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Longevity Risk and the Implications for Norwegian Annuity Providers

A quantitative analysis of longevity risk in light of the Solvency II Standard
Formula and Internal Model

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Abstract

A significant part of the liabilities of Norwegian insurance companies and annuity providers consists of lifetime pension products. These pension products are subject to what is referred to as longevity risk, i.e. the risk that individuals who receive pension annuities live longer than anticipated. Insurance companies and annuity providers are required to hold capital buffers today to cover for the uncertainty in future cash flows. The calculation of the size of these buffers is regulated by a European-wide framework known as Solvency II. The Solvency II framework proposes two solutions for determining Solvency Capital Requirements (SCR); the Standard Formula and an Internal Model. For longevity risk, the Standard Formula assumes a 20% reduction in mortality rates, while the Internal Model is calculated based on the 99.5% Value-at-Risk on a one-year time horizon. In this thesis, we compare the SCRs calculated by the Standard Formula with the SCRs calculated by an Internal Model, based on mortality projections from the Lee-Carter and the Cairns-Blake-Dowd mortality models. We use a simplified pension product to quantify the difference in capital requirements for Norwegian annuity providers.

We find that the 20% reduction of mortality rates following the Standard Formula leads to higher SCRs than those based on the 99.5% VaR-approach using Internal Models. Furthermore, we find that the Lee-Carter model outperforms the CBD-model in terms of both describing historical Norwegian mortality rates, and in estimating lower SCRs. This implies that insurers and annuity providers should develop Internal Models based on a Lee-Carter model to minimize their SCRs. However, the implication is somewhat offset by the costly process of developing Internal Models. This means that approximations by the Standard Formula may be more expedient for smaller insurers. Larger annuity providers, on the other hand, may benefit from an Internal Model through both reduced capital requirements and a more detailed account of their risk exposure.

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Longevity Risk and the Implications for Norwegian Annuity Providers

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1 Introduction

A large number of countries experience significant changes in their population and might undergo further changes in decades to come. Fertility rates decline while life expectancy seems to continuously increase. This phenomenon is prevalent in western countries such as Norway, where the standard of living is high and a large portion of the population can afford to save for retirement. While the drastic increase in life expectancy is an incredible feat of modern society, it poses a challenge to governmental institutions as the tax-bearing population decreases, relative to the beneficiaries. Furthermore, public pension plans are unlikely to be sufficient to bear the load of an aging population. This means that the individual appeal and importance of other means of preparing for retirement, such as pension insurance, becomes increasingly important. Many of these private pension contracts depend on the random, and thus risky, life-expectancy of the annuity holder.

The uncertainty related to mortality development is known as *longevity risk*, i.e. the risk that individuals who receive pension payouts live longer than expected. Insurers and annuity providers might therefore pay pensions for longer periods than anticipated due to longevity risk. This master thesis considers some of the implications of longevity risks for Norwegian annuity providers in light of the Solvency II Directive implemented on January 1. 2016.

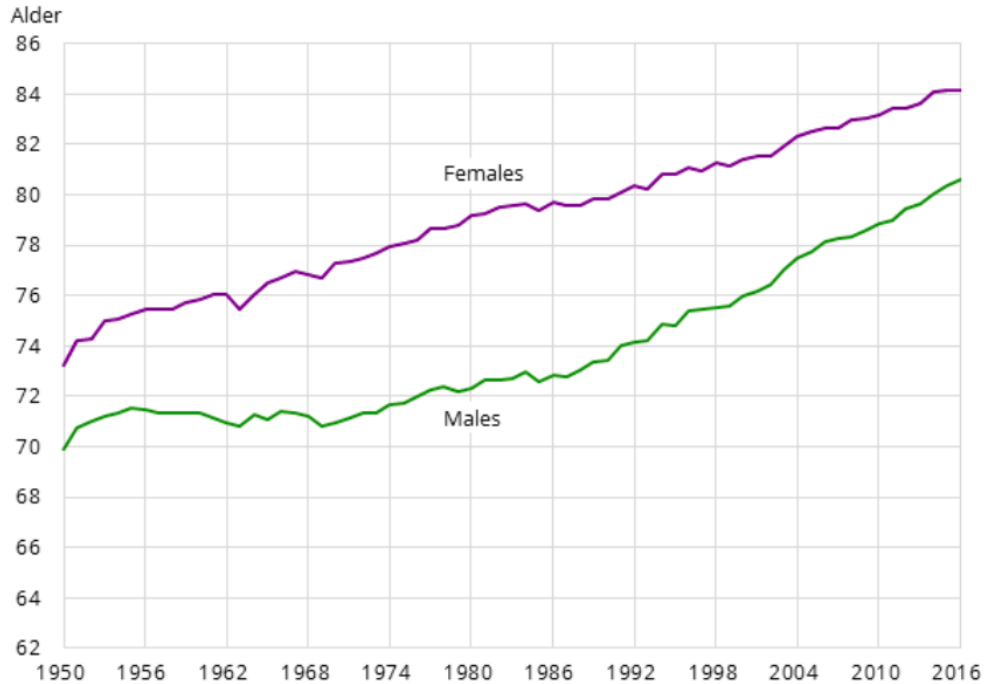


Figure 1: Life expectancy at birth for Norwegian males and females. Source: SSB (2017)

From figure 1 we see that life expectancy for both genders has increased dramatically over time. In the last 50 years, life expectancy at birth has increased with roughly 10 years for both genders (SSB, 2017). We observe that the mortality gap between genders is narrowing. It is not unlikely that the decrease in mortality development will continue as living standards improve.

However, future mortality developments are uncertain. This uncertainty is divided into mortality risk and longevity risk. Mortality risk refers to the short-term mortality risk. Events such as a disease outbreak or a cold winter that could increase mortality rates are examples of events which may incur short-term mortality risk. Longevity risk refers to the long-term mortality changes. In the long term, mortality developments are highly uncertain due to for instance new medical treatment, increased wealth, increased safety precautions, and reductions in carcinogenic habits such as smoking. One could argue that there is a biological constraint of life expectancy for humans, but how quickly the species reach the maximum life expectancy is uncertain.

For annuity providers, the longevity risk is essential because it implies uncertainty in future cash flows. This thesis defines annuity providers as companies that offer annuity products such as pension insurance. The cash flows are uncertain because the number of years with pension payouts depend on how many years the contract holder lives. To mitigate this risk,

annuity providers must hold buffers today that cover uncertainty in the contract holder's future life expectancy. The size of the buffer will thus increase if future mortality rates suddenly decline because the pension payouts will continue longer than anticipated.

The European Insurance and Occupational Pensions Authority (EIOPA) is a financial regulatory organ of the European Union. EIOPA replaced the Committee of European Insurance and Occupational Pension Supervisors (CEIOPS) in 2010. EIOPA is part of the European Union's System of Financial Supervision. In 2009, CEIOPS proposed the Solvency II Directive which is meant to harmonize EU insurance regulation and ensure that insurers hold enough capital to prevent insolvency. The framework consists of 11 submodules that cover a wide range of risks that insurers are exposed to. One of these is longevity risk. To maintain solvency among annuity providers, the Solvency II Directive introduces a capital buffer that all annuity providers must hold in case of unexpected risk development. This capital buffer is known as Solvency Capital Requirement (SCR). The SCR needs to be large enough so that a given insurer or annuity provider is able to cover all their liabilities one year in the future with 99,5% probability. This entails that it should only occur once every 200 years that they fail to uphold their commitments.

In order to determine the size of the SCRs for longevity risk, annuity providers may use either the Standard Formula or an Internal Model. The Standard Formula approach assumes a permanent reduction to all mortality rates across age, while the Internal Model calculates the Value-at-Risk based on a one-year time horizon. The principle behind the Internal Model approach differs from the stipulated reduction of mortality rates across all ages of 20% that the Standard Formula implies. The Standard Formula covers a range of risks by approximation (CEIOPS, 2009). Such approximations are not necessarily accurate compared to the individual annuity provider's risk profile, which is why EIOPA recommend annuity providers to develop internal models. Internal models are tailored to the individual insurance company, and may thus provide a more accurate risk profile.

However, the process of developing an Internal Model is quite costly and time-consuming. According to Jan Hagen with the Financial Supervisory Authority of Norway (Finanstilsynet), there are currently no Norwegian insurance companies who use Internal Models to assess longevity risk (Personal communication, 26.11.2018). This means that the Standard Formula of calculating the SCR for longevity risk is the only method applied in Norway, despite EIOPAs recommendations. The authorization of Internal Models is a complex process which requires extensive consideration from Norwegian financial authorities. As they have no previous experience on the matter, the process is complicated on their side as well.

Annuity providers are incentivized to use the approach that reduces their capital requirements. This would be beneficial as the capital can then be invested elsewhere. However, how much there is to gain from using either the Standard Formula or the Internal Model is uncertain. This thesis aims to compare the Solvency Capital Requirements given the Standard Formula's shock method with Internal Model's one-year Value-at-Risk approach, based on two stochastic mortality models. The two stochastic mortality models we use are the original Lee and Carter (1992)-model with some alterations, and the CBD-model proposed by Cairns et al. (2006).

We thus examine the following two objectives in this thesis:

- Given Norwegian population data, how does the Solvency II Standard Formula compare to an Internal Model in terms of Solvency Capital Requirements?
- Given two stochastic mortality models, how are Solvency Capital Requirements affected by age and gender, and how do the stochastic mortality models compare to one another?

We find that the Standard Formula produces higher SCRs compared to the Internal Model approach for both genders and all ages given Norwegian population data. We find that the exact size of the SCR depends on the age-composition of the portfolios, and that the difference between the Standard Formula and Internal Model increase with higher ages. Furthermore, we find that the Lee-Carter model returns lower SCRs than the CBD-model, for both males and females across all ages. Thus, we conclude that the assumed reduction in mortality rates of 20% across all ages is too prudent and that annuity providers who rationally seek to minimize the size of their SCR should implement an Internal Model. However, the process is costly and time-consuming which partially explains why there are currently no Norwegian insurers or annuity providers with approved internal models.

The thesis is structured in sections where section 2 elaborates on Solvency II and how the capital requirements are determined. Section 3 presents relevant terms related to mortality modeling and reviews relevant literature on the Standard Formula and the Internal Models. The data set is described in section 4, followed by a more detailed account of the Lee-Carter and CBD-model and how they are used to estimate future mortality rates in section 5 and 6. Next, in section 7 we present the simplified pension product used to analyze SCRs between models and portfolios. Section 8 describes Standard Formula approach and the required steps on how to extract the 99,5% value-at-risk estimates. In section 9 we analyze the two approaches using the simplified pension product. Lastly, we conclude our thesis and make recommendations for further research in section 10.

2 Solvency II & Capital Requirements

This section covers the technical provisions of the Solvency II framework and the implications for determining Solvency Capital Requirements. The section elaborates on the two approaches for determining SCRs; the Standard Formula and the Internal Model based on a one-year time horizon.

2.1 Solvency II & Longevity Risk

Annuity providers and insurers are exposed to a wide range of risks such as credit risk, liquidity risk, operational risk, investment risk, mortality risk, and longevity risk. All these risks that insurers take on will have the potential to affect their solvency and financial standing. An insurance company is considered solvent if it has the required capital reserves to cover its liabilities and an additional buffer to cover against risk. EIOPA proposed the Solvency II Directive as a means to harmonize EU insurance regulation, improve customer protection and to offer more realistic modeling and assessment of the various risks that insurance companies are exposed to (Börger, 2010). One particular aspect of Solvency II that holds implications for insurance companies and annuity providers is the life expectancy of annuity holders.

Pre-Solvency II, insurers' capital requirements were determined as a fixed percentage of the mathematical capital reserve, the risk capital. To mitigate the risks that follow increased life expectancy, the Solvency II Directive now determines a risk-based Solvency Capital Requirement as the 99.5% Value-at-Risk of the available capital on a one-year time horizon. This means that insurance companies are required to hold enough capital to cover any losses that might occur over the next year with 99.5% probability.

Risks related to life expectancy and longevity are likely to become increasingly important as the general life expectancy increases, and more so as the longevity of the insured increase. Systematic mortality risk is the risk that mortality rates evolve differently than anticipated (Cairns et al., 2006). Because there is no liquid market where longevity risks can be hedged, nor diversified away, longevity represents a systematic risk.

As the development of Internal Models is sophisticated and costly, CEIOPS determined a scenario-based model which insurers may use to approximate their capital requirements (Börger, 2010). The Standard Formula splits the total risk into several sub-modules for which the individual SCRs are computed (Börger, 2010). Insurance companies are encouraged to

develop internal stochastic models to more accurately assess their risks and subsequent capital requirements. In the long term, one expects mostly small and medium-sized companies to apply the standard model while larger insurers and annuity providers are expected to develop Internal Models. Hence, a reasonable setup and calibration of the Standard Formula is paramount to ensure financial stability of the European insurance market (Börger, 2010)

2.2 Technical Provisions

The Technical Provisions consists of the Best Estimate of Liabilities (BEL) and a Risk Margin (RM) which are used to approximate market value of liabilities if one could sell them in an open market. BEL is the expected market value of an insurers or annuity providers liabilities when accounting for all relevant information from financial markets. The BEL is discounted using the risk-free rate. The Risk Margin is a loading for non-hedgeable risk which “. . . shall cover the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof.” (CEIOPS, 2009). Together with the Best Estimate, they should be “equivalent to the amount insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations” (CEIOPS, 2014). This means that the Risk Margin needs to be large enough for another insurance company to ensure proper closing of the existing portfolio of contracts in case of insolvency.

At time t one may define BEL_t as:

$$BEL_t = \sum_{T \geq 1} \frac{\mathbb{E}_t^{\mathbb{P}}[\hat{C}F_t]}{(1 + i_t)^t} \quad (1)$$

where $\mathbb{E}_t^{\mathbb{P}}$ denotes the expectation under \mathbb{P} given all available information at time t . $\mathbb{E}_t^{\mathbb{P}}[\hat{C}F_t]$ thus represents the estimated value of premiums and payments at every contract year at time-step $t = 1, \dots, T$, while i_t is the risk-free interest rate at time t . In the case of longevity risk, mortality rates affect annuity providers’ calculations of BEL through the expected value of cash flows. In a scenario with lower than expected mortality rates, the expected payout period will be longer because the contract holder will have a higher probability of surviving longer, thus increasing the size of the BEL.

The Risk Margin is computed via the Cost-of-Capital approach (CoC) which means that it should cover the required return in excess of the risk-free rate on assets backing future

Solvency Capital Requirements (Börger, 2010). EIOPA (2014), determines the RM to be:

$$RM = \sum_{t \geq 0} \times \frac{CoC \times SCR_t}{(1 + i_{t+1})^{t+1}} \quad (2)$$

CoC is the cost of capital and SCR_t is the Solvency Capital Requirements at time t . i_t is the annual risk-free rate at time zero, for maturity t .

As previously described, there are two methods of calculating the SCRs, either using the Standard Formula or an Internal Model. For longevity risk, the Standard Formula's SCR depends on the BEL given a 20% shock-reduction in mortality rates. The BEL without the shock is then subtracted, which gives us:

$$SCR^{StandardFormula} = (BEL_0 | longevity\ shock) - BEL_0 \quad (3)$$

The SCR for the Internal Model is determined empirically by sample paths based on the stochastic mortality models. Here, the one-year time horizon is simulated, and the 99.5% worst case scenario is added back to the data set. The projected mortality estimates from $t = 1$ will then estimate how a 1 in 200-year event may change the mortality predictions set at $t = 0$. The SCR for the Internal Model is given by:

$$SCR^{InternalModel} = \frac{BEL_1 - CF_1}{1 + i_{(0,1)}} - BEL_0 \quad (4)$$

The BEL_1 and the CF_1 represent the Best Estimate of Liabilities and cash flows from premiums and payouts at $t = 1$ given a 99.5% worst case scenario. BEL_0 denotes the Best Estimate of Liabilities given central mortality projections at $t = 0$, and $i_{(0,1)}$ the interest rate from $t = 0$ to $t = 1$.

3 Mortality Modeling

To determine SCRs related to longevity risk, one must determine the likelihood that a given annuity holder is still alive at time t . This section covers relevant terms related to mortality probabilities followed by a description of relevant methods to forecast mortality rates. Next, we discuss different mortality models and which of these are applicable to the Norwegian population.

