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Co-Moments of Truth

Is the Pricing of Higher-Order Co-Moments Robust Across Portfolio Sorting Methodologies?

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Abstract

The discovery rate of pricing factors has increased substantially in the last decades. Whereas the number of factors discovered was about one per annum in the period 1980 – 1991, it has risen to about 18 per year in the last decade (Harvey, Liu, & Zhu, 2016).

This thesis investigates whether the proposed factors co-skewness and co-kurtosis are in fact priced in equity markets, and how sensitive the pricing of these factors are to the portfolio sorting methodology. Just as the market beta represents an asset´s co-variance with the market, relative to the variance of the market, the higher-order co-moments, co-skewness and co-kurtosis, are analogous to non-linear variations of the market beta. Given the esoteric nature of these concepts, we also include a more ad-hoc measure of skewness, FMAX, which is a proxy for lottery demand.

We review the pricing of higher-order co-moments with new methods of portfolio sorting. Intuitively, the choice of test assets should not matter, as a pricing model should price all assets, not just subsets of assets. However, Daniel and Titman (2012) show that sorting on a single factor (HML in their case) effectively eliminates most of the variation independent of that factor. Furthermore, we apply the latest adjustments to the CRSP data supported in the asset pricing literature. More specifically, we use univariate, triple-sorted and industry portfolios in our analysis. To illustrate the effect of the portfolio sorting, we also include the more widely known factors SMB (size), HML (value) and the excess market return in our analysis.

We utilise a Fama-MacBeth regression methodology to find the risk premia for the market, SMB, HML, co-skewness, co-kurtosis and FMAX, in the different portfolio settings. Moreover, we follow up on the study by Chung, Johnson and Schill (2006) and check whether co-skewness and co-kurtosis proxy for the SMB and HML factors.

Our results indicate that all the aforementioned factors are sensitive to the portfolio sorting methodology. Co-skewness and co-kurtosis does seem to add some explanatory power (adjusted R-squared) to the Fama-French model and CAPM, but do not appear to be priced factors. Moreover, we find limited evidence of the SMB and FMAX factors being priced. The only factor that exhibits some consistency across sorting methodologies is the HML (value) factor.

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1. Introduction

Since the inception of the Capital Asset Pricing Model (CAPM) and its subsequent empirical failure (see e.g. Jensen, Black and Scholes (1972)), a large share of asset pricing research has been concerned with finding additional factors, beyond beta, that explain the prices of assets. There is currently a plethora of these proposed factors documented in the finance literature, ranging from market factors and behavioural biases, to firm characteristics, even including firms' political campaign contributions (Harvey, Liu, & Zhu, 2016).

Perhaps the most popular extension to the CAPM is the Fama-French Three-Factor Model, which incorporates a size (SMB) and value (HML) effect (Fama & French, 1993). Fama and French argue that small companies have less access to funding and that firms with a high book-to-market-ratio can be in financial distress, i.e. investors should be rewarded for investing in these firms. This explanation has been directly opposed by several researchers, e.g Daniel and Titman (1993). Apart from the model itself, one of the pivotal contributions of Fama and French is the construction of factor-mimicking portfolios which can be utilised in empirical asset pricing tests. They create a portfolio which, for e.g. the size effect, goes long in a set of stocks with low market capitalisations and short in large caps. Despite Fama and French offering a compelling explanation for why their factors should be priced, the model lacks a grounding in economic theory.

An alternative to the Fama-French model is an extension of the CAPM incorporating systematic higher-order moments, usually in the form of a four-moment CAPM. Where beta is an expression for the systematic second central moment, i.e. (co)variance, the systematic third and fourth moments can also be expressed in a similar manner. As we will show theoretically, the pricing of higher-order moments can be obtained by a Taylor-expansion of an investor's utility function. In terms of empiricism, studies such as Kraus and Litzenberger (1976) and Harvey and Siddique (2000) find empirical evidence of co-skewness being priced, whereas Dittmar (2002) finds evidence that co-kurtosis is priced. Finally, Chung, Johnson and Schill (2006) argue that the value and size effects are proxies for higher-order moments. In general, however, the body of studies supporting these empirical findings appear limited.

More recently, Daniel and Titman (2012), have argued that the way test assets are grouped into portfolios, by similar characteristics, effectively eliminates most of the variation independent of the variable being sorted on. The authors go on to claim that the univariate and bivariate portfolio sorts on e.g. size and book-to-market-ratios, which is a popular choice, has led to false discovery of factors. Moreover, in an extensive meta-analysis of asset pricing factors, Harvey, Liu and Zhu (2016) identify 316 candidate pricing factors, published in top journals, and also argue that most of these are likely false.

Since the seminal papers of Fama & French, and the studies supporting the pricing of higher-order moments, there have been updates in the data material, i.e. in the database of the Center for Research in Securities Prices (CRSP). Additionally, Shumway (1997) discovered a severe selection bias in the CRSP data related to delistings. In light of these findings and the recent criticism of the test asset methodology, we empirically review the pricing of higher-order moments, and also test the Fama-French Three-Factor model to illustrate the arguments of Daniel and Titman (2012). We also include a more *ad-hoc* measure of higher moments, FMAX. Given the findings of Chung et al. (2006) indicating that higher-order moments proxy for the Fama-French Three-Factor model, we review these results with newer data and sample adjustments. Note, however, that Chung et al. (2006) include moments up to the 10^{th} order, while we limit ourselves to the 4^{th} , i.e. co-kurtosis.

We first test these models in univariate-sorted portfolios, and follow Daniel and Titman's (2012) suggestions for portfolios and also create portfolios based on a triple-sorting procedure, in addition to industry-affiliation portfolios. Moreover, we apply the adjustment to the CRSP data that are justified in the asset pricing literature, some of which were documented after the models were first published. Additionally, we utilise both monthly, quarterly and semiannual return frequencies in our empirical tests. The interested reader is referred to the Appendix for other results using different portfolio weighting-methods and return calculations.

This thesis is structured as follows: We begin with a literature review, in Section 2, which first provides a quick recap of CAPM and the Fama-French model. Thereafter, we provide a formal derivation of how higher-order moments enters asset pricing, based on previous literature, and also touch upon behaviouristic explanations. In Section 3, we outline our Fama-MacBeth methodology and expand on the portfolio sorting methodology. Furthermore, in Section 4, we detail our data collection and sample structuring, before providing some descriptive statistics for the samples. In the empirical analysis in Section 5, we test the candidate factors across the different portfolios and discuss the results. Lastly, we evaluate the robustness of our results in Section 6, and finally conclude on the results, in addition to providing suggestions for further research in Section 7.

2. Literature Review

In this section, we will present literature and economic theories relevant to our study. First, we provide a short recap of the CAPM and the Fama-French Three-Factor model. This is followed by a examination of expected utility theory, where we, based on previous papers, derive utility functions for investors taking into account the effects of higher-order moments. We also review criticism of higher-order moments in asset pricing. Finally, we discuss how a behaviouristic model of utility, Cumulative Prospect Theory, can lead to the pricing of skewness and how we can go about measuring it.

2.1 The CAPM and the Fama-French Model

The Capital Asset Pricing Model (CAPM), as developed by Sharpe (1964), Lintner (1965) and Mossin (1966), is one of the main pillars of financial economics. CAPM enjoys widespread use both in academia, and among finance professionals. CAPM's central prediction is that there is a positive linear relationship between market-related risk, beta, and expected returns. In other words, investors receive a risk premium for investing in assets which covary with the market portfolio. Thus, as CAPM only relies on beta to compute expected returns, it is an intuitive and easy model to apply in practice.

Like all models, CAPM is simplification of reality, and it relies on assumptions such as no transaction costs, no informational asymmetry, homogeneous expectations, unlimited borrowing and lending at the risk-free rate etc. However, there is one important assumption which is necessary for the CAPM to obtain, which is of special interest to our thesis, namely that investors have quadratic utility functions or that assets have normally distributed returns (Berk, 1997). Despite the theoretical appeal of the CAPM, a number of anomalies have arisen since its inception. Generally, empirical evidence fails to back up the linear relationship between beta and expected returns. A number of studies have found that low-beta assets have higher risk-adjusted returns than high-beta assets (Frazzini & Pedersen, 2014). Another prediction of the CAPM is that the expected payoff to taking on idiosyncratic risk is zero. Nevertheless, Ang, Hodrick, Xing and Zhang (2006) provide compelling evidence that exposure to idiosyncratic risk is associated with lower returns, which is dubbed the "idiosyncratic volatility puzzle".

Furthermore, a plethora of studies have identified non-market factors which appear to carry a risk premium, i.e. opposing CAPM’s central prediction. These non-market factors can further be divided into common factors (affecting all assets) and firm-specific characteristics factors. Harvey, Liu and Zhu (2016) conduct a survey of proposed asset pricing factors in top finance and economics journals since the invention of CAPM, where they identify 103 different common factors and 202 different characteristics-based factors. Needless to say, there is compelling evidence that CAPM is not sufficient to explain asset prices.

Of the proposed characteristics factors, two of the most prominent are the value and size-premium, as proposed by Fama and French (1992). Fama and French proxy firm size with the observed market capitalisation and value with the book-to-market ratio. Moreover, they form factor-mimicking portfolios that go long in high book-to-market and short in low book-to-market equities for the value premium. For the size premium, they form portfolios that go long in small cap equities and short in large caps. Subsequently, Fama and French (1993) suggest an extension to the classic CAPM by incorporating the size and value factors. This model is known as the Fama-French Three-Factor model. The Fama-French model has also been extended to include other factors, such as e.g. firm profitability (RMW) and capital expenditure (CMA), as described in Fama and French (2014). However, for the purpose of this study, we limit ourselves to testing the Three-Factor model

2.2 Utility theory and the rationale for higher moments

The key element of asset pricing models is the pricing of risk. Markets yield higher returns than savings accounts and T-bills, but also comes at the price of higher volatility. Consequently, it is only reasonable that this risk is compensated for (i.e. priced, yielding the investor a premium). In the CAPM, this is represented by the market beta, effectively reflecting the asset´s sensitivity to the systematic risk, meaning the covariance between the asset and the market, relative to the overall variance of the market. Arditti (1967) identifies several risk variables and their relationship to the required rate of return. He finds it convenient to divide them in to two groups; (a) those directly related to the probability distribution of returns (i.e. moments of the distribution of returns), and (b) those that are intertwined with the financial policies of the company (i.e. dividend-earnings, debt-equity ratios etc.). This study will focus on the former.

The CAPM elegantly incorporates the mean return as well as the variance. Thus, it heavily relies on the assumption that returns follow a distribution that can be completely described by the two first moments alone or quadratic utility. Asset pricing models are founded on the basis of utility functions. Accordingly, the mean and variance should therefore completely describe *E(U)* if we assume the CAPM is correct. Scott & Horvath (1980) states that this is only the case for the *normal*, the *uniform* and the *binomial* distributions. Thus, it may be inadequate to restrict an asset pricing model to the first two moments of the return distribution.

In his 1967 paper, Arditti expands the utility function using Taylor series expansion to illustrate how higher moments enter the utility function. Through what he terms a common-sense result he concludes that a risk averse investor will be reluctant to invest if the investment presents him the possibility, however small, of a substantial loss with only a limited gain. This asymmetry factor is reflected in the skewness measure. From this it follows that risk averters like positive skewness and dislike negative skewness. Arditti (1967) goes on by stating that attention in research has been centred on the second and third moments of return´s distribution because higher-order moments of the returns add little or no additional information about the return´s distributive features. This claim was later refuted by Levy (1969) to some extent. Levy (1969) argues that even if higher moments add no additional information about the distribution they should nonetheless be included in the utility function. Only in the special case of a restricted utility function can they be disregarded. He also adds that if the distribution can be well-described by the first moments, then the higher-order moments are approximate functions of the aforementioned moments. That, however, need not imply that they are small in terms of magnitude. Nonetheless, Levy (1969) comes to the same conclusion as that of Arditti when evaluating skewness preference. Relating it to real world concepts, he concludes that investors will prefer positive asymmetry like that of a lottery´s, and consequently dislike negative asymmetry which further substantiates why there is a market for insurance policies.

As per Levy´s discussion, only two cases "opens" for neglecting higher moments; (a) if all the higher moments tend to zero (i.e. approximately symmetric), or (b) if we assume a cubic utility function, consequently resulting in derivatives of higher orders than three to be zero, so that *U(W)* only depends on the first three derivatives. This notion is also supported by Jean (1971), adding that without a specific form of utility function, we cannot decide on the appropriateness of estimation by investigating the remainder terms of the Taylor series expansion. It is infeasible to evaluate the adequacy of utility functions. However, the distribution of asset returns can be investigated, both theoretically and empirically. Jean (1971) illustrates that leveraged capital structures will result in skewed payments to shareholders (i.e. non-symmetric). Thus, returns are most likely not symmetrically distributed and skewness may affect prices. Hagerman (1978) also confirms this notion, finding empirical evidence that strongly suggests a symmetric distribution is not a reasonable feature to assume for asset returns. To see how skewness can impact the investor´s investment decision, Simonson (1972) presents the following graph, illustrating a typical return distribution for three assets:

Figure 1 – Return distributions (Simonson, 1972)

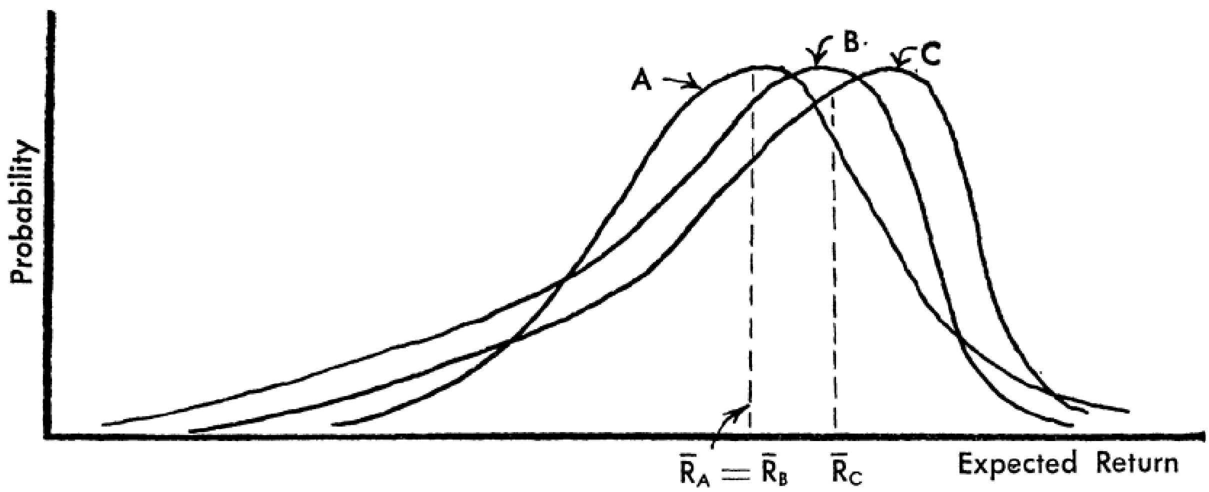


Figure 1 illustrates representative return distributions that the investor is faced with. Distribution A and B have equal mean and variance. Nonetheless, asset A will be favoured over asset B since asset B has a larger downside potential (negative skewness). Similarly, asset C, whose variance and skewness matches that of asset B, will clearly be preferred to asset B due to its higher mean. However, in terms of choosing between asset A and asset C, it is not clear which asset will be favoured. This will depend on whether or not the additional utility related to asset C´s higher mean more than offsets the disutility stemming from its negative skewness. A similar example is presented by Scott and Horvath (1980). Through mathematical derivation they also further ascertain the skewness preference claims of Markovitz (1952), Arditti (1967) and Levy (1969). They conclude that "the preference direction is positive (negative) for positive (negative) values of every odd central moment and negative for every even central moment" pp. 916.

To better understand how higher-order moment preference enter the utility functions that lay the foundation for asset pricing, we present the Taylor series expansion of the utility function. Our derivation and notation follows that of Jean (1971), who´s derivation is similar to the one by Farrar (1962):

Define a time-invariant function, $U(W)$, where U represents the individual investor´s utility and W represents the money-value (wealth). Also, let W be a random variable subject to some statistical distribution. By applying Taylor series expansion around the mean cash flow, $E(W)$, we have

$$U(W) = U[E(W)] + U'[E(W)]\,[W - E(W)] + \frac{U''[E(W)][V-E(W)]^2}{2!} + \frac{U'''[E(W)][V-E(W)]^3}{3!} + \cdots + \frac{U^{(k)}[E(W)][V-E(W)]^k}{k!} + \cdots. \quad (1)$$

We can subsequently take the expected value over W, on both sides to obtain the expected utility,

$$E[U(W)] = U[E(W)] + U'[E(W)]\,E[W - E(W)] + \frac{U''[E(W)]}{2!}E[W - E(W)]^2 + \frac{U'''[E(W)]}{3!}E[W - E(W)]^3 + \cdots + \frac{U^{(k)}[E(W)]}{k!}E[W - E(W)]^k + \cdots . \quad (2)$$

The first expression, $U[E(W)]$, reflects the utility function evaluated around the mean cash flow. The second term will be zero since $E[W-E(W)]$ is zero. More interestingly, the third term is the product of a constant, $\frac{U''[E(W)]}{2!}$, and the variance of the cash flows. Conversely, the remainder terms are also a constant multiplied by a higher-order moment around the mean of W.

In his paper, Jean (1971) also derives the risk premium for any given moment. We do not include his derivations, but present his conclusions:

Define $\boldsymbol{w} \equiv W - E(W)$

Then the risk premia for asset *i* can be expressed as

$\frac{[E(R_M)-R_f]}{\sigma^2(R_M)} E(\boldsymbol{w}_i \boldsymbol{w}_M) = \frac{[E(R_M)-R_f]}{\sigma^2(R_M)} Cov(R_i, R_M)$ for the second moment (variance)

$\frac{[E(R_M)-R_f]}{m_3(R_M)} E(\boldsymbol{w}_i \boldsymbol{w}_M^2) = \frac{[E(R_M)-R_f]}{m_3(R_M)} Cos(R_i, R_M, R_M)$ for the third moment (skewness)

$\frac{[E(R_M)-R_f]}{m_n(R_M)} E(\boldsymbol{w}_i \boldsymbol{w}_M^{n-1})$ then represent the risk premia for the n^{th} moment

In this case, the n^{th} moment is defined as $m_n = E[R_M - E(R_M)]^n$

Since return distributions empirically have shown to be non-symmetrical and therefore not completely described by the mean and variance, the higher moments should thus be considered, according to Levy (1969), Jean (1971) and Scott & Horvath (1980). However, they all emphasise that this is conditional upon whether the utility function permits it or not. Kraus and Litzenberger (1976) corroborate on this. To establish an exact ordering of risky portfolios using the mean, variance and skewness of the returns, one generally have to assume that the investor has a cubic utility function. Obviously, this is necessary to enable derivatives up to the third order. However, the suitability of such a utility function is rather questionable if we are to assume that the utility function should exhibit the traits of a risk averting individual. Kraus and Litzenberger (1976) refer to Arrow (1971), who establish three desirable properties of the utility function:

(1) Positive marginal utility of wealth (i.e. more is better than less, so $U'(W) > 0$)
(2) Decreasing marginal utility for wealth (i.e. risk aversion, so $U''(W) < 0$)
(3) Non-increasing absolute risk aversion (i.e. no reduction in nominal amount invested in risky assets as the wealth increases, so $-\frac{U''(W)}{U'(W)} \leq 0$)

Without imposing condition (3) the utility function would imply that the risky asset portfolio is an inferior good. These desired properties are only upheld by the *logarithmic*, the *power* and the *negative exponential* utility functions. The aforementioned functions are non-polynomials and thus the commonly used quadratic utility function does not comply with the set of conditions. Furthermore, the cubic utility function will also fall short in this regard, as Levy (1969) proves that it only exhibits decreasing marginal utility for wealth for a limited domain of positive wealth levels. As such, choosing the "correct" utility function may be infeasible and consequently boil down to a problem of minimising the unrealistic behavioural implications it entails, as a "best estimate". Not unlike any mathematical model, the expected utility theory is merely an abstract simplification of the complex reality. Thus, its validity is hardly measurable and there is no guarantee for its reliability. Nonetheless, despite the previous derivations´ unreliability concerning real world application, they serve an important role of revealing the possibility of higher-order moment pricing.

Having introduced higher-order moments to the utility function, we need a market relation to obtain an asset pricing model that we can test empirically. Fortunately, Rubinstein (1973) derives this market relation assuming a separable cubic utility function:

Assume a perfect and competitive securities market consisting of I investors and J securities. Let W_i represent the present (positive) wealth of investor i, and let s_{ij} reflect the investor´s dollar-investment in security j. The rational investor will maximise his expected utility of wealth, $E_i[U_i(\tilde{W}_i)]$, subject to his budget constraint, $W_i = \sum_j s_{ij}$.

To corroborate:

- U_i is investor i´s continuously differentiable measurable utility-of-wealth function for which $U_i' > 0$
- R_j (random variable) is unity plus the rate of return of security j
- $\tilde{W}_i = \sum_j s_{ij} R_j$ (random variable) is investor i´s future wealth
- E_i is an expectation operator reflecting investor i´s subjective assessments

By applying the exact Taylor series expansion around $E_i(\tilde{W}_i)$ and subsequently taking the expectation over W_i on both sides yields the same result as Equation 2 by Jean (1971), however presented more generalised:

$$E_i\left[U_i\left(\widetilde{W}_i\right)\right] = \sum_{n=0}^{\infty} \frac{U_i^{(n)}}{n!} m_{in} \tag{3}$$

where:

- $U_i^{(n)}$ is the n^{th} derivative of U_i evaluated at $E_i(\tilde{W}_i)$
- $m_{in} = E_i[\tilde{W}_i - E_i(\tilde{W}_i)]^n$ is the n^{th} central moment of $\tilde{W}_i$

Further, define the n^{th} co-moment $\sigma_{in}(R_j,\tilde{W}_i) = E_i[(R_i - E_i(R_i))(\tilde{W}_i - E_i(\tilde{W}_i))^{n-1}]$ for $n \geq 2$ and $\theta_{in} \equiv \frac{-U_i^{(n)}}{(n-1)!E_i[U_i'(\widetilde{W}_i)]}$. Rubinstein (1973) forms the Lagrangian and the first-order conditions to maximise expected utility. He then subtracts a hypothetical asset *k* from the resulting solution. Assuming that *k* is the risk-free asset and imposing homogenous subjective probabilities he then derives the following fundamental theorem:

$$E\left(R_j\right) = R_F + \sum_{n=2}^{\infty} \theta_{in}\sigma_n\left(R_j, \widetilde{W}_i\right)\ for\ all\ j\ (securities) \tag{4}$$

More generally, if we do not define asset *k* and do not assume homogenous subjective probabilities then the necessary equilibrium conditions follow:

$$\frac{\sum_i E_i\left(R_j\right)}{I} = \frac{\sum_i E_i(R_k)}{I} + \frac{\sum_i \theta_{in}\sigma_{in}\left(R_j - R_k, \widetilde{W}_i\right)}{I}\ for\ all\ j,k. \tag{5}$$

According to Rubinstein (1973):

> … co-moments are the appropriate *individual measures of security risk* since the co-moments reflect the contribution of a marginal increase in the holdings of a security to the corresponding central moments of individual future wealth, which are the appropriate measures of portfolio risk in parameter-preference models. Each co-moment is weighted by the ratio θ_{in} reflecting the corresponding *individual measure of risk aversion* (pp. 65).

Again, letting *k* be the risk-free asset and assuming homogenous subjective probabilities. Moreover, recognising that each sum over *i* reduces to *I* multiplied by the summand. Finally, since all assets must be held in equilibrium we equate to a function of the future value of the (market) $\tilde{W}_i$ portfolio:

$$E[R_j] = R_F + \sum_{i=2}^{N} \lambda_i b_{ij} \tag{6}$$

Equation (6) represents the generalised version of the security market line, but rather than using one co-moment there are an arbitrary number. Chung et al. (2006) clarifies, explaining that "b_{ij} is the *i*th-order systematic co-moment between R_j and R_M, and λ_i is the market measure of risk aversion for the *i*th co-moment" (pp. 926)

For example, when assuming a separable cubic utility function, Rubinstein (1973) derives the following market relation:

$$E(R_j) = R_F + \lambda_2 Cov(R_j, R_M) + \lambda_3 Cos(R_j, R_M, R_M) \tag{7}$$

where Cos is co-skewness, R_M denotes the market portfolio return, and λ_2 and λ_3 are market measures of risk aversion. Kraus and Litzenberger (1976) derive an equation of the same form.

Kraus and Litzenberger (1976) also develop the empirical model to test equation (7). The explicit empirical model is given by:

$$\bar{r}_i = \alpha + \lambda_1 \beta_i + \lambda_2 \gamma_i + u_i \; for \; i = 1,2,\ldots,N \; where \; \bar{r}_i = \frac{\overline{R_\imath - R_f}}{\bar{R}_f},$$
$$\beta_i = \frac{\sum_t (r_{it} - \bar{r}_i)(r_{Mt} - \bar{r}_M)}{\sum_t (r_{Mt} - \bar{r}_M)^2}, \gamma_i = \frac{\sum_t (r_{it} - \bar{r}_i)(r_{Mt} - \bar{r}_M)^2}{\sum_t (r_{Mt} - \bar{r}_M)^3} \tag{8}$$

R_i is the dividend-adjusted return for the *i´*th asset, and R_M is the market portfolio return. Furthermore, R_f is the risk-free return and u_i is the error term assumed to be a random independently distributed variable. The $\bar{r}_i$ is the risk-free rate deflated excess rate of return of asset *i*. In this three-moment version of the CAPM, the constant term α should be zero to correspond to the risk-free rate intercept (R_f) in Sharpe-Lintner version of the CAPM. The factor premiums, meaning the estimated coefficients λ_1 and λ_2, should in sum be equal to the market risk premium ($\lambda_1 + \lambda_2 = r_M$). More precisely, λ_1 should be positive, and λ_2 should have the opposite sign as that of the skewness of the market portfolios.

2.3 Criticism of higher-order moments in asset pricing

Two main arguments persist as to not include higher-order moments than four in asset pricing. The first refers to the argument made by Arditti (1967), as we have previously mentioned. He argues that most research has been developed around the first three moments since moments above this add little or no additional information about the distribution of the returns. This is even more true for moments above kurtosis. Levy (1969) corrected him from a purely theoretical point of view, arguing to include all higher-order moments in the utility functions, as long as they exist. Though from a more pragmatic point of view, to include them in an asset pricing model would simply be a question of whether we have reason to believe they are priced. This brings us to the next premise. The interpretation of kurtosis has not been without disputes in the academic community. Several claims as to how to interpret it has been refuted, and only purely mathematical or vague intuitive interpretations persists. As it seems that the academic community "struggles" in some sense with the interpretation, it would be unreasonable to assume that investors exhibit any clearer intuition of the measure. Thus, one could make the argument that moments above the kurtosis lacks intuition. Consequently, any consistent pricing of these higher-order moments will be very unlikely.

