-0.521	8.088	-0.064	5.13	-0.001	-0.008	-0.007
					[-0.244]	[-1.851]	[-1.651]
Q5	-0.548	8.851	-0.062	3.24	-0.014	-0.026	-0.025
					[-2.464]	[-4.578]	[-4.763]
Q1 - Q5	0.826	6.082	0.136		0.015	0.025	0.025
	[1.487]				[2.776]	[4.448]	[4.699]

TABLE B.5: SUBSAMPLE MILLENNIUM, JANUARY 2000 TO DECEMBER 2017, PORTFOLIOS SORTED BY IVOL

We form value- and equal-weighted portfolios every month sorted by idiosyncratic volatility relative to FF-3 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of December 1998 to December 2017, whilst return calculations are from the period of January 2000 to December 2017, 216 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.389	5.234	0.074	64.62	-0.004	-0.003	-0.003
					[-3.192]	[-2.449]	[-2.482]
Q2	0.220	7.293	0.030	17.99	-0.007	-0.009	-0.009
					[-2.113]	[-2.627]	[-2.776]
Q3	0.211	8.164	0.026	8.59	-0.008	-0.012	-0.013
					[-2.075]	[-3.264]	[-3.514]
Q4	0.422	10.601	0.040	5.49	-0.009	-0.013	-0.014
					[-2.210]	[-3.114]	[-3.118]
Q5	-0.390	11.178	-0.035	3.31	-0.016	-0.023	-0.024
					[-2.955]	[-4.200]	[-4.932]
Q1 - Q5	0.780	9.400	0.083		0.012	0.019	0.021
	[1.380]				[1.991]	[3.208]	[3.946]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.607	5.084	0.119	64.62	-0.003	-0.003	-0.003
					[-0.556]	[-1.350]	[-1.347]
Q2	0.237	6.685	0.035	17.99	-0.007	-0.011	-0.011
					[-2.549]	[-4.274]	[-4.358]
Q3	0.413	7.563	0.055	8.59	-0.005	-0.011	-0.012
					[-1.304]	[-3.521]	[-3.615]
Q4	0.219	9.087	0.024	5.49	-0.009	-0.016	-0.017
					[-2.376]	[-4.382]	[-4.486]
Q5	0.002	10.385	0.000	3.31	-0.012	-0.021	-0.022
					[-2.150]	[-4.241]	[-4.386]
Q1 - Q5	0.606	7.819	0.077		0.010	0.018	0.019
	[1.107]				[1.877]	[3.303]	[3.598]

TABLE B.6: SUBSAMPLE MILLENNIUM, JANUARY 2000 TO DECEMBER 2017, PORTFOLIOS SORTED BY TVOL

We form value- and equal-weighted portfolios every month sorted by total volatility estimated using a 24-month rolling window. The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of December 1998 to December 2017, whilst return calculations are from the period of January 2000 to December 2017, 216 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Total Volatility							
Q1	0.440	5.293	0.083	55.98	-0.004	-0.003	-0.003
					[-1.943]	[-1.442]	[-1.405]
Q2	0.227	6.877	0.033	23.92	-0.007	-0.008	-0.008
					[-2.221]	[-2.646]	[-2.734]
Q3	0.019	8.633	0.002	10.53	-0.011	-0.014	-0.015
					[-3.212]	[-4.092]	[-4.211]
Q4	0.282	9.676	0.029	5.90	-0.01	-0.013	-0.015
					[-2.592]	[-3.538]	[-3.330]
Q5	-0.325	10.758	-0.030	3.68	-0.016	-0.022	-0.023
					[-2.935]	[-4.226]	[-4.993]
Q1 - Q5	0.765	8.863	0.086		0.012	0.02	0.021
	[1.314]				[1.984]	[3.116]	[3.761]
Panel B: Equally Weighted Portfolios Sorted by Total Volatility							
Q1	0.585	4.803	0.122	55.98	-0.001	-0.003	-0.003
					[-0.368]	[-1.244]	[-1.246]
Q2	0.566	6.441	0.088	23.92	-0.003	-0.007	-0.007
					[-1.132]	[-3.464]	[-3.573]
Q3	0.168	7.928	0.021	10.53	-0.008	-0.014	-0.015
					[-2.056]	[-3.796]	[-3.906]
Q4	0.256	9.127	0.028	5.90	-0.009	-0.015	-0.016
					[-2.189]	[-4.354]	[-4.296]
Q5	-0.064	10.555	-0.006	3.68	-0.013	-0.023	-0.023
					[-2.531]	[-4.765]	[-5.022]
Q1 - Q5	0.649	8.083	0.080		0.012	0.02	0.021
	[1.183]				[2.356]	[3.808]	[4.180]