3.1 Mortality Probabilities

When analyzing mortality in light of Solvency II and longevity risk, the probability that an individual will survive t additional years is necessary to determine the expected cash flows, and thus determine the SCR. The one-year death probability, or initial mortality rate, is described by $q_{x,t}^{(g)}$ which represents the probability that a person of gender g and age x at time t will die during year t . The probability of a person surviving a year $p_{x,t}^{(g)}$ is thus given by:

$$p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)} \tag{5}$$

The probability that a person survives this year and the next year, can thus be calculated by multiplying the probability that he survives this year, by the probability that he survives the next. Mathematically, this scenario can be written as $p_{x,0}^{(g)} * p_{x,1}^{(g)}$.

To calculate estimated cash flows, it is necessary to determine the probability that a contract holder is still alive at time t . As described by Kaplan and Meier (1958), this can be expressed through the cumulative survival probability, $S_{x,t}^{(g)}$, determined by:

$$S_{x,t}^{(g)} = \prod_{t_0 \leq t} p_{x,t}^{(g)} \tag{6}$$

Here, the cumulative survival probability, $S_{x,t}^{(g)}$, equals the product of all yearly survival probabilities, $p_{x,t}^{(g)}$. $S_{x,t}^{(g)}$ represents a decreasing vector as time and age increase. The decrease will be steeper for higher ages as the mortality rates are higher for individuals who near a biological constraint.

3.2 Mortality Projections

To generate mortality projections Booth and Tickle (2008) discuss the three general approaches to mortality forecasting. They are Expectation, Extrapolation, and Explanation. In practice, the distinctions between these approaches are often not clear-cut, but we present them in their general form.

Expectations is a subjective method of mortality modeling which to a large degree relies on expert opinion. Expert opinions give grounds for assuming high and low scenarios where a value is assumed for a future date with a specified path (Pollard, 1987). Otherwise, expert opinion is used to adjust trends in age or cause-of-death specific trends (Waldron, 2005). For US population data, expectation modeling has been found to forecast smaller reductions in mortality than extrapolative methods, and observed mortality.

Extrapolation modeling in mortality forecasting assumes that future trends will be a continuation of past trends. This is usually a sound assumption in mortality forecasting, although there are exceptions where mortality rates have increased in certain age groups such as during the AIDS-epidemic and wars. Extrapolative methods range from simple extrapolation based on two factors, age-period, and age-cohort, to three-factor modeling including age, period and cohort (APC). Cohort effects are “period effects that are differentially experienced through age-specific exposures or susceptibility to that event” (Keyes et al., 2010). These processes are independent of aging. This could, for instance, be age groups that were greatly affected by disease or epidemics in a given time period. The age effect is a change in variable values which occurs for all cohorts regardless of period (Blanchard et al., 1977). Period effects are time-specific events that affect the population regardless of age and cohort. Time series methods are prevalent in extrapolative forecasting. Time series modeling is stochastic, allowing for calculation of a probabilistic prediction interval (Booth and Tickle, 2008).

Explanation modeling in mortality forecasting relies on structural or causal epidemiological causes of death, such as known risk factors and disease (Booth and Tickle, 2008). These models often incorporate medical knowledge, behavioral information, and environmental change to predict mortality. One example is the relationship between GDP per capita and mortality (Preston, 2007), where lower GDP per capita is associated with higher mortality rates.

All three general approaches are prevalent in earlier literature, but the current general academic consensus is that extrapolative approaches are the most expedient for long-term forecasting (Booth and Tickle, 2008). For this reason, extrapolative methods will be the focus of this master thesis. However, medical advancements, for instance, may invalidate long-term

forecasts as they may affect time trends of mortality.

3.3 Stochastic Mortality Modeling

Stochastic mortality models are designed with different features in mind, and it is not given that a model will have an equally good fit for different populations. Thus, we discuss properties of a selection of stochastic mortality models to determine which may be suitable for an analysis of longevity risk for the Norwegian population.

3.3.1 Mortality Models

Stochastic mortality models extrapolate either the central rate of mortality or the initial mortality rate (Plat, 2009). The central mortality is given by the number of deaths divided by the associated exposure and can be described as:

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}} = \frac{\text{Number of deaths during calendar year } t \text{ aged } x}{\text{Average population during calendar year } t \text{ aged } x} \quad (7)$$

Recall that the initial mortality rate $q_{x,t}$ is the probability that a person at time t , age x dies within the next year. The link between the central mortality rate and the initial mortality rate is given by approximation:

$$q_{x,t} \approx 1 - e^{-m_{x,t}} \quad (8)$$

The perhaps most well-known stochastic mortality model which relies on central mortality rates is the Lee-Carter model (1992):

$$\ln(m_{x,t}) = \alpha_x + \beta_x k_t \quad (9)$$

Here, α_x represents overall changes in mortality for age x . β_x captures age-specific developments, while k_t represents the time effect. Certain disadvantages of the single-factor model have been discussed in academic literature such as in Cairns et al. (2009). They argue that because it is a one-factor model, it results in perfect correlation between mortality improvements for all ages. Furthermore, the model may give a poor fit to historical data if there are

cohort effects in the data set. However, it is parsimonious with a proven track record and is one of the most used mortality models in Europe to assess longevity risk. (EIOPA, 2017).

There is a wide range of academic literature that adds to the original Lee-Carter model such as Lee and Miller (2001), Brouhns et al. (2002), Booth et al. (2002), Renshaw and Haberman (2003), and De Jong and Tickle (2006). These adaptations of the original model often try to solve one or more of the aforementioned issues with the model, but they are unable to solve all the issues (Plat, 2009).

When fitting mortality models for large age ranges, one may observe cohort effects as humps in the mortality projections over time (Plat, 2009). Cairns et al. (2009) observe that the fitted cohort effects for Wales, England and the US seem to have a trend in the year of birth. According to Plat (2009), this may suggest that the cohort effect compensates for a lack of a second age-period effect while trying to capture the cohort-effect in the data. Cairns et al. (2006) developed the Cairns-Blake-Dowd (CBD) model which because of multiple factors obtain a non-trivial correlation structure while maintaining a relatively parsimonious model structure. Furthermore, the model is particularly applicable for higher age groups, meaning that it is pertinent to assess longevity risk in pension products.

Plat (2011) incorporated the beneficial elements from several of these proposed models. His proposed model accounts for cohort-effects and it has enough stochastic factors so that it has a non-trivial correlation structure. Furthermore, the model is suitable for full age-ranges as it incorporates the same α_x term as the original Lee-Carter model. Lastly, Plats model follows the Currie (2011) structure so that it is robust. The Plat (2011) model quantifies the central mortality rate $m_{x,t}$ as:

$$\ln(m_{x,t}) = \alpha_x + k_t^1 + k_t^2(\bar{x} - x) + k_t^3(\bar{x} - x)^+ + \gamma_{t-x} \quad (10)$$

Where $(\bar{x} - x)^+ = \max(\bar{x} - x, 0)$. The model has four stochastic factors but maintains a relatively simple structure. The α_x term, is similar to that of the original Lee-Carter model as it ensures that the shape of the mortality curve for ages is true to the historical data. The k_t factors and the γ_{t-x} factor can be fitted using ARIMA processes. ARIMA processes are described in section 5.1.3. The model accounts for cohort effects and it is a robust model although it does have issues with lack of smoothing. Plat (2011) propose leaving out the k_t^3 term if fitting the model to higher age groups (60+) as the variable is meant to capture dynamics of mortality at lower age groups that are more often affected by for instance drug use, violence and diseases such as AIDS. Although the Plat model has several advantages,

he does not pay sufficient attention to the plausibility of the mortality estimates. The model lacks a term for old ages, and Börger et al. (2014) finds this parameter to be significant. This is because the volatility of old ages where there are only a few observations is underestimated when there is no old-age parameter (Börger et al., 2014). Furthermore, the Plat model is specific in its goal to directly model the trend and its tail-distribution and it measures both longevity risk and insurance risk (Richards et al., 2014).

To determine which models are the most suitable to assess longevity risk and compare SCRs from the Standard Formula and an Internal Model, we require that the Internal Models work within a Value-at-Risk framework based on a one-year time horizon.

3.3.2 One-year Time Horizon

The One-year risk for annuity providers is comprised of two modules where the first is the risk that the next-year mortality rates will be different from the expected mortality rates, resulting in higher payouts than expected. The second is that the one-year mortality rates will affect expected mortality rates beyond the next year, leading to higher capital requirements than expected. For insurance products exposed to longevity risk, the second module is the most important. The first risk component is regular stochastic variation around the projection of the best estimate (Plat, 2011). The second risk component relates to the risk of changes in the projections of the Best Estimate of Liabilities for future years.

To best illustrate the importance of both risk components we may apply the classic example of a cure for cancer. In a scenario where a cure for cancer is developed, it would not likely affect next year's mortality rates, as it would take time to distribute a cure to a large enough number of people so that the mortality rates for the entire population would be affected. However, one would expect a significant impact on future mortality rates. To quantify the Value-at-Risk for longevity risk, one needs to include both risk components properly.

Stochastic spot models require sample-path simulation to empirically derive a distribution for $p_{x,t}$ (Richards et al., 2014). Recall that $p_{x,t}$ is the probability of survival for age x at time t . However, by using approximations, one may avoid the nested simulations with spot models (Cairns et al., 2011). Nested simulations are simulations where the model components themselves are dependent on stochastic simulations for different scenarios, which adds complexity. Spot models account for anticipated changes in mortality by including an assumption of future mortality trends. In most spot models, the trend-assumption is fixed, and the scenarios of realized mortality are derived as random deviations from this fixed trend-

assumption (Börger, 2010). Thus, liabilities at $t = 1$ will always be founded in the same trend-assumption as that of $t = 0$. Spot models thus only account for the first component of longevity risk, not the second. The issue of a fixed trend-assumption has been circumvented in academic literature, see Cox et al. (2010), Biffis (2005) and Hári et al. (2008). Yet, Börger (2010) argues that spot models are not directly applicable to the Solvency II framework.

Börger (2010) and Cairns et al. (2011) work with what is often referred to as forward stochastic mortality models. Forward mortality models are models which produce multi-year survival probabilities as their output (Richards et al., 2014). Forward models directly specify their output as a distribution of survival probabilities, $p_{x,t}$. The advantages of forward models are that they avoid the need for nested stochastic simulations and allow for simultaneous evolution of realized mortality and changes in the trend parameters. Forward models usually add complexity, but Börger (2010) argues that they are the most applicable to the Solvency II framework.

Forward models are to some degree better suited for Value-at-Risk mortality modeling such as the one necessary for internal Solvency II models. However, one may use spot models to simulate the one-year Value-at-Risk central death rate, and this is also EIOPA's suggested method for annuity providers developing an internal model.

We find that the Lee-Carter and CBD-models are the most suitable models for this thesis. These models are spot-models, but we circumvent the need for nested simulations. We find that both models provide a better fit to the Norwegian population than the respective alternatives. Furthermore, the models are parsimonious and well-suited and applicable in a Value-at-Risk framework on a one-year time horizon. This is unsurprising as Lee-Carter and CBD are the most widely used mortality models in Europe to assess longevity risk (EIOPA, 2017).

4 Data

This thesis relies on empirical Norwegian mortality data acquired from the Human Mortality Database. The Norwegian one-year death probabilities are available from years 1846-2014. The exposures, deaths, and death-probabilities are given in gender-specific period tables.

Age x runs from 0 to 110+, and are measured at end-of-year dates. For our models, we set the maximum age to 100 to avoid an issue where few samples of extraordinary age distort the estimates with high volatility. This means that $x \in X = \{0, 1, \dots, 100\}$

Mortality models are quite sensitive, and while more data points usually provide better estimates, we do not want our estimates to be affected by events such as World War II. This is because such events can be seen as anomalies that do not represent the trend changes in mortality which we attempt to measure. The mortality rates are higher in this period as a high number of young men perished early. To obtain a more stable sample, we thus base our modeling on data starting in 1970 so that $t \in T = \{1970, 1971, \dots, 2014\}$.

Gender g indicates whether the individual is male or female, such that $g \in G = \{M, F\}$. We discriminate between men and women as the time-trend might differ significantly between sexes. We thus have access to a data set consisting of $(X \times T)$ matrices with exposure rates and death rates by gender. $m_{x,t}$ follows the number of deaths over the exposure for all $x \in X$ and $t \in T$.

Figure 2 and 3 below illustrate the log mortality rates for Norwegian males and females respectively, from 1900-2014. We observe the humps as World War I & II where especially male mortality rates were affected. From the data, we observe linearity in the log-mortality rates for both genders, especially in recent years. The colors indicate different log mortality rates, ranging from 0 to -10, where red indicates a high death probability, and blue a low death probability.

Male Log-Mortality Rates from 1900 to 2014

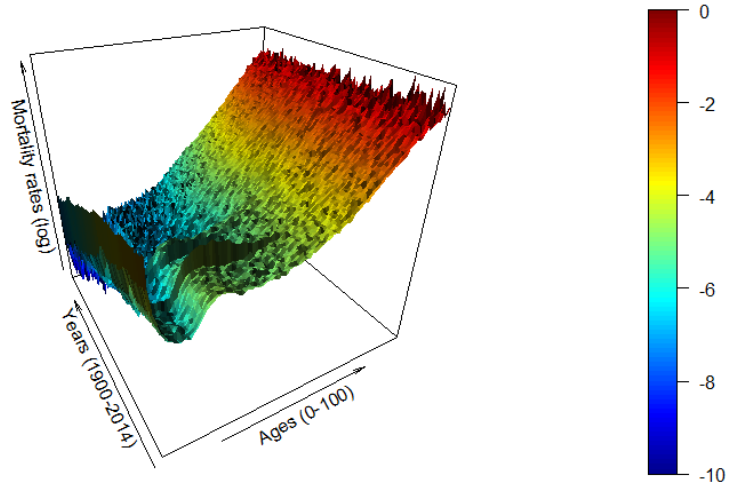


Figure 2: Norwegian male log mortality as a function of age and years. Years run from 1900 to 2014, and ages from 0 to 100.

Female Log-Mortality Rates from 1900 to 2014

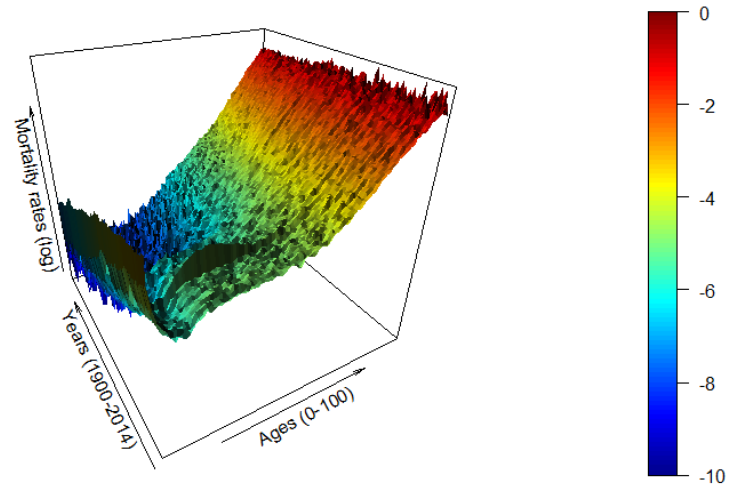


Figure 3: Norwegian female log mortality as a function of age and years. Years run from 1900 to 2014, and ages from 0 to 100.

5 Lee-Carter

This section further discusses the Lee-Carter model and how variables are estimated. Furthermore, the section covers how the model fits Norwegian mortality data, model calibration, and a residual analysis.

5.1 Model Description

5.1.1 The Lee-Carter Model

The Lee-Carter model is a stochastic mortality model based on a factor analytic approach. The Lee-Carter model employs empirical data to create stochastic mortality forecasts by extrapolation. Lee and Carter (1992) assume that the one-year log mortality rates depend on both *age* and *time*. The age-dependency is captured by $\alpha_x^{(g)}$ and $\beta_x^{(g)}$ while $k_t^{(g)}$ captures the time-trend. x denotes the age of an individual. Genders are either *male* or *female* $g \in G = \{M, F\}$, and *time* is denoted $t \in T$. The Lee-Carter model is given by:

$$\ln m_{x,t}^{(g)} = \alpha_x^{(g)} + \beta_x^{(g)} k_t^{(g)} + \epsilon_{x,t}^{(g)} \quad (11)$$

$m_{x,t}^{(g)}$ describes the logarithmically transformed age-specific central rate of death. $\alpha_x^{(g)}$ is a constant specific to every age x that captures the general pattern of mortality by age. $\beta_x^{(g)}$ defines the relative change in mortality by age, while $k_t^{(g)}$ quantifies the evolution of mortality over time as a one-dimensional and time-dependent process. $\epsilon_{x,t}^{(g)}$ is the error-term assumed to be $\epsilon_{x,t}^{(g)} \stackrel{i.i.d.}{\sim} N(0, \Sigma)$ (Lee and Carter, 1992).