2.4 Behavioural Asset Pricing and the FMAX factor

The asset pricing literature we have reviewed thus far all have one thing in common. That is, the underlying assumption of rational investors. In classical asset pricing models, we usually assume that investors are rational and risk-averse. This assumption has come under stern scrutiny in recent decades. The perhaps most prominent alternative explanation to classical utility theory, is prospect theory, or cumulative prospect theory.

Prospect theory was first proposed by behavioural psychologists Kahneman and Tversky (1979), where the authors heavily criticise expected utility theory as a *positive* economic theory. One of the main points of prospect theory is that people think of risky outcomes in relation to a reference point, as opposed to their final wealth. Moreover, Kahneman and Tversky (1979) argue that people value gains and losses of equal size differently, assigning the pain of loss to be roughly twice the size of the pleasure of a gain. Consequently, under prospect theory investors exhibit *loss-aversion*, as opposed to *risk-aversion*.

Furthermore, the authors argue that people tend to overweight the probability of extreme outcomes. For example, under prospect theory, people are *risk-seeking* over gains with low probability, which can explain the observed demand for lotteries. A lottery ticket yields a negative expected wealth, but also provides a very small probability of drastically increasing wealth. Apart from prospect theory, several behavioural biases also contradict the rationality of investors, such as *overconfidence*, *herd behaviour*, *confirmation bias* etc.

Like all theories of choice, prospect theory is not without its weaknesses. Firstly, in applying prospect theory to asset pricing, it can be shown that prospect theory in its original form violates first-order stochastic dominance, which violates the axioms of expected utility theory. Therefore, Tversky and Kahneman (1992) propose a modification to expected utility theory, called Cumulative Prospect Theory (CPT). Under CPT, the aforementioned probability weighting is applied to the cumulative probability function instead of individual outcomes. There are also further reasons to question the applicability of CPT in asset pricing. As described in e.g. Kahneman (2011), the development of CPT is based on a series of lab experiments. One can argue that the test subjects in these experiments are not representative of finance professionals, and thus do not suffice to explain the behaviour of the marginal price setters in capital markets. Additionally, one can surmise that irrational investors should be competed out of the capital markets.

Despite these concerns, CPT has received a lot of attention in behavioural finance and asset pricing. For example, Benartzi and Thaler (1995) show that the observed equity premium is more consistent with CPT than classical expected utility models, whereas Kumar (2009) finds evidence of lottery demand in the stock market, and that individual investors behave less rationally than institutional investors. Cumulative Prospect Theory has wide applications in finance in general. However, in the behavioural asset pricing literature, one notion is of particular interest to our study, namely the pricing of skewness in asset returns. Thus far, we have argued that investors have a preference for positive skewness due to the decreased left-tail risk caused by a positively skewed distribution. Barberis and Huang (2008) show that, theoretically, the probability weighting properties of CPT lead to a demand for positive skewness. Moreover, the authors link this positive skewness to lottery demand, a point which is also corroborated by Kumar (2009).

Therefore, it might be that a preference for positive skewness is caused by the long right tail in positively skewed asset return distributions, which offers a low probability of a large gain. Whether skewness preferences are caused by rational investors concerned with the risk of ruin, or irrational investors who like lotteries is challenging to assess, as lottery demand is very difficult to quantify. Nonetheless, several authors have made attempts at finding a good proxy for lottery demand. Perhaps the most prominent of these proxies is the MAX-factor, as proposed by Bali, Cakici and Whitelaw (2011). In short, the MAX-factor is simply the average of a stock's five highest returns in the previous month, where the idea is that a high realisation of the MAX-factor is a proxy for lottery payoff. Although such a factor could easily proxy for other known asset pricing factors, e.g. momentum, the authors demonstrate that high-MAX-stocks have lower returns than low-MAX stocks when controlling for the most common pricing factors.

Bali, Brown, Murray and Tang (2017) further show that the MAX-factor can explain one of the most persistent pricing anomalies in finance, namely the low-beta anomaly. Moreover, the authors create a factor portfolio in the spirit of Fama and French, called FMAX. This factor portfolio is available at Turan Bali's website, and we include it as a pricing factor in the analysis section of this paper. A final argument in favour of a more ad-hoc approach to skewness, i.e. lottery-stocks, is simply the esoteric nature of higher-order co-moments. The concept of higher-order co-moments is difficult to grasp, and we believe it is not widely understood. As such, it is difficult to believe all investors consider the co-skewness, co-kurtosis etc. of their investments.

3. Methodology

3.1 The Fama-MacBeth procedure

To determine the risk premia of co-skewness, co-kurtosis, SMB, HML and FMAX, we utilise the Fama-MacBeth regression methodology, as developed by Fama and Macbeth (1973). The method is popular in empirical asset pricing and consists of a two-pass regression technique. In the first pass, factor loadings for assets (portfolios) are found through rolling time-series regression for each asset against the proposed risk factors. Thereafter, in the second pass, the risk premium at each time *t* is found by cross-sectionally regressing the portfolio returns against the factor loadings from the first pass. Finally, the *T* set of coefficients are averaged across time, which gives the final estimate of a factor's risk premium.

The motivation behind the Fama-MacBeth procedure, is to correct for cross-sectional correlation between assets. This is an important aspect of asset pricing studies, as equities are susceptible to the same cross-sectional shocks. For example, if at time *t*, a macroeconomic announcement unexpectedly indicates the economy is dropping into a recession, we do not expect two assets *i's* and *j's* reaction to be uncorrelated. Rather, we generally expect their reactions to be highly correlated, of course depending on the asset.

In spite of its intuitive nature and correction for cross-sectional correlation, the Fama-MacBeth method fails to adjust for autocorrelation. Although this is not a major concern, as equities tend to exhibit low or zero autocorrelation, we address autocorrelation further in Section 6.3. Moreover, the Fama-MacBeth procedure has a widely-known errors-in-variables (EIV) problem. This EIV problem arises from the fact that the explanatory variables in the second pass regression are the estimated coefficients from the first pass, which are subject to measurement errors, and thus deviate from "true coefficients". Consequently, the standard errors of the cross-sectional regression may be biased. In the context of Fama-MacBeth regressions, the EIV problem is specifically addressed by Shanken (1992). However, the correction proposed by Shanken is beyond the scope of this thesis.

We rely heavily on the Fama-MacBeth methodology, but make some adjustments to the method for the purpose of our study. First of all, in the original paper, the authors only examine the market beta, and consequently the market risk premium. We expand our model to also include the Fama-French risk factors SMB and HML (Fama & French, 1992), in addition to co-skewness, co-kurtosis and FMAX.

Moreover, in Fama & MacBeth (1973), for the first step, the authors use an initial estimation period for each portfolio's beta with the market of 5 years. In the subsequent 5 years (the testing period), the portfolio betas are kept constant from year t to year $t + 5$. With our approach, in step 1, a portfolio's coefficient for a given factor, for each time *t*, is computed utilising exactly 5 years of trailing returns. As such, we run N x (T – 5 yrs.) time-series regressions, where N is the number of portfolios. To illustrate, for monthly returns in the univariate size-sorted portfolios, we run 25 x (1020-60) = 25,500 time-series regressions in the sample period 1931-2016. The regression we run in the first pass is illustrated in Equation 9.

$$R_{i,t} = a_i + \beta_{i,t} R_{M,t-\tau} + s_{i,t}\, SMB_{t-\tau} + h_{i,t}\, HML_{t-\tau} + e_{i,t} \tag{9}$$

Where τ = t – 5 years and each coefficient is portfolio *i*'s sensitivity to each risk factor. The perhaps most striking feature of Equation 9 is that we do not estimate higher-order co-moment beta coefficients by means of regression. Instead, we follow Chung et al. (2006) in their definition of higher-order systematic co-moments, with a slight adjustment and compute co-moment of order *n* according to Equation 10. Chung et al. (2006) argue that their *non-centered* measure of higher-order co-moments are more reliable. However, in their study, they also estimate higher-order co-moments up to order 10. The denominator in Equation 10 can tend toward 0 for odd *centred* co-moments when calculating co-moments beyond the 4th order (co-kurtosis). Our only odd co-moment is the 3rd (co-skewness), and therefore centring the co-moments does not affect the reliability of our estimates in any notable manner. Moreover, *centred* co-moments are closer to intuition and theory, e.g. when n = 2 in Equation 10, the expression evaluates to the market beta.

$$\beta_{i,n,t} = \frac{\sum_{\tau}^{T} (R_{i,t-\tau} - \bar{R}_J)(R_{i,t-\tau} - \bar{R}_M)^{n-1}}{\sum_{\tau}^{T} (R_{M,t-\tau} - \bar{R}_M)^{n}} \tag{10}$$

Where β is the n^{th} co-moment coefficient for portfolio *i* at time *t*. Note, however, that for practical reasons we chose to rename the co-skewness and co-kurtosis coefficients γ and δ, respectively, in accordance with Kraus & Litzenberger (1976).

In the second step, the coefficient estimates are utilised as independent variables in a cross-sectional regression. For each time period *t*, we regress the portfolio returns at time *t* against the time *t* - 1 coefficient estimates to find the risk premium for each factor. This produces the following set of cross-sectional regression equations:

$$R_{i,t} = \alpha_t + \lambda_t^{(\beta)}\hat{\beta}_{i,t-1} + \lambda_t^{(s)}\hat{s}_{i,t-1} + \lambda_t^{(h)}\hat{h}_{i,t-1} + \lambda_t^{(\gamma)}\hat{\gamma}_{i,t-1} + \lambda_t^{(\delta)}\hat{\delta}_{i,t-1} + \varepsilon_{i,t} \qquad (11)$$

Where λ is the factor risk premium, and the explanatory variables are the estimated coefficients from Equation 9 and 10.

We run this cross-sectional regression with different specifications, adding and excluding factors. For example, the first cross-sectional regression for a given return interval includes SMB, HML and market beta, whereas the second include the aforementioned factors in addition to the third and fourth co-moment coefficients. Finally, we have specifications including the FMAX factor as well. Also note that we run the cross-sectional returns of portfolio *i* at time *t*, versus the *t* - 1 factor loading. Hence, we explain *conditional* expected returns with the proxy for *ex ante* factor loadings conditioned by the information available at time *t*. If instead using *ex-post* factor loadings, one would assume investors could perfectly predict the factor loadings a period in advance.

Our final estimate of the risk premium is then the average of each time period's risk premium, i.e.:

$$\hat{\lambda} = \bar{\lambda}_T \qquad (12)$$

Finally, the standard error of the factor risk premium is computed as:

$$S.E(\hat{\lambda}) = \frac{\sigma(\hat{\lambda}_t)}{\sqrt{T}} \tag{13}$$

We can then use a t-test to test the significance of the final risk premia estimates. However, we also use the Newey-West adjustment for our standard error estimates and significance testing (Newey & West, 1987). Additionally, we test the joint significance of the SMB and HML factors, and the 3rd and 4th co-moment with a Wald test.

3.2 Portfolio sorting

In empirically evaluating the performance of asset pricing models, one essentially has two options with regards to test assets. Either, you utilise the entire universe of assets, or group assets into portfolios based on certain criteria, usually a firm characteristic such as book-to-market or size. Intuitively, the choice should not matter to a great extent, as an asset pricing model should price all assets, not just subsets of assets. However, utilising portfolios as test assets is a common practice in empirical asset pricing. The motivation behind creating portfolios as test assets, is to address the EIV concerns when using estimated betas as regressors, which is outlined in Section 3.1. This reasoning was first proposed by Blume (1970):

> The reason is that if an investor's assessments of α_i and $\beta_i$were unbiased and the errors in these assessments were independent among the different assets, his uncertainty attached to his assessments of $\bar{\alpha}$ and $\bar{\beta}$, merely weighted averages of the $\alpha_i's$ and $\beta_i's$, would tend to become smaller, the larger the number of assets in the portfolios and the smaller the proportion in each asset. Intuitively, the errors in the assessments of α_i and β_i would tend to offset each other. (pp. 156)

Put in other words, Blume's point is that by grouping assets with similar βs (factor loadings) together, one reduces the estimation error of β, as the estimation errors offset each other. Furthermore, he argues that portfolios as test assets increases the precision of the risk premium estimates and therefore, he only examines portfolios with 20 assets or more in his study.

Similar arguments have also been made in influential asset pricing papers, such as Black, Jensen and Scholes (1972), and Fama and MacBeth (1973). Moreover, according to Ang, Liu and Schwarz (2017) "The majority of modern asset pricing papers testing expected return relation in the cross section now use portfolios" (pp. 2). Additionally, grouping assets into portfolios reduces the computational capacity needed to perform the analysis.

Despite the abovementioned advantages of utilising portfolios as test assets, the portfolio approach has come under scrutiny in recent years. Several recent studies have pointed to the sort methodology as the driver of the significance of SMB and HML compared to other candidate risk factors. Daniel and Titman (2012) present an interesting conundrum; a plethora of factors have been proposed and tested as significant, as explanations for the value-effect

(HML), and these factors generally exhibit low correlation with each other. This implies that the models have different pricing kernels, which in and of itself is not a theoretical problem, given incomplete markets. However, the different models must project the proposed factors into the asset return space identically. Although Daniel and Titman (2012) do not directly test the implication of the models' projection of various factors into asset return space, they argue it is unlikely it would be satisfied. Moreover, the authors argue that the culprit of the significance of seemingly contradictory asset pricing models is the sorting methodology employed in testing these models. Through grouping assets with similar characteristics together in portfolios, one effectively eliminates any variation that is independent of said characteristics. Thus, one lacks the statistical power to reject the proposed risk factor when it is being sorted on. Consequently, the authors conclude:

> ... even if the loadings on a proposed factor are only loosely correlated with the expected returns of the individual assets in the economy, the sorting procedure will result in a set of test portfolios that exhibit a strong relationship between loadings on the proposed factor and expected returns (Daniel & Titman, 2012, pp. 109).

The authors move on to propose a triple-sort procedure, whereby one first sort equities on the basis of size, thereafter on BTM, and finally on the equities' 60-month market beta. Alternatively, they suggest testing asset pricing models on industry-affiliation portfolios. Lewellen, Nagel and Shanken (2010) argue along the same lines in favour of industry portfolios. Other researchers argue in favour of abandoning the portfolio sorting methodology entirely. Ang, Liu and Schwarz (2017), directly oppose the view of Blume (1970) presented earlier. They argue that grouping equities into portfolios destroys information, and show both analytically and empirically that more accurate estimates of factor *loadings* do not result in more precise factor *risk premia* estimates. On the contrary, they argue that using portfolios as test assets reduces the precision of *risk premia* estimates.

Taking into account the criticisms of the portfolio approach, we utilise several different sorting methodologies. Due to limited computational capacity, we opted not to employ the individual stock methodology of Ang, Liu and Schwarz (2017).

To check the robustness of SMB, HML, and higher-order risk premia across portfolios, we perform our analysis on three different sorting methodologies. Our univariate portfolio sorts are based on size and book-to-market, and we form 25 portfolios for each characteristic. We chose 25, as this is a common number of test assets in comparable studies. Additionally, although we do want to maximise cross-sectional variation in each of the factors, we note that there are between 6-10 assets in each portfolio in the first ten years of our sample period. Hence, a larger number of portfolios, such as the 50 portfolios used by Chung et al. (2006) could potentially make our factor loading estimates prone to estimation error. Moreover, following Daniel and Titman (2012) we create portfolios based on their triple sort and also use industry portfolios provided by Kenneth French (2017). For the triple sort, we first sort into three size portfolios. Each size portfolio is then split into three new portfolios based on book-to-market. Finally, the nine size and BTM portfolios are each split into 5 portfolios based on market beta. I.e., we are left with 45 size, BTM and beta sorted portfolios. Further details about the portfolio selection can be found in the data section of this paper.

4. Data

4.1 Description of Data

Our main sources of data are The Center for Research in Security Prices (CRSP) and the CRSP/Compustat Merged (CCM) database. We retrieve monthly price data, an adjustment factor to correct prices for distributions, index returns, shares outstanding, Standard Industrial Classification (SIC) codes, CRSP share codes, delisting codes, delisting returns and the bid/ask-spread for all U.S. equities in the CRSP universe from 30. June 1926 – 30. June 2016 (CRSP, 2017). Moreover, we retrieve book value per share from CCM, for all securities in the CCM database (CRSP/Compustat Merged, 2017). Furthermore, we collect the CRSP PERMNO, from both databases, which is a unique 5-digit code identifying securities. In addition to identifying securities within our dataset, we use the PERMNOs to match firms' book values from CCM with the data items retrieved from CRSP. When downloading CCM data, one is faced with the choice of the quality of links that connects data in CRSP and CCM. We chose only the most well-documented link types, "LC" and "LU", which correspond to links where the link research is complete, or unresearched links that match by the unique identifier CUSIP. This is the default setting in CRSP (CRSP, 2013).

We also collect monthly U.S. risk-free rates, small-minus-big (SMB) and high-minus-low (HML) factors, in addition to returns on 17 industry portfolios from the Kenneth French Data Library (French, 2017). All Kenneth French data is collected on a monthly basis from July 1926 – June 2016. Finally, we retrieve the FMAX factor portfolio from the website of Turan Bali from July 1970 – June 2016 (Bali, 2017).

The result of our data collection is a sample consisting of 1081 months of price data for 31,566 U.S. companies in total. However, the first 10 years of the sample is used for pre-formation betas and beta estimation on the portfolio level. Thus, we effectively have 80 years, or 960 months of return observations.

We initially attempted to include daily and weekly data in the analysis as well. This would have been interesting, as CRSP in 2006 added daily stock prices in the period 1926-1962 (Houlihan & Treuthart, 2011). Therefore, this daily data has not been examined to a great extent. However, the combination of selecting and calculating portfolios; calculating rolling higher-order co-moments; and rolling betas for each period is very computationally demanding. Thus, utilising daily or weekly data is simply too computationally demanding to complete within a reasonable amount of time.

4.2 Sample Structuring

4.2.1 Basics and Characteristics Calculation

In order to obtain excess returns of the portfolios, we subtract the 30-day risk-free rate provided by French, from the raw portfolio returns. Moreover, we divide prices by CRSP's adjustment factor to obtain dividend- and stock split adjusted returns.

The market value of each equity is calculated as the product of the price and common shares outstanding at 30. June for each year *t*. Both CRSP and CCM provide data for common shares outstanding. However, we notice that CRSP's figures appear to be more reliable and there are fewer missing values. Although the numbers are largely comparable, number of shares outstanding in CCM exhibit large variance for certain shares, despite no change in the adjustment factor for shares outstanding. This leads us to believe there is a greater frequency of mistyping in CCM, and thus we opt for common shares outstanding from CRSP.

In order to compute the book-to-market ratio for each security, we divide the CCM book equity from the fiscal year ending in calendar year $t - 1$ by the market value as of December 31 in year $t - 1$.

We calculate each equity's beta as the regression slope between excess stock returns and excess market returns for 5 years leading up to June in year *t*. Thus, an equity must have 60 months of consecutive return data to be included in a portfolio. We apply this 60-month requirement to portfolios that are not sorted on beta as well, in order to ensure the assets included in the various portfolios are comparable.

Finally, we select portfolios at the end of June in year *t*, and examine returns from July year *t* through June year $t + 1$ before portfolios are rebalanced at the end of June year $t + 1$ and so forth. The reason for selecting portfolios in June, following Fama and French (1992), is to ensure that all firms' book values are known at the time of portfolio selection. The book value of a firm with fiscal year end in December is not public until that firm publishes their annual report in year $t + 1$. The fact that all firms are required to file their 10 – K reports within 60 to 90 days of their fiscal year end to the U.S. Securities and Exchange Commission (SEC, 2009) would call for portfolio selection to occur at the end of March in year *t*. However, as noted by Fama and French (1992), "… on average 19.8 % of firms do not comply" pp. 429. This point has also been backed up by newer research, e.g. Dalton et al. (2013) find that 8.5 % of firms filed their 10 – K reports late in the period 2000-2007.

4.2.2 Share Type, Industry and Characteristics Exclusions

We keep only equities with share codes 10 and 11, which corresponds to U.S. ordinary common shares. Following Lo and Wang (Lo & Wang, 2003), we note that this excludes ADRs, REITs, closed-end funds and similar assets whose turnover is usually low and might be difficult to interpret in the usual sense (pp. 220).

Moreover, following e.g. Fama and French (1992) financial firms (SIC codes 6000 – 6999) are eliminated from our sample. We also drop regulated utilities (SIC codes 4900 – 4949), following Covas and Den Haan (2011) and Bhojraj et al (2009).

We find that some of the stocks in our sample has a negative book value. Given the fact that listed companies have a limited liability structure, negative book values can be difficult to interpret. According to Brown, Lajbcygier and Li (2008): "… most empirical research in accounting and finance… exclude negative BE[book equity] stocks" pp.98. Therefore, we follow the norm and do the same, and remove negative-BTM stocks, which account for less than 1 % of our sample.

4.2.3 Liquidity

Many of the aforementioned adjustments also implicitly adjust our sample for illiquidity. For example, requiring 60 months of returns exclude many small firms with thin trading. Similarly, keeping only U.S. ordinary common shares also removes several illiquid assets.

When collecting price data from CRSP, the bid/ask-average is also reported, if there was no trade in an equity in a given month. Although the bid/ask-average can be seen as a proxy for the price in a given month, it is by no means a perfect proxy. When buying (selling) a stock, you have to pay the asking (bid) price. Therefore, the bid/ask average does not represent the true price an investor could achieve in the market, and thus return calculations based on bid/ask-averages do not represent true returns.

Nonetheless, we do wish to utilise the information conveyed by bid/ask-averages in some meaningful manner. Unfortunately, there is little guidance in previous literature with regards to how one should handle CRSP bid/ask averages. Consequently, we create our own restrictions for when we allow a bid/ask-average to proxy for a market clearing price.

The ratio of bid/ask-spreads divided by the bid/ask-averages must not exceed 2 %. This rule ensures that the bid/ask-average is close to the actual price an investor could achieve in the market. We do, however, make one exception from this rule. If a bid/ask-average, which does not meet the aforementioned requirement, appears at month t, we keep it if the prices at $t - 1$ and $t + 1$ are market clearing prices. We make this exception, because if not, the 60-months of valid returns required to be included in a portfolio would be violated for a large number of shares. This argument is particularly important in periods where market-wide variation in liquidity results in less trading across all assets.

Applying the same line of reasoning, we also exclude penny stocks (price below $1) from our sample, which tend to be illiquid and are noisy due to large bid/ask-spreads.

4.2.4 Survivorship and Delisting Bias

In order to avoid survivorship bias in the sample, it is essential to include the delisting return when a stock stops trading. Delistings are usually the result of major firm-specific events, such as bankruptcy and mergers & acquisitions. Therefore, delistings are often associated with large returns, both positive and negative. Delisting returns, along with a delisting code corresponding to the reason behind a delisting, is provided by CRSP. However, as discovered by Shumway (1997), the CRSP delisting returns are missing to a much greater extent when the delisting is caused by poor performance than when it is caused by events that lead to positive returns. Consequently, there is a *delisting bias* in the CRSP data. If ignored, the average return of companies in distress (which are more likely to be delisted due to default) would tend to be overstated. To illustrate the potential severity of not correcting for this bias, Shumway and Warther (1999) find that the empirically observed size effect on the NASDAQ exchange is in fact a result of the CRSP delisting bias. When correcting for the bias, they find no evidence of a size effect. Thus, we follow Shumway (1997), Acharya & Pedersen (2005) and many other recent studies in adjusting for this bias. The correction is performed by setting the delisting return to -30 % when the CRSP delisting code is 500, 520, 551-574, and 580, most of which are related to performance reasons. Although it is by no means a perfect correction, the return is set to -30 %, as this is the average delisting returns found by Shumway (1997) for the aforementioned delisting codes, and is the approach suggested by the author.

4.2.5 Portfolios

After performing the aforementioned calculations and adjustments, we sort the equities into portfolios. We create 25 portfolios sorted independently on size and book-to-market-ratio, and 45 portfolios triple-sorted on size, BTM and beta. We also utilise the industry portfolios from Kenneth French's webpage (French, 2017).

The size breakpoints for the portfolios are based on the full sample. This deviates somewhat from the approach of Fama and French (1992), which use breakpoints based on a subsample of only stocks listed on the NYSE. However, this would result in unequal number of assets across the portfolios and according to Bali, Engle & Murray (2016) "…most analyses … in the empirical asset pricing literature use breakpoints calculated using the full [CRSP] sample of stocks" pp. 188. Additionally, the authors show that the choice between using size breakpoints based on NYSE or the full CRSP sample has very limited impact.

When calculating returns, we rebalance weights for each period based on the availability of returns information. For a given period, if an equity in a portfolio is missing a return, the total market value of the portfolio is re-calculated, excluding the equity in question. Thus, we ensure a correct weight for the remaining equities in the portfolio when calculating the portfolio return.

4.3 Descriptive statistics and the normality assumption

TABLE 1 **Summary Statistics of Portfolio Returns**

	Monthly	Quarterly	Semiannual	Monthly	Quarterly	Semiannual
	A. Size-Sorted Portfolios			B. Book-To-Market-Sorted Portfolios		
Number of portfolio-period observations	25,500	8,500	4,250	13,800	4,600	2,300
Mean	.0076	.0264	.0499	.0077	.0251	.0489
Variance	.0066	.0335	.0540	.0041	.0168	.0317
Skewness	2.0710	4.6100	2.4220	-.0188	.3920	.7190
Kurtosis	29.730	53.510	22.230	6.215	4.951	5.717
Jarque-Bera statistic	777,286 **	933,506 **	69,657 **	5,944 **	847 **	906 **
Kolmogorov statistic	.0905**	.1276**	.0963**	.0357**	.0445**	.0604**
	C. Triple-Sorted Portfolios			D. Industry-Sorted Portfolios		
Number of portfolio-period observations	24,840	8,280	4,140	15,300	5,100	2,550
Mean	.0078	.0255	.0495	.0074	.0241	.0478
Variance	.0050	.0197	.0376	.0043	.0187	.0333
Skewness	.3160	.6020	.9810	.7690	2.9800	1.0470
Kurtosis	8.071	6.031	7.409	14.660	36.860	10.710
Jarque-Bera statistic	27,033 **	3,669 **	4,018 **	88,170 **	251,132 **	6,783 **
Kolmogorov statistic	.0502**	.0604**	.0707**	.0621**	.0946**	.0697**

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 25 equal-sized, equally-weighted portfolios (Panel A and B), 45 equal-sized, equally-weighted portfolios (Panel C), and 15 equally-weighted portfolios (Panel D). Portfolios are formed end-of-June each year, using all CRSP common stocks (Panel A, B and C), or stocks exclusively from NYSE, AMEX and NASDAQ (Panel D), excluding financials and utilities. In Panel A the portfolios are sorted on market value, consisting of returns from July 1931 to June 2016. In panel B the portfolios are sorted on book-to-market-ratio, consisting of returns from July 1970 to June 2016. Portfolios in Panel C are triple-sorted on size, book-to-market and market beta (in that order), and consist of returns from July 1970 to June 2016. Lastly, Panel D consists of portfolios sorted on their four digit SIC code, consisting of returns from July 1931 to June 2016. The breakpoints for the size portfolios are determined end-of-June year *t*. The book-to-market-ratios are computed using market value in December year *t-1* and book value at fiscal year-end in calendar year *t-1*. Market betas are obtained by estimating the CAPM regression equation for excess returns, using 5 years of trailing returns. SIC codes for fiscal year ending in calendar year *t-1* are obtained from Compustat when available. Otherwise, CRSP SIC codes are used for June of year *t*. The sample used in each column is comprised of returns for the entire set of portfolios.