APPENDIX C PENNY STOCKS

TABLE C.1: PORTFOLIOS SORTED BY IVOL WITH PENNY STOCK DEFINED AS LESS THAN NOK 5

We form value- and equal-weighted portfolios every month sorted by idiosyncratic volatility relative to FF-3 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.452	5.600	0.081	57.90	-0.005	-0.003	-0.003
					[-4.131]	[-2.995]	[-3.170]
Q2	0.416	6.961	0.060	19.20	-0.005	-0.008	-0.008
					[-2.034]	[-2.876]	[-3.084]
Q3	0.411	7.892	0.052	11.28	-0.006	-0.01	-0.01
					[-2.096]	[-3.378]	[-3.552]
Q4	1.128	9.270	0.122	7.45	-0.001	-0.005	-0.006
					[-0.226]	[-1.437]	[-1.507]
Q5	-0.226	11.290	-0.020	4.16	-0.016	-0.023	-0.023
					[-3.957]	[-5.462]	[-5.841]
Q1 - Q5	0.678	9.087	0.075		0.012	0.019	0.020
	[1.403]				[2.557]	[4.376]	[4.872]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.608	5.290	0.115	57.90	-0.003	-0.003	-0.003
					[-0.940]	[-1.603]	[-1.637]
Q2	0.488	6.226	0.078	19.20	-0.004	-0.008	-0.008
					[-1.439]	[-2.820]	[-2.940]
Q3	0.560	7.614	0.074	11.28	-0.004	-0.01	-0.01
					[-1.466]	[-3.264]	[-3.571]
Q4	0.926	8.332	0.111	7.45	-0.002	-0.008	-0.008
					[-0.498]	[-2.286]	[-2.345]
Q5	-0.120	10.279	-0.012	4.16	-0.014	-0.022	-0.022
					[-3.196]	[-5.215]	[-5.376]
Q1 - Q5	0.728	7.781	0.094		0.012	0.019	0.019
	[1.670]				[2.689]	[4.154]	[4.207]

TABLE C.2: PORTFOLIOS SORTED BY TVOL WITH PENNY STOCK DEFINED AS LESS THAN NOK 5

We form value- and equal-weighted portfolios every month sorted by total volatility estimated using a 24-month rolling window. The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Total Volatility							
Q1	0.515	5.538	0.093	49.70	-0.004	-0.002	-0.002
					[-2.420]	[-1.592]	[-1.598]
Q2	0.238	6.449	0.037	24.08	-0.006	-0.008	-0.008
					[-2.973]	[-3.406]	[-3.629]
Q3	0.454	8.204	0.055	13.59	-0.007	-0.011	-0.011
					[-2.148]	[-3.248]	[-3.540]
Q4	0.588	9.141	0.064	7.75	-0.007	-0.01	-0.01
					[-1.742]	[-2.595]	[-2.674]
Q5	-0.157	10.652	-0.015	4.87	-0.015	-0.021	-0.022
					[-3.549]	[-5.198]	[-5.827]
Q1 - Q5	0.672	8.712	0.077		0.012	0.019	0.019
	[1.406]				[2.379]	[3.987]	[4.639]
Panel B: Equally Weighted Portfolios Sorted by Total Volatility							
Q1	0.687	4.887	0.141	49.70	0.000	-0.002	-0.002
					[-0.057]	[-0.839]	[-0.853]
Q2	0.649	6.223	0.104	24.08	-0.002	-0.006	-0.006
					[-0.939]	[-2.776]	[-2.943]
Q3	0.489	7.591	0.064	13.59	-0.005	-0.011	-0.011
					[-1.655]	[-3.357]	[-3.593]
Q4	0.643	8.408	0.076	7.75	-0.005	-0.01	-0.01
					[-1.480]	[-3.142]	[-3.283]
Q5	-0.035	10.432	-0.003	4.87	-0.013	-0.022	-0.022
					[-3.361]	[-5.889]	[-6.127]
Q1 - Q5	0.723	8.199	0.088		0.013	0.02	0.02
	[1.646]				[3.212]	[4.956]	[5.168]

APPENDIX D RETURN REQUIREMENTS

TABLE D.1: PORTFOLIOS SORTED BY IVOL, RETURN REQUIREMENTS OF 8 MONTHS

We form value- and equal-weighted portfolios every month sorted by idiosyncratic volatility relative to FF-3 estimated using a 24-month rolling window. The portfolio with lowest (highest) idiosyncratic volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.506	5.529	0.092	58.32	-0.004	-0.003	-0.003
					[-3.970]	[-2.738]	[-2.744]
Q2	0.267	7.048	0.038	19.28	-0.007	-0.01	-0.01
					[-2.521]	[-3.350]	[-3.448]
Q3	0.387	7.766	0.050	11.51	-0.007	-0.01	-0.011
					[-1.866]	[-3.041]	[-3.200]
Q4	0.815	9.897	0.082	6.58	-0.005	-0.01	-0.01
					[-1.122]	[-2.331]	[-2.338]
Q5	-0.117	10.887	-0.011	4.30	-0.015	-0.022	-0.022
					[-3.448]	[-4.951]	[-5.395]
Q1 - Q5	0.623	8.685	0.072		0.011	0.019	0.020
	[1.282]				[2.197]	[3.830]	[4.361]
Panel B: Equally Weighted Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
Q1	0.829	5.125	0.162	58.32	-0.002	-0.002	-0.002
					[0.251]	[-0.990]	[-1.010]
Q2	0.474	6.539	0.073	19.28	-0.005	-0.01	-0.01
					[-1.985]	[-3.897]	[-4.013]
Q3	0.693	7.370	0.094	11.51	-0.003	-0.009	-0.009
					[-0.856]	[-2.894]	[-2.962]
Q4	0.700	8.663	0.081	6.58	-0.005	-0.011	-0.012
					[-1.383]	[-3.457]	[-3.494]
Q5	0.229	10.039	0.023	4.30	-0.010	-0.02	-0.02
					[-2.301]	[-4.905]	[-5.042]
Q1 - Q5	0.599	7.343	0.082		0.011	0.018	0.018
	[1.382]				[2.705]	[4.637]	[4.796]