Recall that the model is recommended by EIOPA (2017) and is favored for its parsimonious model structure. However, the Lee-Carter model is a single-factor model with some inherent drawbacks. One of these is that it gives a quite poor model fit if there is a cohort-effect in the historical data. Furthermore, the average improvement of mortality for age x , captured by the variable β_x is proportionate to uncertainty in future mortality. This implies that future changes in mortality cannot be seen independently of past changes. This is particularly prevalent in higher ages, where the original model fails to predict significant improvements in mortality. The predicted uncertainty in future mortality is lower at higher ages than what has been observed. The model thus fails to emulate the observed volatility in mortality for higher age groups.

5.1.2 Maximum Likelihood Estimation

Lee and Carter (1992) proposed Singular Value Decomposition (SVD) as a means to estimate the age- and time-dependent variables of their model. The method is similar to Ordinary Least Squares (OLS)–estimation, in that it solves a minimization problem. This is done by iteratively updating every parameter until the difference between the probability of the fitted and saturated model is minimal under constraints that ensure identification.

However, SVD relies on the homoscedasticity assumption. This entails that all variables have the same finite variance, i.e. the variance σ for all $x \in X$ and $t \in T$ is constant. As pointed out by Brouhns et al. (2002), this assumption is not likely to hold because there is reason to believe that the variance is *not* constant. The log-mortality rates should be more volatile for older ages compared to younger ages. This seems reasonable, as for instance improvements in medical treatment could greatly affect the older population. On the other hand, events such as cold winters with a higher flue exposure could negatively affect the short-term logarithmic mortality rates. Furthermore, the sample size in the older population is relatively smaller, which could result in larger confidence intervals. Therefore, we will rely on maximum likelihood estimation when determining the variables α_x , β_x and k_t .

Brouhns et al. (2002) propose that the number of deaths $D_{x,t}$ can be modeled as a Poisson distribution

$$D_{x,t} \sim \text{Poisson}(E_{x,t}m_{x,t}) \text{ with } m_{x,t} = e^{(\alpha_x + \beta_x k_t)} \quad (12)$$

Recall that $E_{x,t}$ denotes the exposure, $m_{x,t}$ the central mortality rate and α_x , β_x , and k_t are interpreted as in the classical Lee-Carter model.

Instead of resorting to SVD for estimating α_x , β_x and k_t , Brouhns et al. (2002) propose maximizing the log-likelihood, based on the following model:

$$L(\alpha, \beta, k) = \sum_{x,t} \{D_{x,t}(\alpha_x + \beta_x k_t) - E_{x,t}e^{(\alpha_x + \beta_x k_t)}\} + C \quad (13)$$

The maximization can be solved through generalized non-linear models using standard statistical software. L represents log-likelihood and C is a constant. We rely on this method in our estimation and have implemented the generalized non-linear models through the *gnm* R-package (Turner and Firth, 2018).

5.1.3 Forecasting

The Lee-Carter model assumes that only k_t varies with time, while α_x and β_x are held constant over time. Variations in k_t are estimated through ARIMA models. ARIMA modeling is one of the most used approaches for forecasting time series. ARIMA models can be explained through three classification parameters, p , d , and q . In the original paper from 1992, Lee & Carter finds that a random walk with drift best describes k_t (ARIMA 0,1,0) for the US population from 1933-1987.

- p is the number of autoregressive terms
- d is the number of nonseasonal differences needed for stationarity
- q is the number of lagged forecast errors in the prediction equation

$$AR(p) : X_k = a_n X_{k-1} + a_2 X_{k-2} + \dots + a_p X_{k-p} + \sigma \varepsilon_k \quad (14)$$

$$MA(q) : X_k = \sigma(\varepsilon_k + b_1 \varepsilon_{k-1} + \dots + b_q \varepsilon_{k-q}) \quad (15)$$

$$ARMA(p, q) : X_k = a_k X_{k-1} + \dots + a_p X_{k-p} + \sigma(\varepsilon_k + b_1 \varepsilon_{k-1} + \dots + b_q \varepsilon_{k-q}) \quad (16)$$

$$ARIMA(p, d, q) : X_k = d + a_k X_{k-1} + \dots + a_p X_{k-p} + \sigma(\varepsilon_k + b_1 \varepsilon_{k-1} + \dots + b_q \varepsilon_{k-q}) \quad (17)$$

An AR(p) model uses the correlation between historical data to estimate future values. The moving average (MA(q)) term captures more rapid changes in the data set. In mortality forecasting, for instance, a short time period with increased mortality rates could be captured by the MA(q) term. In western countries, however, mortality rates are likely to steadily decline over time. This trend may be captured by the drift term p .

5.2 Model Fit

Next, the estimated Lee-Carter variables of the fitted model for Norwegian males and females from 1970 to 2014 are discussed.

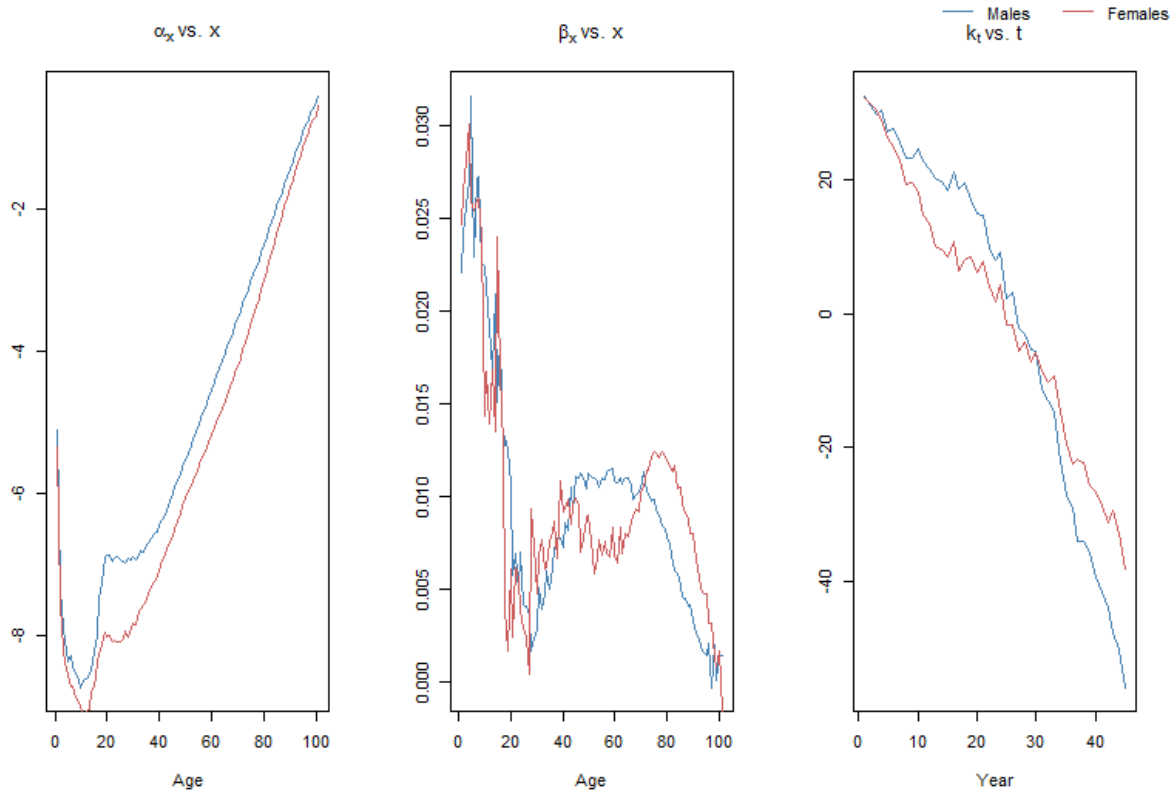


Figure 4: Estimated α_x , β_x and k_t based on the Lee-Carter model. The light blue lines and the light red lines represent males and females, respectively

From the left figure, we observe the estimated α_x values for ages 0 to 100. α_x can be interpreted as the average logarithmic mortality rates. We find that mortality falls sharply from birth until age 10. From ages from 10 to 100 the logarithmic mortality rates steadily increase. Interestingly, we observe the accident hump in the early twenties. This is the phenomenon that young adults tend to take more risks which leads to higher mortality rates, which is well documented. We observe the accident hump for both genders, although it is more prevalent for males. Furthermore, we observe that males have higher death probabilities than females throughout, which is what we would expect as female mortality rates have historically been lower than male mortality rates.

β_x represents how sensitive mortality rates across age x are to changes over time. From the middle graph, we observe the greatest population benefits for ages 1 to 5. One explanation of this could be linked to the reduction in sudden infant death syndrome (SIDS), which was prevalent in the 1980s, but has since been dramatically reduced. Furthermore, we observe high volatility in the β_x variable when age is less than 60. This can be explained through higher variations in the mortality rates for the younger Norwegian population. After age

80, we observe that the sensitivity to changes over time is reduced. This is expected as the sensitivity to changes over time will decrease after a certain threshold, due to biological constraints.

Lastly, estimates of the variable k_t is represented in the rightmost figure. Recall the interpretation of k_t as the overall changes in mortality over time. We observe a decreasing trend, meaning that the general population experiences a decrease in mortality rates. We observe a more rapid decline in male mortality rates compared to female rates. One explanation of this could be the reduction in carcinogenic habits such as smoking, where a higher percentage of males were exposed, relative to females.

5.3 Model Calibration

The purpose of this subsection is to find an appropriate ARIMA time series calibration for k_t in the Lee-Carter model.

We address whether a unit root is present in the time series. This can be statistically proven by applying the augmented Dickey-Fuller test. This tests if a unit root is present in an autoregressive model. This corresponds to the first main version of the Dickey-Fuller test, described by:

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{18}$$

If $\rho = 1$, there is unit root present while ε_t denotes the error term. If unit root is present then the series is non-stationary and must be differentiated. The null- and alternative-hypothesis of the Dickey-Fuller test are:

$$H_0 : \rho = 1 \quad \text{versus} \quad H_a : \rho \neq 1 \tag{19}$$

The Dickey-Fuller test is applied on the k_t series from the fitted model and the test-statistics are 1.063 and 0.472 for males and females, respectively. The critical value of the Dickey-Fuller test at a 5%-significance level is -1.95 . The result of the test is that there is no statistical evidence to reject H_0 . From this we can draw the conclusion that there is a presence of unit root, and thereby use first differences. Equation 11 thus becomes:

$$\Delta y_t = (\rho - 1)y_{t-1} + u_t = \delta y_{t-1} + \varepsilon_t \quad (20)$$

We thus find that d in ARIMA(p, d, q) should equal 1. Next, we proceed by optimizing the parameters p and q .

Earlier empirical research shows that different parameters p and q fits different populations. For instance, Renshaw and Haberman (2006) found that an ARIMA(0,1,0) process best describes the Dutch population, while Richards et al. (2014) used an ARIMA (3,1,3) process to model the population in England and Wales.

To differentiate mortality models the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC) are tools to conclude which model best fits the historical data. Both tools estimate the relative quality of each model. Both AIC and BIC estimate the information lost by a given model, where a low value indicates smaller losses of information and thus the preferred model.

$$BIC = \ln(n)k - 2 \ln(\hat{L}) \quad (21)$$

$$AIC = 2k - 2 \ln(\hat{L}) \quad (22)$$

N represents the number of observations, k the number of free parameters and \hat{L} the maximized likelihood.

Table 1: Lee-Carter ARIMA processes for k_t . Log-likelihood, AIC and BIC are provided for each ARIMA process. The processes are then ranked by BIC.

ARIMA	Log-likelihood		AIC		BIC		Rank (BIC)	
	Males	Females	Males	Females	Males	Females	Males	Females
(0,1,0)	-97.34	-97.12	198.68	198.23	202.25	201.8	(1)	(3)
(0,1,1)	-96.86	-93.52	199.71	193.05	205.07	198.4	(6)	(1)
(1,1,0)	-96.55	-93.82	199.1	193.64	204.45	198.99	(4)	(2)
(1,1,1)	-96.22	-93.46	200.43	194.92	207.57	202.05	(8)	(5)
(1,1,2)	-92.31	-93.45	194.62	196.9	203.54	205.82	(3)	(7)
(2,1,0)	-95.05	-93.39	198.1	194.78	205.23	201.93	(5)	(4)
(2,1,1)	-92.23	-93.3	194.45	196.6	203.37	205.52	(2)	(6)
(2,1,2)	-92.03	-93.29	196.06	198.58	206.77	209.29	(7)	(8)

Table 1 shows that the original ARIMA(0,1,0) process performs well for Norwegian males and that the ARIMA(0,1,1) model best fits the female population. As in Cairns et al. (2009),

the models are ranked by BIC, because AIC is often too liberal and prefers models that may be too complex. Still, we find that BIC and AIC yield similar results.

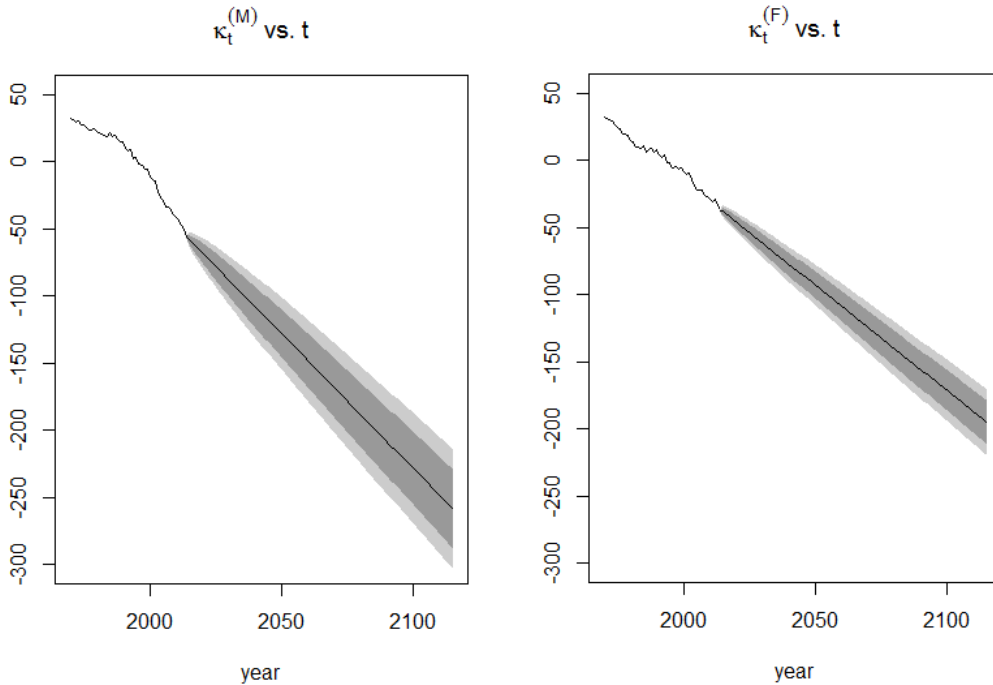


Figure 5: k_t forecasts for males on the left and females on the right. The grey shades represent the 95% confidence interval

From Figure 5 we observe uncertainty in the estimated k_t forecasts for males and females based on the ARIMA processes previously described. We see that the male model indicates larger uncertainty through wider confidence intervals. This is not surprising when analyzing historic male mortality, which tends to be associated with higher uncertainty. Furthermore, we see that the model predicts a more rapid decline in mortality rates for males than for females. Recall from Figure 1 that the difference between male and female life expectancy has narrowed, which is what we would expect from this model going forward. k_t forecasts for different ARIMA processes and mortality simulations are available in Appendix A.2 and A.3, respectively.

5.4 Model Analysis

Lastly, we do a residual analysis of the fitted model. The analysis indicate how well the model accounts for the underlying model assumptions.