* Significant at the 5 % level

** Significant at the 1 % level

4.3.1 Descriptive statistics

The descriptive statistics paint a fairly consistent picture across portfolio sorts (and thus sample period lengths). The mean equally-weighted excess return for the different samples are closely formed around 0.75 % (monthly), 2.5 % (quarterly) and 4.9 % (semiannual). This is despite the fact that the equities in the samples are substantially different in the different sorts. Variance, however, shows discrepancies. The size-sorted portfolios stick out with significantly higher variance than the three other sorts. Since the means are relatively similar across all four sorts, this leads us to believe that a major cause of the larger variance stems from the broader period it covers. The horizon entails one more financial crisis, which was classified as the worst depression of modern history (i.e. *the Great Depression*). The same is true for the industry-sorted portfolio returns. This has translated into a higher skewness, seeing that the size portfolios are the only sample where skewness is substantially above 1. The exception is quarterly returns in the industry-sorted portfolios. Another type of exception is the negative skewness for monthly BTM-sorted returns. Even though the skewness is below zero historically, given the low negative deviation from zero, it might as well have a true skewness that is positive. Thus, we would expect a negative risk premium (i.e discount) as Scott & Horvath (1980) have argued a positive preference for positive skewness, a feature the market seems to exhibit. The size-sorted portfolios have higher kurtosis than the rest, unsurprisingly. Both monthly, quarterly and semiannual returns have double-digit kurtosis. Same goes for industry-sorted portfolios, though at lower levels. When comparing the size and BTM-sorted portfolios, our results very much align with those of Chung et al. (2006). The size-sorted descriptive statistics are higher, in general, compared to the BTM-sorted portfolios. They also find small but negative skewness for return frequencies of monthly or shorter.

4.3.2 Normality

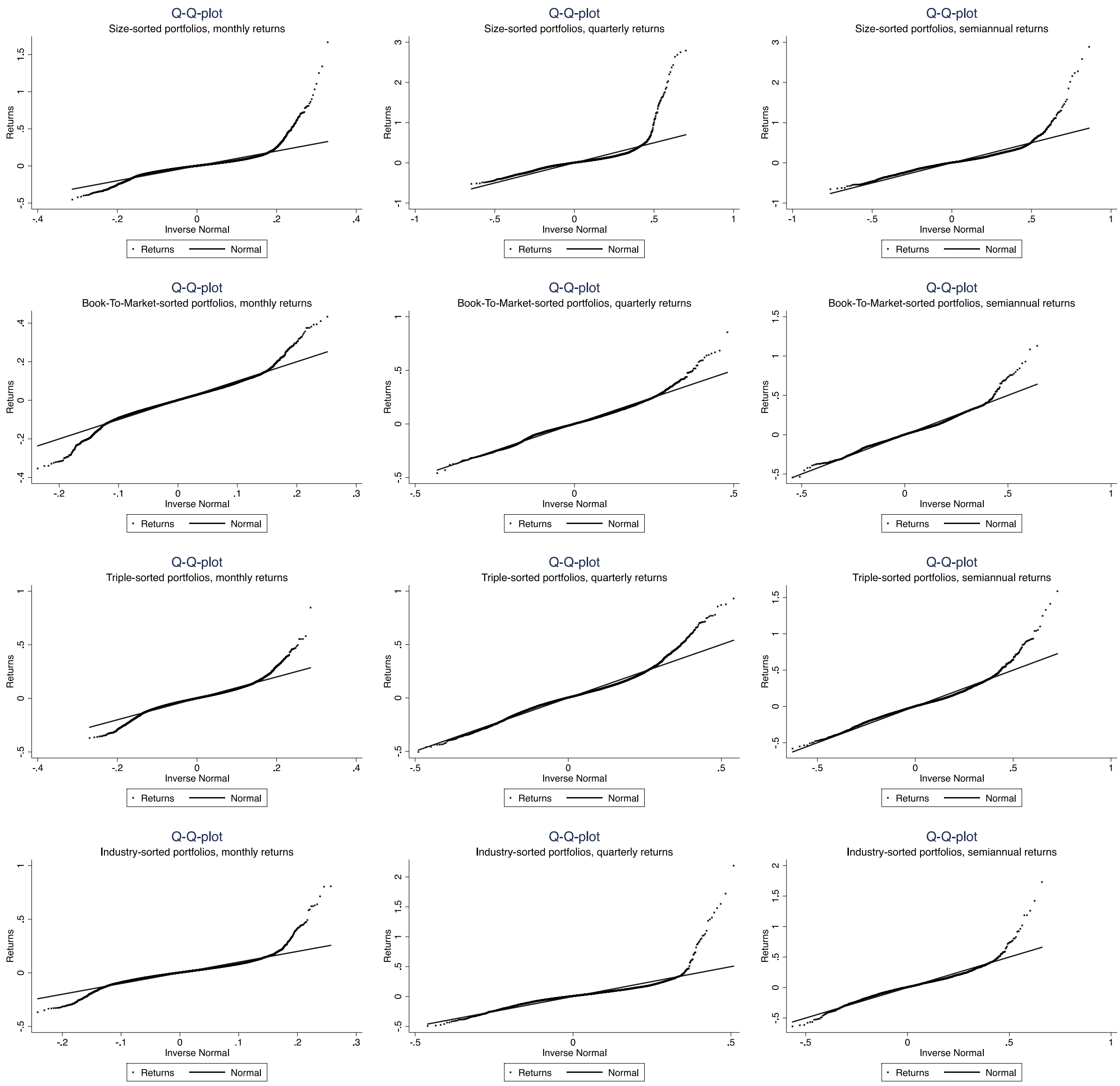
This study investigates a thesis that relies on the predisposition of non-normally distributed asset returns. As explained in Section 2.2, the CAPM implicitly assume normally distributed asset returns (or quadratic utility functions). For there to be a reason to include higher-order moments into the CAPM, there must also exist such higher moments. As such, asset returns must deviate from the normal distribution for there to be pricing of higher-order moments in the first place. A crucial part of this study is therefore to establish whether asset returns are in fact normally distributed or not, before any pricing models are worth testing.

To test the null-hypothesis of normal returns against non-normality, several tests can be utilised. The most popular, due to its power for a given significance, is the Shapiro-Wilk test. Unfortunately, the test is not recommended for sample sizes above 2000 because the probability of Type I-errors becomes rather substantial. This is an issue for most tests of normality. Other extensions of the Shapiro-Wilk test have been suggested, such as the Shapiro-Francia test. However, our samples are far larger than the recommended sample size limits. This is not to say that that is not the case for the tests we choose to use as well. However, opting for the Kolmogorov-Smirnov test is due to two reasons. First, it is a fairly common test in research studies of this sort, making our results more comparable to previous studies on the subject. Second, the test is non-parametric. That is, it does not utilise specific parameters of the sample distribution but rather deviations of the sample distribution from a continuous cumulative distribution function. In this case, the empirical sample distribution and the cumulative distribution function of the normal distribution. Since it has been argued that moments above kurtosis still convey some information about the distribution, we find it convenient to utilise a test like this in addition to the well-known, parametric test of Jarque-Bera. The Jarque-Bera test on the other hand relies solely on the two parameters, skewness and excess-kurtosis, where the two parameters are compared with the same parameters of the normal distribution (i.e. both are zero). As such, we find the two tests, one parametric, one non-parametric, to be sufficient. Due to the large probability of falsely rejecting normality, we find visual inspection of Q-Q-plots to be necessary as well.

4.3.3 Results

Both the Jarque-Bera and the Kolmogorov-Smirnov test statistics are significant at the 1 % level for all frequencies. Thus, both formal tests indicate that the null-hypothesis of normally distributed returns should be rejected. Visual inspection of Figure 2 strongly supports this as well. All returns show substantial deviations from the normal distribution´s quintiles. The deviations are most substantial in the monthly return data where deviations are to be found in both of ends. For the quarterly and semiannual returns, deviations from the normal distribution quintiles are in the upper end. This finding adds further support to the large amount of studies criticising the assumption of normally distributed returns, such as Hagerman (1978). These results fit very well within our hypothesis and allow us to move on to test the potential pricing of higher-order moments.

Figure 2 – Q-Q-plots for normality testing



5. Analysis

To enable a clear understanding of the estimates in the upcoming tables, we will quickly review how they are obtained. Firstly, factor coefficients are obtained using *time-series* regressions (or computed manually), utilising exactly *5 years* of prior data. This is performed with a *rolling* time window, for each asset (portfolio) i. Secondly, returns are then regressed on the factor coefficients obtained from the time-series regression, where coefficients *one period prior* is used. These second-pass regressions are done *cross-sectional* for each time t, across all test assets (portfolios). The coefficients from the second-pass are then *averaged* to obtain the final estimate, which are the ones presented in the tables. Thus, the estimates for any labelled variable represent that variable´s (i.e. factor´s) premium. To review the regression equations please refer to Appendix 9.1.1.

The Fama-MacBeth procedure has an additional benefit not yet mentioned. By obtaining $\lambda^{(k)} = E[mf^{(k)}]$, where m is the pricing kernel, and $f^{(k)}$ is the return of factor k, we have a measure of factor k´s correlation with the true discount factor. As such, testing $\lambda^{(k)} = 0$ asks *if factor k is priced*. The usual regression coefficient in a multiple regression, $\beta^{(k)}$, is a regression coefficient of m on $f^{(k)}$. Consequently, testing $\beta^{(k)} = 0$ will simply answer *whether factor k helps explain the price of an asset, to a greater extent than other factors*. Since we want to test asset pricing models with different factors, the fundamental question is therefore whether the factors are priced. This implies $\lambda^{(k)}$ is the most appropriate measure.

5.1 Univariate portfolios

5.1.1 Size-sorted portfolios

TABLE 2 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moments
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	.0030	.0024*	.0007			.296
			(1.26)	(2.12)	(.44)			[4.9*]
		(II)	.0044	.0029*	.0008	-.0010		.314
			(1.56)	(2.31)	(.47)	(-.56)		[3.5*]
		(III)	.0019	.0031*	.0027	-.0036	.0044	.326
			(.36)	(1.98)	(1.29)	(-1.01)	(.73)	[2.5]
		(IV)	.0074*					.159
			(1.99)					
		(V)	.0028			.0018		.227
			(.76)			(.95)		
		(VI)	.0060			-.0016	-.0014	.250
			(.92)			(-.41)	(-.21)	[2.5]
Quarterly	320	(I)	-.0024	.0099**	.0006			.364
			(-.39)	(2.97)	(.16)			[6.0**]
		(II)	.0042	.0115**	.0019	-.0035		.376
			(.62)	(3.16)	(.45)	(-.99)		[4.5*]
		(III)	-.0028	.0075	.0061	-.0123	.0163	.387
			(-.17)	(1.29)	(.86)	(-.87)	(.67)	[2.4]
		(IV)	.0302**					.220
			(3.01)					
		(V)	.0258*			-.0050		.265
			(2.44)			(-1.15)		
		(VI)	.0173			-.0065	.0035	.306
			(.80)			(-.79)	(.14)	[2.5]
Semiannual	160	(I)	.0116	.0171**	.0055			.370
			(1.12)	(2.74)	(.87)			[6.1**]
		(II)	.0265*	.0207**	.0207*	-.0118		.381
			(1.98)	(3.02)	(2.01)	(-1.65)		[4.4*]
		(III)	.0502	.0194	.0083	.0027	-.0376	.386
			(1.54)	(1.51)	(.52)	(.16)	(-.98)	[2.7]
		(IV)	.0364**					.205
			(2.86)					
		(V)	.0612***			-.0255**		.258
			(3.59)			(-3.19)		
		(VI)	.0318			-.0183	.0170	.293
			(1.01)			(-1.28)	(.47)	[2.5]

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 25 equal-sized, equally-weighted portfolios. Portfolios are formed end-of-June each year, using all CRSP common stocks, excluding financials and utilities. The portfolios are sorted on market value, consisting of returns from July 1931 to June 2016. The breakpoints for the size portfolios are determined end-of-June year *t*. Fo each period, portfolio returns are regressed on a combination of the factor loadings of SMB *(s)* and HML *(h)*, and the co-moment coefficients of variance *(β)*, skewness *(γ)*, and kurtosis *(δ)*. The market, SMB and HML loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB and HML factors. The co-moment coefficients are manually computed as the co-moment between the portfolio excess return and the market excess return, divided by the latter's moment of the same order, using 5 years of trailing data (see Equation 10). As such, the market beta being regressed on differs between equations (I)-(III) and (IV)-(VI). The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equations (I)-(III)), and the co-skewness and co-kurtosis premium estimates (equation (VI)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

The Fama-French Three-Factor Model with co-moment extensions

Looking at the original Fama-French Three-Factor Model, equation (I), the first interesting thing to notice is the consistent significance of the size factor premium across return frequencies. The size premium is significant at the 1 % level both for quarterly and semiannual return frequencies, and below the 5 % for monthly returns. Moreover, the t-statistic for the set of factors across equations (I) to (III) changes both sign and magnitude for some specifications and return frequencies. Furthermore, the size factor premium t-statistics are fairly robust for all specifications, across all return frequencies. This is one of the issues pointed out by Daniel and Titman (2012). Since the portfolios are sorted on size, we are effectively maximising the cross-sectional variation in the SMB factor and will consequently lack statistical power to reject the SMB factor. With the exception of equation (II) with semiannual returns, the two additional factors, market and HML, show no statistical significance. Thus, based on portfolios sorted on size, they do not appear to be priced in a statistical sense. Only in one instance are all three factors significant, namely semiannual returns with co-skewness coefficient included.

The inclusions of co-skewness and co-kurtosis coefficients, γ and δ, add little explanatory power to the model, evident by the low increase in the mean adjusted R-squared. Effectively, approximately 1 % additional variation can be explained by each of the co-moment coefficients (adjusted for the number of explanatory variables). However, neither are near any statistical significance in equations (II) and (III), and this applies for all return frequencies. Since Chung et al. (2006) have argued that SMB and HML might proxy for higher-order moments, we test their joint significance in all three model specifications, (I)-(III). The two factors are jointly significant in the original Three-Factor model. However, the significance level decreases to 5 %, from 1%, for quarterly and semiannual returns when co-kurtosis is included. Joint significance is then disappearing when co-kurtosis is added as well. While this is not anywhere near clear evidence, Chung et al. (2006) show that most of this drop in joint significance can be attributed to the systematic moment and not the fact that additional variables are added in general. Because no return frequency show evidence of all three factors being priced in equation (I), subsequently testing whether SMB and HML proxy for higher-order moments makes less sense. Consequently, we do not find any conclusive evidence for their hypothesis for the size sorted portfolios. Instead, the results of the size sorted portfolios rather indicate that the Three-Factor model does not price all the factors it proposes as explanatory for asset prices. Thus, we move on to test CAPM which has a broader base of theoretical support.

The Capital Asset Pricing Model with co-moment extensions

The original CAPM, represented by equation (IV), give evidence of market risk being priced, when considering no other factors. The longer return frequencies yield higher significance, as with the previous model. Nonetheless, the CAPM fails to explain the majority of variation observed in asset returns. Only 16 % to 20 % of the variation in asset returns can be attributed to the historical market beta, depending on the return frequency. This notion does not support our hypothesis since we have argued that the higher the order of the moment, the less additional information is conveyed. As such, if moments of the return distribution are to explain all of the variation, a substantial amount of co-moment coefficients would consequently have to be added. However, we have also argued that moments above kurtosis are unlikely to be priced at all. Thus, we do not expect the inclusion of co-skewness (γ) and co-kurtosis (δ) to give a large increase in the R-squared. Equations (V) and (VI) supports this notion. Although their inclusion yields a more substantial increase in the R-squared when compared to the Fama-French Three-Factor model, the expanded CAPM only explains approximately 30 % of the variation at best. This is less than the original Fama-French Three-Factor model. For the longer return frequencies, the market risk premium ($\lambda^{(\beta)}$) remains significant when co-skewness is included (equation (V)). The co-skewness premium ($\lambda^{(\gamma)}$) itself is only significant for semiannual returns. When the co-kurtosis coefficient is included, none of the premiums are significant. There are two seemingly contradictory effects at play here. On the one hand, co-kurtosis is not significant, nor are the two other factors. This evidence, coupled with the lack of joint significance of co-skewness and co-kurtosis, goes against the hypothesis of a higher-order CAPM. However, given that the other factors become insignificant and that the mean adjusted R-squared increases, this would indicate that some additional variation is explained by co-kurtosis. This is in line with the results of Chung et al. (2006). We also note that the reduction in t-statistics could be due to a multicollinearity effect, as this could inflate the standard errors. We would like to note that the significance in the 3-moment CAPM (equation (V)) for semiannual returns goes to show how looking at only one return frequency could be insufficient in determining risk premia of pricing factors.

General comments

We have, for simplicity and space purposes, chosen not to report the constant term in the tables. However, in our sample, the average historical value-weighted market excess return is 0.66 % (monthly), 2.13 % (quarterly) and 4.19 % (semiannual). The constant terms are therefore substantial, and in most specifications, significant, which is likely due to low effects and limited explanatory power. However, if the models were correctly pricing the assets, this term should be zero. It therefore seems that neither the Fama-French model nor the CAPM and their extensions are sufficient pricing models.

Our findings support the theoretical findings of Scott and Horvath (1980). When translated to the co-skewness coefficient, their implicit theoretical prediction of the sign of the premium would be negative. This is because a positive change in skewness is preferred by the investors and consequently will yield a discount (negative premium) in the required return. Although the majority is not significant, all but two specifications have negative co-skewness premiums. Accordingly, the co-kurtosis premiums should be positive. This also seems to be the case, though not conclusively. The co-kurtosis premium is, however, never statistically significant.

The longer return frequencies offer more significance and higher explanatory power. We believe this to be caused by the lower amount of noise in the return data.

5.1.2 Book-To-Market-Sorted Portfolios

TABLE 3 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moments
Book-To-Market-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	*SMB* *s*	*HML* *h*	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0029	-.0001	.0055***			.218
			(-.81)	(-.06)	(3.33)			[3.7*]
		(II)	-.0055	-.0013	.0059***	.0023		.224
			(-1.36)	(-.63)	(3.55)	(.95)		[2.7]
		(III)	-.0144*	-.0028	.0071**	-.0043	.0147	.232
			(-2.09)	(-1.15)	(2.98)	(-.79)	(1.57)	[2.3]
		(IV)	-.0076					.088
			(-1.93)					
		(V)	-.0102*			.0031		.142
			(-2.39)			(1.19)		
		(VI)	-.0075			-.0031	.0079	.165
			(-1.06)			(-.50)	(.78)	[2.1]
Quarterly	164	(I)	-.0005	.0057	.0124**			.242
			(-.06)	(1.17)	(2.82)			[3.8*]
		(II)	.0026	.0066	.0117*	-.0013		.248
			(.26)	(1.24)	(2.18)	(-.32)		[3.1]
		(III)	.0196	.0083	-.0002	-.0060	-.0138	.265
			(.88)	(1.12)	(-.03)	(-.54)	(-.50)	[2.7]
		(IV)	-.0105					.099
			(-.95)					
		(V)	-.0079			-.0069		.135
			(-.69)			(-1.81)		
		(VI)	-.0125			-.0118	.0067	.185
			(-.48)			(-.94)	(.19)	[2.3]
Semiannual	82	(I)	.0057	.0061	.0226**			.286
			(.40)	(.81)	(2.85)			[4.4*]
		(II)	.0169	-.0000	.0351*	-.0071		.294
			(.61)	(-.01)	(2.20)	(-.45)		[3.5*]
		(III)	.0064	.0001	.0309	.0297	-.0272	.302
			(.15)	(.01)	(1.60)	(.71)	(-.41)	[2.5]
		(IV)	.0044					.137
			(.26)					
		(V)	.0124			-.0046		.199
			(.49)			(-.33)		
		(VI)	.0188			.0048	-.0164	.230
			(.44)			(.21)	(-.32)	[2.5]

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 25 equal-sized, equally-weighted portfolios. Portfolios are formed end-of-June each year, using all CRSP common stocks, excluding financials and utilities. The portfolios are sorted on book-to-market-ratio, consisting of returns from July 1970 to June 2016. The book-to-market-ratios are computed using market value in December year t-1 and book value at fiscal year-end in calendar year *t-1* . For each period, portfolio returns are regressed on a combination of the factor loadings of SMB *(s)* and HML *(h)* , and the co-moment coefficients of variance *(β)* , skewness *(γ)* , and kurtosis *(δ)* . The market, SMB and HML loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB and HML factors. The co-moment coefficients are manually computed as the co-moment between the portfolio excess return and the market excess return, divided by the latter's moment of the same order, using 5 years of trailing data (see Equation 10). As such, the market beta being regressed on differs between equations (I)-(III) and (IV)-(VI). The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equations (I)-(III)), and the co-skewness and co-kurtosis premium estimates (equation (VI)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

The Fama-French Three-Factor Model with co-moment extensions

As evident from Table 3, when sorting on book-to-market ratio, the HML factor is now the most significant factor across return frequencies and specifications. Similar to Table 2, the sorting procedure entails more consistent t-statistics for the factor that is sorted on. Moreover, the SMB factor is no longer significant in any of the regression specifications. In contrast to the results from the size-sorted portfolios, the original Fama-French Three-Factor model (I) yields lower explanatory power for the BTM-sorted portfolios. Note, however, that this sample is smaller due to book equity data limitations in the CMM database. In this sample, the historical excess market return is 0.767 % (monthly), 2.506 % (quarterly), 4.886 % (semiannual).

In line with the previous table are the insignificance of the two other factors not being sorted on. One specification does, however, yield negative significant market risk premium ($\lambda^{(\beta)}$), namely equation (III). Since this is monthly returns with co-skewness and co-kurtosis included it may very well be a result of noisy data. Therefore, it could possibly be a false positive, which we would expect one out of 20 times, given a 5 % significance level. The value-premium is less significant for the longer return frequencies and becomes insignificant when both co-moment coefficients are added. This notion supports the hypothesis of Chung et al. (2006). The inclusion of co-skewness and co-kurtosis adds less explanatory power to the model than in the case of size-sorted portfolios. The initially lower R-squared of the model makes it less surprising that the joint significance of SMB and HML is lower than in the previous table. However, joint significance is reduced when co-moment coefficients are included just as we would expect if SMB and HML were proxies for higher-order moments. The lack of a complete set of significant factors in the Fama-French Three-Factor model makes the hypothesis of Chung et al. (2006) obsolete yet again.

The Capital Asset Pricing Model with co-moment extensions

The original CAPM (IV) does not suffice to explain the variation in returns for the BTM-sorted sample. Market risk premiums are insignificant across return frequencies and negative for monthly and quarterly returns. The market beta proves to explain as little as 8.8 % of monthly asset returns. This is interesting since it counterintuitively indicates that market risk does not explain a great deal of the variation in asset returns. It is therefore not surprising that CAPM and its co-moment extensions yields lower explanatory power for this sample. However, the inclusion of the co-moment coefficients adds approximately 5 % additional explanatory power to the model. The co-moment coefficients are nonetheless not jointly significant. As such, the BTM-sorted sample does not provide evidence of a higher-order moment CAPM. This further goes against Chung et al.´s (2006) hypothesis of HML and SMB being proxies for higher-order moments. However, as noted earlier, when sorting on a single factor, Daniel and Titman (2012) argue that the test lacks statistical power to reject the proposed factors.

Interesting in this sample is that it seems longer return frequencies have somewhat lower t-statistics in general. This does not align with what we found in the size-sorted sample. In general, the signs of the co-moment coefficients align with what we would expect. However, certain specifications yield negative market and co-kurtosis premiums.