TABLE D.2: PORTFOLIOS SORTED BY TVOL, RETURN REQUIREMENTS OF 8 MONTHS

We form value- and equal-weighted portfolios every month sorted by total volatility estimated using a 24-month rolling window. The portfolio with lowest (highest) total volatility is given by Q1 (Q5), buying Q1 and selling Q5 yields the long-short portfolio given by Q1-Q5. Mean and standard deviation are measured in monthly percentages, and apply to excess, simple return. Column 6 to 8 report the alpha generated for every portfolio when controlling for systematic risk given by CAPM, FF-3 or FF-5. Note that alphas are given in decimals and not percentages. Robust t-statistics Newey and West (1987) in brackets. Results are based on monthly data using a sample period of July 1991 to December 2017, whilst return calculations are from the period of August 1993 to December 2017, 293 months.

Portfolio	Mean	Ex-post Std. Dev.	Sharpe Ratio	% Mkt Share	CAPM Alpha	FF-3 Alpha	FF-5 Alpha
Panel A: Value-Weighted Portfolios Sorted by Total Volatility							
Q1	0.495	5.492	0.090	51.37	-0.004	-0.002	-0.002
					[-2.420]	[-1.592]	[-1.598]
Q2	0.357	6.590	0.054	24.42	-0.006	-0.008	-0.008
					[-2.973]	[-3.406]	[-3.629]
Q3	0.351	8.033	0.044	12.69	-0.007	-0.011	-0.011
					[-2.148]	[-3.248]	[-3.540]
Q4	0.729	9.440	0.077	6.83	-0.007	-0.01	-0.01
					[-1.742]	[-2.595]	[-2.674]
Q5	-0.293	10.376	-0.028	4.70	-0.015	-0.021	-0.022
					[-3.549]	[-5.198]	[-5.827]
Q1 - Q5	0.788	8.301	0.095		0.012	0.019	0.019
	[1.732]				[2.379]	[3.987]	[4.639]
Panel B: Equally Weighted Portfolios Sorted by Total Volatility							
Q1	0.778	4.917	0.158	51.37	0.000	-0.002	-0.002
					[-0.057]	[-0.839]	[-0.853]
Q2	0.785	6.227	0.126	24.42	-0.002	-0.006	-0.006
					[-0.939]	[-2.776]	[-2.943]
Q3	0.475	7.632	0.062	12.69	-0.005	-0.011	-0.011
					[-1.655]	[-3.357]	[-3.593]
Q4	0.735	8.894	0.083	6.83	-0.005	-0.01	-0.01
					[-1.480]	[-3.142]	[-3.283]
Q5	0.203	10.069	0.020	4.70	-0.013	-0.022	-0.022
					[-3.361]	[-5.889]	[-6.127]
Q1 - Q5	0.575	7.472	0.077		0.013	0.02	0.02
	[1.286]				[3.212]	[4.956]	[5.168]

APPENDIX E DERIVATION OF THE OMITTED VARIABLE BIAS

The concept of omitted variable bias can be viewed as the following (Bienz, 2018):

The true relationship is given by (E.1):

$$y = x_1\beta_1 + x_2\beta_2 + \varepsilon \tag{E.1}$$

However, if we do not include x_2 in our estimation we get the following relationship (E.2):

$$y = x_1\beta_1 + \eta \tag{E.2}$$

Which lead us to the following biased estimate of the coefficient (E.3):

$$\hat{\beta} = (x_1'\, x_1)^{-1} x_1' y$$

$$\hat{\beta} = (x_1'\, x_1)^{-1} x_1' (\, \beta_1 x_1 + \beta_2 x_2 + \varepsilon)$$

$$\hat{\beta} = \underbrace{(x_1'\, x_1)^{-1} x_1' x_1}_{1} \beta_1 + \underbrace{(x_1'\, x_1)^{-1} x_1' x_2}_{\lambda} \beta_2 + \underbrace{(x_1'\, x_1)^{-1} x_1'\, \varepsilon}_{0}$$

$$\hat{\beta} = \beta_1 + \beta_2\lambda \tag{E.3}$$