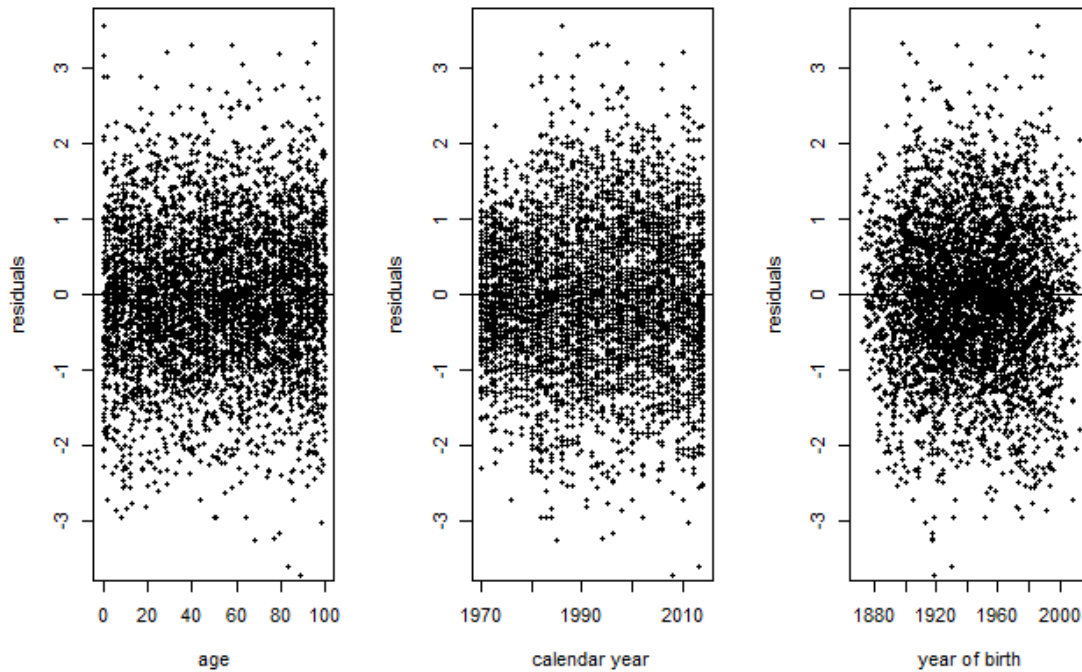


Figure 6: Residual plots of the fitted male Lee-Carter model. See Appendix A.4 for the female residual analysis.

From figure 6 we observe no significant patterns. The first figure represents the residuals for all age groups, and we observe no indication that the model predictions are exaggerated. Furthermore, from the second figure, we observe homoscedasticity, meaning that the variance seems constant throughout the time series. Lastly, the final figure attempts to uncover cohort effects. The residuals seem to cluster in the center. This is, however, not an issue, as the data set is only from 1970 to 2014. It is expected that the residuals are not as consistent for the population born in the 1800s, as the number of observations are fairly limited within our data set.

6 CBD-model

6.1 Model Description

The Cairns-Blake-Dowd (CBD) model was introduced by Cairns et al. (2006). Recall that it is a two-factor model and one of the most well-known variants of the Lee-Carter model. The CBD-model looks at the linearity of the logit one-year death probabilities at higher ages. It assumes that the logit function of the one-year death probabilities in a given year is a linear function of age. The intercept and slope-parameters are thus treated as stochastic processes across years. The original CBD-model has been expanded by, among others, Cairns et al. (2009), who include a quadratic age term and a cohort effect term to create three variations of the original model. A description of model 7 may be found in Appendix B.1. However, we find that the original model has the best fit for Norwegian data among the CBD-variants. The original CBD-model has the following structure:

$$\text{logit } q_{x,t} = \log \frac{q_{x,t}}{1 - q_{x,t}} = \beta_x^{(1)} k_t^{(1)} + \beta_x^{(2)} k_t^{(2)} \quad (23)$$

The model assumes simple parametric forms for $\beta_x^{(2)}$ and $\beta_x^{(1)}$ such that:

$$\beta_x^{(1)} = 1, \quad (24)$$

and

$$\beta_x^{(2)} = (x - \bar{x}) \quad (25)$$

Here, $\bar{x} = n_a^{-1} \sum_i x_i$ is the mean age in the sample range so that

$$\text{logit } q_{x,t} = k_t^{(1)} + k_t^{(2)}(x - \bar{x}). \quad (26)$$

The mortality index $k_t^{(1)}$ can be interpreted as the mortality curve after a logit transformation. Reductions in $k_t^{(1)}$ coincide with improvement in mortality across all ages. The $k_t^{(1)}$ index is similar to Lee-Carter's α , as it represents the general improvement in mortality over time. The second mortality index, $k_t^{(2)}$, can be interpreted as the slope of the mortality curve across all ages in a given year. An increase in $k_t^{(2)}$ would result in mortality at lower ages improves more rapidly than older ages, and vice versa. The $k_t^{(2)}$ variable thus only accounts for changes

across all ages other than the mean age of the data set. In this thesis, the mean age of the CBD-model is set at 70, meaning that changes in $k_t^{(2)}$ will have no effect on the mortality rate of 70-year-olds.

The two-factor CBD-model does not have identification problems (Cairns et al., 2006). The projections for future mortality rates are estimated as a bivariate random walk with drift for $k^{(1)}$ and $k^{(2)}$. Both drift terms and correlation factors are estimated from the observed data.

The CBD-model is effectively a Gompertz-model (sigmoid curve) fitted for each year t . The CBD-model is a two-factor model, compared to the single-factor Lee-Carter model, it does not impose perfect correlation in mortality across ages (Pitacco et al., 2009). Because age is modeled as a continuous variable, the CBD-model is limited when it comes to modeling mortality for individuals aged less than 40 (Peters et al., 2012).

6.2 Model Calibration

Similar to the Lee-Carter model, the choice of ARIMA model is important because it will vary between populations. The CBD-model can be described through independent ARIMA models, where the ARIMA processes for $k_t^{(1)}$ and $k_t^{(2)}$ differ. Like in the Lee-Carter model, we distinguish the different ARIMA processes by applying the AIC and BIC standards. We find that for $k^{(1)}$ and $k^{(2)}$ an ARIMA(2,1,1) and ARIMA(1,1,0) process best fits the males, while an ARIMA(1,1,0) and ARIMA(0,1,1) best fits for females. A full analysis of the ARIMA process can be found in Appendix B.2.

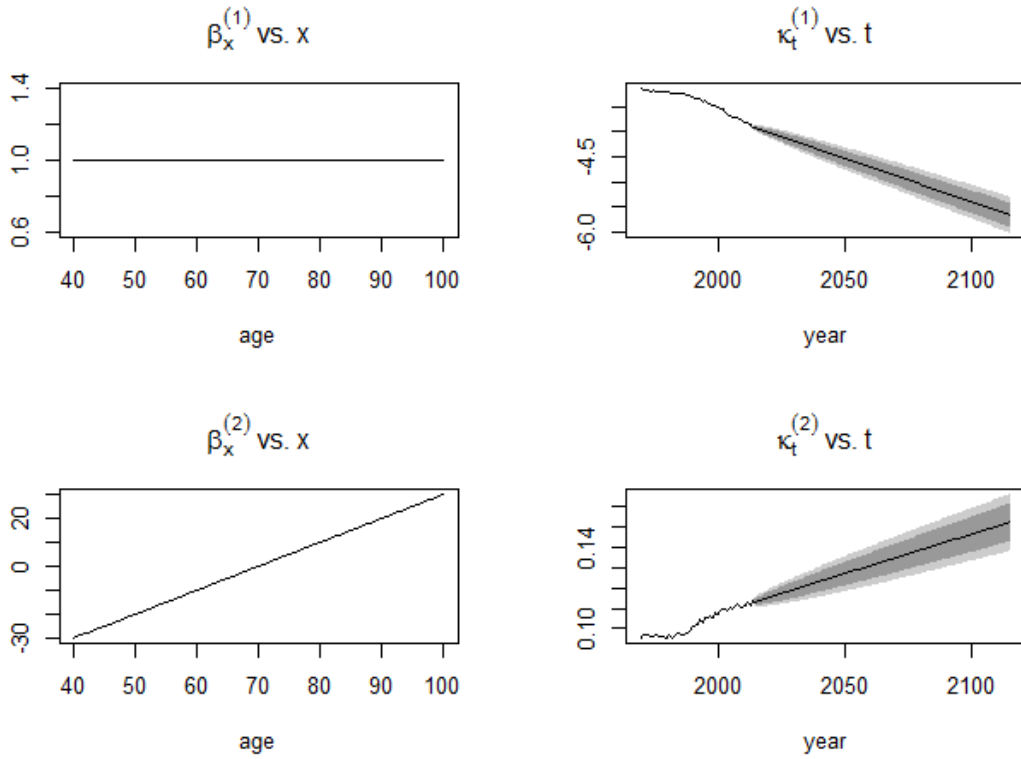


Figure 7: Forecasted CBD variables. Here represented by $\beta_x^{(1)}$, $\beta_x^{(2)}$, $k_t^{(1)}$ and $k_t^{(2)}$

Figure 7 shows the CBD variables for males. We observe that $k_t^{(1)}$'s downward slope returns fairly slim confidence intervals. This comes as a result of low volatility in the recent reduction in mortality rates. On the other hand, $k_t^{(2)}$ is steadily increasing with large confidence intervals. Recall that an increase in $k_t^{(2)}$ represents that the younger population benefit more from mortality improvements compared to the older population. The large confidence interval can be explained by the high uncertainty within the death rates of the older population.

6.3 Model Analysis

Next, we analyze the goodness of fit for the CBD-model. This gives an indication of how well the model is able to explain the historical data, and thus give an indication of how reliable the forecasts are.

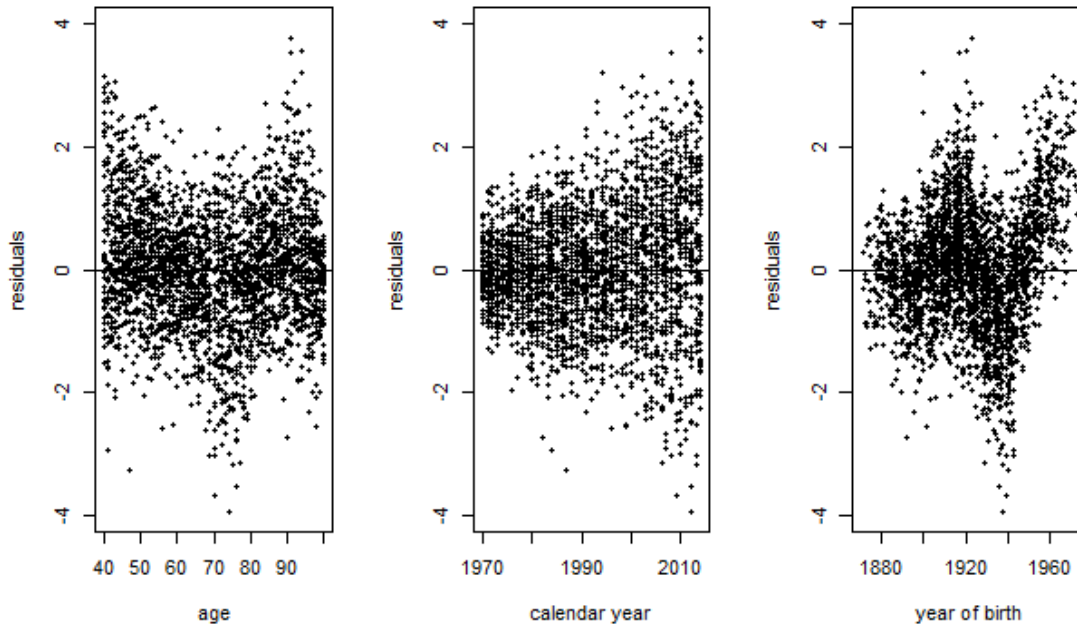


Figure 8: Residual analysis of the CBD-model for Norwegian males. See Appendix B.4 for female residual plots.

From the leftmost figure, we do not see the plots as an unambiguous cloud. This is an indication that the fitted lines fail to correctly explain logarithmic mortality for a large portion of the Norwegian population. Furthermore, the second graph indicates heteroscedasticity, where the residuals seemingly increase as time passes. Heteroscedasticity generally produces consistent, but not efficient predictions. Lastly, the third graph does not produce an even picture of the residuals. We see that the residuals are dependent on the year of birth. This can be explained through different predictions of the older and younger mortality improvements.

From the goodness of fit analysis, we see indications that the CBD-model is less suited for the Norwegian population. The extensions of the CBD-model, model 7 and 8 produced similar residual outputs. It seems evident that Norwegian annuity providers should rely on other means of mortality predictions. Still, it is of interest to continue forward with the model, as it provides good indications of model risk. Annuity providers that rely on Internal Models will be subject to model risk, which is typically present when decisions rely on a single model's outputs. This will be prevalent for assessing longevity risk as a company will only rely on one mortality model when implementing an Internal Model. Testing more than one model could thus emphasize the importance of selecting an appropriate mortality model.

7 Pension Products

If social security and employment-based are insufficient to provide a desired retirement, one needs to rely on private savings. One way to do this is by acquiring pension insurance with an annuity provider. The principle behind pension insurance is that the annuity holder pays a premium over a given period. The annuity provider is then free to manage these funds as their own until the annuity holder reaches retirement age and the funds are paid back to the insured as annuities. These payments continue until the contract runs out, or the client dies. Premiums and payments are thus dependent on the life expectancy of the annuity holders.

A normal pension insurance plan occurs in two phases; a payment phase and payout phase. These phases are mathematically illustrated below:

$$E(PV_0^\pi) = -\pi \sum_{\tau=0}^{x_r-x-1} d^\tau S_{x+\tau,t+\tau} \quad \text{and} \quad E(PV_0^\rho) = \rho \sum_{\tau=x_r-x}^{\infty} d^\tau S_{x+\tau,t+\tau} \quad (27)$$

Here, x denotes the annuity holders current age, while x_r is the retirement age. In Norway, the normal retirement age is 67 years. The policy premium π is the yearly sum paid by the annuity holder from the contracts beginning to attained retirement age x_r . These premiums are discounted by d^τ at time τ based on the risk-free rate.

Recall that $S_{x,t}$ represents the cumulative survival probability of the contract holder at age x , at time t . $S_{x+\tau,t+\tau}$ thus represents the likelihood that the contract holder will still be alive when age is $x + \tau$ at time $t + \tau$. When the annuity holder reaches retirement age, x_r , a yearly payment denoted ρ is made to the annuity holder. These payouts are similarly discounted and multiplied by survival probabilities to get the present value of the sum of payouts.

The present value of the insurance plan depends on the contractual year. If $x < x_r$ the contract is in the payment phase, while in the opposite case where $x > x_r$, the contract is in the payout phase where payouts in the form of pensions will be paid to the annuity holder. In the former case, $E(PV_0^\rho) > E(PV_0^\pi)$ and the insurance policy will represent a liability to the annuity provider.

In this thesis, we assume pension insurance contracts end when the annuity holder dies. When put together, the equation (28) represents the sum present value of the pension insurance contract:

$$E(PV_0) = -\pi \sum_{k=0}^{x_r-x-1} d^k S_{x+k,t+k} + \rho \sum_{k=x_r-x}^{\infty} d^k S_{x+k,t+k} \quad (28)$$

Where k represents the contract time and runs from 0 to infinity because there is no theoretical maximum length of the contract, however, $S_{x+k,t+k}$ will tend towards zero as the probability of survival decrease for higher ages.

Insurance premiums and payouts are usually determined by setting ρ based on the individual customer profile and then solve for π . This is accomplished by setting equation 28 equal to zero and solve for π :

$$\pi = \frac{\rho \times \sum_{k=x_r-x}^{\infty} d^k S_{x+k,t+k}}{\sum_{k=0}^{x_r-x-1} d^k S_{x+k,t+k}} \quad (29)$$

The variable π is referred to as an equivalence prize and denotes the premium which the client must pay for the present value of the contract to be equal to zero.

8 Standard Formula and Internal Model Methodology

In this section, we elaborate on how to calculate the mortality rates based on the Standard Formula and the Internal Model. Recall that mortality projections are necessary to compute the estimated cash flows from the pension contracts. The methods are applied to the Lee-Carter and the CBD-model in section 9.

8.1 Standard Formula

The Standard Formula approach to calculating the SCR for longevity risk sets the SCR based on the change in BEL given a one-off permanent shock to mortality rates. This shock is meant to represent the sudden decrease in mortality rates, similar to the one in 200-year scenario. The longevity shock as stipulated in the Standard Formula is a permanent reduction of mortality rates of 20% for all ages (EIOPA, 2014). This shock is meant to represent a systematic change in mortality rates, which means that it does not account for sample risk (Börger, 2010).