5.2 Portfolio sorts suggested by Daniel & Titman

5.2.1 Triple-Sorted Portfolios

TABLE 4 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moments
Triple-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0018	.0028	.0025			.218
			(-.60)	(1.72)	(1.63)			[7.4**]
		(II)	-.0026	.0017	.0042**	.0014		.224
			(-.77)	(1.04)	(2.84)	(.61)		[4.9*]
		(III)	-.0020	.0014	.0032	-.0004	.0006	.232
			(-.36)	(.76)	(1.69)	(-.09)	(.09)	[3.8*]
		(IV)	.0015					.088
			(.47)					
		(V)	-.0051			.0059		.142
			(-1.53)			(1.68)		
		(VI)	-.0001			.0057	-.0062	.165
			(-.01)			(.95)	(-.78)	[3.6*]
Quarterly	164	(I)	-.0059	.0106*	.0077			.242
			(-.72)	(2.30)	(1.69)			[8.6**]
		(II)	.0004	.0116*	.0083	-.0012		.248
			(.05)	(2.53)	(1.63)	(-.31)		[7.0**]
		(III)	-.0050	.0103	.0048	-.0061	.0119	.265
			(-.33)	(1.96)	(.67)	(-.58)	(.55)	[4.2*]
		(IV)	.0073					.099
			(.71)					
		(V)	.0086			-.0066		.135
			(.79)			(-1.58)		
		(VI)	-.0068			-.0103	.0164	.185
			(-.36)			(-1.49)	(.73)	[3.5*]
Semiannual	82	(I)	.0025	.0146*	.0140			.286
			(.22)	(2.03)	(1.71)			[7.4**]
		(II)	.0035	.0155	.0324	.0006		.294
			(.17)	(1.78)	(1.75)	(.05)		[5.8**]
		(III)	.0140	.0255*	.0274	.0153	-.0273	.302
			(.42)	(2.41)	(1.33)	(.81)	(-.62)	[3.6*]
		(IV)	.0092					.137
			(.62)					
		(V)	.0246			-.0125		.199
			(1.05)			(-1.06)		
		(VI)	-.0109			-.0260	.0475	.230
			(-.28)			(-1.66)	(1.11)	[2.8]

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 45 equal-sized, equally-weighted portfolios. Portfolios are formed end-of-June each year, using all CRSP common stocks, excluding financials and utilities. The portfolios are triple-sorted on market value, book-to-market-ratio and market beta (in that order), consisting of returns from July 1970 to June 2016. The breakpoints for the size portfolios are determined end-of-June year *t*. The book-to-market-ratios are computed using market value in December year *t-1* and book value at fiscal year-end in calendar year *t-1*. Market betas are obtained by estimating the CAPM regression equation for excess returns, using 5 years of trailing returns. For each period, portfolio returns are regressed on a combination of the factor loadings of SMB *(s)* and HML *(h)*, and the co-moment coefficients of variance *(β)*, skewness *(γ)*, and kurtosis *(δ)*. The market, SMB and HML loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB and HML factors. The co-moment coefficients are manually computed as the co-moment between the portfolio excess return and the market excess return, divided by the latter's moment of the same order, using 5 years of trailing data (see Equation 10). As such, the market beta being regressed on differs between equations (I)-(III) and (IV)-(VI). The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equations (I)-(III)), and the co-skewness and co-kurtosis premium estimates (equation (VI)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

Moving on to the triple-sorted portfolios following Daniel and Titman (2012), as described in Section 3.2, the results change notably compared to the single-sorted portfolios. As expected, the significance of the SMB and HML premia taken separately in Table 4, are less pronounced than in Table 2 and 3. Examining first the Fama-French Three-Factor model (I), we see that none of that factors are significant at the 5 % level for the monthly return frequency, whereas the SMB premium is significant at the 5 % level for quarterly and semiannual returns. Perhaps more interestingly, is that both SMB and HML separately are significant at the 10 % level, and have a stronger joint significance than in the single-sorted portfolios.

In contrast to the single-sorted portfolios, the joint significance of SMB and HML is sustained when adding co-skewness and co-kurtosis, i.e. in (II) and (III). Moreover, there is virtually no indication that co-skewness or co-kurtosis is priced, when controlling for the SMB and HML factors. The CAPM with co-moment extensions (IV – VI) paint a slightly different picture. Although neither co-skewness nor co-kurtosis is individually significant, they are jointly significant at the 5 % level for the monthly and quarterly return frequencies, a commonly observed feature when multicollinearity is present. However, the four-moment CAPM yields less explanatory power than the Fama-French Three-Factor model.

The results we have presented so far gives a clear indication that the critique of Daniel and Titman (2012) is valid. Sorting independently on one factor indeed appears to eliminate a lot of the cross-sectional variation of other factors.

5.2.2 Industry-Sorted Portfolios

TABLE 5 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moments
Industry-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	-.0033	-.0000	.0023*			.237
			(-1.39)	(-.02)	(2.01)			[2.5]
		(II)	-.0054	.0014	.0026*	.0024		.268
			(-1.87)	(.90)	(2.00)	(1.24)		[2.6]
		(III)	.0079	.0060*	.0020	.0046	-.0173*	.301
			(1.12)	(2.57)	(.91)	(1.25)	(-2.09)	[2.5]
		(IV)	-.0021					.109
			(-.94)					
		(V)	-.0017			.0010		.145
			(-.63)			(.60)		
		(VI)	.0064			.0026	-.0101	.189
			(.99)			(.71)	(-1.30)	[1.8]
Quarterly	320	(I)	-.0100	.0015	.0077*			.229
			(-1.69)	(.46)	(2.07)			[2.6]
		(II)	-.0051	.0027	.0119**	-.0020		.252
			(-.64)	(.66)	(2.86)	(-.41)		[2.6]
		(III)	-.0280	-.0027	.0095	-.0097	.0328	.294
			(-1.29)	(-.33)	(1.04)	(-.48)	(.96)	[2.6]
		(IV)	-.0062					.096
			(-1.12)					
		(V)	-.0086			.0010		.135
			(-1.11)			(.21)		
		(VI)	-.0232			-.0206	.0391	.197
			(-1.20)			(-.88)	(1.14)	[2.0]
Semiannual	160	(I)	-.0064	.0011	.0128*			.239
			(-.66)	(.20)	(1.98)			[2.6]
		(II)	.0002	.0012	.0099	-.0012		.268
			(.02)	(.17)	(.95)	(-.14)		[2.5]
		(III)	-.0638	-.0325	-.0470*	-.0557	.1203	.309
			(-1.27)	(-1.59)	(-2.41)	(-1.90)	(1.81)	[2.4]
		(IV)	-.0076					.110
			(-.82)					
		(V)	-.0027			-.0015		.165
			(-.22)			(-.24)		
		(VI)	-.0046			-.0303	.0327	.215
			(-.14)			(-1.54)	(.75)	[2.0]

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 15 equally-weighted portfolios obtained from Kenneth French's Data Library. Portfolios are formed end-of-June each year, using stocks from NYSE, AMEX and NASDAQ, excluding financials and utilities. The portfolios are sorted on their four digit SIC code, consisting of returns from July 1931 to June 2016. For each period, portfolio returns are regressed on a combination of the factor loadings of SMB *(s)* and HML *(h)*, and the co-moment coefficients of variance *(β)*, skewness *(γ)*, and kurtosis *(δ)*. The market, SMB and HML loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB and HML factors. The co-moment coefficients are manually computed as the co-moment between the portfolio excess return and the market excess return, divided by the latter's moment of the same order, using 5 years of trailing data (see Equation 10). As such, the market beta being regressed on differs between equations (I)-(III) and (IV)-(VI). The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equations (I)-(III)), and the co-skewness and co-kurtosis premium estimates (equation (VI)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

In a similar fashion to the triple-sorted portfolios, we also expect utilising industry portfolios will yield results that are different to the univariate-sorted portfolios, as there is not a clear factor structure in the portfolio construction process. Nonetheless, we do expect there to be some systematic differences in value and size across industries, e.g. financials (utilities) tend to have a low (high) book-to-market ratio (hence, the exclusion).

As evident from Table 5, in the Fama-French Three-Factor regression (I), the value effect is statistically significant across all return frequencies, although less pronounced than in the book-to-market sorted portfolios. Adding co-skewness to the Three-Factor model does seem to impact the significance of the size premium somewhat. Utilising monthly returns, the co-skewness premium is positive, but not significant. Moreover, the significance of the value premium remains unchanged. For the quarterly and semiannual returns, the co-skewness premium is negative and insignificant. However, for the semiannual returns, adding the co-skewness premium leads to the value premium becoming insignificant, whereas it leads to an increase for the quarterly returns.

When adding co-kurtosis to the regression (III), the value premium becomes statistically insignificant for the monthly and quarterly return frequencies, but negative and significant for the semiannual return frequency. Moreover, in the monthly return frequency, the size premium suddenly becomes statistically significant, and the co-kurtosis premium appears to be negative and priced in this instance. However, given the fact that the co-kurtosis premium ($\lambda^{(\vartheta)}$) is not significant in (VI), and of the opposite sign to what we would expect from theory, we believe this might be a false positive. Furthermore, the size and value premiums do not appear to be jointly significant in any of the specifications, in contrast to the triple-sorted portfolios. Additionally, adding co-skewness and co-kurtosis appears to increase explanatory power somewhat over the Fama-French Three-Factor model.

In further contrast to the triple-sorted portfolios, the CAPM with co-moment extensions (IV – VI) yield no statistical significance whatsoever, neither separately nor jointly.

5.3 Proxying lottery demand with FMAX factor

In this section, we will repeat the analysis in Section 5.1 and 5.2, utilising the same portfolios, but substituting higher-order moments with the FMAX-factor, as described in Section 2.4. Tables 6 and 7 (8 and 9) are analogous to tables 2 and 3 (4 and 5) earlier in the analysis section. They contain regression results for the CAPM (III), Fama-French Three-Factor model (I), Fama-French Three-Factor + FMAX (II) and R_M + FMAX (IV), for monthly, quarterly and semiannual returns. It should be noted, however, since FMAX is a function of the daily returns in the previous month, the monthly return frequencies have the most intuitive interpretation.

From tables 8 (triple-sorted) and 9 (industry-sorted), it is clear that triple-sorted and industry sorted portfolios alter the results notably, also with respect to the FMAX-factor.

5.3.1 **Univariate Portfolios**

TABLE 6 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	.0020	.0023	-.0004		.309
			(.50)	(1.40)	(-.16)		[5.1*]
		(II)	-.0036	-.0009	.0056***	-.0024	.228
			(-1.04)	(-.51)	(3.36)	(-.81)	[3.5*]
		(III)	-.0076				.128
			(-1.93)				
		(IV)	-.0052			-.0073*	.139
			(-1.31)			(-2.37)	
Quarterly	164	(I)	-.0079	.0120*	-.0052		.379
			(-.80)	(2.50)	(-.86)		[5.5*]
		(II)	-.0066	.0098*	-.0015	.0038	.400
			(-.73)	(2.17)	(-.25)	(.47)	[4.7*]
		(III)	.0320*				.203
			(2.14)				
		(IV)	.0187			.0223*	.319
			(1.61)			(2.23)	
Semiannual	82	(I)	.0137	.0165*	-.0010		.389
			(.85)	(2.05)	(-.11)		[5.6*]
		(II)	.0115	.0162*	-.0013	.0163	.398
			(.71)	(2.01)	(-.14)	(1.50)	[4.5*]
		(III)	.0504*				.214
			(2.46)				
		(IV)	.0295			.0368*	.286
			(1.61)			(2.29)	

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 25 equal-sized, equally-weighted portfolios. Portfolios are formed end-of-June each year, using all CRSP common stocks, excluding financials and utilities. The portfolios are sorted on market value, consisting of returns from July 1970 to June 2016. The breakpoints for the size portfolios are determined end-of-June year *t*. For each period, portfolio returns are regressed on a combination of the factor loadings of market excess return *(β)*, SMB *(s)*, HML *(h)* and FMAX *(f)*. The loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB, HML and FMAX factors, in different specifications. The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equation (I) & (II)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

Firstly, we again notice that the critique of Daniel and Titman (2012) seems very valid. In Table 6 (size-sorted portfolios), the size premium is statistically significant, apart from the monthly returns frequency, whereas in Table 7 (BTM-sorted portfolios), the value premium is consistently statistically significant. Adding the FMAX-factor to the Fama-French Three-Factor model does not seem to alter the statistical significance of the Fama-French factors to a great extent, and the FMAX factor is insignificant in these specifications. The exception is the monthly returns in Table 6, where the value premium becomes significant at the 0.1 % level when adding FMAX. Additionally, the size and value premia are jointly significant both before and after adding the FMAX-factor.

TABLE 7 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor *Book-To-Market-Sorted Portfolios*

Return Frequency	Periods	Equation	Market (R_M) β	SMB *s*	HML *h*	FMAX *f*	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0029	-.0001	.0055***		.218
			(-.81)	(-.06)	(3.33)		[3.7*]
		(II)	-.0036	-.0009	.0056***	-.0024	.228
			(-1.04)	(-.51)	(3.36)	(-.81)	[3.5*]
		(III)	-.0076				.088
			(-1.93)				
		(IV)	-.0052			-.0073*	.139
			(-1.31)			(-2.37)	
Quarterly	164	(I)	-.0005	.0057	.0124**		.242
			(-.06)	(1.17)	(2.82)		[3.8*]
		(II)	-.0002	.0066	.0124**	-.0039	.255
			(-.02)	(1.40)	(2.82)	(-.63)	[3.5*]
		(III)	-.0105				.099
			(-.95)				
		(IV)	.0022			-.0134*	.163
			(.25)			(-2.19)	
Semiannual	82	(I)	.0057	.0061	.0226**		.286
			(.40)	(.81)	(2.85)		[4.4*]
		(II)	.0092	.0072	.0208**	-.0061	.306
			(.69)	(1.04)	(2.69)	(-.57)	[3.7*]
		(III)	.0044				.137
			(.26)				
		(IV)	.0214			-.0101	.210
			(1.39)			(-.90)	

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 25 equal-sized, equally-weighted portfolios. Portfolios are formed end-of-June each year, using all CRSP common stocks, excluding financials and utilities. The portfolios are sorted on book-to-market-ratio, consisting of returns from July 1970 to June 2016. The book-to-market-ratios are computed using market value in December year *t-1* and book value at fiscal year-end in calendar year *t-1*. For each period, portfolio returns are regressed on a combination of the factor loadings of market excess return *(β)*, SMB *(s)*, HML *(h)* and FMAX *(f)*. The loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB, HML and FMAX factors, in different specifications. The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equation (I) & (II)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

In concurrence with Bali et al. (2017), we find evidence of FMAX being priced in equation (IV). For the monthly returns, the FMAX-factor is negative, significant at the 5 % level, and very close to being significant at the 1 % level. Interestingly, for the quarterly returns, FMAX is positive and significant in the size portfolios, but negative and significant in the BTM-portfolios. The quarterly FMAX-factor is also negative and significant at the 5 % level, while the semiannual one is insignificant.

5.3.2 Triple-Sorted and Industry Portfolios

TABLE 8 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor ***Triple-Sorted Portfolios***

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0018	.0028	.0025		.297
			(-.60)	(1.72)	(1.63)		[7.4**]
		(II)	-.0014	.0028	.0024	-.0017	.311
			(-.47)	(1.77)	(1.59)	(-.65)	[6.4**]
		(III)	.0015				.141
			(.47)				
		(IV)	.0009			-.0017	.211
			(.28)			(-.62)	
Quarterly	164	(I)	-.0059	.0106*	.0077		.310
			(-.72)	(2.30)	(1.69)		[8.6**]
		(II)	-.0038	.0090*	.0087*	-.0029	.325
			(-.45)	(2.07)	(2.05)	(-.43)	[6.8**]
		(III)	.0073				.173
			(.71)				
		(IV)	.0047			-.0000	.238
			(.46)			(-.00)	
Semiannual	82	(I)	.0025	.0146*	.0140		.294
			(.22)	(2.03)	(1.71)		[7.4**]
		(II)	.0051	.0141*	.0110	-.0031	.306
			(.45)	(2.08)	(1.40)	(-.28)	[6.5**]
		(III)	.0092				.154
			(.62)				
		(IV)	.0147			.0011	.208
			(1.09)			(.09)	

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 45 equal-sized, equally-weighted portfolios. Portfolios are formed end-of-June each year, using all CRSP common stocks, excluding financials and utilities. The portfolios are triple-sorted on market value, book-to-market-ratio and market beta (in that order), consisting of returns from July 1970 to June 2016. The breakpoints for the size portfolios are determined end-of-June year t. The book-to-market-ratios are computed using market value in December year t-1 and book value at fiscal year-end in calendar year t-1. Market betas are obtained by estimating the CAPM regression equation for excess returns, using 5 years of trailing returns. For each period, portfolio returns are regressed on a combination of the factor loadings of market excess return *(β),* SMB *(s),* HML *(h)* and FMAX *(f).* The loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB, HML and FMAX factors, in different specifications. The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equation (I) & (II)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

Similar to what we observed in the former half of the analysis, in the results from the triple-sorted portfolios, the size premium appears to be priced to a varying extent, whereas the value premium appears to be priced in the industry portfolios. The value premium in Table 9 is always significant at the 10 % level, whereas it is consistently significant at the 5 % level in Table 5 (industry-sorted portfolios). This is most likely due to the sample size. As the FMAX-factor is only available from 1970, the sample size in Table 9 is almost half as that of the sample in Table 5. Furthermore, the Fama-French factors are jointly significant in specifications (I) & (II) in Table 8 across all return frequencies, but insignificant in Table 9.

TABLE 9 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Industry-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0030	.0009	.0031		.257
			(-.81)	(.39)	(1.83)		[2.5]
		(II)	-.0060	.0018	.0031	-.0016	.303
			(-1.55)	(.79)	(1.79)	(-.54)	[2.5]
		(III)	-.0020				.128
			(-.56)				
		(IV)	-.0016			-.0019	.194
			(-.41)			(-.69)	
Quarterly	164	(I)	-.0129	.0032	.0118*		.223
			(-1.41)	(.65)	(2.13)		[2.5]
		(II)	-.0004	.0014	.0091	.0009	.289
			(-.04)	(.30)	(1.83)	(.13)	[2.3]
		(III)	-.0074				.092
			(-.91)				
		(IV)	.0021			.0003	.192
			(.24)			(.05)	
Semiannual	82	(I)	-.0151	.0035	.0196*		.260
			(-1.13)	(.43)	(2.29)		[2.6]
		(II)	-.0160	.0024	.0172*	-.0085	.309
			(-1.26)	(.30)	(2.36)	(-.78)	[2.5]
		(III)	-.0089				.138
			(-.68)				
		(IV)	-.0031			-.0077	.220
			(-.22)			(-.65)	

NOTE. - The sample consists of monthly, quarterly and semiannual returns (non-annualised) of 15 equally-weighted portfolios obtained from Kenneth French's Data Library. Portfolios are formed end-of-June each year, using stocks from NYSE, AMEX and NASDAQ, excluding financials and utilities. The portfolios are sorted on their four digit SIC code, consisting of returns from July 1970 to June 2016.For each period, portfolio returns are regressed on a combination of the factor loadings of market excess return *(β),* SMB *(s),* HML *(h)* and FMAX *(f)*. The loadings are computed with a 5-year rolling regression for each point in time, where portfolio excess returns are regressed on the excess market return, and the SMB, HML and FMAX factors, in different specifications. The final coefficient estimates presented in this table are the mean coefficients across the sample and represent the factor premium. The coefficients are reported with their respective t-statistics in parentheses. The joint test is a Wald test for joint significance of the SMB and HML premium estimates (equation (I) & (II)).

* Two-tail significance at the 5 % level

** Two-tail significance at the 1 % level

As with the single-sorted portfolios, adding FMAX to the Fama-French model, does not appear to alter the significance of the Fama-French factors notably. However, the mean adjusted R-squared does increase by a few percentage points when adding FMAX. Perhaps the most interesting result in tables 8 and 9 is that the pricing of FMAX in equation (IV), as we observed in the single-sorted portfolios, seems to have vanished completely. Thus, we have reasons to believe that the pricing of the FMAX-factor is also subject to the portfolio-sorting critique of Daniel and Titman (2012), and that FMAX might not be robust across different kinds of test assets.

6. Robustness

6.1 Normality of residuals

To enable hypothesis testing of the estimations conducted in this study, we rely on the assumption of normally distributed errors. We evaluate whether our estimates exhibit this necessary condition by predicting the residuals for the regressions and test for normality. However, as previously discussed, the Jarque-Bera and the Kolmogorov-Smirnov tests have a strong tendency to reject normality for large samples. Thus, we opt for visual inspections as well. Almost all the formal tests reject normality. Only 4 out of the 120 regressions have residuals where formal tests could not reject normality, either by JB-test statistic, KS-test statistic, or both. A similar figure also reflects the number of rejections on a 5 % confidence level while the rest of the residuals where rejected at 1 %. By inspecting Q-Q-plots of the residuals we find that the residuals are not sufficiently normally distributed for inference. Residual deviations from the normal distribution are more prominent in the upper quintile with larger positive residuals, albeit they are evident in both ends of the residual distributions, generating the well-known S-shape.

6.2 Cross-sectional correlation

Some of the motivation behind the Fama-MacBeth procedure is to correct for cross-sectional correlation between assets. This is an important aspect of asset pricing studies, as equities are susceptible to the same shocks in the cross-section. For example, if at time t, a macroeconomic announcement unexpectedly indicates the economy is dropping into a recession, we do not expect asset i's and j's reaction to be uncorrelated. Rather, we generally expect their reactions to be highly correlated, of course depending on the asset. Nonetheless, Cochrane (2005) show explicitly that the Fama-MacBeth procedure delivers standard errors that are appropriately corrected for cross-sectional dependence and we are therefore not concerned with this issue.

6.3 Autocorrelation and Heteroscedasticity

For inference, we need the standard errors to be correct to avoid making Type I- or Type II-errors unnecessarily. If the standard errors are biased in either direction, this will pave way for incorrect inference that could otherwise be avoided. Heteroscedasticity and autocorrelation will affect the standard errors in this unwanted manner.

In spite of its intuitive nature and correction for cross-sectional correlation, the Fama-MacBeth procedure fails to adjust for autocorrelation. Although this is not a major concern, as equities tend to exhibit zero autocorrelation, we address and visually inspect for autocorrelation.

We believe that it is unlikely that the error terms will be correlated more than a year back in time. Consequently, we evaluate the autocorrelation for time horizons of 12 months, 4 quarters or 2 half-years, depending on the return frequency. By inspecting correlograms (autocorrelation plots) there are signs of autocorrelation for the most part of the estimated regressions. Testing for heteroscedasticity, we first plot the fitted values against the squared residuals. A majority of the scatterplots reveal substantial heteroscedasticity. This is also the case when we address the heteroscedasticity with the more formal method of the simplified White´s test. We therefore choose to report our results with Newey-West heteroskedastic and autocorrelation adjusted standard errors.

6.4 Errors-in-Variables

The Fama-MacBeth procedure has a widely-known errors-in-variables (EIV) problem. This EIV problem arises from the fact that the explanatory variables in the second pass regression are the estimated βs from the first pass, which are subject to measurement errors, and thus deviate from “true βs”. Consequently, the standard errors of the cross-sectional regression may be (downward)biased. In the context of Fama-MacBeth regressions, the EIV problem is specifically addressed by Shanken (1992) who derives a formal way of adjusting for this. Moreover, using the Generalised Method of Moments (GMM) approach would provide standard errors corrected for this issue. However, this is beyond the scope of this master thesis and we will therefore suggest employing these as further research on the topic.

6.5 Multicollinearity

When we are regressing on multiple co-moments, we obviously must expect multicollinearity. This does not cause our coefficients to be biased, but incremental changes to the input data would result in substantially different estimators, even changing their signs. In addition to affecting the estimators, multicollinearity can also affect the standard errors by inflating them (O'Brien, 2007).

When such multicollinearity is present, one will often find joint significance for the affected estimators despite these estimators being individually insignificant. This was, however, not a prominent feature in our data.

On the issue of the standard errors, Rubinstein (1973) argues that even though the estimators become insignificant due to multicollinearity one cannot conclude that the variance and skewness have no effect on the expected return. As Rubinstein (1973) puts it; " The insignificance is attributable to the purely statistical problem of multicollinearity rather than any economic factor."

6.6 Consistency

We believe that by employing several nuances of the same data we can utilise the data better. By this we mean that the data can give a better understanding of the world, despite the results being insignificant. To elaborate, we utilise the same data, but we use different sorting methodologies on that data. We then use different methodologies to calculate the return, both simple and logarithmic. Moreover, different return frequencies are used, and portfolio returns are calculated on a value-weighted basis as well as an equally-weighted basis. This allows us to investigate the robustness of our results. If results are fairly consistent across several or all of these features, then it provides a strong indication, even though the results may themselves be insignificant.

Because we do not find any results showing strong signs of relevance, checking the same results across all combinations of methodologies are not necessary. Despite this, a quick review of the results indicate that the data is at least somewhat consistent. We also note that the predicted signs of the co-skewness and co-kurtosis are highly consistent across the aforementioned combinations of features in the data. Refer to Appendix 9.2 for these results.

7. Closing remarks

7.1 Conclusions

Throughout this paper, we have demonstrated that the choice of test assets has a great impact on the results of empirical asset pricing tests. We originally set out to study how robust the pricing of higher-order moments is across portfolio sorting methodologies. Nonetheless, we have also discovered that one of the most well-documented asset pricing models in finance, the Fama-French model, also appears to be prone to the critique put forth by Daniel & Titman (2012).

We find limited evidence that our measures of higher-order moments, co-skewness and co-kurtosis, in addition to the more *ad-hoc* measure FMAX, are priced in the stock market. Only in the size-sorted portfolios, with semiannual returns, in the regression containing only beta and co-skewness, do we find co-skewness to be statistically significant ((V) in Table 2). Similarly, the only instance in which we found statistical significance for co-kurtosis was in the industry-sorted portfolios, with monthly returns, when adding co-skewness and co-kurtosis to the Fama-French Three-Factor model (III in Table 5). However, in this case, the co-kurtosis is negative, i.e. the opposite sign of what is predicted by theory. The FMAX measure appears to be priced in the portfolios sorted on a single factor, but we find a statistically significant FMAX of the opposite signs. However, as previously stated, the monthly returns frequencies have the most intuitive interpretation for the FMAX-factor, where we find, in concurrence with Bali et al. (2017) a negative FMAX-factor. Nonetheless, when testing the FMAX-factor in the triple and industry-sorted portfolios, the FMAX-factor is nowhere near being statistically significant.

In the size- and book-to-market-sorted portfolios, we initially find that the SMB and HML factors are jointly significant, but they are no longer significant when adding co-skewness and co-kurtosis to the regression. However, in the triple sorted portfolios, the SMB and HML factors remain jointly significant despite adding both co-skewness and co-kurtosis. Moreover, in the industry-sorted portfolios we do not find any evidence of the Fama-French factors being jointly significant. We also find that co-skewness and co-kurtosis are jointly significant for the monthly and quarterly return frequencies in the triple-sorted portfolios, but not in any other instances.

Given the lack of evidence, and the degree to which statistical significance is impacted by the portfolio sorting methodology, we believe neither the higher-order moments, co-skewness and co-kurtosis, nor FMAX to be priced in the market. The existing literature that supports the pricing of higher-order moments is limited, and the reason we reach an opposite conclusion does not necessarily only stem from using different portfolio sorting methodologies. Some of the adjustments we have made were not mainstream in the asset pricing literature at the time several of these studies were published.

We believe that stock returns are non-normal, as corroborated by the normality statistics in Table 1. Furthermore, we believe investors both should be, and are more concerned with extreme downside deviation than CAPM suggests. However, historical co-skewness and co-kurtosis are not necessarily good measures of *ex-ante* market expectations of co-skewness and co-kurtosis. As pointed out previously, there are also a number of issues with the Fama-MacBeth methodology, but improving on this methodology has been outside the scope of this thesis. Therefore, it could very well be the case that higher-order moments are priced in the market, but that our study suffers from measurement errors in this respect. The manner in which extreme downside deviation is priced is yet to discovered, if at all, and is perhaps a line of research that deserves more attention.

Finally, the most interesting result from this study has been the notable observed impact portfolio sorting methodologies has on the statistical significance of proposed asset pricing factors. Of all our tested factors, HML, or the value effect, shows the greatest promise, as it is the one that is most consistently statistically significant, or close to being significant, across the portfolio sorting methodologies. Nonetheless, the statistical significance of the HML factor also varies greatly with the portfolio sorting, and only in the case of book-to-market-sorted portfolios does it meet the hurdle suggested by Harvey, Liu and Zhu (2016), namely a t-statistic greater than 3. As previously mentioned, Harvey, Liu and Zhu (2016), find 316 different factors, when only surveying top journals, and argue that most of these are likely false on the grounds of data mining and regression methodologies. We also find it highly unlikely that such a vast array of factors are priced in the market, and believe that portfolio sorting methodologies might have added to the false discovery rate.

7.2 Suggestions for further research

As commented on throughout this thesis, our methodology does not come without its flaws. Correcting the errors-in-variables (EIV) issue in the Fama-MacBeth methodology has been beyond the scope of this thesis. However, several methods exist to mitigate this issue. A common procedure to reduce the EIV concerns in Fama-MacBeth, is the correction for potential time-series correlation of the variance-covariance matrix suggested by Shanken (1992).

Although the Fama-MacBeth procedure is still widely used in asset pricing research, Generalised Method of Moments (GMM), as developed by Hansen (1982), is currently seen as superior to the Fama-MacBeth methodology. According to Cochrane (2005), GMM is "the easy and elegant way to account for the effects of "generated regressors" such as the β in the cross-sectional regression…" (pp. 241). Consequently, repeating the analysis with GMM could yield better estimates of risk premia.

In addition to improving upon the regression methodology, improvements can also be made in terms of the measurement of ex ante expectations of higher-order moments. Historical skewness and kurtosis are not necessarily good measures what the market expects these parameters to be. Rehman and Vilkov (2012) and Conrad, Dittmar and Ghysels (2013) are both papers that have employed a methodology where they find the implied *ex ante* skewness from equity options and use it in asset pricing tests. Interestingly, these studies find a negative and positive relation between ex ante skewness and subsequent returns, respectively. According to Schneider, Wagner and Zechner (2017), the different conclusions reached in the two aforementioned studies, is due to the construction of the skewness measures. Another issue with this approach is the liquidity of options. Utilising equity options for finding ex ante skewness means you have to restrict the sample to equities which have a healthy option liquidity, i.e. most equities with small market capitalisations would have to be excluded. Nonetheless, utilising ex ante expectations of higher-order moments appear to be a line of research which is gaining in popularity.

Finally, we believe several asset models should be reviewed critically in light of the criticisms of the testing methodology. Given the vast array of existing asset pricing factors, perhaps future research efforts should to a greater extent focus on how we go about testing these candidate factors.

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9. Appendix

9.1 Regression Equations

9.1.1 Tables 2 to 5 (Co-moment coefficients)

(I) First-pass (time-series):

$R_{i,t} = a_i + \beta_{i,t} R_{M,t} + s_{i,t} SMB_t + h_{i,t} HML_t + e_{i,t}$

Second-pass (cross-sectional):

$R_{i,t} = \alpha_t + \lambda_t^{(\beta)} \hat{\beta}_{i,t-1} + \lambda_t^{(s)} \hat{s}_{i,t-1} + \lambda_t^{(h)} \hat{h}_{i,t-1} + \varepsilon_{i,t}$

(II) First-pass (time-series):

$R_{i,t} = a_i + \beta_{i,t} R_{M,t} + s_{i,t} SMB_t + h_{i,t} HML_t + e_{i,t}$

$$\hat{\gamma}_{i,t} = \frac{\sum_{j=\tau}^{t} (R_{i,j} - \bar{R}_{i,t})(R_{M,j} - \bar{R}_{M,t})^2}{\sum_{j=\tau}^{t} (R_{M,j} - \bar{R}_{M,t})^3} \ where\ \bar{R}_t = \frac{1}{t-\tau} \sum_{j=\tau}^{t} R_j$$

Second-pass (cross-sectional):

$R_{i,t} = \alpha_t + \lambda_t^{(\beta)} \hat{\beta}_{i,t-1} + \lambda_t^{(s)} \hat{s}_{i,t-1} + \lambda_t^{(h)} \hat{h}_{i,t-1} + \lambda_t^{(\gamma)} \hat{\gamma}_{i,t-1} + \varepsilon_{i,t}$

(III) First-pass (time-series):

$R_{i,t} = a_i + \beta_{i,t} R_{M,t} + s_{i,t} SMB_t + h_{i,t} HML_t + e_{i,t}$

$$\hat{\gamma}_{i,t} = \frac{\sum_{j=\tau}^{t} (R_{i,j} - \bar{R}_{i,t})(R_{M,j} - \bar{R}_{M,t})^2}{\sum_{j=\tau}^{t} (R_{M,j} - \bar{R}_{M,t})^3} \ where\ \bar{R}_t = \frac{1}{t-\tau} \sum_{j=\tau}^{t} R_j$$

$$\hat{\delta}_{i,t} = \frac{\sum_{j=\tau}^{t} (R_{i,j} - \bar{R}_{i,t})(R_{M,j} - \bar{R}_{M,t})^3}{\sum_{j=\tau}^{t} (R_{M,j} - \bar{R}_{M,t})^4} \ where\ \bar{R}_t = \frac{1}{t-\tau} \sum_{j=\tau}^{t} R_j$$

Second-pass (cross-sectional):

$R_{i,t} = \alpha_t + \lambda_t^{(\beta)} \hat{\beta}_{i,t-1} + \lambda_t^{(s)} \hat{s}_{i,t-1} + \lambda_t^{(h)} \hat{h}_{i,t-1} + \lambda_t^{(\gamma)} \hat{\gamma}_{i,t-1} + \lambda_t^{(\delta)} \hat{\delta}_{i,t-1} + \varepsilon_{i,t}$

(IV) First-pass (time-series):

$$\hat{\beta}_{i,t} = \frac{\sum_{j=\tau}^{t} (R_{i,j} - \bar{R}_{i,t})(R_{M,j} - \bar{R}_{M,t})}{\sum_{j=\tau}^{t} (R_{M,j} - \bar{R}_{M,t})^2} \ where\ \bar{R}_t = \frac{1}{t-\tau} \sum_{j=\tau}^{t} R_j$$

Second-pass (cross-sectional):

$R_{i,t} = \alpha_t + \lambda_t^{(\beta)} \hat{\beta}_{i,t-1} + \varepsilon_{i,t}$

(V) First-pass (time-series):

$$\hat{\beta}_{i,t} = \frac{\sum_{j=\tau}^{t} (R_{i,j} - \bar{R}_{i,t})(R_{M,j} - \bar{R}_{M,t})}{\sum_{j=\tau}^{t} (R_{M,j} - \bar{R}_{M,t})^2} \ where\ \bar{R}_t = \frac{1}{t-\tau} \sum_{j=\tau}^{t} R_j$$

$$\hat{\gamma}_{i,t} = \frac{\sum_{j=\tau}^{t} (R_{i,j} - \bar{R}_{i,t})(R_{M,j} - \bar{R}_{M,t})^2}{\sum_{j=\tau}^{t} (R_{M,j} - \bar{R}_{M,t})^3} \ where\ \bar{R}_t = \frac{1}{t-\tau} \sum_{j=\tau}^{t} R_j$$

Second-pass (cross-sectional):

$R_{i,t} = \alpha_t + \lambda_t^{(\beta)} \hat{\beta}_{i,t-1} + \lambda_t^{(\gamma)} \hat{\gamma}_{i,t-1} + \varepsilon_{i,t}$

(VI) First-pass (time-series):

$$\hat{\beta}_{i,t} = \frac{\sum_{j=\tau}^{t}\left(R_{i,j} - \bar{R}_{i,t}\right)\left(R_{M,j} - \bar{R}_{M,t}\right)}{\sum_{j=\tau}^{t}\left(R_{M,j} - \bar{R}_{M,t}\right)^2} \; where \; \bar{R}_t = \frac{1}{t-\tau}\sum_{j=\tau}^{t} R_j$$

$$\hat{\gamma}_{i,t} = \frac{\sum_{j=\tau}^{t}\left(R_{i,j} - \bar{R}_{i,t}\right)\left(R_{M,j} - \bar{R}_{M,t}\right)^2}{\sum_{j=\tau}^{t}\left(R_{M,j} - \bar{R}_{M,t}\right)^3} \; where \; \bar{R}_t = \frac{1}{t-\tau}\sum_{j=\tau}^{t} R_j$$

$$\hat{\delta}_{i,t} = \frac{\sum_{j=\tau}^{t}\left(R_{i,j} - \bar{R}_{i,t}\right)\left(R_{M,j} - \bar{R}_{M,t}\right)^3}{\sum_{j=\tau}^{t}\left(R_{M,j} - \bar{R}_{M,t}\right)^4} \; where \; \bar{R}_t = \frac{1}{t-\tau}\sum_{j=\tau}^{t} R_j$$

Second-pass (cross-sectional):

$$R_{i,t} = \alpha_t + \lambda_t^{(\beta)}\hat{\beta}_{i,t-1} + \lambda_t^{(\gamma)}\hat{\gamma}_{i,t-1} + \lambda_t^{(\delta)}\hat{\delta}_{i,t-1} + \varepsilon_{i,t}$$

9.1.2 Tables 6 to 9 (FMAX factor)

(I) First-pass (time-series):

$$R_{i,t} = a_i + \beta_{i,t}R_{M,t} + s_{i,t}SMB_t + h_{i,t}HML_t + e_{i,t}$$

Second-pass (cross-sectional):

$$R_{i,t} = \alpha_t + \lambda_t^{(\beta)}\hat{\beta}_{i,t-1} + \lambda_t^{(s)}\hat{s}_{i,t-1} + \lambda_t^{(h)}\hat{h}_{i,t-1} + \varepsilon_{i,t}$$

(II) First-pass (time-series):

$$R_{i,t} = a_i + \beta_{i,t}R_{M,t} + s_{i,t}SMB_t + h_{i,t}HML_t + f_{i,t}FMAX_t + e_{i,t}$$

Second-pass (cross-sectional):

$$R_{i,t} = \alpha_t + \lambda_t^{(\beta)}\hat{\beta}_{i,t-1} + \lambda_t^{(s)}\hat{s}_{i,t-1} + \lambda_t^{(h)}\hat{h}_{i,t-1} + \lambda_t^{(f)}\hat{f}_{i,t-1} + \varepsilon_{i,t}$$

(III) First-pass (time-series):

$$R_{i,t} = a_i + \beta_{i,t}R_{M,t} + e_{i,t}$$

Second-pass (cross-sectional):

$$R_{i,t} = \alpha_t + \lambda_t^{(\beta)}\hat{\beta}_{i,t-1} + \varepsilon_{i,t}$$

(IV) First-pass (time-series):

$$R_{i,t} = a_i + \beta_{i,t}R_{M,t} + f_{i,t}FMAX_t + e_{i,t}$$

Second-pass (cross-sectional):

$$R_{i,t} = \alpha_t + \lambda_t^{(\beta)}\hat{\beta}_{i,t-1} + \lambda_t^{(f)}\hat{f}_{i,t-1} + \varepsilon_{i,t}$$

9.2 Other results

9.2.1 Simple returns – Value-weighted assets vs. value-weighted market

Table 2 (Size-sorted) – co-moment coefficients

TABLE 2 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	.0050*	.0027*	.0003			.279
			(2.11)	(2.42)	(.22)			[4.8*]
		(II)	.0043	.0025*	.0026	-.0000		.302
			(1.40)	(1.97)	(1.60)	(-.02)		[3.3]
		(III)	.0029	.0019	.0036	-.0047	.0057	.317
			(.67)	(.97)	(1.77)	(-1.07)	(.81)	[2.0]
		(IV)	.0075**					.216
			(2.60)					
		(V)	.0091**			-.0013		.247
			(2.59)			(-.73)		
		(VI)	.0089			-.0063	.0034	.263
			(1.47)			(-1.49)	(.45)	[1.8]
Quarterly	320	(I)	.0032	.0089**	.0030			.342
			(.52)	(2.63)	(.81)			[5.8**]
		(II)	.0115	.0111**	.0060	-.0064		.351
			(1.63)	(3.06)	(1.49)	(-1.91)		[4.4*]
		(III)	.0001	.0067	.0087	-.0240	.0283	.362
			(.01)	(1.16)	(1.35)	(-1.33)	(1.02)	[2.3]
		(IV)	.0247**					.209
			(2.69)					
		(V)	.0216*			-.0039		.245
			(2.29)			(-1.03)		
		(VI)	.0125			-.0117	.0126	.280
			(.71)			(-1.18)	(.60)	[2.2]
Semiannual	160	(I)	.0048	.0132*	.0097			.334
			(.46)	(2.18)	(1.56)			[5.4*]
		(II)	.0140	.0151*	.0228*	-.0056		.341
			(1.14)	(2.38)	(2.32)	(-.99)		[4.0*]
		(III)	-.0063	.0041	.0133	-.0115	.0271	.353
			(-.19)	(.33)	(.80)	(-.70)	(.66)	[2.7]
		(IV)	.0259*					.179
			(2.32)					
		(V)	.0470**			-.0213**		.221
			(3.01)			(-3.18)		
		(VI)	.0095			-.0309*	.0443	.259
			(.34)			(-2.04)	(1.39)	[2.3]

Table 3 (BTM-sorted) – co-moment coefficients

TABLE 3 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
BTM-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	.0001	.0024	.0019			.155
			(.02)	(1.43)	(1.24)			[2.4]
		(II)	-.0011	.0033	.0016	.0010		.163
			(-.30)	(1.71)	(1.03)	(.44)		[2.3]
		(III)	-.0040	.0008	.0020	-.0008	.0072	.176
			(-.72)	(.29)	(1.00)	(-.18)	(.89)	[2.2]
		(IV)	.0056					.072
			(1.61)					
		(V)	.0011			.0038		.086
			(.28)			(1.87)		
		(VI)	-.0087			-.0003	.0135	.105
			(-1.22)			(-.06)	(1.54)	[1.5]
Quarterly	164	(I)	-.0131	.0035	.0090*			.168
			(-1.75)	(.91)	(2.08)			[2.9]
		(II)	-.0212**	.0006	.0047	.0050		.172
			(-2.64)	(.14)	(.99)	(1.23)		[2.7]
		(III)	-.0294	-.0015	.0081	.0064	.0054	.186
			(-1.67)	(-.26)	(1.11)	(.96)	(.28)	[2.6]
		(IV)	-.0024					.050
			(-.28)					
		(V)	-.0100			-.0031		.075
			(-1.11)			(-.62)		
		(VI)	-.0201			.0003	.0028	.118
			(-1.05)			(.04)	(.13)	[2.0]
Semiannual	82	(I)	-.0025	.0049	.0114			.158
			(-.31)	(.93)	(1.56)			[3.1]
		(II)	-.0159	.0050	.0106	.0135		.164
			(-1.34)	(.82)	(1.14)	(1.50)		[2.9]
		(III)	-.0204	-.0046	.0191	.0202	-.0019	.170
			(-.52)	(-.36)	(1.36)	(.73)	(-.04)	[2.6]
		(IV)	.0073					.044
			(.62)					
		(V)	-.0059			.0085		.064
			(-.43)			(.92)		
		(VI)	.0220			.0211	-.0412	.091
			(.58)			(1.35)	(-.98)	[1.7]

Table 4 (Triple-sorted) – co-moment coefficients

TABLE 4 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Triple-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0024	.0033*	.0012			.290
			(-.89)	(2.30)	(.81)			[7.6**]
		(II)	-.0011	.0034*	.0009	-.0009		.300
			(-.37)	(2.23)	(.60)	(-.48)		[5.3**]
		(III)	-.0015	.0043*	.0000	-.0000	-.0007	.308
			(-.35)	(2.07)	(.00)	(-.00)	(-.12)	[3.8*]
		(IV)	.0047					.170
			(1.52)					
		(V)	.0023			.0020		.202
			(.64)			(.95)		
		(VI)	.0047			.0055	-.0064	.214
			(.78)			(1.40)	(-.90)	[2.3]
Quarterly	164	(I)	-.0046	.0103**	.0054			.292
			(-.69)	(2.65)	(1.24)			[8.2**]
		(II)	-.0051	.0104**	.0065	.0037		.298
			(-.72)	(2.77)	(1.29)	(1.22)		[6.3**]
		(III)	-.0166	.0086	.0069	.0021	.0139	.305
			(-1.23)	(1.83)	(1.08)	(.43)	(1.00)	[3.7*]
		(IV)	.0075					.158
			(.82)					
		(V)	.0086			-.0045		.206
			(.91)			(-1.19)		
		(VI)	-.0013			-.0060	.0087	.228
			(-.08)			(-.98)	(.54)	[3.0]
Semiannual	82	(I)	-.0025	.0154*	.0131			.273
			(-.28)	(2.34)	(1.63)			[7.3**]
		(II)	-.0159	.0156*	.0319*	.0131		.284
			(-1.21)	(2.09)	(2.16)	(1.65)		[5.9**]
		(III)	.0084	.0295*	.0175	.0189	-.0293	.291
			(.31)	(2.61)	(.92)	(.91)	(-.77)	[4.0*]
		(IV)	.0036					.127
			(.31)					
		(V)	.0060			-.0015		.152
			(.41)			(-.23)		
		(VI)	.0010			-.0125	.0134	.182
			(.03)			(-.55)	(.30)	[2.5]

Table 5 (Industry-sorted) – co-moment coefficients

TABLE 5 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Industry-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	-.0021	-.0008	.0021			.259
			(-.84)	(-.49)	(1.43)			[2.7]
		(II)	-.0020	.0023	.0033*	-.0002		.300
			(-.68)	(1.24)	(1.97)	(-.10)		[2.7]
		(III)	-.0043	-.0027	.0020	.0038	.0060	.327
			(-.69)	(-.83)	(.82)	(.76)	(.56)	[2.4]
		(IV)	-.0042					.125
			(-1.61)					
		(V)	-.0008			-.0006		.181
			(-.25)			(-.27)		
		(VI)	-.0051			-.0010	.0075	.217
			(-.65)			(-.19)	(.73)	[1.9]
Quarterly	320	(I)	.0032	-.0026	.0086*			.271
			(.47)	(-.65)	(2.04)			[2.8]
		(II)	-.0000	-.0030	.0110*	.0085		.297
			(-.00)	(-.64)	(2.19)	(1.72)		[2.6]
		(III)	-.0157	-.0113	.0132	.0229	.0010	.327
			(-.75)	(-1.25)	(1.54)	(.85)	(.03)	[2.3]
		(IV)	-.0094					.128
			(-1.48)					
		(V)	-.0036			.0021		.186
			(-.43)			(.46)		
		(VI)	-.0342			.0043	.0329	.235
			(-1.77)			(.23)	(1.14)	[2.1]
Semiannual	160	(I)	-.0016	-.0031	.0136*			.284
			(-.16)	(-.38)	(1.99)			[2.9]
		(II)	.0071	.0010	.0092	-.0022		.316
			(.48)	(.12)	(1.03)	(-.22)		[2.6]
		(III)	-.0296	-.0181	-.0432	-.0022	.0401	.355
			(-.60)	(-.90)	(-1.85)	(-.07)	(.59)	[2.4]
		(IV)	-.0096					.109
			(-.96)					
		(V)	.0080			-.0127		.190
			(.71)			(-1.93)		
		(VI)	.0064			-.0109	.0050	.249
			(.18)			(-.41)	(.10)	[2.4]

Table 6 (Size-sorted) – FMAX factor

TABLE 6 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	.0053	.0026	.0001		.292
			(1.37)	(1.68)	(.03)		[5.1*]
		(II)	.0030	.0030	.0002	.0001	.315
			(.80)	(1.89)	(.10)	(.02)	[4.5*]
		(III)	.0077*				.212
			(1.99)				
		(IV)	.0063			.0050	.215
			(1.50)			(1.45)	
Quarterly	164	(I)	.0023	.0102*	.0025		.366
			(.23)	(2.20)	(.40)		[5.7**]
		(II)	.0052	.0092*	.0041	.0022	.388
			(.55)	(2.13)	(.77)	(.29)	[5.0*]
		(III)	.0274*				.202
			(1.98)				
		(IV)	.0262*			.0202*	.291
			(2.23)			(2.14)	
Semiannual	82	(I)	.0035	.0132	.0083		.346
			(.22)	(1.72)	(1.01)		[4.9*]
		(II)	.0013	.0129	.0103	.0048	.360
			(.08)	(1.68)	(1.25)	(.41)	[4.6*]
		(III)	.0312				.173
			(1.86)				
		(IV)	.0205			.0245	.243
			(1.20)			(1.65)	

Table 7 (BTM-sorted) – FMAX factor

TABLE 7 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Book-To-Market-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	.0001	.0024	.0019		.155
			(.02)	(1.43)	(1.24)		[2.4]
		(II)	.0026	.0032	.0013	.0009	.164
			(.73)	(1.90)	(.85)	(.36)	[2.3]
		(III)	.0056				.072
			(1.61)				
		(IV)	.0013			-.0013	.076
			(.34)			(-.47)	
Quarterly	164	(I)	-.0131	.0035	.0090*		.168
			(-1.75)	(.91)	(2.08)		[2.9]
		(II)	-.0159*	.0040	.0075	-.0055	.180
			(-2.09)	(1.07)	(1.79)	(-1.02)	[2.9]
		(III)	-.0024				.050
			(-.28)				
		(IV)	-.0037			-.0122*	.089
			(-.43)			(-2.23)	
Semiannual	82	(I)	-.0025	.0049	.0114		.158
			(-.31)	(.93)	(1.56)		[3.1]
		(II)	-.0008	.0059	.0096	-.0033	.190
			(-.10)	(1.11)	(1.41)	(-.42)	[2.8]
		(III)	.0073				.044
			(.62)				
		(IV)	.0043			-.0092	.100
			(.38)			(-.99)	

Table 8 (Triple-sorted) – FMAX factor

TABLE 8 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Triple-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0024	.0033*	.0012		.290
			(-.89)	(2.30)	(.81)		[7.6**]
		(II)	-.0015	.0034*	.0014	-.0007	.300
			(-.55)	(2.38)	(1.00)	(-.30)	[7.1**]
		(III)	.0047				.170
			(1.52)				
		(IV)	.0025			-.0008	.197
			(.84)			(-.31)	
Quarterly	164	(I)	-.0046	.0103**	.0054		.292
			(-.69)	(2.65)	(1.24)		[8.2**]
		(II)	-.0038	.0095*	.0068	-.0046	.304
			(-.55)	(2.54)	(1.65)	(-.74)	[7.3**]
		(III)	.0075				.158
			(.82)				
		(IV)	.0080			.0005	.209
			(.92)			(.08)	
Semiannual	82	(I)	-.0025	.0154*	.0131		.273
			(-.28)	(2.34)	(1.63)		[7.3**]
		(II)	-.0010	.0158*	.0113	-.0057	.287
			(-.11)	(2.40)	(1.45)	(-.55)	[6.8**]
		(III)	.0036				.127
			(.31)				
		(IV)	.0033			-.0043	.189
			(.30)			(-.41)	

Table 9 (Industry-sorted) – FMAX factor

TABLE 9 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Industry-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0016	.0005	.0012		.256
			(-.40)	(.20)	(.52)		[2.5]
		(II)	.0019	-.0039	.0000	.0011	.298
			(.48)	(-1.36)	(.01)	(.36)	[2.4]
		(III)	-.0048				.127
			(-1.22)				
		(IV)	.0013			-.0015	.196
			(.33)			(-.50)	
Quarterly	164	(I)	.0150	-.0004	.0072		.270
			(1.39)	(-.07)	(1.14)		[2.7]
		(II)	.0171	-.0058	-.0017	.0073	.339
			(1.61)	(-.98)	(-.30)	(.91)	[2.4]
		(III)	-.0095				.129
			(-.98)				
		(IV)	.0144			.0017	.246
			(1.35)			(.23)	
Semiannual	82	(I)	.0083	-.0008	.0152		.316
			(.61)	(-.06)	(1.68)		[3.0]
		(II)	.0038	.0097	.0135	.0070	.380
			(.30)	(.66)	(1.47)	(.51)	[2.7]
		(III)	-.0064				.121
			(-.44)				
		(IV)	.0039			-.0035	.241
			(.30)			(-.28)	

9.2.2 Log returns – Equal-weighted assets vs. value-weighted market

Table 2 (Size-sorted) – co-moment coefficients

TABLE 2 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	.0031	.0014	-.0000			.296
			(1.32)	(1.26)	(-.03)			[5.1*]
		(II)	.0030	.0014	.0001	-.0002		.312
			(.96)	(1.13)	(.04)	(-.11)		[3.5*]
		(III)	.0018	.0019	.0012	-.0054	.0054	.325
			(.36)	(1.24)	(.60)	(-1.13)	(.80)	[2.5]
		(IV)	.0055					.154
			(1.62)					
		(V)	.0008			.0018		.225
			(.20)			(.86)		
		(VI)	.0035			-.0019	-.0003	.246
			(.57)			(-.36)	(-.04)	[2.5]
Quarterly	320	(I)	-.0011	.0057	-.0016			.364
			(-.21)	(1.76)	(-.43)			[6.3**]
		(II)	-.0022	.0046	-.0003	.0019		.377
			(-.28)	(1.30)	(-.07)	(.45)		[4.3*]
		(III)	.0046	.0066	.0032	-.0043	-.0007	.389
			(.31)	(1.25)	(.54)	(-.46)	(-.04)	[2.5]
		(IV)	.0192*					.210
			(2.11)					
		(V)	.0138			-.0038		.258
			(1.32)			(-.70)		
		(VI)	.0108			-.0154	.0113	.306
			(.56)			(-1.48)	(.47)	[2.7]
Semiannual	160	(I)	.0051	.0077	.0042			.367
			(.59)	(1.32)	(.74)			[6.4**]
		(II)	.0261*	.0112	.0122	-.0170**		.379
			(2.17)	(1.74)	(1.63)	(-2.67)		[4.7*]
		(III)	.0344	.0086	.0012	.0039	-.0296	.382
			(1.18)	(.79)	(.10)	(.19)	(-.74)	[2.8]
		(IV)	.0180					.199
			(1.58)					
		(V)	.0367*			-.0204*		.255
			(2.59)			(-2.56)		
		(VI)	.0069			-.0450*	.0517	.290
			(.25)			(-2.10)	(1.38)	[2.6]