The simplified approach to the Standard Formula given a reduction in current and future mortality rates can be described as

$$m_{x,t}^{Shock} = m_{x,t} \times (1 - f) \quad (30)$$

where $m_{x,t}$ is the central mortality projection and f is the shock to current and future mortality rates. Recall that central mortality rates are given by the average number of people who died at time t at age x , divided by the number of people age x alive at time t . EIOPA (2014), specifies that the shock factor, f , is 20%.

The Standard Formula to longevity risk differs from the Value-at-Risk approach, described in section 8.2, in that it assumes a 20% reduction to mortality rates for all age groups. This could lead to mortality rates that are too low for young ages, and thus capital requirements that seem too low. Furthermore, because of the general 20% reduction in mortality, the capital requirements seem excessive for ages 70 and up. However, an insurers portfolio is likely to consist of different age groups and for certain groups, the shock-approach may be expedient. For new and growing annuity portfolios, the shock-approach seems reasonable according to Richards and Currie (2009).

Börger (2010) argues that the shock approach has structural shortcomings, given the too low capital requirements for young annuity holders, and too high capital requirements for older annuity holders. He suggests that the shock factor should be age-dependent with more stress to younger ages, and less towards older ages. However, in a scenario with a balanced portfolio, one might argue that the difference in capital requirements for younger and older ages would offset one another. Whether this would work in practice is entirely dependent on the profile of the liabilities by age in the portfolio.

8.2 Internal Model

8.2.1 Methodology

In this section, we elaborate on the Value-at-Risk framework as it is considered by the Norwegian Financial Authorities. According to Jan Hagen with the Norwegian Financial Supervisory Authorities (Finanstilsynet), the method generally allows complete modeling freedom (Personal communication, 26.11.2018). The only requirement is that the model must fulfill the calibration target of a 99.5% VaR over a one-year time horizon.

Richards et al. (2014) provide a one-year framework for determining the Value-at-Risk applicable to Solvency II. Their approach differs from those of Börger (2010) and Plat (2011) as they attempt to model the trend and its tail-distribution directly. Richards et al. (2014) present a more general framework that is applicable to a wider range of stochastic projection models. The framework has a sole focus on longevity trend risk in annuities and pension payouts.

In short, we find the Value-at-Risk by employing a stochastic mortality model to simulate one additional year of mortality rates. To estimate how the central projection might change on a one-year horizon, this process is repeated several times over to generate a data set of projected one-year mortality rates. Among the simulated one-year projected mortality rates we find the sample path with the 99.5% lowest mortality rates. This one-year projection is then fed into the original model. Lastly, a central projection is estimated based on mortality rates from t_{low} to t_{high+1} . A more detailed recounting of the method follows.

We employ a data set consisting of ages from x_{low} which corresponds low years, to x_{high} , which corresponds to high years. The data set runs from years t_{low} to t_{high} , i.e. 1970 to 2014. This data set includes the deaths at each age and in each year $D_{x,t}$ and the corresponding exposures in the population, $E_{x,t}$. For this process, we require the exposures at the beginning of the year $t_{(high+1)}$. As in Richards et al. (2014), we assume that the exposure, $E_{x,t}$, is equal for t_{high} and t_{high+1} .

Next, we employ a statistical model to fit the data set. The model provides output for the central mortality rate $ln m_{x,t}$ where x is age in years and t is time in calendar years. The projections from this model can be used to calculate life-expectancies and annuity factors at different ages.

We use the statistical model to simulate paths for the data $ln m_{x,t_{high+1}}$. This is one year additional to our data set. These simulated paths may contain either volatility and uncertainty, or both. However, on a one-year time horizon, volatility is most likely the source of uncertainty and should thus be included (Richards et al., 2014). See section 8.2.2 for further analysis on the risk assessment. We find the approximate binomial probability of death in year t_{high+1} , $q_{x,t_{high+1}}$, by approximation of $q \approx 1 - e^{-m_{x,t}}$.

The next step is to simulate the number of deaths in the year additional to our data set t_{high+1} for every age. We do this as a binomial random variable where population counts are the $E_{x,t_{high+1}}$ and the binomial probabilities are the approximation of $q \approx 1 - e^{-m_{x,t}}$. This provides the simulated death counts at each age. We repeat this process 10000 times

to form a data set with 10000 simulated mortality rates for t_{high+1} at all ages. Next, we find the sample path with the 99.5% lowest mortality rates by ranking them based on how many deaths each path will generate. From the 10000 simulated paths, the path with the 50th lowest number of deaths is chosen.

Next, we append the simulated path to the original data, leaving us with what a single simulation of the 99.5% lowest mortality data would be in one year. The statistical model is refit to the combined data set before recalculating the life expectancies and annuities.

The model assumes zero-sum immigration/emigration, and that immigrants share the same life expectancy as the Norwegian population. This assumption is essential because the number of deaths in t_{high+1} is calculated based on the exposure from t_{high} . This is not necessarily a realistic assumption. Western countries are experiencing surges of immigration from countries with lower GDP per Capita and life expectancy which could reasonably affect the life expectancy in the receiving country. This is something that annuity providers will need to factor in depending on their portfolio, but for the purpose of this thesis, the assumption is reasonable.

8.2.2 Risk assessment of the VaR approach

As briefly stated in section 8, the VaR approach is subject to two types of risk, trend risk and volatility (Richards et al., 2014). Trend risk is risk associated with the general development of mortality rates, whether they increase or decrease long-term. This uncertainty is a parameter risk, as it is linked to uncertainty in the parameters of the mortality model. Volatility, on the other hand, is not a parameter risk, but uncertainty additional to the trend risk. Volatility is risk associated with temporary fluctuations around the trend component. This risk could be associated with for instance extreme weather, crop failures, or a disease outbreak. It is thus a short-term risk, and it would make intuitive sense that it is of less importance for annuity providers with longer horizons. Still, volatility plays an important role in the Value-at-Risk framework.

The difference between trend risk and volatility can be illustrated by a drift model with mortality index k in year $t + 1$. The relationship with the previous year's index may be described as

$$k_{t+1} = k_t + C + \varepsilon_{t+1} \tag{31}$$

Where k_t is the mortality index as in the Lee-Carter model at time t , and C is a drift constant. ε_t is the independent, and identically distributed error term with zero mean variance σ_ε^2 . Of the three parameters that are estimated, the trend risk lies in the uncertainty of the drift constant denoted σ_C . A simulation should be able to exclude or include any one of the three parameters independently. Distinguishing between the parameters is essential to annuity providers to adequately address the one-year uncertainty.

The drift model is a simple and restricted subset of a full ARIMA model for k_t (Richards et al., 2014). This means that there is a noise process, but that k_{t+1} is related to one or more earlier values as an autoregressive moving average. Thus, there will be more parameters to estimate with each of them with a standard error corresponding to σ_C . As in the simple drift model, σ_ε^2 is the volatility while trend risk is associated with the standard error of $\ln \hat{m}_{x,t}$. This risk stems from the uncertainty in the ARIMA parameter estimates.

An interesting point about the Value-at-Risk approach and longevity risk is that longevity risk is a long-term risk primarily characterized by parameter risk. However, when simulating mortality on a one-year horizon, the uncertainty is to a large degree made up by volatility. There are several ways in which the one-year volatility may be affected. First, a shorter observation period will create more uncertainty, thus a larger variance. On a one-year horizon, the effect may cause the model to overestimate the central projections following the one in 200 year event. Second, sample size will play a large role if for instance higher age groups are not well represented in the data set. With an Internal Model, insurers may be tempted to rely on their own portfolio as model inputs. This, in turn, may increase model volatility and again, the VaR approach might represent an even worse case than the 99.5% scenario.

9 Application

This section describes how the mortality models are applied to the Solvency II framework to determine SCRs. We use a simplified pension product to compare the two options for determining SCRs. The capital requirements from the Standard Formula and the Internal Model are then compared. The SCRs are based on mortality forecasts from the Lee-Carter and the CBD-model described in section 5 and 6.

9.1 Simplified Pension Product

The simplified pension product used in this analysis falls within the category of guaranteed products, similar to those offered by Norwegian insurers. It replicates a product where the annuity holder pays a premium until retirement, followed by pension payouts until they die. The product is affected by longevity risk because of uncertainty related to future cash flows, and the fact that future mortality rates will influence today's estimation of discounted liabilities.

The pension product is defined by the pension payouts, premiums, and the retirement age. For simplicity and a predominant focus on longevity risk, we disregard individual risk assessment of policyholders, such as level of education, city, family history etc. Therefore, payouts are equal for all annuity holders and are set at 1000 NOK. The payouts will thus only vary in correlation with mortality. The premiums depend on the number of years until the Norwegian retirement age of 67 years.

The cash flows are discounted as suggested by the Financial Supervisory Authority of Norway. Recall from section 2.2 that the technical provisions should use an interest curve that reflects the risk-free rate. Norwegian financial authorities refer to the risk-free rates provided by EIOPA. The simplified product runs for a longer period than the rates currently available and are thus kept constant when EIOPA rates no longer apply. More information on the discount factors can be found in Appendix C.

9.2 Projected Mortality Rates

Next, we compare the projected mortality rates for the Standard Model and the Internal Model. We begin with the projected mortality rates based on Lee-Carter projections before we analyze projections from the CBD-model. Recall that the Best Estimate of Liabilities and thus the SCRs rely on future cash flows, which for longevity risk are estimated based on the probability that the contract holder is still alive. To analyze how the estimated cash flows are affected by mortality rates, we thus examine the projected mortality rates of individuals at age 85, because the cumulative probability of survival will be drastically reduced in these age groups.

Lee-Carter Mortality rates for Norwegian males at age 85

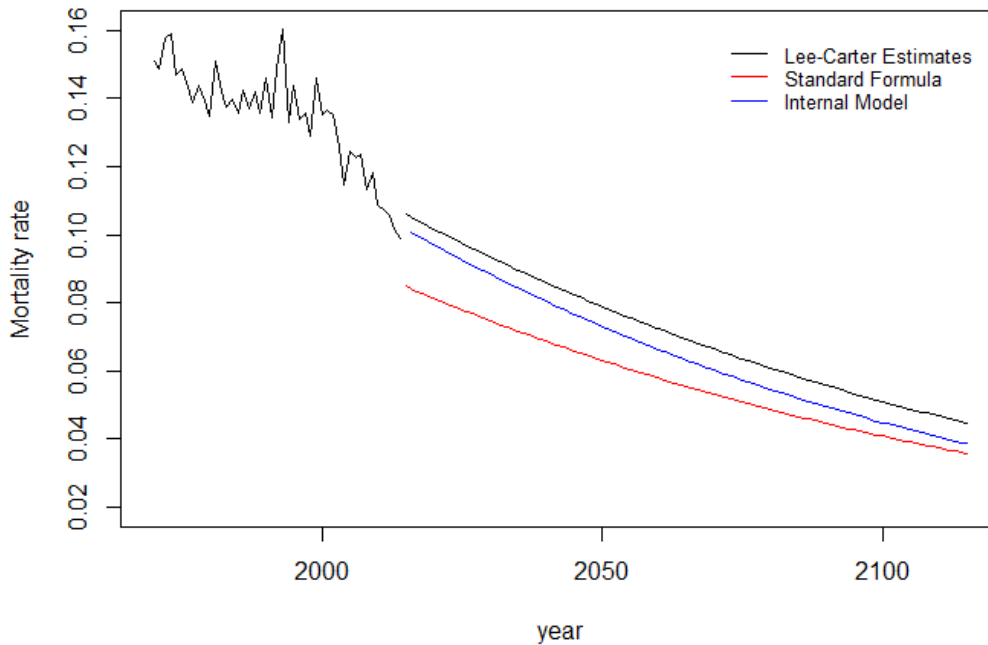


Figure 9: Lee-Carter male mortality rates at age 85. The graph displays observed mortality rates and include central mortality projections. The black line is the Lee-Carter central mortality projection and the blue line represents the Internal Model based on a VaR approach on a one-year time horizon. The red line represents Standard Formula and the 20% shock to Lee-Carter mortality rates.

From figure 9, we unsurprisingly observe that the central Lee-Carter mortality estimates for Norwegian males at age 85 are higher than those of the Standard Formula and the Internal Model. Recall that the Standard and Internal Models represent how mortality rates change based on a one in 200 year worst case scenario. This illustrates the long-term effects on mortality projections from a single year of significantly reduced mortality rates.

The Lee-Carter model projects decreasing mortality rates. Thus, the 20% shock reduction in mortality rates has a greater impact on higher rates early in the projection. The graph does not allow us to make predictions of the effects on the simplified portfolio because not all ages are represented. However, the low initial mortality projections from the Internal Model may indicate that the Standard Formula could result in higher SCRs than those from the Internal Model.

CBD Mortality rates for Norwegian males at age 85

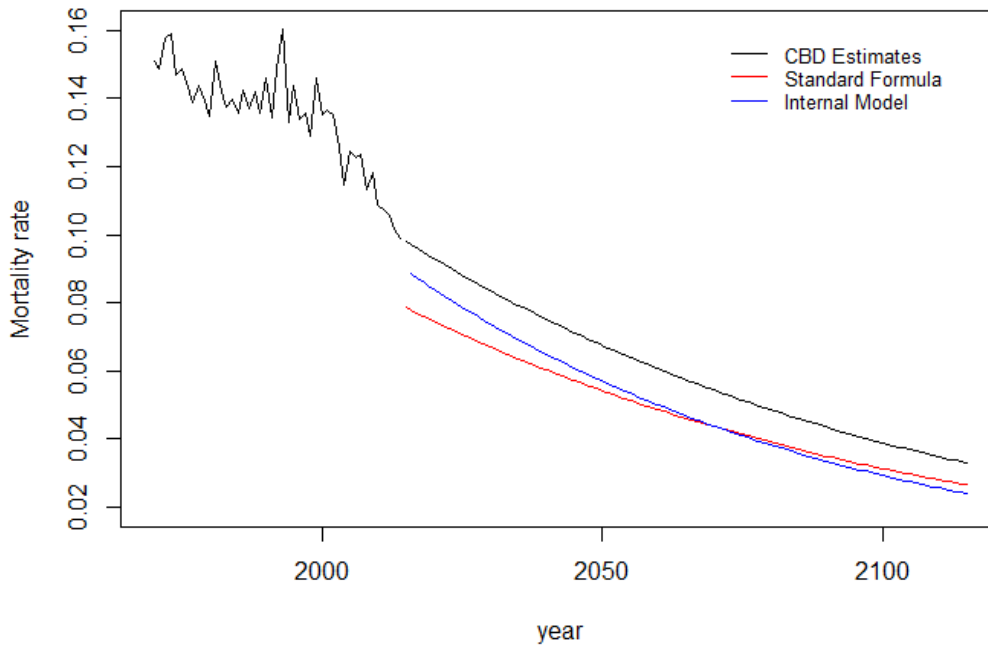


Figure 10: CBD male mortality rates at age 85. The graph displays observed mortality rates and include central mortality projections. The black line is the CBD central mortality projection and the blue line represents the Internal Model based on a VaR approach on a one-year time horizon. The red line represents Standard Formula and the 20% shock to CBD mortality rates.

From the CBD-model we observe similar effects as in the Lee-Carter Model. As the projections continue we observe that the Standard and Internal model projections converge and eventually cross. It is interesting that the CBD central mortality estimates are lower than those of the Lee-Carter model. This leads to smaller mortality decreases from the Standard Formula shock. Comparing the CBD mortality rates to the Lee-Carter mortality rates, we also observe that the Internal Model for CBD projects lower mortality rates, both initially and over time. This can be explained by the larger variance embedded in the CBD-model, as seen in Appendix B.3, simulated mortality rates based on variations in k_t^1 and k_t^2 .

9.3 Results

9.3.1 Best Estimate of Liabilities

The Best Estimate of Liabilities of the simplified portfolios has been calculated by applying the formulas described in section 2.2. In short, the cumulative probability of survival is multiplied by the cash flows. BEL_0 is the discounted sum of cash flows using the interest rate as suggested by EIOPA. The Best Estimate of Liabilities at time zero is denoted BEL_0 . The BEL based on Lee-Carter and CBD-projections are then determined for portfolios with different age compositions. Next, the BEL given the Standard Formula (BEL^{SF}) and BEL given an Internal Model (BEL^{IM}) are calculated based on the mortality assumptions described in section 8. From this, we find that the BEL^{IM} is consistently lower than the BEL^{SF} . This is because the cumulative survival probabilities of the Internal Models are lower than those of the Standard Formula.