Table 3 (BTM-sorted) – co-moment coefficients

TABLE 3 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
BTM-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0035	-.0007	.0055***			.219
			(-1.04)	(-.37)	(3.38)			[3.7*]
		(II)	-.0072	-.0020	.0063***	.0032		.225
			(-1.62)	(-.92)	(3.75)	(1.16)		[2.7]
		(III)	-.0141*	-.0030	.0073**	-.0051	.0145	.233
			(-2.05)	(-1.26)	(3.24)	(-.62)	(1.28)	[2.4]
		(IV)	-.0086*					.083
			(-2.44)					
		(V)	-.0103*			.0022		.144
			(-2.32)			(.83)		
		(VI)	-.0091			-.0068	.0113	.163
			(-1.38)			(-.78)	(.97)	[2.2]
Quarterly	164	(I)	-.0073	.0024	.0134**			.244
			(-1.07)	(.50)	(3.08)			[4.0*]
		(II)	-.0043	.0023	.0130*	-.0018		.251
			(-.44)	(.44)	(2.56)	(-.31)		[3.3]
		(III)	.0231	.0069	-.0011	.0006	-.0298	.266
			(1.21)	(.97)	(-.15)	(.04)	(-1.08)	[2.7]
		(IV)	-.0280**					.088
			(-2.85)					
		(V)	-.0133			-.0060		.139
			(-1.15)			(-1.13)		
		(VI)	-.0079			-.0050	-.0096	.188
			(-.38)			(-.30)	(-.30)	[2.5]
Semiannual	82	(I)	-.0072	.0007	.0271**			.288
			(-.75)	(.10)	(3.25)			[4.7*]
		(II)	.0027	-.0045	.0322**	-.0055		.291
			(.11)	(-.56)	(2.67)	(-.33)		[3.7*]
		(III)	-.0023	-.0023	.0351*	.0267	-.0272	.303
			(-.05)	(-.18)	(2.00)	(.64)	(-.42)	[2.6]
		(IV)	-.0095					.136
			(-.68)					
		(V)	.0059			-.0092		.198
			(.27)			(-.68)		
		(VI)	.0067			-.0007	-.0063	.227
			(.16)			(-.03)	(-.12)	[2.5]

Table 4 (Triple-sorted) – co-moment coefficients

TABLE 4 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Triple-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Skewness γ	Kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0044	.0015	.0030*			.299
			(-1.53)	(.95)	(2.00)			[7.4**]
		(II)	-.0051	.0014	.0030*	.0003		.305
			(-1.85)	(.88)	(2.04)	(.18)		[6.9**]
		(III)	-.0050	.0013	.0031*	.0090	-.0069	.313
			(-1.82)	(.82)	(2.14)	(.58)	(-.44)	[6.7**]
		(IV)	-.0040					.113
			(-1.01)					
		(V)	.0030			-.0042		.115
			(.43)			(-.83)		
		(VI)	-.0316			.0130	.0107	.110
			(-1.56)			(.46)	(.29)	[1.0]
Quarterly	164	(I)	-.0141	.0053	.0094*			.316
			(-1.83)	(1.20)	(2.10)			[9.0**]
		(II)	-.0138	.0055	.0088*	-.0016		.321
			(-1.94)	(1.25)	(2.00)	(-.34)		[8.0**]
		(III)	-.0077	.0064	.0075	.0109	-.0212	.339
			(-1.16)	(1.48)	(1.73)	(.39)	(-.73)	[7.6**]
		(IV)	-.0099					.108
			(-.75)					
		(V)	-.0227			.0035		.109
			(-.67)			(.18)		
		(VI)	-.0268			.0139	-.0114	.114
			(-.39)			(.36)	(-.16)	[1.2]
Semiannual	82	(I)	-.0107	.0057	.0188*			.302
			(-1.10)	(.83)	(2.11)			[7.8**]
		(II)	-.0051	.0069	.0156	-.0057		.321
			(-.52)	(.98)	(1.87)	(-.71)		[7.7**]
		(III)	-.0010	.0081	.0149	-.0259	-.0074	.335
			(-.11)	(1.15)	(1.89)	(-1.02)	(-.22)	[7.4**]
		(IV)	-.0298					.101
			(-1.56)					
		(V)	-.0019			-.0166		.105
			(-.07)			(-1.20)		
		(VI)	.0316			-.0525	-.0055	.103
			(.46)			(-.61)	(-.04)	[1.1]

Table 5 (Industry-sorted) – co-moment coefficients

TABLE 5 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions *Industry-Sorted Portfolios*

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Skewness γ	Kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	-.0051*	-.0004	.0021			.239
			(-2.18)	(-.28)	(1.78)			[2.5]
		(II)	-.0046	.0005	.0019	.0003		.254
			(-1.76)	(.36)	(1.49)	(.18)		[2.5]
		(III)	-.0016	.0012	.0019	.0017	-.0048	.261
			(-.44)	(.81)	(1.33)	(.36)	(-.87)	[2.5]
		(IV)	-.0034					.087
			(-1.96)					
		(V)	-.0017			.0001		.120
			(-.62)			(.08)		
		(VI)	.0056			.0008	-.0077	.159
			(.95)			(.17)	(-.99)	[1.7]
Quarterly	320	(I)	-.0131*	-.0004	.0069			.238
			(-2.22)	(-.13)	(1.80)			[2.6]
		(II)	-.0153*	.0002	.0068	.0033		.245
			(-2.15)	(.06)	(1.67)	(.89)		[2.6]
		(III)	-.0136	.0015	.0026	-.0017	.0015	.251
			(-1.20)	(.30)	(.50)	(-.19)	(.11)	[2.4]
		(IV)	-.0089					.081
			(-1.91)					
		(V)	-.0125			.0038		.110
			(-1.82)			(.86)		
		(VI)	-.0030			-.0045	-.0019	.159
			(-.17)			(-.34)	(-.08)	[1.8]
Semiannual	160	(I)	-.0107	-.0042	.0090			.241
			(-1.14)	(-.73)	(1.33)			[2.5]
		(II)	-.0136	-.0032	.0074	.0077		.248
			(-1.22)	(-.51)	(.98)	(1.28)		[2.4]
		(III)	-.0216	-.0098	-.0007	-.0070	.0200	.252
			(-1.42)	(-1.23)	(-.06)	(-.34)	(.72)	[2.1]
		(IV)	-.0125					.094
			(-1.59)					
		(V)	-.0065			-.0011		.142
			(-.60)			(-.17)		
		(VI)	.0090			-.0234	.0080	.176
			(.29)			(-1.08)	(.18)	[1.8]

Table 6 (Size-sorted) – FMAX factor

TABLE 6 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	.0034	.0016	-.0008		.311
			(.88)	(.99)	(-.32)		[5.2*]
		(II)	.0012	.0016	-.0004	-.0029	.337
			(.33)	(1.02)	(-.16)	(-.80)	[4.4*]
		(III)	.0037				.122
			(.72)				
		(IV)	.0045			.0026	.233
			(1.04)			(.68)	
Quarterly	164	(I)	-.0061	.0079	-.0051		.377
			(-.71)	(1.73)	(-.87)		[5.9**]
		(II)	-.0048	.0059	-.0021	.0043	.405
			(-.58)	(1.43)	(-.37)	(.52)	[5.1*]
		(III)	-.0009				.192
			(-.09)				
		(IV)	.0170			.0180	.318
			(1.53)			(1.80)	
Semiannual	82	(I)	.0045	.0074	.0026		.387
			(.35)	(.95)	(.29)		[6.4**]
		(II)	.0039	.0079	.0028	.0053	.395
			(.29)	(1.01)	(.33)	(.52)	[5.4*]
		(III)	.0103				.211
			(.58)				
		(IV)	.0113			.0151	.281
			(.74)			(1.03)	

Table 7 (BTM-sorted) – FMAX factor

TABLE 7 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Book-To-Market-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	.0001	.0024	.0019		.219
			(.02)	(1.43)	(1.24)		[2.4]
		(II)	.0026	.0032	.0013	.0009	.229
			(.73)	(1.90)	(.85)	(.36)	[2.3]
		(III)	.0056				.083
			(1.61)				
		(IV)	.0013			-.0013	.133
			(.34)			(-.47)	
Quarterly	164	(I)	-.0131	.0035	.0090*		.244
			(-1.75)	(.91)	(2.08)		[2.9]
		(II)	-.0159*	.0040	.0075	-.0055	.256
			(-2.09)	(1.07)	(1.79)	(-1.02)	[2.9]
		(III)	-.0024				.088
			(-.28)				
		(IV)	-.0037			-.0122*	.163
			(-.43)			(-2.23)	
Semiannual	82	(I)	-.0025	.0049	.0114		.288
			(-.31)	(.93)	(1.56)		[3.1]
		(II)	-.0008	.0059	.0096	-.0033	.307
			(-.10)	(1.11)	(1.41)	(-.42)	[2.8]
		(III)	.0073				.136
			(.62)				
		(IV)	.0043			-.0092	.205
			(.38)			(-.99)	

Table 8 (Triple-sorted) – FMAX factor

TABLE 8 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Triple-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0044	.0015	.0030*		.299
			(-1.53)	(.95)	(2.00)		[7.4**]
		(II)	-.0037	.0017	.0031*	-.0044	.313
			(-1.36)	(1.05)	(2.06)	(-1.63)	[6.5**]
		(III)	-.0040				.113
			(-1.01)				
		(IV)	-.0007			-.0041	.208
			(-.24)			(-1.50)	
Quarterly	164	(I)	-.0141	.0053	.0094*		.316
			(-1.83)	(1.20)	(2.10)		[9.0**]
		(II)	-.0122	.0027	.0111*	-.0106	.329
			(-1.59)	(.67)	(2.56)	(-1.51)	[7.1**]
		(III)	-.0099				.108
			(-.75)				
		(IV)	-.0058			-.0086	.234
			(-.64)			(-1.17)	
Semiannual	82	(I)	-.0107	.0057	.0188*		.302
			(-1.10)	(.83)	(2.11)		[7.8**]
		(II)	-.0090	.0057	.0187*	-.0172	.310
			(-.95)	(.86)	(2.12)	(-1.46)	[7.2**]
		(III)	-.0298				.101
			(-1.56)				
		(IV)	-.0018			-.0155	.206
			(-.15)			(-1.28)	

Table 9 (Industry-sorted) – FMAX factor

TABLE 9 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Industry-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0050	.0005	.0031		.261
			(-1.36)	(.25)	(1.85)		[2.5]
		(II)	-.0075	.0014	.0031	-.0034	.310
			(-1.94)	(.62)	(1.75)	(-1.19)	[2.5]
		(III)	-.0044				.093
			(-1.53)				
		(IV)	-.0028			-.0041	.196
			(-.74)			(-1.45)	
Quarterly	164	(I)	-.0169	.0015	.0115*		.236
			(-1.85)	(.30)	(2.04)		[2.5]
		(II)	-.0080	.0003	.0090	-.0044	.298
			(-.90)	(.05)	(1.84)	(-.59)	[2.3]
		(III)	-.0103				.065
			(-1.47)				
		(IV)	-.0050			-.0050	.200
			(-.57)			(-.70)	
Semiannual	82	(I)	-.0184	-.0011	.0176		.265
			(-1.42)	(-.13)	(1.96)		[2.6]
		(II)	-.0179	-.0025	.0159*	-.0127	.302
			(-1.45)	(-.31)	(2.12)	(-1.17)	[2.4]
		(III)	-.0202				.110
			(-1.81)				
		(IV)	-.0109			-.0153	.227
			(-.83)			(-1.26)	

9.2.3 Log returns – Value-weighted assets vs. value-weighted market

Table 2 (Size-sorted) – co-moment coefficients

TABLE 2 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	.0051*	.0014	-.0001			.278
			(2.23)	(1.28)	(-.05)			[4.9*]
		(II)	.0028	.0006	-.0001	.0014		.301
			(.90)	(.53)	(-.04)	(.84)		[3.6*]
		(III)	.0038	.0011	.0002	-.0024	.0020	.312
			(.75)	(.70)	(.09)	(-.74)	(.35)	[2.5]
		(IV)	.0052					.145
			(1.72)					
		(V)	.0011			.0016		.211
			(.34)			(.98)		
		(VI)	.0024			-.0024	.0021	.232
			(.41)			(-.65)	(.32)	[2.4]
Quarterly	320	(I)	.0031	.0039	.0021			.343
			(.56)	(1.19)	(.56)			[6.0**]
		(II)	.0065	.0042	.0038	-.0037		.353
			(.95)	(1.20)	(.93)	(-1.03)		[4.3*]
		(III)	.0127	.0066	.0065	-.0129	.0024	.368
			(.91)	(1.27)	(1.13)	(-1.23)	(.12)	[2.5]
		(IV)	.0147					.202
			(1.73)					
		(V)	.0104			-.0032		.240
			(1.16)			(-.74)		
		(VI)	.0069			-.0151	.0131	.281
			(.42)			(-1.43)	(.62)	[2.4]
Semiannual	160	(I)	.0003	.0047	.0070			.335
			(.04)	(.83)	(1.23)			[5.5*]
		(II)	.0177	.0075	.0129	-.0110		.345
			(1.52)	(1.19)	(1.79)	(-1.82)		[4.1*]
		(III)	-.0180	-.0054	.0089	-.0188	.0418	.353
			(-.59)	(-.51)	(.72)	(-.82)	(.96)	[2.6]
		(IV)	.0092					.178
			(.85)					
		(V)	.0283*			-.0179*		.224
			(2.11)			(-2.55)		
		(VI)	-.0166			-.0581*	.0836*	.263
			(-.60)			(-2.46)	(2.19)	[2.4]

Table 3 (BTM-sorted) – co-moment coefficients

TABLE 3 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
BTM-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0007	.0015	.0019			.156
			(-.21)	(.90)	(1.23)			[2.5]
		(II)	-.0026	.0015	.0023	.0022		.163
			(-.59)	(.82)	(1.40)	(.88)		[2.3]
		(III)	.0024	.0015	.0006	.0083	-.0110	.178
			(.36)	(.70)	(.27)	(1.29)	(-1.26)	[2.3]
		(IV)	.0008					.048
			(.24)					
		(V)	-.0046			.0043		.081
			(-1.08)			(1.92)		
		(VI)	.0005			.0093	-.0114	.107
			(.07)			(1.40)	(-1.28)	[1.8]
Quarterly	164	(I)	-.0117	.0033	.0063			.171
			(-1.66)	(.88)	(1.50)			[3.1]
		(II)	-.0199*	.0000	.0014	.0061		.175
			(-2.18)	(.01)	(.30)	(1.13)		[2.7]
		(III)	-.0243	-.0019	.0049	.0156	-.0068	.187
			(-1.41)	(-.33)	(.74)	(1.22)	(-.30)	[2.6]
		(IV)	-.0076					.041
			(-.99)					
		(V)	-.0080			-.0042		.079
			(-.75)			(-.62)		
		(VI)	-.0192			.0065	-.0032	.118
			(-1.07)			(.45)	(-.13)	[2.1]
Semiannual	82	(I)	.0030	.0045	.0088			.163
			(.35)	(.86)	(1.19)			[3.3]
		(II)	-.0146	.0015	.0082	.0161		.165
			(-1.19)	(.26)	(.96)	(1.65)		[2.9]
		(III)	.0036	-.0010	.0115	.0688	-.0710	.171
			(.09)	(-.08)	(.74)	(1.95)	(-1.24)	[2.6]
		(IV)	.0013					.043
			(.11)					
		(V)	.0043			-.0007		.059
			(.30)			(-.07)		
		(VI)	.0371			.0529*	-.0844	.093
			(.98)			(2.22)	(-1.76)	[1.7]

Table 4 (Triple-sorted) – co-moment coefficients

TABLE 4 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Triple-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0046	.0023	.0017			.291
			(-1.74)	(1.61)	(1.16)			[7.5**]
		(II)	-.0052	.0017	.0025	.0008		.297
			(-1.47)	(1.09)	(1.73)	(.45)		[5.4**]
		(III)	-.0035	.0019	.0016	.0006	-.0017	.303
			(-.69)	(1.06)	(.88)	(.14)	(-.27)	[4.1*]
		(IV)	-.0000					.138
			(-.01)					
		(V)	-.0037			.0029		.194
			(-1.13)			(1.29)		
		(VI)	.0023			.0072	-.0114	.211
			(.44)			(1.29)	(-1.55)	[3.1]
Quarterly	164	(I)	-.0106	.0059	.0076			.297
			(-1.62)	(1.57)	(1.76)			[8.4**]
		(II)	-.0140	.0059	.0090	.0047		.302
			(-1.70)	(1.64)	(1.85)	(1.03)		[6.1**]
		(III)	-.0154	.0049	.0068	.0038	.0038	.308
			(-1.10)	(1.07)	(1.10)	(.54)	(.24)	[3.7*]
		(IV)	-.0041					.154
			(-.48)					
		(V)	-.0034			-.0034		.210
			(-.33)			(-.62)		
		(VI)	-.0148			-.0094	.0153	.231
			(-.95)			(-1.03)	(.89)	[3.2*]
Semiannual	82	(I)	-.0104	.0082	.0162			.275
			(-1.33)	(1.27)	(1.93)			[7.7**]
		(II)	-.0226	.0081	.0284*	.0114		.283
			(-1.72)	(1.20)	(2.37)	(1.38)		[6.0**]
		(III)	-.0047	.0213	.0155	.0140	-.0198	.293
			(-.18)	(1.94)	(.97)	(.55)	(-.47)	[4.0*]
		(IV)	-.0078					.127
			(-.68)					
		(V)	-.0067			-.0005		.149
			(-.48)			(-.08)		
		(VI)	-.0030			-.0324	.0277	.184
			(-.10)			(-.99)	(.56)	[2.6]

Table 5 (Industry-sorted) – co-moment coefficients

TABLE 5 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with Co-moment Expansions
Industry-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	Co-skewness γ	Co-kurtosis δ	Mean Adjusted R^2 [Joint Test]
Monthly	960	(I)	-.0045	-.0017	.0017			.260
			(-1.83)	(-1.13)	(1.22)			[2.7]
		(II)	-.0081*	-.0014	.0032*	.0046*		.292
			(-2.47)	(-.86)	(2.03)	(2.21)		[2.6]
		(III)	.0005	-.0017	-.0002	.0014	-.0041	.323
			(.07)	(-.67)	(-.08)	(.34)	(-.46)	[2.4]
		(IV)	-.0054*					.127
			(-2.22)					
		(V)	-.0065*			.0032		.174
			(-2.16)			(1.64)		
		(VI)	.0023			.0032	-.0086	.215
			(.32)			(.92)	(-1.06)	[1.9]
Quarterly	320	(I)	-.0026	-.0049	.0089*			.273
			(-.42)	(-1.24)	(2.20)			[2.9]
		(II)	-.0155	-.0091*	.0099*	.0145**		.295
			(-1.73)	(-2.08)	(2.12)	(2.69)		[2.7]
		(III)	-.0296	-.0173*	.0107	.0192	.0081	.324
			(-1.50)	(-2.05)	(1.36)	(1.38)	(.30)	[2.3]
		(IV)	-.0169**					.131
			(-2.67)					
		(V)	-.0178*			.0072		.184
			(-2.13)			(1.51)		
		(VI)	-.0418*			.0066	.0278	.233
			(-2.19)			(.56)	(1.13)	[2.0]
Semiannual	160	(I)	-.0115	-.0088	.0101			.287
			(-1.15)	(-1.13)	(1.55)			[2.9]
		(II)	-.0042	-.0084	.0114	-.0017		.316
			(-.29)	(-.94)	(1.29)	(-.21)		[2.7]
		(III)	-.0122	-.0162	-.0436*	-.0003	.0106	.361
			(-.25)	(-.86)	(-1.99)	(-.01)	(.13)	[2.5]
		(IV)	-.0220*					.119
			(-2.14)					
		(V)	-.0013			-.0111		.190
			(-.10)			(-1.67)		
		(VI)	-.0004			-.0067	-.0004	.260
			(-.01)			(-.20)	(-.01)	[2.4]

Table 6 (Size-sorted) – FMAX factor

TABLE 6 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Size-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	.0068	.0015	.0004		.291
			(1.87)	(.98)	(.18)		[5.2*]
		(II)	.0032	.0021	.0005	-.0025	.314
			(.91)	(1.31)	(.24)	(-.71)	[4.5*]
		(III)	.0069				.109
			(1.54)				
		(IV)	.0063			.0032	.211
			(1.72)			(.90)	
Quarterly	164	(I)	.0012	.0056	.0041		.366
			(.14)	(1.26)	(.68)		[6.0**]
		(II)	.0035	.0052	.0045	.0014	.390
			(.40)	(1.29)	(.81)	(.17)	[5.2*]
		(III)	.0184				.195
			(1.43)				
		(IV)	.0225*			.0148	.293
			(2.04)			(1.56)	
Semiannual	82	(I)	-.0041	.0061	.0116		.349
			(-.33)	(.82)	(1.40)		[5.2*]
		(II)	-.0067	.0059	.0125	-.0066	.361
			(-.51)	(.81)	(1.47)	(-.59)	[5.0*]
		(III)	.0115				.171
			(.74)				
		(IV)	.0089			.0071	.241
			(.64)			(.50)	

Table 7 (BTM-sorted) – FMAX factor

TABLE 7 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Book-To-Market-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0007	.0015	.0019		.156
			(-.21)	(.90)	(1.23)		[2.5]
		(II)	.0016	.0023	.0013	-.0001	.165
			(.47)	(1.39)	(.85)	(-.05)	[2.3]
		(III)	.0008				.048
			(.24)				
		(IV)	.0008			-.0025	.075
			(.21)			(-.91)	
Quarterly	164	(I)	-.0117	.0033	.0063		.171
			(-1.66)	(.88)	(1.50)		[3.1]
		(II)	-.0134	.0044	.0045	-.0039	.187
			(-1.88)	(1.16)	(1.11)	(-.66)	[3.0]
		(III)	-.0076				.041
			(-.99)				
		(IV)	-.0080			-.0138*	.083
			(-1.01)			(-2.47)	
Semiannual	82	(I)	.0030	.0045	.0088		.163
			(.35)	(.86)	(1.19)		[3.3]
		(II)	.0019	.0058	.0082	-.0040	.202
			(.22)	(1.11)	(1.15)	(-.46)	[3.0]
		(III)	.0013				.043
			(.11)				
		(IV)	-.0006			-.0113	.096
			(-.05)			(-1.19)	

Table 8 (Triple-sorted) – FMAX factor

TABLE 8 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Triple-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0046	.0023	.0017		.291
			(-1.74)	(1.61)	(1.16)		[7.5**]
		(II)	-.0035	.0024	.0020	-.0031	.302
			(-1.38)	(1.66)	(1.42)	(-1.23)	[7.1**]
		(III)	-.0000				.138
			(-.01)				
		(IV)	.0006			-.0030	.192
			(.19)			(-1.15)	
Quarterly	164	(I)	-.0106	.0059	.0076		.297
			(-1.62)	(1.57)	(1.76)		[8.4**]
		(II)	-.0094	.0050	.0091*	-.0116	.308
			(-1.45)	(1.38)	(2.18)	(-1.77)	[7.6**]
		(III)	-.0041				.154
			(-.48)				
		(IV)	.0016			-.0068	.208
			(.19)			(-1.01)	
Semiannual	82	(I)	-.0104	.0082	.0162		.275
			(-1.33)	(1.27)	(1.93)		[7.7**]
		(II)	-.0089	.0085	.0155	-.0170	.286
			(-1.13)	(1.32)	(1.86)	(-1.55)	[7.5**]
		(III)	-.0078				.127
			(-.68)				
		(IV)	-.0084			-.0168	.183
			(-.79)			(-1.52)	

Table 9 (Industry-sorted) – FMAX factor

TABLE 9 Fama-MacBeth Regression Results for the Fama-French Model and the CAPM with FMAX factor
Industry-Sorted Portfolios

Return Frequency	Periods	Equation	Market (R_M) β	SMB s	HML h	FMAX f	Mean Adjusted R^2 [Joint Test]
Monthly	492	(I)	-.0040	-.0006	.0013		.257
			(-1.03)	(-.23)	(.61)		[2.5]
		(II)	-.0006	-.0037	.0002	-.0025	.300
			(-.16)	(-1.34)	(.10)	(-.79)	[2.4]
		(III)	-.0045				.135
			(-1.18)				
		(IV)	-.0010			-.0050	.199
			(-.26)			(-1.62)	
Quarterly	164	(I)	.0090	-.0023	.0101		.267
			(.99)	(-.41)	(1.69)		[2.8]
		(II)	.0102	-.0079	.0009	.0003	.338
			(1.16)	(-1.45)	(.17)	(.04)	[2.3]
		(III)	-.0163				.133
			(-1.72)				
		(IV)	.0074			-.0043	.242
			(.83)			(-.61)	
Semiannual	82	(I)	-.0015	-.0052	.0180*		.314
			(-.12)	(-.43)	(2.10)		[3.0]
		(II)	.0052	-.0028	.0167	.0004	.373
			(.45)	(-.23)	(1.93)	(.03)	[2.6]
		(III)	-.0153				.127
			(-1.09)				
		(IV)	.0007			-.0113	.249
			(.06)			(-.90)	

9.3 Residuals

9.3.1 Formal tests of normality – Tables 2 to 5

Results of Formal Residual Normality Tests

Reference	Return Frequency	Equation	Kolmogorov-Smirnov statistic	Jarque-Bera statistic
Table 2	Monthly	(I)	.0552**	501764.72**
		(II)	.0499**	304878.22**
		(III)	.0491**	359755.06**
		(IV)	.0757**	22611100.**
		(V)	.0659**	38624472.**
		(VI)	.0588**	8461175.**
	Quarterly	(I)	.0625**	369095.16**
		(II)	.0623**	423181.72**
		(III)	.0561**	565174.19**
		(IV)	.0855**	1355909.75**
		(V)	.0808**	1118933.5**
		(VI)	.0726**	749133.94**
	Semiannual	(I)	.0539**	9200.12**
		(II)	.0538**	10539.33**
		(III)	.0557**	12889.24**
		(IV)	.0879**	86971.89**
		(V)	.0791**	65468.84**
		(VI)	.0754**	44073.79**
Table 3	Monthly	(I)	.0304**	2849.78**
		(II)	.0304**	2715.51**
		(III)	.0294**	2594.95**
		(IV)	.0405**	15059.31**
		(V)	.0361**	4894.14**
		(VI)	.0317**	4044.19**
	Quarterly	(I)	.0411**	2128.55**
		(II)	.0398**	1608.74**
		(III)	.0349**	1044.22**
		(IV)	.0515**	6017.06**
		(V)	.0426**	2736.97**
		(VI)	.0365**	1776.44**
	Semiannual	(I)	.0423**	697.97**
		(II)	.04**	539.75**
		(III)	.0346*	350.62**
		(IV)	.059**	1899.5**
		(V)	.0535**	1606.37**
		(VI)	.0516**	1519.79**
Table 4	Monthly	(I)	.047**	40807.21**
		(II)	.0464**	38732.32**
		(III)	.0461**	38637.22**
		(IV)	.057**	84636.44**
		(V)	.048**	41993.59**
		(VI)	.0491**	41797.67**
	Quarterly	(I)	.0486**	7388.54**
		(II)	.0441**	6475.**
		(III)	.0446**	5701.52**
		(IV)	.0548**	7171.82**
		(V)	.0521**	6547.45**
		(VI)	.0519**	6610.51**
	Semiannual	(I)	.0454**	1045.12**
		(II)	.0441**	622.84**
		(III)	.0426**	634.49**
		(IV)	.0626**	2126.41**
		(V)	.0575**	1871.56**
		(VI)	.0528**	1776.44**
Table 5	Monthly	(I)	.0469**	9445.62**
		(II)	.0418**	9335.8**
		(III)	.0415**	9335.38**
		(IV)	.0505**	7754.33**
		(V)	.0454**	7197.75**
		(VI)	.0405**	7034.5**
	Quarterly	(I)	.0374**	1820.4**
		(II)	.0389**	1746.14**
		(III)	.0397**	1083.96**
		(IV)	.0438**	1567.89**
		(V)	.0392**	1232.52**
		(VI)	.0372**	1135.92**
	Semiannual	(I)	.0402**	173.72**
		(II)	.0309*	158.56**
		(III)	.0272*	206.67**
		(IV)	.0381**	205.08**
		(V)	.0389**	144.25**
		(VI)	.0285*	179.22**