Table 2: Best Estimate of Liabilities. BEL_0 , BEL^{SF} , and BEL^{IM} for the Lee-Carter and CBD mortality models, calculated for simplified portfolios consisting of 50, 60, 70, and 80 year old Norwegian males and females.

Best Estimate of Liabilities, Males						
Portfolio age	Lee Carter			Cairns-Blake-Dowd		
	BEL_0	BEL^{SF}	BEL^{IM}	BEL_0	BEL^{SF}	BEL^{IM}
Age = 50	5284	5738	5492	6285	6815	6560
Age = 60	8739	9399	9000	9797	10580	10166
Age = 70	10289	11087	10523	11344	12281	11780
Age = 80	5856	6630	6020	6821	7705	7247

Best Estimate of Liabilities, Females						
Portfolio age	Lee Carter			Cairns-Blake-Dowd		
	BEL_0	BEL^{SF}	BEL^{IM}	BEL_0	BEL^{SF}	BEL^{IM}
Age = 50	6153	6556	6382	7192	7656	7429
Age = 60	10060	10639	10360	11110	11810	11431
Age = 70	11930	12646	12272	12814	13690	13194
Age = 80	7234	7982	7513	8018	8894	8392

Because the contracts are in the payment phase until retirement age, we find that the BEL_0 increases until age 67 before it decreases with age. Furthermore, there is significant variation in the BEL_0 between the Lee-Carter and the CBD-model. Recall that the interpretation of BEL_0 is what you could sell your liabilities for in an open market at time 0. This is interesting because this means that the true BEL at time = 0 is highly dependant on the mortality model.

The difference between BEL_0 from the Lee-Carter projections and the BEL_0 from the CBD projections is between 10% and 19%, depending on ages in this simplified pension product.

Furthermore, we find that the BEL is higher for the CBD-model than the Lee-Carter model, regardless of Standard or Internal model. This is because the CBD-model produces lower central mortality projections than the Lee-Carter model. However, this does not necessarily infer that the Lee-Carter model is preferable to the CBD-model. First, the Solvency Capital Requirements need to be determined.

9.3.2 Solvency Capital Requirements

Table 3: Solvency Capital Requirements. SCR^{SF} and SCR^{IM} for the Lee-Carter and CBD mortality models. The SCRs are calculated for simplified portfolios consisting of 50, 60, 70, and 80 year old Norwegian males and females.

Solvency Capital Requirement, Males				
Portfolio age	Lee Carter		Cairns-Blake-Dowd	
	SCR^{SF}	SCR^{IM}	SCR^{SF}	SCR^{IM}
Age = 50	454	207	530	275
Age = 60	659	260	782	368
Age = 70	798	254	937	436
Age = 80	774	164	884	426

Solvency Capital Requirement, Females				
Portfolio age	Lee Carter		Cairns-Blake-Dowd	
	SCR^{SF}	SCR^{IM}	SCR^{SF}	SCR^{IM}
Age = 50	403	229	465	238
Age = 60	579	300	700	322
Age = 70	716	342	876	380
Age = 80	748	279	876	374

From table 3, we find that the Internal model consistently returns lower Solvency Capital Requirements than the Standard Formula for all ages. Recall that SCR for the Standard Formula can be calculated as the difference between BEL^{SF} and BEL_0 . Furthermore, the difference between the Internal Model and the Standard Formula seemingly increase with age. This is especially prevalent for the Lee-Carter mortality projections where the BEL^{SF} is relatively stable when age increases.

When comparing the capital requirements for males and females, we find that the SCR^{SF} for females increase with age throughout. For males, on the other hand, the capital requirements

peak at ages around 70, before they decline. This is because females are expected to live longer than males, and the cumulative survival probability is still high in the 70-80 age range.

When comparing the Lee-Carter model to the CBD-model we find that the increase in SCRs across age for the Standard Formula are relatively similar. However, we find that the CBD-model returns roughly 15% higher capital requirements. Comparing the Internal Models, however, we notice that the Lee-Carter Model outperforms the CBD-model with regards to minimizing SCRs, especially at high ages. For 80-year-old males, the Internal Model capital requirement is 2.6 times higher for the CBD-model compared to the Lee-Carter model. This is because the CBD-model predicts lower mortality rates for higher ages, compared to Lee-Carter.

Overall, this leads us to conclude that the Internal Model is superior to the Standard Formula for both genders and mortality models. Furthermore, we find that the Lee-Carter-model is superior to the CBD-model for all ages if one seeks to minimize SCRs, regardless of approach. This is in line with EIOPAs expectations and part of the purpose of Solvency II. By developing Internal Models, insurers will have lower capital requirements and gain a better understanding of their risk exposure, and Solvency II incentivizes this.

9.3.3 SCR as a percentage of Best Estimate of Liabilities

To better quantify the differences between the mortality models and the two approaches, we compare the relative percentages. As briefly discussed, the true BEL_0 is highly uncertain as the deviation in BEL_0 for the two mortality models is between 10% and 19% for ages 50 to 80. We thus wish to identify the Solvency Capital Requirements as a percentage of the Best Estimate of Liabilities. We obtain this estimate by dividing the SCR^{SF} and SCR^{IM} by BEL_0 . We compare the Standard Formula and the Internal Model using the Lee-Carter and the CBD-model.

Lee-Carter: SCR as percentage of BEL_0

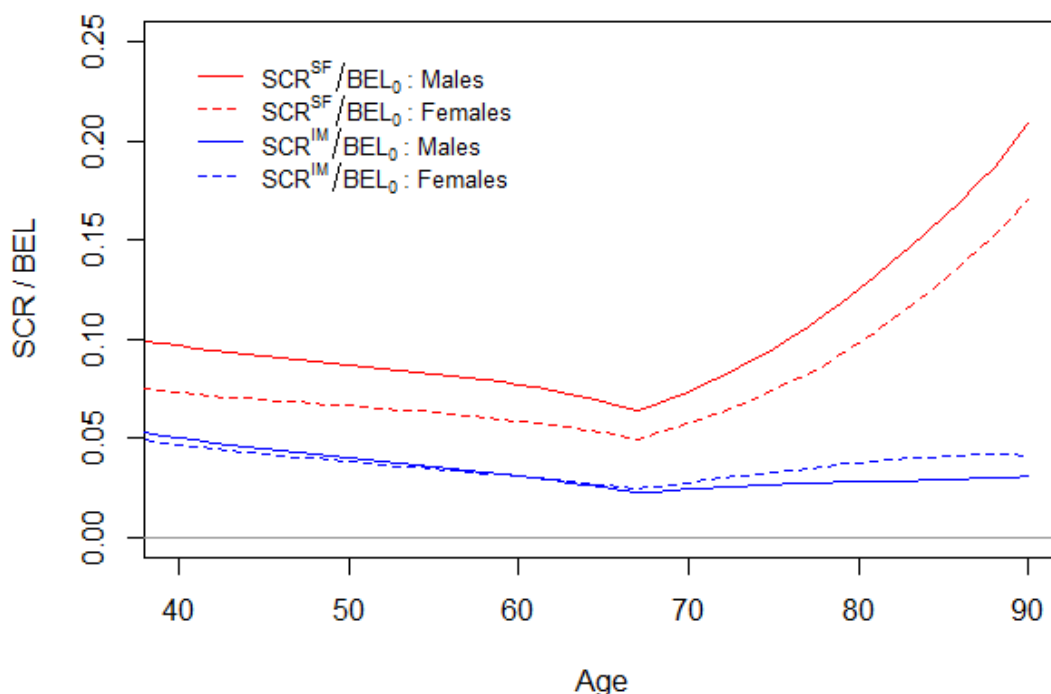


Figure 11: SCR as a percentage of BEL_0 using the Lee-Carter mortality model. The red and blue lines represent SCR^{SF} and SCR^{IM} as a percentage of BEL_0 , respectively. The solid lines represent Norwegian male mortality rates, while the dashed lines refer to female mortality rates.

From Figure 11 we find that the Standard Formula yields higher SCRs as a percentage of BEL_0 for all ages compared to the Internal Model. The turning point at age 67 can be explained through the different payment phases. For the ages after retirement, all cash flows are payouts to the contract holder. The size of both BEL_0 and the SCRs are reduced after retirement, but SCR as a percentage of BEL_0 increases.

We find that the relationship between the Standard Formula and Internal Model diverge early and that the difference continues to increase with age. This is caused by the linearity of the 20% shock that the Standard Formula stipulates. Because expected mortality increases with age, the reduction of mortality rates followed by the shock becomes increasingly large. The sudden increase in gap size between the two approaches is thus because mortality rates are expected to increase drastically from around age 70. By age 80, the difference in SCRs between models for males is roughly 10% as a percentage of BEL_0 . It seems evident that the shock is indeed too prudent, and that the higher age groups lead to larger buffers than

the younger population.

Comparing males and females, we find that for the Standard Formula, the capital requirements are roughly 2-3% lower for females throughout. This is because males have higher death probabilities and the aforementioned effect of the shock approach, where the shock has a greater impact on higher mortality rates. For the Internal Model, the SCRs as a percentage of BEL_0 are nearly coincidental until around age 67. The SCRs for females then makes up a larger portion of the BEL_0 . This is because the Lee-Carter model projects relatively lower mortality rates with the VaR approach for females compared to males.

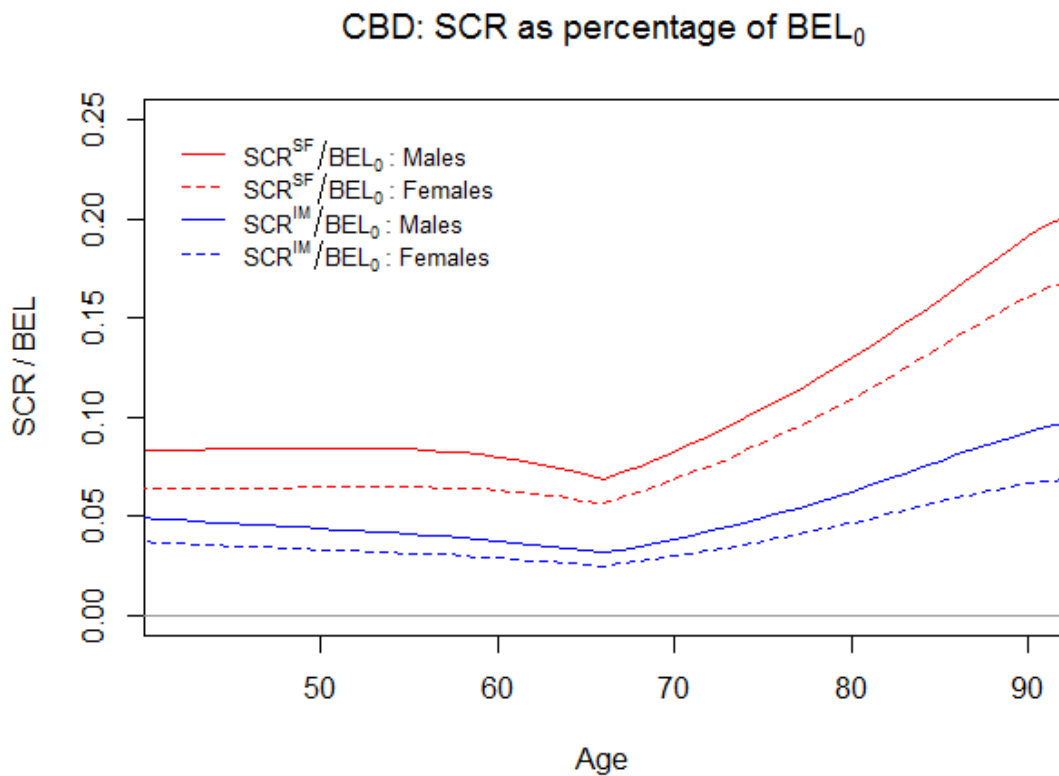


Figure 12: SCR as a percentage of BEL_0 using the CBD-model. The red and blue lines represent SCR^{SF} and SCR^{IM} as a percentage of BEL_0 , respectively. The solid lines represent Norwegian male mortality rates, while the dashed lines refer to female mortality rates.

Figure 12 illustrates the comparison between the Standard Formula and the Internal Model based on the CBD-model. We find similar results as with the Lee-Carter model. The Standard Formula yields consistently higher SCRs as a percentage of BEL_0 than the Internal Model for all ages and both genders. There is an expected and noticeable increase in difference between the two methods from around age 67. However, the gap is less dramatic than in the Lee-Carter model. This is because the CBD-model yields lower mortality projections for

the Internal Model, and thus a reduced gap between the Standard Formula and the Internal Model.

Interestingly, the SCR as a percentage of BEL_0 for females is consistently lower than for males for both the Standard Formula and the Internal Model. This is because there is less uncertainty associated with female mortality estimates, ref appendix B.3. The 1 in 200 year worst case scenario then has a relatively smaller impact on the SCR as a percentage of BEL_0 .

9.3.4 Standard Formula compared to Internal Model

Furthermore, we wish to elaborate on the differences in SCRs from the Standard Formula and Internal Model. This has two purposes. First, to quantify the difference in capital requirements between the Standard Formula and the Internal Model. Second, to compare the Lee-Carter mortality model to the CBD mortality model.

To assess this, we examine the difference between the SCR^{SF} and the SCR^{IM} as a percentage of BEL_0 . ΔSCR may be expressed as:

$$\Delta SCR = SCR^{SF} - SCR^{IM} \tag{32}$$

The difference between the SCRs represents the benefit of using an Internal Model compared to the Standard Formula. To compare this metric across the Lee-Carter and CBD mortality models, we divide ΔSCR by BEL_0 .

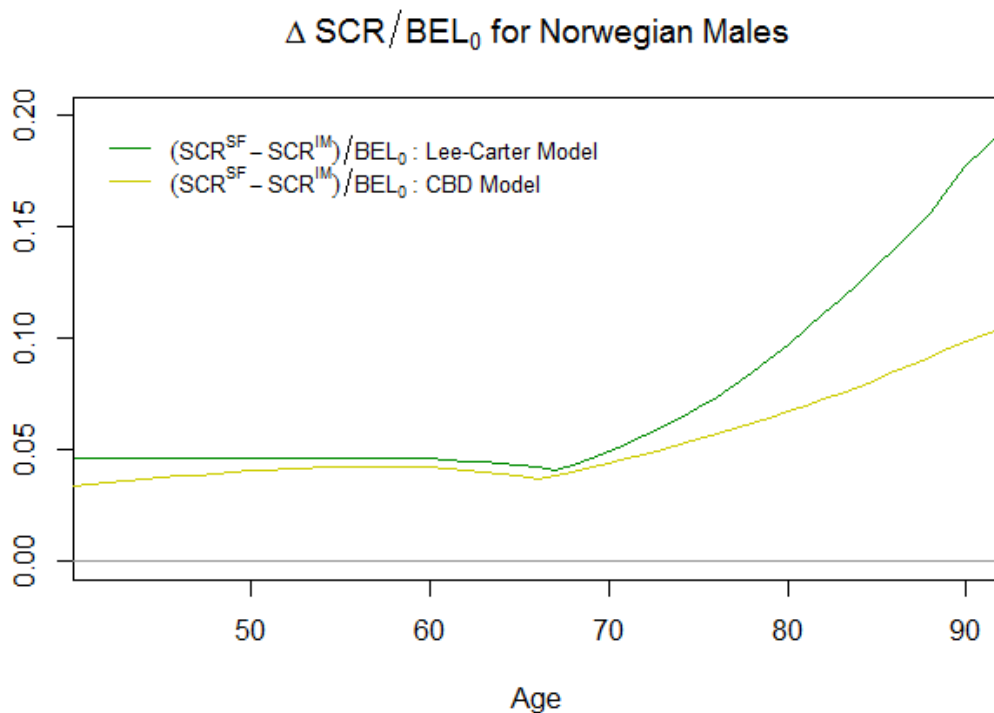


Figure 13: Difference in SCR between Standard Formula and Internal Model as a percentage of BEL_0 for Norwegian Males using the CBD and Lee-Carter models. The green line represents Lee-Carter and the yellow represents CBD.

We find that the ΔSCR is strictly positive. This is because both Internal Models consistently produce lower capital requirements compared to the Standard Formula. We observe that the benefit of using an Internal Model is roughly 4-5% of BEL_0 until retirement, before drastically increasing. This is because the 20% decrease in mortality rates highly affects older ages.