* Significance at the 5 % level
** Significance at the 1 % level

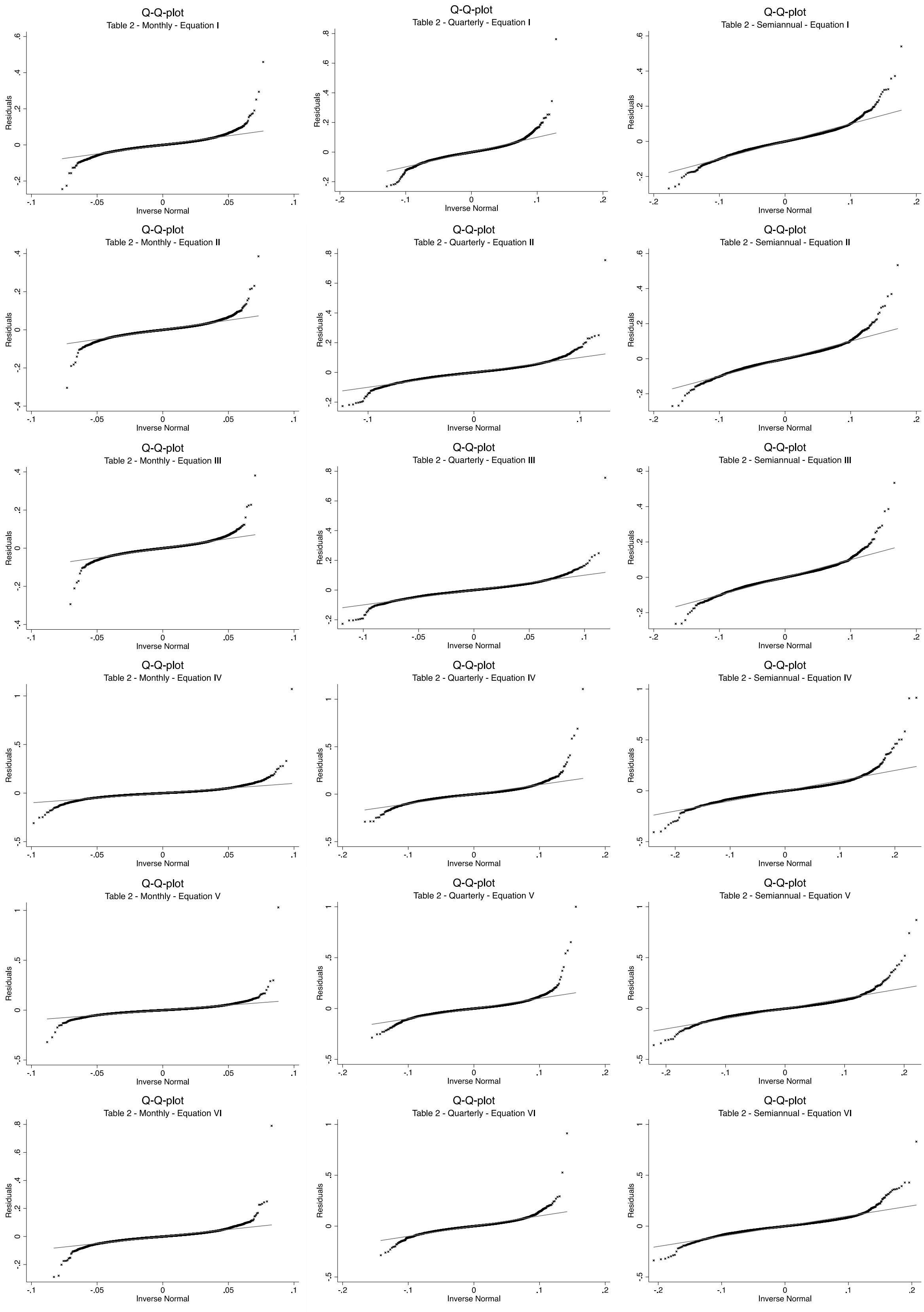
9.3.2 Formal tests of normality – Tables 6 - 9

Results of Formal Residual Normality Tests ***continued***

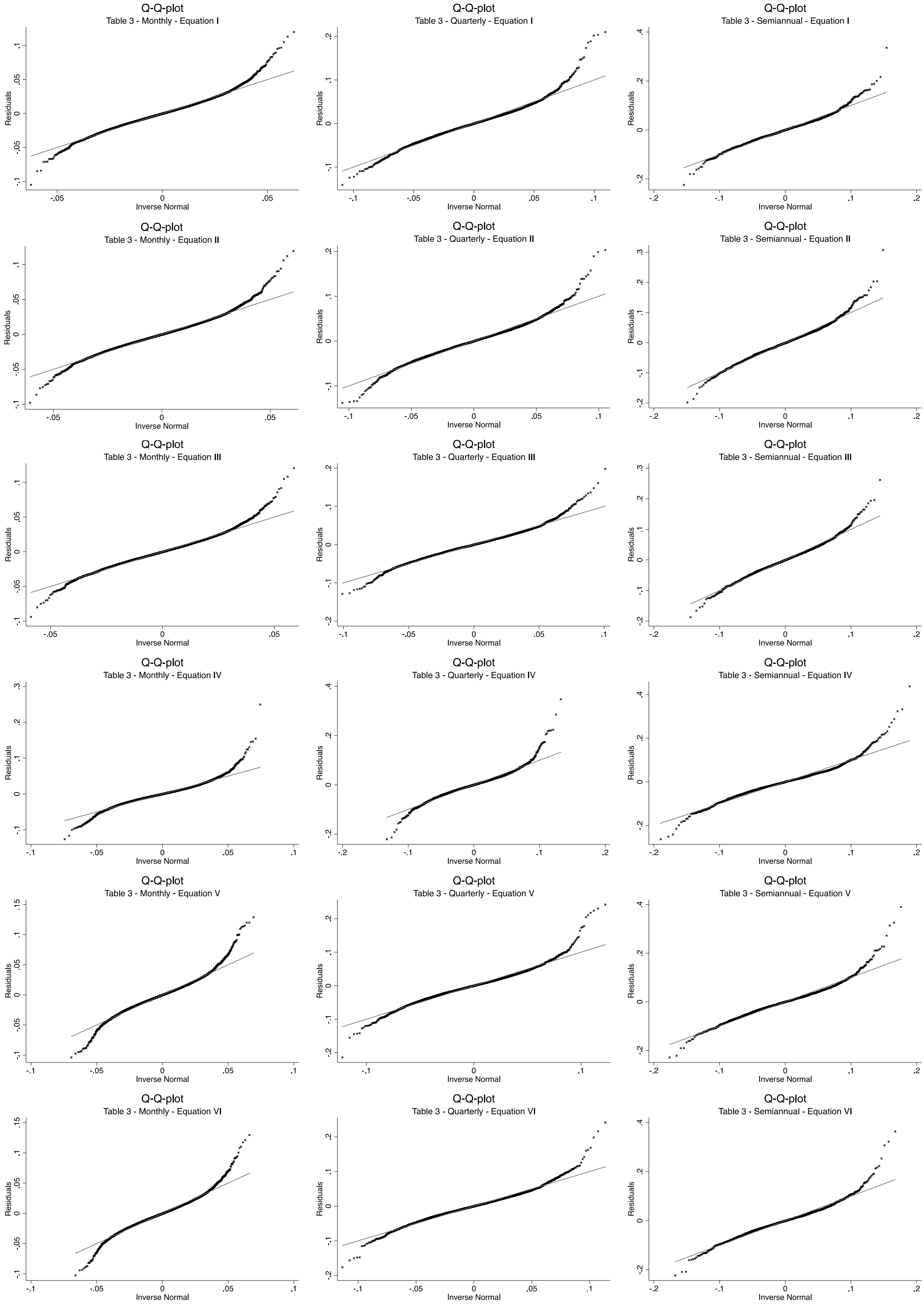
Reference	Return Frequency	Equation	Kolmogorov-Smirnov statistic	Jarque-Bera statistic
Table 6	Monthly	(I)	.0444**	33730.66**
		(II)	.0587**	35416.11**
		(III)	.0802**	287.1**
		(IV)	.0583**	42293.36**
	Quarterly	(I)	.0518**	6597.82**
		(II)	.1282**	81373.21**
		(III)	.0875	28.28**
		(IV)	.0574**	10320.22**
	Semiannual	(I)	.0394**	596.56**
		(II)	.1045**	9241.22**
		(III)	.1121	6.51
		(IV)	.058**	2020.02**
Table 7	Monthly	(I)	.0304**	2849.78**
		(II)	.0425**	11241.44**
		(III)	.0405**	15059.31**
		(IV)	.0368**	18753.03**
	Quarterly	(I)	.0411**	2128.55**
		(II)	.0623**	4692.13**
		(III)	.0515**	6017.06**
		(IV)	.0474**	13821.53**
	Semiannual	(I)	.0423**	697.97**
		(II)	.0614**	3948.9**
		(III)	.059**	1899.5**
		(IV)	.0523**	1304.65**
Table 8	Monthly	(I)	.047**	40807.21**
		(II)	.053**	41303.2**
		(III)	.057**	84636.44**
		(IV)	.0475**	56860.23**
	Quarterly	(I)	.0486**	7388.54**
		(II)	.0793**	39859.79**
		(III)	.0548**	7171.82**
		(IV)	.0549**	6928.55**
	Semiannual	(I)	.0454**	1045.12**
		(II)	.0703**	2760.17**
		(III)	.0626**	2126.41**
		(IV)	.0536**	1631.51**
Table 9	Monthly	(I)	.0439**	2825.29**
		(II)	.0593**	6345.67**
		(III)	.0459*	32.68**
		(IV)	.0383**	2255.**
	Quarterly	(I)	.0426**	928.83**
		(II)	.0707**	4658.24**
		(III)	.0586	1.61
		(IV)	.0331**	292.96**
	Semiannual	(I)	.0446*	107.05**
		(II)	.0678**	421.87**
		(III)	.1067	.89
		(IV)	.0479**	93.63**

* Significance at the 5 % level
** Significance at the 1 % level

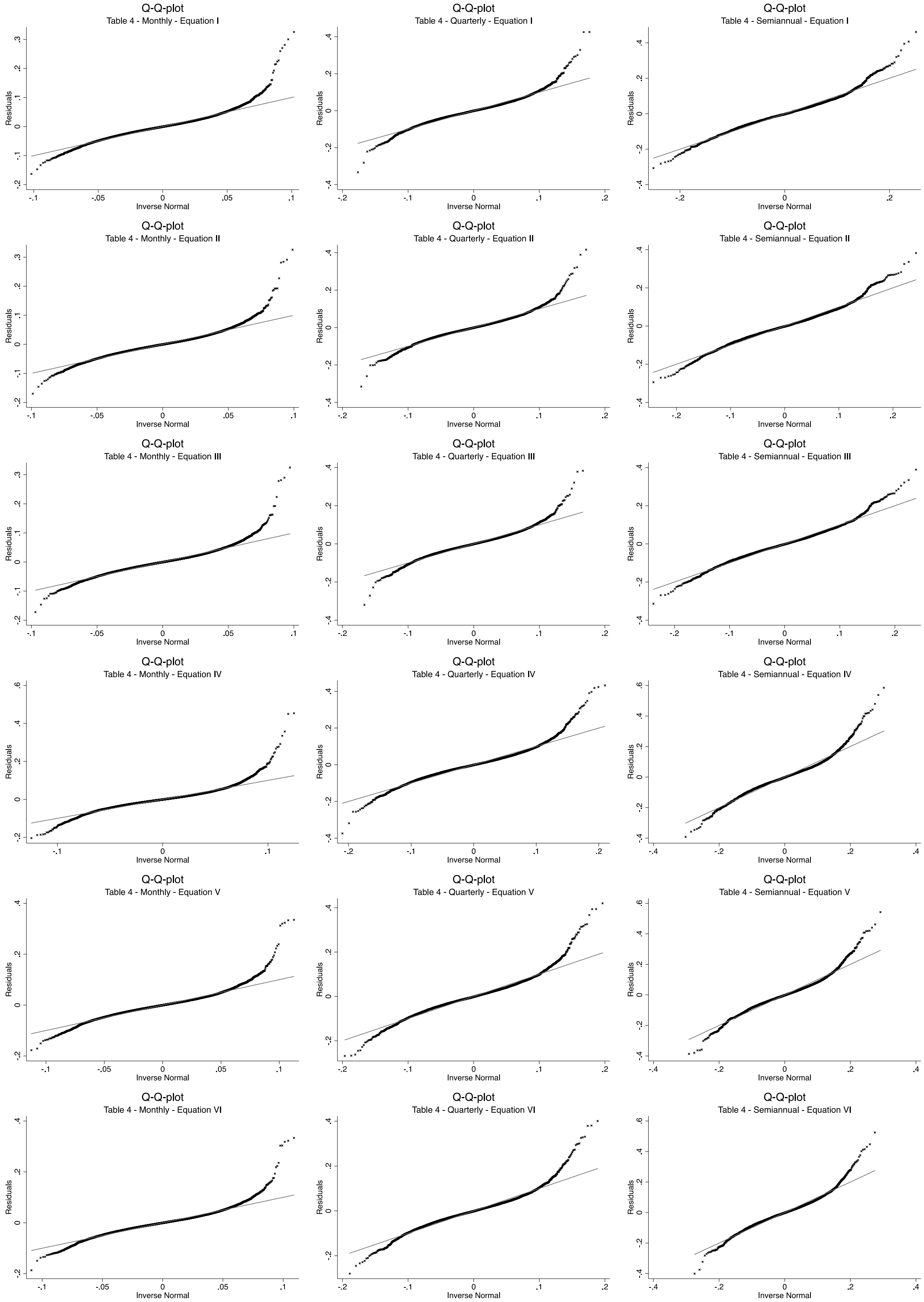
9.3.3 Table 2 (Size-sorted)



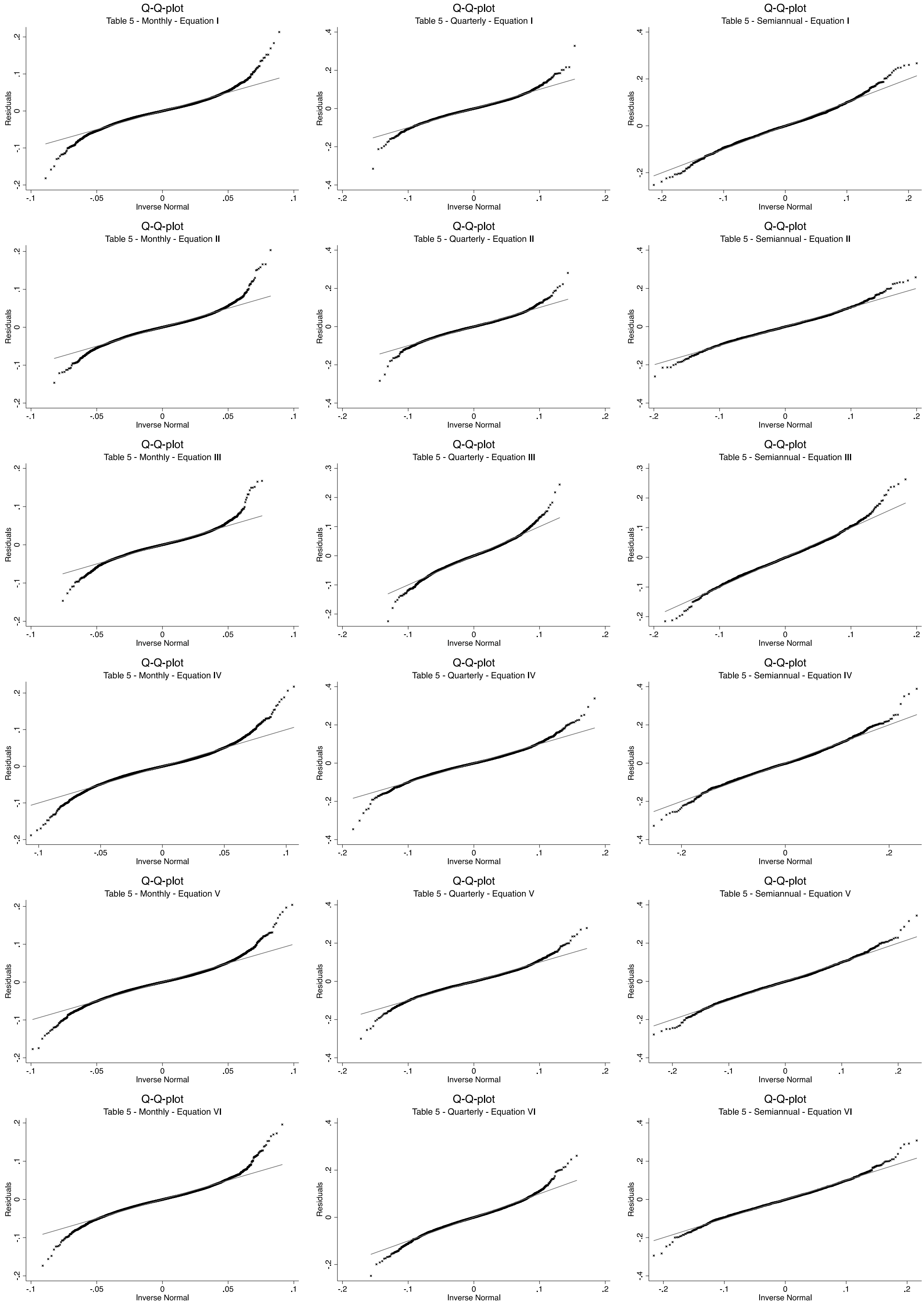
9.3.4 Table 3 (BTM-sorted)



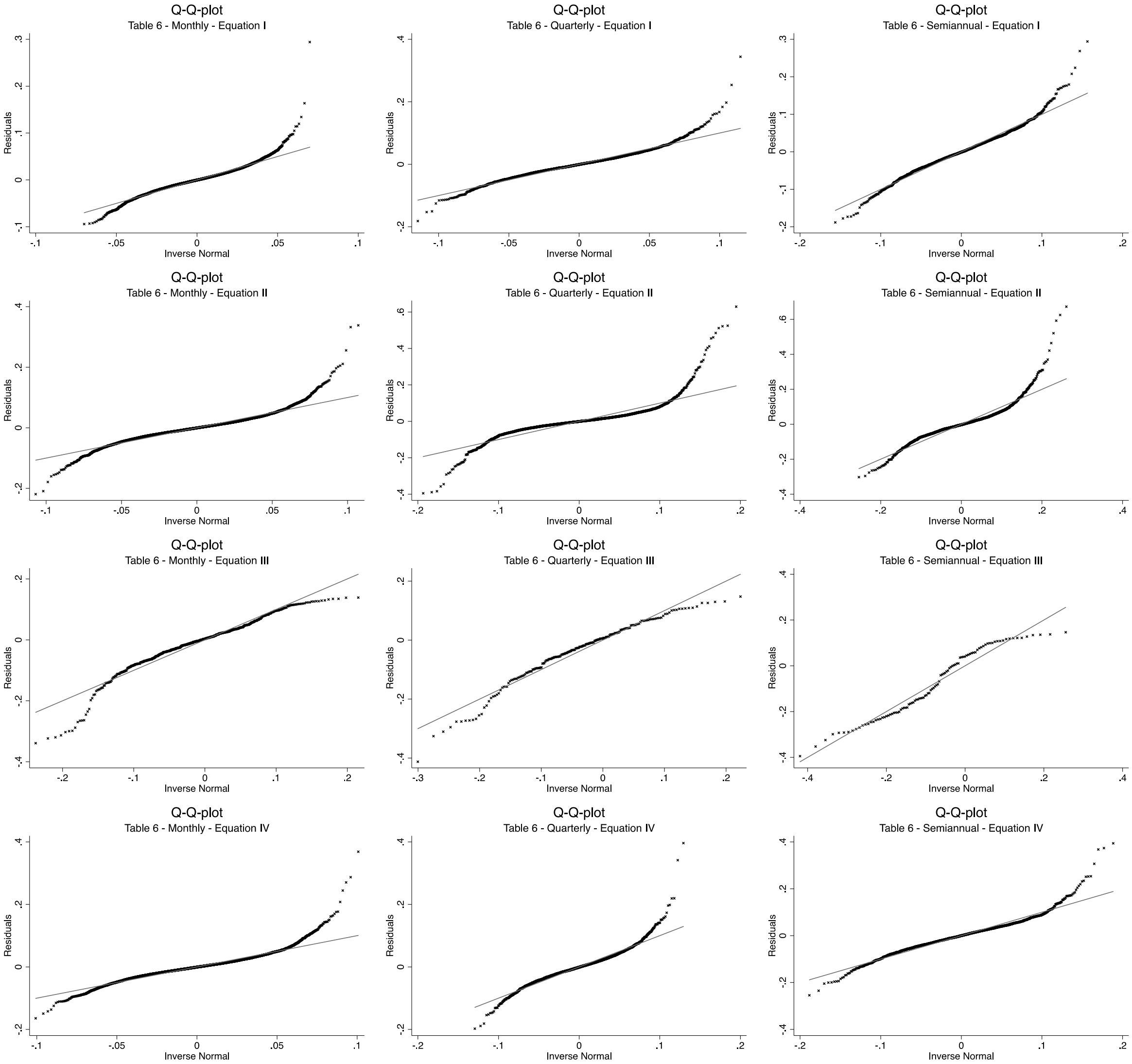
9.3.5 Table 4 (Triple-sorted)



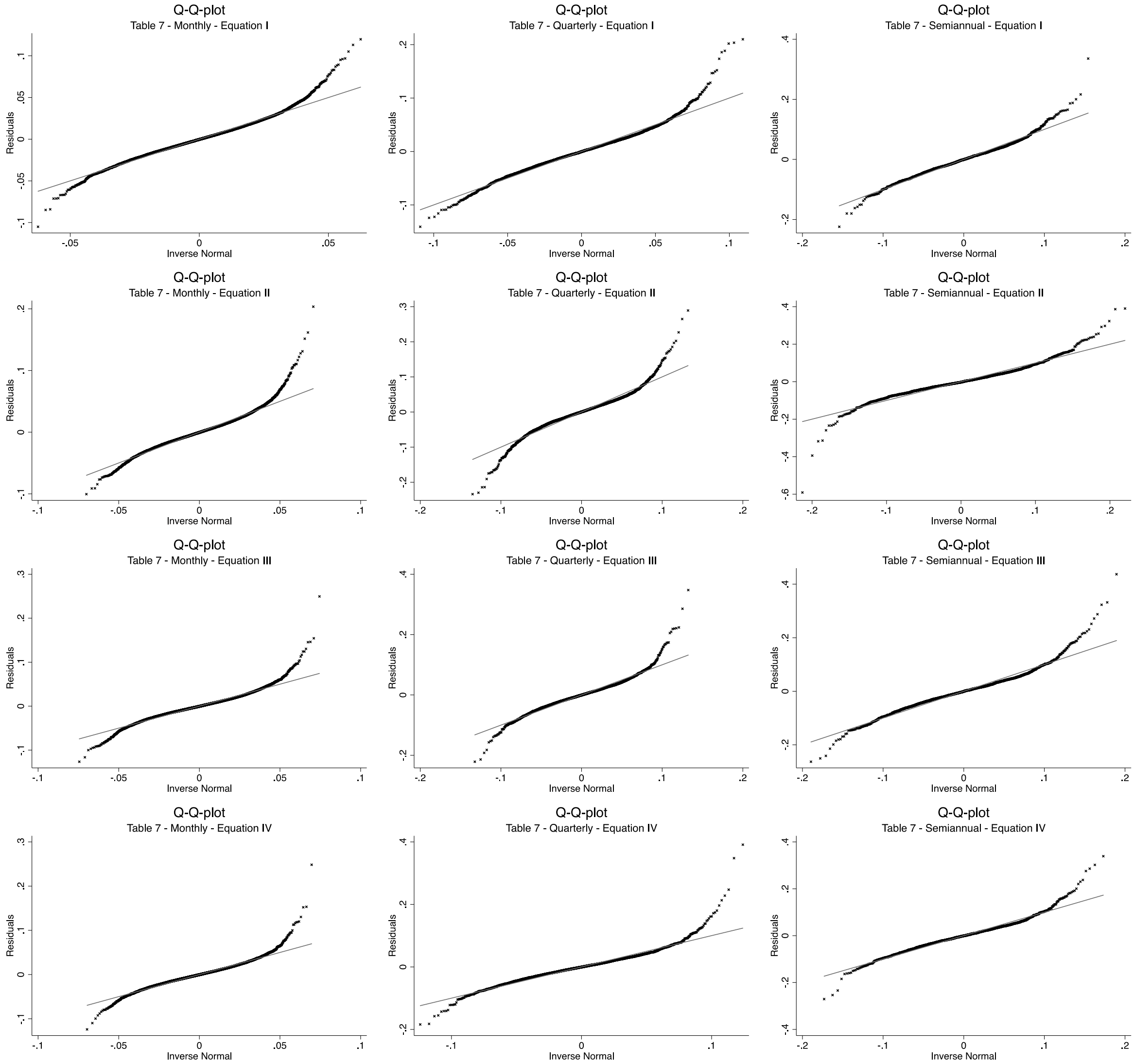
9.3.6 Table 5 (Industry-sorted)