Comparing the Lee-Carter to the CBD-model, we see that for Norwegian males, the benefit of using an Internal Model is consistently greater for the Lee-Carter Model. It is interesting that the gap between the models steadily increases after retirement age at 67. This is because the mortality rates projected by the CBD-model, in general, tend to project lower mortality rates for older ages, compared to the Lee-Carter Model.

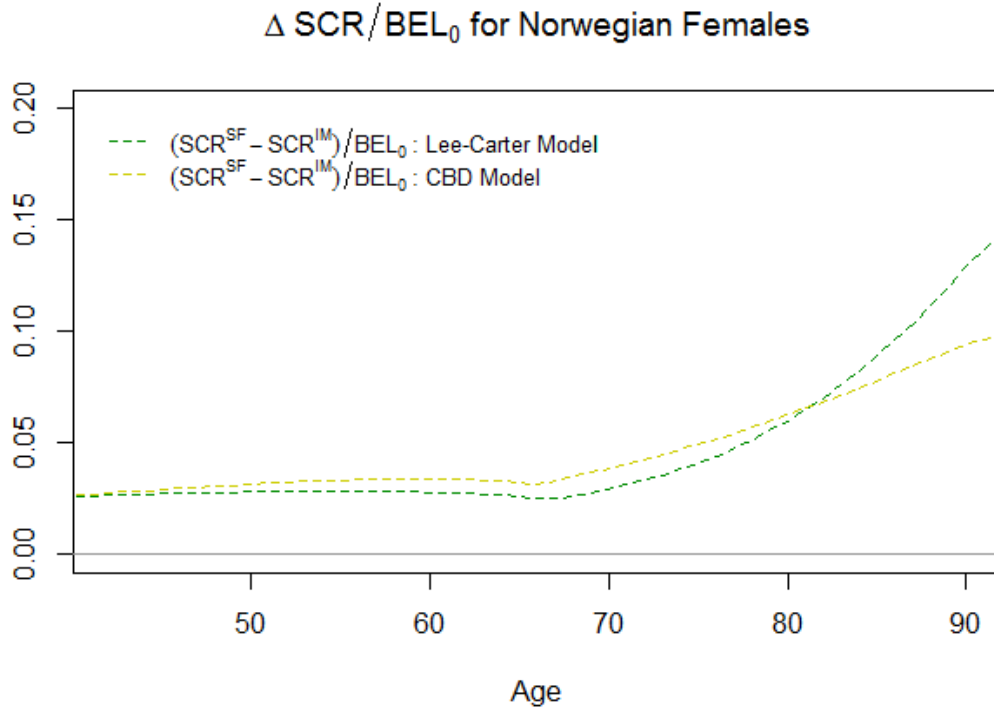


Figure 14: Difference in SCR between Standard Formula and Internal Model as a percentage of BEL_0 for Norwegian Females using the CBD and Lee-Carter models. The green line represents Lee-Carter and the yellow represents CBD.

In Figure 14 we see that for Norwegian females, the benefit of developing an Internal Model is roughly 2-3% of BEL_0 until retirement age. After retirement age, the benefit steadily increases towards 10-15%. Comparing the Lee-Carter to the CBD-model, we see that ΔSCR as a percentage of BEL_0 is roughly the same for both mortality models until age 83. This is because the difference between SCR^{SF} and SCR^{IM} is greater for the CBD-model than the Lee-Carter model as a percentage of BEL_0 . Then the Lee-Carter becomes the preferred model. However, the SCRs in absolute terms are lower for the Lee-Carter model throughout.

We have thus found that Internal Models return lower capital requirements for Norwegian males and females than the Standard Formula. For ages 40-67, the difference varies between 2.5% and 5%, and increases to between 10% and 18% when age nears 90. In terms of relative SCRs as a percentage of BEL_0 , the Lee-Carter model is strictly better for males, while for females, the CBD-model is preferable until age 83, before the Lee-Carter model is favorable. However, when considering SCRs in absolute terms, the Lee-Carter model is always preferable. These findings are consistent with previous literature on other European populations. For further readings, we refer to Börger (2010) and Richards et al. (2014).

10 Conclusion

This thesis has answered the research question of how annuity providers' Solvency Capital Requirements are affected by applying either the Standard Formula or an Internal Model when assessing longevity risk. Based on two stochastic mortality models we compared the Standard Formula from the Solvency II framework with an Internal Model based on a Value-at-Risk approach on a one-year time horizon. The mortality models used in this thesis are the Lee-Carter model with some adaptations and the original Cairns-Blake-Dowd model. We based the analysis on Norwegian mortality data collected from the Human Mortality Database. To compare the methods for calculating the capital requirements, we used a simplified pension product, similar to those offered by Norwegian insurers.

From the results, we conclude that the Standard Formula with its 20% reduction in mortality rates results in SCRs that are higher than those estimated by the Internal Models. This means that annuity providers are incentivized to develop Internal Models as they would rationally seek to minimize their SCRs to free capital for other investments. However, it is worth noting that the Standard Formula accounts for more risks than longevity risk alone. The Standard Formula accounts for deviations in the mortality trend, meaning that some level of adverse selection often found with annuity holders is covered.

The Internal Model, on the other hand, does not cover such risks, merely uncertainty in the trend development. Yet, we find the Standard Formula to exaggerate the size of its mortality shock assumption, and that if mortality development follows either the Lee-Carter estimates or the CBD-estimates, annuity providers should develop Internal Models. Furthermore, we find that factors such as the age and gender composition of a portfolio affects the mortality estimates and thus the size of the SCRs. If a portfolio consists of older annuity holders, the size of the SCR will be relatively larger when determined by the Standard Formula, than for any of our two Internal Models. Our findings are in line with what Börger (2010) concludes; that a mortality shock of 20% is too prudent from an annuity providers perspective.

However, the individual annuity providers' need for developing Internal Models will likely differ. One might argue that development of Internal Models is beneficial as it may give annuity providers better assessment of their risk exposure. This could, in turn, create a better understanding of how to mitigate the longevity risk. Furthermore, insurers with large portfolios may benefit from developing Internal Models, while smaller insurers may find it too costly. Our results indicate that the Standard Formula provides prudent, but not unreasonable mortality estimates and Solvency Capital Requirements.

We have thus found that the Solvency II Standard Formula produces low mortality estimates, and thus generates larger Solvency Capital Requirements. Our findings imply that annuity providers benefit from developing Internal Models. However, the process is time-consuming and costly which may outweigh the benefit for smaller annuity providers.

In this thesis, we have employed two stochastic mortality models to compare SCRs with the standard model. It would be interesting to investigate further if different stochastic models would yield different results for different populations. Furthermore, our thesis' comparisons are based on a single simplified annuity product, which may be representative for a single genre of annuity products. However, most insurance companies offer several differentiated products which are affected by longevity risk and mortality risk. These may include death insurance where a payout is made after the annuity holders' death. Further investigation of the effects of mortality development on these products would be interesting. Lastly, our estimates are based on Norwegian data for the entire population, which is not necessarily representative for the specific insurance portfolios. Thus, estimation based on portfolio specific data would be highly relevant to the topic of quantifying the benefits of developing an Internal Model.

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Appendices

A Lee-Carter

A.1 Single Value Decomposition

In their original paper, Lee and Carter (1992) proposed Singular Value Decomposition (SVD, Appendix A.1) as a means to estimate the age- and time-dependent variables of their model. The method is similar to Ordinary Least Squares (OLS)- estimation, in that it solves a minimization problem. This is done by iteratively updating every parameter until the difference between the probability of the fitted and saturated model is minimal under constraints that ensure identification. Alternatively, one could use other estimation methods such as Maximum Likelihood or Weighted Least Squares. An in-depth description of these methods can be found in Wilmoth (1993).

Due to the lack of regressors in the Lee-Carter model, it cannot be treated as a simple regression model where the error terms are independent and identically distributed with zero-mean variance. OLS would suggest minimization of

$$\sum_x \sum_t \{ \ln m_{x,t} - \alpha_x - \beta_x k_t \} \quad (33)$$

even though one cannot estimate α_x and $\beta_x k_t$ with only $\ln m_{x,t}$ -observables and no regressors. To cope with this potential issue, Lee and Carter (1992) use Single Value Decomposition (SVD) to estimate the model. The model is thus subject to constraints so that $\sum_x \beta_x = 1$ and $\sum_t k_t = 0$. This is done to avoid any transformations of the parameters, as a transformation of any constant, C would result in identification problems. Lee and Carter (1992) used SVD to estimate the model and the constraints on $\sum_x \beta_x = 1$ and $\sum_t k_t = 0$ suggest that α_x denotes the average of $\ln m_{x,t}$ over time t , because

$$\sum_t \ln m_{x,t} = \sum_t \{ \alpha_x + \beta_x k_t + \epsilon_{x,t} \} = (t+1) \alpha_x + \beta_x \sum_t k_t + \sum_t \epsilon_{x,t} \quad (34)$$

Here, t represents the observation time period measured in years from 1970-2014, under the

assumption that $\sum_x \beta_x = 1$ and $\sum_t k_t = 0$. When rearranging the above equation (34) we find that estimate of α_x is given by

$$\hat{\alpha}_x = \frac{1}{t+1} \sum_t \ln m_{x,t} \quad (35)$$

Lee and Carter (1992) show that the observables can be arranged so that

$$\ln m_{x,t} - \hat{\alpha}_x = \sum_{i=1}^r \rho_i U_{x,i} V_{t,i} \quad (36)$$

Here, $r = \text{rank} [\ln m_{x,t} - \hat{\alpha}_x]$ and ρ_i for $i = 1, 2, \dots, r$ represents the singular vectors in increasing order while $U_{x,i}$ and $V_{t,i}$ are the matching left and right singular vectors. Lee and Carter (1992) proved that $\text{SVD} [\ln m_{x,t} - \hat{\alpha}_x] \approx \rho_1 U_{x,1} V_{t,1}$ and so that $\hat{\beta}_x = U_{x,1}$ and $\hat{k}_t = \rho_1 V_{t,1}$.

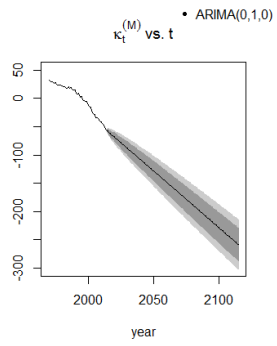
There are three formal steps required to estimate the variables as described by Lee and Carter (1992). The first step is to apply the SVD-methodology followed by adjusting the \hat{k}_t -estimates so that there is equality between the number of deaths implied by the model and the observed number of deaths in a given time-period. This is ensured by replacing \hat{k}_t with \tilde{k}_t such that

$$\sum_x D_{x,t} = \sum_x \{E_{x,t} e^{(\hat{\alpha}_x + \hat{\beta}_x \tilde{k}_t)}\} \quad (37)$$

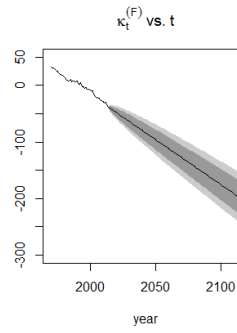
This step is done to reduce differences between the observed number of deaths and the number of deaths implied by the model. \hat{k}_t 's are likely to result in incorrect future mortality rates if not adjusted. Lastly, the Box-Jenkins methodology is employed to estimate the dynamics of the inferred factor \tilde{k}_t . The Box-Jenkins methodology uses ARIMA models to select an appropriate fit of a time series and consequently uses the fitted model to create forecasts.

A.2 k_t Forecasts

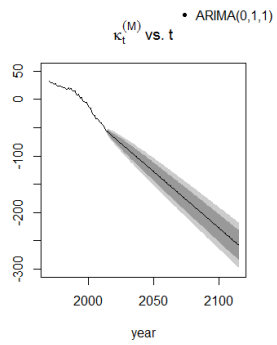
k_t forecasts for different ARIMA processes for males and females are presented below.



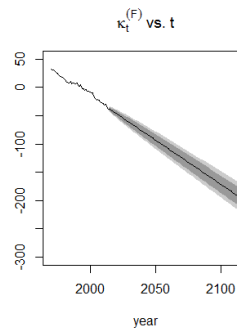
(a) Male ARIMA(0,1,0)



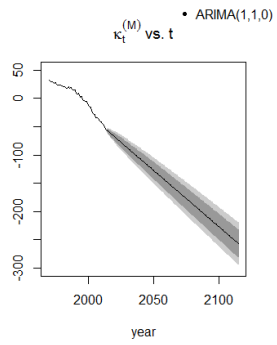
(b) Female ARIMA(0,1,0)



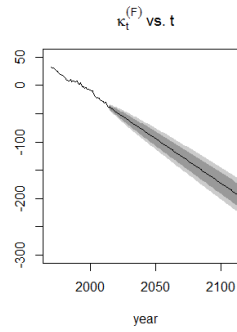
(c) Male ARIMA(0,1,1)



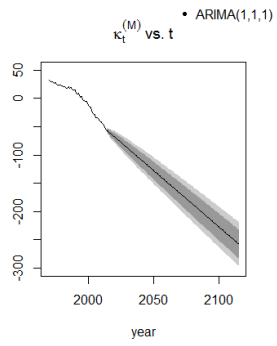
(d) Female ARIMA(0,1,1)



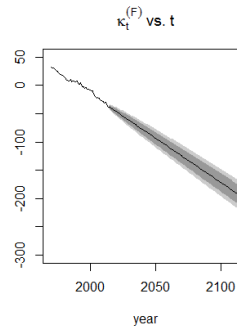
(e) Male ARIMA(1,1,0)



(f) Female ARIMA(1,1,0)



(g) Male ARIMA(1,1,1)



(h) Female ARIMA(1,1,1)

A.3 Simulated Mortality Rate Forecasts

Next, we present simulated forecasts for 85 year old Norwegian males and females. This is based on variations in the k_t from ARIMA(0,1,0) and ARIMA(0,1,1) processes for males and females, respectively.

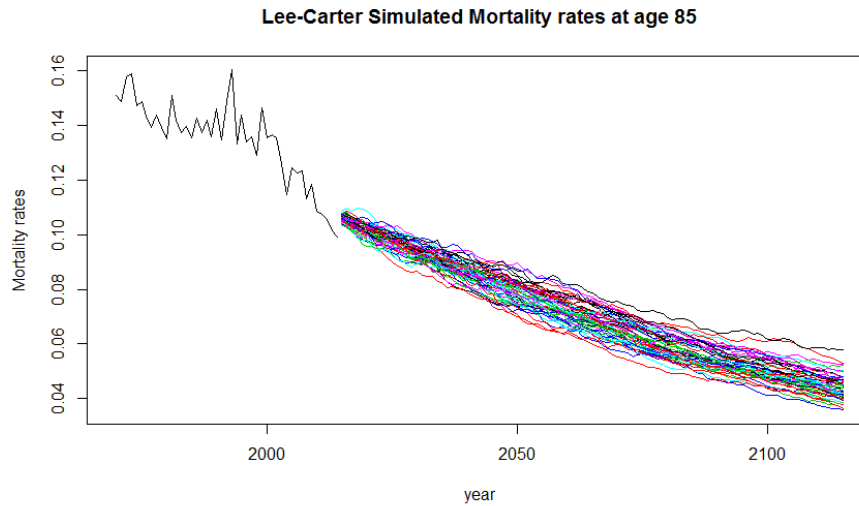


Figure 15: Lee-Carter simulated forecast for males at age 85. 50 simulations are displayed

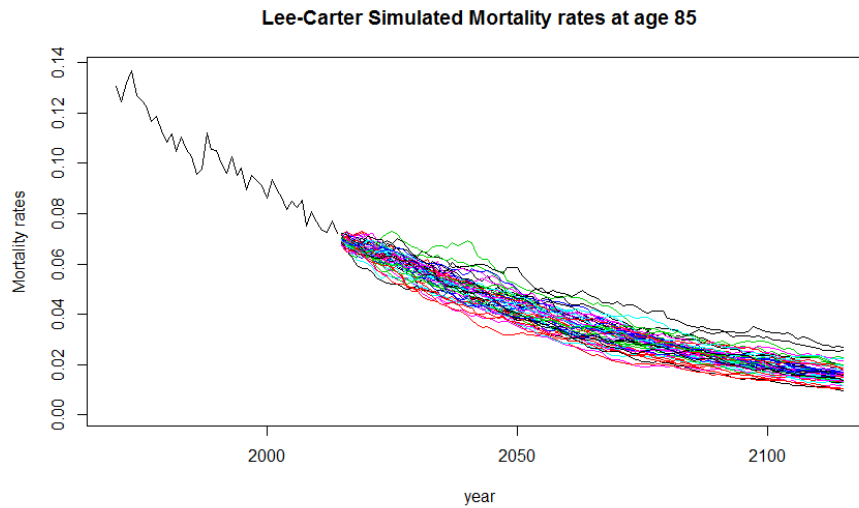


Figure 16: Lee-Carter simulated forecast for females at age 85. 50 simulations are displayed

A.4 Female Residual Plots

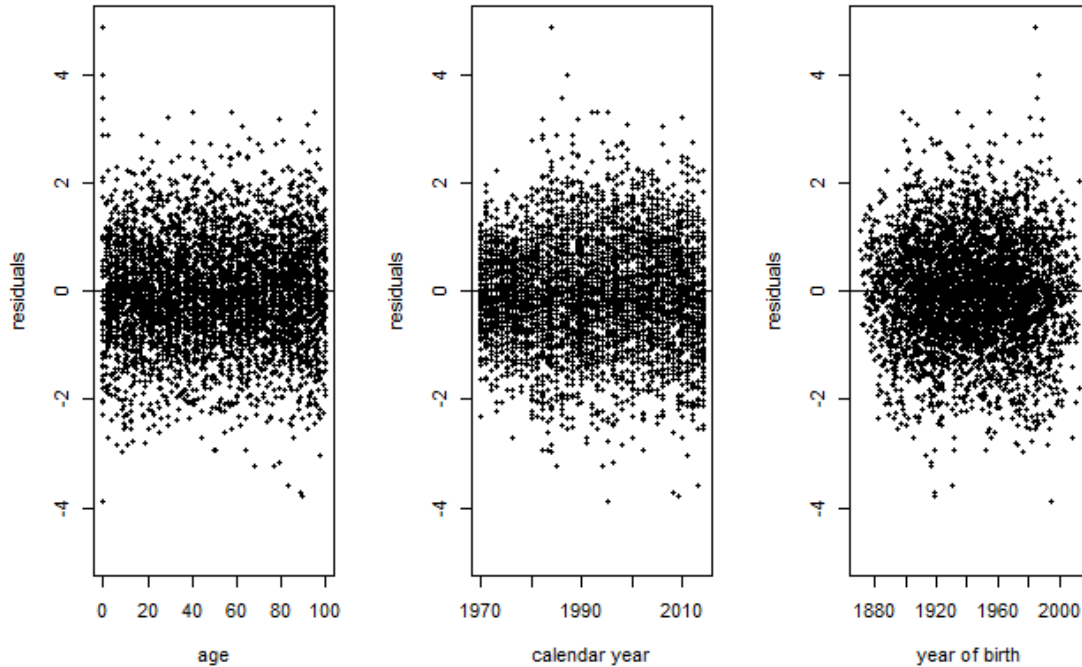


Figure 17: Female residual plots of the fitted Lee-Carter model

From figure 17 we observe no significant patterns. As with the males, the female model seems to fit the historical data well.

In the first figure we observe the residuals for all age groups, and we observe no indication that the model predictions are exaggerated. We do, however, observe high deviations for infants, and see that the model fails to consistently predict infant mortality rates. Furthermore, from the second figure we observe homoscedasticity, meaning that the variance seems constant throughout the time series. Lastly, recall that the last figure figure attempts to uncover cohort effects. The residuals seem to cluster in the center. As with males, however, this is not an issue as the data set is only from 1970 to 2014. It is expected that the residuals are not as consistent for the population born in the 1800's, as the number of observations are fairly limited within our data set.

B CBD

B.1 CBD: Model 7

The original CBD-model has been expanded by, amongst others, Cairns et al. (2009), who include a quadratic age term and a cohort effect term to create three variations of the original model. There are three expanded models of the original CB- model. In mortality modeling literature, the CBD-model is often referred to as model 5. The expansions are thus called model 6, 7 and 8. Of the models 6-8, model 7 had the best fit to the Norwegian population.

As described by Cairns et al. (2009), the model is defined as

$$\text{logit } q(t, x) = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + k_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)} \quad (38)$$

where as in the CBD-model, \bar{x} is the mean age in the sample, and $\hat{\sigma}_x^2$ is the mean value of the quadratic age term $(x - \bar{x})^2$.

Moreover, parameter constraints on the cohort parameter $\gamma_{t-x}^{(4)}$ is applied to ensure model identification.

$$\sum_c \gamma_c = 0, \sum_c c\gamma_c = 0, \sum_c c^2\gamma_c = 0, \quad (39)$$

The constraints ensures that $\gamma_{t-x}^{(4)}$ fluctuates around 0 and has no linear and quadratic curvature (Cairns et al., 2009).

B.2 ARIMA processes

We wish to provide our analysis of the bivariate ARIMA processes in the CBD-model. In the following tables, we provide evidence that ARIMA(2,1,1) and ARIMA(1,1,0) fits $k_t^{(1)}$ and $k_t^{(2)}$ best for males respectively, and that ARIMA(1,1,0) and ARIMA(0,1,1) fits $k_t^{(1)}$ and $k_t^{(2)}$ best for females, respectively.

ARIMA	Log-likelihood		AIC		BIC		Rank (BIC)	
	Males	Females	Males	Females	Males	Females	Males	Females
(0,1,0)	107.71	113.49	-211.42	-222.98	-207.85	-219.41	(2)	(5)
(0,1,1)	108.44	116.79	-210.87	-227.58	-205.52	-222.23	(6)	(2)
(1,1,0)	108.92	117.36	-211.84	-228.74	-206.48	-223.39	(4)	(1)
(1,1,1)	109.29	117.42	-210.59	-226.83	-203.45	-219.7	(8)	(4)
(1,1,2)	113.01	117.97	-216.01	-225.93	-207.09	-217.01	(3)	(8)
(2,1,0)	110.38	117.47	-212.77	-226.95	-205.63	-219.81	(5)	(3)
(2,1,1)	113.67	118.38	-217.33	-226.76	-208.41	-217.84	(1)	(7)
(2,1,2)	114.01	120.31	-216.02	-228.62	-205.32	-217.91	(7)	(6)

ARIMA	Log-likelihood		AIC		BIC		Rank (BIC)	
	Males	Females	Males	Females	Males	Females	Males	Females
(0,1,0)	234.9	232.59	-465.8	-461.17	-462.23	-457.61	(8)	(3)
(0,1,1)	239.64	239.14	-473.28	-472.29	-467.92	-466.93	(2)	(1)
(1,1,0)	241.23	238.12	-476.46	-470.25	-471.1	-464.9	(1)	(2)
(1,1,1)	241.23	239.29	-474.46	-470.48	-467.32	-463.45	(3)	(3)
(1,1,2)	242.8	239.65	-475.6	-469.31	-466.67	-460.39	(5)	(5)
(2,1,0)	241.23	238.67	-474.46	-469.35	-467.32	-462.21	(3)	(4)
(2,1,1)	241.23	239.29	-472.45	-468.59	-463.54	-459.66	(6)	(6)
(2,1,2)	242.89	239.66	-473.77	-467.32	-463.07	-456.61	(7)	(8)

B.3 Simulated Mortality Rate Forecasts

Next, we provide a simulation of forecasted mortality rates from the CBD-model for 85 year old Norwegian males and females. From the graphs below, we observe more volatility than what we have seen from the Lee-Carter model. This is especially prevalent for Norwegian males, where 50 projections show much uncertainty. As shown in section 9, however, the central projections seem to follow a more conservative pattern. From the high variance it is not surprising that the Value-at-Risk from a one-year time horizon produce lower mortality estimates compared to the Lee-Carter model. This is because a worst case scenario in next year's mortality rates will have more affect when the model predicts higher uncertainty.

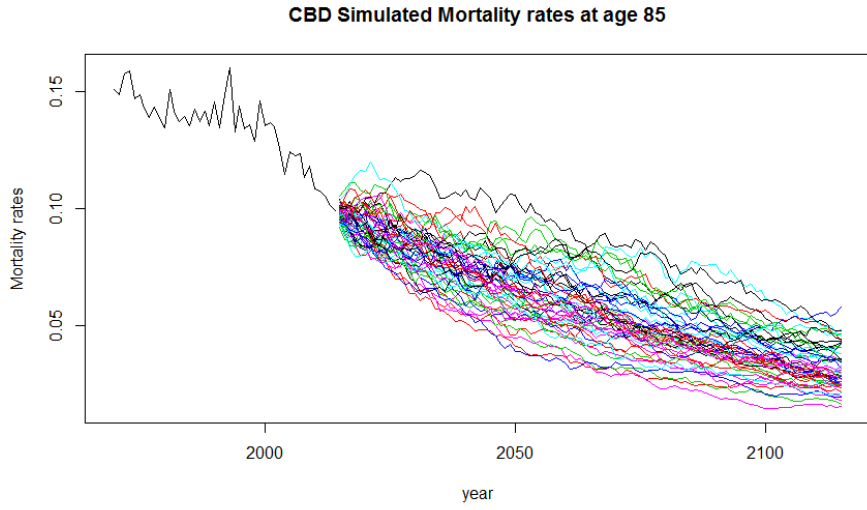


Figure 18: Lee-Carter simulated forecast for males at age 85. 50 simulations are displayed

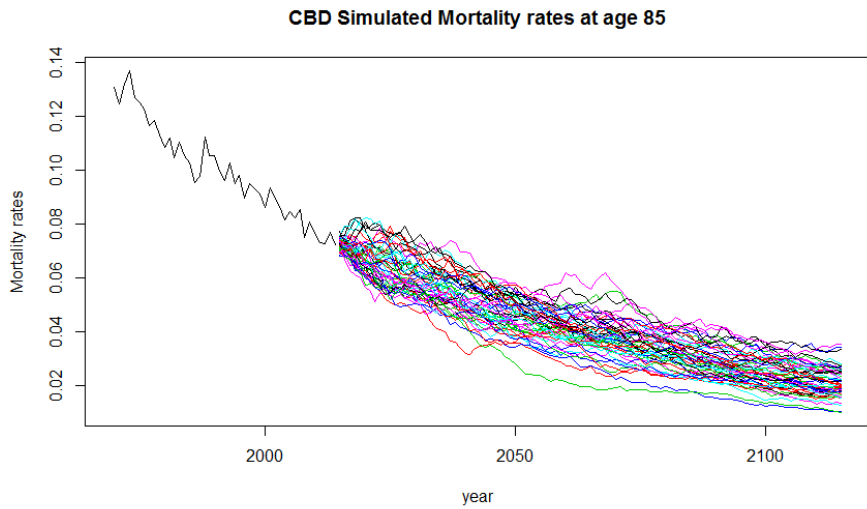


Figure 19: Lee-Carter simulated forecast for males at age 85. 50 simulations are displayed

B.4 Female Residual Plots

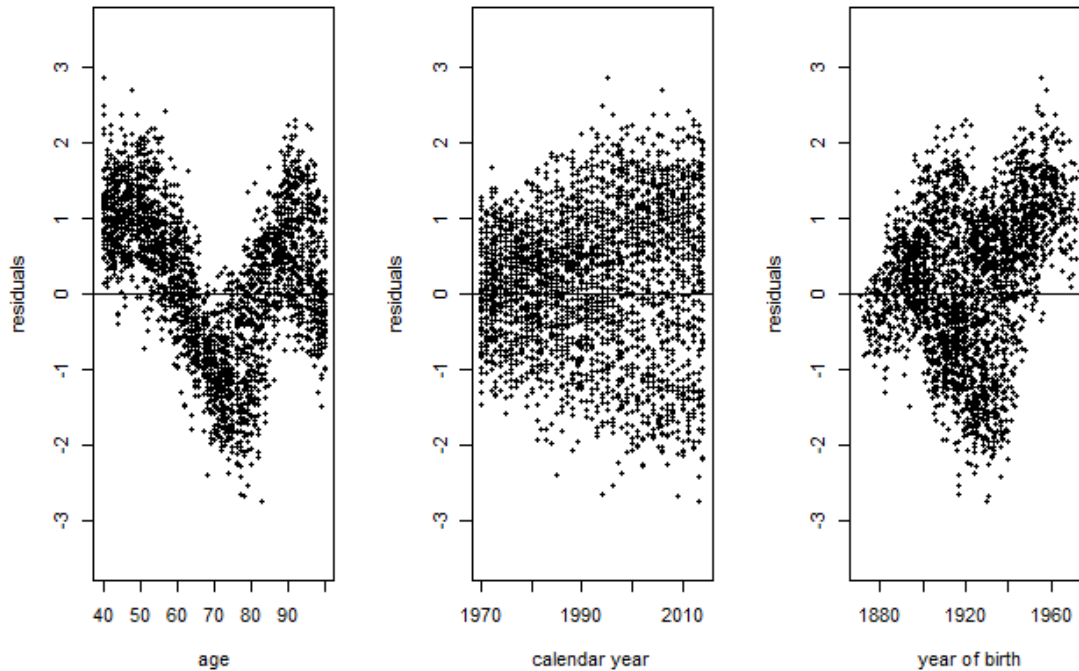


Figure 20: Residual analysis of the CBD-model

From the first figure we see that the female CBD residuals follow a distinct pattern. This is a clear indication that fitted line fails to correctly explain logarithmic mortality at a large portion of the Norwegian population. Furthermore, the second graph indicates heteroscedasticity, where the residuals seemingly increase as time passes. Heteroscedasticity generally produces consistent, but not efficient predictions. For females, the heteroscedasticity is not as clear as for the male model analysis. Lastly, the third graph does not produce an even picture of the residuals. We see that the residuals are dependent on the year of birth. This can be explained through different predictions of the older and younger mortality improvements.

C Risk-Free Interest Rates

Lastly, we provide the risk-free interest rates used in this master thesis. These are used to calculate technical provisions for (re)insurance obligations. In line with the Solvency II Directive, EIOPA publishes these rates on a monthly basis. The ones presented below represent the Norwegian risk-free interest rates from November 2018.

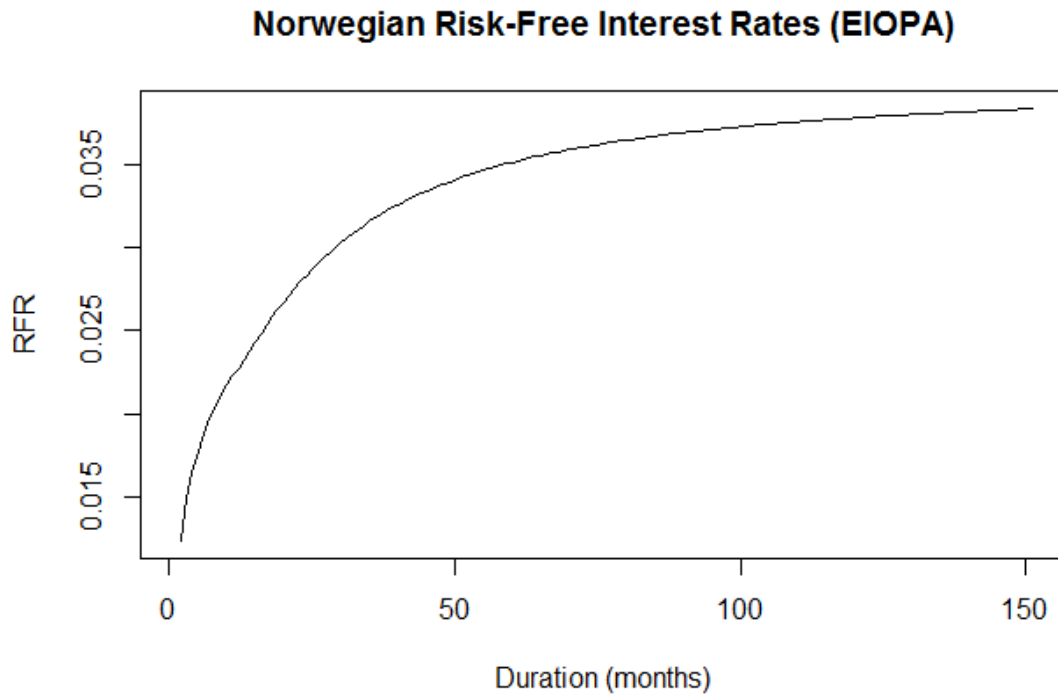


Figure 21: Norwegian Risk-free interest rates from EIOPA (2018).