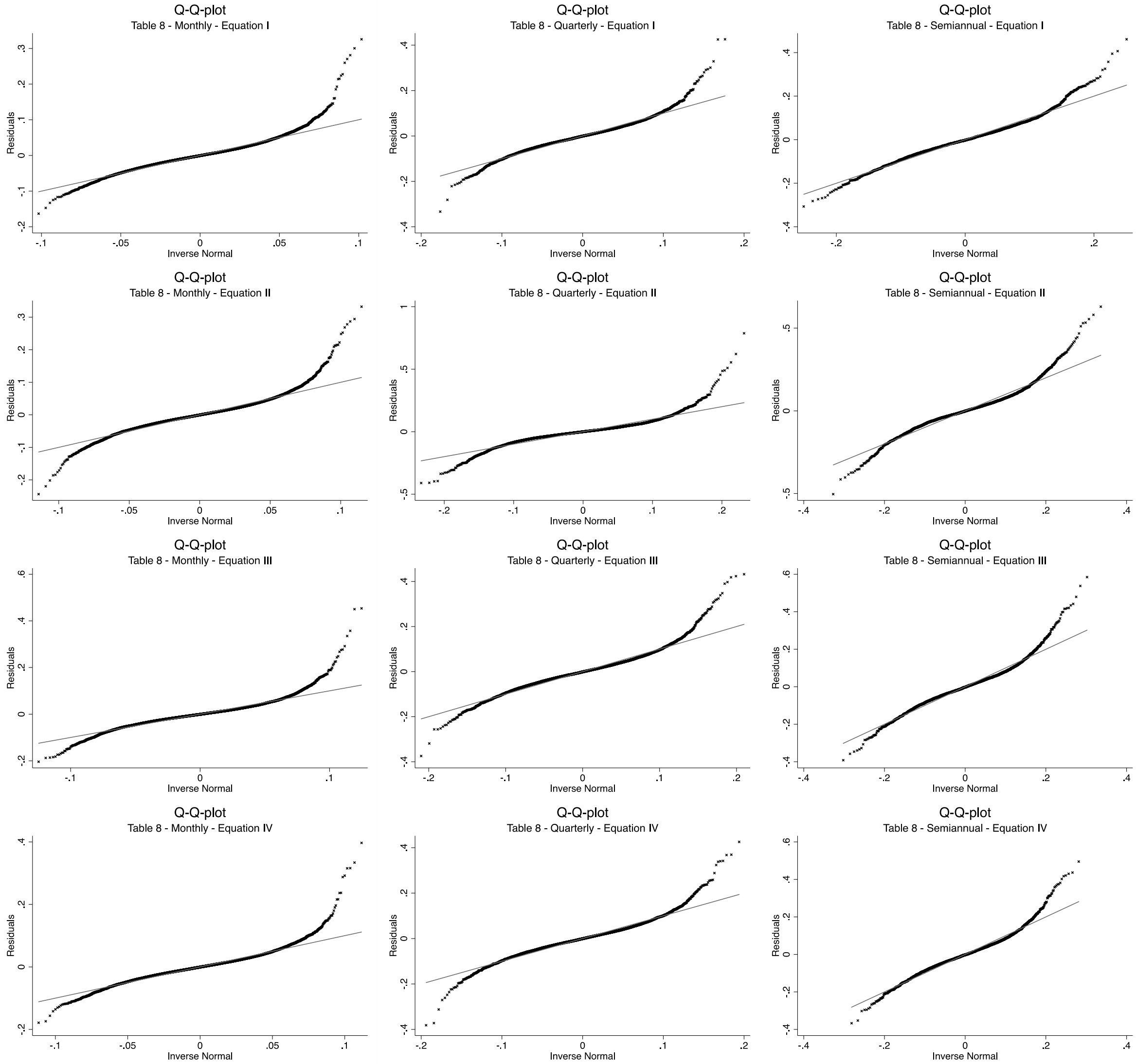
9.3.7 Table 6 (Size-sorted)



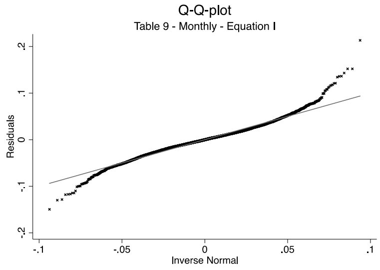
9.3.8 Table 7 (BTM-sorted)

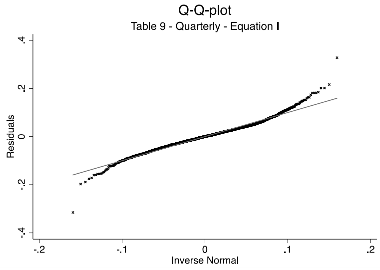


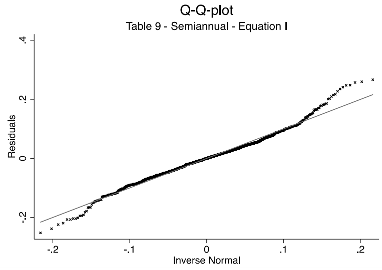
9.3.9 Table 8 (Triple-sorted)

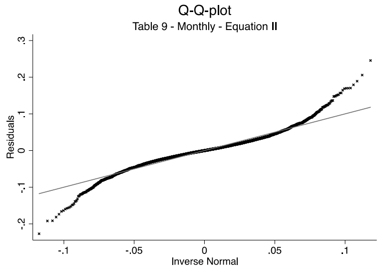


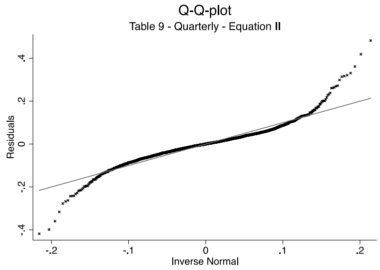
9.3.10 Table 9 (Industry-sorted)

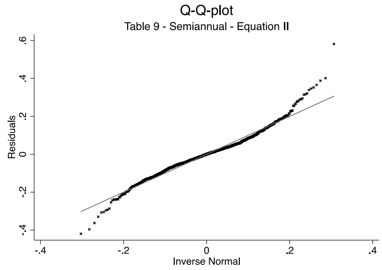


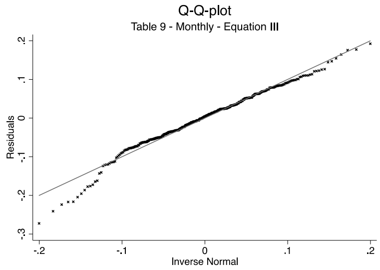


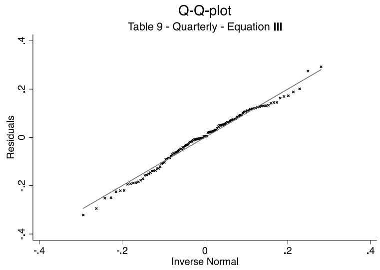


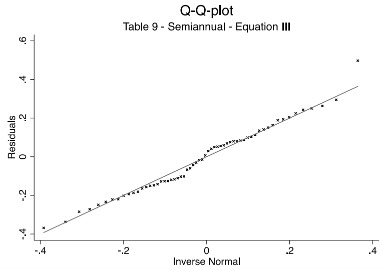


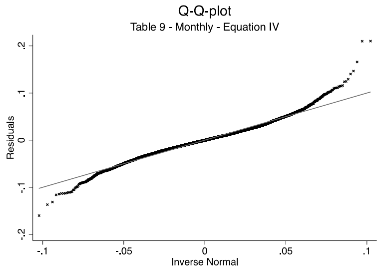


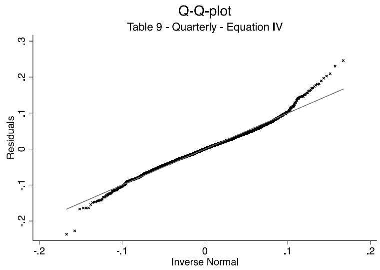


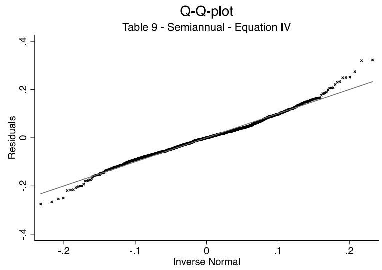












9.4 Normality of returns

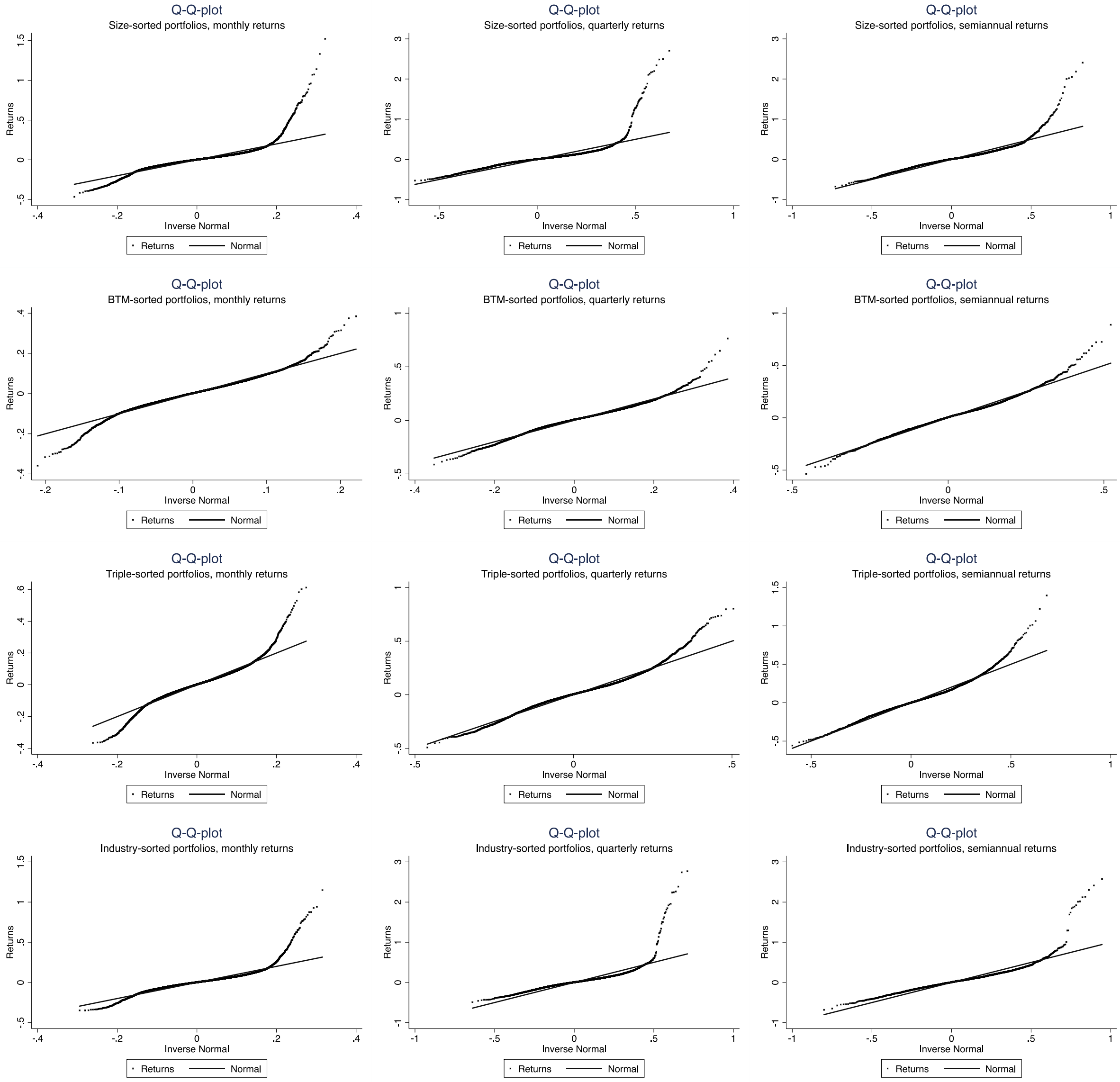
9.4.1 Simple returns – Value-weighted

Summary Statistics

TABLE 1 **Summary Statistics of Portfolio Returns**

	Monthly	Quarterly	Semiannually	Monthly	Quarterly	Semiannually
	A. Size-Sorted Portfolios			B. Book-To-Market-Ratio-Sorted Portfolios		
Number of port-folio-period observations	25,500	8,500	4,250	13,800	4,600	2,300
Mean	.0072	.0249	.0470	.0054	.0169	.0333
Variance	.0064	.0310	.0493	.0032	.0110	.0215
Skewness	1.9690	4.3380	1.9810	-.1870	.0978	.3830
Kurtosis	28.710	49.520	17.120	5.728	5.320	4.864
Jarque-Bera statistic	718,561 **	793,022 **	38,106 **	4,359 **	1,039 **	389 **
Kolmogorov statistic	.0862**	.1219**	.0864**	.0341**	.0466**	.0483**
	C. Triple-Sorted Portfolios			D. Industry-Sorted Portfolios		
Number of port-folio-period observations	24,840	8,280	4,140	15,300	5,100	2,550
Mean	.0067	.0218	.0424	.0104	.0360	.0722
Variance	.0046	.0173	.0333	.0064	.0362	.0676
Skewness	.1240	.3830	.7620	1.7610	4.5830	2.5760
Kurtosis	7.386	5.433	6.182	21.110	50.840	20.940
Jarque-Bera statistic	19,972 **	2,245 **	2,147 **	217,032 **	504,188 **	37,022 **
Kolmogorov statistic	.0440**	.0563**	.0617**	.0821**	.1193**	.0858**

Q-Q-plots


Q-Q-plot
Size-sorted portfolios, monthly returns
Q-Q-plot
Size-sorted portfolios, quarterly returns
Q-Q-plot
Size-sorted portfolios, semiannual returns
Q-Q-plot
BTM-sorted portfolios, monthly returns
Q-Q-plot
BTM-sorted portfolios, quarterly returns
Q-Q-plot
BTM-sorted portfolios, semiannual returns
Q-Q-plot
Triple-sorted portfolios, monthly returns
Q-Q-plot
Triple-sorted portfolios, quarterly returns
Q-Q-plot
Triple-sorted portfolios, semiannual returns
Q-Q-plot
Industry-sorted portfolios, monthly returns
Q-Q-plot
Industry-sorted portfolios, quarterly returns
Q-Q-plot
Industry-sorted portfolios, semiannual returns
Returns
Inverse Normal
Returns
Normal

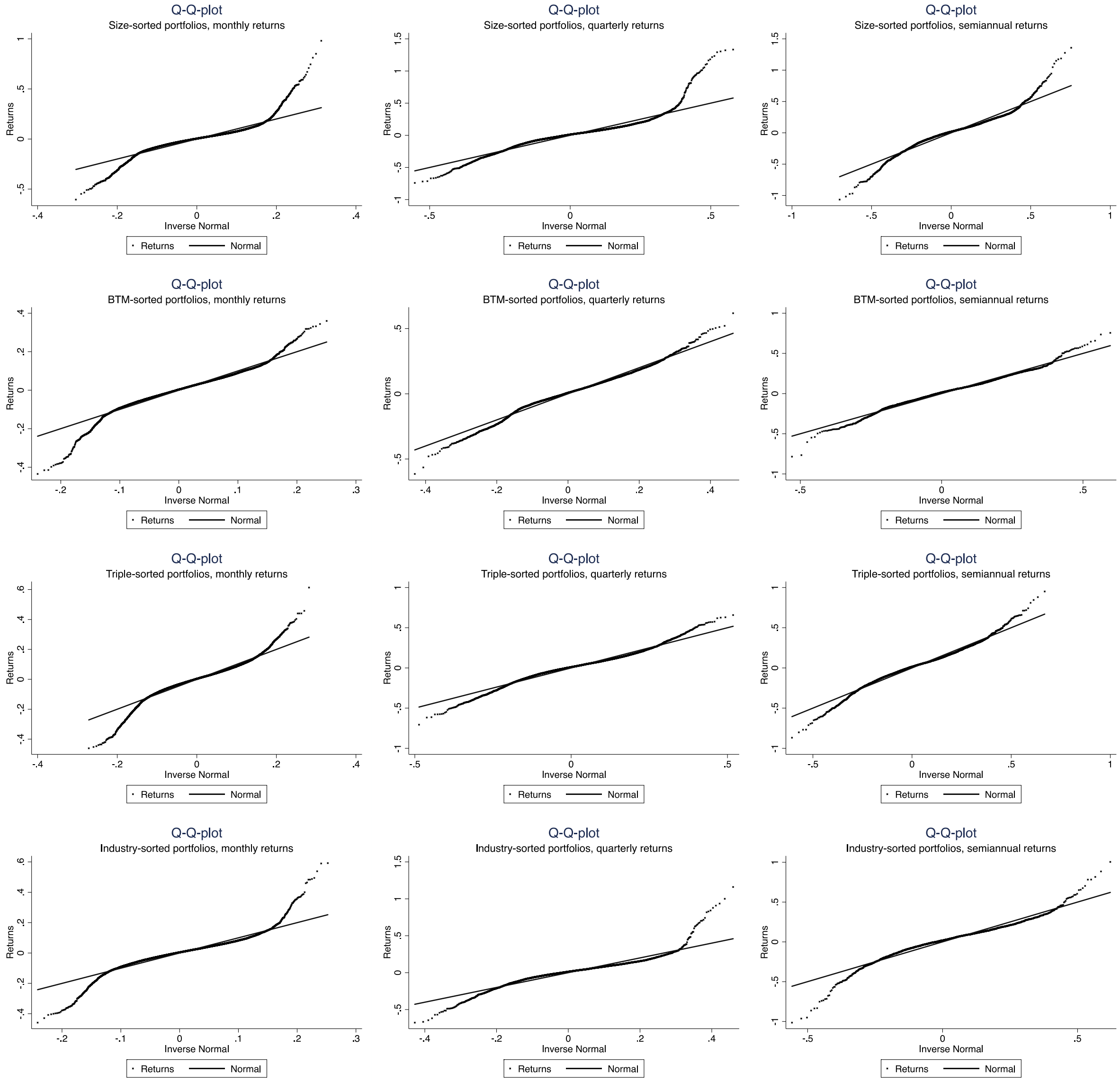
9.4.2 Log returns – Equal-weighted

Summary Statistics

TABLE 1 Summary Statistics of Portfolio Returns

	Monthly	Quarterly	Semiannually	Monthly	Quarterly	Semiannually
	A. Size-Sorted Portfolios			B. Book-To-Market-Ratio-Sorted Portfolios		
Number of port-folio-period observations	25,500	8,500	4,250	13,800	4,600	2,300
Mean	.0045	.0134	.0268	.0056	.0168	.0335
Variance	.0061	.0237	.0433	.0042	.0161	.0286
Skewness	.2450	.8410	-.0722	-.5320	-.2590	-.1820
Kurtosis	13.400	12.330	6.970	6.817	4.375	4.442
Jarque-Bera statistic	115,163 **	31,844 **	2,795 **	9,029 **	414 **	212 **
Kolmogorov statistic	.0767**	.0826**	.0701**	.0417**	.0545**	.0388**
	C. Triple-Sorted Portfolios			D. Industry-Sorted Portfolios		
Number of port-folio-period observations	24,840	8,280	4,140	15,300	5,100	2,550
Mean	.0053	.0159	.0317	.0053	.0158	.0317
Variance	.0049	.0187	.0333	.0042	.0157	.0308
Skewness	-.3590	-.2450	-.1830	-.2500	.2340	-.5840
Kurtosis	7.452	5.033	5.014	10.580	11.130	7.178
Jarque-Bera statistic	21,047 **	1,508 **	723 **	36,760 **	14,103 **	1,999 **
Kolmogorov statistic	.0555**	.0612**	.0478**	.0636**	.0818**	.0715**

Q-Q-plots


Q-Q-plot
Size-sorted portfolios, monthly returns
Q-Q-plot
Size-sorted portfolios, quarterly returns
Q-Q-plot
Size-sorted portfolios, semiannual returns
Q-Q-plot
BTM-sorted portfolios, monthly returns
Q-Q-plot
BTM-sorted portfolios, quarterly returns
Q-Q-plot
BTM-sorted portfolios, semiannual returns
Q-Q-plot
Triple-sorted portfolios, monthly returns
Q-Q-plot
Triple-sorted portfolios, quarterly returns
Q-Q-plot
Triple-sorted portfolios, semiannual returns
Q-Q-plot
Industry-sorted portfolios, monthly returns
Q-Q-plot
Industry-sorted portfolios, quarterly returns
Q-Q-plot
Industry-sorted portfolios, semiannual returns
Returns
Inverse Normal
Returns
Normal

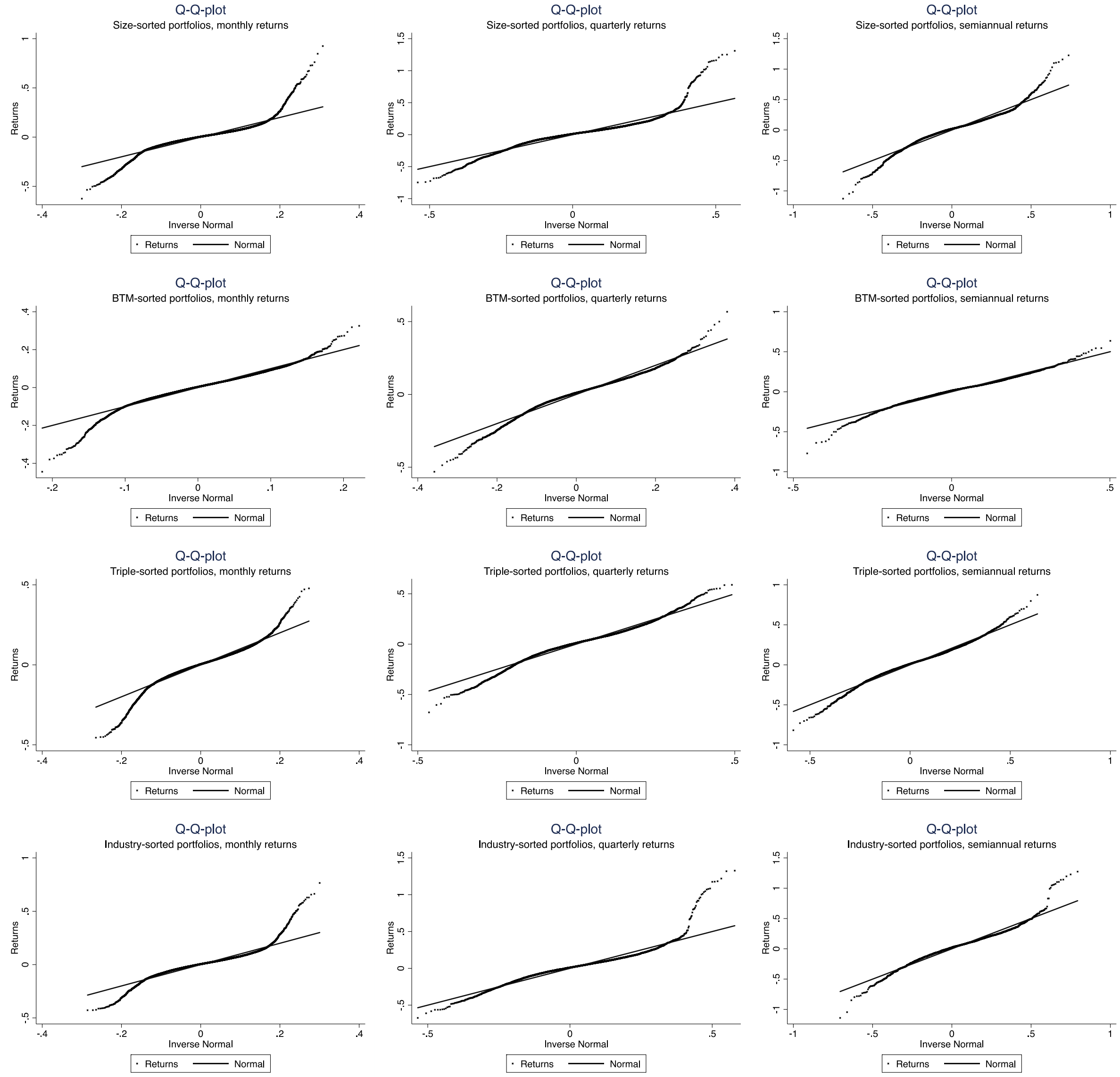
9.4.3 Log returns – Value-weighted

Summary Statistics

TABLE 1 Summary Statistics of Portfolio Returns

	Monthly	Quarterly	Semiannually	Monthly	Quarterly	Semiannually
	A. Size-Sorted Portfolios			B. Book-To-Market-Ratio-Sorted Portfolios		
Number of portfolio-period observations	25,500	8,500	4,250	13,800	4,600	2,300
Mean	.0042	.0126	.0252	.0038	.0113	.0226
Variance	.0059	.0227	.0414	.0033	.0110	.0207
Skewness	.1600	.7270	-.1820	-.6090	-.5090	-.3790
Kurtosis	13.540	12.150	6.719	6.564	4.978	4.801
Jarque-Bera statistic	118,134 **	30,423 **	2,472 **	8,157 **	948 **	366 **
Kolmogorov statistic	.0755**	.0847**	.0681**	.0418**	.0617**	.0469**
	C. Triple-Sorted Portfolios			D. Industry-Sorted Portfolios		
Number of portfolio-period observations	24,840	8,280	4,140	15,300	5,100	2,550
Mean	.0044	.0132	.0264	.0074	.0222	.0443
Variance	.0047	.0169	.0305	.0059	.0247	.0498
Skewness	-.4990	-.3750	-.2320	.3310	.9620	.0887
Kurtosis	7.384	4.934	4.693	11.560	11.840	6.540
Jarque-Bera statistic	20,918 **	1,485 **	532 **	46,956 **	17,375 **	1,335 **
Kolmogorov statistic	.0525**	.0619**	.0491**	.0710**	.0713**	.0543**

Q-Q-plots


Q-Q-plot
Size-sorted portfolios, monthly returns
Q-Q-plot
Size-sorted portfolios, quarterly returns
Q-Q-plot
Size-sorted portfolios, semiannual returns
Q-Q-plot
BTM-sorted portfolios, monthly returns
Q-Q-plot
BTM-sorted portfolios, quarterly returns
Q-Q-plot
BTM-sorted portfolios, semiannual returns
Q-Q-plot
Triple-sorted portfolios, monthly returns
Q-Q-plot
Triple-sorted portfolios, quarterly returns
Q-Q-plot
Triple-sorted portfolios, semiannual returns
Q-Q-plot
Industry-sorted portfolios, monthly returns
Q-Q-plot
Industry-sorted portfolios, quarterly returns
Q-Q-plot
Industry-sorted portfolios, semiannual returns
Returns
Inverse Normal
Returns
Normal